

PM SHRI KENDRIYA VIDYALAYA DHENKANAL

2024-25

Important Question Practice MATHS-12th

CH-1 Relation and functions

Q.1 Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Show that the relation R transitive? Write the equivalence class [0]

Q.2 Show that the relation S in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in \mathbb{Z}, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Also find the equivalence class [1]. Show that the relation R defined by $(a, b) R (c, d) \Leftrightarrow a + d = b + c$ on the set $N \times N$ is an equivalence relation. Also, find the equivalence classes [(2,3)] and [(1,3)].

Q.3 Show that the relation R in the set R of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ neither reflexive nor symmetric nor transitive.

Q.4 Let $f : \{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$ and $g : \{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be functions defined as $f(2) = 3, f(3) = 4, f(4) = f(5) = 5$ and $g(3) = g(4) = 7$ and $g(5) = g(9) = 11$. Find $g \circ f$.

Q.5 Determine whether each of the following relations are reflective, symmetric, and transitive.

(i) Relation R on the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as

$$R = \{(x, y) : 3x - y = 0\}$$

(ii) Relation R on the set N of all natural numbers defined as

$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$

(iii) Relation R on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined as

$$R = \{(x, y) : y \text{ is divisible by } x\}$$

(iv) Relation R on the set Z of all integers by x}

$$R = \{(x, y) : x - y \text{ is an integer}\}$$

Q.6 Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = ax + b$, where $a, b \in \mathbb{R}, a \neq 0$ is a bijection.

Q.7 Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \cos x$ for all $x \in \mathbb{R}$, is neither one-one nor onto.

Q 8. Let $A = \mathbb{R} - \{2\}$ and $B = \mathbb{R} - \{1\}$. If $f: A \rightarrow B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$

show that f is bijective.

Q.9 Classify the following functions as injection, surjection, or bijection:

(i) $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^2$

(ii) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x$

(iii) $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3$

(iv) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(x) = x^3$

(v) $f: \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = |x|$

(vi) $f: \mathbb{Z} \rightarrow \mathbb{Z}$, defined by $f(x) = x^2 + x$

Q.10 Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. State whether f is one-one or not.

CH-2 Inverse Trigonometry

Q.1 For the principal values, evaluate the following:

$$\begin{aligned} \text{(i)} \quad \sin^{-1} \frac{1}{2} - 2 \sin^{-1} \frac{1}{\sqrt{2}} & \qquad \qquad \qquad \text{(ii)} \quad \sin^{-1} \left(-\frac{1}{2} \right) + 2 \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) \\ \text{(iii)} \quad \tan^{-1}(-1) + \cos^{-1} \left(-\frac{1}{\sqrt{2}} \right) & \qquad \qquad \qquad \text{(iv)} \quad \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) + \cos^{-1} \left(\frac{\sqrt{3}}{2} \right) \end{aligned}$$

Q.2 Evaluate each of the following:

$$\begin{aligned} \text{(i)} \quad \sin^{-1} \left(\sin \frac{\pi}{3} \right) & \quad \text{(ii)} \quad \cos^{-1} \left(\cos \frac{2\pi}{3} \right) & \quad \text{(iii)} \quad \tan^{-1} \left(\tan \frac{\pi}{4} \right) \\ \text{(iv)} \quad \sin^{-1} \left(\sin \frac{2\pi}{3} \right) & \quad \text{(v)} \quad \cos^{-1} \left(\cos \frac{7\pi}{6} \right) & \quad \text{(vi)} \quad \tan^{-1} \left(\tan \frac{3\pi}{4} \right) \end{aligned}$$

Q.3 Express each of the following in the simplest form:

$$\begin{aligned} \text{(i)} \quad \left\{ \frac{\sqrt{1-\cos x}}{1+\cos x} \right\}, -\pi < x < \pi & \quad \text{(ii)} \quad \tan^{-1} \left(\frac{\cos x}{1+\sin x} \right), -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \text{(iii)} \quad \tan^{-1} \left(\frac{\cos x}{1-\sin x} \right), -\frac{\pi}{2} < x < \frac{\pi}{2} & \quad \text{(iv)} \quad \tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right), -\frac{\pi}{4} < x < \frac{\pi}{4} \end{aligned}$$

Q.4 Prove that :

$$\begin{aligned} \text{(i)} \quad \tan^{-1} \left\{ \frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right\} &= \frac{\pi}{4} - \frac{x}{2}, \text{ if } \pi < x < \frac{3\pi}{2} \\ \text{(ii)} \quad \cot^{-1} \left\{ \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right\} &= \frac{\pi}{2} - \frac{x}{2}, \text{ if } \frac{\pi}{2} < x < \pi \end{aligned}$$

Q.5 Prove that :

$$\text{(i)} \quad \sin[\cot^{-1}\{\cos(\tan^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+2}} \quad \text{(ii)} \quad \cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$$

Q.6 Write each of the following in the simplest form:

$$\begin{aligned} \text{(i)} \quad \sin^{-1} \left\{ x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2} \right\} & \quad \text{(ii)} \quad \tan^{-1} \left\{ x + \sqrt{1+x^2} \right\}, x \in \mathbf{R} \\ \text{(iii)} \quad \tan^{-1} \left\{ \sqrt{1+x^2} - x \right\}, x \in \mathbf{R} & \quad \text{(iv)} \quad \tan^{-1} \left\{ \frac{\sqrt{1+x^2}-1}{x} \right\}, x \neq 0 \end{aligned}$$

$$(v) \tan^{-1} \left\{ \frac{\sqrt{1+x^2} + 1}{x} \right\}, x \neq 0$$

$$(vi) \tan^{-1} \sqrt{\frac{a-x}{a+x}}, -a < x < a$$

$$(vii) \tan^{-1} \left\{ \frac{x}{a + \sqrt{a^2 - x^2}} \right\}, -a < x < a$$

$$(viii) \sin^{-1} \left\{ \frac{x + \sqrt{1-x^2}}{\sqrt{2}} \right\}, -1 < x < 1$$

$$(ix) \sin^{-1} \left\{ \frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right\}, 0 < x < 1$$

$$(x) \sin \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$$

CH-3 Matrices

Q.1 Find x, y, a and b if $\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$

Q.2 Two farmers Ram Kishan and Gurcharan singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in Rs) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B

| September sales (in Rs) | | | |
|-------------------------|--------|--------|-----------------|
| Basmati | Permal | Naura | |
| 10,000 | 20,000 | 30,000 | Ram Kishan |
| 50,000 | 30,000 | 10,000 | Gurcharan Singh |

$$A = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix}$$

| Basmati | Permal | Naura | |
|---------|--------|--------|-----------------|
| 5,000 | 10,000 | 6,000 | Ram Kishan |
| 20,000 | 10,000 | 10,000 | Gurcharan Singh |

$$B = \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$$

Find :

- (i) What were the combined sales in September and October for each farmer in each variety.
- (ii) What was the change in sales from September to October ?
- (iii) If both farmers receive 2% profit on gross rupees sales, compute the profit for each farmer and for each variety sold in October.

Q.3 If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, find

- (i) $A + B$ and $B + C$
- (ii) $2B + 3A$ and $3C - 4B$.

Q.4 Let $A = \begin{bmatrix} 0 & -\tan(\alpha/2) \\ \tan(\alpha/2) & 0 \end{bmatrix}$ and I be the identity matrix of order 2.

Show that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Q.5 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I_2 = O$

Q.6 If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$, then prove by principle of mathematical induction that

$$A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in \mathbb{N}.$$

Q.7 If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1} .

Also Solve the Questions of the Chapter -1,2,3 From the given Five CBSE Previous Year Question Papers

NOTE:- Solve all these questions in A-4 size papers
