

SUBJECT : MATHEMATICS

Relations & Functions and Inverse Trigonometric FunctionsMCQ

- The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x - 4$
 - Bijjective
 - Surjective but not Injective
 - Injective but not Surjective
 - Neither Surjective nor Injective
- The value of $\cos^{-1}(-1) - \sin^{-1}(1)$ is
 - π
 - $\frac{\pi}{2}$
 - $\frac{3\pi}{2}$
 - $-\frac{3\pi}{2}$
- Let R be a relation on the set N of Natural numbers defined by nRm if n divides m . Then R is
 - Reflexive and symmetric
 - Transitive and Symmetric
 - Equivalence relation
 - Reflexive, Transitive but not Symmetric.
- Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$, then the pre image of 10 is
 - $\{3\}$
 - $\{-3\}$
 - $\{3, -3\}$
 - $\emptyset$
- A set A has 3 elements and a set B has 4 elements. The number of injective function that can be defined from A to B is
 - 144
 - 12
 - 24
 - 64
- Maximum number of equivalence relation on the set $A = \{1, 2, 3\}$ are
 - 1
 - 2
 - 3
 - 5
- The domain of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sqrt{x^2 - 3x} + 2$ is
 - $(-\infty, 1] \cup [2, \infty)$
 - $(-\infty, 1]$
 - $[2, \infty)$
 - $[1, 2]$
- The domain of the function defined by $f(x) = \sin^{-1} x + \cos x$ is
 - $[-1, 1]$
 - $[-1, \pi + 1]$
 - $(-\infty, \infty)$
 - ϕ

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- Both A and R are true and R is the correct explanation of A.
- Both A and R are true but R is not the correct explanation of A.
- A is true but R is false.
- A is false but R is true.

9. Assertion (A): The relation $f: \{1, 2, 3, 4\} \rightarrow \{x, y, z, p\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function.

Reason (R): The function $f: \{1, 2, 3\} \rightarrow \{x, y, z, p\}$ such that $f = \{(1, x), (2, y), (3, z)\}$ is one-one.

10. Assertion (A): $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = 5x$ is injective but not surjective
Reason (R): If co-domain $\neq$ range, then the function is not surjective

11. Assertion (A) : The relation R in the set $\{1,2,3\}$ given by $R = \{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ is an equivalence relation
Reason (R) : A relation R is said to an equivalence relation if R is reflexive, symmetric and transitive.
12. Assertion (A) : Let $A = \{1, 2, 3,4,5\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is 32.
Reason (R) : The number of functions that can be defined from $A = \{1, 2, 3,4,5\}$ to $B = \{a, b\}$ is 32
13. Assertion(A) : $\sin^{-1}(x) = (\sin x)^{-1}$
Reason (R) : Any value in the range of principal value branch is called principal value of that inverse trigonometric function.
14. Assertion(A): The relation $R = \{(1, 2)\}$ on the set $A = \{1, 2, 3\}$ is transitive.
Reasoning (R): A relation R on a non-empty set A is said to be transitive if $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$, for all $a, b, c \in A$.

VERY SHORT ANSWER

- Find the principal value of $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$.
- Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.
- Find the principal value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.
- Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \cos x$, show that f is neither one-one nor onto.

SHORT ANSWER

- Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $9x^2 + 6x - 5$. Show that $f: \mathbb{N} \rightarrow S$ where S is the range of f is bijective.
- Let T be the set of all triangles in a plane The R is defined by $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2, T_1, T_2 \in T\}$. Show that R is an equivalence relation.

LONG ANSWER

- Let $\mathbb{N}$ be the set of natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b) R (c, d)$ iff $ad = bc$ for all $a, b, c, d \in \mathbb{N}$. Show that R is an equivalence relation.
- Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto.
- Let R be a relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a,b)R(c,d)$ iff $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.
- Let $A = \{1,2,3,\dots,9\}$ and R be the relation in $A \times A$ defined by $(a,b)R(c,d)$ if $a+d = b+c$ for $(a,b),(c,d)$ in $A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $\{(2,5)\}$.
- Show that the relation S in the set $A = \{\alpha \in \mathbb{R} : 0 \leq \alpha \leq 12\}$ $\alpha \beta S \iff (\alpha - \beta) \in \mathbb{Z}$ is an equivalence relation. Find the set of all elements related to 2.

CBQ

- Consider the mapping $f: A \rightarrow B$ is defined by $f(x) = \frac{x-1}{x-2}$ such that f is a bijection .
Based on the above information, solve the following questions:

Domain of f is:

(A). $\mathbb{R} - \{2\}$

(B). $\mathbb{R}$

(C). $\mathbb{R} - \{1, 2\}$

(D). $\mathbb{R} - \{0\}$

(ii). Range of f is: a. $\mathbb{R}$

(A). $\mathbb{R}$

(B). $\mathbb{R} - \{1\}$

(C). $\mathbb{R} - \{0\}$

(D). $\mathbb{R} - \{1, 2\}$

(iii). If $g: \mathbb{R} - \{2\} \rightarrow \mathbb{R} - \{1\}$ is defined by $g(x) = 2f(x) - 1$, then $g(x)$ in terms of x is:

(A). $\frac{x+2}{x}$

(B). $\frac{x+1}{x-2}$

(C). $\frac{x-2}{x}$

(D). $\frac{x}{x-2}$

(iv). The function g defined above, is:

(A) one-one

(B). many-one

(C). into

(D). None of these

MATRICES AND DETERMINANTS

1. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true?

A) $a_{ij} = \begin{cases} 0, & \text{if } i=j \\ 1, & \text{if } i \neq j \end{cases}$

B) $a_{ij} = 1, \forall i, j$

C) $a_{ij} = 0, \forall i, j$

☒ D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i=j \end{cases}$

2. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is:

A. $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

B. $30 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

C. $\frac{1}{30} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

D. $\frac{1}{30} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

3. For any square matrix A , $(A - A^T)^T$ is always:

A. An identity matrix

B. A null matrix

☒ C. A skew symmetric matrix

D. A symmetric matrix

4. If $A \cdot (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to:

☒ A. 12

B. 9

C. 3

D. 27

5. Let, A be the area of a triangle having vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) . Which of the following correct?

A. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$

B. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

C. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$

D. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

6. If area of triangle is 35 sq units with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$. Then k is

- (A) 12 (B) -2 (C) -12, -2 (D) 12, -2

7. Matrices A and B will be inverse of each other only if

- (A) $AB = BA$ (B) $AB = BA = 0$ (C) $AB = 0, BA = I$ (D) $AB = BA = I$

8. If A is a square matrix such that $A^2 = A$, then the value of $(I+A)^2 - 3A$ is, where I is an identity matrix

- (a) -I (b) I (c) A (d) $A - I$

9. If A_{ij} is the cofactor of the elements a_{ij} of the determinant

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

then the value of $a_{32} \cdot A_{32}$ is

- (a) -110 (b) 0 (c) -22 (d) 110

$$5 \times (-1)^5 \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$$

$$5 \times (-1) (8 - 30)$$

$$5 \times (-1) (-22)$$

$$5 \times 22$$

$$110$$

10. If $A = \begin{bmatrix} -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ then $|\text{adj}(\text{adj} A)|$

- (a) 14^4 (b) 14^3 (c) 14^2 (d) 14

11. A is a square matrix of order 3×3 such that $|A| = 5$ then $|3A|$ is equal to

- (a) 45 (b) 27 (c) 81 (d) 135

12. If the matrix A is both symmetric and skew symmetric, then

- (A) A is a diagonal matrix (B) A is a zero matrix (C) A is a square matrix (D) None of these

SHORT ANSWER

1. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find a matrix X such that $A^2 - 5A + 4I + X = 0$

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

2. Express $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ as the sum of symmetric and skew symmetric Matrices

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

3. Find the equation of the line joining the points $(3, 1)$ and $(9, 3)$ using determinants

LONG ANSWER

1. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the system of linear equations:

$$x + y + z = 6, y + 3z = 7 \text{ and } x - 2y + z = 0.$$

2. Solve the system of equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \text{ find } A^{-1}, \text{ using } A^{-1} \text{ solve system of equations } 2x-3y+5z=11, 3x+2y-4z=-5$$

$$x+y-2z=-3.$$

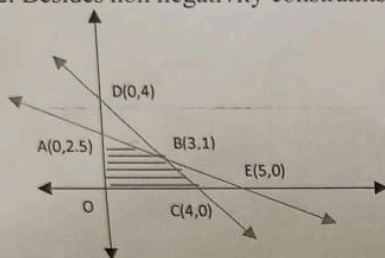
4. Using matrix method, solve. $8x - 4y + z = 5$, $10x + 6z = 4$, $8x + y + 6z = 5$
5. The sum of three numbers is 6. If we multiply the third number by 3 and add second number to it, we get 11. By adding the first and third number we get double of second number represent it algebraically and find the numbers using matrix method.

LINEAR PROGRAMMING PROBLEM

1. The linear inequalities or equations or restrictions on the variables of a linear programming problem are called:

- (A) a constraint (B) Decision variables (C) Objective function (D) None of the above

2. Besides non negativity constraints, the figure given below is subject to which of the following constraints



- A. $x + 2y \leq 5$; $x + y \leq 4$
 B. $x + 2y \geq 5$; $x + y \leq 4$
 C. $x + 2y \geq 5$; $x + y \geq 4$
 D. $x + 2y \leq 5$; $x + y \geq 4$

3. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called

- A. Feasible solutions (B) Constraints
 C. Optimal solutions (D) Infeasible solutions

4. The Corner points of the feasible region determined by a set of constraints are P(0,5), Q(3,5), R(5,0) and S(4,1) and the objective function is $Z = ax + 2by$ where $a, b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is

- (A) $a-5b=0$ (B) $a-3b=0$ (C) $a-2b=0$ (D) $a-8b=0$

5. Minimize $Z = 5x + 10y$ subject to the constraints;

$$x + 2y \leq 120; x + y \geq 6; x - 2y \geq 0; x, y \geq 0$$

6. Maximize $Z = 3x + 9y$, Subject to constraints

$$x + 3y \leq 60, \quad x + y \geq 10$$

$$x \leq y, \quad x, y \geq 0$$

Solve the above L.P.P graphically.