गणित

Mathematics

कक्षा/Class: XII

2024-25

विद्यार्थी अध्ययन सामग्री

Student Support Material

केन्द्रीय विद्यालय संगठन

Kendriya Vidyalaya Sangathan

<u>संदेश</u>

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना केन्द्रीय विद्यालय संगठन की सर्वोच्च वरीयता है। हमारे विद्यार्थी, शिक्षक एवं शैक्षिक नेतृत्व कर्ता निरंतर उन्नति हेतु प्रयासरत रहते हैं। राष्ट्रीय शिक्षा नीति 2020 के संदर्भ में योग्यता आधारित अधिगम एवं मूल्यांकन संबन्धित उद्देश्यों को प्राप्त करना तथा सीबीएसई के दिशा निर्देशों का पालन, वर्तमान में इस प्रयास को और भी चुनौतीपूर्ण बनाता है।

केन्द्रीय विद्यालय संगठन के पांचों **आंचलिक शिक्षा एवं प्रशिक्षण संस्थान** द्वारा संकलित यह 'विद्यार्थी सहायक सामग्री' इसी दिशा में एक आवश्यक कदम है । यह सहायक सामग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है । केन्द्रीय विद्यालय संगठन की 'विद्यार्थी सहायक सामग्री' अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री-संकलन की विशेषज्ञता के लिए जानी जाती है और अन्य शिक्षण संस्थान भी इसका उपयोग परीक्षा संबंधी पठन सामग्री की तरह करते रहे हैं । शुभ-आशा एवं विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर सतत मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुंचाएगी ।

शुभाकांक्षा सहित ।

निधि पांडे
आयुक्त, केन्द्रीय विद्यालय संगठन

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CLASS-XII

(2024-25)

One Paper **Max Marks: 80**

No.	Units	No. of Periods	Marks
I.	Relations and Functions	30	08
II.	Algebra	50	10
III.	Calculus	80	35
IV.	Vectors and Three - Dimensional Geometry	30	14
V.	Linear Programming	20	05
VI.	Probability	30	08
	Total	240	80
	Internal Assessment		20

Unit-I: Relations and Functions

1. Relations and Functions **15 Periods**

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and ontofunctions.

2. Inverse Trigonometric Functions **15 Periods**

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices **25 Periods**

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness ofinverse, if it exists; (Here all matrices will have real entries).

2. Determinants **25 Periods**

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability **20 Periods**

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, $like$ $\sin^{-1}$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives **10 Periods**

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima andminima (first derivative test motivated geometrically and second derivative test given as a provabletool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals 20 Periods

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}},$$

$$\int \frac{(px+q)dx}{ax^2+bx+c}, \int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx, \int \sqrt{ax^2 + bx + c}\, dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals 15 Periods

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations 15 Periods

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx} + py = q \ where \ p \ and \ q$ are functions of x or constants

$\frac{dx}{dy} + px = q \ where \ p \ and \ q$ are functions of y or constants

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors **15 Periods**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry **15 Periods**

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equationof a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1. Linear Programming **20 Periods**

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability **30 Periods**

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

MATHEMATICS (Code No. - 041)

QUESTION PAPER DESIGN CLASS - XII

(2024-25)

Time: 3 hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, givingdescriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a differentway.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combiningelements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overallweightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper. However, 33% internal choices will be given in all the section.

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

RELATIONS AND FUNCTIONS

KEY POINTS:-

1. RELATIONS:

- The set of all ordered pairs (a , b) of element $a \in A, b \in B$ is called the cartesian product of set A and set B and is denoted by A x B.

  A x B = { (a,b); a∈ A , b∈ B}
- A relation R from A to B is a subset of A x B .
- Let A and B be any two non-empty finite sets containing m and n elements respectively,

  i) number of ordered pairs in A x B is mn.

  ii) total number of subsets of A x B is 2^{mn}.

  iii) total number of relations from A to B is 2^{mn}.
- If there is no relation between the elements of a set or sets , then the relation is called empty relation. R =∅ .
- A relation R in a set A is called universal relation , if every element of A is related to each of the elements of A , R = A x A.
- A relation R on a set A is said to be reflexive if every element of A is related to itself. R is reflexive if (a,a) ∈ R for every a ϵ A.
- A relation R on a set A is said to be a symmetric relation, if (a,b) ∈ R⇒ (b,a) ∈ R , a, b ∈ A.
- A relation R on a set A is said to be a transitive relation, if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R.
- A relation which is reflexive, symmetric and transitive , is called an equivalence relation .
- Let R be an equivalence relation on a set A. Let a ∈ A. Then the set of all those elements of A which are related to A is called equivalence class denoted by [a] or Cl {a}.

2. FUNCTIONS:

- Let A and B be two non-empty sets. Then, a subset f of A x B is a function that associates each element of A to a unique element of B.
- Let f : A→ B then the set A is called the domain of f and the set B is known as its co-domain. The set of images of elements of set A is known as the range of f. Clearly Range is a subset of Co-domain.
- A function f : A→ B is a one-one function or an injection , if

$$f(x) = f(y) \Rightarrow x = y \text{ for all } x,y \in A \text{ or}$$

$$x \neq y \Rightarrow f(x) \neq f(y) \text{ for all } x,y \in A$$

- A function f : A→ B is an onto function or a surjection, if range(f) = co-domain (f). So each element in B must have atleast one pre-image in A.
- A function f : A→ B is a many-one function if there exists at least two or more elements of A having the same f image in B.i.e.,

  $$f(x) = f(y) \text{ but } x \neq y.$$

  So many-one function cannot be injective.
- If f : A→ B is such that there exists atleast one element in co-domain which is not the image of any element in domain, then f(x) is into. So into function can not be surjective.
- If a set A has m elements and set B has n elements, then the number of functions possible from A to B is n^m.
- If a set A has m elements and set B has n elements,

  if n≥ m, then the number of injective functions or one-one functions is given by $\frac{n!}{(n-m)!}$ and

  if n<m then 0.
- If a set A has m elements and set B has n elements, the number of onto functions from A to B is $\sum_{r=1}^{n}(-1)^{n-r}\, {}^{n}C_r\, r^m$,if m≥ n and 0 , if m<n
- If there is a bijection between two sets A and B then both sets will have the same number of elements. If n(A) = n(B) i.e., m = n, then the number of bijective functions = n!, if m ≠ n then 0.
- Algebraic operations on functions:

  if f and g are real valued functions of x with domain set A and B respectively then

  i) $(f\pm g) = f(x) \pm g(x)$ (domain is A∩B)

  ii) (f.g) (x) = f(x). g(x) (domain is A∩B)

  iii) $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$ (domain is A∪B and g(x) ≠ 0)

  iv) f=g if and only if A=B and f(x)=g(x) for all x in A.

Functions	**Domain**	**Range**
Algebraic functions:		
i) x^n ($n \in N$)	R	R, if n is odd $R^+ \cup \{0\}$, if n is even
ii) $1/x^n$ ($n \in N$)	R – {0}	R – {0}, if n is odd

		R^+, if n is even
iii) $x^{1/n}$ ($n \in N$)	R, if n is odd $R^+ \cup \{0\}$, if n is even	R, if n is odd $R^+ \cup \{0\}$, if n is even
iv) $1/x^{1/n}$ ($n \in N$)	$R - \{0\}$, if n is odd R^+, if n is even	$R - \{0\}$, if n is odd R^+, if n is even
Trigonometric functions:		
i) sin x	R	[- 1 , + 1]
ii) cos x	R	[- 1 , + 1]
iii) tan x	$R - (2k+1)\frac{\pi}{2}$, $k \in Z$	R
iv) cot x	$R - k\pi$, $k \in Z$	R
v) sec x	$R - (2k+1)\frac{\pi}{2}$, $k \in Z$	$(-\infty, -1] \cup [1, \infty)$
vi) cosec x	$R - k\pi$, $k \in Z$	$(-\infty, -1] \cup [1, \infty)$
Inverse Trigonometric functions:		
i) $\sin^{-1}x$	[- 1 , + 1]	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
ii) $\cos^{-1}x$	[- 1 , + 1]	$[0, \pi]$
iii) $\tan^{-1}x$	R	$(-\frac{\pi}{2}, \frac{\pi}{2})$
iv) $\cot^{-1}x$	R	$(0, \pi)$
v) $\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \{\frac{\pi}{2}\}$
vi) $\mathrm{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$
Exponential functions:		
i) e^x	R	R^+

ii) $e^{1/x}$	R – { 0 }	R^+ - { 1 }
iii) a^x , a > 0	R	R^+
iv) $a^{1/x}$, a > 0	R – { 0 }	R^+ - { 1 }
Logarithmic functions:		
i) $\log_a x$ (a > 0, a≠1)	R^+	R
ii) $\log_x a$ (a > 0, a≠1)	R^+ - { 1 }	R – { 0 }
Integral part or greatest integer functions:		
i) [x]	R	I
ii) 1/x	R – [0,1)	1/n , n ϵZ– { 0 }
Fractional part functions:		
i) { x }	R	[0 , 1)
ii) 1/ { x }	R - Z	(1, ∞)
Modulus functions:		
i) | x|	R	R^+ U {0}
ii) 1/ | x|	R - {0}	R^+
Signum function:		
$\frac{\|x\|}{x}$, x ≠ 0 0 , x = 0	R	{ -1 , 0 , 1}
Constant function:		
f(x) = c	R	{ c }

MCQ:

1. If a relation R on the set {1,2,3,4] be defined by R= {(1,2)}, then R is

a) Reflexive b) Transitive c) Symmetric d) None of these

2. Which of the following functions from Z to Z are one-one and onto?

a) $f(x) = x^3$ b) $f(x) = x+2$ c) $f(x)= 2x + 1$ d) $f(x) = x^2 + 1$

3. The function $f: R \to R$ given by $f(x) = \cos x, x \in R$ is

a) one-one but not onto b) onto but not one-one

c) one-one and onto d) neither one-one nor onto

4. Greatest integer function f(x) =[x] is

a) one-one b) many-one c) both (a) & (b) d) none of these

5. The maximum number of equivalence relations on the set A = {1, 2, 3} are

a) 1 b) 2 c) 3 d) 5

6. If $R = \{(x, y) : x^2 + y^2 = 4 , x,y \epsilon Z\}$ is a relation of Z , then the domain of R is

a) {0,1,2} b) {-2 ,0,2} c) {-2, -1,1,2} d) { 1,2}

7. Consider the non-empty set consisting of children in a family and a relation R defined as a R b if a is sister of b. Then R is

a) symmetric but not transitive b) transitive but not symmetric

c) neither symmetric nor transitive d) both symmetric and transitive

8. Number of relations that can be defined on the set A = {a, b, c, d} is

a) 2^3 b) 4^4 c) 4^2 d) 2^{16}

9. Let R be the relation in the set Z of all integers defined by R = {(x, y): x – y is an integer}. Then R is

a) Reflexive b) Transitive c) Symmetric d) an equivalence relation

10.Let S be the set of all real numbers. Then the relation R = {(a, b): 1 + ab > 0} on S is

a) reflexive, symmetric but not transitive b) reflexive, transitive but not symmetric

c) reflexive, symmetric and transitive d) both symmetric and transitive but not reflexive

Assertion –reasoning:

a) Both A and R are true and R is the correct explanation of A

b) Both A and R are true and R is not the correct explanation of A

c) A is true but R is false

d) A is false but R is true.

11. Assertion(A): a relation R ={ |a-b|< 2 } defined on the set A ={ 1,2,3,4,5} is reflexive.

Reason(R): A relation R on the set A is said to reflexive if for (a,b) ϵ R and (b,c) ϵ R we have (a,c) ϵ R

12. Assertion(A): Let A= {2,4,6} , B={3,5,7,9} and defined a function f = { (2,3),(4,5), (6,7)} from A to B ,then f is not onto.

Reason(R): A function f: A→ B is said to be onto, if every element of B is the image of some element of A under f.

13. Assertion(A): The smallest integer function f(x) is one-one.

Reason(R): A function is one-one if f(x) = f(y) ⇒ x = y.

14. Assertion(A): The function f: R→ R, f(x) = |x| is not one-one.

Reason(R): The function f(x) = |x| is not onto.

Long questions:

15. If f : N → N be the function defined by f(x) = $4x^3$ +7, check whether f is a one-one and onto function or not.

16. show that the function f: R → { xϵ R : -1 < x <1 } defined by f(x) = $\frac{x}{1+|x|}$, xϵ R is one-one and onto function.

17. A function f: [-4,4] → [0,4] , given by f(x)= $\sqrt{16 - x^2}$. Show that f is an onto function but not one-one. Further find all possible values of a, f(a) = $\sqrt{7}$

18. Consider f : R – { - $\frac{4}{3}$ } → R – { $\frac{4}{3}$ } given by f(x) = $\frac{4x+3}{3x+4}$. show that f is one-one and onto .

19. Let A = {1,2,3,........,9} and the relation R on the set A x A defined by (a,b) R (c,d) ↔ a +d = b +c for all (a,b),(c,d) ∈ A xA. Prove that R is an equivalence relation. Also find [(2,5)].

20. Show that the relation R on the set A ={ x ∈ Z ; 0 ≤ x ≤ 12} , given by R = { (a,b) : |a - b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

Hint or Answer keys of Selected Questions

1 – b	4 - b	7 - b	10 – a	13 - d
2 – b	5 - d	8 - d	11 – c	14 - b
3 – d	6 - b	9 - d	12 – d	

17.	$a = 3, -3$
19.	[(2,5)] = {(1,4), (2,5), (3,6),(4,7),(5,8), (6,9)}.
20.	Transitive: (a,b) ∈ R ⇒ |a-b|=4k (multiple of 4)⇒(a – b) = ± 4k ....(1) (b,c) ∈ R ⇒ |b-c|=4m (multiple of 4) ⇒(b – c) = ± 4m ....(2) Adding (1) and (2) we get, a – c = ± 4(k + m), multiple of 4. so, (a,c) ∈ R. Relation is transitive . The set of all elements related to 1= { 1, 5, 9}

Practice questions:

1. Let A = R – {3} and B = R- {1}. Find the value of a such that the function f: A→ B defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also , check whether the given function is one-one or not.

2. Let N denote the set of all natural numbers and R be the relation on N x N defined by

(a , b) R (c,d) imply that ad(b+c) = bc(a+d).

Check whether R is an equivalence relation or not on N x N.

3. Show that the relation R on the set A of points in a plane, given by

R = { (P,Q) :Distance of the point P from the origin is same as the distance of the point Q from the origin},

is an equivalence relation. Further show that the set of all points related to a point P ≠ (0,0) is the circle passing through P with origin as centre.

4. Show that f : N → N defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{if n is odd and} \\ \frac{n}{2}, \text{if n is even} \end{cases}$$

is many-one and onto function.

5. Test whether the relation R on Z defined by R = {(a,b): $|a - b| \leq 5$} is reflexive, symmetric, and transitive.

6. Show that the relation R in the set A={1,2,3,4,5}, given by R = {(a,b) : $|a - b|$ is divisible by 2}, is an equivalence relation . Show that all the elements of {1,3,5} are related to each other and all the elements of {2,4} are related to each other , but no elements of {1,3,5} is related to any element of {2,4}.

7. Let R be the equivalence relation in the set A = {0,1,2,3,4,5} given by R ={ (a,b): 2 divides (a – b)}. Write the equivalence class [0].

8. Let R = {(a, a^3): a is a prime number less than 5} be a relation. Find the range of R.

9. Check whether the function f : N → N given by f(x) = $9x^2 + 6x - 5$ is one-one and onto or not.

10. If R = {(x, y) : x + 2y =8 } is a relation on N, then write the range of R.

11. Check whether a function f : R → $[-\frac{1}{2}, \frac{1}{2}]$ defined as f(x) = $\frac{x}{1+x^2}$ is one-one and onto or not.

12. Let L be the set of all lines in XY plane and R be the relation on L defined as

$$R = \{ (L_1, L_2): L_1 \text{ is parallel to } L_2\} .$$

Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

13. Prove that a function f: [0,∞) → [-5, ∞) defined as f(x) = $4x^2 + 4x - 5$ is both one-one and onto.

14. Let f : R – { $-\frac{4}{3}$ } → R given by f(x) = $\frac{4x}{3x+4}$. show that f is one-one and onto .

15. Show that the signum function f: R→ R , given by , $f(x) = \begin{cases} 1, \text{if } x > 0 \\ 0, \text{if } x = 0 \\ -1, \text{if } x < 0 \end{cases}$

is neither one-one nor onto.

Inverse Trigonometric Functions

KEY POINTS :

What is an Inverse Function?

If y=f(x) and x=g(y) are two functions such that f (g(y)) = y and g (f(y)) = x, then f and g are said to be inverse of each other, written as $g = f^{-1}$ and $f = g^{-1}$.

If y = f(x) then $x = f^{-1}(y)$

The inverse trigonometric functions are the inverse functions of the trigonometric functions written as $\cos^{-1} x$, $\sin^{-1} x$, $\tan^{-1} x$, $\cot^{-1} x$, $\operatorname{cosec}^{-1} x$, $\sec^{-1} x$.

If y = sin x , then $x = \sin^{-1} y$, similarly for other trigonometric functions, inverse trigonometric functions are written.

Domain & Range of Inverse Trigonometric Functions

Functions	**Domain**	**Range**
$\sin^{-1} x$	[-1,1]	$[-\frac{\pi}{2},\frac{\pi}{2}]$
$\cos^{-1} x$,	[-1,1]	$[0,\pi]$
$\tan^{-1} x$	R	$(-\frac{\pi}{2},\frac{\pi}{2})$
$\cot^{-1} x$	R	$(0,\pi)$
$\sec^{-1} x$	R – (-1,1)	$[0,\pi]-\{\frac{\pi}{2}\}$
$\operatorname{cosec}^{-1} x$	R – (-1,1)	$[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$

You may note:

$\sin^{-1}(\sin\theta)=\theta$, $\cos^{-1}(\cos\theta)=\theta$ within the domain,

$\sin^{-1}(-x)= -\sin^{-1}(x)$, $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$, $\tan^{-1}(-x)=-\tan^{-1}x$.

$\sin^{-1} x + \cos^{-1} x=\frac{\pi}{2}$, $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$

MCQ WITH ANSWER

1. One branch of $\cos^{-1} x$ other than the principal value branch corresponds to:

(a) $[\frac{\pi}{2},\frac{3\pi}{2}]$ (b)$[\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$ $(c)(-\pi, 0)$ $(d)\ [2\pi, 3\pi]$

Ans . (d)

2. The principal value of $\cos^{-1}(\cos(-680^0)$ is:

(a) $\frac{2\pi}{9}$ (b) $-\frac{2\pi}{9}$ (c) $\frac{34\pi}{9}$ $\frac{(d)\pi}{9}$

Ans. (a)

3. If $\cos^{-1} x - \sin^{-1} x=0$, then the value of x is:

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Ans. (c)

4. The value of $\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right)$ is:

(a) $\frac{13\pi}{6}$ (b) $\frac{7\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

Ans.(d)

5. The value of $\sin^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is:

(a) $\frac{\pi}{2}$ (b)- $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d)none of these

Ans. (d)

ONE MARK QUESTIONS WITH ANSWERS

1. Find the value of $\sin^{-1}(\sin\frac{3\pi}{5})$. Ans . $\frac{2\pi}{5}$

2. Evaluate $\cos^{-1}(\cos\frac{7\pi}{6})$. Ans. $\frac{5\pi}{6}$

3. Which is greater tan1 or $\tan^{-1}1$. Ans. tan1> $\tan^{-1}1$

4. $\sin^{-1}(\sin\frac{2\pi}{3})=\frac{2\pi}{3}$, state true or false. Ans. False

5. Domain of function $\sin^{-1}x$ is (-1,1), state true or false. Ans. False

6. If $\tan^{-1}x=\sin^{-1}\frac{1}{\sqrt{2}}$, then x is ___ . Ans. 1

7. Find the value of $\cot(\tan^{-1}a+\cot^{-1}a)$ Ans. 0

Assertion- Reason Question

Assertion(A): Principal value branch of x is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Reason(R): In this branch the function is bijective

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true and R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

Ans: (b)

THREE LEVELS OF GRADED QUESTIONS

Level I (Very Short Type Question)

1. Find the principal value of $\sin^{-1}(\frac{-1}{2})$. Ans. - $\frac{\pi}{6}$

2. Find the principal value of $\cos^{-1}(\frac{\sqrt{3}}{2})$. Ans. $\frac{\pi}{6}$

3. Find the principal value of $\tan^{-1}(-\sqrt{3})$. Ans: $\frac{4\pi}{3}$.

4. Find the value of $\tan^{-1}(1)+\sin^{-1}(\frac{-1}{2})$. Ans: $\frac{\pi}{12}$.

5. Find the value of $\tan^{-1}(\sqrt{3})-\cot^{-1}(-\sqrt{3})$. Ans:- $\frac{\pi}{2}$.

6. Find the value of $\sin(\pi/3-\sin^{-1}(\frac{-1}{2})$. Ans: 1.

7. Find the principal value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$. Ans:- $\frac{\pi}{3}$.

Level II (SHORT TYPE QUESTION)

1. Find the value of $\tan^{-1}(1)+\cos^{-1}(\frac{-1}{2})+\sin^{-1}(\frac{-1}{2})$ Ans: $\frac{3\pi}{4}$.

2. Find the value of $\tan \{ \sin^{-1}(\frac{3}{5})+\cot^{-1}(\frac{3}{2}) \}$ Ans: $\frac{17}{6}$.

3. Write in simplest form: $\tan^{-1}\left(\frac{cosx-sinx}{cosx+sinx}\right)$ Ans: $\frac{\pi}{4}$- x.

4. Show that $\sin^{-1}(\frac{3}{5})+-\sin^{-1}(\frac{8}{17})= \cos^{-1}(\frac{84}{85})$

5. Prove that $\cot^{-1}[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}]=\frac{x}{2}$, $0 < x < \frac{\pi}{2}$

6. Prove that $\tan^{-1}(x)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Level III (Long Type Question)

1. If $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, find the value of x. Ans: x= 0, ½.

2. Show that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{60}=\pi$

3. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$,then show that xy+yz+zx=1

4. Prove that $\cos(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2})=\frac{6}{5\sqrt{13}}$

5. Solve: $\cos(\tan^{-1}x)=\sin(\cot^{-1}¾)$ Ans : x= ¾.

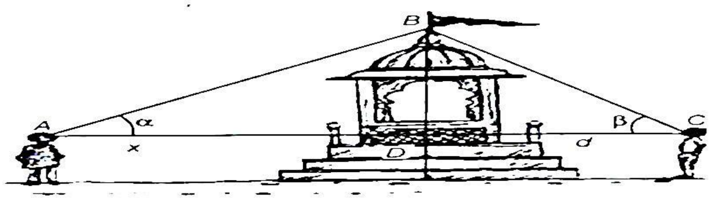
6. Simplify: $\sin^{-1}\left(\frac{3x+4\sqrt{1-x^2}}{5}\right)$ Ans: $\cos^{-1}$ ⅗ $-\cos^{-1}$ x.

7. Prove that $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]=\sqrt{\left(\frac{x^2+1}{x^2+2}\right)}$

8. If $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c = \pi$ prove that a + b + c = abc

Case based Question

Two men on either side of a temple of 30 m high observe its top at the angles of elevation α and β respectively,



The distance between the two men is $40\sqrt{3}$ m and the distance between the first person A and the temple is $30\sqrt{3}$ m.

Answer the following questions using the above information.

(i)	$\angle CAB = \alpha =$	$(a)\left(\frac{2}{\sqrt{3}}\right)$	(b) $\left(\frac{1}{2}\right)$	(c) (2)	(d) $\left(\frac{\sqrt{3}}{2}\right)$
(ii)	$\angle CAB = \alpha =$	$(a)\left(\frac{1}{5}\right)$	(b) $\left(\frac{2}{5}\right)$	(c) $\left(\frac{4}{5}\right)$	(d) $\left(\frac{\sqrt{3}}{2}\right)$
(iii)	$\angle BCA = \beta =$	$(a)\left(\frac{1}{2}\right)$	(b) (2)	(c) $\left(\frac{1}{\sqrt{3}}\right)$	(d) $(\sqrt{3})$
(iv)	$\angle ABC =$	$(a)\frac{\pi}{4}$	(b) $\frac{\pi}{6}$	(c) $\frac{\pi}{2}$	(d) $\frac{\pi}{3}$

Ans:. (i) (b) (ii) (d) (iii) (d) (iv) (c)

MATRICES

KEY POINTS:

Matrix: An ordered rectangular array of numbers or functions.

Matrix Order: A matrix having m rows and n columns is read as a matrix of order m by n & written as m x n.

Row Matrix: A matrix of order having only one row and any number of columns.

Column Matrix: A matrix having only one column and any number of rows.

Square matrix: A matrix of order m × n is called square matrix if m = n.

Zero matrix: $A = [a_{ij}]_{m\times n}$ is called a zero matrix, if $a_{ij} = 0$ for all i and j.

Diagonal matrix: A square matrix $[a_{ij}]_{m\times n}$ is said to be diagonal, if $a_{ij} = 0$ for $i \neq j$.

Scalar matrix: A diagonal matrix $A = [a_{ij}]_{m\times n}$ is said to be scalar, if $a_{ij} = k$ for $i = j$ for some real number k.

Unit matrix (Identity matrix): A diagonal matrix $A = [a_{ij}]_n$ is a unit matrix, if $a_{ij} = 1$ for $i = j$

Addition/Subtraction of Matrices

The order of two matrices should be same for addition or subtraction operations to be performed.

$A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$

Then, $A + B = [a_{ij} + b_{ij}]_{m \times n}$

Properties:

- *A+B = B+A (Commutative Property of Addition)*
- *(A+B) + C = A + (B+C)[Associative Property of Addition]*
- *k(A+B) = kA+ kB, where k is any scalar*
- *A + (-A) = **O** where **O** is zero matrix of same order*

Scalar Multiplication: If A is a matrix of order mxn then scalar multiplication of A by a scalar k, which is another matrix of order m x n denoted by

(kA), further if C=(kA), then $c_{ij} = k\, a_{ij}$,i=1,2,3,......m and j=1,2,3,......n.

As for example if A =$\begin{bmatrix} 4 & 8 \\ 7 & 1 \end{bmatrix}$ then 5A = $\begin{bmatrix} 20 & 40 \\ 35 & 5 \end{bmatrix}$

Matrix Multiplication- The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let A = $[a_{ij}]$ be an m × n matrix and B = $[b_{jk}]$ be an n × p matrix. Then the product of the matrices A and B is the matrix C of order m × p. To get the $(i, k)^{th}$ element c of the matrix C, we take the i^{th} row of A and k^{th} column of B, multiply them element-wise and take the sum of all these products.

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$ then $AB = C = [c_{ik}]_{m\times p}$ where $c_{ik} = \sum_{j=1}^{n} a_{ij}b_{jk}$

Some properties of matrix multiplication

- *AB ≠ BA (In general matrix multiplication is not commutative)*
- *(AB)C = A(BC) (Associative Property of Multiplication)*
- *A(B + C) = AB + AC (Distributive Property of Multiplication)*

Transpose of a Matrix

If a matrix is obtained from any given matrix A, by interchanging rows and columns, it is called the transpose of A and is denoted by A' or A^T.

If $A = [a_{ij}]_{mxn}$ and $A' = [b_{ij}]_{nxm}$ then $b_{ij} = a_{ji}$, ∀i, j

Properties:

- *(A')' = A.*
- *(kA)' = kA'*
- *(A + B)' = A' + B'*
- *(AB)' = B' A'*

Symmetric and Skew Symmetric Matrix

1. A is a symmetric matrix if A' = A.
2. A is a skew symmetric matrix if A' = - A.

For example: A = $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ is a symmetric matrix and A = $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$ is a skew symmetric matrix

*Any square matrix can be represented as the sum of a symmetric and a skew-symmetric matrix in an unique way.i.e for any matrix A ,

A = ½ (A+A^T) +½ (A-A^T), where ½ (A+A^T) is a symmetric matrix and ½ (A-A^T) is skew symmetric.

Inverse of Matrix:

If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse matrix of A and is denoted by A^{-1}**.**

Example:

$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ be two matrices.

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -6+6 \\ 2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Similarly $BA = I$

Hence, B is the inverse of A or in other words,

B = A^{-1} and A is the inverse of B, i.e., A = B^{-1}

Properties of Invertible Matrices

Let A and B be n×n invertible matrices (nonsingular). Then

- If A is non-singular, then so is A^{-1} and $(A^{-1})^{-1} = A$.
- If A and B are non-singular matrices, then AB is non-singular and $(AB)^{-1} = B^{-1} A^{-1}$.
- If A is non-singular then $(A^T)^{-1} = (A^{-1})^T$.
- Inverse of a matrix if exists is unique.

MCQs (1 MARK QUESTIONS)

1. How many 2x2 order matrices can be formed with entries 0 and 1 only.

   (a) 8 (b) 16 (c) 4 (d) 32

2. If the order of matrix A is m×p. and the order of B is p×n. Then the order of matrix AB is

   (a) n × p (b) m × n (c) n × p (d) n × m

3. If A is a square matrix such that $A^2 = A$, then $(I – A)^3 + A$ is equal to

   (a) I (b) 0 (c) I – A (d) I + A

4. If A and B are symmetric matrices of the same order, then (AB'–BA') is a

   (a) Skew symmetric matrix (b) Null matrix (c) Symmetric matrix (d) None of these

5. If A = $\begin{bmatrix} 2 & x-3 & x-2 \\ 3 & -2 & -1 \\ 4 & -1 & -5 \end{bmatrix}$ is a symmetric matrix then x is

   (a) 3 (b) 6 (c) 8 (d) 0

6. If A is a square matrix and if A is skew symmetric then which of the following is true.

   (a) $(A+A^T) = O$ (b) $(A - A^T) = O$ (c) $(AA^T) = I$ (d) $(A+A^T) = I$

7. If A= $\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 – kA – 5I = 0$, then k =

   (a) 5 (b) 3 (c) 7 (d) None of these

8. If matrix A = $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = pA$, then the value of p is

   (a) 1 (b) 2 (c) -4 (d) 4

9. If the sum of all the elements of a 3 × 3 scalar matrix is 9, then the product of all its elements is:

   (a) 0 (b) 9 (c) 27 (d) 729

TWO MARKS QUESTIONS

1. If A and B are symmetric matrices, show that AB is symmetric, if AB=BA.

   Let A = $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ B= $\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ then show that (AB) ′ = B ′ A ′.

2. A= $\begin{bmatrix} \sin x & cosx \\ -cosx & \sin x \end{bmatrix}$ then verify that AA' =I

3. If $\begin{bmatrix} cos\frac{2\pi}{5} & -sin\frac{2\pi}{5} \\ sin\frac{2\pi}{5} & cos\frac{2\pi}{5} \end{bmatrix}^k$ = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,then find the least integral value of k.

4. Consider the following information regarding the number of men and women worker in three factories I, II and III.

Factory No	Men Workers	Women Workers
I	20	25
II	15	30
III	40	50

Represent the above information in the form of a 3 x 2 matrix. What does the entry in the second row and second column represent?

5. If one given matrix A is both symmetric and also skew-symmetric, then find A.
6. What possible orders can a matrix have if it has 24 elements?
7. Prove that the principal diagonal of any skew symmetric matrix is zero.
8. Two schools A and B want to award their selected students on the values of Honesty, Hardwork and Punctuality. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of Rs.2200. School B wants to spend Rs.3100 to award its 4, 1 and 3 students on the respective values. The total amount of award for one prize on each value is Rs.1200. Convert this problem in matrix form.
9. If A = $\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ then express it as the sum of a symmetric and a skew symmetric matrix.

THREE MARKS QUESTIONS

1. If A is a square matrix such that $A^2 = I$ then find the simplified value of $(A - I)^3 + (A + I)^3 - 7I$.
2. In a legislative assembly election, a political group hired a public relations firm to promote its candidate in three ways: telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

   $A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Telephone} \\ \text{house calls.} \\ \text{letters} \end{matrix}$

The number of contacts of each type made in two cities X and Y is given by B

$$\begin{matrix} & \text{telephone} & \text{house calls} & \text{letters} \end{matrix}$$

$$B=\begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix}$$

Find the total amount spent by the group in two cities X and Y.

3. If $F(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that F(x)F(y)=F(x+y)
4. Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $A^2 + 4A - 42I = O$. Hence find A^{-1}.
5. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that CD – AB = O.
6. Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then what will be value of $\alpha + \beta$.
7. Find matrix A if, $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$.
8. Find The matrix X where $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} X \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

9.If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O. Hence\ find\ A^{-1}$.

FIVE MARKS QUESTIONS

1. If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$ then find the value of (a + x) – (b + y).
2. A trust invested some money in two types of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs. 2800 as interest. However, if trust had interchanged money in bonds, they would have got Rs. 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation.

3. Find matrix A such that

$$\begin{bmatrix} 2 & -2 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

ANSWERS(1 MARK QUESTIONS)

1.(b) 2. (b) 3.(a) 4.(a) 5(b) 6(a) 7(a) 8(d) 9.(a)

HINT FOR 2 MARKS QUESTIONS

3. K=2.

5. i) $\begin{bmatrix} 20 & 25 \\ 15 & 30 \\ 40 & 50 \end{bmatrix}$, ii) No of women workers in II factory is 30.

7. 1x24,2x12,3x8,4x6,6x4,8x3,12x2,24x1

8. A =-A^T thus a_{ij}= - a_{ji} also for the diagonals a_{ii}=-a_{ii} ,so 2a_{ii}=0 or a_{ii}=0

9. AX = B, where A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$& B = $\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$, solving x = 300, y = 400 & z = 500.

HINT FOR 3 MARKS QUESTIONS

1. 8A-7I.
4. A^{-1}= $\frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$.
5. D = $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$
6. 8
7. $\frac{-1}{4}\begin{bmatrix} -35 & -18 \\ 25 & 10 \end{bmatrix}$
8. $\begin{bmatrix} -4 & -3 \\ 6 & 4 \end{bmatrix}$
9. $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$.

HINT FOR 5 MARKS QUESTIONS

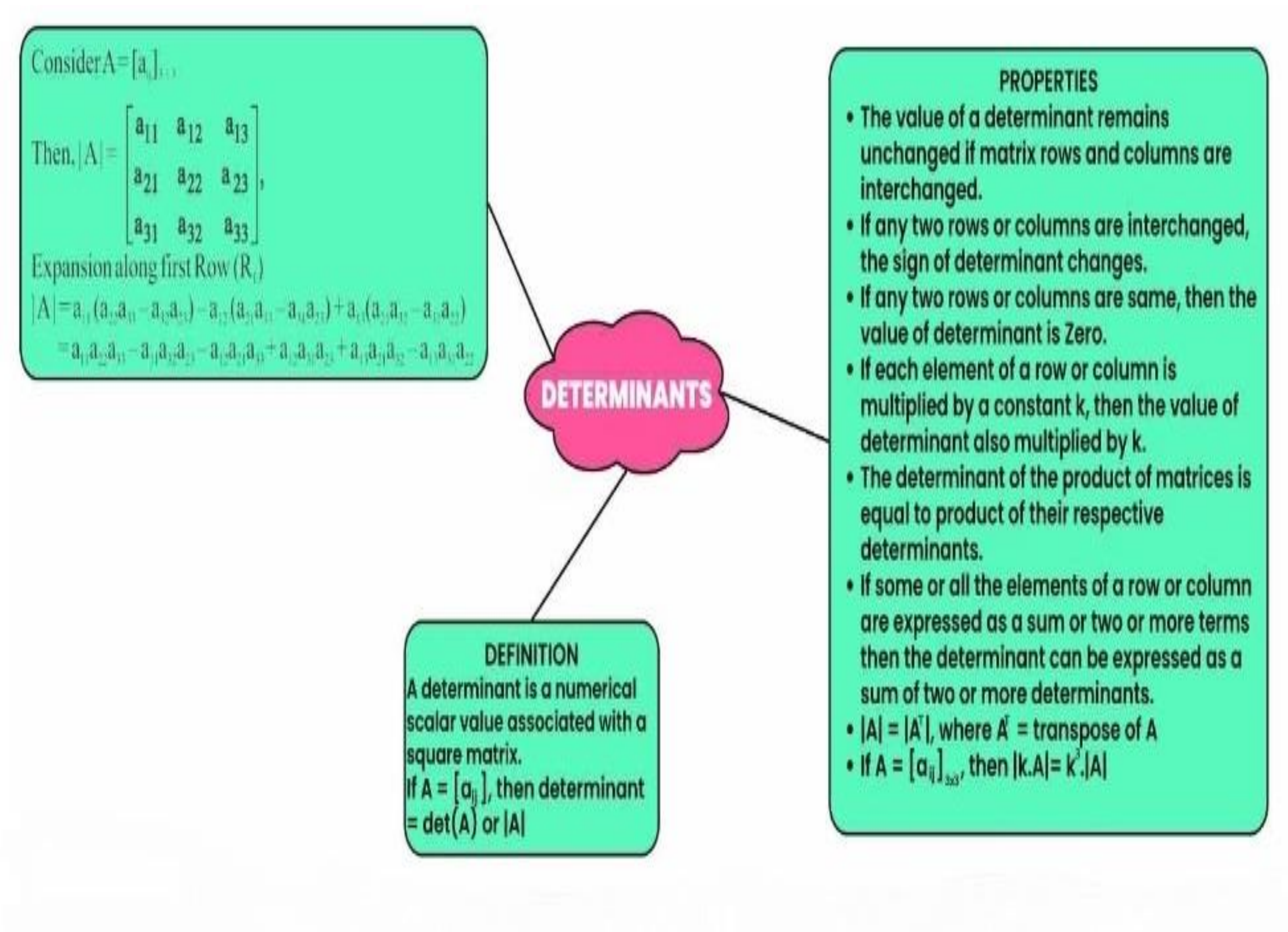
1. AA^{-1} = I; then comparing the matrices and getting linear equations and solve, hence (a + x) – (b + y) = 3
2. ₹ 10000 and ₹ 15000
3. $\begin{bmatrix} 1 & -2 \\ 3/2 & 2 \end{bmatrix}$

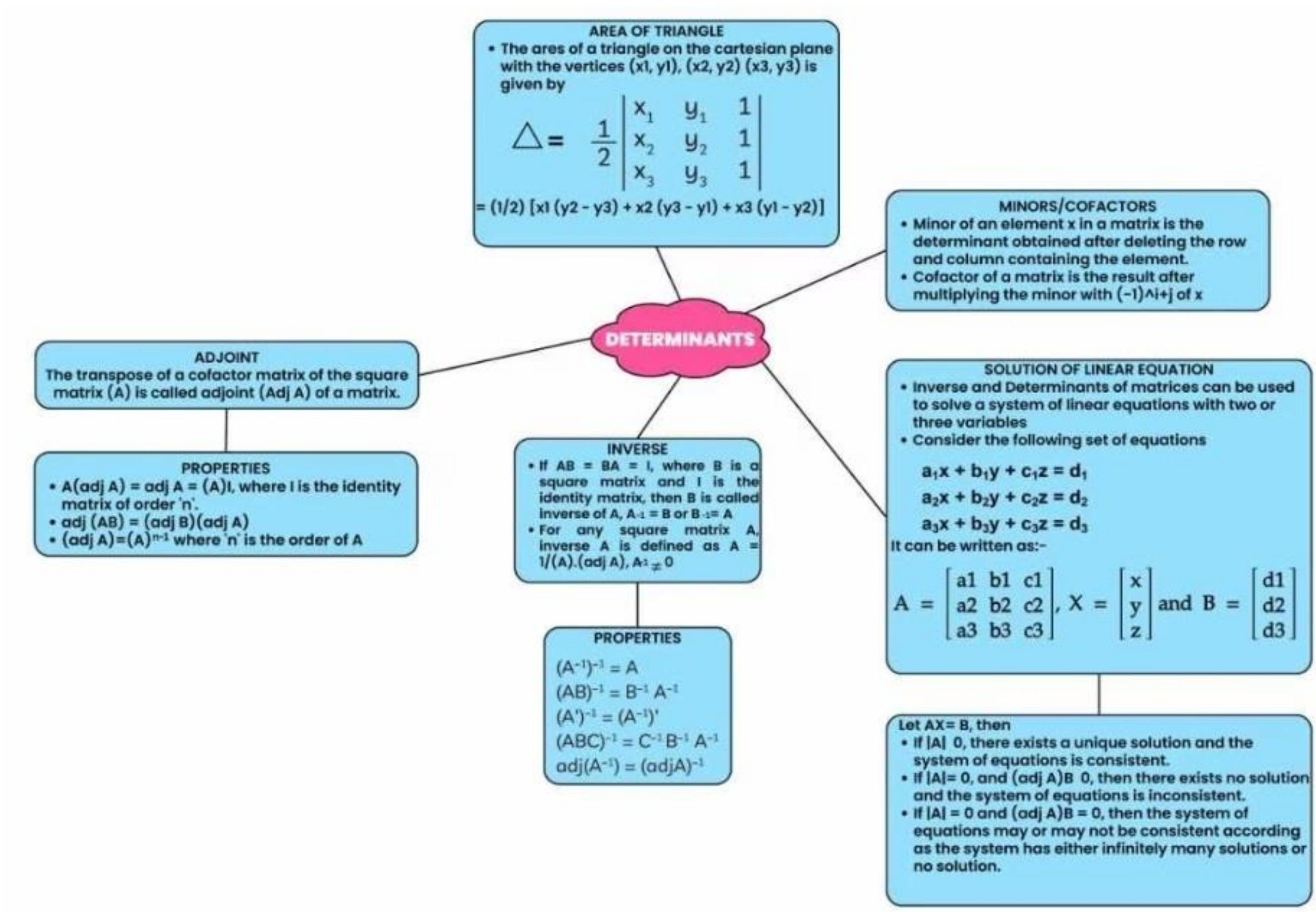
<u>**DETERMINANTS**</u>

Conceptual notes:

- Every square matrix can be associated with a number (real or complex), is known as determinant.
- Determinant is a a real valued function whose domain is the set of all square matrix.
- For a square matrix A, its determinant is denoted by det (A) or $|A|$.Here, |A| denotes determinant value, not modulus of A.
- Matrix being an arrangement of numbers has no value, but determinant has a fixed value. Determinants are not defined for non-square matrix.
- A Determinant corresponding to matrix A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ of order 2× 2 is

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}.$$





Multiple Choice Questions:

1. If for a matrix A= $\begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α

   (A) ± 3 (B) -3 (C) ± 1 (D) 1

2. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then value of x =

   (A) ±3 (B) -3 (C) ±2 (D) 2

3. If $\begin{vmatrix} 16 & x \\ x & 4 \end{vmatrix} = \begin{vmatrix} 8 & 32 \\ 2 & 8 \end{vmatrix}$ then the value(s) of x is/are

   (A) 8 (B) -8 (C) ± 8 (D) 32

4. If A is square matrix of order 3 x 3 such that $|A| = 4$, then the value of $|3A|$ is

   (A) 9 (B) 36 (C) 108 (D) None of these

5. If A is a square matrix of order 3 x 3, then $|kA|$ is equal to

   (A) $3k$ (B) $k|A|$ (C) $k^2|A|$ (D) $k^3|A|$

6. Let A be a non-singular square matrix of order 3 x 3. The $|adjA|$ is

   (A) $|A|$ (B) $|A|^2$ (C) $|A|^3$ (D) $3|A|$

7. If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, then the value of the determinant of 3AB is

(A) -9 (B) -27 (C) -81 (D) 81

8. If $A^2 - A + I = O$, then the inverse of A is

(A) A^{-2} (B) $1 - A$ (C) O (D) A

9. The value of $(A^{-1})^T$ is

(A) $(A^T)^{-1}$ (B) A^{-1} (C) I (D) A^T

10. If the value of a third order determinant is 12, then the value of the determinant formed by replacing each element by its cofactor will be

(A) 12 (B) 24 (C) 144 (D) 1728

11. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

(A) $x = 0$(B) $x = 1$ (C) $x = 2$ (D) $x = 3$

12. For what value of k, the matrix $\begin{bmatrix} k & 1 \\ 2 & -3 \end{bmatrix}$ will be the adjoint of the matrix $A = \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$?

(A) $k = -3$ (B) $k = 1$ (C) $k = 2$ (D) $k = 5$

Assertion-Reasoning Questions:

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of a.

(c) A is true but R is false.

(d) A is false but R is true.

(i) **Assertion (A):** If A is a square matrix of order 3 x 3, such that |A| = 9, then $|A^{-1}| = \frac{1}{9}$.

Reason (R): For a square matrix A of order n x n, $|A^{-1}| = \frac{1}{|A|}$.

(ii) **Assertion (A):** $\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12}$,

where a_{ij}, is element of a determinant and A_{ij} is cofactor of a_{ij}.

Reason (R): For $\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$, $a_{11}+A_{21} + a_{12}+A_{22} = 0$

(iii) **Assertion (A):** Points $(1, 7), (3, 5) and$ $(4, 4)$ are collinear.

Reason(R): Area of the triangle formed by the points (1, 7), (3, 5) and (4, 4) is 0.

(iv) **Assertion(A): A=** $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ **and** $A^{-1}=kA$ then $k=\frac{1}{19}$

Reason(R) : $|A^{-1}| = \frac{1}{|A|}$

Case Based Questions:

1. Two schools A and B want to award their selected students on the values of Honesty, Hard work and Punctuality. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to its 3, 2 and 1 students respectively with total award money of ₹ 2200. School B wants to spend Rs. 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school A). The total amount of award for one prize on each value is Rs. 1200.

Based on the above information, answer the following questions:

(a) Write the system of linear equation in variables x, y and z.

(b) Write the matrix equation represented by the above situation.

(c) Find the values of x, y and z.

OR

Find the value of $\frac{x+y}{z}$.

2. On his birthday Rahul decided to donate some money to the children of an orphanage home. If there were 8 children less, everyone would have got Rs.10 more. Howeverif there were16 children more,everyone would have got Rs. 10 less.Using matrix method find the number of children and amount distributed by Rahul.

3. Gautam buys 5 pens, 3 bags and 1 instrument boxe and pays a sum of Rs.160. From the sameshop, Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of Rs.190. Also, Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of Rs. 250.

Based on the above information, answer the following questions.

(i) Convert the given above situation into a matrix equation of the form AX = B.

(ii) Find A^{-1}

(iii) Using A^{-1} find the cost of a pen, a bag and an instrument box.

4. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements while others are rewarded based on their financial needs. Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria in the session 2023-24, the school offered monthly scholarship of Rs. 3000 each to some girl students and Rs. 4000 each to meritorious achievers in academics as well as sports. In all 50 students were given the scholarships and monthly expenditure incurred by the school on scholarship was Rs180000.

Based on the above information answer the following questions.

a. Express the given information algebraically using matrices.
b. Check whether the system of matrix equations so obtained is consistent or not.
c. Find the number of scholarships of each kind given by the school using matrices.
d. Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school?

5. In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

Based on the above information, answer the following questions.

i) Total units of type I produced for boys? ii) Total production of each type for boys? iii) Total production of each type for girls?

6. Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs.100 and Rs. 50 each respectively. The numbers of articles sold are given as

School /Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	3)	40

Based on the information given above, answer the following questions:

(i)What is the total money (in Rupees) collected by the school DPS?

(ii) What is the total amount of money (in Rs.) collected by schools CVC and KVS?

(iii)What is the total amount of money collected by all three schools DPS, CVC and KVS?

(iv)If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?

(v)How many articles (in total) are sold by three schools?

Very Short Answer Type Questions:

(i) If A is a 3 × 3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

(ii) If for any 2× 2 square matrix A, $A(adjA) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

(iii) If $|A| = 3$ and $A^{-1} = \begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$ find A.

(iv) Find A^{-1} if $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

(v) A square matrix A is invertible if A is non-singular. Now if A= $\begin{bmatrix} 2 & p & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$ then for which value of p, A^{-1} exists?

Short Answer Type Questions:

(i) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

(ii) If A = $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that $A.(adjA) = |A|\text{I}$.

(iii) Verify A(adj. A) =(adj. A)A = (det A). I for A = $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

(iv) The monthly incomes of two brothers Sirish and Srijan are in the ratio 3:4 and the monthly expenditures are in the ratio 5:7. Each brother saves Rs. 15000 per month. Using matrix find their monthly income.

<u>Long Answer Type Questions</u>:

(i) If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = O$.

(ii)The sum of three numbers is 2. If we subtract the second number from twice the first number, we get 3. By adding double the second number and the third number we get 0. Represent it algebraically and find the numbers using matrix method.

(iii) Solve the following system of equations, using matrices:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

(iv)If A = $\begin{bmatrix} 1 & cotx \\ -cotx & 1 \end{bmatrix}$ then show that $A'A^{-1}$ = $\begin{bmatrix} -cos2x & -sin2x \\ sin2x & -cos2x \end{bmatrix}$

(v)Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the following system of equations:

x– y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1

(vi)An amount of Rs 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income from these investments is Rs 358. If the total annual income from first two investments is Rs 70 more than the income from the third, find the amount of each investment by the matrix method.

(Vii) Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that $A^{-1} = \frac{A^2-3I}{2}$.

<u>Self-practice requisitions</u>

(i) The matrix $\begin{bmatrix} 4+3k & 3 \\ 1+2k & 2 \end{bmatrix}$ is singular matrix, for k equal to

(a) 0 (b) -1 (c) 1 (d) no value of k

(ii) The values of x for which $\begin{vmatrix} 6 & -2 \\ 2 & 4 \end{vmatrix} = x^2 - 12x$ are

(a) $-2,14$ (b) $2, -14$ (c) $-2, -14$ (d) None of these

(iii) Three vegetable shopkeepers A, B and C are using polythene bags. Handmade bags which are prepared by old age home workers and newspapers' envelope as carry bags. It is found that A, B and C are using (20, 30, 40), (30, 40, 20) and (40, 20, 30) polythene bags, handmade bags, and newspaper envelopes respectively. The shopkeepers A, B, and C spent 250, 270 and 200 on these bags respectively.

Based on the above information answer the following:

(a) What is the cost of one handmade bag?

(b) What is the cost of one polythene bag?

(c) Which vegetable shopkeeper is better, based on the social condition?

OR

Which vegetable shopkeeper is better, based on the environmental condition?

(iv) Raja purchases 3 pens, 2 pencils and 1 mathematics instrument box and pays 41 to the shopkeeper. His friends, Daya and Anil purchases 2 pens, 1 pencil, 2 instrument boxes and 2 pens, 2 pencils and 2 mathematical instrument boxes respectively. Daya and Anil pays 29 and 44 respectively.

Based on the above information answer the following

(a) Find the cost of one pen.

(b) Find the cost of one pen and one mathematical instrument box.

(c) Find the cost of one pencil and one mathematical instrumental box.

OR

Find the cost of one pen, one pencil and one mathematical instrumental box.

(v) If A is a square matrix satisfying $A'A = I$, then write the value of $|A|$.

(vi) If A is a square matrix satisfying AA' = I, then write the value of |A|.

(vii) If A is a 3×3 matrix, $|A| \neq 0$ and $|3A| = k|A|, then$ find the value of k.

(viii) If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, find the value of x.

(ix) For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

(x) Show that for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, $A^3 - 6A^2 + 5A + 11I = 0.$ hence, find A^{-1}.

Answer-Key for the Multiple Choice Questions:

1) $(A) \pm 3$ 2) (d) 2 3) (C) ± 8 4) (C) 108 5) (D) $k^3|A|$ 6) (B) $|A|^2$

7) (C) -81 8) (B) $1 - A$ 9) (A) $(A^T)^{-1}$ 10) (C) 144 11) (D) $x = 3$ 12) (D) $k = 5$

Answer-Key for the Assertion-Reasoning Questions:

(i) (a) (ii) (c) (iii) (a) (iv) (b)

Answer-Key for the Case Based Questions: ,

1) x=300,y=400,z=500. **2)** $x = 32, y = 30.$ **3)** $x = 10, y = 20, z = 50.$ **4)** x=20,y=30.**5.** (i)165, (ii) 437,(iii) 430 **6.** (i) Total money collected by the school DPS is Rs. 7000, (ii) Rs.14000, (iii)Rs.21000, (iv) Rs.21000, (v) 330.

Very Short Answer Type Questions:

(i)k=-1 (ii) $|A| = 8$ (iii)$3\begin{bmatrix} \frac{2}{3} & 1 \\ \frac{5}{3} & 3 \end{bmatrix}$ (iv)$\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$. (v) p≠ $\frac{-8}{5}$.

Answer-Key for the Short Answer Type Questions:

(i) $k = \frac{1}{19}$. (iii) $adj\ A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$, det A=0. (iv) Rs.30000 & Rs.15000.

Answer-Key for the Long Answer Type Questions:

(i) $A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$ (ii)x = 1, y = – 2, z = 2 (iii) x=2 ,y=3,z=5 (v) x =3, y = -2 & z =-1.

(vi) x=1000, y=2200,z=1800

(Vii) $A^{-1} = \frac{A^2-3I}{2}$.

Answer-Key for the self-practice questions:

(i) (d) no value of k (ii) (a) -2,14 (iv) x=2, y=15, z=5. (v) $|A|=\pm 1$ (vii)K = 9. (viii) $x = \frac{1}{2}$.

(ix) $A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$

CONTINUITY& DIFFERENTIABILITY

KEY POINTS:

- A function f is said to be continuous at x = a if

  Left hand limit = Right hand limit = (value of the function at x = a)

  i.e. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

  i.e. $\lim_{x \to a} f(x) = f(a)$

- Properties of continuous function:

  1. If f and g are two continuous functions at a point a, then

     (i) f + g is continuous at a (ii) f – g is continuous at a

     (iii) f.g is continuous at a (iv) $\frac{f}{g}$ is continuous at a , provided $g(a) \neq 0$

     (v)the composition functions $fog\ and\ gof$ are continuous at a.

  2. Every polynomial function is continuous
  3. Modulus function $(|f|)$ is continuous.
  4. sin x and cos x are continuous in R.
  5. tan x, cot x, sec x, cosec x are continuous in R except the points where these functions are undefined.
  6. Every logarithmic and exponential function is a continuous function.

- A function is said to be **differentiable** at x = a, if

  $Lf'(a) = Rf'(a)$ i.e $\lim_{h \to 0} \frac{f(a-h)-f(a)}{-h} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

- Rule

  $\frac{d}{dx}(ku) = k\frac{du}{dx}$ $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

  $\frac{d}{dx}(u.v) = u\frac{d}{dx}v + v\frac{d}{dx}u$ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

- **Formulae**

  (i) $\frac{d}{dx}(x^n) = n\,x^{n-1}$, $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$, $\frac{d}{dx}\left(\sqrt{x}\right) = -\frac{1}{2\sqrt{x}}$

  (ii) $\frac{d}{dx}(x) = 1$ (iii) $\frac{d}{dx}(c) = 0,\ c \in R$

  (iv) $\frac{d}{dx}(a^x) = a^x \log a,\ a > 0,\ a \neq 1.$ (v) $\frac{d}{dx}(e^x) = e^x.$

  (vi) $\frac{d}{dx}(\log_a x) = \frac{1}{x \log a}$, a > 0, a ≠ 1**,** x>0 (vii) $\frac{d}{dx}(\log x) = \frac{1}{x}$, x > 0

(viii) $\frac{d}{dx}$ ($\log_a$| x |) = $\frac{1}{x \log a}$, a > 0, a ≠ 1, x ≠ 0

(ix) $\frac{d}{dx}$ (log | x |) = $\frac{1}{x}$, x ≠ 0

(x) $\frac{d}{dx}$ (sin x) = cos x, ∀ x ∈ R.

(xi) $\frac{d}{dx}$ (cos x) = – sin x, ∀ x ∈ R.

(xii) $\frac{d}{dx}$ (tan x) = $\sec^2$x, ∀ x ∈ Domain.

(xiii) $\frac{d}{dx}$ (cot x) = – cosec^2x, ∀ x ∈ Domain.

(xiv) $\frac{d}{dx}$ (sec x) = sec x tan x, ∀ x ∈ Domain.

(xv) $\frac{d}{dx}$ (cosec x) = – cosec x cot x, ∀ x ∈ Domain.

(xvi) $\frac{d}{dx}$ ($\sin^{-1}$x) = $\frac{1}{\sqrt{1-x^2}}$. $(-1 \leq x \leq 1)$

(xvii) $\frac{d}{dx}$ ($\cos^{-1}$x) = $\frac{-1}{\sqrt{1-x^2}}$. $(-1 \leq x \leq 1)$

(xviii) $\frac{d}{dx}$ ($\tan^{-1}$x) = $\frac{1}{1+x^2}$, ∀ x ∈ Domain

(xix) $\frac{d}{dx}$ ($\cot^{-1}$x) = $\frac{-1}{1+x^2}$, ∀ x ∈ Domain.

(xx) $\frac{d}{dx}$ ($\sec^{-1}$x) = $\frac{1}{|x|\sqrt{x^2-1}}$ ∀ x ∈ Domain

(xxi) $\frac{d}{dx}$ (cosec^{-1}x) = $\frac{-1}{|x|\sqrt{x^2-1}}$ ∀ x ∈ Domain

(xxii) $\frac{d}{dx}$ (| x |) = $\frac{x}{|x|}$, x ≠ 0

- **Differentiation of Implicit Function**- If two variables are expressed by some relation the one is called implicit function of the other.

To find $\frac{dy}{dx}$ $in\ f(x, y) = k$, (k is a constant) differentiating both side w.r.t x, then collect all the $\frac{dy}{dx}$ and find this.

For Example : $y = cos\ x + sin\ sin\ (xy) \Rightarrow \frac{dy}{dx} = -sinx + cos\ cos\ (xy)\ \frac{d}{dx}(xy)$

$$\Rightarrow \frac{dy}{dx} = -sin\ x + cos\ cos\ (xy)\ \left(y + x\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} - x\ cos(xy)\frac{dy}{dx} = -sin\ x + y\ cos(xy)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-sin\ x\ +\ y\ cos(xy)}{1\ -\ x\ cos(xy)}$$

- **Logarithmic Differentiation:** Logarithmic Differentiation are used for differentiating of functions which consists of the product or quotients of a number of functions and/or the function is of type $[f(x)]^{g(x)}$.

  Method – If $y = [f(x)]^{g(x)}$, by taking log and using properties of log, we get

  $log\ y = g(x)\ log\ f(x)$, differentiating both side w.r.t x , we have

  $\frac{1}{y}\frac{dy}{dx} = g'(x)logf(x) + g(x).\frac{1}{f(x)}f'(x)$

  $\Rightarrow \frac{dy}{dx} = y[g'(x)logf(x) + g(x).\frac{1}{f(x)}f'(x)]$

- **Parametric Form :** Sometimes we come across the function when both x and y are expressed in terms of another variables say t i.e., $x = \phi(t) and\ y = \psi\ (t)$. This form is called parametric form and t is called parameter.

$$\frac{dy}{dx} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}}$$

QUESTIONS FOR PRACTICE

1 MARK QUESTIONS

1. The function f(x) = $[x]$ is continuous at

   (a) 4 (b) -2 (c) 1 (d) 1.5

2. If $f(x) = \begin{cases} 3x-5, x \le 5 \\ 2k, x > 5 \end{cases}$,

   is continuous at x=5 then k is

   (a) 5 (b) 10 (c) 15 (d) 20

3. If y = log tan $\sqrt{x}$ then the value of $\frac{dy}{dx}$ is

   (a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{sec^2\sqrt{x}}{\sqrt{x}tan\sqrt{x}}$ (c) $2\, sec^2\sqrt{x}$ (d) $\frac{sec^2\sqrt{x}}{2\sqrt{x}\, tan\sqrt{x}}$

4. If y = (cos x^2)^2 then $\frac{dy}{dx}$ is

   (a) -4 x sin $2x^2$ (b) – x sin x^2 (c) -2 x sin $2x^2$ (d) - x cos $2x^2$

5. If y = $\cot^{-1}(x^2)$ then the value of $\frac{dy}{dx}$ is

   (a) $\frac{2x}{1+x^2}$ (b) $\frac{2x}{\sqrt{1+4x}}$ (c) $\frac{-2x}{1+x^4}$ (d) $\frac{-2x}{\sqrt{1+x^2}}$

6. If x = t^2 and y = t^3 then $\frac{d^2y}{dx^2}$ is equal to

   (a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$

7. Derivative of x^2 w.r.t x^3 is

   (a) $\frac{1}{x}$ (b) $\frac{2}{3x}$ (c) $\frac{2}{3}$ (d) $\frac{3x}{2}$

8. If $f(x) = \begin{cases} \frac{kx}{|x|}\ if\ x < 0 \\ 3\ if\ x \ge 0 \end{cases}$

   is continuous at x = 0, then the value of k is

   (a) – 3 (b) 0 (c) 3 (d) any real number

9. The set of all points where the function $f(x) = x + |x|$ is differentiable, is

   (a)(0, ∞) (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

10. The function $f(x) = x|x| is$:

    (a) continuous and differentiable at x = 0

    (b) continuous but not differentiable at x = 0

    (c) not continuous but differentiable at x = 0

    (d) neither continuous nor differentiable at x = 0

In the following questions (11 - 14), a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

11. **Assertion(A)** : f(x) = $[x]$ not continuous at x = 2

    Reason(R) : f(x) = $[x]$ not differentiable at x = 2

12. **Assertion(A)** : The value of x for which the function $f(x) = \frac{x^2-3x-4}{x^2+3x-4}$ is not continuous at – 4 and 1.

    Reason(R) : The value of x for which the function $f(x) = \frac{g(x)}{h(x)}$ is not continuous given by $h(x) = 0$

13. **Assertion(A)** : If $f(x).g(x)$ is continuous at x = a, then $f(x)$ and $g(x)$ are separately continuous.

    Reason(R) : Any function $f(x)$ is said to be continuous at x = a if $f(x) = f(a)$

14. **Assertion(A)** : $|sinx|$ is a continuous function.

    Reason(R) : If $f(x)$ and $g(x)$ are both continuous functions, then $f(x)og(x)$ is also continuous.

2-MARKS QUESTIONS

15. If the function$f(x) = \begin{cases} \frac{1-cos4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$

    is continuous at x=0 then find the value of k.

16. Find the value of m, for which the function

    $$f0(x) = \begin{cases} m(x^2 - x), & x > 0 \\ \cos cos\, x, & x \leq 0 \end{cases}$$

    is continuous at $x = 0$.

17. Show that the function $f(x) = |x - 3|\ x \in R$, is continuous but not differentiable at $x = 3$.

18. If $f(x) = \begin{cases} x^2, if\ x \geq 1 \\ x, if\ x < 1 \end{cases}$

    then show that f is not differentiable at x = 1

19. For what value of k is the following function is continuous at x=2

$$f(x) = \begin{cases} 2x + 1, if\ x < 2 \\ k, if\ x = 2 \\ 3x - 1, if\ x > 2 \end{cases}$$

20. If $x\ sin\ (a+y)+sina\ cos(a+y)=0,$ then prove that $\frac{dy}{dx}=\frac{sin^2(a+y)}{sina}$

3-MARKS QUESTIONS

21. If $y=x^{sinx}+sinx^{cosx}$ then find $\frac{dy}{dx}$.

22. If $y=x^x$, then prove that $\frac{d^2y}{dx^2}-\frac{1}{y}(\frac{dy}{dx})^2-\frac{y}{x}=0$

23. If $x\sqrt{1+y}+y\sqrt{1+x}=0\ ,prove\ that\ \frac{dy}{dx}=\ \frac{-1}{(1+x)^2}$

24. Differentiate $(\frac{1}{\sqrt{1-x^2}})\ w.r.t\ (2x\sqrt{1-x^2})$

25. If $f(x)=\begin{cases} 3ax+b\ ifx<1 \\ 11\ ifx=1 \\ 5ax-2b\ if\ x>1 \end{cases}$

is continuous at x = 1, find a and b.

26. If x=$\sqrt{a^{sin^{-1}}t}$, y=$\sqrt{a^{cos^{-1}}t}$. Show that $\frac{dy}{dx}=-\frac{y}{x}$

5-MARKS QUESTIONS

27. If $x=a(cost+tsint)\ and\ y=a(sint-tcost), 0<t<\frac{\pi}{2}$, find $\frac{d^2x}{dt^2},\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

28. If y=$(tan^{-1}x)^2$. Show that $(x^2+1)^2\frac{d^2y}{dx^2}+2x(x^2+1)\frac{dy}{dx}=2$.

4-MARKS QUESTIONS (CASE BASED)

29. The use of electric vehicles will be curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V:

$V(t)=\frac{1}{5}t^3-\frac{5}{2}t^2+25t-2$, where t represents the time and t = 1, 2, 3,….. corresponds to year 2001,2002,2003,…respectively.

Based on the above information , answer the following questions

(i) Can the above function be used to estimate the number of vehicles in the year 2000? Justify.

(ii) Is the function continuous? Justify.

(iii) Find the slope of the tangent to the curve at t = 3,(i.e derivative V w.r.t time t at t=3)

30. A potter made a vessel, where the shape of the pot is based on $f(x)=|x-3|+|x-2|$, where f(x) represents the height of the pot.

(i) When x > 4 what will be the height in terms of x?

(ii) Will the derivative vary with x?

(iii) What is $\frac{dy}{dx}$ at x = 3 ?

(iv) if the potter is trying to make a pot using

the function $f(x) = [x]$, will he get pot or not ? Justify.

SOLUTION

1) (d) 1.5 2)(a) 5 3)(d) $\frac{sec^2\sqrt{x}}{2\sqrt{x}\,tan\,\sqrt{x}}$ 4)(c) -2 x sin $2x^2$ 5)(c) $\frac{-2x}{1+x^4}$ 6)(b) $\frac{3}{4t}$

7)(b) $\frac{2}{3x}$ 8)(a) – 3 9)(c) $(-\infty, 0) \cup (0, \infty)$ 10)(a) 11) (b) 12)(a)

13)(d) 14) (a) 15) K= 1 16) no value of m exists 17) continuous but not differentiable at $x = 3$. 18) f(x) is not differentiable at x = 1. 19) k=5.

21) Let y= $(\cos x)^{\sin x} + (\sin x)^{\cos x}$

y = v + u (let) , proceed now

22) Given, $y = x^x$ Taking log on both sides, we get $logy = x.logx$, now differentiate

23)x$\sqrt{1+y}$ = $-y\sqrt{1+x}$; square it, simplify and bring $y = -\frac{x}{1+x}$ and then differentiate.

24) $\frac{1}{2}$ 25) $a = 3, b = 2$

26) Here $xy = a^{(sin^{-1}t + cos^{-1}t)/2} = a^{\pi/4}$ Differentiating both side w.r.t x, we have

$x\frac{dy}{dx} + y.1 = 0 \Longrightarrow \frac{dy}{dx} = -\frac{y}{x}$ 27) Here $\frac{dy}{dx} = \frac{sec^3t}{at}$

29) (i) no. (ii) Since the function is polynomial, hence it is continuous. (iii)$\frac{77}{5}$

30) (i) $f(x) = 2x - 5\ for\ x > 4$ (ii) derivative varies with x (iv)pot is not possible.

APPLICATION OF DERIVATIVES

KEY POINTS:

RATE OF CHANGE OF QUANTITIES

1. The rate of change of area of a circle(A) with respect to radius ‘r’ $= \frac{dA}{dr}$

   As we know that area of circle (A) = $\pi r^2, \frac{dA}{dr} = 2\pi r$

2. The rate of change of area of a circle(A) with respect to time ‘t’= $\frac{dA}{dt}$

   As we know that area of circle (A) = $\pi r^2, \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

3. The rate of change of volume of a cube (v) with respect to time ‘t’ $= \frac{dV}{dt}$

   If edge of cube is x, then V= x^3 and so $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

4. The rate of change of surface area of a cube (S) with respect to time ‘t’ $= \frac{\mathrm{dS}}{\mathrm{dt}}$

   If edge of cube is x, then S= $6x^2$ and $\frac{\mathrm{dS}}{\mathrm{dt}} = 12 \frac{dx}{dt}$

5. The perimeter P of a rectangle (of length x and breadth y) = $P = 2(x + y)$

   Therefore, the rate of change of perimeter with respect to time ‘t’, $\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$

6. The area A of a rectangle (of length x and breadth y)= $A = xy$

   Therefore, the rate of change of area with respect to time ‘t’ $= \frac{dA}{dt} = \frac{dx}{dt}y + \frac{dy}{dt}x$

7. The volume of a sphere (V) $= \frac{4}{3}\pi r^3$,(r is the radius of sphere)

   Therefore, the rate of change of the volume of a sphere (V) with respect to time ‘t’= $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

8. The surface area of a sphere (S) $= 4\pi r^2$,(r is the radius of sphere)

   Therefore, the rate of change of the surface area of a sphere (S) with respect to time ‘t’= $\frac{dS}{dt} =$

   $8\pi r \frac{dr}{dt}$

9. The volume of a cone of radius ‘r’ and height ‘h’= $\frac{1}{3}\pi r^2 h$

   Therefore, the rate of change of the volume of a cone (V) with respect to time‘t’= $\frac{dV}{dt} = \frac{2}{3}\pi r \frac{dh}{dt}$

10. The marginal cost at a given value , of the total cost C (x) in Rupees associated with the production of x units $= C'(x)$

11. The marginal revenue at a given value , of the total revenue R (x) in Rupees received from the sale of x units of a product $= R'(x)$

INCREASING AND DECREASING FUNCTIONS

DEFINITION (A)

Let I be an interval contained in the domain of a real valued function, then

1. f is said to be increasing function, if $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in$ I
2. f is said to be strictly increasing function, if $x_1 < x_2 \Rightarrow \mathrm{f(x_1) < f(x_2)}$ for all $x_1, x_2 \in$ I
3. f is said to be decreasing function, if $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in$ I
4. f is said to be strictly decreasing function, if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ I

DEFINITION (B)

Let f be continuous function on $[a, b]$ and differentiable on (a, b), then

1. f is said to be strictly increasing in $[a, b]$ if $f'(x) > 0$ for each $x \in (a, b)$.
2. f is said to be strictly decreasing in $[a, b]$ if $f'(x) < 0$ for each $x \in (a, b)$.
3. f is said to be constant function in $[a, b]$ if $f'(x) = 0$ for each $x \in (a, b)$.

Note: If a function f is strictly increasing on $(\boldsymbol{a}, \boldsymbol{b})$ and strictly decreasing on $(\mathbf{b}, \mathbf{c})$, then f is neither increasing nor decreasing on $(\mathbf{a}, \mathbf{c})$

PROCESS FOR SOLVING THE PROBLEMS

ALGORITHM 1: For strictly increasing function

Step I Let $x_1 < x_2$

Step II Find $f(x_1)$

Step III Find $f(x_2)$

Step IV Show that $\mathrm{f(x_1) < f(x_2)}$

ALGORITHM 2: For strictly decreasing function

Step I Let $x_1 < x_2$

Step II Find $f(x_1)$

Step III Find $f(x_2)$

Step IV Show that $\mathrm{f(x_1) > f(x_2)}$

ALGORITHM 3: For increasing function

Step I Obtain the function and let it as $f(x)$

Step II Find $f'(x)$

Step III observe if $f'(x) > 0$

ALGORITHM 4: For decreasing function

Step I Obtain the function and let it as $f(x)$

Step II Find $f'(x)$

Step III observe if $f'(x) < 0$

TO FIND THE INTERVAL IN WHICH FUNCTION IS INCREASING OR DECREASING

ALGORITHM 5: For increasing function

Step I obtain the function and let it as $f(x)$

Step II Find $f'(x)$

Step III Put $f'(x) > 0$ and solve the in-equation.

ALGORITHM 6: For decreasing function

Step I obtain the function and let it as $f(x)$

Step II Find $f'(x)$

Step III Put $f'(x) < 0$ and solve the in-equation

MAXIMA AND MINIMA

Maximum value of a function: Let $f\ (x)$ be a real valued function defined on$[a, b]$. Then $f(x)$ is said to have maximum value in $[a, b]$ if there exist a point 'c' in $[a, b]$ such that $\mathbf{f(x) \leq f(c)}$ for all $x \in [a, b]$.

Minimum value of a function: Let $f\ (x)$ be a real valued function defined on$[a, b]$. Then $f(x)$ is said to have minimum value in $[a, b]$ if there exist a point 'c' in $[a, b]$ such that $\mathbf{f(x) \geq f(c)}$ for all $x \in [a, b]$.

LOCAL MAXIMA AND LOCAL MINIMA

A function $f\ (x)$ is said to attain a local maximum at $x = a$ if there exists a neighbourhood $(a - \delta, a + \delta)$ of a such that $\mathbf{f(x) < f(a)}$ for all $\in (a - \delta, a + \delta), x \neq a$.

A function $f\ (x)$ is said to attain a local minimum at $x = a$ if there exists a neighbourhood $(a - \delta, a + \delta)$ of a such that $\mathbf{f(x) > f(a)}$ for all $\in (a - \delta, a + \delta), x \neq a$.

FIRST DERIVATIVE TEST FOR LOCAL MAXIMA AND MINIMA

WORKING RULE

Step I obtain the function and let it as $f(x)$

Step II Find $f'(x)$

Step III Put $f'(x) = 0$, to get the values of x (Known as **CRITICAL POINTS**)

Step IV observe the behavior of function at critical points say c

x	Take a value slightly < c	Take a value slightly > c	Nature of point
Sign of $f'(x)$	$+ve$	$-ve$	Maximum
Sign of $f'(x)$	$-ve$	$+ve$	Minimum
Sign of $f'(x)$	$+ve$	$+ve$	Neither maximum nor minimum
Sign of $f'(x)$	$-ve$	$-ve$	Neither maximum nor minimum

NOTE: When $f'(c) = 0$, but $f'(x)$ does not change sign as we move from left to right through p, then f has a point of inflexion at $x = c$.

SECOND DERIVATIVE TEST FOR FINDING MAXIMA AND MINIMA AT A POINT

WORKING RULE

Step I obtain the function and let it as $f(x)$

Step II Find $f'(x)$

Step III Put $f'(x) = 0$, to get the values of x (Known as **CRITICAL POINTS**)

Step IV Find $f''(x)$

Step V Find $f''(x)$ at required Critical point/points i.e c

Step VI If $f''(x)$ at c i.e $f''(c) < 0$, then f has maxima at c and maximum value of function is $f(c)$.

Step VII If $f''(x)$ at c i.e $f''(c) > 0$, then f has minima at c and minimum value of function is $f(c)$.

MULTIPLE CHOICE QUESTIONS (1 MARK)

1. If the rate of change of area of a circle is equal to the rate of change of its diameter , then its radius is

   (i) $\frac{2}{\pi}$ unit (ii) $\frac{1}{\pi}$ unit

   (iii) $\frac{\pi}{2}$ unit (iv) π unit

2. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is

   (i) π cm/sec (ii) 2π cm/sec

   (iii) $\frac{1}{2}\pi$ cm/sec (iv) $\frac{1}{4}\pi$ cm/sec

3. The radius of a circular plate is increasing at the rate of 0.01 cm/sec. The rate of increase of its area when the radius is 12 cm, is

   (i) $144\pi cm^2/sec$ (ii) $2.4\pi cm^2/sec$

   (iii) $0.24\pi cm^2/sec$ (iv) $0.024\pi cm^2/sec$

4. The radius of a sphere is increasing at the rate of 0.2 cm/s . The rate at which the volume of the sphere increases when radius is 15 cm is

   (i) $12\pi cm^3/s$ (ii) $180\pi cm^3/s$

   (iii) $225\pi cm^3/s$ (iv) $3\pi cm^3/s$

5. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 5$ is decreasing is

   (i) $(-1, \infty)$ (ii) $(-\infty, -2)$

   (iii) $(-2, -1)$ (iv) $[-1,1]$

6. The function $f(x) = x^3 + 3x$ is increasing in the interval

   (i) $(-\infty, 0)$ (ii) $(0, \infty)$

   (iii) R (iv) $(0,1)$

7. The function $f(x) = x^3 - 27x + 5$ is increasing when

   (i) $x < -3$ (ii) $|x| > 3$

   (iii) $x \leq -3$ (iv) $|x| \geq 3$

8. The function $f(x) = x^2e^{-x}$ is strictly increasing at

   (i) $(-\infty, \infty)$ (ii) $(0,2)$

   (iii) $(2, \infty)$ (iv) $(-\infty, 0)$

9. If the sides of a square are decreasing at the rate of 1.5 cm/s, the rate of decrease of its perimeter is

   (i) 1.5 cm/s (ii) 6 cm/s

   (iii) 3 cm/s (iv) 2.25 cm/s

10. The rate of change of surface area of a sphere with respect to its radius ‘r, when r= 4 cm is

(i) $64\pi\ cm^2/cm$ (ii) $48\pi\ cm^2/cm$

(iii) $32\pi\ cm^2/cm$ (iv) $16\pi\ cm^2/cm$

ANSWERS (MCQ)

1) (ii) 2) (i) 3) (iii) 4) (ii) 5) (iii) 6) (iii) 7) (ii) 8) (ii) 9) (ii) 10) (iii)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)

1. A spherical ball of salt is dissolved in water in such a manner that the rate of decrease of volume at any instant is proportional to the surface. Palak claims that the radius of salt ball is decreasing at a constant rate.
   Is Palak's claim right? Show steps to justify your answer.
2. The volume of a cube is increasing at a constant rate. Show that the increase in surface area is inversely proportional to the length of edge of the cube.
3. The amount of pollution content added in air in a city due to x diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$.
   Find the marginal increase in pollution content when Ravi uses 3 diesel vehicles.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1. Find the intervals in which the function $f(x) = \frac{logx}{x}$ is strictly increasing or decreasing.
2. Find the absolute maximum and minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$ on the interval $[1,2]$
3. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.
4. Find the absolute maximum and minimum values of the function f given by $f(x) = 4x - \frac{1}{2}x^2$, x∈ $[-2, \frac{9}{2}]$
5. Find the maximum and minimum values of the function f given by $f(x) = sinx + cosx, x \in (0, \frac{\pi}{2})$

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. It is given that $\mathrm{f(x) = x^4 - 62x^2 + ax + 9}$ attains local maximum value at $x = 1$. Find the value of 'a ', hence obtain all other points where the given function $f(x)$ attains local maximum or minimum values.
2. The perimeter of a rectangle metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of the cylinder so formed is maximum.

3. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and circle is minimum?

CASE BASED QUESTIONS

1. In order to set up a rainwater harvesting system, a tank to collect rainwater is to be dug. The tank should have a square base and a capacity of $250m^3$. The cost of land is Rs. 5000 per m^2 and cost of digging increases with depth and for the whole tank, it is $40000h^2$,where h is the depth of the tank in meters. x is the side of the square base of the tank in meters.

   Based on above information, answer the following questions

   (i) Find the total cost C of digging the tank in terms of x.

   (ii) Find $\frac{dC}{dx}$

   (iii) (a) Find the value of x for which cost C is minimum.

   OR

   (b) Check whether the cost function $C(x)$ is expressed in terms of x is increasing or not, where $x > 0$.

2. Read the following passage and answer the question given below.

The relation between the height of the plant (y in cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$., where $x$is the number of days exposed to the sunlight for $x \leq 3$.

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days? What will be the height of the plant after 2 days?

SELF PRACTICE QUESTIONS

1. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 $m^3/hour$. Then the depth of the wheat is increasing at the rate of

   (a) $1\ m^3/hour$ (b) 0.1 $m^3/hour$ (c) 1.1 $m^3/hour$ (d) 0.5 $m^3/hour$ [1]

2. A circular disc of radius 3 cm is being heated. Due to expansion its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm. [2]

3. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/s. How fast is the area decreasing when the two equal sides are equal to the base? [2]

4. Show that the function $f(x) = tan^{-1}(sinx + cosx), x > 0$ is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$. [3]

5. Find the intervals in which $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$is strictly increasing or strictly decreasing. [3]

6. Find the intervals in which $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is increasing or decreasing. [3]

7. A tank with rectangular base and rectangular sides , open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs. 70 per square meters for the base and Rs. 45 per square meter for sides. What is the cost of least expensive tank? [5]

8. A window is in the form of a rectangle surmounted by a semi circular opening. The total perimeter of the window is 10 m. find the dimension of the window to admit maximum light through the whole opening. [5]

INTEGRALS

IMPORTANT POINTS:

1. $\int \frac{p(x)}{g(x)} dx$ *when* $\deg p(x) \geq \deg g(x)$, *then* $\int \frac{p(x)}{g(x)} dx = \int \left\{ q(x) + \frac{r(x)}{g(x)} \right\} dx$

$q(x) = \textbf{Quotient}$ and $r(x) = \textbf{Remainder}$

2. Idea to apply Substitution method

For the function of type $e^{f(x)}$, $\log f(x)$, $\sin f(x)$, $\tan^{-1} f(x)$ or any ***trigonometric/ITF***, put $f(x) = t$ and solve.

3. Use rationalization method for the following functions:

$\frac{1}{1 \pm \sin x}$; $\frac{1}{1 \pm \cos x}$; $\frac{\sin x}{1 \pm \sin x}$; $\frac{\cos x}{1 \pm \cos x}$; $\frac{1}{\sec x \pm 1}$; $\frac{1}{\operatorname{cosec} x \pm 1}$; $\frac{\sec x}{\sec x \pm 1}$ and $\frac{\operatorname{cosec} x}{\operatorname{cosec} x \pm 1}$

Apply trigonometric identity, divide separately and integrate.

4. <u>FUNDAMENTAL THEOREM OF CALCULUS</u>

Let f be continuous on $[a, b]$. If F is any antiderivative for f on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

5. <u>PROPERTIES OF DEFINITE INTEGRALS</u>

(i) $\int_a^b f(x)dx = \int_a^b f(t)dt$

(ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(iii) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(iv) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

(v) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(vi) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

(vii) $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, if\ (2a-x) = f(x) \\ 0, \quad if\ (2a-x) = -f(x) \end{cases}$

(viii) $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, if\ f(x)\ is\ an\ even\ function\ i.e.\ f(-x) = f(x) \\ 0, \quad if\ f(x)\ is\ an\ odd\ function\ i.e.\ f(-x) = -f(x) \end{cases}$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 7

Integrals

Integration

It is the inverse of differentiation. Let, $\frac{d}{dx}F(x)=f(x)$. Then $\int f(x)dx=F(x)+c$, 'c': constant of integral. These integrals are called indefinite or general integrals.

Properties of indefinite integrals are

(i) $\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$, (ii) $\int kf(x)dx=k\int f(x)dx$,

For *eg* : $\int(3x^2+2x)dx=x^3+x^2+c$ where k is real.

Some standard integrals

(i) $\int x^n dx=\frac{x^{n+1}}{n+1}+c, n\neq -1$ like, $\int dx=x+c$

(ii) $\int \cos x\,dx=\sin x+c$ (iii) $\int \sin x\,dx=-\cos x+c$

(iv) $\int \sec^2 x\,dx=\tan x+c$ (v) $\int \operatorname{cosec}^2 x\,dx=-\cot x+c$

(vi) $\int \sec x\tan x\,dx=\sec x+c$ (vii) $\int \operatorname{cosec} x\cot x\,dx=-\operatorname{cosec} x+c$

(viii) $\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x+c$ (ix) $\int\frac{dx}{\sqrt{1-x^2}}=-\cos^{-1}x+c$

(x) $\int\frac{dx}{1+x^2}=\tan^{-1}x+c$ (xi) $\int\frac{dx}{1+x^2}=-\cot^{-1}x+c$

(xii) $\int e^x dx=e^x+c$ (xiii) $\int a^x dx=\frac{a^x}{\log a}+c$

(xiv) $\int\frac{dx}{x\sqrt{x^2-1}}=\sec^{-1}x+c$ (xv) $\int\frac{dx}{x\sqrt{x^2-1}}=-\operatorname{cosec}^{-1}x+c$

(xvi) $\int\frac{1}{x}dx=\log|x|+c$

Integration by partial fractions

A rational function of the form $\frac{P(x)}{Q(x)}(Q(x)\neq 0)=T(x)+\frac{P_1(x)}{Q(x)}$, $P_1(x)$ has degree less than that of $Q(x)$. We can integrate $\frac{P_1(x)}{Q(x)}$ by expressing it in the following forms –

(i) $\frac{px+q}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}, a\neq b.$

(ii) $\frac{px+q}{(x+a)^2}=\frac{A}{x-a}+\frac{B}{(x-a)^2}$

(iii) $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}=\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$

(iv) $\frac{px^2+qx+r}{(x-a)^2(x-b)}=\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$

(v) $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}=\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$

Second fundamental theorem of integral calculus

Let f be a continuous function of x defined on $[a,b]$ and let F be another function such that $\frac{d}{dx}F(x)=f(x)\,\forall x\in$ domain of f, then $\int_a^b f(x)dx=[F(x)+c]_a^b=F(b)-F(a)$. This is called the definite integral of f over the range $[a,b]$, where a and b are called the limits of integration, a being the lower limit and b be the upper limit.

Example

$$\int_{-\pi/4}^{\pi/4}\sin^2 x\,dx$$
$$=2\int_0^{\pi/4}\sin^2 x\,dx$$
$$=2\int_0^{\pi/4}\left(\frac{1-\cos 2x}{2}\right)dx$$
$$=\int_0^{\pi/4}(1-\cos 2x)dx$$
$$=\left[x-\frac{\sin 2x}{2}\right]_0^{\pi/4}$$
$$=\frac{\pi}{4}-\frac{1}{2}$$

First fundamental theorem of integral calculus

Let the area function be defined by $A(x)=\int_a^x f(x)dx\,\forall x\geq a$, where f is continuous on $[a,b]$ then $A'(x)=f(x)\,\forall x\in[a,b]$.

Some special type of integrals

(i) $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c.$

(ii) $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+c.$

(iii) $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c.$

Integration by parts

$$\int f_1(x)f_2(x)dx=f_1(x)\int f_2(x)dx-\int\left[\frac{d}{dx}f_1(x)\int f_2(x)dx\right]dx$$

Integration of some special functions

(i) $\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$ (ii) $\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

(iii) $\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$ (iv) $\int\frac{dx}{\sqrt{x^2-a^2}}=\log\left|x+\sqrt{x^2-a^2}\right|+c$

(v) $\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$ (vi) $\int\frac{dx}{\sqrt{x^2+a^2}}=\log\left|x+\sqrt{x^2+a^2}\right|+c.$

Integration by substitution

The method in which we change the variable to some other variable is called the method of substitution

$\int\tan x\,dx=\log|\sec x|+c$ $\int\cot x\,dx=\log|\sin x|+c$

$\int\sec x\,dx=\log|\sec x+\tan x|+c$ $\int\operatorname{cosec} x\,dx=\log|\operatorname{cosec} x-\cot x|+c.$

MCQ

INDEFINITE INTEGRALS

Q1 $\int \sec^{-1} x\, dx = ?$

a) $x \sec^{-1} x + \log |x + \sqrt{x^2 - 1}| + C$ b) $x \sec^{-1} x + \log |x - \sqrt{x^2 - 1}| + C$

c) $x \sec^{-1} x + \log |x + \sqrt{x^2 - 1}| + C$ d) $x \sec^{-1} x - \log |x + \sqrt{x^2 - 1}| + C$

Q2 Let [x] denote the greatest integer less than or equal to x. Then, $\int_{-1}^{1} [x]\, dx =$

a)2 b)-1 c) 0 d) $\frac{1}{2}$

Q3 $\int e^x \sec x(1 + \tan x) dx$ is equal to

(a) $e^x \cos x + c$ (b) $e^x \sec x + c$

(c) $e^x \sin x + c$ (d) $e^x \tan x + c$

Q. 4 $\int x^x(1 + logx)\, dx$ is equal to

(a) x^x logx + C (b) $\frac{x^x}{\log x}$+ C (c) x^x + C (d) x^x + 1 + C

Q. 5 $\int e^{log\ sinx}\ dx$ is equal to

(a) sin x+C (b) cos x+C (c) –cos x+C (d) –sin x+C

Q. 6 $\int \cot^2 x\, dx$ equals to

(a) cot x – x + C (b) cot x + x + C (c) –cot x + x + C (d) –cot x – x + C

Q. 7 $\int e^x$ (log $sinx$ + cot x) dx

(a) e^x log $sinx$ + C (b) e^x cot x + C (c) e^x tan x + C (d) e^x (log cos$x$ - cot x) + C

Q. 8 $\int \frac{(cos2x - cos2\theta)}{(cosx - cos\theta)} dx$ is equal to where θ is constant

(a) 2(sin x + x cos θ) + C (b) 2(sin x – x cos θ) + C

(c) 2(sin x + 2x cos θ) + C (d) 2(sin x – 2x cos θ) + C

Q.9 If $\int \sec^2(7 - 4x)dx = a \tan (7 - 4x) + C$, then value of a is

(a) 7 (b) –4 (c) 3 (d) –1/4

Q. 10 The anti-derivative of $\sqrt{x} + \frac{1}{\sqrt{x}}$ equals …..

(a) $\frac{1}{3}x^{1/3} + 2x^{1/2}$+C (b) $\frac{2}{3}x^{2/3} + \frac{1}{2}x^{1/2}$+C

(c) $\frac{3}{2}x^{3/2} + 2x^{1/2}$+C (d) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2}$+C

Q. 11 $\int \frac{(10x^9 + 10^x \log_e 10)}{x^{10} + 10^x} dx$= …….

(a) $10^x - 10^x$+c (b) $10^x + 10^x$+c (c) $(10^x + x^{10})^{-1} + c$ (d) $\log(x^{10} + 10^x)$+c

Q. 12 $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

(a) tan x + cot x + C (b) tan x + cosec x + C (c) − tan x + cot x + C (d) tan x + sec x + C

Q. 13 $\int \frac{e^x(1+x)}{cos^2(xe^x)} dx$

(a) $-\cot(ex^x) + c$ (b) $\tan(xe^x) + c$ (c) $\tan(e^x) + c$ (d) $\cot(e^x) + c$

Q. 14 $\int \frac{dx}{x(x^2+1)}$ = ...

(a) $\log|x| - \frac{1}{2}\log(x^2+1) + c$ (b) $\log|x| + \frac{1}{2}\log(x^2+1) + c$

(c) $-\log|x| + \frac{1}{2}\log(x^2+1) + c$ (d) $\frac{1}{2}\log|x| + \log(x^2+1) + c$

Q. 15 $\int \sqrt{1+x^2}\ dx$

(a) $\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log\left(x+\sqrt{1+x^2}\right) + c$ (b) $\frac{2}{3}(1+x^2)^{3/2} + c$

(c) $\frac{2}{3}x(1+x^2)^{3/2} + c$ (d) $\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right) + c$

Q. 16 $\int x^2 e^{x^3}\ dx = \cdots$

(a) $\frac{1}{3}e^{x^3} + c$ (b) $\frac{1}{3}e^{x^2} + c$ (c) $\frac{1}{2}e^{x^3} + c$ (d) $\frac{1}{2}e^{x^2} + c$

MCQ

DEFINITE INTEGRALS

Q. 1 $\int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}} \mathrm{dx}$ =...

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) π

Q. 2 $\int_{-1}^{1} |1-x|\ dx = \cdots$

(a) 3 (b) 2 (c) -2 (d) 1

Q. 3 $\int_{-1}^{1} \log\left(\frac{2+x}{2-x}\right)\ dx = \cdots$

(a) e (b) 0 (c) 1 (d) 2

Q. 4 $\int_0^{\pi/2} \log(cotx)\ dx = \cdots$

(a) $\pi/4 \log tanx$ (b) $\pi/8 \log 2$ (c) 0 (d) $\pi/8 \log 8$

Q. 5 $\int_{-\pi/2}^{\pi/2} x^{2022}\ \sin^{-1} x\ dx = \cdots$

(a) 2022/2021 (b) 2021/2022 (c) 1 (d) 0

Q. 6 $\int_0^a f(x)dx = -2020, then\ \int_0^a f(a-x)dx = \cdots$

(a) 0 (b) -2020 (c) 2020 (d) 1

Q. 7 $\int_{-5}^{5} |x| dx = \cdots$

(a) 0 (b) 25/2 (c) 25 (d) 50

Q. 8 $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

(a) 2 (b)1 (c) π (d) $\pi/2$

Q. 9 $\int_0^{\pi/2} \cos x\, e^{sinx}\, dx = \cdots$

(a) 0 (b) e (c) e-1 (d) $e^{\pi/2}$

Q. 10 $\int_0^1 \frac{dx}{1+x^2}$

(a) 1 (b) 0 (c) $\pi/2$ (d) $\pi/4$

Q. 11 $\int_0^{\pi/4} \sin 2x\ \ dx = \cdots$

(a) 1 (b) 0 (c) ½ (d) $\pi/2$

Q. 12 $\int_0^3 \frac{dx}{9+x^2}$

(a) $\pi/12$ (b) $\pi/2$ (c) $\pi/4$ (d) π

Q. 13 $\int_e^{e^2} \frac{dx}{xlogx}$

(a) 0 (b) 2 (c) log2 (d) 1

Q. 14 $\int_0^2 \sqrt{4-x^2}\ \ dx$

(a) π (b) 2π (c) $\pi/2$ (d) $\pi/4$

Q. 15 If $\int_0^1 (3x^2+2x+k)dx = 0\ , then\ find\ the\ value\ of\ k.$

(a) 1 (b) 2 (c) -2 (d) 4

SHORT – ANSWER QUESTIONS

INDEFINITE INTEGRALS

Q. 1 Integrate $\frac{cos\sqrt{x}}{\sqrt{x}}$ with respect to x.

Q. 2 Evaluate: $\int sec^3 x\, tan\, x\, dx$

Q. 3 Evaluate: $\int \frac{sec^2 x\ \ dx}{3\ + tan\, x}$

Q. 4 Evaluate: $\int \frac{cos\, 2x\ \ dx}{(cos\ +\ sin\, x)^2}$

Q. 5 Evaluate: $\int \frac{x\ +\ cos\, 6x}{3\, x^2\ +\ sin\, 6x} dx$

Q. 6 Evaluate: $\int \frac{dx}{sin^2\, x\ cos^2\, x}$

Q. 7 Evaluate: $\int \frac{sec^2\, x\, dx}{cos\, ec^2\ x}$

Q. 8 Evaluate: $\int \frac{e^{2x}\ -\ 1}{e^{2x}\ +\ 1} dx$

Q. 9 Evaluate: $\int \frac{3\, x\ +\ 2}{(x\ -\ 1)\,(2\, x\ +\ 3)} dx$

Q. 10 Evaluate: $\int x\, e^{x}\, dx$

Q. 11 Integrate $\sqrt{1 - 4\,x - x^2}$ with respect to x.

Q. 12 Evaluate: $\int \frac{dx}{\sqrt{x^2\, +\, 4\,x\, +\, 10}}$

Q. 13 Evaluate: $\int \frac{dx}{x^2\, -\, 6\,x\, +\, 13}$

Q. 14 Evaluate: $\int \frac{dx}{\sqrt{6\, -\, 4\,x\, -\, x^2}}$

Q. 15 Evaluate: $I = \int \frac{(3\,x\, -\, 1)\, dx}{(x\, +\, 2)^2}$

SHORT – ANSWER QUESTIONS

DEFINITE INTEGRALS

Q. 1 Evaluate: $\int_2^4 \frac{x}{x^2\, +\, 1} dx$

Q. 2 Evaluate: $\int_0^1 \frac{tan^{-1}\, x}{1\, +\, x^2} dx$

Q. 3 Evaluate: $\int_0^{\pi/4} tan\, x\, dx$

Q. 4 If $\int_0^a 3x^2\, dx = 8$, then find ‘a’.

Q. 5 Evaluate: $\int_{-1}^1 x^{17} cos^4 x\, dx$

Q. 6 Evaluate: $\int_0^{\frac{\pi}{2}} \frac{sin\, x\, -\, cos\, x}{1\, +\, sin\, x\, cos\, x} dx$

Q. 7 Evaluate: $I = \int_0^{\frac{\pi}{2}} \frac{dx}{1\, +\, \sqrt{cot\, x}}$

Q. 8 Evaluate: $\int_0^1 x\, (1 - x)^n\, dx$

Q. 9 Evaluate: $\int_2^8 \frac{\sqrt{x}}{\sqrt{10\, -\, x}\, +\, \sqrt{x}}\, dx$

Q. 10 Evaluate: $\int_2^5 \frac{\sqrt{x}}{\sqrt{7\, -\, x}\, +\, \sqrt{x}} dx$

Q. 11 Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{tan\, x}}{1\, +\, \sqrt{tan\, x}}\, dx$

Q. 12 Evaluate: $\int_0^{\frac{\pi}{4}} log(1 + tan\, x)\, dx$

Q. 13 Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x\, +\, sin\, x}{1\, +\, cos\, x} dx$

Q. 14 Evaluate: $\int_0^{\frac{\pi}{2}} log\left(\frac{3\, +\, 5\, sin\, x}{3\, +\, 5\, cos\, x}\right) dx$

Q. 15 Evaluate: $\int_2^8 |x - 5|\, dx$

LONG – ANSWER QUESTIONS

INDEFINITE INTEGRALS

Q. 1 Integrate $\frac{x}{(x-1)^2(x+2)}$ with respect to x.

Q. 2 Evaluate: $I = \int \frac{1}{1+e^x} dx$

Q. 3 Evaluate: $I = \int \frac{(x^2+1)dx}{(x^2+4)(x^2+25)}$

Q. 4 Find: $\int \frac{(x+2)\ dx}{\sqrt{x^2+2x+3}}$

Q. 5 Evaluate:$\int \frac{(5x+3)\ dx}{\sqrt{x^2+4x+10}}$

Q. 6 Evaluate $\int e^x \frac{(x^2+1)}{(x+1)^2} dx$

Q. 7 Evaluate $\int e^x (\frac{1-\sin x}{1-\cos x}) dx$

Q. 8 Evaluate $\int \frac{dx}{3x^2+13x-10}$

Q. 9 Evaluate: $I = \int \frac{2x\ \ dx}{(x^2+1)(x^2+3)}$

Q. 10 Evaluate: $I = \int \frac{x\,dx}{(x^2+1)(x-1)}$

Q. 11 Evaluate: $I = \int \frac{1-x^2}{x(1-2x)} dx$

Q. 12 Evaluate: $I = \int \frac{x^2\ dx}{(x^2+1)(x^2+4)}$

Q. 13 Evaluate:$I = \int \frac{dx}{x(x^8+1)}$

Q. 14 Integrate $e^x(\frac{1+\sin x}{1+\cos x})$ with respect to x.

Q. 15 Evaluate: $\int \left\{log(logx) + \frac{1}{(logx)^2}\right\} dx$

LONG – ANSWER QUESTIONS

DEFINITE INTEGRALS

Q. 1 Evaluate: $\int_1^4 |x-1| + |x-2| + |x-3|$ dx.

Q. 2 Evaluate: $\int_0^{\pi} log(1+\cos x)dx$

Q. 3 Evaluate: $\int_0^{\pi} \frac{x\ \tan x}{\sec x + \tan x} dx$

Q. 4 Evaluate: $I = \int_0^{\frac{\pi}{2}} logsin\left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} logcos\, x\, dx$

Q. 5 Evaluate: $\int_{-1}^{2} |x^3 - x|$

Q. 6 Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)dx$

Q. 7 Evaluate: $I \ = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$

Q. 8 Evaluate: $I = \int_0^1 cot^{-1}(1 - x + x^2)dx$

Q. 9 Evaluate: $I = \int_0^{\pi/4}(\sqrt{tan\, x} + \sqrt{cot\, x})dx$

Q. 10 Evaluate: $I = \int_{-2}^{2} \frac{x^2}{1 + 5^x} dx$

ASSERTION – REASON: INTEGRALS

Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion.

(c) Assertion is correct, Reason is incorrect.

(d) Assertion is incorrect, Reason is correct.

Read the text carefully and answer the questions:

1

Assertion : $\int x\, e^{x^2} dx = \frac{1}{2} e^{x^2} + c$

Reason : To solve above integral put $x^2 = t$.

2

Assertion : $\int 3x^2(\cos x^3 + 8)dx = \sin x^3 + 8x^3 + C$

Reason : The above integration is solved using substitution method.

3 **Assertion** : $\int_0^{\frac{\pi}{2}} \sqrt{tan\, x}\, dx = 0$

Reason : $tan\, x = t^2$ makes the integrand in I as a rational function.

4 **Assertion** : $\int_{-2}^{2} log\left(\frac{1 + x}{1 - x}\right) dx = 0$

Reason : If f is an odd function, then $\int_{-a}^{a} f(x)\, dx = 0$

5 **Assertion** : If the derivative of the function x is $\frac{d}{dx}(x) = 1$, then the anti – derivative is $\int 1\, dx = x + C$

Reason : If $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$ then the corresponding integral of the function is $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$.

4. **Assertion** : If $\frac{d}{dx}\int f(x)dx = f(x)$, then $\int f(x)dx = f'(x) + C$, where C is an arbitrary constant.

Reason **:** Process of differentiation and integration are inverse of each other.

5. **Assertion** **:** Geometrically, derivative of a function is the slope of the tangent to the corresponding curve at a point.

Reason **:** Geometrically, indefinite integral of a function represents a family of curves parallel to each other.

ANSWER KEY(MCQ INDEFINITE INTEGRALS)

1) (d) 2) (c) 3) (c) 4) (d) 5) (a) 6) (a) 7) (d) 8) (d) 9) (d) 10) (a) 11) (b) 12) (a) 13) (a) 14) (a) 15) (b)

ANSWER KEY(MCQ DEFINITE INTEGRALS)

1) (c) 2) (c) 3) (b) 4) (c) 5) (d) 6) (b) 7) (c) 8) (a) 9) (c) 10) (d) 11) (c) 12) (a) 13) (c) 14) (a) 15) (c)

SHORT – ANSWER QUESTIONS

DEFINITE INTEGRALS

Q. 6 Use property $\int_0^a f(x)dx = \int_0^a f(a-x)$

Q. 8 I $=\int_0^1 x(1-x)^n$

$=\int_0^1 (1-x)(1-(1-x))^n dx$; use property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Q. 9 use property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Q. 10 We know that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$,

$I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}} dx$ ……………….(i) $I = \int_2^5 \frac{\sqrt{2+5-x}}{\sqrt{7-(2+5-x)}+\sqrt{2+5-x}} dx$

Q. 11 $\mathrm{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\mathrm{dx}}{1+\sqrt{\tan\left(\frac{\pi}{6}+\frac{\pi}{3}-\mathrm{x}\right)}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\mathrm{dx}}{1+\sqrt{\cot \mathrm{x}}}$

Q. 12 $I = \int_0^{\frac{\pi}{4}} log\left\{1 + tan\left(\frac{\pi}{4} - x\right)\right\} dx$

$=\int_0^{\frac{\pi}{4}} log\left\{1 + \frac{1-tan\, x}{1+tan\, x}\right\}$

Q. 13 $\int_0^{\frac{\pi}{2}} \frac{x+sin\, x}{1+cos\, x} dx = \int_0^{\frac{\pi}{2}} \frac{x+2\, sin\frac{x}{2} cos\frac{x}{2}}{2\, cos^2\frac{x}{2}} dx$

Q. 14 Using $\log\frac{a}{b}$ =loga – logb and use $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Q. 15 $I = \int_2^8 |x-5|\, dx = \int_2^5 |x-5|\, dx + \int_5^8 |x-5|\, dx$

LONG – ANSWER QUESTIONS

INDEFINITE INTEGRALS

Q. 1 Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$.

x = A(x-1) (x+2) +B(x+2) + C $(x-1)^2$

Q. 2 Put $e^x = t \implies e^x\,dx = dt$

dx=$\frac{dt}{e^x} \implies$ dx = dt/t $\quad I = \int \frac{1}{(1+t)}\frac{dt}{t}$

Q. 3 Here we have all even powers of x, so let $x^2 = y$

Let $\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{y+1}{(y+4)(y+25)}$

$\frac{y+1}{(y+4)(y+25)} = \frac{A}{y+4} + \frac{B}{y+25}$

Q. 4 x+2 =A (2x+2) +B.

Q. 5 $5x + 3 = \lambda(\frac{d}{dx}(x^2+4x+10)) + \mu$

$5x + 3 = \lambda(2x+4) + \mu$

Q. 6 $I = \int \left(\frac{x^2-1+1+1}{(x+1)^2}\right) e^x dx = \int \left(\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right) e^x dx = = \int \left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right) e^x dx$

Q. 7 $I = \int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx = \int e^x \left(\frac{1-\sin x}{2\sin^2\frac{x}{2}}\right) dx \quad = \int e^x \left(\frac{1}{2\sin^2\frac{x}{2}} - \frac{2\sin(x/2)\cos(x/2)}{2\sin^2\frac{x}{2}}\right)$

$= \int e^x \left(\frac{1}{2}\cos e\, c^2\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right)\right) d = e^x \cot(x/2) +$

Q. 9 On substituting $x^2 = t$ *then* 2xdx=dt $\quad I = \int \frac{dt}{(t+1)(t+3)}$

Q. 10 Let $\frac{x}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+c}{(x^2+1)}$

Q. 11 $\frac{1-x^2}{x(1-2x)}$ *is an improper rational function, which can be rewritten as*

$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{-\frac{x}{2}+1}{x(1-2x)} = \frac{-\frac{x}{2}+1}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$

Q. 13 On Multiplying and dividing by x^7 in numerater and denominater.

I= $\int \frac{x^7 dx}{x^8(x^8+1)}$

Put x^8 =t $\implies 8x^7\,dx = dt \implies x^7\,dx = dt/8$ so I = $\frac{1}{8}\int \frac{dt}{t(t+1)}$

Q. 14 $\int e^x(\frac{1+\sin x}{1+\cos x})dx = \int e^x(\frac{1}{1+\cos x} + \frac{\sin x}{1+\cos x})dx = \int e^x (\frac{1}{2\cos^2 x/2} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2 x/2})\,dx$

$= \int e^x (1/2\sec^2 x/2 + \tan\frac{x}{2})\,dx \quad = e^x \tan\frac{x}{2} + C$

Q. 15 Put t = $\log x$ then x=e^t, dx=e^tdt

LONG – ANSWER QUESTIONS

DEFINITE INTEGRALS

Q. 1 $Let\ f(x) = [|x-1| + |x-2| + |x-3|],$

$$f(x) = [|x-1| + |x-2| + |x-3|] = \begin{cases} 4-x, & if\ 1 < x < 2 \\ x, & if\ 2 < x < 3 \\ 3x-6, & if\ 3 < x < 4 \end{cases}$$

Q. 3 I=$\int_0^{\pi} \frac{(\pi-x)\tan(\pi-x)}{sec(\pi-x)+tan(\pi-x)} dx$

Q. 4 $I = \int_0^{\frac{\pi}{2}} logsin\left(\frac{\pi}{2}-x\right) dx$ …………(i)

$= \int_0^{\frac{\pi}{2}} logcos\,x\,dx$ ………………(ii) by using property

Adding (i) and (ii) $2I = \int_0^{\frac{\pi}{2}} logcos\,x\,dx + \int_0^{\frac{\pi}{2}} logcos\,x\,dx$

Q. 5 $\left|x^3 - x\right| = \begin{cases} x^3 - x, if x^3 - x \geq 0 \\ -(x^3 - x), if x^3 - x < 0 \end{cases}$, x^3-x=0⇒x = 0 and x = ±1

$\int_{-1}^{2} |x^3 - x|$ dx = $\int_{-1}^{0}$ $(x^3 - x)$ dx + $\int_{0}^{1}$ - $(x^3 - x)$ dx + $\int_{1}^{2}$ $(x^3 - x)$ d = 11/4

Q. 6 $$\int_0^{\frac{\pi}{2}} (2\,logsin\,x - logsin\,2\,x)dx = \int_0^{\frac{\pi}{2}} log\left(\frac{sin^2 x}{2\sin x \cos x}\right) dx = \int_0^{\frac{\pi}{2}} log\left(\frac{tan\,x}{2}\right) dx$$

$$I = \int_0^{\frac{\pi}{2}} log\left(\frac{tan\left(\frac{\pi}{2}-x\right)}{2}\right) dx \quad or \quad I = \int_0^{\frac{\pi}{2}} log\left(\frac{cot\,x}{2}\right) dx$$

$$2I = \int_0^{\frac{\pi}{2}} log\left(\frac{tanx}{2}\right) log\left(\frac{cot\,x}{2}\right) dx = \int_0^{\frac{\pi}{2}} log\left(\frac{1}{4}\right) dx = \frac{\pi}{4} log\left(\frac{\pi}{4}\right)$$

Q. 7 I $=\int_0^{\pi} \frac{x\sin x}{1+\sin x} dx$………….(1) I $=\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)} dx$ $=\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\sin x} dx$………(2)

Q. 8 $$I = \int_0^1 Cot^{-1}(1 - x + x^2)dx$$

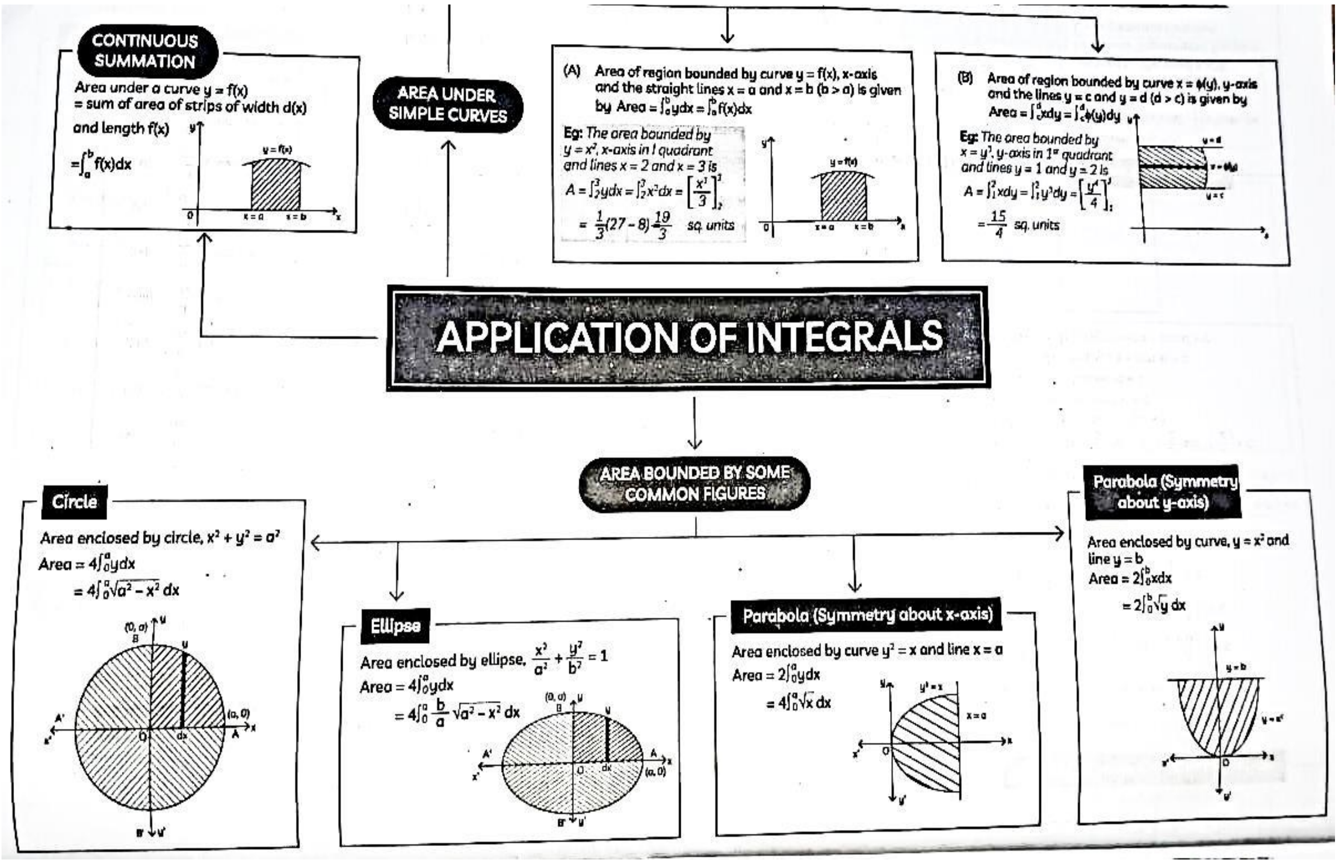
$=\int_0^1 tan^{-1}\left\{\frac{(1-x)+x}{1-x(1-x)}\right\} dx = \int_0^1 tan^{-1}(1-x)dx + \int_0^1 tan^{-1} x\,dx$

Q. 9 I $=\int_0^{\pi/4} (\sqrt{tan\,x} + \sqrt{cot\,x})dx$ Consider I_1 $=\int \frac{sin\,x+cos\,x}{\sqrt{sin\,x.cos\,x}} dx$ $=\int \frac{dt}{\sqrt{\frac{1-t^2}{2}}}$

Q. 10 Use property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx,$ $I = \int_{-2}^{2} \frac{x^2}{1+5^{-x}} dx$

ASSERTION – REASON **1) (a) 2)(a) 3)(a) 4) (d) 5) (b)**

APPLICATION OF INTEGRATION



SL NO.	MCQs (Solutions provided)

Q1. Area of the region bounded by the curve $y^2 = 4x, y$ −axis and the line $y = 3$ is:

a) 2 sq. units b) $\frac{9}{4}$ sq. units c) $\frac{9}{3}$ sq. units d) $\frac{9}{2}$ sq. units

Q2. The area of the region bounded by the curve $y = sinx$ between the ordinates $x = 0$ and $x = \frac{\pi}{2}$ and the x-axis is:

a) 2 sq. units b) 4 sq. units c) 3 sq. units d) 1 sq. unit

Q3. Area bounded by the curves $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = -1$ is:

a) -9 sq. units b) $-\frac{15}{4}$ sq. units c) $\frac{15}{4}$ sq. units d) $\frac{17}{4}$ sq. units

Q4. Area of the region bounded by the curve $y = cosx$ between $x = 0$ and $x = \pi$ is:

a) 2 sq. units b) 4 sq. units c) 3 sq. units d) 1 sq. unit

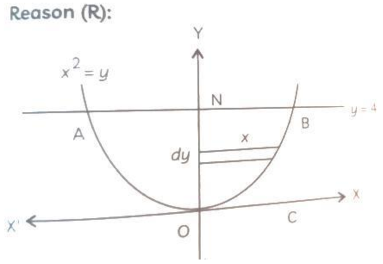
Q5. If we draw a rough sketch of the curve $y = \sqrt{x-1}$ in the interval $[1, 5]$, then the area under the curve between the lines $x = 1$ *and* $x = 5$ is:

a) $\frac{32}{3}$ sq. units b) $\frac{16}{3}$ sq. units c) $\frac{32}{9}$ sq. units d) $\frac{16}{9}$ sq. units

Q6. **Assertion-Reasoning:**

Assertion (A): The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is $\frac{3}{32}$ sq. unit

Reason (R):



Choose the correct answer out of the following choices:

a. Both A and R are correct and R is the correct explanation of A.

b. Both A and R are correct and R is not the correct explanation of A.

c. A is true but R is false.

d. A is false but R is true.

MCQs (For self- practice)

Q7. The area of the region bounded by the lines $y = x, x = 0, x = 3$ and x-axis is:

a) $\frac{1}{5}$ sq. units b) $\frac{9}{4}$ sq. units c) $\frac{9}{2}$ sq. units d) $\frac{4}{5}$ sq. units

Q8. Which of the following CLOSEST to the area under the parabola $y = 4x^2$, bounded by the x-axis and the lines $x = -1$ and $x = -2$?

a) 6 sq. units b) 8 sq. units c) 9 sq. units d) 12 sq. units

Q9. The area of the region bounded by the curves $y^2 + 6x = 0$ and $y^2 + 4x = 4$ is:

a) $\frac{4}{\sqrt{3}}$ sq. units b) $\frac{8}{\sqrt{3}}$ sq. units c) $4\sqrt{3}$ sq. units d) none of these

Q10. The area of the region made by the lines: $|x| + |y| = 1$ is

a) 15 sq. units b) 2 sq. units c) 3 sq. units d) 4 sq. units

Q11. The area made by $y = |x|, |x| = 1$ and x-axis is:

a) 1 sq. units b) 2 sq. units c) 3 sq. units d) 4 sq. units

Q12. $y = |x - 1|$ and $y = 1$ make a region with an area of:

a) 0.5 sq. units b) 1 sq. units c) 2 sq. units d) 4 sq. units

Q13. The area of the region bounded $y = |sinx|, x\ axis\ and\ |x| = \frac{\pi}{2}$ is:

a) 1 sq. units b) 0.5 sq. units c) 2 sq. units d) none of these

Q14. The area of the region bounded by $y = \sqrt{8 - x^2}$ and $y = |x|$ is:

a) π sq. units b) 1.5π sq. units c) 2π sq. units d) $\frac{3}{2}(\pi + 1)\ \pi$ sq. units

Q15. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is:

a) $\frac{20}{3}$ sq. units b) $\frac{32}{3}$ sq. units c) $\frac{16}{3}$ sq. units d) none of these

Q16. The area between the curves $y = x^2\ and\ y = x^3$ is:

a) $\frac{1}{12}$ sq. units b) $\frac{1}{6}$ sq. units c) $\frac{1}{24}$ sq. units d) none of these

SA Type (Solution provided)

Q1. Find the area between the parabola $y^2 = 6x$ and its latus rectum.

Q2. Find the area bounded by $y = tanx, y = 0$ and $x = \frac{\pi}{4}$ in the first quadrant.

Q3. Find the area bounded by the lines $x = 0, y = 0$ and $x + y + 2 = 0$.

Q4. Find the area of the region bounded by $y^2 = 4x, x = 1, x = 4$ and the x- axis in the first quadrant.

Q5. Determine the area enclosed between the curve $y^2 = 4x - x^2$ and the x-axis.

SA Type (For self-practice)

Q6. Find the area of the region bounded by the curve $y^2 = 4x\ and\ x = 3$

Q7. Find the area of the region bounded by the curve $y^2 = 4x, y\ axis\ and\ y = 3$.

Q8. Find the area of the region bounded by the curve $x = 4y - y^2\ and\ y - axis$.

Q9. Find the area enclosed by the circle $x^2 + y^2 = a^2$.

Q10. Sketch the graph of the curve $y = \sqrt{x + 1}, 0 \leq x \leq 4$ and determine the area of the region enclosed by the curve, x-axis and the lines $x = 0\ and\ x = 4$.

LA Type (Solution provided)

Q1. Using integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0, 2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$

Q2. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

Q3. Using integration, find the area of the region in the first quadrant enclosed by the x axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Q4. Using integration, find the area of the region enclosed by the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.

Q5. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m.

LA Type (For self-practice)

Q6. Using integration, find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$

Q7. Find the area of the region $\{(x, y): y \geq x^2, y \leq |x|\}$

Q8. Using integration, find the area of the region bounded by the curves $y = x^2 + 2, y = x, x = 0$ and $x = 3$

Q9. Using integration, find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

Q10. Using integration,find the area of the region bounded by the curve $|x| + |y| = 1$

Case Study Based Type

Q1. A mirror in the shape of an ellipse represented by $\frac{x^2}{9} + \frac{y^2}{4}$ was hanging on the wall. Arun and his sister were playing with ball inside the house, even their mother refused to do so. All of sudden, ball hit the mirror and got a scratch in the shape of line represented by $\frac{x}{3} + \frac{y}{2} = 1$.

Based on the above information answer the following

(i)**Assertion(A):** The point of intersection of ellipse and scratch (straight line) are (4, 2) and (3, 0)

Reason(R): The point(s) where the lines intersect is called intersection point.

Choose the correct answer out of the following choices:

a. Both A and R are correct and R is the correct explanation of A.
b. Both A and R are correct and R is not the correct explanation of A.
c. A is true but R is false.

d. A is false but R is true.

(ii)Find the value of $\frac{2}{3}\int_0^9 \sqrt{9 - x^2}\, dx$

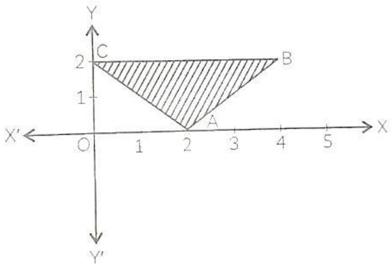
(iii)Find the value of $2\int_0^3 (1 - \frac{x}{3}) dx$

(iv)Area of the smaller region bounded by the mirror and scratch is:

a)$3\left(\frac{\pi}{2}+1\right) sq.units$ b) $\left(\frac{\pi}{2}+1\right) sq.units$

c) $\left(\frac{\pi}{2}-1\right) sq.units$ d) $3\left(\frac{\pi}{2}-1\right) sq.units$

Q2. **Look at the shaded region on the graph and answer the following questions:**



(i)The shaded region is bounded by the lines:

a. $y = x - 2, x = 2, x - axis$

b. $x = y - 2, y = 2 \; and \; x - axis$

c. $y = |x - 2| \; and \; y = 2$

d. $x = |y - 2| \; and \; x = 2$

(ii) which of the following is not a corner point of the bounded region? **(self-practice)**

a. (0, 2) b. (2, 0) c. (4, 2) d) (2, 4)

(iii) The area A of the shaded region is given by

a) $A = \int_0^4 2\,dx - \int_0^2 (2 - x)\,dx - \int_2^4 (x - 2)\,dx$

b) $A = \int_0^4 2\,dx - \int_0^2 (x - 2)\,dx - \int_2^4 (2 - x)\,dx$

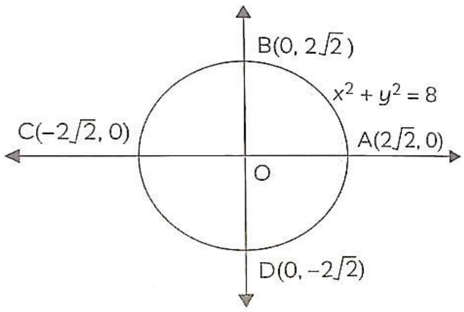
c) $A = \int_0^4 2\,dx + \int_0^2 (x - 2)\,dx - \int_2^4 (2 - x)\,dx$

d) $A = \int_0^4 2\,dx + \int_0^2 (x - 2)\,dx + \int_2^4 (2 - x)\,dx$

(iv) The area of the shaded region is: **(self-practice)**

a. 2 sq. units b) 4 sq. units c) 6 sq. units d) 8 sq. units

Q3. **A farmer has a field in the shape of a circle, in which he wishes to grow four varieties of vegetables as shown below:**



(i) Evaluate: $\int \sqrt{8-x^2}\,dx$.

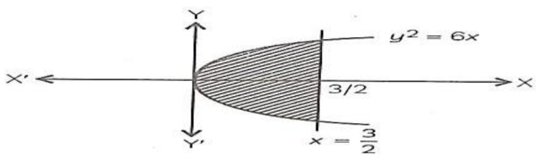
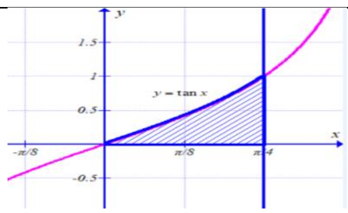
(ii) What is the area of the first quadrant of the field? **(self-practice)**

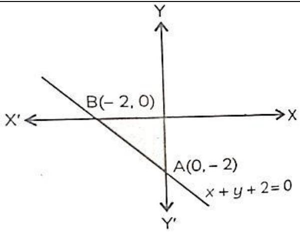
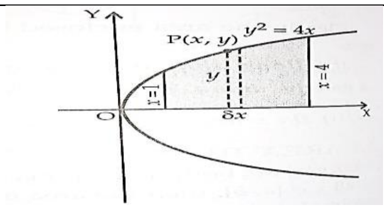
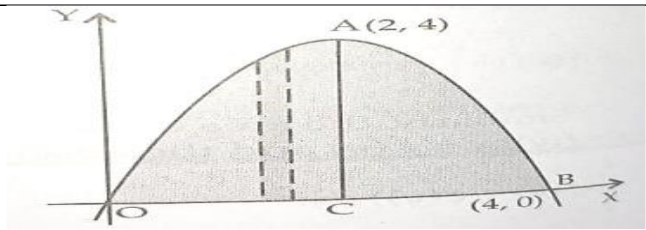
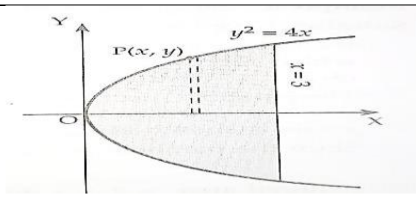
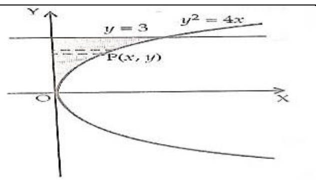
(iii) Using integration, find the area of the field.

ANSWER KEY (MCQs)

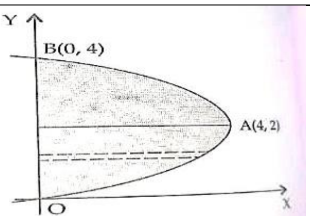
1) (b) 2) (d) 3) (d) 4) (a) 5)(a) 6) (d) 7) (c) 8) (c) 9) (b) 10) (b)

11) (a) 12) (b) 13) (c) 14) (c) 15) (c) 16(a)

LA Type (with solutions)	
Q1.	 $4a = 6 \rightarrow a = \frac{3}{2}$ So, the equation of the latus rectum is $x = \frac{3}{2}$ So, the area of the shaded region $= 2 \times \int_0^{3/2} y dx = 2 \times \int_0^{3/2} \sqrt{6x} dx$
Q2.	 Required area of the shaded part= $\int_0^{\pi/4} tanx\ dx = [\log(secx)]_0^{\pi/4} = log\sqrt{2}$

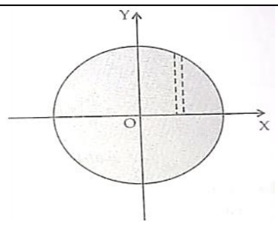
Q3.	 Line $x + y + 2 = 0$ intersect the coordinate axes at $A(0, -2)$ and $B(-2, 0)$ So, the area of the shaded part= $\left\|\int_0^{-2} \|y\| dx\right\| = \left\|\int_0^{-2} \|(-2 - x)\| dx\right\| = \|4 - 2\| = 2$
Q4.	 Req. area= $\int_1^4 y dx = \int_1^4 2\sqrt{x} dx = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 = \frac{4}{3}[8 - 1] = \frac{28}{3}$ sq. units
Q5.	 $\boldsymbol{y} = 4x - x^2 \rightarrow x^2 - 4x = -y \rightarrow (x - 2)^2 = -(y - 4)$ which represents a downward parabola with the vertex at (2, 4) Req. area= $\int_0^4 y dx = \int_0^4 (4x - x^2) dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = \frac{32}{3}$ sq. units
SA Type (Self-Practice answer key)	
Q6.	 Req area=2 $\int_0^3 y dx = 2\int_0^3 2\sqrt{x} dx = 8\sqrt{3}$ sq. units
Q7.	 Req area= $\int_0^3 x dy = \int_0^3 \frac{y^2}{4} dy = \frac{9}{4}$ sq. units

Q8.



Req area= $2\int_0^2 x\,dy = 2\int_0^2 (4y - y^2)dy = \frac{32}{3}$ sq. units

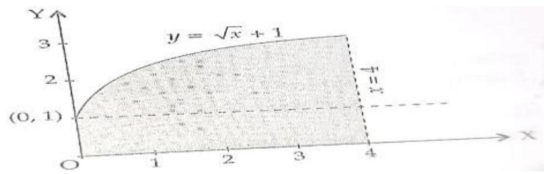
Q9.



$x^2 + y^2 = a^2 \rightarrow y = \sqrt{a^2 - x^2}$

Req area= $4\int_0^4 y\,dx = 4\int_0^4 \sqrt{a^2 - x^2}\,dx = \pi a^2$ sq. units

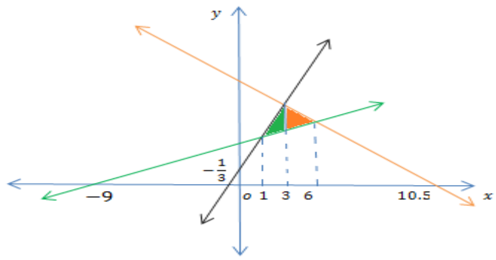
Q10. $y = \sqrt{x} + 1 \rightarrow \sqrt{x} = y - 1 \rightarrow (y-1)^2 = x$ which represents a right-hand parabola with the vertex at (0, 1)



Req area= $\int_0^4 (\sqrt{x} + 1)dx = \frac{28}{3}$ sq. units.

LA Type (with solutions)

Q1.



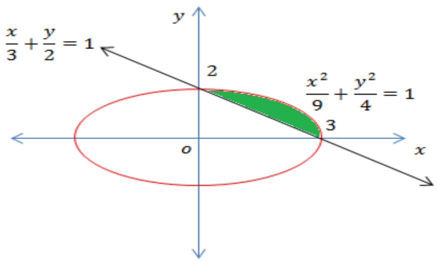
$3x - 2y + 1 = 0 \ldots (i) \quad 2x + 3y - 21 = 0 \ldots (ii) \quad x - 5y + 9 = 0 \ldots (iii)$

Solving (i) and (ii), (ii) & (iii), (iii) & (i), we get respectively $x = 3, x = 6$ and $x = 1$.

$$\text{The required area is} := \int_1^3 y_i dx + \int_3^6 y_{ii} dx - \int_1^6 y_{iii} dx$$

$$= \frac{1}{2}\int_1^3 (3x+1)dx + \frac{1}{3}\int_3^6 (-2x+21)dx - \frac{1}{5}\int_1^6 (x+9)dx$$

Q2.

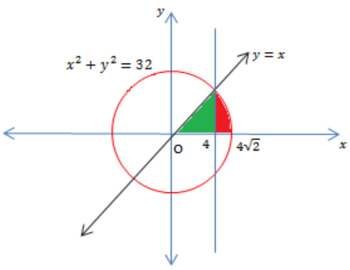


$\frac{x^2}{9}+\frac{y^2}{4}=1 \ldots (i)$ In first quadrant $y=\frac{2}{3}\sqrt{9-x^2}$

$$\frac{x}{3}+\frac{y}{2}=1 \ldots (ii) \quad \Rightarrow y=\frac{-2}{3}x+2$$

$$\text{Area}=\int_0^3 y_e dx-\int_0^3 y_l dx \;=\frac{2}{3}\int_0^3 \sqrt{9-x^2}dx-\int_0^3\left(\frac{-2}{3}x+2\right)dx$$

Q3.



$$x^2+y^2=32 \quad \Rightarrow x^2+y^2=\left(4\sqrt{2}\right)^2 \ldots (i)$$

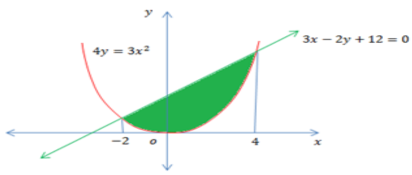
Solving(i) and (ii), we get $x=\pm 4$ In first quadrant $x=4$

$$\text{area}=\int_0^4 y_l dx+\int_4^{4\sqrt{2}} y_c dx \quad =\int_0^4 x dx+\int_4^{4\sqrt{2}}\sqrt{\left(4\sqrt{2}\right)^2-x^2 dx}$$

Q4.

$$4y=3x^2 \ldots (i)$$

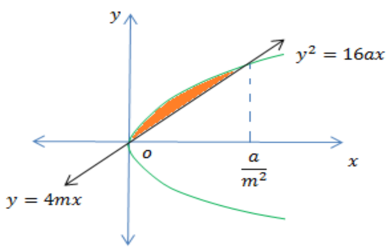
$$3x-2y+12=0 \ldots (ii)$$



Solving(i) and (ii), we get $x=-2,4$

$$\text{Area}=\int_{-2}^4 y_l dx-\int_{-2}^4 y_p dx \;=\int_{-2}^4\left(\frac{3}{2}x+6\right)dx-\frac{3}{4}\int_{-2}^4 x^2 dx$$

Q5.

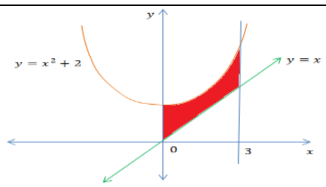
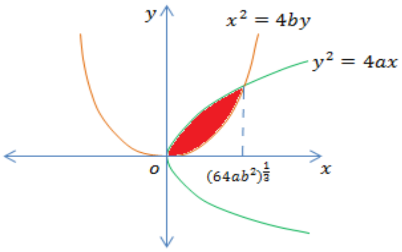
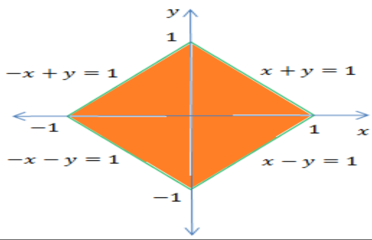


$$y^2=16ax \ldots (i) \quad y=4mx \ldots (ii)$$

Solving(i) and (ii), we get $x=0,\frac{a}{m^2}$

The required area is :

$$=\int_0^{\frac{a}{m^2}} y_p dx-\int_0^{\frac{a}{m^2}} y_l dx=\int_0^{\frac{a}{m^2}} 4\sqrt{a}\sqrt{x}dx-\int_0^{\frac{a}{m^2}} 4mx dx$$

LA Type (Self- Practice with answer key)	
Q6.	$y = x^2 \ldots (i) \quad y = \lvert x \rvert \ldots (ii)$ Solving(i) and (ii), we get $x = -1,0,1$, $\quad Area = 2\left[\int_0^1 y_l dx - \int_0^1 y_p dx\right]$
Q8.	$y = x^2 + 2 \ldots (i)$ $y = x \ldots (ii)$ Area $= \int_0^3 (x^2+2)dx - \int_0^3 x dx$ 
Q9.	 $y^2 = 4ax \ldots (i)$ $x^2 = 4by \ldots (ii)$ Solving(i) and (ii), we get $x = 0, (64ab^2)^{\frac{1}{3}}$ $Area = \int_0^{(64ab^2)^{\frac{1}{3}}} y_i dx - \int_0^{(64ab^2)^{\frac{1}{3}}} y_{ii} dx$
Q10.	$\lvert x \rvert + \lvert y \rvert = 1$ *The required area is :* $= 4\int_0^1 y dx \quad = 4\int_0^1 (1-x)dx$ 
CASE BASED STUDY TYPE	
Q1.	(i) (d) A is false but R is true (ii) $I = \frac{2}{3}\int_0^3 \sqrt{9-x^2}dx = \frac{2}{3}\int_0^3 \sqrt{3^2-x^2}dx = \frac{2}{3}\left[\frac{1}{2}\sqrt{9-x^2} + \frac{9}{2}sin^{-1}(\frac{x}{3})\right]_0^3$ (iii) $I = 2\int_0^3 \left(1-\frac{x}{3}\right)dx = 2\left[x - \frac{x^2}{6}\right]_0^3$=3 (iv) Area=$\frac{2}{3}\int_0^9 \sqrt{9-x^2}\,dx - 2\int_0^3 \left(1-\frac{x}{3}\right)$ dx]=$\frac{3\pi}{2} - 3 = 3\left(\frac{\pi}{2}-1\right)$ sq. units
Q2.	(i) (c) Equation of three lines are $y = 2-x, y = 2$ and $y = x-2$ (ii) self-try (iii) (a) (iv) self-try
Q3.	(i) $\int \sqrt{8-x^2}\,dx$ $= \frac{x}{2}\sqrt{8-x^2} + 4sin^{-1}(\frac{x}{2\sqrt{2}})$ (ii) Self-try (iii) Area of the field=4 $\times \int_0^{2\sqrt{2}} ydx = 4 \times 2\pi = 8\pi$ sq. units

CHAPTER: DIFFERENTIAL EQUATION

Basic Concepts

Definition: An equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called a **differential equation**.

Example: $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, $\left(\frac{dy}{dx}\right)^2 + \frac{d^2y}{dx^2} = \sin x$ etc.

Order of a differential equation:

Order of a differential equation is defined as the **order of the highest order** derivative of the dependent variable with respect to the independent variable involved in the given differential equation.

Differential Equation	Order
$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 10\,x$	Order is 2
$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 = \sqrt{1 + \frac{dy}{dx}}$	Order is 3
$x\,dy + y\,dx = 0$	Order is 1

Degree of a differential equation:

The degree of a differential equation is defined only when a differential equation is polynomial equation in derivatives, i.e., y′, y″, y‴ etc.

By the degree of a differential equation, when it is a polynomial equation in derivatives, we mean **the highest power (positive integral index) of the highest order derivative involved** in the given differential equation.

Differential Equation	Degree
$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 10\,x$	Degree is 1
$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 = \sqrt{1 + \frac{dy}{dx}}$ It is not in polynomial form so we have to square both	Degree is 2

side $$\Rightarrow \left(\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2\right)^2 = 1 + \frac{dy}{dx}$$ Now it is a polynomial Equation. If we expand the LHS we get that the highest power in $\frac{d^3y}{dx^3}$ will be 2.	
$x\,dy + y\,dx = 0$	Degree is 1
$\frac{d^3y}{dx^3} + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$	Degree is 1

CASE WHEN THE DEGREE OF THE DIFFERENTIAL EQUATION CAN'T BE DEFINED

$$\frac{d^2y}{dx^2} + sin\left(\frac{dy}{dx}\right) = x$$

$\Rightarrow$ In this case the equation is not represented as a polynomial equation in its derivatives. So, the degree is not defined.

Note: Order and Degree (if defined) of any differential equation are always positive integers.

General and Particular Solutions of a Differential Equation

Solution of differential equations is the relation between the variables of a differential equation that satisfies the given differential equation. All the solutions of a differential equation are obtained by integrating the differential equation.

Note: A differential equation generates a family of curves that satisfies it. For an example:

$y = x^2 \Rightarrow \frac{dy}{dx} = 2x$ & $y = x^2 + c \Rightarrow \frac{dy}{dx} = 2x, where\ c\ is\ a\ constant.$

So, we observe that by solving a D.E we get a family of curves.

General solution:

The solution which contains arbitrary constants is called the general solution (primitive) of the differential equation.

Particular solution:

The solution free from arbitrary constants i.e., the solution obtained from the general solution by giving particular values to the arbitrary constants is called a particular solution of the differential equation.

Methods of Solving First Order, First Degree Differential Equations

- **Method of variables separation**
- **Homogeneous differential equations**
- **Linear differential equations**

Method of variables separation

$\frac{dy}{dx} = f(y)g(x)$	$\frac{dy}{f(y)} = \frac{dx}{g(x)}$ Then Integration

Homogeneous differential equations

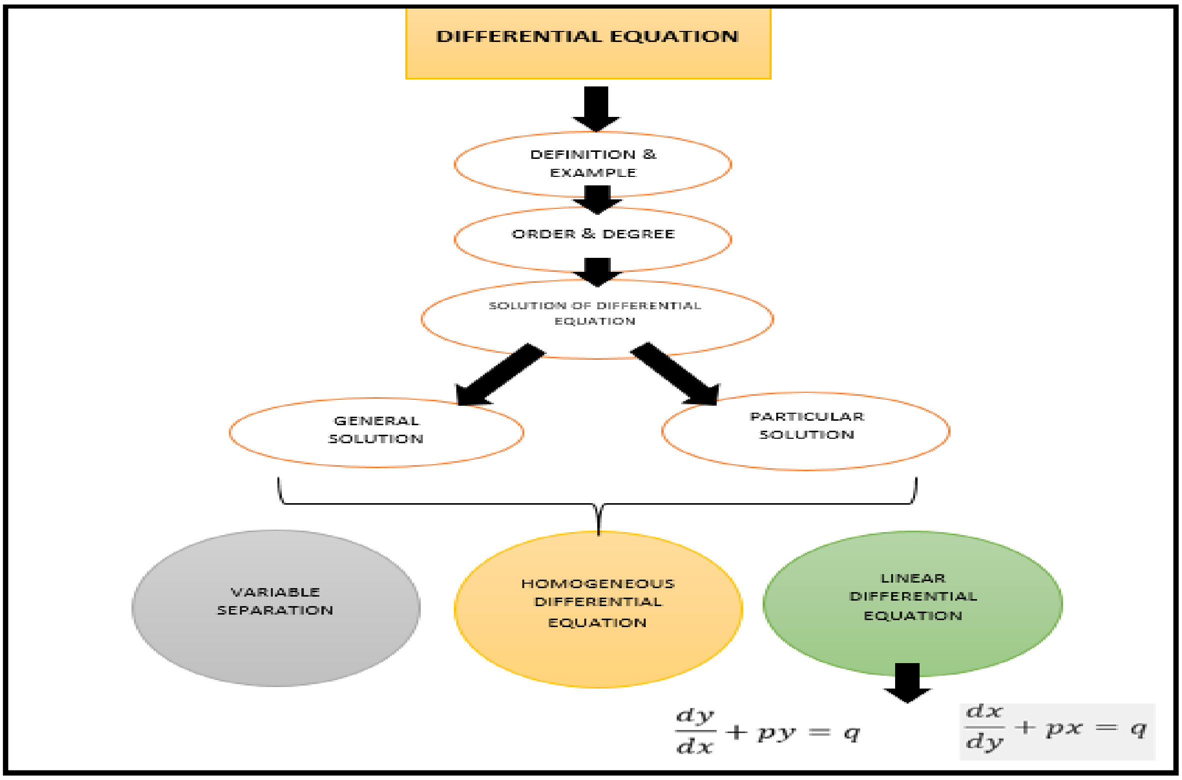
A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogenous if F(x, y) is a homogenous function of degree zero.

- To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$ we substitute $y = vx$
- To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = h\left(\frac{x}{y}\right)$ we substitute $x = vy$

Linear differential equations

Form	Description	Type	Integrating Factor	Solution
$\frac{dy}{dx} + py = q$	$p\ and\ q\ are\ either\ constants\ or\ function\ of\ x\ only$	Linear in y	$e^{\int p\,dx}$	$y \times IF = \int (q \times IF)\ dx + c$
$\frac{dx}{dy} + px = q$	$p\ and\ q\ are\ either\ constants\ or\ function\ of\ y\ only$	Linear in x	$e^{\int p\,dy}$	$x \times IF = \int (q \times IF)\ dy + c$

MIND MAPPING OF DIFFERENTIAL EQUATION



PROBLEMS ON DIFFERENTIAL EQUATIONS

MCQ

1. The degree of the differential equation $\frac{dy}{dx} + \cos x = x$ is –

   a) 1 b) 2 c) 0 d) undefined

2. The degree of the differential equation $xy\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$ is –

   a) 2 b) 1 c) 0 d) undefined

3. The degree of the differential equation $y'' + y^2 + e^{y'} = 0$ is –

   a) 2 b) 1 c) 0 d) undefined

4. The order of the differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^{200} = 10$ is –

   a) 2 b) 1 c) 0 d) 3

5. The sum of the order and degree of the differential equation $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$ is –

   a) 2 b) 1 c) 4 d) 3

6. The degree of the differential equation

   $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + sin\left(\frac{dy}{dx}\right) + 1 = 0$ is

a) 2 b) 1 c) 3 d) undefined

7. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is –

a) $tan^{-1}y = tan^{-1}x + c$ b) $tan^{-1}y \,.\, tan^{-1}x = c$ c) $tan^{-1}y = x + \frac{x^3}{3} + c$ d) $y + \frac{y^3}{3} = x + \frac{x^3}{3} + c$

8. The integrating factor of the differential equation $\frac{dy}{dx} - y = \cos x$ is –

a) e^x b) e^{2x} c) e^{-x} d) e^y

9. The integrating factor of the differential equation $y\,dx - (x + 2y^2)dy = 0$ is –

a) $\frac{1}{x}$ b) e^x c) $\frac{2}{y}$ d) $\frac{1}{y}$

10. The general solution of the differential equation $e^x dy + (ye^x + 2x)dx = 0$ is –

a) $xe^y + x^2 = c$ b) $xe^y + y^2 = c$ c) $ye^x + x^2 = c$ d) $ye^y + x^2 = c$

11. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is –

a) $4e^{3x} + 3e^{-4y} = 7$ b) $e^{3x} + 3e^{-4y} = 7$ c) $3e^{3x} + 4e^{-4y} = 7$ d) $4e^{3x} - 3e^{-4y} = 7$

12. The general solution of the differential equation $\frac{y\,dx - x\,dy}{y} = 0$ is –

a) $xy = c$ b) $x = cy^2$ c) $y = cx$ d) $y = cx^2$

13. The sum of the degree and order of the differential equation $\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}$ is –

a) 1 b) 2 c) 4 d) undefined

14. The sum of the degree and order of the differential equation $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \left(c\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$ is –

a) 1 b) 2 c) 4 d) undefined

15. The degree of the differential equation $y = px + \sqrt{a^2p^2 + b^2}$, $\left(where\ p = \frac{dy}{dx}\right)$is –

a) 1 b) 2 c) 4 d) undefined

16. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ –

a) 1 b) 2 c) 4 d) undefined

17. The general solution of the differential equation $\frac{dy}{dx} = (x + y + 1)^2$ is –

a) $tan^{-1}(x + y + 1) = x + c$ b) $x + y + 1 = \tan c$

c) $\tan y = x + y + c$ d) $y + x = c$

18. The general solution of the differential equation $\frac{dy}{dx} = \frac{1}{cos(x+y)}$ is –

a) $y = \sec\left(\frac{x+y}{2}\right) + c$ b) $y = \tan\left(\frac{x+y}{2}\right) + c$ c) $\tan y = x + y + c$ d) $y + x = c$

19. If m is the order and n is the degree of the differential equation $y''' + y'' + y' = \sin y$, then m^n is –

a)2 b) 3 c) 1 d) 9

20. The order of the differential equation $(x + 2y^2)dx + (y + 2x^2)dy = 0$ is –

a)2 b) 4 c) 1 d) 9

VSA

2 marks each

1. Solve: $(x-1)\frac{dy}{dx} = 2xy$ $[Answer: 2x + 2\log|x-1| = \log y + c]$
2. Solve: $xydy = (y-1)(x+1)dx$ $[Answer: y - x = \log|x| - \log|y-1| + c]$
3. Solve: $\sqrt{1+x^2}\, dy + \sqrt{1+y^2}\, dx = 0$ $\left[Answer: \left(y+\sqrt{1+y^2}\right)\left(x+\sqrt{1+x^2}\right) = c\right]$
4. Solve: $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ $\left[Answer: \sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}-1}\right| = c\right]$
5. Solve: $(y+xy)dx + (x - xy^2)dy = 0$ $\left[Answer: \log xy + x - \frac{y^2}{2} = c\right]$
6. For the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$. Find the solution curve passing through the point$(1,-1)$. $[Answer: y - x + 2 = \log|x^2(y+2)^2|]$
7. Solve: $\frac{dy}{dx} = y\cot 2x$ $\left[Answer: \log y = \frac{1}{2}\sin 2x + c\right]$
8. Solve: $x(x\, dy - y\, dx) = y\, dx$ $\left[Answer: \left|\frac{y}{x}\right| = ce^{-\frac{1}{x}}\right]$
9. Solve: $\frac{dy}{dx} = \frac{1}{sin^4x + cos^4x}$ $\left[Answer:\ y = \frac{1}{\sqrt{2}}tan^{-1}\left(\frac{\tan x - \cot x}{\sqrt{2}}\right) + c\right]$
10. Solve: $sin^{-1}\left(\frac{dy}{dx}\right) = x + y$ $[Answer:\ x = \tan(x+y) - \sec(x+y) + c]$
11. Solve: $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ $\left[Answer:\ y\sqrt{1-x^2} + x\sqrt{1-y^2} = c\right]$
12. Find the equation of the curve passing through the point $(1,1)$ whose differential equation is $x\, dy = (2x^2+1)\, dx\ (x \neq 0)$ $[Answer:\ y = \mathrm{x}^2 + log\,|x| + c]$
13. Solve: $\frac{dy}{dx} = (4x+y+1)^2$ $\left[Answer: \frac{4x+y+1}{2} = \tan\ (2\mathrm{x} + 2\mathrm{c})\right]$
14. Find the I.F. of D.E. $(1+y^2)dx = (tan^{-1}y - x)dy$ $\left[Answer:\ e^{tan^{-1}y}\right]$
15. Solve: $y\, dx + (x - y^3)dy = 0$ $\left[Answer:\ xy = \frac{y^4}{4} + c\right]$

SA(3 marks each)

1. Suppose the growth of a population is proportional to the number present. If the population of a colony doubles in 25 days, in how many days will the population becomes triple?

$[Answer:\ 25\log_2 3]$

2. $\frac{dy}{dx} - 3y\cot x = \sin 2x\ ; y\left(\frac{\pi}{2}\right) = 2$ $[Answer:\ y = 4sin^3x - 2\ sin^2x]$

3. $(x - \sin y)dy + (\tan y)dx = 0; y(0) = 0$ $[Answer:\ 2x = \sin y]$

4. Show that the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$ is a homogeneous differential equation. Hence solve it. $[Answer:\ \mathrm{x}^2 - y^2 = c^2(x^2 + y^2)^2]$

5. Solve: $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ $\left[Answer:\ x + ye^{\frac{x}{y}} = c\right]$

6. Solve : $x\ sin\left(\frac{y}{x}\right)\frac{dy}{dx} + x - ysin\left(\frac{y}{x}\right) = 0, y(1) = \frac{\pi}{2}$ $\left[Answer:\ \log|x| = cos\left(\frac{y}{x}\right)\right]$

7. Solve: $y\ \left\{x\ cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}dx - x\left\{y\ sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}dy = 0$

$\left[Answer:\ \left|xy\ cos\left(\frac{y}{x}\right)\right| = k, x \neq 0\ \&k > 0\right]$

8. Solve: $3e^x\tan y + (1 - e^x)sec^2y\ dy = 0$ $[Answer:\ (e^x - 1)^3 = c\tan y]$

9. Solve: $(x - y)(dx + dy) = (dx - dy); y(0) = -1$ $[Answer:\ x - y = e^{x+y+1}]$

10. Solve: $x\frac{dy}{dx} - y = (x + 1)e^{-x}$ $[Answer:\ y\ log\ x = (\log x)^2 + c\ ]$

CASE STUDY BASED QUESTIONS ON DIFFERENTIAL EQUATIONS

11. In a college hostel accommodating 1000 students, one of the hostellers came in carrying covid, and the hostel was isolated. The rate at which the virus spreads is assumed to be proportional to the product of the number of infected students and remaining students. There are 50 infected students after 4 days.

Based on the above fact answer the following questions.

i) If $n(t)$ denotes the number of infected students at any time t, then find the maximum value of $n(t)$.

ii) Find the most general solution of the differential equation so formed in the above situation. **OR** Find the value of $n(4)$.

$Answer: i)\ 1000$ $ii)\ \frac{1}{1000}\log\left|\frac{n}{1000 - n}\right| = \lambda t + c$ $ii)\ or$ 50

12. It is known that, if the interest is compounded continuously, the principal changes at the

rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be $r\%$ per annum.

Based on the above information answer the following questions.

i) At what interest rate will $\text{Rs.}\,100$ double itself in 10 years.

ii) How much will Rs. 1000 be worth at 5% interest after 10 years?

OR

If the interest is compounded continuously at 5% per annum, in how many years will Rs. 100 double itself? $[\text{Use } \ln 2 = 0.6931; \mathrm{e}^{0.5} = 1.648]$

Answers: i) 6.931% ii) 1648 ii) OR 13.862 years

ANSWER: MCQ

1. a) 2. b) 3. d) 4. d) 5. d) 6. d) 7. a) 8. c) 9. d) 10. c) 11. a)12. c) 13. c) 14. c) 15 b) 16 d) 17. a) 18 b) 19. b) 20. d)

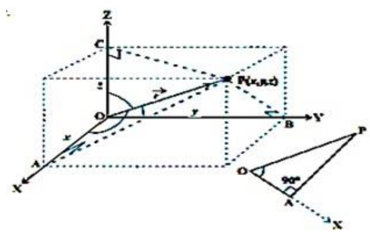
CHAPTER-10: VECTORS

IMPORTANT CONCEPTS TO BE REMEMBERED:

- **Scalar** is a quantity that has only magnitude like length, mass, time, temperature, work, etc.
- **Vector** is a quantity that has magnitude as well as direction like displacement, velocity, force, weight, etc.
- A directed line segment AB is a vector denoted as $\overrightarrow{AB}$
- **Zero vector** or null vector is a vector whose magnitude is zero and direction is undefined, i.e. whose initial and terminal points are coincident. $\overrightarrow{AA} = \vec{0}$
- **Unit vector ($\hat{a}$):** a vector whose magnitude is unity, $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$
- **Equal vectors:** if two vectors $\vec{a}$ and $\vec{b}$ have same direction and equal magnitude, *i.e.* $\vec{a}=\vec{b}$
- **Co-initial Vector:** two or more vectors having the same initial point are called co-initial vectors.
- **Collinear Vectors:** two or more vectors are said to be collinear if they are parallel to the same line, irrespective of their magnitudes and directions.
- **Negative of a vector:** a vector whose magnitude is the same as that of a given vector, but direction is opposite that of it, is called negative of the given vector.
- **Position Vector:** vector $\overrightarrow{OP}$ having origin point O (0, 0, 0) as initial point and point P(x, y, z) as terminal point is called the position vector of point P with respect to origin.

Magnitude of $\overrightarrow{OP}$ = $\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2}$

- **Direction Cosines:** as in figure, angle α, β, γ made by the vector $\vec{r}$ with the positive directions of x, y and z-axes respectively, are called its direction angles and the cosine values of these angles *i.e.* $l = cos\alpha, m = cos\beta$ and n=$cos\gamma$ are called direction cosines of vector $\vec{r}$

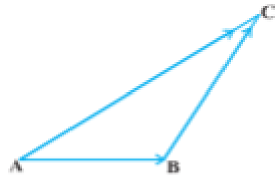


- **Direction Ratios :** the coordinate of point P(as in above figure) may also be expressed as (lr, mr, nr). The numbers lr, mr, nr are proportional to the direction cosines are called as direction ratios of vector $\vec{r}$ and denoted as a, b, c.

NOTE: $l^2 + m^2 + n^2 = 1$ but $a^2 + b^2 + c^2 \neq 1$ in general.

- **Addition of Vectors:** in general, if we have two vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ then to add them, they are positioned so that the initial point of one coincides with the terminal point of the other. (as in figure)

As $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ since $\overrightarrow{AC}$ = - $\overrightarrow{CA}$



So, we have, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AA} = \vec{0}$, this means that when the sides of a triangle are taken in order, it leads to zero.

Parallelogram law of vector addition- if we have two vectors $\vec{a}$ and $\vec{b}$ represented by the two adjacent sides of a parallelogram in magnitude and direction, then their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram through their common point.

- **Properties of vector addition**:

(i)Commutative property : $\vec{a}+\vec{b} = \vec{b}+\vec{a}$

(ii) Associative property :$(\vec{a}+\vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

- **Components of a vector:** if the position vector of point P(x, y, z) is

$\overrightarrow{OP}$= $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. This form of any vector is called its component form. Here x, y and z are called as the scalar components of $\vec{r}$ and $x\hat{\imath}, y\hat{\jmath}$ and $z\hat{k}$ are called vector components of $\vec{r}$.

- **Vector joining two points:** if P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) are any two points, then the vector joining P and Q is the vector $\overrightarrow{PQ} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$ and the magnitude of $\overrightarrow{PQ}$ = $|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- **Section Formula**: Let P and Q are two points represented by the position vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ and point R divides PQ in m : n

(i) Internally $\overrightarrow{OR} = \frac{m\overrightarrow{OQ}+n\overrightarrow{OP}}{m+n}$ (ii) Externally $\overrightarrow{OR} = \frac{m\overrightarrow{OQ}-n\overrightarrow{OP}}{m-n}$

- **Scalar product of two vectors:**

  $\vec{a}.\vec{b} = |\vec{a}|\ |\vec{b}|\ cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$

- **Vector product of two vectors:**

  $\vec{a} \times \vec{b} = |\vec{a}|\ |\vec{b}|\ sin\theta\ \hat{n}$, where $\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$

- **Projection** of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

- **Area of a parallelogram having adjacent side** $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$ sq. unit

- **Area of a triangle having vertices A,B and C** is $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$ sq. unit

- $|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$

Q. NO	ONE MARK QUESTIONS

1. If $\vec{a} = \hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{k} + \hat{\imath}$, find a unit vector in the direction of $\vec{a} + \vec{b} - \vec{c}$

   $(i)\ \hat{\imath}$ (ii) $\hat{k}$ (iii) $\hat{\jmath}$ (Iv) $2\hat{\jmath}$

2. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 2\sqrt{5}$ then find the value of $|\vec{a} \times \vec{b}|$

   $(i)\ \pm 4$ (ii) 4 (iii) -4 (Iv) $\sqrt{26}$

3. The value of x, y and z if $vectors$ $x\hat{\imath} - 2\hat{\jmath} + z\hat{k} = 3\,\hat{\imath} - y\hat{\jmath} - 2\hat{k}$ are

   (i) 3,2,-2 (ii) 3,2,1 (iii) 3,2,2 (iv) -2,3,2

4. The area of parallelogram with adjacent sides as $\vec{a}$ and $\vec{b}$ is is given by

   (i) $|\vec{a} \times \vec{b}|$ (ii) $\frac{1}{2}|\vec{a} \times \vec{b}|$ (iii) $\frac{1}{4}|\vec{a} \times \vec{b}|$ (iv) $\frac{1}{8}|\vec{a} \times \vec{b}|$

5. The value of μ for which the vectors $3\,\hat{\imath} - 6\hat{\jmath} + \hat{k}$ is parallel to $2\,\hat{\imath} - 4\hat{\jmath} + \mu\hat{k}$ is

   (i) 2/3 (ii) 3/2 (iii) 5/2 (iv) 2/5

6. The scalar projection of the vector $3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ on $5\hat{\imath} + 12\hat{\jmath}$ is

   (i) 7/√14 (ii) 7/14 (iii) 3/13 (iv) 7/2

7. The vector having initial point and terminal points as $(2,5,0)$ and $(-3,7,4)$ respectively is (i) $-\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ (ii) $5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ (iii) $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ (iv) $\hat{\imath} + \hat{\jmath} + \hat{k}$

8. Write the value of($\hat{\imath}$ x $\hat{\jmath}$) .$\hat{k}$ + ($\hat{\jmath}$ x $\hat{k}$). $\hat{\imath}$ + ($\hat{\imath}$ x $\hat{k}$). $\hat{\jmath}$

   (i) 0 (ii) 1 (iii) 2 (iv) 3

9. If $\vec{a}$ is a $unit$ $vector$ and $(\vec{x} + \vec{a}).(\vec{x} - \vec{a}) = 24, then$ $|\vec{x}|$is

   (i) 16 (ii) 4 (iii) 2 (iv) 5

10. The projection of vector $\vec{a}$ on $\vec{b}$ is $given$ by

    (i) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ (ii) $\frac{\vec{a}\times\vec{b}}{|\vec{a}|}$ (iii) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ (iv) $\frac{\vec{b}\times\vec{a}}{|\vec{a}|}$

11. The scalar product of vector $\vec{a}\ and\ \vec{b}, where\ \theta\ is\ angle$ between $\vec{a}\ and\ \vec{b}$ is given by

(i) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta$ (ii) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|sin\theta$ (iii) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|$ (iv) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|tan\theta$

12. Two vectors $\vec{a}\ and\ \vec{b}$ are perpendicular to each other then

(i) $\vec{a}.\vec{b} = 1$ (ii) $\vec{a}.\vec{b} = 0$ (iii)$\vec{a} \times \vec{b} = 0$ (iv) None of these

13. The value of λ for which the vector $2\ \hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are perpendicular to each other is

(i) 0 (ii) 1 (iii) 3/2 (iv) -5/2

14. The vector of 5 magnitude in the direction of $\hat{\imath} + \hat{\jmath} + \hat{k}$ is

(i) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{3}$ (ii) $5(\hat{\imath} + \hat{\jmath} + \hat{k})$ (iii) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$ (iv) $5\left(\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}\right)$

15. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$,$\vec{b} = 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{c} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ then $2\vec{a} + \vec{b} - \vec{c} =$

(i) (ii) (iii) (iv)

16. If $\overrightarrow{OA} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$, $\overrightarrow{OB} = 6\hat{\imath} - \hat{\jmath} + 4\hat{k}$ then vector $\overline{AB}\ is$

(i)$6\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ (ii) $6\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ (iii) $6\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ (iv) $6\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

17. The value of $(\hat{\imath}.\hat{\imath}) + (\hat{\jmath} \times \hat{\jmath}) + (\hat{k}.\hat{k})$ is

(i) 0 (ii) 1 (iii) 2 (iv) 2+$\hat{k}$

18. If θ is the angle between vectors $\vec{a}\ and\ \vec{b}$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, then the value of θ is

(i) 0 (ii) π (iii) $\pi/4$ (iv) $\pi/2$

19. Find the area of a parallelogram (in sq. units) having A(-1,1/2,4) , B(1,1/2,4) ,C(1,-1/2,4) and D (-1,-1/2,4) as the vertices .

(i) 2 (ii) 1 (iii) 0 (iv) 4

20. The position vector of the point which divides the line joining the points with position vectors $2\vec{a} - 3\vec{b}\ and\ \vec{a} + \vec{b}$ in the ratio 3:1 internally is

(i) $\frac{3\vec{a}-2\vec{b}}{2}$ (ii) $\frac{7\vec{a}-8\vec{b}}{4}$ (iii) $\frac{3\vec{a}}{4}$ (iv) $\frac{5\vec{a}}{4}$

2 MARKS QUESTIONS

Q.NO.	QUESTIONS
1	Find the unit vector in the direction of $\overrightarrow{PQ}$,where P and Q are the points (1,2,3) and (4,5,6) respectively.
2	Find the value of p for which $\vec{a}$ = 3$\hat{\imath}$ +2$\hat{\jmath}$ +9$\hat{k}$ and $\vec{b}$ = $\hat{\imath}$ +p$\hat{\jmath}$ + 3$\hat{k}$ are parallel vectors.
3	If $\|\vec{a}\|$ = $\sqrt{3}$, $\|\vec{b}\|$ = 2 and $\vec{a}$. $\vec{b}$ = 3 , find the angle between $\vec{a}$ and $\vec{b}$
4	Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}$. $\vec{b}$ = 8 and $\vec{b}$ =2$\hat{\imath}$ +6$\hat{\jmath}$ + 3$\hat{k}$
5	If $\vec{a}$. $\vec{a}$ = 0 and $\vec{a}$. $\vec{b}$ = 0 ,what can you conclude about the vector $\vec{b}$?
6	Find the value of x for which x ($\hat{\imath}$ +$\hat{\jmath}$ + $\hat{k}$) is a unit vector.
7	If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{\imath}$, $\frac{\pi}{4}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$, then find θ .
8	Find the direction cosines of the vector $\hat{\imath}$ +2$\hat{\jmath}$ + 3$\hat{k}$.
9	Find the area of the parallelogram whose diagonals are 4$\hat{\imath}$ -$\hat{\jmath}$ - 3$\hat{k}$ and $-2\hat{\imath}$ +$\hat{\jmath}$ - 2$\hat{k}$
10	If two vectors $\vec{a}$ and $\vec{b}$ are such that $\|\vec{a}\|$ =2, $\|\vec{b}\|$ = 1 and $\vec{a}$. $\vec{b}$ = 1, then find the value of (3$\overrightarrow{a}$ - 5 $\vec{b}$) . (2$\overrightarrow{a}$ + 7 $\vec{b}$) .

3 MARKS QUESTION

Q.NO.	QUESTIONS
1	Find the value of γ if (2$\hat{\imath}$ +6$\hat{\jmath}$ +14$\hat{k}$) x ($\hat{\imath}$ - γ $\hat{\jmath}$ +7$\hat{k}$) = $\vec{0}$
2	If $\vec{a}$ = 2$\hat{\imath}$ +2$\hat{\jmath}$ +3$\hat{k}$, $\vec{b}$ = $-\hat{\imath}$ +2$\hat{\jmath}$ +$\hat{k}$ and $\vec{c}$ = 3$\hat{\imath}$ +$\hat{\jmath}$ are such that $\vec{a}$ +β $\vec{b}$ is perpendicular to $\vec{c}$,then find β
3	Find $\|\vec{x}\|$, if for a unit vector $\vec{a}$, ($\vec{x}$ - $\vec{a}$)($\vec{x}$ + $\vec{a}$) =20.
4	If the points (-1 , -1, 2), (2 , m, 5) , and (3, 11, 6) are collinear, find the value of m.
5	If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a}$ + $\vec{b}$ + $\vec{c}$ = $\vec{0}$, find the value of $\vec{a}$. $\vec{b}$ + $\vec{b}$. $\vec{c}$ + $\vec{c}$. $\vec{a}$
6	Let $\vec{a}$ = $\hat{\imath}$ +4$\hat{\jmath}$ +2$\hat{k}$, $\vec{b}$ = 3$\hat{\imath}$ -2$\hat{\jmath}$ +7$\hat{k}$ and $\vec{c}$ = 2$\hat{\imath}$ -$\hat{\jmath}$ + 4$\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}$. $\vec{d}$ =15
7	If $\|\vec{a}\|$ =2, $\|\vec{b}\|$ =5 and $\|\vec{a}$ x $\vec{b}\|$ = 8 , find $\vec{a}$. $\vec{b}$
8	Find a vector of magnitude 9 , which is perpendicular to both the vectors 4$\hat{\imath}$ -$\hat{\jmath}$ + 3$\hat{k}$ and

$-2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

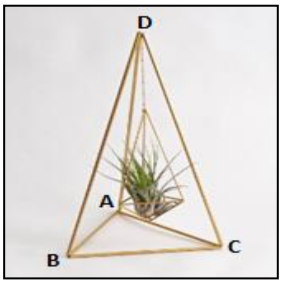
9 Find the values of x and y if the vectors $\vec{a} = 3\hat{\imath} + x\hat{\jmath} - \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} + y\,\hat{k}$ are mutually perpendicular vectors of equal magnitude.

10 Show that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ form the sides of a right angled triangle.

4 MARKS QUESTION

Q.NO.	QUESTIONS

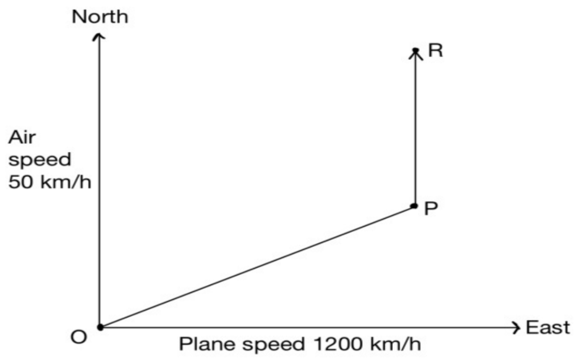
1 Ginni purchased an air plant holder which is in the shape of a tetrahedron. Let A, B, C and D are the coordinates of the air plant holder where A=(1,1,1), B=(2,1,3), C=(3,2,2) and D=(3,3,4).



Based on the above information, answer the following questions.

(i) Find the vector $\overrightarrow{AB}$. (ii) Find the vector $\overrightarrow{AC}$. (iii) Find the area of ΔABC. **OR**, Find the unit vector along $\overrightarrow{AD}$

2 A plane started from airport situated at O with a velocity of 1200 km/h towards east. Air is blowing at a velocity of 50 km/h towards north as shown in the figure. As a result, for 2 hour, the plane travelled with the resultant velocity in direction OP as shown in the figure. Then from P to R plane travelled 1 hour keeping velocity of 1200 km/h and finally landed at R.



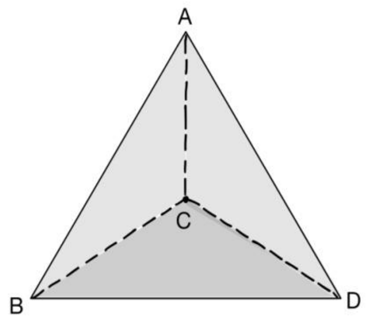
Based on the above information, answer the following questions (Take O as the origin, $\hat{\imath}$ along east and $\hat{\jmath}$ along north).

(i) Find the velocity vector along direction $\overrightarrow{OP}$ and $\overrightarrow{PR}$.

(ii) Find the position vector of point P.

(iii) Find the unit vector along $\overrightarrow{OR}$.**OR** Find the net direction of travel of plane from O to R with east.

3 A pyramid shaped greenhouse is to be constructed in the form of a pyramid ABCD as shown in the figure.



Let its angular points are A(0,1,2), B(3,0,1), C(4,3,6) and D(2,3,2) and G be the point of intersection of the medians of ΔBCD.

Based on the above information, answer the following questions.

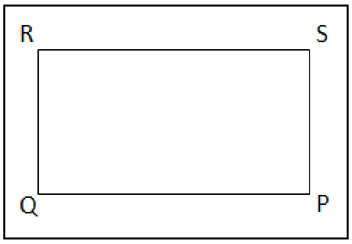
(i) Find the position vector of point G.

(ii) Find the unit vector along $\overrightarrow{AG}$.

(iii) Find the magnitude of the sum of $\overrightarrow{AC}$ and $\overrightarrow{CB}$.**OR** Find $|\overrightarrow{AB} \times \overrightarrow{AC}|$.

4 Solar panels are to be installed on a slanting roof. A surveyor determines the coordinates of the four corners of the roof where solar panels are mounted. Suppose

the points are labeled as P(6,8,4), Q(21,8,4), R(21,16,10) and S(6,16,10).

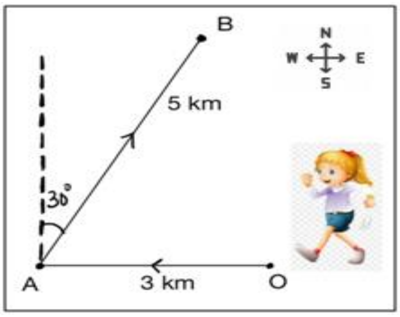


Based on the above information, answer the following questions.

(i) Find the scalar components of vectors $\overrightarrow{PQ}$ and $\overrightarrow{SP}$.

(ii) Find a unit vector perpendicular to the surface of the roof.

5 A girl walks 3 km towards west to reach point A and then walks 5 km in a direction $30°$ east of north and stops at point B. Let the girl starts from O (origin) and take $\hat{\imath}$ along east and $\hat{\jmath}$ along north.



Based on the above information,

Answer the following questions.

(i) Find the scalar components of $\overrightarrow{AB}$.

(ii) Find the unit vector along $\overrightarrow{AB}$.

ANSWERS (MCQ) :

1) (iii) 2) (ii) 3) (i) 4) (i) 5) (i) 6) (iII) 7) (iII) 8) (ii) 9) (iv) 10) (ii)

11) (i) 12) (ii) 13) (iv) 14)(iv) 15) (i) 16) (ii) 17) (iii) 18) (iii) 19) (i) 20) (iv)

ANSWER: 2 MARKS QUESTION

Q.NO.	QUESTIONS
1	The unit vector in the direction of $\overrightarrow{PQ}$ is = $\frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$
2	$\frac{1}{3}$
3	$\frac{\pi}{6}$
4	$\frac{8}{7}$
5	$\vec{b}$ is any vector
6	$\pm\frac{1}{\sqrt{3}}$
7	$\frac{\pi}{3}$
8	$\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$
9	$\frac{15}{2}$ sq. units
10	0

ANSWER: 3 MARKS QUESTION

Q.NO.	QUESTIONS
1	-3
2	8
3	$\sqrt{21}$
4	8
5	$\frac{-3}{2}$
6	$\frac{1}{3}(160\hat{\imath} - 5\hat{\jmath} + 70\hat{k})$
7	6
8	$-3\hat{\imath} + 6\hat{\jmath} + 6\hat{k}$

9 x = $\frac{-31}{12}$, y = $\frac{41}{12}$

10 $\vec{a}$. $\vec{b}$ = 0, so triangle is a right angled triangle.

ANSWERS OF 4 MARKS QUESTIONS

Q.NO. ANSWERS

1 (i) $\overrightarrow{AB} = (2-1)\hat{\imath} + (1-1)\hat{\jmath} + (3-1)\hat{k} = \hat{\imath} + 2\hat{k}$.

(ii) $\overrightarrow{AC} = (3-1)\hat{\imath} + (2-1)\hat{\jmath} + (2-1)\hat{k} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$.

(iii) Ar$(\Delta ABC) = \frac{1}{2}\sqrt{14}$ sq. units. **OR** Unit vector along $\overrightarrow{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{2\hat{\imath}+2\hat{\jmath}+3\hat{k}}{\sqrt{17}}$

2 (i) Velocity vector along $\overrightarrow{OP} = (1200\hat{\imath} + 50\hat{\jmath})$ km/h.

Velocity vector along $\overrightarrow{PR} = (1200 + 50)\hat{\jmath} = (1250\hat{\jmath})$ km/h.

(ii) $\overrightarrow{OP} = (1200\hat{\imath} + 50\hat{\jmath}) \times 2 = 2400\hat{\imath} + 100\hat{\jmath}$.

(iii) Unit vector along $\overrightarrow{OR} = \frac{\overrightarrow{OR}}{|\overrightarrow{OR}|} = \frac{(2400\hat{\imath}+1350\hat{\jmath})}{\sqrt{7582500}} = \frac{(2400\hat{\imath}+1350\hat{\jmath})}{150\sqrt{337}} = \frac{16}{\sqrt{337}}\hat{\imath} + \frac{90}{\sqrt{337}}\hat{\jmath}$.

OR $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = (2400\hat{\imath} + 100\hat{\jmath}) + (1250\hat{\jmath}) = 2400\hat{\imath} + 1350\hat{\jmath}$

So, net direction of travel of plane from O to R with east = $\left(\frac{1350}{2400}\right) = \left(\frac{45}{8}\right)$

3 (i) P.V. of G = $\frac{(3+4+2)}{3}\hat{\imath} + \frac{(0+3+3)}{3}\hat{\jmath} + \frac{1+6+2}{3}\hat{k} = 3\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

(ii) Unit vector along $\overrightarrow{AG} = \frac{\overrightarrow{AG}}{|\overrightarrow{AG}|} = \frac{3\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{11}}$.

(iii) $\left|\overrightarrow{AC} + \overrightarrow{CB}\right| = \sqrt{11}$. ***OR*** $\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{4 + 256 + 100} = \sqrt{360} = 6\sqrt{10}$.

4 (i) Scalar components of $\overrightarrow{PQ}$ are 15, 0, 0 & Scalar components of $\overrightarrow{SP}$ are 0, 8, 6.

(ii) unit vector is $-\frac{3}{5}\hat{\jmath} + \frac{4}{5}\hat{k}$.

5 (i) So, scalar components of $\overrightarrow{AB}$ are 2.5, 2.5$\sqrt{3}$.

(ii) Unit vector along $\overrightarrow{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\frac{5}{2}(\hat{\imath}+\sqrt{3}\hat{\jmath})}{\sqrt{\frac{25}{4}+\frac{75}{4}}} = \frac{1}{2}\left(\hat{\imath} + \sqrt{3}\hat{\jmath}\right)$

PRACTICE QUESTIONS

1. Let $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + c\hat{k}$. The value of c if $|\vec{a}| = 5$ is

(a)0 (b)$2\sqrt{3}$ (c) 1 (d)12

2. If $\vec{a} = (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k})$ then the value of h so that $h\vec{a}$ may be unit vector is

(a)$\frac{1}{5}$ (b)$\frac{1}{\sqrt{3}}$ (c)$\frac{1}{3\sqrt{5}}$ (d)$\frac{1}{5\sqrt{3}}$

3. If $\overrightarrow{AB} = (2\hat{\imath} + \hat{\jmath} - 3\hat{k})$ and A(1,2,-1) is the given point, then the coordinates of B are

(a) (3,-3,4) (b) (3,3,4) (c) (-3,-3,-4) (d) (3,3,-4)

4. If $|\vec{a} \times \vec{b}| = 4$, $\vec{a}.\vec{b} = 2$, then $|\vec{a}|^2|\vec{b}|^2 =$

(a)6 (b)20 (c)8 (d)2

5. If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 3 and $\frac{2}{3}$ respectively such that $\vec{a}\times\vec{b}$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

6. The vector in the direction of the vector $\vec{a}\hat{\imath} - \widehat{2\jmath} + 2\hat{k}$ that has a magnitude 9 is:

(a) $\hat{\imath} - \widehat{2\jmath} + 2\hat{k}$ (b)$\frac{1}{3}(\hat{\imath} - \widehat{2\jmath} + 2\hat{k})$

(c) $3(\hat{\imath} - \widehat{2\jmath} + 2\hat{k})$ (d)$9\,\hat{\imath} - \widehat{2\jmath} + 2\hat{k}$

7. The position vectors of the points *A*, *B*, *C* are $2\hat{\imath} + \hat{\jmath} - \hat{k}, 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ *and* $\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ respectively. These points :

(a) form an isosceles triangle (b) form a right triangle

(c) are collinear (d) form a scalene triangle

8. The projection of the vector $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ is:

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{5}{\sqrt{2}}$ (c) $\frac{\sqrt{5}}{6}$ (d) $\frac{\sqrt{6}}{5}$

9. If θ is the angle between any two vectors $\vec{a}$ *and* $\vec{b}$, then $|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}|$ when θ is:

(a) $\frac{\pi}{3}$ (b)$\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) none of these

10. If $\hat{a}, \hat{b}, \hat{c}$ are mutually perpendicular unit vectors, then value of $\left|\hat{a}+\hat{b}+\hat{c}\right|$ is:

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

11. The vector $\vec{b}$=3$\hat{\imath}$+4$\hat{k}$ is to be written as the sum of a vector $\vec{\alpha}$ parallel to $\vec{a}=\hat{\imath}+\hat{\jmath}$ and a vector $\vec{\beta}$ perpendicular to $\vec{a}$. Then $\vec{\alpha}$ is:

(a) $\frac{3}{2}(\hat{\imath}+\hat{\jmath})$ (b) $\frac{2}{3}(\hat{\imath}+\hat{\jmath})$ (c) $\frac{1}{2}(\hat{\imath}+\hat{\jmath})$ (d) $\frac{1}{3}(\hat{\imath}+\hat{\jmath})$

12. $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them then $cos\frac{\theta}{2}=$

(a) $\frac{1}{2}|\vec{a}+\vec{b}|$ (b) $|\vec{a}+\vec{b}|$ c) $|\vec{a}-\vec{b}|$ (d) $\frac{1}{2}|\vec{a}-\vec{b}|$

13. Let $\vec{a}, \vec{b}$, and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Find $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

14. **In the following questions (Q. No. 10) A statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choice as follows:**

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion(A): The area of parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$.

Reason(R): If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then the area of a triangle, then the area of triangle can be obtained by evaluating $|\vec{a} \times \vec{b}|$.

15. Find the direction ratios and direction cosines of the vector $\vec{a}$ = (5$\hat{\imath}$ - 3$\hat{\jmath}$ + 4$\hat{k}$).

16. If $\vec{a}$ = x$\hat{\imath}$+2$\hat{\jmath}$-z$\hat{k}$ and $\vec{b}$ = 3$\hat{\imath}$-y$\hat{\jmath}$+$\hat{k}$ are two equal vectors, then write the value of y^x+5z.

17. Find a unit vector parallel to the sum of the vectors ($\hat{\imath}$+$\hat{\jmath}$+$\hat{k}$) and (2$\hat{\imath}$-3$\hat{\jmath}$+5$\hat{k}$).

18. $If\ \vec{a}=2\hat{\imath}-2\hat{\jmath}+\hat{k}$, $\vec{b}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$ and $\vec{c}=-\hat{\imath}+2\hat{k}$, then find the value of $|\hat{a}-\hat{b}+2\hat{c}|$ .Find direction ratio of the vector $\hat{a}-\hat{b}+2\hat{c}$.

19. The sum of two unit vectors is a unit vector. Show that the value of their difference is $\sqrt{3}$.

20. Show that the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ is equally inclined to the axes OX, OY, OZ

21. If $|\vec{a}\ | = 10, |\vec{b}\,| = 1$ and $|\vec{a}\,.\vec{b}\,| = 6$, then find $|\vec{a}\ \times\ \vec{b}\,|$

22. If $\vec{a}$ = $3\hat{\imath}$+$4\hat{\jmath}$+$5\hat{k}$ and $\vec{b}$ = $2\hat{\imath}$+$\hat{\jmath}$-$4\hat{k}$, then express $\vec{b}$ in the form $\vec{b}$ = $\vec{b}_1$+$\vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

23. Using vector show that the points A(-2,3,5), B(7,0,-1), C(-3,-2,-5) and D(3,4,7) are such that AB and CD intersect at P(1,2,3).

24. Find the area of a parallelogram whose adjacent sides are given by vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.

25. Rohan is walking around a triangular park. The vertices of the park are given by the position vectors (-$\hat{\jmath}$-$2\hat{k}$), ($3\hat{\imath}$+$\hat{\jmath}$+$4\hat{k}$) and ($5\hat{\imath}$+$7\hat{\jmath}$+$\hat{k}$). Show that the park is in right triangular shape. Also find its other two angles.

26. Find the position vector of the point which divides the join of the points ($2\vec{a}$-$3\vec{b}$) and ($3\vec{a}$-$2\vec{b}$) in the ratio, (i) internally, (ii) externally.

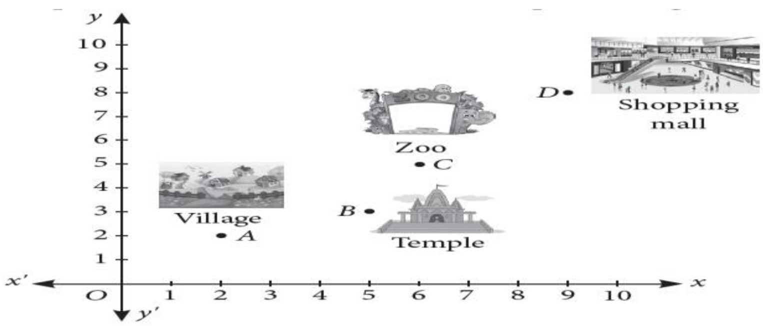
27. On the week days, every morning Piya first drops her son to his school and then she goes to her office. Let her house, the school and the office are represented by the position vectors (-$2\vec{a}$+$3\vec{b}$+$5\vec{c}$), ($\vec{a}$+$2\vec{b}$+$3\vec{c}$) and ($7\vec{a}$-$\vec{c}$). Show that for any $\vec{a}$, $\vec{b}$ and $\vec{c}$ the house, the school and the office are on the same straight path.

28. $\vec{a}$, $\vec{b}$, and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$, and $\vec{c}$.

29. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1$ = $3\hat{\imath}$+$\hat{\jmath}$-$2\hat{k}$ and $\vec{d}_2$ = $\hat{\imath}$-$3\hat{\jmath}$+$4\hat{k}$

CASE BASED QUESTION

30 Ishaan left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Ishaan at different places is given in the following graph.



Based on the above information, answer the following questions.

(i) Find position vector of *B*

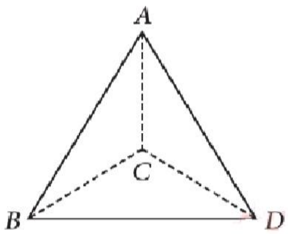
(ii) Find the vector $\overrightarrow{BC}$ in terms of $\hat{\imath}, \hat{\jmath}$

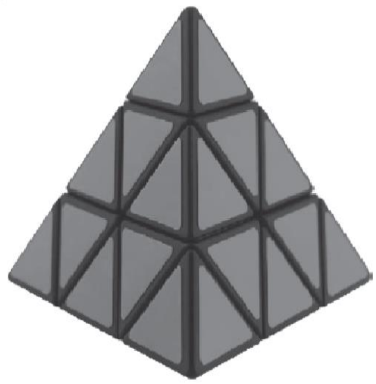
(iii) Find the length of vector $\overrightarrow{AD}$

OR

If $\vec{M}$ =4 *j*-3*k*, then find unit vector along $\vec{M}$

31 A building is to be constructed in the form of a triangular pyramid, *ABCD* as shown in the figure.





Let its angular points are *A*(0, 1, 2), *B*(3, 0, 1), *C*(4, 3, 6) and *D*(2, 3, 2) and *G* be the point of intersection of the medians of □*BCD*.

Based on the above information, answer the following questions.

(i) Find the coordinates of point *G* are

(ii) Find the length of vector $\overrightarrow{AG}$

(iii) Find the Area of Δ*ABC* (in sq. units)

OR

(iii) Find the sum of lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

ANSWERS: PRACTICE MATERIALS

1) (b) 2) (c) 3)(d) 4(b) 5) (c) 6) (c) 7) (a) 8) (c) 9(b) 10) (c) 11(a) 12) (a) 13(b) 14(c)

Q. NO	ANSWER	MARKS
15	The direction ratios are (5, -3, 4)	
	∴ The direction cosines are = $\frac{5}{5\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}} = \frac{1}{\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$	2

16 $\therefore y^x+5z = (-2)^3 + 5(-1) = -8 - 5 = -13$ 2

17 The required unit vector = $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{3\hat{\imath}-2\hat{\jmath}+6\hat{k}}{\sqrt{9+4+36}} = \frac{3\hat{\imath}-2\hat{\jmath}+6\hat{k}}{7} = \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$ 2

18 $\left|\hat{a} - \hat{b} + 2\hat{c}\right| = \sqrt{4+25+1} = \sqrt{30}$ & $d.r = -\frac{2}{\sqrt{30}}. -\frac{5}{\sqrt{30}}, -\frac{1}{\sqrt{30}}$ 2

19 $|\vec{a}| = 1, |\vec{b}| = 1, |\vec{a}+\vec{b}| = 1$ 2

$(|\vec{a}+\vec{b}|)^2 + (|\vec{a}-\vec{b}|)^2 = 2\left\{|\vec{a}|^2 + |\vec{b}|^2\right\} = 4$

$(|\vec{a}-\vec{b}|)^2 = 3$ or $|\vec{a}-\vec{b}| = \sqrt{3}$

21 $(|\vec{a}.\vec{b}|)^2 + (|\vec{a}\times\vec{b}|)^2 = \left\{|\vec{a}|^2.|\vec{b}|^2\right\}$ or $(|\vec{a}\times\vec{b}|)^2 = 64$ or $|\vec{a}\times\vec{b}| = 8$ 2

22 $\vec{b} = (-\frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath} - \hat{k}) + (\frac{13}{5}\hat{\imath} + \frac{9}{5}\hat{\jmath} - 3\hat{k}) = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ is the required expression. 3

23 To prove P intersects $\overrightarrow{AB}$ and $\overrightarrow{CD}$, show that A,P,B are collinear as well as C,P,D. P is a common to $\overrightarrow{AB}$ and $\overrightarrow{CD}$ and so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ intersect at P. 3

24 Area = $|\vec{a}\times\vec{b}| = 15\sqrt{2}$ Sq. units 3

25 Let the position vectors of the vertices A, B and C of the triangular park is 3

$\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{BC}}{|\overrightarrow{AB}||\overrightarrow{BC}|} = \frac{(3\hat{\imath}+2\hat{\jmath}+6\hat{k}).(2\hat{\imath}+6\hat{\jmath}-3\hat{k})}{7\times7} = \frac{6+12-18}{49} = 0$

$\therefore \theta = \frac{\pi}{2}$, Therefore, the park is in right triangular shape.

The other two angles are $\frac{\pi}{4}, \frac{\pi}{4}$ i.e., 45° and 45°

26 $\frac{12}{5}\vec{a} - \frac{13}{5}\vec{b}$ And $-5\vec{b}$ 3

27 The position vectors of the house, the school and the office are 3

$\vec{A} = (-2\vec{a}+3\vec{b}+5\vec{c})$, $\vec{B} = (\vec{a}+2\vec{b}+3\vec{c})$ and $\vec{C} = (7\vec{a}-\vec{c})$

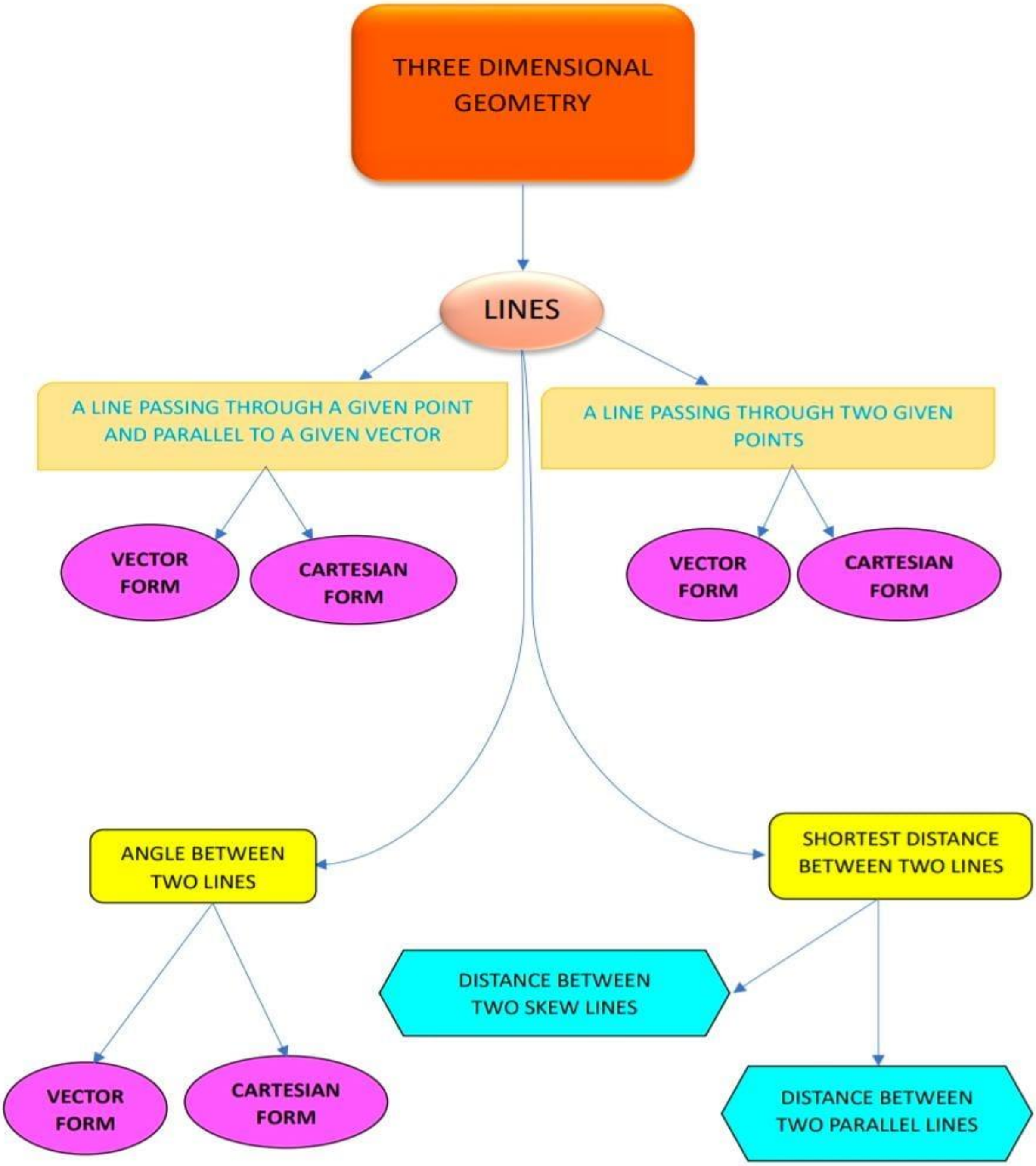
Now prove $\vec{A}, \vec{B}, \vec{C}$ are collinear for the required result

29 area = $5\sqrt{3}$ sq. units 3

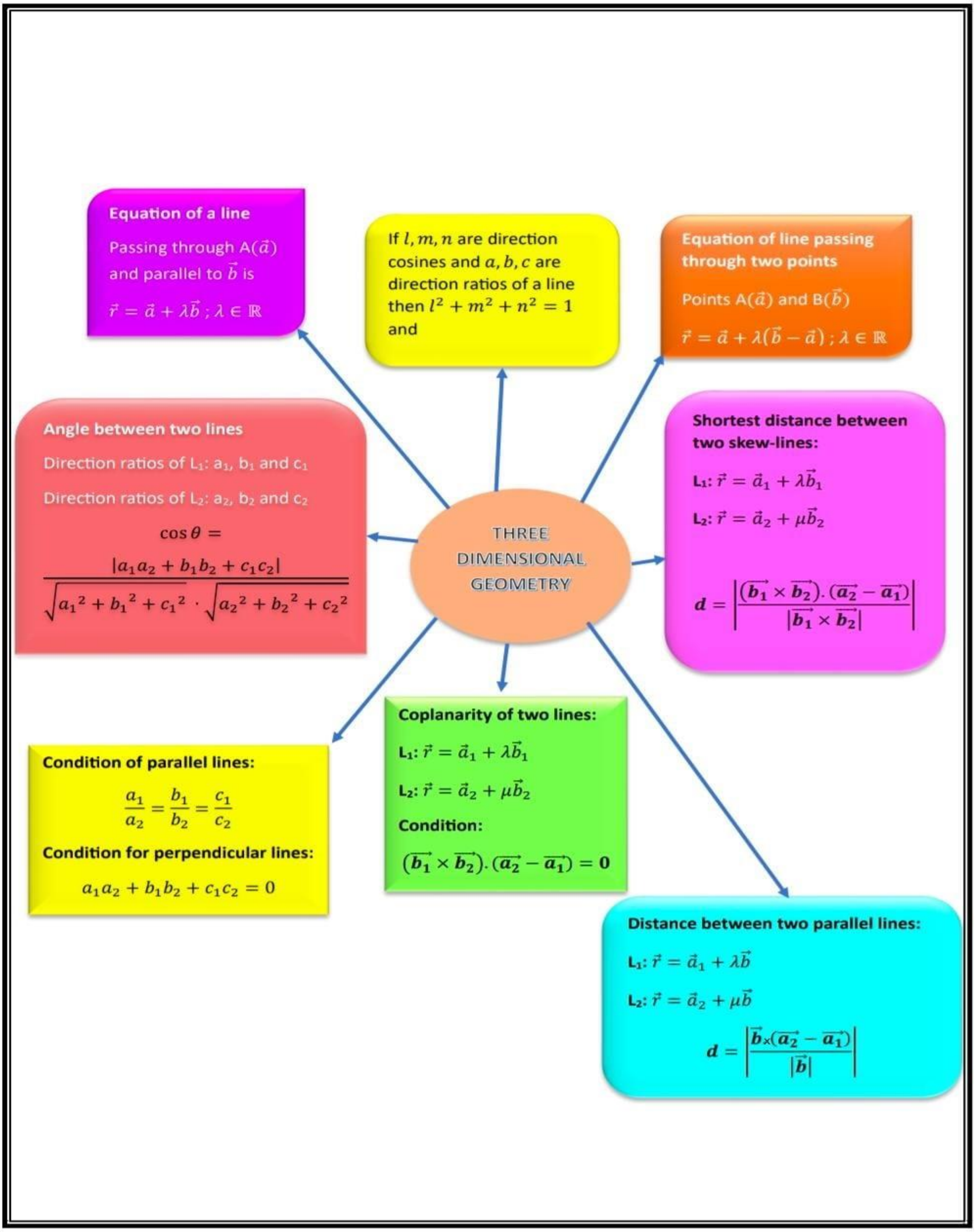
30 $(i)\ 5\hat{\imath} + 3\hat{\jmath}$ $(ii)\ \hat{\imath} + 2\hat{\jmath}$ (iii) $\sqrt{85}$ units **OR** $\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$ 4

31 (i) (3, 2, 3) (ii) $\sqrt{11}$ units (iii) $3\sqrt{10}$ sq. unit **OR** 9.32 units 4

CONCEPT MAPPING:



CONCEPT MAPPING:



KEY POINTS:

Direction Cosines of a line:

A directed line l passing through origin making angles α, β, γ with x, y and z axes respectively are called direction angles. Cosine of these angles namely $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction cosines of the directed line l. Direction cosines of a line are denoted by l, m and n where $l = \cos\alpha$, $m = cos\,\beta$, $n = cos\,\gamma$. If l, m and n are the direction cosine of a line then $l^2 + m^2 + n^2 = 1$.

Direction ratios of a line:

Any three numbers which are proportional to the direction cosines of a line are called the direction ratios of the line. Direction ratios of a line are denoted as a,b,c .

$l = ak, m = bk, n = ck, k$ being a constant. $l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$; $m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$ and $n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$. The sign to be taken for l, m and n depend on the desired sign of k, either a positive or negative. The direction ratios of the line segment joining the points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) may be taken as $x_2 - x_1, y_2 - y_1, z_2 - z_1$.

Skew lines: Skew lines are lines in space which are neither parallel nor intersecting. They lie in different planes.

Equation of a line in Space

Equation of a line passing through a point and parallel to a vector:

- **Vector form:-** Let a line passes through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$ .Let P be any point on this line with position vector $\vec{r}$. Then the equation of the line in vector form is given by $\vec{r} = \vec{a} + \mu\,\vec{b}$,where μ is some scalar.
- **Cartesian form:-** In cartesian form the equation of the line is given by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ where the line passes through the point (x_1, y_1, z_1) and a,b,c are the direction ratios of the line.

Equation of a line passing through two points:

- **Vector form:-** Let a line passes through two points with position vector $\vec{a}$ and $\vec{b}$ .Let P be any point on this line with position vector $\vec{r}$ .Then the equation of the line in vector form is given by $\vec{r} = \vec{a} + \mu\,(\vec{b} - \vec{a})$, where μ is a scalar.

- **Cartesian form:**- Let the line passes through the points (x_1, y_1, z_1) and (x_2, y_2, z_2). Then the equation of the line is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Angle Between Two lines:

➢ The angle between the two lines

The acute angle θ between the lines $\vec{r} = \overrightarrow{a_1} + \mu\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + k\overrightarrow{b_2}$ is given by $\cos\theta = \frac{|\overrightarrow{b_1}.\overrightarrow{b_2}|}{|b_1||b_2|}$

And the obtuse angle between them = $\pi - \theta$

Distance Formula:

1) Distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2) (a) The Shortest Distance between the Skew Lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by

$$d = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$$

(b) The Shortest Distance between the Skew Lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is
$$d = \left|\frac{\begin{vmatrix} (x_2-x_1) & (y_2-y_1) & (z_2-z_1) \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-b_1a_2)^2+(b_1c_2-b_2c_1)^2+(c_1a_2-a_1c_2)^2}}\right|$$

3) The distance between the parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is given by

$$d = \left|\frac{\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\vec{b}|}\right|$$

MCQ QUESTIONS

1. If the cartesian equation of a line $\frac{(3-x)}{5} = \frac{(y+4)}{7} = \frac{(2z-6)}{4}$, write its vector equation.

   (a) $\vec{r} = (\hat{i} - 4\hat{j} + 6\hat{k}) + \mu\,(5\hat{i} + 7\hat{j} + 2\hat{k})$ (b) $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \mu\,(-5\hat{i} + 7\hat{j} + 2\hat{k})$

   (c) $\vec{r} = (3\hat{i} + 4\hat{j} + 3\hat{k}) + \mu\,(-5\hat{i} + 7\hat{j} + 2\hat{k})$ (d) $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \mu\,(5\hat{i} + 7\hat{j} + 2\hat{k})$

2. Find the foot of the perpendicular drawn from the point $(2, -3, 4)$ on the y-axis.

(a) $(2,0,4)$ (b) $(0,3,0)$ (c) $(0,-3,0)$ (d) $(-2,0,-4)$

3. If a line makes angles 90° and 60° with the positive direction of x and y axes, find the angle which it makes with positive direction of z -axis.

   (a) $\pi/3$ (b) $\pi/4$ (c) $\pi/6$ (d) 0

4. The distance of a point P (a, b, c) fom x-axis is

   (a) $\sqrt{a^2+b^2}$ (b) a^2+b^2 (c) a^2+c^2 (d) $\sqrt{b^2+c^2}$

5. Write direction cosines of a line parallel to z-axis.

   (a) 1,0,0 (b) 0,0,1 (c) 1,1,0 (d) $-1,-1,-1$

6. If the lines $\frac{(x-2)}{3}=\frac{(y-1)}{1}=\frac{(4-z)}{k}$ and $\frac{(x-1)}{k}=\frac{(y-4)}{2}=\frac{(z-5)}{-2}$ are perpendicular, find the value of k .

   (a) -2/5 (b) -2/7 (c) 4 (d) 2/7

7. The equation of a line is $\frac{(x-1)}{-2}=\frac{(y+3)}{3}=\frac{(z+2)}{6}$, find the direction cosines of a line parallel to the given line.

   (a) $-2/7, 3/7, 6/7$ (b) $2/7, -3/7, -6/7$ (c) $-2, 3, 6$ (d) $2, -3, -6$

8. Direction ratio of line joining (2, 3, 4) and (−1, −2, 1), are:

   (a) (−3, −5, −3) (b) (−3, 1, −3) (c) (−1, −5, −3) (d) (−3, −5, 5)

9. The vector equation for the line passing through the points (–1, 0, 2) and (3, 4, 6) is:

   (a) $i + 2k + \lambda(4i + 4j + 4k)$ (b) $i - 2k + \lambda(4i + 4j + 4k)$

   (c) $-i + 2k + \lambda(4i + 4j + 4k)$ (d) $-i + 2k + \lambda(4i - 4j - 4k)$

10. If a line has direction ratios 2, – 1, – 2, determine its direction cosines:

   (a) ⅓, ⅔, −⅓ (b) ⅔, −⅓, −⅔ (c) −⅔, ⅓, ⅔ (d) None of the above

MCQ ANSWERS

1) (b) 2) (c) 3) (c) 4) (d) 5) (b) 6) (a) 7) (a) 8) (a) 9) (c) 10) (b)

Assertion Reason Type Questions

In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices:

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.

(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.

(c) Assertion is correct statement but Reason is wrong statement.

(d) Assertion is wrong statement but Reason is correct statement.

1. **Assertion:** If the cartesian equation of a line is $\frac{(x-5)}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ then its vector form is

$\vec{r} = 5\,i - 4\,j + 6k + \lambda(3\,i + 7j + 2k)$.

Reason: The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{(x+3)}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{(x+3)}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

2. **Assertion:** The pair of lines given by $\vec{r} = i - j + \lambda(2\,i + k)$ and $\vec{r} = 2\,i - k + \upsilon(i + j - k)$ intersect

   Reason: Two lines intersect each other, if they are not parallel and shortest distance = 0.

3. **Assertion:** If a line passes through a point whose position vector is $a^{\rightarrow} = i - 4j - 2k$ and is parallel to $b^{\rightarrow} = 2i + 2j + 3k$ then, its equation is

   $$\vec{r} = i - 4j - 2k + \mu(2i + 2j + 3k)$$

   Reason: If a line passes through a point whose position vector is $\vec{a}$ and is parallel to the vector $\vec{b}$ then, its equation is $\vec{r} = \vec{a} + \mu\,\vec{b}$

4. **Assertion:** The three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$ *and* $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$ *and* $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

   Reason: The line through the points (1, –1, 2) and (3, 4, –2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

5. **Assertion:** The angle between the straight lines
   $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{3}$ is 90°

   Reason: Skew-lines are lines in different planes which are parallel and intersecting.

<u>ANSWERS(Assertion Reason Type Questions)</u>

1) (c) 2) (a) 3)(a) 4)(b) 5(c)

<u>2 MARKS QUESTIONS</u>

1. If α, β, Υ are the direction angles of a line, find the value of $sin^2\alpha + sin^2\beta + sin^2\,\Upsilon$
2. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.
3. Find the vector and Cartesian equation of the line passing through points $(3, -2, -5)$ and $(5, -4, 6)$.
4. Check whether the lines passing through $(1, 1, 2)$ and $(3,5,1)$ is parallel to the line through $(4, 2, -1)$ and $(2, -2\,, 0)$
5. The points A $(1,2,3)$, B $(-1, -2, -3)$and C $(2,3,2)$ are the vertices of a parallelogram, then find the equation of CD.

ANSWERS OF 2 MARKS QUESTIONS

1.	2
2.	The lines are: $\frac{x}{1}=\frac{y}{-1}=\frac{z}{1/k}$ and $\frac{x-1}{2}=\frac{y+1}{1/2}=\frac{z-1}{-1}$ after applying condition k = 2
3	Cartesian : $\frac{x-3}{2}=\frac{y+2}{-2}=\frac{z+5}{11}$ Vector : $\vec{r}=\left(3\hat{\imath}-2\hat{\jmath}-5\hat{k}\right)+\mu\left(2\hat{\imath}-2\hat{\jmath}+11\hat{k}\right)$
4	Show that d.r. of lines are proportional
5	First find coordinates of $D\ as\ (4,7,8)$ then equation.

LONG ANSWER (TYPE 1) QUESTIONS (3 marks questions)

1) Find the equation of the line passing through the point P $(-1,3,-2)$ and perpendicular to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$
2) Find the acute angle between the two lines by $\frac{x-4}{3}=\frac{y+3}{4}=\frac{z+1}{5}$ by $\frac{x-1}{4}=\frac{y+1}{-3}=\frac{z+10}{5}$
3) Find the shortest distance between the lines whose vector equations are
$$\vec{r}=(1-t)\hat{\imath}+(t-2)\hat{\jmath}+(3-2t)\hat{k}$$
$$\vec{r}=(s+1)\hat{\imath}+(2s-1)\hat{\jmath}-(2s+1)\hat{k}$$
4) Find the shortest distance between the lines $\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=5\hat{\imath}-2\hat{\jmath}+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$. Find the point of intersection if exists.
5) Show that the points A (2, 3, – 4), B (1, – 2, 3) and C (3, 8, – 11) are collinear
6) The cartesian equation of a line is 6x – 2 = 3y + 1 = 2z – 2. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, – 1, – 1) which are parallel to the given line.

ANSWERS OF (LONG ANSWER TYPE 1) QUESTIONS (3 marks)

1. Equation of required line is $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$
2. Using appropriate formula we get $\cos\theta=\frac{1}{2} \Rightarrow \theta=\pi/3$
3. Shortest distance, $d=\frac{8}{\sqrt{29}}$
4. The S.D. is 0 ∴ lines are intersecting.pt of intersection is $(-1,-6,-12)$
5. Show that direction ratios of AB and BC are proportional. That means AB is parallel to BC. Hence, A, B, C are collinear points.
6. DR's of the line are (1, 2, 3).∴ Direction cosines of the line are $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$

   Cartesian : $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ $Vector$: $\vec{r}=(2\hat{\imath}+\hat{\jmath}+\hat{k})+\lambda(\hat{\imath}+2\hat{\jmath}+3\hat{k})$

LONG ANSWER (TYPE 2)QUESTIONS (5 marks questions)

1) (a) Find the image of the point $(1,6,3)$ in the line: $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

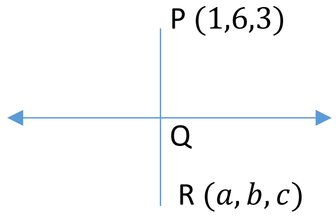
(b) Also, find the length of the segment joining the given point and its image.

2) Show that the lines $\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath})$ and $\vec{r} = (4\hat{\imath} - \hat{k}) + \mu(2\hat{\imath} + 3\hat{k})$ intersect each other. Find their point of intersection.

3) Prove that the line through A (0, – 1, – 1) and B(4, 5, 1) intersects the line through C (3, 9, 4) and D (- 4, 4, 4).

ANSWERS OF (LONG ANSWER TYPE 2) QUESTIONS

1



Q is the foot of the perpendicular from P

Therefore, Q is (λ ,2 λ +1,3 λ +2)

PQ is perpendicular to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, so we have

$(\lambda - 1)1 + (2\lambda + 1 - 6)2 + (3\lambda + 2 - 3)3 = 0$ or $\lambda = 1$ *so that* Q $= (1,3,5)$

Using midpoint formula $\frac{a+1}{2} = 1, \frac{b+6}{2} = 3, \frac{c+3}{2} = 5 \Rightarrow a = 1, b = 0, c = 7$,∴ image is (1,0,7)

Required distance PR $= 2\sqrt{13}$ using distance formula

2 The two given lines will intersect if

$(\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath}) = (4\hat{\imath} - \hat{k}) + \mu(2\hat{\imath} + 3\hat{k})$ for some particular values of λ *and* μ

Equating coefficients of $\hat{\imath}$ *and* $\hat{\jmath}$ and solving we get, $\lambda = 1$ and $\mu = 0$

Substituting in the coefficient of $\hat{k}$, the equation is satisfied. ∴ the two lines intersect

Putting λ = 1 in the first line, the point of intersection is $(4,0,-1)$

3 Equation of line passing through (0,-1,-1) and (4,5,1) is $\vec{r} = \left(-\hat{\jmath} - \hat{k}\right) + \lambda(4\hat{\imath} + 5\hat{\jmath} + \hat{k})$

Equation of line passing through (3,9,4) and (-4,4,4) is $\vec{r} = \left(3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}\right) + \lambda(-4\hat{\imath} + 4\hat{\jmath} + \widehat{4k})$

Now solve as above in Q.2

CASE STUDY BASED QUESTIONS

1. Two aeroplanes A and B are flying along the lines $\vec{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k})$ respectively.

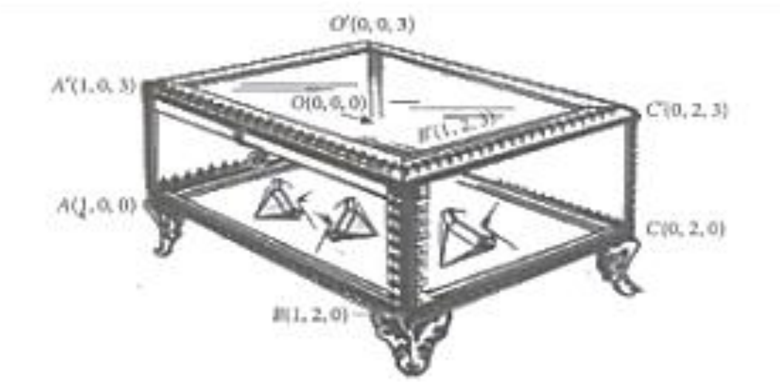
Based on the above information answer the following questions:

(a) Find the cartesian equation of the line along which aeroplane A is flying

(b) What are the direction cosines of the line of flight of the 2^{nd} aeroplane

(c) Find the shortest distance between the given lines.

2. In a diamond exhibition, a diamond is covered in cubical glass box having coordinates O$(0, 0, 0)$, A$(1, 0, 0)$, B$(1, 2, 0)$, C$(0, 2, 0)$, O'$(0,0,3)$, A'$(1, 0, 3)$, B'$(1, 2, 3)$ and C$(0, 2, 3)$.

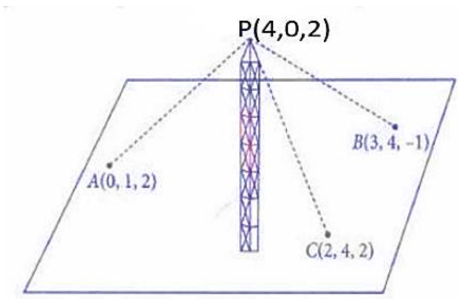


Based on the above information answer the following questions:

(a) Find direction ratios of OA

(b) Find the cartesian and vector equation of the diagonal OB′

3. A mobile tower stands at the top of a hill. Consider the surface on which tower stands as a plane having points A$(0, 1,2)$, B$(3, 4, -1)$ and C$(2, 4, 2)$ on it. The mobile tower is tied with 3 cables from the points A, B and C such that it stands vertically on the ground. The peak of the tower is at the point (4,0,2), as shown in the figure. Let N$(2, -3,1)$ be the foot of the perpendicular from the point P.



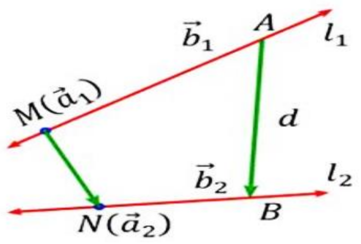
Based on the above information answer the following questions:

(a) Find the equation of the line BC

(b) Find the equation of the perpendicular line drawn from the peak of tower to the foot of the perpendicular where N$(2, -3, 1)$ is the foot of the perpendicular.

(c) Find the height of the tower.

4. A student drew 2 skew lines as shown below with their points through which they pass and their directions $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$. The equations of these two lines l_1 and l_2 are given by $\frac{x+2}{2} = \frac{y}{3} = \frac{z-2}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$



Based on the above information, answer the following questions:

(a) Find the vector $\overrightarrow{b_1} \times \overrightarrow{b_2}$

(b) Find the shortest distance between these two lines l_1 and l_2.

ANSWERS OF CASE STUDY BASED QUESTIONS

1(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

1(b) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

1(c) 0

2(a) 1,0,0

2(b) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and Vector equation: $\vec{r} = \mu(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ where μ is a scalar.

3(a) $\frac{x-2}{1} = \frac{y-4}{0} = \frac{z-2}{-3}$

3(b) $\frac{x-4}{-2} = \frac{y}{-3} = \frac{z-2}{-1}$

3(c) $\sqrt{14}$ units

4(a) $5\hat{\imath} - \hat{\jmath} - 7\hat{k}$

4(b) $\sqrt{3}$ units

Linear Programming Problems

The term 'Programming' means planning and it refers to a particular plan of actions from amongst several alternatives for maximizing profit or minimizing cost etc. The term 'Linear' means that all inequations or equations used and the function to be maximized or minimized are linear.

So, linear programming deals with the optimization (maximization or minimization) of a linear function of a number of variables subject to a number of conditions on the variables, in the form of linear inequations or equations in variables involved.

Few important terms related to LPP :

Objective function: A linear function z = ax + by, (where a and b are constants) which has to be maximized or minimized according to a set of given conditions, is called as linear objective function. Variables x and y are called **Decision variables**.

Constraints: The restrictions in the form of linear inequalities on the variables of linear programming problems are called constraints. The condition x >= 0 y >= 0 are called non-negative constraints.

Solution: A set of values of variables x, y which satisfies the constraints of LPP, is called a solution of LPP.

Feasible Solution: A set of values of the variables x, y is called a feasible solution of a LPP, if it satisfies the constraints and non-negativity restrictions of the problem.

Infeasible Solution: If the system of constraints has no point which satisfies all the constraints and non-negativity restrictions.

Theorem 1

Let R be the feasible region (convex polygon) for a linear programming problem and let Z = *ax* + *by* be the objective function. When Z has an optimal value (maximum or minimum), where the variables *x* and *y* are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Theorem 2

Let R be the feasible region for a linear programming problem, and let

Z = *ax* + *by* be the objective function. If R is **bounded** , then the objective function Z has both a **maximum** and a **minimum** value on R and each of these occurs at a corner point (vertex) of R.

If R is **unbounded**, then a maximum or a minimum value of the objective function may not exist. However, if it exists, it must occur at a corner point of R.

Solving linear programming problem using Corner Point Method.

The method comprises of the following steps:

- Find the feasible region of the linear programming problem and determine its corner

Points (vertices) either by inspection or by solving the two equations of the lines intersecting

at the point.

- Evaluate the objective function Z=ax+ by at each corner point. Let M and m, respectively

denote the largest and smallest values of these points.

(I) When the feasible region is bounded, M and m are the maximum and minimum value of Z.

(II) In case, the feasible region is unbounded, we have:

(a) M is the maximum value of Z , if the open half plane determined by ax+ by > M has no point in common with the feasible region Otherwise, Z has no maximum value.

(b) Similarly, m is the minimum value of Z, if the open half plane determined by ax+ by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

MCQ (Each questions carries 1 mark)

1. Solution set of the inequality 3x+ 5y < 4 is ....

(a) an open half-plane not containing the origin

(b) an open half-plane containing the origin

(c) the whole xy- plane not containing the line 3x+ 5y=4

(d) a closed half plane containing the origin

2. Which of the following points satisfies both the in equations 2x+y≤10 and x+2y≥ 8?

(a) (-2,4) (b) (3,2) (c) (-5,6) (d) (4,2)

3. The corner points of the bounded feasible region determined by a system of linear constraints are (0,3), (1,1)and (3,0). Let Z = px + qy, where p,q >0. The condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is

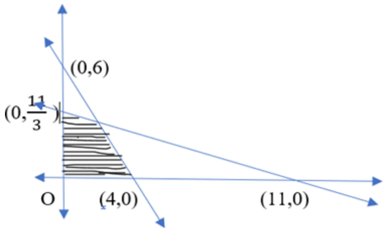
(a) p=2q (b) p=q/2 (c) p=3q (d) p=q

4. The feasible region satisfied by the constraints

$x + y \leq 5$, $x \leq 4$, $y \leq 4$, $x \geq 0$, $y \geq 0$, $5x + y \geq 5$, $x + 6y \geq 6$ is bounded by

(a)4 straight lines (b) 5 straight lines (c) 6 straight lines (d) 7 straight lines

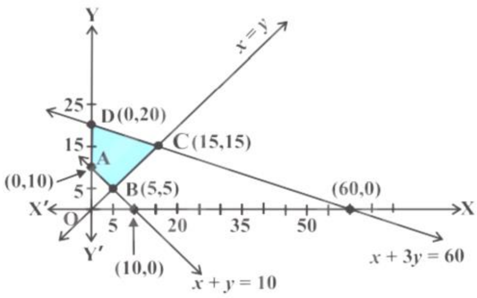
5. For the following feasible region, the linear constraints are : $x \geq 0, y \geq 0, 3x + 2y \leq 12$, $x + 3y \leq 11$



(a) $x \geq 0, y \geq 0, 3x + 2y \geq 12, x + 3y \geq 11$ (b) $x \geq 0, y \geq 0, 3x + 2y \leq 12, x + 3y \geq 11$

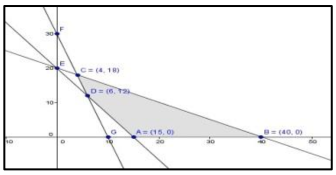
(c) $x \geq 0, y \geq 0, 3x + 2y \leq 12, x + 3y \leq 11$ (d) None of these

6. Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function Z = 3x + 9y maximum?



(a) Point B (b) Point C (c) Every point on line segment CD (d) Point D

7. In the given graph, the feasible region for a LPP is shaded. The objective function $Z = 20x + 10y$, will be minimum at:



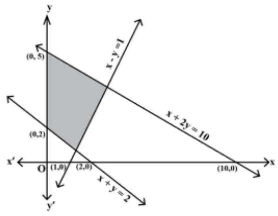
(a) (15,0) (b) (40,0) (c)(4,18) (d)(6,12)

8. Corner points of the feasible region for an LPP are $(0,2), (3,0), (6,0), (6,8)$ and $(0,5)$.

Let $Z = 4x + 6y$ be the objective function. The minimum value of Z occurs at:

(a) (0,2) only (b) (3,0) only (c) The mid-point on the line segment joining the point (0,2) and (3,0) only (d)Any point on the line segment joining the points (0,2) and (3,0).

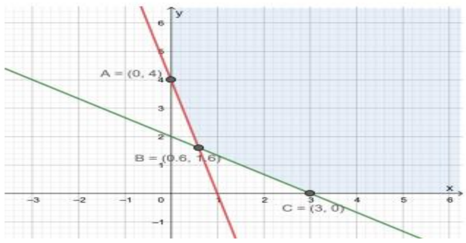
9. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



Which of the following is not a constraint to the given Linear Programming Problem?

(a) x+y≥2 (b) x + 2y ≤10 (c) x-y≥1 (d) x- y ≤1

10. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function Z = 4x + 6y occurs at



(a)(0.6, 1.6) $only$ (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only

(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)

SA (This section comprises of short answer type questions of 3 marks each)

11. Solve the following Linear Programming Problem graphically:

Minimize: z = x + 2y ,

subject to the constraints: x + 2y ≥100, 2x -y≤0, 2x +y ≤200, x,y≥0

12. Solve the following Linear Programming Problem graphically:

Maximize: z= - x +2 y

subject to the constraints: x≥3, x +y ≥5, x +2y ≥6, y≥0

13. Solve the following Linear Programming Problem graphically:

Maximize Z = 400x + 300y

subject to $x + y \leq 200$, $x \leq 40$, $x \geq 20$, ≥ 0

14. Solve the following Linear Programming Problem graphically:

Minimize Z = 5x + 10y

subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$

15. Solve the following Linear Programming Problem graphically:

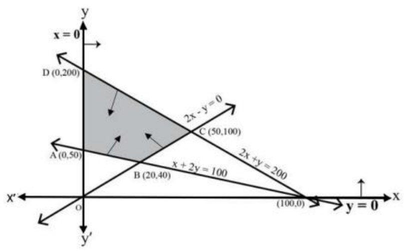
Maximize Z = 3x + 9y

subject to the constraints $x + y \geq 10$, $x + 3y \leq 60$, $x \leq y$, $x \geq 0$, $y \geq 0$

MCQs(Answers)

1. (b) 2. (d) 3. (b) 4. (b) 5. (c) 6. (c) 7. (d) 8. (d) 9. (c) 10. (d)

11. Minimize: z = x + 2y , subject to the constraints: $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$

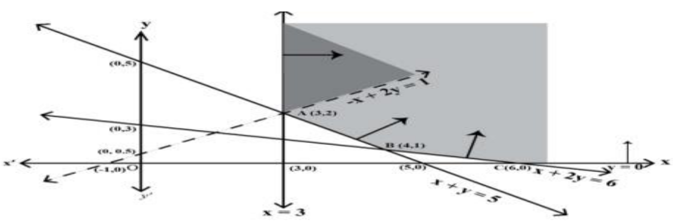


Corner Point	Z= x +2y	
A(0,50)	100	Min
B(20,40)	100	Min
C(50,100)	250	
D(0,200)	400	

The minimum value of Z is 100 at all the points on the line segment joining the points A(0,50) and (20,40).

12. Maximize: z= - x +2 y ,

subject to the constraints: $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$

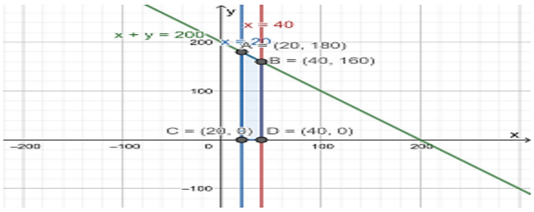


Corner Point	z= - x +2 y	
A(3,2)	1	(may or may not be the maximum value)
B(4,1)	-2	
C(6,0)	-6	

Since the feasible region is unbounded, Z=1 may or may not be the maximum value. Now, we drawq the graph of – x+2y>1 and we check whether the resulting open half-plane has any point/s, in common with the feasible region or not. Here, the resulting open half plane has points in common with the feasible region.

Hence, Z =1 is not the maximum value. We conclude, Z has no maximum value .

13. We have Z= 400x +300y, subject to x + y ≤ 200, ≤ 40, $x \geq 20$, $y \geq 0$. The corner points of the feasible region are C(20,0), D(40,0), B(40,160), A(20,180)

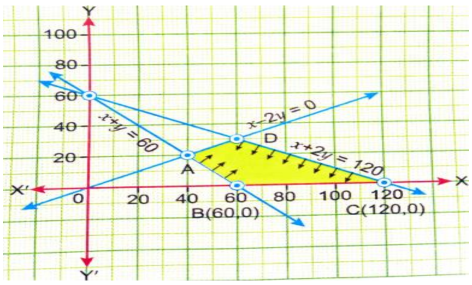


Corner Point	Z = 400x + 300y
A(20,180)	62000
B(40,160)	64000 maximum
C(20,0)	8000
D(40,0)	16000

Maximum profit occurs at x= 40, y=160 and the maximum profit = Rs 64,000

14. We have minimize Z = 5x + 10y

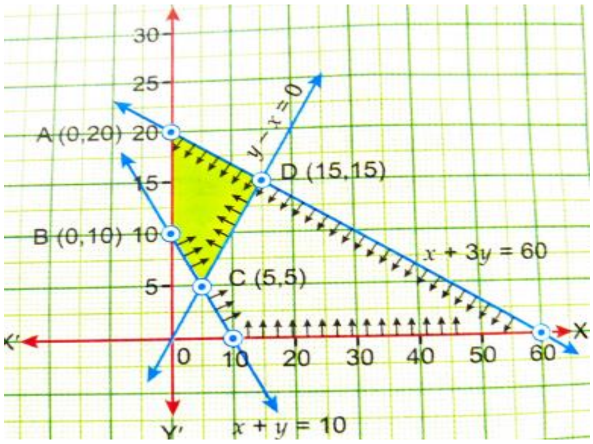
subject to $x + 2y \leq 120$, x +y ≥ 60, x-2y≥0, $x \geq 0$, $y \geq 0$



Corner Point	Z = 5x + 10y
A(40,20)	400
B(60,0)	300 Min
C(120,0)	600
D(60,30)	600

So, Z_{min}= 300 at (60,0)

15. We have maximize Z = 3x + 9y subject to the constraints $x + y \geq 10$, $x + 3y \leq 60$, $x \leq y$, $x \geq 0$, $y \geq 0$

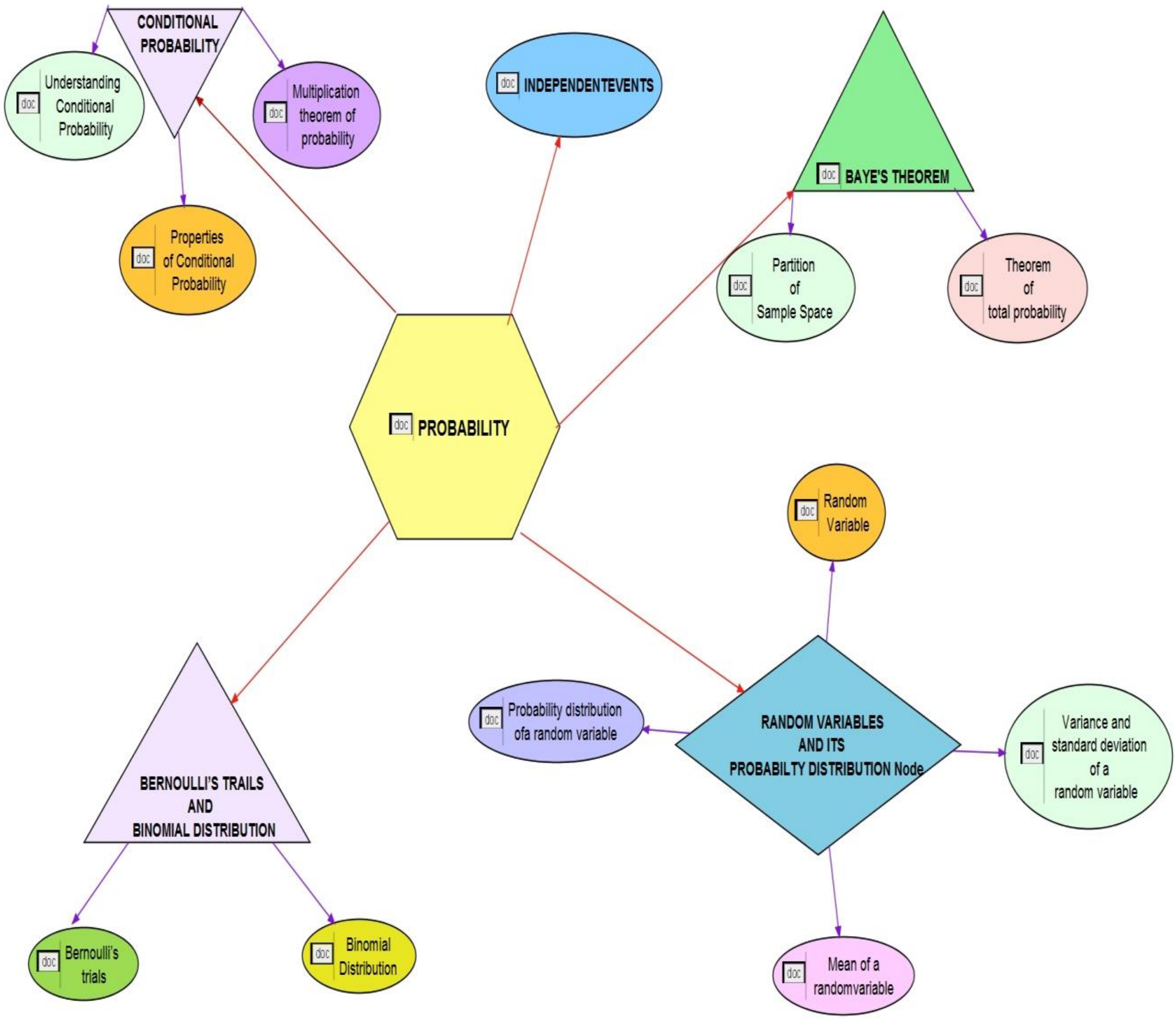


Corner Points	Z = 3x + 9y
A(0,20)	180 max
B(0,10)	90
C(5,5)	60
D(15,15)	180 max

So, Z_{max}= 180 at infinitely many points lying on the line joining points (0,20) and (15,15).

PROBABILITY

CONCEPT MAP:



KEY POINTS:

Conditional Probability

If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the event E under the condition that the event F has occurred, written as P (E/F), is given by

$$P(E/F) = P(E \cap F)/ P(F) \; ; P(F) \neq 0$$

Properties of Conditional Probability

Let E and F be events associated with the sample space S of an experiment. Then:

(i) P (SIF) = P(FIF) = 1

(ii) P[(A∪B)/F] = P(AIF) + P(BIF)−P [(A∩BI F)], where A and B are any two events associated with S.

(iii) P ($\bar{E}$IF) = 1−P (E/F)

Multiplication Theorem on Probability

Let E and F be two events associated with a sample space of an experiment. Then

P (E∩F) = P(E) P (F/E), P (E) ≠ 0

= P (F) P (E/F), P (F) ≠ 0

If E, F and G are three events associated with a sample space, then

P(E∩F∩G) = P(E) P (F/E) P (G/E∩F)

Independent Events

Let E and F be two events associated with a sample space S. If the probability of occurrence of one of them is not affected by the occurrence of the other, then we say that the two events are independent. Thus, two events E and F will be independent, if

(a) P (FIE) = P (F), provided P (E) ≠ 0

(b) P (E/F) = P(E), provided P (F) ≠ 0

Using the multiplication theorem on probability, we have

(c) P(E/F) = P(E) P (F)

Three events A, B and C are said to be mutually independent if all the following conditions hold:

P(A∩B)=P(A) P (B)

P(A∩C)=P(A) P (C)

P(B∩C) = P(B) P (C) and

P(A∩B∩C) = P (A) P (B) P (C)

Partition of a Sample Space

A set of events $E_1, E_2, ..., E_n$ is said to represent a partition of a sample space S if

(a) $E_i \cap E_j$ =, i$\neq j$, I, j = 1, 2, 3........, n

(b) $E_1 U E_2 U ... U E_n$ = S, and

(c) Each E_i, ≠ 0, i. e, P (E_i) > 0 for all i = 1, 2, ..., n

Theorem of Total Probability

Let ($E_1, E_2, ..., E_n$} be a partition of the sample space S. Let A be any event associated with S, then

P(A) = $\sum_{i=1}^{n} \mathrm{P}(E_i)\mathrm{P}(\mathrm{A}/E_i)$

Bayes' Theorem

IF $E_1, E_2, ..., E_n$ are mutually exclusive and exhaustive events associated with a sample space, and A is any event of non zero probability, then

P(EIA) = P(E_i)P(AIE_i) /$\sum_{i=1}^{n} \mathrm{P}(E_i)\mathrm{P}(\mathrm{A}/E_i)$

Random Variable and its Probability Distribution

A random variable is a real valued function whose domain is the sample space of a random experiment.The probability distribution of a random variable X is the system of numbers

X	x_1	x_2	……………………	x_n
P(X)	p_1	p_2	…………………….	p_n

where $p_i > 0$, i=1, 2,..., n , $\Sigma p_i = 1$,

1 MARK QUESTIONS

1 Two events A and B are such that $P(B) \neq 0$, then $P(A/\bar{B}).\frac{1-P(B)}{1-P\left(\frac{A}{B}\right)} =$

$a)\ P(B)$ b) $P(A)$ c)$P(A \cap B)$ d)None of these

2 If two events are A and B, then the probability of happening only one event of them exactly

$a)\ P(A) + P(B) - P(A \cap B)$ $b)\ P(\overline{A}) + P(\overline{B}) - 2P(\overline{A} \cap \overline{B})$

$c)\ P(A) + P(B) - P(A \cup B)$ $d) P(A/\bar{B}) + P(B/\bar{A})$

3 Given P (A) = ½, P (B) = p and P (A∪B) = 3/5. The value of p for which A and B are mutually exclusive.

$a)\ \frac{1}{5}$ $b) \frac{1}{15}$ $c)\ \frac{1}{10}$ $d)\ \frac{1}{7}$

4 The probability of having 53 Mondays in a leap year chosen randomly

a) $\frac{1}{7}$ b) $\frac{3}{7}$ c)$\frac{4}{7}$ d) $\frac{2}{7}$

5 If the probability for A to fail in an examination is 0.2 and that for B is 0.3. Then the probability that neither fails.

a) 0.57 b) 0.58 c) 0.56 d) 0.65

6 If A and B are independent events, and P(A)= ½, P(B) =1/3, then

P ($\overline{A} \cup \overline{B}$)=

a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) $\frac{4}{3}$

7 A random variable X is specified by the following distribution.

X	2	3	4
P (X = r)	0.3	0.4	0.3

the mean of this distribution is

a) 2 b) 2.7 c) 3.5 d) 3

8 Two dice are thrown. It is known that the sum of numbers on the dice is less than 6, the probability of getting a sum 3 is

(a) 1/18 (b) 1/10 (c) 2/5 (d) 1/5

9 Events A and B are not independent if

(a) $P(A \cap B) = P(A/B)\, P(B)$ (b) $P(A \cap B) = P(B/A)\, P(A)$

(c) $P(A \cap B) = P(A)+P(B)$ (d) $P(A \cap B) = P(A)P(B)$

10 If two events are independent, then

(a) they must be mutually exclusive (b) the sum of their probabilities must be 1

(c) both(a) and (b) are correct (d) none of the above

11 If $P(A \cap B) = 0.15$, $P(B') = 0.10$, then $P(A/B) =$

(a) 1/3 (b) ¼ (c) 1/6 (d) 1/5

12 A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, the probability of getting exactly one red ball is

(a) 45/196 (b) 135/392 (c) 15/56 (d) 15/29

13 If $P(A) = 1/2$, $P(B) = 0$, then $P(A|B)$ is

(a) 0 (b) ½ (c) not defined (d) 1

14 If A and B are any two events such that P(A) + P(B) – P (A and B) = P(A), then

(a) $P(B|A) = 1$ (b) $P(A|B) = 1$ (c) $P(B|A) = 0$ (d) $P(A|B) = 0$

15 If A and B are two events such that $P(A) \neq 0$ and $P(B \mid A) = 1$, then

(a) $A \subset B$ (b) $B \subset A$ (c) $B = \phi$ (d) $A = \phi$

16 The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(a) 0 (b) 1/3 (c) 1/12 (d) 1/36

17 Two events A and B will be independent, if

(a) A and B are mutually exclusive (b) $P(A' \cap B') = [1 - P(A)]\,[1 - P(B)]$

(c) $P(A) = P(B)$ (d) $P(A) + P(B) = 1$

18 If $P(A|B) > P(A)$, then which of the following is correct:

(a) $P(B|A) < P(B)$ (b) $P(A \cap B) < P(A) \cdot P(B)$ (c) $P(B|A) > P(B)$ (d) $P(B|A) = P(B)$

19 Events A and B are said to be mutually exclusive iff

(a) $P(A \cap B) = P(A) + P(B)$ (b) $P(A \cap B) = P(A)P(B)$ (c) $A \cap B = \phi$ (d) None of these

20 If A and B are independent events, then which of the following is not true

(a) P(A|B) = P(A) (b)P(B|A) = P(B) (c)P(B|A) = P(A|B) (d)None of these

2MARKS QUESTIONS

1 If for any two events A and B, P(A) = $\frac{4}{5}$ and P(A∩ $B) = \frac{7}{10}$, then find $P\left(\frac{B}{A}\right)$

2 A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then find the probability that both the balls are white.

3 A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

4 Let A and B be two events. If $P(A) = 0.2,\ P(B) = 0.4, P(A \cup B) = 0.6$ then find $P\left(\frac{A}{B}\right)$

5 A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

6 To test the quality of electric bulbs produced in a factory, two bulbs are randomly selected from a large sample without replacement. If either bulb is defective, the entire lot is rejected. Suppose a sample of 200 bulbs contains 5 defective bulbs. Find the probability that the sample will be rejected.

7 A fair die is rolled. Consider the events A={1, 3, 5}, B={2, 3} and C={2, 3, 4, 5}.
Find (i) $P(A/B)$ (ii) $P(B/A)$

8 A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not .

9 10% of the bulbs produced in a factory are red colour and 2 % are red and defective. If one bulb is picked at random, determine the probability of its being defective if it is red.

10 The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get plumbing contract is $\frac{4}{7}$. If the probability of getting at least one contract is $\frac{2}{3}$, what is the probability that he will get both ?

3 MARKS QUESTIONS

1 A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome ?

2 Bag I contains 3 red and 4 black balls while another bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag II .

3 The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k

(ii) Find P(1$\leq X < 3)$

(iii) Find E(X), the mean of X

4 Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male ? Assume that there are equal number of males and females

5 A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine :

(i) k (ii) P$(X < 3)$ (iii) P$(X > 6)$

6 A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag, one ball is drawn. Find the probability that the ball drawn is red.

7 Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed. 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O ?

8 An electric assembly consists of two sub-systems say A and B. From previous testing procedures, the following probabilities are assumed to be known

P(A fails)=0.2, P(B fails alone)=0.15, P(A and B fail)=0.15

Evaluate the following probabilities

(i) P(A fails/B has failed)

(ii) P(A fails alone)

9 A machine operates if all of its three components function. The probability that the first component fails during the year is 0.14, the second component fails is 0.10 and the third component fails is 0.05. What is the probability that the machine will fail during the year ?

10 Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting a first class is 0.25 and that of a boy getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

5 MARKS QUESTIONS

1 Two boxes containing candies are placed on a table. The boxes are labeled B_1 and B_2. Box B_1 contains 7 cinnamon candies and 4 ginger candies. Box B_2 contains 3 cinnamon candies and 10 pepper candies. The boxes are arranged so that the probability of selecting box B_1 is 1⁄3 and the probability of selecting box B_2 is 2⁄3. Suresh is blindfolded and asked to select a candy. He will win a color TV if he selects a cinnamon candy. What is the probability that Suresh will win the TV (that is, he will select a cinnamon candy)?

2 Three companies A, B and C supply 25%, 35% and 40% of the notebooks to a school. Past experience shows that 5%, 4% and 2% of the notebooks produced by these companies are defective. If a notebook was found to be defective, what is the probability that the notebook was supplied by A?

3 If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces acceptable items, find the probability that the machine is correctly set up.

4 Of the students in a college, it is known that 60% reside in hostels and 40% are day scholars (not residing in hostels). Previous year results report that 30% of all students who reside in hostels attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade. What is the probability that the student is a hostler?

5 An urn contains 5 red and 5 black balls. A ball is drawn at random, its color is noted and is returned to the urn. Moreover, 2 additional balls of the color drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?

6 Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean of the number of aces.

7 Colored balls are distributed in three bags as shown in the following table

Bag	Red	White	Black
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from bag I.

8 A factory has three machine X, Y, Z producing 1000, 2000, 3000 bolts per day respectively. The machine X produced 1% defective bolts, Y produce 1.5% and Z produce 2% defective bolts. At the end of a day, a bolt is drawn at random and is found defective. What is the probability that the defective bolt is produced by the machine X?

9 Two cards are drawn in succession from a well shuffled deck of 52 cards, the first card being replaced, before the second is drawn. Let X denote the number of spades drawn. Find the probability distribution of X?

10 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six.

CASE BASED QUESTIONS

1 A coach is training three players. He observed that player A hits 4 times the shot in 5 tries, player B hits 3 times the shot in 4 and player C is able to hit twice the shot in 3 tries.

Based on above information, answer following questions

i) A,B,C all hit. It is a shot. What is the probability that the shot is hit by B only?

ii) If A,B,C all try , what is the probability that it was hit by none?

iii) Find the probability that the shot was hit by exactly 2 players.

2 Read the following passage and answer the questions given below:

In an Office three employees Jayant, Sonia and Olivia process a calculation in an excel form. Probability that Jayant, Sonia, Olivia process the calculation respectively is 50%, 20% and 30% . Jayant has a probability of making a mistake as 0.06 , Sonia has probability 0.04 to make a mistake and Olivia has a probability 0.03 . Based on the above information, answer the following questions.

(i)Find the probability that Sonia processed the calculation and committed a mistake.

(ii)Find the total probability of committing a mistake in processing the calculation.

(iii)The boss wants to do a good check. During check, he selects a calculation form at random from all the days. If the form selected at random has a mistake, find the probability that the form is not processed by Jayant.

3 Tiki has started late for college. She is running towards Laboni bus-stop. To reach college she has to change buses from either Hidco Crossing or Dharmatala. For that she would take either bus A or bus B.Probability of getting into bus A, B are 3/7 , 4/7 . If she gets on bus A coming from Karunamoyee , she would get bus 1 or 2 from Hidco crossing. Probability of getting bus 1 from Hidco crossing is $\frac{2}{5}$, probability of getting bus 2 from Hidco crossing is $\frac{3}{5}$. If she gets on bus B from Quality crossing and gets bus 1 or bus 3 from Dharmatala. Probability of getting bus 1 from Dharmatala is $\frac{1}{3}$, probability of getting bus 3 from Dharmatala is 2/3

i)Tiki reaches college by bus 1. What is the probability that she caught bus B?

ii)What is the probability that she reaches college by bus 2?

4 Rubiya, Thaksh, Shanteri, and Lilly entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both. Spinners have numbers 1 to 9 on those: Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:

- Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player!
- You win a photo frame if Spinner A lands on a value greater than 4.
- You win an earplug if you get even in spinner A or odd in spinner B.

i)Thaksh spun both the spinners, A and B in one of his turns. What is the probability that Thaksh wins a music player in that turn?

ii) Lilly spun spinner A in one of her turns. What is the probability that the number she got is even given that it is a multiple of 3?

iii) Rubiya spun both the spinners. What is the probability that she wins a photo frame only?

Or

As Shanteri steps up to the screen, the game administrator reveals that she would see either Spinner A or Spinner B for her turn, the probability of seeing Spinner A on the screen is 65%, while that of Spinner B is 35%. What is the probability that Shanteri wins an earplug?

5. A doctor is to visit a patient. From past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

Based on the above information, answer the following questions.

(i) When the doctor arrives late, what is the probability that he comes by metro?

(ii) When the doctor arrives late, what is the probability that he comes by cab?

(iii) When the doctor arrives late, what is the probability that he comes by bike?

(iv)When the doctor arrives late, what is the probability that he comes by other means of transport?

<u>SOLUTIONS OF 1 MARK QUESTIONS</u>

1)(d) 2)(b) 3) (c) 4)(d) 5)(c) 6)(c) 7)(d) 8)(d) 9)(c) 10)(d) 11)(c) 12)(c)

13)(c) 14)(d) 15)(a) 16)(d) 17)(b) 18)(c) 19)(c) 20)(c)

SOLUTIONS 2 MARKS QUESTIONS

1 $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{7}{10}}{\frac{4}{5}} = \frac{35}{40} = \frac{7}{8}$$

2 $$P(Both\ balls\ are\ white) = \frac{{}^3C_2}{{}^9C_2} = \frac{\frac{3!}{2!1!}}{\frac{9!}{2!7!}} = \frac{1}{12}$$

3 $P(A \cap B) \neq P(A)\,.P(B)$ Hence, A and B are not independent events.

4 $P(A/B) = P(A \cap B)/P(B) = 0/0.4 = 0$

5 $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

6 Required probability $= P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}).P\left(\frac{\bar{B}}{\bar{A}}\right) = 1 - \frac{195}{200} \times \frac{194}{199} = \frac{197}{3980}$

7 $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{\frac{1}{6}}{\frac{1}{3}}=\frac{1}{2}$ & $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{6}}{\frac{1}{2}}=\frac{1}{3}$

8 Independent

9 The required probability $=\frac{1}{5}$

10 17/105

SOLUTION OF 3 MARKS QUESTIONS

1 Let X denote the number of milk chocolate drawn.

X	P(X)
0	$\frac{4}{6}\times\frac{3}{5}=\frac{12}{30}$
1	$(\frac{2}{6}\times\frac{4}{5})\times 2=\frac{16}{30}$
2	$\frac{2}{6}\times\frac{1}{5}=\frac{2}{30}$

Most likely outcome is getting one chocolate of each type.

2 using Bayes' theorem, we have $P\left(\frac{E_2}{A}\right)=\frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}=\frac{\frac{1}{2}\times\frac{5}{11}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}}=\frac{35}{68}$

3 $(i)\sum P(X_i)=1 \ \Rightarrow k=1$ (ii)P(1≤ $X<3)=\frac{5}{6}$ $(iii)\ E(X)=\sum p_i.x_i=\frac{5}{3}$

4 Taking events as usual $P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}=\frac{\frac{1}{2}\times\frac{5}{100}}{\frac{1}{2}\times\frac{5}{100}+\frac{1}{2}\times\frac{1}{400}}=\frac{20}{21}$

5 (i) $\sum p_i=1 \ \Rightarrow \ k=\frac{1}{10}$ (ii) P($X<3)=\frac{3}{10}$ (iii) P($X>6)=\frac{17}{100}$

6 Let $E_1=$ Selecting bag I, $E_2=$Selecting bag II, $A=$ Drawing a red ball

Required probability= P(A)=$P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)=\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{2}{6}=\frac{19}{42}$

7 Let $E_1=$ A person with blood group O is selected, $E_2=$ A person with other blood groups is selected, $A=$ A left handed person is selected

Hence, required probability is $P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}=\frac{\frac{3}{10}\times\frac{6}{100}}{\frac{3}{10}\times\frac{6}{100}+\frac{7}{10}\times\frac{10}{100}}=\frac{9}{44}$

8 Let E= A fails, F=B fails, then P(A fails/B has failed)=$P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}=\frac{0.15}{0.30}=\frac{1}{2}$ and

P(A fails alone)=$P(E\cap\overline{F})=P(E)-P(E\cap F)=0.2-0.15=0.05$

9 ∴ Required probability= $P(A\cup B\cup C)=1-P(\overline{A\cup B\cup C})$

$=1-P(\overline{A}\cap\overline{B}\cap\overline{C})=1-P(\overline{A}).P(\overline{B}).P(\overline{C})=1-(0.86)(0.90)(0.95)=0.2647$

10 E_1=A boy is chosen from the class, E_2=A girl is chosen from the class

A=the students get first class marks

P(A)=$P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)=\frac{2}{3}\times 0.28+\frac{1}{3}\times 0.25=0.27$

SOLUTIONS OF 5 MARKS QUESTIONS

1 Let A be the event of drawing a cinnamon candy, B_1 be the event of selecting box B_1, B_2 be the event of selecting box B_2 .

P(A) = P(A ∩ B_1) + P(A ∩ B_2)= P(A|B_1) P(B_1) + P(A|B_2) P(B_2)= $(\frac{7}{11})$ $(\frac{1}{3})$ + $(\frac{3}{11})$ $(\frac{2}{3})$= $\frac{13}{33}$.

2 Let A, B and C be the events that notebooks are provided by A, B and C respectively.

Let D be the event that notebooks are defective

P(A|D) =$\frac{P(D|A)P(A)}{P(D|A)P(A)+P(D|B)P(B)+P(D|C)P(C)}$ = $\frac{0.05*0.25}{0.05*0.25+0.04*0.35+0.02*0.4}$ =$\frac{125}{345}$ = $\frac{25}{69}$.

3 Let A be the event that the machine produces 2 acceptable items.

Also let B_1 represent the event of correct set up and B_2represent the event of incorrect setup.

Therefore P(B_1|A) =$\frac{P(B_1)\,P(A|B_1)}{P(B_1)\,P(A|B_1)\ +P(B_2)P(A|B_2)}$ = $\frac{0.8\times 0.9}{0.8\times 0.9\ +0.2\times 0.4}$ =$\frac{72}{80}$ = $\frac{9}{10}$.

4 Let E_1: a student is residing in hostel E_2: a student is a day scholar & E: a student attains A grade Then P(E_1/ E) =$\frac{P(E\,/\,E_1)\,P(E_1)}{P(E\,/\,E_1)\,P(E_1)+P(E\,/\,E_2)P(E_2)}$=$\frac{0.6\,X\,0.3}{0.6\,X\ 0.3+0.4\,X\,0.2}$=$\frac{18}{26}$=$\frac{9}{13}$

5 Let E_1: first draw gives a red ball, E_2: first draw gives a black ball

Let A: The second draw gives a red ball.

Required probability = P(A) = P(A/E_1)P(E_1)+P(A/E_2)P(E_2) = $\frac{7}{12}$ x $\frac{1}{2}$ + $\frac{5}{12}$ x $\frac{1}{2}$ = $\frac{1}{2}$

6 If X is the number of aces drawn ∴ The Probability Distribution of X is given by

X	0	1	2
P(X)	188/221	32/221	1/221

Mean = 34/221

7 Let E_1 , E_2 & E_3 Bag I,II &III is selected respectively and A = A black ball and a red ball is drawn, Then Using Baye's Theorem $P\left(\frac{E_1}{A}\right)=\frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{2}{21}+\frac{1}{3}\times\frac{2}{11}}=\frac{\frac{1}{5}}{\frac{1}{5}+\frac{2}{21}+\frac{2}{11}}=\frac{231}{551}$

8 Let E_1: Bolt is manufactured by machine 'X', E_2: Bolt is manufactured by machine Y and E_3: Bolt is manufactured by machine Z,

Required probability = P (E_3/E) = $\frac{1/6\times 1/100}{1/6\times 1/100+1/3\times 3/200+½\times 2/100} = \frac{1/6}{1/6+½+1}$ = 1/10

9 Hence, the probability distribution of x is

X	0	1	2
P(X)	9/16	3/8	1/16

10 Let E = the event that the man reports that six occurs in the throwing of a dice.

$S_1 =$ event of getting six, $S_2 =$ event of getting no six

Req. Prob. = $P(S_1/E)=P(S_1)P(E/S_1)/(P(S_1)P(E/S_1)+P(S_2)P(E/S_2)$ = 3/8

SOLUTION OF CASE BASED QUESTIONS

1 i) The probability that the shot is hit by B only=$(1-\frac{4}{5})\frac{3}{4}(1-\frac{2}{3})=\frac{3}{60}=\frac{1}{20}$

ii)If A,B,C all try , the probability that it was hit by none is $(1-\frac{4}{5})(1-\frac{3}{4})(1-\frac{2}{3})=\frac{1}{60}$

iii) The req probability= $(\frac{4}{5})(\frac{3}{4})(1-\frac{2}{3}) + (1-\frac{4}{5})(\frac{3}{4})(\frac{2}{3}) + (\frac{4}{5})(1-\frac{3}{4})(\frac{2}{3}) = \frac{26}{60} = \frac{13}{30}$

2 (i)The probability = $\frac{0.04\,X\,\frac{20}{100}}{0.06\,X\,\frac{50}{100}+0.04\,X\,\frac{20}{100}+0.03\,X\,\frac{30}{100}} = \frac{8}{47}$

(ii) Required Probability= $P\left(\frac{D}{A}\right)P(A)+P\left(\frac{D}{B}\right)P(B)+P\left(\frac{D}{C}\right)P(C)$= .047

(iii) the probability = 1-probability that the form has a mistake and is processed by Jayant = 1- $\frac{0.06\,X\,\frac{50}{100}}{0.06\,X\,\frac{50}{100}+0.04\,X\,\frac{20}{100}+0.03\,X\,\frac{30}{100}} = \frac{17}{47}$

3 i)If Tiki reaches college by bus 1 , the probability that she caught bus B is

$= \frac{\frac{4}{7}X\frac{1}{3}}{\frac{3}{7}X\frac{2}{5}+\frac{4}{7}X\frac{1}{3}} = \frac{20}{38} = \frac{10}{19}$

ii)The probability that she reaches college by bus 2

$=\frac{4}{7}\times\frac{3}{5}=\frac{12}{35}$

4 (i) $\frac{1}{81}$ ii) $\frac{1}{9}$ iii) $\frac{5}{9}$ Or $\frac{107}{180}$

5 (i) $\frac{2}{7}$ (ii) $\frac{5}{14}$ (iii) $\frac{1}{6}$ (iv) $\frac{4}{21}$

SAMPLE QUESTION PAPERS WITH BLUE PRINTS AND MARKING SCHEME

KENDRIYA VIDYALAYA SANGATHAN

Blue-Print

Sample Paper 1

Class-XII **Subject-Mathematics (041)**

Units and Chapters	MCQ	Assertion/Reasoning	2 Marks	3 Marks	5 Marks	4 marks Case Based	Total
1. Relation & Function & I.T. Functions	1	-	1	-	1	-	8(3)
2. Matrices and Determinants	5	1	-	-	-	1	10(6)
3. Calculus	5	-	2	4	2	1	35(14)
4. Vector Algebra & 3-D Geometry	4	1	2	-	1	-	14(8)
5. Linear programming Problem	2	-	-	1	-	-	5(4)
6. Probability	1	-	-	1	-	1	8(3)
Total	**18**	**2**	**5**	**6**	**4**	**3**	**80(38)**

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 1

CLASS – XII | **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) | **TIME – 03 HOURS**

General Instructions:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question paper contains **- five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
3. **Section A** has **18 MCQ's and 02 Assertion-Reason** based questions of 1 mark each.
4. **Section B** has **5 Very Short Answer (VSA)-type** questions of 2 marks each.
5. **Section C** has **6 Short Answer (SA)-type** questions of 3 marks each.
6. **Section D** has **4 Long Answer (LA)-type** questions of 5 marks each.
7. **Section E** has **3 source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
9. Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. let Z denote the set of integers , then the function f: Z→ $Z \; defined \; as \; f(x) = x^3$-1 is

   (A) both on-one and onto (B) one-one but not onto

   (C) onto but not one-one (D) neither one-one nor onto

2. if A = $[a_{ij}]$ $is \; a \; diagonal \; matrix, then \; which \; of \; the \; following \; is \; true$?

   (A) $a_{ij} = \begin{cases} 0 \; if \; i = j \\ 1 \; if \; i \neq j \end{cases}$ (B) $a_{ij} = 1$, $\forall \; i, j$

   (C) $a_{ij} = 0 \; if \; i \neq j$ & $a_{ij} \neq 0 \; if \; i = j$ (D)$a_{ij} = 1$, $\forall \; i, j$

3. Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ be a square matrix such that $adj \; A = A$. Then $(p + q + r + s) \; is \; equal \; to$

   (A) 2p (B) 2q (C) 2r (D)0

4. If A and B are symmetric matrix of the same order , then $(AB' - BA')$ is a

   (A) Skew Symmetric matrix (B)Null matrix (C) Symmetric matrix (D) None of these

5. If $x, y \in R, then\ the\ determinant\ \Delta = \begin{vmatrix} cosx & -sinx & 1 \\ sinx & cosx & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval

(A)$[-\sqrt{2}, \sqrt{2}]$ (B)$[-1,1]$ (C)$[-\sqrt{2}, 1]$ (D)$[-1, \sqrt{2}]$

6. The area of the triangle with vertices (-3,0), (3,0) and (0,k) is 9 square units. The value of k will be

(A) 9 (B)3 (C) -9 (D) 6

7. The number of points at which the function $f(x) = \frac{1}{x-[x]}$ is not continuous is

(A) 1 (B)2 (C) 3 (D) None of these

8. Differential coefficient of $sec(tan^{-1}x)$ with respect to x is

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\frac{x}{1+x^2}$ (C)$x\sqrt{1+x^2}$ (D)$\frac{1}{\sqrt{1+x^2}}$

9. The function $f(x) = tanx - x$

(A) Always increases (B)Always decreases

(C)Never increases (D)Sometimes increases and sometime decreases

10. $\int_{a+c}^{b+c} f(x)dx\ \ is\ equal\ to$

(A) $\int_a^b f(x-c)dx$ (B) $\int_a^b f(x+c)dx$ (C)$\int_a^b f(x)dx$ (D)$\int_{a-c}^{b-c} f(x)dx$

11. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents a family of

(A) Straight lines (B)Circles (C)Parabolas (D)Ellipses

12. The angle between the vectors $\hat{\imath} - \hat{\jmath}\ and\ \hat{\jmath} - \hat{k}$ is

(A)$\frac{\pi}{3}$ (B)$\frac{2\pi}{3}$ (C)$\frac{-\pi}{3}$ (D)$\frac{5\pi}{6}$

13. If $|\vec{a}|$ = 8, $|\vec{b}|$ = 3 and $|\vec{a} \times \vec{b}|$ = 12 then $\vec{a} \cdot \vec{b}$ is

(A) $6\sqrt{3}$ (B)$8\sqrt{3}$ (C)$12\sqrt{3}$ (D)None of these

14. The equation of x-axis in space are

(A)$x = 0,\ y = 0$ (B)$x = 0,\ z = 0$ (C)$x = 0$ (D)$y = 0, z = 0$

15. If a line makes equal acute angles with coordinate axes , then direction cosines of the line is

(A)1,1,1 (B)$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (C)$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ (D)$\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$

16. The corner point of the feasible region determined by the system of linear constraints are (0,10), (5,5), (15,15) and (0,20) . let Z = px+ qy , where p,q >0.

    Condition on p and q so that the maximum of z occurs at both the points (15,15) and (0,20) is

    (A)p = q (B)p = 2q (C)q = 2p (D)q = 3p

17. the linear function which is to be optimized in the Linear Programming Problem is known as

    (A)constraints (B)optimal solution (C)objective function (D)decision variables

18. Let A and B be two events such that P(A) = 0.6 , P(B) = 0.2 and $P\left(\frac{A}{B}\right) = 0.5$ then $P(\frac{A'}{B'})$ equals

    (A)1/10 (B)3/10 (C)3/8 (D)6/7

In Questions number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true

19. Assertion (A) : Every scalar matrix is a diagonal matrix

    Reason(R) : In a diagonal matrix , all the diagonal elements are zero.

20. Assertion (A) : Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

    Reason(R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$

SECTION B

This section comprises very short answer(VSA) type questions of 2 marks each.

21. Find the value of tan^{-1}(-1) + $sin^{-1}\left(-\frac{1}{2}\right) + cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.

22. (a) for what value of μ is the function defined by

$$f(x) = \begin{cases} \mu(x^2 - 2x) & if\ x < 0 \\ x + 1 & if\ x \geq 0 \end{cases}$$

    Continuous at x= 0 ?

    OR (b) Find $\frac{dy}{dx}$ $if\ x = a(\theta - sin\theta)\ and\ y = a(1 + cos\theta)$

23. (a) Evaluate: $\int_0^{\pi/2} \frac{sinx}{1+cos^2x}\, dx$

OR (b) Evaluate: $\int_0^4 |x-1|\, dx$

24. Find the area of the parallelogram whose diagonals are $4\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $-2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

25. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) If $f(x) = \begin{cases} 3ax + b & if x < 1 \\ 11 & if x = 1 \\ 5ax - 2b, & if x > 1 \end{cases}$ is continuous at x = 1, find a and b

OR (b) If $y = x^{sinx} + (sinx)^{cosx}$ $then\ find \frac{dy}{dx}$

27. (a) It is given that $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a ', hence obtain all other points where the given function $f(x)$ attains local maximum values.

OR (b) A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

28. Find : $\int \frac{x^3}{x^4+3x^2+2}\, dx$

29. (a) Find: $\int e^{tan^{-1}x} \left(\frac{1+x+x^2}{1+x^2}\right) dx$

OR (b) Evaluate : $\int_0^{\pi/4} \log(1 + tanx)\, dx$

30. Solve the linear programming problem graphically

Maximize $Z = 510\,x + 675\,y$

subject to the constraints:

$x + y \leq 300$; $2x + 3y \leq 720$; $x \geq 0, y \geq 0$

31. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive.

SECTION D

This section comprises long answer(LA) type questions of 5 marks each.

32. Show that the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1. Also find the equivalence class [3]

33. (a) Find the critical points and hence find absolute maximum and minimum values of a function f given by $f(x) = 12x^{4/3} - 6x^{\frac{1}{3}},\ x \epsilon [-1,1]$.

OR (b) A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

34. Using integration Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}\, y$ and the circle $x^2 + y^2 = 4$.

35. (a) Find the distance of a point (2,4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

OR (b) Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study -1

1. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and unskilled themselves.

A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below:

$$P(X = x) = \begin{cases} kx^2, & for\ x = 1,2,3 \\ 2kx, & for\ x = 4,5,6 \\ 0, & otherwise \end{cases}$$

Where x denotes the number of hours.

Based on the above information, answer the following questions:

(i)	Express the probability distribution given above in the form of a probability distribution table.	1
(ii)	Find the value of k.	1
(iii)	(a) Find the mean number of hours spent by the student. **OR** (b) Find $P(1<X<6)$.	2

Case Study - 2

2. It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be $r\%$ per annum.

Based on the above information answer the following questions.

iii)	At what interest rate will Rs. 100 double itself in 10 years.	2
iv)	(a) How much will Rs. 1000 be worth at 5% interest after 10 years?	2
	OR (b) If the interest is compounded continuously at 5% per annum, in how many years will Rs. 100 double itself?	2
	[Use $\ln 2 = 0.6931; e^{0.5} = 1.648$]	

Case Study -3

3. The monthly income of two sisters Ojaswini and Tejaswini are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. Each sister saves ₹ 15,000 per month.

a)	Write the information in the matrix equation.	1
b)	Is the system of equation consistent?	1
c)	Find the monthly income of both sisters by matrix method.	2
	OR Find the monthly expenditure of both sisters by matrix method.	

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE PAPER 1

MARKING SCHEME

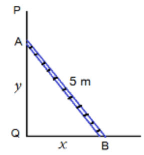
CLASS – XII **SUB : MATHEMATICS (041)**

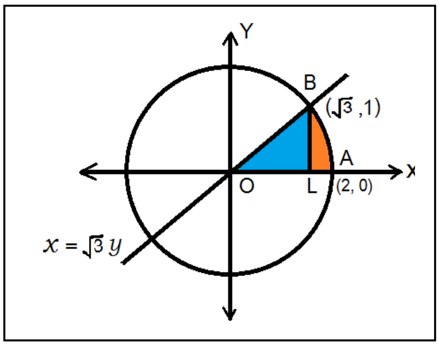
MCQ ANSWERS

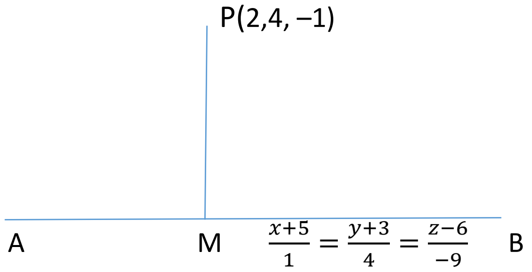
1.(A) 2.(C) 3.(A) 4.(A) 5.(A) 6.(B) 7.(D) 8.(A) 9.(B) 10.(C)

11.(C) 12.(B) 13.(C) 14.(D) 15.(B) 16.(D) 17.(C) 18.(C) 19.(C) 20.(A)

Q.NO	ANSWER	VALUE POINTS
21)	For each value of tan^{-1}(-1), $sin^{-1}\left(-\frac{1}{2}\right)$ $and\ cos^{-1}$ $(\frac{-1}{\sqrt{2}})$ For final correct answer	3x$\frac{1}{2}$ $\frac{1}{2}$
22)	(a) LHL = 0, RHL =1 = f(0) Equating and finding the value of μ as no such value of μ exists **OR(b)** Finding the values of $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ Finding $\frac{dy}{dx}$	1 1 1.5 0.5
23)	(a) Putting cosx = t so that -sinxdx = dt and limits of t will be 1 to 0 $\therefore I = \int_1^0 \frac{-dt}{1+t^2}$ = $\frac{\pi}{4}$ after simplification **OR** Writing I = $\int_0^4 \|x-1\|\ dx = \int_0^1 \|x-1\|\ dx + \int_1^4 \|x-1\|\ dx$ = $\int_0^1 -(x-1)dx + \int_1^4 (x-1)\ dx$ For correct answer	1 1 1 1
24)	Finding adjacent sides of the parallelogram as vectors $\vec{a}$ and $\vec{b}$ Finding area of the parallelogram using $\|\vec{a} \times \vec{b}\|$	1 1
25)	$\|\vec{a}+\vec{b}+\vec{c}\|^2 = (\vec{a}+\vec{b}+\vec{c}) . (\vec{a}+\vec{b}+\vec{c})$ and using $\vec{a}, \vec{b}, and \vec{c}$ as a unit vector For correct answer -3/2	1 1
26)	LHL = 3a + b, RHL = 5a – 2b and f(1) = 11 Equating all and getting the values of a and b as 3 and 2 respectively	1.5 1.5
27)	(a) Finding $f'(x)$ $and\ equating\ f'(a) to\ 0\ to\ find\ the\ value\ of\ a = 120$ Then $f(x) = x^4 - 62x^2 + 120x + 9$ and finding other points where the given function $f(x)$ attains local maximum values.	1.5 1.5

	OR (b) Let PQ be the wall. At certain time t, let AB be the position of the ladder such that $\text{QB} = x$ and $\text{AQ} = y$  Then $x^2 + y^2 = 5^2 \quad \dots\dots..(1)$ Diff. both sides with respect to t, we get $\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\Rightarrow \frac{dy}{dt} = -\frac{2x}{2y} \times \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = -\frac{x}{y} \times 2 \text{ cm/s} \Rightarrow \frac{dy}{dt} = -\frac{2x}{y} \text{ cm/s} \quad \dots\dots..(2)$ When $x = 4\,m$, then from $(1), y = \sqrt{5^2 - 4^2} = 3 \text{ m}$ Putting these values of x and y in equation (2), we find $\frac{dy}{dt} = -\frac{2 \times 4\,m}{3\,m} \text{ cm/s} = -\frac{8}{3} \text{ cm/s}$ Thus, the rate of decrease of height on the wall is $\frac{8}{3}$ cm/s	1.5 1.5
28)	Let $I = \int \frac{x^3}{x^4+3x^2+2}\,dx$ Putting $x^2 = t$ *so that* $2xdx = dt$ *and* $\therefore I = \int \frac{t.dt/2}{t^2+3t+2}$ Finding correct integral by partial fraction or any other method	1.5 1.5
29)	(a) *let* $I = \int e^{tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ Putting $x = \tan t$ *so that* $dx = sec^2 t\, dt$ and $\therefore I = \int e^t\left(\frac{1+tant+tan^2 t}{1+tan^2 t}\right)sec^2 t\, dt$ $= \int e^t(tant + sec^2 t)\, dt = e^t(tant) + \text{C}$ **OR (b)***taking* $\int_0^{\frac{\pi}{4}} \log(1 + tanx)\; dx$ as Integral I and applying the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ Adding both integral and finding the value of I as $\frac{\pi}{8}\log 2$	1.5 1.5 1.5 1.5
30)	For correct feasible region For corner point, corresponding value of Z and finding solution	1.5 1.5
31)	Let E1 = Event that the person has a disease. E2 = Event that the person is healthy. $\therefore P(E_1) = 0.1\% = \frac{0.1}{100} = \frac{1}{1000}$ *and* $P(E_2) = 1 - \frac{1}{1000} = \frac{999}{1000}$ A = Event that the test result is positive. $\therefore P(A\|E_1) = 99\ \% = \frac{99}{100}\quad P(A\|E_2) = 0.5\% = \frac{0.5}{100} = \frac{5}{1000}$ $\therefore$ By Bayes' Theorem, $P(E_1\|\,A) = \frac{P\,(E_1)\,.P\,(A\|E_1)}{P\,(E_1).P\,(E_1)+P\,(E_2).P\,(E_2)} = \frac{\frac{1}{1000}\times\frac{99}{100}}{\frac{1}{1000}\times\frac{99}{100}+\frac{999}{1000}\times\frac{5}{1000}} = \frac{22}{133}$	1 2

32)	For showing relation reflexive For showing relation symmetric For showing relation transitive Finding the set of all elements related to 1 and [3]	1 1 1.5 1.5
33)	(a) Critical points are the points where $f'(x) = 0$ or $f'(x)$ does not exist. $after\ solving\ critical\ points\ are\ x = 0\ and\ x = \frac{1}{8}$ $finding\ the\ value\ of\ f(x)$ at critical and boundary points and deciding absolute maximum is 18 which occurs at x = -1 and absolute minimum is -9/4 which occurs at x = 1/8 **OR** (b) Let x = side of the square to be cut-off So that Volume of the box , V= (45-2x)(24-2x)x Taking first derivative of Volume to zero and finding the value of critical point x = 5cm, 18cm and rejecting 18 cm , 2^{nd} derivative of V = (-)ve so Volume is maximum at x = 5 cm Thus Side of the square to be cut-off and Maximum volume = 2450 cm^3.	1 1 3 1.5 2 1.5
34)	For the points of intersection, we solve equations of given circles The point of intersection are $(\sqrt{3}, 1)$ and $(-\sqrt{3}, -1)$ The rough sketch of the given curve is as follows:  The required area = Area of the shaded region OBALO = Area of OBLO + Area of BLAB $= \int_0^{\sqrt{3}} (y \text{ of line})\ dx + \int_{\sqrt{3}}^{2} (y \text{ of circle})\ dx$ $= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}\ dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\ dx$	1 1.5 1.5

<table>
<tr><td></td><td>For integrating and finding the area $\frac{\pi}{3}$ sq. units</td><td>1</td></tr>
<tr><td>35)</td><td>(a)

Let M be the foot of the perpendicular from given point to given line
Taking the general point $(\mu-5, 4\mu-3, -9\mu+6)$ on the line AB and taking this is the coordinate of M.
The d. r. of PM = $\mu-7, 4\mu-7, -9\mu+7$
d.r. of AB = 1,4,-9
since AB⊥PM
$\therefore 1(\mu-7)+4(4\mu-7)-9(-9\mu+7)=0$
$\mu=1$
∴ Coordinate of M = (-4,1,-2) and so PM = $\sqrt{46}$ units
OR
(b) Let the d.r. of the required line is a, b, c
Since required line is perpendicular to given two line so
$3a-16b+7c=0$ and $3a+8b-5c=0$
Solving and getting the direction ratio
Getting the equation of the required line</td><td>1.5
1.5
1
1
2
1.5
1.5</td></tr>
<tr><td>36)</td><td>(i)
<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>P(X)</td><td>k</td><td>4k</td><td>9k</td><td>8k</td><td>10k</td><td>12k</td></tr></table>(ii) $\sum P(X)=1$
$\Rightarrow k=\frac{1}{44}$
(iii) (a) Mean = $\sum XP(X)=\frac{190}{44}$
OR (b) P(1<X<6) =P(2)+P(3)+P(4)+P(5) = 31/144</td><td>1
1
2
2</td></tr>
<tr><td>37)</td><td>Now, as per question
$\frac{dP}{dt}=r\%$ of P OR $\int\frac{1}{P}dP=\frac{r}{100}\int dt \Rightarrow \log P=\frac{r}{100}t+C \ldots\ldots\ldots\ldots(1)$
Given that when $t=0$ then $\log P_0 = C$ After solving $\log\frac{P}{P_0}=\frac{r}{100}t$</td><td>1</td></tr>
</table>

	(i) when $t = 10$ then $\mathrm{P} = 2P_0$ so $\log\frac{2P_0}{P_0} = \frac{r}{100} \times 10 \quad \therefore r = 6.931$ (ii) (a)$\log\frac{\mathrm{P}}{P_0} = \frac{r}{100}t \Rightarrow \log\frac{\mathrm{P}}{1000} = \frac{5}{100} \times 10 \Rightarrow \frac{\mathrm{P}}{1000} = e^{1/2} \Rightarrow P = Rs. 1648$ $(b) \log\frac{\mathrm{P}}{P_0} = \frac{r}{100}t \Rightarrow \log\frac{200}{100} = \frac{5}{100}t \Rightarrow \log 2 = \frac{t}{20}t = 20\ log2 = 13.86\ years$	1 2 2
38)	(i) Let the monthly income of Ojaswini and Tejaswini are 3x and 4x and their expenditures are 5y and 7y. So the equations are $3x - 5y = 15000\ and\ 4x - 7y = 15000$ In matrix form $\begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$ or $AX = B$ (ii) $\|A\| = -1 \neq 0$ so the system is consistent (iii) Solving by matrix method and getting x = 30000 and y = 15000 $(a) \therefore Monthly\ income\ of$ Ojaswini and Tejaswini are ₹90,000 and ₹ 1,20,000 **OR** (b) Monthly expenditure of Ojaswini and Tejaswini are ₹75,000 and ₹ 1,05,000	1 1 2 2 2

KENDRIYA VIDYALAYA SANGATHAN

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Sample Paper 2

Class-XII **Subject-Mathematics**

S.No./ Unit	Topics	MCQ (1M)	ARQ (1M)	VSA (2M)	SA (3M)	LA (5M)	CASE BASED (4M)	TOTAL
1	RELATIONS AND FUNCTIONS	-		-	-	1		**8(3)**
	INVERSE TRIGONOMETRIC FUNCTIONS	1		1	-	-		
2	MATRICES	2		-	-	1		**10(6)**
	DETERMINANTS	-		1	-	-		
3	CONTINUITY & DIFFERENTIABILITY	3		2(1)	-	-		**35(15)**
	APPLICATION OF DERIVATIVES	-		1		-	**2**	
	INTEGRALS	2		-	-	-		
	APPLICATION OF INTEGRALS	-		-	1	1		
	DIFFERENTIAL EQUATIONS	2		-	1	-		
4	VECTORS	2	1	2	-	-		**14(8)**
	3-DIMENTIONAL GEOMETRY	2		-	-	1		
5	LINEAR PROGRMMING	2		-	1	-		**5(3)**
6	PROBABILITY		1	-	1	-	**1**	**8(3)**
	TOTAL	**18**	**2**	**5**	**6**	**4**	**3**	**80(38)**

**Number written in the bracket is the number of questions.

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 2

CLASS – XII **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

GENERAL INSTRUCTION:

1. **This question paper contains five sections A,B,C,D and E . Each section is compulsory. However , there are internal choices in some questions.**
2. **Section A has 18 MCQS and 02 Assertion-Reason based questions of 1 mark each.**
3. **Section B has 5 very short answer (VSA)-type questions of 2 marks each.**
4. **Section C has 6 short answer (SA) questions of 3 marks each.**
5. **Section D has 4 long answer (LA) type questions of 5 marks each.**
6. **Section E has 3 source based /case based/integrated units of assessment (4 marks each) with sub parts.**

Section – A

Q.1) If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix}=\begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, the value of x is

a)3 b) ± 3 c) ± 6 c)6

Q.2) The domain of $\cos^{-1}(3x-2)$is

(a) $(\frac{1}{3},2)$ b)$[\frac{1}{3},1]$ c)[-1,1] d)$[\frac{-1}{3}, \frac{1}{3}]$

Q.3) If $ax+\frac{b}{x}\geq c$ for all positive x where a,b >0

a)ab $<c^2/4$ b)ab $\geq c^2/4$ c)ab ≥ c/4 d)none of these

Q.4) Let A be a square matrix of order 3 such that adj(4A)=λ(adj A);Then the value of λ is

a) 4 b)8 c)12 d) 16

Q.5) The area of a triangle with vertices (-3,0),(3,0) and (0,k) is 9 sq unit .The value of k is

a) 9 b) 3 c) -3 d) 6

Q.6) The set of points of discontinuity of the function f(x)=2x-[x] is a)Q   b)R   c)Z   d)W

Q.7)If the function is f(x)= $\begin{cases} \frac{x^3-a^3}{x-a} & , x \neq a \\ b, & x = a \end{cases}$ is continuous at x=a then b is equal to

a) a^2 b)$2a^2$ c)$3a^2$ d)$4a^2$

Q.8)If y=$\tan^{-1}\left(\frac{sinx+cosx}{cosx-sinx}\right)$, then $\frac{dy}{dx}$ is equal to

a)$\frac{1}{2}$ b)0   c)1   d)None of these

Q.9) If x=at^2 ; y=2at,then $\frac{d^2y}{dx^2}$ =

a)$\frac{-1}{t^2}$ b)$\frac{1}{2at^3}$ c)$\frac{-1}{t^3}$ d)$\frac{-1}{2at^3}$

Q.10)Degree of the differential equation $\frac{d^2y}{dx^2}+\sqrt{\frac{dy}{dx}}$=0 is

a)1   b)2   c)3   d)4

Q.11)The Integrating factor of the differential equation (x log x)$\frac{dy}{dx}$ +y=2 log x ,is given by

a)log(log x)   b)e^x c) log x   d)x

Q.12)$\int 2^{x+2}\,dx$ is equal to

a)2^{x+2} +c   b) 2^{x+2}log2 +c   c) $(\frac{2^{x+2}}{log2})$+c   d)$\frac{2.2^x}{log2}$ +c

Q.13)$\int_0^{\pi/2} log\ \left(\frac{4+3sin\ x}{4+3cosx}\right) dx\ is$

a)2   b)$\frac{3}{4}$ c)0   d)-2

Q.14) If the diagonals of a parallelogram are represented by the vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}+3\hat{j}-4\hat{k}$, then its area in square unit is:

a)$5(3)^{1/2}$ b)$6(3)^{1/2}$ c) $(42)^{1/2}$ d) $(28)^{1/2}$

Q.15) Objective function of a L.P.P is

a) a constraint   b) a function to be optimize   c)a relation between the variable   d) none of these

Q.16)if the constraint in a LPP are changed

a)the problem is to be re-evaluated b)solution is not defined

c) the objective function has to be modified d) the change in constraint is ignored.

Q17) If α is the angle between two vectors $\vec{a}$ and $\vec{b}$ then$\vec{a}.\vec{b}$ ≥ 0 only when

(a) 0 < α < π/2 (b) 0 ≤ α ≤ π/2 (c) 0 < α < π (d) 0 ≤ α ≤ π

Q 18)If a line make angles a, b, c with the co-ordinate axes respectively, then, cos(2a)+cos(2b)+cos(2c) = ?

(a) 2 (b) -1 (c) 1 (d)- 2

Q.19) **Assertion(A):** 20 persons are sitting in a row.Two of these person are not at random. The probability that the two selected person are not together is 0.9.

Reason(R): If $\overline{A}$ denotes the negation of an event A, then P($\overline{A}$)=1-P(A)

a) Both A and R are true and R is the correct explanation of A
b) Both A and R are true but R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

Q.20) **Assertion(A):** if the vectors $\overrightarrow{AB}$ =3 $\hat{\imath}$ + 4$\hat{\jmath}$ and $\overrightarrow{AC}$= 5$\hat{\imath}$ - 2$\hat{\jmath}$ + 4$\hat{k}$ are sides of a triangle ABC then the length of the median AD through A is $\sqrt{33}$.

Reason(R): if AD is the median of triangle ABC then $\overrightarrow{AB}$ + $\overrightarrow{AC}$ = 2$\overrightarrow{AD}$

a) Both A and R are true but R is the correct explanation of A
b) Both A and R are true but R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

Section -B

Q.21)Find the value of $\cos^{-1}(\cos\frac{5\pi}{3})+\sin^{-1}(\sin\frac{5\pi}{3})$

Or Find the value of $\sin^{-1}(\cos\frac{43\pi}{5})$

Q.22) Find the dimensions of the rectangle with perimeter 36 cm. which will generate maximum volume when resolved about one of its sides

Or If f(x)=2x+cos x + b; b∈R, find the interval for which f(x) is strictly increasing.

Q.23) If $e^y(x+y)=1$ then show that $\frac{d^2y}{dx^2}=(\frac{dy}{dx})^2$

Q.24) If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=7$, then find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Or If the vertices A, B, C of a triangle ABC are (1,2,3), (-1,0,0), (0,1,2) respectively, find ∠ABC

Q.25) Find the vector and Cartesian equation of the line through the point (1,2, -4) and perpendicular to the lines $\vec{r}=(3\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-10\hat{j}+7\hat{k})$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$

Section-C

Q.26) Find a matrix A such that 2A-3B+5C=0

Where B=$\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and C =$\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 0 \end{bmatrix}$

Or If A = $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$; then find the value of A^2-5A

Q.27) The volume of a cube is increasing at a rate of 9 cubic centimetre per second. How fast is the surface area increasing when the length of an edge is 10 centimetres?

Q.28) $\int \frac{1}{\sqrt{sin^3x sin(x+\alpha)}}dx$

Q.29) Solve the differential equation $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dy}{dx}=1$, (x≠0)

Q.30) Solve the following linear programming problem graphically

Minimizing z=200x + 500y

Subject to the constrains X+2y≥10; x≥0, y≥0

Q.31) A fair die is rolled . Consider the events E={1,3,5},F={2,3} and G={2,3,4,5}

Find:

1) P(E/F) and P(F/E)
2) P(E/G) and P(G/E)
3) P[(E U F)/G] and P[(E ∩F)/G]

Section-D

Q.32) Let N denotes the set of all natural numbers and R be the relation on NxN Defined by (a,b) R (c,d) ⇔ad(b+c) =bc(a+d).Prove that R is an equivalence relation

Or Show that the function: f:R→{ x∈R :-1≤ x ≤ 1}defined by f(x)= $\frac{x}{1+|x|}$, x+R is both one-one and onto function.

Q.33) use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve a system of equations

x-y+2z=1, 2y-3z=1, 3x-2y+4z=2

Q.34) Using method of integration find the area of the region in the quadrant enclosed by the x-axis, the line y= $\sqrt{3}$x and $x^2+y^2=9$

Q.35)Find the equation of the line through A(5, -3, -2) And through the intersection point of the lines:

$\frac{x-2}{1} = \frac{y-2}{3} = \frac{z-4}{4}$ and $\frac{x-4}{3} = \frac{y-2}{4} = \frac{z+3}{-3}$

Or Find the coordinates of the foot of the perpendicular drawn from point A(5,4,2) to the line $\vec{r}$ =(î+3ĵ+k̂) +λ(2î+3ĵ-k̂) also find the image of A in this line.

SECTION-E

Q.36) In the survey of a town it was found that 6% of people with blood group O are left handed and 10% of those with other blood group are left handed. 30% of the people have blood group O. Based o the above information answer the following questions :

A) Probability of selecting a left handed person given that he/she has blood group O

(i) 0.3 (ii) 0.6 III)0.1 (iv)0.06

B) Probability of selecting a left handed person given that he/she dosn't have blood group O

I) (i) 0.06 (ii)0.01 III)0.6 (iv) 0.1

C) Probability of selecting a left handed person is

(i) 0.088 (ii) 0.08 III)0.88 (iv) 0.80

D) The probability that a randomly selected person is right handed

I)0.88 II)104/125 III)114/125 IV)114/250

Q.37) the use of electric vehicles will curb air pollution in the long run . The electric vehicles is increasing every year and estimated electric vehicles in use at any time tis given by the function :

$V(t) = \frac{1}{5}t - \frac{5}{2}t^2 + 25t - 2$

Where t represents the time t=1,2,3,…. correspond to year 2001,2002,2003,….respectively.

Answer the following

I) Can the above function be used to estimate number of vehiclesin the year 2000?justify

II) Prove that the function V(t) is an increasing function .

Q.38) A company x units of output at a total cost of $C=\frac{1}{3}x^3 - 18x^2 + 160x$.the average cost(AC) is the cost per unit and marginal cost is the rate of change of C with respect to x. Based on the above information answer the following questions

A)The average cost(AC) is given by:

I)$\frac{x^2}{3} - 18x + 160$ II)$x^2 - 36x + 160$ (iii) $\frac{x^3}{3} - 18x + 160$ (iv) none of the above

B)The output at which average cost is equal to marginal cost , is

I)27 units II)18 units III)9units IV) 36units

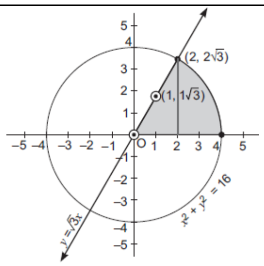
C) The output at which marginal cost is minimum

I)27units II)18units III)16units IV) 12units

D)The output at which AC is minimum

I)27units II)18units III)9units IV)12units

	KENDRIYA VIDYALAYA SANGATHAN **SAMPLE PAPER 2** **MARKING SCHEME** **CLASS – XII** **SUB :MATHEMATICS (041)** **MARKING SCHEME**	
Q.NO	**ANSWER**	**VALUE POINTS**
MCQ	**1)(C) 2)(B) 3)(B) 4)(D) 5)(B) 6)(C) 7)(C) 8)(C) 9)(D) 10)(B)** **11)(C) 12)(C) 13)(C) 14)(A) 15)(B) 16)(A) 17)(B) 18)(B) 19)(A) 20)(A)**	
21	$Cos^{-1}Cos\left(\frac{6\pi-\pi}{3}\right)+Sin^{-1}Sin(\frac{6\pi-\pi}{3})$ Result= 0 OR $Sin^{-1}Cos\left(\frac{40\pi+3\pi}{5}\right)= Sin^{-1}Cos\left(8\pi+\frac{3\pi}{5}\right)= Sin^{-1}Cos\left(\frac{3\pi}{5}\right)= Sin^{-1}Sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)$ Result=$\frac{-\pi}{10}$	**1** **1** **1** **1**
22.	Development of proper Function V=πl^2b L=12, b=6	1 1
23	To find correct $\frac{dy}{dx}$ To find correct$\frac{d^2y}{dx^2}$ To show the result correctly after Simplification	½ ½ 1
24	To get correct expression for $(\vec{a}+\vec{b}+\vec{c})^2$ To get correct value of $\vec{a}.\vec{b}$+$\vec{b}.\vec{c}$+$\vec{c}.\vec{a}$	1 1
25	To get correct perpendicular vector from the cross product of (3î -10ĵ +7k̂) and (3î +8ĵ -5k̂) To get correct vector and Cartesian Equation	1 (1/2+1/2)
26	To assume A with proper order To get correct value of A=$\begin{bmatrix}-8 & 3 & 5\\ -13 & -1 & -9\end{bmatrix}$ Or To get correct value of A^2 To get correct value of 5A To get correct value of A^2-5A	1 2 1 1 1
27	To assume correct function To find out correct value of the differentiation To get the correct result =3.6cm^2/s	1 1

		1
28	To convert the expression in the form $=\int \frac{dx}{sin^2x\sqrt{\frac{\sin(x+\alpha)}{sinx}}}$ To assume z=$\frac{\sin(x+\alpha)}{sinx}$ To get correct integration	1 1 1
29	To convert the equation in the form $\frac{dy}{dx}$+Py=Q To get correct integrating factor To get the correct result	1 1 1
30	To draw the lines correctly To get correct feasible region and the vertices To get correct value of z	1 1 1
31	P(E/F)=1/2, P(F/E)=1/3 P(E/G)=1/2,P(G/E)=2/3 P(E∪ $F/G) = 3/4$,P(E∩ F/G)=1/4	½+1/2 ½+1/2 ½+1/2
32	To show reflexivity correctly To show symmetricity correctly To show transitivity correctly To conclude properly OR To show one one properly(considering three different cases) To show onto properly(considering two cases)	1 1 2 1 3 2
33	To get proper multiplication as AB=I To write X=A^{-1}B To get proper values of x,y,z	2 1 2
34	 To get diagram properly Putting $y=\sqrt{3}x$ in $x^2+y^2=16$ we get $x^2+(\sqrt{3}x)^2=16$ $\Rightarrow \quad 4x^2=16 \quad \Rightarrow \quad x = \pm 2$ $\therefore \quad y=\pm 2\sqrt{3}.$ Therefore, intersecting point of circle and line is $(\pm 2, \pm 2\sqrt{3})$ To get correct area=4πsq.unit	1 2 2

35	To express the general point of the 1st line and 2nd line correctly To get correct point of intersection To get correct equation of the line	1 3 1
36	I)d II)b III)a IV)c	1 1 1 1
37	I)V(t)=$\frac{1}{5}$t $-\frac{5}{2}t^2+$25t$-$2 to estimate no of vehicles in the year 2000 we need to know the value at t=0 which cannot be determined by V(t) as it is defined for t=1,2,3.... II)V'(t)=$\frac{3}{5}$ {$(t-\frac{25}{6})^2$+$\frac{875}{36}$}>0 for all t then V(t) is an increasing function	2 2
38	I)A II)A III)B IV)A	1 1 1 1

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Sample Paper 3

Class-XII **Subject-Mathematics (041)**

Units and Chapters	MCQ	Assertion/Reasoning	2 Marks	3 Marks	5 Marks	4 marks Case Based	Total
1. Relation & Function & I.T. Functions	1	-	1	-	1	-	8(3)
2. Matrices and Determinants	6		-	-	-	1	10(6)
3. Calculus	4	1	2	4	2	1	35(14)
4. Vector Algebra & 3-D Geometry	4	1	2	-	1	-	14(8)
5. Linear programming Problem	2	-	-	1	-	-	5(4)
6. Probability	1	-	-	1	-	1	8(3)
Total	**18**	**2**	**5**	**6**	**4**	**3**	**80(38)**

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SAMPLE QUESTION PAPER 3

CLASS – XII **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

General Instructions:

1.This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section-A(20x1=20)

1 Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is

(a) Reflexive and symmetric (b) Transitive and symmetric

(c) Equivalence (d) Reflexive, transitive but not symmetric

2 If AB = A and BA =B, where A and B are square matrices , then

(a) $B^2 = B$ and $A^2 = A$ (b) $B^2 \neq B$ and $A^2 = A$

(c) $A^2 \neq A$ and $B^2 = B$ (d) $A^2 \neq A$ and $B^2 \neq B$

3 Given that A is a square matrix of order 3 and $|A| = -2$, then $|\,adj(2A)|$ is equal to

(a) -26 (b) +4 (c) -28 (d)28

4 If the area of triangle is 40 sq units with vertices (1,-6), (5,4) and (k,4). then k is

(a) 13 (b) -3 (c) -13,-2 (d) 13,-3

5 If $x=t^2$, $y=t^3$ then $\frac{dy}{dx}$ is

(a) $\frac{3t}{2}$ (b) $\frac{3t^2}{2}$. (c) $\frac{3}{2}$ (d) $\frac{2}{3t}$

6 The anti-derivative of $\operatorname{cosec}^2 2x$ is

(a) sec 2x (b) tan x +cot x +C (c) $\frac{1}{4}$(tan x -cot x+C) (d) tan x -cotx+C

7 The rate of change of area of a circle with respect to its radius r at r=6 is

(a)10π (b)12π (c)11π (d)8π

8 Choose the correct option :

(a) Every scalar matrix is an identity matrix

(b) Every square matrix whose each element is 1 is an identity matrix

(c) Every scalar matrix is a diagonal matrix

(d) Every diagonal matrix is a scalar matrix

9 Corner points of the feasible region for an LPP are (0,2), (3,0), (6,0), (6,8) and (0,5). Let F=4x+6y be the objective function. Maximum of F – Minimum of F =

(a)60 (b) 48 (c)42 (d)18

10 Which of the following function is decreasing in (0, $\frac{\pi}{2}$)

(a) sin 2x (b) tan x (c) cos x (d) cos 3x

11 The area of the quadrilateral ABCD where A (0,4,1), B (2,3,-1), C(4,5,0) and D(2,6,2) is equal to

(a) 9 sq units (b) 18 sq units (c) 27 sq units (d) 81 sq units

12 The value(s) of p for which the vectors joining (3,p,2), (1,0,5) and (1,0, -2) , (0,-p,-4) are orthogonal is (are)

(a) 1 (b) $\frac{1}{2}$ (c) 2 or -2 (d)1 or -1

13 The reflection of the point (1,-2,3) in the XY- plane is

(a) (1,-2,-3) (b) (-1,2,-3) (c) (-1,-2,3) (d) (1,2,3)

14 Which among the following is an intersecting point of the two lines x-1=y=5-z and x+2=y+3=z

(a)(1,2,3) (b)(2,1,4) (c)(3,0,-1) (d)(-1,2,1)

15 The equation of the line joining the points (1, 2) and (3, 6) is

(a) $y = 2x$ (b) $x = 3y$ (c) $y = x$ (d) $4x - y = 5$

16 The corner points of the feasible region determined by the following system of linear inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0,0), (5,0), (3,4), (0,5). Let $Z = px + qy$, where $p,q > 0$. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is

(a) $p = q$ (b) $p = 2q$ (c) $p = 3q$ (d) $q = 3p$

17 A flashlight has 8 batteries of which 3 are dead. If two batteries are selected without replacement and tested then probability that both are dead is

(a) $\frac{33}{56}$ (b) $\frac{9}{64}$ (c) $\frac{1}{14}$ (d) $\frac{3}{28}$

18 The minor M_{ij} of an element a_{ij} of a determinant is defined as the value of the determinant obtained after deleting the

(a) j^{th} row of the determinant

(b) i^{th} column and jth row of the determinant

(c) i^{th} row and jth column of the determinant

(d) i^{th} row of the determinant

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19 The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees.

Assertion (A): The marginal revenue when $x = 5$ is 66.

Reason (R): Marginal revenue is the rate of change of total revenue with respect to the number of items sold at an instance.

20 Assertion (A): The area of parallelogram with the diagonals a and b is $|\vec{a} \times \vec{b}|$.

Reason (R): If a and represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating $\frac{1}{2}|\vec{a} \times \vec{b}|$.

Section B (5x2=10)

21 Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ in simplest form , $\frac{-3\pi}{2} < x < \frac{\pi}{2}$.

22 Whether the function f: R→ R defined by $f(x)=x^3-x$, has any critical point/s or not ?If yes, then find the point/s.

Or Prove that the function f given by $f(x) = |x - 1|$, $x \in R$ is not differentiable at x=1

23(a) Find $\int x^2 \tan^{-1} x \, dx$

23(b) Or Integrate: $\int \sin 3x \cos 2x \, dx$.

24 Find the direction ratio and direction cosines of a line parallel to the line whose equations are $6x-2 = 3y+ 1 = 2z - 4$.

25 Find the vector equation of the line joining (1, 2, 3) and (–3, 4, 3) and show that it is perpendicular to the z-axis.

Section-C (6X3=18)

26(a) If $y = x^{\sin x}$, find $\frac{dy}{dx}$.

26(b) Or Solve the differential equation $xy \log x \, dy - y^3 \, dx=0$.

27(a) Find the intervals of increasing and decreasing nature of the function $f(x)=x^3+6x^2+9x-8$.

27(b) Or A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is 8m3.If building of tank costs Rs.70 per sq. metre for the base and Rs. 45 per sq. metre for sides, that is the cost of least expensive tank?

28 Evaluate : $\int \frac{x^2}{x^4+x^2-2} dx$.

29(a) Find $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$

29(b) Or Find the area of the region bounded by y2 = 9x, x = 2, x = 4 and the x-axis in the first quadrant.

30 Find the maximum value of the objective function

$Z = 5x + 10y$

subject to the constraints

$x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, $y \ge 0$.

31 In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$ What is the probability that the student knows the answer given that he answered it correctly?

Section-D (4X5=20)

32(a) Find the subsets of the set of real numbers in which the following function is (a) increasing (b)decreasing , $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$.

32(b) **Or** Find the ratio of the volume of the largest cone that can be inscribed in sphere of radius R and the volume of the sphere.

33 Find the area of the triangular region whose sides are y=2x+1 ,y =3x +1 , x=4 .

34(a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

34(b) **Or** Find the foot of perpendicular from the point (3, -1, -11) to line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

35) Let N be the set of all natural numbers and R be a relation on NxN defined by (a,b)R(c,d) if and only if ad= bc for all (a,b), (c,d) in NxN. Show that R is an equivalence relation on NxN. Also, find a pair which is related to (2,6) .

Section-E (3X4=12)

36) A helicopter moves on a path in such a way that at any point (x,y) of the path the derivative of ordinate w. r.t. abscissa is twice the slope of the line – segment joining the point of contact to the point (−4,−3) .

(i) Write The differential equation according to the given condition.

(ii) Find the solution of the differential equation.

(iii) If the helicopter passes through the point (−2, 1) , then find the equation of the path.

37) Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, and SUV cars. The sales figure for 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, and 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, and 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, and 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan,5 SUV cars in 2020.

  

Based on the above information, answer the following questions.

(i) The matrix summarizing sales data for 2019 is

(a) $\begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 120 & 50 & 15 \\ 100 & 30 & 5 \\ 95 & 40 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$ (d)) $\begin{bmatrix} 120 & 50 & 10 \\ 200 & 50 & 6 \\ 90 & 40 & 2 \end{bmatrix}$

(ii) Suppose dealer A sells two types of Hatchback cars Indica and Alto in 2019 and showroom price for Indica and Alto are Rs 600000 and 500000 respectively. If one-third of dealer A's 2019 profit of Rs 60000000 is from Hatchback, express this in matrix form.

(iii) Calculate the increase in sales of Hatchback cars by A from 2019 to 2020 if it sells 100 Indica and 200 Alto in 2020.

Or Calculate the sales of Sedan and SUV cars by A in 2019 .

38) Rubiya, Thaksh, Shanteri, and Lilly entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both. Spinners have numbers 1 to 9 on those: Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:

- Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player!
- You win a photo frame if Spinner A lands on a value greater than 4.
- You win an earplug if you get even in spinner A or odd in spinner B.

i)Thaksh spun both the spinners, A and B in one of his turns. What is the probability that Thaksh wins a music player in that turn?

ii) Lilly spun spinner A in one of her turns. What is the probability that the number she got is even given that it is a multiple of 3?

iii) Rubiya spun both the spinners. What is the probability that she wins a photo frame only?

Or

As Shanteri steps up to the screen, the game administrator reveals that she would see either Spinner A or Spinner B for her turn, the probability of seeing Spinner A on the screen is 65%, while that of Spinner B is 35%. What is the probability that Shanteri wins an earplug?

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SAMPLE PAPER 3

MARKING SCHEME

CLASS – XII **SUB : MATHEMATICS (041)**

Section-A(20x1=20)		
	MCQ ANSWERS **1) (d) 2)(a) 3) (d) 4) (b) 5) (a) 6) (c)7)(b) 8) (c) 9) (a) 10) (c)** **11) (a)12) (c) 13) (a) 14)(b) 15)(a) 16) (d) 17) (d) 18)(c) 19(a) 20 (a)**	
21	$\frac{\cos x}{1-\sin x}=\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}=\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$ Therefore $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)=\frac{\pi}{4}-\frac{x}{2}$	
22	critical points are $\frac{1}{\sqrt{3}}$ $-\frac{1}{\sqrt{3}}$. **Or** At x =1 , LHD = -1, RHD =1 So not differentiable at x=1	
23	(a) $\frac{x^3}{3}\tan^{-1}x - \frac{x^2}{6} + \frac{1}{6}\log(x^2+1)+C$ Or (b) $\frac{cos^3 x}{3}+\frac{cos^5 x}{5}+ C$	
24)	the line has direction ratios 1,2,3 & d.c. of the line: $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$.	
25)	Equation of line is $\vec{r}= \hat{i}+2\hat{j}+3\hat{k}+\lambda(-4\hat{i}+2\hat{j})$ and of Z-axis is $\vec{r} = \hat{k}$	
26	(a) $\frac{dy}{dx}=x^{\sin x}(\cos x \log x+\frac{\sin x}{x})$. Or (b) $-\frac{1}{y} = \frac{1}{2x^2} + C$	
27)	(a) f is increasing in $(-\infty,-3)\cup(-1,\infty)$ & f is decreasing in (-3,-1). Or (b) The cost of least expensive tank =Rs 70 x 4+45(8+8)=Rs 1000.	
28)	$\frac{\sqrt{2}}{3}\tan^{-1}x+\frac{1}{6}\log\frac{x+1}{x-1}+ C$.	
29	(a)$\frac{1}{\sqrt{2}}\log(2+\sqrt{2})$. Or (b) $4(4-\sqrt{2})$ sq unit	

30)	Z has a maximum value 600 at x=60,y=30 and at x=0,y=60 [at all points of AB].	
31)	$\frac{12}{13}$	
32)	(a) So f is Increasing in the subset (1,2)U (3,∞) and f is decreasing in the subset (−∞,1)U(2,3). **Or** (b) $\frac{1}{2\sqrt{2}}$	
33)	Required area=$\int_0^4 (3x+1)dx$ -$\int_0^4 (2x+1)dx$=8 sq unit	
34	(a) the angle between $\vec{a}$ and $\vec{b}$ =$\frac{\pi}{3}$ Or (b) foot of the perpendicular $(-\frac{118}{29}, -\frac{119}{29}, -\frac{149}{29})$	
35)	A pair which is related to (2,6) is (3,9).	
36)	(i) $\frac{dy}{dx}$=2y+3x+4 (ii) y+3=C (x+4) (iii) y=2x+5.	
37)	(i) (a) (ii) The matrix form of AX=B where A=$\begin{bmatrix}1 & 1\\6 & 5\end{bmatrix}$, X=$\begin{bmatrix}x\\y\end{bmatrix}$ and B= $\begin{bmatrix}120\\200\end{bmatrix}$. (ii) 140000000. **Or** The sales of Sedan and SUV cars by A Rs 40000000	
38)	(i) $\frac{1}{81}$ (ii) $\frac{1}{9}$ (iii) $\frac{5}{9}$ or $\frac{107}{180}$	

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Sample Paper 4

Class-XII **Subject-Mathematics (041)**

UNITS	NAME OF THE CHAPTERS	SECTION - A (Objective Type) (1 Mark each)		SECTION - B (VSA) (2 MARKS EACH)	SECTION - C (SA) (3 MARKS EACH)	SECTION - D (LA) (5 MARKS EACH)	SECTION - E (CBQ) (4 MARKS EACH)	TOTAL
		MCQ	ARQ					
UNIT-I (RELATIONS AND FUNCTIONS)	RELATIONS AND FUNCTIONS		1(1)			5(1)		8(3)
	INVERSE TRIGONOMETRIC FUNCTIONS			2(1)				
UNIT-II (ALGEBRA)	MATRICES	4(4)				5(1)		10(6)
	DETERMINANTS	1(1)						
UNIT-III (CALCULUS)	CONTINUITY AND DIFFERENTIABILITY	2(2)		2(1)	3(1)			35(16)
	APPLICATION OF DERIVATIVES		1(1)		3(1)		4(1)	
	INTEGRALS	2(2)		2(1)	3(1)	5(1)		
	APPLICATION OF INTEGRALS				3(1)			
	DIFFERENTIAL EQUATIONS	2(2)			3(1)			
UNIT-IV (VECTORS AND 3D)	VECTORS	1(1)		2(1)			4(1)	14(6)
	THREE-DIMENSIONAL GEOMETRY	2(2)				5(1)		
UNIT-V (LPP)	LPP	2(2)			3(1)			5(3)
UNIT-VI (PROBABILITY)	PROBABILITY	2(2)		2(1)			4(1)	8(4)
TOTAL		18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

KENDRIYA VIDYALAYA SANGATHAN

EXCELLENCE SERIES SAMPLE QUESTION PAPER 4

CLASS – XII **MAX.MARKS – 80**

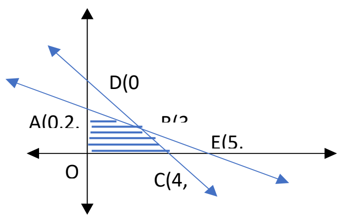
SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

General Instructions:

1. This question paper contains 38 questions divided into three sections- A, B, C & D
2. All questions are compulsory.
3. Section - A contains **20 very short answer type (VSA)** of 1 mark each.
4. Section - B contains **5 short answer type (SA-I)** questions of 2 marks each.
5. Section - C contains **6 short answer type (SA-II)** of 3 marks each.
6. Section -D contains **4 long answer type questions (LA)** of 5 marks each.
7. Section -E contains **3 case based questions (CBQ)** of 4 marks each.

SECTION-A

1	If $A = [a_{ij}]$ is an identity matrix, then which of the following is true? (A) $a_{ij} = \begin{cases} 0, \text{if } i = j \\ 1, \text{if } i \neq j \end{cases}$ (B)$a_{ij} = 1, \forall i, j$ (C)$a_{ij} = 0, \forall i, j$ (D) $a_{ij} = \begin{cases} 0, \text{if } i \neq j \\ 1, \text{if } i = j \end{cases}$	1
2	If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is : (A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) 30 $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D)$\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$	1
3	For any square matrix A, $(A - A^t)^t$ is always : (A)An identity matrix (B)A null matrix (C)A skew symmetric matrix (D) A symmetric matrix	1
4	If $A.(\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $\|A\| + \|\text{adj } A\|$ is equal to : (A)12 (B)9 (C) 3 (D)27	1
5	Let, A be the area of a triangle having vertices $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3). Which of the following is correct? (A) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$ (B) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$	1

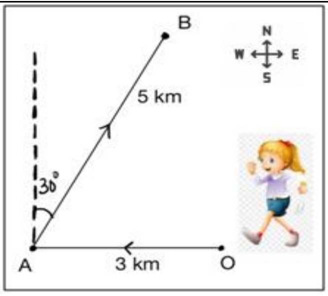
	(C) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm\frac{A}{2}$ (D) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$	
6	The value of k for which the function $f(x) = \begin{cases} x^2, x \geq 0 \\ kx, x < 0 \end{cases}$ is differentiable at $x = 0$ is : (A)1 (B)2 (C)Any real number (D)0	1
7	If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is : (A) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (B)$\sec^2\left(\frac{\pi}{4} - x\right)$ (C)$\ln\left\|\sec\left(\frac{\pi}{4} - x\right)\right\|$ (D)$-\ln\left\|\sec\left(\frac{\pi}{4} - x\right)\right\|$	1
8	$\int 2^{x+2}dx$ is equal to : (A)$2^{x+2} + c$ (B)$2^{x+2}\ln 2 + c$ (C)$\frac{2^{x+2}}{\ln 2} + c$ (D)$2.\frac{2^x}{\ln 2} + c$	1
9	$\int_0^2 \sqrt{4 - x^2}\,dx$ equals : (A)$2\ln 2$ (B)$-2\ln 2$ (C)$\frac{\pi}{2}$ (D)π	1
10	What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3\cos y = \sqrt{y}$? (A)3 (B) 2 (C)6 (D)Not defined	1
11	$x\ln x\frac{dy}{dx} + y = 2\ln x$ is an example of a : (A)Variable separable diff equation. (B) Homogeneous diff l equation. (C)First order linear diff equation. (D) Diff equation whose degree is not defined.	1
12	Besides non negativity constraints, the figure given below is subject to which of the following constraints  (A)$x + 2y \leq 5$; $x + y \leq 4$ (B)$x + 2y \geq 5$; $x + y \leq 4$ (C)$x + 2y \geq 5$; $x + y \geq 4$ (D)$x + 2y \leq 5$; $x + y \geq 4$	1
13	In ΔABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is the mid-point of BC, then $\overrightarrow{AD}$ is equal to : (A)$4\hat{i} + 6\hat{j}$ (B)$2\hat{i} - 2\hat{j} + 2\hat{k}$ (C)$\hat{i} - \hat{j} + \hat{k}$ (D)$2\hat{i} + 3\hat{k}$	1
14	If the point $P(a, b, 0)$ lies on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$, then (a, b) is :	1

	(A) $(1, 2)$ (B) $\left(\frac{1}{2}, \frac{2}{3}\right)$ (C) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (D) $(0, 0)$	
15	If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is not true? (A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ (C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ (D) $\cos\alpha + \cos\beta + \cos\gamma = 1$	1
16	The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called : (A) Feasible solutions (B) Constraints (C) Optimal solutions (D) Infeasible solutions	1
17	If $P(A \cap B) = \frac{1}{8}$ and $P(A') = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to : (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$	1
18	If A and B are independent events, then which of the following is not true? (A) A' and B are independent events. (B) A and B' are independent events. (C) A' and B' are independent events. (D) None of these	1
	Question number 19 and 20 are Assertion and Reason based question. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answers from the codes A, B C and D as given below. A. Both A and R are true and R is the correct explanation of A. B. Both A and R are true but R is not the correct explanation of A. C. A is true and R is false. D. A is false and R is true.	
19	Assertion(A): The relation $R = \{(1, 2)\}$ on the set $A = \{1, 2, 3\}$ is transitive. Reasoning (R): A relation R on a non-empty set A is said to be transitive if $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$, for all $a, b, c \in A$.	1
20	Assertion(A): The function $f(x) = (x + 2)e^{-x}$ is strictly increasing on $(-1, \infty)$. Reasoning (R): A function $f(x)$ is strictly increasing if $f'(x) > 0$.	1
	SECTION-B	
21	Find the principal value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$. **OR** Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.	2
22	If $x = a\tan^3\theta$ and $y = a\sec^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$. .	2

23	Evaluate : $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$ **OR** Evaluate $\int \sqrt{1 - \sin 2x}\, dx, \frac{\pi}{4} < x < \frac{\pi}{2}$.	2
24	If $\|\vec{a}\| = 2, \|\vec{b}\| = 7$ and $\vec{a} \times \vec{b} = -3\hat{\imath} + \hat{\jmath} + 2\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.	2
25	Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$. Find the probability distribution of X. **OR** A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let, A be the event "number obtained is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.	2
SECTION-C		
26	If $(\cos y)^x = (\sin x)^y$, then find $\frac{dy}{dx}$.	3
27	Find the intervals in which the function f(x) = $3x^4 - 4x^3 - 12x^2 + 5$ is (I) strictly increasing (II) strictly decreasing	3
28	Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{16 + 9\sin 2x} dx$ **OR** Prove that $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$, and hence evaluate $\int_0^1 x^2 (1 - x)^n\, dx$.	3
29	Find the area of the region $\{(x, y): y \geq x^2, y \leq \|x\|\}$ **OR** If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq. units, then using integration, find the value of m.	3
30	Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 0$ when $x = 1$. **OR** Find the particular solution of the differential equation $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$, given that $y(1) = \frac{\pi}{2}$.	3
31	Solve the above L. P. P graphically : Maximize $Z = 3x + 9y$ Subject to constraints $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$ $x, y \geq 0$	3

SECTION-D		
32	Let $\mathbb{N}$ be the set of natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)\ R\ (c, d)$ iff $ad = bc$ for all $a, b, c, d \in \mathbb{N}$. Show that R is an equivalence relation. **OR** Show that the function $f: \mathbb{N} \to \mathbb{N}$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto.	5
33	If $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix}$, then find A^{-1} and hence solve the system of system of linear equations: $x + y + z = 6,\ \ y + 3z = 7$ and $x - 2y + z = 0$.	5
34	Evaluate: $\int_1^4 [\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert]\, dx$	5
35	Find the co-ordinates of the foot of the perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining points $B(0, -1, 3)$ and $C(2, -3, -1)$. Hence find the image of the point A in the line BC.	5
SECTION-E		
36	Read the following passage and answer the questions given below: In an Office three employees Jayant, Sonia and Olivia process a calculation in an excel form. Probability that Jayant, Sonia, Olivia process the calculation respectively is 50%, 20% and 30% . Jayant has a probability of making a mistake as 0.06, Sonia has probability 0.04 to make a mistake and Olivia has a probability 0.03. Based on the above information, answer the following questions. I. Find the probability that Sonia processed the calculation and committed a mistake. II. Find the total probability of committing a mistake in processing the calculation. III. The boss wants to do a good check. During check, he selects a calculation form at random from all the days. If the form selected at random has a mistake, find the probability that the form is not processed by Jayant.	1 1 2
37	A girl walks 3 km towards west to reach point A and then walks 5 km in a direction 30° east of north and stops at point B. Let the girl starts from O (origin) and take $\hat{\imath}$ along east and $\hat{\jmath}$ along north.	

Based on the above information, answer the following questions.



(I) Find the scalar components of $\overrightarrow{AB}$. 1

(II) Find the unit vector along $\overrightarrow{AB}$. 1

(III)Find the position vector of point B. 2

38 In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 cubic m. The cost of land is Rs 5000 per sq m and cost of digging increase with depth and for the whole tank it is 40,000 h^2 , where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

Based on the above information answer the following questions:

I. Find the total cost C of digging the tank in terms of x. 1

II. Find $\frac{dC}{dx}$. 1

III. Find the value of x for which cost C is minimum 2

OR

Check whether the cost function C(x) expressed in terms of x increasing or not, where x >0.

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 4

CLASS – XII **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

MARKING SCHEME

1-20	**1)(D) 2)(A) 3)(C) 4)(A) 5)(B) 6)(D) 7)(A) 8)(C) 9)(D) 10)(B)** **11)(C) 12)(A)13)(D) 14)(C) 15)(D) 16)(B) 17)(A 18)(D) 19)(A) 20)(D)**	
21	$\frac{\pi}{6}$ **OR** $-\frac{\pi}{3}$	
22	(-1)/(54 a)	
23	$\log\left\|\tan x + \sqrt{\tan^2 x + 4}\right\| + c$ **OR** $-\cos x - \sin x + c$	
24	$\sin^{-1}\left(\pm\frac{1}{\sqrt{14}}\right)$	
25	X: x_1, x_2, x_3, x_4; P(X): $\frac{30}{122}$, $\frac{30}{183}$, $\frac{30}{61}$, $\frac{30}{305}$ **OR** $P(A) = \frac{1}{2}, P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{6}$ So $P(A \cap B) \neq P(A).P(B)$ so dependent	
26	$\frac{dy}{dx} = \frac{\log\cos y - y\cot x}{\log\sin x + x\tan y}$	
27	f(x) is strictly decreasing in$(-\infty, -1) \cup (0,2)$ f(x) is strictly increasing in $(-1,0) \cup (2, \infty)$	
28	$\frac{1}{30}\log 4$ **OR** $\left[\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right]$	
29	$y = \frac{1}{3}$ sq. units **OR** $m = 2$	

30	$\tan^{-1} y = x + \frac{x^3}{3} - \frac{4}{3}$ **OR** the complete sol^n is : $\cos\left(\frac{y}{x}\right) = \log\lvert x\rvert$	
31	Maximum value of Z is 180 and which is at any point on the line segment joining B and C.	
SECTION-D		
32	R is Reflexive , symmetric and transitive and so equivalence on $\mathbb{N} \times \mathbb{N}$: **OR** f is one-one but f is not onto	
33	$\therefore A^{-1} = \frac{adjA}{\lvert A\rvert} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ and $x = \frac{7}{3}, y = 2, z = \frac{5}{3}$	
34	$\int_1^4 [\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert]\, dx = \int_1^4 \lvert x-1\rvert\, dx + \int_1^4 \lvert x-2\rvert\, dx + \int_1^4 \lvert x-3\rvert\, dx$ $= 4.5 + 2.5 + 2.5 = 9.5$	
35	Coordinates of foot of perpendicular is (- 2, 1, 7) and image is (-3,-6,10).	
SECTION-E		
36	(i) $\frac{8}{47}$ (ii) 0.047 (iii) $\frac{17}{47}$	
37	(i) So, scalar components of $\overrightarrow{AB}$ are 2.5, $2.5\sqrt{3}$. (ii) Unit vector along $\overrightarrow{AB} = \frac{1}{2}\left(\hat{\imath} + \sqrt{3}\hat{\jmath}\right)$. $(iii)\overrightarrow{OB} = -\frac{1}{2}\hat{\imath} + \frac{5\sqrt{3}}{2}\hat{\jmath}$.	
38	(i) C = $5000\, x^2 + 2500000000/x^4$ (ii) $\frac{dC}{dx}$= $10000x - 10000000000/x^5$ (iii)(a)For C to be minimum , $\frac{dC}{dx}$=0 $\Rightarrow 10000x - 10000000000/x^5 = 0 \Rightarrow x = 10$ OR (b) $\frac{dC}{dx}$ = $10000x - 100000\ 00000/x^5$ Which is < 0 for 0<x<10. So, the cost function C(x) is not increasing where 0<x<10	

KENDRIYA VIDYALAYA SANGATHAN

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Sample Paper 5

Class-XII **Subject-Mathematics (041)**

Chapters	MCQ	Assertion/ Reasoning	2 Marks	3 Marks	5 Marks	Case Based	Total
1. Relation & Function & I.T. Functions	1	-	1	-	1	-	8(3)
2. Matrices and Determinants	5	-	-	-	1	-	10(6)
3. Calculus	5	1	2	4	1	2	35(15)
4. 3-D Geometry	1	1	1	-	1	-	9(4)
Vector Algebra	3	-	1	-	-	-	5(4)
5. L.P.P.	2	-	-	1	-	-	5(3)
6. Probability	1	-	-	1	-	1	8(3)
Total	**18**	**2**	**5**	**6**	**4**	**3**	**80(38)**

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 5

CLASS – XII **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

General Instructions:

- This Question paper contains – five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
- **Section A** has **18** MCQ's and **02** Assertion-Reason based questions of **1** mark each.
- **Section B** has **5** Very Short Answer (VSA)-type questions of **2** marks each.
- **Section C** has **6** Short Answer (SA)-type questions of **3** marks each.
- **Section D** has **4** Long Answer (LA)-type questions of **5** marks each.
- **Section E** has **3** source based/case based/passage based/integrated units of assessment of **4** marks each with sub-parts.

	Section-A **(Multiple Choice Questions)** **Each question carries 1 mark**
1	If A, B are symmetric matrices of some order, then AB-BA is a (a) Skew symmetric matrix (b) Symmetric matrix (c) Zero matrix (d) Identity matrix
2	If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then the value of (x+y+z) is (a) 6 (b) -6 (c) 0 (d) can't be determined
3	If for matrix A=$\begin{bmatrix} \propto & -2 \\ -2 & \propto \end{bmatrix}$,$\lvert A^3 \rvert$=125, then the value of $\propto$ is: (a)± 3 (b) -3 (c) ± 1 (d) 1
4	The interval in which $y = -x^3 + 3x^2 + 2021$ is increasing in: (a) (-∞, ∞) (b) (0,2) (c) (2,∞) (d) (-2,∞)
5	Let R be a relation in the set N given by R={(a,b): a=b-2,b>6} then (a) (2,4)∈R (b) (3,8)∈R (c) (6,8)∈R (d) (8,7)∈R
6	The order and degree of the differential equation- $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dy^2}$ is (a) 2,1 (b)1,2 (c) 2,Not defined (d) 2,2
7	If the objective function Z=ax+y is minimum at (1,4) and its minimum value is 13, then value is 13, then value of a is: (a) 1 (b) 9 (c) 4 (d)13
8	If $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines of the vector $\vec{a}$ then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to: (a) 3 (b) 0 (c) 2 (d) -1

9	The value of $\int\left(x^2-\frac{1}{x^2}\right)^2 dx$ is: (a) $\frac{x^5}{5}+\frac{1}{3x^3}+2x+C$ (b) $\frac{x^5}{5}-\frac{1}{3x^3}-x+C$ (c) $\frac{x^5}{5}-\frac{1}{3x^3}-2x+C$ (d) $\frac{x^5}{5}-\frac{1}{x^3}-2x+C$
10	For what value of K, the matrix $\begin{bmatrix}2-K & 3\\ -5 & 1\end{bmatrix}$ is not invertible (a) 17 (b) 2 (c) 13 (d) None of these
11	In a linear programming problem, the constraints on the decision variables x and y are x-3y≥0, y≥0, 0≤x≤3, the feasible region: (a) Is not in the first quadrant, (b) is bounded in the first quadrant, (c) is unbounded in the first quadrant (d) does not exists
12	The position vector of the point which divides the join of points $2\vec{a}$-$3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3:1 internally is: (a) $\frac{5\vec{a}}{4}$ (b) $\frac{3\vec{a}-2\vec{b}}{2}$ (c) $\frac{7\vec{a}-8\vec{b}}{2}$ (d) $\frac{3\vec{a}}{4}$
13	For A=$\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ then $14A^{-1}$ is given by: (a) $14\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$ (b) $2\begin{bmatrix}2 & -1\\ 1 & -3\end{bmatrix}$ (c) $2\begin{bmatrix}-3 & -1\\ 1 & -2\end{bmatrix}$ (d) $\begin{bmatrix}4 & -2\\ 2 & 6\end{bmatrix}$
14	If A and B are two events such that P(A)=$\frac{1}{2}$, P(B)=$\frac{1}{3}$ and P(A/B)=$\frac{1}{4}$ then P(A'∩B') equals (a) $\frac{1}{12}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{3}{16}$
15	The integrity factor of the differential equation: $x\frac{dy}{dx}-y=2x^2$ is (a) e^{-x} (b) $\frac{1}{x}$ (c) x (d) None of these
16	The angle between the lines 2x=3y=-z and 6x=-y=-4z is: (a) 0° (b) 45° (c) 90° (d) 30°
17	The maximum value of the function f(x)=4.$\sin x\cos x$ is: (a) 2 (b) 4 (c) 1 (d) 8
18	The value of x if x($\hat{\imath}+\hat{\jmath}+\hat{k}$) is a unit vector, is: (a) $\pm\sqrt{3}$ (b) $\pm\frac{1}{\sqrt{3}}$ (c)$\pm\frac{1}{3}$ (d) ± 3
	ASSERTION REASON BASED QUESTIONS The following questions 19 and 20 consist of two statements-Assertion(A) and Reason (R). Answer the questions selecting the appropriate option given below: (a) Both A and R are true, and R is the correct explanation for A. (b) Both A and R are true, and R is not the correct explanation for A. (c) A is true but R is false. (d) A is false but R is true.
19	Assertion(A): The acute angle between the lines $\vec{r}=\hat{\imath}+\hat{\jmath}+2\hat{k}+\lambda(\hat{\imath}-\hat{\jmath})$ and the x-axis is π/4. Reason(R): The acute angle θ between the lines $\vec{r}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}+\lambda(a_1\hat{\imath}+b_1\hat{\jmath}+c_1\hat{k})$ and $\vec{r}=x_2\hat{\imath}+y_2\hat{\jmath}+z_2\hat{k}+\mu(a_2\hat{\imath}+b_2\hat{\jmath}+c_2\hat{k})$ is given by

	$\cos\theta = \frac{\lvert a_1a_2+b_1b_2+c_1c_2\rvert}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$
20	Assertion(A): If $y = \sin^{-1}(6x\sqrt{1-9x^2})$ then $\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$ Reason(R): $\sin^{-1}\left(6x\sqrt{1-9x^2}\right)$=3$\sin^{-1}(2x)$
	Section-B **Each question carries 2 marks each**
21	Find the value of $\tan^{-1}(\frac{-1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$ **Or,** Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$
22	The side of an equilateral triangle is increasing at the rate of 30 cm/s. At what rate is its area increasing when the side of the triangle is 30 cm? **Or,** A stone is dropped into a quiet lake and waves move in a circle at a speed of 3.5 cm/sec. At the instant when the radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing?
23	Let $\vec{a}, \vec{b}\ and\ \vec{c}$ be three vectors such that $\lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 4, \lvert\vec{c}\rvert = 5$ and each one of them being perpendicular to the sum of the other two, find $\lvert\vec{a}+\vec{b}+\vec{c}\rvert$
24	Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
25	Find the shortest distance between the lines- $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}\ and\ \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$
	Section-C **This section comprises of short answer type questions (SA) of 3 marks each**
26	Solve the following linear programming problem graphically minimize z=3x+5y Subject to constraints- x+3y≥ 3, x+y≥2, x≥0, y≥0
27	Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces. **Or,** The probabilities of two students A and B coming to the school in time are 3/7 and 5/7 respectively. Assuming that the events 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time.
28	Find $\int x\sin^{-1}x\,dx$ **Or,** Find $\int\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$
29	Solve the differential equation: $(x^2+xy)\,dy - (x^2+y^2)\,dx = 0$ **Or**, Solve the differential equation: $(1+x^2)\,dy + 2xy\,dx = \cot x\,dx \quad (x \neq 0)$
30	Draw a rough sketch of the curve 4y-2=x and x^2=4y and find the area bounded by these two using integration.
31	If$x = a(\theta+\sin\theta), y = a(1-\cos\theta)$ then find $\frac{d^2y}{dx^2}$ at $\theta = \pi/2$
	Section-D **This section comprises of long answer type questions (LA) of 5 marks each**

32	Determine the product of $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and then use this to solve the system of equations x-y+z=4; x-2y-2z=9; 2x+y+3z=1
33	Find the vector equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ **Or**, Solve that the lines: $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect each other. Also find their point of intersection.
34	Evaluate $\int_0^{\pi/2} \log \sin x \, dx$ **Or**, Evaluate $\int_0^{\pi} \frac{x\,dx}{a^2 \cos^2 x + b^2 \sin^2 x}$
35	Show that the relation R in the set A={1,2,3,4,5} given by R= {(a,b):|a-b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.
	<u>Section-E</u> **This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.**
36	In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.  Based on the above information answer the following questions- (i) The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics. (ii) The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics. (iii) The probability that the selected student has failed in at least one of the two subjects.

37	The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation: $y = 4x - \frac{1}{2}x^2$ Where x is the number of days exposed to sunlight.  Based on the above information answer the following questions: (i) Find the rate of the plant with respect to sunlight. (ii) What is the number of days it will take for the plant to grow to the maximum height? (iii) If the height of the plant is 7/2 cm, find the number of days it has been exposed to the sunlight.
38	Megha wants to prepare a handmade gift box for her friend's birthday at home. For making the lower part of box, she takes a square piece of cardboard of side 20 cm.  Based on the above information, answer the following- If x can be the length of each side of the square cardboard which is to be cut off from corners of the square piece of side 20 cm. (i) What should be the side of square to be cut off so that volume of the box is maximum? (ii) The maximum value of the volume?

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 5

CLASS – XII **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

Marking Scheme

	Section-A **For each correct option- 1 mark** **1(a),2(c),3(a),4(b),5(c),6(d),7(d),8(d),9(c),10(b),11(a),12(d),13(d),14(c),15(b), 16(c),17(a), 18(b),19(a),20(c)**	
21	**Section-B** Value=$\frac{-\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4}$ $=\frac{-\pi}{12}$ **Or,** Value=$\frac{-\pi}{6}+\frac{\pi}{6}$ =0	1 1 1 1
22	A=$\frac{\sqrt{3}}{4}a^2$ $(\frac{dA}{dt})_{a=30}=45\sqrt{3}$ cm^2/s **Or,** A=πr^2 $(\frac{dA}{dt})_{r=7.5}=52.5\pi$ cm^2/s	0.5 1.5 0.5 1.5
23	$\vec{a}.(\vec{b}+\vec{c})=\vec{b}.(\vec{c}+\vec{a})=\vec{c}.(\vec{a}+\vec{b})=0$ $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=5\sqrt{2}$	0.5 1.5
24	Correct figure Area=πab	0.5 1.5
25	S.D.=$\left\lvert\frac{(\vec{a_2}-\vec{a_2}).(\vec{b_1}\times\vec{b_2})}{\lvert\vec{b_1}\times\vec{b_2}\rvert}\right\rvert$ $=2\sqrt{29}$	0.5 1.5
26	**Section-C** Drawing correct graph For showing unbounded feasible solution region For finding minimum value	 1 1 1
27	X=0,1,2 X: 0, 1, 2 P(x): $\frac{144}{169}$, $\frac{24}{169}$, $\frac{1}{169}$ **Or,** Required probability=$\frac{3}{7}\left(1-\frac{5}{7}\right)+\frac{5}{7}\left(1-\frac{3}{7}\right)$ $=\frac{6}{49}+\frac{20}{49}=\frac{26}{49}$	0.5 2.5 2 1

28	$\int x\sin^{-1}x\,dx = \sin^{-1}x\int x\,dx - \left[\frac{d}{dx}\left(\sin^{-1}x\int x\,dx\right)\right]dx$ $= \frac{1}{4}(2x^2-1)\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} + C$ **Or,** $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = \int\left(1 - \frac{4x^2+10}{(x^2+3)(x^2+4)}\right)dx$ $= \int\left(1+\frac{2}{x^2+3}-\frac{6}{x^2+4}\right)dx$ $= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$	1 2 0.5 1.5 1
29	$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$ Putting y=vx, $\frac{dy}{dx} = V + x\frac{dV}{dx}$ Solving and getting- $(x-y)^2=Cxe^{-y/x}$ **Or,** $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cot x}{1+x^2}$ I.F.= $1+x^2$ Req. Soln. $y(1+x^2)=\log\lvert\sin x\rvert+C$	0.5 1 1.5 0.5 1 1.5
30	Rough Sketch Using integration and getting area enclosed= 9/8 sq. units	1 2
31	$\frac{dx}{d\theta} = 2a\cos^2\frac{\theta}{2}, \frac{dy}{d\theta} = 2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ $\frac{dy}{dx} = \tan\frac{\theta}{2}, \frac{d^2y}{dx^2} = \frac{1}{4a}\sec^4\frac{\theta}{2}$ Value=1/a	1 1.5 0.5
32	**Section-D** AB=8I B^{-1}=1/8A X=B^{-1}C x=3,y=2,z=1	 2 1 2
33	$\vec{r} = (\hat{\imath}+2\hat{\jmath}-4\hat{k}) + \lambda(b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k})$---1 Line 1 is perpendicular to the two given lines $\frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$ $\therefore \vec{r} = (\hat{\imath}+2\hat{\jmath}-4\hat{k}) + \lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ **Or,** For showing shortest distance between two lines=0 i.e, lines are intersecting. For finding point of intersection- x=(2λ+1);y=(3λ+2);z=(4λ+3) from 1st equation of line Putting into 2nd equation of line and getting λ=-1 Required point(-1,-1,-1)	1 3 1 2 1 1.5 0.5
34	Using property of definite integral Adding and simplifying- Getting 2I=I-$\frac{\pi}{2}\log 2$ I=-$\frac{\pi}{2}\log 2$ **Or,**	1 1 2.5 0.5

	Using property of definite integral Adding and simplifying Getting I=π$\int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2+b^2 \tan^2 x}$ =$\frac{\pi^2}{2ab}$	1 1 2 1
35	For proving Reflexive & Symmetric For proving transitive For writing equivalence class of R	1+1 2 1
36	**Section-E** (i) P(E/M)=5/7 (ii) P(M/E)=1/2 (iii) P(E∪M)=3/5	 1 1 2
37	(i) 4-x (ii) 4 (iii) 1	1 1 2
38	(i) x=10/3 cm (ii) Maximum Volume=16000/27 cm^3	2 2

KENDRIYA VIDYALAYA SANGATHAN

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Sample Paper 6

Class-XII **Subject-Mathematics (041)**

CHAPTERS	MCQ	A/R QNS	2 M	3 M	5 M	CBQ	TOT
RELATIONS AND FUNCTIONS	1		1		1		**3**
INVERSE TRIGONOMTRIC FUNCTIONS		1					**1**
MATRICES	3						**3**
DETERMINANTS	2				1		**3**
CONTINUITY AND DIFFERENTIABLIT Y	1		1	1			**3**
APPLICATION OF DERIVATIVE	2					1	**3**
INTEGRALS	2		1	1	1		**5**
APPLICATION OF INTEGRALS			1	1			**2**
DIFFERENTIAL EQUATIONS	2			1			**3**
VECTOR ALGEBRA	1		1				**2**
THREE DIMENSIONAL GEOMETRY	1	1			1	1	**4**
LINEAR PROGRAMMING	2			1			**3**
PROBABLITY	1			1		1	**3**
TOTAL	**18**	**2**	**5**	**6**	**4**	**3**	**38**

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 6

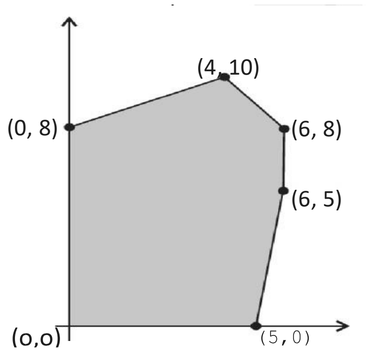
<u>CLASS</u> – XII **<u>MAX.MARKS</u> – 80**

<u>SUB</u>. – MATHEMATICS (Code – 041) **<u>TIME</u> – 03 HOURS**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
2. Section **A** has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section **B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section **C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section **D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section **E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Q. No	Question <u>SECTION – A</u> (Multiple Choice Questions) Each question carries 1 mark.	Marks
1	The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is: (a) 27 (b) 18 (c) 81 (d) 512	1
2	If A = [a_{ij}] is a symmetric matrix of order n, then) $a_{ij} = 1/a_{ij}$ for all i,j (b) $a_{ij} \neq 0$ for all i,j $a_{ij} = a_{ji}$ for all i,j (d) $a_{ij} = 0$ for all i, j	1
3	Let A be a non singular square matrix of order 3 × 3. Then |*adj* A| is equal to (a) $\|A\|$ (b) $\|A\|^2$ (c) $\| A\|^3$ (d) $3\|A\|$	1
4	The area of a triangle with vertices (–3, 0), (3, 0) and (0, *k*) is 9 sq. units. The value of *k* will be (a) 9 (b) 3 (c) – 9 (d) 6	1
5	If A and B are invertible matrices, then which of the following is not correct? (a) *adj* A = $\|A\|. A^{-1}$ (b) $\det(A)^{-1} = [\det (A)]^{-1}$ (c) $(AB)^{-1} = B^{-1} A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
6	The function *f* (*x*) = [*x*], where [*x*] denotes the greatest integer function, is continuous at (a) 4 (b) – 2 (c) 1 (d) 1.5	1

7	Differentiation of($\tan^{-1} x)^2$ is (a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\frac{2\tan^{-1}x}{1+x^2}$ (c) $x\sqrt{1+x^2}$ (d) $\frac{1}{\sqrt{1+x^2}}$	1
8	The rate of change of the area of a circle with respect to its radius *r* at *r* = 6 cm is (a) 10 π (b) 12 π (c) 8 π (d) 11 π	1
9	On which of the following intervals is the function *f* given by $f(x) = x^{100} + \sin x - 1$ decreasing ? (a) (0,1) (b) $(\frac{\pi}{2}, \pi)$ (c) $(0,\frac{\pi}{2})$ (d) None of these	1
10	$\int e^x$ (sec x + tan x) is equal to (a) $e^x \cos x + c$ (b) $e^x \sec x + c$ (c) $e^x \sin x + c$ (d) $e^x \tan x + c$	1
11	The value of $\int_{-a}^{a} \sin^3 x\, dx$ is equal to (a) a (b) a/3 (c) 1 (d) 0	1
12	The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is (a) 3 (b) 2 (c) 1 (d) not defined	1
13	A homogeneous differential equation of the from $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution. (a) *y* = *vx* (b) *v* = *yx* (c) *x* = *vy* (d) *x* = *v*	1
14	If $\vec{a}$ is a nonzero vector of magnitude '*a*' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if (a) $\lambda = 1$ (b) $\lambda = -1$ (c) $a = \lvert \lambda \rvert$ (d) $a = \frac{1}{\lvert\lambda\rvert}$	1
15	The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the *x*-axis are given by (a) (2, 0, 0) (b) (0, 5, 0) (c) (0, 0, 7) (d) (0, 5, 7)	1
16	The feasible solution for a LPP is shown in given figure. Let Z=3x-4y be the objective function. Minimum of Z occurs at a) (0,0) b) (0,8) c) (5,0) d) (4,10)  	1

17	Region represented by x≥ 0,y≥ 0 is: (a) First quadrant (b) Second quadrant (c) Third quadrant (d) Fourth quadrant	1
18	If A and B are two events such that P(A)+P(B)- P(A and B)=P(A), then (a) P(B/A) =1 (b) P(A/B) =1 (c) P(A/B) =0 (d) P(B/A) =0	1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19	A: The Principal value of $\cot^{-1}(-\sqrt{3}) + \tan^{-1}(1) + \sec^{-1}(2/\sqrt{3})$ is equal to $\frac{5\pi}{4}$. R: Domain of $\cot^{-1} x$ and $\sin^{-1} x$ are respectively $(0, \pi)$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.	1
20	A: The following straight lines are perpendicular to each other. $$\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4} \ and \ \frac{1-x}{-1} = \frac{y+2}{2} = \frac{3-z}{3}$$ R: Let line L-l passes through the point $(x_1, y_1, z_1,)$ and parallel to the vector whose direction ratios are a_1, b_1, and c_1, and let line L- 2 passes through the point $(x_2, y_2, z_2,)$ and parallel to the vector whose direction ratios are a_2, b_2, and c_2, · Then the lines L-1 and L-2 are perpendicular if $a_1.a_2 + b_1.b_2 + c_1.c_2 = 0$	1

SECTION – B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21	Check whether the relation R in the set **R** of real numbers, defined as R = {(*a*, *b*) : $a \leq b^2$} is transitive.	2
22	Find $\frac{dy}{dx}$ of the function $y^x = x^y$. **Or**, Find the values of *k* so that the function *f* is continuous at the indicated point $$f(x) = \begin{cases} kx + 5, if & x \leq 2 \\ x - 1, if & x > 2 \end{cases} \ at x = 2$$	2
23	Evaluate: $\int x/(x+1)(x+2)\,dx$	2
24	Find the area of the region in the first quadrant enclosed by X-axis , line x = $\sqrt{3}$ y and the circle $x^2 + y^2 = 4$. **OR** Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.	2
25	If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.	2

	SECTION C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
26	If $y = (\tan^{-1}x)^2$, show that $(1+x^2)^2\ y_2 + (2x)(1+x^2)\ y_1 = 2$	3
27	Evaluate $\int (\sin x \sin 2x \sin 3x)\, dx$ **OR** Evaluate: $\int_{-5}^{5} \lvert x+2 \rvert dx$.	3
28	Find the area of the region bounded by the curve $y = \sin x$ between x = 0 and x = 2π .	3
29	Solve the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$. OR Find the general solution of $(e^x + e^{-x})\, dy - (e^x - e^{-x})\, dx = 0$.	3
30	Solve the following Linear Programming Problem graphically: Maximize and Minimize Z = x + 2y subject to the constraints: x + 2y ≥ 100, 2x - y ≤ 0, 2x + y ≤ 200, x ≥ 0, y ≥ 0.	3
31	Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X). **OR** The random variable X can take only the values 0,1,2,3. Given that P (X = 0) P = (X = 1)= p and P (X = 2)= P (X = 3) such that $\sum pi\ xi = 2 \sum pi\ xi$. Find the value of p.	3
	SECTION D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
32	Show that the function f : R→R defined by $f(x) = \frac{x}{x^2+1}$, ∀ x∈R is neither one-one nor onto. **OR** Let f: W → W be defined by : $f(n) = \{n-1, if\ n\ is\ odd\ n+1, if\ n\ is\ even$. Show that f is one-one and onto .	5
33	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1} . Using A^{-1} Solve the following system of equations by matrix method. 2*x* – 3*y* + 5*z* = 11; 3*x* + 2*y* –4 *z* = -5; *x* + *y* - 2*z* = -3	5
34	Evaluate $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$. **OR** Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x}$ dx .	5
35	Find the shortest distance between the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \alpha(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + \widehat{2k})$ **OR** Find the cartesian equation of the line which passes through the point (-2,4,-5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.	5

SECTION E

(This section comprises of 3 case-study based questions with two sub-parts. First two case study questions have three subparts of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.)

36 Read the following text and answer the following questions, on the basis

of the same:

The relation between the heights of the plant (y in cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

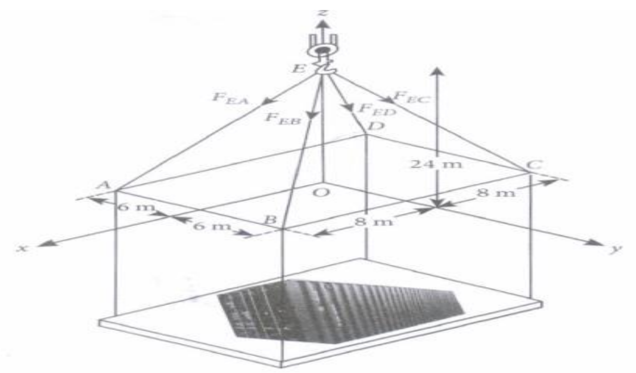
(i) Find the rate of growth of the plant with respect to sunlight. 1

(ii) Is this function satisfy the condition of second order derivative? 1

(iii) What is the number of days it will take for the plant to grow to the maximum height? 2

Or (iii) What is the maximum height of the plant?

37 A pillar is said to be constructed on a field. Radhe is an Engineer for that project . This was Radhe's first project after completing his Engineering. He draws the following diagram of that pillar for the approval.



Consider the following diagram, where the forces in the cable are given.

(i) Write the coordinates of A and B. 1

(ii) Write the coordinates of C and D. 1

(iii) Find the equation of the line along the cable AD. 2

OR

Find the sum of the distances OA, OB and OC.

38	One day, a sangeet mahotsav is to be organised in an open area of Rajasthan. In recent years, it has rained only 6 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time. If leap year is considered, then answer the following questions.  (i) Find the probability that the weatherman predict rain. (ii) Find the probability that it will rain on the chosen day, if weatherman predict rain for that day.	 2 2

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 6

CLASS – XII **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

Marking Scheme

Q. No.	Question **SECTION – A**	Marks
1-20	**1)(c) 2)(c) 3)(a) 4)(c) 5) (d) 6) (d) 7) (b) 8)(b) 9) (c) 10) (a)** **11)(c) 12)(d) 13)(a) 14)(a) 15)(a) 16)(c) 17) (c) 18) (c) 19) (a) 20)(b)**	
21	Not transitive	
22	$\frac{dy}{dx} = \frac{y}{x}\left(\frac{y-x\log y}{x-y\log x}\right)$ **Or** k = -2	
23	log I(x + 2)^2/(x + 1) I + C	
24	Required area = $\int_0^{\sqrt{3}} \frac{1}{\sqrt{3}}$ x dx + $\int_{\sqrt{3}}^{2} \sqrt{4-x^2}$ dx = $\frac{\pi}{3}$ sq unit **Or**, Area bounded by the ellipse = $4\int_0^4 \frac{3}{4}\sqrt{16-x^2}$ dx = 12 π sq unit.	
25	$(\vec{a}\ .\ \vec{b} + \vec{b}\ .\ \vec{c} + \vec{c}\ .\ \vec{a}) = -\frac{3}{2}$	
	SECTION C	
26	$y = (\tan^{-1}x)^2$, differentiating both sides w.r.t. x twice to get the result	
27	$-\frac{\cos 2x}{8} + \frac{\cos 6x}{24} - \frac{\cos 4x}{16} + C$ Or, $\int_{-5}^{5} I\ x + 2\ I$ dx = $\int_{-5}^{-2} -(x+2)$dx + $\int_{-2}^{5}(x+2)$ dx = 29	
28	Required area = $\int_0^{\pi} I \sin x$ I dx + $\int_{\pi}^{2\pi} I \sin x$ I dx = 4 sq unit	
29	y = $\frac{e^{tan^{-1}x}}{2}$+ C $e^{-tan^{-1}x}$ **Or**, y = log I ($e^x + e^{-x}$) I + C	
30	The maximum value of Z is 400at (0,200) and minimum value of Z is 100 at all the points on the line segment joining (0,50) and (20,40)	
31	probability distribution is **X**: 2, 3, 4, 5, 6 **P(X)**: $\frac{1}{15}$, $\frac{2}{15}$, $\frac{3}{15}$, $\frac{4}{15}$, $\frac{1}{3}$ Expectation of X = E(X) = ΣXP(X)= 2 x $\frac{1}{15}$+3 x $\frac{2}{15}$+4 x $\frac{3}{15}$+5 x $\frac{4}{15}$+6 x $\frac{1}{3}$ = $\frac{2+6+12+20+30}{15}$ = $\frac{70}{15}$ **Or**, P = $\frac{3}{8}$	

	SECTION D	
32	Since f(1) = $\frac{1}{2}$ and f(-1) = $\frac{-1}{2}$, but 1 ≠ -1**, f is not one-one.** And there is no real number x such that f(x) equals any positive real number. Hence, **f is not onto**. **Or**, show that f(n) = f(m) => n = m so f is one-one. For onto show codomain = range	
33	$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ Solving by matrix method , x = 1, y= 2, z = 3	
34	Let I = $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$ ....(i) I = $\int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}\, dx$ or I = $\int_0^{\pi} \frac{\pi - x}{1+\sin x}\, dx$ ....(ii) On adding eqs. (i) and (ii) ,we get 2 I = $\int_0^{\pi} \frac{\pi}{1+\sin x}\, dx$ after solving I = π **Or**, Let I = $\int_0^{\frac{\pi}{2}} \frac{sin^2 x}{\sin x+\cos x}$ dx -------------------- (i) = $\int_0^{\frac{\pi}{2}} \frac{sin^2(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)+\cos(\frac{\pi}{2}-x)}$ dx or I = $\int_0^{\frac{\pi}{2}} \frac{cos^2 x}{\sin x+\cos x}$ dx ....(ii) Adding equn. (i) and (ii) 2 I =$\int_0^{\frac{\pi}{2}} \frac{1}{\sin x+\cos x}$ dx after solving I = $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$	
35	Shortest distance = $\frac{\sqrt{2}}{2}$ units **Or**, $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$ is the equation of the required line.	
	SECTION E	
36	(i) 4−*x* cm/day. ii) Yes, the function satisfies the condition of the second-order derivative because the second derivative is a constant value. iii) it will take 4 days for the plant to grow to the maximum height. **Or**,Therefore, the maximum height of the plant is 8 cm.	
37	i) The coordinates of point A and B are (8,10,0) and (-6,4,0) respectively. ii) The coordinates of point C and D are (15,-20,0) and (0,0,30) respectively. iii) $\frac{x}{4} = \frac{y}{5} = \frac{30-z}{15}$ **Or**, the sum of the distances OA,OB,OC = $\sqrt{164}+\sqrt{52}+25$ = $(2\sqrt{41}+2\sqrt{13}+25)$units	
38	(i) 0.2098 (ii) 0.0625	

KENDRIYA VIDYALAYA SANGATHAN

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Sample Paper 7

Class-XII **Subject-Mathematics (041)**

Chapters	1 mark	2 marks	3 marks	5 marks	4 marks	Total questions	Total marks
Relation and Function	1				1 (2+2)	5	8
ITF	1	1 OR				3	
Matrix	3 (AR)			1		8	10
Determinant	2					2	
Continuity	1					1	35
Derivative	1	1	1			6	
Appl of derivative	1 (AR)	2 (OR)			1 (1+1+2)	9	
Integration	1		1 (OR)	1 (OR)		9	
Appl. Of Integral				1		5	
Diff Equation	2		1 (OR)			5	
Vector	1	1	1			7	14
3d Geometry	3			1 (OR)		7	
LPP	2		1 (OR)			5	5
Probability	1	1	1		1(1+1+2)	8	8
Total	20	5	6	4	3	38	80

KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 7

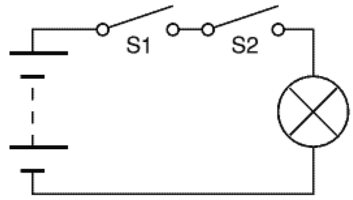
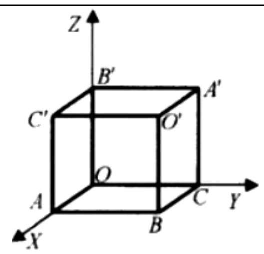
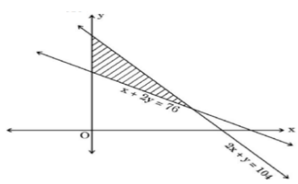
CLASS – XII **MAX.MARKS – 80**

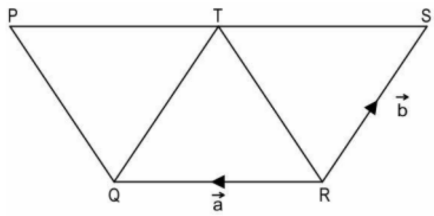
SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

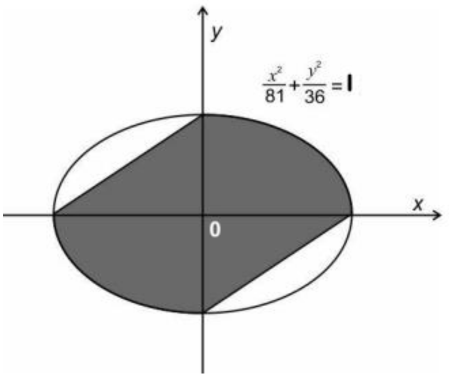
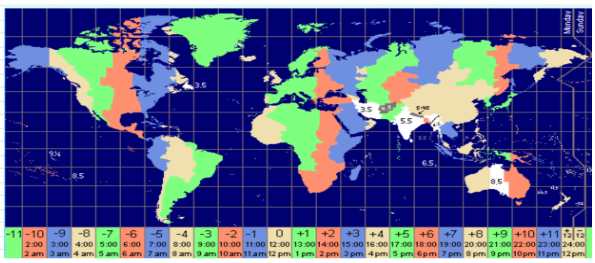
General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts

	SECTION A (20 X 1)	
1	If $\begin{pmatrix} 1 & 5 & 3 \\ 4 & 6 & 2 \end{pmatrix}$ $and\ B = \begin{pmatrix} 5 & 3 \\ 1 & -1 \\ 2 & 4 \end{pmatrix}$ then value of $\sum_{i=1}^{3} a_{2i} b_{i1} =$ (a) 30 (b) 16 (c) 14 (d) 32	
2	Let A= {1,2,3,…30}. A relation R is defined in A X A by $(a,b)R(c,d) \Leftrightarrow a+d=b+c$ then number of elements related (1,3) **(a) 26** **(b) 17** **(c) 28** **(d) 21**	
3	Let $A = \begin{pmatrix} 2 & 3 \\ \alpha & 0 \end{pmatrix} = P + Q$ where P is symmetric matrix and Q is skew symmetric matrix and |Q| = 9 then value of α may be (a) 9 (b) 3 (c) 0 (d) 4	
4	Which one of the following points is at a distance of 5 units from (1,5,3) and lies on the line $\frac{x-1}{2} = \frac{y-5}{6} = \frac{z-3}{3}$ (a) $(2, 8, \frac{9}{2})$ (b) $\left(\frac{17}{7}, \frac{65}{7}, \frac{36}{7}\right)$ (c) (6,5,3) (d) (0,2,4)	
5	If $A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$ Then $\|A^2 - 2A\| =$ (a) 5 (b) 25 (c) -5 (d) -25	
6	The number of real solution of the equation $\begin{vmatrix} sinx & cosx & cosx \\ cosx & sinx & cosx \\ cosx & cosx & sinx \end{vmatrix} = 0$ lying in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is (a) 0 (b) 1 (c) 2 (d) 3	
7	If $f(x) = \frac{1}{1-x}$ then the set of points of discontinuity of $f(f(f(x)))$ is (a) {1} (b) {0,1} (c) {-1,1} (d) Ø	

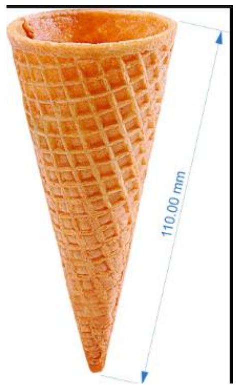
8	$\int_{-2023}^{2023} x^{2023}dx =$ (a) 0 (b) 2023 (c) -1 (d) $\frac{2023}{2024}$	
9	Let $S_n(x) = 1 + x + x^2 + x^3 + \cdots x^n, \lvert x \rvert < 1$ then $S_\infty{}'(\frac{1}{2})$ (where f ' (x) denotes derivative of f(x)) (a) 4 (b) 1 (c) $\frac{1}{2}$ (d)	
10	Let $y = y(x)$ be the solution of differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2, y(0) = 0$, then y(1) = (a) $tan\frac{5}{4}$ (b) $tan\frac{3}{2}$ (c) $tan\frac{1}{2}$ (d) $tan\frac{\pi}{4}$	
11	The general solution of the differential equation $ydx - xdy = 0$; is of the form (a) $xy = c$ (b) $x = c\,y^2$ (c) $y = cx$ (d) $y = cx^2$	
12	Two switches S_1, S_2 have respectively 80% and 90% chances of working. The probabilities that circuit of the figure will work  (a) $\frac{18}{25}$ (b) 49/50 (c) 20/25 (d) 35/50	
13	$sin\left(cos^{-1}\left(tan\frac{\pi}{4}\right)\right) =$ (a) 0 (b) π (c) 2π (d) $-\pi$	
14	The figure represents a unit cube with one corner at origin. The direction cosines of the vector $\overrightarrow{OO'}$ is  (a) $\langle\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle$ (b) $\langle-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\rangle$ (c) $\langle-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle$ (d) $\langle\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{3}}\rangle$	
15	For which value of $\vec{a}$ the equation $\vec{a}.\hat{\imath} = \vec{a}.(\hat{\imath} + \hat{\jmath})$ =$\vec{a}.(\hat{\imath} + \hat{\jmath} + \hat{k})$ is satisfied (a) $\hat{\imath}$ (b) $\hat{\imath} + \hat{\jmath}$ (c) $\hat{\imath} + \hat{\jmath} + \hat{k}$ (d)) $\hat{\jmath}$	
16	The corner points of feasible region of the LPP $2x + y \leq 7, x + y \leq 4, x \geq 0, y \geq 0$ is (a) (0,0) , (4,0), (7,1), (0,4) (b) (o,0), (7/2,0), (3,1), (0,4) (c) (0,0) , (0,7), (3,1), (0,4) (d) (0,0), (7/2,0), (3,1, (4,0)	
17	Of the following , which group of constraints represents the feasible region  a) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$ (b) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$ (c) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$ (d) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$	

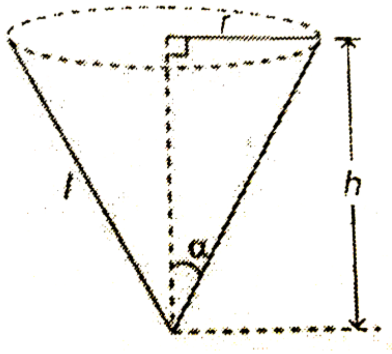
18	The direction ratios of the line $3x + 1 = 6y - 2 = 1 - z$ are (a) $(3, 6, 1)$ (b) $(3, 6, -1)$ (c) $(2, 1, 6)$ (d) $(2, 1, -6)$	
The following questions consist of two statements – Assertion (A) and Reason (R). Answer these questions selecting the appropriate option given below: **(a) Both A and R are true and R is the correct explanation for A.** **(b) Both A and R are true and R is not the correct explanation for A.** **(c) A is true but R is false.** **(d) A is false but R is true.**		
19	**Assertion (A):** The maximum value of the function $f(x) = x^3, x \in [-1, 1]$, is attained at its stationary point, x = 0. **Reason (R) :** for maximum value of a function $f(x)$ at a point $f'(x) = 0$ at the point	
20	**Assertion (A):** Adj(AdjA) = $\|A\|^{n-2}A$ for any square matrix A **Reason (R) :** A.AdjA = |A|I_n	
Section B		
21	Draw the graph of $\boldsymbol{y = sin^{-1}(x-1)}$ in the principal range OR Find the value of $\boldsymbol{tan^2}(\boldsymbol{sec^{-1}4}) + \boldsymbol{cot^2}(\boldsymbol{cosec^{-1}5})$	**2**
22	Find $dy\ dx$ if $= (e^{secx} + x)^4$.	**2**
23	At what point of the ellipse $16x^2 + 9y^2 = 400$ does the ordinate decrease at the same rate at which abscissa increases.	**2**
24	If $f(x) = e^{(cos^2x + cos^4x + cos^6x + \cdots \infty) log_e 2}$, find whether f(x) will increase or decrease in $\left(0, \frac{\pi}{2}\right)$. OR Find the interval in which the function is increasing or decreasing $(x) = log_e(1+x) - \frac{x}{1+x}$.	**2**
25	QRST and QRTP are parallelograms. Using the vectors shown for $\overrightarrow{RQ}$ and $\overrightarrow{RS}$, prove that the area of QRST is equal to the area of QRTP.  	**2**
Section C		
26	If $y = \sin(2sin^{-1}x)$ then prove that $(1 - x^2)y_2 - xy_1 + 4y = 0$	**3**
27	Evaluate: $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$ **Or** Evaluate: $\int_0^{\pi} \frac{x}{1+sinx} dx$	**3**
28	The probability that a married man watches a certain T.V. show is 0.4 and the probability that a married woman watches the show is 0.5. The probability that a man watches the show, given that his wife does, is 0.7. Find the probability that married couples watch the show collectively	**3**
29	Solve the following linear programming problem graphically: $Maximize\ .Z = x + 2y$ Subject to the constraints : $x + y \leq 50, 3x + y \leq 90, x \geq 0, y \geq 0$. **Or** Solve the following Linear Programming Problem graphically: Maximize $z = 2.5x + 1.5y + 410$ Subject to $x \leq 60, y \leq 50, 60 \leq x + y \leq 100, x \geq 0, y \geq 0$	**3**

30	If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath}$ *and* $\vec{c} = \hat{\imath} - \hat{\jmath} - \hat{k}$. Let $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$ then find $\|\vec{r}\|$.	3
31	Solve the differential equation $ydx + (x - y^2)dy = 0$ **OR** Solve differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$	3
	Section D	
32	If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ then find product AB. Use this solve the following system of equations $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$	5
33	$Evaluate \int_0^{\frac{\pi}{2}} logsinxdx$ **OR** $Evaluate \int \frac{x^2+1}{(x^2+2)(x^2+4)} dx$	5
34	 Given Figure in the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Find the area of shaded region with the help of integral.	5
35	Find the coordinate of the image of the point Q (1,6,3) w.r.t the line $\vec{r} = (\hat{\jmath} + 2\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ **OR** An eagle is flying along the line $\frac{x-5}{1} = \frac{y-1}{-1} = \frac{z-8}{-3}$ and a snake is crawling along a path $x = 1 + 2s, y = -4 + s, z = 8 - 2s$. Will the eagle able to catch the snake? If yes find the coordinate of the point where the snake will be caught.	5
	SECTION E	
36	The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone. A relation R is defined on the set U = {All people on the Earth} such that R = {(x, y)| the time difference between the time zones x and y reside in is 6 hours}.  Based on this information answer the following question	4

(i) Is the relation is reflexive? **1**
(ii) Check whether the relation is symmetric. **1**
(iii) Is the relation is Transitive **2**

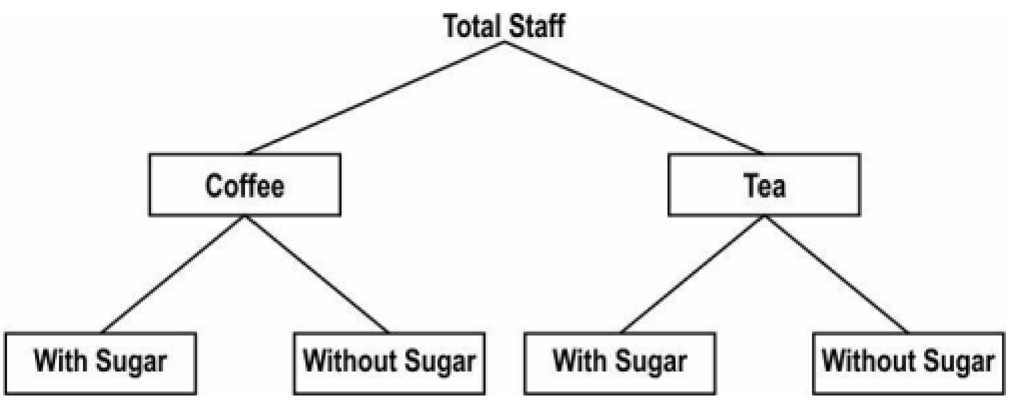
37 Priyanka is very fond of ice cream cone. She selected an icecream of slant height of 110 mm as shown in figure. She wants to calculate the criteria for maximum volume of cream. Help her by answering the following questions





(a) If α is the semi vertical angle of cone. Find radius and height of cone **1**
(b) Find volume of cone as a function of α **1**
(c) Find value of α for maximum volume of cream. **2**

38 A school conducted a survey of their school staff to find their beverage preferences. Each of them picked either tea or coffee as their first preference and then with sugar or without sugar as their second preference as shown in the below tree diagram. Based on the information answer the question that follow



Some of the
insights from the survey are given below.

♦ 60% percent of the staff prefer coffee.
♦ 90% of those who prefer coffee prefer it with sugar.
♦ 20% of those who prefer tea prefer it without sugar.

i) What is the probability that a person selected randomly from the staff prefers a beverage with sugar? **2**
ii) ii) What is the probability that a person from the staff selected at random prefers coffee given that it is without sugar? **2**

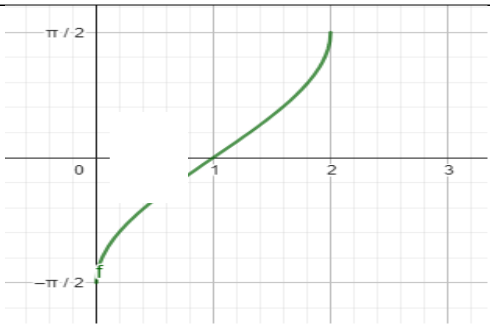
KENDRIYA VIDYALAYA SANGATHAN

SAMPLE QUESTION PAPER 7

CLASS – XII **MAX.MARKS – 80**

SUB. – MATHEMATICS (Code – 041) **TIME – 03 HOURS**

Marking Scheme

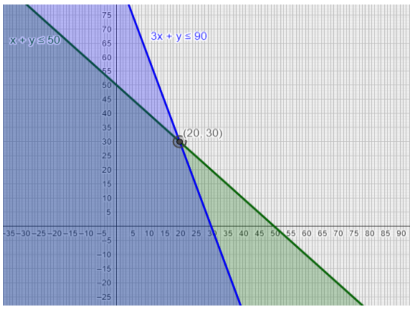
1-20	**1) (a) 2)(c) 3)(a) 4) (b) 5)(b) 6)(c) 7)(b) 8)(a) 9)(a) 10)(b)** **11)(c) 12)(a) 13)(a) 14)(a) 15)(a) 16)(b) 17)(d) 18)(d) 19)(d) 20)(a)**	
	Section B	
21)	 OR $tan^2(sec^{-1}4) + cot^2(cosec^{-1}5)$ $= (sec^2(sec^{-1}4) - 1) + (cosec^2(cosec^{-1}5) - 1)$ $= (16 - 1) + (25 - 1)$ $= 39$	**2** **1** **0.5** **0.5**
22)	$\frac{dy}{dx} = 4\,(e^{secx} + x)^3(e^{secx} + x)'$ $= 4\,(e^{secx} + x)^3(e^{secx}secxtanx + 1)$	**1** **1**
23)	$16\left(2x\frac{dx}{dt}\right) + 9\left(2y\frac{dy}{dt}\right) = 0$ $\Rightarrow 32x\frac{dx}{dt} - 18y\frac{dx}{dt} = 0 \quad \left(\because \frac{dx}{dt} = -\frac{dy}{dt} \neq 0\right)$ $\Rightarrow 32x - 18y = 0$ $\Rightarrow y = \frac{16x}{9}$ Putting the value of y in $16x^2 + 9y^2 = 400$, we get $x = \pm 3, y = \pm\frac{16}{3}$ points are (3,16/3) $(-3, -\frac{16}{3})$	**0.5** **0.5** **0.5** **0.5**
24)	$f(x) = e^{\left(\frac{cos^2x}{1-cos^2x}\right)log_e 2} = e^{cot^2x\,log_e 2} = 2^{cot^2x}$ $f'(x) = 2^{cot^2x}(log2)(2cotx)(-cosec^2x) < 0\ in\ \left(0, \frac{\pi}{2}\right)$ So f(x) will decrease in given interval OR $f(x) = log_e(1 + x) - \frac{x}{1+x} \Rightarrow f'(x) = \frac{1}{1+x} - \frac{1}{(1+x)^2} = \frac{x}{(1+x)^2}$ Increasing interval $(0, \infty)$ Decreasing interval $(-1,0)$	**1** **0.5** **0.5** **1** **0.5** **0.5**

25)	Ar(QRST) = $\lvert\vec{b}\times\vec{a}\rvert$ ar(QRTP) = $\lvert\overrightarrow{RT}\times\overrightarrow{RQ}\rvert$ $=\lvert(\vec{b}+\vec{a})\times\vec{a}\rvert$ $=\lvert\vec{b}\times\vec{a}+\vec{a}\times\vec{a}\rvert$ $=\lvert\vec{b}\times\vec{a}\rvert$	**0.5** **0.5** **1**
	Section C	
26)	$y=\sin(2sin^{-1}x)$ $\Rightarrow y_1=\cos(2sin^{-1}x).\frac{2}{\sqrt{1-x^2}}$ $\Rightarrow (1-x^2){y_1}^2=4\cos^2(2sin^{-1}x)$ $\Rightarrow (1-x^2){y_1}^2=4(1-y^2)$ $\Rightarrow\ (1-x^2)y_2-xy_1+4y=0$	**1** **0.5** **0.5** **1**
27)	$\mathrm{I}=\int\frac{\cos x}{(1-\sin x)\,(2-\sin x)}\,dx$ Let $\sin x=t\ \ \therefore \cos x\,dx=dt$ $\therefore \mathrm{I}=\int\frac{1}{(1-t)(2-t)}\,dt\ldots\ldots\ldots\ldots\ldots.(1)$ Let $\frac{1}{(1-t)(2-t)}=\frac{\mathrm{A}}{(1-t)}+\frac{\mathrm{B}}{(2-t)}\ldots\ldots\ldots\ldots\ldots(2)$ $\Rightarrow\therefore$ A $=1$ and B $=-1$ Thus from (1) and (2), we get $\mathrm{I}=\int\left[\frac{1}{(1-t)}+\frac{-1}{(2-t)}\right]dt$ $=\int\frac{1}{1-t}\,dx-\int\frac{1}{2-t}\,dx$ $=-\log\lvert 1-t\rvert+\log\lvert 2-t\rvert+\mathrm{C}$ $=\log\left\lvert\frac{2-t}{1-t}\right\rvert+\mathrm{C}=\log\left\lvert\frac{2-\sin x}{1-\sin x}\right\rvert+\mathrm{C}$ Or $I=\int_0^{\pi}\frac{x}{1+\sin x}dx=\int_0^{\pi}\frac{\pi-x}{1+\sin x}dx$ $2I=2\pi\int_0^{\frac{\pi}{2}}\frac{1}{1+\sin x}dx=\pi\int_0^{\frac{\pi}{2}}\frac{1-\sin x}{\cos^2 x}dx$ $=\pi\int_0^{\frac{\pi}{2}}(\sec^2 x-\sec x\tan x)dx$ $=\pi[\tan x-\sec x]_0^{\frac{\pi}{2}}=\pi\left[\frac{\sin x-1}{\cos x}\right]_0^{\frac{\pi}{2}}=2\pi\Rightarrow I=\ \pi$	0.5 0.5 0.5 0.5 1 **1** **1** **0.5** **0.5**
28)	M = man watches TV show W = his wife watches TV show. P(M) = 0.4, P(W) = 0.5 P(M/W) = 0.7 $P(M\cap W)=P\left(\frac{M}{W}\right)P(W)$ $=0.7\times 0.5=0.35$	**1** **0.5** **1** **0.5**

29) drawing graph and shading **2**

drawing per line 0.5 and shading correctly 0.5 each

finding intersection pt (20,30)

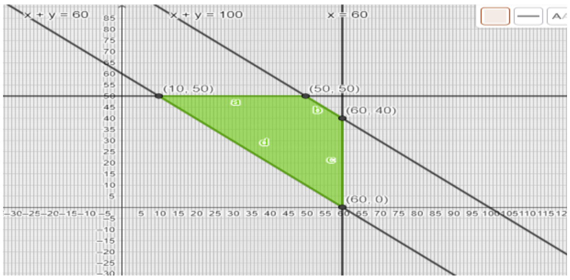


pt	X+2y
(0,0)	0
(30,0)	30
(20,30)	80
(0,50)	100 max

1

Or

Similar approach



pt	2.5x+1.5y+410
(10,50)	510
(50,50)	610 max
(60,40)	620
(60,0)	90

30) Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{r} \times \vec{a} = \hat{\imath}(-y - 2z) + \hat{\jmath}(z + x) + \hat{k}(2x - y)$, **0.5**

$\vec{c} \times \vec{a} = \widehat{3\imath} + \widehat{3k}$ **0.5**

$\vec{r}.\vec{b} = 0 \Rightarrow x - y = 0$ **0.5**

$\therefore 2x - y = 3, \quad z + x = 0, -y - 2z = 3$

Solving we get x=3, y= 3 z = -3 **1**

Hence $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$ **0.5**

31) $ydx + (x - y^2)dy = 0$ **)**

$\Rightarrow \quad y\frac{dx}{dy} + x = y^2$ linear in x **1**

$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{y} = y$ spotting p and Q **0.5**

Int factor = $e^{\int \frac{1}{y} dy} = y$ **1**

General solution $y.x = \int y^2 dy = \frac{y^3}{3} + c$ **0.5**

Or

$xdy - ydx = \sqrt{x^2 + y^2}dx$

$\Rightarrow xdy = \left(y + \sqrt{x^2 + y^2}\right)dx$

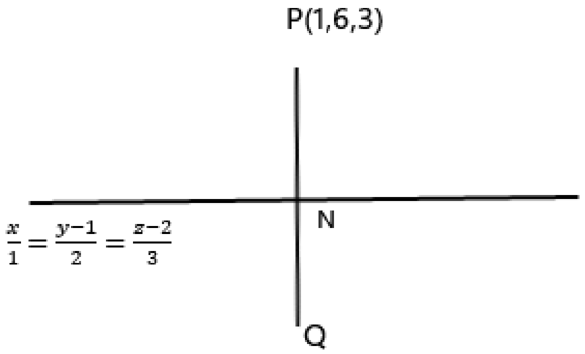
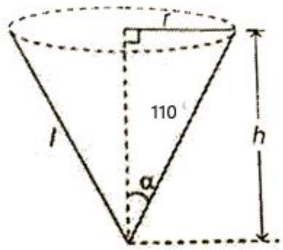
$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}$ **0.5**

Putting y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ **0.5**

$v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$

$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$ **1**

	$\Rightarrow \log\left(v+\sqrt{1+v^2}\right) = logx + logc$ $\Rightarrow v+\sqrt{1+v^2} = cx$ $\Rightarrow \frac{y}{x}+\frac{\sqrt{y^2+x^2}}{x} = cx$	**0.5** **0.5**
	Section D	
32)	AB=6I $\Rightarrow A^{-1} = \frac{B}{6}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ Solution $x = 2, y = -1, z = 4$	**2** **1** **1** **1**
33)	Let $I = \int_0^{\frac{\pi}{2}} logsinxdx$ Using prop. $I = \int_0^{\frac{\pi}{2}} logcosxdx$, Adding $2I = \int_0^{\frac{\pi}{2}} logsinx\ cosxdx = \int_0^{\frac{\pi}{2}} logsin2x/2)2dx$ $\therefore 2I = \frac{1}{2}\int_0^{\pi} logsintdt - \frac{\pi}{2}log2 \quad substituting\ t = 2x$ $2I = \frac{1}{2}\times 2\int_0^{\frac{\pi}{2}} logsintdt - \frac{\pi}{2}log2$ $using\ property \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \quad f(2a-x) = f(x)$ $\therefore 2I = I - \frac{\pi}{2}log2 \Rightarrow I = -\frac{\pi}{2}log2$ $I = \int \frac{x^2+1}{(x^2+2)(x^2+4)}dx$ Let $x^2 = y$, then $\frac{x^2+1}{(x^2+2)(x^2+4)} = \frac{y+1}{(y+2)(y+4)}$ Let $\frac{y+1}{(y+2)(y+4)} = \frac{A}{y+2}+\frac{B}{y+4}$ this gives $A = -\frac{1}{2}, B = \frac{3}{2}$ $\therefore I = -\frac{1}{2}\int\frac{1}{x^2+2}dx + \frac{3}{2}\int\frac{1}{x^2+4}dx$ $\Rightarrow I = -\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{4}\tan^{-1}\left(\frac{x}{2}\right)+c$	**1** **1** **1** **1** **1** **1** **1** **1** **2**
34)	*For writing the equation of line passing through (-9,0) and (0,6)* $-\frac{x}{9}+\frac{y}{6} = 1$ *Area of shaded region=* $2(\int_{-9}^{0} y_{st\ line} + \int_0^9 y_{ellipse})$ $= 2\left(\int_{-9}^{0}\frac{2}{3}(9+x)dx + \int_0^9 \frac{2}{3}\left(\sqrt{9-x^2}\right)dx\right)$	**1** **1.5**

	$=2\left(\frac{2}{3}\left(9x+\frac{x^2}{2}\right)_{-9}^{0}\right)+2\times\frac{2}{3}\left(\frac{x}{2}(\sqrt{81-x^2}+\frac{81}{9}sin^{-1}(\frac{x}{9})\right)_{0}^{9}$ $=2\times 27+2\times\frac{27\pi}{2}=27(\pi+2)sq\ units$	**1.5** **1**
35)	Equation of line in cartesian form $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$  Any point N on the line N ($\lambda,2\lambda+1,3\lambda+2$) D.r of PN ($\lambda-1,2\lambda-5,3\lambda-1$), but PN IS PERPENDICULAR TO GIVEN LINE $\therefore 1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0\Rightarrow\lambda=1$ Hence N(1,3,5). Now N is midpoint of P and Q, where Q($\alpha,\beta,,\gamma$) is image of P, $\therefore\frac{\alpha+1}{2}=1,\frac{\beta+6}{2}=3,\frac{\gamma+3}{2}=5\rightarrow\alpha=1,\beta=0,\gamma=7$ Therefore image of P is (1,0,7)	**1** **1** **1** **0.5** **0.5** **1**
	SECTION E	
36)	The relation can be defined as $xRy\ iff\ \lvert x_T-y_T\rvert\le 6, wher\ x_T=time\ at\ x's\ place$ (i) As $\lvert x_T-x_T\rvert=0$ so relation is reflexive (ii) If $(x,y)\in R\Rightarrow\lvert x_T-y_T\rvert\le 6\Rightarrow\lvert y_T-x_T\rvert\le 6\Rightarrow(y,x)\in R$ symmetric (iii) Let $(x,y)\in R,\ (y,z)\in R\Rightarrow\lvert x_T-y_T\rvert\le 6,\ \lvert y_T-z_T\rvert\le 6\nRightarrow\lvert x_T-z_T\rvert\le 6$ so this relation is not transitive give example	 **1** **1** **2**
37)	(a) r=110 $sin\alpha\quad h=110cos\alpha$ from figure (b) V = $\pi r^2h=\pi(110)^3sin^2\alpha cos\alpha$ (c) $\frac{dV}{d\alpha}=\pi(110)^3(2sin\alpha cos^2\alpha-sin^3\alpha)$ For max or min $\frac{dV}{d\alpha}=0$ $\Rightarrow 2sin\alpha cos^2\alpha-sin^3\alpha=0$ $\Rightarrow tan^2\alpha=2$ $\Rightarrow\alpha=tan^{-1}\sqrt{2}$  	**1** **1** **1** **1**
38) (i)	Takes P(S), P(C) and P(T) as the probabilities that a person selected randomly from the staff prefers sugar, coffee and tea respectively. Finds P(T) = P(C′) = 1 − 0.6 = 0.4. Finds P(S|T) = 1 − 0.2 = 0.8. Uses theorem on total probability and finds the probability that a randomly selected staff prefers a beverage with sugar as: P(S) = P(C) × P(S|C) + P(T) × P(S|T) $=0.6\times 0.9+0.4\times 0.8=0.86$ or $\frac{86}{100}$ or $\frac{43}{50}$	 **1** **1**

(ii)	Uses the sum of probabilities = 1 and finds the following probabilities: ◆ P(without sugar|coffee) = 1 − 0.9 = 0.1 ◆ P(tea) = 1 − 0.6 = 0.4 Uses Bayes' theorem to find the probability that a staff selected at random prefers coffee given that it is without sugar, P(coffee|without sugar) as: $\frac{P(\text{coffee}) \times P(\text{without sugar}\mid\text{coffee})}{P(\text{coffee}) \times P(\text{without sugar}\mid\text{coffee}) + P(\text{tea}) \times P(\text{without sugar}\mid\text{tea})}$ $= \frac{0.6 \times 0.1}{0.6 \times 0.1 + 0.4 \times 0.2}$ (Award 0.5 marks if only the formula for Bayes' theorem is written correctly.) Simplifies the above expression and finds the required probability as $\frac{6}{14}$ *or* $\frac{3}{7}$.	**0.5** **0.5** **1**

तत् त्वं पूषन् अपावृणु

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