केन्द्रीय विद्यालय संगठन
Kendriya Vidyalaya Sangathan

गणित
Mathematics

कक्षा/Class: IX
2024-25

विद्यार्थी अध्ययन सामग्री
Student Support Material

संदेश

    विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना केन्द्रीय विद्यालय संगठन की सर्वोच्च वरीयता है। हमारे विद्यार्थी, शिक्षक एवं शैक्षिक नेतृत्व कर्ता निरंतर उन्नति हेतु प्रयासरत रहते हैं। राष्ट्रीय शिक्षा नीति 2020 के संदर्भ में योग्यता आधारित अधिगम एवं मूल्यांकन संबन्धित उद्देश्यों को प्राप्त करना तथा सीबीएसई के दिशा निर्देशों का पालन, वर्तमान में इस प्रयास को और भी चुनौतीपूर्ण बनाता है।

    केन्द्रीय विद्यालय संगठन के पांचों **आंचलिक शिक्षा एवं प्रशिक्षण संस्थान** द्वारा संकलित यह 'विद्यार्थी सहायक सामग्री' इसी दिशा में एक आवश्यक कदम है । यह सहायक सामग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है । केन्द्रीय विद्यालय संगठन की 'विद्यार्थी सहायक सामग्री' अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री-संकलन की विशेषज्ञता के लिए जानी जाती है और अन्य शिक्षण संस्थान भी इसका उपयोग परीक्षा संबंधी पठन सामग्री की तरह करते रहे हैं । शुभ-आशा एवं विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर सतत मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुंचाएगी ।

शुभाकांक्षा सहित ।

निधि पांडे
आयुक्त, केन्द्रीय विद्यालय संगठन

LIST OF CONTRIBUTORS

S.N.	NAME OF CHAPTER	NAME OF THE TEACHERS WHO HAVE BEEN ALLOTTED THE FOLLOWING CHAPTERS	NAME OF K.V.	NAME OF REVIEWING MEMBER	NAME OF K.V.
1	NUMBER SYSTEMS	MRS. MAITREYEE TRIPATHI	K.V. HAPPY VALLEY SHILLONG	MRS. RAJNI SHARMA	K.V. EAC UPPER SHILLONG
2	POLYNOMIALS	MRS. MAITREYEE TRIPATHI	K.V. HAPPY VALLEY SHILLONG	MRS. RAJNI SHARMA	K.V. EAC UPPER SHILLONG
3	COORDINATE GEOMETRY	MRS. RAJNI SHARMA	K.V. EAC UPPER SHILLONG	MR. VED PRAKASH	K.V. HAPPY VALLEY SHILLONG
4	LINEAR EQUATIONS IN TWO VARIABLES	MRS. RAJNI SHARMA	K.V. EAC UPPER SHILLONG	MR. VED PRAKASH	K.V. HAPPY VALLEY SHILLONG
5	INTRODUCTION TO EUCLID'S GEOMETRY	MR. VED PRAKASH	K.V. HAPPY VALLEY SHILLONG	MRS. MAITREYEE TRIPATHI	K.V. HAPPY VALLEY SHILLONG
6	LINES AND ANGLES	MR. VED PRAKASH	K.V. HAPPY VALLEY SHILLONG	MRS. MAITREYEE TRIPATHI	K.V. HAPPY VALLEY SHILLONG
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12	STATISTICS	MR. S K GAUTAM	K.V. HAPPY VALLEY SHILLONG	MRS. RAJNI SHARMA	K.V. EAC UPPER SHILLONG

INDEX

Curriculum

MATHEMATICS (IX-X) (CODE NO. 041)
Session 2024-25

 The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in the Focus Group on Teaching of Mathematics which is to meet the emerging needs of all categories of students. For motivating the teacher to relate the topics to real life problems and other subject areas, greater emphasis has been laid on applications of various concepts. The curriculum at Secondary stage primarily aims at enhancing the capacity of students to employ Mathematics in solving day-to-day life problems and studying the subject as a separate discipline. It is expected that students should acquire the ability to solve problems using algebraic methods and apply the knowledge of simple trigonometry to solve problems of height and distances. Carrying out experiments with numbers and forms of geometry, framing hypothesis and verifying these with further observations form inherent part of Mathematics learning at this stage. The proposed curriculum includes the study of number system, algebra, geometry, trigonometry, mensuration, statistics, graphs and coordinate geometry, etc. The teaching of Mathematics should be imparted through activities which may involve the use of concrete materials, models, patterns, charts, pictures, posters, games, puzzles and experiments. Objectives The broad objectives of teaching of Mathematics at secondary stage are to help the learners to:

• consolidate the Mathematical knowledge and skills acquired at the upper primary stage;

 • acquire knowledge and understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles and symbols and underlying processes and skills;

• develop mastery of basic algebraic skills;

 • develop drawing skills;

• feel the flow of reason while proving a result or solving a problem;

 • apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method;

• to develop ability to think, analyze and articulate logically;

 • to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases; • to develop necessary skills to work with modern technological devices and mathematical software's.

 • to develop interest in mathematics as a problem-solving tool in various fields for its beautiful structures and patterns, etc.

 • to develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics;

 • to develop interest in the subject by participating in related competitions;

 • to acquaint students with different aspects of Mathematics used in daily life;

• to develop an interest in students to study Mathematics as a discipline.

COURSE STRUCTURE CLASS –IX

Units	Unit Name	Marks
I	NUMBER SYSTEMS	10
II	ALGEBRA	20
III	COORDINATE GEOMETRY	04
IV	GEOMETRY	27
V	MENSURATION	13
VI	STATISTICS	06
	TOTAL	80

MATHEMATICS QUESTION PAPER DESIGN CLASS – IX (2024-25)

Time: 3 Hrs.   **Max. Marks: 80**

S.NO.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3	**Analysing:** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating**: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	TOTAL	80	100

INTERNAL ASSESSMENT	20 MARKS
Pen Paper Test and Multiple Assessment (5+5)	10 MARKS
Portfolio	05 MARKS
Lab Practical (Lab activities to be done from the prescribed books)	05 MARKS

The changes for classes IX-XII year-end Board Examinations (2024-25)

CLASS -(IX-X)		
Particulars	Academic Session 2023-24	Academic Session 2024-25 (No change from previous academic session)
Composition of question paper for year-end examination/ Board Examination (Theory)	• Competency Focused Questions in the form of MCQs/ Case Based Questions, Source-based Integrated Questions or any other type = 50% • Select response type questions (MCQ) = 20% • Constructed response questions (Short Answer/Long Answer Type Questions, as per existing pattern) = 30%	• Competency Focused Questions in the form of MCQs/Case Based Questions, Source-based Integrated Questions or any other type = 50% • Select response type questions (MCQ) = 20% • Constructed response questions (Short Answer/Long Answer Type Questions, as per existing pattern) = 30%

NUMBER SYSTEMS

Key Points:

The number system is the system of writing numbers that we use in everyday life. In mathematics, numbers are classified into different categories based on their properties and uses.

Types of Numbers

1. **Natural Numbers (N)**: These are the counting numbers. -
    o   Example: 1, 2, 3, 4, ...
2. **Whole Numbers (W)**: Natural numbers along with zero.
    o   Example: 0, 1, 2, 3, ...
3. **Integers (Z)**: Positive and negative whole numbers, including zero.
    o   Example: ..., -3, -2, -1, 0, 1, 2, 3, ...
4. **Rational Numbers (Q)**: Numbers that can be expressed as a fraction of two integers, where the denominator is not zero.
    o   Example: 1/2, -3/4, 5, ...

Irrational Numbers: Numbers that cannot be expressed as fractions and have non-terminating and non-repeating decimal representations.

-   Example: √2, π, ...

⬜ **Real Numbers (R)**: All rational and irrational numbers together.

-   Example: All numbers on the number line.
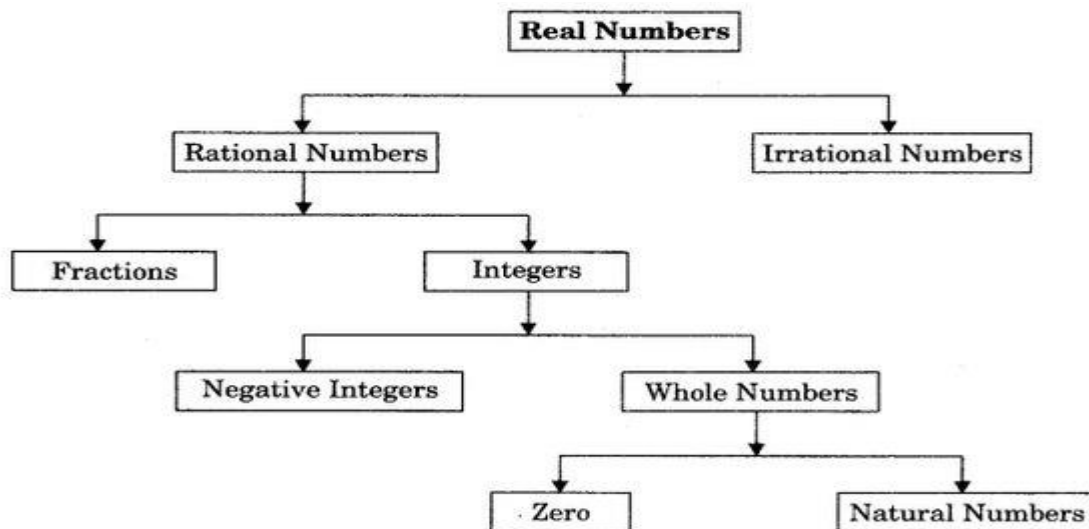
*There are infinitely many rational numbers between any two given rational number.

Decimal Expansion of Real Numbers

Types of Decimal Expansions

1. **Terminating Decimals**:
   - Terminating decimals are decimal numbers that end, i.e., they have a finite number of digits after the decimal point.
   - Example: 0.25, 1.5, 3.75

2. **Non-Terminating, Repeating Decimals (Rational Numbers)**:
   - Rational numbers have decimal expansions that either terminate or repeat indefinitely.
   - Example: 1/3 = 0.333…, 2/7 = 0. 285714285714..

3. **Non-Terminating, Non-Repeating Decimals (Irrational Numbers)**:
   - Irrational numbers cannot be expressed as fractions and have decimal expansions that neither terminate nor repeat.
   - Example: √2 ≈ 1.414213562…, π ≈ 3.141592653589…

Representation of Irrational numbers in number line
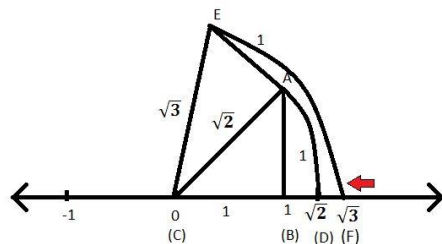
Rationalization: Rationalization of irrational numbers refers to the process of manipulating an expression involving irrational numbers so that the irrational part (usually a square root) is eliminated from the denominator, resulting in a rational expression.

Rationalize $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times 1$$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}}$$

$$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\frac{1}{2+\sqrt{5}} = \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$\frac{1}{2+\sqrt{5}} = \frac{2-\sqrt{5}}{2^2-(\sqrt{5})^2}$$

$$\frac{1}{2+\sqrt{5}} = \frac{2-\sqrt{5}}{4-5}$$

$$\frac{1}{2+\sqrt{5}} = \frac{2-\sqrt{5}}{-1}$$

$$\frac{1}{2+\sqrt{5}} = \sqrt{5}-2$$

Laws of exponents

Product Rule	$a^m \times a^n = a^{m+n}$
Quotient Rule	$a^m \div a^n = a^{m-n}$
Power of a Power Rule	$(a^m)^n = a^{mn}$
Power of a Product Rule	$(ab)^m = a^m b^m$
Power of a Quotient Rule	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
Zero Exponent Rule	$a^0 = 1$
Negative Exponent Rule	$a^{-m} = \dfrac{1}{a^m}$
Fractional Exponent Rule	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Case Study Question:

Real numbers are numbers which include both rational and irrational numbers. Rational numbers are the numbers which can be expressed in the form of p/q where p and q are integers and q is not equal to zero. Irrational numbers are numbers that cannot be expressed as a ratio of two integers.

Q1- Every rational number is a.

      (a) Whole number
      (b) Natural number
      (c) Integers.
      (d) Real numbers.

Q2-The product of two irrational numbers is.

(a)Always rational

(b). Always irrational.

(c)Always integer.

(d) Sometimes rational sometimes irrational.

Q3- Between two rational numbers,

(a). there is no rational number

(b) there is exactly one rational number.

(c) there are infinitely many irrational numbers.

(d) there are no irrational numbers.

Q4- The sum of a rational and an irrational number is.

(a)Rational

(b) Irrational

(c) Both rational and irrational.

(d)None of the above.

Q5-Which of the following is an irrational number?

(a) 3.14
(b) 3.141414…
(c) 3.144444…
(d) 3.141141114…

COMPETANCY BASED QUESTIONS

Q1- A number line consists of an infinite number of points. Points on it are associated with a rational number.

Khushi says: " A point on a number line can represent different forms of a rational number."
Akash says: "I think each point represents a unique number"

Who is correct? Give an example to support your argument.
Q2

Deep draws the spiral of irrational numbers below on a paper.
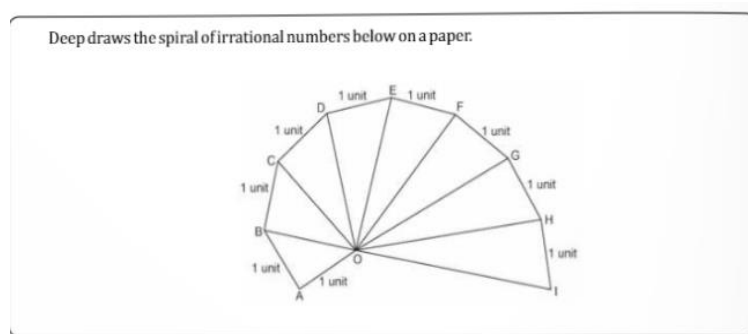
What is length of OE in the spiral?

Q3- Which of the following statements are correct?
 (i) Every irrational number can be written as a fraction.
 (j) Every irrational number can be represented with help of decimals
 (k) Every rational number can be represented as terminating decimal
 (l) Every rational number can be represented as an integer

Q4-Can we write 0 in the form of p/q?

a. Yes

b. No

c. Cannot be explained

d. None of the above

Q5-The number obtained on rationalising the denominator of 1/ ($\sqrt{7}$ – 2) is

a. ($\sqrt{7}$+2)/3
b. ($\sqrt{7}$-2)/3
c. ($\sqrt{7}$+2)/5
d. ($\sqrt{7}$+2)/45
ANSWERS
Case based study:
 Q1-D
Q2-D
Q3-C
Q4-B
Q5-D
CBQS
Q1-FULL CREDIT: both statements are correct as 0.5 can be shown as1/2,8/16 so on
    PARTIAL CREDIT: explanation for any one statements.
Q2-$\sqrt{5}$
Q3-j
 Q4-A
Q5-A

CHAPTER-2
POLYNOMIALS

• Definition: An algebraic expression consisting of variables and coefficients, involving addition, subtraction, multiplication, and non-negative integer exponents.

• Types: Monomial, binomial, trinomial based on the number of terms.

2. **Degree of a Polynomial:**

• The highest power of the variable in a polynomial determines its degree.

• E.g $2x^3+3x+5$ has degree 3

• Degree of a polynomial can be 0 (constant), 1 (linear), 2 (quadratic), 3 (cubic), etc.

•Degree of Zero polynomial is 'Not defined'

• Degree of non-zero constant polynomial is 0.

Classification by Terms	
monomial	*one term: 12, 4x, x^2, -5xy*
binomial	*two terms: 2x - 1, x^2 - 4*
trinomial	*three terms: x^2 + 2x + 1*
polynomial - *one or more terms:* polynomial means "many", but it can also be one term.	
The ending of these words "nomial" is Greek for "part".	
Classification by Degree	
Linear - *degree of 1 or 0:* 3x + 1 or 12	
Quadratic - *degree of 2:* $2x^2$ - x + 7	
Cubic - *degree of 3:* $3x^3$ + $4x^2$ + 3x + 5	

Zero (or Root) of a Polynomial: For a polynomial $P(x)P(x)P(x)$, a number r is called a zero (or root) of the polynomial if $P(r)=0$

Example:

Consider the polynomial $P(x)= x^3-4x-x+4$

Here for x=1, $P(1)=1^3-4×1-1+4=1-4-1+4=0$

So x=1 is zero (root) of the polynomial

Factorisation of Polynomials

Any polynomial of the form p(a) can also be written as p(a) = g(a) × h(a) + R(a)

Dividend = Quotient × Divisor + Remainder

p(a) =g(a) x h(a) if the remainder is zero, that is, the polynomial p(a) is a product of two other polynomials h(a) and g(a). For instance, $3a + 6a^2 = 3a \times (1+2a)$.

A polynomial may also be expressed as the product of two or more polynomials.

Algebraic identities

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
    $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
    $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Questions

Q1-Which one is not a polynomial

(a) $4x^2 + 2x - 1$

(b) $y + \dfrac{3}{y}$

(c) $x^3 - 1$

(d) $y^2 + 5y + 1$

Q2- The zero of the polynomial $p(x) = 2x + 5$ is

(a) 2

(b) 5

(c) $\dfrac{2}{5}$

(d) $-\dfrac{5}{2}$

Q3- The number of zeros of $x^2 + 4x + 2$

   (a) 1

(b) 2

(c) 3

(d) none of these

Q4-The polynomial of type $ax^2 + bx + c$, $a = 0$ is of type

   (a) linear

(b) quadratic

(c) cubic

(d) Biquadratic

Q5-If value of 104 × 96 is

   (a) 9984

(b) 9469

(c) 10234

(d) 11324

Short answer type questions:

  Q1-If $p(x) = x^2 - 2\sqrt{2}x + 1$, then find value of $p(2\sqrt{2})$.

Q2-If a + b + c = 9 and ab + bc + ca = 26, find a² + b² + c²

Q3-Compute the value of $9x^2 + 4y^2$ if $xy = 6$ and $3x + 2y = 12$

Case based study question 1

Ankur and Ranjan start a new business together. The amount invested by both partners together is given by the polynomial $p(x) = 4x^2 + 12x + 5$, which is the product of their individual shares.

Q1-Coefficient of x^2 in the given polynomial is
(a) 2 (b) 3 (c) 4 (d) 12
Q2-**Total amount invested by both, if x = 1000 is**
(a) 301506 (b)370561 (c) 4012005 (d)490621

Q3-**The shares of Ankur and Ranjan invested individually are**
(a) (2x + 1),(2x + 5)(b) (2x + 3),(x + 1) (c) (x + 1),(x + 3) (d) None of the above

Q4-**Name the polynomial of amounts invested by each partner.**
(a) Cubic (b) Quadratic (c) Linear (d) None of these

Case based study question 2

A school is organizing a fundraising event to support a local charity. The students are divided into three groups: Group A, Group B, and Group C. Each group is responsible for collecting donations from different areas of the town.

Group A consists of 30 students and each student is expected to collect 'x' amount of money. The polynomial representing the total amount collected by Group A is given as $A(x) = 2x^2 + 5x + 10$. Group B consists of 20 students and each student is expected to collect 'y' amount of money. The polynomial representing the total amount collected by Group B is given as $B(y) = 3y^2 - 4y + 7$.

Group C consists of 40 students and each student is expected to collect 'z' amount of money. The polynomial representing the total amount collected by Group C is given as $C(z) = 4z^2 + 3z - 2$.

Q1. What is the coefficient of x in the polynomial A(x)?
(a) 2
(b) 5
(c) 10
(d) 0

Q2. What is the degree of the polynomial B(y)?
(a) 2
(b) 3
(c) 4
(d) 1

Q3. What is the constant term in the polynomial C(z)?
(a) 4
(b) 3
(c) -2
(d) 0

Q4. What is the sum of the coefficients of the polynomial A(x)?
(a) 2
(b) 5
(c) 10
(d) 17

Q5. What is the total number of students in all three groups combined?
(a) 30
(b) 20
(c) 40
(d) 90

ANSWERS-

Q1-B

Q2-d

Q3-b

Q4-b

Q5-a

Short answer type

Q1-1

Q2-29

Q3-72

Competency based question

Q1-C

Q2-C

Q3-A

Q4-C

Case based study -2

Q1-B

Q2-B

Q3-C

Q4-C

Q5-C

COORDINATE GEOMETRY

KEY POINTS:

• Co-ordinate Geometry is the branch of Mathematics in which we study the position of any object lying in a plane, called the Cartesian plane.

• In Cartesian system; there are two mutually perpendicular straight lines xx' and yy' intersecting at origin O.

• These mutually perpendicular straight lines, known as x-axis and y-axis, divides the plane into four quadrants.

• The coordinates of a point is the position of the point in Cartesian plane and are determined by perpendicular distance from x-axis and y-axis.

• The perpendicular distance of a point from y-axis is called abscissa (x-coordinate) and from x-axis is called ordinate (y-coordinate).

• Any point in the Cartesian plane is shown by P(a, b) where (a, b) are coordinates of point P.

abscissa (x)	ordinate (y)	Position of point
positive (+)	positive (+)	Quadrant I
positive (+)	negative (−)	Quadrant IV
negative (−)	negative (−)	Quadrant III
negative (−)	positive (+)	Quadrant II

• The coordinate of a point on x-axis is of the form (x, 0) and on y-axis is of the form (0, y).

• If x-coordinate of two or more points are same, then the line joining these points is parallel to y-axis.

• If y-coordinate of two or more points are same, then the line joining these points is parallel to x-axis.

NOTE: If a point lie on x-axis or y-axis then it does not lie in any quadrant.

• The mirror image of a point is just a reflection of this point about one of the axes. Mirror image about x-axis: sign of abscissa remains same but sign of ordinate changes.

MIND MAP
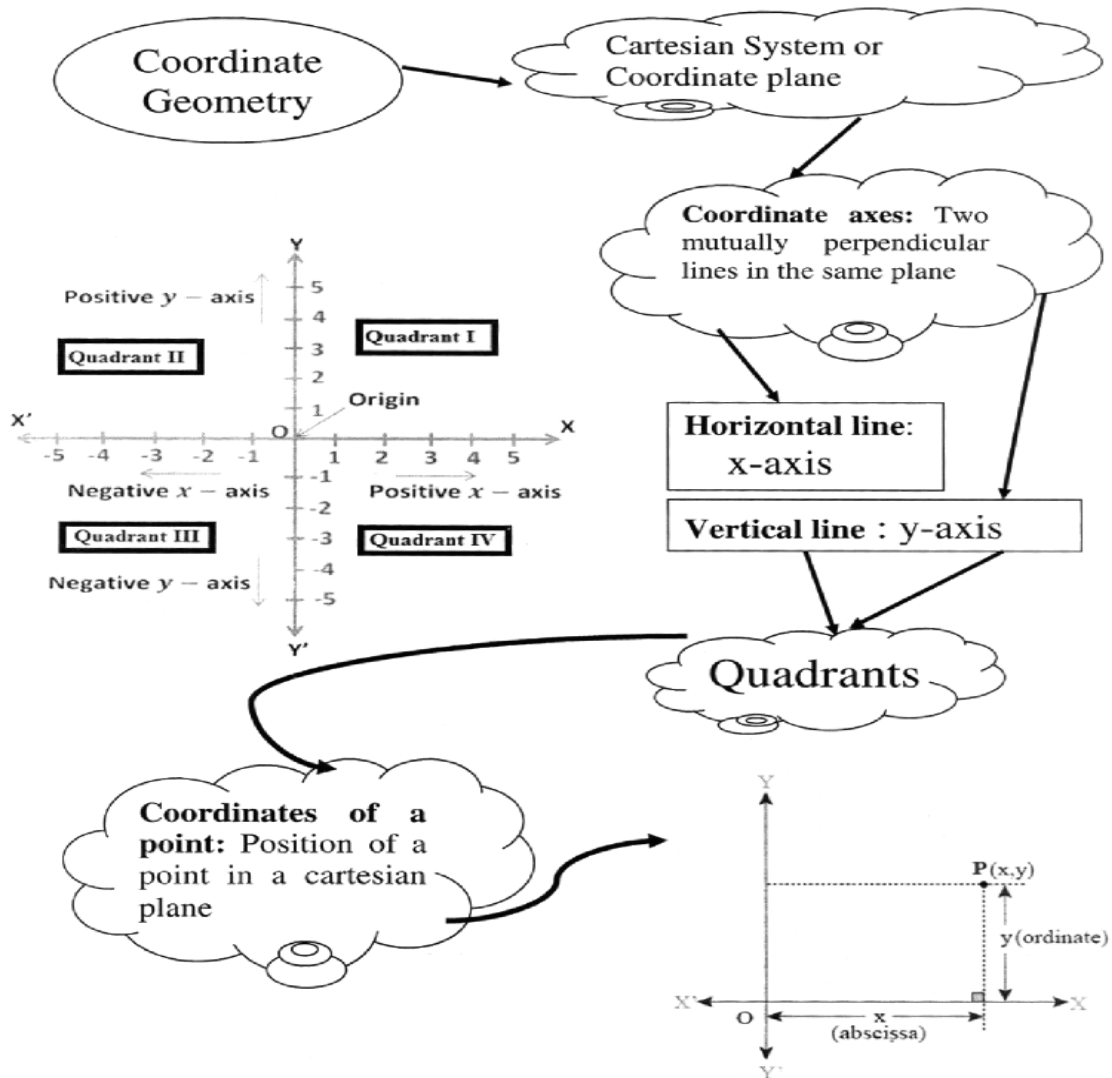
Multiple Choice Questions:

1. The abscissa of a point is the distance of the point from

    (a)   x-axis   (b) y-axis
    (c) origin   (d) None of these

2. The y-coordinate of a point is the distance of that point from

    (a)   x-axis   (b) y-axis
    (b)   origin   (d) None of these

3. **If both the coordinates of a point are negative then that point will lie in**

   (a) First quadrant   (b) Second quadrant

   (c) Third quadrant   (d) Fourth quadrant

4. **If abscissa of a point is zero then that point will lie**

   (a) on x-axis   (b) on y-axis

   (c) at origin   (d) in Ist quadrant

5. **If x > 0 and y < 0, then the point (x, − y) lies in**
   (a) I quadrant   (b) II quadrant
   (c) III quadrant   (d) IV quadrant

6. **Point (a, 0) lies**

   (a) on x-axis   (b) on y-axis

   (c) in third quadrant   (d) in fourth quadrant

7. **The signs of abscissa and ordinate of a point in the second quadrant are respectively.**
   (a) + , +   (b) − , −
   (c) − , +   (d) + , −

8. **The ordinate of a point is positive in**

   (a) I and IV quadrants   (b) I quadrant only

   (c) I and II quadrants   (d) I and III quadrants

9. **The point which lies on y-axis at a distance of 10 units in the negative direction of y-axis is**

   (a) (10, 0)   (b) (0, 10)

   (c) (−10, 0)   (d) (0, −10)

10. **The end points of a line lies in I quadrant and III quadrant. The line may pass through**
    (a) origin   (b) negative x-axis
    (c) positive y-axis   (d) quadrant II

11. **The angle formed between the coordinate axes is**

(a) Zero angle   (b) Right angle   (c) Acute angle   (d) Obtuse angle

Very Short answer questions :

12. The origin is the only common point of the two axes.   (TRUE / FALSE)

13. In a graph paper, the vertical axis is usually known as the x-axis. (TRUE / FALSE)

14. On a graph paper, the positions of the points (1,4), (4,1) coincide. (TRUE / FALSE)

15. On a graph paper, the horizontal axis is usually known as the x-axis. (TRUE / FALSE)

16. The coordinate axes divide the plane into four parts, each part is called _____.

17. It the coordinates of a point are (–2, 5), then its ordinate is _____ and its abscissa is _____.

18. The point (200, –111) lies in the _____ quadrant.

19. The abscissa of any point on the y-axis is _____.

20. The ordinate of any point on the x-axis is _____.

21. The points (0, 0), (0, 4) and (4, 0) form a/an _____ triangle.

Short Answer Type Questions:

22. Plot following points on the graph paper

A(3, 1), B(6, 0), C(0, 6), D(-3, 0), E(-4, 3), F(-2, -4), G(0, -5), H(3, -6), P(7, -3) and Q(7, 6).

23. Identify the quadrant in which each of the points P , Q , R and S lie in .

	Abscissa	Ordinate
P	2	2
Q	-4	3
R	5	-4
S	- 1	- 5

24. Three vertices of a rectangle are (3,2), (-4,2), and (-4,5). Plot these points and find the coordinates of the fourth vertex.

25. Write coordinates of each of the following points marked in the graph paper:
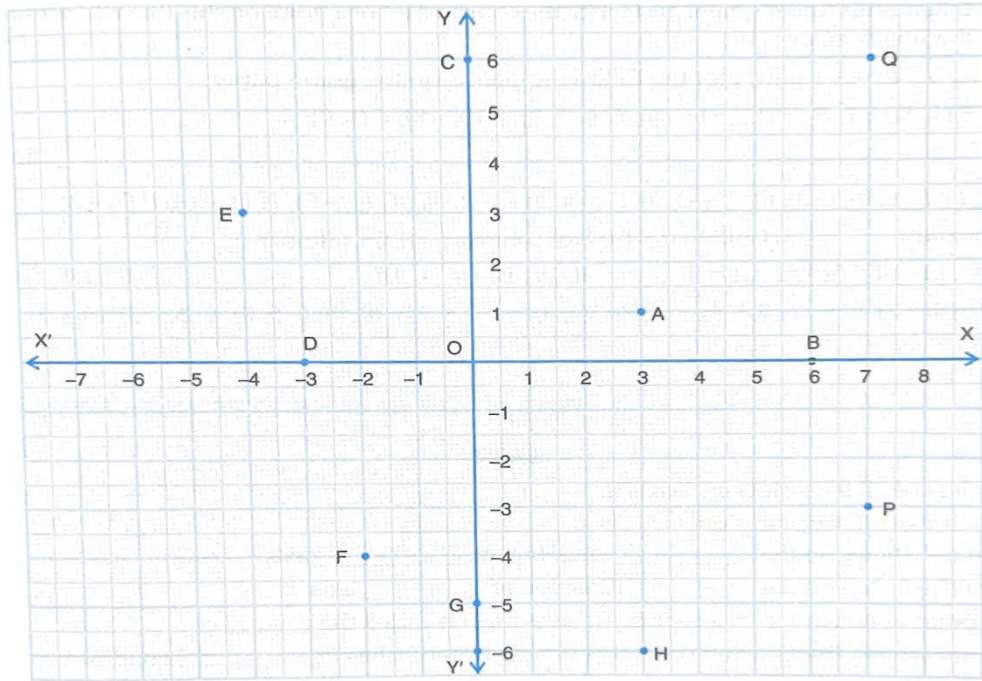
Hints and Answers

1. (b) y-axis

2. (a) x-axis

3. (c) Third quadrant

4. (b) y-axis

5. (d) IV quadrant

6. (a) on x-axis

7. (c) -, +

8. (c) I and II quadrant

9. (d) (0, -10)

10. (a) origin

11. (b) Right angle

12. True

13. False

14. False

15. True

16. Quadrant

17. 5, -2

18. IV quadrant

19. 0

20. 0

21. Isosceles triangle

22.

23. P – I quadrant, Q – II quadrant, R – IV quadrant, S – III quadrant

24.
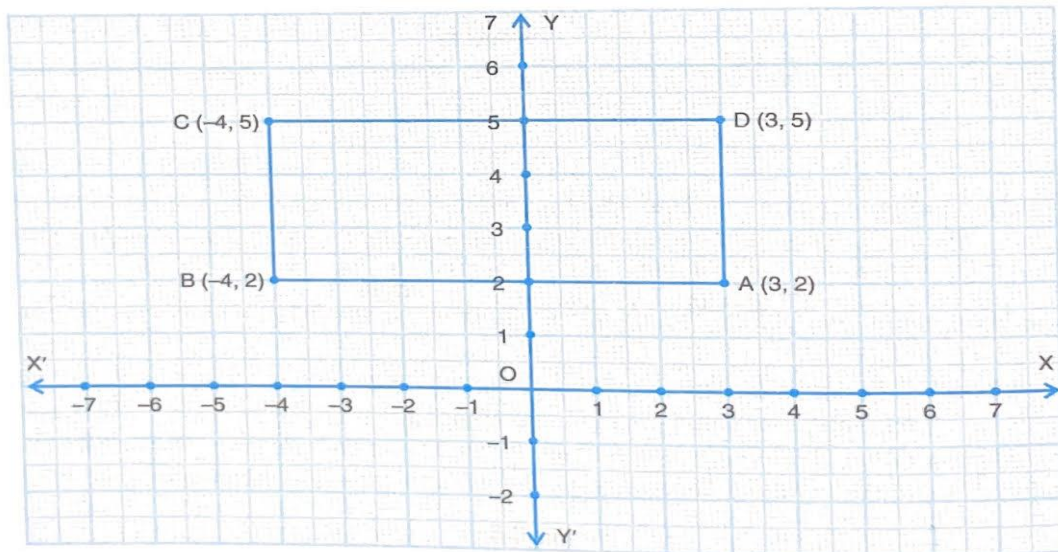
25. A (3, 1), B (6, 0), C (0, 6), D (-3, 0), E (-4, 3), F (-2, -4), G (0, -5), H (3, -6) & P(7, -3).
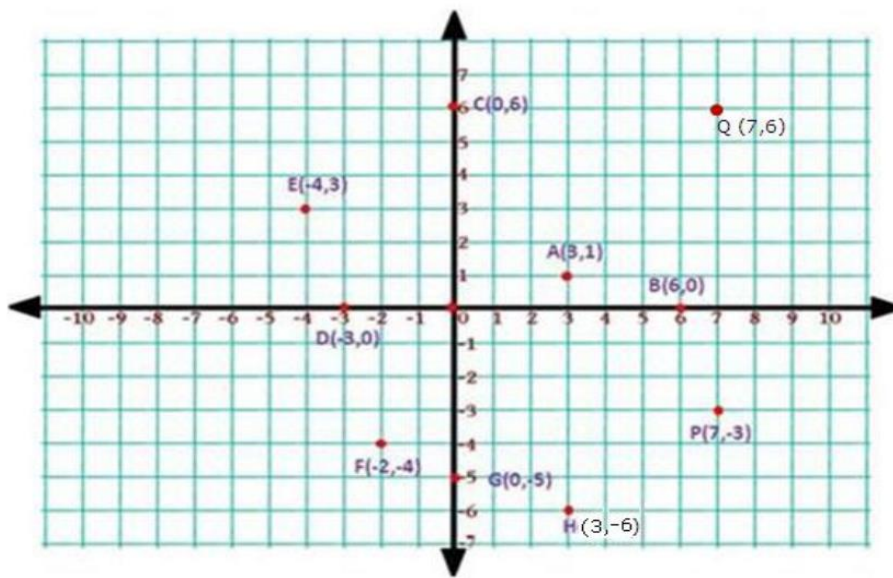
LINEAR EQUATIONS IN TWO VARIABLES

Key points

- **Linear equation in one variable:** An equation which can be written in the form $ax + b = 0$, where a, b are real numbers and $a \neq 0$ is called a linear equation in one variable.

- **Linear equation in two variables:** An equation which can be written in the form $ax + by + c = 0$, where a, b and c are real numbers and a, $b \neq 0$, is called a linear equation in two variables.

  Linear equation in one variable has a unique solution.

  $ax + b = 0 \Rightarrow x = \dfrac{b}{a}$

- Linear equation in two variables has infinitely many solutions.

- The graph of every linear equation in two variables is a straight line.

- Every point on the line satisfies the equation of the line.

- Every solution of the equation is a point on the line. Thus, a linear equation in two variables is represented geometrically by a line whose points make up the collection of solutions of the equation.

Graph

- The pair of values of x and y which satisfies the given equation is called solution of the linear equation in two variables.

  Example: $x + y = 4$

  Solutions of equation $x + y = 4$ are (0, 4) (1, 3) (2, 2) (4, 0) and many more.
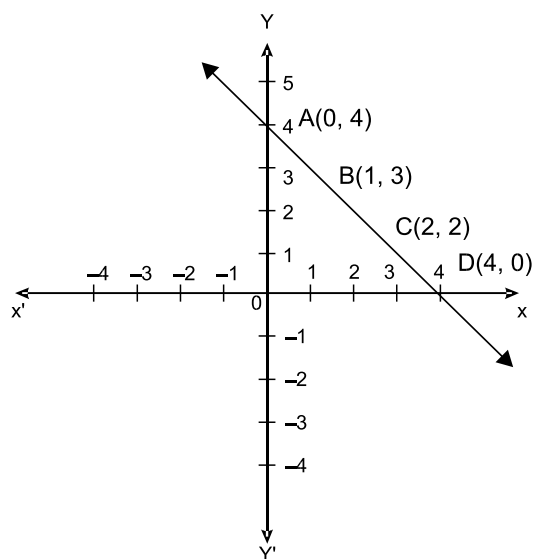
MIND MAP
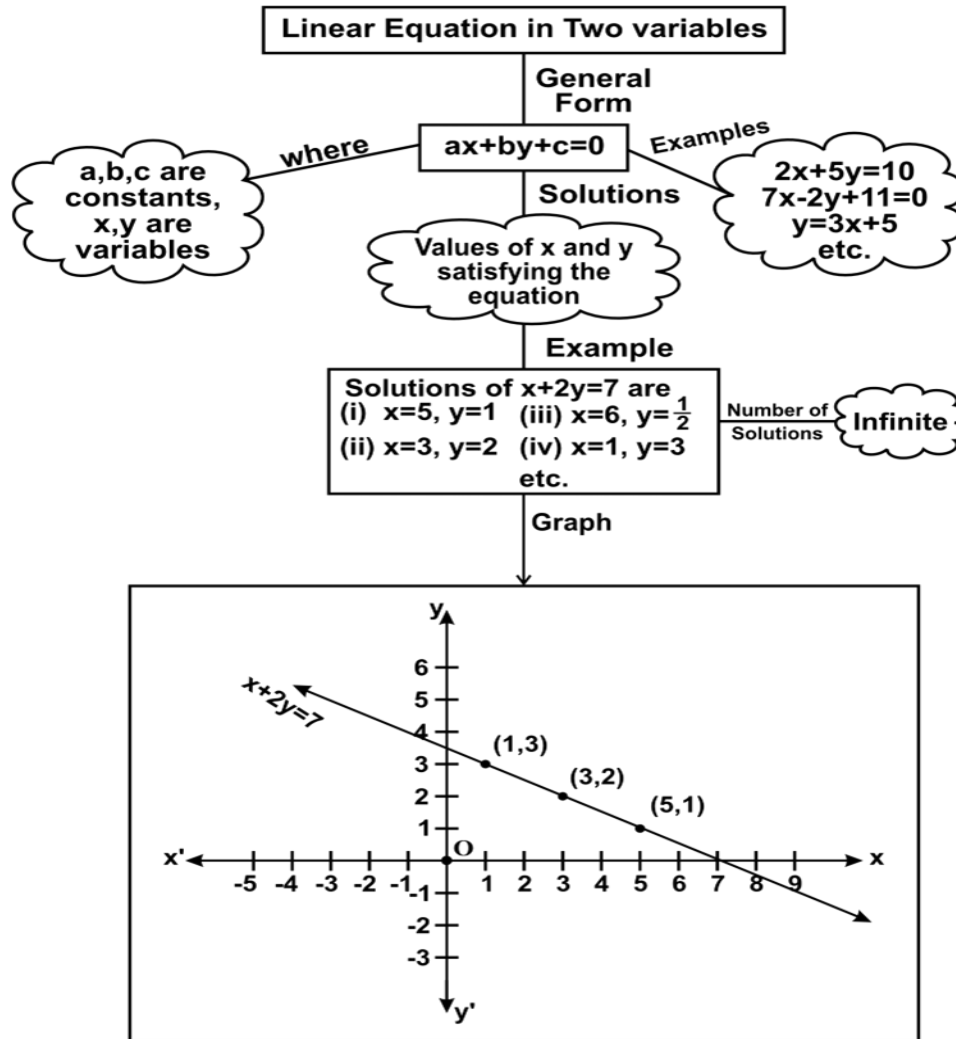
Multiple Choice Questions:

1. **Which of the following is not a linear equation?**
   (a) $3x + 3 = 5x + 2$
   (b) $x^2 + 5 = 3x - 5$
   (c) $\frac{7}{3}x - 5 = 4x - 3$
   (d) $(x + 2)^2 = x^2 - 8$

2. **Which of the following is not a linear equation in two variables?**
   (a) $2x + 3y = 5$
   (b) $3t + 2s = 6$
   (c) $ax^2 + by = c$
   (d) $ax + by = c$

3. **A linear equation in two variables has maximum**
   (a) Only one solution
   (b) Two solutions
   (c) Infinite solutions
   (d) None of these

4. **The graph of $ax + by + c = 0$ is**

(a) a straight line parallel to x-axis   (b) a straight line parallel to y-axis

(c) a general straight line   (d) None of these

5.   **If $x = 1$, $y = 1$ is a solution of equation $9ax + 12ay = 63$, then the value of a is**

   (a)   3   (b) 0

   (b)   −3   (d) 4

6.   **The equation of x-axis is**

   (a) $x = k$ (b) $x = 0$

   (c) $y = k$ (d) $y = 0$

7.   **Any point on the line $y = x$ is of the form**

   (a) $(a, 0)$ (b) $(0, a)$

   (c) (a, a) (d) $(a, −a)$

8.   **The equation $x = 0$ represents –**

   (a)   x-axis   (b) y-axis

   (c)   a line parallel to x-axis   (d) a line parallel to y-axis

9.   **Which of the linear equation has solution as $x = 2$, $y = 3$?**

   (a) $2x + y = 8$ (b) $x + 2y = 8$

   (c) $x + y = 8$ (d) $−x + y = 8$

10. **The graph of $2x + 3y = 6$ is a line which meets the y-axis at the point.**

   (a) $(2, 0)$ (b) $(3, 0)$

   (c) $(0, 2)$ (d) $(0, 3)$

Very Short answer questions:

11. The equation of a line parallel to x-axis is _____ $= a$, where a is any non-zero real number.

12. The equation of a line parallel to y-axis is _____ $= a$, where a is any non-zero real number.

13. The graph of every linear equation in two variables is a _____.

14. An equation of the form $ax + b = 0$, where a, b are real numbers and $a ≠ 0$, in the variable x, geometrically represents _____.

15. The coefficient of x in the linear equation $2(x + y) − x = 7$ is _____

16. The linear equation $7x + 9y = 8$ has a unique solution. (TRUE / FALSE)

17. All the points (2, 0), (–3, 0), (4, 2) lie on the x-axis. (TRUE / FALSE)

18. The line parallel to y-axis at a distance of 5 units to the left of y-axis is given by the equation $x = -5$. (TRUE / FALSE)

19. The graph of every linear equation in two variables need not be a line. (TRUE / FALSE)

20. The graph of the linear equation $x + 2y = 5$ passes through the point (0, 5). (TRUE / FALSE)

Short Answer Type Questions:

21. Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case:

(i) -2x + 3y = 12   (ii) x – y/2 – 5 = 0   (iii) 2x + 3y = 9.35   (iv) 3x = -7y

(v) 2x + 3 = 0   (vi) y – 5 = 0   (vii) 4 = 3x   (viii) y = x/2

22. Write two solutions for each of the following equations:
(i) 3x + 4y = 7   (ii) x = 6y   (iii) x + πy = 4   (iv) 2/3x – y = 4

23. Check which of the following are solutions of the equation 2x – y = 6 and which are not:

(i) (3 , 0)   (ii) (0 , 6)   (iii) (2 , -2)   (iv) (√3, 0)   (v) (1/2 , -5)

24. If x = -1, y = 2 is a solution of the equation 3x + 4y = k, find the value of k.

Long answer type questions:

25. Write $3y = 8x$ in the form of $ax + by + c = 0$. Write x in terms of y. Find any two solutions of the equation. How many solutions you can find out?

26. Sarika distributes chocolates on the occasion of children's Day. She gives 5 chocolates to each child and 20 chocolates to adults. If number of children is represented by 'x' and total distributed chocolates as 'y'.

(i)   Write it in the form of linear equation in two variables.

(ii)   If she   distributed 145 chocolates in total, find number of children?

1. (b) $x^2 + 5 = 3x - 5$

2. (c) $ax^2 + by = c$

3. (c) Infinite solutions

4. (c) a general straight line

5. (a) 3

6. (d) $y = 0$

7. (c) (a, a)

8. (b) y-axis

9. (b) $x + 2y = 8$

10. (c) (0, 2)

11. Y

12. X

13. Straight line

14. a point on number line

15. 1

16. False

17. False

18. True

19. False

20. False

21. (i) $- 2x + 3y - 12 = 0$

    Comparing the given equation with $ax + by + c = 0$

    We get, $a = - 2$; $b = 3$; $c = -12$

    (ii) $x - y/2 - 5 = 0$, $a = 1$; $b = -1/2$, $c = -5$

    (iii) $2x + 3y - 9.35 = 0$, $a = 2$; $b = 3$; $c = -9.35$

    (iv) $3x + 7y = 0$, $a = 3$; $b = 7$; $c = 0$

(v) 2x + 0y + 3 = 0,   a = 2 ; b = 0 ; c = 3

(vi) 0x + y − 5 = 0,   *a = 0; b = 1; c = -5*

(vii) 3x + 0y − 4 = 0,   a = 3; b = 0; c = -4

(viii) x − 2y + 0 = 0,   a = 1; b = -2; c = 0

22.   *(i) (1, 1) and (2, 1/4)*

(ii) (0, 0) and (6, 1)

(iii) (0, 4/ π) and (4, 0)

(iv) (0, -4) and (3, -2)

23.   (i) Yes   (ii) No   (iii) Yes   (iv) No   (v) Yes

24.   (-1, 2) is the solution of 3x + 4y = k, so it satisfies the equation.

Substituting x = -1 and y = 2 in 3x + 4y = k, we get

$\Rightarrow$ 3 (− 1) + 4(2) = k

$\Rightarrow$ − 3 + 8 = k

$\Rightarrow$k = 5

25.   8x − 3y + 0 = 0;   x = $\frac{3y}{8}$

(0, 0) (3, 8)

Infinitely many solutions.

26.   (i) 5x + 20 = y

   (iii) 25

INTRODUCTION TO EUCLID'S GEOMETRY

Basic Concepts and Important Points

1. Postulates: The basic facts which are taken, for granted, without proof and which are specific to geometry are called postulates.

2. Axioms: The basic facts which are taken for granted, without proof and which are used throughout in the mathematics are called axioms.

3. Theorem: The conclusions obtained through logical reasoning based on previously proved results and some axioms constitute a statement known as a theorem or a proposition.

4. Point: A point is represented by a fine dot made by a sharp pencil on a sheet of paper.

5. Plane: The surface of a smooth wall or the surface of a sheet of paper or the surface of a smooth black board are close examples of a plane.

6. Line: A line is breadthless length e.g.. if we fold a piece of paper, the crease in the paper represents a geometrical straight line. The edge of a ruler, the edge of the top of a table, the meeting place of two walls of a room are some examples of a geometrical straight line.

7. Incidence Axioms:

   Axiom 1: A line contains infinitely many points.

   Axiom 2: Through a given point, infinitely many lines can pass through.

   Axiom 3: In given two points A and B, there is one and only one line that contains both the points.

8. Collinear Points: Three or more points are said to be collinear, if there is a line which contains all of them.

9. Concurrent Lines: Three or more lines are said to be concurrent, if there is a point which lies on all of them.

10. Intersecting Lines: Two lines which meet at one point are said to be intersecting lines. The common point is called the 'point of intersection'.

   Note: Two distinct lines cannot have more than one point in common.

11. Parallel Lines: Two lines l and m in a plane are said to be parallel lines, if l ∩ m = φ. If l and m are two parallel lines in a plane, we can write l ∥ m.
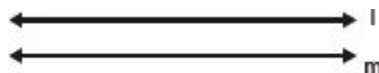
12. Parallel Axiom: If l is a line and P is a point not on line l, there is one and only one line m which passes through P and is parallel to l.
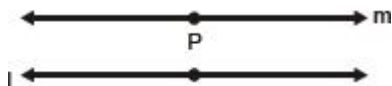
13. Two lines which are both parallel to the same line, are parallel to each other.

14. If l, m, n are three lines in the same plane such that l intersects m and n ∥ m, then l intersects n also.

15. If l and m are intersecting lines, l ∥ p and q ∥ m, then p and q also intersect.

16. If lines AB, AC, AD and AE are parallel to a line l, then points A, B, C, D and E are collinear.

17. **Line Segment:** In given two points A and B on a line l, the connected part (segment) of the line with end points at A and B, is called the line segment AB.

18. **Interior Point of a Line Segment:** A point P is called an interior point of a line segment AB, if P ε AB but P is neither A nor B.

19. **Congruence of Line Segments:** Two line segments AB and CD are congruent, if the trace-copy of one can be superposed on the other so as to cover it completely and exactly.

20. **Line Segment Length Axiom:** Every line segment has a length. It is measured in terms of 'metre' or Its parts.

21. **Congruent Line Segment Length Axiom:** Two congruent line segments have equal length and conversely, two line segments of equal length are congruent,

   i.e., $AB \cong CD \Leftrightarrow l(AB) = l(CD)$.

22. **Line Segment Addition Axiom:** If C is any interior point of a line segment AB, then

$$l(AB) = l(AC) + l(CD).$$
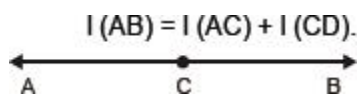
23. **Line Segment Construction Axiom:** Given a point O on a line l and a positive real number r, there are exactly two points P_1 and P_2 on l, on either side of O such that

   $l(OP_1) = l(OP_2) = r$ cm.
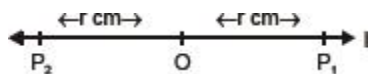
24. **Distance between Two Points:** The distance between two points P and Q is the length of the line segment joining them and it is denoted by PQ.

25. **Betweenness:** Point C is said to lie between the two points A and B, if

   (a) A, B and C are collinear points and

   (n) AC + CB = AB.

26. **Mid-point of a Line Segment:** Given a line segment AB, a point M is said to be the mid-point of AB, if M is an interior point of AB such that AM = MB.
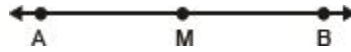
Line through M, other than line AB is called the bisector of the segment AB.

27. **Opposite Rays:** Two rays AB and AC are said to be opposite rays if they are collinear and point A is the only common point of these two rays.

   Note: Two rays or two line segments or a line segment and a ray (line) are said to be parallel, if the lines containing them are parallel.

28. **Euclid's Five Postulates:**

   (a) A straight line may be drawn from any one point to any other point.

   (b) A terminated line can be produced indefinitely.

   (c) A circle can be drawn with any centre and any radius.

   (d) All right angles are equal to one another.

   (e) If a straight line falling on two straight lines makes the interior angles on the same side of it, taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

29. **Some Euclid's axioms:**

   (a) Things which are equal to the same thing are equal to one another.

   (b) If equals are added to equals, the wholes are equal.

   (c) If equals are subtracted from equals, the remainders are equal.

   (d) Things which coincide with one another are equal to one another.

   (e) The whole is greater than the part.

   (f) Things which are double of the same things are equal to one another.

   (g) Things which are halves of the same things are equal to one another.

30. A system of axioms is called consistent, if it is impossible to deduce from these axioms a statement that contradicts any axioms or previously proved statement.

CHAPTER-6

LINES AND ANGLES

Basic Definitions and Terms

The ideal way to begin with the chapter is to understand and learning the important definitions and terms of the chapter. Mentioned below are some vital definitions of varied types of lines and angles.

Line Segment

If a line has two ends, then it is called a line segment.

Ray

That part of the line with one endpoint is called a ray.

Non-Collinear Point

It is an essential concept of class 9 lines and angles, which says that collinear points are present in a line having three or more points. Else, they are called non-collinear points. In simpler terms, non-collinear points do not allow a single line to be formed through them.
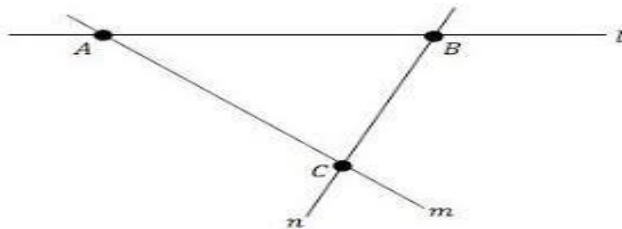
In the aforementioned picture, no single line through points A, B, C is formed because they are non-collinear, rather separate lines *l,m,n* can be seen.

Now let us have a look at the types of angles that we will be studying in this chapter.

Angle

When two rays originate from the same endpoint, it is called an Angle. Arms are the two rays that make the base for an angle. The endpoints where the two rays meet is known as the Vertex.
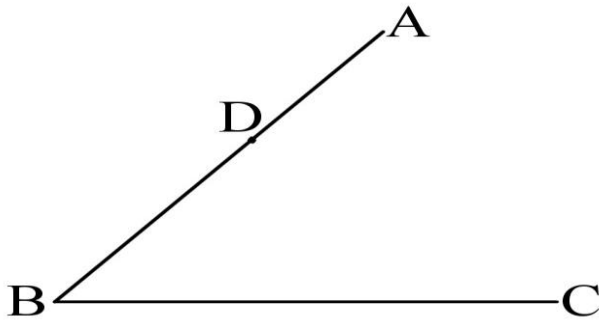
There are various types of angles, explained below are the important ones according to the class 9 lines and angles chapter

Acute Angle

All the angles that measure between the range of 0 to 90 degrees are called an acute angle.
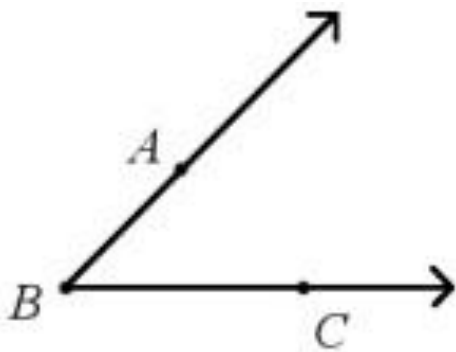
Right Angle

As per the class 9 lines and angles chapter, an angle which measures exactly 90 degrees is called a Right Angle.

90°
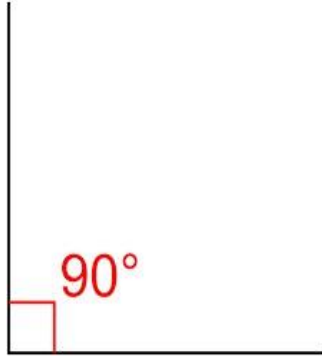
Obtuse Angle

This is an angle greater than 90 degrees but smaller than 180 degrees.

The picture given below depicts a comparison between the 3 main types of angles.

ACUTE   RIGHT   OBTUSE
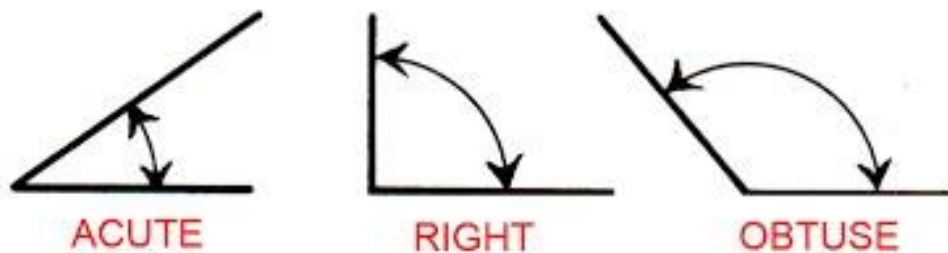
Reflex Angle

A reflex angle is greater than 180 degrees but less than 360 degrees.
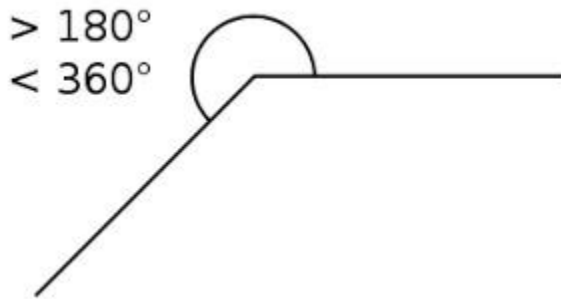
Complementary Angles

Two angles having a sum of 90 degrees are called complementary angles.
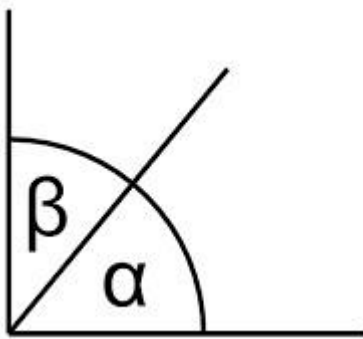
Supplementary Angles

It happens to be an important concept of class 9 lines and angles. Supplementary angles are those angles which have some of 180 degrees.
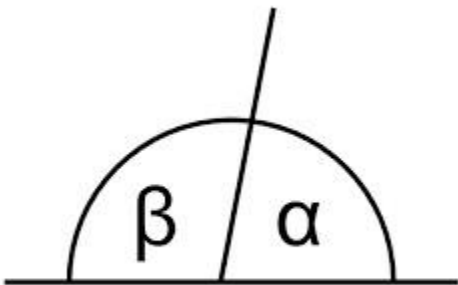
Adjacent Angles

Two or more angles that have a common vertex are called adjacent angles.
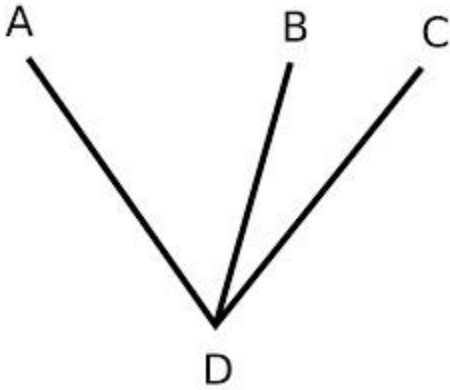
Linear Pair Angles

It is a fundamental concept of this chapter and it says that if two non-common arms say, *a* and *b* can form a line, they will be called line pair angles.

Vertically Opposite Angles

When two lines, say, AB and CD intersect one another, they are called vertically opposite angles.
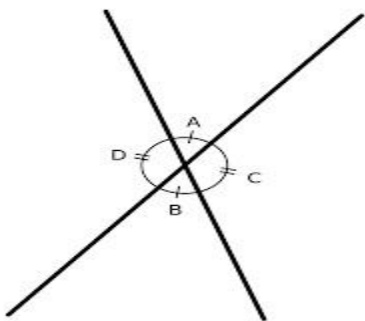
If there are two lines parallel to one another and have the same length of perpendicular on the surface of the line, it is called the distance between two parallel lines. The vertically opposite angle is an elementary concept of lines and angles.

Transversal Lines

A transversal line is one that intersects two or more lines at distinct points.
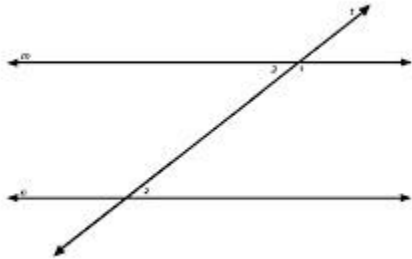
Axioms of Lines and Angles

For all of us to solve the questions of the lines and angles class 9, there are some axioms that have been laid down. All the steps that are involved in the solution of a question must be in accordance with these axioms.

Line Pair Axiom

The non-common arms of the angle will always form a line whenever the sum of two adjacent angles is more than 180 degree.

Corresponding Angles Axiom

When two lines are intersected by a transversal forming two corresponding angles in pairs, they will be parallel to one another.

Consecutive Interiors

They are the angles that lie on the same transversal's side. They are also called allied or co-interior angles.

Theorems of Lines and Angles

Just as the axioms of the lines and angles are important while solving a question, similarly, one must keep in mind these theorems.

Theorem 1

When two lines that have a pair of alternate interior angles that are equal are intersected by a transversal, then the two lines are always parallel.

Theorem 2

A pair of interior angles on the same transversal's side on two parallel lines will be supplementary whenever a transversal intersects.

Theorem 3

If two interior angles are formed on the same side of a transversal and are supplementary to each other and have a transversal interesting the two parallel lines, then those two lines are parallel.

Theorem 4- Sum of all the angles of the triangle is 180 degrees.

Theorem 5- The sum of two interior opposite angles of a triangle is always equal to the exterior angle of the triangle.

SOME QUESTIONS AND THEIR SOLUTIONS

QUESTION 1: In figure, lines XY and MN intersect at 0. If ∠POY = 90° , and a : b = 2 : 3. find c.
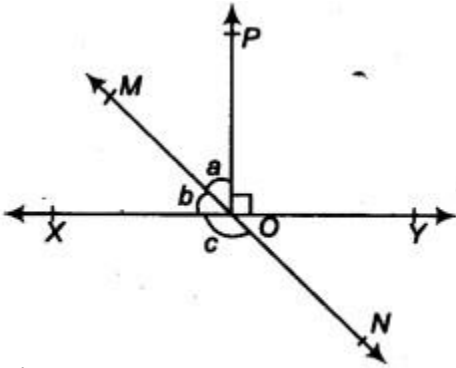
Solution:
Since XOY is a straight line.
∴ b+a+∠POY= 180°
But ∠POY = 90° [Given]
∴ b + a = 180° − 90° = 90° ...(i)
Als3a/2 + A = 90°

⇒ 5a/2 = 90°

⇒ a = (90/5)x 2= 36 = 36°

From (ii), we get

b = (3/2) x 36° = 54°

Since XY and MN interstect at O,

∴ c = [a + ∠POY] [Vertically opposite angles]

or c = 36° + 90° = 126°

o a : b = 2 : 3 ⇒ b = 3a/2 ...(ii) Now from (i) and (ii), we get

Thus, the required measure of c = 126°.

QUESTION 2: In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that: ∠ROS= 1/2 (∠QOS- ∠POS)

Solution 2:

POQ is a straight line. [Given]
∴ ∠POS + ∠ROS + ∠ROQ = 180°

But OR ⊥ PQ
∴ ∠ROQ = 90°

⇒ ∠POS + ∠ROS + 90° = 180°

⇒ ∠POS + ∠ROS = 90°

⇒ ∠ROS = 90° − ∠POS ... (1)

Now, we have ∠ROS + ∠ROQ = ∠QOS

⇒ ∠ROS + 90° = ∠QOS

⇒ ∠ROS = ∠QOS − 90° ......(2)

Adding (1) and (2), we have

2 ∠ROS = (∠QOS − ∠POS)
∴ ∠ROS = 1/2(∠QOS−∠POS)

QUESTION 3: In figure, find the values of x and y and then show that AB || CD.

Solution: y = 130° ...(1)

[Vertically opposite angles]
Again, PQ is a straight line and EA stands on it.
∠AEP + ∠AEQ = 180° [Linear pair]
or 50° + x = 180°
⇒ x = 180° − 50° = 130° ...(2) From (1) and (2), x = y
As they are pair of alternate interior angles.
∴AB || C

TRIANGLES

KEY CONCEPTS:

• Two figures are congruent if they are of the same shape and of the same size.

• Two circles of the same radii are congruent.

• Two squares of the same sides are congruent.

• Two triangles are congruent if their corresponding parts are congruent.

• If two triangles ABC and PQR are congruent under the correspondence A ↔ P, B ↔ Q and C ↔ R, then symbolically, it is expressed as Δ ABC ≅ Δ PQR.

Some congruence rules are
1. SAS(Side-Angle-Side) Congruence Rule:
Two triangles are congruent if two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle. 2.ASA(Angle-Side-Side) Congruence Rule: Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.
3. AAS Congruence Rule: Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

MULTIPLE CHOICE QUESTIONS

Q1   In triangles ABC and DEF, AB = FD and ∠A = ∠D. The two triangles will be congruent by SAS axiom if

       (a)   BC = EF
       (b)   AC = DE
       (c)   AC = EF
       (d)   BC = DE

Q2   In the given figure, PS is the median then ∠QPS? a) 40º

       b) 90º
       c) 80º
       d) 50º
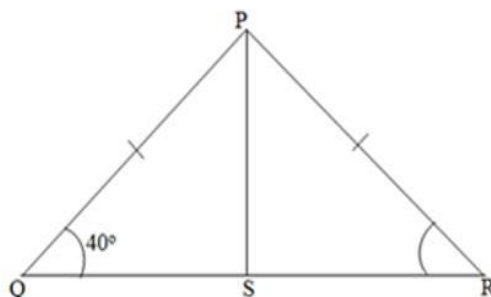
Q3   In triangles ABC and DEF, AB = FD and ∠A = ∠D. The two triangles will be congruent by ASA axiom if:

a) ∠A = ∠F

b) ∠C = ∠D

c) ∠B = ∠F

d) None

Q4   In two triangles DEF and PQR, if DE = QR, EF = PR and FD = PQ, then

a) ΔDEF ≅ ΔPQR

b) ΔFED ≅ ΔPRQ

c) ΔEDF ≅ ΔRPQ

d) ΔPQR ≅ ΔEFD

Q 5   Two sides of a triangle are of length 5 cm and 1.5 cm. The length of the third side of the triangle cannot be:

a) 3.6 cm

b) 4.1 cm

c) 3.8 cm

d) 6.9 cm

Q 6   In ΔABC, ∠C = ∠A and BC = 4 cm and AC = 5 cm, then find length of AB. (a) 5 cm (b) 3 cm
        (c)   4 cm
        (d)   2.5 cm

Q 7   It is given that ΔABC ≅ ΔFDE and AB = 5 cm, ∠B = 40° and ∠A = 80°. Then which of the following is true?
        (a) DF = 5 cm, ∠F = 60°
        (b)   DF = 5 cm, ∠E = 60°
        (c)   DE = 5 cm, ∠E = 60°
        (d)   DE = 5 cm, ∠D = 40°

Congruent triangles
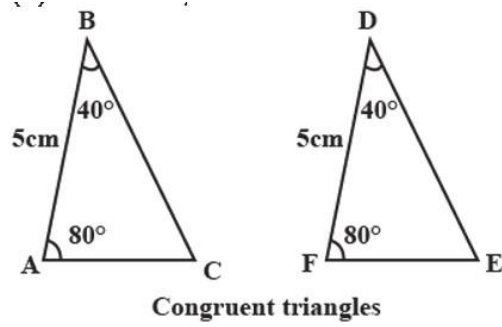
Q 8   In triangles ABC and PQR, AB = AC, ∠C = ∠P and ∠B = ∠Q. The two triangles are:

a) Isosceles but not congruent

b) Isosceles and congruent

c) Congruent but not isosceles

d) Neither congruent nor isosceles

Q 9   In a triangle ABC, ∠B = 35° and ∠C = 60°, then

a) ∠A = 80°

b) ∠A = 85°

c) ∠A = 120°

d) ∠A = 145°

Q10   If in ΔPQR, PQ = PR then:

a) ∠P = ∠R

b) ∠P = ∠Q

c) ∠Q = ∠R

d) None of these

OBJECTIVE TYPE QUESTIONS

1.In Δ ABC, AB=AC and ∠B=40°. Find ∠C.

2.Write correct symbolic form of congruency if AB=QR, BC=PR and CA=PQ.

3.The_____is the largest side of a right-angled triangle.

4.What does 'R' stands for in RHS congruence?

5.Choose the correct answer

If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent. _____ (SAS, ASS, SSS)

6.Choose the correct answer

If two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent. _____ (ASA, ASS, SSS).

7.Two circles are congruent if their _____are same.

8.State true/false

Two angles opposite to the two equal sides of an isosceles triangle are also equal.

9.State true/false

If there are two right-angled triangles then they will be congruent if their hypotenuse and any one side are equal.

10.State true/false

If the two angles and a side of one triangle is respectively equal to the two angles and a side of another triangle, then they are called congruent triangles.

CASE STUDIES/ SOURCE BASED INTEGRATED QUESTIONS
<u>**QUADRILATERAL BY PAPER FOLDING**</u>

The Maths teacher gave students some coloured papers in the shape of a quadrilateral. She asked them to make a parallelogram from the quadrilateral ABCD using paper folding. She made the following parallelogram.
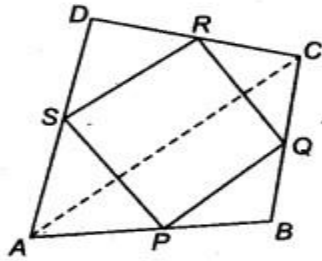
How can a parallelogram be formed by using paper folding?

    a)   Joining the sides of quadrilateral.

    b)   Joining the mid-points of the sides of quadrilateral.

    c)   Joining the various quadrilaterals.

    d)   None of these.

Which of the following is true?

            a) PQ = AC   b) PQ= ½ AC   c) 3PQ= AC   d) PQ=2RS

(iii)   Why paper folding method is adopted by the teacher?

(iv)   Which of the following is the correct combination?

            a) PS=AC   b) QR=AC   c) PS=QR   d) PS=RS

(v)   Explain the geometrical principle underlying the activity and justify.

II   **MOHAN'S WHEAT FARM**

A farmer Mohan have a field ABCD. Field ABCD is in the shape of a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. The farmer wants to crop wheat in area PQRS and mustard crop in rest of the region.
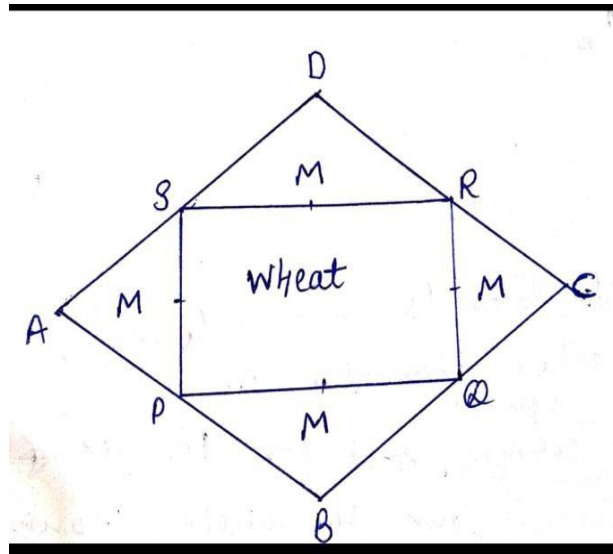
(i)   Which shape is formed by the region of wheat crop?

(ii)   Find the diagonal AC if the side of rhombus field 12m and diagonal BD is 20m?

(iii)   The farmer wants to fence the field with the wire, and then what is the length of wire required if side of rhombus is 12 m?

(iv)   Find all the three angles of the field if the corner angle A is 70°

(v)   If the side of the rhombus field is 20 m, then what length of fencing wire is required for two complete round of the field?

SHORT ANSWER TYPE QUESTIONS

Q 1.   ABC is an isosceles triangle with AB = AC. Drawn AP $\perp$ BC to show that $\angle$B = $\angle$C.

Q 2   ABC and DBC are two isosceles triangles on the same base BC (see the given figure). Show that $\angle$ABD = $\angle$ACD.
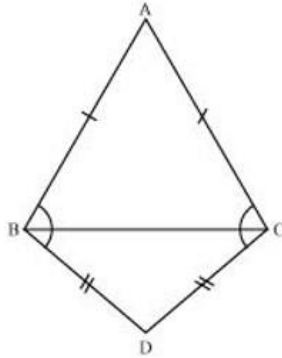
Q 3   AD and BC are equal perpendiculars to a line segment AB (See the given figure). Show that CD bisects AB.
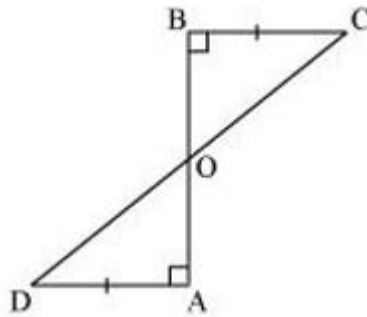
Q 4   ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.
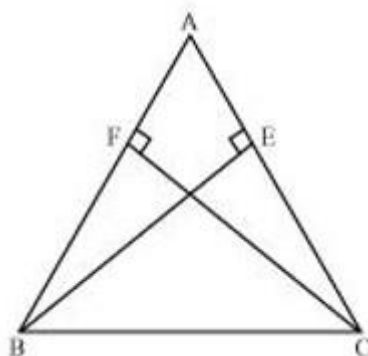
Q 5   Find the $\angle$P, $\angle$R, $\angle$N and $\angle$M if ΔLMN $\cong$ ΔPQR.
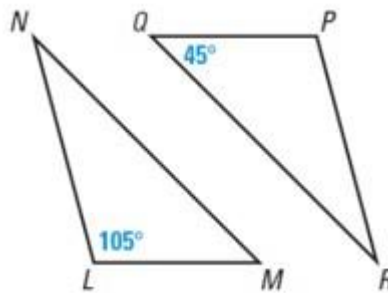
LONG ANSWER TYPE QUESTIONS

Q 1   Show that in a right-angled triangle, the hypotenuse is the longest side.

Q 2   How many equilateral triangles of side 1 cm as you can make in the following figures? Which has more number of triangles?
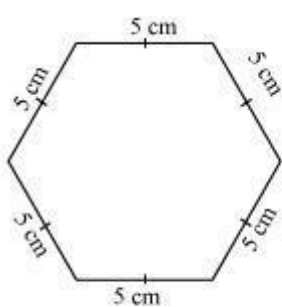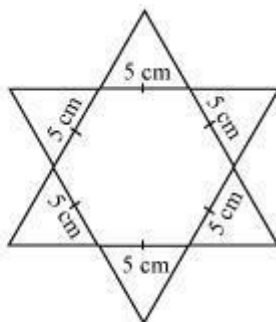
(I)   (II)

Q 3   Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see the given figure). Show that:

   (i)ΔABM ≅ ΔPQN

   (ii)ΔABC ≅ ΔPQR
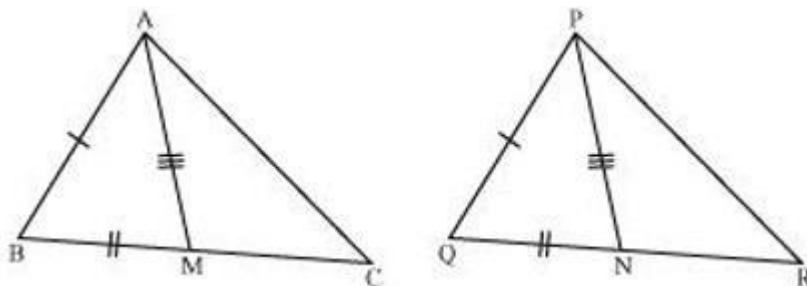
Q 4   In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the given figure). Show that:

   (i)   ΔAMC ≅ ΔBMD

   (ii)   ∠DBC is a right angle.

   (iii)   ΔDBC ≅ ΔACB

Q 5   ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see the given figure). Show that ∠BCD is a right angle.
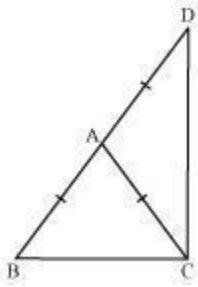
Q 6   Show that the angles of an equilateral triangle are 60º each.

Q 7   BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Q 8   If AB = AC, CH = CB and HK || BC and ∠CAX=137
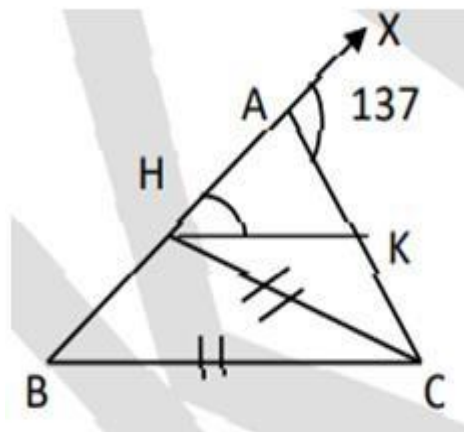
   1) Calculate the measure of ∠ACB.

   2) Calculate the measure of ∠ CHK.

   3) Calculate the measure of ∠AHK.

   4) Sum of measure of ∠CBH and ∠AHK

Q 9   From the adjoining figure ABD. Angle ABC = x and angle CAD = y.

AC = BC, AC = AD
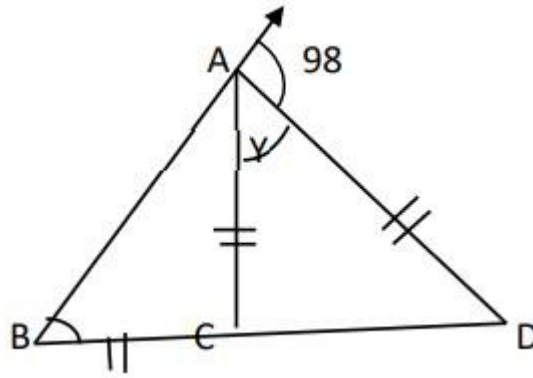
A. Find the value of x.

B. Find the value of y.

C. Find measure of ∠ ADC.

D. Find measure of ∠ BAD.

SOLUTIONS TO MCQ

1.   b
2.   b
3.   c
4.   b
5.   d
6.   c
7.   b
8.   a
9.   b
10.   c

SOLUTIONS OF OBJECTIVE TYPE QUESTIONS

1.   40°
2.   ABC -QRP
3.   Hypotenuse
4.   Right angle
5.   SAS
6.   ASA
7.   Radii
8.   True
9.   True
10.   True

SOLUTIONS TO SHORT ANSWER TYPE QUESTIONS

1 In ΔAPB and ΔAPC,

   ∠APB = ∠APC (Each 90º)

   AB =AC (Given)

   AP = AP (Common)

   ∴ ΔAPB ≅ ΔAPC (Using RHS congruence rule)

   ⇒ ∠B = ∠C (By using CPCT)

2 Let us join AD.

   In ΔABD and ΔACD,

   AB = AC (Given)

   BD = CD (Given)

   AD = AD (Common side)

   ∴ ΔABD ≅ ΔACD (By SSS congruence rule)

   ⇒ ∠ABD = ∠ACD (By CPCT)

3 In ΔBOC and ΔAOD,

   ∠BOC = ∠AOD (Vertically opposite angles)

   ∠CBO = ∠DAO (Each 90º)

   BC = AD (Given)

   ∴ ΔBOC ≅ ΔAOD (AAS congruence rule)

   ∴ BO = AO (By CPCT)

   ⇒ CD bisects AB.

	SOLUTIONS TO LONG ANSWER TYPE QUESTIONS
1.	Let us consider a right-angled triangle ABC, right-angled at B. In ΔABC, ∠A + ∠B + ∠C = 180° (Angle sum property of a triangle) ∠A + 90º + ∠C = 180° ∠A + ∠C = 90° Hence, the other two angles have to be acute (i.e., less than 90º). ∴ ∠B is the largest angle in ΔABC. ⇒ ∠B > ∠A and ∠B > ∠C ⇒ AC > BC and AC > AB [In any triangle, the side opposite to the larger (greater) angle is longer.] Therefore, AC is the largest side in ΔABC. However, AC is the hypotenuse of ΔABC. Therefore, hypotenuse is the longest side in a right-angled triangle.

5. If Δ LMN ≅ ΔPQR, then

∠L=∠P ,∠M =∠Q

∠N =∠R

So,

∠L=∠P = 105°

∠M =∠Q = 45°

∠M + ∠N + ∠L = 180° (Sum of three angles of a triangle is 180°)

45° + 105° + ∠N = 180°

∠N = 180°- 45° + 105°

∠N = 30°

∠N = ∠R = 30°

2.
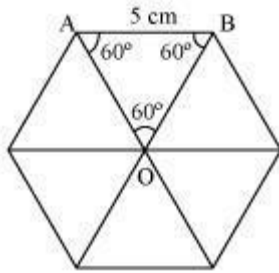
Area of ΔOAB $= \dfrac{\sqrt{3}}{4}(\text{side})^2 = \dfrac{\sqrt{3}}{4}(5)^2$

$= \dfrac{\sqrt{3}}{4}(25) = \dfrac{25\sqrt{3}}{4}$ cm²

Area of hexagonal-shaped *rangoli* $= 6 \times \dfrac{25\sqrt{3}}{4} = \dfrac{75\sqrt{3}}{2}$ cm²

Area of equilateral triangle having its side as 1 cm $= \dfrac{\sqrt{3}}{4}(1)^2 = \dfrac{\sqrt{3}}{4}$ cm²

Number of equilateral triangles of 1 cm side that can be filled

in this hexagonal-shaped *rangoli* $= \dfrac{\dfrac{75\sqrt{3}}{2}}{\dfrac{\sqrt{3}}{4}} = 150$

Star-shaped *rangoli* has 12 equilateral triangles of side 5 cm in it.
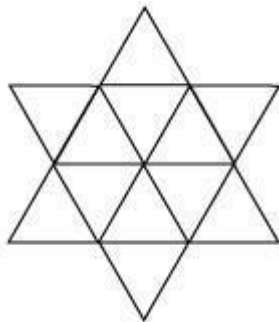
Area of star-shaped *rangoli* = $12 \times \dfrac{\sqrt{3}}{4} \times (5)^2 = 75\sqrt{3}$

Number of equilateral triangles of 1 cm side that can be filled

in this star-shaped *rangoli* $= \dfrac{75\sqrt{3}}{\dfrac{\sqrt{3}}{4}} = 300$

Therefore, star-shaped *rangoli* has more equilateral triangles in it.

3. (i) In ΔABC, AM is the median to BC.

∴ BM = $\frac{1}{2}$ BC

In ΔPQR, PN is the median to QR.

∴ QN = $\frac{1}{2}$ QR

However, BC = QR

∴ $\frac{1}{2}$ BC = $\frac{1}{2}$ QR

⇒ BM = QN … (1)

In ΔABM and ΔPQN,

AB = PQ (Given)

BM = QN [From equation (1)]

AM = PN (Given)

∴ ΔABM ≅ ΔPQN (SSS congruence rule)

∠ABM = ∠PQN (By CPCT)

∠ABC = ∠PQR … (2)

(ii) In ΔABC and ΔPQR,

AB = PQ (Given)

∠ABC = ∠PQR [From equation (2)]

BC = QR (Given)

⇒ ΔABC ≅ ΔPQR (By SAS congruence rule)

4. (i) In ΔAMC and ΔBMD,

AM = BM (M is the mid-point of AB)

∠AMC = ∠BMD (Vertically opposite angles)

CM = DM (Given)

∴ ΔAMC ≅ ΔBMD (By SAS congruence rule)

∴ AC = BD (By CPCT)

And, ∠ACM = ∠BDM (By CPCT)

(ii)   ∠ACM = ∠BDM

However, ∠ACM and ∠BDM are alternate interior angles.

Since alternate angles are equal,

It can be said that DB || AC

⇒ ∠DBC + ∠ACB = 180º (Co-interior angles)

⇒ ∠DBC + 90º = 180º

⇒ ∠DBC = 90º

(iii)   In ΔDBC and ΔACB,

DB = AC (Already proved)

∠DBC = ∠ACB (Each 90°)

BC = CB (Common)

∴ ΔDBC ≅ ΔACB (SAS congruence rule)

5   AB = AC (Given)

⇒ ∠ACB = ∠ABC (Angles opposite to equal sides of a triangle are also equal) In ΔACD,

AC = AD

⇒ ∠ADC = ∠ACD (Angles opposite to equal sides of a triangle are also equal)

In ΔBCD,

∠ABC + ∠BCD + ∠ADC = 180º (Angle sum property of a triangle)

⇒ ∠ACB + ∠ACB +∠ACD + ∠ACD = 180º

⇒ 2(∠ACB + ∠ACD) = 180º

⇒ 2(∠BCD) = 180º
⇒ ∠BCD = 90º

6   consider that ABC is an equilateral triangle.

Therefore, AB = BC = AC

AB = AC

$\Rightarrow \angle C = \angle B$ (Angles opposite to equal sides of a triangle are equal) Also,

AC = BC

$\Rightarrow \angle B = \angle A$ (Angles opposite to equal sides of a triangle are equal)

Therefore, we obtain

$\angle A = \angle B = \angle C$

In $\triangle ABC$,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle A + \angle A = 180°$

$\Rightarrow 3\angle A = 180°$

$\Rightarrow \angle A = 60°$

$\Rightarrow \angle A = \angle B = \angle C = 60°$

7
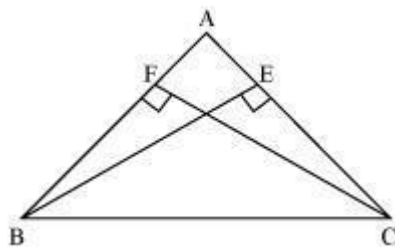
In $\triangle BEC$ and $\triangle CFB$,

$\angle BEC = \angle CFB$ (Each 90°)

BC = CB (Common)

BE = CF (Given)

$\therefore \triangle BEC \cong \triangle CFB$ (By RHS congruency)

$\Rightarrow \angle BCE = \angle CBF$ (By CPCT)

$\therefore AB = AC$ (Sides opposite to equal angles of a triangle are equal) Hence, $\triangle ABC$ is isosceles

8.   1.1 $\angle ABC = \angle ACB$ (AB = AC) $\angle CAX =$

2$\angle ABC$, $\angle ACB = 68.5$

1.2 $\angle CHK = 180 - 137 = 43$
1.3) AHK = 68.5
1.4) $\angle CBH + \angle AHK = 68.50 + 68.50 = 137$

9.   1 $\angle ACD = 2x = \angle ADC$

98 = x + 2x, x = 98/3

2.   y = 180 - 2x - 2x = $180 - 4x = 180 - 398/3 = 148/3$

3.   196/3

4. 82^0

QUADRILATERALS

KEY CONCEPTS

1.Quadrilateral: It is a closed figure bounded by four line segments.

⬚ A quadrilateral has four sides, four angles and four vertices.
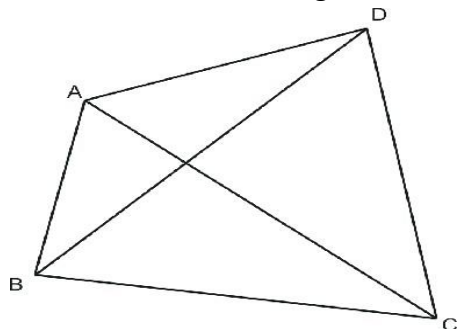
(i)   Two pairs of opposite sides e.g. AB & CD, AD & BC.

(ii)   Two pairs of opposite angles e.g. ∠A & ∠C, ∠B & ∠D.

(iii)   Four pairs of adjacent sides, AB & BC, BC & CD, CD & AD, AD & AB.

(iv)   Line segment joining the opposite vertices are called diagonals.

   e.g. AC & BD.

⬚ The sum of angles of a quadrilateral is 360°.

2. Types of Quadrilateral:

Parallelogram: A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram.

In a parallelogram,

•   A diagonal divides it into two congruent triangles.

•   Opposite sides are equal.

•   Opposite angles are equal.

•   Diagonals bisect each other.

Rectangle: A parallelogram whose one of the angles is a right angle is called a rectangle.

Rhombus: A parallelogram having all sides equal is called a rhombus.

Diagonals of a rhombus bisect each other at right angles. **Square:** A square is a rhombus whose one angle is a right angle.

Diagonals of a square bisect each other at right angles and are equal.

Trapezium : A trapezium is a quadrilateral whose one pair of opposite sides is parallel.

Kite: A quadrilateral in which two pairs of adjacent sides are equal.

Mid-point Theorem

The line segments joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.
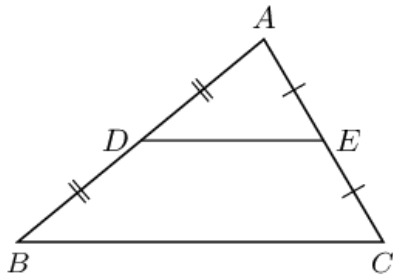
e.g. In ΔABC, D and E are the mid-points of AB and AC respectively.

Then DE ∥ BC and DE = ½ BC.

Converse: A line through the mid-point of a side of a triangle and parallel to another side bisects the third side.

The quadrilateral formed by joining the mid-points of the sides of a quadrilateral, in order, is a parallelogram.

Q1   The three angles of a quadrilateral are 75°, 90° and 105°. The fourth angle is

a) 90°   b) 95°   c) 105°   d) 120°

Q2   In the given figure, ABCD is a parallelogram in which a pair of opposite angles is given. The value of x is
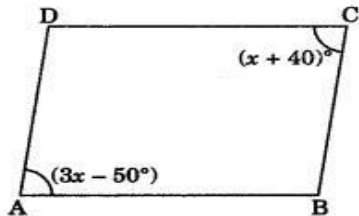
a) 40°   b) 45°   c) 50°   d) 55°

Q3   In a parallelogram ABCD, if $\angle A$ = 75°, then $\angle B$ = ____.

a) 75°   b) 105°   c) 15°   d) 95°

Q4   Which of the following statement is correct for a square?

a)   Diagonals are equal and bisect each other at right angles.

b)   Diagonals are unequal and do not bisect each other.

c)   Diagonals are equal and do not bisect each other.

d)   Diagonals are unequal and bisect each other at right angles.

Q 5   The lines joining the mid-points of the adjacent sides of a quadrilateral enclose a

_____.

Square   b) rectangle   c) rhombus   d) parallelogram

Q 6   Two adjacent angles of a rhombus are (3x- 40°) and (2x +20°).
The measurement of the greater angle is

160°   b) 100°   c) 80°   d) 120°

Q 7   The quadrilateral formed by joining the mid-points of the

consecutive sides of a rectangle is a _____.

Square   b) rhombus   c) parallelogram   d) trapezium

Q 8   In quadrilateral ABCD, AP and BP are bisectors of $\angle A$ and $\angle B$
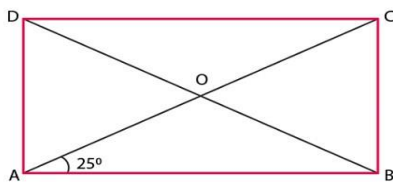respectively, then the value of x is

A
D
130°
$x°$ P
60°
B
C
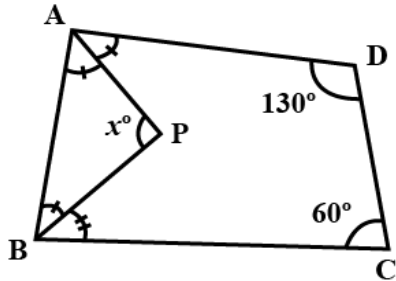
a) 60°   b) 85°   c) 95°   d) 100°

Q 9   Points D, E are the mid-points of sides AB and AC of ΔABC. If the length of the line
segment DE= 6.5 cm, then the length of side BC is equal to

a) 6.5 cm   b) 26 cm   c) 13 cm   d) 5.5 cm

Q 10   A diagonal of a rectangle is inclined to one side of the rectangle at
25°. The acute angle between the diagonals ($\angle AOD$) is _

D
C
O
25°
A
B

a) 55°   b) 50°   c) 40°   d) 25°

Q 11   ABCD is a rhombus such that $\angle ACB = 40°$. Then
$\angle ADB$ is ____.

a) 40°   b)45°   c)50°   d) 60°

D
O
A
40°
C
B
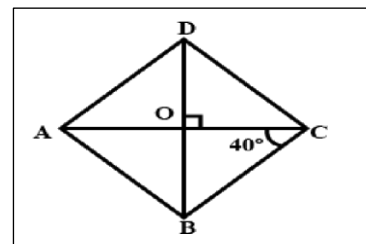
Q 12   The line segment joining the mid-points of the two sides of a triangle is parallel to the third side and _____ of it.

   a) Half   b) one-third   c) one-fourth   d) equal

Q 13   If the diagonals of a quadrilateral bisect each other at right angles, then the quadrilateral is a _____.

   Trapezium   b) rhombus   c) parallelogram   d) rectangle

Q 14   Two angles of a quadrilateral are 50° and 80° and other two angles are in the ratio 8:15, then the remaining two angles are

   a) 50°, 130° b) 80°, 150° c) 70°, 160° d) 60°, 170°

Q 15   ΔPQR is formed by joining the mid-points of sides BC, CA and AC respectively. If ΔABC is an equilateral triangle with side 12 cm, then the length of PQ is

   a) 6 cm   b) 12 cm   c) 3 cm   d) 16 cm

SHORT ANSWER TYPE QUESTIONS

Q 1   In the given figure, ABCD is a rhombus, OA= 4 cm and OD= 3 cm. find the perimeter of the rhombus.
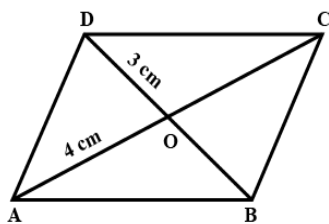
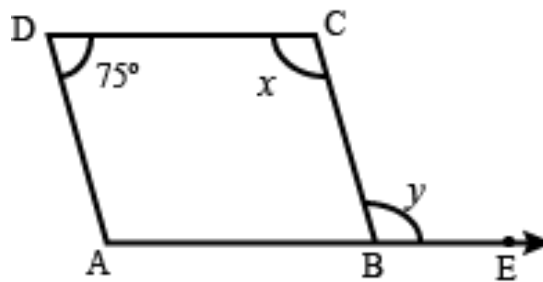Q 2   In the given figure, ABCD is a parallelogram. Find the measure of the angles x and y.
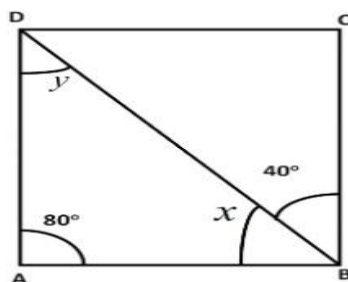
Q3   The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.
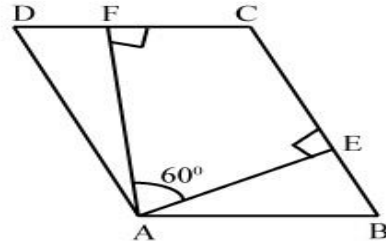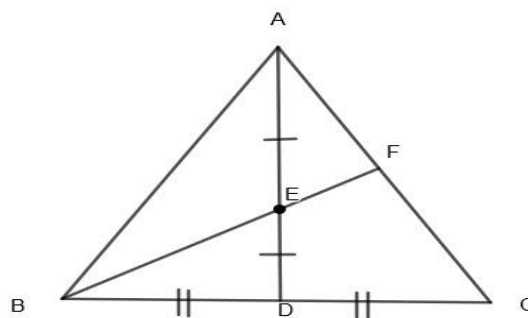

Q4   ABCD is a trapezium in which AB∥DC and $\angle A = \angle B = 45°$. Find the other two angles of the trapezium.

Q5   ABCD is a parallelogram in which $\angle ADC = 75°$ and side AB is produced to point E as shown in the figure. Find x + y.


Q6   In the given figure, ABC is triangle, AD is a median and E is the mid-point of AD. BE is joined and produced to intersect AC in a point F. Prove that AF = $\frac{1}{3}$ AC.
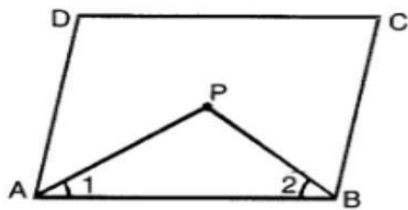
Q 7   In a parallelogram, ABCD, AB= 10cm and AD= 6cm. The bisector of $\angle A$ meets DC in E. AE and BC produced meet at F. Find the length of CF.
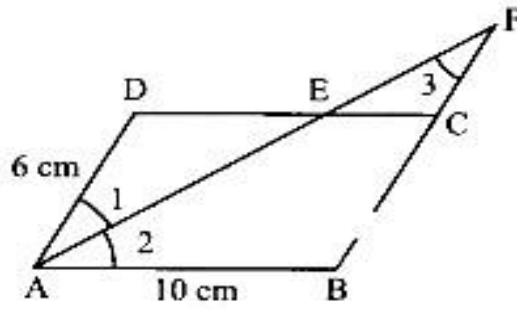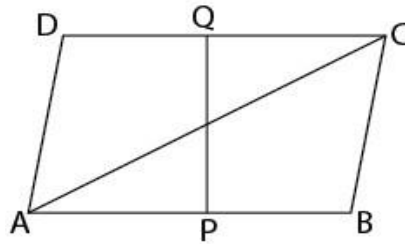

Q 8   In the given figure, ABCD is a parallelogram; E is the mid-point of AB and DE bisects $\angle D$. **Prove that** BC=BE.
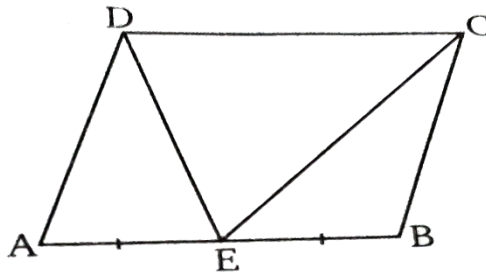
Q 9   In a parallelogram, ABCD, bisectors of consecutive angles A and B intersect at P. Prove that $\angle APB$= 90°
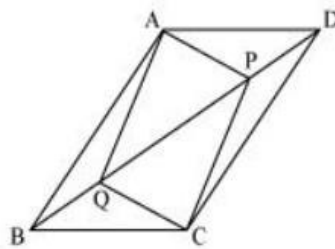
Q 10   Points P and Q have been taken on opposite sides of AB and CD of a parallelogram
        ABCD such that AP = CQ. (see fig.) Show that AC and PQ bisect each other at O.

LONG ANSWER TYPE QUESTIONS

Q 1   In the given figure, ABCD is a parallelogram. Two points P and Q are taken on
        diagonal BD such that DP=BQ. Show that:

        i) ΔAPD≅ ΔCBQ   iii) ΔAQB ≅ ΔCPD

        ii) AP = CQ   iv) AQ = CP

Q 2   In ΔABC and Δ DEF, AB=DE, AB ‖ DE, BC=EF and BC ‖ EF.
        Vertices A, B and C are joined to vertices D, E and F respectively.
        (See fig.)

        Show that (i) quadrilateral ABED is a parallelogram.
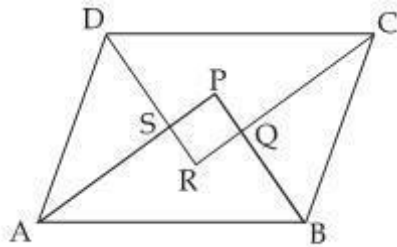
          ii)quadrilateral BEFC is a parallelogram.

        iii) AD ‖ CF and AD = CF. iv)   Quadrilateral ACFD is a

        parallelogram.

        v) AC = DF vi) ΔABC ≅
        ΔDEF
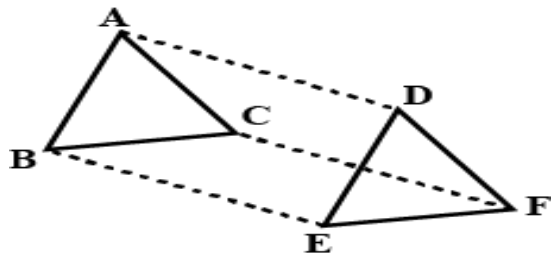
Q 3   Show that bisectors of angles of a parallelogram form a rectangle.


Q 4   Prove that the figure formed by joining the mid-points of the pairs of adjacent sides of a quadrilateral is a parallelogram.

Q 5   If ABCD is a rhombus, show that $AC^2 + BD^2 = 4 AB^2$.

Q 6   A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

Q 7   ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well $\angle C$. Show that (i) ABCD is a square (ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Q 8   A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.
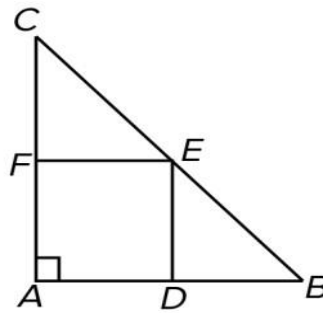
SOLUTIONS TO MCQ

Ans 1   a) 90°

Ans 2   b) 45°

Ans 3   c) 105°

Ans 4   a) Diagonals are equal and bisect each other at right angles.

Ans 5   d) Parallelogram

Ans 6   100°   [3x- 40° + 2x+20°=180°, 5x-20°=180°, x=100°]

Ans 7   c) Rhombus

Ans 8   c) 95°   [$\angle A + \angle B$ =360°- (130°+60°)= 170° (angle sum prop.)

                ½ $(\angle A + \angle B)$ +x =180°, x=95°]

Ans 9   c)13 cm   [BC= 2 x DE = 2 x 6.5 = 13]

Ans 10   a) 50°   [AC=BD ∴ OA=OB ⇒ $\angle OAB = \angle OBA$ =25°,

$\angle AOB$ =130°,

$\angle AOD$ = 180°- 130°= 50°

Ans 11

© 50°   [In ΔBOC, $\angle CBO$ = 180°- (90°+ 40°) = 50°

$\angle CBO = \angle ADB$ = 50°(Alt. in. ∠s)

Ans 12   a) Half

Ans 13   b) Rhombus

Ans 14   b) 80°, 150°   [50°+80°+8x+15x= 360°, x= 10]

Ans 15   a) 6 cm   [by mid-point Thm. PQ= ½ AB, PQ=QR=RP =6]

SOLUTIONS TO CASE STUDIES/ SOURCE BASED

INTEGRATED QUESTIONS

Ans I   (b) Joining the mid-points of the sides of quadrilateral.
(i)

ii)   (b) PQ = ½ AC

iii)   It is an easy, effective and accurate method. Activity based learning of students.

iv)   (c) PS=QR

v)   Principle of Mid-point theorem is used.

Ans II   (i)   Rectangle

ii)   BC=12 m, OB=10m (diagonals bisect), $OC^2 = 12^2 - 10^2$,

OC=√144 − 100

OC= 2√11, AC= 2x 2√11 = 4√11

iii)   4 x 12 = 48 m

iv) $\angle A = \angle C = 70°$ (opp. $\angle's$ are equal), $\angle C + \angle D = 180°$ (adj. angles supplementary), $\angle B = \angle D = 180° - 70° = 110°$

V)   20 x 4 = 80m, for two rounds, 80 x 2 = 160 m

Ans

III   (i)   d)$2\overline{\sqrt{13}}$ [$6^2 + 4^2 = 52$, diagonal = $\sqrt{52} = 2\sqrt{13}$]

(ii)   (c) Kite, rectangle, Parallelogram

iii)   (c) 3

iv)   a) Opposite sides of a parallelogram are equal.

v)   b) Congruent

Ans

IV(i)   Trapezium

ii) $\frac{AH}{AF} = \frac{BC}{BE} = \frac{1}{3}$[Three parallel lines making equal intercepts on any transversal will make equal intercepts on other transversal also]

iii) $\frac{AH}{HF} = \frac{BC}{CE} = \frac{5}{20} = \frac{1}{4}$.

iv) $\angle$ AHC = $\angle$ GFE = 70° (corresponding angles).

SOLUTIONS TO LONG ANSWER TYPE QUESTIONS

Ans 1   In ΔAPD and ΔCBQ, DP=BQ, $\angle$ADP=$\angle$ CBQ (Alt. $\angle$s), AD=BC (Opp sides of ‖gm) ∴ ΔAPD ≅ ΔCBQ by SAS. ∴ AP=CQ (cpct)

Similarly, ΔAQB ≅ ΔCPD ∴ AQ=CP Hence APCQ is a ‖gm

Ans 2   In quadrilateral ABED, AB=DE and AB‖ DE, ∴ ABED is a ‖gm---1

In quadrilateral BEFC, BC=EF and BC‖ EF, ∴ BEFC is a ‖gm---2

From 1 & 2, AD=CF & AD ‖ CF. ∴ ACFD is a ‖gm.

AC=DF. In ΔABC and ΔDEF, AB=DE, BC=EF, CA=FD.

∴ By SSS, ΔABC ≅ ΔDEF.

Ans 3   In ΔADS, ∠ADS + ∠DAS = ½ ∠D+½ ∠A= ½ (∠D+∠A) =½
180°=90°

Also, ∠ADS + ∠DAS + ∠ASD= 180°, 90°+∠ASD = 180°

⟹   ∠ASD = 90° Thus ∠ASD=∠PSR= 90° (VOA)

Similarly, ∠SRQ = ∠RQP = ∠QPS=90° ∴   PQRS is a rectangle.

Ans 4
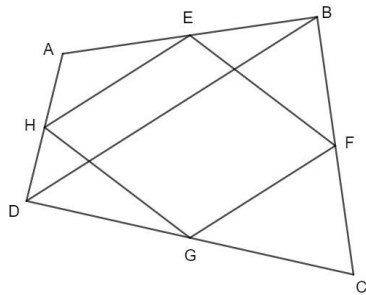
Construction: Join BD.

In ΔBCD, G and F are the mid pts of CD and BC respectively. ∴ GF ∥ BD & GF=

½ BD------ 1

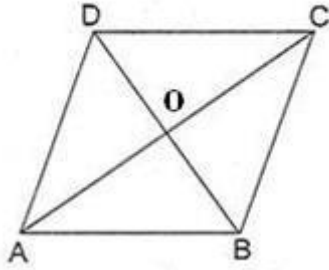In ΔABD, E and H are the mid pts of AB and AD respectively.

∴ EH ∥ BD & EH= ½ BD------ 2

From 1 & 2, EH ∥ GF & EH= GF. ∴ EFGH is a ∥gm

Ans 5

Diagonals of a rhombus bisect each other at rt. angles.

∴ OA = ½ AC, OB= ½ BD

∠AOB=90° ∴ AB²= OA²+OB² ⟹ AB² = (½ AC)²+ (½ BD)²   AB²= ¼ AC²+ ¼ BD² ∴ 4 AB²= AC²+ BD².

Ans 6
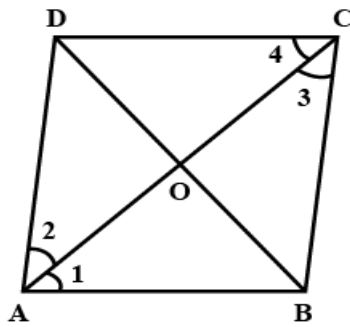
Given: AC bisects ∠A. ∴ ∠1= ∠2------(1),   ∠2 = ∠3 (alter. ∠s)------(2)

From 1 & 2, ∠1= ∠3. ∴ AB=BC (sides opp. to equal angles)

Also, AB=CD & AD=BC (Opp. sides of ∥gm)

⟹ ABCD is a ∥gm in which sides AB=BC=CD=AD

Ans 7
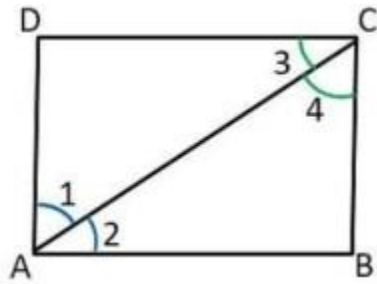
In ΔABC and ΔADC, ∠1= ∠2, ∠3= ∠4, AC=AC. So, ΔABC ≅ ΔADC by AAS

∴ BC=CD, AB=AD (sides opp. to equal angles)

∴ AB=BC=CD=AD. Also ABCD is a rectangle. ∴ ABCD is a square.

Ans 8   Let, ∠A=90°, AB=AC , ∠A=∠C= 45° (∠s opp. to equal sides)

In rt ΔBDE & ΔFCE, ∠CFE = ∠EDB =90°, ∠ C = ∠B = 45°

⟹ DE=EF (Sides square) ∴ By AAS, ΔDBE ≅ ΔFEC

∴ BE=CE (cpct) ∴ E bisects BC.

Ans 9   PS=QR (opp sides of ∥gm) , PB=AR (∵ ½ PS= ½ QR)

∴ PARB is a ∥gm, ∴ PA ∥ BR

∵ B is mid pt of PS and BR ∥ PA ∴ C is the mid pt of SD.

∴ SC = CD. ∵ D is mid pt of QC ∴ QD=CD

Thus, SC=CD=QP. QS=12 cm ∴ CD $= \frac{1}{3}$ QS= $\frac{1}{3}$ x 12 = 4 cm

Ans 10
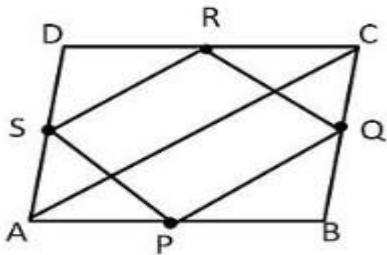
In ΔABC, P is the mid pt of AB and Q is the mid pt of BC.

∴ PQ ∥ AC and PQ = ½ AC------1

In ΔADC, R is the mid pt of CD and S is the mid pt of AD.

∴ SR ∥ AC and SR = ½ AC------2

From 1 & 2 PQRS is a ∥gm.

Now AB=BC (sides of rhombus) ∴ ½ AB= ½ BC ⟹PB=BQ

In ΔPBQ, PB = BQ ∴ ∠QPB=∠ PQB

Now in ΔAPS and ΔCQR, AP=CQ, AS=CR, PS=QR ∴ΔAPS ≅ΔCQR by SSS. ∴ ∠ APS = ∠CQR cpct Now ∠ APS + ∠SPQ + ∠BPQ = 180° And ∠ CQR + ∠PQR + ∠PQB = 180°

∠ APS + ∠SPQ + ∠BPQ = ∠ CQR + ∠PQR + ∠PQB But ∠APS=∠CQR and ∠BPQ= ∠PQB ∴ ∠SPQ = ∠PQR

∠SPQ+ ∠PQR= 180°, ∠SPQ+∠SPQ = 180°
∴ ∠SPQ = 90° PQRS is a ∥ gm with one angle 90°. ∴ PQRS is a rectangle.

CIRCLE

KEY CONCEPTS

1. Recall and review the definition and basic terms related to Circle.

2. To appreciate the theorems

   a. Equal chords of a circle subtend equal angles at the centre.

   b. If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

   c. The perpendicular from the centre of a circle to a chord bisects the chord.

   d. The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord

   e. There is one and only one circle passing through three given non-collinear points

   f. Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).

   g. Chords equidistant from the centre of a circle are equal in length.

   h. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

   i. Angles in the same segment of a circle are equal.

   j. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

   k. The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.

   If the sum of a pair of opposite angles of a quadrilateral is

180º, the   quadrilateral is cyclic.

MULTIPLE CHOICE QUESTIONS

Q1   AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the Centre of the circle is:
  (a) 17 cm (b) 15 cm (c) 4 cm (d) 8 cm

Q2   AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is:
  (a) 6 cm (b) 8 cm (c) 10 cm (d) 12 cm

Q3   ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and ∠ADC = 140º, then ∠BAC is equal to:
  (a) 80º (b) 50º (c) 40º (d) 30º

Q4   In Figure, if ∠OAB = 40º, then ∠ACB is equal to:
(a) 80º (b) 50º   (c) 40º (d) 30º
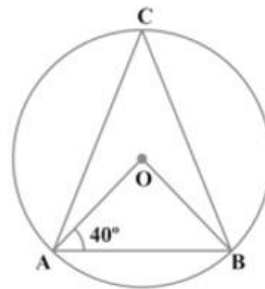
Q 5   The radius of a circle is 13 cm and the length of one of its chords is 10 cm. The distance of the chord from the centre is

(a) 6 cm (b) 8 cm (c) 10 cm (d) 12 cm

Q 6   AD is diameter of a circle and AB is a chord. If AD = 50 cm, AB = 48 cm, then the distance of AB from the centre of the circle is:
  (a) 6 cm (b) 8 cm (c) 5 cm (d) 7 cm

Q 7   In a circle, O is a centre of a circle, A, B and C are the points on a circle, such that ∠ABC=20 º, then ∠AOC is equal to:

(a) 10_0 (b) 20_0 (c) 30_0 (d) 40_0

Q 8   A chord is at a distance of 8 cm from the centre of a circle of radius 17 cm. The length of the chord is:

(a) 25 cm (b) 12.5 cm (c) 30 cm (d) 9 cm

Q 9   The radius of a circle which has a 6 cm long chord, 4 cm away from the centre of the circle is:

(a) 9 cm (b) 8 cm (c) 10 cm (d) 5 cm

Q10   In the given figure,
    O is the centre of the circle.
        ∠ AOB=∠ COD=55 º and CD=5.5 cm, then AB is equal
        ∠ to (a) 9 cm (b) 8 cm (c) 10 cm (d) 5.5 cm
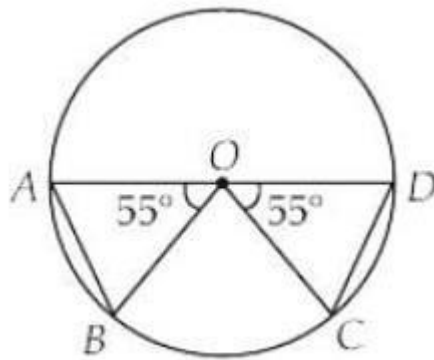
CASE STUDIES/ SOURCE BASED QUESTIONS

Q 1   Q1 Three girls Reshma,   Salama and Mandeep are playing a game by standing on a circle of radius 5 metre. Reshma throws a ball to Salma, Salma to Mandeep and Mandeep to Reshma. The distance between Resham to Mandeep is 6 metre and Reshma to salma is 8 metre. If O is the centre of the circle then answer the following questions.

(i)   Diameter of the circle:
       (a) 6 m (b) 8 m (c) 10 m (d) 12 m

(ii)   Measure of ∠ MRS:
       (a)   180° (b)   90° (c) 100° (d)   80°

(iii) Area of Triangle RMS:
       (a)10 m$_2$ (b)20 m$_2$ (c)24 m$_2$ (d) 40 m$_2$

(iv)   Length of longest chord of a circle : (a)6 m (b)
       8 m (c) 10 m (d) 12 m

(v)   What is the distance between Mandeep and Salma? (a) 6 m (b)
       8 m (c) 10 m (d)   12 m

Q 2   As class IX teacher entered in the class, she told students to do some practice in a circle chapter. She draws two line segment AB and BC so that AB = 8 cm, BC = 6 cm and draw a circle through the three points A, B and C.
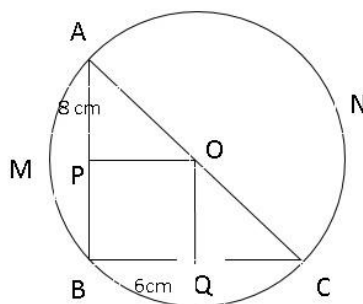
i.   Dilip drew AB and BC as per the figure.
ii.   He drew perpendicular OP and OQ to the line segment AB and BC respectively.

(i)   What you called the line segment AC?
       (a) Arc (b) Diameter (c) Radius (d) Chord

(ii)   What is the measure of ∠ ABC?
                (a)60° (b) 90° (c)45° (d 75°

(iii)   What do you call the region enclosed by minor arc AB and Chord AB?
                (a) Arc (b)Sector (c)Major segment (d)Minor segment

(iv)   What do you call the region enclosed by major arc AB and chord AB? (a) Arc (b)
                Sector (c) Major segment (d) Minor segment

(v)   What is the radius of the circle?
                  (a)   4 cm (b) 3 cm (c) 7 cm (d) 5 cm

Q 3   There was a circular park in Defence colony at Delhi. For fencing purpose Poles A, B, C and D were installed at the circumference of the park. Ram tied wires from A to B, B to C and C to D and D to A. He managed to measure the ∠A = 100° and ∠ D = 80°.The point O is at the centre of the park.Now answer the following questions:
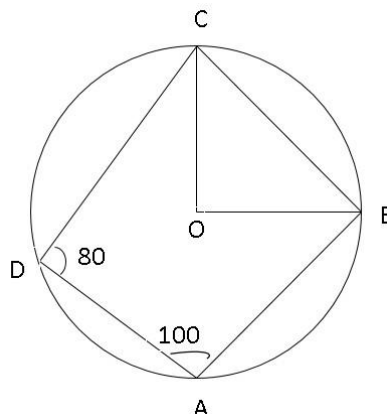
(i)   What is the value of ∠ABC?
               (a) 80° (b) 100° (c) 90° (d) 70°

(ii)   What is the value of ∠BCD?
               (a) 80° (b) 100° (c) 90°(d) 70°

(iii)   What is the special type of quadrilateral ABCD?
               (a) Square (b)Rectangle (C)Cyclic quadrilateral(d) Trapezium

(iv)   What is the property of cyclic quadrilateral? (a)
                    Opposite angles are supplementary
              (b) Adjacent angles are equal
               (C) Opposite angles are equal
               (D) Adjacent angles are complementary

(v)   What do you call the region enclosed by the radii OB, OC and the minor arc BC?
               (a)Segment (b)Arc (c)Chord (d) Sector

Q 4   Government of India is working regularly for the growth of handicapped persons. For this, three STD booths situated at A, B, C as shown in the figure, which are operated by handicapped persons. These three booths are equidistant from each other as shown in the figure
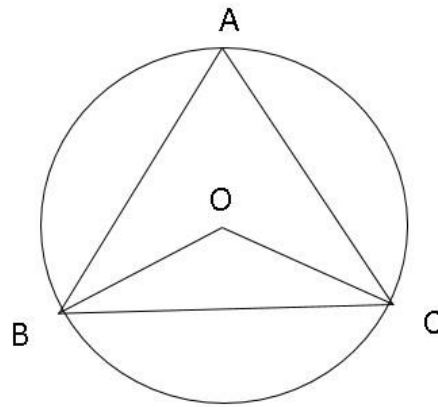
(i)   Which type of Δ ABC in the given figure?
(a) Equilateral triangle(b) Isosceles triangle (c) Right angled triangle

(ii)   Measure of ∠ABC is
(a) 45° (b) 60° (c) 30° (d) 90°

(iii)   if AB = 6 m, the value of BC+CA is (a) 10 (b)
12 (c) 14 (d) 16

(iv)   Measure of ∠BOC is
(a) 90° (b) 100° (c) 120° (d) 50°

(v)   Value of ∠OBC + ∠OCB is
(a) 60° (b) 30° (c) 45° (d) 90°

Q 5   A farmer has a circular garden as shown in the picture below. He has a different type of trees, plants and flower plants in his garden. In the garden, there are two mango trees A and B at a distance of AB
= 10 m. Similarly, he has two Ashoka trees at the same distance of
10 m as shown at C and D. AB subtends ∠AOB = 120° at the centre O. The perpendicular distance of AC from centre is 5 m. The radius of the circle is 13 m.
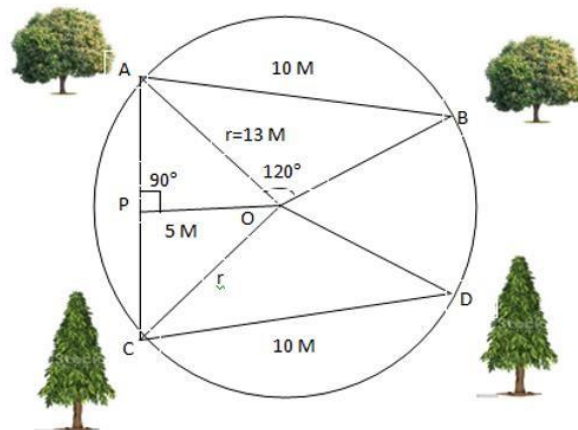
(i)   What is the value of ∠AOB?
(a)   60° (b) 120° (c) 100° (d) 80°

(ii)   What is the distance between mango tree A and ashoka tree C? (a) 12 cm (b)24 cm (c) 13 cm(d) 15 cm

(iii)   What is the value of ∠OAB?
(a) 60° (b) 120° (c) 30° (d) 90°

(iv)   What is the value of ∠COD?
        (a) 60° (b) 120° (c) 30° (d) 90°

(v)   What is the value of ∠ODC?
        (a) 90° (b) 120° (c) 60° (d) 30°

OBJECTIVE TYPE QUESTIONS (OTHER THAN MCQs)

Write True or False and justify your answer in each of the following:

Q 1   Two chords AB and CD of a circle are each at distances 4 cm from the centre. Then AB = CD.

Q 2   Two chords AB and AC of a circle with centre O are on the opposite sides of OA. Then ∠OAB = ∠OAC .

Q 3   If AOB is a diameter of a circle and C is a point on the circle, then $AC^2 + BC^2 = AB^2$.

FILL IN THE BLANKS

Q 4   Two congruent circles with centres O and O' intersect at two points A and B. Then ∠ AOB =_____ .

Q 5   A circle of radius 3 cm can be drawn through two points A, B.The maximum possible length of AB =_____.

Q 6   Segment of a circle is the region between an arc and _____ of the circle.

Q 7

In Figure, if ∠ABC = 20º, then find the ∠AOC.
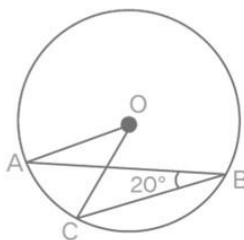
Q 8   In Figure, if AOB is a diameter of the circle and AC = BC, then find the ∠CAB.
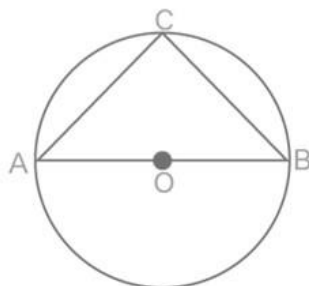
Q 9   AB is a chord of a circle with radius 'r'. If P is any point on the circle such that ∠APB is a right angle, then find AB.

Q 10   What is the measure of the angle subtended by a semi-circle at centre?

SHORT ANSWER TYPE QUESTIONS

Q 1   Prove that Equal chords of a circle subtend equal angles at the centre.

Q 2   If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Q 3   A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.
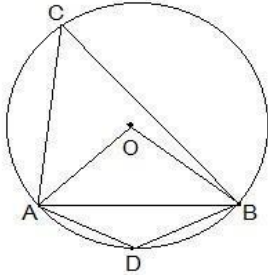
Q 4   In Figure, $\angle$ ABC = 69°, $\angle$ ACB = 31°, find $\angle$ BDC.
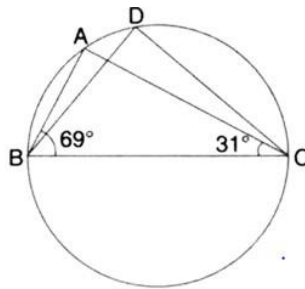
Q 5   In Figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle$ BEC = 130° and $\angle$ ECD = 20°. Find $\angle$ BAC.
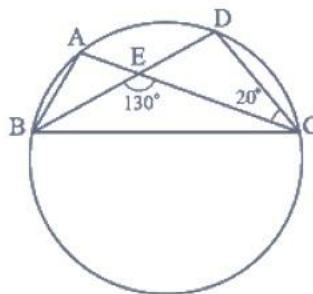
Q 6   In the given figure, O is the centre of the circle with chords AP and BP being produced to R and Q respectively. If $\angle$QPR = 35°, find the measure of $\angle$AOB.
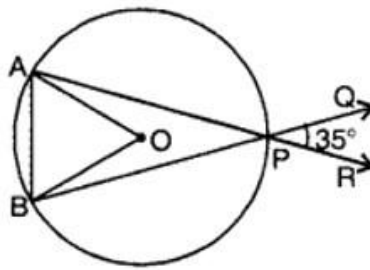
Q 7   In the figure, PQRS is a cyclic quadrilateral. Find the value of x.
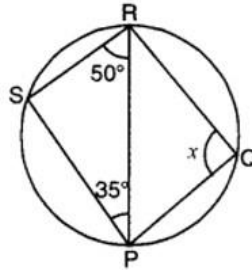
Q 8   Prove that a cyclic parallelogram is a rectangle.
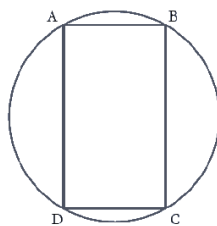
Q 9   Two circles intersect at two points A and B. AD and AC are diameters to the two circles (see Figure). Prove that B lies on the line segment DC.
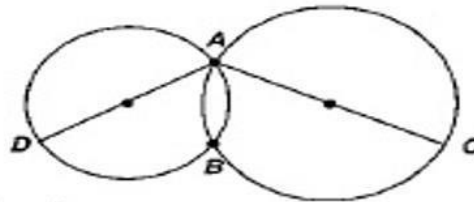
Q 10   If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Figure).
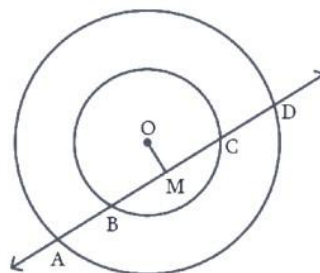
LONG ANSWER TYPE QUESTIONS

Q 1   Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Q 2   ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If ∠ DBC = 55° and ∠ BAC = 45°, find ∠ BCD.
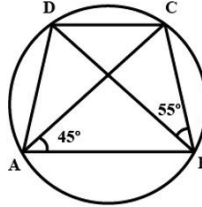
Q 3   △ ABC and △ ADC are two right triangles with common hypotenuse AC. Prove that ∠ CAD = ∠ CBD.

Q 4   ABCD is a parallelogram. The circle through A, B and C intersects CD (produce if necessary) at E. Prove that AE = AD.

Q 5   If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that AQ = 1/ 2 (BC + CA + AB)
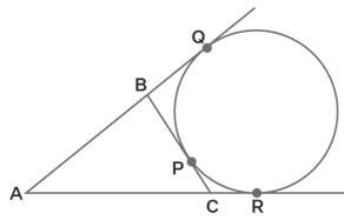
Q 6   ∠ PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠ OPR.
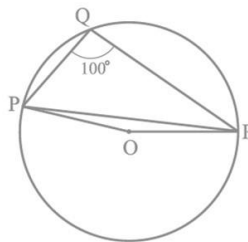
Q 7   ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠BCD. Further, if AB = BC, find ∠ ECD.
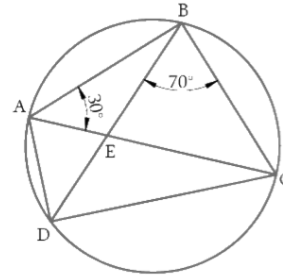
Q 8.   If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

SOLUTIONS TO MCQ

Ans 1   (C) $17^2-15^2=8^2$, $8÷2=4$cm

Ans2   (C) $12^2+16^2=20^2$, $20÷2=10$cm

Ans3   (B) ∠ABC = 180º - 140º, ∠BAC + 90º + 40º = 180º, ∠BAC = 50º

Ans4   (B) ∠ABO=40 º. ∠O=100, ∠C=100÷2=50

Ans5   (D) $13^2-5^2=12^2$, 12cm

Ans6   (D) $25^2-24^2=7^2$, 7cm

Ans7   (A) ∠AOC = 2 × 20° , ∠AOC = 40°

Ans8   (C) $17^2-8^2=15^2$, 15+15=30cm

Ans9   (D) $4^2+3^2=5^2$, 5cm

Ans10

(D) OA = OD, OB = OC, ∠ AOB = ∠ COD,Δ AOB= Δ COD (By SAS)

AB = CD = 5.5 cm

SOLUTIONS TO CASE STUDIES/ SOURCE BASED INTEGRATED QUESTIONS

1   (i)c,(ii)b,(iii)c,(iv)c,(v)c

2   b,b,d,c,d

3   b,a,c,a,d

4   A,b,b,c,c

5   B,b,c,a,d

1   True

2   True

3   True

4   ∠AO'B

5   6 cm

6   chord

7   We know that Angle at the centre of the circle is twice the angle at the <u>circumference</u> subtended by the same arc. ∠AOC = 2 ∠ABC

It is given that ∠ABC = 20º
Substituting the values
∠AOC = 2 × 20º
∠AOC = 40º
Therefore, ∠AOC is equal to 40º.

8   We know that
<u>Diameter</u> subtends a right angle to the circle
∠BCA = 90º … (1)
It is given that
AC = BC
As the angles opposite to equal sides are equal
∠ABC = ∠CAB … (2)
In triangle ABC using the <u>angle sum property</u>
∠CAB + ∠ABC + ∠BCA = 180º
From equations (1) and (2)
∠CAB + ∠CAB + 90º = 180º
2∠ CAB = 180 º - 90 º
2∠ CAB = 90 º
Dividing both sides by 2
∠CAB = 45º, Therefore, ∠CAB is equal to 45º.

9   It is given to us that, ∠ APB = 90°.
We know that a right angle is formed inside a circle when a triangle is inscribed inside a semicircle.
Thus, the chord used to form the right angle has to be the diameter of the circle.
Thus, chord AB is the diameter of the circle, that is, it is equal to 2r.

10   Angle subtended by a diameter/ semicircle on any point of a circle is 90°

SOLUTIONS TO SHORT ANSWER TYPE QUESTIONS

1   Using equal angles at the centers and the fact that circles are congruent, we prove the statement using Side-Angle-Side (SAS criteria) and corresponding parts of congruent triangles (CPCT).
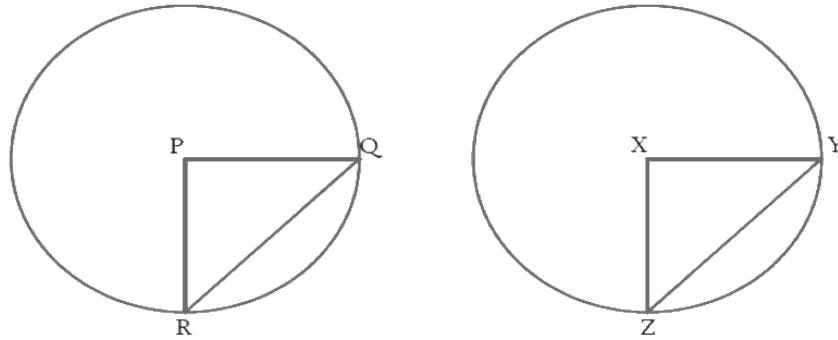
Draw chords QR and YZ in two congruent circles as shown above. Join the radii PR, PQ, and XY, XZ respectively.

Given that chords subtend equal angles at the center.   So, $\angle$ QPR = $\angle$ YXZ.

We need to prove that chords are equal, that is, QR = YZ Since the circles are congruent, their radii will be equal.

PR = PQ = XZ = XY

Consider the two triangles ΔPQR and ΔXYZ.

PQ = XY (Radii are equal)

$\angle$ QPR = $\angle$ YXZ (Chords subtend equal angles at center)

PR = XZ (Radii are equal)

By SAS criteria, ΔPQR is congruent to ΔXYZ.

So, QR = YZ (Corresponding parts of congruent triangles)

Hence proved if chords of congruent circles subtend equal angles at their center then the chords are equal.

2
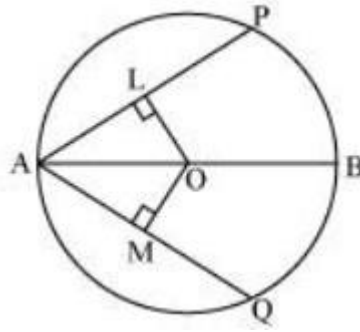
Given: **AB** is the diameter of the circle with centre **O.AP** and **AQ** are two intersecting chords of the circle such that ∠**PAB** = ∠**QAB**

Proof: In △**AOL** and △**AOM**
∠**OLA** = ∠**OMB** (each **90∘**)
OA = OA (common line)
∴ ∠**OAL** = ∠**OAM** (∠ **PAB** = ∠**QAB)**

∴ △ **AOL** ≅ △ **AOM** by AAS congruence criterion
⇒**OL = OM** by CPCT
⇒ Chords **AP** and **AQ** are equidistant from centre **O**
⇒ **AP=AQ**(chords which are equidistant from the centre are equal.)

3
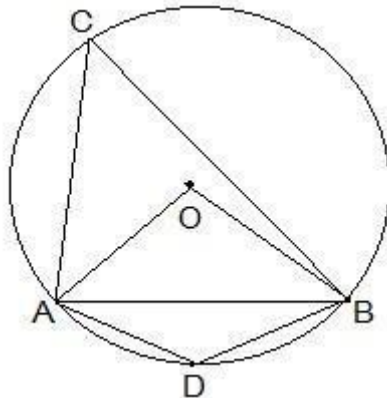
The chord AB is equal to the radius of the circle.
OA and OB are the two radii of the circle.
From ΔOAB.
AB = OA = OB = radius of the circle.
ΔOAB is an equilateral triangle.
∴ AOC = 60°
And, ACB = ½ AOB
So, ACB = ½ × 60° = 30°
Now, ACBD is a cyclic quadrilateral,
ADB +ACB = 180° (Since they are the opposite angles of a cyclic quadrilateral)

So, ADB = 180°-30° = 150°
So, the angle subtended by the chord at a point on the minor arc and also at a point on the major arc is 150° and 30° respectively.

4   Consider the ΔABC, the sum of all angles will be 180°.
     ∠ABC + ∠BAC + ∠ACB = 180°
     69° + ∠BAC + 31° = 180°
     ∠BAC = 180° -100°
     = 80°
     We know that angles in the same segment of a <u>circle</u> are equal. So, ∠BDC =
     ∠BAC = 80°

5   Consider the straight-line BD. As the line AC intersects with the line BD, the
     sum of two adjacent angles so formed is 180°.
     Therefore, ∠BEC + ∠DEC = 180°
     130° + ∠DEC = 180°
     ∠DEC =180° - 130° = 50°
     Consider the ΔDEC, the sum of all angles will be 180º.
     ∠DEC + ∠EDC + ∠ECD = 180°
     50° + ∠EDC + 20° = 180°
     ∠EDC = 180° - 70° = 110°
     ∴∠BDC = ∠EDC = 110°
     We know that angles in the same segment of a <u>circle</u> are equal. ∴∠BAC = ∠BDC
     = 110°

6   ∠APB = ∠RPQ = 35° [vertically opposite angles]
     Now, ∠AOB and ∠APB are angles subtended by an arc AB at centre and at the
     remaining part of the circle.
     ∴ ∠AOB = 2∠APB
          = 2 × 35°
          = 70°

7   In ΔPRS, by using angle sum property, we have
     ∠PSR + ∠SRP + ∠RPS = 180°
     ∠PSR + 50° + 35º = 180°
     ∠PSR = 180° − 85º = 95°
     Since PQRS is a cyclic quadrilateral
     ∴ ∠PSR + ∠PQR = 180°
     [∵ opposite angles of a cyclic quadrilateral are supplementary]
     95° + x = 180°
     x = 180° − 95°
     x = 85°

8   Let ABCD be the cyclic parallelogram.
     We know that opposite angles of a parallelogram are equal.
     ∠A = ∠C and ∠B = ∠D   ... (1)
     We know that the sum of either pair of opposite angles of a cyclic
     <u>quadrilateral</u> is 180°.
     ∠A + ∠C = 180°
     ∠A + ∠A = 180° (From equation (1))
     2∠A = 180°
     ∠A = 90°
     We know that if one of the <u>interior angles</u> of a parallelogram is 90°, all the
     other angles will also be equal to 90°.

     Since all the angles in the parallelogram are 90°, we can say that parallelogram

ABCD is a <u>rectangle</u>

9   Join AB. ∠ ABD = 90° (Angle in a semicircle) ∠ ABC = 90° (Angle in a semicircle) So, ∠ ABD + ∠ ABC = 90° + 90° = 180° Therefore, DBC is a line. That is B lies on the line segment DC.

10   Draw a perpendicular from the center of the circle OM to the line AD. We can see that BC is the chord of the smaller circle, and AD is the chord of the bigger circle. We know that <u>perpendicular</u> drawn from the center of the circle <u>bisects</u> the chord. ∴ BM = MC ... (1) and, AM = MD ... (2)
Subtracting (2) from (1), we obtain
AM − BM = DM − CM
∴ AB = CD

SOLUTIONS TO LONG ANSWER TYPE QUESTIONS

1   Given :
An arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.
To prove :POQ=2PAQ
To prove this theorem we consider the arc AB in three different situations, minor arc AB, major arc AB and semi-circle AB.
Construction :
Join the line AO extended to B.
Proof :
BOQ=OAQ+AQO   .....(1)
Also, in △ OAQ,
OA=OQ   [Radii of a circle]
Therefore,
∠ OAQ=∠ OQA   [Angles opposite to equal sides are equal]
∠ BOQ=2∠ OAQ   ....(2)
Similarly, BOP=2∠ OAP   ....(3)
Adding 2 & 3, we get,
∠ BOP+∠ BOQ=2(∠ OAP+∠ OAQ)
∠ POQ=2∠ PAQ   ....(4)
For the case 3, where PQ is the major arc, equation 4 is replaced by Reflex angle,
∠ POQ=2∠ PAQ
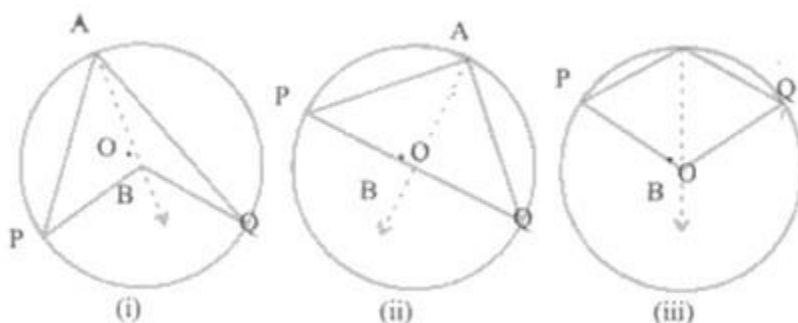
2
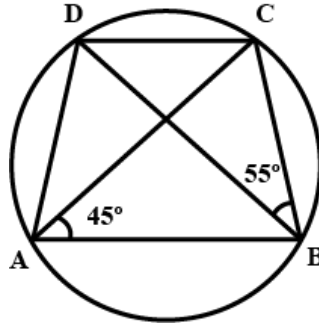
∠ CAD = ∠ DBC = 55° (Angles in the same segment) Therefore, ∠ DAB = ∠ CAD + ∠ BAC = 55° + 45° = 100° But ∠ DAB + ∠ BCD = 180° (Opposite angles of a cyclic quadrilateral) So, ∠ BCD = 180° − 100° = 80°

3
We know that, the sum of all angles in a triangle is 180°.

If the sum of pair of opposite angles in a quadrilateral is 180°, then it is a cyclic quadrilateral.
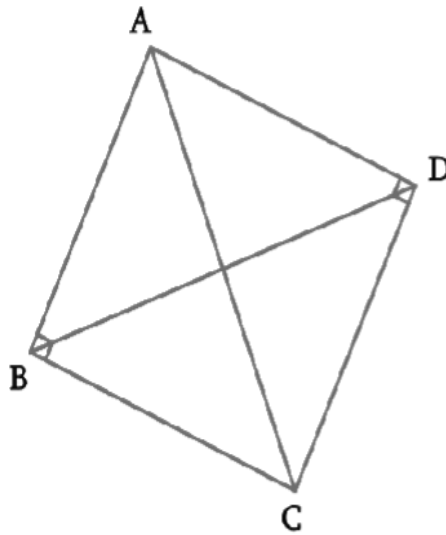
Consider ΔABC,

∠ABC + ∠BCA + ∠CAB = 180° (Angle sum property of a triangle)

90° + ∠BCA + ∠CAB = 180°

∠BCA + ∠CAB = 90°.....(1)

Consider ΔADC,

∠CDA + ∠ACD + ∠DAC = 180° (Angle sum property of a triangle)

90° + ∠ACD + ∠DAC = 180°

∠ACD + ∠DAC = 90°.....(2)

Adding Equations (1) and (2), we obtain

∠BCA + ∠CAB + ∠ACD + ∠DAC = 180°

(∠ BCA +∠ ACD) + (∠ CAB + ∠DAC) = 180°

∠BCD + ∠DAB = 180°.....(3)

However, it is given that

∠B + ∠D = 90° + 90° = 180°.....(4)

From Equations (3) and (4), it can be observed that the sum of the measures of opposite angles of quadrilateral ABCD is 180°. Therefore, it is a cyclic quadrilateral.

Since it is a cylclic quadrilateral the below figure can be drawn.
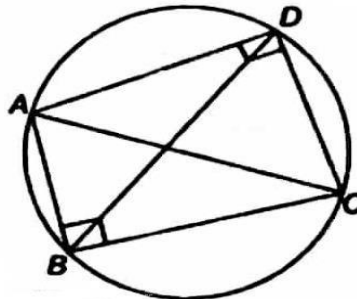
Consider chord CD. ∠CAD and ∠CBD are formed on the same segment CD.

∠CAD = ∠CBD (Angles in the same segment are equal)

4
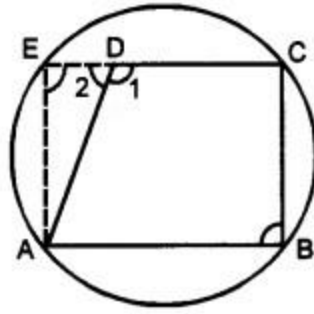
Given : ABCD is a parallelogram. Circle through A, B and C intersects CD produced in E.

To Prove: AE = AD

Proof : ABCE is a cyclic quadrilateral.

∴∠B + ∠E = 180°

…(i) ABCD is a parallelogram.

∴∠B = ∠1 … (ii)

Also, ∠1 + ∠2 = 180° [linear pair]

∠B + ∠2 = 180° …(iii) [using (ii)]

Now, from (i) and (iii), we have

∠B + ∠E = ∠B + ∠2

∠E = ∠2 In ΔDE, we have

∠E = ∠2

⇒ AD = AE [side opposite to equal angles of a A]

5   If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal., BQ = BP CP = CR, and AQ = AR Now, 2AQ = AQ + AR = (AB + BQ) + (AC + CR) = AB + BP + AC + CP = (BP + CP) + AC + AB = BC + CA + AB i.e., AQ = 1/ 2 (BC + CA + AB).
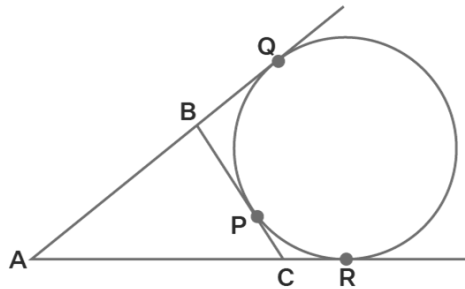
6   The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle and the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Mark any point on the major arc side (opposite side to point Q) as S.
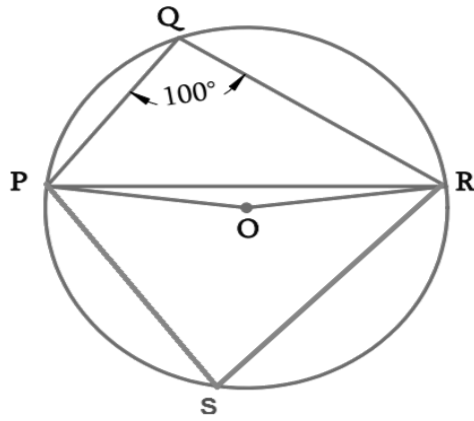
Since all points P, Q, R, S lie on the circle, PQRS becomes a cyclic quadrilateral.

We know that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Therefore,

∠PQR + ∠PSR = 180°

100° + ∠PSR = 180°

∠PSR = 180° - 100° = 80°

We know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

Therefore,

∠POR = 2∠ PSR = 2 × 80° = 160°

Consider the ΔOPR. It is an isosceles triangle as OP = OR = Radius of the circle.

Thus, ∠OPR = ∠ORP .Sum of all angles in a triangle is 180°.

Therefore,

∠OPR + ∠POR + ∠ORP = 180°

7.
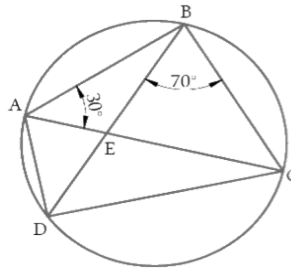
In the triangles, ABD and BCD, ∠CAD = ∠CBD = 70°. (Angles in the same segment are equal)

Hence, ∠BAD = ∠CAB + ∠DAC

= 30° + 70° = 100°

Thus, ∠BAD = 100°

Since ABCD is a cyclic quadrilateral, the sum of either pair of opposite angles of a cyclic quadrilateral is 180º.

∠BAD + ∠BCD = 180°

∠BCD = 180° - 100°

= 80°

Thus, ∠BCD = 80°

Also given AB = BC.

So, ∠BCA = ∠BAC = 30° (Base angles of isosceles triangle are equal)

∠ECD = ∠BCD - ∠BCA

= 80° - 30°

= 50°

Thus, ∠ECD = 50°

8.
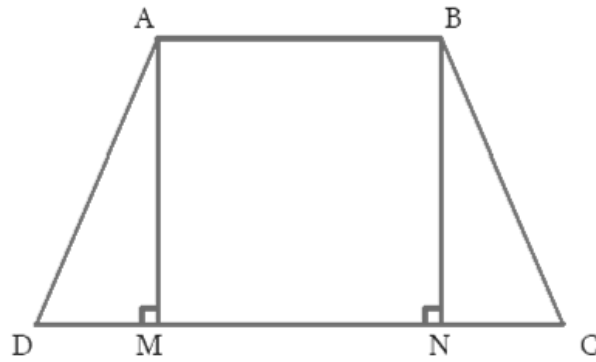
Consider ΔAMD and ΔBNC

AD = BC (Given)

∠AMD = ∠BNC (90°)

AM = BN (Perpendicular distance between two parallel lines is same)
By RHS congruence, ΔAMD ≅ ΔBNC.

Using CPCT, ∠ADC = ∠BCD.....(1)

∠BAD and ∠ADC are on the same side of transversal AD.

∠BAD + ∠ADC = 180°

∠BAD + ∠BCD = 180° [From equation(1)]

This equation proves that the sum of opposite angles is supplementary. Hence, ABCD is a cyclic quadrilateral.

Area of Triangle

- $\frac{1}{2}$ ×BASE ×HEIGHT
- When Three sides are given, then we can find the

Area of a Triangle — by Heron's Formula

The formula given by Heron about the area of a triangle, is also known as Hero's formula. It is stated as:

AREA OF TRIANGLE = $\sqrt{s(s-a)(s-b)(s-c)}$

Semi perimeter (s) = $\frac{a+b+c}{2}$

Long Type Questions

Q.1 Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm.

Q.2: A triangular park ABC has sides 120m, 80m and 50m. A gardener Rashmi has to put a fence

all around it and also plant grass inside. How much area does she need to plant? Find the cost.

of fencing it with barbed wire at the rate of Rs.20 per metre leaving a space 3m wide for a gate on one side.

Q.3: The sides of a triangular plot are in the ratio of 3: 5: 7 and its perimeter is 300 m. Find its area.

Q.4 An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area

of the triangle.

Q.5 Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

Answer: (1) $8\sqrt{30}$**cm²** **(2) Rs. 4940** **(3)** $1500\sqrt{3}$ **m²** **(4)** $9\sqrt{15}$ **cm²** **(5)** $21\sqrt{11}$ **cm²**

Short Type Questions
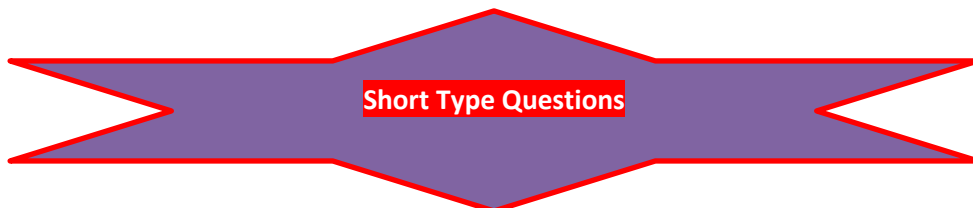
Q.1 Find the area of an equilateral triangle with side $6\sqrt{3}$ cm.

Q.2 If each side of an equilateral triangle of area A is doubled, then find the area of new triangle.

Q.3 The side of a triangle are 16 cm,30 cm and 34 cm. Find its area.

Q.4 Find the area of an equilateral triangle with perimeter 12 cm.

Q.5 The sides of a triangle are 10 cm, 12 cm and 14 cm respectively, then find its semiperimeter.

Answer (1) $27\sqrt{3}$ **cm²** **(2) 4 times the area of the original triangle** **(3)** 240 **cm²** **(4)** $4\sqrt{3}$ **cm²**

(5) 18

Very Short Type Questions

Q.1 The sides of a triangle are 10 cm, 12 cm and 14 cm respectively, then its area is _____cm².

Q.2 The perimeter of a triangular field is 450 m and its sides are in the ratio 13: 12 :5. then the area of the triangle is _____ m².

Q.3 If the sides of a triangle are 18 cm and 10 cm and its perimeter is 42 cm, then the area of the triangle is _____

Q.4 An isosceles triangle has perimeter 20 cm and each of the equal sides is 6 cm, then its area is ___.

Q.5 If two sides of a triangle are 5 cm, 12 cm and the perimeter is 30 cm , then the area of the triangle

is ____.

Multiple Choice Questions (MCQ)

Q.1 Area of triangle is equal to:

a. Base x Height

b. 2(Base x Height)

c. 1⁄2(Base x Height)

d. 1⁄2 (Base + Height)

Q.2 If the perimeter of an equilateral triangle is 180 cm. Then its area will be:

a. 900 cm^2

b. 900√3 cm^2

c. 300√3 cm^2

d. 600√3 cm^2

Q.3 The sides of a triangle are 122 m, 22 m and 120 m respectively. The area of the triangle is:

a. 1320 sq.m

b. 1300 sq.m

c. 1400 sq.m

d. 1420 sq.m

Q.4 The area of triangle with given two sides 18cm and 10cm respectively and perimeter equal to 42 cm is:

a. 20√11 cm^2

b. 19√11 cm^2

c. 22√11 cm^2

d. 21√11 cm^2

Q.5 The sides of a triangle are in the ratio 12: 17: 25 and its perimeter is 540cm. The area is:

a. 1000 sq.cm.

b. 5000 sq.cm.

c. 9000 sq.cm.

d. 8000 sq.cm.

Answer: 1(c)   2.(b)   3 (a)   4 (d)   5(c)

Q.1 Director of DAV public school planned to fix a signal board, indicating **SCHOOL AHEAD** across main

road. It is an equilateral triangular shaped with side 24 cm. Principal of the school calls the monitor

Brajesh of class IX and asked the following questions:

Q.1 What is the perimeter of board?

   (a) **72 cm**

   (b) 82 cm

   (c) 90 cm

   (d) 100 cm

Q.2 Find area using Heron's formula.

   (a) $150\sqrt{3}$ cm²

   (b) $160\sqrt{3}$ cm²

   (c) $170\sqrt{3}$ cm²

   (d) **$144\sqrt{3}$ cm²**

Q.3 Sum of all sides is called----

   (a) **Volume**

   (b) Area

   (c) **Perimeter**

   (d) None of these

Q.4 If all three sides were 20 cm then perimeter will be -----

(a) **30 cm**

(b) 40 cm

(c) 50 cm

(d) **60 cm**

Answers: 1.(a)   2.(d)   3.(c) 4.(d)

Q.2 Gaurav has umbrella which is made by stitching 10 triangular pieces of cloth of two different designs,

each piece measuring 20 cm,50 cm and 50 cm.

Q.1 Perimeter of triangle is

(a) **120 cm**   (b) 150 cm   (c) 180 cm   (d) 200 cm

Q.2 Semi perimeter of triangle is -----

(a) 50 cm   (b) **60 cm**   (c) 40 cm   (d) 30 cm

Q.3 Area of triangular piece in cm² is ----

(a) $100\sqrt{6}$ cm²

(b) $500\sqrt{6}$ cm²

(c) **$200\sqrt{6}$ cm²**

(d) $600\sqrt{6}$ cm²

Q.4   Area of 5 triangular piece in cm² is ----

(a) $500\sqrt{6}$ cm²

(b) $1000\sqrt{2}$ cm²

(c) **$1000\sqrt{6}$ cm²**

(d) $290\sqrt{6}$ cm²

Answer: 1.(a)   2.(b)   3.(c)   4.(c)

CHAPTER- 11

SURFACE AREAS AND VOLUMES

CURVED SURFACE AREA OF A CONE = $\pi r l$

TOTAL SURFACE AREA OF A CONE = $\pi r(l + r)$

SLANT HEIGHT OF A CONE(l) = $\sqrt{h^2 + b^2}$

SURFACE AREA OF SPHERE = $4\pi r^2$

CSA OF SPHERE = $4\pi r^2$

SPHERE

VOLUME OF SPHERE = $\frac{4}{3}\pi r^3$

Curved Surface Area of a Hemisphere = $2\pi r^2$

Total Surface Area of a Hemisphere = $3\pi r^2$
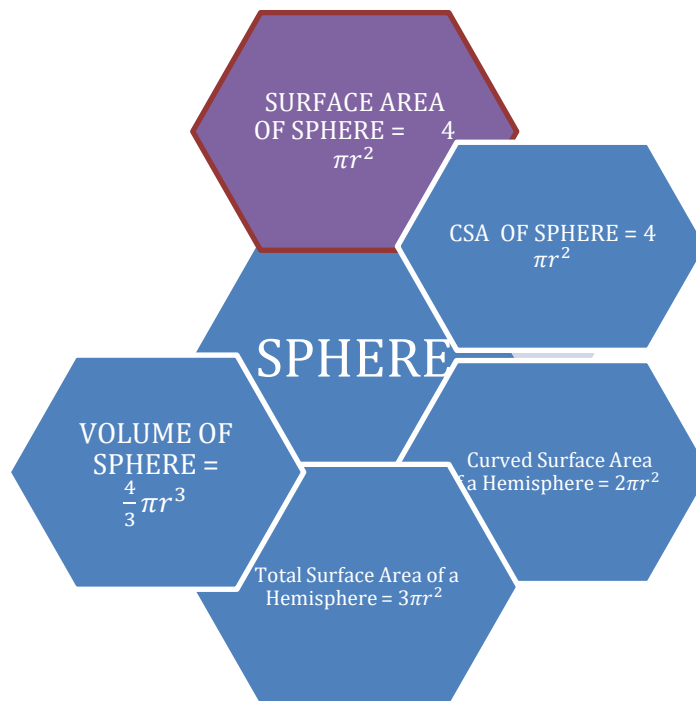
- **VOLUME OF HEMISPHERE =** $\frac{2}{3}\pi r^3$

Q.1 What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use π = 3.14).

Q.2 A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is
17.6 m, find the cost of painting it, given the cost of painting is Rs.5 per 100 cm^2 .
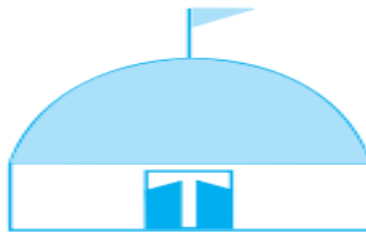
Q.3 The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the
ratio of surface areas of the balloon in the two cases.

Q.4 A right circular cylinder just encloses a sphere of radius r .

Find (i) surface area of the sphere,
(ii) curved surface area of the cylinder,
(iii) Ratio of the areas obtained in (i) and (ii).

Q.5 Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with
surface area S'. Find the (i) radius r' of the new sphere, (ii) ratio of S and S'.

Answer: (1) 63 m (2) Rs. 24640 (3) 1:4 (4) 3r and 1:9 (5) (i) $4\pi r^2$ (ii) $4\pi r^2$ (iii) 1:1

SHORT TYPE QUESTIONS

Q.1 A conical tent is 10 m high and the radius of its base is 24 m. Find slant height of the tent.

Q.2 The hollow sphere, in which the circus motorcyclist performs his stunts, has a diameter of 7 m. Find the
area available to the motorcyclist for riding.

Q.3 The height of a cone is 15 cm. If its volume is 1570 cm^3 , find the radius of the base. (Use π = 3.14)

Q.4 The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of

the volume of the earth is the volume of the moon?

Q.5 A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

Answer: (1) 26 m	(2) 154 m²	(3) r = 10 cm	(4) 1:64	(5) 89.8 cm³

VERY SHORT TYPE QUESTIONS

Q.1 Fin

d the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.

Q.2 Find the surface area of a sphere of radius 7 cm.

Q.3 Find the total surface area of a hemisphere of radius 10 cm. (Use π = 3.14)

Q.4 Find the volume of a sphere of radius 11.2 cm.

Q.5 Find the surface area of a sphere of diameter 14 cm.

Answer: (1) 220 cm²	(2) 616 cm²	(3) 942 cm²	(4)5887.32 cm³	(5) 616 cm²

MULTIPLE CHOICE QUESTIONS (MCQ)

Q.1 What is the total surface area of a hemisphere of radius r?
 (a) $4\pi r^2$
(b) πr^2
(c) $2\pi r^2$
(d) $3\pi r^2$

Q.2 If a right circular cone has radius 4 cm and slant height 5 cm then what is its volume?
(a) 16 π cm³
(b) 14 π cm³
(c) 12 π cm³
(d) 18 π cm³

Q.3 The surface area of a sphere of radius 14 cm is:
a. 1386 sq.cm
b. 1400 sq.cm
c. 2464 sq.cm
d. 2000 sq.cm

Q.4 If the radius of a sphere is doubled, then what is the ratio of their surface area?
(a) 1: 2
(b) 2: 1
(c) 1: 4
(d) 4: 1

Q.5 If slant height of the cone is 21cm and diameter of base is 24 cm. The total surface area of cone is:
 a. 1200.77 sq.cm
b. 1177 sq.cm

c. 1222.77 sq.cm

d. 1244.57 sq.cm

Answer: (1) d	(2) a	(3) c	(4) c	(5) d

CASE STUDY BASED QUESTIONS

Q.1 Today is Shalu's birthday. Her parents went to the market and bought a cake and 12 birthday caps. All caps are like a cone. Shalu's mother gave a cap to each child. All of Shalu's friends enjoyed a lot there. After dinner, Shalu's father called all her friends and asked them some questions.

(a) If the diameter of a cone is 14 cm and its slant height is 9 cm. The radius of cone is-

    (i) 28 cm

    (ii) 14 cm

    (iii) 7 cm

    (iv) 23 cm

(b) If the diameter of a cone is 14 cm and its slant height is 9 cm. The curved surface area of cone is-

    (i) 126 cm^2

    (ii) 178 cm^2

    (iii) 188 cm^2

    (iv) 198 cm^2

(c) If the radius of a cone is 8 cm and its slant height is 10cm. The height of cone is-

    (i) 6 cm

    (ii) 7 cm

    (iii) 8 cm

    (iv)None of these

(d) The diameter of a cone is 8 cm and its volume is 48 π cm^3.What is its height?

    (i) 7 cm

    (ii) 8 cm

    (iii) 9cm

    (iv) None of these

Answer: (a) ii	(b) iv	(c) i	(d) iii

Q.2 Traffic cone are used outdoor during road work in various situations such as traffic redirection, advance warning of hazards or the prevention of traffic. A traffic cone has the radius of 12 cm and height 16 cm.

Answer the following questions based of the above data.

(a) What is the slant height of the traffic cone?

(i) 10 cm

(ii) 20 cm

(iii) 30 cm

(iv) 40 cm

(b) What will be the total surface area of of the traffic cone?

(i) 1204.76 cm^2

(ii) 1203.76 cm^2

(iii) 1206.76 cm^2

(iv) 1205.76 cm^2

(c) What will be the curved surface area of the each traffic cone?

(i) 753.6 cm^2

(ii) 756.3 cm^2

(iii) 735.6 cm^2

(iv) None of these

(d) Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm.

(i) 210 cm^2

(ii) 202 cm^2

(iii) 220 cm^2

(iv) None of these

Answer: (a) ii (b) iv   (c) i   (d) iii

Summary

In this chapter, you have studied the following points:

1. Curved surface area of a cone = πrl

2. Total surface area of a right circular cone = πrl + πr 2, i.e., πr (l + r)

3. Surface area of a sphere of radius r = 4 π r^2

4. Curved surface area of a hemisphere = 2πr^2

5. Total surface area of a hemisphere = 3πr^2

6. Volume of a cone = $\frac{1}{3}$ πr^{2}h

7. Volume of a sphere of radius r = $\frac{4}{3}$ π r^3

8. Volume of a hemisphere = $\frac{2}{3}$ πr^3

CHAPTER- 12

STATISTICS

In this chapter, you have studied the following points:

1. How data can be presented graphically in the form of bar graphs, histograms   and frequency polygons.

Bar Graph-

A bar graph is a pictorial representation of data in which usually bars of uniform width are drawn with

equal spacing between them on one axis (say, the x-axis), depicting the variable. The

values of the variable are shown on the other axis (say, the y-axis) and the heights of

the bars depend on the values of the variable.

Histogram- This is a form of representation like the bar graph, but it is used for continuous class intervals.

Frequency Polygon-There is yet another visual way of representing quantitative data and its frequencies.

This is a polygon.

2. In Histogram areas of the rectangles erected are proportional to the corresponding frequencies. **However, since the widths of the rectangles are all equal, the lengths of the rectangles are proportional to the frequencies.**

3. Frequency polygons can also be drawn independently without drawing histograms. For this,
     we require the mid-points of the class-intervals used in the data. These mid-points of the class-
     Intervals are called class-marks.

$$\text{Class-Mark} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Q.1 A family with a monthly income of ` 20,000 had planned the following expenditures per month under
various heads:

Heads	Expenditure (in thousand rupees)
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Draw a bar graph for the data above.

Q.2   The following table gives the marks scored by 100 students in an entrance examination.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Students(frequency)	4	10	16	22	20	18	8	2

Represent this data in the form of **a histogram**

Q.3   Consider the marks, out of 100, obtained by 51 students of a class in a test, given in table.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	Total
No. of Students	5	10	4	6	7	3	2	2	3	9	51

Draw a **frequency polygon** corresponding to this frequency distribution table.

Q.4   The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:

Number of balls	Team A	Team B
1-6	2	5
7-12	1	6
13-18	8	2
19-24	9	10
25-30	4	5
31-36	5	6
37-42	6	3
43-48	10	4
49-54	6	8
55-60	2	10

Represent the data of both the teams on the same graph by **frequency polygons.**
[Hint : First make the class intervals continuous.]

Q.5 A random survey of the number of children of various age groups playing in a park was found as follows:

Age (in years)	Number of children
1-2	5
2-3	3
3-5	6
5-7	12
7-10	9
10-15	10
15-17	4

Draw a **histogram** to represent the data above.

SHORT TYPE QUESTIONS
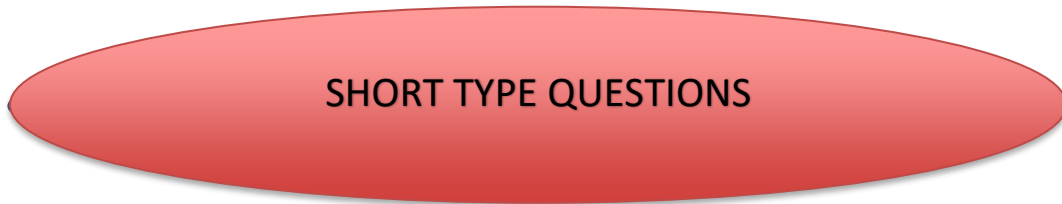
Q.1 The following is the distribution of weights (in kg) of 50 persons:

Weight (in kg)	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
No. of persons	12	8	5	4	5	7	6	3

Draw a **histogram** for the above data.

Q.2 Given below are the seats won by different political parties in the polling outcome of a state assembly

Elections

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

Draw a **bar graph** to represent the polling results.

Q.3 Construct a **frequency polygon** for the following data**:**

Weight (in kg)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
No. of persons	2	4	6	8	9	6	5	3	1

Q.4 Draw a **histogram** for the following data:

Class interval	25-29	30-34	35-39	40-44	45-49	50-54
Frequency	5	15	23	20	10	7

Q.5 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the

number of letters in the English alphabet in the surnames was found as follows:

Number of letters	Number of surnames
1-4	6
4-6	30
6-8	44
8-12	16
12-20	4

Draw a **histogram** to depict the given information

Q.1

100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the

number of letters in the English alphabet in the surnames was found as follows:

Number of letters	Number of surnames
1-4	6
4-6	30
6-8	44
8-12	16
12-20	4

Write the class interval in which the maximum number of surnames lie.

Q.2 The following table gives the life times of 400 neon lamps:

Life time (in hours)	Number of lamps
300-400	14
400-500	56
500-600	60
600-700	86
700-800	74
800-900	62
900-1000	48

How many lamps have a life time of more than 700 hours?

Q.3 The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is

represented in the following table:

Length (in mm)	Number of leaves
118-126	3
127 - 135	5
136-144	9
145 - 153	12
154-162	5
163 - 171	4
172-180	2

Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

Q.4 Given below are the seats won by different political parties in the polling outcome of a state assembly

Elections

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

Which political party won the maximum number of seats?

Q.5 What is the class mark of the class interval 120-130.

Q.1 The class-mark of the class 120-160 is :
 (a) 130
(b) 135
(c) 140
(d) 145

Answer: c

Q.2 In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is:
a. 6
b. 7
c. 8
d. 12

Answer: b

Solution:
Consider x and y as the upper and lower class limit in a frequency distribution.
It is given that
Mid value of a class=10
(x + y)/2 = 10
x + y = 20 …. (1)
Width of a class=6
x - y = 6 …. (2)
By adding both the equations
2x = 20 + 6
2x = 26
Dividing both sides by 2
x = 13
Substitute x value in equation (1)
13 + y = 20
By further calculation
y = 20 - 13
y = 7
Therefore, the lower limit of the class is 7.

 Q.3 In the class intervals 10-20, 20-30, 30-40, the number 30 is included in:
 (a) 30-40
(b) 20-30
(c) both the intervals
(d) none of these

Answer: a

Q.4 The width of each of five continuous classes in a frequency distribution is 5 and the lower class limit of the
       lowest class is 10. The upper class limit of the highest class is:
(a) 35
(b) 15
(c) 25
(d) 40

Answer: a

Solution

We have:

Class width = 5

Lower class limit of the lowest class = 10

Now,

Upper class limit of the highest class = 10 + 5 × 5 = 35

Q.5 Let m be the midpoint and 'u' the upper class limit of a class in a continuous frequency distribution. The lower limit of the class is

(a) 2m + u

(b) 2m – u

(c) m – u

(d) m – 2u

Answer: b

CASE STUDY BASED QUESTIONS

Q.1 A group of students decided to make a project on Statistics. They are collecting the heights (in cm) of their 51 girls of Class IX-A, B and C of their school. After collecting the data, they arranged the data in the following frequency distribution table form:

Height (in cm)	Number of girls
135 - 140	4
140 - 145	7
145 - 150	18
150 - 155	11
155 - 160	6
160 - 165	5

Based on the information, answer the following questions:

(a) The class interval with highest frequency is :

(i) 145-150   (ii) 150 -155

(iii) 140-145   (iv) 155-160

(b) What is the width of the class?

(i) 10   (ii) 15   **(iii) 5**   (iv) none of these

(c) How many students of the height 150 cm and below are there?

(i) 40   **(ii) 29**

(iii) 18   (iv) 22

(d) How many students of the height 145 cm and above are there?

(i) 40   (ii) 29   (iii) 18   (iv) 22

(e) How many students of the height more than 145 cm but less than 155 are there?

(i) 40

(iii) 18

(ii) 29

(iv) 22

Answer: (a) i (b) iii   (c) ii (d) i (e) ii

Q.2 The COVID-19 pandemic, also known as the coronavirus pandemic, is an ongoing pandemic of coronavirus disease 2019 (COVID-19) caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2).It was first identified in December 2019 in Wuhan, China.

During survey, the ages of 80 patients infected by COVID and admitted in the one of the City hospital were recorded and the collected data is represneted in the less than cumulative frequency distribution table.

Based on the information, answer the following questions:

Age (in yrs)	No. of patients
5 - 15	6
15 - 25	11
25 - 35	21
35 - 45	23
45 - 55	14
55 - 65	5

a) The class interval with highest frequency is:

(i) 45-55   **(ii) 35-45**   **(iii) 25-35**   **(iv) 15-25**

(b) Which age group was affected the least?

(i) 35-45   **(ii) 25-35**

(iii) 55-65   **(iv) 45-55**

(c) Which are group was affected the most?

(i) 35-45   **(ii) 25-35**

(iii) 15-25   **(iv) 45-55**

(d) How many patients of the age 45 years and above were admitted?

(i) 61   **(ii) 19**   **(iii) 14**   **(iv) 23**

(e) What is the width of the class?

(i) 10   **(ii) 15**   **(iii) 5**   **(iv) None of these**

Answer: (a) ii   (b) iii   (c) i   (d) ii   (e) i

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