केन्द्रीय विद्यालय संगठन
Kendriya Vidyalaya Sangathan

भौतिक विज्ञान
Physics

कक्षा/Class: XI

2024-25

विद्यार्थी सहायक सामग्री

Student Support Material

<u>संदेश</u>

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना केन्द्रीय विद्यालय संगठन की सर्वोच्च वरीयता है। हमारे विद्यार्थी, शिक्षक एवं शैक्षिक नेतृत्व कर्ता निरंतर उन्नति हेतु प्रयासरत रहते हैं। राष्ट्रीय शिक्षा नीति 2020 के संदर्भ में योग्यता आधारित अधिगम एवं मूल्यांकन संबन्धित उद्देश्यों को प्राप्त करना तथा सीबीएसई के दिशा निर्देशों का पालन, वर्तमान में इस प्रयास को और भी चुनौतीपूर्ण बनाता है।

केन्द्रीय विद्यालय संगठन के पांचों **आंचलिक शिक्षा एवं प्रशिक्षण संस्थान** द्वारा संकलित यह 'विद्यार्थी सहायक सामाग्री' इसी दिशा में एक आवश्यक कदम है । यह सहायक सामग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है । केन्द्रीय विद्यालय संगठन की 'विद्यार्थी सहायक सामग्री' अपनी गुणवत्ता एवं परीक्षा संबंधी सामाग्री-संकलन की विशेषज्ञता के लिए जानी जाती है और अन्य शिक्षण संस्थान भी इसका उपयोग परीक्षा संबंधी पठन सामग्री की तरह करते रहे हैं । शुभ-आशा एवं विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर सतत मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुंचाएगी ।

शुभाकांक्षा सहित ।

निधि पांडे

आयुक्त,

केन्द्रीय विद्यालय संगठन

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विषय सूची
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Class XI – 2023-24 (Theory) Time: 3 hrs. Max Marks: 70

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Total		**160**	**70**

Unit I:Physical World and Measurement 08 Periods

Chapter–2: Units and Measurements

Need for measurement: Units of measurement; systems of units; SI units, fundamental and derived units. significant figures. Dimensions of physical quantities, dimensional analysis and its applications.

Unit II: Kinematics 24 Periods

Chapter–3: Motion in a Straight Line

Frame of reference, Motion in a straight line, Elementary concepts of differentiation and integration for describing motion, uniform and non- uniform motion, and instantaneous velocity, uniformly accelerated motion, velocity - time and position-time graphs. Relations for uniformly accelerated motion (graphical treatment).

Chapter–4: Motion in a Plane

Scalar and vector quantities; position and displacement vectors, general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors, Unit vector; resolution of a vector in a plane, rectangular components, Scalar and Vector product of vectors.

Motion in a plane, cases of uniform velocity and uniform acceleration- projectile motion, uniform circular motion.

Unit III:Laws of Motion 14 Periods

Chapter–5: Laws of Motion

Intuitive concept of force, Inertia, Newton's first law of motion; momentum and Newton's second law of motion; impulse; Newton's third law of motion.

Law of conservation of linear momentum and its applications.

Equilibrium of concurrent forces, Static and kinetic friction, laws of friction, rolling friction, lubrication.

Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on a banked road).

Unit IV:Work, Energy and Power 14 Periods

Chapter–6: Work, Energy and Power

Work done by a constant force and a variable force; kinetic energy, work- energy theorem, power.

Notion of potential energy, potential energy of a spring, conservative forces: non-conservative forces, motion in a vertical circle; elastic and inelastic collisions in one and two dimensions.

Unit V:Motion of System of Particles and Rigid Body 18 Periods

Chapter–7: System of Particles and Rotational Motion

Centre of mass of a two-particle system, momentum conservation and Centre of mass motion. Centre of mass of a rigid body; centre of mass of a uniform rod. Moment of a force, torque, angular momentum, law of conservation of angular momentum and its applications.

Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions.

Moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects (no derivation).

Unit VI:Gravitation 12 Periods

Chapter–8: Gravitation

Kepler's laws of planetary motion, universal law of gravitation.
Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy and gravitational potential, escape speed, orbital velocity of a satellite.

Unit VII:Properties of Bulk Matter 24 Periods

Chapter–9: Mechanical Properties of Solids

Elasticity, Stress-strain relationship, Hooke's law, Young’s modulus, bulk modulus, shear modulus of rigidity (qualitative idea only), Poisson's ratio; elastic energy.

Chapter–10: Mechanical Properties of Fluids

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes), effect of gravity on fluid pressure.

Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its simple applications.

Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.

Chapter–11: Thermal Properties of Matter

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity.

Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wein's displacement Law, Stefan's law .

Unit VIII: Thermodynamics 12 Periods

Chapter–12: Thermodynamics

Thermal equilibrium and definition of temperature, zeroth law of thermodynamics, heat, work and internal energy. First law of thermodynamics,

Second law of thermodynamics: gaseous state of matter, change of condition of gaseous state -isothermal, adiabatic, reversible, irreversible, and cyclic processes.

Unit IX:Behavior of Perfect Gases and Kinetic Theory of Gases

Chapter–13: Kinetic Theory

Kinetic theory of gases - assumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.

Unit X:Oscillations and Waves **26 Periods**

Chapter–14: Oscillations

Periodic motion - time period, frequency, displacement as a function of time, periodic functions and their applications.

Simple harmonic motion (S.H.M) and its equations of motion; phase; oscillations of a loaded spring- restoring force and force constant; energy in S.H.M.
Kinetic and potential energies; simple pendulum derivation of expression for its time period.

Chapter–15: Waves

Wave motion: Transverse and longitudinal waves, speed of travelling wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats.

QUESTION PAPER DESIGN

Theory (Class: XI)

Maximum Marks: 70 **Duration: 3 hrs.**

S No.	Typology of Questions	Total Marks	Approx Percentage
1	**Remembering**: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding**: Demonstrate understanding of facts andideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	27	38 %
2	**Applying**: Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	22	32%
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating**: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	21	30%
	Total Marks	70	100
	Practical	30	
	Gross Total	100	

Note:

- **Competency Focused Questions in the form of MCQs/ Case Based Questions, Source-based Integrated Questions or any other type = 50%**
- **Select response type questions (MCQ) = 20%**
- **Constructed response questions (Short Answer Questions/Long Answer type Questions, as per existing pattern) = 30%.**

CLASS 11TH STUDY MATERIAL 2024-25

Unit I Physical World and Measurement

Chapter 2: Units and Measurement

GIST OF THE CHAPTER:

(I) Measurement:

- Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called unit.
- Measurement of a physical quantity is expressed by a number (or numerical measure) accompanied by a unit.
- M=n x u
- The units for the fundamental or base quantities are called fundamental or base units.
- The units of all other physical quantities can be expressed as combinations of the base units. Such units obtained for the derived quantities are called derived units.
- A complete set of the units, both the base units and derived units, is known as the system of units
- Systems of Unit:
  - ✓ In CGS system they were centimetre, gram and second respectively.
  - ✓ In FPS system they were foot, pound and second respectively.
  - ✓ In MKS system they were metre, kilogram and second respectively.
  - ✓ The system of units which is at present internationally accepted for measurement is the Système Internationale d' Unites (French for International System of Units), abbreviated as SI.
- There are seven fundamental quantities.

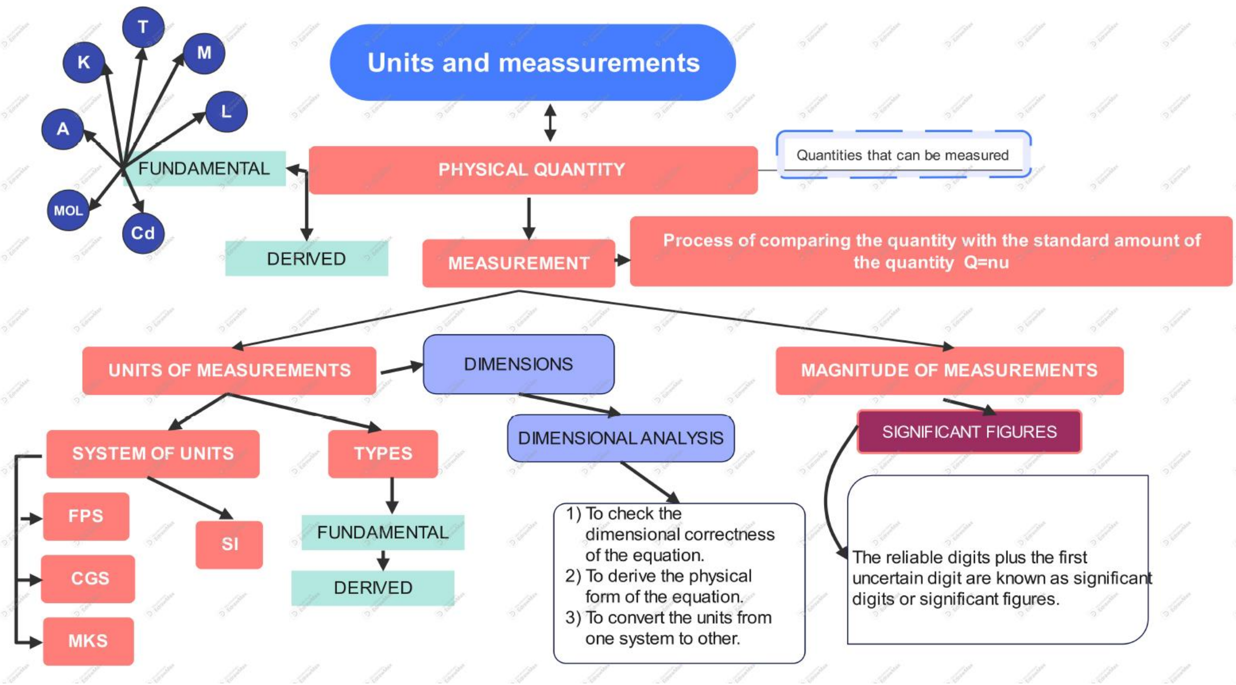
Physical quantity	Unit	Symbol
Fundamental quantities		
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Supplementary quantities		
Plane angle	radian	rad
Solid angle	steradian	sr

(II) DIMENSIONS AND DIMENSIONAL ANALYSIS:

- The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.
- An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity.
- The dimensions of base quantities and combination of these dimensions describe the nature of physical quantities. Dimensional analysis can be used to check the dimensional consistency of equations, deducing relations among the physical quantities, etc. A dimensionally consistent equation need not be actually an exact (correct) equation, but a dimensionally wrong or inconsistent equation must be wrong.
- The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions. This is called the principle of homogeneity of dimensions. [A+B] = [A] = [B]

- **LIMITATIONS OF DIMENSIONAL AYALYSIS.**

o Subtraction and addition of parameters cannot be reflected in dimensional analysis.
o Dimensional analysis cannot confirm the validity of a relationship of the physical quantities.
o It is impractical for the correlation of more than three parameters.
o The dimensional analysis cannot determine the nature of the unknown physical quantities.
o Data obtained from a large number of experiments may be undetermined.



Multiple Choice Questions:

1. If frequency (F), velocity (v) and density (D) are considered as fundamental units, the dimensional formula for momentum will be:
   (a) DVF^2
   (b) DV^2F^{-1}
   (c) $D^2V^2F^2$
   (d) DV^4F^{-3}
2. The equation of the stationary wave is $y = 2a\sin(\frac{2\pi ct}{\lambda})\cos\left(\frac{2\pi x}{\lambda}\right)$, which of the following statements is wrong?
   (a) The unit of ct is same as that of λ
   (b) The unit od x is same as that of λ
   (c) The unit of $\frac{2\pi ct}{\lambda}$ is same as that of $\frac{2\pi x}{\lambda}$
   (d) The unit of $\frac{c}{\lambda}$ is same as that of $\frac{x}{\lambda}$
3. During a short interval of time the speed v in m/s of an automobile is given by$v = at^2 + bt^3,$ where the time t is in seconds. The units of a and b are respectively,
   (a) ms^2; ms^4
   (b) s^3/m; s^4/m
   (c) m/s^2; m/s^3
   (d) m/s^3; m/s^4
4. The term (1/2)ρv^2 occurs in Bernoulli's equation, with ρ being the density of a fluid and v its speed. The dimensions of this term are
   (a) $[M^{-1}\,L^5\,T^2]$
   (b) $[M^1\,LT^2]$
   (c) $[M\,L^{-1}\,T^{-2}]$
   (d) $[M^{-1}\,L^9\,T^{-2}]$
5. If velocity v, acceleration A and force F are chosen as fundamental quantities, then the dimensional formula of angular momentum in terms of v,A, F would be
   (a) FA^{-1}v
   (b) Fv^3A^{-2}
   (c) Fv^2A^{-1}
   (d) $F^2v^2A^{-1}$
6. The largest mass (m) that can be moved by a flowing river depends on velocity and acceleration due to gravity (g). The correct relation is
   (a) $m \propto \frac{\rho^2 v^4}{g^2}$
   (b) $m \propto \frac{\rho v^6}{g^2}$
   (c) $m \propto \frac{\rho v^4}{g^3}$
   (d) $m \propto \frac{\rho v^6}{g^3}$
7. Which of the following sets have different dimensions?
   (a) Pressure, Young's modulus, stress
   (b) Emf, potential difference, electric potential
   (c) Heat, work, energy
   (d) Dipole moment, electric flux, electric field
8. Which one of the following pairs of quantities has the same dimension?

(a) Force and work
(b) Momentum and impulse
(c) Pressure and force
(d) Surface tension and stress

9. The time dependence of a physical quantity p is given by $p = p_0 \exp(-\alpha t^2)$, where α is a constant and t is the time. The constant α
(a) Is dimensionless
(b) Has dimensions $[T^{-2}]$
(c) Has dimensions $[T^2]$
(d) Has dimensions of p.

10. According to Newton, the viscous force acting between liquid layers of area A and velocity gradient (Δv/Δx) is given by
$$F = -\eta A \frac{\Delta v}{\Delta x}$$
Where η is constant called coefficient of viscosity. The dimensional formula of η is:
(a) $[ML^{-2}T^{-2}]$
(b) $[M^0L^0T^0]$
(c) $[ML^2T^{-2}]$
(d) $[ML^{-1}T^{-1}]$

11. The significant figures of the number 6.0023 are
(a) 1
(b) 5
(c) 4
(d) 2

12. A cube has a side of length 1.2×10^{-2} m. Calculate its volume.
(a) 1.7×10^{-6} m^3
(b) 1.73×10^{-6} m^3
(c) 1.0×10^{-6} m^3
(d) 1.732×10^{-6} m^3

13. The sum of 6.2g, 4.33g, 17.456g expressed in appropriate number of significant figures is
(a) 28.0g
(b) 27.9g
(c) 27.98g
(d) 27.986g

14. The number of significant digits in 0.020800m is
(a) 2
(b) 3
(c) 6
(d) 5

15. Round off 15.65 upto 3 digits
(a) 15.7
(b) 15.6
(c) 15.60
(d) 15.70

Answers: MCQ

1.d	2. d	3.d	4.c	5.b
6.d	7.d	8.b	9.b	10.d
11.b	12.a	13.a	14.d	15.b

Detailed answer:

1. $[momentum] = [f]^a[v]^b[d]^c$
   $$(M^1\, L^1\, T^{-1}) = T^{-a}[L^bT^{-b}][M^cL^{-3c}]$$
   $$on\ solving\ c = 1, b = 4, a = -3$$
   Thus Option d
2. $dimension\ of\left(\frac{2\pi x}{\lambda}\right) = dimension\ of(\frac{2\pi ct}{\lambda})$
   $dimension\ of\left(\frac{x}{\lambda}\right) = dimension\ of(\frac{ct}{\lambda})$
   $thus\ option: d$

3. $[v] = [at^2]$
   $[v] = [bt^3]$
   $thus\ option\ d$
4. $[(1/2)\rho v^2] = M^1L^{-3}L^2T^{-2}$ thus option c
5. $[momentum] = [f]^a[v]^b[a]^c$
   $(M^1\, L^2\, T^{-1}) = [M^1\, L^1\, T^{-2}]^a[L^bT^{-b}][L^cT^{-2c}]$
   On solving a=1, b= 3, c=-2
   Thus option b
6. $(M^1\, L^0\, T^0) = [\, L^1\, T^{-1}]^a[M^bL^{-3b}][L^cT^{-2c}$
   On solving a= 6, b=1, c= -1
   Thus option d
7. $p = p_0 \exp(-\alpha t^2)$, exponent is a natural noOption d
8. Option b
9. Option b
10. Option d
11. Option b
12. Option a
13. Option a
14. Option d
15. Option b

Assertion And Reasoning Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a. *If both assertion and reason are true and reason is a correct explanation of the assertion*
b. *If both assertion and reason are true but reason is not a correct explanation of the assertion*
c. *If assertion is true but reason is false.*
d. *If assertion and reason both are false*

16. **Assertion:** Dimensional constants are the quantities whose values are constant.
Reason: Dimensional constants are dimensionless.
17. **Assertion:** When we change the unit of measurement of a quantity, its numerical value changes.
Reason: Smaller the unit of measurement smaller is its numerical value.
18. **Assertion:** Number of significant figures in 0.006 is one and that in 0.600 is one
Reason: This is because zeros are not significant.
19. **Assertion:** Energy can be divided by volume.
Reason: Dimensions for energy and volume are different.
20. **Assertion:** If L=2.331cm, B=2.1cm, then L+B=4.4 cm
Reason: The least number of significant figures in any number of problems determines the number of significant figures in the answer of addition or subtraction.
21. **Assertion:** Pressure has same dimensions as that of Young's modulus.
Reason: Work and Torque has same dimensions.
22. **Assertion:** All dimensionally correct equations are numerically correct.
Reason: A dimensionless quantity always has a unit.
23. **Assertion:** In the equation F = a+ b x, the unit of a is newton.
Reason: The principle of homogeneity of dimensions is based on the fact that only the physical quantities of the same kind can be added, subtracted or compared.
24. **Assertion:** If power of an engine depends on mass, angular speed, torque and angular momentum, then the formula of power is not derived with the help of dimensional method.
Reason: In mechanics, if a particular quantity depends on more than three quantities, then we cannot derive the formula of the quantity by the dimensional method.
25. **Assertion:** The number of significant figures in 80200 is 3
Reason: All the zeros are not significant.

Answers of 1mark assertion and reasoning questions:

16.c 17.b 18.d 19. 20.a 21.b 22.d 23.a 24.a 25.c

Short Questions(2 marks)

26. Give the dimensional formula of (i) Surface tension (ii) power
27. State the principle of homogeneity of dimensions. Test the dimensional homogeneity of the following equation:

$$h = h_o + v_o t + \frac{1}{2} g t^2$$

28. Out of formulae (i) $y = a \sin\frac{2\pi t}{T}$ and (ii) $y = \mathrm{a} \sin vt$ for the displacement y of a particle undergoing a certain periodic motion, rule out the wrong formula on dimensional grounds. [where a = maximum displacement of the particle, v= speed of the particle, T = time period of motion.

29. When white light travels through glass the refractive index $\mu \frac{velocity\ of\ light\ in\ air}{velocity\ of\ light\ in\ glass}$ is found to vary with wavelength as $\mu = A + \frac{B}{\lambda^2}$where A and B are constants. Using the principle of homogeneity of dimensions, determine the SI unit in which A and B must be expressed.
30. If length, time and energy are fundamental units, find the dimension of mass.
31. The radius of a solid sphere is 5.74 cm. Find its surface area to appropriate number of significant figures.
32. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Find the area and volume of the sheet to correct significant figures.
33. The photograph of a house occupies an area of 1.75 cm^2 on a 35mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement?
34. 5.74 g of a substance occupies1.2 cm^3. Express its density keeping significant figures in view.
35. What are the limitations of dimensional analysis.

Answers of 2mark questions:

26. (a) $[surface\ tension] = \frac{[force]}{[length]} = \frac{MLT^{-2}}{L} = [MT^{-2}]$

(b) [Power] = $\frac{[work]}{[time]} = [\ ML^2T^{-3}]$

27. Statement: All terms of any physical relation must have the same dimensions.

$$[h] = [L]$$
$$[h_o] = [L]$$
$$[v_o t\ ] = [L]$$
$$[\frac{1}{2}gt^2] = [L]$$

28. (i) ${2\pi t}/{T}$ have dimensions $M^0L^0T^0$ Thus y = a Hence correct.

(ii) vt have dimensions $M^0L^1T^0$ not dimensionless, so it is wrong.

29. $[\mu] = [A] = M^0L^0T^0$ thus A has no unit. Also $[B] = [\lambda^2]$ so its unit is m^2

30. m≡[F][LT^{-2}] ≡[$FL^{-1}T^2$]

31. $A = 4\pi r^2 = 4\pi(5.74)^2 = 72.16\ cm^2$

Expressing to three significant figures, A = 72.2 cm^2

32. A=Lx B= 4.25517m^2

Expressing to four significant figures, A = 4.255 m^2

Volume = area x thickness = 0.0855m^3

33. Area magnification = Area of image/ Area of object = 1.55 x 10^4 /1.75 = 8857

Linear magnification = $\sqrt{8857} = 94.1$

34. density= mass/ vol= 5.74/1.2 = 4.7833= 4.8 gm/cc

35. Limitations of dimensional ayalysis**.**

i) Subtraction and addition of parameters cannot be reflected in dimensional analysis.
ii) Dimensional analysis cannot confirm the validity of a relationship of the physical quantities.
iii) It is impractical for the correlation of more than three parameters.

iv) The dimensional analysis cannot determine the nature of the unknown physical quantities.
v) Data obtained from a large number of experiments may be undetermined.

Short Question(3 Marks)

36. Find the dimensional formula of (i) acceleration due to gravity (ii) gravitational constant
37. Using the dimensional analysis, check whether the following equation is correct or not: $T = 2\pi\sqrt{\frac{R^3}{GM}}$
38. If the time period (T) of vibration of a liquid drop depends on surface tension (S) and radius (r) of the drop, and density (ρ) of the liquid, derive an expression for T using dimensional analysis.
39. The frequency 'f' of vibration of a stretched string depends upon:
    (i) Its length 'l'
    (ii) The mass per unit length 'm'
    (iii) The tension'T' in the string.

    Obtain dimensionally an expression for frequency 'f'
40. An artificial satellite is revolving around a planet of mass M and radius R, in a circular orbit period of a satellite around a common central body, square or the period of revolution T is proportional to the cube of the radius of the orbit r. Show using dimensional analysis, that $T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$
41. A cubical object has an edge length of 1.00 cm. If a cubical box contained a mole of cubical objects, find its edge length (one mole = 6.02 x 10^{23} units)
42. A grocer's balance shows the mass of an object as 2.500 kg. Two gold pieces of masses 21.15 g and 21.17 g are added to the box. What is (a) the total mass in the box and the difference in the masses of the gold pieces to the correct number of significant figures?
43. The surface tension of water is 72 dyne/cm. What would be its value in SI units.
44. An electric bulb has a power of 500W. Express it in cgs units
45. In SI units, the value of Stefan's constant is $\sigma = 5.67 \times 10^{-8} J/sm^2K^4$. Find its value in CGS system.

Answers of 3 marks questions

36. (i) [g]=[vel]/[time] = $[LT^{-2}]$
(ii) [G]= [FORCE]$[R^2]$/ [MASS][MASS]=$[M^{-1}L^3T^{-2}]$
37.LHS= [T]
RHS = [$2\pi\sqrt{\frac{R^3}{GM}}$]= [[L3]/ [M-1L3T-2][M]]1/2= [T]
38.$T \propto \rho^a r^b \sigma^c$
$T = k\rho^a r^b \sigma^c$
$[T] = [ML^{-3}]^a [L]^b [MT^{-2}]^c$
$[T] = [M^{a+c} L^{-3a+b} T^{-2c}]$
a + c = 0
-3a+b = 0
-2c= 1

a = ½ , c = - ½ b = 3/2

$T = \sqrt{\frac{\rho r^3}{\sigma}}$

39. $f \propto l^a t^b m^c$

$f = k l^a t^b m^c$

$[T^{-1}] = [M^0 L^1 T^0]^a [M^1 L^1 T^{-2}]^b [M^1 L^{-1} T^0]^c$

On solving b=1/2 c=-1/2 a= -1 thus $f = \frac{k}{l}\sqrt{\frac{t}{m}}$

40. $T \propto r^{3/2}$

$T \propto r^{\frac{3}{2}} R^y g^x$

$[L^0 M^0 T^1] = [L^{3/2} M^0 T^0][L^1 T^{-2}]^X [L^1]^Y$

On solving x = - ½ and y = 1

$T = k\, r^{3/2} g^{-1/2} R^1$

$T = \frac{k}{R}\sqrt{\frac{r^3}{g}}$

41. The volume of cubical object = 1 cu cm= $1 \times 10^{-6} m^3$

The volume of a mole of them = 6.02×10^{23} cm^3= 6.02×10^{17} m^3

Edge length = $\sqrt[3]{6.02 \times 10^{17}}$ = 8.4×10^5 m^3

42. Total mass= 2.500+0.02115+0.02117= 2.54232kg

Total mass with appropriate significant figure=2.542 kg

Difference= 0.02kg

43.[surface tension] = MT^{-2}

$n_2 = 72 \left[\frac{1g}{1kg}\right]\left[\frac{1s}{1s}\right]^{-2}$ = 0.072

72 dyne/cm= 0.072 N/m

44.[power]= $[ML^2T^{-3}]$

$n_2 = 500\left[\frac{1kg}{1g}\right]\left[\frac{1m}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-3}$ = 5×10^9 erg/s

45.$\sigma = 5.67 \times 10^{-8} \frac{J}{Sm^2 K^4}$

$\sigma = 5.67 \times 10^{-8} \times 10^7 \times 100^{-2}\ erg/scm^2K^4$

Long Questions (5marks)

46. (a) Distinguish between dimensional and non-dimensional constants.
(b) Reynold number N (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N is a function of the density of the liquid 'ρ' , its average speed 'v' and coefficient of viscosity 'η'. Given that N is also directly proportional to 'D' (the diameter of the pipe), show using dimensional analysis, $N \propto \frac{\rho v D}{\eta}$
47. Assuming that the mass M of the largest stone that can be moved by a flowing river depends upon 'v' the velocity, 'ρ' the density of water and on 'g' , the acceleration due to gravity. Show that M varies with the sixth power of the velocity of flow.
48. Define dimensions. Find the dimensional formula of angular velocity and torque. A gas bubble, form an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$, where p is the static pressure, d is the density of water and E is the total energy of the explosion. Find the values of a, b, c

49. Using the principle of homogeneity, identify the correct equation:

(a) $T = 2\pi\sqrt{g/l}$

(b) $T = 2\pi\sqrt{\frac{l}{g}}$

50. Find the dimensional formula of Planck's constant. Find the dimensions of (a/b) in the equation: $P = \frac{a-t^2}{bx}$ where P is pressure, x is distance and t is time.

51. Deduce the dimensional formula for universal gas constant.
The velocity of sound waves 'v' through a medium may be assumed to depend on:
(a) the density of the medium 'd'
(b) The modulus of elasticity 'E'

Deduce by the method of dimensions the formula for the velocity of sound. Take dimensional constant K = 1.

Answers of 5 marks questions:

46. (a) The quantities which have dimensions as well as constant value are called dimensional constants Eg: Planck's constant, The quantities with contant value but no dimension are called nondimensional constant. Eg: $\pi, sin\theta, cos\theta$
(b) $N = k\,\rho^a v^b \eta^c D$
$[M^0L^0T^0] = [M^1L^{-3}]^a[L^1T^{-1}]^b[M^1L^{-1}T^{-1}]^c[L^1]$
$[M^0L^0T^0] = [M^{a+c}L^{-3a+b-c+1}T^{-b-c}]$
On solving, a=1, b=1, c=-1
47. $M = k\,v^a\rho^b g^c$
$[M^1L^0T^0] = [L^1T^{-1}]^a[M^1L^{-3}]^b[L^1T^{-2}]^c$
$[M^1L^0T^0] = [M^bL^{a-3b+c}T^{-a-2c}]$
On solving, a= 6 b= 1, c= -3

$$\therefore M \propto v^6$$

48. Dimensions: The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

[angular velocity]= [angle/time]= [T^{-1}]
[torque]= [force][distance]=[$M^1L^2T^{-2}$]
$[T] = p^a d^b E^c$
$[T] = [M^1L^{-1}T^{-2}]^a[M^1L^{-3}]^b[M^1L^2T^{-2}]^c$
$[T] = [M^{a+b+c}L^{-a-3b+2c}T^{2a-2c}]$
On equating the powers and solving it,
a= -5/6 , b=1/2 c=1/3
49. Dimensions of LHS= [T]
Dimensions of RHS of equation 1 =$\left[\sqrt{\frac{g}{l}}\right] = \left[\frac{g}{l}\right]^{1/2} = \left[\frac{LT^{-2}}{L}\right]^{1/2} = T^{-1}$

Thus, $T = 2\pi\sqrt{g/l}$ is dimensionally incorrect.

Dimensions of RHS of equation 2 $=\left[\sqrt{\frac{l}{g}}\right] = \left[\frac{l}{g}\right]^{1/2} = \left[\frac{L}{LT^{-2}}\right]^{1/2} = T^1$

Thus, $T = 2\pi\sqrt{\frac{l}{g}}$ is dimensionally correct.

50. $[E] = [h\nu]$

$[h] = [E/\nu] = \frac{[M^1L^2T^{-2}]}{[T]^{-1}} = [M^1L^2T^{-1}]$

$$P = \frac{a - t^2}{bx}$$

[a] = [t^2] = [T^2]

$$[P] = \left[\frac{a}{bx}\right]$$

$$[P][x] = \left[\frac{a}{b}\right]$$

$$[M^1L^{-1}T^{-2}][L^1] = \left[\frac{a}{b}\right]$$

$$[M^1T^{-2}] = \left[\frac{a}{b}\right]$$

51. $[R] = \left[\frac{PV}{nT}\right]$ = [ML^{2}T^{-2}K^{-1}mol^{-1}]

$$v = k\, d^a E^b$$

$[L^1T^{-1}] = [M^1L^{-3}]^a[M^1L^{-1}T^{-2}]^b$

$[L^1T^{-1}] = [M^{a+b}L^{-3a-b}T^{-2b}]$

On solving, a= -1/2 b= ½

Case Based Questions

Case study-1

52. The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity. The dimensions of base quantities and combination of these dimensions describe the nature of physical quantities. Dimensional analysis can be used to check the dimensional consistency of equations, deducing relations among the physical quantities, etc. A dimensionally consistent equation need not be actually an exact (correct) equation, but a dimensionally wrong or inconsistent equation must be wrong. The magnitudes of physical quantities may be added together or subtracted from one another only if they have the same dimensions. This is called the principle of homogeneity of dimensions. [A+B] = [A] = [B]

i. The equation of state of gas is expressed as $\left(P + \frac{a}{V^2}\right)(V - b) = nRT$, where P= pressure, V= volume, T= temperature and n, a,b,R are constants. The dimensions of a will be

(a) $M^1L^1T^{-1}$

(b) $M^1L^5T^{-2}$

(c) L^{-3}
(d) L^{6}

ii. The force is given in terms of time t and displacement x by the equation $F = AcosBx + CsinDt$ .The dimensional formula of D/B
(a) $M^0L^0T^0$
(b) $M^0L^0T^{-1}$
(c) $M^0L^{-1}T^0$
(d) M^0LT^{-1}

iii. In $x = a + bt + ct^2$ where x is in metre and t in second; then the unit of c is
(a) m
(b) m/s
(c) m/s^2
(d) s^2

iv. In the equation $P = \frac{b-x^2}{at}$, the dimension of b is
(a) L^2/s
(b) L^2
(c) L^3
(d) $M^1L^{-1}T^{-2}$

Case study-2

53. The reliable digits plus the first uncertain digit are known as significant digits or significant figures. In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures. In addition, or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

(i) The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is
(a) 663.821
(b) 664
(c) 663.8
(d) 663.82

(ii) The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is
(a) 1.6048g/cm^3
(b) 1.69 g/cm^3
(c) 1.7 g/cm^3
(d) 1.695 g/cm^3

(iii) The number of significant figures in 0.06900 is
(a) 5
(b) 4
(c) 2
(d) 3

(iv) The number 2.745 and 2.735 on rounding off to 3 significant figures will give
(a) 2.75 and 2.74
(b) 2.74 and 2.73
(c) 2.75 and 2.73
(d) 2.74 and 2.74

Case study-3

54. The dimensional method is very convenient way of finding the dependence of physical quantity on other physical quantities of a given system. This method has its own limitations in a complicated situation, it is often not easy to guess the factors on which a physical quantity will depend. Secondly this method gives no information about the dimension-less proportionality constant. Thirdly this method is used only if a physical quantity depends on the product of other physical quantities. Fourthly this method will not work if a physical quantity depends on another quantity being a trigonometric or exponential function. Finally, this method does not give complete information in case where a physical quantity on more than three quantities in problems in mechanics.

(i) Expression of physical quantity in terms of powers of fundamental physical quantity is known as
(a) Dimensional formula
(b) Basic physical quantity
(c) Derived physical quantity
(d) Dimensional analysis

(ii) The smallest value that can be measured by the measuring instrument is called
(a) Minor length
(b) Small measuring value
(c) Least count
(d) Relative length

(iii) Dimensional analysis can be applied to

(a) to check the correctness of a physical equation.
(b) to derive the relationship between different physical quantities.
(c) to convert a physical quantity from one system of units to other.
(d) All of the above

(iv) Method of dimensional analysis uses the principle of
(a) Heterogeneity
(b) Homogeneity
(c) Power's law
(d) Associativity

(v) Dimensional analysis will not work if a physical quantity depends on another quantity like
(a) Fraction value
(b) Trigonometric function
(c) Parabolic function
(d) Square root of a quantity

Answers of case-based questions:

52(i) (b) 52 (ii) (d) 52(iii) (c) 52(iv) (b)

53(i)(c)	53 (ii) (c)	53(iii) (b)	53(iv) (d)
54(i)(a)	54(ii) (c)	54(iii) (b)	54(iv) (b)

Competency- Based Questions:

55. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity X as follows:

$$[position] = [X^{\alpha}]$$
$$[speed] = [X^{\beta}]$$
$$[acceleration] = [X^{p}]$$
$$[linear\ momentum] = [X^{q}]$$
$$[force] = [X^{r}]$$

(a) $\alpha + p = 2\beta$
(b) $p + q - r = \beta$
(c) $p - q + r = \alpha$
(d) $p + q + r = \beta$

56. In a particular system of units, a physical quantity can be expressed in terms of the physical quantity can be expressed in terms of the electric charge e, electron mass me, Planck's constant h, and Coulomb's constant $k = \frac{1}{4\pi\varepsilon_o}$, where ε_o is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^{\alpha}[m_e]^{\beta}[h]^{\gamma}[k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ is ___________

57. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement(s) is / are correct?
(a) The dimension of force is L^{-3}
(b) The dimension of energy is L^{-2}
(c) The dimension of power is L^{-5}
(d) The dimension of linear momentum is L^{-1}

58. The density of a material in SI units is 128kg/m3. In certain units in which the unit of length is 25cm and the unit of mass is 50g, the numerical value of density of the material is
(a) 640
(b) 410
(c) 40
(d) 16

59. If surface tension(S), moment of inertia (I) and Planck's constant (h) were to be taken as the fundamental units, the dimensional formula for linear momentum would be
(a) $S^{3/2}I^{1/2}h^{0}$
(b) $S^{1/2}I^{1/2}h^{0}$
(c) $S^{1/2}I^{1/2}h^{-1}$
(d) $S^{1/2}I^{3/2}h^{-1}$

Answers Of Competency Based Questions:

55. [X]= X^{α} $\therefore \frac{P}{V} = TIME$
$[V] = X^{\beta}$
$[a] = X^{p}$ $\therefore \frac{v}{a} = time$
. [p]= X^{q}
. [F]= X^{r}
$\left(\frac{position}{speed}\right) = \left(\frac{speed}{acceleration}\right) = \left(\frac{p}{F}\right)$
$\left(\frac{X^{\alpha}}{X^{\beta}}\right) = \left(\frac{X^{\beta}}{X^{p}}\right) = \left(\frac{X^{q}}{X^{r}}\right)$
On solving $p + q - r = \beta$

56. $B = [e^{\alpha}][m_e]^{\beta}[h^{r}][k^{\delta}]$
$[M^1\, T^{-2}\, A^{-1}] = [A^{\alpha}\, T^{\alpha}\,][M^{\beta}\,][M^1\, L^2\, T^{-1}]^{\gamma}[M^1\, L^3\, A^{-2}\, T^{-4}]^{\delta}$
$[M^1\, T^{-2}\, A^{-1}] = [M^{\beta+\gamma+\delta}\, L^{2r+3\delta}\, T^{\alpha-r-4\delta}\, A^{\alpha-2\delta}]$
On solving $\alpha = 3\ \beta = 2\ \gamma = -3 \therefore \alpha + \beta + \gamma + \delta = 4$
57. [M]=[mass]= $[M^0 L^0\, T^0]$
[Angular momentum]= $[M^1\, L^2\, T^{-1}]$
[length]= [L]
$[M^1\, L^2\, T^{-1}] = [M^0\, L^0\, T^0]$
$[L^2]$= [T]
[power]= $[M^1\, L^1\, T^2][\, L^1\, T^{-1}] = [M^1\, L^2\, T^{-3}] = [\, L^{-4}\,]$
Similarly [work]= $[\, L^{-2}\,]$
[force]= $[\, L^{-3}\,]$
[linear momentum]= $[\, L^{-1}\,]$
58. 128 kg/m^3= $\frac{125(50g)(20)}{25cm^4\ 4^3} = 40\ unit$
59. $p = kS^{a}I^{b}h^{c}$
$[M^1\, L^1\, T^{-1}] = [M^1\ \ T^2][M^1\, L^2\,][M^1\, L^2\, T^{-1}]$
$a + b + c = 1$
$2b + 2c = 1$
$-2a - c = -1$
a=1/2 b=1/2 c=0
$p = kS^{1/2}I^{1/2}h^{0}$

Self-Assessment Question:

60. A dimensionless quantity is constructed in terms of electronic charge e, permittivity of free space ε_0, Planck's constant h, and speed of light c. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_o^{\beta}h^{\gamma}c^{\delta}$ and n is a non-zero integer, then $(\alpha, \beta, \gamma, \delta)$ is given by
(A) (2n, −n, −n, −n)
(B) (n, −n, −2n, −n)
(C) (n, −n, −n, −2n)
(D) (2n, −n, −2n, −2n)
Ans: OPTION A

61. Young's modulus of elasticity Y is expressed in terms of three derived quantities, namely, the gravitational constant G, Planck's constant h and the speed of light c, as $Y = c^{\alpha} h^{\beta} G^{\gamma}$. Which of the following is the correct option?
(A) $\alpha = 7, \beta = -1, \gamma = -2$
(B) $\alpha = -7, \beta = -1, \gamma = -2$
(C) $\alpha = 7, \beta = -1, \gamma = 2$
(D) $\alpha = -7, \beta = 1, \gamma = -2$
Ans: OPTION A

Multimedia:

SUB TOPIC	TYPE	LINK
Significant figure	Video	https://youtu.be/eMl2z3ezlrQ https://youtu.be/iorZdz4dsBU https://youtu.be/xHgPtFUbAeU
Rounding off the nos.	Reference	https://chem.libretexts.org/Bookshelves/General_Chemistry/Chem1_(Lower)/04%3A_The_Basics_of_Chemistry/4.06%3A_Significant_Figures_and_Rounding
Dimensional analysis	Video	https://youtu.be/sk9BUMBK6hU?si=fudbNLkc2uOPEIsU
Significant figure	quizizz	https://quizizz.com/admin/quiz/668cdacc617fe69496e20823?source=quiz_share
Dimensional analysis	quizizz	https://quizizz.com/admin/quiz/613a21357a830e001da20ac4?source=quiz_share

Unit II: Kinematics

Chapter–3: Motion in a Straight Line

GIST OF THE CHAPTER:

Elementary concepts of differentiation and integration for describing motion, Frame of reference, Motion in a straight line, uniform and nonuniform motion, and instantaneous velocity, uniformly accelerated motion, velocity - time and position-time graphs. Relations for uniformly accelerated motion (graphical treatment).

Kinematics

Motion in One Dimension : Position
Position of any object is the location of object. Position is completely expressed by two factors : Its distance from the observer and its direction with respect to observer.

That is why position is characterised by a vector known as position vector.

Frame of Reference: It is a system to which a set of coordinates along with time are attached and with reference to which observer describes any event.

Rest and Motion
If a body does not change its position as time passes with respect to frame of reference, it is said to be at **rest.**
And if a body changes its position as time passes with respect to frame of reference, it is said to be in **motion.**
Rest and motion are relative terms. It depends upon the frame of references. Nobody can exist in a state of absolute rest or of absolute motion.

***One dimensional motion:-** The motion of an object is said to be one dimensional motion if only one out of three coordinates specifying the position of the object change with time. In such a motion an object move along a straight line path.
***Two dimensional motion:-** The motion of an object is said to be two dimensional motion if two out of three coordinates specifying the position of the object change with time. In such motion the object moves in a plane.
***Three dimensional motion:-** The motion is said to be three dimensional motion if all the three coordinates specifying the position of an object change with respect to time ,in such a motion an object moves in space.

Distance and Displacement

(1) Distance : It is the actual path length covered by a moving particle in a given interval of time.
(i) Its a scalar quantity.
(ii) Dimension : $[M^0L^1T^0][M^0L^1T^0]$
(iii) Unit : metre (S. I.)

(2) Displacement : Displacement is the change in position vector i.e., A vector joining initial to final position.
(i) Displacement is a vector quantity
(ii) Dimension : $[M^0L^1T^0][M^0L^1T^0]$
(iii) Unit : metre (S. I.)
(iv) If $\overrightarrow{s_1}, \overrightarrow{s_2}, \overrightarrow{s_1}, \overrightarrow{s_2}, \ \overrightarrow{s_3}, \overrightarrow{s_3}, \ \overrightarrow{s_4}, \overrightarrow{s_4},$ .... are the displacements of a body then the total (net) displacement is the vector sum of the individuals.

$$\overrightarrow{s_{net}} = \overrightarrow{s_1} + \overrightarrow{s_2} + \overrightarrow{s_3} + \overrightarrow{s_4} + \cdots \dots \dots \dots \dots \dots \dots \dots \dots$$

(3) Comparison between distance and displacement :

S. No.	Distance	Displacement
1.	For a moving particle distance can never be negative or zero Distance > 0	For a moving particle displacement can be negative or zero i.e., Displacement > = or < 0
2.	It is the actual path length covered	Its magnitude is the shortest length between initial and final position.
3.	For a moving particle, distance can never decrease with time.	For a moving particle, displacement can decrease or increase. Decrease in displacement with time means body is moving towards the initial position.

4. The magnitude of displacement is less than or equal to the actual distance travelled by the object in the given time interval.

Magnitude of Displacement ≤ Actual distance

Speed and Velocity:

(1) Speed : Rate of distance covered with time is called speed.

(i) It is a scalar quantity having symbol ‘ vv ‘.

(ii) Dimension : $[M^0 L^1 T^{-1}][M^0 L^1 T^{-1}]$

(iii) Unit : metre/second (S.I.), cm/second (C. G. S.)

Types of speed :

(a) Uniform speed : When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed.

(b) Non-uniform (variable) speed : In non-uniform speed particle covers unequal distances in equal intervals of time.

(c) Average speed : The average speed of a particle for a given interval of time is defined as the ratio of distance travelled to the time taken.

$$Average\ Speed = \frac{Total\ distance\ travelled}{Total\ time} = v_{av} = \frac{\Delta s}{\Delta t}$$

(d) Instantaneous speed : It is the speed of a particle at particular instant. When we say ..speed.., it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (*i.e.* $\Delta t \to 0 \Delta t \to 0$). Thus

$$Instantaneous\ speed\ v = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

(2) Velocity : Rate of change of position *i.e.*, rate of displacement with time is called velocity.

(i) It is a vector quantity having symbol $\vec{v}\vec{v}$

(ii) Dimension: $[M^0 L^1 T^{-1}][M^0 L^1 T^{-1}]$

(iii) Unit : metre/second (S. I.), cm/second (C. G. S.)

Types of Velocity

(a) Uniform velocity: A particle is said to have uniform velocity, When a particle covers equal displacements in equal intervals of time i.e. if magnitudes as well as direction of its velocity remains same.

Uniform velocity is possible only when the particles moves in same straight line without reversing its direction.

(b) Non-uniform velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

(c) Average velocity : It is defined as the net displacement divided by time taken by the body.

$$Average\ Velocity = \frac{Net\ displacement}{Total\ time} = \overrightarrow{v_{av}} = \frac{\overrightarrow{\Delta r}}{\Delta t}$$

(d) Instantaneous velocity : Instantaneous velocity is defined as rate of change of

position vector of particles with time at a certain instant of time.
The instantaneous velocity is average velocity for infinitesimally small time interval (*i.e.*$\Delta t \rightarrow 0 \Delta t \rightarrow 0$). Thus

$$Instantaneous\ velocity\ \vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

❖ Instantaneous velocity is always directed tangential to the path followed by the particle.

❖ A particle may have constant instantaneous speed but variable instantaneous velocity.

❖ The magnitude of instantaneous velocity is equal to the instantaneous speed.

❖ If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

❖ If displacement is given as a function of time, then time derivative of displacement will give velocity.

S. No.	**Average Speed**	**Average velocity**
1.	Average speed is scalar.	Average velocity is a vector.
2.	Same units (m/s) and dimensions: $[M^0L^1T^{-1}][M^0L^1T^{-1}]$	Same units (m/s) and dimensions: $[M^0L^1T^{-1}][M^0L^1T^{-1}]$
3.	For a given time interval average speed can have many values depending on path followed.	For a given time interval average velocity is single valued. .
4.	If after motion body comes back to its initial position then $Average\ Speed = v_{av} > 0$ $Average\ Speed = v_{av} > 0$	If after motion body comes back to its initial position then $Average\ Velocity = \overrightarrow{v_{av}} = 0$
5.	For a moving body average speed can never be negative or zero.	For a moving body average velocity can be negative or zero

Instantaneous velocity = =*Speed:- It is rate of change of distance covered by the body with respect to time.

Speed = Distance travelled /time taken

Speed is a scalar quantity .Its unit is meter /sec. and dimensional formula is $[M^0L^1T^{-1}]$ .It is positive or zero but never negative.

*Uniform Speed:- If an object covers equal distances in equal intervals of time than the speed of the moving object is called uniform speed. In this type of motion, position – time graph is always a straight line.

*Instantaneous speed:-The speed of an object at any particular instant of time is called instantaneous speed. In this measurement, the time $\Delta t \rightarrow 0$.

When a body is moving with uniform speed its instantaneous speed = Average speed = uniform speed.

*Velocity:- The rate of change of position of an object in a particular direction with respect to time is called velocity. It is equal to the displacement covered by an object per unit time.

Velocity =Displacement /Time

Velocity is a vector quantity, its SI unit is meter per sec. Its dimensional formula is $[M^0L^1T^{-1}]$. It may be negative, positive or zero.

*When a body moves in a straight line then the average speed and average velocity are equal.

*Acceleration:- The rate of change of velocity of an object with respect to time is called its acceleration.

Acceleration = Change in velocity /time taken

It is a vector quantity, Its SI unit is meter/ $sec.^2$ and dimension is $[M^0L^1T^{-2}]$, It may be positive ,negative or zero.

*Positive Acceleration:- If the velocity of an object increases with time, its acceleration is positive .

*Negative Acceleration :-If the velocity of an object decreases with time, its acceleration is negative . The negative acceleration is also called retardation or deacceleration.

*Formulas of uniformly accelerated motion along straight line:-

For accelerated motion,

$V = u + at$

$S = ut + {}^1/_2\, at^2$

$V^2 = u^2 + 2as$

$Sn = u + {}^a/_2(2n - 1)$

For deceleration motion

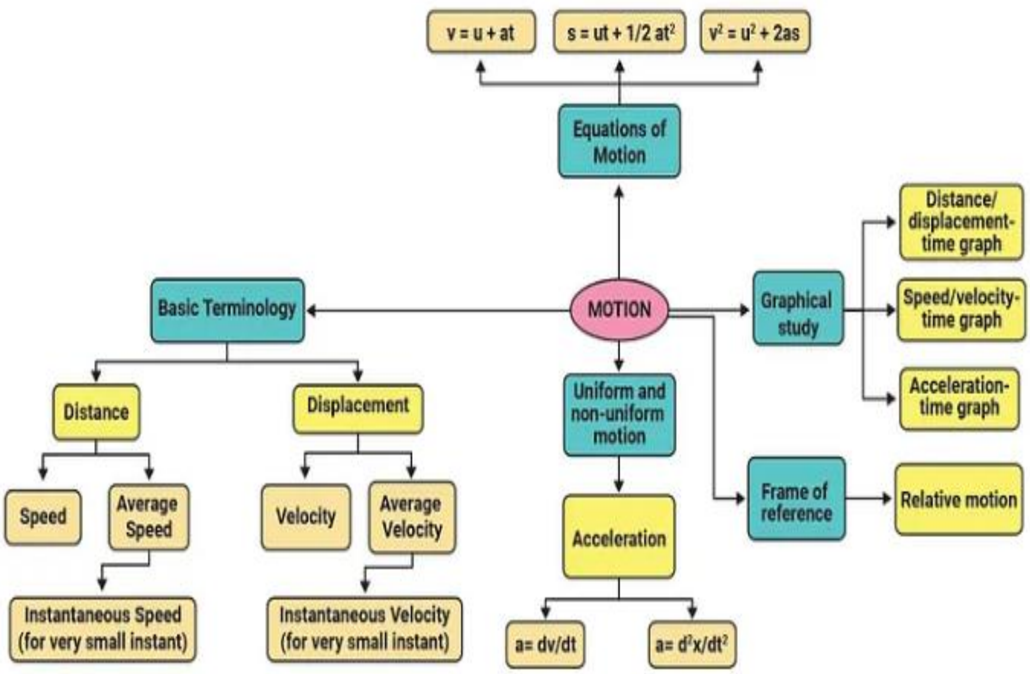
$v = u - at$

$S = ut - {}^1/_2\, at^2$

$V^2 = u^2 - 2as$

$Sn = u - {}^a/_2(2n - 1)$

*Free fall :- In the absence of the air resistance all bodies fall with the same acceleration towards earth from a small height. This is called free fall. The acceleration with which a body falls is called gravitational acceleration (g).Its value is 9.8 m/sec^2.



Very Short Answer Type Questions (1 marks **)**

Q1. What does the slope of v-t graph indicate ?
Ans : Acceleration

Q2. Under what condition the average velocity equal to instantaneous velocity?
Ans :For a uniform velocity.

Q.3. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s
Ans : 54 m/sec.

Q4. Give an example when a body moving with uniform speed has acceleration.
Ans : In the uniform circular motion.

Q5. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are h_1 and h_2 respectively what is h_1/h_2.
Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q6. State the essential condition for the addition of the vector.
Ans : They must represent the physical quantities of same nature.

Q7. What is the angle between velocity and acceleration at the peak point of the projectile motion ?
Ans : 90^0.

Q8. What is the angular velocity of the hour hand of a clock ?
Ans : $W = 2\pi/12 = \pi/6$ rad h^{-1},

Q9. What is the source of centripetal acceleration for earth to go round the sun ?
Ans. Gravitation force of the sun.

Q10. What is the average value of acceleration vector in uniform circular motion .
Ans : Null vector .

Short Answer Type Question (2 Marks **)**

Q11. Derive an equation for the distance travelled by an uniform acceleration body in n^{th} second of its motion.
Ans. $S_n = u + \frac{a}{2}(2n-1)$

Q12. The velocity of a moving particle is given by $V=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s
Ans. Differentiation of the given equation eq. w.r.t. time
We get $a = 18 + 18t$
At $t = 2$ sec.
$a= 54$ m/sec^2.

Q13. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.
Ans : We know that the horizontal range
$R = u^2 \sin 2\Theta/g$
For maximum range $\Theta = 45^0$,
$R_{max} = u^2/g$
and Height
$H = u^2 \sin^2\Theta/2g$
For $\Theta = 45^0$
$H = u^2/4g$ = 1/4 of the R_{max}.

Q14. State the parallelogram law of vector addition. Derive an expression for

magnitude and direction of resultant of the two vectors.
Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{R} = \vec{A} + \vec{B}$$

And $R = (A^2 + B^2 + 2AB \cos\Theta)^{1/2}$

And $\tan \beta = B \sin\Theta / (A + B \cos\Theta)$,

Where Θ is the angle between vector $\vec{A}$ and vector $\vec{B}$, And β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

Q15. A gunman always keeps his gun slightly tilted above the line of sight while shooting. Why,
Ans. Because bullet follow parabolic trajectory under constant downward acceleration.
Q16. Derive the relation between linear velocity and angular velocity.
Ans : Derive the expression

$$V = r\,\omega$$

Q17. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane .
Q18. The greatest height to which a man can a stone is h, what will be the longest distance upto which he can throw the stone ?
Ans: we know that

$$H_{max.} = R_{max}/2$$
$$\text{So} \quad h = R/2$$
$$\text{Or} \quad R = 2h$$

Short Answer Questions (3 Marks)
Q19. If 'R' is the horizontal range for Θ inclination and H is the height reached by the projectile, show that R(max.) is given by

$$R_{max} = 4H$$

Q20. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles Θ_1 and Θ_2 projections for the same horizontal range. Such that $(\Theta_1 + \Theta_2) = 90^{0.}$
Q21. Prove that there are two values of time for which a projectile is at the same height . Also show that the sum of these two times is equal to the time of flight.
Q22.: Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.
Q23. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. (g = 9.8 m/sec^2)
Q24. Find the value of λ so that the vector $\vec{A} = 2\hat{\imath} + \lambda\,\hat{\jmath} + \hat{k}$ and $\vec{B} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are perpendicular to each.
Q25.. Show that a given gun will shoot three times as high when elevated at angle of 60^0 as when fired at angle of 30^0 but will carry the same distance on a horizontal plane.
Long Answer Question (5 Marks)
Q26. Draw velocity- time graph of uniformly accelerated motion in one dimension. From the velocity – time graph of uniform accelerated motion, deduce the equations of motion in distance and time.
Q27. (a) With the help of a simple case of an object moving with a constant velocity show that the area under velocity – time curve represents over a given time interval.

(a) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

Q28. Establish the following vector inequalities :

(i) $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

(ii) $|\vec{a} - \vec{b}| \leq |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q29. What is a projectile ? show that its path is parabolic. Also find the expression for :

(i) Maximum height attained and

(ii) Time of flight

Q30. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.

HOTS

Q31. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If

$|\vec{A} \times \vec{B}| = \sqrt{3}\,(\vec{A}.\vec{B})$, calculate the value of angle Θ.

Ans : 60^0

Q32. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity of the river.

Ans : 6 km/h.

Q33. A cricket ball is hit at 45^0 to the horizontal with a kinetic energy E. calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component $u\mathrm{Cos}45^0$ is present on highest point.)

Q34. Speed of two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant.

Ans : 1 : 16

Q35. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that $t_1t_2 = 2R/g$

ans : for equal range the particle should either be projected at an angle Θ and (90 - Θ) ,

then $t_1 = 2u\,\mathrm{Sin}\Theta/g$

$t_2 = 2u\,\mathrm{Sin}(90 - \Theta)/g = 2u\,\mathrm{Cos}\Theta/g$

$t_1t_2 = 2R/g$.

<u>Multiple Choice Question</u>

Q 36 For the motion with uniform velocity, the slope of the velocity-time graph is equal to

a. 1 m/s
b. Zero
c. 10m/s
d. 5m/s

Q37. An object is moving with constant velocity then its

a) acceleration is 1
b) acceleration is constant
c) acceleration is zero
d) Non uniform acceleration
Ans: c) acceleration is zero

Q38 An object travels 24m with a speed of 6m/s and 48m with a speed of 8m/s. Find its average speed.
a) 72m/s
b) 4m/s
c) 7.2m/s
d) 6m/s

Ans:7.2m/s

Q39 The slope of velocity time graph gives
a) distance
b) displacement
c) acceleration
d) speed
Ans: c) acceleration

Q 40 The displacement time graph for two particles A and b are straight lines inclined at 30 and 60 degrees respectively.with time axis. The ratio of velocities V_A :V_B is
a) 1:2
b) 1: $\sqrt{3}\sqrt{3}$
c) 11:1
d) 1:3
Ans: c) 11:1

Q 41. If the displacement of an object is proportional to square of time, then the object is moving with
a) constant acceleration
b) uniform motion
c) constant velocity
d) non uniform motion
Ans: a) constant acceleration

Q 42 The numerical ratio of average speed to average velocity is
(a) always less than one
(b) always equal to one
(c) always more than one
(d) equal to or more than one
Ans: (d) equal to or more than one

Q 43 A ball is dropped from height h. The velocity attained by the ball just before reaching the ground is
(a) $\sqrt{2gh}$
(b) $\sqrt{gh}$
(c) $\sqrt{3gh}$

(d)zero

Ans: (a)$\sqrt{2gh}$

Q 44 What is the acceleration if the body starts from rest and travels a distance of 's' m in 2 seconds?

a. 2/3 s m/s2
b. 3/2 s m/s2
c. 1/3 s m/s2
d. 1/2s m/s2

Ans: d s/2 m/s2

Q 45 An athlete finishes a round of circular track of radius R in 40 sec. What is his displacement at the end of 2 min 20 sec?

a. 2R
b. 2πR
c. 7πR
d. Zero

Ans : (d)**2R**

Q 46 A man leaves home for a cycle ride and comes back home after an half-an-hour ride covering a distance of one km. What is the average velocity of the ride?

a. 10 kms^{-1}
b. ½ kms^{-1}
c. 2 kms^{-1}
d. Zero

Ans: (d) Zero

Q 47 A body is moving with uniform acceleration described as 30 m in the first 5 sec and 65 m next 5 sec. Its initial velocity will be:

1. 4 m/s
2. 2.5 m/s
3. 5.5 m/s
4. 11 m/s

Ans: 2

Case Based Questions (4 MARKS)

Q 47 Case 01

Free Fall:A body released near the surface of the earth is accelerated down under the influence of force of gravity. In the absence of air resistance,all bodies fall with the same acceleration near the suraface of the earth.This motion of body falling towards the earth from a small height is called free fall.The acceleration with a body falls is called acceleration due to gravity and is denoted by g.When the body falls freely under gravity the action of gravity ,its velocity and the value of g is taken as positive while it is thrown vertically upwards ,its velocity and the valu g is taken as negative.

1. A stone is dropped from the top of a tower 50 m high. Simultaneously another stone is thrown upwards from the ground with a speed of 20 m.s^{-1}. Calculate the time at which both the stone cross each other.
   2.5s ii) 5 s ii) 10s iv) 20s
2. When a body is falling freely under gravity, its velocity at highest point is
   i) zero
   ii) 1
   iii) 9.8m/s
   iv) 10m/s
3. A cricket ball is dropped from a height of 20m.Find its final velocity.
   i) 20m/s
   ii) 2m/s
   iii) 200m/s
   iv) 10m/s

Answers: 1) 2.5 s 2) zero 3) 20m/s

Q 48 Case 02

When an object moves along a straight line with uniform acceleration, it is possible to relate its velocity, acceleration during motion and the distance covered by it in a certain time interval by a set of equations known as the equations of motion. For convenience, a set of three such equations are given below:

v = u + at

s = ut + ½ at2

2a s = v2 – u 2

Where u is the initial velocity of the object which moves with uniform acceleration a for time t, v is the final velocity and s is the distance travelled by the object in time t.

i. Equation of motions are applicable to motion with
   a) uniform acceleration b) non uniform acceleration
   c) constant velocity d) none of these
ii. The distance travelled by a body is directly proportional to the square of time taken its acceleration
   a) increases b)decreases c) becomes zero d) remains constant
iii. The brakes applied to a car produce an acceleration of 10 m/s2 in the opposite direction to the motion. If the car takes 1 s to stop after the application of brakes, calculate the distance traveled during this time by car.
iv. An object is dropped from a tower falls with a constant acceleration of 10 m/s2. Find its speed 10 s after it was dropped
v. A bullet hits a target with a velocity of 10 m/s and penetrates it up to a distance of 5 cm. Find the deceleration of the bullet in the target

Answers . i)a ii) d iii) 5m iv) 100m/s v) -10m/s2

Competency Based Questions:

Q 49 A body moves in a straight line along Y-axis. Its distance y (in metre) from the origin is given by $y = 8t – 3t^2$. The average speed in the time interval from t = 0 second to t = 1 second is

(1) – 4 ms^{-1} (2) zero 3) 5 ms^{-1} 4) 6 ms^{-1}

Ans: AV Vel = $\frac{8X1-3X1X1}{1}$ = 5m/s

Q 50 Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t2. The time taken by her to walk up on the moving escalator will be:

1. $\frac{t1\ t2}{t2-t1}$ 2. $\frac{t1\ t2}{t2+t1}$ 3. $t2-t1$. 4. $\frac{t2+t1}{2}$

Ans: Velocity of preeti w.r.t elevator v1= d/t1

Velocity of elevator w.r.t ground v2 =d/t2

Velocity of preeti w.r.t ground v = v1+ v 2

d/t= d/t1+d/t2

t= $\frac{t1\ t2}{t2+t1}$

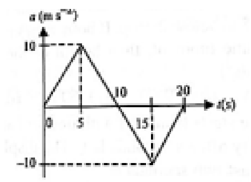
Q 51 Two bodies of different masses ma and mb are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances is

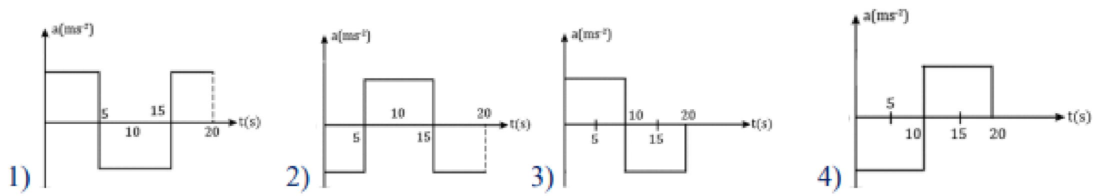
1) a:b 2) b:a 3) $\sqrt{a}$: $\sqrt{b}$ 4) a^2:b^2

Ans: h= ½ gt^2 and t = $\sqrt{2gh}$

ta = $\sqrt{\frac{2a}{g}}$ tb = $\sqrt{\frac{2b}{g}}$ ta/tb = $\sqrt{a}$: $\sqrt{b}$

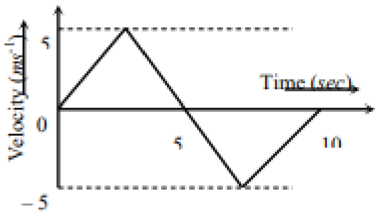
Q 52 Plot the acceleration –time graph of the velocity – time graph given in the figure.



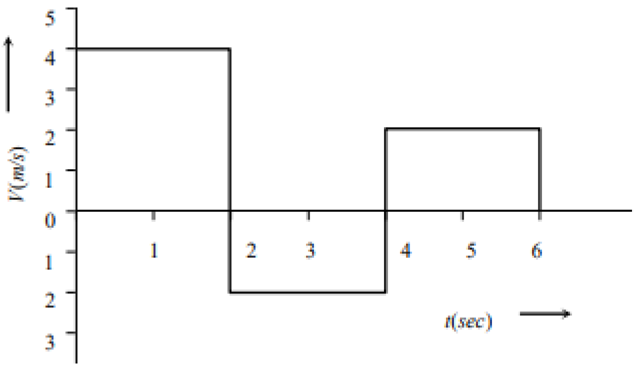


Ans: 1. From 0 to 5 acceleration is positive, 5 to 10s, acceleration is negative and from 10 to 20s, acceleration is positive

Q 53 The υ − t plot of a moving object is shown in the figure. The average velocity of the object during the first 10 seconds is



(1)0 m/s (2)2.5 ms–1 (3)5 ms–1 (4)2 ms–1
Ans : (1) Since total displacement is zero, hence average velocity is also zero.
Q 54. The area under acceleration-time graph gives
(1)Distance travelled (2) Change in acceleration (3) Force acting (4) Change in velocity
Ans:(4) Acceleration – time graph represents the objects change in velocity.
Acceleration = $\Delta v / \Delta t$
Area between acceleration – time graph gives:
$a \times \Delta t = \Delta v / \Delta t \times \Delta t = \Delta v$
Q 55. The velocity-time graph of a body moving in a straight line is shown in the figure. The displacement and distance travelled by the body in 6 sec are respectively
1. 16m, 8m 2. 8m, 16m 3. 8m, 8m 4. 16m,16m



Ans: 2 .Displacement = Summation of all the area with sign
= 8-4+4
∴ Displacement = 8 m
Distance =Summation of all the areas without sign
= 8+4+4 =16m
Q 56 The displacement y (in meter) of a body varies with time t (in second) as $y=2/3\ t^2 + 16t + 2$. How long does the body take to come to rest?
Ans: 3) Velocity= dy /dt = -4 /3 t+ 16 .For body to be at rest, v = 0
0= -4 /3 t+ 16 and solving t= 12s

Self-Assessment

1. A body starts from point P and moves to Q. If the body returns to the same point (P), find i) displacement ii) distance iii) velocity iv) average speed.
2. A man runs across the roof top of a tall building and jumps horizontally with the hope of landing on the roof top of the next building which is of lower height than the first. If his speed is 9m/s, the horizontal distance between the two buildings is 10m and the height difference is 9m, will he be able to land on the next building take g = 10m/s2.
3. On a 60 km straight road, a bus travels the first 30km with uniform speed of 30km/hr. How fast must the bus travel the next 30km so as to have average speed of 40km/hr for the entire trip

Unit II: Kinematics

Chapter– 4: Motion in a Plane

GIST OF THE CHAPTER:

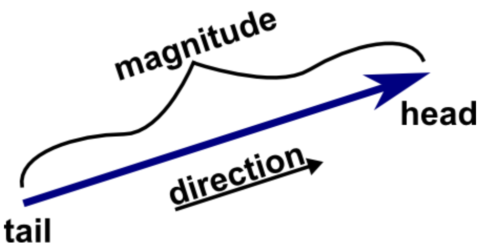
Scalar and vector quantities; position and displacement vectors, general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors, Unit vector; resolution of a vector in a plane, rectangular components, Scalar and Vector product of vectors.
Motion in a plane, cases of uniform velocity and uniform acceleration, projectile motion, uniform circular motion.

Scalar And Vector Quantities

Scalar quantity: A scalar is a physical quantity that has only a magnitude.
Examples: Mass, Length, Time, Temperature, Volume, Density etc.

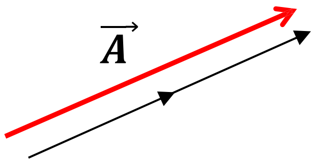
Vector quantity: A vector is a physical quantity that has both a magnitude and a direction.

Representation of a vector



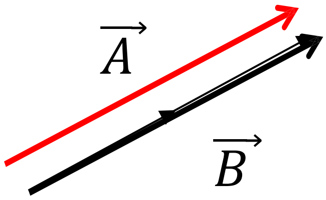
- Vector diagrams are shown using an arrow.
- The length of the arrow represents its magnitude.
- The direction of the arrow shows its direction.
- They are also represented by a single capital letter with an arrow above it.
- Magnitude (Modulus) of a Vector
  Magnitude of a $\vec{A} = |\vec{A}| = A$

$$|\vec{A}| = 2\ units$$



TYPES OF VECTORS

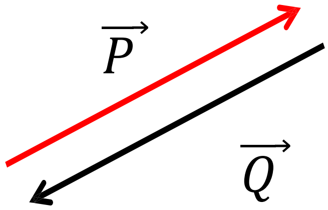
Parallel Vectors: Two vectors are said to be parallel vectors, if they have same direction.



Equal Vectors: Two parallel vectors are said to be equal vectors, if they have same magnitude.

$$If\ |\vec{A}| = |\vec{B}|\ then\ \vec{A} = \vec{B}\ and\ \vec{A} \parallel \vec{B}$$

Anti-parallel Vectors: Two vectors are said to be anti-parallel vectors, if they are in opposite directions.



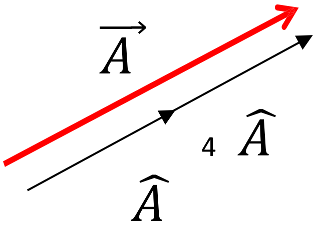
Negative Vectors: Two anti-parallel vectors are said to be negative vectors, if they have same magnitude.

$$If\ \vec{P} = -\vec{Q}\ Then\ |\vec{P}| = |-\vec{Q}|\ and\ \vec{P} \parallel -\vec{Q}$$

Zero Vectors or Null Vectors: A Zero vector or Null Vector is a vector of zero magnitude

$$\vec{P} + (-\vec{P}) = \vec{0} \quad \& \quad |\vec{0}| = 0,\ also, \quad \vec{P} + (\vec{0}) = \vec{P} = \vec{0} + (\vec{P})$$

Example: When the initial and final positions of a moving object coincide, the displacement is a ″null vector″.
Unit vectors: A unit vector is a vector that has a magnitude of

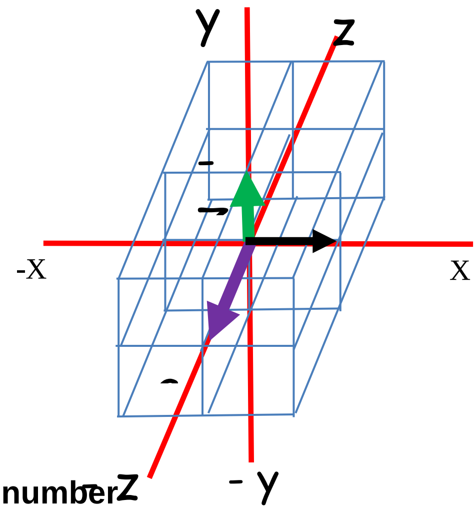


exactly 1 and drawn in the direction of given vector.

- It lacks both dimension and unit.
- Its only purpose is to specify a direction in space.

Unit Vector along axes

- Unit Vector Along X-axis = $\hat{\imath}$
- Unit Vector Along Y-axis = $\hat{\jmath}$
- Unit Vector Along Z-axis = $\hat{k}$
- $|\hat{\imath}|$ =1
- $|\hat{\jmath}|$ =1
- $|\hat{k}|$ =1

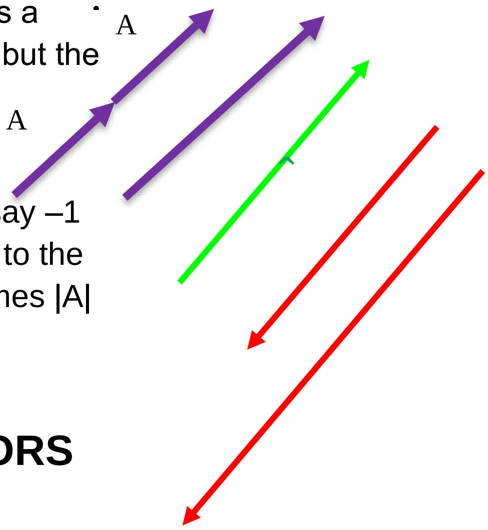


Multiplication of a Vector with real number

- Multiplying a vector A with a positive number λ gives a vector whose magnitude is changed by the factor λ but the direction is the same as that of A.

$$If \quad 2\,x\ \vec{A} = 2\vec{A} \quad Then\ |2\vec{A}| = 2|\vec{A}|$$

- Multiplying a given vector A by negative numbers, say –1 and –1.5, gives vectors whose direction is opposite to the direction of A and whose magnitude is –1 or -1.5 times |A| as shown in the figure.



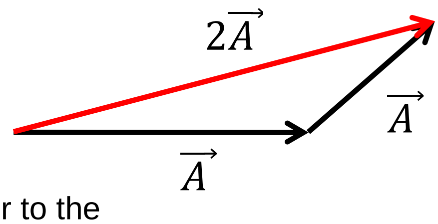
ADDITION OF VECTORS

(i) Only vectors of same nature can be added.

(ii) The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$

$$\vec{R} = \vec{A} + \vec{B}$$

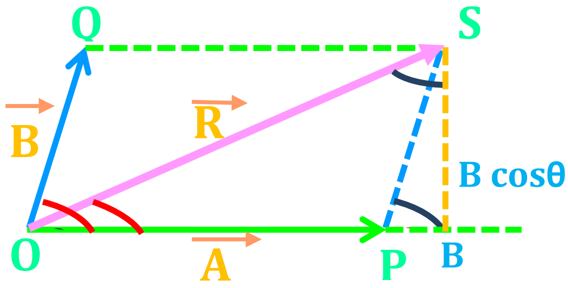
The Triangle Law



- When two vectors are joined head to tail
- The sum vector points from the tail of the first vector to the head of the second vector.
- Draw the resultant vector by completing the triangle

The Parallelogram Law

If the two vectors are joined along two adjacent sides of a parallelogram with coinciding tails, then the diagonal drawn from the point where tails coincide represents the sum of two vectors in magnitude and direction, with its tail at point of coincidence of the two vectors.



$$R = \sqrt{A^2 + 2AB\cos\theta + B^2}$$
$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Where Θ is the angle between vector $\overrightarrow{A}$ and vector $\overrightarrow{B}$, And $\boldsymbol{\alpha}$ is the angle which R makes with the direction of $\overrightarrow{A}$.

R is maximum if Θ = 0 and minimum if Θ = 180 ^{0}.

Properties of addition of vectors

Vector addition is commutative $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Vector addition is associative, $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

Subtraction of two vectors:-

(i) Only vector of same nature can be subtracted.

(ii) Subtraction of $\overrightarrow{B}$ from $\overrightarrow{A}$ = vector addition of $\overrightarrow{A}$ and $(-\overrightarrow{B})$,

$\overrightarrow{R} = \overrightarrow{A} - \overrightarrow{B} = \overrightarrow{A} + (\overrightarrow{-B})$

Where
$$R = \sqrt{A^2 + 2AB\cos(180-\theta) + B^2}$$
$$\tan\alpha = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)}$$

Where Θ is the angle between vector $\overrightarrow{A}$ and vector $\overrightarrow{B}$, And $\boldsymbol{\alpha}$ is the angle which R makes with the direction of $\overrightarrow{A}$.

(iii) Vector subtraction is not commutative $(\vec{A} - \vec{B}) \neq (\vec{B} - \vec{A})$

(iv) Vector subtraction is not associative,

$$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$$

Rectangular components of a vector in a plane :- If $\overrightarrow{A}$ makes an angle Θ with x-axis and $\overrightarrow{A}_x$ and $\overrightarrow{A}_y$ be the rectangular components of $\overrightarrow{A}$ along X-axis and Y- axis respectively, then

$$\overrightarrow{A} = \overrightarrow{A}_x + \overrightarrow{A}_y = A_x\,\hat{i} + A_y\,\hat{j}$$

$$Here\ A_x = A\cos\theta \text{ and } A_y = A\sin\theta\ also, \quad A = \sqrt{{A_x}^2 + {A_y}^2}$$

$$\tan\theta = \frac{A_x}{A_y}$$

Dot product or scalar product : - The dot product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A}.\vec{B}$ is a scalar , which is equal to the product of the magnitudes of A and B and the Cosine of the smaller angle between them.
If Θ is the smaller angle between A and B, then

$$\vec{A}.\vec{B} = AB\cos\theta$$

(i) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ii) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(iii) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Cross or Vector product :-

The cross product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A} \times \vec{B}$ is a vector , which is equal to the product of the magnitudes of A and B and the sine of the smaller angle between them.
If Θ is the smaller angle between A and B, then

$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$

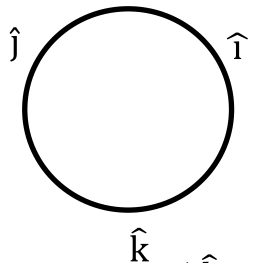
Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

(ii) $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$

(iii) If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$



$$\vec{A} \times \vec{B} = (A_xB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Projectile motion : - Projectile is the name given to anybody which once thrown in to space with some initial velocity, moves thereafter under the influence of gravity alone without being propelled by any engine or fuel. The path followed by a projectile is called its trajectory.

Assumptions of Projectile Motion:
(1) There is no resistance due to air.
(2) The effect due to curvature of earth is negligible.
(3) The effect due to rotation of earth is negligible.
(4) For all points of the trajectory, the acceleration due to gravity ëgí is constant in magnitude and direction.

- Path followed by the projectile is parabola.
- Velocity of projectile at any instant t,

$$v = \sqrt{u^2 - 2ugt\sin\theta + g^2t^2}$$

- Horizontal range $R = \dfrac{u^2 \sin 2\theta}{g}$

  For maximum range Θ =45⁰, $R_{max} = u^2/g$

- Flight time

$$T = \frac{2u\sin\theta}{g}$$

- Height

$$H = \frac{u^2\sin^2\theta}{2g}$$

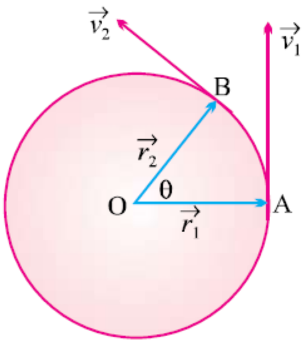
  For maximum height Θ =90⁰ $H_{max.} = u^2/2g$

Uniform Circular Motion

Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.

(i) Displacement : The change of position vector or the displacement of the particle from position A to position B is given by

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1} \; and \;\; \Delta r = 2r\sin\frac{\theta}{2}$$



(ii) Distance : The distance covered by the particle during the time t is given as

d = length of the arc AB

(iii) Angular displacement Vector (θ) : The angle swept by the position vector of body moving on a circle from some reference line is called angular displacement.

- Angular displacement is an **axial vector quantity**.
- Its direction depends upon **the sense of rotation** of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of **angular displacement vector.**
- Relation between linear displacement and angular displacement

$$\Delta\vec{r} = \vec{\theta} \, X \, \vec{r}$$

(iv) Angular velocity (ω) :

Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement. It is also an axial vector quantity. Its direction is the same as that of Angular displacement Δθ.

$$Angular\ velocity\ \omega = \frac{angle\ swept}{time\ taken} = \frac{\Delta\theta}{\Delta t};\quad \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Dimension : $[M^0 L^0 T^{-1}]$
- Units: Radians per second (rad/s) or Degree per second.
- For uniform circular motion Angular velocity ω remains constant.

(v) Change in velocity :. The change in velocity vector of the particle which is performing uniform circular motion is given as

$$\Delta\vec{v} = \vec{v_2} - \vec{v_1}\ and\ \ \Delta v = 2v\sin\frac{\theta}{2}$$

Relation between linear velocity and angular velocity in vector form is given as:

$$\vec{v} = \vec{\omega} X \vec{r}$$

(vi) Angular acceleration (α) :
Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

$$Angular\ acceleration\ \alpha = \frac{change\ in\ angular\ velocity}{time\ taken} = \frac{\Delta\omega}{\Delta t};$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

- Units : rad /s^2
- Dimension : $[M^0 L^0 T^{-2}]$
- Relation between linear acceleration and angular acceleration

$$\vec{a} = \vec{\alpha} X \vec{r}$$

- For uniform circular motion since $\vec{\omega}$ is constant so $\alpha =$ 0.

Centripetal Acceleration

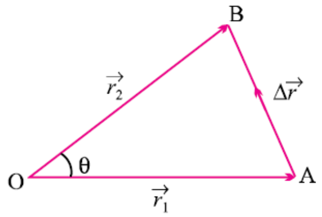
(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.
(2) It always acts on the object along the radius towards the centre of the circular path.
(3) Magnitude of centripetal acceleration

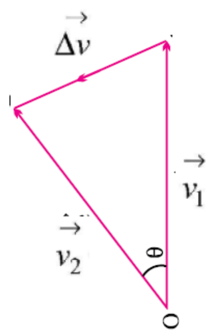
$$|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{1}{\Delta t}\frac{v|\Delta\vec{r}|}{r}$$

$$= \frac{v}{r}\ \frac{|\Delta\vec{r}|}{\Delta t}$$

$$|\vec{a}| = \frac{v^2}{r}$$

$$\left(\therefore v = \frac{|\Delta\vec{r}|}{\Delta t}\right)$$





Very Short answer type questions (1 marks)

Q 01. Under what condition the average velocity equal to instantaneous velocity?

Ans : For a uniform velocity.

Q 02. The position coordinate of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's velocity at t=2s

Ans : $x = 6 + 18t + 9t^2 \Rightarrow v = \frac{dx}{dt} = \frac{d}{dt}(6 + 18t + 9t^2) = 18 + 18t$

$$v\ (at\ t = 2s) = 18 + 18x2 = 54\ m/s$$

Q 03. Give an example when a body moving with uniform speed has acceleration.

Ans : In the uniform circular motion.

Q 04. Two balls of different masses are thrown vertically upward with same initial velocity. Height attained by them are h_1 and h_2 respectively what is h_1/h_2.?

Ans : 1/1, because the height attained by the projectile is not depend on the masses.

Q 05. State the essential condition for the addition of the vector.

Ans : They must represent the physical quantities of same nature.

Q 06. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Ans : 90^{0}. Acceleration at the peak point remain downward and vertical component become zero so, at the peak point ne velocity is horizontal component of velocity.

Q 07. What is the angular velocity of the hour hand of a clock ?

Ans : W = 2π/12 = π/6 rad h^{-1},

Q 08. What is the source of centripetal acceleration for earth to go round the sun ?

Ans. Gravitation force of the sun.

Q 09. What is the average value of acceleration vector in uniform circular motion

Ans : Null vector .

Q 010. Will the displacement of a particle change on changing the position of origin of the coordinate system ?

Ans: The displacement will not change.

Q 011. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero?

Ans : No. (highest point of vertical upward motion under gravity)

Q 012. A projectile is fired with Kinetic energy 1 kJ. If the range is maximum, what is its Kinetic energy, at the highest point?

Ans : here = 1 KJ = 1000 J, and for Max. range θ = 45°
At the highest Point,

$$K.E. = \frac{1}{2}mu^2\cos^2\theta = 1000\text{ J x}\frac{1}{2} = 500\text{J}$$

Q 013. Write an example of zero vector.

Ans: The velocity vectors of a stationary object is a zero vectors.

Q 014. When is the magnitude of ($\vec{A}$ + $\vec{B}$) equal to the magnitude of ($\vec{A}$ - $\vec{B}$)?

Ans: When ($\vec{A}$) is perpendicular to ($\vec{B}$).

Q 015. What is the maximum number of component into which a vector can be resolved?

Ans: Infinite.

Q 016. A body projected horizontally moves with the same horizontal velocity although it moves under gravity. Why?

Ans: Because horizontal component of gravity is zero so along horizontal direction the velocity remains same.

Q 017. What is the angle between velocity and acceleration at the highest point of a projectile motion ?

Ans: 90°.

Q 018. When does (i) height attained by a projectile maximum ? (ii) horizontal range is maximum ?

Ans: height is maximum at θ = 90
Range is maximum at θ = 45.

Q 019. A particle is in clockwise uniform circular motion the direction of its acceleration is radially inward. If sense of rotation or particle is anticlockwise then what is the direction of its acceleration?

Ans: Radial inward.

Q 020. A train is moving on a straight track with acceleration a. A passenger drops a stone. What is the acceleration of stone with respect to passenger ?

Ans: $\sqrt{a^2 + g^2}$ where *g* = acceleration due to gravity.

<u>Short Answer type question (2 marks)</u>

Q 021. Derive the distance travelled by a body moving with uniform acceleration in n^{th} second of its motion.

Ans. $S_n = u + \frac{a}{2}(2n - 1)$

Q 022. The displacement of a moving particle is given by $x=6+18t+9t^2$ (x in meter, t in seconds) what is it's acceleration at t=2s?

Ans. Differentiation of the given equation eq. w.r.t. time

We get $v = \frac{dx}{dt} = \frac{d}{dt}(6 + 18t + 9t^2) = 18 + 18t$

$$a = \frac{dv}{dt} = \frac{d}{dt}(18 + 18t) = 18$$

$$a\ (at\ t = 2s) = 18\ m/s$$

Q 023. What are positive and negative acceleration in straight line motion ?

Ans. If speed of an object increases with time, its acceleration is positive. (Acceleration is in the direction of motion) and if speed of an object decreases with time its acceleration is negative (Acceleration is opposite to the direction of motion).

Q 024. Can a body have zero velocity and still be accelerating ? If yes gives any situation.

Ans. Yes, at the highest point of vertical upward motion under gravity.

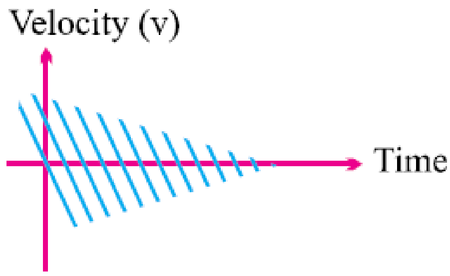
Q 025. The displacement of a body is proportional to t^3, where t is time elapsed. What is the nature of acceleration -time graph of the body ?

Ans.:

$$as\ s\ \alpha\ t^3 \Rightarrow\ s = k\ t^3\ \ \Rightarrow velocity\ v = \frac{ds}{dt} = 3k\ t^2$$

$$\Rightarrow acceleration\ a = \frac{dv}{dt} = 6kt$$

⇒ Motion is nonuniformly accelerated motion, *a* - *t* graph is slant straight-line.



Q 026. Suggest a suitable physical situation for the following graph.

Ans.: A ball thrown up with some initial velocity rebounding from the floor with reduced speed after each hit.

Q 027. An object is in uniform motion along a straight line, what will be position time graph for the motion of object, if x_0 is position at $t = 0$ and

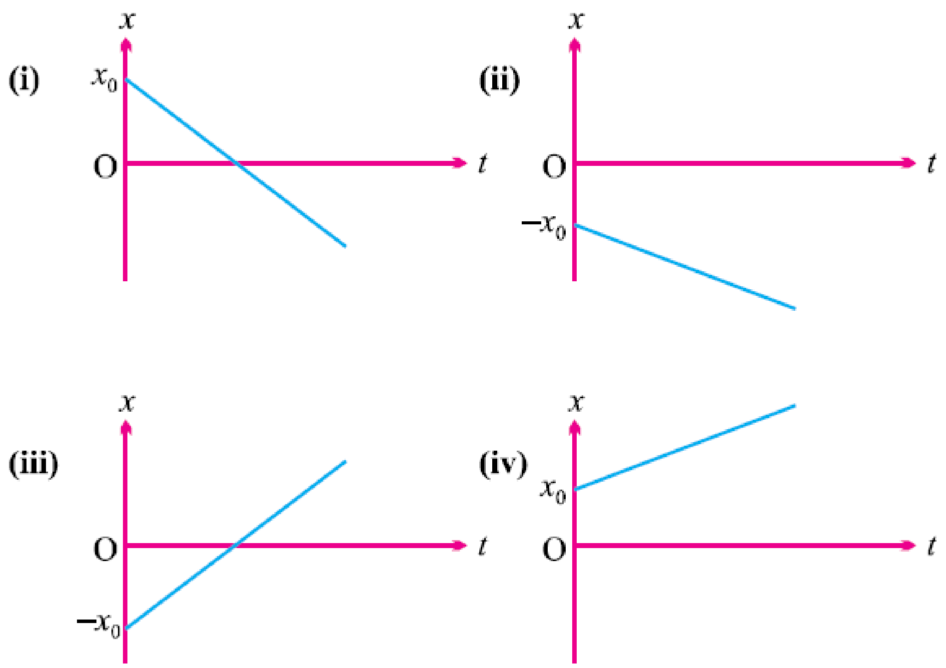
(i) x_0 = positive, v = negative is constant.

(ii) both x_0 and v are negative | v | is constant.

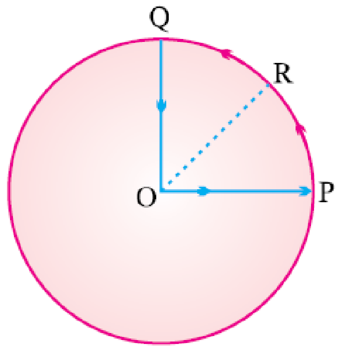
(iii) x_0 = negative, v = positive is constant.

(iv) both x_0 and v are positive is constant.

Ans.: If velocity is constant then x-t graph remain slant straight line.



Q 028. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path OPRQO as shown. If he maintains constant speed of 10 m/s, what is his acceleration at point R in magnitude & direction?



$$Ans : Centripetal\ acceleration = a = \frac{v^2}{r} = \frac{10^2}{1000} = 0.1\ ms^{-2}\ along\ RO$$

Q 029. What will be the effect on horizontal range of a projectile when its initial velocity is doubled keeping angle of projection same ?

$$Ans:\ Range\ R = \frac{u^2\ \sin 2\theta}{g} \Longrightarrow range\ becomes\ 4\ times$$

Q 030. The greatest height to which a man can throw a stone is h. What will be the greatest distance upto which he can throw the stone?

$$Ans:\ height\ H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow H_{max} = \frac{u^2}{2g} = h$$

$$\Rightarrow Max. Range\ R = \frac{u^2}{g} = \ 2h$$

Q 031. A person sitting in a train moving at constant velocity throws a ball vertically upwards. How will the ball appear to move to an observer?

(i) Sitting inside the train

(ii) Standing outside the train

Ans: (i) Vertical straight line motion

(ii) Parabolic path.

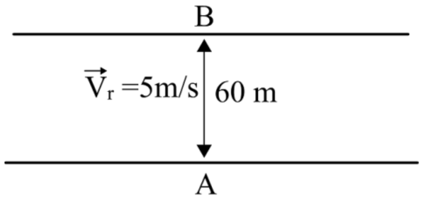
Q 032. Show that when the horizontal range is maximum, height attained by the body is one fourth the maximum range in the projectile motion.

Ans : We know that the horizontal range

$$Ans: R = \frac{u^2 \ \sin 2\theta}{g} \Rightarrow R_{max} = \frac{u^2}{g} \ when \ \theta = 45^0$$

$$Height \ H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow H_{max} = \frac{u^2}{4g} = \frac{R_{max}}{4} \qquad when \ \theta = 45^0$$

Q 033. State the parallelogram law of vector addition along with formula. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at a distance of 60 m in 5 sec. His velocity in still water should be



Ans. The addition of two vector $\vec{A}$ and $\vec{B}$ is resultant $\vec{R}$ then $\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$

And according to parallelogram law $R = \sqrt{A^2 + B^2 + 2ABcos\theta}$ and $\tan\beta = \frac{B \ sin\theta}{A + Bcos\theta}$

And β is the angle which $\vec{R}$ makes with the direction of $\vec{A}$.

If a person crosses a river while it's flowing at a certain velocity and reaches a point directly across the river in a given amount of time,

then β =90^0 => $A + Bcos\theta = 0$ => $cos\theta = -\frac{A}{B} = -\frac{v_{river}}{v_{man}} = -\frac{5}{v_{man}}$

$$here \ R = \ v_{net} = \frac{d}{t} = \frac{60}{5} = 12 \ m/s \ and \ A = v_{man} \ and \ B = \ v_{river} = 5m/s$$

using the above formula:

$$v_{net} = \sqrt{(v_{man})^2 + (v_{river})^2 + 2v_{man}v_{river}cos\theta}$$

$$12 = \sqrt{(v_{man})^2 + (5)^2 + 2v_{man}x5x\left(-\frac{5}{v_{man}}\right)} => 144 = (v_{man})^2 + 25 - 50$$

$$(v_{man})^2 = 144 + 25 = 169 => \ v_{man} = 13 \ m/s$$

Q 034. Write vector the relation between linear velocity and angular velocity.

Ans : The expression is

$$\vec{v} = \vec{\omega} \text{ x } \vec{r}$$

Q 035. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane.

Ans: see in topic rectangular components of a vector

Short answer questions (3 marks)

Q 036. A car travels along a straight road with an initial velocity of 20 m/s. It accelerates uniformly at 2 m/s² for 10 seconds. Calculate:

(a) The final velocity of the car.

(b) The total distance covered by the car during this time.

Ans:

(a) Using the equation of motion v=u+at

$$v = u + at => v = 20 + 2X10 = 40m/s$$

(b) Using the equation $s = ut + \frac{1}{2}at^2$

$=> s = 20x10 + \frac{1}{2}x2x10x10 = 200 + 100 = 300m$

Q 037. A body is projected at an angle Θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles Θ_1 and Θ_2 projections for the same horizontal range. Such that $(\theta_1 + \theta_2) = 90^{0}$.

$$Ans: R = \frac{u^2 \sin 2\theta_1}{g} = \frac{u^2 \sin 2\theta_2}{g} \Rightarrow \sin 2\theta_1 = \sin 2\theta_2$$

$$\Rightarrow either\ 2\theta_1 = 2\theta_2\ or\ 2\theta_1 = 180^o - 2\theta_2 because\ \sin 2\theta_2 = \sin(180^o - 2\theta_2)$$

$$\Rightarrow 2\theta_1 + 2\theta_2 = 180^o \Rightarrow \theta_1 + \theta_2 = 90^o$$

Q 038. Is it possible that there are two values of time for which a projectile is at the same height? If yes, show that the sum of these two times is equal to the time of flight.

$Ans: At\ any\ time\ t\ height\ of\ projectile\ is\ given\ by \qquad h = usin\theta\ t + \frac{1}{2}gt^2$

Since the equation is quadratic therefore it is possible that there are two values of time for which a projectile is at the same height. Using quadratic formula,

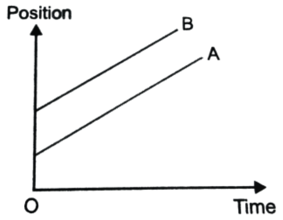
$$t = \frac{-usin\theta \pm \sqrt{(usin\theta)^2 - 4x\frac{1}{2}gx(-h)}}{2x\frac{1}{2}g} => t_1 = \frac{-usin\theta + \sqrt{(usin\theta)^2 + 2gh}}{g}$$

$$and\ t_2 = \frac{-usin\theta - \sqrt{(usin\theta)^2 + 2gh}}{g}$$

$$=> t_1 + t_2 = \frac{2usin\theta}{g} = time\ of\ flight$$

Q 039. Draw position –time graphs of two objects , A and B moving along straight line, when their relative velocity is zero.

Ans:-



Q 040. A plane is flying north with a velocity of 250 km/h, while a wind is blowing from the west at 50 km/h. Determine the resultant velocity of the plane with respect to the ground in terms of magnitude and direction.

Ans: Given: Represent velocities as vectors:
Velocity of the plane (northward): $\vec{v}_{plane} = 250km\ h^{-1}\ north = +250km\ h^{-1}$
Velocity of the wind (westward): $\vec{v}_{wind} = 50km\ h^{-1}\ west = -50km\ h^{-1}$
(since northward and eastward is taken as positive direction)
Vector addition to find resultant velocity:

$$\vec{v}_{resultant} = \vec{v}_{plane} + \vec{v}_{wind}$$

Magnitude and direction of the resultant velocity: = $|\vec{v}_{resultant}|$

$$|\vec{v}_{resultant}| = \sqrt{|\vec{v}_{plane}|^2 + |\vec{v}_{wind}|^2} \quad as\ angle\ between\ \vec{v}_{plane}\ and\ \vec{v}_{wind} is\ 90^o$$

$$|\vec{v}_{resultant}| = \sqrt{(+250)^2 + (-50)^2} = \sqrt{62500 + 2500} = 255.1km\ h^{-1}$$

Direction: The direction of the resultant velocity can be found using $\tan\beta = \frac{B\ sin\theta}{A+Bcos\theta}$

$$\tan\beta = \frac{\vec{v}_{plane}\ sin90^o}{\vec{v}_{wind} + \vec{v}_{plane}cos90^o} = \frac{+250}{-50} = -5 => \beta = \tan^{-1}(-5) = -78.69^o$$

Since the direction β is negative, it indicates westward direction relative to north (or 101.31^o east of north).

Conclusion:

The resultant velocity of the plane with respect to the ground is approximately 255.1 km/h at an angle of about 101.31^o east of north. This means the plane is moving northeastward relative to the ground due to the combined effect of its own velocity and the wind.

Q 041. A stone of mass 0.2 kg is tied to a string of length 1 meter and swung in a horizontal circle at a constant speed of 4 m/s.
(a) Calculate the tension in the string.
(b) Determine the magnitude of the stone's centripetal acceleration.

Ans:

Given: m is the mass of the stone, m=0.2 kg
v is the speed of the stone, v=4 m/s
r is the radius of the circular path, r=1 m (length of the string)

(a) Calculation of tension in the string:

To find the tension T in the string, we consider the forces acting on the stone in circular motion. The tension provides the centripetal force necessary to keep the stone moving in a circle.

In uniform circular motion, the centripetal force $F_c = \frac{mv^2}{r}$

$$F_c = \frac{mv^2}{r} = \frac{0.2X4X4}{1} = 3.2N$$

Therefore, the tension in the string T = the centripetal force Fc=3.2 N
(b) Calculation of centripetal acceleration:

Centripetal acceleration $a_c = \frac{v^2}{r} = \frac{4X4}{1} = 16ms^{-2}$
Therefore, the magnitude of the stone's centripetal acceleration is $16ms^{-2}$.

Q 042. When the angle between two vectors of equal magnitudes is 2π/3, prove that the magnitude of the resultant is equal to either.

Ans: According to parallelogram law $R = \sqrt{A^2 + B^2 + 2ABcos\theta}$
and $\tan\beta = \frac{B\,sin\theta}{A+Bcos\theta}$ Here β is the angle which $\vec{R}$ makes with the direction of $\overrightarrow{A}$.

$$here\ \theta = \frac{2\pi}{3}\ and\ A = B\ then\ R = \sqrt{A^2 + A^2 + 2\,A^2\cos\frac{2\pi}{3}}\ and\ \cos\frac{2\pi}{3} = -\frac{1}{2}$$

$$=> R = \sqrt{A^2 + A^2 - A^2} = A$$

Q 043. A ball thrown vertically upwards with a speed of 19.6 m/s from the top of a tower returns to the earth in 6s. find the height of the tower. (g = 9.8 m/sec^2)

$$Ans: At\ any\ time\ t\ displacement\ of\ projectile\ is\ given\ by \qquad h = ut + \frac{1}{2}gt^2$$

$$Here, t = 6s, \qquad h = -H, u = +19.6ms^{-1}, and\ g = -9.8ms^{-2}$$

$$so, -H = 19.6X6 + \frac{1}{2}X(-9.8)X6^2 = 117.6 - 176.4 = -58.8m => H = 58.8m$$

Q 044. Find the value of λ so that the vector $\vec{\boldsymbol{A}} = 2\hat{\imath} + \lambda\,\hat{\jmath} + \hat{k}$ and $\vec{\boldsymbol{B}} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are perpendicular to each.

$Ans: If\ two\ vectors\ are\ perpendicular\ then\ \vec{\boldsymbol{A}}.\vec{\boldsymbol{B}} = 0$

$=> \ (2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}).(4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}) = 0 => 4 - 2\lambda - 2 = 0 => \lambda = 1$

Q 045. Show that a given gun will shoot three times as high when elevated at angle of 60^0 as when fired at angle of 30^0 but will carry the same distance on a horizontal plane.

$$Ans:\ height\ H = \frac{u^2\sin^2\theta}{2g} \Rightarrow H_{60} = \frac{3u^2}{8g}\ and\ H_{30} = \frac{u^2}{8g} \Rightarrow H_{60} = 3H_{30}$$

$$R = \frac{u^2\ \sin 2\theta}{g} \Rightarrow R_{60} = \frac{\sqrt{3}u^2}{2g}\ and\ R_{30} = \frac{\sqrt{3}u^2}{2g}\ because\ \sin(2x60) = \sin(2x30)$$

<u>Long answer question (5 marks)</u>

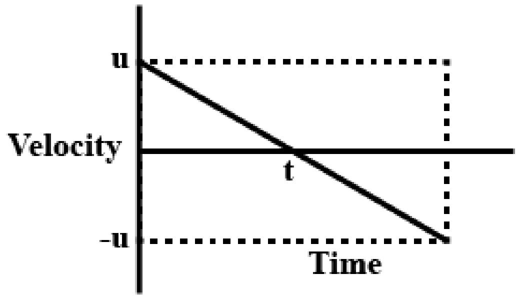
Q 046. Draw velocity- time graph of

(i) motion of an object thrown vertically upward.

(ii) projectile motion.

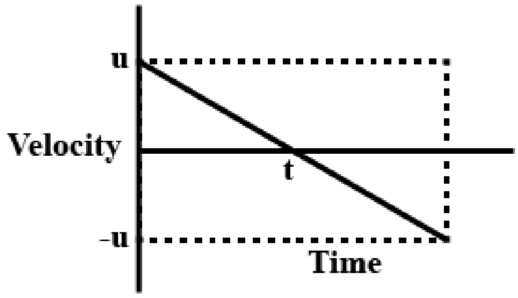
(iii) motion of an object thrown horizontally from height h.

Ans: (i) motion of an object thrown vertically upward.



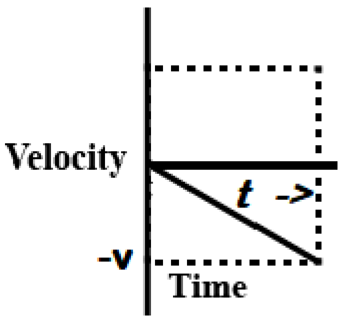
(ii) projectile motion.

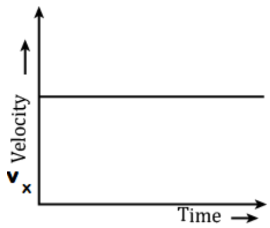
(a) for vertical component of velocity (b) for horizontal component of velocity



(iii) motion of an object thrown horizontally from height h.

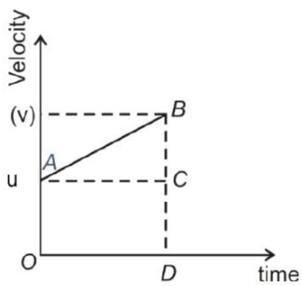
(a) for vertical component of velocity (b) for horizontal component of velocity





Q 047. (a) With the help of a simple case of an object moving along straight line with a constant acceleration show that the area under velocity – time curve represents distance covered over a given time interval.

Ans:



The area under velocity – time curve = area OABCDO=

$$areaOABCDO = \frac{1}{2}(OA + BD)OD = \frac{1}{2}(u + v)t$$

$$areaOABCDO = \frac{1}{2}(u + u + at)t$$

$areaOABCDO = \ ut + \frac{1}{2}at^2 = s = distance\ covered$

(b) A car moving with a speed of 126 km/h is brought to a stop within a distance of 200m. calculate the retardation of the car and the time required to stop it.

$Ans: u = 126kmh^{-1} = \frac{126x5}{18}ms^{-1} = 35ms^{-1}\,; v = 0ms^{-1}\ and\ s = 200m$

For the retardation using 3rd equation of motion

$v^2 = u^2 + 2as => \ 0^2 = 35^2 + 2a(200) => a = -\frac{1225}{400} = -3.0625ms^{-2}$

For the time using 1st equation of motion

$$v = u + at => 0 = 35 + \left(-\frac{1225}{400}\right)t => t = \frac{35(400)}{1225} = 11.43\ s$$

Q 048. Establish the following vector inequalities :

$| \vec{a} + \vec{b} | \leq |\vec{a}| + |\vec{b}|$

$| \vec{a} - \vec{b} | \leq |\vec{a}| + |\vec{b}|$

When does the equality sign apply.

Q 049. A football is kicked with an initial velocity of 28 m/s at an angle of 30 degrees above the horizontal. Calculate:

(a) The maximum height reached by the football.

(b) The total time taken for the football to return to the ground.

Ans:

Analyze the projectile motion of the football.

Given:

- Initial velocity u = 28 m/s

- Angle θ = 30 degrees above the horizontal

- Acceleration due to gravity g= 9.8 m/s² (assuming downward direction)

The maximum height reached by the football

Using the formula for projectile motion:

$$height\ H = \frac{u^2 \sin^2\theta}{2g} = \frac{(20)^2 \sin^2 30}{2X9.8} = 20m$$

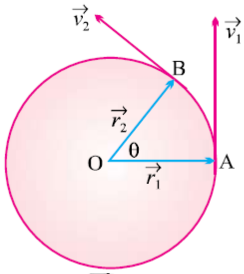
So, the maximum height reached by the football is approximately 20 meters.

The total time taken for the football to return to the ground

$$T = \frac{2u \sin\theta}{g} = \frac{2x28x \sin 30}{9.8} = \frac{2x28x1}{2X9.8} = 2.857s = 2.9s$$

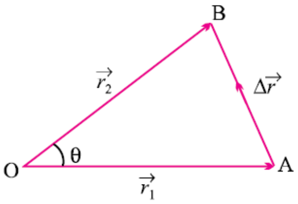
Therefore, the total time taken for the football to return to the ground is approximately 2.9 seconds.

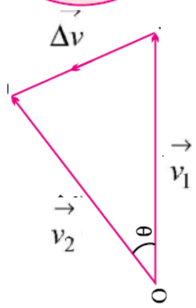
Q 050. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r. explain how it acts along the radius towards the centre of the circular path.



Ans: Centripetal Acceleration

(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.
(2) Magnitude of centripetal acceleration





$$|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{1}{\Delta t}\frac{v|\Delta\vec{r}|}{r} = \frac{v}{r}\frac{|\Delta\vec{r}|}{\Delta t}$$

$$|\vec{a}| = \frac{v^2}{r}$$

$$\left(\therefore v = \frac{|\Delta\vec{r}|}{\Delta t}\right)$$

Competency Based Questions

Q 051. $\vec{A}$ and $\vec{B}$ are two vectors and Θ is the angle between them, If $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A}.\vec{B})$, calculate the value of angle Θ .

Ans : 60^0

$$|\vec{A}X\vec{B}| = AB\sin\theta = \sqrt{3}\vec{A}.\vec{B} = \sqrt{3}AB\cos\theta => \tan\theta = \sqrt{3} => \theta = 60^0$$

Q 052. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity the river.

Ans : 6 km/h.

$$here\ v_{net} = 10\ kmh^{-1} and\ v_{boat} = 8\ kmh^{-1}\ and\ v_{river} = ?$$

If a boat is sent across a river then $\theta = 90^0$

using the parallelogram formula:

$$v_{net} = \sqrt{(v_{boat})^2 + (v_{river})^2 + 2v_{boat}v_{river}cos90^0}$$
$$10 = \sqrt{(v_{river})^2 + (8)^2} => 100 = (v_{river})^2 + 64$$
$$(v_{river})^2 = 100 - 64 = 36 => v_{river} = 6\ kmh^{-1}$$

Q 053. A cricket ball is hit at 45^0 to the horizontal with a kinetic energy E. Calculate the kinetic energy at the highest point.

Ans : E/2.(because the horizontal component uCos45^0 is present on highest point.)

Q 054. Speeds of the two identical cars are u and 4u at a specific instant. The ratio of the respective distances at which the two cars stopped from that instant with same acceleration.

Ans : 1 : 16

$$v^2 = u^2 + 2as => 0^2 = u^2 + 2as_1 \; and \; 0^2 = (4u)^2 + 2as_2$$

$$\frac{s_1}{s_2} = \frac{u^2/2a}{(4u)^2/2a} = \frac{1}{16}$$

Q 055. A projectile can have the same range R for two angles of projection. If t_1 and t_2 be the time of flight in the two cases, then prove that $t_1 t_2 = \frac{2R}{g}$

Ans : for equal range the particle should either be projected at an angle Θ or at an angle (90 - Θ) ,

$$Then\; t_1 = \frac{2u\sin\theta}{g} \quad and\; t_2 = \frac{2u\sin(90^0 - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$so, t_1 t_2 = \frac{2u\sin\theta}{g} \text{ x } \frac{2u\cos\theta}{g} = \frac{2u^2\sin 2\theta}{g^2} = \frac{2}{g} \text{x} \frac{u^2\sin 2\theta}{g} = \frac{2R}{g}$$

Multiple Choice Questions

Q 056. If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are respectively 5, 4 and 3 respectively, then the angle between $\vec{A}$ and $\vec{C}$ is

A) $\sin^{-1}\left(\frac{3}{5}\right)$ B) $\cos^{-1}\left(\frac{3}{5}\right)$ C) $\cos^{-1}\left(\frac{4}{5}\right)$ D) $\sin^{-1}\left(\frac{4}{3}\right)$

Ans. Option (B) is correct

Since the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are respectively 5, 4 and 3 respectively, therefore angle between $\vec{B}$ and $\vec{C}$ is 90^O $because$ $5^2 = 4^2 + 3^2$

And according to parallelogram law,

$$\tan\beta = \frac{B\,sin90}{C + Bcos90} = \frac{4}{3} => \sin\beta = \frac{4}{5} \; and \; \cos\beta = \frac{3}{5}$$

Q 057. If $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{B} = \hat{\imath} + \hat{\jmath}$ find the magnitude of component of $\vec{A}$ along $\vec{B}$ is

(A) $\frac{5}{\sqrt{2}}$ (B) $\frac{3}{\sqrt{2}}$ C) $\frac{1}{\sqrt{2}}$ D) $\sqrt{2}$

Ans. Option (A) is correct.

Q 058. A particle is projected making an angle 60° with the horizon, with K amount of kinetic energy. The kinetic energy of the particle at the highest position is

(A) 0 (B)$\frac{K}{2}$ (C) $\frac{K}{4}$ (D) K

Ans. Option (C) is correct.

$$Initial\; KE = \frac{1}{2}mu^2 = K; KE\; at\; top = \frac{1}{2}m(u\cos 60°)^2 = \frac{K}{4}$$

Q 059. Which quantity of a projectile remains unchanged?

(A) Momentum
(B) Kinetic energy
(C) Vertical component of velocity
(D) Horizontal component of velocity
Ans. Option (D) is correct.
Explanation: There is no acceleration in horizontal direction. Hence, the horizontal

component of velocity remains unchanged.

Q 060. Which one of the following is the correct expression of centripetal acceleration? Here ν is frequency and R is radius of circular path.

(A) $4\pi^2\nu^2R$ (B) $\frac{4\pi^2\nu^2}{R}$ (C) $2\pi\nu R$ (D) $\frac{2\pi\nu}{R}$

Ans. Option (A) is correct. Explanation: ac = ω^2R and Putting $\omega = 2\pi R$

Q 061. Three nonzero vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ satisfy the relation $\vec{P}.\vec{Q} = 0$ and $\vec{P}.\vec{R} = 0$. The vector $\vec{P}$ is parallel to

(A) $\vec{Q}$ (B) $\vec{R}$ (C) $\vec{Q} \times \vec{R}$ (D) $\vec{Q}.\vec{R}$

Ans. Option (C) is correct. Explanation: $\vec{P}$ *is perpendicular to* $\vec{Q}$ *and* $\vec{P}$ *is perpendicular to* $\vec{Q}$ *so* $\vec{P}$ *is parellel to* $\vec{Q} \times \vec{R}$.

Q 062. Two bullets are fired simultaneously horizontally and with different speeds from the same place. Which bullet will hit the ground first?

(A) The slower one
(B) The faster one
(C) Both will reach simultaneously
(D) Depends on the masses

Q 063. A boat which has the speed of $5\ kmh^{-1}$ in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water is

(A) $3\ kmh^{-1}$
(B) $4\ kmh^{-1}$
(C) $\sqrt{41}\ kmh^{-1}$
(D) $1\ kmh^{-1}$

Q 064. Forces F_1 and F_2 act on a point mass in two mutually perpendicular directions. The resultant force on the point mass will be

(a) $F_1 + F_2$ (b) $F_1 - F_2$ (c) $\sqrt{{F_1}^2 + {F_2}^2}$ (d) ${F_1}^2 + {F_2}^2$

Q 065. Which of the following remains constant for a projectile fired from the Earth?

(A) kinetic energy
(B) momentum
(C) horizontal component of velocity
(D) vertical component of velocity

Q 066. Which of the following is the largest, when the height attained by the projectile is the greatest?

(A) Horizontal range
(B) Time of flight (T)
(C) Angle of projectile with horizontal direction
(D) none of these

Q 067. The following forces are acting on a particle

(i) (2i + 3j – 2k) N
(ii) (3i + j – 3k) N
(iii) (-5i – 2j + k) N

the particle will move in

(A) x-y plane
(B) x-z plane
(C) y-z plane
(D) along x-axis

Q 068. If $\vec{A}.\vec{B} = AB$ then ………… is the angle between $\vec{A}$ and $\vec{B}$.

(A) 0°
(B) 30°
(C) 60°
(D) 90°

Q 069. If $\vec{A} X \vec{B} = AB$, then ……….. is the angle between $\vec{A}$ and $\vec{B}$.

(A) 0°
(B) 30°
(C) 60°
(D) 90°

Q 070. When a ball is projected upwards, then its acceleration at the highest point is equal to ………

(A) Zero
(B) Infinity
(C) Acceleration due to gravity
(D) One

Q 071. Angle between velocity and acceleration vectors in the following cases are given below. Match the correct pairs.

List I	List II
a) Vertically projected body	e) 900
b) For freely dropped body	f) changes from point to point
c) For projectile	g) zero
d) In uniform circular motion	h) 1800

(A) a → h, b → g, c → f, d → e.
(B) a → f, b → g, c → h, d → e.
(C) a → e, b → f, c → h, d → g.
(D) a → g, b → h, c → e, d → f

Q 072. A body is projected horizontally from the top of a cliff with a velocity of 9.8m/s. What time elapses before horizontal and vertical velocities become equal? Take g = 9.8m/s^2.

(A) 9.8s (B) 0s (C) 10s (D) 1s

Q 073. A body travels along the circumference of a circle of radius 2 m with a linear velocity of 6 m/s. Then its angular velocity is

(A) 6 rad /s
(B) 3 rad /s
(C) 2 rad / s
(D) 4 rad / s

Q 074. One radian is equal to?

(A) 57.7°
(B) 53.7°
(C) 59.3°
(D) 57.3°

Q 075. If a body A of mass M is thrown with velocity v at angle of 30° to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60° to the horizontal, the ratio of the horizontal ranges of A and B will be

(A) 1: 3
(B) 1: 1
(C) √3: 1
(D) 1 : √3

Q 076. A particle has initial velocity (3i + 4j) and has acceleration (0.4i + 0.3j). Its speed after 10s is

(A) 7unit
(B) 7 √2 unit
(C) 7 / √2 unit
(D) √2 unit

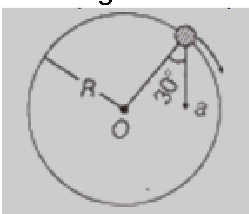
Q 077. The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is:

(A) 60°
(B) 15°
(C) 30°
(D) 45°

Q 078. A missile is fired for maximum range with an initial velocity of 20 m/s. If g=10 m/s^2, the range of the missile is:

(A) 50 m
(B) 60 m
(C) 20 m
(D) 40 m

Q 079. In the given figure, a = 15m/s^2 represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is:

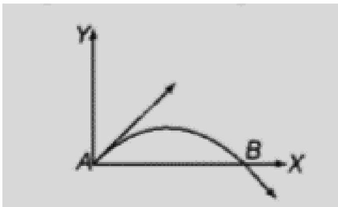


(A) 4.5 m/s
(B) 5.0 m/s
(C) 5.7 m/s
(D) 6.2 m/s

Q 080. A particle is moving such that its position coordinates (x, y) are (2m, 3m) at time t = 0, (6m, 7m) at time t = 2s and (13m, 14m) at time t = 5s. Average velocity vector (v) from t = 0 to t = 5s is

(A) (13i + 14j)/5
(B) (7i + 7j)/3
(C) 2(i + j)
(D) (11 i + 11 j)/5

Q 081. The velocity of a projectile at the initial point A is (2i + 3j) m/s. Its velocity (in m/s) at point B is:



(A) -2i+3j
(B) -2i-3j
(C) 2i-3j
(D) 2i+3j

Q 082. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is:

(A) $\theta = \tan^{-1}\frac{1}{4}$
(B) $\theta = \tan^{-1} 4$
(C) $\theta = \tan^{-1} 4$
(D) $\theta = 45^0$

Q 083. The x and y coordinates of the particle at any time are x=5t-2t2 and y=10t respectively, where x and y are in the meters and t is in seconds. The acceleration of the particle at t=2s is :

(A) 0
(B) 5 m/s2
(C) -4 m/s2
(D) -8 m/s2

Q 084. A body of mass 0.2 kg is rotated along a circle of radius 0.5 m in the horizontal plane with uniform speed 3 m/s. The centripetal force acting on that body is:

(A) 1.8 N
(B) 3.6 N
(C) 18 N
(D) 36 N

Q 085. Two equal forces (P each) act at a point inclined to each other at an angle of 120°. The magnitude of their resultant is

(A) P/2 (B) P/4 (C) P (D) 2P

Q 086. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be

(A) 60 m (B) 71 m (C) 100 m (D) 141 m

Q 087. It is found that $\vec{A} + \vec{B} = \vec{A}$. This necessarily implies,

(A) $\vec{B}$ = 0
(B) $\vec{A}$, $\vec{B}$ are antiparallel
(C) $\vec{A}$, $\vec{B}$ are perpendicular
(D) $\vec{A} - \vec{B} < 0$

Q 088. The angle between $\vec{P} = \hat{\imath} + \hat{\jmath}$ and $\vec{Q} = \hat{\imath} - \hat{\jmath}$ is

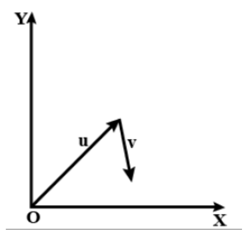
(A) 45°
(B) 90°
(C) –45°
(D) 180°

Q 089. Which one of the following statements is true?

(A) A scalar quantity is the one that is conserved in a process.
(B) A scalar quantity is the one that can never take negative values.
(C) A scalar quantity is the one that does not vary from one point to another in space.
(D) A scalar quantity has the same value for observers with different orientations of the axes.

Q 090. Figure shows the orientation of two vectors u and v in the XY plane.

If $\vec{u} = a\hat{\imath} + b\hat{\jmath}$ and $\vec{v} = p\hat{\imath} + q\hat{\jmath}$
which of the following is correct?



(A) a and p are positive while b and q are negative.
(B) a, p and b are positive while q is negative.
(C) a, q and b are positive while p is negative.
(D) a, b, p and q are all positive

Q 091. Two particles are projected in air with speed v_0 at angles θ1 and θ2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then

(A) angle of projection : θ1 > θ2
(B) angle of projection : θ1 = θ2
(C) time of flight : T1 < T2
(D) time of flight : T1 = T2

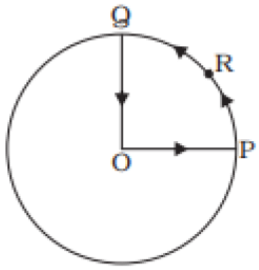
Q 092. For a particle performing uniform circular motion, choose the incorrect statement from the following.

(A) Magnitude of particle velocity (speed) remains constant.
(B) Particle velocity remains directed perpendicular to radius vector.
(C) Direction of acceleration keeps changing as particle moves.
(D) Magnitude of acceleration does not remain constant.

Q 093. For two vectors $\vec{P}$ and $\vec{Q}$, $|\vec{P}+\vec{Q}| = |\vec{P}-\vec{Q}|$ is always true when

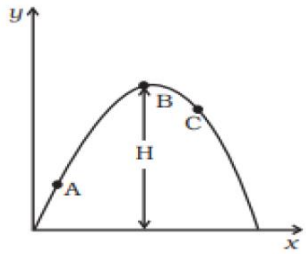
(A) when either $\vec{P}$ or $\vec{Q}$ is zero
(B) when $\vec{P} \perp \vec{Q}$
(C) when $\vec{P}$ and $\vec{Q}$ are parallel or anti parallel
(D) when both (A) and (B)

Q 094. A cyclist starts from centre O of a circular park of radius 1km and moves along the path OPRQO as shown Fig. If he maintains constant speed of $10 ms^{-1}$, what is his acceleration at point R?



(A) $10\ m/s^2$
(B) $0.1\ m/s^2$
(C) $0.01\ m/s^2$
(D) $1\ m/s^2$

Q 095. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Fig., where x and y indicate horizontal and vertical directions, respectively. What will be the direction of velocity and acceleration at point B.



(A) +x and +y
(B) +x and –y
(C) –x and +y
(D) –x and –y

Answers (Multiple Choice Questions):

Q.NO.	ANS	Q.NO.	ANS	Q.NO.	ANS	Q.NO.	ANS
56	B	66	B	76	B	86	C
57	A	67	C	77	A	87	A
58	C	68	A	78	D	88	B
59	D	69	D	79	C	89	D
60	A	70	C	80	D	90	B
61	C	71	A	81	C	91	A
62	C	72	D	82	B	92	D
63	A	73	B	83	C	93	D
64	C	74	D	84	B	94	B
65	C	75	B	85	A	95	B

Assertion & Reason Type Questions

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is true

Q 096. Assertion (A): When a 30 dyne force is inclined to y-axis at an angle 60°, the vertical and horizontal components of the force are 15 dyne and $15\sqrt{3}$ dyne respectively.

Reason (R): When a vector A is inclined to y-axis at an angle θ, the vertical and horizontal components of the vector are A cos θ and A sin θ respectively.

Ans. Option (A) is correct.

Q 097. Assertion (A): A projectile is projected at an angle θ and then at an angle (90° – θ), keeping the initial velocity same. In both the cases, the range will be same.

Reason (R): Range of a projectile = R =$\frac{u^2 \sin 2\theta}{g}$

Ans. Option (C) is correct.

Q 098. Assertion (A): For uniform circular motion, the displacement and acceleration are directed towards the centre along the radius.

Reason (R): Centripetal acceleration is represented as $\frac{\omega^2}{R}$

Ans. Option (D) is correct.

Q 099. ASSERTION: A physical quantity cannot be called as a vector if its magnitude is zero.

REASON: A vector has both magnitude and direction.

Ans. Option (D) is correct.

Q 0100. ASSERTION: The scalar product of two vectors can be zero.

REASON: If two vectors are perpendicular to each other, their scalar product will be zero.

Ans. Option (A) is correct.

Q 0101. ASSERTION: If dot product and cross product of $\vec{P}$ and $\vec{Q}$ are zero, it implies that one of the vector $\vec{P}$ and $\vec{Q}$ must be a null vector.

REASON: Null vector is a vector with zero magnitude.

Ans. Option (C) is correct.

Q 0102. ASSERTION: If $\vec{P}.\vec{Q} = \vec{P}.\vec{R}$.Then $\vec{P}$ may not always be equal to $\vec{R}$.

REASON: The dot product of two vectors involves cosine of the angle between the two vectors.

Ans. Option (A) is correct.

Q 0103. ASSERTION: The maximum horizontal range of projectile is proportional to square of its initial velocity.

REASON: The maximum horizontal range of projectile is equal to maximum height attained by projectile.

Ans. Option (C) is correct.

Q 0104. ASSERTION: Minimum number of non-equal vectors in a plane required to give zero resultant is three.

REASON: If $\overrightarrow{P} + \overrightarrow{Q} + \overrightarrow{R} = 0$, then they must lie in one plane.

Ans. Option (C) is correct.

Q 0105. ASSERTION: When range of a projectile is maximum, its angle of projection may be 45° or 135°.

REASON: Whether θ is 45° or 135° value of range remains the same, only the sign changes.

Ans. Option (A) is correct.

Q 0106. 8. ASSERTION: If there were no gravitational force, the path of the projected body always is a straight line.

REASON: Gravitational force makes the path of projected body always parabolic.

Ans. Option (C) is correct.

Q 0107. ASSERTION: A body of mass 1 kg is making 1 rps in a circle of radius 1 m. Centripetal acceleration of the body is $4\pi^2$ m/s^2.

REASON: Centripetal acceleration is given by a $=v^2/r$.

Ans. Option (A) is correct.

Q 0108. ASSERTION: Centripetal acceleration is always directed towards the centre.

REASON: In nonuniform circular motion acceleration is always directed towards the centre.

Ans. Option (C) is correct.

Case Study Question

Q 0109. Read the following text and answer any 4 of the following questions on the basis of the same:

Projectile motion

When the football travels through the air, it always follows a parabolic path because the movement of the ball in the vertical direction is influenced by the force of gravity. As the ball travels up, gravity slows it down until it stops for a moment at its peak height; the ball then comes down and gravity accelerates it until it hits the ground.

When a footballer kicks a ball he has to consider 3 factors:

- The velocity at which the ball leaves his feet
- The angle of kick
- The rotation of the ball

The rotation of the ball determines how the ball will slow down in flight. The velocity of the ball and the angle of the kick are the major factors those determine:

- How long the ball will remain in air (hangtime)
- How high the ball will go
- How far will the ball go

When the ball leaves the footballer's foot, it moves in two directions - horizontally and vertically. If the ball is kicked at a steep angle, then it will have more velocity in the

vertical direction than in the horizontal direction - the ball will go high, have a long hang-time, but travel a short distance. If the ball is kicked at a shallow angle, it will have more velocity in the horizontal direction than in the vertical direction - the ball will not go very high, will have a short hang-time, but will travel a long distance.
The footballer decides on the best angle in view of his field position.

Q. 1. A footballer kicks a ball with velocity 60 m/s at an angle a. For which of the following values of a, the hang-time will be highest?
(A) 15° (B) 30° (C) 45° (D) 60°
Ans. Option (D) is correct.
Explanation: Hang time = Time of flight = T =(2u sina)/g, u remaining constant, T increases as sin a increases. Hence, for a = 60°, T will be maximum.

Q. 2. Ball A is kicked at an angle 30° and ball B is kicked at an angle 60° with same velocity. Which one will have larger vertical component of velocity?
Ans. Ball B will have larger vertical component of velocity. Vertical component is u sin θ . u remaining constant, vertical component increases as sin θ increases. Hence, the ball B will have larger vertical component of velocity.

Q. 3. As the angle of the kick is increased (keeping velocity constant), the distance traveled by the ball increases to a maximum at 45° and then decreases. Why?
Ans. Horizontal range R = $\frac{u^2 \sin 2\theta}{g}$. Value of $\sin 2\theta$ is maximum when 2θ = 90°, i.e., at θ = 45°. Beyond that angle, the value of $\sin 2\theta$ decreases. For this reason the distance traveled by the ball increases to a maximum at 45° and then decreases.

Q. 4. A player can impart a maximum speed 40 m/s to a football. At which angle he should kick the ball to pass it on to another player of his team standing a distance 80m?
Ans. Range = R = $\frac{u^2 \sin 2\theta}{g}$ Or, 80 =(1600 sin 2 θ)/10 Or, 800 = 1600 sin 2 θ
Or, sin 2θ =1/2 Or, 2 θ = 30° or, θ = 15°

Q. 5. State a difference between projectile and a free fall.
Ans. Free fall starts moving down ward with no initial velocity whereas projectile starts with an initial upward or horizontal velocity.

Q 0110. Read the following text and answer any 4 of the following questions on the basis of the same:

Self-Assessment

Q 01. Forces F_1 and F_2 act on a point mass in two mutually perpendicular directions. The resultant force on the point mass will be
(a) $F_1 + F_2$ (b) $F_1 - F_2$ (c) $\sqrt{{F_1}^2 + {F_2}^2}$ (d) ${F_1}^2 + {F_2}^2$

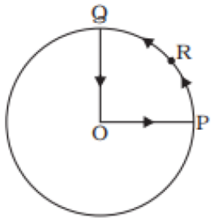
Q 02. Which of the following remains constant for a projectile fired from the Earth?
(A) kinetic energy

(B) momentum
(C) horizontal component of velocity
(D) vertical component of velocity

Q 03. For two vectors $\vec{P}$ and $\vec{Q}$, $|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$ is always true when

(A) when either $\vec{P}$ or $\vec{Q}$ is zero
(B) when $\vec{P} \perp \vec{Q}$
(C) when $\vec{P}$ and $\vec{Q}$ are parallel or anti parallel
(D) when both (A) and (B)

Q 04. A cyclist starts from centre O of a circular park of radius 1km and moves along the path OPRQO as shown Fig. If he maintains constant speed of $10ms^{-1}$, what is his acceleration at point R?



(A) 10 m/s^2
(B) 0.1 m/s^2
(C) 0.01 m/s^2
(D) 1 m/s^2

Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is true

Q 05. Assertion (A): For uniform circular motion, the displacement and acceleration are directed towards the centre along the radius.

Reason (R): Centripetal acceleration is represented as $\frac{v^2}{R}$

Q 06. ASSERTION: A physical quantity cannot be called as a vector if its magnitude is zero.

REASON: A vector has both magnitude and direction.

Q 07. ASSERTION: The scalar product of two vectors can be zero.

REASON: If two vectors are perpendicular to each other, their scalar product will be zero.

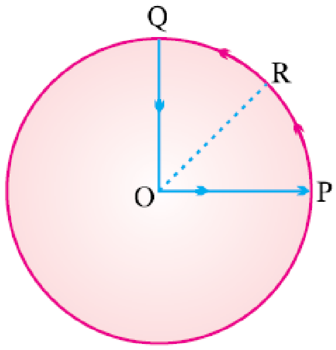
Q 08. What is the angle between velocity and acceleration at the peak point of the projectile motion ?

Q 09. The x and y coordinates of the particle at any time are $x=5t-2t^2$ and $y=10t$ respectively, where x and y are in the meters and t is in seconds. Calculate acceleration of the particle at t=2s.

Q 010. Find the value of λ so that the vector $\vec{\boldsymbol{A}} = 2\hat{\imath} + \lambda\,\hat{\jmath} + \hat{k}$ and $\vec{\boldsymbol{B}} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are perpendicular to each.

Q 011. A projectile is fired with Kinetic energy 1 kJ. If the range is maximum, what is its Kinetic energy, at the highest point?

Q 012. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path OPRQO as shown. If he maintains constant speed of 10 m/s, what is his acceleration at point R in magnitude & direction?



Q 013. A football is kicked with an initial velocity of 28 m/s at an angle of 30 degrees above the horizontal. Calculate:

(a) The maximum height reached by the football.

(b) The total time taken for the football to return to the ground.

Q 014. A boat is sent across a river with a velocity of 8km/h. if the resultant velocity of boat is 10 km/h , then calculate the velocity the river.

Answers Of Self-Assessment

Ans.01 Option (C) is correct. $Resultant\ force = \sqrt{F_1{}^2 + F_2{}^2}\ because\ angle = 0$

Ans.02 Option (C) is correct. horizontal component of velocity remains constant for a projectile.

Ans.03 Option (B) is correct. (B) when $\vec{P} \perp \vec{Q}$ then$|\vec{P} + \vec{Q}| = |\vec{P} - \vec{Q}|$.

Ans.04 Option (B) is correct. $Acceleration = \frac{v^2}{r} = \frac{10X10}{1000} = 0.1\ ms^{-2}$.

Ans.05 Option (D) is correct. For uniform circular motion, the displacement is directed along tangent to circular path and acceleration is directed towards the centre along the radius. Also centripetal acceleration is represented as $\frac{v^2}{R}$

Ans.06 Option (D) is correct. A physical quantity may be called as a vector even if its magnitude is zero.

Ans.07 Option (A) is correct.

Ans.08 90^0. Acceleration at the peak point remains downward and vertical component become zero so, at the peak point ne velocity is horizontal component of velocity.

Ans.09 Net acceleration is $5\ ms^{-2}$ because $v_x = 5 - 4t\ and\ v_y = 10 => a_x = 5\ and\ a_y = 0$

Ans.010 $Ans: If\ two\ vectors\ are\ perpendicular\ then\ \vec{\boldsymbol{A}}.\vec{\boldsymbol{B}} = 0$

$=> (2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}).(4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}) = 0 => 4 - 2\lambda - 2 = 0 => \lambda = 1$

Ans.011 Ans : here = 1 KJ = 1000 J, and for Max. range $\theta = 45°$

At the highest Point,

$$K.E. = \frac{1}{2}mu^2\cos^2\theta = 1000\text{ J x}\frac{1}{2} = 500\text{J}$$

Ans.012 $Ans : Centripetal\ acceleration = a = \frac{v^2}{r} = \frac{10^2}{1000} = 0.1\ ms^{-2}\ along\ RO$

Ans.013 Analyze the projectile motion of the football.

Initial velocity u = 28 m/s

Angle θ = 30 degrees above the horizontal

Acceleration due to gravity g= 9.8 m/s² (assuming downward direction)

The maximum height reached by the football

Using the formula for projectile motion:

$$height\ H = \frac{u^2\sin^2\theta}{2g} = \frac{(20)^2\sin^2 30}{2X9.8} = 20m$$

So, the maximum height reached by the football is approximately 20 meters.

The total time taken for the football to return to the ground

$$T = \frac{2u\sin\theta}{g} = \frac{2\text{x}28\text{x}\sin 30}{9.8} = \frac{2\text{x}28\text{x}1}{2\text{x}9.8} = 2.857s = 2.9s$$

Therefore, the total time taken for the football to return to the ground is approximately 2.9 seconds.

Ans.014 Ans : 6 km/h.

$$here\ \ v_{net} = 10\ kmh^{-1} and\ v_{boat} = 8\ kmh^{-1}\ and\ \ v_{river} = ?$$

If a boat is sent across a river then $\theta = 90^o$

using the parallelogram formula:

$$v_{net} = \sqrt{(v_{boat})^2 + (v_{river})^2 + 2v_{boat}v_{river}cos90^o}$$

$$10 = \sqrt{(v_{river})^2 + (8)^2} => 100 = (v_{river})^2 + 64$$

$$(v_{river})^2 = 100 - 64 = 36 => \ v_{river} = 6\ kmh^{-1}$$

Unit III: Laws of Motion

Chapter 5: Laws of Motion

GIST OF THE CHAPTER:

Force:

1. Force: it is the push or pull that brings the change in (i) change in magnitude of velocity (ii) the change in direction of motion (iii) the change in shape
2. Force is the cause of translatory motion. It is measured in Newton and has the dimension MLT^{-2}
3. The inherent property of anybody to resist any change in its state of motion is called inertia. The heavier the body, the inertia is and the lighter the body lesser the inertia.
4. Unbalanced external forces acting on a body or system can only bring a change in the state of motion.

OR

Force: It is an external agency that changes or tries to change the state of rest or motion of a body or the direction of motion of the body. It is a vector quantity. Its SI unit is Newton (N). Some important forces in nature are:

(a) Gravitational force
(b) Electromagnetic force
(c) Interatomic or intermolecular forces
(d) Nuclear force

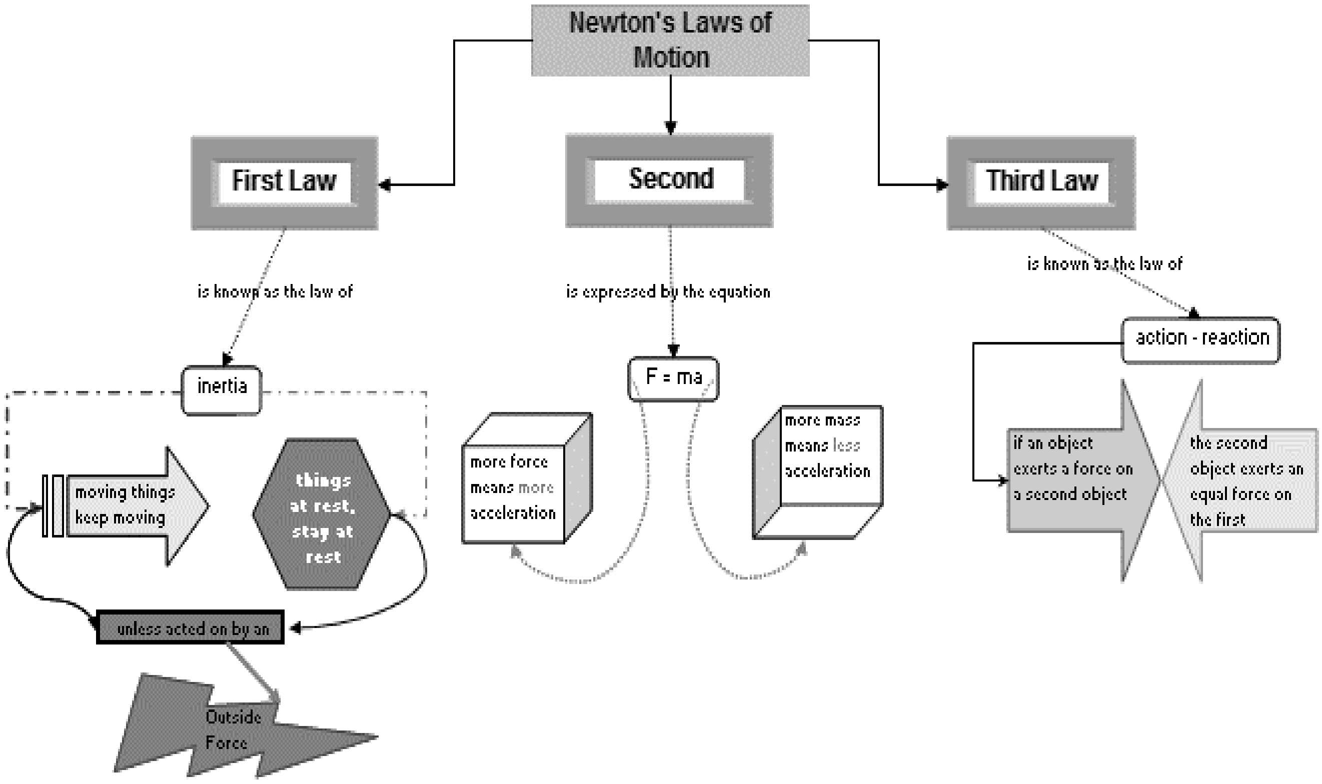
Inertia: It is the property of a body by which it continues to be in a state of rest or uniform motion along a straight path unless an external unbalanced force acts on the body. Inertia is of three types:

(a) Inertia of rest
(b) Inertia of motion
(c) Inertia of direction

Inertia to linear motion is measured by the mass of the body. Larger the mass; greater is the inertia of the body i.e. it is more difficult to change the state of rest or uniform motion of the body.
In the absence of friction between a passenger and the bus, the passenger will not move with the bus when it starts. As a result, he will hit the back of the bus (Inertia of rest).
Likewise, when the brakes are applied to the moving bus in the above situation, the passenger will hit the front of the bus (Inertia of motion).
If the bus takes a turn to the left, the passenger will the thrown towards the right (Inertia of direction). This is because of the directional inertia of the body of the passenger.

Linear Momentum: It is the quantity of motion present in a body. Mathematically, it is measured as the product of mass and velocity **v** of the body.
Momentum $\mathbf{p} = m\,\mathbf{v}$

It is a vector in the direction of velocity. Its SI unit is kg ms^{-1} or Ns. Dimensionally momentum is MLT^{-1}.



Newton's Laws of Motion:

First Law: A body at rest or in uniform motion remains in its state till an unbalanced external force acts on it.
Second Law: the rate of change of linear momentum of a body is directly proportional to the applied force and takes place in the direction in which force is acting. Thus F= k dp /dt = k ma

or

The total unbalanced external force acting on a mass it the product of its mass m and acceleration a

i.e. F= ma

Third law: for every action there is an equal and opposite reaction.

Momentum is the product of mass and velocity P= mv. Its is the outcome of force acting on body. It is measured in kg m/s and has the dimensions of MLT^{-1}

Principle Of Conservation Of Linear Momentum

It states that in the absence of any external applied force, total linear momentum of a system remains conserved.

Proof-

We know that,

$$\vec{F} = m\vec{a}$$

or, $F = mdv/dt$

or, $F = dp/dt$

if, $F = 0$

$dp/dt = 0$

or, **p = Constant** (differentiation of constant is zero)

or, $\mathbf{P_{initial} = P_{final}}$

Impulse: force acting for a short duration is called impulsive force. It is $I = \int F\,dt$ or change in momentum i.e. $I = P_2 - P_1$. The area under the F-t graph gives impulse.

When no external force acts on a body or system, momentum remains conserved. If Fe = 0, P= constant.

Action and Reaction never act on the same body.

When a bullet of mass m is fired with the velocity v, the gun of mass M will recoil with a velocity $V = -mv/M$

Apparent weight in a lift

(i) Stationary or lift in uniform motion Apparent Weight = Actual Weight

(ii) Lift accelerated up by a

Apparent weight = actual weight + Ma = M (g +a)

(iii) Lift accelerated down by a

Apparent weight = actual weight - Ma = M(g-a)

(iv) In free fall a=g

Apparent weight = actual weight - Mg = M(g-g) =0

(v) When a> g, while falling

Apparent weight is negative; therefore, the mass m is said to be in contact with the roof.

Concurrent Forces

The forces acting at the same point of a body are called concurrent forces.

Equilibrium of Concurrent Forces: For a number of concurrent forces acting on a body in equilibrium, the forces can be represented by a closed polygon taken in order or the resultant force is zero.

F1+ F2 + **F3+……….**+**Fn=**0

The Free Body Diagrams

A diagram for each body in a system indicating all the forces exerted on the body by the remaining parts of the system and the external agents is called free-body diagram.

In mechanics, we usually come across systems consisting of a number of bodies / particles under the
action of a number of forces due to their mutual interaction; gravitational forces, frictional force, force due to strings, springs (called tension); supports etc.

Solution of such problems can be simplified by using the following steps:

1. Draw a diagram showing various parts of the system with links, supports etc.

2. Select any convenient part of the system.

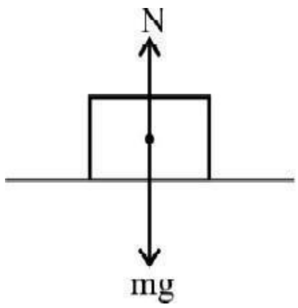
3. Draw a separate diagram for the part selected above showing all the forces on it by the remaining parts of the system and also by external agents. This diagram is called the free-body diagram.

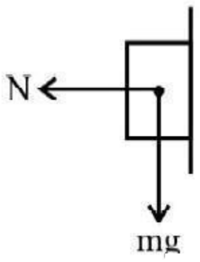
4. Remember to include the information about the magnitude and direction of forces which are given
`and which you are sure of. The remaining forces are treated as unknown. Use Newton's second law of motion to get an equation of motion for the chosen part i.e. equate the net force acting in a particular direction to the product of mass and acceleration of the body in that particular direction.

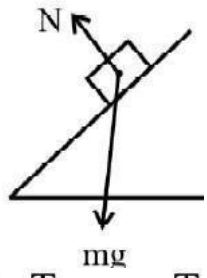
5. Follow the same procedure for any other part of the system to get more equations. You need as many independent equations to solve the problem as the number of unknowns.

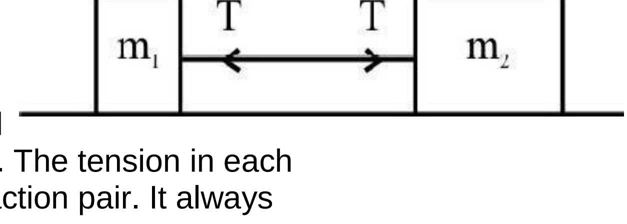
6. Solve the equations obtained for different parts of the system to obtain the values of the desired unknowns or to get a desired equation.

Following points must be kept in mind while drawing the free body diagram (FBD)
The reaction force (called normal reaction) always acts normally on the surface in contact or on which the body is placed. (See diagram below)







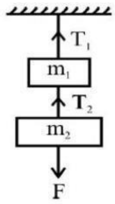


Tension in a string arises due to the restoring forces caused by intermolecular forces of interaction. It is the force exerted by one part of the string on the other part. The tension in each branch of the string must form action - reaction pair. It always acts towards the support (Hand / pulley).

In the above system, the tension on m1 is towards m2, and that on m2 is towards m1.

A pulley is used to change the direction of force to a more convenient direction. For a light (massless); frictionless pulley with on inextensible string passing over it without any kinks, the tension on either side of the string is the same.
In a FBD; take into account all the forces acting on the body.

(ii) Inter Connected Bodies suspended from a Rigid support

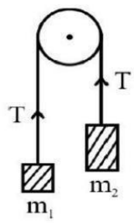


When two masses m_1 and m_2 are suspended vertically from a rigid, non-yielding support using strings as shown and a pulling force F is applied to pull the mass m_2 down, we have

$T_2 = F + m_2 g$

and $T_1 = F + (m_1 + m_2)\, g$

(iii) For two interconnected bodies with a rope passing over a smooth support or smooth light pulley;

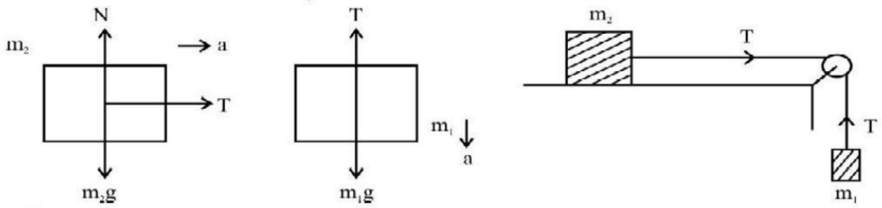


$$a = \frac{(m_2 - m_1)\, g}{(m_1 + m_2)}$$

$$T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Note that $a < g$ for the system.

(iv) For two masses m_1 and m_2 attached to the ends of a string passing over a smooth pulley with mass m_2 on a smooth horizontal surface, we have.



$$a = \frac{m_1 g}{m_1 + m_2} \quad (< g)$$

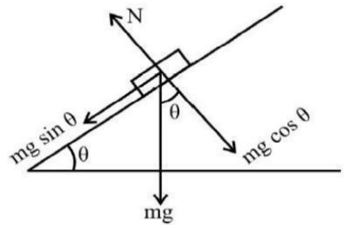
$$T = \frac{m_1 m_2}{m_1 + m_2}\, g$$

Equation of motion of m_1 in $m_2 g - T = m_1 a$

Equation of motion of m_2 is $T = m_2 a$

(v) For a body on a smooth inclined plane, acceleration down the plane is given by

$a = g \sin\theta$



Friction:

Whenever a body moves or tends to move over the surface of another body, a force comes into play to opposes their relative motion. This force is known as force of friction. It opposes motion and acts parallel to the surface of contact of bodies.

Frictional force may sometimes act in the direction of motion of the body. The following examples illustrate the situations where the force of friction 'acts' in the direction of motion of the object.

For a man walking due north, the frictional force also acts due north.

In a bicycle; the driving forces are connected to the rear wheel. The direction of frictional force on the rear wheel at point of contact with the ground is in the direction of motion whereas that on the front wheel is opposite to the direction of motion.

Static Friction:

The force of friction which comes into play between two bodies before one object actually begins to move over the other is called static friction (fs). Static friction is a self-adjusting force (both in magnitude as well as direction). It is always equal and opposite to the applied force as long as there is no relative motion.

`

Limiting Friction :

The maximum force of static friction which comes into play when a body just starts moving over the surface of another body is called limiting friction or the maximum force of static friction fs^{max}. The force of friction never exceeds fs^{max}.

Kinetic Friction:

The force of friction which comes into play when a body is in motion over the surface of another body is called kinetic or dynamic friction. It is denoted by f_k and is less than limiting friction.

Laws of Limiting Friction

- The force of limiting friction depends on the nature of the two surfaces in contact and their state of roughness.
- The force of limiting friction acts tangential to the surfaces in contact and in a direction opposite to that of the applied force.
- The force of limiting friction between any two surfaces is independent of the shape and the area of contact so long as the normal reaction remains unchanged.
- The force of limiting friction between two given surfaces is directly proportional to the normal reaction between the surfaces.

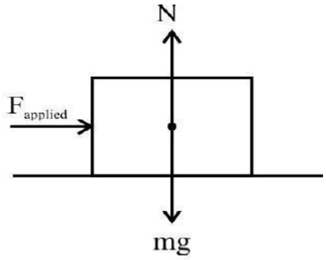
f is directly proportional to N or $f = \mu_s N$; where μs is coefficient of limiting friction. coefficient of limiting friction =Limiting Friction(f_s^{max})/Normal Reaction(N)

and coefficient of kinetic friction μk=fk/N

We have $\mu_k < \mu_s$ because fk is always less than f_s^{max}.

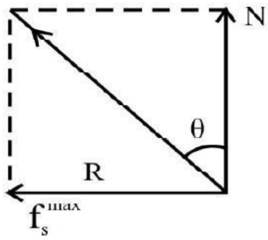
Angle of Friction

It is the angle which the resultant of the force of limiting friction and the normal reaction makes with the normal reaction.



We have $\tan\theta = \frac{f_s^{max}}{N}$

$= \mu_s$



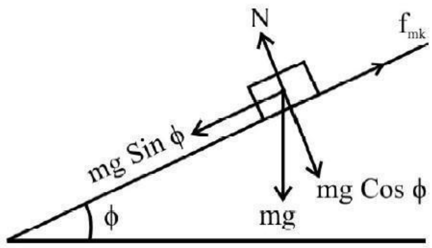
Angle of Repose

It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down the plane. Fig. shows forces acting on a body of mass m about to slide down the inclined plane.

We have, $f_{ms} = mg\sin\phi$

$N = mg\cos\phi$

$\therefore \frac{f_{ms}}{N} = \mu_s = \tan\phi$

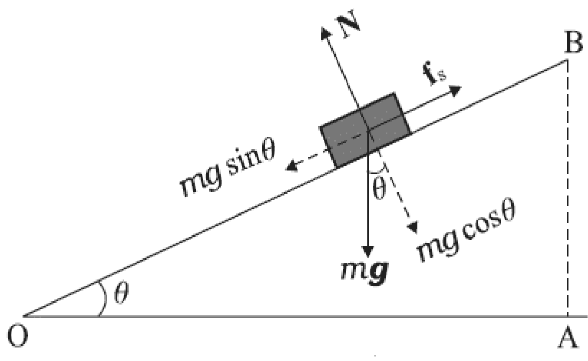


the frames of reference which are not accelerated and in which Newton's laws hold good are called inertial frames.
The frames of reference which are accelerated and in which Newton's laws do not hold good are called non-inertial frames.
The earth is not a perfect inertial frame. But we take it as inertial.
Friction is an opposing force acting tangentially on a body.
Static friction is a self-adjusting force. ($f_s \leq \mu_s N$). the maximum value is called limiting friction.

The friction experienced under motion is called kinetic friction ($f_k = \mu_k N$) and is less than the limiting friction.
Frictional force depends on (i) the nature of the surface (ii) normal reaction $f = \mu N$
Friction is independent of the area of contact.
The coefficient of static, kinetic, and rolling frictions μ_s, μ_k, and μ_r are related as $\mu_s > \mu_k > \mu_r$
The coefficient of static friction = tangent of the angle of friction, $\mu_s = \tan\theta$
The angle of friction is defined as the angle between the resultant of the normal

reaction and the frictional force with the normal reaction.

The maximum angle α to which an inclined plane be tilted horizontally so that any mass on its top cannot slide is called the angle of repose also μs= tan θ

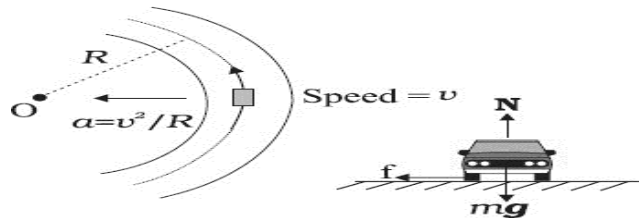


Polishing, lubricating and use of ball bearings reduce friction.

Acceleration of a body coming down an inclined plane θ which frictional coefficient μ is a = g (sin θ – μ cos θ)

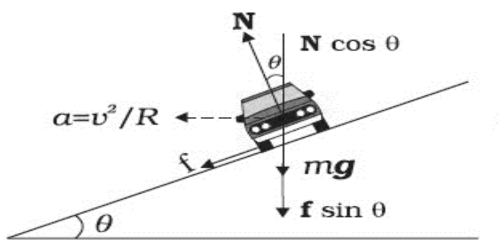
The minimum force required to take a mass m up an inclined plane θ with frictional coefficient u is F= mg(sin θ + μ cos θ)

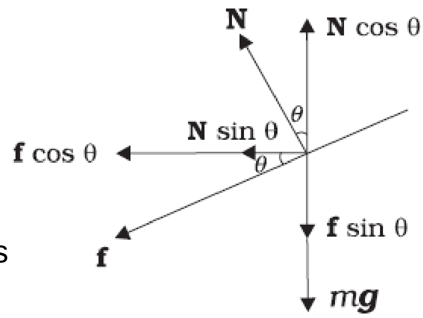
The maximum speed with which a vehicle can negotiate a curve is v max = √(μ r g) where μ is coefficient of friction and r is radius of path.



In a banked curved ɵ with friction, the Maximum velocity is given by

$$v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1-\mu \tan\theta)}}$$





Pseudo force: the force that comes into existence on anybody due to the acceleration of the frame in which it is placed is call pseudo force. Ex. Centrifugal force.

a. The accelerated frames of reference are called non inertial frames. In them Fe + Fp = ma, where Fe is external force and Fp is pseudo force.

Equilibrium of Concurrent Forces

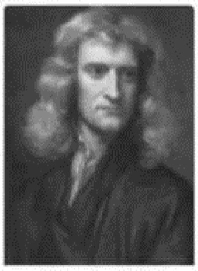
If the number of forces act at the same point, they are called concurrent forces. The condition or the given body to be in equilibrium under the number of forces acting on the body is that these forces should produce zero resultant.

The resultant of the concurrent forces acting on a body will be zero if they can be represented completely by the sides of a closed polygon taken in order.

BRAIN MAP

CLASS XI

LAWS OF MOTION



SIR ISSAC NEWTON (1643-1727)

LAWS OF MOTION AND THEIR CONSEQUENCES

Newton's 1st Law
A body continues its state of rest or motion until unless an external force is acted on it.

Inertia of Direction
The property due to which a body cannot change its direction of motion by itself.

Inertia of Motion
The tendency of a body to remain in a state of uniform motion in a straight line.

Inertia of Rest
The property of a body due to which it cannot change its state of rest by itself.

Newton's 2nd Law
The rate of change of linear momentum of a body is directly proportional to the external force applied on the body in the direction of force.

Introduces the Concept of Force

Force = mass × acceleration
$F = ma$, SI unit ≡ N

Newton's 3rd Law
To every action there is always an equal and opposite reaction
$\vec{F}_{AB} = -\vec{F}_{BA}$

Impulse
A large force acts for a very short duration of time producing a finite change in momentum.
$J = F \times t = \Delta p$

Conservation of Linear Momentum
Total momentum of an isolated system of interacting particles is conserved if there is no external force acting on it. $P_{\text{initial}} = P_{\text{final}}$

Friction
Resistance offered to the relative motion between the surface of the two bodies in contact.

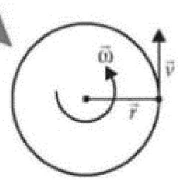
- Acceleration of a body sliding down a rough inclined plane, $a = g(\sin\theta - \mu\cos\theta)$
- Angle of friction, $\theta = \tan^{-1}(\mu_s)$.
- Angle of repose, $\alpha = \tan^{-1}(\mu_s)$.
- Numerically, $\alpha = \theta$

Types of Friction
(a) Static friction : acts when a body is at rest on application of a force, $f_s = F_{ext}$
(b) Limiting friction : acts when a body is just at the verge of movement, $f_l = \mu_s N$
(c) Kinetic friction : acts when a body is actually sliding, $f_l = \mu_s N$

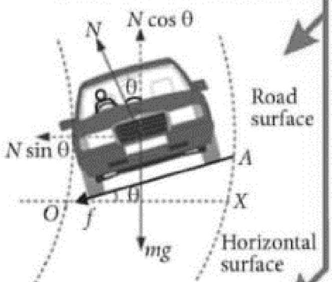
CIRCULAR MOTION IN HORIZONTAL PLANE

Circular Motion
A body moving in a circular path is called circular motion.
Centripetal force $F_c = \frac{mv^2}{r}$

Non-uniform circular motion
Acceleration, $\vec{a} = \vec{a}_T + \vec{a}_C = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$
$a = \sqrt{a_T^2 + a_C^T}$

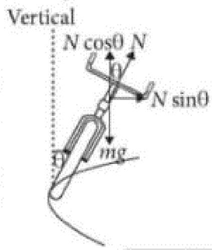


Motion of a Car on Banked Road
- Maximum velocity on banked road to avoid skidding
$v_{\max} = \sqrt{\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s \tan\theta)}}$
- Optimum speed on a smooth banked road
$v_0 = \sqrt{rg\tan\theta}$



Motion of a Car on Level Road
Maximum velocity on a curved road to avoid skidding
$v_{\max} = \sqrt{\mu_s rg}$

Bending of a Cyclist on a circular turn
Angle of bending
$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$



Problem Solving Techniques in Mechanics
- Make free body diagram of each element showing all external forces acting on it.
- Analyse the magnitude and direction of pseudo forces if there is any.
- For equilibrium of concurrent forces use
$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0$
$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$ (Lami's theorem)
- For horse cart type system
$a = \frac{F_x - f}{M_H + M_{cart}}$ F_x = horizontal component of reaction, f = frictional force

BRAIN MAP

CLASS XI

NEWTON'S LAWS OF MOTION

Problem Solving Strategies

- *Identify the unknown forces and accelerations.*
- *Draw FBD of bodies in the system.*
- *Resolve forces into their components.*
- *Apply* $\Sigma\vec{F} = M\vec{a}$ *in the direction of motion.*
- *Apply* $\Sigma\vec{F} = 0$ *in the direction of equilibrium.*
- *Write constraint relation if exists.*
- *Solve equations* $\Sigma\vec{F} = M\vec{a}$ and $\Sigma\vec{F} = 0$.

LAWS OF MOTION AND THEIR CONSEQUENCES

Newton's 1st Law

A body continues its state of rest or motion until unless an external force is acted on it.

- Inertia of rest
- Inertia of motion
- Inertia of direction

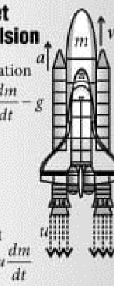
Pseudo Force

For non-inertial frame of reference

$\vec{F}_{ext} + \vec{F}_{Pseudo} = M\vec{a}$

$\vec{F}_{pseudo} = -M\vec{a}_{frame}$

Rocket Propulsion



Acceleration $a = \frac{u}{m}\frac{dm}{dt} - g$

Instantaneous velocity $v = u\log_e\left(\frac{m_0}{m_r}\right) - gt$

Burn out speed $v_{max} = u\log_e\left(\frac{m_0}{m_r}\right)$

Thrust $F = -u\frac{dm}{dt}$

Newton's 2nd Law

The rate of change of linear momentum of a body is directly proportional to the external force applied on the body in the direction of force.

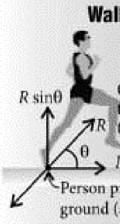
$F = \frac{dp}{dt} = ma$

Newton's 3rd Law

To every action there is always an equal and opposite reaction.

$\vec{F}_{AB} = -\vec{F}_{BA}$

Walking



Ground pushes the person (reaction)

Person presses the ground (action)

Horse Cart Type System

For horse cart type system

$a = \frac{F_x - f}{M_H + M_{cart}}$

F_x = horizontal component of reaction force

f = frictional force

FRICTION

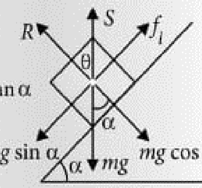
The motion resisted by a bonding between the body and the surface in contact represented by single force called

Angle of Friction (θ) and Angle of Repose (α)

$S = \sqrt{R^2 + f_l^2}$

$\tan\theta = \frac{f_l}{R} = \mu_s = \tan\alpha$

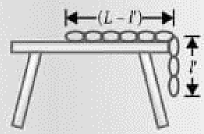
∴ Numerically, $\theta = \alpha$



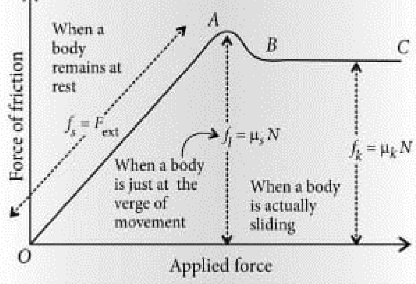
Maximum Length of Hanging Chain

Length of a chain hanging in air

$l' = \frac{\mu L}{1 + \mu}$

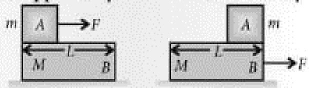


Types of Friction v/s Applied Force



Motion of Two Bodies One Resting on the Other

Force *F* applied to upper body	Force *F* applied to lower body



Force F applied to upper body

- **When there is no friction**
  - $a_A = F/m$; $a_B = 0$
  - *A* will fall from *B* after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Combined system will move together with $a = F/(M + m)$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = \frac{MF - \mu_k mg(m + M)}{mM}$
  - *A* will fall from *B* after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mML}{MF - \mu_k mg(m + M)}}$

Force F applied to lower body

- **When there is no friction**
  - $a_B = F/M$ and $a_A = 0$
  - *A* will fall from *B* (backward) after time *t*
  - $\therefore t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Both the bodies will move together
  - $a = \frac{F}{M + m}$ and $f_l = \mu_s mg$
  - Pseudo force on the body *A*, $F' = ma = \frac{mF}{m + M}$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = -\left[\frac{F - \mu_k g(m + M)}{M}\right]$
  - *A* will fall from *B* (backward) after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F - \mu_k g(m + M)}}$

Multiple Choice Questions:

Q 1 .Which of the following is known as law of inertia?

(A). Newton's first law of motion.

(B). Newton's Second law of motion.

(C). Newton's third law of motion.

(D). law of conservation of mass.

Q 2.Mohan jumps from his school bus while it is in running state, then Mohan fall in

(A). forward direction.

(B). backward direction.

(C). running in straight direction

(D). none of these.

Q 3.For a body moving with constant speed in a horizontal circle which of the following remains constant?

(A). velocity

(B). acceleration

(C). centripetal force

(D). kinetic energy

Q 4.A particle of mass 10 kg is moving in straight line, if its displacement x with time t is given by $x = (t^3 - 2t - 10)x = (t^3 - 2t - 10)$m, then the force acting on it at the end of 4 sec is

(A). 24 N

(B). 240 N

(C). 300N

(D). 1200 N

Q 5. A light string passing over a smooth light pulley connects two blocks of masses m and M vertically. If the acceleration of the system is g/8, then the ratio of masses is

(A). 8/1

(B). 9/7

(C). 4/3

(D). 5/3

Q 6.A body of mass m collides against a wall with velocity v and rebounds with same speed. Its change of momentum is

(A). 2 mv

(B). mv

(C). - mv

(D). zero

Q 7.The static force of friction

(A). remains the same with increase in the applied force.
(B). decrease with the increase in the applied force.
(C). increase with increase in the applied force.
(D). increase with the decrease in the applied force.

Q 8 A 100 *g* iron ball with having velocity 10 *m/s* collides with a wall at an angle 30° and rebounds with the same angle. If the period of contact between the ball and wall is 0.1 *second*, then the force experienced by the wall is

(a) 10 *N* (b) 100 *N*
(c) 1.0 *N* (d) 0.1 *N*

Q 9. A car moves at a speed of 20 m/s on a banked road and describes an arc of a circle of radius $40\sqrt{3}$ m. The angle of banking in degrees is ($g = 10\ m/s^2$)

(A). 25

(B). 60

(C). 30

(D). 45

Competency-Based Question:

Q 10.Sand is being dropped on a conveyor belt at the rate of M kg/s. The force necessary to keep the belt moving with a constant velocity v m/s will be

(A). Mv

(B). 2 Mv

(C). Mv/2

(D). zero

Q 11.A block is kept on a frictionless inclined plane with angle of inclination x. The plane is given an acceleration a to keep the block at rest. Then a is equal to

(A). g

(B). g tan x

(C). g/ tan x

(D). g/sin x

Q 12.A 60 *kg* body is pushed with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficient of static friction and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is

(a) $6 m/s^2$ (b) $4.9 m/s^2$

(c) $3.92 m/s^2$ (d) $1 m/s^2$

Q 13.A bullet of mass 0.05 kg moving with a speed of 80 m/s enters a wooden block and is stopped after a distance of 0.40 m. the average resistive force exerted by the block on the bullet is

(A). 300 N

(B). 20 N
(C). 400N
(D). 40 N

Q 14.An object of mass 5 kg is attached to the hook of a spring balance and the balance is suspended vertically from the roof of a lift. The reading on the spring balance, when the lift is going up with an acceleration of 0.25 ms^{-2} (g= $10m/s^2$)

(A). 51.25 N

(B). 48.75 N

(C). 52.75 N

(D). 47.25N

Q 15.Maximum value of static friction is called

(a)Limiting friction (b)Rolling friction
(c)Normal reaction (d)Coefficient of friction

Q 16.A bullet of mass 20 g has an initial speed of 1 m/s just before it starts penetrating a mud wall of thickness 20 cm. if the wall offers a mean resistance of 2.5 x 10^{-2} N, the speed of the bullet after emerging from the other side of wall is close to

(A). 0.4 m/s
(B). 0.1 m/s

(C). 0.3 m/s
(D). 0.7 m/s

Q 17.A block of mass 2 kg rests on a horizontal surface. If a horizontal force of 5 N is applied on the block, coefficients of static and kinetic frictions are 0.4 and 0.2 respectively. the frictional force on it is

(A). 20 N

(B). zero

(C). 8 N

(D). 5 N

Q 18.A block of mass 2 kg is lying on an inclined plane, inclined to the horizontal at 30^0. The coefficient of friction is 0.7, then magnitude of frictional force acting on the block will be

(A). 1109 N

(B). 11.9 N

(C). 0.119 N

(D). 119 N

Q 19.A bullet of mass 0.1 *kg* is fired with a speed of 100 *m*/*sec*, the mass of gun is 50 *kg*. The velocity of recoil is

(a) 0.2 *m*/*sec* (b) 0.1 *m*/*sec*
(c) 0.5 *m*/*sec* (d) 0.05 *m*/*sec*

Q 20.On the horizontal surface of a truck (μ = 0.6), a block of mass 1 *kg* is placed. If the truck is accelerating at the rate of $5 m/sec^2$ then frictional force on the block will be

(a) 5 *N* (b) 6 *N*
(c) 5.88 *N* (d) 8 *N*

Q 21.A jet plane flies in the air because

(a) The gravity does not act on bodies moving with high speeds
(b) The thrust of the jet compensates for the force of gravity
(c) The flow of air around the wings causes an upward force, which compensates for the force of gravity
(d) The weight of air whose volume is equal to the volume of the plane is more than the weight of the plane

Q 22.A player caught a cricket ball of mass 150 *gm* moving at a rate of 20 *m/s*. If the catching process be completed in 0.1 *s*, then the force of the blow exerted by the ball on the hands of the player is

(a) 0.3 *N* (b) 30 *N*
(c) 300 *N* (d) 3000 *N*

Q 23.When a body is moving on a surface, the force of friction is called

(a) Static friction (b) Dynamic friction
(c) Limiting friction (d) Rolling friction

Q 24.Which one of the following is not used to reduce friction

(a) Oil (b) Ball bearings
(c) Sand (d) Graphite

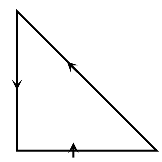
Q 25.A man fires a bullet of mass 200 *g* at a speed of 5 *m/s*. The gun is of one *kg* mass. by what velocity the gun rebounds backwards

(a) 0.1 *m/s* (b) 10 *m/s*
(c) 1 *m/s* (d) 0.01 *m/s*

Q 26.The coefficient of friction μ and the angle of friction λ are related as

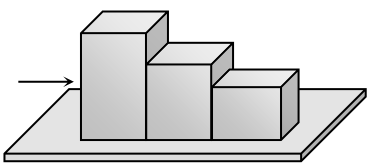
(a) $\sin\lambda = \mu$ (b) $\cos\lambda = \mu$
(c) $\tan\lambda = \mu$ (d) $\tan\mu = \lambda$

Q 27.Three forces starts acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle *ABC* (as shown). The particle will now move with velocity



(a) $\vec{v}$ remaining unchanged
(b) Less than $\vec{v}$
(c) Greater than $\vec{v}$
(d) $\vec{v}$ in the direction of the largest force *BC*

Q 28.Consider the following statements about the blocks shown in the diagram that are being pushed by a constant force on a frictionless table



A. All blocks move with the same acceleration

B. The net force on each block is the same Which of these statements are/is correct

(a) A only (b) B only

(c) Both A and B (d) Neither A nor B

Q 29.The maximum speed that can be achieved without skidding by a car on a circular unbanked road of radius *R* and coefficient of static friction μ, is

(A) μRg (B) $Rg\sqrt{\mu}$

(C) $\mu\sqrt{Rg}$ (D) $\sqrt{\mu Rg}$

Q 30 Two masses 2 *kg* and 3 *kg* are attached to the end of the string passed over a pulley fixed at the top. The tension and acceleration are

(a) $\frac{7g}{8}, \frac{g}{8}$ (b) $\frac{21g}{8}, \frac{g}{8}$

(c) $\frac{21g}{8}, \frac{g}{5}$ (d) $\frac{12g}{5}, \frac{g}{5}$

Q 31.The proper use of lubricants cannot reduce.

(a)Static friction (b) Inertia

(c) Sliding friction (d) rolling friction

Q 32.Which of the following statements is not true ?

(a) The coefficient of friction between two surfaces increases as the surface in contact are made rough

(b) The force of friction acts in a direction opposite to the applied force

(c) Rolling friction is greater than sliding friction

(d) The coefficient of friction between wood and wood is less than 1

Q 33.A 20 *kg* block is initially at rest on a rough horizontal surface. A horizontal force of 75 *N* is required to set the block in motion. After it is in motion, a horizontal force of 60 *N* is required to keep the block moving with constant speed. The coefficient of static friction is

(a) 0.38 (b) 0.44

(c) 0.52 (d) 0.60

Q 34 The optimum speed, for which wear and tear of the tyre is minimum, that can be achieved without skidding by a car on a circular banked road of radius *R* is

(A) $Rgtan\theta$ (B) $Rtan\theta/g$

(C) Rg (D) $\sqrt{Rgtan\theta}$

Q 35.A horizontal force of 129.4 *N* is applied on a 10 *kg* block which rests on a horizontal surface. If the coefficient of friction is 0.3, the acceleration should be

(a) $9.8\,m/s^2$ (b) $10\,m/s^2$

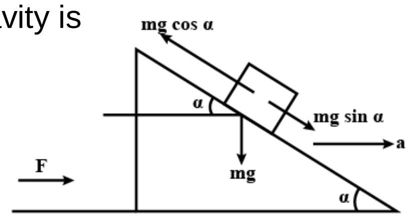
(c) $12.6\,m/s^2$ (d) $19.6\,m/s^2$

Answers (Multiple Choice Questions):

Q.NO.	ANS	Q.NO.	ANS	Q.NO.	ANS	Q.NO.	ANS	Q.NO.	ANS
1.	(A)	8.	(A)	15.	(A)	22.	(B)	29.	(D)
2.	(A)	9.	(C)	16.	(D)	23.	(D)	30.	(D)
3.	(D)	10.	(A)	17.	(D)	24.	(C)	31.	(B)
4.	(B)	11.	(B)	18.	(B)	25.	(C)	32.	(C)
5.	(B)	12.	(D)	19.	(A)	26.	(C)	33.	(A)
6.	(A)	13.	(A)	20.	(A)	27.	(A)	34.	(D)
7.	(C)	14.	(A)	21.	(B)	28.	(A)	35.	(D)

Explanation:

4. (b) We have $x = (t^3 - 2t - 10)$ on differentiating we get v= $3t^2$-2, differentiating it again, we get a=6t.
   So at t=4 sec, a=24 m/s^2.
   So, by F=ma, F = 10 x 24 = 240 N.
5. (b) we have a=(M-m)g/(M+m) , given a=g/8, on solving m/M is 9/7
6. (a) we have $\Delta p = p_2 - p_1$ = -p-p = -2p
8. (a) we know F= (p_2-p_1) / t, = 2mv $\sin 30^0$/t, = 2 x 0.1 x 10 x 0.5/0.1 = 10N.
9. (c) we know, $\tan\theta = v^2/rg$ = 400/ 40√3 x 10 = 1/√3 = tan 30^0, so θ = 30^0.
10. (a) force is the rate of change of momentum (per second), so F = Mv/1 = Mv.
11. (b) the horizontal component of the acceleration due to gravity is g sinα



And for equilibrium maCosα= mgSinα so
a = g tanα

12. (d) Formula Used: f=μN

Given: m=60 kg,μs=0.5,μk=0.4,

Limiting friction=μs=0.5(60g)=300 N

Kinetic friction=μk=0.4(60g)=240 N

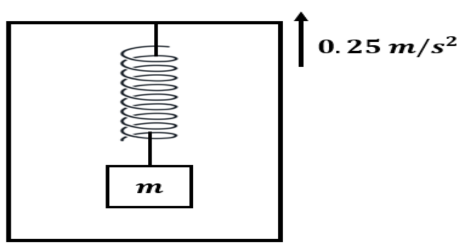
Formula Used: a=Fnet/m

∵F=300 N

As force is applied just to balance static friction and once block started moving applied force remains the same and kinetic friction starts acting on it

Hence, acceleration

a=(300−240)60=1m/s2



1. (a) We have F x s = ½ x 0.05 x 80 x 80 , F = -

160/0.4 = -400 N.

2. (a) On moving upward,

Weight, $W=m(g+a)=5(10+0.25)$ ($\because a=0.25$ m/s2)

$W=5\times10.25=51.25$ N

16. (d) mass of bullet =20 g, Initial speed u=1 ms−1,

Wall thickness s=2 cm

Resistance of wall F=2.5×10−2 N

Deceleration produced due to the mud wall,

a=F/m

From the third equation of motion

v2−u2=2aS⇒v2=u2−2(Fm)S

⇒v2=(1)2−(2)[2.5×10−220×10−3]20100

⇒v2=1−12

⇒v=1√2 m/s=0.7 m/s

17. (d) The frictional force is the hindrance force in the movement of the block. It refers to the force generated by two surfaces that contact and slide against each other. The coefficient of friction signifies determining the roughness of the surface.

Frictional force = μN (coefficient of kinetic friction is used here)

= μmg = 0.4×2×10

f = 8N

frictional force (static) > Horizontal force

Therefore, the frictional force = 5N

18. (b) N = 2 kg * 9.81 m/s² * cos(30°) = 2 kg * 9.81 m/s² * 0.866 = 16.97 N

Then, the frictional force:

f = 0.7 * 16.97 N = 11.879 N = 11.9 N

19. (a) From the law of conservation of momentum,
total initial momentum= final momentum

$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\therefore 0.1 \times 0 + 50 \times 0 = 0.1 \times 100 + 50(-v_2)$

$\Rightarrow 0 = 10 - 50 v_2$

$\therefore v_2 = \frac{10}{50} = 0.2\ ms^{-1}$

20. (a) f the block has to move, it should overcome static frictional force which is μmg=0.6×10=6N

As the truck is moving with an acceleration, a pseudo force acts on the block which is ma=1×5=5N

As this force is not enough for the block to move, an equal frictional force acts in the opposite direction which is equal to 5N

21. (b) The thrust of the jet compensates for the force of gravity

22. (b) mass of cricket ball, m=150g

=0.150Kg[As,1Kg=1000g] ,

Initial velocity of cricket ball, u=20m/s

The final velocity of cricket ball, v=0m/s [As, after catching ball will stop]

Time taken for the catch process, t=0.1s

The magnitude of change in momentum, $\Delta P = mv - mu$

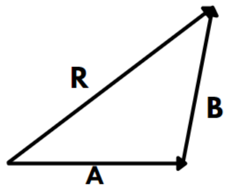
=0.150×0-0.150×20=3$Kg.m/s$

Average force exerted by the ball, $Fav = \Delta P / \Delta t$ [From Newton's second law of motion]
F = 30 N

23. (d) Rolling friction
24. (C)
25. (c) $v = m_b v_b / m_g$ = 0.2 x 5 / 1 = 1 m/s.
26. (c)

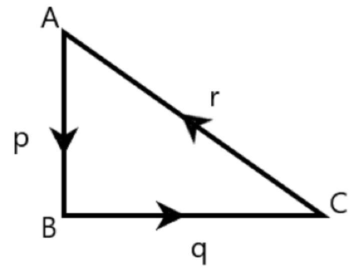
27. (a) As velocity is a vector quantity and we know that vectors have both magnitude and direction, so they cannot be simply added or subtracted algebraically. For finding the resultant of two vectors we use the triangular law of vector addition.

Triangle law of vector addition states that when two simultaneous vectors are represented as two sides of a triangle with their order of magnitude and direction, then the third vector (third side of triangle) represents the magnitude and direction of the resultant vector in opposite order.



TRIANGLE LAW OF VECTOR ADDITION

Three forces acting simultaneously on a particle moving with velocity$\vec{V}$ $V\rightarrow$. These forces are represented in magnitude and direction by the three sides of a triangle ABCABC.



Therefore, the net external force acting on a particle is zero.
Hence, if three vectors are represented by three sides of a triangle taken in the same order, then according to the triangle law of vector addition their resultant is zero. Now, as the resultant of the forces acting on the particle is zero, so the particle's velocity remains unchanged.

28. (a)

29. (d)

30. (d) Both the masses will have same acceleration being attached to the same string. Using the below two formulas for 2 mass system,

T=2m1m2m1+m2g=2×2×32+3g=12g5

a=(m2−m1m1+m2)g=(3−22+3)g=g5

31. (b)

32. (c)

33. (a) Coefficient of static friction μs=F1R=75/mg=75/20×9.8=0.38

34. (b)

35. (d) Mass of block, M=10kg

coefficient of friction,μk=0.3

Force applied, F=129.4N

F−fk=Ma

F−μkMg=Ma

129·4−Ma=μkmg

129·4−10×a=0.3×10×9·8

a=129.4−29.410=10ms−2.

Competency Based Questions

36. A satellite in force-free space sweeps stationary interplanetary dust at a rate dM/dt = av,
Where M is the mass and v the speed of the satellite, and a is a constant. What is the deceleration the satellite experiences.

1) $-av^2$ 2) $-av^2/2M$ 3) $-av^2/M$ 4) $-2av^2/M$

37. A body is moving with an acceleration 'a' under the action of a force 'g'. The weight of the body is

1) g/a 2) $-g^2/a$ 3) g^2/a 4) a^2/g

38. force produces an acceleration of a1 in a body and the same force produces an acceleration of a2 in another body. If the two bodies are combined and the same force is applied on the combination, the acceleration produced in it is

1) a1 + a2 2) 1 2 1 2 a a a a 3) 1 21 2a aa a 4) a1a2

39. A rubber ball falls from a height h and rebounds to a height h/2. A rubber ball of double the mass falling from the same height h rebounds to a height

1) h 2) h/2 3) 3h/4 4) 2h

40. A fat hose pipe is held horizontally by a fireman. It delivers water through a constricting nozzle at 1 liter/sec. If by increasing the pressure, the water is delivered at 2 liter/sec, the fireman now has to

1) push forward twice as hard 2) push forward four times as hard
3) push forward eight times as hard 4) push backward four times as hard

41. Two trains A and B are running in the same directions on parallel tracks such that A is faster than B, packets of equal weight are exchanged between them. Then

1) A will be retarded and B will be accelerated
2) B will be retarded and A will be accelerated
3)There will no change in A but B will be retarded
4) There will no change in B but A will be retarded

42. An object will continue accelerating till

1) the resultant force on it begins to decrease 2) the resultant force on it zero
3) the resultant force is at right angles to its motion
4) the resultant force is increasing continuously

43. A force of constant magnitude starts acting on a moving particle when it is at some point 'P'. Depending on the orientation of the force, the particle may

a) pass through point P at some time later b) not return to point P
c) describe a circular path d) describe a parabolic path

1) a is correct 2) a, b, c, d are correct
3) c only correct 4) d only correct

44. The force exerted by the floor of an elevator on the foot of a person standing there is more than the weight of the person if the elevator is

a) going up and slowing down b) going up and speeding up c) going down and slowing down d) going down and speeding up

1) b and c are correct 2) a and b are correct
3) b and d are correct 4) d only correct

45. A reference frame attached to the earth a) is an inertial frame by definition b) can not be an inertial frame because the earth is revolving around the sun c) is an inertial

frame because Newton's laws are applicable in this frame d) cannot be an inertial frame because the earth is rotating about its axis

1) a, b, c are correct 2) b only correct
3) b and d are correct 4) All are correct

46. a) In a frame of reference S1 , though the net force is zero, the net acceleration is not zero.
b) In a frame of reference S2, though the net force is not zero, the net acceleration is zero.
c) In a frame of reference S3, the net acceleration is zero whenever the net force is zero.

1) S1 and S3 are inertial and S2 is non-inertial 2) S1 and S2 are non-inertial and S3 is inertial
3) S1 , S2 , S3 are non-inertial 4) S1 , S2 , S3 are inertial

47. When train stops suddenly, the passengers in the train feel jerk in the forward direction.
This is because
1) the inertia of rest stops the train on takes the body forward.
2) the upper part of the body continues to be in the state of motion where as the lower part of the body which is in contact with seat comes to rest
3) the lack of seat pushes the body forward 4) due to some reason other than above

Assertion & Reason :

These Questions consist of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.
1) Both (A) and (R) are true and (R) is the correct explanation of (A)
2) Both (A) and (R) are true and (R) is not the correct explanation of (A)
3) (A) is true but (R) is false
4) (A) is false but (R) is true

48. (A) : A player lowers his hands while catching a cricket ball.
(R) : Increase in the time of catch, decreases the impulsive force.
49. (A) : Sportsman runs some distance before taking a long jump
(R) : Because of inertia of motion body remains in state of motion and enables him to have greater velocity to jump.
50. (A) : A body can have acceleration even if its velocity is zero at a given instant of time
(R) : A body is momentarily at rest when it reverses its direction of motion
51. (A) : A rocket moves forward by pushing the surrounding air backwards.
(R) : Rocket derives the necessary thrust to move forward according to Newton’s third law of motion
52. (A) : When stationary bomb explodes into two pieces their speeds are in the inverse ratio of their massess.
(R) : Explosion does not violate Law of conservation of linear momentum .

53. (A) : When bullet is fired from a gun, the gunner should exert force on gun in the direction of motion of bullet.

(R) : The gun recoils when bullet is fired.

54. (A) : A man standing on top of tower throws a ball downwards. He moves up when gravity is absent.

(R) : Momentum of system is conserved in absence of external force.

55. A cork is submerged in water by a spring attached to the bottom of a pail. When the pail is kept in elevator moving with acceleration downwards, the spring length

1) increases 2) decreases

3) remains unchanged 4) data insufficient

56. In a tug of war contest, two men pull on a horizontal rope from opposite sides. The winner will be the man who

1) Exerts greater force on the rope 2) Exerts greater force on the ground

3) Exerts force on the rope which is greater than tension in the rope

4) Makes smaller angles with vertical

57. The linear momentum 'p' of a body moving in one dimension varies with time according to the equation p a bt2 , where a and b are positive constants. The net force on the body is

1) Proportional to t2 2) Proportional to t

3) constant 4) inversely proportional to t

Answer

36.3	37.3	38.3	39.2	40.2
41.1	42.2	43.2	44.1	45.3
46.2	47.2	48.1	49.1	50.1
51.4	52.1	53.1	54.1	55.2
56.2	57.2			

Assertion & Reason Questions:

Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a. Both A and R are true and R is the correct explanation of A.

b. Both A and R are true but R is not the correct explanation of A.

c. A is true but R is false.

d. A is false and R is also false.

Q 36.Assertion**:** Frictional forces are conservative forces.

Reason: Potential energy can be associated with frictional force.

Q 37 Assertion**:** A man in a closed cabin, which is falling freely, does not experience gravity.

Reason: Inertial and gravitational masses have equivalence.

Q 38 Assertion**:** On a rainy day, it is difficult to derive a car or a bus at high speed.

Reason: The value of coefficient friction is lowered due to wetting of the surface.

Q 39 Assertion**:** The apparent weight of a body in an elevator moving with some downward acceleration is less than the actual weight of body.

Reason: The part of the weight is spent in producing downward acceleration, when body is in elevator.
Q 40 Assertion**:** Impulsive force is large and acts for a short time.

Reason: Finite change in momentum should be produced by the force.
Q 41 Assertion**:** A table cloth can be pulled from a table without dislodging the dishes.

Reason: To every action there is an equal and opposite reaction.
Q 42 Assertion**:** A quick collision between two bodies is more violent than a slow collision, even when the initial and final velocities are identical.

Reason: The momentum is greater in first case.
Q 43 Assertion**:** Centripetal force is always required for motion in curved path.

Reason: On a banked curved track, vertical component of the normal reaction provides the necessary centripetal force.
Q 44 Assertion**:** For equilibrium of concurrent forces acting at a point, net force must be zero.

Reason: For a point mass object to be in equilibrium, its linear acceleration must be zero..
Q 45 Assertion**:** On a banked curved track, without friction, horizontal component of normal reaction provides the necessary centripetal force.

Reason: Centripetal force is not always required for turning.

Answers (Assertion & Reason Questions):

Q.NO.	36	37	38	39	40	41	42	43	44	45
ANS	d	b	a	b	a	b	c	c	A	C

Short / Long Answer Type Questions:
Instructions: Q. No. 46 to 50 carry 2 marks each, Q. No. 51 to 55 carry 3 marks each and Q. No. 56 &56 carry 5 marks each.

Q 46 A man is at rest in the middle of a pond on perfectly frictionless ice.
How can he get himself to the shore of the pond?

Ans. He can get himself to the shore if he throws away his shirt or anything in his possession in a direction opposite to the desired direction of motion or by spitting in the forward direction or by blowing air from his mouth.

Q 47 Describe why you hold your gun next to your shoulder while deer hunting.
Ans. Because of 3rd Law (Action-Reaction law)
When you pull the gun's trigger, it forces the bullet out of the gun, but at the same time, the gun is forced in the opposite direction of the bullet (towards you). Your shoulder is a new force that is introduced to keep your gun from flying away from you.

Q 48 Using Newton's laws explain why heavier objects require more force than lighter objects to move or accelerate them.

Ans. Because of 2nd Law
Something with more mass moving at the same acceleration as a lighter object

would require more force to change its speed or change its direction according to the formula F=m x a derived from the 2nd law.

Q 49

a) Two objects having different masses have some momentum. Which one of them will move faster?

Ans. The object with a smaller mass will move faster (mv=constant so, v α 1/m)

b) A table is lying on the floor of a room. Is some force of friction acting on it?

Ans. No (There is no relative motion or tendency of motion)

Q 50 Is it correct to say that the banking of roads reduces the wear and tear of the tires of automobiles? If yes explain.

Ans. Yes, if the road is not banked, then the necessary centripetal force will be provided by the force of friction between tires and the road. On the other hand; when the road is banked, a component of the normal reaction provides the necessary centripetal force, which reduces wear and tear.

Q 51 Explain how each of Newton's laws affects a game of Tug of War.

Ans. First Law: The rope will stay in the same place until the tugging starts (a new force is introduced) Second Law: We could measure a team's force that they can pull the rope with based on their body masses and the acceleration that they are causing the rope to move at.

Third: One team pulls the rope towards themselves with a certain amount of force and the opposing team is also putting force on the rope. The same amount of force is applied from the ground to the people as they are put on the ground.

Q 52 a. According to Newton's third law of motion, every force is accompanied by an equal (in magnitude) and opposite (in direction) force called reaction, then how can a movement take place?

Ans. As the action and reaction never act on the same body, the motion is possible.

b. You can move a brick easily by pushing it with your foot on a smooth floor, but, if you kick it, then your foot is hurt. Why?

Ans. As F x t remains constant, so if t is reduced, then F will be increased and hence hurt our foot.

Q 53 What provides the centripetal force in the following cases?

(i) Electron revolving around the nucleus.

(ii) Earth revolves around the sun.

(iii) Car taking a turn on a banked road.

(i) Ans. It is provided by the electrostatic force of attraction between the electron and the nucleus.

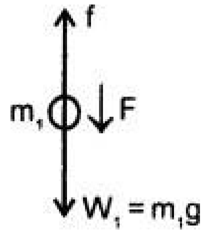
(ii) The gravitational force of attraction between Earth and Sun.

(iii) A component of the reaction of the road.

Q 54 Two bodies of different masses m1 and m2 are falling from the same height. If resistance offered by the air is the same for both the bodies, then will they reach the

Earth simultaneously? (Assume m1 > m2)

Ans. Let f be the resistance offered by the air for both bodies.



If F = Net force acting on the body of mass m, i.e. weight W1 = m1g,

Then F = W1 – f

= m1g – f …(1)

If a1 = acceleration produced in the body of mass m, then

$$a_1 = \frac{F}{m_1} = \frac{m_1g - f}{m_1} = g - \frac{f}{m_1} \quad .... (2)$$

Similarly if a2 be the acceleration produced in the body of mass m2, then a2 = g – f/m2……. (3)

Now as m1 > m2, it is clear from equations (2) and (3) that a2 < a1 or a1 > a2. Thus, we conclude that the body having larger acceleration will reach earlier, so the body of larger mass will reach the Earth earlier than the body of smaller mass.

Q 55 A bus moving with a velocity of 60 kmh^{-1} weighs 50 tons. Calculate the force required to stop it in 10s.

Ans. m = mass of bus = 50 tones = 50 × 1000 kg

u = initial velocity of bus = 60 kmh^{-1} = 60 × 5/18 ms^{-1} = 50/3 ms^{-1}v = final velocity of bus = 0

t = 10 s, a =? and retarding force, F =?

a = (0-50/3)/10 = - 5/3 ms^{-2}.

So, F = ma

= 50000 x (-5/3)

= - 250000/3

= - 8.33 x 10^4 N

It is a retarding force, which opposes the motion.

Q 56 A machine gun placed horizontally in front of a target fires 600 bullets per minute into the target. The average mass of the bullet is 50 gm and the bullets are fired at a speed of 800 ms^{-1}. Calculate the average force applied to the target.

(i) A hunter has a machine gun that can fire 50 g bullets with a velocity of 150 ms^{-1}. A 60 kg tiger springs at him with a velocity of 10 ms^{-1}. How many bullets must the hunter fire into the tiger so as to stop him in his track?

Ans. (i) Mass of each bullet, m = 50 gm = $50x10^{-3}$ kg.v = velocity of each bullet = 800 ms^{-1}

No. of bullet fired, n = 600,t = 60s,

average force exerted on the target = fav =?

∴ p1 = initial momentum of each bullet

= mv = 50x10^{-3} × 800 = 40 kg ms^{-1}∴ Total momentum of 600 bullets per min.,Pi = nP1

= 600 × 40 = 24000 kg ms^{-1}.
As the bullets come to rest after striking the target,

∴pf = final momentum of bullets = 0

Thus, change in momentum ΔP = pi – pf = 24000 kg ms^{-1}But by definition, Impulse = average force × time
= change in momentum or
Fav × t = Δp
∴ Fav = Δp/t=24000/60 =400N

(ii) m = mass of bullet = 50 gm = 0.050 kg M = mass of tiger = 60 kg
v = velocity of bullet = 150 ms^{-1}V
= velocity of tiger = – 10 ms^{-1}
(∵ it is coming from opposite direction)

Let n = no. of bullets fired per second at the tiger so as to stop it.

∴ pi = 0 before firing …. (i) pf = n(mv) + MV…. (ii)

∴ According to the law of conservation of momentum,

$$p_i = p_f$$

or

$$0 = n(mv) + MV$$

or

$$n = -\frac{MV}{mv} = \frac{-60\times(-10)}{0.050\times150} = 80$$

Case Study Based Questions:

Case 1. Topic: Newton's 2nd law of motion:
According to Newton's second law of motion, F = ma, where F is force required to produce an acceleration a in a body of mass m. if a = 0, then F = 0 i. e. no external force is required to move a body uniformly along a straight line. If a force act on a body for t seconds, the effect of force is given by impulse = F x t = change in linear momentum of body.
With the help of passage given above, choose the appropriate alternative for each of following questions:

(i). A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that the ball is turned back with a velocity of 20 m/s. if duration on contact between the ball and bat is 0.01 s the impulse of force is

(A) 7.4 Ns.

(B) 4.8 Ns.

(C) 1.2 Ns.

(D) 4.7 Ns.

(ii). Average force exerted by the bat is

(A) 480 N. (B) 120 N. (C) 1200 N. (D) 840 N.

(iii). The retardation of ball is

(A) 1600 m/s^2.

(B) 320 m/s^2.

(C) 3200 m/s^2.

(D) 160 m/s^2.

(iv). The force acting on the object whose linear momentum changes by 20 kg m/s in 10 s is

(A) 2 N.
(B) 20 N.
(C) 200 N.
(D) 0.2 N.

(v). An impulsive force of 100 N acts on an object for 1 s. The change in its linear momentum is

(A) 10 Ns.

(B) 100 Ns.

(C) 1000 Ns.

(D) 1 Ns.

Case 2. TOPIC: FRICTION

Friction between any two surfaces in contact is the force that opposes the relative motion between them. The force of limiting friction (F) between any two surfaces in contact is directly proportional to the normal reaction R between them i.e. F α R or F = μR, where μ is coefficient of limiting friction. If x is angle of friction then μ = tan x. With the help of passage given above, chose the appropriate alternative for each of following questions:
(i). The force of 49 N is just able to move a block of wood weight 10 kg on a rough horizontal surface. The coefficient of friction is

(A) 0.5

(B) 4.9

(C) 10/49

(D) 49/9.8

(ii). The angle of friction for coefficient of friction 1 is

(A) 34^0 26’ (B) $30^{0’}$ (C) 26^0 34’ (D) 45^0.
(iii). What would be coefficient of friction if angle of friction is 30^0

(A) $\sqrt{}3$

(B) 5.77
(C) 1.577
(D) 0.577

(iv). A horizontal force of 1.2 kgf is applied on a 1.5 kg block which rests on a horizontal surface. If the coefficient of friction is 0.3, force of friction is

(A) 0.45 kgf.

(B) 1.2 kgf.

(C) 1.5 kgf.

(D) 0.3 kgf.

(v). The acceleration produced in a block in the above question is

(A) 9.8 m/s^2

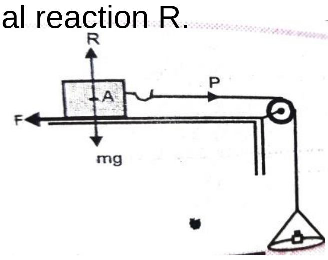
(B) 0.3 m/s^2

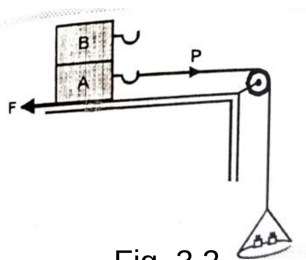
(C) 1.5 m/s^2

(D) 4.9 m/s^2

<u>Case 3</u>. TOPIC: Free Body Diagram

To verify the laws of limiting friction, we take two exactly identical rectangular blocks of wood A and B, each of same weight mg. each block is provided with a hook on one side. The block is placed on a horizontal table provided with a frictionless pulley on one side, fig 3.1. One end of a string is attached to the hook of the block. The string is then passed over the pulley and a pan is attached to the free end of the string. Any number of weights can be added to the pan.
We adjust the weights in the pan till the block just begins to move. The applied force P at this stage gives us a measure of force of limiting friction F. All blocks are similar. And all weights are similar. Carefully observe the blocks and number of weights in the pan and answer following question.
Block applies force equal to its weight on the table and in return table provides normal reaction R.





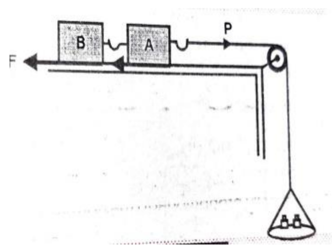


Fig. 3.1 Fig. 3.2 Fig. 3.3

(i). Refer fig 3.1 and 3.2 it was observed that when only one block was place, it could be just moved by placing one weight in pan and when two block were placed one over the other, then two weights were required to just move the blocks. This gives that

(A) frictional force is directly proportional to the normal reaction.

(B) frictional force is directly proportional to the square of normal reaction.

(C) frictional force is inversely proportional to the normal reaction.

(D) frictional force is inversely proportional to the square of normal reaction.

(ii). It is evident from fig 3.1 and 3.2 that

(A) R = mg and F= P and frictional force is directed in the direction of applied force.

(B) R = mg and F= P and frictional force is directed opposite to direction of applied force.

(C) R = mg and F=1/ P and frictional force is directed in the direction of applied force.
(D) R = mg and F= 1/ P and frictional force is directed opposite to direction of applied force.
(iii). This question refer to fig. 3.3, it was observed experimentally that same force was required to just bring the blocks in motion as it was needed in case of fig 3.2. this suggests that

(A) force of friction is independent of the blocks.

(B) force of frictions is always constant.

(C) force of friction depends on the total area of contact.

(D) force of limiting friction is independent of the apparent area of contact between the bodies, so long as the normal reaction R between them remains the same.

(iv). Take the rectangular block A. make one surface of A smooth and well-polished and the opposite surface of A very rough. Find the force of limiting friction by putting weights when (i) rough surface is in contact with the table and (ii) when smooth surface is in contact with the table. We observe that more weight is required in case (i) than in case (ii). This suggests that

(A) force of friction is directly proportional to the area of contact.

(B) force of friction is always equal to applied force.
(C) force of friction is independent of nature of surface.

(D) force of friction is more between rough surfaces than between smooth surfaces.

(v). If we take two blocks, one of wood and other of metal of same weight, Friction in either case is different. This proves that

(A) force depends on the weight of the block.

(B) force of friction is always opposite to the applied force

(C) force of frictions depends on the nature of material of the bodies in contact.

(D) force of friction is directly proportional to the surface area of body.

Case 4. TOPIC: Weight in lift or elevator

The person of 60 kg is standing on a platform scale balance kept on the floor of an elevator Cab is capable of motion in up or down direction with either a uniform velocity or a constant acceleration. The value of velocity as well as acceleration may be adjusted at any suitable value. Newton's second law of motion can be applied for the motion of elevator cab. Take g = 10 m/s^2.
(i). Observed weight of the person, when elevator is at rest, is

(A) 600 N

(B) 300 N

(C) 900 N

(D) 120 N

(ii). Observed weight of the person, when elevator cab is moving upward with a constant velocity of 2 m/s, will be

(A) 600 N (B) 450 N (C) 800 N (D) 500 N

(iii). Observed weight of the person, when elevator cab is going upward with a constant acceleration of $\frac{g}{4}$, will be

(A) 200 N

(B) 750 N

(C) 450 N

(D) 800 N

(iv). Observed weight of the person, when elevator cab is going downward with a acceleration of $\frac{g}{4}$, will be

(A) 600 N

(B) 300 N

(C) 450 N

(D) 750 N

(v). If the elevator cab suddenly goes out of order and starts falling freely under gravity then observed weight of the person will be

(A) 60 kg

(B) 120 N

(C) 30 N

(D) Zero

Answers (Case study based questions):

	Ans (i)	Ans (ii)	Ans (iii)	Ans (iv)
Case 1	B	A	C	A
Case 2	A	D	D	A
Case 3	A	B	D	D
Case 4	A	A	B	C
Case 5	C	D	D	B

Self-Assessment

Two Mark Questions:

1. Name the physical quantity which gives a measure of quantity of motion contained in a moving body. Also draw the graph showing the variation of this physical quantity with velocity for an object with constant mass.

2. Give two important applications of Newton's second law.

3. What do you understand by impulse? Use it to explain why does a cricket player lower his hands while catching a ball?

4. State and explain Newton's third law. Identify the action-reaction pairs in case of firing of a gun.

5. Comment on -"Newton's first law is contained in Second law".

6. State the law of conservation of linear momentum and derive it by using Newton's second law.

7. Derive an expression to show that the recoil velocity of gun is directly proportional to the velocity of the bullet.

8. A balloon with mass M is descending down with an acceleration a, where a<g. What mass m of its contents must be removed so that it starts moving up with acceleration a? [Ans: 2Ma/(g+a)]

9. What are concurrent forces? Write the condition under which the body will said to be in equilibrium.

10. Two masses M and m are connected at the two ends of an inextensible string. The string passes over a frictionless pulley. Calculate the acceleration of the masses.

11. What is friction. A heavy box is kept (at rest) on a table. Will there be friction in the given case, if yes, name the type of the friction involved here.

12. Explain why the static friction is called as self-adjusting force.

13. Using the knowledge of different types of friction, draw a plot to show the variation of force of friction with the applied force.

14. A physical quantity X is obtained when limiting friction is numerically divided by the normal reaction. Identify the quantity X and write its SI unit.

15. Define angle of Repose and hence deduce its relation with co-efficient of static friction.

16. The distance travelled by a moving body is directly proportional to time. Is any external force acting on it?

Three Mark Questions:

17. A vehicle is moving on a horizontal road with speed v. If the coefficient of friction between the tyres and the road is μ, show that the shortest distance in which the vehicle can be stopped is $v^2/2\mu g$.

18. Find the expression for the work done against friction when a body is made to slide up an inclined plane.

19. By giving three advantages and disadvantages, show that the friction is a necessary evil.

20. Why is it easier to pull a roller than to push it? Explain.

21. Explain why (i) China wares are wrapped in straw papers before packing (ii) shockers are used in vehicles.

22. Why does a cyclist lean inward when moving along a curved path. Determine the angle through which the cyclist bends from the vertical to negotiate a curve.

23. A horse has to apply more force to start a cart than to keep it moving explain.

24. What is meant by banking of roads? Explain the need for it. Obtain an expression for the maximum speed with which a vehicle can safely negotiate a curved road banked at angle θ. The coefficient of friction between road and wheels is μ.

25. **(i)** Find the expression for the recoil velocity of a gun.

(i) A person of mass m is standing in a lift. Write expression for his apparent weight when the lift is

(a) moving upward with uniform acceleration 'a' (b) moving downward with uniform acceleration 'a' (a<g) and (c) falls freely.

Numerical

26. Forces of $5\sqrt{2}$N and $6\sqrt{2}$N are acting on a body of mass 1000kg at an angle 60 degrees to each other. Find the acceleration and distance covered by the mass after 10s.

[**Ans**: 0.01349m/s^2; 0.6745m]

27. A body of mass m moves along the X-axis such that its position coordinate at any instant t is $x=at^4-bt^3+ct$; where a, b, c are constants. What is the force acting on the particle at any time t.

28. A bullet of mass 100g moving with 20m/s strikes a wooden plank and penetrates up to 20cm. calculate the resistance (reaction force) offered by the wooden plank. [**Ans**: 100N]

29. A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of12m/s. If the mass of the ball is 0.15kg, determine the impulse imparted to the ball (assuming linear motion). [**Ans**: -3.6Ns]

30. A ball moving with a momentum p strikes against a wall at an angle of 45 degrees and is reflected at the same angle. Calculate the change in momentum. [**Ans**: -2p cos45]

31. The velocity of a 2 kg object initially moving at (–2i+3j–5k) m/s changes to (i+3j+4k) m/s after 3 s. Calculate the magnitude of the force acting on the body. [**Ans**: $2\sqrt{10}$N]

32. A force of 49N is just sufficient to pull a block of wood weighing 10kg on a rough horizontal surface.

33. Calculate the coefficient of friction and angle of friction. [**Ans**: 0.5; 26degrees]A block slides down an incline of 30degree with an acceleration equal to g/4. Find the coefficient of kinetic friction. [**Ans**: $1/(2\sqrt{3})$]

34. Find the maximum speed at which a car can take turns around a curve of a 30m radius on a level road, if the coefficient of friction between tires and road is 0.4. [**Ans**: 11m/s]

35. For traffic moving at 60kmph, what should be the correct angle of banking of the road if the radius of the curve is 0.1km. [**Ans**: 15.5 degrees]

36. An object of mass 3 kg is at rest. Now a force of $F = 6t^2i + 4t\,j$ is applied on the object. Calculate the velocity of object at t = 3s. [**Ans**: 18i+6j]

37. A body of mass M hits normally a rigid wall with velocity V and bounces back with the same velocity. The impulse experienced by the body is……. [**Ans**: 2MV]

38. One end of a string of length l is connected to a particle of mass m and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in a circle with speed v, what will be the net force on the particle (directed towards center; T represents the tension in the string).

[**Ans**: T]

39. A person of mass 60 kg is inside a lift of mass 940 kg and presses the button on control panel. The lift starts moving upwards with an acceleration 1.0 m/s2 . If g = 10 m s–2, calculate the tension in the supporting cable. [**Ans**: 11000N]

40. A man weighs 80 kg. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of 5 m/s^2 . What would be the reading on the scale ? (g = 10 m/s^2)

[**Ans**: 1200N]

Unit IV Work, Energy and Power

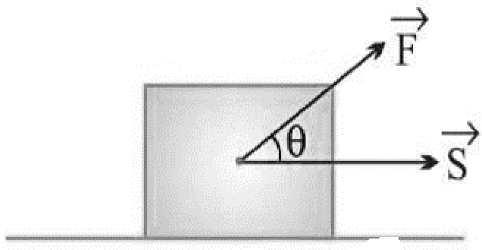
Chapter 6: Work, Energy and Power

GIST OF THE CHAPTER:

Work done by a constant force and a variable force; kinetic energy, work-energy theorem, power. Notion of potential energy, potential energy of a spring, conservative forces, non-conservative forces, motion in a vertical circle; elastic and inelastic collisions in one and two dimensions**.**

Work : Work is done when the force acting on an object produces displacement in it in the direction of force or in the direction of component of force.

It is scalar. S I Unit of work is Joule (J) or N.m and dimension is $[M^1 L^2 T^{-2}]$



$$W = F.S = FS\cos\theta$$

Work is positive if the force and displacement are in the same direction, negative if they are opposite, and zero if the force is perpendicular to the displacement.

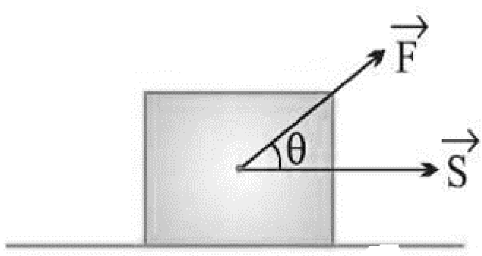
Condition for work done

1. There must be displacement of object.

2. There must be force or component of force in the direction of displacement.

Work done by a constant force

Work done on an object by a constant force is defined as the product of the magnitude of the force and the displacement of the object in the direction of the force.
Mathematically, it is represented as $W = F.S = FS\cos\theta$
Generally, the constant force acting on a body does not act in the direction in which the body moves. Fig. shows such a situation. The force F making an angle with the horizontal pulls the block through displacement S.



The component of the force F in the direction of displacement is F cosθ. Therefore, work done by the constant force F on the block is

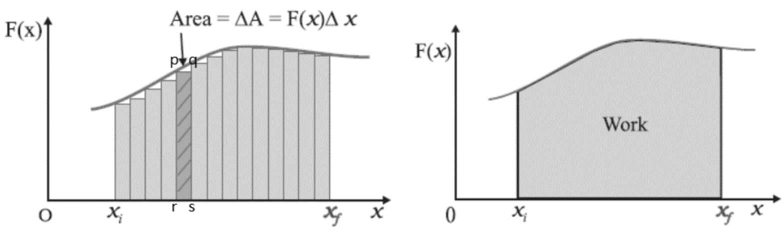
Work done, W = (F cosθ)S = FS cosθ

θ is the angle between the directions of F and S. Although force and displacement are vector quantities, work is a scalar quantity.
e.g. Work done by or against gravitational force is work done by a constant force.

Work done by a variable force

Suppose a variable force F acts on a body along the fixed direction, say x – axis. The magnitude of the force F depends on x, as shown in fig. Let us calculate the work done when the body moves from the initial position x_i to the final position x_f under the force F. The displacement can be divided into a large number of small equal displacements dx During a small displacement dx, the force F can be assumed to be constant.
Then the work done,

W = Fdx = Area of rectangle pqrs



Adding area of all the rectangles,

We get the total work done

$$W = \sum_{x_i}^{x_f} F\,dx \text{ = Sum of area of all the rectangles}$$

In the limit when $\delta x \to 0$, the number of rectangles tends to be infinite, but the above summation approaches a definite integral whose value is equal to the area under the curve. Thus the total work done is

$$W = \sum_{x_i}^{x_f} F\,dx = \int_{x_i}^{x_f} F\,dx$$

Which is equal to area under the force and displacement curve.

Energy

Energy of a body is defined as the capacity or ability of body to do work.
Different types of energy : Heat energy, light energy, chemical energy, electrical energy, internal energy, nuclear energy.

It is scalar and SI Unit is Joule and CGS unit is erg.
Practical units
Calorie (1 cal = 4.2J)
Kilowatt hour (1 KWh = 3.6 x 10^6J)
Electron volt (1eV = 1.6 x10^{-19}

Kinetic energy
Kinetic energy is the energy that an object possesses due to its motion.

$$K\,E = \frac{1}{2} mv^2$$

Work-energy theorem
Statement: It states that the work done on an object by the net force acting on it is equal to the change in kinetic energy.

$$W = K_f - K_i = \Delta K$$

Notion of potential energy
Potential energy is the energy stored in an object due to its position, condition, or configuration.
potential energy is associated with the position of an object within a force field or the arrangement of parts of a system. Potential energy is also known as mutual energy or energy of configuration. The energy of a system can be altered by the application of a force. Examples of bodies possessing PE due to position (i) water stored in a reservoir (ii) lifted weight. Examples of bodies possessing PE due to configuration (i) The coiled spring of a watch or gramophone. (ii) A stretched bow (iii) Compressed gas (iv) Compressed or elongated spring.
There are different types of potential energy gravitational potential energy, elastic potential energy, Electrostatic potential energy and chemical potential energy.
(i) **Energy gravitational potential energy**: It is the potential energy associated with gravitational force between the objects. This type of potential energy depends on an object's height above the ground and the gravitational force acting on it.

$$P\,E = mgh$$

(ii) **Elastic potential energy:** This type of potential energy is stored in objects that can be stretched or compressed, such as springs or rubber bands. The formula for elastic potential energy in a spring is

$$P\,E = \frac{1}{2} kx^2$$

Where k is spring constant, x is is the displacement from the equilibrium position.
(iii) Chemical Potential Energy: This type of potential energy is stored in the bonds between atoms and molecules. It can be released during chemical reactions.
(iv) Electrostatic potential energy. It is the potential energy associated with electrostatic force.
In case the two electric charges are alike, the electrostatic PE is positive and in the case of two unlike charges, it is negative.

Potential energy of a spring
potential energy in a spring is

$$P\,E = \frac{1}{2} kx^2$$

Where k is spring constant, x is the displacement from the equilibrium position.

Conservative forces
A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.
e. g. Gravitational force.

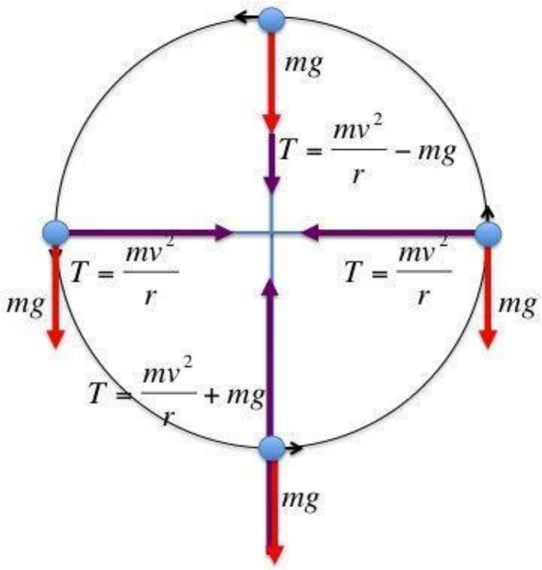
Non-conservative forces
A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.
e.g. frictional force.

Power

Power of a body is defined as the rate at which the body can do the work.
Average power (Pav) = W/ t
The instantaneous power is defined as the limiting value of the average power as time interval approaches zero,
The work done by a force F for a displacement dx is
dW = F.dx
dW/dt = Fdx/dt
P = F dx/dt
P = F.V
IT is scalar. S I Units: watt or joule/sec
Dimension: $[P] = [ML^2T^{-3}]$
Practical Units: Kilowatt (kW), Megawatt (MW) and Horsepower (hp)
Relations between different units: 1 watt = 1 joule/sec = 10^7 erg/sec
1hp = 746 watt

Motion in a vertical circle

Vertical circular motion refers to the movement of an object along a circular path in a vertical plane. This type of motion involves both centripetal force, which acts towards the center of the circle, and gravitational force, which acts downward. The object's speed and tension in the string vary at different points of the circle due to the influence of gravity. At the top of the path, the object experiences reduced tension or force, while at the bottom, the tension or force is at its maximum. Analyzing vertical circular motion often requires applying principles of energy conservation and Newton's laws of motion.



Elastic collision: Collision between two or more particles is the interaction for a short interval of time in which they apply relatively strong forces on each other.

As a result there is change in momentum and energy.
In a collision physical contact of two bodies is not necessary.
types of collisions

1. Elastic collision - The collisions in which both the momentum and the kinetic energy of the system remains conserved are called elastic collisions. In an elastic collision all the forces involved are conservative forces. Total energy remains conserved.
e.g. In gases, when atoms or molecules collide elastically, they exchange energy and momentum without any loss in kinetic energy.

2. Inelastic collision
Collisions in which only the momentum remains conserved but kinetic energy does not remain conserved are called inelastic collisions. In an inelastic collision some or all the involved forces are non conservative forces. Total energy of the system remains conserved.
e. g. Car Crashes: In most car accidents, the cars crumple and deform, leading to a significant loss of kinetic energy which is converted into sound, heat, and deformation energy.
3. Perfectly Inelastic or Plastic Collision
If the two bodies stick together after the the collision is said to be perfectly inelastic. In this type of collision, the loss of kinetic energy is maximum but not complete.
Examples : (i) Mud thrown on a wall.
(ii) A man jumping on to a moving cart.
(iii) The collision between a bullet and a block of wood such that the bullet remains embedded in the wood
(iii) Collision between two putty balls.
(iv) Collision between proton and electron.
4**. Superplastic or Explosive Collision**
In such a collision, there is an increase in kinetic. There is a release of potential energy on an impact.
Examples : (i) Bursting of a cracker when it hits the floor forcefully.
(ii) Collision of a trolley with another trolley may release a compressed spring.

Elastic collision in one dimension
If the colliding molecules displace along the same straight-line path both before and after the collision, the collision is said to be one dimensional.

In a one-dimensional elastic collision, the relative velocity of approach before collision is equal to the relative velocity of separation after collision.
If two objects of mass m_1 and m_2 displacing with velocities u_1 and u_2 respectively collide head on such that v_1 and v_2 be their respective velocities after collision, then,

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2 u_2}{m_1 + m_2}$$

$$v_2 = \frac{2m_1 u_1}{m_1 + m_2} + \frac{(m_2 - m_1)}{m_1 + m_2}u_2$$

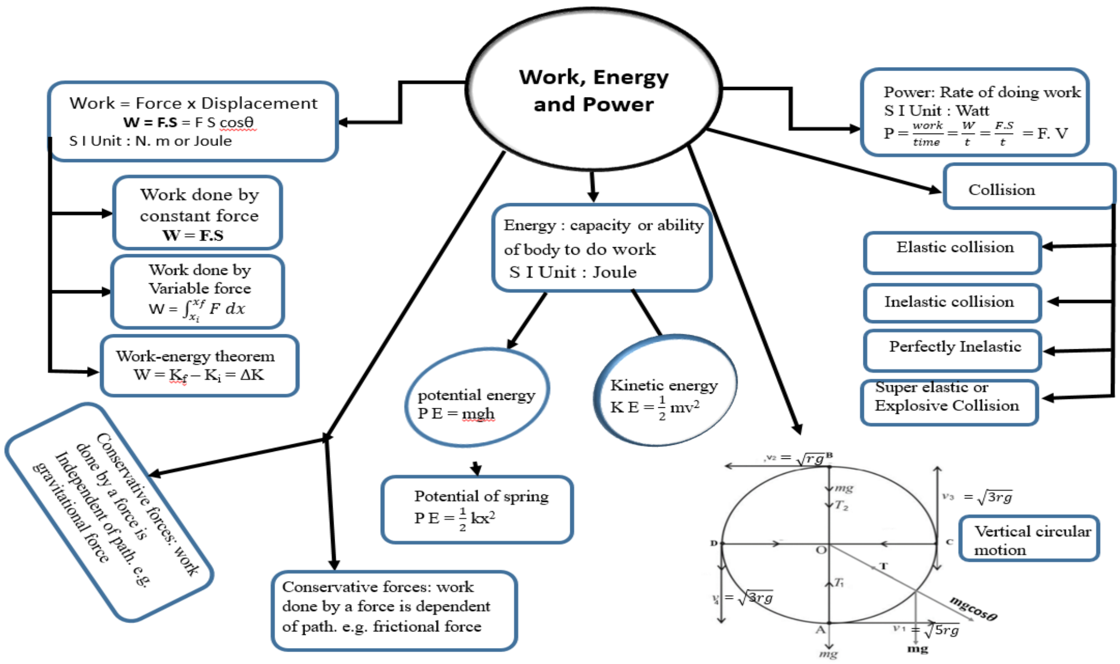
Inelastic collision in one dimension

In a one-dimensional inelastic collision, two objects collide and stick together, moving as a single unit after the collision. In this type of collision, momentum is conserved, but kinetic energy is not; some kinetic energy is converted into other forms of energy such as heat or deformation. The final velocity of the combined object can be calculated using the conservation of momentum. This type of collision is often studied to understand real-world impacts where objects do not bounce apart perfectly. Inelastic collisions are common in everyday situations like car crashes and sports impacts.

Elastic collision in Two dimension

In a two-dimensional elastic collision, two objects collide and bounce off each other in a plane. Both kinetic energy and momentum are conserved in the system, requiring the consideration of both the x and y components of momentum. The objects' final velocities can be determined by solving equations that account for these conserved quantities. The collision angles and speeds are influenced by the masses and initial velocities of the objects

MIND



MIND MAP

Multiple Choice Questions

1. A ball moves in a frictionless inclined table without slipping. The work done by the table surface on the ballis
   (a) Negative (b) Zero (c) Positive (d) None of the options
2. A body of mass 25 kg is moved parallel to the ground, through a distance of 1.5 m. The work done against gravitational force is
   (a) 50 J (b) -9.8 J (c) 20 J (d) zero
3. When work done on a particle is positive, then its
   (a) K E increases (b) K E decreases

(c) K E remains constant (d) K E increases and K E decreases both

4. According to work-energy theorem, the work done by the net force on a moving particle is equal to the change in its
   (a) kinetic energy (b) potential energy
   (c) linear momentum (d) angular momentum
5. Unit of energy is
   (a) kwh (b) joule (c) electron volt (d) All of these
6. What a body moves with constant along a circle The work done by the force is
   (a) No work is done (b) no acceleration is produced in it
   (c) Its velocity remains constant (d) None of the above
7. If two particles are brought near one another, the potential energy of the system will
   (a) increase (b) decrease (c) remains the same (d) equal to the K.E
8. When two spheres of equal masses undergo perfect inelastic collision with one of them at rest, after collision they will move
   (a) opposite to one another (b) in the same direction (c) together (d) at right angle to each other
9. If you lift a suitcase from the ground and keep it
   the table, the work done by you does not depend on
   (a) the path taken by suitcase
   (b) weight of suitcase
   (c) frame of reference
   (d) none of the above
10. In an elastic collision, what is conserved?
    (a) Kinetic energy (b) Momentum (c) Both (a) and (b) (d) Neither (a) nor (b)
11. A vehicle is moving on a rough road in a straight line with uniform velocity. Then
    (a) no force is acting on the vehicle
    (b) a force must act on the vehicle
    (c) an acceleration is being produced in the vehicle
    (d) no work is being done on the vehicle
12. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest
    (a) at the instant just before the body hits the earth
    (b) at the highest position of the body
    (c) it remains constant all through
    (d) at the instant just after the body is projected.
13. The energy that will be ideally radiated by a 100 kW transmitter in 1 hour is
    (a) 36×10^5 J (b) 36×10^7 J (c) 36×10^9 J (d) 1×10^5 JA
14. If force $\vec{F} = (\hat{i} + 5\hat{j} + 7\hat{k})$ acts on a particle and displaces it through $\vec{d} = (6\hat{i} + 9\hat{k})$. Calculate the work done if the force is in newton and displacement is in metre.
    (a) 69 J (b) 90 J (c) 50 J (d) 60 J
15. A person is holding a bucket by applying a force of 10 Newton. He moves a horizontal distance of 5 m and then climbs up a vertical distance of 10 m. Find the total work done by him.
    (a) 100 J (b) 2000 J (c) 50 J (d) 60 J
16. Which of the following is not a conservative force?
    (a) Gravitational force (b) Frictional force (c) Spring force (d) None

of these

17. A force (F = 15 + 0.50 x) acts on a particle in the x-direction, where F is in newton and x in metre. Find the work done by this force during a displacement from x = 0 to x = 2.0 m.
(a) 31 J (b) 0 J (c) 21 J (d) 50 J
18. During the perfectly elastic collision, which of the following is conserved?
(a) Linear momentum of the each body is conserved.
(b) Kinetic energy of the each body is conserved.
(c) Linear momentum of the system is conserved.
(d) None of the above
19. An electric heater of rating 1000 W is used for 5 hrs per day for 20 days. The electrical energy utilised is:
(a) 150 kWh (b) 200 kWh (c) 100 kWh (d) 300 kWh
20. The potential energy of a long spring when stretched by 2 cm is U. If the spring is stretched by 8 cm the potential energy stored in it is:
(a) U/4 (b) 4 U (c) 8 U (d) 16 U

Answer

1. (b) Zero
2. (d) zero, W = FS cosθ, θ = 90
3. (a) K E increases
According to work energy theorem
Work = K_f - K_i, when work done is positive.
This shows $K_f > K_i$.
4. (a) kinetic energy
5. (d) All of these
6. (a) No work is done
7. (a) increase
8. (c) together
9. (a) the path taken by suitcase
10. (c) Both (a) and (b)
11. (b) a force must act on the vehicle
12. (a) at the instant just before the body hits the earth
13. (b) 36 x 10^7 J, W = P x t = 100 x 1000 x 60 x 60 = 36 x 10^7 J
14. (a) 69 J, W = $F_x S_x + F_y S_y + F_z S_z$ = 6 + 63 = 69J
15. (a) 100 J, for W = FS cosθ for horizontal θ = 90 and vertical θ = 0, W = 10 x 10 = 100J.
16. (b) Frictional force
17. (a) 31 J, use W = $\int_{x_i}^{x_f} F\,dx$
18. (c) Linear momentum of the system is conserved.
19. (c) 100 kWh , time = 5 x 20 =100, P = 100 x 1000 = 100000 = 100kWh
20. (d) 16 U, use P E = $\frac{1}{2}$ kx^2

<u>Assertion And Reason Type Questions</u>

In the following questions, a statement of assertion (A) is followed by a statement of reason(R).Mark the correct choice as:
(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If the assertion is true but the reason is false.
(d) If the assertion and reason both are false.

1. **Assertion**: Work done depends on angle between force and displacement
   Reason: When the force retards the motion of a body, the work done is zero.
2. A**ssertion**: If momentum of a body increases by 50% its kinetic energy will increase by 125%.
   Reason: Kinetic energy is proportional to square of velocity.\
3. **Assertion :** A spring has potential energy, both when it is compressed or stretched.
   Reason : In compressing or stretching, work is done on the spring against the restoring force.
4. **Assertion :** The change in kinetic energy of a particle is equal to the work done on it by the net force.
   Reason : Change in kinetic energy of particle is equal to work done only in case of a system of one particle.
5. **Assertion:** Kinetic energy of a body is quadrupled, when its velocity is doubled.
   Reason : Kinetic energy is proportional to square of velocity. If velocity is doubled the K.E. will be quadrupled.
6. **Assertion:** The earth moving around the sun in a circular orbit is acted upon by a force and hence work must be done on the earth by this force.
   Reason: It is necessary that the work done in the motion of a body over a closed loop is zero for every force in nature.
7. **Assertion:** A chemical reaction is basically a rearrangement of atoms. If the total energy of the reactants is more than the products of the reaction, heat is released and the reaction is said to be an exothermic reaction.
   Reason: Energy spent against friction does not follow the law of conservation of energy.
8. Assertion: When a body moves along a circular path, no work is done by the centripetal force.
   Reason: The centripetal force is used in moving the body along the circular path and hence no work is done.
9. **Assertion**: Kinetic energy is conserved at every instant of elastic collision.
   Reason: No deformation of matter occurs in elastic collision.
10. **Assertion:** A particle strikes head on with another stationary particle such that the first particle comes to rest after collision. The collision should necessarily be elastic.
    Reason: In elastic collision, there is a loss of momentum of the system of particles.
11. **Assertion:** Work done in case of circular motion is zero.
    Reason: Power developed in circular motion is always zero.

Answer
1. (c) If the assertion is true but the reason is false.
2. (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
3. (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

4. (c) Change in kinetic energy = work done by net force. This relationship is valid for particle as well as system of particles.
5. (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
6. (d) If the assertion and reason both are false.
7. (c) If assertion is true but reason is false.
8. (c) If the assertion is true but the reason is false.
9. (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
10. (d) If the assertion and reason both are false.
11. (c) If the assertion is true but the reason is false.

Case Study Based Questions

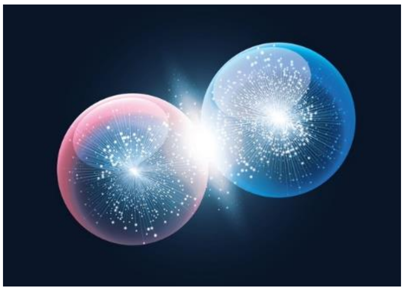
Work energy theorem states that – change in kinetic energy of a body is equal to the work done by the net force. In deriving the theorem, it is assumed that force is effective only in changing the KE. When the force and displacement are in same direction, KE increases and work done is positive. When the force and displacement are in opposite direction, KE decreases and work done is negative. When the body is in uniform motion, KE does not change and work done by centripetal force is zero.
Examples : (1) A car moving in with constant velocity is accelerated. Hence there is an increase in velocity. So kinetic energy is increased by the positive work done by the engine. (2) A ball dropped from a height H. Work done by gravity increases the velocity of the ball and thus kinetic energy increases.3) Brakes are applied in a motor bike and it comes to rest after a few seconds. Here work done by the break is negative. So with decrease in velocity, kinetic energy decreases.

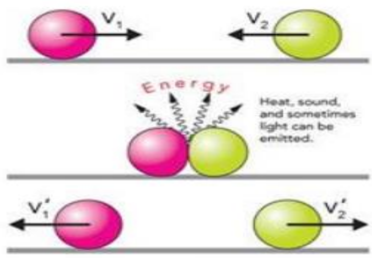
1. When does kinetic energy increase according to the work-energy theorem?
   (a) When force and displacement are perpendicular.
   (b) When force and displacement are in the same direction.
   (c) When force and displacement are in opposite directions.
   (d) When there is no force acting on the body.
2. what happens to the kinetic energy of a car moving with constant velocity when it is accelerated?
   (a) Kinetic energy decreases due to negative work done by the engine.
   (b) Kinetic energy increases due to positive work done by the engine.
   (c) Kinetic energy remains constant due to zero work done by the engine.
   (d) Kinetic energy fluctuates randomly due to work done by the engine.
3. What is the work done by centripetal force on a body moving in uniform circular motion?
   (a) Positive
   (b) Negative
   (c) Zero
   (d) Varies with speed
4. A 10 kg ball is dropped from a height of 15 m. Calculate the increase in kinetic energy just before it hits the ground.
   (a) 1,250 J
   (b) 1,350 J
   (c) 1,470J
   (d) 1,500 J

5. A 2 kg object moving with a velocity of 10 m/s is brought to rest by a constant force over a distance of 4 m. Calculate the magnitude of the force.
   (a) 12.5 N
   (b) 15 N
   (c) 25 N
   (d) 50 N

Source Based Questions

An elastic collision is a collision in which there is no net loss in kinetic energy in the system as a result of the collision. Both momentum and kinetic energy are conserved quantities in elastic collisions. The molecules of a gas collide with one another and with the container. The collisions of a neutron with an atom is well known. In a nuclear reactor, fast neutrons produced in the fission of uranium atom have to be slowed down. They are, therefore, made to collide with hydrogen atom. The term collision does not necessarily mean that a particle or a body must actually strike another. In fact, two particles may not even touch each other and yet they are said to collide if one particle influences the motion of the other. When two bodies collide, each body exerts an equal and opposite force on the other. The fundamental conservation law of physics are used to determine the velocities of the bodies after the collision. Collision may be elastic or inelastic collision.





1. Which conservation law is NOT typically used to analyse elastic collisions?
   (a) Conservation of energy
   (b) Conservation of momentum
   (c) Conservation of angular momentum
   (d) Conservation of charge
2. What happens to the total kinetic energy in an elastic collision?
   (a) It is completely lost
   (b) It is partially converted into other forms of energy
   (c) It remains constant
   (d) It doubles
3. What fundamental physics principle is used to determine the velocities of bodies after a collision?
   (a) Hooke's Law
   (b) Newton's Third Law
   (c) Conservation of momentum
   (d) Thermodynamics
4. Considering the conservation laws, which scenario best illustrates a violation of the conservation of momentum in an elastic collision?
   (a) Two particles exert forces on each other without any net external force.
   (b) External forces act on the colliding system during the interaction.

(c) The collision occurs in a vacuum.
(d) The particles undergo a perfectly inelastic collision.

5. When fast neutrons produced in the fission of a uranium atom collide with hydrogen atoms, what physical quantity primarily determines the efficiency of these collisions in slowing down the neutrons?
   (a)The charge of the hydrogen atoms
   (b) The relative mass difference between the neutrons and hydrogen atoms
   (c) The volume of the container
   (d) The initial velocity of the uranium atoms

Answers of Case Study Based Questions

1. (b) When force and displacement are in the same direction.
2. (b) Kinetic energy increases due to positive work done by the engine.
3. (c) Zero
4. (c) 1470 J, K E = mgh
5. (c) 25 N

Source Based Questions

1. (d) Conservation of charge
2. (c)It remains constant
3. (c) Conservation of momentum
4. (b) External forces act on the colliding system during the interaction.
5. (b) The relative mass difference between the neutrons and hydrogen atoms

Very Short Answer Questions (2 Marks)

1. When is the work done by a force positive, negative and zero?
   Ans. Work done, W= F.S = F S cosθ. Here θ is the angle between force (F) and displacement (S). (i) If θ < 90°, work done is positive (ii) If θ > 90°, work done is negative (iii) If θ = 90°, work done is Zero.
2. A force F -5i +6j- 4k acting on a body produces a displacement S =6i +5k. What is the work done?
   Ans: Work done, W= F.S = (5i +6j- 4k).(6i +0 + 5k) = 30 + 0 - 20 = 10 units.
3. A car is moving with a constant speed in a straight line. Is there any net work done?
   Ans. No, because net work done is equal to the change in kinetic energy. In this case, there is no change in kinetic energy.
4. Give three examples of conservative forces.
   Ans: A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions. Examples are gravitational force, elastic forces and electrostatic forces.
5. What is non-conservative force? Write two examples.
   Ans: A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions. e.g. frictional force, viscous force, strong nuclear force.
6. Write the properties of conservative forces.
   Ans.: Properties of Conservative Forces (i) Work done by or against a conservative force depends only on the initial and final positions of the body. (ii)

Work done by or against a conservative force does not depend upon the nature of the path between initial and final positions of the body. (iii) Work done by or against a conservative force in a round trip is zero. (iv) The work done by a conservative force is completely recoverable.

7. Write the properties of elastic collision.
   Ans: (i) The momentum is conserved (ii) K E conserved (iii) The mechanical energy is not converted into any other form of energy. (iv) Forces involved are of conservative forces.
8. Write the properties of inelastic collision.
   Ans: (i) The momentum is conserved (ii) K E not conserved (iii) The mechanical energy is converted into any other form of energy. e.g. heat, light etc. (iv) Forces involved are of nonconservative forces.

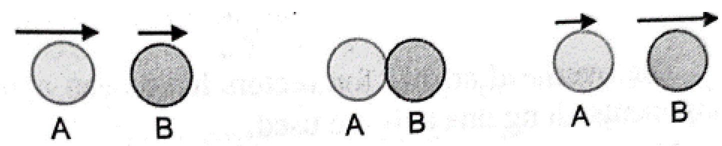
Long Answer Questions (5 Marks)

1. Derive an expression for final velocities in elastic collision

OR

show that velocity of approach equal to velocity of separation

Ans : Consider two bodies A and B of masses m_1 and m_2 moving with initial velocities u_1 and u_2 along the same straight line . If u_1 is greater than u_2, After some, these bodies are colliding each other. After collision their velocities become v_1 and v_2 respectively.



As linear momentum is conserved in any collision

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Since K E is also conserved in an elastic collision

$$\frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2$$

$$m_1 u_1^2 - m_1 v_1^2 = m_2 v_2^2 - m_2 u_2^2$$

$$m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2) \quad \ldots\ldots\ldots\ldots\ldots(4)$$

Dividing equation (4) by (2), we get

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The above equation is,
Relative velocity of approach = Relative velocity of separation
From equation (5)

$$v_2 = u_1 + v_1 - u_2$$

put this value in equation (1)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 (u_1 + v_1 - u_2)$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 u_1 + m_2 v_1 - m_2 u_2$$

$$(m_1 - m_2) u_1 + 2 m_2 u_2 = (m_1 + m_2) v_1$$

$$v_1 = \frac{(m1 - m2)\, u1}{m1 + m2} + \frac{2\, m2\, u2}{m1 + m2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

This is final velocity of body A after collision
Similarly, we can find v_2 by putting value of v_1 from equation (5) in equation (1)

$$v_2 = \frac{(m2 - m1)u2}{m1 + m2} + \frac{2\, m1\, u1}{m1 + m2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

This is final velocity of body B after collision
Case (i)
When two bodies of equal masses collide

Let $m_1 = m_2 = m$ (say)
From equation (6)

$$v_1 = \frac{2\, m\, u2}{2m} = u2$$ = velocity of body of m_2 before collision

$$v_1 = u2$$

From equation (7)

$$v_2 = \frac{2\, mu1}{2\, m} = u1$$

$$v_2 = u1$$

When two bodies of equal mases suffer one dimensional elastic collision their velocity get exchanged after the collision.
Case (ii)
When a body collides against a stationary body of equal mass
$m_1 = m_2 = m$ (say)
Then, $v_1 = 0$ and $v_2 = u1$
Case (iii)
When a light body collides against a massive stationary body
Here $m_1 << m_2$ (m1 neglecting)
And $u_2 = 0$
Then

$$v_1 = -\frac{m2\, u1}{m2} = u1 \quad \text{and} \quad v_2 = 0$$

Case (iv)
When massive body collides against a light stationary body
Here $m_1 >> m_2$ and

$$u_2 = 0 \quad \text{and } m_2 = 0$$

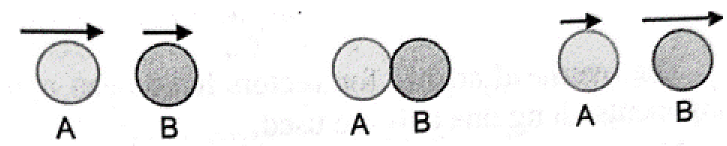
$$v_1 = -\frac{m1\, u1}{m1} = u1$$

$v_2 = \frac{2\,m1u1}{m1} = 2\,u1$

2. (a) Explain inelastic collision in one dimension

(b) Derive the formula for loss of kinetic energy in perfectly inelastic collision.

Ans: Consider two bodies A and B of masses m_1 and m_2 moving with initial velocities u_1 and u_2 along the same straight line. If u_1 is greater than u_2, After some, these bodies are colliding each other with some loss of K E. Therefore collision is inelastic. Let v_1 and v_2 are velocities of the two bodies after collision.



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \text{………………………(1)}$$

As the two bodies from one system which is closed and isolated. Therefore, total momentum before collision equal to total momentum after collision.

But K E is not conserved.

Knowing m_1, m_2, u_1, u_2 and one of the final velocities, we can calculate the other final velocity.

Loss of K E

If collision to be perfectly inelastic, after collision both bodies continue to move with same straight line with common velocity V. Both the bodies stick to each other as show in fig.

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\,V$$

$$V = \frac{m1\,u1 + m2\,u2}{m1 + m2} \quad \text{………………………….. (2)}$$

Loss of K E in this collision

$\Delta K = K_i -- K_f$

Initial K E (K_i) $= \frac{m_1 u_1{}^2}{2} + \frac{m_2 u_2{}^2}{2}$

Final K E (K_f) $= \frac{(m_1 + (m_2)V^2}{2}$

Loss K E, $\Delta K = \frac{m_1 u_1{}^2}{2} + \frac{m_2 u_2{}^2}{2} - \frac{(m_1 + m_2)\,V^2}{2}$

Put the value of V from equation (2)

$$\Delta K = \frac{m_1 u_1{}^2}{2} + \frac{m_2 u_2{}^2}{2} - \frac{(m_1 + (m_2)}{2} \left(\frac{m1\,u1 + m2\,u2}{m1 + m2}\right)^2$$

$$\Delta K = \frac{m1\,m2\,(u1 - u2)^2}{2\,(m1 + m2)}$$

As $u_1 > u_2$

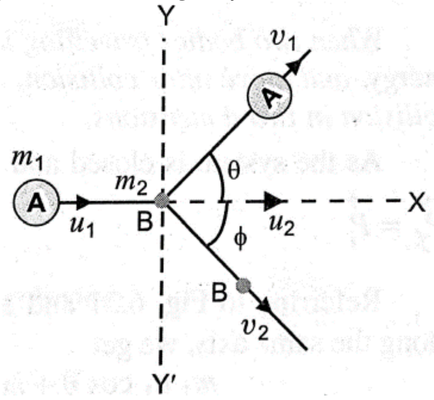
So ΔK comes out to be positive. This appears as the sound, light and heat energies.

3. Describe elastic and inelastic collision in two dimensions.

Ans: Elastic collision in two dimensions

when two bodies travelling initially along the same straight line collide without loss of kinetic energy and move along different directions in a plane after collision, the

collision is said to be elastic collision in two dimensions.
Consider m_1 and m_2 are the masses of two bodies A and B moving initially along X-axis with velocities u_1 and u_2 respectively. When $u_1 > u_2$, the two bodies collide. After collision, let the body A move with a velocity v_1 at an angle θ with X-axis. Let the body B move with a velocity v_2 at an angle φ with X-axis as shown in Fig.



Here, θ is known as angle of scattering and φ is known as the angle of recoil. As the collision is elastic, kinetic energy is conserved.

Total KE. after collision = Total K.E. before collision

$$\frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2 \quad \ldots\ldots(2)$$

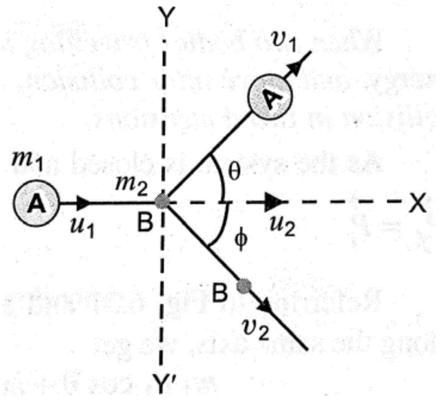
As linear momentum is conserved in elastic collision, therefore, along X-axis. total linear momentum after collision equal to total linear momentum before collision.

$$m_1 v_1 \cos\theta + m_2 v_2 \cos\varphi = m_1 u_1 + m_2 u_2 \quad ,,,,,,,,,,,,,(3)$$

Along Y-axis, linear momentum before collision is zero (as both the bodies are moving along X-axis), And after collision, total linear momentum along Y-axis is $m_1 v_1 \sin\theta - m_2 v_2 \sin\varphi$

$$m_1 v_1 \sin\theta - m_2 v_2 \sin\varphi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

From equation (1), (2) and (3), we have to calculate four variables v_1
v_2 and θ and φ which is not possible. We have, therefore, to measure experimentally any one parameter. The rest of the three parameters can be calculated from equation (1), (2) and (3).
Inelastic collision in two dimensions
When two bodies travelling initially along the same straight line collide involving some loss of kinetic energy, and move after collision, along different directions in a plane, the collision is said to be inelastic collision in two dimensions.
As the system is closed and isolated, the total linear momentum of the system remains constant
Total momentum after collision = Total momentum before collision
along X-axis to initial momentum



Total linear momentum after collision equal to total linear momentum before collision along X axis.

$m_1 v_1 \cos\theta + m_2 v_2 \cos\varphi = m_1 u_1 + m_2 u_2$,,,,,,,,,,,,,(1)

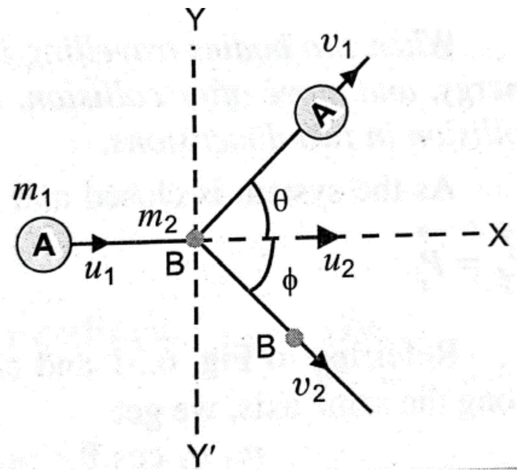
Total linear momentum after collision equal to total linear momentum before collision along Y axis.

$m_1 v_1 \sin\theta - m_2 v_2 \sin\varphi = 0$ …………………..(2)

Knowing the value of m_1, m_2, u_1, u, θ and φ we can calculate final velocities v_1 v_2.

Perfectly inelastic collision in two dimensions

The perfectly inelastic collision between two bodies of masses m_1 and m_2. Is shown in following fig. The body of mass m_2 is moving initially with velocity u_2 along X-axis. The body of mass m_1 is moving with velocity u_1 at an angle θ with X-axis. After the collision at O, the two bodies stick to each other and move with a common velocity V at an angle φ with X-axis. As the system is closed and isolated, the total linear momentum of the system remains constant.



Total linear momentum after collision equal to total linear momentum before collision along X axis.

$m_1 u_1 \cos\theta + m_2 u_2 = (m_1 + m_2) V \cos\varphi$ …………………..(3)

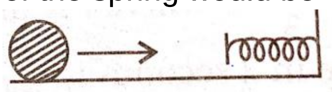
Total linear momentum after collision equal to total linear momentum before collision along Y axis.

$m_1u_1 \sin\theta + 0 = (m_1 + m_2) V \cos\theta$ ……………………………(4)

Knowing the value of m_1, m_2, u_1, u, θ we can calculate final velocity V and its direction (i. e φ).

Self-Assessment

1. A mass of 0.5 kg moving with a speed of 1.5 m/s on a horizontal smooth surface, collides with a nearly weightless spring of force constant k = 50 N/m. The maximum compression of the spring would be



   (a) 0.15m (b) 0.12m (c) 1.5m (d) 0.5m
2. An electric lift with a maximum load of 2000 kg (lift + passengers) is moving up with a constant speed of 1.5 m s, The frictional force opposing the motion is 3000 N. The minimum power delivered by the motor to the lift in watts is (g= 10 m s)
   (a) 23000W (b) 20000W (c) 34500W (d) 23500W
3. Two springs A and B having spring constant K_A and K_B ($K_A = 2K_B$) are stretched by applying force of equal magnitude. If energy stored in spring A is E_A then energy stored E_B in B will be
   (a) 2E (b) E/4 (c) E/2 (d) 4E
4. A body projected vertically from the earth reaches a height equal to earth's radius before returning to the earth. The power exerted by the gravitational force is greatest
   (a) at the highest position of the body
   (b) at the instant just before the body hits the earth
   (c) it remains constant all through
   (d) at the instant just after the body is projected
5. A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?
   (a) 950 J (b) 900 J (c) 850 J (d) 875 J
6. A particle of mass 500 g is moving in a straight line with velocity $v = bx^{5/2}$, The work done by the force during its displacement from x = 0 to x = 4m is
   t (Take $b=0.25\ m^{-3/2}\ s^{-1}$).
   (a) 2J b) 4J (c) 8J (d) 16 J
7. For an object in vertical circular motion, how does the kinetic energy at the highest point compare to the kinetic energy at the lowest point?
   (a) It is the same (b) It is half (c) It is twice (d) It is one fourth
8. A roller coaster car of mass mmm is at the top of a vertical loop of radius RRR. If the car barely makes it through the loop, what is the normal force exerted by the track on the car at the top of the loop?
9. (a) Zero (b) mg (c) 2mg (d) mg/2

Answer

1. (a) 0.15m

The KE of mass is converted into energy required to compress a spring which is given by

$\frac{1}{2}$ mv^2 = $\frac{1}{2}$ kx^2

x = $\sqrt{\frac{mv^2}{k}}$ = $\sqrt{\frac{0.5\ x\ (1.5)^2}{50}}$ = 0.15m

2. (c) 34500W
   Given that,
   Mass of lift + passengers, m = 2000 kg
   Speed, v = 1.5 m/s
   Frictional force, F= 3000N
   Power delivered, P = Force x velocity
   Force acting
   F = mg + f
   F=2000 × 10 + 3000
   F= 23000 N
   P= 23000 × 1.5=34500 W
3. (a) 2E
   Energy
   E = $\frac{1}{2}$ kx^2 = $\frac{F^2}{2\ k}$
   Given $K_A = 2K_B$
   Therefore $E_B = 2\ E_A$
4. (b) at the instant just before the body hits the earth
   Power, **F.v** = Fv cosθ
   Just before hitting the earth = 0°. Hence, the power exerted by the gravitational force is greatest at the instant just before the body hits the earth.
5. (b) 900 J
   x = 3t^2 + 5 ,
   m = 2 kg, v = $\frac{dx}{dt}$ = 6t, at t = 0, v = 0 when t = 5 , v = 30ms^{-1}
   W = $\frac{1}{2}$ mv^2 - $\frac{1}{2}$ mu^2 = 900J
6. (d) 16 J
   v = bx$^{5/2}$
   x = 0, v_i = 0
   for x = 4m, v_i = 0.25 x (4)$^{5/2}$ = 8
   using W = $\frac{1}{2}$ mv^2 - $\frac{1}{2}$ mu^2 = 16J
7. (b) It is half
   At the highest point, the potential energy is maximum and the kinetic energy is minimum. At the lowest point, the potential energy is minimum and the kinetic energy is maximum. By energy conservation:
   $KE_{lowest} = KE_{highest} + 2mgR$
8. If the car barely makes it through the loop, the normal force at the top is zero. At this point, the only force providing the centripetal force is the weight of the car
   mg = $\frac{mv^2}{r}$ = $\sqrt{rg}$

Thus, the normal force N is zero.

Unit V : System Of Particles & Rigid body

Chapter 7: System Of Particles & Rotational Motion

GIST OF THE CHAPTER:

Important formula

1.	Position vector of centre of mass of two particles system $\overrightarrow{R} = \frac{m_1\overrightarrow{r_1}+m_2\overrightarrow{r_2}}{m_1+m_2}$
2.	velocity vector of centre of mass of two particles system $\vec{V} = \frac{m_1\overrightarrow{v_1}+m_2\overrightarrow{v_2}}{m_1+m_2}$
3.	Acceleration vector of centre of mass of two particles system $\vec{a} = \frac{m_1\overrightarrow{a_1}+m_2\overrightarrow{a_2}}{m_1+m_2}$
4.	Force acting on the centre of mass, $\vec{F} = M\overrightarrow{a_{CM}}$
5.	Momentum of the centre of mass, $\vec{p} = M\overrightarrow{V_{CM}}$
6.	Torque = force × perpendicular distance from the axis of rotation $\vec{\tau} = \vec{r} \times \vec{F} = rF\ sin\ sin\ \theta$
7.	Work done by the torque = torque × angular displacement W $=\tau\theta$
8.	Angular momentum = linear momentum × perpendicular distance from the axis of rotation $\vec{L} = \vec{r} \times \vec{p}$ For a particle of mass m moving with a uniform speed v along a circle of radius r is $L = mvr$
9.	Torque = rate of change of angular momentum $\vec{\tau} = \frac{d\vec{L}}{dt}$
10.	Power = torque × angular velocity $P = \tau\omega$

11.	(table below)

	Linear Motion		Rotational Motion
1.	Distance/displacement (s)	1.	Angle or angular displacement (θ)
2.	Linear velocity $v = \frac{ds}{dt}$	2.	Angular velocity $\omega = \frac{d\theta}{dt}$
3.	Linear acceleration $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	3.	Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
4.	Mass (m)	4.	Moment of inertia (I)
5.	Linear momentum $P = mv$	5.	Angular momentum $L = I\omega$
6.	Force $F = ma$	6.	Torque $\tau = I\,\alpha$
7.	Also, force $F = \frac{dp}{dt}$	7.	Also, torque $\tau = \frac{dL}{dt}$
8.	Translational K.E $= \frac{1}{2}mv^2 = \frac{p^2}{2m}$	8.	Rotational K.E $= \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$
9.	Work done, $W = Fs$	9.	Work done, $W = \tau\,\theta$
10.	Power $P = Fv$	10.	Power $= \tau\,\omega$
11.	Linear momentum of a system is conserved when no external force acts on the system. (Principle of conservation of linear momentum)	11.	Angular momentum of a system is conserved when no external torque acts on the system. (Principle of conservation of angular momentum)
12.	Equations of Translational motion (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$, where the symbols have their usual meaning.	12.	Equations of Rotational motion (i) $\omega_2 = \omega_1 + \alpha t$ (ii) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ (iii) $\omega_2^2 - \omega_1^2 = 2\,\alpha\,\theta$, where the symbols have their usual meaning.
13.	Distance travelled in nth second $S_{nth} = u + \frac{a}{2}(2n-1)$	13.	Angle traced in nth second $\theta_{nth} = \varphi_1 + \frac{\alpha}{2}(2n-1)$

12. Moment of inertia of bodies, axis of rotation and values

1.	Uniform rod of length l	perpendicular to rod through its centre	$\frac{1}{12}Ml^2$
2.	Uniform rectangular lamina of length l and breadth b	perpendicular to lamina and through its centre	$M\left(\frac{l^2+b^2}{12}\right)$
3.	Uniform circular ring of radius R	perpendicular to its plane and through the centre	MR^2
4.	Uniform circular ring of radius R	Diameter	$MR^2/2$
5.	Uniform circular disc of radius R	perpendicular to its plane and through the centre	$\frac{1}{2}MR^2$
6.	Uniform circular disc of radius R	Diameter	$\frac{1}{4}MR^2$
7.	Hollow cylinder of radius R	Axis of cylinder	MR^2
8.	Solid cylinder of radius R	Axis of cylinder	$\frac{1}{2}MR^2$
9.	Hollow sphere of radius R	Diameter	$\frac{2}{3}MR^2$
10.	Solid sphere of radius R	Diameter	$\frac{2}{5}MR^2$

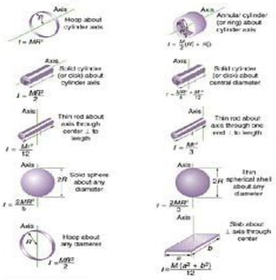
13.	Radius of gyration $K = \sqrt{\frac{I}{M}}$ When all the particles are of same mass, $$K = \sqrt{\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}}$$
14.	Rotational kinetic energy, $K.E = \frac{1}{2} \times I\omega^2$
15.	Total K.E= translational K.E +rotational K.E = $\frac{1}{2} \times mv^2 + \frac{1}{2} \times I\omega^2$
16.	Torque = M.I × angular acceleration, $\tau = I\alpha$
17.	Angular momentum = M.I × angular velocity, L= $I\omega$
18.	In the absence of any external torque, the angular momentum is conserved L= $I\omega$, a constant $I_1\omega_1 = I_2\omega_2$, ω_1, ω_2 are the angular velocities $I_1 \frac{2\pi}{T_1} = I_2 \frac{2\pi}{T_2}$, T_1, T_2 are the time periods

Systems of particles and Rotational Motion

Moment / Inertia

Inertia of rotational motion, M.I.,

$$I = \sum_{i=1}^{n} m_i r_i^2$$



Theorem of parellel axes

$I = I_{cm} + mr^2$

I_{cm} = M.I.about the parallel axis through the centre of mass

Theorem of perpendicular axes

$I_z = I_x + I_y$

I_x, I_y & I_z moments of inertia about perpendicular axes x, y and z respectively.

Moment of Inertia of some regular shaped body

1) Rod $I_{cm} = \frac{mI^2}{12}$

$I_{one\ edge} = \frac{mI^2}{3}$

2) Ring $I_{cm} = mR^2$

$I_{diameter} = \frac{mR^2}{2}$

3) Disc $I_{cm} = \frac{mR^2}{2}$

$I_{diameter} = \frac{mR^2}{4}$

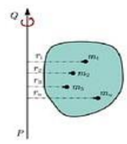
4) Solid Sphere

$I_{diameter} = 2/5\ mR^2$

Radius of Gyration

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \ldots\ldots r_n^2}{n}}$$

$$K = \sqrt{\frac{I}{n}}$$



Equation of rotaional motion

i) $\omega_2 = \omega_1 + \alpha t$

ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

iii) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

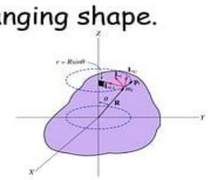
Torque or couple or moment of force

$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta\ \hat{n}$

$\tau_{rotational} = I\alpha$

Rigid Body

A body with perfectly definite and unchanging shape.



Angular momentum

$L = I\,\omega = mvr = r \times mv$

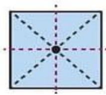
$L = r \times p = rp\sin\theta$

Centre of Mass

The point where the whole mass of the system is supposed to be concentrated

Position of centre of mass of the system

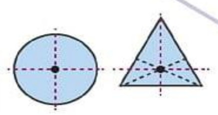
$$V_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i}$$



- Position of centre of mass of an object changes in translatory motion but remains unchanged in rotatory motion
- Position of centre of mass depends upon shape, size, distribution of mass of the body.

Velocity of centre mass of the system

$$V_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{\sum_{i=1}^{n} m_i}$$



Acceleration of centre of mass of the system

$$a_{cm} = \sum_{i=1}^{n} \frac{m_i a_i}{\sum m_i}$$

Centre of mass for rigid body is independent of the state i.e., rest or motion of the body

Multiple Choice Questions

1 The motion of the centre of mass depends on
(a) total external forces (b) total internal forces
(c) sum of (a) and (b) (d) None of these

2 The centre of mass of a rigid body lies
(a) inside the body (b) outside the body
(c) neither (a) nor (b) (d) either (a) or (b)

3 The sum of moments of all the particles in a system about the centre of mass is always
(a) maximum (b) minimum
(c) infinite (d) zero

4 If the resultant of all external forces is zero, then velocity of centre of mass will be
(a) zero (b) constant
(c) either (a) or (b) (d) neither (a) nor (b)

5 Centre of mass of the earth and the moon system lies
(a) closer to the earth
(b) closer to the moon
(c) at the mid-point of line joining the earth and the moon
(d) cannot be predicted

6 Position vector of centre of mass of two particles system is given by
(a) $\vec{R} = \frac{m_1\vec{r_1} - m_2\vec{r_2}}{m_1+m_2}$
(b) $\vec{R} = \frac{m_1\vec{r_1} \times m_2\vec{r_2}}{m_1+m_2}$
(c) $\vec{R} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1+m_2}$
(d) $\vec{R} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{r_1+r_2}$

7 The time rate of change of angular momentum of a particle is equal to
(a) force (b) acceleration
(c) torque (d) linear momentum

8 The wide handle of screw is based upon
(a) Newton's second law of motion
(b) law of conservation of linear momentum
(c) turning effect of force
(d) None of these

9 Which of the following is an expression for power
(a) $P = \tau\omega$ (b) $P = I\alpha$ (c) $P = I\omega$ (d) $P = \tau\alpha$

10 According to the principle of conservation of angular momentum, if moment of inertia of a rotating body decreases, then its angular velocity
(a) decreases (b) increases
(c) remains constant (d) becomes zero

11 When a mass is rotating in a plane about a fixed point, its angular momentum is directed along the

(a) radius of orbit
(b) tangent to the orbit
(c) line parallel to plane of rotation
(d) line perpendicular to plane of rotation

12 During somersault, a swimmer bends his body to
(a) increase moment of Inertia
(b) decrease moment of Inertia
(c) decrease the angular momentum
(d) reduce the angular velocity

13 Two identical particles move towards each other with a velocity $2v$ and v respectively. The velocity of centre of mass is
(a) v (b) $\frac{v}{3}$ (c) $\frac{v}{2}$ (d) zero
(b)

14 Moment of inertia of a circular wire of mass M and radius R about its diameter is

(a) $\frac{MR^2}{2}$ (b) MR^2 (c) 2MR^2 (d) $\frac{MR^2}{4}$

15 The moment of inertia of a uniform circular disc of radius 'R' and mass 'M' about an axis passing from the edge of the disc and normal to the disc is

(a) MR^2 (b) $\frac{MR^2}{2}$ (c) $\frac{3MR^2}{2}$ (d) $\frac{7MR^2}{2}$

16 A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child, if he folds his hands back and thereby reduces his moment of inertia to (2/5) times the initial value? Assume that the turntable rotates without friction.
(a) 40 rpm (b) 45 rpm c) 55 rpm (d) 100 rpm

17 A wheel is rotating at 900 r.p.m about its axis when the power is cut off. It comes to rest in 1 minute. The angular retardation in $\frac{rad}{s^2}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{6}$

18 The radius of gyration of a thin rod of length *l* about an axis passing through one of its ends and perpendicular to it is

(a) $\frac{l}{\sqrt{2}}$ (b) $\frac{l}{3}$ (c) $\frac{l}{\sqrt{3}}$ (d) $\frac{l}{4}$

19 Two bodies of masses 1 kg and 2 kg are lying on x-y plane at (1,2) and (- 1 ,3) respectively. What are the coordinates of centre of mass?

(a) (2 ,-1) (b) $(\frac{8}{3}, \frac{-1}{3})$ (c) $(\frac{-1}{3}, \frac{8}{3})$ (d) None of these

20 Three identical spheres of mass M each are placed at the corners of an equilateral triangle of side 2m. Taking one of the corners as the origin, the position vector of the centre of mass is
(a) $\sqrt{3}\ (\hat{\imath} - \hat{\jmath})$ (b) $\frac{\hat{\imath}}{\sqrt{3}} + \hat{\jmath}$ (c) $\frac{\hat{\imath}+\hat{\jmath}}{3}$ (d) $\frac{\hat{\jmath}}{\sqrt{3}} + \hat{\imath}$

21 A force F = $5\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ acts on a particle whose position vector is r= $\hat{\imath} - 2\hat{\jmath} + \hat{k}$. What is the torque about the origin

(a) $8\hat{\imath} + 10\hat{\jmath} + 12\hat{k}$ (b) $8\hat{\imath} + 10\hat{\jmath} - 12\hat{k}$

(c) $8\hat{\imath} - 10\hat{\jmath} - 8\hat{k}$ (d) $10\hat{\imath} - 10\hat{\jmath} - 10\hat{k}$

22 The torque required to stop a wheel of M.I 5×10^{-3} kgm^2 from a speed of 20 rad/s in 10s is

(a) 1Nm (b) 0.1 Nm (c) 0.01Nm (d) 0.001 Nm

23 A circular disc X of radius R made from an iron plate of thickness t and another disc Y of radius 4R is made from an iron plate of thickness t/4 then the relation between moment of inertia of X and Y is

(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$ (c) $I_Y = I_X$ (d) $I_Y = 64I_X$

24 If the radius of the earth contracts to half of its present value the duration of day will be

(a) 48 hours (b) 24 hours (c)12 hours (d) 6 hours

25 A bomb at rest explodes into large number of parts the center of mass of a system

(a) describes a parabola
(b) describes a straight line
(c) continues to be at the same point at which it was before explosion
(d) none of these

MCQ ANSWERS

1 (a) total external forces
2 (d) either (a) or (b)
3 (d) zero
4 (b) constant
5 (a) closer to the earth (Since C.M lies closer to the heavy mass)
6 (c) $\vec{R} = \frac{m_1\vec{r_1}+m_2\vec{r_2}}{m_1+m_2}$
7 (c) torque ($\vec{\tau} = \frac{d\vec{L}}{dt}$)
8 (c) turning effect of force
9 (a) $P = \tau\omega$
10 (b) increases (To conserve angular momentum)
11 (d) line perpendicular to plane of rotation
12 (b) decrease moment of Inertia
13 (c) $\frac{v}{2}$

$\vec{v} = \frac{m_1\vec{v_1}+m_2\vec{v_2}}{m_1+m_2} = \frac{m\times 2v+m(-v)}{2m} = \frac{v}{2}$

14 (a) $\frac{MR^2}{2}$

Moment of inertia of a ring about an axis passing through the centre and perpendicular to its plane $I = MR^2$

Using perpendicular axis theorem, moment of inertia about the diameters AB and CD

$I_{AB} + I_{CD} = I = MR^2$

$2I_{AB} = MR^2$ or $I_{AB} = \frac{MR^2}{2}$

15 (c) $\frac{3MR^2}{2}$

Moment of inertia of a disc about an axis passing through the centre and perpendicular

to the plane of the disc $I = \frac{MR^2}{2}$
According to the parallel axis theorem, moment of inertia about the given axis of rotation $=\frac{MR^2}{2} + MR^2 = \frac{3MR^2}{2}$

16 (d) 100 rpm
from the law of conservation of angular momentum,
$I_1\omega_1 = I_2\omega_2 \rightarrow \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{I_1 40}{\frac{2}{5}I_1} = \frac{200}{2} = 100rpm$

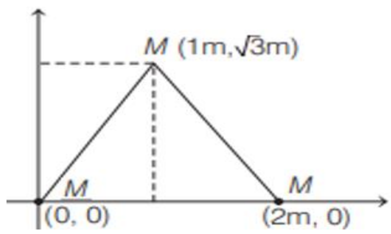
17 (a) $\frac{\pi}{2}$
Given t=60 s, $v_0 = \frac{900}{60} = 15/s$, and $\omega_0 = 2\pi v_0 = 30\pi\ rads^{-1}$, $\omega = 0$
Using $\omega = \omega_0 + \alpha t$, we get 0=30$\pi$ +α 60
Angular acceleration $\alpha = -\frac{\pi}{2}\ rads^{-2}$
Angular retardation $= \frac{\pi}{2}\ rads^{-2}$

18 (c) $\frac{l}{\sqrt{3}}$
Moment of inertia of a thin rod of length l about an axis passing through one of its ends and perpendicular to it is, $I = \frac{Ml^2}{3}$
if K is the radius of gyration, then $I = MK^2$
$MK^2 = \frac{Ml^2}{3}$ or $K = \frac{l}{\sqrt{3}}$

19 (c) $(\frac{-1}{3}, \frac{8}{3})$

Coordinates of the center of mass
$x = \frac{m_1x_1+m_2x_2}{m_1+m_2} = \frac{1\times1+2\times(-1)}{3} = \frac{1}{3}$
$y = \frac{m_1y_1+m_2y_2}{m_1+m_2} = \frac{1\times2+2\times3}{3} = \frac{8}{3}$

20 (b) $\frac{\hat{\imath}}{\sqrt{3}} + \hat{\jmath}$



The x and y coordinates of the centre of mass

$$x = \frac{\sum m_i x_i}{\sum m_i} = \frac{M\times 0 + M\times 1 + M\times 2}{M+M+M} = 1$$

$$y = \frac{\sum m_i y_i}{\sum m_i} = \frac{M\times 0 + M\times\sqrt{3} + M\times 0}{M+M+M} = \frac{\sqrt{3}M}{3M} = \frac{1}{\sqrt{3}}$$

21 (a) $8\hat{\imath} + 10\hat{\jmath} + 12\hat{k}$

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 5 & 2 & -5 \end{vmatrix} = \vec{\imath}(10-2) - \vec{\jmath}(-5-5) + \vec{k}(2+10) = 8\hat{\imath} + 10\hat{\jmath} + 12\hat{k}$$

22 (c) 0.01Nm
M.I = 5×10^{-3} kgm^2 $\omega_0 = 20$ rad/s and t=10s, $\omega = 0$
$\tau = I\alpha = I\times\frac{\omega-\omega_0}{t} = 5\times10^{-3}\times\frac{20}{10} = 0.01Nm$

23 (d) $I_Y = 64I_X$
Moment of inertia of a disc $I = \frac{MR^2}{2}$
$I_x = \frac{1}{2}\times(\rho\times\pi R^2\times t)R^2 = \frac{\pi\rho t}{2}\times R^4$ ....(1) (using mass=volume×density)

$I_y = \frac{1}{2} \times \left(\rho \times \pi(4R)^2 \times \frac{t}{4}\right)(4R)^2 = \frac{64\pi\rho t}{2} \times R^4$……….(2)

$\frac{I_x}{I_y} = \frac{1}{64}$

24 (d) 6 hours

Given $R_2 = \frac{R_1}{2}$

$I_1\omega_1 = I_2\omega_2$ ………………(1)

$\frac{2}{5} \times MR_1^2\left(\frac{2\pi}{T_1}\right) = \frac{2}{5} \times MR_2^2(\frac{2\pi}{T_2})$

$T_2 = \left(\frac{R_2^2}{R_1^2}\right)T_1 = \frac{T_1}{4} = \frac{24}{4}$=6 hours

25 (c) continues to be at the same point at which it was before explosion

Assertion And Reason Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) both A and R are false

26 Assertion: if the ice on the polar caps of the earth melts, the length of the day will increase.
Reason: Moment of inertia of the earth increases as ice on polar cap melts.

27 Assertion: A shell at rest explodes, the centre of mass of the fragments remains at rest.
Reason: In explosion the linear momentum of the system is always conserved.

28 Assertion: Value of the radius of gyration of a body depends on the axis of rotation.
Reason: Radius of gyration is the root mean square distance of the particles of the body from the axis of rotation.

29 Assertion: To unscrew a rusted nut we need a wrench with longer arm.
Reason: Wrench with longer arm reduces the torque of the arm.

30 Assertion: Position of the centre of mass is independent of the reference frame.
Reason: Centre of mass is same for all the bodies.

31 Assertion: A person standing on a rotating platform suddenly stretches his arms, the platform slows down.
Reason: A person by stretching his arms, increases the moment of inertia and decreases the angular velocity.

32 Assertion: The angular speed of a planet around the sun increases, when it is closer to the sun
Reason: The total angular momentum of the planet and sun system does not remain constant.

33 Assertion: Two circular discs of equal masses and thickness made of different material, will have the same moment of inertia about their central axis of rotation.
Reason: Moment of inertia depends upon the mass of the body only.

answers

26. a 27. b 28. b 29. c 30. c 31. a 32. c 33. d

<u>Short Answer Questions (2 Marks)</u>

34 Define centre of mass? Where does the centre of mass of a ring of a homogeneous mass distribution lie?

Centre of mass of a system is a point where the entire mass of the system is supposed to be concentrated.

The centre of mass of the ring lies at the centre of the ring.

35 Show that the centre of mass of a system moves with constant velocity when no external force acts on the system?

If an external force $\vec{F}$ acts on a system of mass M produces an acceleration $\overrightarrow{a_{CM}}$ in its centre of mass, then $\vec{F} = M\overrightarrow{a_{CM}}$.

In the absence of any external force $M\overrightarrow{a_{CM}} = 0$ which means velocity of the centre of mass is constant.

36 State the conditions for equilibrium of a rigid body?

For translation equilibrium of a rigid body, the vector sum of all the forces acting on it must be zero.

For rotational equilibrium of a rigid body, the vector sum of all the torque acting on it must be zero.

37 What is a rigid body? State factors on which the position of centre of mass of a rigid body depends?

A body is said to be a rigid body if it does not undergo any change in its size and shape however large the external force acts on it.

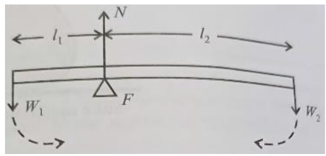
or a rigid body is one in which distance between the constituent particles remains fixed even after the application of external forces.

The position of the centre of mass of a body depends on

(a)the geometrical shape of the body

(b)the distribution of mass in the body

38 State and explain the principle of moments?



When a body is in rotational equilibrium the sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about that point.

Consider a uniform rod free to rotate about a point O.

W_1 and W_2 are two weights hung at distances l_1 and l_2 from the point O.

anticlockwise moment about O = $F_1 \times d_1 = W_1 \times l_1$

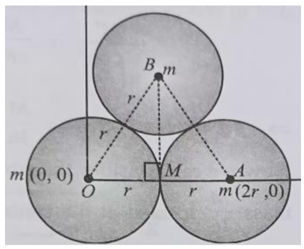
clockwise moment about O = $F_2 \times d_2 = W_2 \times l_2$

39 A 3kg particle is located on the X axis at x_1 = - 4m and a 5kg particle is on the X axis at x_2=2m find the coordinate of centre of mass?

$m_1 = 3kg \quad m_2 = 5kg \quad x_1 = -4m \qquad x_2 = 2m$

$x = \frac{m_1x_1+m_2x_2}{m_1+m_2} = -\frac{1}{4}$

40 Three identical spheres kept on a horizontal floor are touching each other. The mass of each sphere is m and its radius is r find the position of the center of mass of this system?



$x = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3} \qquad y = \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}$

Coordinates of O, A and B are (0,0), (2r,0), (r,$r\sqrt{3}$)

Substituting the values, we get

$x =$ **r and** $\boldsymbol{y = \frac{r}{\sqrt{3}}}$

41 To maintain a rotor at a uniform angular speed of 200 rad/s, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine? (Assume 100%

efficiency)
Angular speed of the rotor, ω = 200 rad/s
Torque required,τ = 180 Nm
The power of the rotor P = τω = 180× 200 = 36 kW
Hence, the power required by the engine is 36 kW.

42 The car of mass 1500 kg moves in a circular race track of radius 50 m with a speed of 40 m/s. What is the magnitude of angular momentum relative to the centre of the track?

$m = 1500kg, r = 50\ m, v = 40m/s$

$L = mvr$=$1500 \times 40 \times 50$ =$3 \times 10^6 \frac{kgm^2}{s}$

43 A rod rotates 30 times in a minute about an axis passing through its centre and perpendicular to the rod. If the kinetic energy of the rod is 200J, find the M.I about the axis of rotation?

K.E = 200J , angular velocity $\omega = 2\pi f$ =$2 \times \pi \times \frac{30}{60}$ = π rad/s

$K.E = \frac{1}{2} \times I\omega^2$ or $I = \frac{2K.E}{\omega^2} = \frac{2\times 200}{\pi^2}$ = $40kgm^2$

44 The angular displacement of a particle is given by$\theta = 5t^2 + 6t - 8$ .Find the angular speed at t=2 seconds?

Angular speed $\omega = \frac{d\theta}{dt}$ = $\frac{d(5t^2+6t-8)}{dt} = 10t + 6$

At t=2s, $\omega = 26\ units$

45 Calculate the angular momentum of the earth rotating about its axis? Take $I = \frac{2MR^2}{5}$,where $M = 6 \times 10^{24}kg\ , R = 6400km$

$L = I\omega$ = $\frac{2MR^2}{5} \times \frac{2\pi}{T}$ = $7.14 \times 10^{33}kgm^2/s$ T=86400 s

46 A planet revolves around a massive star in a highly elliptical orbit. Is its angular momentum constant over the entire orbit. Give reason?

A planet revolves around the star under the effect of gravitational force since the force is radial, and does not contribute towards torque. Thus, in the absence of an external torque, angular momentum of the planet remains constant.

Short answer questions (3marks)

47 A ring of diameter 0.4m and of mas10 kg is rotating about its axis at the rate of 2100rpm. Find the moment of inertia, angular momentum and rotational kinetic energy of the ring?

$R = 0.2m, M = 10kg, f = 2100rpm = 35\ Hz$

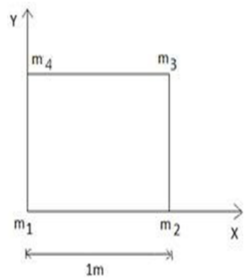
$\omega = 2\pi f$ =$220\ rad/s$

(a) $I = MR^2$= $0.4\ kgm^2$

(b) $L = I\omega = 88 \frac{kgm^2}{s}$

(c) Rotational K.E=$\frac{1}{2} \times I\omega^2$= 9680 J

48 Four particles of mass 1kg, 2kg, 3kg and 4kg are placed at the four vertices A, B, C and D of square of side 1m. Find the position of centre of mass of the particle?



$x = \frac{m_1x_1+m_2x_2+m_3x_3+m_4x_4}{m_1+m_2+m_3+m_4}$, y= $\frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4}{m_1+m_2+m_3+m_4}$

Thus, centre of mass (0.5m, 0.7m)

49 Show that angular momentum of a satellite of mass M_s revolving around the earth

having mass M_e in an orbit of radius r is $\sqrt{GM_eM_S^2r}$?

Suppose that the satellite revolves around the earth with an orbital speed v.
The gravitational force between the earth and the satellite provides the necessary centripetal force to revolve in the circular orbit.

So the centripetal force $F = \frac{M_S v^2}{r}$ ………….(1)

Gravitational force between the satellite and the earth is

$$F = \frac{GM_eM_S}{r^2} \quad \text{…………(2)}$$

From (1) and (2) we get $\frac{M_S v^2}{r} = \frac{GM_eM_S}{r^2}$

$$v = \sqrt{\frac{GM_e}{r}}$$

Angular momentum $L = M_S vr = \sqrt{GM_eM_S^2r}$

50 Derive the equations of rotational motion where the symbols have their usual meanings?

To derive (i) $\omega = \omega_0 + \alpha t$ (ii) $\theta - \theta_0 = \omega_0 t + \frac{1}{2} \times \alpha t^2$ (iii) $\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$

51 What will be the duration of the day if the earth suddenly shrinks to 1/64 of its original volume, mass remaining unchanged? Given Moment of inertia of earth= $\frac{2}{5} \times MR^2$.

Let T_1 and T_2 be the periods of revolution before and after its contraction; and R_1 and R_2 be the radii before and after contraction.

According to the principle of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2 \quad \text{……………….(1)}$$

$$\frac{2}{5} \times MR_1^2\left(\frac{2\pi}{T_1}\right) = \frac{2}{5} \times MR_2^2\left(\frac{2\pi}{T_2}\right)$$

$$T_2 = \left(\frac{R_2^2}{R_1^2}\right)T_1 \quad \text{…………………(2)}$$

As the earth shrinks to 1/64 of its volume,

$$\frac{4}{3} \times \pi R_2^3 = \frac{1}{64} \times \frac{4}{3}\pi R_1^3$$

$$\frac{R_2}{R_1} = \frac{1}{4} \quad \text{……………(3)}$$

Substituting (3) in (2),

$$T_2 = \left(\frac{1}{4}\right)^2 \times 24 = 1.5 \text{ hours}$$

52 Using expression for power and kinetic energy of rotational motion, derive the relation $\tau = I\alpha$?

In rotational motion, power $P = \tau\omega$ ……….(1)

and $K.E = \frac{1}{2} \times I\omega^2$

Also power =rate of change of kinetic energy

$$P = \frac{d\left(\frac{I\omega^2}{2}\right)}{dt} = \frac{1}{2} \times 2\omega \times \frac{d\omega}{dt}$$

$$P = I\omega\alpha \quad \text{……………..(2)}$$

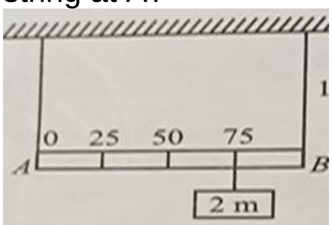
From (1) and (2) we get, $\tau\omega = I\omega\alpha$

$$\tau = I\alpha$$

Case Based Questions

53 Why do we need spanners of different sizes? A spanner is used to tighten a nut on a bolt. Nuts and Bolts are of different sizes and need different magnitudes of torque to do the tightening process precisely. Torque is product of the magnitude of the force and

perpendicular distance of the line of action of the force from the axis of rotation.
Based on the above information, answer the following questions?
(a)While applying force, if we increase the perpendicular distance of the line of action of the force from the axis of rotation then the torque
(i) increases
(ii) decreases
(iii) remain unchanged
(iv) cannot be predicted.
(b)If sum of the clockwise torque equals sum of the anticlockwise torque, then angular acceleration is
(i)constant
(ii)zero,
(iii)negative
(iv)positive
(c)A force F = $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ N acts at a point $\hat{4\imath} + 3\hat{\jmath} - \hat{k}$ m, then the magnitude of torque about the point $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ m will be $\sqrt{x}$ Nm, Find the torque?
OR
Shown in the figure is rigid and uniform 1metre long rod AB held in a horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass m and has another weight of mass 2m hung at a distance of 75 cm from A. Find the tension in the string at A?



ans (a) Increases
(b) zero
(c) $\sqrt{195}\, Nm$
or
mg

54 The moment of a force is the rotational analogue of force in linear motion, the quantity angular momentum is the rotational analogue of linear momentum. Angular momentum is also called as moment of (linear) momentum.

Consider a particle of mass m and linear momentum v at a position r relative to the origin O. The angular momentum L of the particle with respect to the origin O is defined as, $\vec{L} = \vec{r} \times \vec{p}$ or $L = rp \sin\theta$ where θ is the angle between $\vec{r}$ and $\vec{p}$.

The time rate of change of the angular momentum of a particle is equal to the torque acting on it and given by $\vec{\tau} = \frac{d\vec{L}}{dt}$

This is the rotational analogue of the equation $\overrightarrow{F} = \frac{d\vec{p}}{dt}$ which expresses Newton's second law for the translational motion of a single particle.
(a) Angular momentum is
(i) Vector
(ii) Scalar
(iii)Tensor
(b) The total angular momentum of the system is conserved if

(i) total external force become zero
(ii) total external torque become zero
(iii) total linear momentum become zero
(iv) None of these

(c)State and prove the law of conservation of angular momentum?

or

Prove that the rate of change of angular momentum equals torque?

Ans (a) Vector (b) (ii) total external torque become zero

Long answer type questions (to be practiced by the student)

55 Define the centre of mass of a system? Derive expression for the position, velocity and acceleration of centre of mass of a two-particle system?

56 What do you mean by radius of gyration? Show that the radius of gyration is equal to the root mean square distance of the particles from the axis of rotation?

57 Explain moment of a force? Show that torque is given by the product of force and the perpendicular distance from the point or axis of rotation?

58 Define moment of inertia? The moment of inertia of a uniform circular ring about an axis passing through the centre and perpendicular to the plane is MR^2. Hence write the expression for moment of inertia (i) about its diameter (ii) about a tangent perpendicular to the plane of the ring and (iii)about a tangent in the plane of the ring?

(i) $\frac{MR^2}{2}$ (ii) $2MR^2$ (iii) $\frac{3MR^2}{2}$

<u>Competency Based Questions</u> (From JEE and NEET exams)

59 Three objects A (solid sphere), B (thin circular disc) and C (circular ring) each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axis. The amount of work W required to bring them to rest, would satisfy the relation

(a) $W_C > W_B > W_A$
(b) $W_A > W_B > W_C$
(c) $W_A > W_C > W_B$

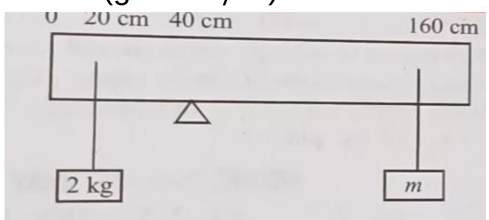
(d) $W_B > W_A > W_C$

60 Two particles of mass 5kg and 10kg respectively are attached to the two ends of a rigid rod of length 1m with negligible mass. the center of mass of the system from the 5kg particle is nearly at a distance of

(a) 50cm
(b) 67cm
(c) 80cm
(d) 33cm

61 A uniform rod of length 200cm and mass 500 g is balanced on a wedge placed at 40 cm mark. The mass of 2kg is suspended from the rod at 20 cm and another unknown mass m is suspended from the rod at 160 cm mark as shown in figure. Find the value of m such that the rod is in equilibrium? (g =$10m/s^2$)

(a) $\frac{1}{12}kg$
(b) $\frac{1}{2}kg$
(c) $\frac{1}{3}\ kg$
(d) $\frac{1}{6}kg$



62 A disc of radius 2m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20 cm/s, how much work is needed to stop it?

Answers for competency based questions

59 (a) $W_C > W_B > W_A$

Work done equals change in rotational kinetic energy

$$\Delta W = \frac{1}{2} \times I\omega^2$$

Angular speed being same, the work done depends on moment of inertia.

M.I for solid sphere= $\frac{2MR^2}{5}$, M.I for disc= $\frac{MR^2}{2}$, M.I for ring = MR^2 ,

$W_C > W_B > W_A$

60 (b) 67cm

Taking the 5kg mass at the origin then,

$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{5 \times 0 + 10 \times 100}{15} = \frac{200}{3} = 66.67cm$

61 (a) $\frac{1}{12}kg$

The rod will be in equilibrium at the net torque is zero.

Torque of the 2kg mass about the wedge = torque due to the mass of the rod +torque due to the mass m

$20 \times 0.2 = 5 \times 0.6 + 10m \times 1.2$

$1 = 12m$

Or $m = \frac{1}{12}kg$

62 Work done to stop the disc = change in total kinetic energy of the disc.

Final K.E = zero

Initial K.E = translational K.E + Rotational K.E

$= \frac{1}{2} \times mv^2 + \frac{1}{2} \times I\omega^2 = \frac{1}{2} \times mv^2 + \frac{1}{2} \times mR^2(\frac{v}{R})^2 = \frac{3mv^2}{4}$

=3J

Unit VI Gravitation

Chapter 8: Gravitation

GIST OF THE CHAPTER:

Kepler's laws of planetary motion,

Universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy and gravitational potential

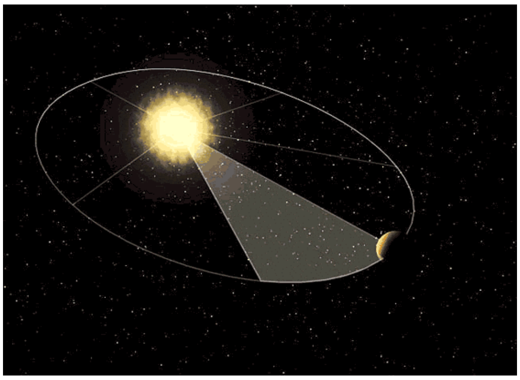
There is a tendency of all material objects to be attracted towards the earth. Italian Physicist Galileo (1564-1642) who recognized the fact that all bodies, irrespective of their masses, are accelerated towards the earth with a constant acceleration.

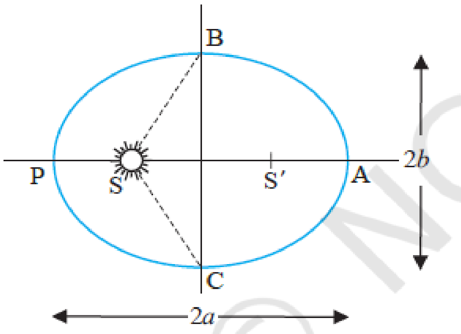
'Geocentric' model -(Ptolemy about 2000 years ago) - All celestial objects, stars, the sun and the planets, all revolved around the earth.

'Heliocentric' model – (Aryabhatta ,5th century A.D.) (Nicolas Copernicus 1473-1543)- All the planets moved in circles around a fixed central sun.

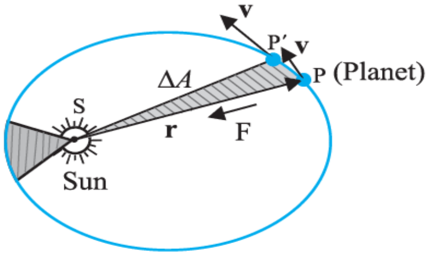
KEPLER'S LAWS OF PLANETARY MOTION

1. Law of orbits : All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.





2. Law of areas : The line that joins any planet to the sun sweeps equal areas in equal
intervals of time. i.e. dA/dT= constant.



3. Law of periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet. i.e T^2/R^3=constant

Animation of Kepler's Law
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HYPERLINK "https://www.sciencephoto.com/media/901940/view/kepler-s-2nd-law-of-planetary-motion" https://www.sciencephoto.com/media/901940/view/kepler-s-2nd-law-of-planetary-motion

A **central force** is such that the force on the planet is along the vector joining the Sun and the planet.

Geometrical Meaning of Angular momentum and Keplers's Second law

Let the Sun be at the origin and let the position and momentum of the planet be denoted by **r** and **p** respectively. Then the area swept out by the planet of mass m in time interval △*t* is △**A** given by

△**A** = ½ (**r** × **v**△*t*)

△**A** /△t =½ (**r** × **p**)/m, (since **v** = **p**/m)

= **L** / (2 m)

where **v** is the velocity, **L** is the angular momentum equal to (**r** × **p**).

since L =constant i.e.

△**A** /△t= constant

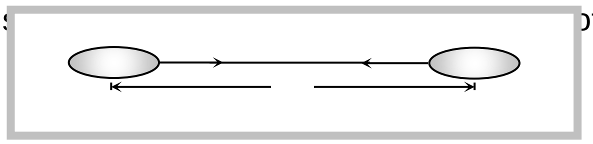
This is Kepler's second law.

UNIVERSAL LAW OF GRAVITATION

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

The direction of the force is along the line joining the particles.

Thus the magnitude of the gravitational force *F* that two particles of masses m_1 and m_2 s[...]other is given by $F \propto \frac{m_1 m_2}{r^2}$



or

EMBED Equation.3 = unit vector from *A* to *B*

EMBED Equation.3 = unit vector from *B* to *A*,

EMBED Equation.3= gravitational force exerted on body *A* by body *B*

EMBED Equation.3= gravitational force exerted on body *B* by body *A*

Vector form : According to Newton's law of gravitation

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r}_{21} = \frac{-Gm_1m_2}{r^3}\vec{r}_{21} = \frac{-Gm_1m_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

Here negative sign indicates that the direction of $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$.

Similarly $\vec{F}_{21} = \frac{-Gm_1m_2}{r^2}\hat{r}_{12} = \frac{-Gm_1m_2}{r^3}\vec{r}_{12} = \frac{-Gm_1m_2}{|\vec{r}_{12}|^3}\vec{r}_{12}$

$= \frac{Gm_1m_2}{r^2}\hat{r}_{21}$ [$\because \hat{r}_{12} = -\hat{r}_{21}$]

$\therefore$ It is clear that $\vec{F}_{12} = -\vec{F}_{21}$. Which is **Newton's third law of motion**.

Here G is constant of proportionality which is called 'Universal gravitational constant'.

If $m_1 = m_2$ and $r = 1$ then $G = F$

i.e. universal gravitational constant is equal to the force of attraction between two bodies each of unit mass whose centres are placed unit distance apart.

Important points:

(i) The value of G in the laboratory was first determined by Cavendish using the torsional balance.

(ii) The value of G is 6.67×10^{-11} $N\text{–}m^2\ kg^{-2}$ in S.I. and 6.67×10^{-8} $dyne\text{-}\ cm^2\text{-}g^{-2}$ in C.G.S. system.

(iii) Dimensional formula $[M^{-1}L^3T^{-2}]$.

(iv) The value of G does not depend upon the nature and size of the bodies.

(v) It also does not depend upon the nature of the medium between the two bodies.

(vi) As G is very small hence gravitational forces are very small, unless one (or both) of the masses is huge.

ACCELERATION DUE TO GRAVITY OF THE EARTH

The force of attraction exerted by the earth on a body is **called gravitational pull or gravity**.

We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

The acceleration produced in the motion of a body under the effect of gravity is called **acceleration due to gravity, it is denoted by *g*.**

Consider a body of mass m is lying on the surface of earth then gravitational force on the body is given by

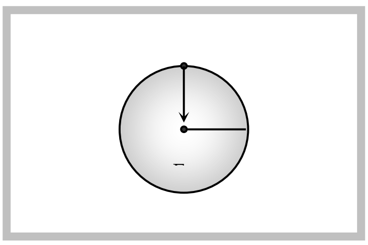
$$F = \frac{GMm}{R^2} \quad .....(i)$$

Where M = mass of the earth and R = radius of the earth.

If g is the acceleration due to gravity, then the force on the body due to earth is given by

Force = mass × acceleration

or $F = mg$ .....(ii)



From (i) and (ii) we have $mg = \frac{GMm}{R^2}$

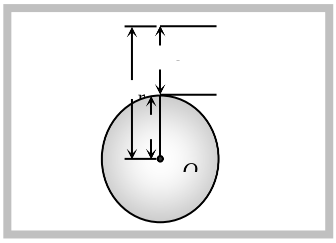
$$\therefore \quad g = \frac{GM}{R^2} \quad .....(iii)$$

$\Rightarrow \quad g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$ [As mass (M) = volume ($\frac{4}{3}\pi R^3$) × density (ρ)]

$\therefore \quad g = \frac{4}{3}\pi\rho G R$ …..(iv)

Variation in *g* With Height.

Acceleration due to gravity at the surface of the earth



$$g = \frac{GM}{R^2} \quad .....(i)$$

Acceleration due to gravity at height *h* from the surface of the earth

$$g' = \frac{GM}{(R+h)^2} \quad .....(ii)$$

From (i) and (ii) $g' = g\left(\frac{R}{R+h}\right)^2$ .....(iii)

$= g\frac{R^2}{r^2}$ .....(iv) [As $r = R + h$]

Important Points

(i) As we go above the surface of the earth, the value of *g* decreases because $g' \propto \frac{1}{r^2}$.

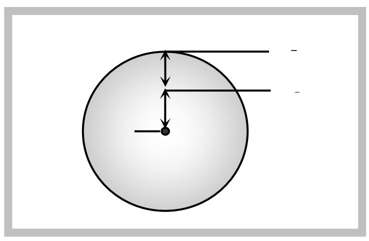
(ii) If $r = \infty$ then $g' = 0$, *i.e.*, at infinite distance from the earth, the value of *g* becomes zero.

(iii) If $h << R$ *i.e.*, height is negligible in comparison to the radius then from equation (iii) we get

$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2} = g\left[1-\frac{2h}{R}\right]$ [As $h << R$]

Variation in *g* With Depth.

Acceleration due to gravity at the surface of the earth



$$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho G R \quad …..(i)$$

Acceleration due to gravity at depth *d* from the surface of the earth

$$g' = \frac{4}{3}\pi\rho G(R-d) \quad …..(ii)$$

From (i) and (ii) $g' = g\left[1-\frac{d}{R}\right]$

GRAVITATIONAL POTENTIAL.

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point *i.e.,*

$$V = -\frac{W}{m} = -\int \frac{\vec{F}.\vec{dr}}{m} = -\int \vec{I}.\vec{dr} \qquad \left[\text{As } \frac{F}{m} = I\right]$$

$$\therefore \quad I = -\frac{dV}{dr}$$

i.e., negative gradient of potential gives intensity of field or potential is a scalar function of position whose space derivative gives intensity. Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

Important points

(i) It is a scalar quantity because it is defined as work done per unit mass.

(ii) Unit : *Joule/kg* or m^2/sec^2

(iii) Dimension : $[M^0L^2T^{-2}]$

(iv) If the field is produced by a point mass then

$$V = -\int I\, dr = -\int \left(-\frac{GM}{r^2}\right) dr \qquad \left[\text{As } I = -\frac{GM}{r^2}\right]$$

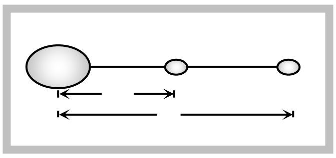
$$\therefore \quad V = -\frac{GM}{r} + c \qquad [\text{Here } c = \text{constant of integration}]$$

Assuming reference point at ∞ and potential to be zero there we get

$$0 = -\frac{GM}{\infty} + c \Rightarrow c = 0$$

$$\therefore \quad \text{Gravitational potential } V = -\frac{GM}{r}$$

(v) **Gravitational potential difference** : It is defined as the work done to move a unit mass from one point to the other in the gravitational field. The gravitational potential difference in bringing unit test mass m from point A to point B under the gravitational influence of source mass M is



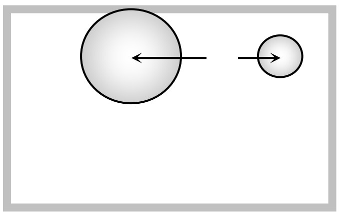
$$\Delta V = V_B - V_A = \frac{W_{A\to B}}{m} = -GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Gravitational Potential Energy.

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = \int_{\infty}^{r} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GMm}{r}$$



This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

Important Points

(i) Potential energy is a scalar quantity.

(ii) *Unit* : Joule

(iii) *Dimension* : $[ML^2T^{-2}]$

(iv) Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

(v) As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

(vi) If $r = \infty$ then it becomes zero (maximum)

(vii) In case of discrete distribution of masses

Gravitational potential energy $U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$

(viii) If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$ or $\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

ESCAPE VELOCITY.

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$W = \int_R^{\infty} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow \quad W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity, then kinetic energy which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \quad \frac{1}{2}mv_e^2 = \frac{GMm}{R} \quad \Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \quad v_e = \sqrt{2gR} \qquad [\text{As } GM = gR^2]$$

or $v_e = \sqrt{2 \times \frac{4}{3}\pi\rho GR \times R}$ $\Rightarrow$ $v_e = R\sqrt{\frac{8}{3}\pi G\rho}$ [As $g = \frac{4}{3}\pi\rho GR$]

Important points

(i) Escape velocity is independent of the mass and direction of projection of the body.

(ii) Escape velocity depends on the reference body. Greater the value of (M/R) or (gR) for a planet, greater will be escape velocity.

(iii) For the earth as $g = 9.8 m/s^2$ and $R = 6400 km$

$$\therefore v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 km/\sec$$

Case:

If a body is project with velocity greater than escape velocity $(v > v_e)$ then by conservation of energy.

Total energy at surface = Total energy at infinite

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(v')^2 + 0$$

i.e., $(v')^2 = v^2 - \frac{2GM}{R}$ $\Rightarrow$ $v'^2 = v^2 - v_e^2$ [As $\frac{2GM}{R} = v_e^2$]

$\therefore$ $v' = \sqrt{v^2 - v_e^2}$

i.e, the body will move in interplanetary or inter stellar space with velocity $\sqrt{v^2 - v_e^2}$.

ORBITAL VELOCITY OF SATELLITE.

Satellites are natural or artificial bodies describing orbit around a planet under its gravitational attraction.

Moon is a natural satellite while INSAT-1B is an artificial satellite of earth.

Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth.

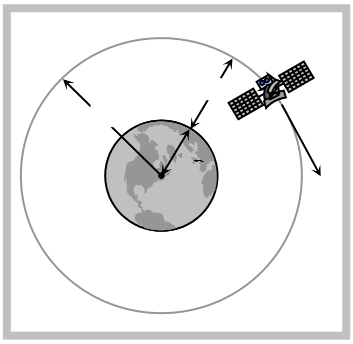
Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$\Rightarrow$ $v = \sqrt{\frac{GM}{r}}$

$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$ [As $GM = gR^2$ and $r = R + h$]



Important points:

(i) Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit *i.e.,* satellites of deferent masses have same orbital velocity, if they are in the same orbit.

(ii) Orbital velocity depends on the mass of central body and radius of orbit.

(iii) For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite $\left(v \propto 1/\sqrt{r}\right)$.

(iv) Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \quad \therefore \; v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$ [As $h = 0$ and $GM = gR^2$]

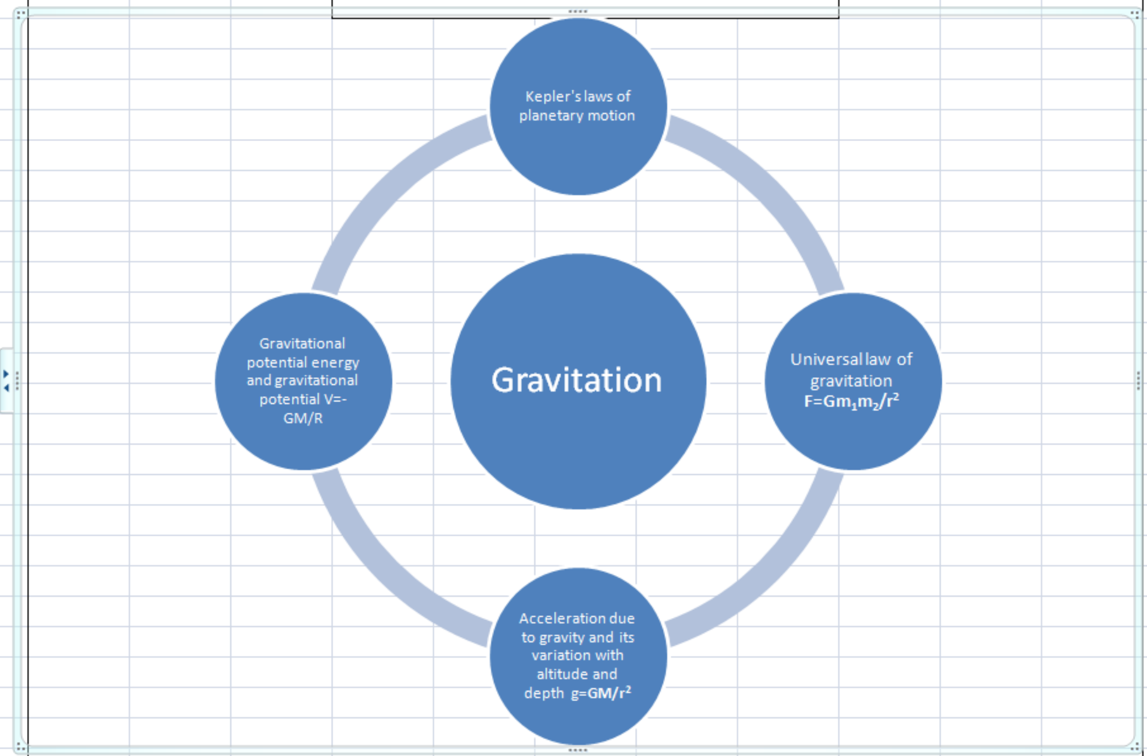
For the earth $v = \sqrt{9.8 \times 6.4 \times 10^6} = 7.9\,km/s \approx 8\,km/sec$

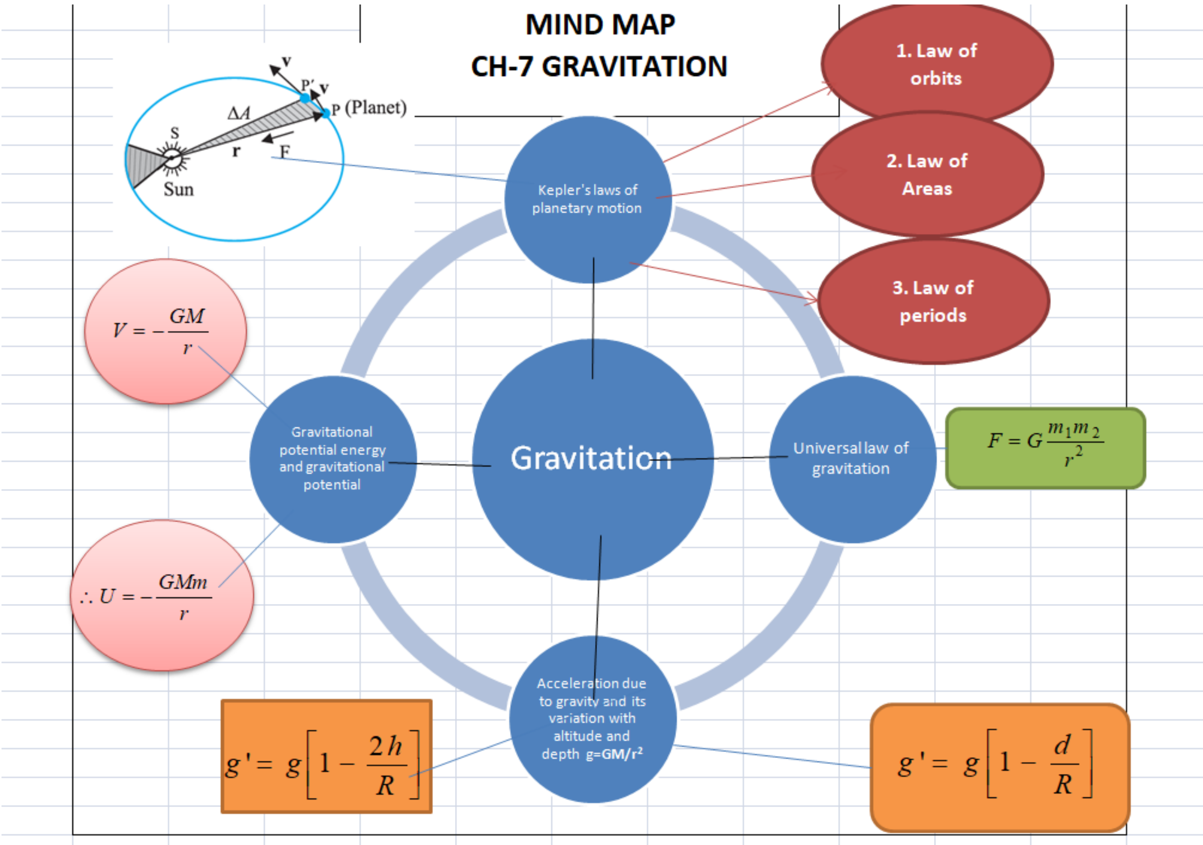
(v) Close to the surface of planet $v = \sqrt{\frac{GM}{R}}$ [As $v_e = \sqrt{\frac{2GM}{R}}$]

$\therefore$ $v = \frac{v_e}{\sqrt{2}}$ *i.e.,* $v_{escape} = \sqrt{2}\; v_{orbital}$

It means that if the speed of a satellite orbiting close to the earth is made $\sqrt{2}$ times (or increased by 41%) then it will escape from the gravitational field.

MIND MAP
CH-7 GRAVITATION





Competency Based Questions (MCQ)

Kepler's law

Q 1. The distance of a planet from the sun is 5 times the distance between the earth and the sun. The Time period of the planet is **[UPSEAT 2003]**

(a) $5^{3/2}$ *years* (b) $5^{2/3}$ *years* (c) $5^{1/3}$ *years* (d) $5^{1/2}$ *years*

Ans: (a) According to Kepler's law $T \propto R^{3/2}$ $\therefore$ $T_{planet} = (5)^{3/2} T_{earth} = 5^{(3/2)} \times 1\, year$ $= 5^{3/2}$ *years* .

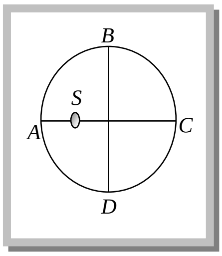
Q 2. In planetary motion the areal velocity of position vector of a planet depends on angular velocity (ω) and the distance of the planet from sun (*r*). If so the correct relation for areal velocity is

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$ (c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

Ans : (c) $\frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m} = \frac{1}{2}\omega r^2$ [As Angular momentum $L = mvr$ and $v = r\omega$]

$\therefore \frac{dA}{dt} \propto \omega r^2$.

Q 3. The planet is revolving around the sun as shown in elliptical path. The correct option is



(a) The time taken in travelling *DAB* is less than that for *BCD*

(b) The time taken in travelling *DAB* is greater than that for *BCD*

(c) The time taken in travelling *CDA* is less than that for *ABC*

(d) The time taken in travelling *CDA* is greater than that for *ABC*

Ans : (a) When the planet passes nearer to sun then it moves fast and vice-versa. Hence the time taken in travelling *DAB* is less than that for *BCD.*

Q 4. The distance of Neptune and Saturn from sun are nearly 10^{13} and 10^{12} meters respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio

(a) $\sqrt{10}$ (b) 100 (c) $10\sqrt{10}$ (d) $1/\sqrt{10}$

Ans : (c) Kepler's third law $T^2 \propto R^3$ $\therefore \frac{T_{Neptune}}{T_{Saturn}} = \left(\frac{R_{Neptune}}{R_{Saturn}}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10\sqrt{10}$.

Q 5. A satellite *A* of mass *m* is at a distance of *r* from the centre of the earth. Another satellite *B* of mass 2*m* is at distance of 2*r* from the earth's centre. Their time periods are in the ratio of

(a) 1 : 2 (b) 1 : 16 (c) 1 : 32 (d) $1:2\sqrt{2}$

Ans : (d) Time period does not depend upon the mass of satellite, it only depends upon the orbital radius.

According to Kepler's law $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \frac{1}{2\sqrt{2}}$.

Newton's law of gravitation

Q 6. The gravitational force between two objects does not depend on

(a) Sum of the masses (b) Product of the masses

(c) Gravitational constant (d) Distance between the masses

Ans : (a) $F = \frac{\text{Gravitational constant} \times \text{product of the masses}}{(\text{Distance between the masses})^2}$.

Q 7. The mass of the moon is about 1.2% of the mass of the earth. Compared to the gravitational force the earth exerts on the moon, the gravitational force the moon exerts on earth

[SCRA 1998]

(a) Is the same (b) Is smaller (c) Is greater (d) Varies with its phase

Ans : (a) Earth and moon both exerts same force on each other.

Q 8. Three identical point masses, each of mass 1*kg* lie in the *x*-*y* plane at points (0, 0), (0, 0.2*m*) and (0.2*m*, 0). The net gravitational force on the mass at the origin is

(a) $1.67 \times 10^{-9}(\hat{j}+\hat{j})N$ (b) $3.34 \times 10^{-10}(\hat{i}+\hat{j})N$

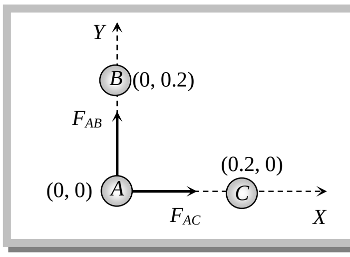
(c) $1.67 \times 10^{-9}(\hat{i}-\hat{j})N$ (d) $3.34 \times 10^{-10}(\hat{i}+\hat{j})N$

Ans : (a) Let particle A lies at origin, particle B and C on y and x-axis respectively

$$\overrightarrow{F_{AC}} = \frac{G m_A m_B}{r_{AB}^2}\hat{i} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(0.2)^2}\hat{i} = 1.67 \times 10^{-9}\,\hat{i}\,N$$

Similarly $\overrightarrow{F_{AB}} = 1.67 \times 10^{-9}\,\hat{j}\,N$

$\therefore$ Net force on particle *A* $\vec{F} = \vec{F}_{AC} + \vec{F}_{AB} = 1.67 \times 10^{-9}(\hat{i}+\hat{j})\,N$



Acceleration due to gravity

Q 9. Acceleration due to gravity on moon is 1/6 of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_m) and moon (ρ_e) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$ then radius of moon R_m in terms of R_e will be

(a) $\frac{5}{18}R_e$ (b) $\frac{1}{6}R_e$ (c) $\frac{3}{18}R_e$ (d) $\frac{1}{2\sqrt{3}}R_e$

Ans : (a) Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR \;\therefore\; g \propto \rho R$ or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e}.\frac{R_m}{R_e}$ [As $\frac{g_m}{g_e} = \frac{1}{6}$ and $\frac{\rho_e}{\rho_m} = \frac{5}{3}$ (given)]

$$\therefore \frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6} \times \frac{5}{3} \quad \therefore \quad R_m = \frac{5}{18}R_e$$

Q 10. A spherical planet far out in space has a mass M_0 and diameter D_0. A particle of mass *m* falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to

(a) GM_0/D_0^2 (b) $4mGM_0/D_0^2$ (c) $4GM_0/D_0^2$ (d) GmM_0/D_0^2

Ans : (c) We know $g = \frac{GM}{R^2} = \frac{GM}{(D/2)^2} = \frac{4GM}{D^2}$

If mass of the planet $= M_0$ and diameter of the planet $= D_0$. Then $g = \frac{4GM_0}{D_0^2}$.

Q 11. The moon's radius is 1/4 that of the earth and its mass is 1/80 times that of the earth. If *g* represents the acceleration due to gravity on the surface of the earth, that on the surface of the moon is

(a) $\frac{g}{4}$ (b) $\frac{g}{5}$ (c) $\frac{g}{6}$ (d) $\frac{g}{8}$

Ans : (b) Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\therefore \frac{g_{moon}}{g_{earth}} = \frac{M_{moon}}{M_{earth}} \cdot \frac{R_{earth}^2}{R_{moon}^2} = \left(\frac{1}{80}\right)\left(\frac{4}{1}\right)^2$$

$$g_{moon} = g_{earth} \times \frac{16}{80} = \frac{g}{5}.$$

Q 12. If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earth's surface would

(a) Decrease by 2% (b)Remain unchanged (c)Increase by 2% (d) Increase by 1%

Ans : (c) We know $g \propto \frac{1}{R^2}$ [As R decreases, *g* increases]

So % change in $g = 2$ (% change in R) $= 2 \times 1\% = 2\%$

$\therefore$ acceleration due to gravity increases by 2%.

Q 13. Mass of moon is $7.34 \times 10^{22} kg$. If the acceleration due to gravity on the moon is 1.4*m*/*s*^{2}, the radius of the moon is $(G = 6.667 \times 10^{-11} Nm^2/kg^2)$

[AFMC 1998]

(a) $0.56 \times 10^4 m$ (b) $1.87 \times 10^6 m$ (c) $1.92 \times 10^6 m$ (d) $1.01 \times 10^8 m$

Ans : (b) We know $g = \frac{GM}{R^2}$ $\therefore$ $R = \sqrt{\frac{GM}{g}} = \sqrt{\frac{6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{1.4}}$

$= 1.87 \times 10^6 m$.

***Q* 14.** The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is

(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$ (b) $g_1 : g_2 = R_1 R_2 : \rho_1 \rho_2$

(c) $g_1 : g_2 = R_1 \rho_2 : R_2 \rho_1$ (d) $g_1 : g_2 = R_1 \rho_1 : R_2 \rho_2$

Ans : (d) Acceleration due to gravity $g = \frac{4}{3}\pi\rho G R$ $\therefore$ $g_1 : g_2 = R_1 \rho_1 : R_2 \rho_2$.

Variation in g with height

***Q* 15.** The acceleration of a body due to the attraction of the earth (radius *R*) at a distance 2*R* from the surface of the earth is (*g* = acceleration due to gravity at the surface of the earth)

(a) $\frac{g}{9}$ (b) $\frac{g}{3}$ (c) $\frac{g}{4}$ (d) *g*

Ans : (a) $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+2R}\right)^2 = \frac{1}{9}$ $\therefore$ $g' = \frac{g}{9}$.

***Q* 16.** The height of the point vertically above the earth's surface, at which acceleration due to gravity becomes 1% of its value at the surface is (Radius of the earth = *R*)

(a) 8*R* (b) 9*R* (c) 10 *R* (d) 20*R*

Ans : (b) Acceleration due to gravity at height h is given by $g' = g\left(\frac{R}{R+h}\right)^2$

$$\Rightarrow \frac{g}{100} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{10} \Rightarrow h = 9R.$$

Q 17. At surface of earth weight of a person is 72 *N* then his weight at height *R*/2 from surface of earth is (*R* = radius of earth) **[CBSE PMT 2000; AIIMS 2000]**

(a) 28*N* (b) 16*N* (c) 32*N* (d) 72*N*

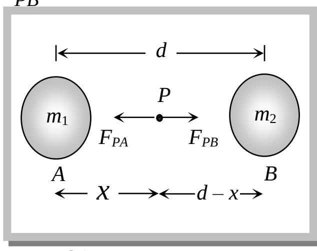
Ans : (c) Weight of the body at height *R*, $W' = W\left(\frac{R}{R+h}\right)^2 = W\left(\frac{R}{R+\frac{R}{2}}\right)^2$

$$= W\left(\frac{2}{3}\right)^2 = \frac{4}{9}W = \frac{4}{9} \times 72 = 32\,N.$$

Q 18. If the distance between centres of earth and moon is *D* and the mass of earth is 81 times the mass of moon, then at what distance from centre of earth the gravitational force will be zero

(a) *D*/2 (b) 2*D*/3 (c) 4*D*/3 (d) 9*D*/10

Ans : (d) If *P* is the point where net gravitational force is zero then $F_{PA} = F_{PB}$



$$\frac{Gm_1 m}{x^2} = \frac{Gm_2 m}{(d-x)^2}$$

By solving $x = \frac{\sqrt{m_1}\,d}{\sqrt{m_1} + \sqrt{m_2}}$

For the given problem $d = D$, $m_1 =$ earth, $m_2 =$ moon and $m_1 = 81 m_2$

$$\therefore m_2 = \frac{m_1}{81}$$

So $x = \frac{\sqrt{m_1}\,D}{\sqrt{m_1} + \sqrt{m_2}} = \frac{\sqrt{m_1}\,D}{\sqrt{m_1} + \sqrt{\frac{m_1}{81}}} = \frac{D}{1 + \frac{1}{9}} = \frac{9D}{10}$

<u>Variation in g with depth</u>

Q19. Weight of a body of mass *m* decreases by 1% when it is raised to height *h* above the earth's surface. If the body is taken to a depth h in a mine, change in its weight is

(a) 2% decrease (b) 0.5% decrease (c) 1% increase (d) 0.5% increase

Ans : (b) Percentage change in *g* when the body is raised to height h, $\frac{\Delta g}{g} \times 100\ \% = \frac{2h \times 100}{R} = 1\%$

Percentage change in *g* when the body is taken into depth *d*, $\frac{\Delta g}{g} \times 100\ \% = \frac{d}{R} \times 100\ \% = \frac{h}{R} \times 100\ \%$ [As $d = h$]

$\therefore$ Percentage decrease in weight $= \frac{1}{2}\left(\frac{2h}{R} \times 100\right) = \frac{1}{2}(1\%) = 0.5\%$.

Q 20. The depth at which the effective value of acceleration due to gravity is $\frac{g}{4}$ is (R = radius of the earth)

(a) *R* (b) $\frac{3R}{4}$ (c) $\frac{R}{2}$ (d) $\frac{R}{4}$

Ans : (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \frac{3R}{4}$

Q 21. Assuming earth to be a sphere of a uniform density, what is the value of gravitational acceleration in a mine 100 *km* below the earth's surface (Given *R* = 6400*km*)

(a) $9.66\, m/s^2$ (b) $7.64\, m/s^2$ (c) $5.06\, m/s^2$ (d) $3.10\, m/s^2$

Ans : (a) Acceleration due to gravity at depth *d*, $g' = g\left[1 - \frac{d}{R}\right] = g\left[1 - \frac{100}{6400}\right]$

$= 9.8\left[1 - \frac{1}{64}\right] = 9.8 \times \frac{63}{64} = 9.66\, m/s^2$.

Q 22. The depth *d* at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface, is [*R* = radius of the earth]

[MP PMT 1999]

(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$ (c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

Ans : (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{d}{R} = 1 - \frac{1}{n} \Rightarrow d = \left(\frac{n-1}{n}\right)R$

Gravitational potential

Q 23. In some region, the gravitational field is zero. The gravitational potential in this region

(a) Must be variable (b) Must be constant
(c) Cannot be zero (d) Must be zero

Ans : (b) As $I = -\frac{dV}{dx}$, if $I = 0$ then *V* = constant.

Q 24. The gravitational potential due to the earth at infinite distance from it is zero. Let the gravitational potential at a point *P* be $-5\, J/kg$. Suppose, we arbitrarily assume the gravitational potential at infinity to be + 10 J/kg, then the gravitational potential at *P* will be

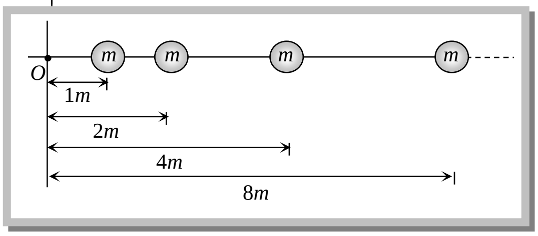
(a) $-5\ J/kg$ (b) $+5\ J/kg$ (c) $-15\ J/kg$ (d) $+15\ J/kg$

Ans : (b) Potential increases by $+10\, J/kg$ every where so it will be $+10-5=+5J/kg$ at P

Q 25. An infinite number of point masses each equal to *m* are placed at *x* =1. *x* = 2, *x* = 4, *x* = 8 ……… What is the total gravitational potential at *x* = 0

(a) $-Gm$ (b) $-2Gm$ (c) $-4Gm$ (d) $-8Gm$

Ans : (b) Net potential at origin $V = -\left[\frac{Gm}{r_1} + \frac{Gm}{r_2} + \frac{Gm}{r_3} + ...\right]$



$= -Gm\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right] = -Gm\left(\frac{1}{1-\frac{1}{2}}\right) = -2Gm$

Potential Energy

Q 26. Energy required to move a body of mass *m* from an orbit of radius $2R$ to $3R$ is

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$ (c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

Ans : (d) Work done = Change in potential energy $= U_2 - U_1 = \left[-\frac{GMm}{r_2}\right] - \left[-\frac{GMm}{r_1}\right]$

$= -\frac{GMm}{3R} + \frac{GMm}{2R} = \frac{GMm}{6R}$.

Q 27. A body of mass *m kg.* starts falling from a point 2*R* above the earth's surface. Its kinetic energy when it has fallen to a point '*R*' above the earth's surface [*R*-Radius of earth, *M*-Mass of earth, *G*-Gravitational constant]

(a) $\frac{1}{2}\frac{GMm}{R}$ (b) $\frac{1}{6}\frac{GMm}{R}$ (c) $\frac{2}{3}\frac{GMm}{R}$ (d) $\frac{1}{3}\frac{GMm}{R}$

Ans : (b) When body starts falling toward earth's surface its potential energy decreases so kinetic energy increases.

Increase in kinetic energy = Decrease in potential energy

Final kinetic energy – Initial kinetic energy = Initial potential energy – Final potential energy

Final kinetic energy – 0 $= \left(-\frac{GMm}{r_1}\right) - \left(-\frac{GMm}{r_2}\right)$

$\therefore$ Final kinetic energy $= \left(\frac{-GMm}{R+h_1}\right) - \left(-\frac{GMm}{R+h_2}\right) = \left(-\frac{GMm}{R+2R}\right) - \left(-\frac{GMm}{R+R}\right) = -\frac{GMm}{3R} + \frac{GMm}{2R} = \frac{1}{6}\frac{GMm}{R}$.

Q 28. A body of mass *m* is taken from earth surface to the height *h* equal to radius of earth, the increase in potential energy will be **[CPMT 1971, 97; IIT-JEE 1983; CBSE PMT 1991;**

(a) mgR (b) $\frac{1}{2}mgR$ (c) $2mgR$ (d) $\frac{1}{4}mgR$

Ans : (b) Work done $= \frac{mgh}{1+h/R}$, If $h = R$ then work done $= \frac{mgR}{1+R/R} = \frac{1}{2}mgR$.

Q 29. If mass of earth is *M,* radius is *R* and gravitational constant is *G*, then work done to take 1 *kg* mass from earth surface to infinity will be

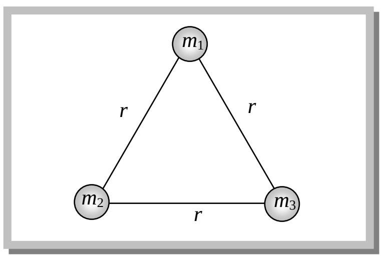
(a) $\sqrt{\frac{GM}{2R}}$ (b) $\frac{GM}{R}$ (c) $\sqrt{\frac{2GM}{R}}$ (d) $\frac{GM}{2R}$

Ans : (b) Work done $= U_{final} - U_{initial} = U_{\infty} - U_R = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$ [As $m = 1kg$]

Q 30. Three particles each of mass 100 *gm* are brought from a very large distance to the vertices of an equilateral triangle whose side is 20 *cm* in length. The work done will be

(a) $0.33\times10^{-11} J$ (b) $-0.33\times10^{-11} J$ (c) $1.00\times10^{-11} J$ (d) $-1.00\times10^{-11} J$

Ans : (d) Potential energy of three particles system



$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_1m_3}{r_{13}}$

Given $m_1 = m_2 = m_3 = 100\ gm$ and $r_{12} = r_{23} = r_{13} = 2cm$

$\therefore\ U = 3\left[\frac{-6.67\times10^{-11}\times(10^{-1})\times(10^{-1})}{20\times10^{-2}}\right] = -1.00\times10^{-11}\ Joule$.

Escape velocity

Q 31. For a satellite escape velocity is $11\,km/s$. If the satellite is launched at an angle of 60^o with the vertical, then escape velocity will be

(a) $11\,km/s$ (b) $11\sqrt{3}\,km/s$ (c) $\frac{11}{\sqrt{3}}km/s$ (d) $33\,km/s$

Ans : (a) Escape velocity does not depend upon the angle of projection.

Q **32.** The escape velocity from the earth is about $11\,km/s$. The escape velocity from a planet having twice the radius and the same mean density as the earth, is

(a) 22 *km/s* (b) 11 *km/s* (c) 5.5 *km/s* (d) 15.5 *km/s*

Ans : (a) $v_e = \sqrt{\frac{2Gm}{R}} = \sqrt{\frac{8}{3}\pi\rho G R^2}$ $\therefore v_e \propto R$ if ρ = constant. Since the planet having double radius in comparison to earth therefore the escape velocity becomes twice *i.e.* $22\,km/s$.

Q **33.** If the radius of earth reduces by 4% and density remains same then escape velocity will

(a) Reduce by 2% (b) Increase by 2%

(c)Reduce by 4% (d) Increase by 4%

Ans : (c) Escape velocity $v_e \propto R\sqrt{\rho}$ and if density remains constant $v_e \propto R$

So if the radius reduces by 4% then escape velocity also reduces by 4%.

Self-Assessment

1. Derive an expression for the variation of acceleration due to gravity (i) with altitude
   above the surface of the Earth (ii) with depth below the surface of Earth.
2. Show that the acceleration due to gravity at a height h above the surface of the Earth has the same value as that at depth d = 2h below the surface of the earth.
3. Define gravitational potential energy. Derive the expression for the same.
4. Define escape velocity .Derive an expression for the escape velocity of a satellite projected from the surface of the earth.
5. Define orbital velocity of a satellite. Derive an expression for the orbital velocity of a satellite.
6. State and explain Kepler's laws of planetary motion.

Animations

1. Animation of Kepler's Law

https://www.flippingphysics.com/kepler-first.html
https://www.sciencephoto.com/media/901940/view/kepler-s-2nd-law-of-planetary- motion

2. **Universal law of Gravitation**
   https://giphy.com/gifs/apple-gravity-threadless-l1J9z5SmyhtdybtaU
3. **https://phet.colorado.edu/en/simulations/gravity-and-orbits**
4. **https://phet.colorado.edu/en/simulations/filter?subjects=physics&type=html**
5. **Why is gravitational potential energy always negative?**
   https://www.youtube.com/watch?v=qveyVsOahI8

Quizzes

https://quizizz.com/admin/quiz/5ebfb9e2766a71001cc30eb7/gravitation-

Unit VII Properties of Bulk Matter

Chapter 9: Mechanical Properties Of Solids

GIST OF THE CHAPTER:

Mechanical Properties of Solids:

Interatomic Force

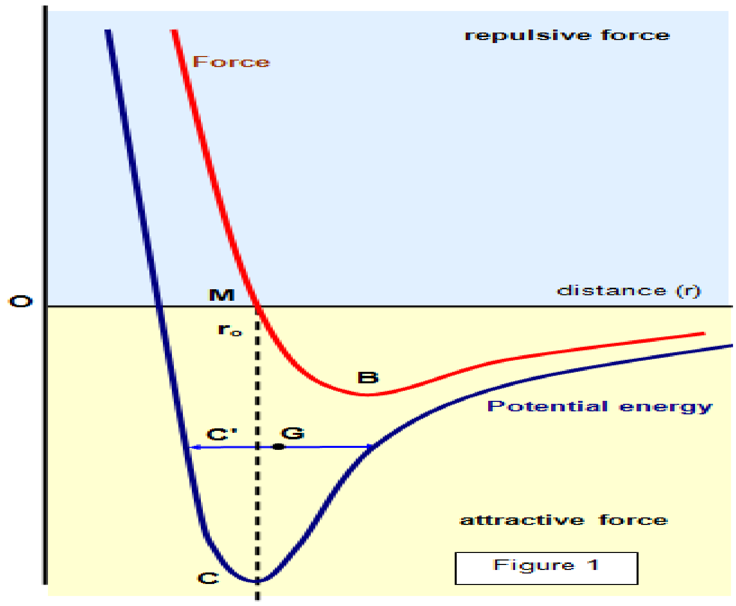
The forces between the atoms due to electrostatic interaction between the charges of the atoms are called interatomic forces.

(1) When two atoms are brought close to each other to a distance of the order of

10^{-10} m, attractive interatomic force is produced between two atoms.

(2) This attractive force increases continuously with decrease in r and becomes maximum for one value of r called critical distance.

(3) When the distance between the two atoms becomes r_0, the interatomic force will be zero. This distance r_0 is called normal or equilibrium distance.



(4) When the distance between the two atoms further decreased, the interatomic force becomes repulsive in nature and increases very rapidly.

(5) The potential energy U is related with the interatomic force F by the following relation.

$F = dU/dr$

When the distance between the two atoms becomes r_0, the potential energy of the system of two atoms becomes minimum (i.e., attains maximum negative value hence the two atoms at separation r0 will be in a state of equilibrium.

Intermolecular Forces

The forces between the molecules due to electrostatic interaction between the charges of the molecules are called intermolecular forces. These forces are also called Vander Waal forces and are quite weak as compared to interatomic forces.

Elasticity: The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

Hooke's law:- when a wire is loaded within elastic limit, the extension produced in wire is directly proportional to the load applied.

OR

Within the elastic limit, stress α strain

Stress/ Strain = constant

Stress : Restoring force set up per unit area when deforming force acts on the body is called stress.

Stress = Restoring force/ Area

S.I Unit of stress = N/m^2 or Pascal (Pa)

Dimensional formula = $M^1 L^{-1} T^{-2}$

Types of stress: -

Tensile stress: When there is an increase in dimension of the body along the direction of force

Tangential stress: When deforming force acts tangential to the surface of body

Strain: The ratio of change in dimension to the original dimension is called strain

It has no unit.

Types of strain:

Longitudinal strain = change in length / original length

Volumetric strain = change in volume / original volume

Shearing Strain = $\varphi = \Delta L/L$, where ΔL = displacement of the face on which force is applied and L is the height of the face

Hooke's Law: - Within elastic limit, stress α strain = Constant (Modulus of Elasticity)

Modulus of elasticity are of 3 types.

(1) Young's Modulus (Y) = Normal stress / Longitudinal Strain

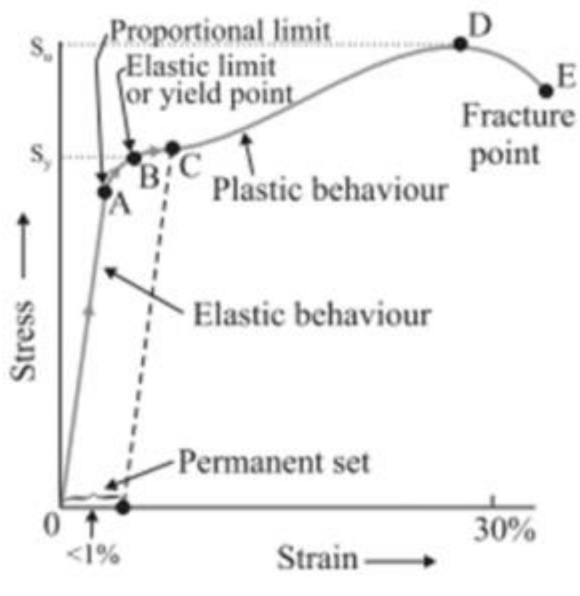
2) Bulk Modulus (K) = Normal Stress/ Volumetric Strain

(3) Modulus of rigidity modulus (η) = Tangential stress/ Shearing Strain

Compressibility: The reciprocal of bulk modulus of a material is called its compressibility.

Compressibility = 1/K

Stress – Strain curve



The region O To A – In the graph above, we can see that region O to A is a straight line or linear; this implies that Hooke's law is obeyed in this region.
The region A to B- In regions A to B, the stress applied and the strain produced are not proportional to each other. So, we can conclude that if the exerted force is removed, the body will return to its initial dimension.

The point B-The point B in the curve is known as the yield point or the elastic limit of the curve. The tensile stress corresponding to the yield point is named yield stress and denoted by Sy. Further, as we increase the load, the stress starts exceeding the yield stress. This means now even if there is a small change in stress, the strain will increase rapidly.

Region B to D- In this region, the strain increases very quickly even if we change the stress by a small amount. If we withdraw the exerted force at point C between B and D, the body will not return to its initial dimension. Hence, we can conclude that if the applied tensile stress is null, the strain will not be equal to zero. This is a deformation produced in the body, and we call this deformation **plastic deformation.** At this point, the material is said to have a permanent set.

The point D and E- In the graph, the "D point" is called the "ultimate tensile strength of the material". Beyond point D, additional strain is produced in the material; even if the applied tensile strength is reduced, fracture occurs. Point E is defined as a fracture point.
If the distance between point E and point D is not much, then the material is called brittle material.
If point E and point D are far apart from each other, then the material is known as ductile material.

Poisson's ratio - The ratio of lateral strain to longitudinal strain is called Poisons ratio

Elastic Energy: In stretching a wire work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

$\therefore$ Energy stored in wire U $= \frac{1}{2}\frac{YA\,l^2}{L}\frac{1}{2}\frac{YA\,l^2}{L} = \frac{1}{2}Fl\frac{1}{2}Fl$ where l = change in length.

Energy stored on per unit volume of wire = $\frac{1}{2}\ \frac{1}{2}$ stress x strain

BRAIN MAP

CLASS XI

MECHANICAL PROPERTIES OF SOLIDS AND FLUIDS

PROPERTIES OF SOLIDS

STRESS AND STRAIN

- Stress = $\frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$
- Strain = $\frac{\text{Change in configuration}}{\text{Original configuration}}$

HOOKE'S LAW

Stress ∝ Strain

or Stress = $E \times$ Strain,

(E = modulus of elasticity)

Young's modulus

$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

Bulk modulus,

$B = \frac{\text{Normal stress}}{\text{Volumetric strain}}$

Compressibility, $k = 1/B$

Modulus of rigidity

$G = \frac{\text{Shearing stress}}{\text{Shearing strain}}$

Poisson's ratio

$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

RELATION BETWEEN Y, B, G AND σ

- $Y = 3B(1 - 2\sigma)$
- $Y = 2G(1 + \sigma)$
- $\sigma = \frac{3B - 2G}{2G + 6B}$
- $\frac{9}{Y} = \frac{1}{B} + \frac{3}{G}$

ELASTIC POTENTIAL ENERGY

$U = \frac{1}{2} F \times \Delta L = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

P.E. stored per unit volume of stretched wire,

$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$

ELASTICITY AND PLASTICITY

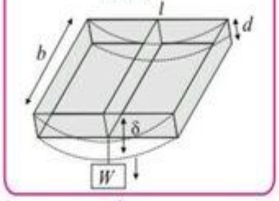
Elasticity : Ability of a body to regain its original shape, on removing deforming force.

Plasticity : The inability of a body to regain its original size and shape on the removal of the deforming forces.

APPLICATION OF ELASTICITY

Designing beams for bridges

The depression in rectangular beam, $\delta = \frac{Wl^3}{4Ybd^3}$



PROPERTIES OF FLUIDS

FLUIDS IN MOTION | FLUIDS AT REST

VISCOSITY

Coefficient of viscosity:

$\eta = \frac{F}{A\left(-\frac{dv}{dx}\right)}$ where $-\frac{dv}{dx}$ is the velocity gradient between two layers of liquid.

BERNOULLI'S THEOREM

Bernoulli's theorem : For the streamline flow of an ideal liquid, the total energy per unit volume remains constant

$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

Basic results on viscosity

Stoke's law : Backward dragging force on a spherical body, $F = 6\pi\eta rv$.

Poiseuille's formula

$Q = \frac{\pi}{8}\frac{Pr^4}{\eta l}$

Reynold's number : Determines nature of fluid flow $R = \frac{\rho vd}{\eta}$

PRESSURE

Pascal's law

The pressure is same at all points inside the liquid lying at the same depth in a horizontal plane.

Gauge pressure = $P - P_0 = h\rho g$.

ARCHIMEDE'S PRINCIPLE

When a body is immersed fully or partly in a liquid at rest, it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.

Apparent weight = $mg\left(1 - \frac{\rho'}{\rho}\right)$

(For fully immersed body)

SURFACE TENSION

Surface tension: The property by which the free surface of liquid at rest tends to have minimum surface area.

Surface energy: Work done against the force of surface tension in forming the liquid surface.

CAPILLARITY

The phenomenon of rise or fall of liquid in a capillary tube is called capillarity.

Height of the liquid within capillary tube

$h = \frac{2S\cos\theta}{a\rho g}$ {Where, θ = angle of contact, ρ = density of liquid, a = radius of tube}

Excess Pressure

In an air bubble

$\Delta P = \frac{2S}{R}$

Inside a soap bubble

$\Delta P = \frac{4S}{R}$

Inside a liquid drop

$\Delta P = \frac{2S}{R}$

Multiple Choice Questions: (1 Mark)

1. Elastomers are the materials which
(a) are not elastic at all (b) have very small elastic range
(c) do not obey Hooke's law (d) None of these

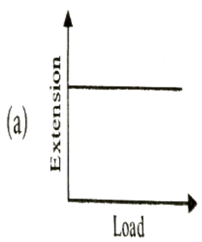
2. When an elastic material with Young's modulus Y is subjected to stretching stress S, elastic energy stored per unit volume of the material is
(a) YS / 2 (b) S2Y / 2 (c) S2 / 2Y (d) S / 2Y

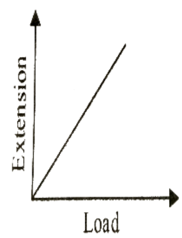
3. Which of the following substances has the highest elasticity?
(a) Steel (b) Copper (c) Rubber (d) Sponge

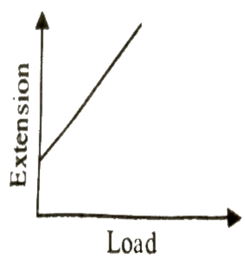
4. The Young's modulus of a wire of length of length L and radius r is Y. If the radius is reduced to r/2 and length is reduced to L/2. Comment on the Young 's modulus of the material:
(a) Y (b) 2Y (c) 4Y (d) Y/2

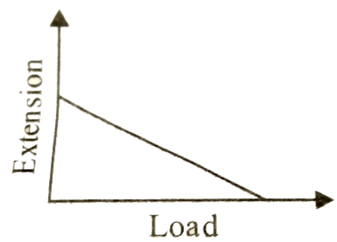
5. A wire of length l and radius R has a mass M. The Modulus of Elasticity of the wire is Y. If the wire is suspended vertically from a fixed point. Find the increase in the length of the wire produced due to its own weight.
(a) $\frac{Mgl\ Mgl}{\pi r^2 Y \pi r^2 Y}$ (b) $2\frac{Mgl}{\pi r^2 Y}2\frac{Mgl}{\pi r^2 Y}$ (c) $\frac{Mgl\quad Mgl}{2\pi r^2 Y 2\pi r^2 Y}$ (d) none of these

6. Within the limit of elasticity, which of the following graph obey Hooke's law?

(a)


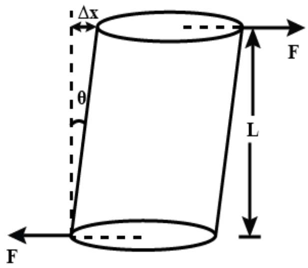
(b)


(c)


(d)


7. There are two wires of same material and same length while the diameter of second wire is two times the diameter of first wire, then the ratio of extension produced in the wires by applying same load will be
(a) 1: 1 (b) 2 : 1 (c) 1 : 2 (d) 4 : 1

8. If two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder as shown in the figure, there is a relative displacement between the opposite faces of the cylinder. The ratio of Δx to L is known as



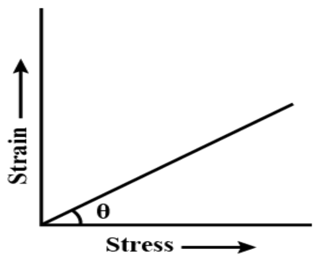
(a) Longitudinal strain (b) Volumetric strain (c) Shearing strain (d) Poission 's ratio

9**.** Consider two cylindrical rods of identical dimensions, one of rubber and the other of steel. Both the rods are fixed rigidly at one end to the roof. A mass M is attached to each of the free ends at the centre of the rods.

A. Both the rods will elongate but there shall be no perceptible change in shape
B. The steel rod will elongate and change shape but the rubber rod will only elongate
C. The steel rod will elongate, without any perceptible change in shape, but the rubber rod will elongate with the shape of the bottom edge tapered to a tip at the centre
D. The steel rod will elongate without any perceptible change in shape, but the rubber rod will elongate and the shape of the bottom edge will change to an ellipse

10. The value of $\tan(90 - \theta)$ in the graph gives

A. Young's modulus of elasticity
B. Compressibility
C. Shear strain
D. Tensile strength



Answers Of MCQ

1.c	**2.c**	**3.a**	**4.a**	**5.c**
6.b	**7.d**	**8.c**	**9.d**	**10.a**

Solution Of MCQ 2

$U = \frac{1}{2} \times \text{stress} \times \text{strain}$

Now, Young's modulus is

$Y = \frac{\text{stress}}{\text{strain}}$

$\therefore \text{strain} = \frac{\text{stress}}{Y}$

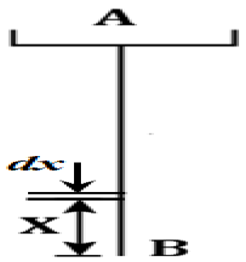
$\therefore U = \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y} = \frac{\text{stress}^2}{2Y} = \frac{S^2}{2Y}$

Answer is option c

SOLUTION OF MCQ 3

Young"s modulus depends upon the nature of the material and id independent of the dimension of material

SOLUTION OF MCQ 5



Consider a small element dx at a distance of x from B.L is the length of the wire ,then the weight of wire of length x ,W= (mg/L)x

The elogation in the element dx will be ,

dl = (mg/LAY)xdx

Therefore total elongation

$=(\text{mg/LAY}) \int_0^L x dx$

= mgL/2AY

=mgL/2∏r²Y

Answer is option c

7. As both have same young modulus it means

Y1=Y2

$FL/A_1l_1=FL/A_2l_2$

As, both have same length and weight, therefore

$r_1^2l_1 = r_2^2l_2$,

$r_1^2l_1 = 4\, r_1^2l_2$

$l_1:l_2=4:1$

Answer is option d

Assertion & Reason

.**Direction** : In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

(c) Assertion is correct but Reason is incorrect.

(d) Assertion is incorrect but Reason is correct

11. **Assertion**: For small deformations, the stress and strain is proportional to each other.

.**Reason :**A class of solids called elastomers does not obey Hooke's law

12. Assertion: For rubber, strain is more as compared to steel.

Reason: Rubber is less elastic than steel

13. **Assertion** A solid sphere placed in the fluid under high pressure is compressed uniformly on all sides

Reason: The volume of the solid sphere will decrease with change of its geometrical shape.

14. **Assertion:** Elongation produced in a body is directly proportional to the applied force.

Reason: With in the elastic limit, stress is inversely proportional to the strain.

15. Assertion: Young's modulus for a perfectly plastic body is zero.

Reason: For a perfectly plastic body, restoring force is zero.

16. **Assertion**: Bulk modulus of elasticity (K) represents incompressibility of the material.

Reason: Bulk modulus of elasticity is proportional to change in pressure.

17. **Assertion:** Young's modulus for a perfectly plastic body is zero.

Reason: For a perfectly plastic body, restoring force is zero.

18. **Assertion:** Strain is unitless quantity

Reason: Strain is equivalent to force .

19. **Assertion**: Ropes are always made of a number of thin wires braided together.

Reason: It helps to ease in manufacturing, flexibility and strength.

20. **Assertion:** Spring balance shows incorrect readings after using it for a long time.

Reason: Spring in the spring balance loses its elastic strength over the period of time.

Answers Of Assertion & Reason

11.b	12.a	13.c	14.c	15.a
16.a	17.a	18.c	19.a	20.a

Very Short Questions (2 Marks)

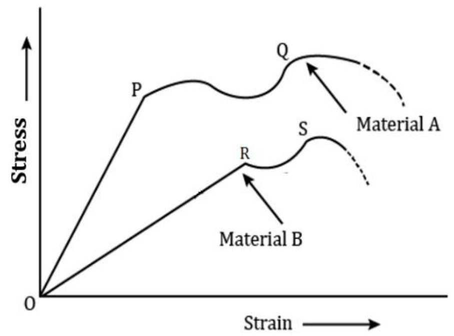
21.Which type of substances are called elastomers ? Give one example.

ANS: Those materials for which stress-strain variation is not a straight line within elastic limit e.g. Rubber.

22. Which is more elastic : water or air ? Why ?

ANS: Water is more elastic than air because bulk modulus of elasticity is reciprocal of compressibility and air is more compressible than water.

23. Stress-strain curve for two wires of material A and B are as shown in Fig.



(a) Which material in more ductile ?
(b) Which material has greater value of young modulus ?
(c) Which of the two is stronger material ?
(d) Which material is more brittle?
Ans. (a) Wire with larger plastic region is more ductile material A.
(b) Young's modulus is ∴ YA > YB
(c) For given strain, larger stress is required for A than that for B. ∴ A is stronger than B
(d) Material with smaller plastic region is more brittle, therefore B is more brittle than A.
24. What is the value of bulk modulus for an incompressible liquid?
Ans : K= $\frac{stress}{strain}$= $\frac{stress}{0}$ = infinite
25. For solids with elastic modulus of rigidity, the shearing force is proportional to shear strain. On what factor does it depend in case of fuilds.
Ans. Rate of Shear Strain
Short Questions(3 marks)
26. Calculate the percentage increase in the length of a wire of diameter 2mm stretched by a force of 1kg F. Young's modulus of the material of wire is 15 X 10^{10} Nm^{-2} .
Ans F = 1 Kg F = 9.8N,
r = = 1mm = 10^{-3}m, Cross section of wire, $\pi r^2 = \pi \times (10^{-3})^2 = \pi \times 10^{-6} m^2$
Y = $\frac{FL}{al}$
$\frac{l}{L} = \frac{F}{aY}$= 2.1 x$10^{-5}$
% increase
= 2.1 x10^{-5}x100 = 0.0021%
27. A truck is pulling a car out of a ditch by means of a steel cable that is 9.1 m long and has a radius of 5 mm. When the car just begins to move, the tension in the cable is 800 N. How much has the cable stretched? (Young's modulus for steel is 2 × 10^{11} Nm^{-2} .
Ans . Given, Length of cable, L=9.1 m r=5 mm=5×10^{-3} m,A=π r^2

Tension in cable, F=800 N

Y=2×10^{11}N/m^2

$\Delta L = \frac{FLFL}{AYAY}$

Substituting the values, we get

$\Delta L=(800\times9.10)/ (3.14\times10^{-3}\times10^{-3}\times5\times5\times2\times10^{11})$

$\Delta L=4.64\times10^{-4}$ m

28. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper.

Ans 1.79:1

Let the Young's Modulus of steel and copper be Ys and Yc respectively

Length of steel wire, Ls = 4.7 m Length of copper wire, Lc = 3.5 m

Area of cross-section of steel wire, As = 3 × 10-5 m^2 Area of cross-section of copper wire, Ac = 4×10^{-5} m^2

Since change in lengths are equal. ∴ ΔLs = ΔLc = ΔL.

As the load is same, Fs = Fc = F

We have, Y = (F/A x L/ΔL)

$\frac{Ys}{Yc} = \frac{LsAc}{LsLc}$

$= \frac{4x4.7x10^{-5}}{3x3.5x10^{-5}}$

= 1.79

29. How does the elasticity of material change on

(a) Increasing the temperature. (b) on heating and cooling gradually.

(c) on hammering.

Ans: (a) Decreases. (b) Decreases. (c) Increases.

30. A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of two atmospheres. What is the bulk modulus of its material in C.G.S. units.

Ans : Bulk modulus B = $\frac{P}{\frac{\Delta V}{V}}$

$\frac{\Delta V}{V}$ = 0.01/100 = $1x10^{-4}$

B= 100 atm /$1x10^{-4}$

= $(100x\ 1.01x10^{-5})/(1x10^{-4})$

= 1.01×10^{11} Nm^{-2}

Case Study

31. The proportional region within the elastic limit of the stress-strain curve is of great importance for structural and manufacturing engineering designs. The ratio of stress and strain, called modulus of elasticity, is found to be a characteristic of the material. Experimental observation show that for a given material, the magnitude of the strain produced is same whether the stress is tensile or compressive. The ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ϵ) is defined as Young's modulus and is denoted by the symbol Y.

Y= σ / ϵ

Since strain is a dimensionless quantity, the unit of Young's modulus is the same as that of stress i.e., **$N\text{-}m^{-2}$** or Pascal (Pa). As steel has more modulus of elasticity than copper brass and aluminium hence steel is preferred in heavy-duty machines and in structural designs. Wood, bone, concrete and glass have rather small Young's moduli. Answer the following.

I) If stress strain changes then young's modulus is
a) Also changes
b) Remains constant
c) Either changes or remains constant depends on amount of stress and strain
d) None of these
II) SI unit of young's modulus is
a) $N\text{-}m^{-2}$
b) Pascal (Pa).
c) $N\text{-}m^{-2}$ or Pascal (Pa).
d) None of these
III) Which of the following is more elastic
a) Aluminum
b) Steel
c) Wood
d) Glass
IV) Why steel is more preferred in heavy industries than copper and brass?

ANSWER

I. b
II. c
III. B
IV. Steel is more preferred in heavy industries than copper and brass because steel has more modulus of elasticity that is higher young's modulus than copper and brass. i.e steel is more elastic than copper and brass.

32 When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force. The restoring force per unit area is known as stress. If F is the force applied normal to the cross–section and A is the area of cross section of the body.
Magnitude of the stress = F/A
The SI unit of stress is $N\text{-}m^{-2}$ or Pascal (Pa) and its dimensional formula is $[ML^{-1} T^{-2}]$.
The restoring force per unit area in this case is called tensile stress. If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as compressive stress. Tensile or compressive stress can also be termed as longitudinal stress. In both the cases, there is a change in the length of the cylinder. The change in the length ΔL to the original length L of the body is known as longitudinal strain.

The restoring force per unit area developed due to the applied tangential force is known as tangential or shearing stress.
I) Restoring force per unit area is called as
a) Stress
b) Strain
c) Modulus of elasticity
d) None of these
II) Ratio of change in dimension to original dimension is called
a) Stress
b) Strain
c) Modulus of elasticity
d) None of these
III) Define shear stress.
IV) Define stress. Give its SI unit and dimension
V) Define strain. Give its SI unit and dimension

ANSWER
I) a
II) b
III) The tangential restoring force per unit area developed known as tangential or shearing stress.
IV) When a body is subjected to a deforming force, a restoring force is developed in the body. This restoring force is equal in magnitude but opposite in direction to the applied force. The restoring force per unit area is known as stress.
If F is the force applied normal to the cross–section and A is the area of cross section of the body.
Magnitude of the stress = **F/A**
The SI unit of stress is **$N\text{-}m^{-2}$** or Pascal (Pa) and its dimensional formula is **$[ML^{-1} T^{-2}]$.**
V) Ratio of change in dimension to original dimension is called strain. As it is ratio of similar quantities so it carries no unit and hence no dimensions.

Long Type Question (5 Marks)
21. A wire of length l and area of cross section A is stretched by the application of a force. If the Young's modulus is Y, what is the work done per unit volume?
Ans : The wire has length l, area of cross-section A made of material constant Y. Let a force F be applied and at any instance, x be the extension associated (x < L), where L is the maximum extension.

At this instant, F= $\frac{AYx}{l}$

W =$\int_0^L F dx$

W = $\frac{AYL^2}{2l}$

W= $\frac{1}{2}\,(Al)\,\frac{YL}{l}\,\frac{L}{l}$

W= $\frac{1}{2}\; Volume\ x\ Stressx\ Strain$

Work done per unit Volume
= $\frac{1}{2}$ x Stress x Strain
22. Describe stress-strain relationship for a loaded steel wire and hence explain the terms: Elastic limit, yield point, tensile strength.

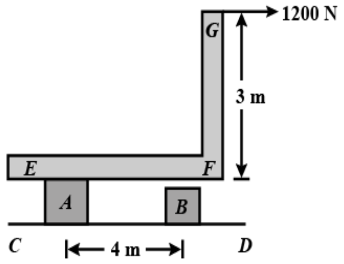
Ans : Refer NCERT Text Book .
23. Define Youngs modulus of elasticity, normal stress and longitudinal strain. Give units of each of them. Derive an expression for the elastic potential energy of awire, when it stretched
Ans: Refer text book for definitions & Ans no. 21

Competency Based Questions
23. In the figure shown. A and B are two short steel rods each of cross-sectional area 5 cm^2. The lower ends of A and B are welded to a fixed plate CD.

The upper end of A is welded to the L – shaped piece EFG, which can slide without friction

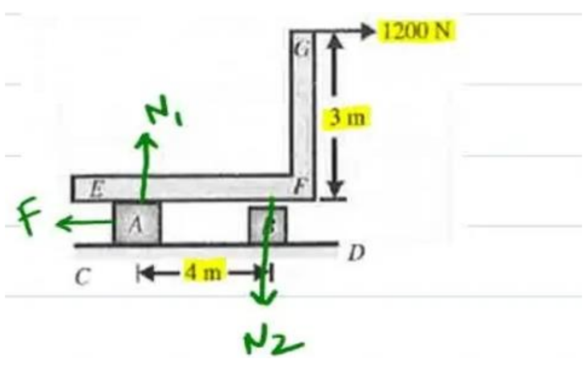
on upper end of B. A horizontal pull of 1200 N is exerted at G as shown.

Neglect the weight of EFG.
Longitudinal stress in B is



A.Tensile in nature and having magnitude 180N/cm^2
B.Tensile in nature and having magnitude 240N/cm^2
C.Compressive in nature and having magnitude 180N/cm^2
D.Compressive in nature and having magnitude 240N/cm^2

solution



Let the Normal reaction at A be N_1 and at b be N_2

On L shaped piece N_$_2$ will act upward and N_$_1$ be downward.

For vertical equilibrium $N_1=N_2$,

Horizontal equilibrium F= 1200N

Rotational equilibrium about E

N_2 x4 = 1200x3

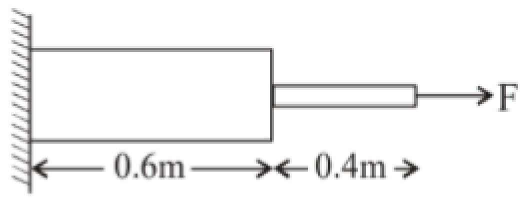
N_2 = 900 N

From the free body diagram

Stress on B=N_2/5=900/5=180 N/cm^2, Tensile in nature.

Ans A

24. Two bars of steel $Y=2\times1011\ N/m2$ are joined together as shown. The area of cross-section of the left bar is 15 $cm2$ and the area of right bar is unknown. The extension in both bars is the same.



a.The area of right bar is 10 $cm2$
b.The stresses in left and right bar are in ratio 3 : 2
c.The decrease in thickness of bar is more for the left
d.The decrease in thickness of bar is more for right bar
solution

$\Delta L_1 = \Delta L_2$

$$\frac{Fx0.6}{Yx15} = \frac{Fx0.4}{YxA}$$

A= 10cm^2

Answer is option a

25. A catapult consists of two parallel rubber strings, each of lengths 10 cm and cross-sectional area 10 mm^2. When stretched by 5 cm, it can throw a stone of mass 100 gm to a vertical height of 25 m. Determine Young's modulus of elasticity of rubber.

Solution

A stretched catapult has elastic potential energy stored in it (Strain energy stored in both the rubber strings)
U= ½ stress x strain x volume
U=$\frac{1}{2}\frac{YAl^2}{L}$x2 (multiply by2 for2 strings)
This energy, when imparted to the stone, it flies off a height 25 m. Energy possessed by the Stone = mgh.
Now, U=mgh⇒YAl^2/L=mgh
Solving , Y = = 9.8×10^7 N/m^2

26.A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bilk modulus of the gas is?

a. 2P/3

b .P

c. 3P/2

d. 2P

Solution

Bulk modulus is given by
K=Stress/Volumetric strain

K=$-\frac{\Delta P}{\Delta V/V}$ =−V$\frac{dP}{dV}$

For isothermal process, PV = Constant.

Differentiating both sides,

⇒PdV+VdP=0

⇒PdV=−VdP

⇒dP/dV=−PV

Now,
K=−V×(dP/dV) =−V(−P/V)

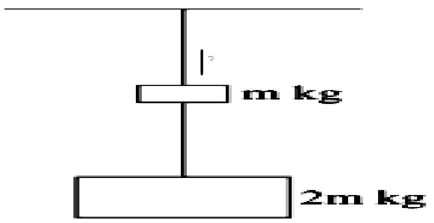
⇒K = P
Hence, the correct option is (b).

Self-Assessment 10Marks

1 Find the value of bulk modulus for an incompressible liquid (2M)

2. Which is more elastic steel or rubber. Explain. (2M)

3. Two wires P and Q of same diameter are loaded as shown in the Figure. The length of wire P is L m and its young's modulus is Y N/m2 while length of wire a is twice that of P and its material has young's modulus half that of P. Compute the ratio of their elongation (2M)



4.An aluminium wire 1 m in length and radius 1 mm is loaded with a mass of 40 kg hanging vertically. Young's modulus of Al is 7.0 × 1010 N/m2. Calculate (a) tensile stress (b) change in length (c) tensile strain and (d) the force constant of such a wire (2M)

5. How much should the pressure on one litre of water be changed to compress it by 0.10%. (2M)

Solutions of Self-Assessment

1. Srain = 0
2. K = Stress/Strain = Stress /0 = ∞
3. Ys $=\frac{F}{A}\frac{l}{\Delta ls}$

   Yr $= \frac{F}{A}\frac{l}{\Delta lr}$

   For same force applied to wires made of steel & rubber of same length and same area of cross section $\Delta ls < \Delta l r$

   $\therefore Y_s > Y_r$

4. $\Delta l\ p = \frac{3mg}{A}\frac{L}{Y}$

   $\Delta l\ Q = \frac{2mg}{A}\frac{2L}{Y/2} = \frac{8mg}{A}\frac{L}{Y}$

   $\Delta l\ p\ /\ \Delta l\ Q = 3/8$

4.(a) tensile Stress

$$\text{Stress} = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{40\times10}{\pi\times(1\times10^{-3})^2} = 1.27\times10^8\ \text{N/m}^2$$

$$\text{(b)}\ \ \Delta L = \frac{FL}{AY} = \frac{40\times10\times1}{\pi\times(1\times10^{-3})^2\times7\times10^{10}} = 1.8\times10^{-3}\ \text{m}$$

$$\text{(c)}\ \ \text{Strain} = \frac{\Delta L}{L} = \frac{1.8\times10^{-3}}{1} = 1.8\times10^{-3}$$

(d) $F = Kx = K\Delta L$, K = Force constant

$$K = \frac{\Delta F}{L} = \frac{40\times10}{1.8\times10^{-3}} = 2.2\times10^5\ \text{N/m}$$

5. V = 1 litre = $10^{-3}\ m^3$

$\Delta V/V = 0.10\% = 0.10/100 = 0.0001$
$K = P/(\Delta V/V)$
$\Rightarrow P = K(\Delta V/V) = 2.2 \times 10^9 \times 0.001$
$P = 2.2 \times 10^6\ Nm^{-2}$

Animated Video : Mechanical properties of Solids
https://www.youtube.com/watch?v=LytaVRxD5tU

Unit VII Properties of Bulk Matter

Chapter 10. Mechanical Properties Of Fluids

GIST OF THE CHAPTER:

***Hydrostatic** refers to the study of fluids at rest and the forces and pressures associated with them
***Fluid**: A substance which begins to flow when a tangential force is applied on it e.g. Liquids and gases.
***Thrust and Pressure of the Fluid**:-
Thrust: The normal (perpendicular) force exerted by liquid at rest on a given surface in contact.
Pressure: The normal force (or thrust) exerted by a liquid at rest per unit area of the surface in contact.

$$P = F / A$$

Pressure is a scalar quantity because at one level Inside the liquid the pressure is exerted equally in all direction.

SI unit = N/m^2 Dimension = $[M^1L^{-1}T^{-2}]$

Pressure plays a crucial role in many everyday situations. Here are some common examples:
Air Pressure in Tires: The pressure inside car, bicycle, and other vehicle tires keeps them inflated and ensures proper contact with the road for safe and efficient travel.
Hydraulic Systems: Car brakes and heavy machinery like excavators use hydraulic systems where fluid pressure is applied to control and move parts.
Water Supply: Water pressure in plumbing systems allows water to flow from taps and showerheads.
Atmospheric Pressure: Weather patterns and human activities, such as breathing, depend on atmospheric pressure. Barometers measure atmospheric pressure to forecast weather.
Soda Cans and Bottles: Carbonated drinks are bottled under high pressure to keep the carbon dioxide dissolved. When opened, the pressure release causes the fizz.
Cooking with Pressure Cookers: These cookers use steam pressure to cook food faster. The pressure increases the boiling point of water, cooking food more quickly.
Syringes and Pipettes: Medical syringes and laboratory pipettes rely on pressure differences to draw in and expel liquids.
Airplanes: The cabins are pressurized to maintain a safe and comfortable environment for passengers and crew at high altitudes where the atmospheric pressure is low.
Balloons: Inflating balloons involves increasing the air pressure inside them, making them expand.
Blood Pressure: The pressure exerted by circulating blood on the walls of blood vessels is a critical measure of cardiovascular health.

Pascal's law:

Pascal's law states that when pressure is applied to a confined fluid, the pressure change is transmitted equally in all directions throughout the fluid.

In simpler terms, if you squeeze a liquid in a closed container, the pressure will increase everywhere inside the container equally, not just where you are squeezing. This principle is the reason why hydraulic systems, like car brakes and hydraulic jacks, work.

Effect of gravity on fluid pressure:

Gravity Increases Pressure with Depth: When you go deeper in a fluid (like water), the pressure increases because the weight of the fluid above you increases. This is why divers feel more pressure the deeper they go underwater.

Fluid Pressure in a Container: In a container filled with fluid, gravity causes the pressure at the bottom to be higher than at the top because the fluid's weight adds to the pressure as you go down.

Hydrodynamics is the study of how liquids (like water) move and the forces that act on them.

Viscosity:

Viscosity is a measure of a fluid's resistance to deformation or flow. It describes how much a fluid resists being moved or stirred.

High Viscosity: Fluids with high viscosity, like honey or oil, resist motion more and flow slowly.

Low Viscosity: Fluids with low viscosity, like water or alcohol, resist motion less and flow easily.

Viscosity is influenced by factors such as temperature and the internal friction between molecules in the fluid.

$$\eta = -\frac{A\,dv}{F\,dx}$$

SI unit poiseuille , Dimension $[\eta] = [ML^{-1}T^{-1}]$

Stokes' law describes the force of friction experienced by a small, spherical object moving through a viscous fluid.

According to Stokes' law, the force of friction (or drag force) on the ball is proportional to the ball's radius, the fluid's viscosity, and the ball's speed..

The formula is:

$$F = 6\,\pi\,\eta\,r\,v$$

Where: **F** is the drag force, **η (eta)** is the fluid's viscosity, **r** is the radius of the sphere, **V** is the velocity of the sphere.

This law helps in understanding how particles settle in fluids and is useful in fields like engineering and physics.

Terminal velocity:

Terminal velocity is the constant speed that an object reaches when the force of gravity pulling it down is balanced by the air resistance pushing it up.

When you drop an object, it starts to fall faster and faster. As it speeds up, the air resistance against it also increases. Eventually, the air resistance becomes equal to the force of gravity, so the object stops accelerating and falls at a steady speed. This steady speed is called the terminal velocity.

For example, when a skydiver jumps out of a plane, they speed up until the air resistance balances their weight. At this point, they fall at a constant speed.

Raindrops falling from clouds accelerate due to gravity. However, they reach terminal velocity when the upward force of air resistance equals the downward force of gravity. The terminal velocity of raindrops depends on their size and shape.

When a parachute opens, it significantly increases its surface area, which increases air resistance. As a result, the parachutist slows down and reaches terminal velocity at a much lower speed than during free fall.
Objects like feathers, leaves, or small paper pieces falling through the air will eventually reach terminal velocity. The terminal velocity of these objects is much lower than that of a skydiver due to their small mass and large surface area.

Expression for terminal velocity: Let a spherical body of radius r falling through a viscous liquid of density σ and coefficient of viscosity η. Let ρ be the density of the body.

(i) Weight of the body acting vertically downward. $W = mg = \frac{4}{3}\pi r^3 \rho g$

(ii) Upward thrust equal to weight $U = \frac{4}{3}\pi r^3 \sigma g$

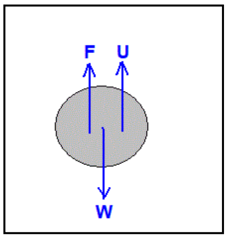
(iii) Force of viscosity F acting in upward direction $F = 6\pi\eta r v$

When body attains terminal velocity: U + F = W

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$v = \frac{2}{9}\frac{r^2 (\rho - \sigma) g}{\eta}$$



Streamline and Turbulent Flow:

Streamline flow, also known as laminar flow, occurs when a fluid flows smoothly in parallel layers with minimal mixing between them.
Characteristics: It's characterized by orderly motion where fluid particles follow predictable paths called streamlines.
Example: When you pour honey or syrup, it flows in a smooth, predictable manner. Water flowing through a narrow pipe with a slow and steady flow also exhibits streamlines flow.

Turbulent Flow:

Turbulent flow is chaotic and irregular motion of a fluid, characterized by mixing and swirling of fluid particles.
Characteristics: It's characterized by eddies, swirls, and rapid changes in velocity and pressure throughout the fluid.
Example: When you stir a cup of coffee vigorously, the motion of the coffee becomes turbulent. Water flowing rapidly over rocks in a river or the air in a turbulent wind gust are also examples of turbulent flow.

Bernoulli's theorem

It is based on principle of conservation of energy applied to ideal liquids in streamline motion.
Statement: For the streamline flow of an ideal liquid, the total energy (sum of pressure energy, potential energy and kinetic energy) per unit mass remains constant at every cross-section, throughout the flow.

$P + (\frac{1}{2})\rho v^2 + \rho g h = \text{constant}$

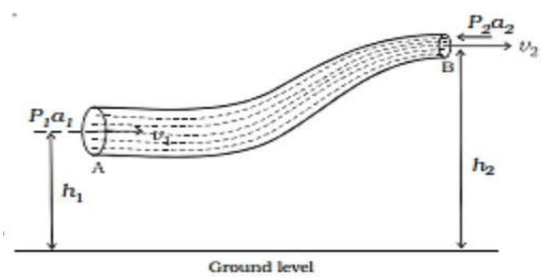


Fig. Bernoulli's theorem

Proof: Consider a streamline along which a fluid element is moving. Let points 1 and 2 be two points along the streamline. The fluid element has a volume dV, a mass dm = ρdV, where ρ is the density of the fluid.
Work-Energy Principle: The work done on the fluid element as it moves from point 1 to point 2 is equal to the change in its mechanical energy.
Work done by the fluid $W_P = (P_1 - P_2)\, dV$
where P_1 and P_2 are the pressures at points 1 and 2, respectively.
Change in kinetic energy of the fluid: $\Delta KE = \frac{1}{2}\rho dV\,(v_2^2 - v_1^2)$
where v_1 and v_2 are the velocities at points 1 and 2
Change in potential energy of the fluid $\Delta PE = \rho\, g\, dV\,(h_2 - h_1)$
where h_1 and h_2 are the heights of points 1 and 2 above some reference level, and g is the acceleration due to gravity.

$$(P_1 - P_2)\, dV = \frac{1}{2}\rho\,(v_2^2 - v_1^2)\, dV + \rho g\,(h_2 - h_1)\, dV$$

$$(P_1 - P_2) = \frac{1}{2}\rho\,(v_2^2 - v_1^2) + \rho g\,(h_2 - h_1)$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\underline{P + \frac{1}{2}\rho v^2 + \rho g h = \text{Constant}}$$

Applications of Bernoulli's Principle

- Venturimeter
- Atomizer or sprayer
- Lift of an aircraft wing
- Curved path of a spinning ball

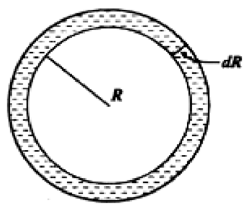
Surface Tension:

Definition: Surface tension is the tendency of the surface of a liquid to contract to the smallest possible area. It arises because molecules at the surface of a liquid experience a net inward force due to unbalanced intermolecular forces.

$$\textbf{Surface Tension} = \frac{Force}{Length}$$

Effect: Surface tension allows small objects, like paper clips or insects, to float on water without sinking. It also causes water droplets to form spherical shapes and enables capillary action, where liquids rise or fall in narrow tubes.
Measurement: Surface tension is measured in units of force per unit length (e.g., dyne per centimeter or newton per meter).

Expression for excess pressure inside a liquid drop/ soap bubble :



Initial surface area = $4\pi R^2$
Final surface area = $4\pi(R + dR)^2 = 4\pi R^2 + 8\pi R\, dR$ (dR^2 is neglected)
Increase in surface area = $8\pi R\, dR$
Work done in enlarging the drop = increase in surface area x surface tension
= $8\pi R\, dR.\, \sigma$
But work done = F X d = p X $4\pi R^2$ X dR
p X $4\pi R^2$ X dR = $8\pi R\, dR.\, \sigma$

Excess pressure, p = $\frac{2\sigma}{R}$

In a soap bubble,

$$p = \frac{4\sigma}{R}$$

<u>Surface Energy:</u>

Definition: Surface energy is the energy required to increase the surface area of a liquid by a unit amount. It is a thermodynamic property that describes the tendency of a liquid to minimize its surface area to reduce its energy.

Relation to Surface Tension: Surface tension is related to surface energy through the equation Surface Tension = $\frac{\text{surface energysurface energy}}{\text{surface area} \quad \text{surface area}}$

Applications: Surface energy influences the wetting and spreading of liquids on solid surfaces, affecting processes like painting, printing, and adhesion in various industries.

<u>Angle Of Contact</u>: It is the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both tangents being drawn at the point of contact of the liquid with the solid.

e.g. * Liquid which wet the surface, angle is acute. Water-glass=8^0

* Liquid which does not wet the surface, angle is obtuse. Mercury-glass=140^o

Silver-water=90^o, Pure water-glass = 0^o

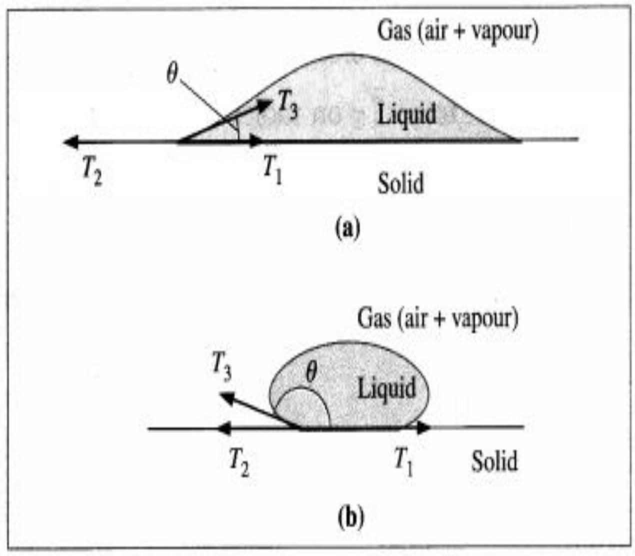
<u>Factors affecting Angle of contact:</u>

- Depends on the nature of the liquid and solid in contact
- Depends on the medium that exists above the free surface of liquid
- Is independent of the inclination of solid to liquid surface.
- Is fixed for a given pair of solid and liquid and surrounding medium.

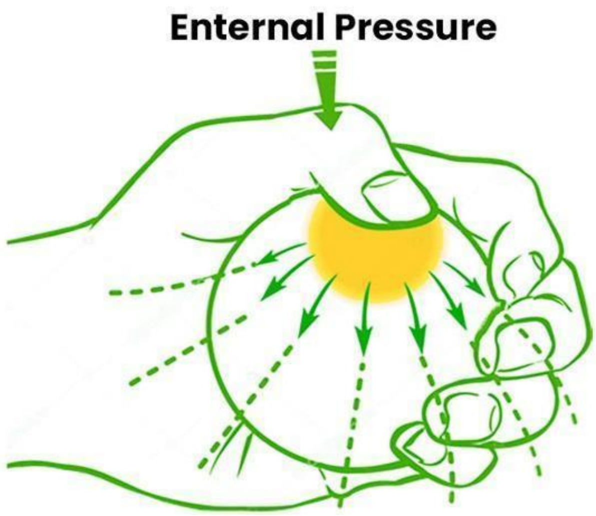
<u>Capillarity and Capillary action:</u> The phenomenon of rise or fall of a liquid in a capillary tube in comparison to the surrounding is called capillarity

Rise of liquid in a capillary tube (Ascent formula)

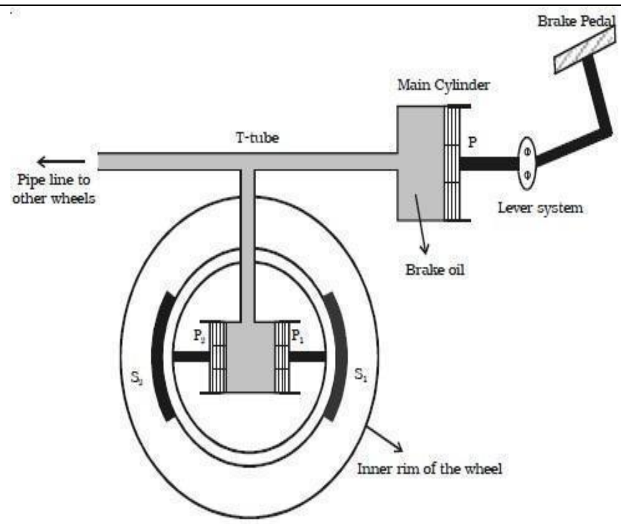
$$h = \frac{2\sigma \cos\theta}{r\rho g}$$



Shape of a drop of (a) a wetting liquid (b) a non-wetting liquid



PASCAL'S LAW



Hydraulic brake

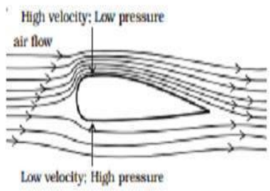


Fig. Lift of an aircraft wing

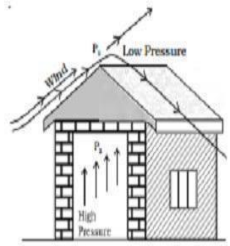


Fig. Blowing of roofs

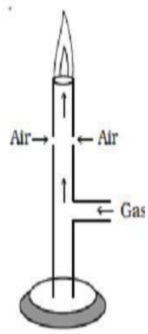


Fig. Bunsen Burner

Applications of Bernoulli's principle

<u>Mind Mapping</u>

PASCAL'S LAW

Pascal's Law states that when pressure is applied to a confined fluid, the pressure change is transmitted equally and undiminished throughout the fluid in all directions. This principle is fundamental in fluid mechanics and has various practical applications, such as in hydraulic systems.

- HYDRAULIC LIFT
- HYDRAULIC BRAKE

Viscosity

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction between layers of a fluid as they move past each other. The higher the viscosity, the more resistant the fluid is to flow; the lower the viscosity, the more easily it flows

Factors Affecting Viscosity:

Temperature: For liquids, viscosity typically decreases with increasing temperature. For gases, viscosity increases with increasing temperature.

Pressure: Generally, an increase in pressure can increase the viscosity of a fluid, although this effect is more significant for gases than liquids.

Composition: The molecular structure and composition of the fluid significantly impact its viscosity.

Examples:

Honey vs. Water: Honey has a higher viscosity than water, meaning it flows more slowly and resists motion more than water.

Motor Oil: The viscosity of motor oil is chosen based on the operating conditions of the engine to ensure proper lubrication and protection.

Terminal velocity

Terminal velocity is the constant speed that a freely falling object eventually reaches when the resistance of the medium (usually air) prevents further acceleration. At terminal velocity, the force of gravity pulling the object downward is balanced by the drag force acting upward, resulting in a net force of zero and, consequently, no further acceleration.

Skydiving: When a skydiver jumps from an airplane, they accelerate until the drag force equals the gravitational force. At this point, they reach terminal velocity and fall at a constant speed.

Rain Droplets: Small rain droplets quickly reach their terminal velocity and fall at a constant, slow speed.

V= QUOTE

Objects in Fluids: Objects sinking in water or other fluids reach terminal velocity when the drag force from the fluid balances their weight.

FLOW OF FLUIDS

Streamline

Streamline flow, also known as laminar flow, is a type of fluid flow in which the fluid travels smoothly or in regular paths the motion of the fluid is orderly, and the layers of fluid slide past each other without mixing.

Blood Flow: In small blood vessels, blood flows in a laminar manner, ensuring

Turbulent

Turbulent flow is a type of fluid flow characterized by chaotic changes in pressure and flow velocity turbulent flow is irregular and involves mixing and swirling motions. This type of flow is common in natural and industrial processes.

Rivers and Streams: Fast-flowing rivers and streams

creating

pids.

<u>BERNOULLI'S PRINCIPLE</u>

Bernoulli's Principle, named after the Swiss mathematician Daniel Bernoulli, states that in a flowing fluid, an increase in velocity occurs simultaneously with a decrease in pressure or potential energy. This principle is fundamental in fluid dynamics and has numerous applications in engineering and science.

$P +$ QUOTE $\rho v^2 + \rho g h = \text{constant}$

<u>Practical Applications Of Bernoulli's principle</u>

<u>Aircraft Wings (Air foil):</u> The shape of an aircraft wing is designed so that the air moves faster over the top surface than the bottom surface. According to Bernoulli's Principle, the pressure on the top surface is lower than on the bottom, creating lift.

<u>Venturi Effect</u>: In a Venturi tube, the fluid velocity increases as it passes through a constricted section, causing a drop in pressure. This principle is used in devices like carburettors and flow meters.

<u>Atomizers</u>: In devices such as perfume sprayers, a high-speed air jet reduces the pressure, drawing the liquid up through a tube and dispersing it as a fine mist.

<u>Multiple Choice Questions</u>

Q.1-Which law states that the pressure exerted anywhere in a confined incompressible fluid is transmitted equally in all directions?

a) Boyle's law　　b) Archimedes' principle

c) Pascal's law　　d) Bernoulli's theorem

Q.2-What is the SI unit of viscosity?

a) Pascal (Pa)　　b) Newton (N)

c) Poise (P)   d) Pascal-second (Pa·s)

Q.3-Surface tension is caused by:
a) Gravitational force   b) Cohesive forces between molecules
c) Centrifugal force   d) Electrostatic force

Q.4-The working of hydraulic machines based on?
a) Pascal's Law   b) Newton's Law of Cooling
c) Law of Gravitation   d) Ideal Gas Law

Q.5- A force F is applied on a uniform rod of cross-section A and a force F' is applied on a uniform rod of cross-section 3A. What is the relation between F and F' if the pressure on both is the same?
a) F/F' = 1/3   b) F/F' = 3
c) F'/F = 1/3   d) F/F' = 1/9

Q.6-Water rises up to a height of 5 cm in a capillary tube of radius 2 mm. what is the radius of the radius of the capillary tube if the water rises up to a height of 10 cm in another capillary?
a) 2 mm   b) 1 mm
c) 3 mm   d) 5 mm

Q.7-In Bernoulli's theorem which of the following is conserved?
(a) Mass   (b) Linear momentum   (c) Energy   (d) Angular momentum

Q.8-If ratio of terminal velocity of two drops falling in air is 3 : 4, then what is the ratio of their surface area?
(a) 2:3   (b) 3:2   (c) 3:4   (d) 4:3

Q.9- Paint-spray gun is based on
(a) Bernoulli's theorem   (b) Archimedes' principle
(c) Boyle's law   (d) Pascal's law

Q.10- If two forces in the ratio 1 : 7 act on two pistons of areas in the ratio 3 : 2, then the pressure exerted by the forces is in ratio
(a) 2 : 21   (b) 3 : 14   (c) 6 : 7   (d) 4 : 21

Q.11- Liquid pressure depends upon
(a) Height of liquid column   (b) Direction
(c) Shape of liquid surface   (d) Area of the liquid surface

Q.12-The spherical shape of raindrop is due to
(a) Density of the liquid   (b) Surface tension
(c) Atmospheric pressure   (d) Gravity

Q.13-The energy of soap bubble of diameter 6cm amd T= 0.04 N/m is nearly \
(a) 0.9×10^{-3} J   (b) 0.4×10^{-3} J
(c) 0.7×10^{-3} J   (d) 0.5×10^{-3} J

Q.14- Angle of contact for the pair of pure water with glass is
(a) Acute   (b) Obtuse
(c) 90^0 (d) 0^0

Q.15- Angle of contact between glass and mercury
(a) 30^0 (b) 90^0

(c) 135^0 (d) 0^0

Q.16- If the diameter of a capillary tube is doubled then height of the liquid that will increase is
(a) Twice (b) Half
(c) Same as earlier (d) None of the above

Q.17-Two capillary tube of different diameter are partially dipped in the same liquid then the rise of liquid
(a Is same in both tubes (b) Is more in the tube of larger diameter
(c) Will not be in smaller diameter tube (d) Is more in the tube of smaller diameter

Q.18- A flow of liquid is Laminar or a streamline is determined by
(a) Reynold's Number (b) Density of fluids
(c) Stoke's law (d) Coefficient of viscosity of liquid

Q.19- Water is flowing through a horizontal pipe in streamline flow, at the narrowest part of the pipe
(a) Velocity is maximum and pressure is minimum
(b) Pressure is maximum and viscosity is minimum
(c) Both the pressure and velocity are maximum
(d) Both the pressure and velocity are minimum

Q.20- When a drop of water splits up into number of drops
(a)Area increases (b) Volume increases
(c)Energy decreases (d) Both (a) and ©

MCQs Answers:

1 .c	2.d	3.b	4.a	5.a	6.c	7.c	8.c	9.a	10.a
11.a	12.b	13.a	14.a	15.c	16.b	17.d	18.d	19.a	20.d

MCQs Answers (Brief Explanation):

Q.5A- Pressure on first rod (P) = F/A
Pressure on the second rod (P') = F'/3A
Given;
P = P'
F/A = F'/3A
F/F' = 1/3.
Q.6A- Since height of capillary rise is inversely proportional to radius of the capillary hence,
$h_1 / h_2 = r_2 / r_1 \Rightarrow$ 5cm / 10cm
$r_2 = r_1 \times h_1 / h_2 = 2 \times 5/10 = 1$ mm
r2=1mm
Q.8 A - Terminal velocity is given by,
$v_T = \frac{2}{9} \frac{r2\,(\rho-\sigma)\,g}{\eta}$ That means, v_T proportional to r_2

Surface area of a drop, $A = 4\pi r^2$
So, v_T proportional A
Given, $v_{T1} / v_{T2} = 3 / 4$
So, $v_{T1} / v_{T2} = A_1 / A_2 = 3 / 4$
Q.10 A - Pressure = Force / Area
The ratio of the forces is F1: F2=1:7
The ratio of the areas is A1: A2 = 3:2
P1= F1 / A1 = k / 3   P2 = F2 / A2 = 2 / 7
P1 / P2 = F1 / A1 X A2 / F2 = k / 3 X 7 / 2
P1 / P2 = 2 / 21
Q.13 A- Energy stored per unit surface area = 2 times of surface tension.
Surface tension is taken two times because a soap bubble has two surfaces.
So,
Energy = 2× surface tension× surface area.
Here, E = energy, S = surface tension and A = surface area in metre squares.
radius = half of diameter = 3cm or 0.03m. $A = 4\pi r^2$
Now, Putting values in the above formula we get,
$E = 2 \times 0.4 \times 4 \times \pi \times (0.03)^2 = 0.009$ joule.= 0.9×10^{-3} J
Q.16 A- $h = 2T\cos\theta / \rho g r$ or $4T\cos\theta / \rho g 2r$ or $4T\cos\theta / \rho g D$
New height $h' = 4T\cos\theta / \rho g 2D$ or $h' = h/2$

Assertion- Reason

Directions : Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct

Q.1- Assertion (A): The pressure at a point in a fluid at rest is the same in all directions.
Reason (R): Fluids at rest cannot sustain shear stress.
Q.2- Assertion : The blood pressure in humans is greater at the feet than at the brain
Reason : Pressure of liquid at any point is proportional to height, density of liquid and acceleration due to gravity.
Q.3- Assertion: The pressure of water reduces when it flows from a narrow pipe to a wider pipe.
Reason: Since for wider pipe area is large, so flow of speed is small and pressure also reduces proportionately
Q.4- Assertion (A): A fluid in a container exerts pressure only on the bottom of the container.
Reason (R): Pressure in a fluid at rest is transmitted equally in all directions
Q.5- Assertion (A): In a streamline flow, the velocity of the fluid particle crossing a particular point is always the same.
Reason (R): In a turbulent flow, the velocity of fluid particles crossing a point is not the same.

Q.6- Assertion (A): Capillary rise is higher for a liquid with greater surface tension.
Reason (R): Capillary rise occurs due to the adhesive forces between the liquid and the walls of the capillary.
Q.7-Assertion (A): Surface tension acts tangentially to the surface of the liquid.
Reason (R): Surface tension is a force that minimizes the surface area of a liquid.
Q.8-Assertion (A): The pressure inside a soap bubble is greater than the pressure outside.
Reason (R): The excess pressure inside the soap bubble is due to the surface tension of the soap film.
Q.9-Assertion (A): Water rises in a capillary tube because of cohesion.
Reason (R): Adhesion between water molecules and the walls of the capillary tube causes the water to rise.
Q.10-Assertion (A): The shape of a liquid meniscus in a capillary tube depends on the relative strengths of cohesive and adhesive forces.
Reason (R): If cohesive forces are stronger than adhesive forces, the meniscus is concave.

Assertions & Reasons Answers:

1.a	2.a	3.a	4.d	5.b	6.b	7.a	8.a	9.d	10.b

Very Short Answer Type Questions (2 Marks)
Q.1 Explain the concept of capillary action with an example.
Q.2 Differentiate between streamline flow and turbulent flow.
Q.3 Write the expression for the pressure at a depth h in a liquid of density d
Q.4 A hydraulic lift is used to lift a car of mass 1000 kg. The area of the cross-section of the piston carrying the car is 1 m^2. What is the pressure exerted on the piston? ($g=9.8 m/s^2$)
Q.5 Calculate the excess pressure inside a soap bubble of radius 5 cm. Surface tension of soap solution is 0.03N/m.
Q.6 A liquid has a surface tension of 0.075N/m. Calculate the excess pressure inside a spherical droplet of this liquid with a radius of 0.5 mm.
Q.7 A cylindrical tank is filled with water to a height of 10 m. What is the pressure at the bottom of the tank? (Density of water = 1000 kg/m^3 ,$g=9.8 m/s^2$)
Q.8 Explain why oil rises through the wick of an oil lamp?
Q.9 State Bernoulli's principle.
Q.10 How does temperature affect the viscosity of liquids and gases?

Answers Of Very Short Questions(numerical only):

Ans .4 Pressure exerted on the piston P is given by: P=F/A
: F=mg F=1000×9.8=9800 N
Therefore, P=9800/1=9800Pa

Ans. 5 For a soap bubble, the excess pressure ΔP is given by: ΔP = 4T/r
ΔP=4×0.03/0.05
ΔP=0.12/0.05

$\Delta P = 2.4$ Pa

Ans. 6 Surface tension T=0.075N/m
Radius of the droplet r=0.5mm = 0.5×10^{-3} m = 0.0005 m
The excess pressure $\Delta P = 2T/r$
$\Delta P = 2\times0.075 / 0.0005$
$\Delta P = 0.15 / 0.0005$
$\Delta P = 300$ Pa
Ans.7
h=10m, $\rho = 1000$ kg/m^3 g=9.8 m/s^2
P = ρgh
= 1000 x 9.8 x 10
= 98000 Pa

Short Answer Type Questions(3 marks)

Q.1 Discuss the variation of fluid viscosity with temperature and pressure, also find the units and dimensions of coefficients of viscosity.
Q.2 State and deduce Stokes' law also state the condition under which Stokes' law is valid.
Q.3 A hydraulic lift has two pistons of different diameters. The diameter of the smaller piston is 5 cm, and the diameter of the larger piston is 30 cm. If a force of 200 N is applied to the smaller piston, calculate the force exerted by the larger piston.
Q.4. State Pascal's law and give an example of its application.
Q.5 Explain why?
(i) Small drops of mercury are spherical and larger ones tend to flattened.
(ii) Sand is drier than clay.
Q.6 On the basis of Bernoulli's principle explains the lift on an aircraft.
Q.7 Derive formula for capillary rise. Calculate the capillary rise of water in a glass tube of radius 0.5 mm. (Surface tension of water = 0.072 N/m, contact angle = 0°, density of water = 1000 kg/m^3)
Q.8 Define angle of contact and on what factors it depends.
Q.9 Derive excess of pressure inside an air bubble.
Q.10 Two soap bubbles have their radii in the ratio 2:3 compare excess of pressure inside these bubbles also compare the work done in blowing these bubbles.

Answers Of Short Questions(numerical only):

Ans 3. Area of smaller piston = $\pi (d1/2)^2 = \pi (5/2)^2 = \pi(2.5)^2 = 19.625$cm^2
Area of larger piston =$\pi (d_2/2)^2 = \pi (30/2)^2 = \pi (15)^2 = 706.5$ cm^2
Force exerted by larger piston = $\frac{\text{Area of larger piston}}{\text{Area of smaller piston}}$ x Force applied to smaller piston
$=\frac{706.5}{19.625}$ x200 N = 7200 N
Ans 7 h = $\frac{2T\cos\theta}{pgt}$
h= $\frac{2\times00.072\times\cos\theta}{1000\times9.8\times0.0005}$ =2.94 cm
Ans 10. $\frac{R1}{R2} = \frac{2}{3}$,

$\frac{P1}{P2} = \frac{4\sigma}{R1}$ X $\frac{R2}{4\sigma}$ = 3:2
Work done in blowing up the bubble

$w_1 = 2 \times 4\pi R^2{}_1 \times \sigma$ $w_2 = 2 \times 4\pi R^2{}_2 \times \sigma$

$\frac{W1}{W1} = \frac{R2\ 1}{R2\ 2} = 4:9$

Long Answer Type QuestionS (5 Marks)

Q.1 Give principle and working of hydraulic brake with a suitable diagram.
Q.2 State and prove Bernoulli's principle for flow of non-viscus, incompressible liquid in streamline flow.
Q.3 (i) Explain why sometimes the light roofs of thatched houses are blown off during storm.
(ii) Derive stokes law dimensionally
Q.4 (i) Derive an expression for the rise in capillary tube of uniform diameter and sufficient length.
(ii) A liquid drop of diameter D breaks up into 27 small drops , find the resulting change in energy. Take surface tension on the liquid as $\sigma\sigma$.
Q.5 Describe how does a body attains terminal velocity when it is dropped from rest to a viscous medium. Derive an expression for terminal velocity.
Q.6 Show that there is always an excess pressure on the concave side of the meniscus of a liquid. Obtain expression for the excess pressure
(i) inside a liquid drop (ii) inside liquid bubble (iii) inside an air bubble.
Q.7 The surface tension of soap solution at 20 °C is 2.50×10^{-2} N m^{-1}. Calculate the excess pressure inside a soap bubble of radius 5 mm of this solution. If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution of relative density 1.20, what would be the pressure inside the bubble? (1 atm = 1.01×10^5 pa.)

Case Study Questions

Read the passage given below and answer the following questions

Case 01

Angle of Contact

The angle of contact, also known as the contact angle, is the angle formed between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid. It is denoted by θ

Wettability:

- **Acute Angle (< 90°):** If the angle of contact is acute, the liquid wets the solid surface. This indicates strong adhesive forces between the liquid and the solid compared to the cohesive forces within the liquid. For example, water on a glass surface.
- **Obtuse Angle (> 90°):** If the angle of contact is obtuse, the liquid does not wet the solid surface. This indicates weaker adhesive forces compared to the cohesive forces within the liquid. For example, mercury on glass.

Factors Affecting Angle of Contact:

- **Nature of the liquid and solid:** Different combinations of liquids and solids have different contact angles.
- **Surface roughness and cleanliness:** A rough or dirty surface can change the contact angle.
- **Impurities in the liquid:** Soluble impurities tend to reduce the contact angle, whereas insoluble impurities tend to increase it.

Measurement of Surface Tension: The angle of contact is crucial in measuring surface tension using methods like the capillary rise method or the drop weight method.

(i) Which of the following statements is true regarding the angle of contact?

a) It is the angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid.
b) It is the angle between the tangent to the solid surface at the point of contact and the liquid surface inside the liquid.
c) It is always 90°.
d) It is the angle between the solid surface and the horizontal.

(ii) If the angle of contact between a liquid and a solid is greater than 90°, the liquid

a) Completely wets the solid. b) Partially wets the solid.
c) Does not wet the solid at all. d) Forms a thin film over the solid

(iii) Which factor does not affect the angle of contact between a liquid and a solid?

a) Nature of the liquid. b) Nature of the solid.
c) Surface tension of the liquid. d) Temperature of the liquid.

(iv) The angle of contact is used to determine:

a) Viscosity of the liquid. b) Density of the liquid.
c) Surface tension of the liquid. d) Boiling point of the liquid.

<u>CASE 2</u>

Bernoulli's Principle states that for an incompressible and non-viscous fluid in a streamline flow, the sum of the pressure energy, kinetic energy per unit volume, and potential energy per unit volume remains constant along a streamline.
For a fluid element moving along a streamline, Bernoulli's equation can be written as:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

P = Pressure energy per unit volume ρ = Density of the fluid
v = Flow velocity of the fluid g = Acceleration due to gravity
h = Height above a reference point

Applications of Bernoulli's Principle:

1. **Airplane Wings:**
The shape of airplane wings is designed so that air flows faster over the top surface than underneath. According to Bernoulli's principle, the pressure above the wing is lower than below, creating lift.
2. **Venturimeter:**
Used to measure the flow rate of a fluid in a pipe. The fluid velocity increases when it passes through a narrow section of the pipe, leading to a decrease in pressure, which can be measured to determine the flow rate.
3. **Atomizer or sprayer:**
Devices like perfume sprayers use Bernoulli's principle. When air flows rapidly over the tube opening, the pressure drops, and the liquid rises into the tube and is sprayed out.
4. **Carburettors in Engines:**
Carburettors mix air with fuel using Bernoulli's principle. The high-speed airflow in the

narrow part of the carburettor decreases pressure, drawing fuel into the air flow.

(i) If the velocity of a fluid doubles, the dynamic pressure component ($\frac{1}{2}\rho v^2$) will:
a) Remain the same b) Double
c) Quadruple d) Halve
(ii) An aircraft wing generates lift due to Bernoulli's principle because:
a) Air pressure is higher on top of the wing.
b) Air speed is higher on top of the wing.
c) Air pressure is higher below the wing.
d) Both b and c.
(iii) Water flows through a horizontal pipe of cross-sectional area A_1 = 0.01 m^2 with a speed of v_1 = 2 m/s. The pipe then narrows to a cross-sectional area A_2=0.005 m^2. What is the speed of water v_2 the narrower section?
a) 1 m/s b) 2 m/s
c) 4 m/s d) 8 m/s
(iv) Air is flowing through a pipe at a speed of 10 m/s10 where the pressure is 500Pa and the density is 1.2 kg/m^3 . If the speed of the air increases to 20 m/s in a narrower section of the pipe, what is the pressure in this section?
a) 380 Pa b) 420 Pa
c) 460 Pa d) 480 Pa

Answers

Case 1

1 .a	2.c	3.d	4.c

Case 2

1.c	2.d	3.c Hint $A_1v_1= A_2v_2$	4.b Hint $P_1+\frac{1}{2}\rho v_1^2 = P+\frac{1}{2}\rho v_2^2$

Competency Bases / Logical Puzzle (With Answers)

1.Logic Puzzle

Question: A container is divided into two sections by a movable, frictionless piston. Initially, the piston is at rest, with one side filled with a fluid of density ρ1 and the other side with a fluid of density ρ2. If ρ1>ρ2 predict the direction in which the piston will move and explain why.

Answer: The piston will move towards the side with the fluid of lower density (ρ2). This is because the side with the higher density fluid (ρ1) exerts a greater pressure on the piston. The pressure difference will cause the piston to move until the pressures on both sides are equal, achieving mechanical equilibrium.

2.Logic Puzzle: The Airplane Wing Mystery

Scenario: You are an aerodynamics engineer investigating the lift on four different airplane wings (A, B, C, and D) using Bernoulli's theorem. Each wing is tested in a wind tunnel where air flows at different speeds (V1, V2, V3, and V4) over the top and bottom surfaces of the wings. The pressure differences between the top and bottom surfaces are measured, and the following clues are given:

Clues:

1. Wing A has a higher airflow speed over the top surface (V1) compared to its bottom surface (V2). The lift generated is the highest.

2. Wing B has equal airflow speeds over the top and bottom surfaces, resulting in no lift.
3. Wing C has a higher airflow speed over the bottom surface (V3) compared to its top surface (V4). The lift generated is negative (downward force).
4. Wing D generates lift, but the lift is less than that of Wing A. The airflow speed over the top surface (V4) is equal to that of the bottom surface of Wing A (V2).

Task: Using the clues provided, determine:

- The airflow speeds over the top and bottom surfaces for each wing.
- The relative pressure differences and lift generated by each wing.
- Identify which wing is A, B, C, and D based on the lift generated.

Solution:

1. **Analyze each clue:**
    - **Wing A**: Higher speed over the top (V1) compared to the bottom (V2), generating the highest lift.
    - **Wing B**: Equal speeds over the top and bottom, generating no lift.
    - **Wing C**: Higher speed over the bottom (V3) compared to the top (V4), generating negative lift.
    - **Wing D**: Generates lift less than Wing A, with V4 (top of Wing D) equal to V2 (bottom of Wing A).
2. **Applying Bernoulli's theorem:**
    - According to Bernoulli's principle, an increase in the speed of the fluid (air) occurs simultaneously with a decrease in pressure.
    - Lift is generated when the pressure on the top surface is lower than the pressure on the bottom surface.
3. **Determine airflow speeds and pressure differences:**
    - For **Wing A**:
        - V1 > V2 (higher speed on top)
        - Highest lift implies maximum pressure difference:
    - For **Wing B**:
        - V = V (equal speed on top and bottom)
        - No lift:
    - For **Wing C**:
        - V3 > V4 (higher speed on bottom)
        - Negative lift:
    - For **Wing D**:

- V4 = V2 (top of Wing D = bottom of Wing A)
- Lift, but less than Wing A

4. **Identify wings based on lift:**
   - **Wing A**: Highest lift (V1 > V2)
   - **Wing B**: No lift (V = V)
   - **Wing C**: Negative lift (V3 > V4)
   - **Wing D**: Lift less than Wing A, V4 = V2
   - v3 must be less than 2 m/s.

In conclusion:

- The drag forces are related as F1<F2<F3.

The velocity v1 is greater than 2 m/s, and v3 is less than 2 m/s.

Unit VII Properties of Bulk Matter

Chapter 11: Thermal Properties Of Matter

GIST OF THE CHAPTER:

● **Heat:** It is a form of energy which produces in us the sensation of hotness or coldness. Heat is the energy in transit.

● **Temperature:** It is the degree of hotness or coldness of a body.

● **Relationship between different temperature scales:**

$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4} = K - \frac{273.15}{5}$$

● **For a constant volume air thermometer, $T = T_o \times P/P_o$**

● **In terms of triple point, $T = T_{tr} \times P/P_{tr}$**

● **Thermal expansion:** The increase in size of a body when it is heated is called thermal expansion. It is of three types –

1. **Linear expansion:** It is the increase in the length of a metal rod on heating

Coefficient of linear expansion (α)= $\frac{increase\ in\ length}{original\ length\ X\ rise\ in\ temperature}$

2. **Superficial expansion:** It is the increase in the surface area of a metal sheet on heating

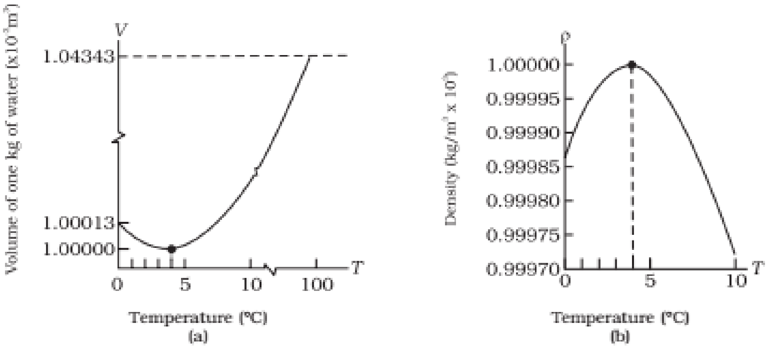
Coefficient of superficial expansion (β) = $\frac{increase\ in\ surface\ area}{original\ surface\ area\ X\ rise\ in\ temperature}$

3. **Cubical expansion:** It is the increase in the volume of a block on heating

Coefficient of cubical expansion (γ) = $\frac{increase\ in\ volume}{original\ volume\ X\ rise\ in\ temperature}$

● **Anomalous expansion of water**: It refers to the unique physical property of water that it expands instead of contracting when it freezes, not like other materials.

Below 4 degrees Celsius, water becomes less dense as it cools further, contrary to most substances which become denser as they cool.



Thermal expansion of water.

● **Specific heat(c):** It is the amount of heat required to raise the temperature of unit mass of the substance by one degree.

$$c = \frac{\Delta Q}{M \, X \, \Delta T}$$

● **Molar Specific heat(C):** It is the amount of heat required to raise the temperature of one mole of the substance by one degree.

$$C = \frac{\Delta Q}{n \, X \, \Delta T}$$

● **Heat capacity(S):** It is the amount of heat required by a body to raise its temperature by one degree.

Heat capacity = Mass x Specific heat

S= m x c

● **Calorimetry:** It is a branch of physics that deals with the measurement of heat.

● **Principle of Calorimetry:** It states that "heat gained by the cold body must be equal to the heat lost by the hot body provided there is no exchange of heat with the surroundings".

Heat gained = Heat lost

● **Change of state:** The transition of a substance from one state to another by heating or cooling.

1. **Melting (solid to liquid on heating)**
2. **Vaporization (liquid to vapour on heating)**
3. **Condensation (vapour to liquid on cooling)**
4. **Freezing (liquid to solid on cooling)**
5. **Sublimation (solid to gas directly on heating)**

● **Melting point:** The temperature at which the solid and liquid state of a substance coexist in thermal equilibrium with each other.

● **Boiling point:** The temperature at which the liquid and vapour state of a substance coexist in thermal equilibrium with each other.

● **Latent heat (L):** The amount of heat required to change the state of unit mass of a substance at constant temperature and pressure.

Q = m L

SI unit of latent heat is J/kg

● **Latent heat of fusion (L_f):** The amount of heat required to change the state of unit mass of a substance from solid to liquid at its melting point.
● **Latent heat of vaporization (L_v):** The amount of heat required to change the state of unit mass of a substance from liquid to vapour at its melting point.

● **Modes of Heat transfer:** Heat can be transfer from one place to another by three different methods-
1. **Conduction**: The process in which heat is transmitted from one part of a body to another at lower temperature through molecular collisions, without any actual flow of matter.

2. Convection: The process in which heat is transmitted from one point of a body to another due to bodily motion of the heated particles of the substance.

3. Radiation: The process in which heat is transmitted from one place to another without heating the intervening medium.

● **Coefficient of thermal conductivity (K):** The quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.

$$Q = K\,A\frac{t(T1 - T2)}{X}$$

If A =1, t =1, T1-T2 = 1, X=1
Then Q = K

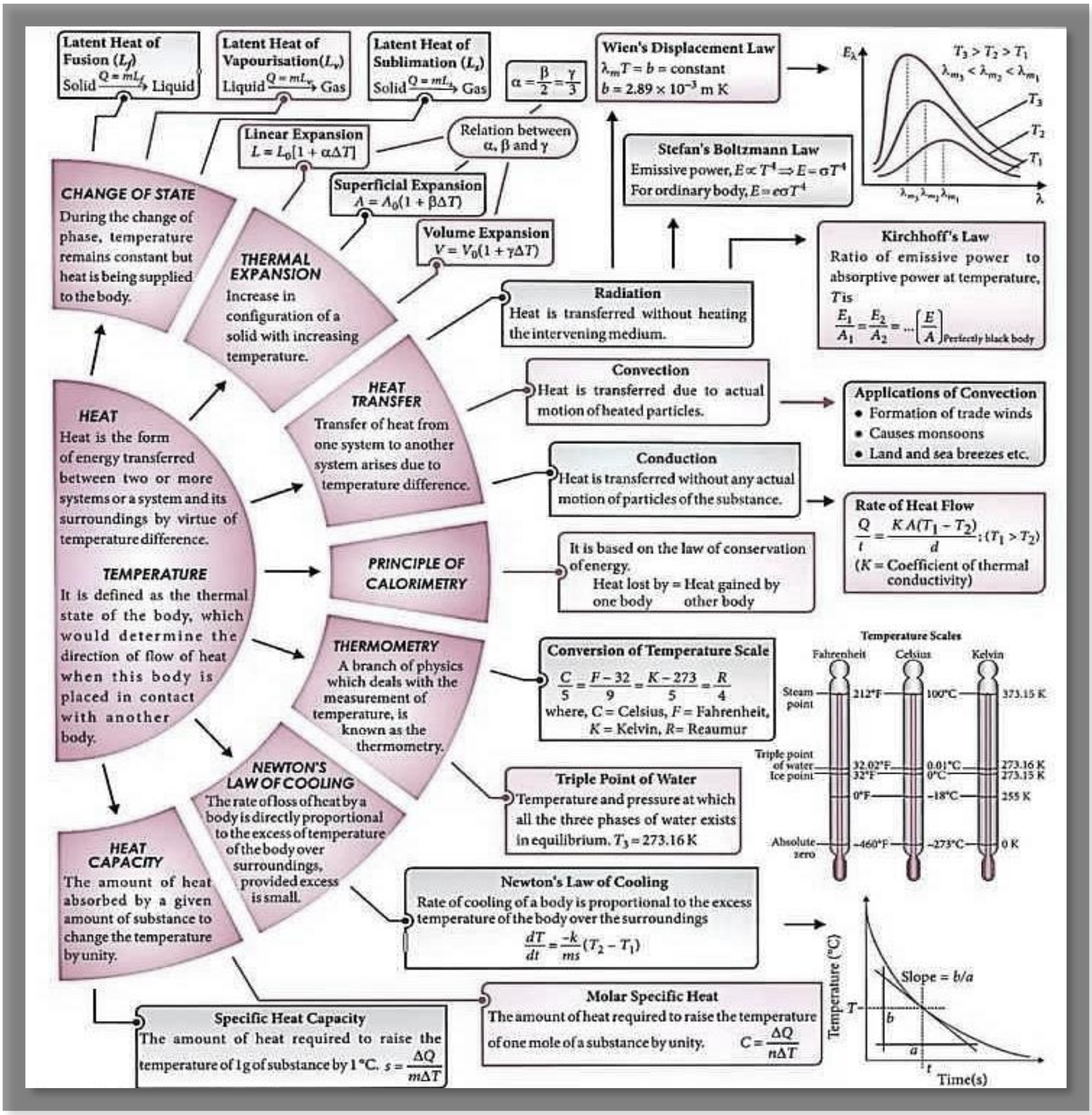
● **Blackbody radiation:** A body which neither reflects nor transmits but absorb whole of the heat radiation incident on it is called a black body.

When a black body is heated to a high temperature, it emits radiations of all the possible wavelengths within a certain wavelength range. These radiations emitted are called black body radiation.
● **Wein's displacement Law:** It states that “the wavelength (λ_m) corresponding to which the energy emitted by a perfect black body is maximum, is inversely proportional to the absolute temperature(T) of the black body”.

$$\lambda m = \frac{b}{T}$$

● **Stefan's- Boltzmann law:** It states that the total amount of energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the body.

$$\mathbf{E = \sigma T^4}$$



Multiple Choice Questions

1. The Fahrenheit and Celsius scales intersect at temperature

a) -30° b) 35° c) -40° d) 45°

2. When a body is heated, the maximum rise will be in its

a) length b) surface area c) volume d) density

3. If temperature of a body is increased by 10°C, its length increases by 1%. What is the percentage change in volume of the body of the same material for 10°C increase in temperature

a) 9% b) 1% c) 5% d) 3%

b)

4. When water is heated from 0^{o}C to 10^{o}C, its volume
a) Decreases continuously
b) First decreases then increases
c) First increases then decreases
d) Increases continuously

5. Two liquids A and B of equal amount are heated for equal intervals of time on identical stoves in identical conditions. After heating temperature of A is found to be less than B. Choose the correct option:
a) A has more specific heat capacity than B
b) A has less specific heat capacity than B
c) A has less latent heat than B
d) A has more latent heat than B

6. Specific heat of a substance is a function of its
a) Mass b) weight c) volume d) molecular structure

7. Three identical rods A, B and C of equal lengths and equal diameters are joined in series as shown in diagram. Their thermal conductivities are 2K, K and K/2 respectively. Find the temperature at two junction points
a) 85.7, 57.1^{o}C
b) 80.85, 50.3^{o}C
c) 77.3, 48.3^{o}C
d) 75.8,49.3^{o}C

8. 50g of ice at 0oC is mixed with 50g of water at 80oC, final temperature of the mixture will be
a) 0 ^{o}C b) 40 ^{o}C c) 60 ^{o}C d)4^{o}C

9. A flask of volume V has some liquid in it. It is found that at different temperatures, the volume if air inside the flask remains constant. If coefficient of linear expansion of the flask is 1/18 of the cubical expansion of liquid, the volume of the liquid in the flask is
a) V/2 b) V/3 c) V/6 d) V/18

10. The ratio of thermal conductivity of two rods of different materials is 5:4. The two rods of same area of cross section and same thermal resistance will have lengths in the ratio
a) 4:5 b) 9:1 c) 1:9 d) 5:4

11. A perfect black body has
a) Absorptive power is 1
b) Absorptive power is 0
c) Absorptive power is 0.5
d) Emissive power is 1

12. The latent heat of vaporization of water is
a) Less than the latent heat of fusion of ice
b) Equal to the latent heat of fusion of ice
c) More than the latent heat of fusion of ice

d) Can be greater or lesser than the latent heat of fusion of ice

13. Three cubes of sides 1,2 and 3 cm are at constant temperature of 1000^{o}C.The amount of heat lost per second by them are in the ratio

a) 1:8:27 b) 27:8:1 c) 1:4:9 d) 9:4:1

14. Two bars of copper having same length but an equal diameter are heated to the same temperature. The change in length will be:

(a) more in thicker bar
(b) more in thinner bar
(c) same for both
(d) determined by the ratio of length and diameter of the bars

15. In the pressure cooker, the cooking is faster because the increase of vapour pressure:

(a) increases melting point
(b) increases boiling point
(c) decreases boiling point
(d) decreases melting point

16. A beaker is completely filled with water at 4°C. It will overflow

(a) when heated but not when cooled
(b) when cooled but not when heated
(c) neither when heated nor when cooled
(d) both when cooled or heated

17. Alcohol is more volatile than water because the

(a) boiling point of alcohol is > than that of water
(b) boiling point of alcohol is < than that of water
(c) melting point of alcohol is < than that of water
(d) melting point of alcohol is > than that of water

18. Two rods A and B of different materials are welded together as sown in the diagram. Their thermal conductivities are K1 and K2. The thermal conductivity of the composite rod will be

a) (K1+K2)/2
b) 3(K1+K2)/2
c) K1+K2
d) 2(K1+K2)

19. On observing light from three different stars P, Q and R it was found that the intensity of violet colour is maximum in the spectrum of P, the intensity of green colour is maximum in the spectrum of R and the intensity of red colour is maximum in the spectrum of Q. If Tp, Tq and Tr are the respective absolute temperatures of P, Q and R then it can be concluded that

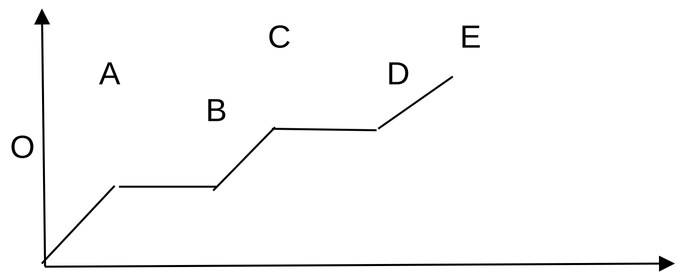
a) Tp>Tr>Tq
b) Tp<Tr<Tq
c) Tp<Tq<Tr
d) Tp<Tq>Tr

20. A cup of coffee cools from 90 to 80^{o}C in time t, when the room temperature is 20^{o}C. The time taken by a similar cup of coffee to cool from 80 to 60^{o}C at room

temperature same at 20°C is
a) 5t/13
b) 13t/10
c) 13t/5
d) 10t/13

21. A spherical black body of radius r radiates power P. Its rate of cooling is Q then
a) P α r
b) P α r^3
c) Q α r^2
d) Q α 1/r

22. A solid material with heat at constant rate and the temperature of the material changes as shown in the graph, from the graph false conclusion drawn is



a) AB and CD of the graph represent phase changes
b) AB represents the change of state from solid to liquid
c) Latent heat of fusion is twice the latent heat of vaporisation
d) CD represent change of state from liquid to vapour

23. A calorimeter of water equivalent 20g contains 180g of water at 25°C. m grams of steam at 100°C is 31°C. The value of m in grams is close to:
(given, Latent heat of water = 540 cal/g, specific heat of water = 1 cal g^{-1} C^{-1})
a) 2
b) 2.6
c) 4
d) 3.2

Answers:

1.c	2.c	3.d	4.b	5.a
6.d	7.a	8.a	9.c	10.d
11.a	12.c	13.c	14.c	15.b
16.d	17.b	18.a	19.a	20.c
21.d	22.c	23.a		

Two Marks Questions
07. Why water is preferred over any other liquid in hot water bottles?

Ans: because specific heat of water is high, it does not cool fast and provides warmth for longer

08. The ice at 0^oC is converted into steam at 100^oC. State the isothermal changes in process.

Ans: Conversion of ice at 0^oC into water at 0^oC, conversion of water at 100^oC into steam at 100^oC

09. Two rods A and B of equal length. Each rod has the ends at temps T1 and T2. What is the condition that will ensure equal rates of flow of heat through the rods A and B?

Ans: Since, $\frac{\Delta Q1}{\Delta t} = \frac{\Delta Q2}{\Delta t}$

So, $K1\,A1\frac{T1-T2}{L1} = K2\,A2\frac{T1-T2}{L2}$

Or $\frac{A1}{A2} = \frac{K1}{K2}$

So, the cross-sectional area of two rods must be in the inverse ratio of thermal conductivities.

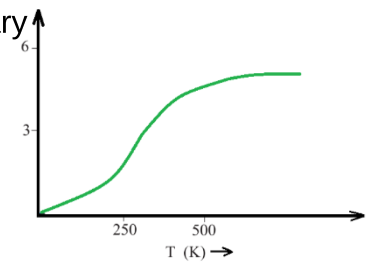
10. A beaker filled with water at 4^oC over flows if the temperature of water increases or decreases. Explain why?

Ans: because water shown anomalous expansion, its density is maximum at 4^oC

11. How does the coefficient of cubical expansion of a substance vary with temperature. Draw γ vs T curve for copper.

Ans: the value of γ first increases and then becomes constant at a high temperature.

γ



12. The difference between lengths of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. Is this possible?

Ans: Yes, this is possible when the lengths of the rods are in the inverse ratio of their coefficients of linear expansion. For the difference in the lengths of the two rods to remain same,

$\Delta l_1 = \Delta l_2$

$l_1\, \alpha_1\, \Delta T = l_2\, \alpha_2\, \Delta T$

$l_1/\alpha_2 = l_2/\alpha_1$

13. How much the temperature of a brass rod should be increased so as to increase its length by 1%? (given α for brass = 0.00002^oC^{-1})

Ans: $\alpha = 0.00002^oC^{-1}$

Initial length $= l_o$

Final length $= l$

Change in temperature =t

Since, the length has to be increased by 1%,

So, change in length = $l - l_0 = 0.01\ l_0$

We know, $\alpha \times l_0 \times t = l - l_0$

or, $\alpha \times l_0 \times t = 0.01\ l_0$

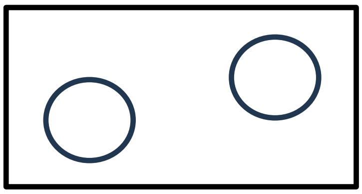
$\Rightarrow 0.00002\ t = 0.01$

$\Rightarrow t = 500^\circ C$

14. Why a body with large reflectivity is a poor emitter?

Ans: A body whose reflectivity is large would absorb less heat. So, a body with large reflectivity is a poor emitter

15. Two large holes are cut in a metal sheet.
If the sheet is heated,
how will the diameters of the holes change?



Ans: When a body is heated, the distance between
Any of its two points increases. Hence, diameters of the two holes will increase.

16. Explain why heating systems based on steam circulation are more efficient in warming a building than those based on circulation of hot water?

Ans: However, both steam and boiling water are at same temperature but each unit mass of steam contains a larger amount of additional heat called latent heat. So, steam loses more heat than boiling water.

17. When 10g of coal is burnt, it raises the temperature of 2 litres of water from 20^oC to 55^oC. Calculate the heat of combustion of fuel.

Ans: Heat produced $Q = m\ c\ \Delta T = 2 \times 10^3 \times 1 \times (55-20) = 70000$ cal
Heat of combustion = Q / M = 70000/ 10 = 7000 cal/g

18. How does the thermal conductance take place in solid and gaseous phase substances?

Ans: In solids, it takes place in two ways:

a) By the motion of free electrons, in case of metals

b) By the lattice vibrations or phonons, in case of non- metals

In gases, it takes place through discrete molecular collisions of gaseous particles/ molecules.

19. Find the thermal capacity of 40 gm of aluminium (given specific heat = 0.2 cal/gm/°C).

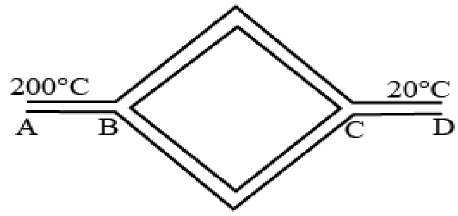
Ans: Thermal capacity = m x c
= 40 × 0.2
= 8 cal^{o}C^{-1}

20. The normal temperature of human body is 98.4^{o}F, calculate this temperature on Celsius and absolute scale.

Ans: $\frac{C}{100} = \frac{F-32}{180}$

Here F = 98.4, on substituting the value and solving we get
C = 36.88^{o}C or 36.9^{o}C (approx.)
For absolute scale, K = C + 273 = 36.9 + 273 = 309.9 K

21. Six identical rods are joined as shown in the diagram, point A and D are maintained at temp 200^{o}C and 20^{o}C respectively.



Find the temperature of junction B.

Ans: The two resistances connected along two paths from B to C are equivalent to 2R each and their parallel combination is R.

Effective thermal resistance between B and D = 2R

So, the resistance of the three parts AB, BC and CD are equal (R)

Therefore, temp difference must be distributed equally = (200 -20)/3 = 60^{o}C

So, temp at point B = 200 – 60 = 140^{o}C

<u>Short Type Questions</u> (3 Marks)

22. Calculate the amount of heat required to convert 1.00 kg of ice at -10^{o}C into steam at 100^{o}C at normal pressure. Specific heat of ice = 2100 J Kg^{-1} K^{-1}, latent heat of fusion of ice = 3.36 x 10^{5}J/kg, specific heat capacity of water= 4200 J Kg^{-1} K^{-1} and latent heat of vaporization of water = 2.25 x 10^6 J/kg

Ans:
Heat required to increase the temp of ice from -10 to 0^{o}C
= smΔT = 2100 x 1 x 10 = 21000 J
Heat required to melt ice to 0^{o}C
= m L = 1x 3.36 x 10^5= 336000 J
Heat required to increase the temp of water from 0 to 100^{o}C
= smΔT = 4200 x 1 x 100 = 420000 J
Heat required to convert to 100^{o}C water to 100^{o}C steam
= m L = 1 x 2.25 x 10^6 J = 2250000 J
So total heat required = 21000 + 336000+ 420000 + 2250000
= 3.027 x 10^6 J

23. The coefficient of volume expansion of glycerin is 49 x 10^{-5} $^oC^{-1}$. What is the fractional change in its density for a 30^oC rise in temperature?

Ans:
$$\gamma = \frac{Vt - Vo}{Vo\Delta T}$$
$$\gamma = \frac{\frac{M}{\rho t} - \frac{M}{\rho o}}{\frac{M\Delta T}{\rho o}}$$
$$\gamma = \frac{\rho o - \rho t}{\rho o\Delta T}$$
So fractional change, $\frac{\rho o - \rho t}{\rho o \Delta T} = \gamma\Delta T = 0.0147$

24. How many grams of ice at -14^oC are needed to cool 200 gm of water from 25 to 10^oC? Given specific heat of ice = 0.5 cal g^{-1} $^oC^{-1}$ and latent heat of ice = 80 cal g^{-1}

Ans:
Heat lost by water in cooling from 25 to 10^o C, Q = m c ΔT
Q = 200 x 1 x (25-10)
Q = 3000 cal

Heat gained by ice at -14^oC to change into water at 10 ^oC
= $(m\ c\ \Delta T)_{ice}$ + mL + $(m\ c\ \Delta T)_{water}$
= 97 m cal
According to principle of calorimetry,
Heat gain = heat lost
97 m = 3000
So, m = 31 g

25. Heat is flowing through a rod of length 25 cm having cross-sectional area 8.80 cm^2. The coefficient of thermal conductivity for the material of the rod is K = 9.2 x 10^{-2} kcal s^{-1} m^{-1} oC^{-1}. The temperature of the ends of the rod are 125^oC and 0^oC in the steady state. Calculate
i) Temperature gradient in the rod
ii) Temperature of a point at a distance of 10 cm from the hot end
iii) Rate of flow of heat

Ans:
i) temp gradient = $(T_2-T_1)/x$ = (0-125)/ 25 = 5^oC
ii) change in temp at a point 10 cm distant from hot end= temp grad x distance = -5 x 10 = -50^oC
so, temp at this point = 125 – 50 = 75 ^oC
iii) rate of flow of heat, Q/t = $KA(T_1-T_2)/x$
= (9.2 x 10^{-2} x 8.80 x 10^{-4} x 125)/0.25
= 4.048 x 10-2 k cal s^{-1}

26. Two spheres made of same substance have diameter in the ratio of 1:2. What will be the ratio of their thermal capacities?

Ans: Thermal capacity = Mass x Specific heat
Due to same material both spheres will have same specific heat

So, Ratio of thermal capacity = m1/m2 = V1 ρ / V2 ρ = V1/V2 = $(R1/R2)^3$ = 1:8

27. A beaker contains 200 gm of water. The heat capacity of the beaker is equal to that of 20 gm of water. The initial temperature of water in the beaker is 20°C. If 440 gm of hot water at 92°C is poured in it, find the final temperature?

Ans:
According to the principle of calorimetry,
Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker
If the final temp is T,

$$\text{Then, } 440 \times (92-T) = 200 \times (T-20) + 20 \times (T-20)$$

On solving we get, T = 68°C

28. Railway lines are laid with gaps to allow for expansion. If the gap between steel rails 66m long be 3.63cm at 10°C, then at what temperature will the line just touch? Coefficient of linear expansion of steel = $11 \times 10^{-6}\,{}^{o}C^{-1}$

Ans: l = 66 m, $\Delta l = 3.63 \times 10^{-2}$ m
$\alpha = 11 \times 10^{-6}\,{}^{o}C^{-1}$
$\Delta l = l\,\alpha\,\Delta T$
So, $\Delta T = \Delta l / l\,\alpha$ = 50°C
Final temperature = initial temperature + ΔT = 10 + 50 = 60°C

29. From what height should a piece of ice fall so that it melts completely? Only one quarter of the heat produced is absorbed by the ice.(latent heat of ice is 3.4×10^5 J/kg and g = 10 m s^{-2})

Ans:
heat produced on falling of the ice = ¼ decrease in P.E
= mgh/4
Heat required to melt the ice = m L
So, mgh/4 = mL
Or, h = 4L/g = $(4 \times 3.4 \times 10^5)/10 = 136 \times 10^3$ m

30. A thin rod having length l_o at 0°C and coefficient of linear expansion α has its two ends maintained at temperature T_1 and T_2 respectively. Find its new length.

Ans: at steady state, $\frac{dQ}{dt} = \frac{KA(T1-T)}{\frac{lo}{2}} = \frac{KA(T-T2)}{\frac{lo}{2}}$

So, $T_1 - T = T - T_2$
Or, $T = (T_1 + T_2)/2$
So new length, $l = l_o(1+\alpha T) = l_o[1+\alpha(T_1+T_2)/2]$

31. If 0.93 watt-hour of energy is supplied to a block of ice weighing 10 gm. What will be the effect on the ice block? (given latent heat of fusion for ice = 80 cal/g)

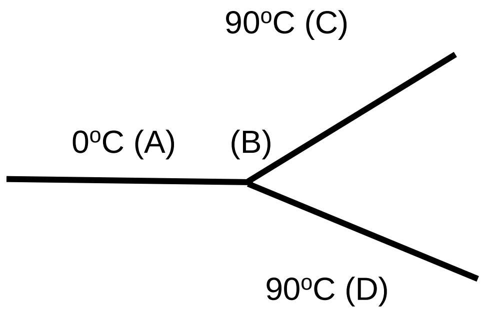
Ans: 1 watt – hour = 3600 J
energy supplied, H = 0.93 x 3600 J = 3348 J
Heat required to melt the block, Q = m L = 10 x 80 x 4.18 = 3344 J
Since, H > Q so ice block just melts completely.

32. Three rods made of same material and having the same cross section have been joined as shown in the diagram. Each rod is of the same length. The left and right ends are kept at 0^oC and 90^oC respectively. What will be the temperature of the junction of the three rods.

Ans: Assuming R be the thermal resistance and T be the temp of the junction then

$$\frac{90-T}{R}+\frac{90-T}{R}=\frac{T-0}{R}$$

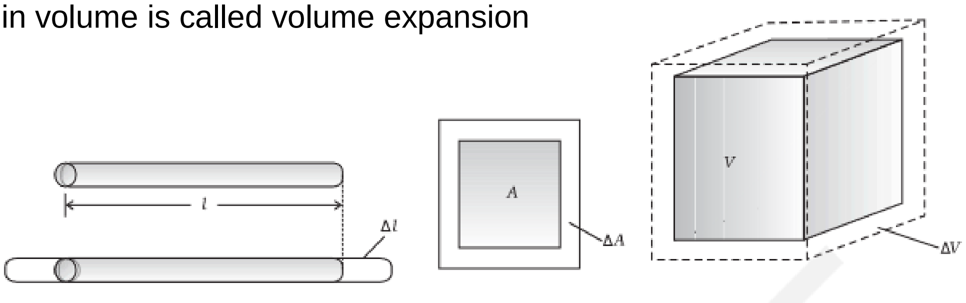
so on solving we get, T = 60^oC



Case Based Questions

33. THERMAL EXPANSION:

The increase in the dimensions of a body due to the increase in its temperature is called thermal expansion.
The expansion in length is called linear expansion.
The expansion in area is called area (superficial) expansion.
The expansion in volume is called volume expansion



Linear expansion area expansion volume expansion

Coefficient of linear expansion, $\alpha = \frac{\Delta l}{l\, \Delta T}$

Coefficient of area or superficial expansion, $\beta = \frac{\Delta A}{A\, \Delta T}$

Coefficient of volume expansion, $\gamma = \frac{\Delta V}{V\, \Delta T}$

The three coefficients are related as $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$

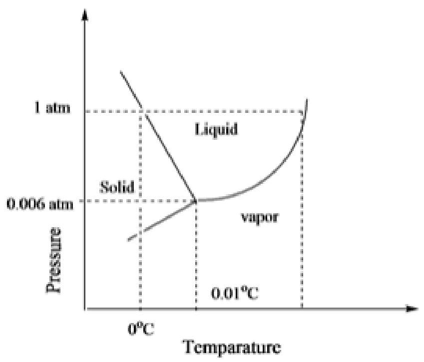
Questions:

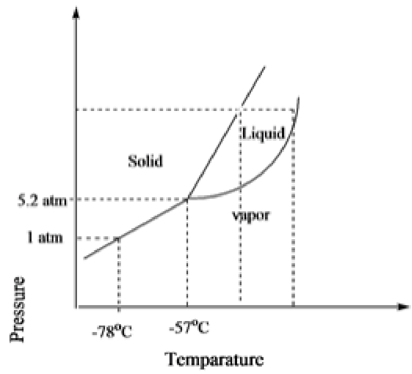
I) A bar of iron is 10 cm at 20^oC, at 19^oC it will be (given $\alpha = 11 \times 10^{6}/^oC$)
a) 11×10^{6} cm longer
b) 11×10^{-5} cm shorter
c) 11×10^{5} cm shorter
d) 11×10^{-6} cm longer
II) Expansion during heating
a) Occurs only in solids

b) Increases the weight of a material
c) Decreases the density of a material
d) Occurs at the same rate for all solids and liquids
III) When a bimetallic strip is heated
a) Doesn't bend at all
b) Get twisted in the form of a helix
c) Bend in the form of an arc with more expandable metal outside
d) Bend in the form of an arc with more expandable metal inside
IV) Water has maximum density at
a) 0^oC
b) 32^oF
c) -4^oC
d) 4^oC

34. Triple Point:

The temperature of a substance remains constant during its change of state (phase change). A graph between the temperature T and the Pressure P of the substance is called a phase diagram or P – T diagram. The following figure shows the phase diagram of water and CO2.





Phase diagram for water phase diagram for CO_2

Such a phase diagram divides the P – T plane into a solid-region, the vapour-region and the liquid-region.
The regions are separated by the curves such as sublimation curve, fusion curve and vaporization curve. The points on sublimation curve represent states in which solid and vapour phases coexist. The point on the sublimation curve represents states in which the solid and vapour phases co-exist. Points on the fusion curve represent states in which solid and liquid phase coexist. Points on the vaporization curve represent states in which the liquid and vapour phases coexist. The temperature and pressure at which the fusion curve, the vaporization curve and the sublimation curve meet and all the three phases of a substance coexist is called the triple point of the substance.

Questions:

I) Absolute zero is the temperature at which
a) Water freezes
b) All substance exists in solid state

c) Molecular motion stops
d) None of the above
II) At what temperature solid, liquid and vapour phases of CO_2 coexist in equilibrium
a) 0^oC
b) -57^oC
c) -78^oC
d) 100^oC
III) A body cools from 80 to 64^oC in 5 minutes and same body cools from 80 to 52^oC in 10 minutes. What is the temperature of surroundings?
a) 24^oC
b) 30^oC
c) 26^oC
d) -26^oC
IV) The sprinkling of water reduces slightly the temperature of a closed room because
a) Temperature of water is less than that of room
b) Specific heat of water is high
c) Water has large latent heat of vaporization
d) Water is a bad conductor of heat

Answers:

50.

I) b	II) c	III) c	IV) d

51.

I) c	II) b	III) a	IV)c

Long Questions

35. a) Define coefficient thermal conductivity. And give its SI unit.

b) Two vessels A and B of different of different materials but having identical shape, size and wall – thickness is filled with ice and kept at the same place. Ice melts at the rate of 100 g min^{-1} and 150 g min^{-1} in A and B respectively. Assuming that heat enters the vessels through the walls only, calculate the ratio of thermal conductivities of their materials.

Ans: a) Coefficient of thermal conductivity (K) is the quantity of heat that flows per unit time through a unit cube of the material when its opposite faces are kept at a temperature difference of one degree.

$$Q = K\,A\frac{t(T1 - T2)}{X}$$

If A =1, t =1, T1-T2 = 1, X=1

Then Q = K

SI unit is J s^{-1} m^{-1} K^{-1}

b) let m1 and m2 be the masses of ice melted in same time t = 1min in vessel A and B then,

$$Q1 = K1\,A\frac{t(T1 - T2)}{X} = m1\,L$$

$$Q2 = K2\,A\frac{t(T1 - T2)}{X} = m2\,L$$

Then, Q1/Q2 = K1/K2 = m1/m2 = 100/150 = 2:3

36. a) Why pendulum of a clock generally run fast in winter and slow in summer?

b) The diameter of iron rods A and B of same length are 2cm and 4cm respectively. They are heated through 100ºC. What is the ratio of increase in length of A to that of B?

c)45 gm of alcohol is needed to completely fill up a weight thermometer at 15◦C. Find the weight of alcohol which will overflow when the weight thermometer is heated to 33◦C. (Given for alcohol, $\gamma = 121 \times 10^{-5}$ /◦C)

Ans:

a) for a pendulum, Time period $T = 2\pi(\frac{l}{g})^{1/2}$

in winters, l decreases as a result T decreases and clock runs fast and vice versa

b) Since increase in length is independent on diameter and depends on length only. For both rods length is same

So, inc in length of A: inc in length of B = 1:1

c) Since, for volume expansion

V'= V (1 + γ ΔT)

ΔV=V γ ΔT

So, Δm = ρ′ ΔV

$$\rho' = \frac{\rho}{1 + \gamma \Delta T}$$

So,

$$\Delta m = \frac{\rho\, \Delta V}{1 + \gamma \Delta T}$$

$$\Delta m = \frac{\rho\, V\, \gamma\, \Delta T}{1 + \gamma \Delta T}$$

Substituting, m=45 g, γ = 121×10^{-5} /◦C and ΔT = 18ºC

We get, Δm = 0.98 gm

37. a) Define the following: absorptive power, emissive power and emissivity

b) Calculate the temperature (in K) at which a perfect black body radiates energy at the rate of 5.67 W cm^{-2}. Given σ = 5.67 x 10^{-8} W m^{-2} K^{-4}

Ans:

a) Absorptive power- It is the ratio of amount of heat energy absorbed in a certain time to the total heat energy incident on it in the same time within a unit wavelength range around the wavelength.

Emissive power- for a body at a given temperature and wavelength, the amount of radiant energy emitted per unit time per unit surface area of the body within a unit wavelength range.

Emissivity- it is the ratio of heat energy radiated per unit time per unit area by the given body to the amount of heat energy radiated per unit time per unit area by a perfect black body of the same temperature.

b) E= 5.67 W cm^{-2} = 5.67 x 10^4 W m^{-2}

σ = 5.67 x 10^{-8} W m^{-2} K^{-4}

According to Srefan's law, E = σ T^4

Substituting all the values we get, T= 1000 K

38. a) A faulty thermometer has its fixed point marked at 10oC and 95oC. The temperature of a body as measured by faulty thermometer is 50°C. find the correct temperature of the body?

c) Is it possible for a body to have negative temperature on kelvin scale and why?

d) Explain why the earth would be inhospitably cold without its atmosphere?

Ans: a) $\frac{C-0}{100} = \frac{x-lower\ fixed\ point}{upper\ fixed\ point-lower\ fixed\ point}$

$$\frac{C}{100} = \frac{50-10}{95-10}$$

C= (40 x 100)/85 = 47.05°C

b) No, because scale has absolute zero as minimum possible temperature.

c) Because there will be no extra heat trapped in the absence of atmospheric gases and the entire amount of heat would be radiated back from the earth's surface.

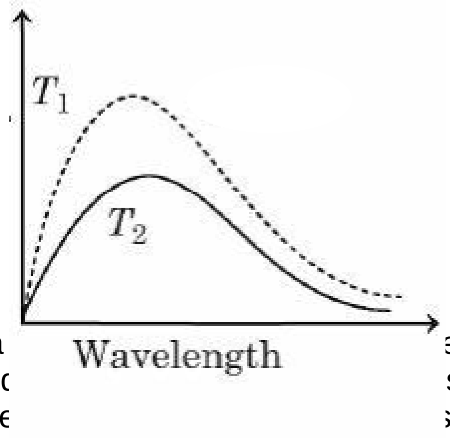
39. a) Draw energy distribution curve for a black body at two different temperatures T1 and T2 (T1>T2).

b) Write any two conclusions drawn from the curve.

c) The surface temperature of a hot body is 1227°C. Find the wavelength at which it radiates maximum energy. Given Wien's constant, b = 0.2898 cm K

Ans: a)

Energy



b) Conclusions:

At each temperature, a ... eat radiation spectrum. The energy is not distributec ... s.

As wavelength increase ... reaches a maximum for a particular wavelength and the decreases.

c)T= 1227 + 273 = 1500 K, b = 0.2898 cm K

by Wien's law, λ_m = b/T = 0.2898/1500 = 19320 X 10^{-8} cm

Matrix Match Type Question

57:

COLUMN I	**COLUMN II**
a) rate of loss of heat from body	i) kirchhof's law
b) good absorbers are good radiator	ii) Wein's displacement law
c) determination of temperature of sun	iii) newton's law of cooling

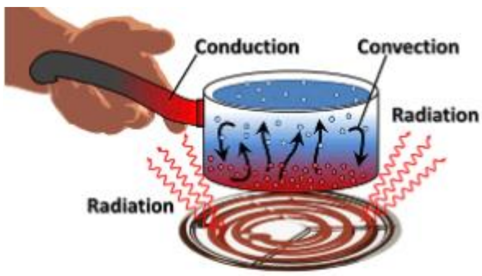
Ans: A-iii, B-i , C-s

CCT Based Questions

1. The diagram shows the different modes of transfer of heat, heat transfer is defined as the movement of heat across the border of the system due to a difference in temperature between the system and its surroundings. The temperature difference exists between the two systems, heat will find a way to transfer from the higher to the lower system.

i) The sea breeze is caused by:
a) conduction
b) convection
c) radiation
d) none of these

ii) At what factor heat absorbed on radiation by the body depends on?
a) distance between body
b) source of heat
c) its colour
d) all of the above



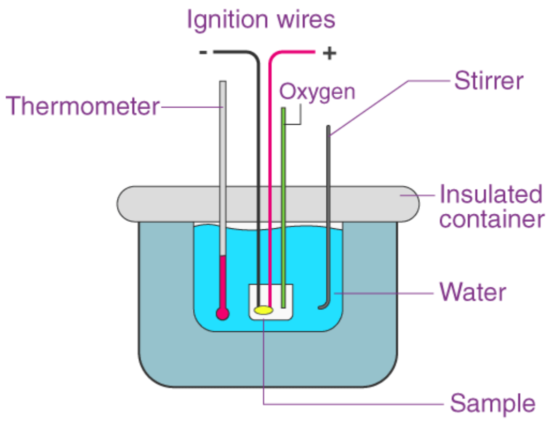
iii) Mass transfer does not take place in-
a) conduction
b) convection
c) radiation
d) none of these

iv) Thermal conductivity of air with rise in temperature:
a) increase
b) decrease
c) constant
d) none of these

2. Calorimetry means measurement of heat. When a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings. A device in which heat measurement can be done is called a calorimeter. It consists of a metallic vessel and stirrer

of the same material, like copper or aluminium. The vessel is kept inside a wooden jacket, which contains heat insulating material. Matter normally exists in three states: solid, liquid and gas. A transition from one of these states to another is called a change of state. Two common changes of states are solid to liquid and liquid to gas (and, vice versa). These changes can occur when the exchange of heat takes place between the substance and its surroundings.

i) At NTP water boils at 100ºC, deep down the mine water will boil at a temperature
a) 100ºC
b) >100ºC
c) < 100ºC
d) Will not boil at all



ii) The saturation vapour pressure of water at 100ºC is
a) 739 mm of mercury
b) 750 mm of mercury
c) 760 mm of mercury
d) 712 mm of mercury

iii) If mass energy equivalence is taken into account, when water is cooled to form ice, the mass of water should
a) Increase
b) Remains unchanged
c) Decrease
d) First increase then decrease

iv) The amount of work which can be obtained by supplying 200 cal of heat is
a) 840 dynes
b) 840 Watts
c) 840 ergs
d) 840 J

Answers

1.
i) b
ii) d
iii) c
iv) a

2.
i) b
ii) c
iii) b
iv) d

QUIZ :Go to the link below to attempt quiz or scan the qr code –

https://quizizz.com/join?gc=90598018

Unit VIII Thermodynamics

Chapter 12. Thermodynamics

GIST OF THE CHAPTER:

THERMODYANAMICS: Thermodynamics is the branch of physics that deals with the concepts of heat and temperature and the inter-conversion of heat and other forms of energy.
The state of a gas in thermodynamics is specified by macroscopic variables such as pressure, volume, temperature, mass, and composition.

Thermodynamic Equilibrium: The state of a system is an equilibrium state if the macroscopic variables that characterize the system do not change in time.

Thermal Equilibrium: Two system are said to be in thermal equilibrium with each other if they have the same temperature.

Adiabatic wall: An insulating wall (can be movable) that does not allow flow of energy (heat) from one to another.

Diathermic wall: A conducting wall that allows energy flow (heat) from one to another.

ZEROTH LAW OF THERMODYNAMICS: Two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other.
It implies that, if A and B are two systems separately in equilibrium with system C, $T_A = T_C$ and $T_B = T_C$. Then $T_A = T_B$ or the systems A and B are also in thermal equilibrium.

Temperature: Is a thermodynamic variable which is equal for two systems in equilibrium

Thermodynamic state variable: A variable which depends on the state of the system not on the path taken to arrive at that state.

Internal energy: The sum of molecular kinetic and potential energies in the frame of reference relative to which the center of mass of the system is at rest. Internal energy is a thermodynamic state variable.

Equation of state: A relation between pressure, volume and temperature for a system is called its equation of state.

Types of Thermodynamics process: -

(1) Isothermal process- process taking place at constant temperature.
(2) Adiabatic process- process where there is no exchange of heat.
(3) Isochoric process-process taking place at constant volume.
(4) Isobaric process- process taking place at constant pressure.
(5) Cyclic process- process where the system returns to its original state.

FIRST LAW OF THERMODYNAMICS: The heat energy (∆Q) supplied to the system goes in partly to increase the internal energy of the system (∆U) and

$$\Delta Q = \Delta U + \Delta W$$

the rest in work on the environment (∆W).

$$\Delta W = P\Delta V$$

Specific heat capacity at constant volume (C_v) and specific heat capacity at constant pressure (C_p):
C_p—C_v = R, R is the universal gas constant

Quasi static process: It is an imaginary process for non-equilibrium states. processes that are sufficiently slow and do not involve accelerated motion of the piston, large temperature gradient, etc., are reasonably approximation to an ideal quasi-static process.

Work done during expansion / compression of gas: - When the volume of gas changes from V1 to V2, the work done is given by W= $\oint_{V1}^{V2} pdV$ = Area under the P-V diagram.

Thermodynamical operations are –

(1) **Isothermal process:** A thermodynamic process that takes place at constant temperature is called an isothermal process.

Equation of state for isothermal process: PV = constant.
- Work done during an isothermal process
Wiso = RT loge$\frac{V2}{V1}$ = 2.303 RT loge$\frac{V2}{V1}$

(2) **Adiabatic process:** A thermodynamic process that takes place in such a manner that no heat enters or leaves the system is called adiabatic process
Equation of state for adiabatic process
(i) PV^{γ} = constant (ii) $TV^{\gamma-1}$=constant

$\frac{P^{\gamma-1}}{T^{\gamma}}$ = constant

Work done during adiabatic change.
$W_{adia} = \frac{R(T1-T2)}{\gamma-1}$

Reversible process: - It is a process in which the system can be retraced to its original state by reversing the conditions.
Irreversible process: - It is a process in which the system cannot be retraced to its original state by reversing the conditions.

SECOND LAW OF THERMODYNAMICS:

Kelvin-Planck statement
No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work. Means it is impossible to derive a continuous supply of work by cooling a body to a temperature lower than of the coldest of its surrounding.

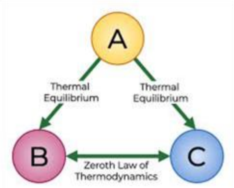
Clausius statement
No process is possible whose sole result is the transfer of heat from a colder object to a hotter object. Means it is impossible for a self-acting machine unaided by any external agency to transfer heat from a body to another body at higher temperature.

Mind map: -

THERMO DYNAMICS

Basic Concepts

- Zeroth Law of Thermodynamics – Two systems in thermal equilibrium with a third system separately are in thermal equilibrium with each other.



Thermodynamics state variable

- A variable which depends on the state of the system not on the path taken to arrive at that state.

Internal energy

- The sum of molecular kinetic and potential energies in the frame of reference relative to which the center of mass of the system is at rest. Internal energy is a thermodynamic state variable.

First Law of Thermodynamics

$\Delta Q = \Delta U + \Delta W$,

$\Delta W = P\Delta V$

Mayer relation

$C_p - C_v = R$

Second Law of Thermodynamics

- Statements

Kelvin-Planck- it is impossible to derive a continuous supply of work by cooling a body to a temperature lower than of the coldest of its surrounding.

Clausius- it is impossible for a self-acting machine unaided by any external agency to transfer heat from a body to another body at higher temperature.

Processes in Thermodynamics

-Reversible and irreversible processes

-Types of processes

(1) Isothermal process- T constant.

$W_{iso} = RT \log_e \frac{V2}{V1}$

(2) Adiabatic process- no exchange of heat.

$W_{adia} = \frac{R(T1-T2)}{\gamma - 1}$

(3) Isochoric process- constant volume.

(4) Isobaric process- constant pressure.

(5) Cyclic process- system returns to its original state. $\Delta U = 0$

I . Multiple Choice Questions

1. Which of the following is incorrect regarding the first law of thermodynamics?
   State A – it is not applicable to any cyclic process
   State B – it is a restatement of principle of conservation of energy
   State C – it introduces the concept of internal energy
   State D- it introduces the concept of internal entropy

   (a) A and D (b) B and C (c) C and A (d) A and B

2. Which of the following parameters does not characterize the thermodynamic state of matter.
   (a) Temperature (b) Pressure (c) Volume (d) work

3. By the Zeroth law of Thermodynamics, the thermodynamic variable whose value is equal for two systems in thermal equilibrium is ……………
   (a) Mass (b) Pressure (c) Volume (d) Temperature

4. In an isobaric process……………
   (a) Volume is constant
   (b) Pressure is constant
   (c) Temperature is constant
   (d) No heat flow

5. Which of the following variables is a thermodynamic state variable?
   (a) Work (b) heat (c) Internal energy (d) None of these

6. If a system goes from initial to final state without changing internal energy, then the heat supplied to system………….
   (a) Is fully utilized for doing work.
   (b) Is partially utilized for doing work by the system.
   (c) Is partially utilized for doing work on the system.
   (d) Not used for doing work

7. Which of the following is NOT true about isothermal expansion of an ideal gas
   (a) There is no change in internal energy
   (b) Heat supplied to the gas equals work done
   (c) The ideal gas equation for the process is $\frac{PV}{T}$ = constant
   (d) The ideal gas equation for the process is PV = constant

8. When steam is converted into water, internal energy of the system
   (a) increases (b) decreases (c) remains constant (d) becomes zero

9. Tell which of the following phenomenon are reversible?
   (a) Water fall.
   (b) Charging of a battery.
   (c) Rusting of iron by chemical change.
   (d) Production of heat by rubbing of hands.

10. the specific heat at constant pressure is more than that at constant volume due to the fact that:
    (a) molecular oscillations are more violent at constant pressure.
    (b) additional work need to be done for allowing expansion of gas at constant pressure.
    (c) there is more intermolecular attraction at constant pressure.
    (d) there is some reason other than those given above.

Answer Key: -

1.a	2.d	3.d	4.b	5.d
6.c	7.c	8.b	9.b	10.b

Explanation of MCQ'S

(1) a, the first law of thermodynamics states the principle of conservation of energy. It introduces the concept of the internal energy. And it is applicable for any cyclic process. The 2^{nd} law of thermodynamics explains the concept of entropy.
(2) d, the work done does not characterize a thermodynamic state of matter. It gives only a specific relationship (depending on path) between two different thermodynamic state.
(3) d, temperature, two system are said to be in thermal equilibrium with each other if they have the same temperature.
(4) b, pressure is constant in isobaric process
(5) c, internal energy is a thermodynamic state variable, meaning it is a property that depends only on the current state of a system and not on the path taken to reach that state. Work and heat are not state variables because they describe energy transfer and depend on the process or path taken.
(6) a, according to first law of thermodynamics
(7) c
(8) b, during a change of phases the temperature does not change, but the internal energy does. The internal energy is the sum of the kinetic energy of the molecules and the chemical potential energy of the molecules.

(9) b, (i) Waterfall: The falling of water is not a reversible process. During the fall of water, a major part of its potential energy is converted into kinetic energy of water. On striking the ground, a part of it is converted into heat and sound energy. It is not possible to convert the heat and sound produced along with the K.E of water into potential energy to make the water rise back to the initial height. Therefore, waterfall is not a reversible process.

(ii) The charging process of a battery is generally reversible because the electrochemical reactions that take place inside batteries are usually reversible.

(iii) Rusting of Iron: In the rusting of iron, the iron gets oxidized by the oxygen from the air. Since it is a chemical change, it is not a reversible process.

(iv) Rubbing hands together to warm them is considered an irreversible process. This is because the friction between the palms of hands causes kinetic energy to convert to heat energy, which is an irreversible process.

(10) b, at constant volume heat supplied is used in increasing the internal energy only, as the work done would be zero.
But in the case of constant pressure, external work needs to be done for allowing expansion of gas at constant pressure. Therefore, more heat is required in the latter.

II. Assertion Reason Question

Two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

- **a)** Both A and R are true, and R is the correct explanation of A
- **b)** b Both A and R are true, and R is NOT the correct explanation of A
- **c)** A is true but R is false.
- **d)** A is false and R is also false.

11. **Assertion**: In isothermal process whole of the heat energy supplied to the body is converted into internal energy.
    Reason: According to the first law of thermodynamics $\Delta Q = \Delta U$.
12. **Assertion:** The specific heat of a gas is an adiabatic process is zero and in an isothermal process is infinite.
    Reason: Specific heat of a gas in directly proportional to change of heat in system and inversely proportional to change in temperature.
13. **Assertion:** First law of thermodynamics does not forbid flow of heat from lower temperature to higher temperature.
    Reason: Heat supplied to a system always equal to the increase in its internal energy.
14. **Assertion:** In an adiabatic process, change in internal energy of a gas is equal to work done on/by the gas in the process.
    Reason: This is because temperature of gas remains constant in an adiabatic process.
15. **Assertion:** In the case of refrigerator, some external work is done on it.
    Reason: The working substance extract heat from cold reservoir
16. **Assertion:** In a cyclic process, there is no change in internal energy
    Reason: Internal energy is a state variable and the entire heat converted into work
17. **Assertion:** Irreversibility arises due to the dissipative forces
    Reason : Spontaneous processes are reversible

18. **Assertion:** A quasi-static process deals with process in equilibrium
    Reason : It is not a good approximation for non-equilibrium process
19. **Assertion:** Zeroth law defines the temperature
    Reason : Temperature is a state variable
20. **Assertion:** Air quickly leaking out of a balloon becomes cool
    Reason: The leaking air undergoes adiabatic expansion.

<u>ANSWER KEY</u>: - (11) d (12) a (13) b (14) d (15) a (16) a (17) c (18) d (19) b (20) a

III. <u>Case Study Questions</u>

21. **Kelvin-Planck statement:** No process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat into work.

    Clausius statement: No process is possible whose sole result is the transfer of heat from a colder object to a hotter object. It can be proved that the two statements above are completely equivalent.

    A thermodynamic process is reversible if the process can be turned back such that both the system and the surroundings return to their original states, with no other change anywhere else in the universe. a reversible process is an idealized motion. A process is reversible only if it is quasi-static (system in equilibrium with the surroundings at every stage) and there are no dissipative effects. For example, a quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is a reversible process.

    The free expansion of a gas is irreversible. The combustion reaction of a mixture of petrol and air ignited by a spark cannot be reversed. Cooking gas leaking from a gas cylinder in the kitchen diffuses to the entire room. The diffusion process will not spontaneously reverse and bring the gas back to the cylinder. The stirring of a liquid in thermal contact with a reservoir will convert the work done into heat, increasing the internal energy of the reservoir. The process cannot be reversed exactly; otherwise, it would amount to conversion of heat entirely into work, violating the Second Law of Thermodynamics. Irreversibility is a rule rather an exception in nature.

    i) The diffusion process is
    a) Reversible process
    b) Irreversible process

    ii) A quasi-static isothermal expansion of an ideal gas in a cylinder fitted with a frictionless movable piston is
    a) Reversible process
    b) Irreversible process

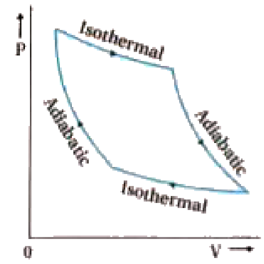
iii) State Kelvin Planck statement.

iv) State Clausius statement.

v) Define reversible processes and irreversible processes of thermodynamics.

22. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following the following questions based on the paragraph.
    (i) What is the final pressure of the gas in A and B?
    (ii) What is the change in internal energy of the gas?
    (iii) What is the change in the temperature of the gas?
    (iv) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface?

23. For an isothermal process, pressure of a given mass of gas varies inversely as its volume. There is no change in the internal energy of an ideal gas in an isothermal process. In an adiabatic process, the system is insulated from the surroundings and heat absorbed or released is zero. The P-V curve for Isothermal and Adiabatic process of an ideal gas is given below. Answer the following question.



    (v) What is the formula for work done in an isothermal process ?
    (vi) The above curve is known as …………
    (vii) Which state variables do change during adiabatic process?
    (viii) When work done will be positive in the case of adiabatic process?

Answer key

21. i) b
    ii) a
    iii) Kelvin-Planck statement states that We cannot construct any device like the heat engine that operates on a cycle, absorbs the heat energy, and completely transforms this energy into an equal amount of work. Some of the heat gets released into the atmosphere. Practically no device bears 100% thermal efficiency.

iv) According to Clausius It is nearly impossible for heat to move by itself from a temperature that is lower in temperature to a reservoir that is at a higher temperature. That is we can say that the transfer of heat can only occur spontaneously from high temperature to temperature. i.e. No process is possible whose sole result is the transfer of heat from a colder object to a hotter object without any external work provided to do it in short we cannot construct a refrigerator that can operate without any input work.

v) A thermodynamic process is said to be reversible if both the system and the surroundings return to their original states, with no other change anywhere else in the universe. On the other hand, an irreversible process can be defined as a process in which the system and surrounding will not return to their original condition once the process is initiated.

22. (i) 0.5 atm
(ii) Zero
(iii) Zero
(iv) No

23. (i) $\mu RT \ln\frac{V_2}{V_1}$
(i) Isothermal
(ii) pressure and volume
(iii) $T_2 < T_1$

Iv. Two Mark Questions

24. What is the significance of second law of thermodynamics?
25. Give two examples for irreversible process?
26. Differentiate heat and temperature?
27. What are the conditions for Work done to be (a) negative and (b) positive in an Adiabatic process?
28. Differentiate isobaric and isochoric processes?
29. Why is the conversion of heat into work not possible without a sink at lower temperature?
30. What are the limitations of the first law of thermodynamics?
31. What happen to the internal energy of a gas during:
    (i) isothermal expansion (ii) adiabatic Expansion?
32. Can we increase the temperature of gas without supplying heat to it?
33. If hot air rises, why is it cooler at the top of mountain than near the sea level?

Answer key

24. The second law of Thermodynamics helps us to determine the direction in which energy can be transformed. It also helps us to predict whether a given process or chemical reaction can occur spontaneously or not.

25. Examples of an irreversible process: (1) Free expansion of a gas (2) All chemical reaction (3) Diffusion of two dissimilar inert gases (4) A gas seeping through a porous plug.

26.

Differences between heat and temperature

Heat	Temperature
It is a form of energy	It is the degree of hotness or coldness of a body
It is measured in joules	It is measured in Kelvin
it is not determined directly by an instrument	it is directly determined by a thermometer
it is a derived quantity	It is a fundamental quantity

27. In adiabatic process $\Delta Q = 0$ therefore when a gas compress adiabatically, (work done on the system) work done will be negative. When a gas expands adiabatically (work done by the system) then work done will be positive

28. Isochoric process-process taking place at constant volume.
Isobaric process- process taking place at constant pressure.

29. For converting heat energy into work continuously, a part of the heat energy absorbed from the source has to be rejected. The heat energy can be rejected only if there is a body whose Temperature is less than that of the source. This body at lower temperature is called sink.

30. The limitations are ---
    a. It does not tell us the directions of heat transfer.
    b. it does not tell us how much of the heat is converted into work.
    c. it does not tell us under what conditions heat is converted into work.

31. (i) In isothermal expansion, temperature remains constant. Therefore, internal energy which is a function of temperature will remain constant.
(ii) for adiabatic change dQ = 0 and hence first law of thermodynamics becomes 0 = dU + dW therefore, dW = -dU
During expansion, work is done by the gas i.e. dW is positive. Hence,

dU must be negative. Thus, in an adiabatic expansion, the internal energy of the system will decrease.

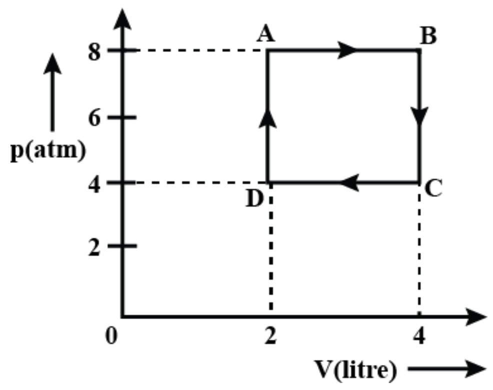
32. Yes, the temperature of gas can be by compressing the gas under Adiabatic condition.

33. Since atmospheric pressure decreases with height, pressure at the top of the mountain is lesser. When the hot air rises up, it suffers adiabatic expansion at the top of the mountain. For an adiabatic change, first law of thermodynamics may be express as
    - ⇨ dU + dW =0 (dQ = 0)
    - ⇨ dW = -dU

    Therefore work done by the air in rising up (dW =+ve) result in decrease in the internal Energy of the air (dU = -ve) and hence a fall in the temperature.

V. Three Mark Questions

34. Explain the relation between heat, work and internal energy?
35. Explain about a system doing work without changing internal energy? State the law governs it?
36. If at 50°C and 75 cm of mercury pressure, a definite mass of gas is compressed (i)slowly (ii) suddenly, then what will be the final pressure and temperature of the gas in each case, if the final volume is one fourth of the initial volume? Given γ = 1.5
37. When is a system said to be in the state of thermodynamic equilibrium?
38. Define internal energy of a gas. Explain whether it is an extensive or intensive variable? How internal energy of a gas can be changed?
39. One mole of an ideal gas undergoes a cyclic process ABCDA as shown in the PV diagram. The net work done in the process is (1 atm = 10^6 dyne cm^{-2})



40. A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sane on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume?
41. In changing the state of a gas adiabatically from an equilibrium state A to

another equilibrium state B, an amount of work equal to 22.3J is done on the system. If the gas is taken from state A to B via process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (take 1 cal = 4.19J)

Answer Key

34. The relationship between heat, work, and internal energy is governed by the first law of thermodynamics, which states that energy can't be created or destroyed, only transformed. The equation for this law is dQ=dU+dW, where:

- dU: is the change in internal energy of the system.
- dQ: is the heat transferred to the system.
- dW: is the work done by the system.

35. In case of isothermal, free expansion, and cyclic process there is no change in the internal energy of the system as system's temperature remains constant in these processes. As internal energy is a function of temperature, there is no change in the internal energy of the system.

First law of thermodynamics:- The heat energy (ΔQ) supplied to the system goes in partly to increase the internal energy of the system (ΔU) and the rest in work on the environment (ΔW).

⇨ dQ=dU+dW

36. (i) When the gas is compressed slowly, the change is isothermal.
Therefore, $P_2V_2 = P_1V_1$
$P_2 = P_1V_1/V_2 = (75x\ V_1/V_1)x4 = 300$ cm of mercury
Temperature remains constant at 50°C

(i) When the gas is compressed suddenly, the change is adiabatic
As per $p_2v_2^{\gamma} = p_1v_1^{\gamma}$
⇨ $p_2 = p_1(v_1/v_2)^{\gamma}$
⇨ 75 x $(4)^{1.5}$ =600 cm of Hg
⇨ Also $T_2v_2^{\gamma-1} = T_1v_1^{\gamma-1}$
⇨ 323x$(4)^{(1.5-1)}$ = 646K
⇨ 646-273 =373°C

37. A system is said to be in the state of thermodynamic equilibrium if the microscope variables describing the thermodynamic state of the system do not change wit time.
(a) Mechanical equilibrium: There is not unbalanced force in its interior or between the system and the surroundings.
(b) Thermal equilibrium: All parts of the system and the surroundings are at the same temperature.
(c) Chemical equilibrium: The system does not undergo any

spontaneous change in its internal structure due to chemical reaction, diffusion, etc.

38. Internal energy is the sum of the kinetic energies and potential energies of the molecular constituents of the system. It does not include the overall kinetic energy of the system. Internal energy is an extensive variable because it depends upon the mass and size (volume) of the system. Intensive variables like temperature, pressure does not depend upon the mass and size of the system. Change in internal energy does not depend on the path of the process. TI depends only on the initial and final states of the system, i.e. $\Delta U = U_f - U_i$.

39. From the P-V diagram,

    Work done, W=area under the P−V diagram W=AB×BC

    W=2litre×4atm

    $W=2\times10^{-3}\times4\times1.03\times10^{5}$ or W=8×100 or W=800J

40. The cylinder is completely insulated from its surroundings. As a result, no heat is exchanged between the system (cylinder) and its surroundings. Thus, the process is adiabatic.

    Initial pressure inside the cylinder $=P_1$

    Final pressure inside the cylinder $=P_2$

    Initial volume inside the cylinder $=V_1$

    Final volume inside the cylinder $=V_2$

    Ratio of specific heats, $\gamma = C_P/C_V = 1.4$

    For an adiabatic process, we have: $p_2 v_2^{\gamma} = p_1 v_1^{\gamma}$

    The final volume is compressed to half of its initial volume.

    $\therefore V_2=V_1/2$

    $P_2V_2^{\gamma}=P_1(V_1/2)^{\gamma}$

    $P_2/P_1=V_2^{-\gamma}/(V_1/2)^{\gamma}$

    $=2^{1.4}=2.64$

    Hence, the pressure increases by a factor of 2.64.

41. In the first case, the process is adiabatic. i.e ΔQ=0

22.3J work is done on the system i.e. ΔW=−22.3J

ΔQ=ΔU+ΔW

0=Δu−22.3 Δu=22.3J

In the later process, the initial and final states are same as those in the former process, Δu will remain same for the later case.

In the later case, net heat absorbed by the heat is 9.35cal ΔQ=9.35×4.2

ΔQ=39.3J ΔW=ΔQ−Δu ΔW=39.3−22.3 ΔW=17J

The net work done in the system in this case is 17J.

VI. Long Questions

42. Show that for an isothermal process work done $w = nRT\ ln\frac{v2}{v1}$ where symbols have their usual meaning.
43. Derive Mayer's equation from the 1st law of thermodynamics.
44. Define an adiabatic process. State two essential condition for such a process to take place. Derive an expression for adiabatic process to take place.
45. What is the need of introducing the second law of thermodynamics? State the kelvin – planck and Claussius statement and show that both the statements are equivalent.
46. Derive an expression for work done in Adiabatic process.

Answer Key

42. For an isothermal process, PV=constant.

At any intermediate stage with pressure P and change in volume from. V to V+dV,

Work done dW=PdV. However, for the entire process **W=** $\oint_{V1}^{V2} pdV$

Where PV = nRT

⇨ P = $\frac{nRT}{V}$

⇨ **W=** $\oint_{V1}^{V2} \frac{nRT}{V} dV$ => $w = nRT\ ln\frac{v2}{v1}$

⇨ For $V_2 > V_1$, W> 0 And for $V_2 < V_1$, W<0

In an isothermal expansion the gas absorbs heat and does work while in an isothermal compression, work is done on the gas by the environment and heat is released.

43. We know that, dQ=dU+dW (First law of thermodynamics)

At a constant volume, heat supplied to 1 mole of ideal gas enclosed in a conducting cylinder fitted with a conducting piston will be utilised to increase the internal energy of the system and dW=0

$\therefore (dQ)_{volume} = dU = C_V dT$

where C_V is molar specific heat of a gas at constant volume.

At a constant pressure heat supplied will be used to increase the internal energy as well to do external work by the gas

$\therefore (dQ)_{pressure} = dU + dW$

For the same rise in the temperature in both the cases at a constant volume and pressure,

$dU = C_V dT$

For 1 mole of gas,

$C_P dT = C_V dT + dW$

where, $dW = Fdx = PAdx = PdV$

where, Adx. is a small change in the volume of the gas.

Also, from PV=RT, at constant pressure,

Pressure, $P(V+dV) = R\ (T+ dT)$ i.e., $PdV=RdT$

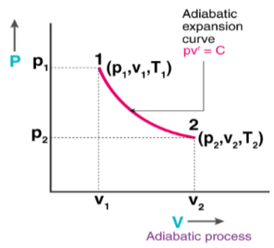
Hence, $C_P dT = C_V dT + RdT$ i.e., $C_P = C_V + R$ This equation is known as Mayer's equation.

44. An adiabatic process is defined as the thermodynamic process in which there is no exchange of heat from the system to its surrounding neither during expansion nor during compression.

The adiabatic process can be either reversible or irreversible. Following are the **essential conditions** for the adiabatic process to take place:

(i)The system must be perfectly insulated from the surrounding.

(ii)The process must be carried out quickly so that there is enough time for heat transfer to take place.



According to first law of thermodynamics, dQ=dU+dW

For 1 mole of gas

$dU = C_v\, dT$

$dW = P\, dV$

For adiabatic Process

$dQ = 0$

$dQ = dU + dW$

$0 = C_v\, dT + P\, dV$

According to Ideal Gas Equation

$PV = RT$

Differentiate both sides, we get

$P\, dV + V\, dP = R\, dT$

$$dT = \frac{P\,dV + V\,dP}{R}$$

$$C_V \frac{P\,dV + V\,dP}{R} + P\,dV = 0$$

$$C_V\,P\,dV + C_V\,V\,dP + RP\,dV = 0$$

$$C_V\,V\,dP + (C_V + R)P\,dV = 0$$

$$C_V\,V\,dP + C_P\,P\,dV = 0 \quad [\,C_P = (C_V + R)\,]$$

$$\frac{C_v V\,dP}{C_v PV} \frac{+ C_P P\,dV}{C_v PV} = 0 \quad [\text{ dividing both side by } C_v PV\ ]$$

$$\frac{dP}{P} \frac{+ C_P\,dV}{C_v\ V} = 0$$

$$\frac{dP}{\rho} + \gamma \frac{dV}{V} = 0$$

$$\int \frac{dP}{P} + \gamma \int \frac{dV}{V} = C$$

$$log_e P + \gamma\, log_e V = C$$

$$log_e PV^{\gamma} = C$$

$$PV^{\gamma} = e^{C}$$
$$PV^{\gamma} = K$$

Where K= constant

This equation gives the relation between P(Pressure) and V(Volume) of Ideal Gas.

45. The first law of Thermodynamic tells the basic definition of internal energy and states the law of conservation of energy. It does not describe any restrictions or possibilities for a process to take place. The second law of Thermodynamic deals with the direction of natural process
(i) Statement of Kelvin: According to this statement it is not possible to make an engine in which the working substance can take heat from the source and change it completely into, work and remain unchanged itself in one complete cycle. It is concluded from this statement that it is not possible to get work continuously from heat source. The sink in necessary and it necessary to release. Some heat to the sink. This statement is based on the principle of Carnot heat engine.

(ii). Statement of Clausius: According to this statement, it is an impossible process that in cyclic process the working substance transfer heat from low temperature object to high temperature object directly in the absence of external work.

It is concluded from this statement that transfer of heat from a low temperature object high temperature object on its own is not possible. For this it is important that external work be done on the working substance. This statement is based on the principle of refrigerator.

Equivalence of Clausius and Kelvin Statements: Let there be an engine violating the Kelvin statement: i.e., on that drains heat and converts it completely into work in a cyclic manner without any other results. Now, pairing it with a reverse Carnot engine.

The newly created engine (consists of two engines) transfers heat from the colder region (sink) to the hotter region (source), which violates the statements of Clausius. Thus, violation of Kelvin statement means violation of Clausius statement, i.e., Clausius statement implies the Kelvin statement. Similarly, we can prove that the Kelvin Statement implies the Clausius statement and hence both the statements are equivalent.

46. In an adiabatic process, change in pressure, volume and temperature takes place under thermal isolation.
For an adiabatic process, $\boldsymbol{PV^{\gamma} = K}$ where $\boldsymbol{\gamma = \frac{C_P}{C_V}}$ is known as the ratio of specific beats of a gas.
For a change of coordinates from (P_1V_1) to (P_2V_2)

$$p_2v_2^{\gamma} = p_1v_1^{\gamma} \;, P = \frac{K}{V^{\gamma}}$$

Work done = w = $\oint_{V1}^{V2} PdV$

i.e. w = $K\oint_{V1}^{V2}\frac{dV}{V^{\gamma}} = \frac{K}{1-\gamma}[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}}]$

$=\frac{1}{1-\gamma}[\frac{K}{V_2^{\gamma-1}} - \frac{K}{V_1^{\gamma-1}}]$

$=\frac{1}{1-\gamma}[\frac{p_2v_2^{\gamma}}{V_2^{\gamma-1}} - \frac{p_1v_1^{\gamma}}{V_1^{\gamma-1}}] = \frac{1}{1-\gamma}[P_2V_2 - P_1V_1]$

W = $\frac{1}{1-\gamma}nR(T_2 - T_1)$

Competency based questions: -

47. A Carnot engine operating between temperatures T_1 and T_2 has efficiency 1/6. When T_2 is lowered by 62 K, its efficiency increases to ⅓. Then T_1 and T_2 are, respectively.
(a) 372 K and 310 K
(b) 372 K and 330 K
(c) 330 K and 268 K
(d) 310 K and 248 K

48. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J $kg^{-1}K^{-1}$)
(a) 4.2 kJ
(b) 8.4 kJ
(c) 84 kJ
(d) 2.1 kJ

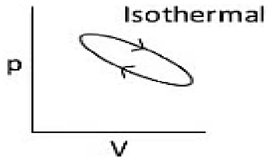
49. The work of 146 kJ is performed in order to compress one-kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7ºC. The gas is (R = 8.3 J mol^{-1} K^{-1})
(a) monoatomic
(b) diatomic
(c) triatomic
(d) a mixture of monoatomic and diatomic

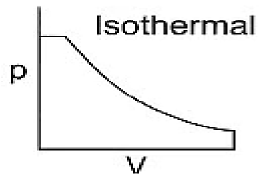
50. One mole of an ideal gas at 300K is expanded isothermally from 1L volume to 10L volume. ΔU for this process is –
(a) 0J
(b) 1260J
(c) 2520J
(d) 5040J

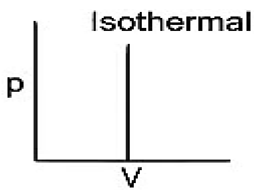
51. For irreversible expansion of an ideal gas under isothermal condition, the correct option is –
(a) $\Delta U=0$ and $\Delta S_{total} \neq 0$
(b) $\Delta U\neq 0$ and $\Delta S_{total} = 0$
(c) $\Delta U=0$ and $\Delta S_{total} = 0$
(d) $\Delta U\neq 0$ and $\Delta S_{total} \neq 0$

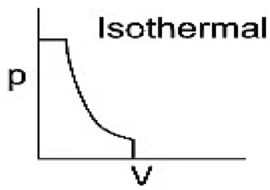
52. A vessel contains 3.2g of dioxygen gas at STP (273.15K and 1 atm). The gas is now transferred to another vessel at constant temperature, where pressure becomes one-third of the original pressure. The volume of new vessel in L is (given molar volume at STP is 22.4 L) –
(a) 67.2
(b) 6.72
(c) 2.24
(d) 22.4

53. Which of the following P-V curve represents maximum work done?

(a)


(b)


(c)


(d)


Answer key: -

47. The efficiency of Carnot engine,
$\eta = 1 - (T_2/T_1)$
$\eta = 1/6$
$T_2/T_1 = 5/6$
$T_1 = 6T_2/5$ ————(1)
As per the question, when T_2 is lowered by 62 K, then its efficiency becomes 1/3
$1/3 = [1 - (T_2 - 62/T_1)]$ $[T_2 - 62/T_1] = 1 - (1/3)$ (Using eq. (1))
$5(T_2 - 62)/6T_2 = 2/3$
$5T_2 - 310 = 4T_2 \Rightarrow T_2 = 310$ K
From equation (1) $T_1 = (6 \times 310)/5 = 372$ K

48. $\Delta Q = ms\Delta T$
Here m = 100 g = 100 x10^{-3} Kg
S = 4184 J $kg^{-1}K^{-1}$ and ΔT = (50 – 30) = 20^oC
ΔQ = 100 x 10^{-3} x 4184 x 20 = 8.4 x 10^3 J
$\Delta Q = \Delta U + \Delta W$
Change in internal energy -
$\Delta U = \Delta Q$ = 8.4 x 10^3 J = 8.4 kJ

49. According to the first law of thermodynamics
$\Delta Q = \Delta U + \Delta W$
For an adiabatic process, $\Delta Q = 0$
$0 = \Delta U + \Delta W$
$\Delta U = -\Delta W$
$nC_v\Delta T = -\Delta W$
$C_v = -\Delta W/n\Delta T$ = -[-146 x 10^3]/[(1 x 10^3) x 7] = 20.8 $Jmol^{-1}K^{-1}$
For diatomic gas, C_v = (5/2)R = (5/2)x8.3 = 20.8 $Jmol^{-1}K^{-1}$
Hence, the gas is diatomic.

50. $\Delta U = nC_v\Delta T$
For isothermal condition $\Delta T = 0$
⇨ ΔU = 0J

51. Answer – (a)
$\Delta U = nC_v\Delta T$
In the case of isothermal process $\Delta T = 0$
⇨ $\Delta U = 0$
In reversible and irreversible expansion $\Delta T = 0$
But the total entropy change of a spontaneous process is not equal to zero.

52. At constant temperature

$$P_1V_1 = P_2V_2$$

$$P_1V_1 = \frac{P_1}{3}V_2 \, [\because P_2 = \frac{P_1}{3}]$$

$$V_2 = 3V_1$$

mole of $O_2(g) = \frac{3.2}{32} = 0.1$ mole

Volume of $O_2(g) = (0.1 \times 22.4)$ L = 2.24 L

At STP (V_1)

$$V_2 = 3V_1 = 3 \times 2.24$$

= 6.72 L

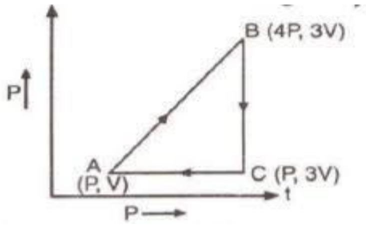
53. Work done under any thermodynamic process can be determined by the area under the P-V curve. As it can be observed maximum area is covered in option (b)

Self-Assessment: -

54. Define internal energy of a gas. Explain whether it is an extensive or intensive variable? How internal energy of a gas can be changed? **(3M)**

55. If no external energy is supplied to an expanding gas, will the gas do any work? If yes, then what will be the source of energy? **(1M)**

56. A sample of an ideal monatomic gas is taken round the cycle ABCA as shown in the figure. What is the work done during the cycle? **(1M)**



57. A system is given 200 calories of heat and it does 600 joules of work. How much does the internal energy of the system change in this process. (J = 4.18 joule/cal)? **(2M)**

58. The volume of steam produced by 1g of water at 100°c is 1650 cm^3 Calculate the change in internal energy during the change of state. (given J = 4.2×10^7 erg cal^{-1}, g = 98 J cm/s^2, latent heat of steam = 540 cal/g) **(3M)**

TOPIC	**TYPE**	**LINK**
1^{st} law of thermodynamics activity	video	https://www.youtube.com/watch?v=FuTjUbW60Fs
laws of thermodynamics	video	https://www.youtube.com/watch?v=tuQHbR5ltAs
thermodynamics	quiz	https://quizizz.com/admin/quiz/61f7fc4b2ce014001d7e0b5f/thermodynamics
	quiz	https://quizizz.com/admin/quiz/5a721b3266a3db001ebe7df1/thermodynamics-laws

Unit IX Behaviour Of Perfect Gases And Kinetic Theory Of Gases

Chapter 13. Kinetic Theory

GIST OF THE CHAPTER:

- **Kinetic theory** of gases relates the three basic features of gases: volume, pressure and temperature. The kinetic theory was developed by Maxwell, Boltzmann and Gibbs in the nineteenth century.
- **Equation of state for a perfect gas:** PV= nRT where n= number of moles of the gas and
$R= N_A k_B$ = universal gas constant. R= 8.31 $Jmole^{-1}K^{-1}$
- To calculate the work done in compressing a gas, you need to use the formula W = - PΔV. When a gas is compressed, work is done on the gas to reduce its volume. The amount of work done can be calculated using the formula W = -PΔV, where W is the work done, P is the pressure applied, and ΔV is the change in volume.
- **Assumptions of Kinetic theory of gases**

(1) The molecules of a gas are identical, spherical and perfectly elastic point masses.
(2) The volume of molecules is negligible in comparison to the volume of gas.
(3) Molecules of a gas move randomly in all direction.

(4) The speed of gas molecules lie between zero and infinity.
(5) Their collisions are perfectly elastic.
(6) The number of collisions per unit volume in a gas remains constant.
(7) No attractive or repulsive force acts between gas molecules.

- **Concept of pressure**: The pressure of the gas is the force that the gas exerts on the container boundaries. The gas molecules move randomly along the given volume. During this movement, they collide with the surface and also with each other.

- $P = (1/3)mnv^2$ This equation is derived from the kinetic theory of gases and represents the pressure exerted by the gas molecules on the walls of the container due to their thermal motion

- **Kinetic interpretation of temperature:** The average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating temperature, a macroscopic measurable parameter of a gas (a thermodynamic variable as it is called) to a molecular quantity, namely the average kinetic energy of a molecule.

- **rms speed of gas molecules:** Since the molecules in gases are in motion, the molecules will also have velocity. Since the molecules differ in their velocity at each instant, thus root mean square velocity of the molecules is usd.

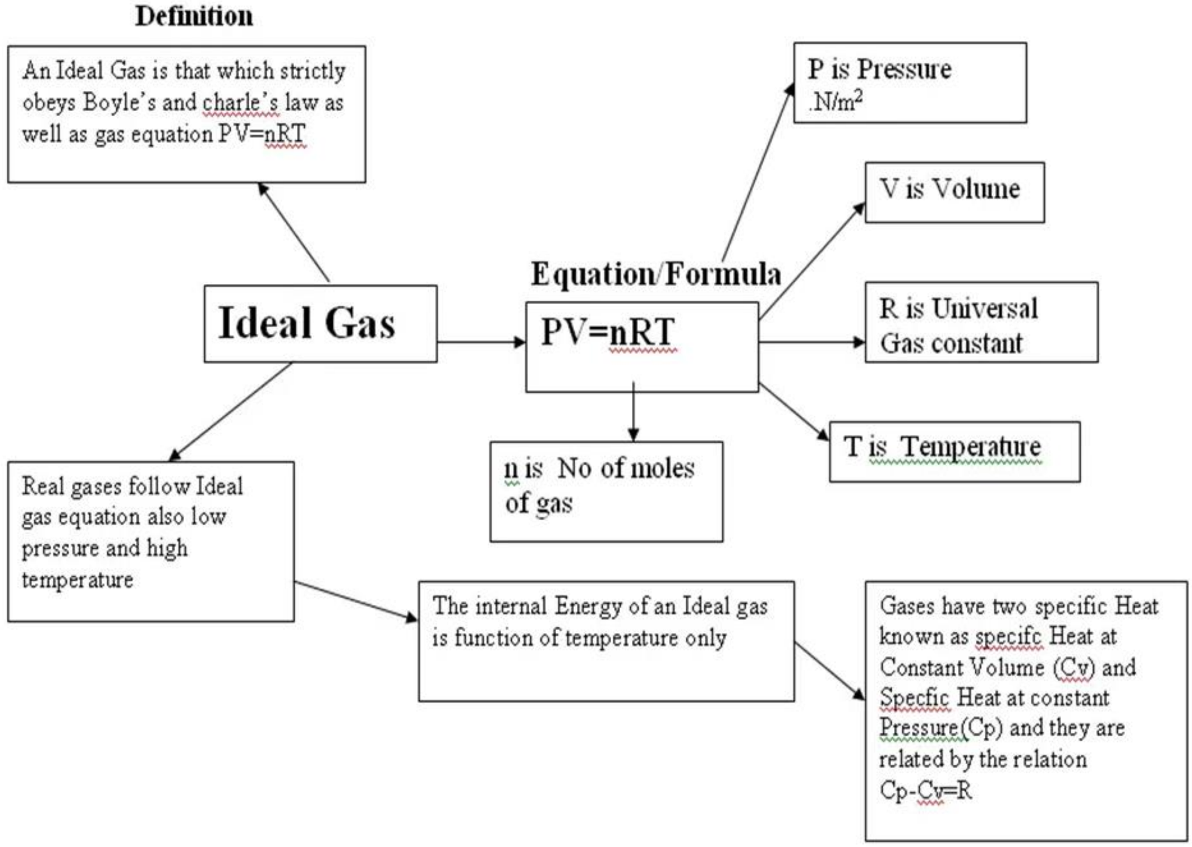
- Formula $V_{rms} = \sqrt{3RT/M}$

Where R= Universal gas constant
T = Temperature
M= Molar mass of gas

- Degrees of freedom: The total number of independent modes (ways) in which a system can possess energy is called the degree of freedom (f).
- The degree of freedom are of three types :
- (i) Translational degree of freedom
- (ii) Rotational degree of freedom
- (iii)Vibrational degree of freedom
- General expression for degree of freedom f = 3N – R
- where N = Number of independent particles, R = Number of independent restriction (1) Monoatomic gas : It can have 3 degrees of freedom (all translational). (2) Diatomic gas : A diatomic molecule has 5 degree of freedom : 3 translational and 2 rotational. (3) Triatomic gas (
- **Law of Equipartition of Energy :** For a dynamic system in thermal equilibrium, the energy of the system is equally distributed amongst the various degrees of freedom and the energy associated with each degree of freedom per molecule is 1/2 kT, where k is Boltzman constant.

- **Mean Free Path**
- Mean free path of a molecule in a gas is the average distance travelled by the molecule between two successive collisions.
- **Avogadro's Law** : Equal volumes of all gases under S.T.P. contain the same number of molecules equalling 6.023×10^{23}.

MIND MAP



Multiple Choice Questions

Q1 The monoatomic molecules have only three degrees of freedom because they can possess:

a) only translatory motion
b) only rotatory motion
c) both translatory and rotatory motion
d) translatory, rotatory and vibratory motion

Q2 Gases deviate from the ideal gas behaviour because their molecules:

a) are polyatomic
b) are of very small size
c) don't attract each other
d) interact with each other through intermolecular forces

Q3 What is the number of degrees of freedom of an ideal diatomic molecule at ordinary temperature?

a) 3
b) 5
c) 7
d) 6

Q4 A sample of gas is at 0°C. To what temperature it must be raised in order to double the rms speed of the molecule

a) 270°C
b) 719°C
c) 1090°C
d) 100°C

Q5 A gas is filled in a container at pressure P. If the mass of molecules is halved and their rms speed is doubled. The resultant pressure would be
a) 2P
b) 4P
c)P/4
d) P/2

Q6 In kinetic theory of gases, it is assumed that:
a) the collisions are not perfectly elastic.
b) the molecular collisions change the density of the gas.
c) the molecules don't collide with each other on the well.
d) between two collisions the molecules travel with uniform velocity.

Q7 At what temperature is the rms velocity of hydrogen molecule equal to that of an oxygen molecule at 47°C.
a) –73K
b) 3K
c) 20 K
d) 80 K

Q8 The work done by (or on) a gas per mole per kelvin is called
a) Universal gas constant
b) Boltzmann's constant
c) Gravitational constant
d) Entropy

Q9 A gas behaves as an ideal gas at
a) low pressure and high temperature
b) low pressure and low temperature
c) high pressure and low temperature
d) high pressure and high temperature

Q10 The root mean square velocity of gas molecules is 10 km/s The gas is heated till its pressure becomes four times. The velocity of gas molecules will be
a) 10 Km/s
b) 20 Km/s
c) 40 Km/s
d) 80 Km/s

Q11 According to kinetic theory of gases at absolute zero
a) Water freezes
b) Liquid helium freezes
c) Molecular motion stops
d) All of the above are correct

Q12 Kinetic theory of gases provide a base for:
a) Both Charle's law and Boyle's law
b) None of these
c) Boyle's law
d) Charle's law

Answers

1.a	2.d	3.b	4.b	5.a
6.d	7.c	8.a	9.a	10.a
11.c	12.a			

Answers With Explanation:

1) a) Reason: Monoatomic molecules have only 3 degrees of freedom because they can only move in three dimensions: left-right, forward-backward, and up-down. They can only translate (move) in these three dimensions, and they cannot rotate or vibrate like diatomic or polyatomic molecules.
The three degrees of freedom of a monoatomic molecule are:
1. Translational motion along the x-axis (left-right)
2. Translational motion along the y-axis (forward-backward)
3. Translational motion along the z-axis (up-down)
Since monoatomic molecules have no bonds to vibrate or rotate, they only have these three translational degrees of freedom.
2) d) Reason: Molecules interact with each other through intermolecular forces of attraction and repulsion.
3) b) Number of degrees of freedom $d=3N-1$
where N is the number of atoms in a molecules
In diatomic molecules, $N=2$
$\Rightarrow d=3(2)-1=5$
Hence diatomic molecule has 5 degrees of freedom (3 translational and 2 rotational).
4) b) To double the RMS speed of molecules, the temperature of the gas must be increased to 4 times the initial temperature. Since the RMS speed is proportional to the square root of the temperature
($v_{rms} \propto \sqrt{T}$), doubling the RMS speed requires a fourfold increase in temperature.
So, if the initial temperature is 0°C (273 K), the temperature must be raised to:
4 × 273 K = 1092 K
or approximately 819°C.
5) Using, $P= 1/3\rho v^2_{rms} =1/3mNv^2_{rms}/V$
$\therefore P \propto mv^2_{rms}$
As m is halved and v_{rms} is doubled then P becomes twice or 2P.
7) c)
RMS velocity of a gas molecule is given by:
$v=\sqrt{3RT/M}$
Let the temperature at which the velocity of hydrogen is equal to the velocity of oxygen at 47^0 C be T
$\sqrt{3RT/2}=\sqrt{3R\times(273+47)/32}$
Squaring both the sides we get
T= 20 K
8) a) The gas constant is a fundamental physical constant that relates the energy of a gas to its temperature, volume, and pressure. It is given as:

R = 8.3145 J/mol/K
The work done by or on a gas per mole per kelvin is called the gas constant (R)
9) a) A gas behaves as an ideal gas at low pressure and high temperature
A gas behaves ideally when its molecules are widely spaced , move freely and don't interact with each other.
10) a)
$PV=1/3\ MV^2_{rms}$
$V^2_{rms}= 3PV/M$
$V^2_{rms}= 3P/\rho$
$V_{rms1}= \sqrt{3P/\rho} = 5$ km/s
$V_{rms2}= \sqrt{4}\ \sqrt{3P/\rho}$
$V_{rms2}= 2 \times 5$ km/s
$V_{rms2}= 10$ km/s
11) c) According to kinetic theory of gases at absolute zero Molecular motion stops
12 a) Kinetic theory of gases provide a base for: Both Charle's law and Boyle's law.
Kinetic theory of gases relates the three basic features of gases: volume, pressure and temperature

Assertion And Reason Questions
Choose the correct option from the following:
a) Both A and R are true and R is the correct explanation of A
b) Both A and R are true but R is NOT the correct explanation of A
c) A is true but R is false
d) A is false and R is also false
13. **Assertion(A):** The kinetic energy of an oxygen molecule will be equal to the kinetic energy of a hydrogen molecule.
Reason (R) : Oxygen and hydrogen gases at the same temperature
the temperature of unit mass of the substance through unit degree.
14 Assertion: A gas can be liquified at any temperature by increase of pressure alone.
Reason: On increasing pressure, the temperature of gas decreases.
15 Assertion: Equal masses of helium and oxygen gases are given equal quantities of heat. There will be a greater rise in the temperature of helium compared to that of oxygen.
Reason: The molecular weight of oxygen is more than the molecular weight of helium.
16 Assertion (A) : The number of degrees of freedom of a linear triatomic molecule is 7.
Reason (R) : The number of degree of freedom depends on number of particle in the system.
17 **Assertion (A) :** Absolute zero is not the temperature corresponding to zero energy.
Reason (R) : The temperature at which no molecular motion exists is called absolute zero temperature.
18 **Assertion (A) :** The ratio of specific heat of a gas at constant pressure and specific heat at constant volume for a diatomic gas is more than that for a monatomic gas.
Reason (R) : The molecules of a monatomic gas have more degree of freedom than

those of a diatomic gas.
19 **Assertion (A) :** Air pressure in a car tyre increases during driving.
Reason (R) : Absolute zero degree temperature is not zero energy temperature.
20 **Assertion (A) :** Specific heat of a gas at constant pressure is greater than its specific heat at constant volume.
Reason (R) : At constant pressure, some heat is spent in expansion of the gas.
21 **Assertion (A) :** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.
Reason (R) : The molecules of a gas collide with each other and the velocities of the molecules change due to collision.
22 Assertion (A) : Mean free path of gas molecules varies inversely as density of the gas.
Reason (R) : Mean free path of gas molecules is defined as the average distance travelled by a molecule between two successive collisions.

Answers to assertion reason questions

13.a	14.d	15.b	16.b	17.a
18.d	19.b	20.a	21.b	22.b

<u>Case Study Question</u>

Q23 The equipartition of kinetic energy was proposed initially in 1843 and more correctly in 1845, by John James Waterston. In 1859, James Clerk Maxwell argued that the kinetic heat energy of a gas is equally divided between linear and rotational energy. In 1876, Ludwig Boltzmann expanded on this principle by showing that the average energy was divided equally among all the independent components of motion in a system. Boltzmann applied the equipartition theorem to provide a theoretical explanation of the Dulong-Petit law for the specific heat capacities of solids.
Law of Equipartition of Energy
According to this law, for any system in thermal equilibrium, the total energy is equally distributed among its various degree of freedom.
And each degree of freedom is associated with energy 1/2 kT
At a given temperature T; all ideal gas molecules no matter what their mass have same average translational kinetic energy; namely, 3/2 kT.
When we measure the temperature of a gas, we are also measuring the average translational kinetic energy of it's molecules.

Answer the following questions

1) At 0 K, which of the following properties of a gas will be zero?
(a) kinetic energy (b) potential energy
(c) vibrational energy (d) density

2) The root mean square velocity of a gas molecule of mass m at a given temperature is proportional to
(a) m^2 (b) m (c) $m^{1/2}$ (d) $m^{-1/2}$

3) An insect is walking on the horizontal surface. The number of degrees of freedom of the insect will be
(a) 1 (b) 2 (c) 3 (d) 6

4) The number of degrees of freedom for a diatomic gas molecule is
(a) 2 (b) 3   (c) 5   (d) 6

Answers of case study

1.a	2.d	3.b	4.c

Case study Question

Q24. For a gas, the relationship between volume and pressure (at constant mass and temperature) can be expressed mathematically as follows. $P \propto (1/V)$ Where P is the pressure exerted by the gas and V is the volume occupied by it. This proportionality can be converted into an equation by adding a constant, k. Charles law states that the volume of an ideal gas is directly proportional to the absolute temperature at constant pressure. The law also states that the Kelvin temperature and the volume will be in direct proportion when the pressure exerted on a sample of a dry gas is held constant. Charles law and Boyle's law applied to low density gas only. The total pressure of a mixture of ideal gases is the sum of partial pressures. This is Dalton's law of partial pressures

1 Which of the following is a assumption of the perfect gas equation?
a) Gas molecules have finite size and interact with each other.
b) Gas molecules have zero size and do not interact with each other.
c) Gas molecules have finite size but do not interact with each other.
d) Gas molecules have zero size and interact with each other.
2 Which of the following statements is a consequence of Charles' Law?
a) The volume of a gas is directly proportional to the temperature.
b) The pressure of a gas is inversely proportional to the temperature.
c) The volume of a gas is inversely proportional to the temperature.
d) The pressure of a gas is directly proportional to the temperature.
3 1. If the temperature of a gas is increased from 200 K to 400 K, what happens to its volume?
a) It decreases by half
b) It remains the same
c) It doubles
d) It triples
4 If the pressure of a gas is increased from 100 kPa to 200 kPa, what happens to its volume?
a) It remains the same
b) It decreases by half
c) It doubles
d) It triples

5. If pressure and temperature are constant, what happens to the volume of a perfect gas if the number of moles is increased?
a) It decreases
b) It remains the same
c) It increases
d) It becomes zero

Answers of case study

1.b	**2.a**	**3.c**	**4.b**	**5.c**

2 Marks Questions

Q25 Given Samples of 1 cm^3 of Hydrogen and 1 cm^3 of oxygen, both at N. T. P. which sample has a larger number of molecules?

Ans: Equal volumes of all gases, at equivalent temperatures and pressures, contain the same number of molecules, according to Avogadro's hypothesis. As a result, the number of molecules in both samples is the same. As a result, the number of molecules in both samples is the same.

Q26 What is Mean free path?

Ans: Mean free path of a molecule in a gas is the average distance travelled by the molecule between two successive collisions

Q27 A tank of volume $0.3m^3$ contains 2 moles of Helium gas at 20 °C. Assuming that helium behaves as an ideal gas, find the total internal energy of the system.

Ans: Given, n = No. of moles = 2,
Temperature T = 273+ 20= 293 K
R = Universal Gas constant = 8.31 J / mole. So we know that,
Total energy of the system
E = nRT Hence, E = x 2 x 8.31x 293 = 7.30 x 10^3 J
Q 28 Air pressure in a car tyre increases during driving? Why?
Ans: This happens because of the action, the temperature of the air inside the tyre rises during driving. According to Charles's law, as the temperature rises, the pressure inside the tyres rises as well.
Q29 Why gases at high pressure and low temperature show large deviation from ideal gas behaviour?

Ans When temperature is low and pressure is high the intermolecular forces become appreciable thus the volume occupied by the molecular is not negligibly small as composed to volume of gas.

Q30 Absolute temperature of a gas is increased four times its original value. What will be the change in rms velocity of its molecules?

Ans: We know that $V_{rms}\ \alpha\ \sqrt{\ }\ T$

So when absolute temperature of the gas is made four times its original value, its rms velocity will become double its initial value i.e change in r.m.s velocity will be equal to its initial value.

Q31 Explain absolute zero of temperature on the basis of kinetic theory of gases.
Ans. Absolute zero of temperature may be defined as that temperature at which the root mean square velocity of the gas molecules reduces to zero. It means, molecular motion ceases at absolute zero.

Q32 On the basis of kinetic theory of gases, explain, how does a gas exert pressure?

Ans: According to kinetic theory, the molecules of a gas are in a state of continuous random motion. They collide with one another and also with the walls of the vessel. Whenever a molecule collides with the wall, it returns with a changed momentum and an equal
momentum is transferred to the wall and thus creating pressure .

Q33 Two different gases have exactly the same temperature. Does this mean that their molecules move with the same r.m.s. speed?

Ans: When the two gases have exactly the same temperature the average kinetic energy per
molecule for each gas is the same. But as the different gases may have molecules of different masses, the r.m.s. speed of molecules of different
gases shall be different.

Q34 On what parameters does the λ (mean free path) depends?

Ans : (i) diameter of molecule as $\lambda \alpha\ 1/d^2$
(ii) Pressure of gas as $\lambda \alpha\ 1/P$

Short Answer Questions (3 Marks)

Q35 Determine the volume of 1 mole of any gas at S. T. P., assuming it behaves like an ideal gas?

Ans: Using the ideal gas equation, PV = nRT
P = Pressure, V = Volume, n =No of moles of gas R =Universal Gas Constant,
T = Temperature
V = nRT /P .
Given n = 1 mole; R = 8: 31 J / mol / K T= 273 K, P = 1.01×10^{5} N/ m^2

$V = 1 \times (8.31) \times 273/ 1.01 \times 10^5$
$V = 22.4 \times 10^{-3}\ m^3$
Hence V = 22.4 l . Since 1 litre =1000 cm^3= $10^3\ m^3$
i.e. at S.T.P., any gas has a volume of 22.4l. (Standard Temperature & Pressure)

Q 36 Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C .
Ans: Given : Volume of the room, V = 25 m^3 ,
Temperature of the room, T = 27° C= 300 K
Pressure in the room, P = 1atm= 1.013×10^5 Pa The ideal gas equation relating pressure (P), Volume (V), and absolute temperature (T) can be written as: $PV = Nk_BT$ where k_B is the Boltzmann constant 1.38×10^{-23} J/K
N is the number of air molecules in the room $N= PV/k_BT$
Substituting all the values,

$N = 6.11 \times 10^{26}$ molecules

Q37 Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contains equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Ans. (a) Yes, because according to Avogadro's hypothesis, equal volume of all the gases has same number of molecules under the condition of same temperature and pressure.

(b) Using $v_{rms} = \sqrt{3kT/m}$, we get $v_{rms} \propto 1/\sqrt{m}$,

i.e. v_{rms} will not be same in the three cases because it depends upon mass of the gas. It will be the largest for neon

Q38 At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the r.m.s. speed of a helium gas atom at –20°C ? (Atomic mass of Ar = 39.9 u and that of He = 4.0 u.)

Ans. R.M.S. speed of argon at temperature T

$v = \sqrt{3RT/M} = \sqrt{3RT/39.9}$

R.M.S. for helium at temperature T= – 20° C or 253K is $v' = \sqrt{3R \times 253/4}$

But v = v' (given)

$\sqrt{3RT/39.9} = \sqrt{3R253/4}$

$T/39.9 = 253/4$

T = 2523.7 K

Q39 A vessel is filled with a gas at a pressure of 76 cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.

Ans:

According to kinetic theory of gases, $PV = 1/3M\, V^2_{rms}$

At constant temperature, V^2_{rms} is constant.

As V is also constant, so $P \propto M$.

When the mass of the gas increase by 50% pressure also increases by 50%,

$\therefore$ Final pressure = = 114 cm of Hg.

Long Answer Type Questions (5 Marks)

Q40 What are the basic assumptions of kinetic theory of gases? On their basis derive an expression for the pressure exerted by an ideal gas.

Ans : Assumptions of Kinetic theory of gases

(1) The molecules of a gas are identical, spherical and perfectly elastic point masses.

(2) The volume of molecules is negligible in comparison to the volume of gas.

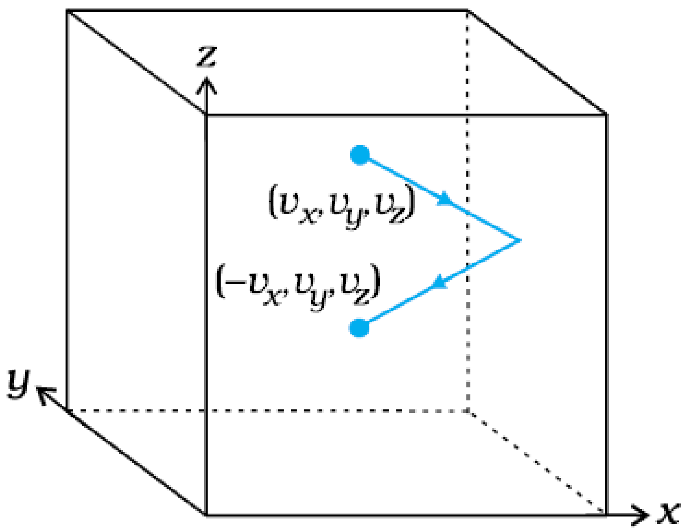
(3) Molecules of a gas move randomly in all direction.

(4) The speed of gas molecules lie between zero and infinity.

(5) Their collisions are perfectly elastic.

(6) The number of collisions per unit volume in a gas remains constant.

(7) No attractive or repulsive force acts between gas molecules.



Consider an ideal gas confined in a cubical container with capacity on each side a.
Volume of the container $V= a^3$
Let V_x ,V_y, V_z be the components of velocity along x, y ,z respectively
n= number of molecules of gas per unit volume.
m= mass of each molecules
A molecule moves with momentum mV_x along the x axis and rebounds with momentum $-mV_x$

Change in momentum $\Delta P_x = -2mV_x$

$F_x= \Delta P_x / \Delta t = mnV_x^2 \, A \, \Delta t / \Delta t$

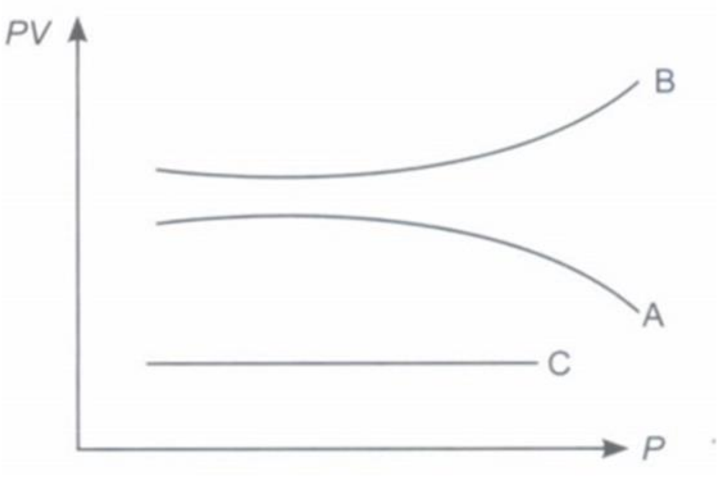
Pressure along x axis $=P_x= F_x/A = = mnV_x^2$
Pressure along y axis $=P_y= F_y/A = = mnV_y^2$
Pressure along z axis $=P_z= F_z/A = = mnV_z^2$

As gas molecules have same properties in all directions, we can write $P_x= P_y=P_z = P$

$P=1/3(P_x+ P_y+P_z)$
$P= 1/3(mnV_x^2 + mnV_y^2+ mnV_z^2)$
$P= 1/3 \, mn(V^2)$
$P=1/3 \, \rho \, V^2$

Concept Based Questions

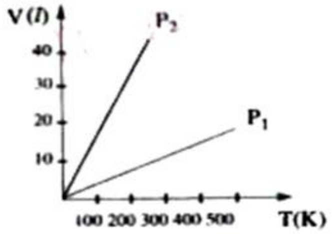
Q41



Above graphs show the variation of product PV with respect to pressure P of given masses of three gases A, B and C. The temperature is kept constant. State with proper arguments which of these gases is ideal?

Answer: Gas C is ideal because PV is constant for this gas. It means gas C obeys Boyles law at all pressures.

Q 42 In the given V-T graph what is the relation between pressure P1 and P2? Explain .



Answer :For an ideal gas , PV=nRT
V= nRT/P
Slope of V-T graph = dV/dT= nR/P
Hence Slope α 1/P
$P_2 < P_1$
Self-assessment questions
Q1 A flask Contains Argon and Chlorine in the ratio of 2:1 by mass. The temperature of the mixtures is 27^0 C the ratio of average kinetic energy per molecule is (1)
(a). 1:2 (b). 2:1 (c). 3:1 (d). 1:1
Ans: (d)
Q2 Root mean square velocity of a particle is V at pressure P. If pressure is increased two times, then the r.m.s. velocity becomes (1)
(a) 5V (b) 3V (c) V (d) 2V

Ans **(c)** RMS velocity does not depend upon pressure
Q3 Why temperature less than absolute zero is not possible? (2)
Ans According to kinetic interpretation of temperature,
Absolute temperature α Mean Kinetic energy of molecules.
As heat is taken out, the temperature falls and hence velocity decreases. At absolute zero, velocity of molecules becomes zero. i.e KE becomes zero. So, no more decrease in KE is possible. Hence, temperature cannot fall further.
Q4 An electric bulb of volume 250 cm^3 was sealed off during manufacture at pressure of 10^{-3}mm of Hg at 27^0 C. Find the number of molecules in the bulb. (3)
Ans 8.05×10^{15} (Hint: PV= nKT)
Q5 Two vessels of the same size are at the same temperature. One of them contains one kg of H_2 gas and the other contains one kg of N_2 gas.
A) Which of the vessels contain more molecules?
B) Which of the Vessels is under greater pressure and why?
C) In which vessel is the average molecular speed greater? How many times the speed of one is greater than that of the other gas molecule? (5)

Ans: A) The vessel containing H_2 gas contains more molecules. It is because, molecular weight of hydrogen is less than that of N_2 gas. (1)
B) P= 2/3 n KE_{mean} Since n of hydrogen is more, pressure exerted by H_2 gas will be more (2)
C) Hint: Take the ratio of rms speeds of both gases $\sqrt{m_N/m_H} = \sqrt{28/2} = \sqrt{14}$ (2)

Unit X Oscillations and Waves

Chapter 14. Oscillations

GIST OF THE CHAPTER

- Different types of motion
- Simple harmonic motion
- Important terms like oscillation, amplitude,time period,angular frequency,phase difference
- Displacement ,velocity and acceleration in SHM
- Energy in SHM : kinetic and potential
- Oscillations due to a spring
- Oscillations of a loaded spring

● **Oscillations -**

To and Fro motion of a body about mean position in regular interval of time, e.g. motion of simple pendulum.

● **Periodic motion**

Motion that repeat itself after regular intervals of time over same path, e.g. motion of planets around sun.

➢ **Time Period (T)**

Time interval during which motion repeat itself, unit-second.

➢ **Frequency (v)**

Number of revolutions of a periodic motion, per unit time.

$V = 1/T\ s^{-1}$

Unit - Hertz

❖ **Periodic function –**

The function which repeats its value in regular interval of time,T

$f(t) = A \cos \omega t$

$f(t) = f(t+T)$

● **Displacement -**

Change in position with respect to mean position or reference point.

● **Simple harmonic motion -**

To and fro motion about mean position under the influence of a restoring fore always acting towards mean position. Restoring force (F)x Displacement

Mathematically,

$F_{net} = -kx$

where, k is known as force constant

$\Rightarrow ma = -- kx$

$\Rightarrow a = -kx/m$

However, $a=-\omega^2x$

$a = -\omega 2x$

where, ω is known as angular frequency $\Rightarrow d^2x/dt^2=-\omega^2x$

This equation is known as the differential equation of S.H.M.

- Characteristics of S.H.M.

1) Displacement x(t) = A cos $(\omega t+\phi)$
Distance of particle from the mean position.

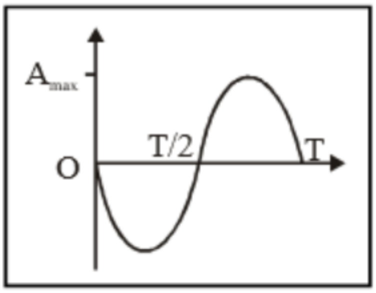
2) Amplitude (A) –
Maximum displacement from mean position.
The amplitude of a particle executing S.H.M. refers to its maximum displacement on either side of the equilibrium position. The amplitude of a particle is represented by A.

3) Phase – State of motion w.r.t. a reference.
The phase of particle executing S.H.M. at any instant refers to its state with respect to its position and direction of motion at that particular instant. It is measured as angle of sine in the equation of S.H.M.

Phase=(ωt+ϕ)=$(\omega t+\phi)$
When
t=0t=0, phase=ϕ;
the constant ϕ_0 is called initial phase of the particle or phase constant.
4) Angular frequency – $\omega = 2\pi v$

5) Velocity (v) – v
At mean position x = 0 , v = A
At extreme position x = A, v = 0

6) Acceleration –



At any instant t, a(t)=ω2A sin(ωt+ϕ)$a(t)=\omega 2A \sin(\omega t+\phi)$

- At any position x, a(x)=ω2x$a(x)=\omega 2x$
- Acceleration is always directed towards the equilibrium position.
- The magnitude of acceleration is minimum at equilibrium position and maximum at extremes.
- |a|min=0$|a|$min=0 at equilibrium position
- |a|max=ω2A$|a|$max=$\omega 2A$ at extremes

Energy in Simple Harmonic Motion –
Particle executing SHM possesses both K.E. total E = K+ U

Kinetic Energy
$K = \frac{1}{2} mv^2$, $K = \frac{1}{2} k(A^2-x^2)$

Potential Energy –
$U = \frac{1}{2} Kx^2$

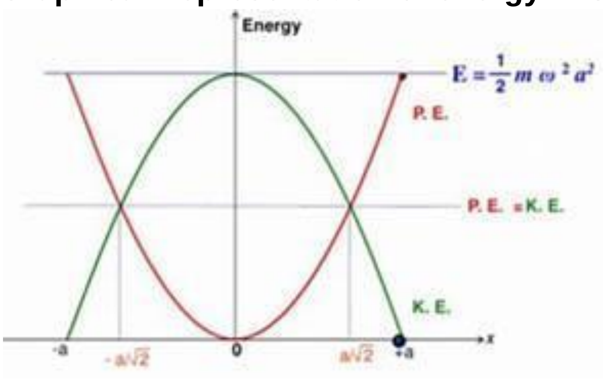
Total Energy
$E = \frac{1}{2} KA^2$, $E = 2\pi^2 mv^2A^2$

Time $T = 2\pi/\omega$

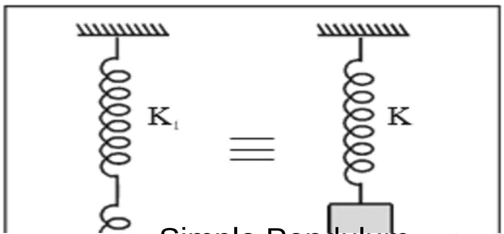
$$\omega = \sqrt{\frac{K}{M}} = \frac{Force\ constant}{M} Inertia$$

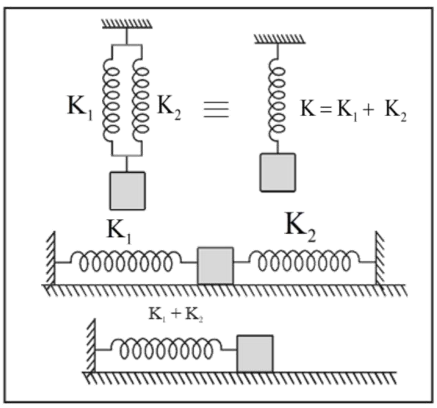
$$T = 2\pi\sqrt{\frac{M}{K}}$$

Graphical Representation of energy in SHM

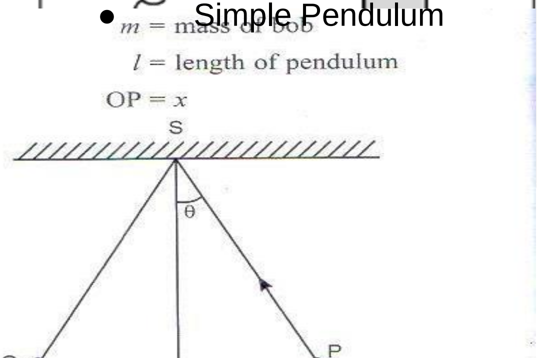


- Some system executing S.H.M.
- Loaded Spring





- Simple Pendulum



BRAIN MAP

OSCILLATIONS

Class XI

Periodic Motion

A motion that repeats itself at regular interval of time is called periodic motion. The displacement is represented by a periodic function of time with time period T. i.e., $f(t) = f(t+T) = f(t+2T) = ...$

Oscillatory Motion

If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point. Such motions are called oscillatory motion.

Simple Harmonic Motion

The motion arises when the force on the oscillating body is directly proportional to its displacement from mean position. Such motion is called simple harmonic motion.

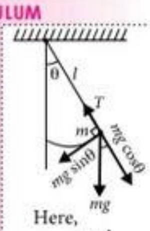
System Excuting SHM

SHM IN SPRING

- Equation of motion
  $$\frac{d^2y}{dt^2} = \frac{-ky}{m} = -\omega^2 y$$
- If the spring is not light but has a definite mass m_s then
  $$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$
- Two bodies of masses m_1 and m_2 are attached through a light spring of spring constant k, the time period of oscillation
  $$T = 2\pi\sqrt{\frac{\mu}{k}} \quad \text{where } \mu = \frac{m_1 m_2}{m_1 + m_2}$$

SIMPLE PENDULUM

- Time period
  $$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$$
- If the length of simple pendulum is very large,
  $$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$



Here, $\alpha = -\frac{mgl}{I}\theta$

where R is the radius of length of pendulum

Dynamic of SHM

FORCE LAW IN SHM

- The force acting on a particle of mass m in SHM is
  $\vec{F} = -m\omega^2\vec{x}$ or $\vec{F} = -k\vec{x}$
  where, $k = m\omega^2$ = force constant
- Linear SHM:
  - Angular velocity, $\omega = \sqrt{\frac{k}{m}}$
  - Time period, $T = 2\pi\sqrt{\frac{m}{k}}$
- Angular SHM:
  - Torque, $\tau = -k\theta$
  - Angular velocity, $\omega = \sqrt{k/I}$
  - Angular acceleration, $\alpha = -\frac{k\theta}{I}$
  - Time period, $T = 2\pi\sqrt{\frac{I}{k}}$

  where I = moment of inertia

ENERGY IN SHM

- Linear SHM:
  - Kinetic energy $(K) = \frac{1}{2}m\omega^2A^2\cos^2\omega t$
  - Potential energy $(U) = \frac{1}{2}m\omega^2A^2\sin^2\omega t$
  - Total energy $(E) = \frac{1}{2}m\omega^2A^2$
- Angular SHM:
  - Kinetic energy $(K) = \frac{1}{2}I\omega^2$
  - Potential energy $(U) = \frac{1}{2}k\theta^2 = \frac{1}{2}I\omega^2\theta^2$
  - Total energy $(E) = \frac{1}{2}I\omega^2\theta_0^2$

Characteristics of SHM

GENERAL EQUATIONS OF SHM

- Linear SHM:
  - Differential equation $\frac{d^2y}{dt^2} + \omega^2 y = 0$
  - Displacement $y = A\sin(\omega t + \phi)$
  - Velocity, $v = \omega\sqrt{A^2 - y^2}$
  - Acceleration, $a = -\omega^2 y$
- Angular SHM:
  - Differential equation
    $$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$$
  - Displacement $\theta = \theta_0\sin(\omega t + \delta)$

DAMPED AND FORCED OSCILLATIONS

- Damped oscillations
  - Angular frequency $(\omega') = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$
  - Mechanical energy $E(t) = \frac{1}{2}kA^2e^{\frac{-bt}{m}}$
  - Amplitude $A' = Ae^{-bt/2m}$
    where b is damping constant.
- Forced oscillations
  - When driving frequency ω_d far from natural frequency ω:
    Amplitude $A' = \frac{F_0}{m(\omega^2 - \omega_d^2)}$
  - When driving frequency ω_d closed to natural frequency ω:
    Amplitude $A' = \frac{F_0}{\omega_d b}$

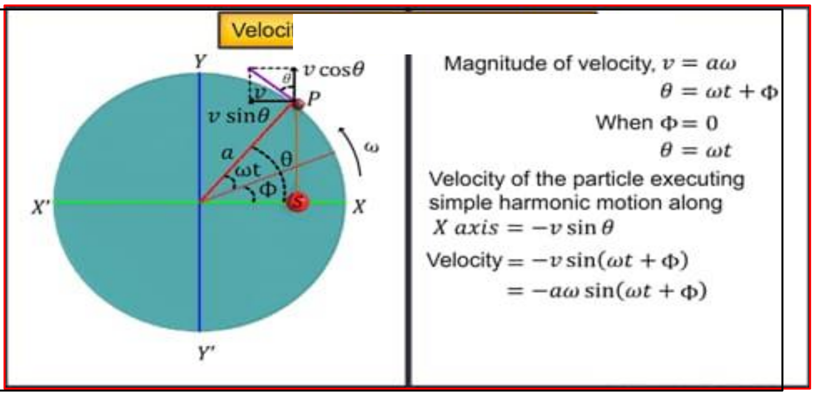
Simple Harmonic Motion

S.H.M. is the simplest form of oscillatory motion

Displacement x(f) = A cos($\omega t+\phi$)
Distance of particle from the mean position.

Phase – The varying quantity ($\omega t+\phi$)
Phase constant or phase angle (ϕ)
Its depend upon velocity(v) & Displacement of particle at t=0

Velocity in S.H.M.



Acceleration in S.H.M.
a =   = dv/dt = - $\omega^2 A\cos(\omega t+\phi)$
$a_{max} = \omega^2 A$

Energy in S.H.M.

Kinetic Energy :
K.E. = ½ mω^2 $A^2\sin^2(\omega t+\phi)$

Potential Energy :
P.E. = ½ mω^2 $A^2\cos^2(\omega t+\phi)$

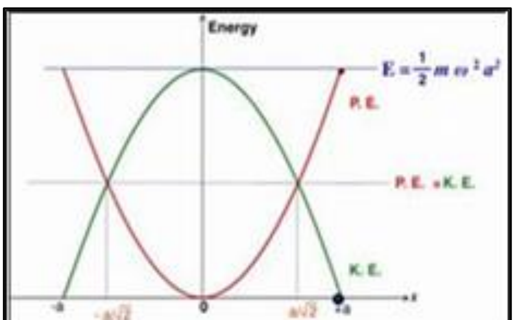
Total Energy : T.E. = K.E. + P.E.
½ m$\omega^2 A^2$ = $2\pi^2$ mv^2A^2

Time T = 2 π/ ω

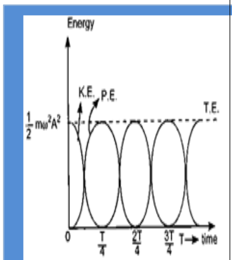
ω = k/m $\sqrt{\frac{K}{M}}$ = $\frac{Force\ constant}{M}$ $Inertia$

T = 2 $\pi\sqrt{\frac{M}{K}}$

Graphical Representation :

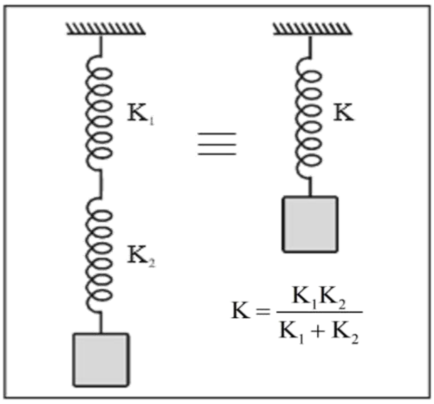


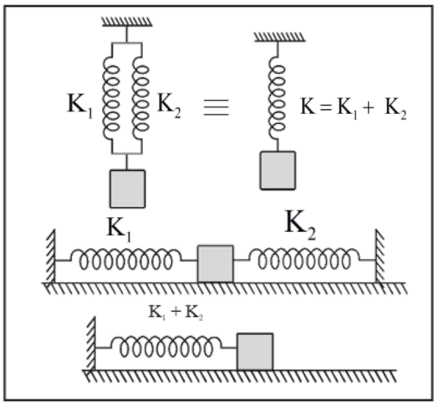
K.E., P.E. & E are function of Displacement X, for a harmonic Oscillator
E = K.E. + P.E.



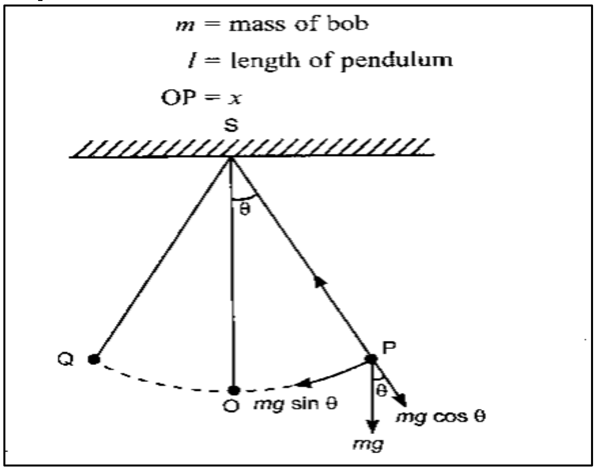
❖ **Application of S.H.M.**

- **Some system executing S.H.M.**
- **Loaded Spring**





- **Simple Pendulum**



<u>**Multiple Choice Questions**</u>

1.Which of the following is the correct definition of oscillation?
a) A linear motion from one point to another
b) A random back-and-forth movement
c) A repetitive to-and-fro motion about an equilibrium position
d) A circular motion around a fixed point

2.Which of the following quantities remains constant in simple harmonic motion?
a) Displacement
b) Velocity
c) Acceleration
d) Total Energy

3.The time period of a simple pendulum depends on which of the following factors?

a) Mass of the pendulum bob
b) Amplitude of the oscillation
c) Length of the pendulum
d) Damping force

4.Which of the following statements about angular frequency (ω) is correct?
a) It is measured in meters per second squared (m/s^2)
b) It is the reciprocal of the time period (T)
c) It remains constant for all types of oscillations
d) It is directly proportional to the amplitude

5.If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 m/s^2 at any time, the angular frequency of the oscillator is equal to
a) 10 rad/s
b) 1 rad/s
c) 100 rad/s
d) 1 rad/s

6.The restoring force in simple harmonic motion is directly proportional to:
a) Displacement
b) Velocity
c) Acceleration
d) Time period

7.A mass-spring system oscillates with a period of 2 seconds. What is the frequency of oscillation?
a) 0.5 Hz
b) 1 Hz
c) 2 Hz
d) 4 Hz

8.The time period of a thin magnet is 4s. If it is divided into two equal halves, then the time period of each part will be:
a) 4s
b) 1s
c) 2s
d) 8s

9.The maximum velocity for particle in SHM is 0.16 m/s and maximum acceleration is 0.64 m/s2. The amplitude is
a) 4 × 10-2m
b) 4 × 10-1m
c) 4 × 10 m
d) 4 × 100 m

10.For a magnet of time period T, magnetic moment is M. If the magnetic moment becomes one fourth of the initial value, then the time period of oscillation becomes
a) Half of initial value
b) One fourth of initial value

c) Double of initial value
d) Four time initial value

Answers –

1.c	2.d	3.c	4.b	5.a
6.a	7.a	8.c	9.a	10.c

<u>Assertion And Reason Questions</u>

Directions: (a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false.

1Assertion : Sine and cosine functions are periodic functions.
Reason : Sinusoidal functions repeats it values after a definite interval of time.

2Assertion : Simple harmonic motion is a uniform motion.
Reason : Simple harmonic motion is not the projection of uniform circular motion

3Assertion : Acceleration is proportional to the displacement. This condition is not sufficient for motion in simple harmonic.
Reason : In simple harmonic motion direction of displacement is also considered.

4Assertion : The graph between velocity and displacement for a harmonic oscillator is a parabola.
Reason : Velocity changes uniformly with displacement in harmonic motion.

5Assertion : All oscillatory motions are necessarily periodic motion but all periodic motion are not oscillatory.
Reason : Simple pendulum is an example of oscillatory motion.

6Assertion : Resonance is special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency of external periodic force and the amplitude of forced vibration is maximum.
Reason : The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.

7Assertion : The graph of total energy of a particle in SHM w.r.t., position is a straight line with zero slope.
Reason : Total energy of particle in SHM remains constant throughout its motion.

8. Assertion : The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is 0.612 s–1.
Reason : The frequency of a second pendulum does not depend upon acceleration

due to gravity.

9.Assertion : Damped oscillation indicates loss of energy.
Reason : The energy loss in damped oscillation may be due to friction, air resistance etc.

10.Assertion : The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%.
Reason : Time period is directly proportional to length of pendulum.

11.Assertion: Average kinetic energy in one oscillation during SHM of a body is ¼ $m\omega^2A^2$.
Reason: Maximum kinetic energy is ½ $m\omega^2A^2$

12.Assertion : The motion of a simple pendulum is simple harmonic for all angular displacement.
Reason : Motion of simple pendulum is independent of the angular displacement.

13Assertion: In a SHM, kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude. Reason: In SHM, kinetic energy is zero when potential energy is maximum.

Answer:

1.a	2.d	3.a	4.d	5.b
6.c	7.a	8.c	9.b	10.c
11.b	12.a	13.b		

2 Marks Questions

Q.1 Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
a. the rotation of earth about its axis.
b. motion of an oscillating mercury column in a U-tube.
Ans- It is periodic but not simple harmonic motion because it is not to and fro about a fixed point.
b. It is a simple harmonic motion because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.

Q.2 State some practical examples of S. H. M.
Ans: Some practical examples of S. H. M. are as follows.
1Motion of piston in a gas-filled cylinder
2Atoms vibrating in a crystal lattice
3 Motion of helical spring

Q.3Are all oscillatory motions simple harmonic? If yes, give an example but if not, why?
Ans- No, all oscillatory motions cannot be simple harmonic. In fact, only those

oscillatory motions are simple harmonic for which force is proportional to displacement and directed towards the mean position.
Q.4 What is the displacement of a particle executing simple harmonic motion?
Ans. For a particle executing mechanical simple harmonic motion, displacement is defined as the term which specifies its position as well as direction with respect to its mean position.

Q.5 Can motion of a satellite around a planet be considered simple harmonic? Why?
Ans. No, periodic motion of a satellite around a planet cannot be considered as simple harmonic. The force acting on satellite is $F = GMm/R^2$
where M = mass of the planet, m = mass of a satellite, G = gravitational constant and r = orbital radius of satellite. As force is not proportional to the displacement, hence motion cannot be simple harmonic.

Q.6 a) What is the significance of phase of a particle in an oscillatory motion?
b)How is simple harmonic motion related to a uniform circular motion?
Ans. a) In an oscillatory motion, the phase of a particle describes the state of motion of the particle at a given instant of time.
b) Simple harmonic motion may be considered as the projection of a uniform circular motion on a diameter of the circle.

Q.7 a) Can an object executing SHM have acceleration without having velocity?
b) Can an ideal simple pendulum be constructed in practice? Give reason too.
Ans.a) Yes, an object executing SHM can have maximum acceleration but zero velocity when situated at its extreme position.
b) We can never design an ideal simple pendulum because neither the pendulum bob can be an infinite, point mass nor the string be massless and inextensible.

Q.8What are the two basic characteristics of a simple harmonic motion?
Ans. The two basic characteristics of a SHM are:
(a) The force (or acceleration) is proportional to its displacement.
(b) The force (or acceleration) is directed towards the mean position of SHM.

Q.9Define force constant of a spring. Give its SI units and dimensional formula.
Ans. Force constant of a spring is defined as the restoring force set up in the spring per unit extension or compression in the spring. Its SI unit is N/m and dimensional formula is [MT-2].

Q.10 a) A spring-mass system oscillating vertically has a period T. What will be the value of time period if the same system is made to oscillate horizontally on a smooth surface?
b) When will the motion of a simple pendulum pendulum be simple harmonic?
Ans. a) The time period of spring-mass system will remain unchanged at $T=2\pi\sqrt{m/k}$.
b) Motion of a simple pendulum pendulum is simple harmonic when its amplitude (or angular amplitude θ) is small enough so that sinθ may have the same value as θ.

3 Marks Questions

Q1. A pendulum of length/is attached with a bob and placed in a lift. What will be period when the lift is
(i) having uniform motion upwards,
(ii) accelerated upwards by a,
(iii) accelerated downward by a?

Ans.
(i) When there is uniform motion there is no change in acceleration.

$$T \quad = 2\pi\sqrt{\frac{l}{g}}$$

(ii) When there is an upward acceleration, g → g + a.

$$T \quad = 2\pi\sqrt{\frac{l}{g+a}}$$

(iii) When there is a downward acceleration, g → g - a.

$$T \quad = 2\pi\sqrt{\frac{l}{g-a}}$$

Q2. Find the expression for the total energy of a particle executing S.H.M.
Ans. P.E. with a S.H.M.

$$\frac{1}{2}\kappa x^2 = \frac{1}{2}m\omega^2x^2$$

kinetic energy with SHM $=\frac{1}{2}$mv2 $=\frac{1}{2}m(\omega\sqrt{A^2 - x^2}\ )2$

$$= \frac{1}{2}m\omega^2\ (A^2 - x^2)$$

where m is mass, A is amplitude, x is any position and ω is angular frequency

Total energy $= \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2\ (A^2 - x^2)$

$$\frac{1}{2}\kappa x^2 = \frac{1}{2}m\omega^2A^2$$

Q3. A 0.2 Kg. of mass hangs at the end of a spring. When 0.02 kg more mass is added to the end of the spring, it stretches 7 cm more. If the 0.02 kg mass is removed, what will be the period of vibration of the system?

Ans. When 0.02 kg is added, there is a stretch of 7 cm, Using mg =Kx, we have

$$K = \frac{0.02\times10}{7\times10^{-2}} = \frac{20}{7} = 2.86\text{N/m}$$

$$\text{Time period } T = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{0.2}{2.86}}$$

= 1.66 sec

Q4. What is Simple Harmonic Motion ? Show that in S.H.M., acceleration is directly proportional to its displacement at a given instant .
Ans. Simple Harmonic Motion :
i) Motion is always directed towards a fixed point or equilibrium point.
ii) Motion being represented by bounded trigonometric functions l
Acceleration is directly proportional to negative of displacement, Equation for S.H.M.

Acceleration = $-\omega^2 x$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

ω is angular frequency (radians/sec) , f is linear frequency unit is hertz

Q.5) A body is executing S.H.M of amplitude 1 m. Its velocity while passing through the mean position, is $10ms^{-1}$. Find its frequency.
Here a = 1 m, v_{max} = 10m/s

$V_{max} = a\omega = 2\,\pi v a$
$= v = \frac{v_{max}}{2\pi a}$
= 1.592Hz

Q.6) In a HCI molecule, we may treat CI to be of infinite mass and H alone be oscillating. If the oscillation of HCI molecule shows a frequency of $9 \times 10^{13}\ s^{-1}$, deduce the force constant.
[Given : Avogadro's number = 6×10^{26} per kg mole.]
Ans. 1 Kg of H has 6×10^{26} atoms.
$m = \frac{1}{6\times10^{26}}$
$v = 9X10^{13}\ s^{-1}$
$v = ½\pi \sqrt{\frac{k}{m}}$
Squaring
$k = 4\,\pi^2 v^2 m$ (putting the values)

$= 5.32\ X\ 10^2 Nm^{-1}$

Q.7) The kinetic energy of a particle vibrating in S.H.M. is 4 J when it passes the mean position. If the mass of the body is 2 kg and the amplitude is 1 m, calculate its time period.
Ans. In its mean position, the kinetic energy of the particle in maximum.
Given -
$E_{Kmax} = 1/2mw^2A^2$
$E_{Kmax} =$ 4J; m=2kg; A = 1m
$4J = ½ \times 2x\ w^2\ x1$
$w = 2,\ T = 2\pi/\omega = \pi\ sec$

Q.8) A harmonic oscillation is represented by y = 0.34 cos(3000 t+0.74), where y and t are in m and s respectively. Deduce : (i) the amplitude, (ii) the frequency and

angular frequency, (iii) the period and (iv) the initial phase.
Ans. Given that, y = 0.34 cos(3000 t + 0.74) While the general expression for displacement is, y=a cos(ωt+Φ_0)
Comparing these two expression,
(i) Amplitude, a = 0.34m.
(ii) Angular frequency

$\omega = 3000\ radian\ sec^{-1}$
Frequency = v
= ω/2π = 1500/π

iii) Period T =
1/ω =π/1500

iii) Initial phase φ_0 = 0.74 rad

Case Study Questions

Question 1:

Simple Harmonic Motion
Simple harmonic motion is the simplest form of oscillation. A particular type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is termed as simple harmonic motion (S.H.M). A body is undergoing simple harmonic motion if it has an acceleration which is directed towards a fixed point, and proportional to the displacement of the body from that point
$a \propto -x \Rightarrow a = -kx$ or $\frac{d^2x}{dt^2} = -kx$

1. Which of the following is not a characteristic of simple harmonic motion?
(a) The motion is periodic.
(b) The motion is along a straight line about the mean position.
(c) The oscillations are responsible for the energy conversion.
(d) The acceleration of the particle is directed towards the extreme position

2. The equation of motion of a simple harmonic motion is
a) $\frac{d^2x}{dt^2} = -\omega 2x$
b) $\frac{d^2x}{dt^2} = -\omega 2t$

c) $\frac{d^2x}{dt^2} = -\omega t$

d) $\frac{d^2x}{dt^2} = -\omega x$

3 Which of the following expressions does not represent simple harmonic motion?

(a) x = Acoswt + Bsinwt
(b) x = Acos(wt + a)
(c) x = Bsin(wt + b)
(d) x = Asinwt $\cos^2$wt

4. The time period of simple harmonic motion depends upon
(a) amplitude
(b) energy
(c) phase constant
(d) mass

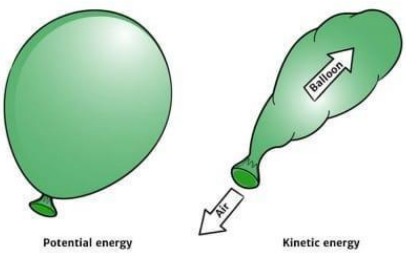
5 Which of the following motions is not simple harmonic?
(a) Vertical oscillations of a spring
(b) Motion of a simple pendulum
(c) Motion of planet around the Sun
(d) Oscillation of liquid in a U-tube

Answer:

1.d	2.a	3.d	4.d	5.c

Case study
Q. 2) Potential energy is the energy stored within an object, due to the object's position, arrangement or state. Potential energy is one of the two main forms of energy, along with kinetic energy. Potential energy depends on the force acting on the two objects

Potential and Kinetic Energy



1. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?
a) kinetic energy b) potential energy
c) mechanical energy d) none of these

2. Work done by a conservative force is positive, if
a) potential energy decreases b) potential energy increases
c) kinetic energy decreases d) kinetic energy increases

3. When does the potential energy of a spring increases?
a) only when spring is stretched b) only when spring is compressed
c) both a and b d) none of these

4. Dimension of k/m is, here k is force constant

a) T^2 b) T^{-2} c) T^1 d) T^{-1}

5.A vehicle of mass 5000kg climbs up a hill of 10 m. The potential energy gained by it

a) 5 J b) 500 J c) 5 × 104 J d) 5 × 105 J

Answer Key:

1.c	2.a	3.c	4.b	5.d

Q.3)A motion that repeats itself at regular intervals of time is called periodic motion. Very often, the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to oscillations or vibrations. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory. The smallest interval of time after which the motion is repeated is called its period. Let us denote the period by the symbol T. Its SI unit is second. The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The relation between n and T is n = 1/T. The unit of n is thus s-1. After the discoverer of radio waves, Heinrich Rudolph Hertz (1857–1894), a special name has been given to the unit of frequency. It is called hertz (abbreviated as Hz).

Answer the following.

1) Every oscillatory motion is periodic motion true or false?

a) True b) False

2) Circular motion is

a) Oscillatory motion b) Periodic motion c) Rotational motion d) None of these

3) Define period. Give its SI unit and dimensions

4) Define frequency of periodic motion. How it is related to time period

5) What is oscillatory motion

Answer key -

1) a

2) b

3) The smallest interval of time after which the motion is repeated is called its period. Its SI unit is second and dimensions are $[T^1]$.

4) Reciprocal of Time period (T) gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion. It is represented by the symbol n. The relation between n and T is n = 1/T i.e. they are inversely proportional to each other. The unit of n is thus s-1 or hertz.

5) Oscillatory motion is type of periodic motion in which body performs periodic to and fro motion about some mean position. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.

Long Questions 5 Marks

Q1. (a) Define SHM. What are its characteristics? At what distance from the mean position in SHM of amplitude r the energy is half kinetic and half potential ?
(b) A spring having a force constant K is divided into three equal parts. What would be force constant for each individual parts?
Ans. (a) Simple harmonic motion is the projection of uniform circular motion on diameter of a circle of reference.
Characteristics of SHM:
(I) Displacement : The displacement of a particle executing SHM at an instant is defined as the distance of a particle from the mean position at that instant.
(II) Amplitude: The maximum displacement on either side of the mean position is called the amplitude of the motion.
(III) Velocity : The velocity of the particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant.
(IV) Acceleration. The acceleration of the particle executing SHM at any instant is defined as the time rate of change of its velocity at that instant.

$$\frac{1}{2}\kappa x^2 = \frac{1}{2}m\omega^2 x^2$$

kinetic energy with SHM $=\frac{1}{2}mv2 \ = \frac{1}{2}m(\omega\sqrt{A^2-x^2}\ )2$

$$= \frac{1}{2}m\omega^2\,(A^2 - x^2)$$

where m is mass, A is amplitude, x is any position and ω is angular frequency

PE= KE

$$\frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2\,(A^2 - x^2)$$

$$x = \pm\frac{A}{\sqrt{2}}$$

b) force constant K = F/y

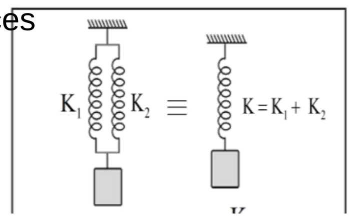
spring is cut into three equal parts therefore displacement = y/3

$$k' = \frac{F}{\frac{y}{3}} = \frac{3F}{y} \quad =3k$$

Q2. Derive the expression for resultant spring constant when two springs having constants k_1 and k_2 are connected (i) in parallel and (ii) in series.

ans : a) when the springs are connected in parallel the extension in them will be same

total restoring force= sum of their restoring forces



$F = F_1 + F_2$

$-kx = -k_1x + (-k_2x)$

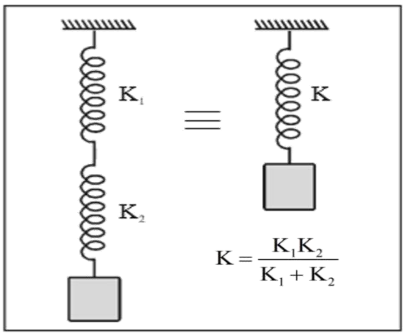
$k = k_1 + k_2$

b) when the springs are connected in series, the restoring force is same in both springs and extensions will be different so

$x = x_1 + x_2$

$\frac{F}{-k} = \frac{(-F)}{k_1} + \frac{-F}{K_2}$

$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{K_2}$



Q3. (i) Derive expression for kinetic energy and potential energies of simple harmonic oscillator. Hence show that the total energy is conserved.
(ii) What is the length of a simple pendulum which ticks in one second?

PE in SHM $\frac{1}{2}\kappa x^2 = \frac{1}{2}m\omega^2 x^2$

kinetic energy with SHM $= \frac{1}{2}mv2 = \frac{1}{2}m(\omega\sqrt{A^2 - x^2})2$

$= \frac{1}{2}m\omega^2 (A^2 - x^2)$

Total energy $= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m\omega^2 (A^2 - x^2)$

$\frac{1}{2}\kappa x^2 = \frac{1}{2}m\omega^2 A^2$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\times 3.14\sqrt{\frac{l}{9.8}}$

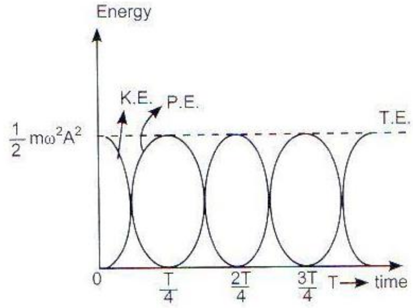
solving $l = 0.25 = 25cm$

Q4) Find the total energy of the particle executing S.H.M and show graphically the variation of P.E. and K.E. with time in S.H.M. What is the frequency of these ernergies with respect to the frequency of the particle executing S.H.M.

Ans. In a S.H.M., with y=A sin ω t, $\frac{1}{2}\kappa y^2 = \frac{1}{2}m\omega^2 y^2$

$= \frac{1}{2}m\omega^2 A^2 sin^2\omega t$

$= \frac{1}{2} mv^2 = \frac{1}{2} m(\frac{dy}{dx})^2$



$=\frac{1}{2} m\omega^2 A^2 cos^2 \omega t$

Total Energy = P.E. + K.E.

$= \frac{1}{2} m\omega^2 A^2$

as $\sin^2 \omega t + cos^2 \omega t = 1$

Since, both P.E. and K.E. are square sinusoidal functions, their frequency will be double of a normal simple harmonic function. So, P.E. and K.E. will have a frequency 2f for a S.H.M. with frequency f. Total energy = $\frac{1}{2}m\omega^2A^2$ is a constant and so, there is no variation.

Q.5) A particle is executing S.H.M. If v_1 and v_2 are the speeds of the particle at distance x_1 and x_2 from the equilibrium position, show that the frequency of oscillations is

$$f = \frac{1}{2\pi} (\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2})1/2$$

The displacement of particle executing S.H.M. is given by
x = a cosωt

Velocity v = $\frac{dx}{dt}$ $or\ v^2 = a^2\ \omega^2 sin^2 \omega t$

$$v_1^2 = \omega^2(a^2 - x_1^2)$$

$$v_2^2 = \omega^2(a^2 - x_2^2)$$

Subtracting $v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$

$$\omega^2 = (\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2})1/2$$

$$\omega = 2\pi f$$

$$f = \frac{1}{2\pi} (\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2})1/2$$

Q.6) A body of mass 5 kg executes S.H.M. of amplitude of 0.5 m. If the force constant is 100 Nm^{-1}, calculate

(i) its time period.

(ii) its maximum kinetic energy, maximum potential energy and total energy.

Ans – m = 5kg, k = 100N/m A = 0.5m

i) Time period T = $2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{5}{100}} = 0.41$ s

ii) Angular Velocity =, $\omega = 2\pi\frac{1}{T} = 4.5\ rad/s$

Maximum K.E. = Ekmax = ½ mv2

½ m ωA2 = 12.50J

Maximum P.E. = ½ kA^2

=12.5J

Total Energy = 12.50J

Q7) A particle of mass 10g is placed in a potential field given by U= 50x2 +100 erg/gm. Calculate the frequency of oscillation.

Ans –

P.E. of 10gm particle is U

$10(50x^2-100)$erg

F = m $(-dU/dx) = \frac{-d}{dx}(500x^2 + 1000)$

= -1000x

= $\frac{d^2x}{dt^2} = -100x$

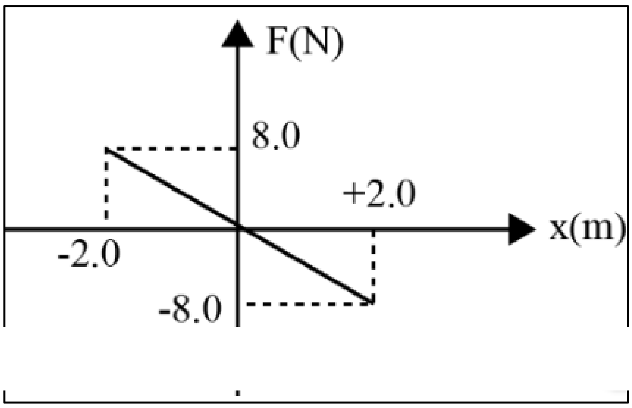
= $\frac{d^2x}{dt^2} = -\omega^2 x = 100\ x$

= $\omega = 10$

= v = $\frac{\omega}{2\pi} = 1.58\ s^{-1}$

Self-Assessment Questions -

Q1) A body of mass 0.01 kg executes simple harmonic motion (S.H.M.) about x = 0 under the influence of a force shown below. Find the period of the S.H.M.



Q.2) What is simple pendulum ? Show that the motion of the pendulum is S.H.M. and hence deduce an expression for the time period of pendulum. Also define second's pendulum.

Q.3) An 8 Kg body performs S.H.M of a amplitude 30cm. The restoring force is 60 N when the displacement is 30 cm. Find (a) time period (b) the acceleration, P.E. and K.E., when displacement is 12cm.

Unit X Oscillations and Waves

Chapter 15. Waves

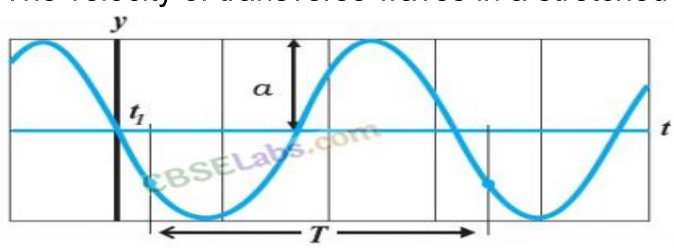
GIST OF THE CHAPTER

● Wave motion
● Longitudinal and transverse waves
● Speed of longitudinal waves
● Newton's formula and Laplace correction
● Factors affecting speed of sound in a gas
● Displacement relation for a progressive wave
● Principle of superposition of waves
● Stationary waves
● Stationary waves in a string fixed at both ends
● Stationary waves in a string closed organ pipe & open organ pipe
● Beats and its applications

• **Waves**
Wave is a form of disturbance which travels through a material medium due to the repeated f periodic motion of the particles of the medium about their mean positions without any actual
transportation of matter.
• **Characteristics of wave**
The characteristics of waves are as follows:
(i) The particles of the medium traversed by a wave execute relatively small vibrations about their mean positions but the particles are not permanently displaced in the direction of propagation of the wave.
(ii) Each successive particle of the medium executes a motion quite similar to its predecessors along/perpendicular to the line of travel of the wave.
(iii) During wave motion only transfer of energy takes place but not that of a portion of the medium.
Waves are mainly of three types: (a) mechanical or elastic waves, (b) electromagnetic waves and (c) matter waves.
• **Mechanical waves**
Mechanical waves can be produced or propagated only in a material medium. These waves are governed by Newton's laws of motion. For example, waves on water surface, waves on strings, sound waves etc.
• **Electromagnetic Waves**
These are the waves which require no material medium for their production and propagation, i.e., they can pass through vacuum and any other material medium. Common examples of electromagnetic

waves are visible light; ultra-violet light; radio waves, microwaves etc.

• Matter waves
These waves are associated with moving particles of matter, like electrons, protons, neutrons etc. Mechanical waves are of two types:
(i) Transverse wave motion, (ii) Longitudinal wave motion,
• Transverse wave motion
In transverse waves the particles of the medium vibrate at right angles to the direction in which the wave propagates. Waves on strings, surface water waves and electromagnetic waves are transverse waves. In electromagnetic waves (which include light waves) the disturbance that travels is not a result of vibrations of particles but it is the oscillation of electric and magnetic fields which takes place at right angles to the direction in which the wave travels.
• Longitudinal wave motion
In these types of waves, particles of the medium vibrate to and fro about their mean position along the direction of propagation of energy. These are also called pressure waves. Sound waves are longitudinal mechanical waves.
• Wavelength
The distance travelled by the disturbance during the time of one vibration by a medium particle is called the wavelength (λ). In case of a transverse wave the wavelength may also be defined as the distance between two successive crests or troughs. In case of a longitudinal wave, the wavelength (λ) is equal to distance from centre of one compression (or refraction) to another.
• Wave Velocity
Wave velocity is the time rate of propagation of wave motion in the given medium. It is different from particle velocity. Wave velocity depends upon the nature of medium.
Wave velocity (υ) = frequency (v) x wavelength (λ)
• Amplitude
The amplitude of a wave is the maximum displacement of the particles of the medium from their mean position.
• Frequency
The number of vibrations made by a particle in one second is called Frequency. It is represented by v. Its unit is hertz (Hz) v =1/T
• Time Period
The time taken by a particle to complete one vibration is called time period.
T = 1/v, it is expressed in seconds.
• The velocity of transverse waves in a stretched string is given by



where T is the tension in the string and μ is the mass per unit length of the string, μ is also called linear mass density of the string. SI unit of μ is kg m-1.
• The velocity of the longitudinal wave in an elastic medium is given by

$$\omega = 2\pi/T$$

where E is the modulus of elasticity of the medium and ρ is the density of the medium. In case of solids, E is Young’s modulus of elasticity (Y), then

• Newton's Formula for the velocity of sound in Air
According to Newton, when sound waves travel in air or in a gaseous media, the change is taking place isothermally and hence, it is found that

$$PV = Nk_B T$$

Speed of sound in air at STP conditions, calculated on the basis of Newton's formula is 280 ms^{-1}. However, the experimentally determined values is 332 ms^{-1}.
According to Laplace, during propagation of sound waves, the change takes place under adiabatic conditions because gases are thermal insulators and compressions and refractions are alternately taking place with a high frequency.

• Factors Influencing Velocity of Sound
The velocity of sound in any gaseous medium is affected by a large number of factors like density, pressure, temperature, humidity, wind velocity etc.
(i) The velocity of sound in a gas is inversely proportional to the square root of density of the gas.
(ii) The velocity of sound is independent of the change in pressure of the gas, provided temperature remains constant.
(iii) The velocity of sound in a gas is directly proportional to the square root of its absolute temperature.
(iv) The velocity of sound in moist air is greater than the velocity of sound in dry air.
(v) If wind flows at an angle θ to the direction of propagation of sound, the velocity of sound is $v + w \cos \theta$, where w is the velocity of wind.

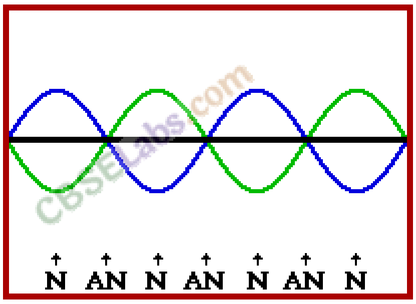
• General Equation of Progressive Waves
"A progressive wave is one which travels in a given direction with constant amplitude, i.e., without attenuation."
As in wave motion, the displacement is a function of space as well as time, hence displacement relation is expressed as a combined function of position and time as:
$y(x, t) = A \sin(kx - \omega t + \Phi)$
We may also choose a cosine function instead of sine function. Here A, K, ω and Φ are four constant for a given wave and are known as amplitude, angular wave number, angular frequency and initial phase angle of given wave.

• Relation between phase and path difference

$$\Delta x = \frac{\lambda}{2\pi} \Delta \phi$$

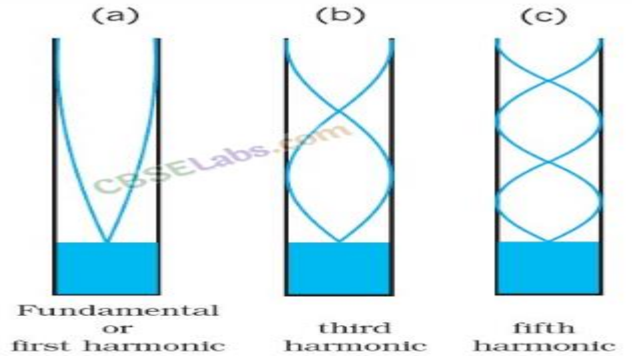
• A wave motion can be reflected from a rigid as well as from a free boundary. A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal but the reflection at an open boundary takes place without any phase change.

• The Principle of Superposition of Wave
When any number of waves meet simultaneously at a point in a medium, the net displacement at a given time is the algebraic sum of the displacements due to each wave at that time.



• Standing waves or Stationary waves

When two sets of progressive wave trains of the same type (i.e., both longitudinal or both transverse) having the same amplitude and time period/frequency/ wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.



• Progressive Waves

1. The disturbance progresses on wards; it being handed over from particle to particle. Each particle executes the same type of vibration as the preceding one, though at a different time.
2. The waves are in the form of crests and troughs, i.e., sine/cosine functions, which move on wards with a definite velocity.
3. Every particle has the same amplitude; which it attains in its own time depending upon the progress of the wave.
4. The phase of every particle varies continuously from 0 to 2π .
5. No particle remains permanently at rest. Twice during each vibration, the particles are momentarily at rest. Different particles attain this position at different times.
6. All the particles have the same maximum velocity which they attain one after another, as the wave advances.
7. There is a regular flow of energy across every plane along the direction of propagation of the wave. The average energy in a wave is half potential and half kinetic.

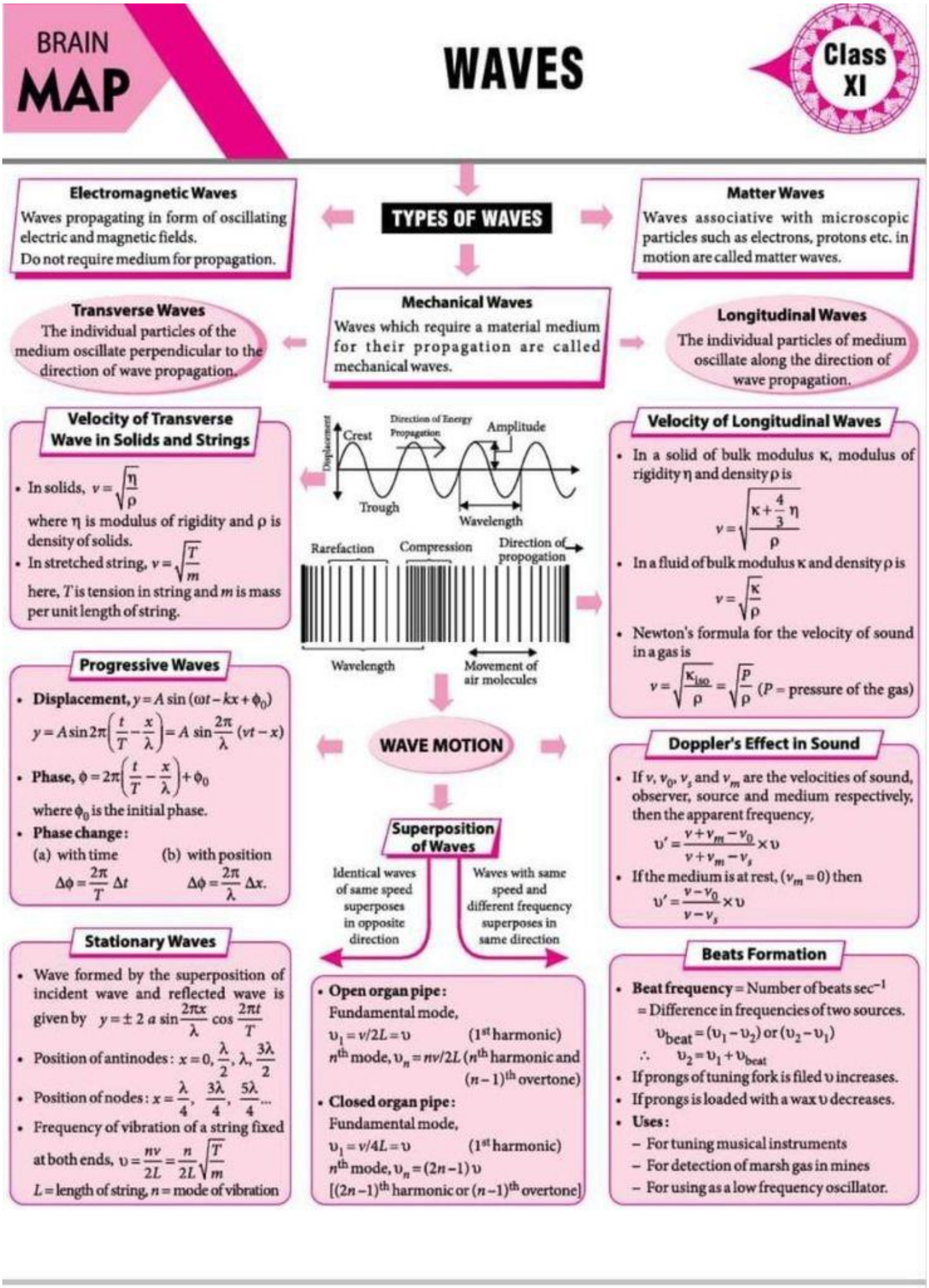
• Stationary Waves

1. The disturbance is stationary, there being no forward or backward movement of the wave. Each particle has its own vibration characteristics.
2. The waves have the appearance of a sine/cosine function, which shrink to a straight line, twice in each vibration. It never advances.
3. Every particle has a fixed allotted amplitude. Some have zero amplitude (nodes) and some have maximum amplitude (antinodes) always. Each partic1eattains this at the same given moment.
4. All the particles in one-half of the waves have a fixed phase and all the particles in the other half of the wave have the same phase in the opposite direction

simultaneously.
5. There are particles -which are permanently at rest (nodes) and all other particles have their own allotted maximum displacement, which they attain simultaneously. These particles are momentarily at rest twice in each vibration, all at the same time.
6. All the particles attain their individual allotted velocities depending upon their positions, simultaneously. Two particles (nodes) in one wave form have zero velocities all the time.
7. There is no flow of energy at all, across any plane. Each particle has its own allotted individual energy. They all attain their values of RE. at one time and all energy becomes KB. at another given time.

• Beats

The phenomenon of regular rise and fall in the intensity of sound, when two waves of nearly equal frequencies travelling along the same line and in the same direction superimpose each other is called beats.
One rise and one fall in the intensity of sound constitutes one beat and the number of beats per second is called beat frequency. It is given as:
$v_b = (v_1 - v_2)$
where v_1 and v_2 are the frequencies of the two interfering waves; v_1 being greater than v_2.



Multiple Choice Questions

1.Which of the following properties of wave does not change with change in medium?

(a) Frequency (b) Wavelength

(c) Velocity (d) Amplitude

2. A tuning fork of frequency 580 Hz is employed to produce transverse waves on a long rope.The distance between nearest crust found to be20 cm. The velocity of wave is

(a) 58 m/s (b)580m/s (c) 20m/s (d) 116m/s

3. A 5.5 meter length of string has a mass of 0.035 kg. If the tension in the string is 77 N, the speed of a wave on string is

(a) 110 m/s (b) 165 m/s (c) 77 m/s (d) 102m/s

4. If the density of oxygen is 16 times that of the hydrogen, what will be the corresponding ratio of their velocities of sound waves?

(a) 1:4 (b)16:1 (c) 4:1 (d) 1:16

5. Transverse waves are generated in uniform steel wires A and B by attaching their free ends to a vibrating source of frequency500 Hz. The diameter of wire A is half that oof B and the tension of wire A is half that of wire B. The velocities of waves in wire A & B are in the ratio.

(a) 1:2 (b)2:1 (c) $1:\sqrt{2}$ (d) $\sqrt{2}:1$

6.The temperature at which speed of sound in air becomes double of its value at 27°C:

(a) 54°C (b) 327°C (c) 927°C (d) -123°C

7.A travelling harmonic wave is represented by the equation $y(x,t)=10^{-3}\sin(50+2x)$ where x &y are in meters and t is in seconds. Which of the following is a correct statement about the wave?

(a) The wave is propogating along the -ve X axis with a speed 25m/s.

(b) The wave is propogating along the +ve X axis with a speed 100m/s.

(c) The wave is propogating along the -ve X axis with a speed 100m/s.

(d) The wave is propogating along the +ve X axis with a speed 25m/s.

8.The phase difference between two waves, represented by

$Y_1=10^{-6}\sin(100t+(x/50)+0.5)$ m

$Y_2=10^{-6}\cos(100t+(x/50)$ m

where x is in metres and t is in second , is approximately

(a) 1.07 rad (b) 2.07 rad (c) 0.5 rad (d) 1.5 rad

9. A wave y=a sin (ωt – kx) , on a string meets with another wave producing a node at x=0. Then the equation of the unknown wave is

(a) y = a sin (ωt + kx) (b) y = -a sin (ωt + kx)

(c) y = a sin (ωt – kx) (d) y= -a sin (ωt – kx)

10. In an experiment with sonometer a tuning fork of frequency 256 Hz resonates with a length of 25 cm and another tuning fork resonates with a length of 16 cm. Tension of the string remaining constant, the frequency of the second tuning fork is

(a) 163.84 Hz (b) 400 Hz (c) 320 Hz (d) 204.8 Hz

11. Five organ pipes are described below. Which one has the highest fundamental frequency?

(a) A 2.3 m pipe with one end open and the other end closed

(b) A 3.3 m pipe with one end open and the other closed

(c) A 1.6 m pipe with both ends open

(d) A 3.0 m pipe with both ends open

12. An organ pipe open at one end is vibrating in first overtone and is in resonance with another pipe open at both ends and vibrating in third harmonic. The ratio of length of two pipes is

(a) 1:2 (b) 4:1 (c) 8:3 (d) 3:8

13.An organ pipe, open at both ends produces 5 beats per second when vibrated

with a source of frequency 200 Hz. The second harmonic of the same pipe produces 10 beats per second with a source of frequency 420 Hz.The frequency of source is
(a) 195 Hz (b) 205 Hz (c) 190 Hz (d) 210 Hz
14.A fork produces 6 beats per sec with another fork of frequency 384. If the prongs of first fork are slightly filed 4 beats per second are produced. The frequency of the first fork after filing is
(a) 390 Hz (b) 378 Hz (c) 380 Hz (d) 388 Hz
15. A stationary wave is set up in a resonance air column of a glass tube partially filed with water by holding a tuning fork near the open end , the open end of the tube is
(a) always a node
(b) always an antinode
(c) sometimes a node and sometimes an antinode
(d) neither a node nor an antinode

Answers Of MCQs

1.a	2.d	3.d	4.b	5.d
6.c	7.a	8.a	9.b	10.b
11.c	12.a	13.b	14.c	15.b

MCQs Answers (Brief Explanation):

2. ANS (d)
$V = v\lambda = 580 \times 0.20 = 116 m/s$
3. ANS (a)
m = 0.035/5.5 Kg/m , T =77N
$V = \sqrt{T/m} = \sqrt{77 X 5.5/0.035} = 110 m/s$
4. ANS (a)
$V_o/V_H = \sqrt{d_H/d_O} = \sqrt{1/16} = 1:4$
5. ANS (d)
$V_A/V_B = \sqrt{T_A/D^2_A} X \sqrt{D^2_B/T_B} = \sqrt{2} : 1$
6. ANS (c)
$V_2/V_1 = \sqrt{T_2/T_1}$ $V_2 = 2V_1$, $T_1 = 300 K$
$2V_1/ V_1 = \sqrt{T_2/300}$
$T_2 = 927°C$
8 ANS (a)
$Y_1 = 10^{-6}\sin(100t+(x/50) +0.5) m$
$Y_2 = 10^{-6} \cos(100t+(x/50) m = 10^{-6}\sin(100t+(x/50) +\pi/2)$
$\Delta\varphi = \pi/2 - 0.05 = 3.14/2 – 0.5 = 1.57 – 0.5 = 1.07$ rad
1. ANS (b)
$Y = Y_1 + Y_2 = a \sin(\omega t – kx) - a \sin(\omega t + kx) = -2a \cos\omega t \sin kx$
At x =0, y=0, a node is formed.
2. ANS (b)
$v_2/ v_1 = L_1/L_2$; $v_2 = 25/16 X 256 = 400$ Hz
3. ANS (c)
Fundamental frequency of a closed organ pipe v =V/4L
Fundamental frequency of a open organ pipe v =V/2L
v will be maximum for an open organ pipe of length 1.6 m
4. ANS (a)
First overtone of a closed pipe =3v = 3V/4L

Third harmonic of an open pipe =3v' =3V/2L'
As two pipes are in resonance L / L' =1:2

5. ANS (b)

Fundamental frequency of open pipe,
f=200+- 5=195 Hz or 205 Hz
Second harmonics of open pipe,
2f = 420+-10= 410 Hz or 430 Hz
f = 205 Hz or 215 Hz

6. ANS (c)

Let frequency of the tuning fork be f
f = 384+- 6= 390 or 378 Hz
On filing the frequency increases & beats produced are 4 so un known frequency is 378 Hz and on filing 380 Hz

Assertions And Reasons

Directions:
In the following questions, a statement of assertion(A) is followed by a statement of reason(R). Mark the correct choice as:
(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false.
16. Assertion : Sound would travel faster on a hot summer day than on a cold winter day.
Reason : Velocity of sound is directly proportional to the square of its absolute temperature.
17.Assertion : The change in air pressure effect the speed of sound.
Reason : The speed of sound in a gas is inversely proportional to square root of pressure.
18. Assertion : In the case of a stationary wave, a person hear a loud sound at the nodes as compared to the antinodes.
Reason : In a stationary wave all the particles of the medium vibrate in phase.
19. Assertion : To hear distinct beats, difference in frequencies of two sources should be less than 10.
Reason : More the number of beats per sec more difficult to hear them.
20. Assertion : Solids can support both longitudinal and transverse waves but only longitudinal waves can propagate in gases.
Reason : For the propagation of transverse waves, medium must also necessarily have the property of rigidity.

ANSWERS

16.c	17.d	18.c	19.b	20.a

Case Study Based Questions

PARAGRAPH 1
If during the propagation of a wave through a medium, the particles of the medium

vibrate simple harmonically about their mean positions, then the wave is said to be plane progressive wave. For mathematical description of a travelling wave, we need a function of both position x and time t. If we wish to describe a sinusoidal travelling wave the corresponding function must also be sinusoidal. If the position of the constituents of the medium is denoted by x, the displacement from the equilibrium position may be denoted by y. A sinusoidal travelling wave is then described by:
$y(x,t) = a\,\mathrm{Sin}(\omega t - kx + \varphi_0)$ The term φ_0= initial phase, a= amplitude, k= angular wave number, ω=angular frequency ,$(\omega t - kx + \varphi_0)$= phase angle. If the wave is travelling along -ve x direction, then $y(x,t) = a\,\mathrm{Sin}(\omega t + kx + \varphi_0)$

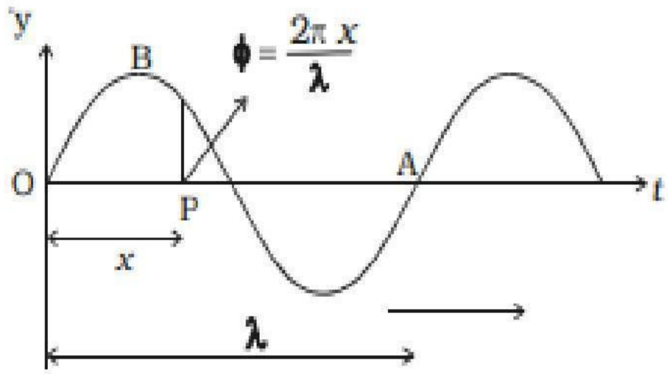


Fig. Plane Progressive wave

QUESTIONS
21.If equation of a sound wave is y=0.0015 sin(62.8x+314t) then its wavelength will be
(a) 0.1 unit (b) 0.2 unit (c) 0.3 unit (d) 2 unit
22.The displacement of a wave travelling in the X- direction is given by
$Y = 10^{-4} \sin(600t - 2x + \pi/3)$
Where x is expressed in metres & t in seconds. The speed of wave motion is
(a) 300 (b)600 (c) 1200 (d) 200
23.The equation of the wave is given by y=10 sin(2πt/30 + α)
If displacement is 5 cm at t=0 then the total phase at t=7.5 s will be
(a) π/3 rad (b) π/2 rad (c) 2π/5 rad (d) 2π/3 rad
24.The equation of progressive wave is y=5sin(100 πt – 0.4 πx),where y & x are in m and t in s.
(1) The amplitude wave is 5m. (2) The wavelength of the wave is 5 m.
(3) The frequency of the wave is 50 Hz. (4) The velocity of the wave is 250 m/s
Which of the above statements are correct
(a) (1),(2) &(3) (b) (2) & (3) (c) (1) & (4) (d) All are correct

PARAGRAPH 2

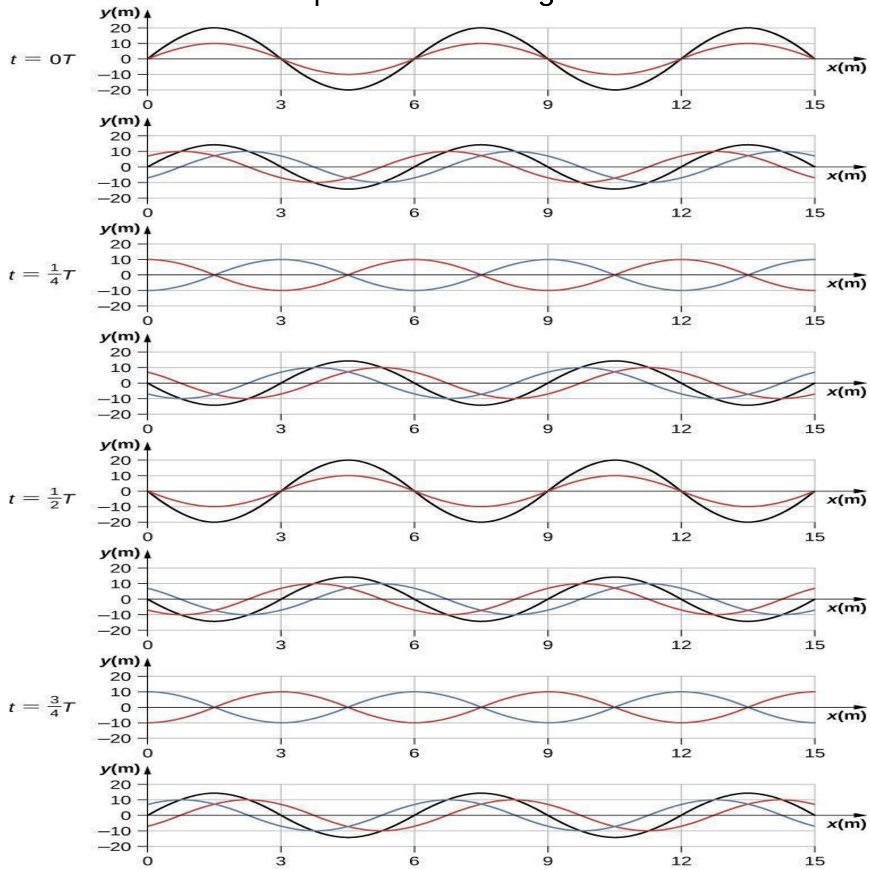
In a string, for example, a wave travelling in one direction will get reflected at one end, which in turn will travel and get reflected from the other end. This will go on until there is a steady wave pattern set up on the string. Such wave patterns are called standing waves or stationary waves. To see this mathematically, consider a wave travelling along the positive direction of x-axis and a reflected wave of the same amplitude and wavelength in the negative direction of x-axis
$Y_1 = A \sin(\omega t - kx)$ (Incident wave)
$Y_2 = A \sin(\omega t + kx)$ (reflected wave)
The stationary wave formed by the superposition of the incident and reflected wave

will be

Y = Y_1+ Y_2 = 2A cos kx sin ωt)

Thus, in this wave pattern, the amplitude varies from point-to-point, but each element of the string oscillates with the same angular frequency ω or time period. There is no phase difference between oscillations of different elements of the wave. The string as a whole vibrates in phase with differing amplitudes at different points. The wave pattern is neither moving to the right nor to the left.

The points at which the amplitude is zero (i.e., where there is no motion at all) are nodes; the points at which the amplitude is the largest are called antinodes.



QUESTIONS

25.The transverse displacement of a string (clamped at its both ends) is given by

y (x,t) = 0.06 sin (2πx/3) cos (120πt)

All the points on the string between two consecutive nodes vibrate with

(a) same frequency (b) same phase

(c) same energy (d) different amplitude

26. Which of the following statements are true for a stationary wave?

(a) Every particle has a fixed amplitude which is different from the amplitude of its nearest particle.

(b) All the particles cross their mean position at the same time.

(c) All the particles are oscillating with same amplitude.

(d) There is no net transfer of energy across any plane.

(e) There are some particles which are always at rest

27.The equation of stationary wave in a stretched string is given by y = 5 sin (πx/3) cos (40πt). The separation between two adjacent nodes is

(a)1.5cm (b) 3 cm (c) 6 cm (d) 4 cm

28. In a resonance tube we get

(a) Stationary longitudinal waves (b) Stationary transverse waves

(c) Progressive longitudinal waves (d) Progressive transverse waves

Answers Case Study Questions

21.a	22.a	23.d	24.d
25.a,b &d	26 a,b,d &e	27.b	28.a

Explanation:

21 ANS (a)

From the equation

$2\pi/\lambda = 62.8$

$\lambda = 0.1$ unit

22 ANS (a)

From the equation $\omega = 600$ rad/s, k= 2 rad/m

$V = \omega/k = 600/2 = 300$ m/s

23 ANS (d)

At t= 0, $5 = 10 \sin\alpha$, $\sin\alpha = ½$ or $\alpha = \pi/6$

Total phase at t= 7.5s, $\varphi = 2\pi \times 7.5/30 + \pi/6 = 2\pi/3$

24 ANS (d)

$y = 5 \sin(100 \pi t – 0.4 \pi x)$,

$y = a \sin(\omega t – kx)$, a= 5m, $\omega = 100\pi$ rad/s, $k = 0.4 \pi\ m^{-1}$

$\lambda = 2\pi/k = 5$m

$v = \omega/2\pi = 50$ Hz

$V = v\lambda = 50 \times 5 = 250$ m/s

27 ANS $K = \pi/3$, $\lambda = 2\pi/k = 6$ cm

The separation between two adjacent nodes is 3 cm.

1 Mark Questions

29. Sound of maximum intensity is heard successively at an interval of 0.2 second on sounding two tuning fork to gather. What is the difference of frequencies of two tuning forks?

ANS. The beat period is 0.2 second so that the beat frequency is $f_b = 1/0.2 = 5$Hz Therefore, the difference of frequencies of the two tuning forks is 5 Hz.

30. Why do tuning forks have two prongs?

ANS. The tuning fork has two prongs because the two prongs of a tuning fork produce resonant vibrations that help to keep the vibrations going for longer.

31. Velocity of sound increases on a cloudy day. Why?

ANS. Velocity of sound increases on a cloudy day, because the air is wet on a cloudy day, it contains a lot of moisture, the density of air is lower, and because velocity is inversely proportional to density, velocity increases.

32. A sitar wire is replaced by another wire of same length and material but of three times the earlier radius. If the tension in the wire remains the same, by what factor will the frequency change?

ANS. Frequency will become 1/3. Since frequency

$\nu \propto \sqrt{(1/m)}\nu \propto \sqrt{(1/m)} = \sqrt{1/\pi r^2 \rho}$

33. A tuning fork A, marked 512 Hz, produces 5 beats per second, where sounded with another unmarked tuning fork B. If B is loaded with wax the number of beats is again 5 per second. What is the frequency of the tuning fork B when not loaded?

ANS. Frequency of tunning fork B is 517 Hz.

34. Why longitudinal waves are called pressure waves?

ANS. Longitudinal waves are called pressure waves because the propagation of longitudinal waves through a medium consists of the variations in the volume and the pressure of the air, these variations in volume and air pressure result in the formation of compressions and rarefactions.

35. At what temperatures (in °C) will the speed of sound in air be 3 times its value at 0°C?

ANS. $v\_2/v\_1 = \sqrt{(T\_2/T\_1)}$, $V_2 = 3 V_1$

$T_2 = 9 \times 273 = 2457$ K

36. When we start filling an empty bucket with water, the pitch of the sound produced goes on changing?

ANS. A bucket may be regarded as a pipe closed at one end. It produces a note of frequency, v= V/4L, where Vis the velocity of sound in air and L is the length of air column, which is equal to depth of water level from open end. As bucket is filled with water, the value of L decreases. Hence pitch of the sound produced goes on changing.

37. A sonometer wire is vibrating in resonance with a tuning fork. Keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be is resonance with the wire?

ANS. Wire of twice the length vibrates in its second harmonic. Thus if the tuning fork resonates at L, it will resonate at 2L

38. Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, why?

ANS. This is due to the fact that gases have only the bulk modulus of elasticity whereas solids have both, the shear modulus as well as the bulk modulus of elasticity.

2 Marks Questions

39**.** A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

ANS. $m = \frac{2\cdot5}{20} = \frac{2\cdot5}{20}$ kg / m , T = 200 N

Speed of transverse jerk is $v = \sqrt{T/m} = 40$ m/s

Time taken by jerk to reach the other end = Distance / speed =0.5 s.

40. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a source of 1237.5 Hz? (sound velocity in air = 330 m s^{-1})

ANS. Fundamental frequency or first harmonics $\nu_1 = \frac{v}{4l}\nu_1 = \frac{v}{4l}$ = 412.5 Hz

Thus answer is third harmonics.

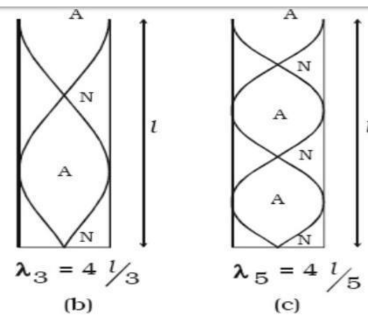
41.Give four differences between transverse & longitudinal waves.

ANS. Page 280 NCERT Physics part II

42. Given below are some examples of wave motion. State in each case if the wave motion is transverse, longitudinal or a combination of both:

(a) Motion of a kink in a longitudinal spring produced by displacing one end of the

spring sideways.
(b) Waves produced in a cylinder containing a liquid by moving its piston back and forth.
(c) Waves produced by a motorboat sailing in water.
(d) Ultrasonic waves in air produced by a vibrating quartz crystal.
ANS. (a) Transverse and longitudinal
(b) Longitudinal
(c) Transverse and longitudinal
(d) Longitudinal
43. Figure shows two vibrating modes of an air column. Find the ratio of frequencies of two modes.



Overtones in a closed pipe

ANS. For mode (b) $L = 3\lambda/4$ or $\lambda = 4L/3$, $v_1 = V/\lambda = 3V/4L$
For mode (c) $L = 5\lambda/4$ or $v_2 = V/\lambda = 5V/4L$
Hence $v_1 : v_2 = 3:5$
44. Estimate the speed of sound in air at standard temperature and pressure. The mass of 1 mole of air is 29.0×10^{-3} kg.
ANS. We know that 1 mole of any gas occupies 22.4 litres at STP. Therefore, density of air at STP is: ρ o = (mass of one mole of air)/ (volume of one mole of air at STP)
$29.0 \text{ X} 10^{-3}$ kg /$22.4\text{X } 10^{-3}$ m^3 = 1.29 kg m–3
45. What are stationary waves? State the necessary condition for formation of stationary waves.
ANS. Definition of stationary wave
Condition for formation of stationary waves
(i) A stationary wave cannot be formed from two independent waves travelling in opposite directions.
(ii) A stationary wave can be produced in a finite medium which has its boundaries
46. Two sitar strings A and B playing the note 'Dha' are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease to 3 Hz. What is the original frequency of B if the frequency of A is 427 Hz ?
ANS. Increase in the tension of a string increases its frequency. If the original frequency of B (v_B) were greater than that of A (v_A), further increase in v_B should have resulted in an increase in the beat frequency. But the beat frequency is found to decrease. This shows that $v_B < v_A$. Since $v_A - v_B = 5$ Hz, and $v_A = 427$ Hz, we get $v_B = 422$ Hz.
47.On the basis of dimensional considerations, write the formula for speed of transverse waves on a stretched string.
ANS. Derive it using dimensions.
48. An incident wave is represented by y(x,t) = 20 sin(2x-4t). Write the expression for

reflected wave:
(i) from a rigid boundary
(ii) from a open boundary
ANS. (i) The wave reflected from a rigid boundary is y= 20 sin (2x + 4t)
(ii) The wave reflected from an open boundary is y = -20 sin (2x+4t)

3 Marks Questions
49. Use the formula $v=\sqrt{\gamma P/\rho}$
to explain why the speed of sound in air
(a) is independent of pressure,
(b) increases with temperature,
(c) increases with humidity.
ANS. (a) We have,

$$v=\sqrt{\gamma P/\rho}$$

Where, Density,

$$\rho=\text{Mass/Volume}=M/V$$

M = Molecular weight of gas
V = Volume of gas
Hence, equation (i) becomes: $v=\sqrt{\gamma PV/M}$ ……(ii)
Now ideal gas equation for n=1 is: PV=RT
For constant T, PV=Constant
Both M and γ are constants, v=Constant
Hence, the speed of sound is independent of the change in the pressure of the gas at a constant temperature.
(b) increases with temperature,
Ans: We have, $v=\sqrt{\gamma P/\rho}$ ……(i)
Now ideal gas equation for n=1 is: PV=RT

$$P=RT/V\text{……(ii)}$$

Substituting (ii) in (i), we get: $v=\sqrt{\gamma RT/V\rho}=\sqrt{\gamma RT/M}$……(iii)
Where Mass M=ρ V is a constant & γ and R are also constants.
We get from equation (iii),

$$V \propto \sqrt{T}$$

.Hence, the sound speed in a gas is directly proportional to the square root of the gaseous medium's temperature, i.e., the sound speed increases with rise in the gaseous medium's temperature and vice versa.
(c) increases with humidity.
Ans: Let v m and v d are the sound speed in moist air and dry air respectively and ρ m and ρ d are the densities of moist air and dry air respectively.
We have ,$v=\sqrt{\gamma P/\rho}$
The speed of sound in moist air is: $v_m=\sqrt{\gamma P/\rho_m}$ ……(i)
The speed of sound in dry air is $v_d=\gamma P/\rho_d$ ……(ii)
On dividing equations (i) and (ii), we get:

$$v_m/v_d=\sqrt{\rho_d/\rho_m}\text{……(iii)}$$

However, the presence of water vapour decreases the density of air, i.e.,

$$\rho_d < \rho_m \Rightarrow v_m > v_d$$

Hence, the speed of sound in moist air is higher than it is in dry air. Thus, in a gaseous medium, the sound speed increases with humidity.

50. Write Newton's formula for speed of sound in a gas. Why &what correction was applied by Laplace in this formula?

ANS. NEWTON'S FORMULA FOR SPEED OF SOUND IN A GAS

Assumption : Sound waves travel through gas under isothermal conditions. Thus temperature of gas remains constant. We have already seen that the sound waves travel in the form of compressions and rarefactions of small volume elements of air. The elastic property that determines the stress under compressional strain is the bulk modulus of the medium .

Thus, the general formula for longitudinal waves in a medium is: $v = \sqrt{B/\rho}$

We can estimate the speed of sound in a gas in the ideal gas approximation. For an ideal gas, the pressure P, volume V and temperature T are related by

$PV = N k_B T$ --------(i)

where N is the number of molecules in volume V, k_B is the Boltzmann constant and T the temperature of the gas (in Kelvin).

Therefore, for an isothermal change it follows from Eq.(1) that

$V\Delta P + P\Delta V = 0$ $P = \frac{-\Delta P}{\Delta v/v}$ $P = \frac{-\Delta P}{\Delta v/v}$

Hence, substituting in Eq. (14.16), we have B = P Therefore, the speed of a longitudinal wave in an ideal gas is given by,

$v = \sqrt{P/\rho}$ -----------(ii)

This relation was first given by Newton and is known as Newton's formula.

According to Newton's formula for the speed of sound in a medium, we get for the speed of sound in air at STP,

$V = \sqrt{(1.013 \times 10^5)/1.293} = 280$ m/s

The result shown is about 15% smaller as compared to the experimental value of 331 m/ s

If we examine the basic assumption made by Newton that the pressure variations in a medium during propagation of sound are isothermal, we find that this is not correct. It was pointed out by Laplace that the pressure variations in the propagation of sound waves are so fast that there is little time for the heat flow to maintain constant temperature. These variations, therefore, are adiabatic and not isothermal.

For adiabatic processes the ideal gas satisfies the relation

PV^{γ} = constant .Differentiating both sides , we get

$P(\gamma V^{\gamma-1})dV + V^{\gamma} dP = 0$

$\gamma P = \frac{-dP}{dv/v} P = \frac{-dP}{dv/v} = B$

where γ is the ratio of two specific heats, C_p/C_v .

The speed of sound is, therefore given by,

$v = \sqrt{\gamma P/\rho}$ ---------(III)

This modification of Newton's formula is referred to as the Laplace correction. For air $\gamma = 7/5$. Now using Eq. (III) to estimate the speed of sound in air at STP, we get a value 331.3 m s–1 , which agrees with the measured speed.

51. A wave travelling along a string is described by,

$y(x, t) = 0.005 \sin(80.0 x - 3.0 t)$,

in which the numerical constants are in SI units (0.005 m, 80.0 rad/m , and 3.0 rad/s). Calculate (a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave.

Also, calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s ?

ANS. On comparing this displacement equation with Eq. y (x, t) = a sin (kx – ωt), we find

(a) the amplitude of the wave is 0.005 m = 5 mm.

(b) the angular wave number k and angular frequency ω are $k = 80.0\ m^{-1}$ and $\omega = 3.0\ s^{-1}$

We, then, relate the wavelength λ to k through Eq

λ = 2π/k

λ = 2π/180 = 7.85 cm

(c) Now, we relate T to ω by the relation T = 2π/ω

T= 2π/3 = 2.09 s and frequency, v = 1/T = 0.48 Hz

The displacement y at x = 30.0 cm and time t = 20 s is given by

y = (0.005 m) sin (80.0 × 0.3 – 3.0 × 20)

= (0.005 m) sin (–36 + 12π)

= (0.005 m) sin (1.699)

= (0.005 m) sin (97°) = 5 mm

52. A pipe, 30.0 cm long, is open at both ends. Which harmonic mode of the pipe resonates a 1.1 kHz source? Will resonance with the same source be observed if one end of the pipe is closed ? Take the speed of sound in air as 330 m /s.

ANS The first harmonic frequency is given by

v1 = v/ λ_1= v/2L (open pipe) where L is the length of the pipe.

The frequency of its nth harmonic is: v_n = n v /2L, for n = 1, 2, 3, ... (open pipe)

First few modes of an open pipe are shown .

For L = 30.0 cm, v = 330 m/ s , v_n = n 330 (m s) /0.6 (m) = $550\ n\ s^{-1}$

Clearly, a source of frequency 1.1 kHz will resonate at v_2 , i.e. the second harmonic.

53. A transverse harmonic wave on a string is described by

$$y(x,t)=3.0\sin(36t+0.018x+\pi/4)$$

Where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave?

If it is travelling, what are the speed and direction of its propagation?

ANS The given equation is the equation of a travelling wave, moving from right to left because it is an equation of the type

$$y(x,t)=A\sin(\omega t+kx+\phi)$$

Here, A=3.0cm, $\omega=36rad^{-1}$, k=0.018cm and ϕ=π/4

Speed of wave propagation is given by v=ω/k = 20ms−1

(b) What are its amplitude and frequency?

Ans: Amplitude of wave, A=3.0cm

Frequency of wave, v=ω/2π=36/2π=5.7Hz

(c) What is the initial phase at the origin?

Ans: Initial phase at origin, ϕ=π/4 rad

(d) What is the least distance between two successive crests in the wave?

Ans: Least distance between two successive crests in the wave,

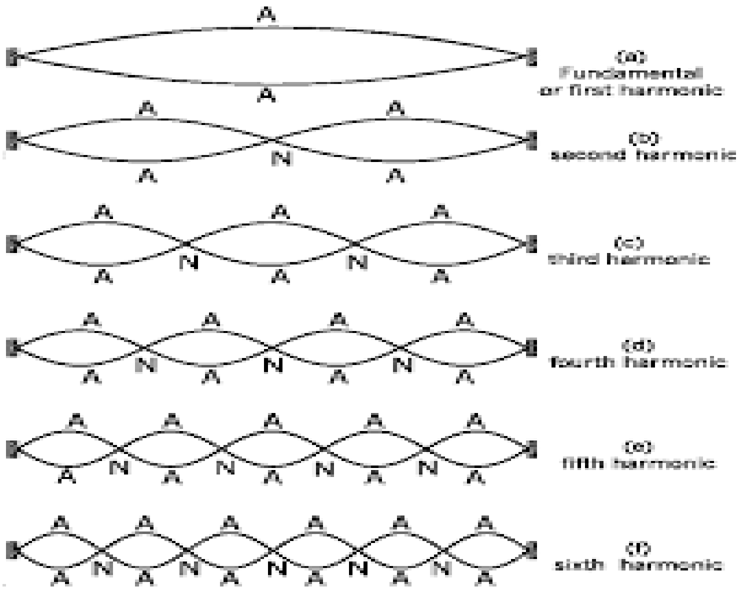
$$\Rightarrow \lambda = 2\pi/k = 2\pi/0.018 = 349cm = 3.49m$$

Long Questions 5 Marks

54. Discuss the formation of harmonics in a stretched string. Show that in the case of stretched string the first four harmonics are 1:2:3:4.

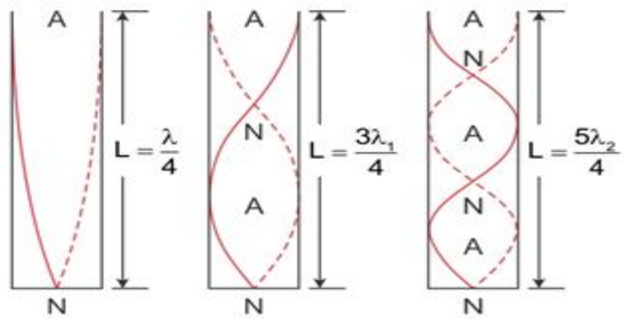
ANS. Consider a wave travelling along the positive direction of x-axis and a reflected

wave of the same amplitude and wavelength in the negative direction of x-axis.
we get: y_1 (x, t) = a sin (kx – ωt)
y_2 (x, t) = a sin (kx + ωt)
The resultant wave on the string is, according to the principle of superposition:
y (x, t) = y_1 (x, t) + y_2 (x, t)
= a [sin (kx – ωt) + sin (kx + ωt)]
Using the familiar trignometric identity Sin (A+B) + Sin (A–B) = 2 sin A cos B
we get, y (x, t) = 2a sin kx cos ωt
The amplitude of this wave is 2a sin kx. Thus, in this wave pattern, the amplitude varies from point-to-point, but each element of the string oscillates with the same angular frequency ω or time period..
The points at which the amplitude is zero (i.e., where there is no motion at all) are nodes;
the points at which the amplitude is the largest are called antinodes.
Let us determine these normal modes for a stretched string fixed at both ends. the positions of nodes (where the amplitude is zero) are given by
sin kx = 0 . which implies kx = nπ; n = 0, 1, 2, 3, ...
Since, k = 2π/λ , we get x =n λ/2 ; n = 0, 1, 2, 3,
the distance between any two successive nodes is λ/ 2.
In the same way, the positions of antinodes (where the amplitude is the largest) are given by the largest value of sin kx , sin k x = 1
which implies kx = (n + ½) π ; n = 0, 1, 2, 3, ...
With k = 2π/λ, we get x = (n + ½) 2 λ ; n = 0, 1, 2, 3, ...
Again the distance between any two consecutive antinodes is λ/2 .
. Taking one end to be at x = 0, the boundary conditions are that x = 0 and x = L are positions of nodes. The x = 0 condition is already satisfied. The x = L node condition requires that the length L is related to λ by L = n λ/2 ; n = 1, 2, 3, ..
Thus, the possible wavelengths of stationary waves are constrained by the relation
λ = 2L/ n ; n = 1, 2, 3, …
with corresponding frequencies v_n = n v/2L , for n = 1, 2, 3, (14.42)
We have thus obtained the natural frequencies - the normal modes of oscillation of the system. The lowest possible natural frequency of a system is called its fundamental mode or the first harmonic. For the stretched string fixed at either end it is given by v = v/ 2L , corresponding to n = 1 of Eq. (14.42). Here v is the speed of wave determined by the properties of the medium
. The n = 2 frequency is called the second harmonic;
n = 3 is the third harmonic and so on. We can label the various harmonics by the symbol v n (n = 1, 2, ...).



55.(a) Discuss the various modes of vibration in a closed end organ pipe. Show that in a closed end organ pipe the harmonics are in the ratio of 1:3:5…..
(b) A meter-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340Hz) when the tube length is 25.5cmor 79.3cm. Estimate the speed of sound in air at the temperature of the experiment.

ANS (a) Normal modes of a closed organ pipe



First mode of vibration, there is one node at closed end and one antinode at the open end.
$L = \lambda_1/4$ or $\lambda_1 = 4L$

Frequency $v_1 = 1/4L\sqrt{\gamma P/\rho} = v$ Fundamental frequency

Second mode of vibration, two nodes & two antinodes
$L = 3\lambda_2/4$ $\lambda_2 = 4L/3$

Frequency $v_2 = 3/4L\sqrt{\gamma P/\rho} = 3v$ =First overtone or third harmonics

Third mode of vibration, three nodes & three antinodes

Frequency $v_3 = 5/4L\sqrt{\gamma P/\rho} = 5v$ = Second overtone or fifth harmonics

(b)Frequency of the turning fork, v=340Hz
Because one end of the supplied pipe is connected to a piston, it will behave as a pipe with one end closed and the other open, as illustrated in the diagram.
Such a system produces odd harmonics. The fundamental note in a closed pipe is given by the relation:
$l1=\lambda/4$
Where,
Length at the pipe, l1=25.5cm=0.255m

$\therefore \lambda = 4l1 = 4 \times 0.255m = 1.02m$

The speed of sound is given by the relation:
v=vλ=340×1.02=346.8m/s

56.What are beats? Prove that the number of beats per second is equal to the difference between the frequencies of the two superimposing waves..

ANS Beats is an interesting phenomenon arising from interference of waves. When two harmonic sound waves of close (but not equal) frequencies are heard at the same time,. We hear audibly distinct waxing and waning of the intensity of the sound, with a frequency equal to the difference in the two close frequencies

Let us consider two harmonic sound waves of nearly equal angular frequency ω1 and ω2 and fix the location to be x = 0 for convenience. Eq. (14.2) with a suitable choice of phase (φ = π/2 for each) and, assuming equal amplitudes, gives

s1 = a cos ω1 t and s2 = a cos ω2 t

The resultant displacement is, by the principle of superposition,

s = s1 + s2 = a (cosω1 t + cos ω2 t)

Using the familiar trignometric identity for cos A + cos B,

we get $2\,acos\left(\frac{\omega_1-\omega_2}{2}\right)t\,cos\left(\frac{\omega_1+\omega_2}{2}\right)t2\,acos\left(\frac{\omega_1-\omega_2}{2}\right)t\,cos\left(\frac{\omega_1+\omega_2}{2}\right)t$

s = [2 a cos ω_b t] cos ω_a t

where $\omega_b = \left(\frac{\omega_1-\omega_2}{2}\right)\left(\frac{\omega_1-\omega_2}{2}\right)$ and $\omega_a = \left(\frac{\omega_1+\omega_2}{2}\right)\left(\frac{\omega_1+\omega_2}{2}\right)$

The amplitude is the largest when the term cos ω_b t takes its limit +1 or –1.

In other words, the intensity of the resultant wave waxes and wanes with a frequency which is $2\omega_b = \omega_1 - \omega_2$. Since ω = 2πv, the beat frequency v_{beat}, is given by

$v_{beat} = v_1 - v_2$

HOT QUESTIONS (CCT)

57. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonics and it resonates with the fundamental frequency of the pipe. If the tension in the pipe is 50 N and speed of sound is 320 m/s, the mass of string is

(a) 5 grams (b) 10 grams (c) 20 grams (d) 40 grams

ANS (b)

For hollow pipe, fundamental frequency is f = V/4l = 320/4X0.8

For string in 2^{nd} harmonic

f = 1/l√T/m =1/0.5 √50X 0.5/m

Equating & solving m= 10 g

58. **A wave travelling along the x-axis is described by the equation y(x, t) = 0.005 cos(αx –β t). If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are**

(a) α = 12.50π, β = π/2.0

(b) α = 25.00π, β = π

(c) α = 0.08/π, β = 2.0/π

(d) α = 0.04/π, β =1.0/π

Answer: (b) α = 25.00π, β = π

The wave travelling along the x-axis is given by

y(x,t) = 0.005 cos(αx – βt)

Therefore, $\alpha = k = 2\pi/\lambda$ As $\lambda = 0.08$ m
$\alpha = 2\pi/0.08 = \pi/0.04 \Rightarrow \alpha = (\pi/4) \times 100 = 25.00\pi$
$\omega = \beta \Rightarrow 2\pi/2 = \pi$
$\therefore \alpha = 25.00\pi$
$\beta = \pi$

59.The length of a sonometer wire is 0.75 m and density 9×10^3 Kg/m^3. It can bear a stress of 8.1×10^8 N /m^2 without exceeding the elastic limit. What is the fundamental frequency that can be produced in the wire? (IIT 1990)
ANS. Let a be the area of cross section of wire then fundamental frequency ,
$v = 1/2L\sqrt{T/m} = 1/2L\ \sqrt{\text{Stress} \times a/a \times 1 \times \rho} = 200$ Hz.

SELF ASSESSMENT

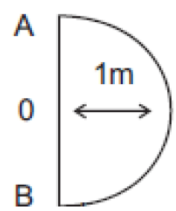
1.An open organ pipe produces a note of frequency 5/2 Hz at 150C, calculate the length of pipe. Velocity of sound at 0^0C is 335 m/s.
2. An incident wave is represented by Y(x, t)=20sin(2x-4t).Write the expression for reflected wave (i) From a rigid boundary (ii) From an open boundary.
3. Explain why
 (i) in a sound wave a displacement node is a pressure antinode and vice- versa
 (ii) The shape of pulse gets- distorted during propagation in a dispersive medium.
4. The equation of a plane progressive wave is **y=10 sin2π(t-0.005 x),** where y & x are in cm & t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.
5. (i) A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?
(ii) A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly exited by a 430 Hz source? Will this same source be in resonance with the pipe if both ends are open? (Speed of sound = 340 m/s).
6. a) Explain the formation of beats.
 b) When a tuning fork of unknown frequency is sounded with another tuning fork whose frequency is 384 Hz, 6 beats per second are produced. When wax is attached to the first fork, then on sounding it with the sound, 4 beats per second are produced. Determine the unknown frequency

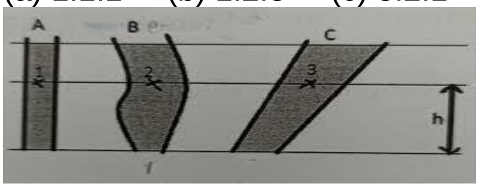
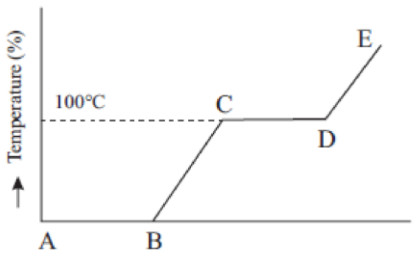
SAMPLE QUESTION PAPER

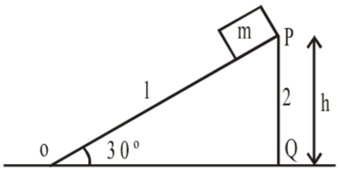
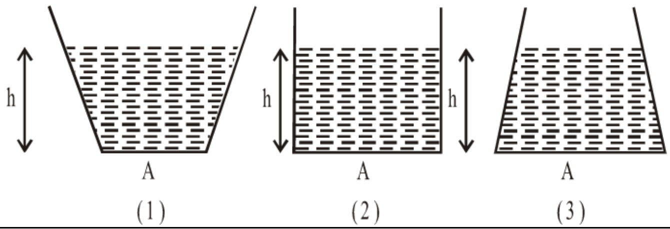
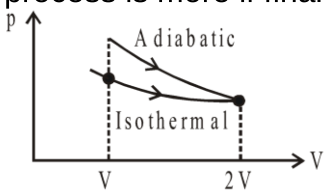
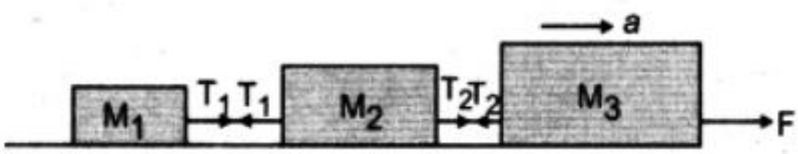
Class-XI SUBJECT-PHYSICS Max Marks 70 Time 3 Hrs.

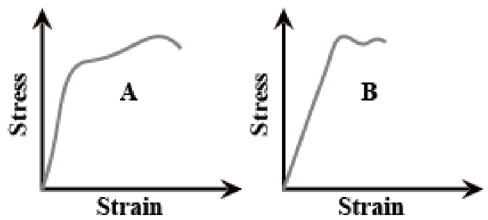
General Instructions:

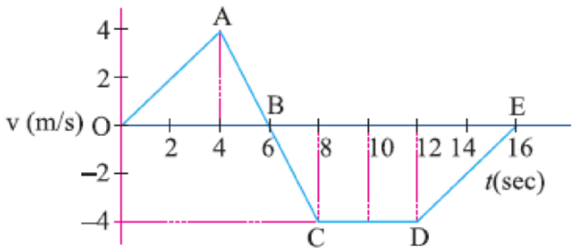
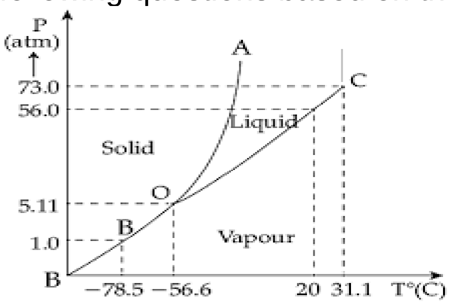
(1) All questions are compulsory. There are 33 questions in all.
(2) This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
(3) Section A contains ten MCQs and four assertion reasoning of 1 mark each.
Section B contains five short answer questions of 2 marks each.
Section-C contains six short answer questions of 3 marks each.
Section D has two case-based questions of 4 marks each.
Section E contains three long answer questions of 5 marks each.
(4) There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

	Section – A All questions are compulsory. In case of internal choices, attempt any one of them.	
1	Convert 5 N to Dynes. (a) 500000 Dynes (b) 500 Dynes (c) 5 Dynes (d) 0.5 Dynes	1
2	Body, thrown upwards with some velocity, reaches the maximum height of 20 m. Another body with double the mass thrown up, with double the initial velocity, will reach a maximum height of (a) 40m (b) 50m (c) 80m (d) 160m	1
3	An object, moving with a speed of 6.25 m/s, is decelerated at a rate given By $\frac{dv}{dt} = -2 \cdot 5\sqrt{v}\frac{dv}{dt} = -2 \cdot 5\sqrt{v}$ Where v is the instantaneous speed. The time taken by the object, to come to rest would be (a)1 s (b) 2 s(c) 4 s (d) 8 (s) (OR) In 1 s, a particle goes from point A to point B moving in a semicircle as shown in figure. The magnitude of the average velocity is (a) Zero (b) 1 m/s (c) 2 m/s (d) 3.14 m/s  	1
4	If the linear momentum is increased by 50%, then kinetic energy will increase by (a) 50% (b) 100%(c) 125% (d) 25%	1
5	Two identical particles move towards each other with velocities 2V and V respectively. The velocity of centre of mass is (a) V (b) V/3 (c) V/2 (d) Zero (OR) Find the torque of a force 7*i* – 3*j*– 5*k* about the origin which acts on a particle whose position vector is *i*+*j*-*k* a)*i*+4*j*– 2*k*. b) 8*i*+2*j*– 10*k*. c) -8*i* - 2*j*– 10*k*. d) 8*i* + *j*–6*k*.	1

6	A mass M splits in to two parts m and (M-m), which are separated by a certain distance. The ratio m/M which maximizes the gravitational force between the parts is (a) 1: 4 (b) 1: 3 (c) 1: 2 (d) 1: 1	1
7	An ideal fluid flow through a pipe of circular cross section made of two sections with diameters 2.5 cm & 3.75 cm. The ratio of the velocities in the two pipes is (a) 9: 4 (b) 3: 2(c) $\sqrt{2}:\sqrt{3}\sqrt{2}:\sqrt{3}(d)\sqrt{3}:\sqrt{2}\sqrt{3}:\sqrt{2}$	1
8	3 containers A, B, and C are filled with water as shown in the figure. What is the relation between the pressures at 1, 2, 3? Assume that all jars are stationery. (a) 1:1:1 (b) 1:2:3 (c) 3:2:1 (d)1:2:1 	1
9	Refer to the plot of temperature versus time showing the changes in the state of ice on heating (not of scale)  Which of the following is correct? (a) The region AB represents ice & water in thermal equilibrium (b) At B water stats boiling (c) At C all the water gets converted into steam (d) C-D represents water & steam in equilibrium at boiling point. <u>(OR)</u> A gas behaves as an ideal gas at (a) low pressure and high temperature(b) low pressure and low temperature (c) high pressure and low temperature(d) high pressure and high temperature	1
10	Two equations of two SHM y = a Sin (ωt–α) and y = a Cos (ωt–α). The phase difference between the two is (a) 0° (b) α° (c) 90° (d) 180°	1
11	At the upper most of a projectile, its velocity and acceleration at an angle of (a) 0° (b) 45° (c) 90° (d) 180°	1
12	The door of a running refrigerator inside a room is left open. The correct statement out of the following ones is (a) The room will be cooled slightly (b) The room will be warmed up gradually (c) The room will be cooled to the temperature inside the refrigerator (d) The temperature of the room will remain unaffected	1
	For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.	

	a) Both A and R are true and R is the correct explanation of A b) Both A and R are true but R is NOT the correct explanation of A c) A is true but R is false d) A is false and R is also false	
13	STATEMENT – 1: Work done by gravitation force in reaching the block at and Q is same. STATEMENT – 2: Gravitational force is conservative in nature.  	1
14	STATEMENT – 1: In the three cases shown in the figure force exerted by liquid on three vessels is same. STATEMENT – 2: Pressure at the bottom in each case is same.  	1
15	STATEMENT – 1: A gas is expanded from a volume V to 2V, first through adiabatic process then through isothermal process. Work done in isothermal process is more if final stage (i.e. pressure and volume) in both case is same.  STATEMENT – 2: Work done by gas is equal to area under P–V curve.	1
16	STATEMENT – 1: When two vibrating tuning forks have f_1 = 300 Hz and f_2 = 350 Hz and held close to each other; beats cannot be heard. STATEMENT – 2:The principle of superposition is valid only when $f_1 – f_2 < 10$ Hz.	1
	SECTION B 2 MARKS	
17	The velocity of water in a river is less on the bank and large in the middle. Why?	2
18	Three blocks of masses m_1 = 10 kg, m_2 = 20 kg are connected by strings on smooth horizontal surface and pulled by a force of 60 N. Find the acceleration of the system and frictions in the string.  (OR) **A constant force acting on a body of mass 3.0 kg changes its speed from 2.0 ms^{-1} to 3.5 ms^{-1} in 25 s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force?**	2

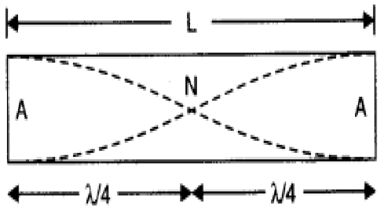
19	Prove that bodies of identical masses exchange their velocities after head-on elastic collision. (OR) A railway carriage of mass 9000 kg moving with a speed of 36 km h^{-1} collides with a stationary carriage of same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision? What type of collision is this?	2
20	The stress-strain graphs for materials A and B are shown in figure.  **The graphs are drawn to the same scale.** **(a)**Which of the materials has the greater Young's modulus ? (b) Which of the two is the stronger material ?	2
21	What is an adiabatic process? What are the essential conditions for adiabatic process. Write an expression for work done during an adiabatic process.	2
	SECTION C 3 MARKS	
22	Why circular roads are banked? Derive an expression for angle of banking for safe circular turn. Consider that coefficient of friction between the tyre and road is μ. (OR) A spring balance is attached to the ceiling of a lift. When the lift is at rest spring balance reads 49 N of a body hang on it. If the lift moves: (i) Downward with an acceleration of 5 ms^{-2}(ii) upward, with an acceleration of 5 ms^{-2} (iii) with a constant velocity.What will be the reading of the balance in each case?	3
23	A body oscillates with SHM according to he equation $x(t)=5\cos(2\pi t+\pi/4)$ where t is in sec and x is in meters .Calculate (i) Displacement at t=0 (ii) Time period (iii) Initial velocity.	3
24	State and prove Bernoullis theorem. Give its limitation. Name any two application of the principle. (OR) Derive an expression for the rise of liquid in a capillary tube and show that the height of the liquid column is inversely proportional to the radius of the tube..	3
25	Define escape velocity obtain an expression for escape velocity of a body from the surface of earth? Does the escape velocity depend on (i) location from where it is projected (ii) the height of the location from where the body is launched? Justify it.	3

26	The velocity time graph of a particle is given by (i) Calculate distance and displacement of particle from given *v-t* graph. (ii) Specify the time for which particle undergone acceleration, retardation and moves with constant velocity.  (iii) Calculate acceleration, retardation from given *v-t* graph. (iv) Draw acceleration-time graph of given *v-t* graph.	3
27	Describe stress- strain relationship for a loaded steel wire and hence explain its various portions briefly.	3
28	Expain the following: (a) The angular velocity of revolution of the earth around the sun increases,when comes closer to sun.Why? (b) When a diver leaves the diving board,he brings his hands and feet closer together in order to make a somersault.Why? (c)It is difficult to open the door by pushing it at the hinge.Why?	3
	Section – D Questions 15 and 16 are Case Study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark	
29	A graph between the temperature T and the Pressure P of the substance is called a phase diagram or P – T diagram. Figure shows P-T graph of CO^2 . Such a phase diagram divides the P – T plane into a solid-region, the vapour-region and the liquid-region. The regions are separated by the curves such as sublimation curve , fusion curve and vaporisation curve . Answer the following questions based on the diagram.  P-T Phase diagram for CO_2	
29.1	At what temperature and pressure can the solid, liquid and vapour phases of CO2 co-exist in equilibrium?	1M
29.2	What is the effect of the decrease of pressure on the fusion and boiling point of CO2 ?	1M
29.3	What are the critical temperature and pressure for CO2? What is their significance?	1M
29.4	Is CO_2 solid, liquid or gas at (a) −70 ^{O}C under 1 atm, (b) −60 ^{O}C under 10 atm, (c) 15 ^{O}C under 56 atm?	1M

30	When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion. The word “uniform” refers to the speed, which is uniform (constant) throughout the motion. Suppose an object is moving with uniform speed v in a circle of radius R Since the velocity of the object is changing continuously in direction, the object undergoes acceleration. Let us find the magnitude and the direction of this acceleration. Thus, the acceleration of an object moving with speed v in a circle of radius R has a magnitude V 2 /R and is always directed towards the centre. This is why this acceleration is called centripetal acceleration (a term proposed by Newton).	
30.1	SI unit of angular velocity is a) rev/sec b) m/s c) m/s^2 d) None of these	1M
30.2	A centripetal acceleration is not a constant vector. True or false? a) True b) False	1M
30.3	Name the physical quantity which remains same in an uniform circular motion	1M
30.4	If both speed of a body and radius of the circular path are doubled, what will be the change in centripetal force ?	1M
	Section – E All questions are compulsory. In case of internal choices, attempt any one	
31	A projectile is fired horizontally with a velocity *u*. Show that its trajectory is a parabola. Also obtain expression for (i) time of flight (ii) horizontal range (iii) velocity at any instant. (OR) Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed *v* along a circular path of radius *r*. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone?	5
32	State first law of thermodynamics. On its basis establish the relation between two molar specific heat for a gas. (OR) What are the basic assumptions of kinetic theory of gases? On their basis derive an expression for the pressure exerted by an ideal gas. Deduce Boyle’s law and Charles law of gases from it.	5
33	Describe the various modes of vibration in case of closed end organ pipe. **A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 k Hz. What is the speed of sound in steel?** (OR) Derive expressions for the kinetic and potential energies of a simple harmonic oscillator. Hence show that the total energy is conserved in S.H.M. All trigonometric functions are periodic, but only sine or cosine functions are used to define SHM. Why?	5

MARKING SCHEME SAMPLE QUESTION PAPER Class-XI SUBJECT-PHYSICS Max. Marks 70 Time 3 Hrs.		
1	A	1
2	C	1
3	B OR C	1
4	C	1
5	C OR C	1
6	C	1
7	A	1
8	A	1
9	D OR A	1
10	C	1
11	C	1
12	B	1
13	A	1
14	D	1
15	A	1
16	C	1
17	In the river water flow in the form of streams. The forces of adhesion is less on the stream in the middle of river than those near the bank. Due to it, the velocity of streams near the bank is least and is maximum in the middle of river.	2M
18	All the blocks more with common acceleration *a* under the force F = 60 N. $F = (m_1 + m_2 + m_3)a$ a = = 1 m/s^2 To determine, T1 →Free body diagram of mass m_1. $T1 = m_1a = 10 \times 1 = 10$ N To determine, T2 – Free body diagram of m_3. $F - T2 = m_3a$ Solving, we get T2 = 30 N OR F=m(v-u)/t F=0.18N	2M 2M
19	For correct derivation OR m1 = 9000 kg, u1 = 36 km/h = 10 m/s m2 = 9000 kg, u2 = 0, v = v1 = v2 = ? By conservation of momentum : m1u1 + m2u2 = (m1 + m2) v v = 5 m/s Total K.E. before collision == 45000 J Total K.E. after collision =	2M

	As total K.E. after collision < Total K.E. before collision ∴Collision is inelastic.	
20	(a)Material A has more Youngs modulus. (b)A is stronger than B.	1M 1M
21	Definition Essential conditions W = nR(T2-T1) / (γ-1)	1/2M 1M 1/2M
22	Correct derivation OR (i) R = m (g – a) weight = 49 N so m = = 5 kg R = 5 (9·8 – 5),R = 24N (ii) R = *m* (*g* + *a*) R = 5 (9·8 + 5),R = 74 N (iii) as *a* = 0 so R = *mg* = 49N	3M
23	(i) $5/\sqrt{2}$ m (ii) 1 s (iii) $10\pi/\sqrt{2}$ m/s	3M
24	Diagram with correct explanation	3M
25	Diagram with correct explanation	3M
26	(i) distance = area of ΔOAB + area of trapezium BCDE = 12 + 28 = 40 m (ii) displacement = area of ΔOAB – area of trapezium BCDE = 12 – 28 = – 16 m (iii) time acc. (0 < t < 4) and (12 < t < 16) retardation (4 < t < 8) constant velocity (8 < t < 12) graph	3M
27	Diagram with correct explanation	3M
28	(a) When the earth comes closer to the sun, its moment of inertia about the axis through the sun decreases. To conserve angular momentum. the angular velocity of the earth increases. (b) By doing this, he decreases his moment of inertia. And, consequently, his angular velocity will increase, his linear velocity will also increase. (c) So we need maximum torque for easy opening or shutting a door by applying minimum force. When the force is applied at the hinges, the line of action of the force passing through the axis of rotation is zero. Thus the force is maximum and torque is minimum near the hinge.	3M
29	(i) C is the triple point of the CO^2 phase diagram. This means that at the pressure and temperature corresponding to this point (i.e., at −56.6 OC and 5.11 atm), the solid phase, liquid phase, and vaporous phases of CO^2 coexist in equilibrium. (ii) The fusion and boiling points of CO^2 decrease with a decrease in pressure. (iii) The critical temperature and critical pressure of CO^2 are 31.1o C and 73 atm respectively. Even if it is compressed to a pressure greater than 73 atm, CO^2 will not liquefy above the critical temperature. (iv) It can be concluded from the P-T phase diagram of CO^2 that: (a) CO^2 is gaseous at −70 ^{O}C, under 1 atm pressure (b) CO^2 is solid at −60	1+1+1+1

	^{O}C, under 10 atm pressure (c) CO^2 is liquid at 15 ^{O}C , under 56 atm pressure	
30	i) a ii)a iii) K.E and speed iv) doubled	1+1+1+1
31	Diagram with correct explanation	5M
32	Diagram with correct explanation	5M
33	For correct derivation. Here, L = 100 cm = 1m, v = 2.53 k Hz = 2.53 x 10^3 Hz When the rod is clamped at the middle, then in the fundamental mode of vibration of the rod, a node is formed at the middle and ant mode is formed at each end. Therefore, as is clear from Fig.  $L = \frac{\lambda}{4}+\frac{\lambda}{4}=\frac{\lambda}{2}$ $\lambda = 2L = 2$ m As $v = \nu\lambda$ ∴ $v = 2.53 \times 10^3 \times 2$ $= \mathbf{5.06 \times 10^3\ ms^{-1}}$ Diagram with correct explanation OR For correct derivation Total energy=1/2 kA^2 All trigonometric functions are periodic. The sine and cosine functions can take value between -1 to +1 only. So they can be used to represent a bounded motion like SHM. But the functions such as tangent, cotangent, secant and cosecant can take value between 0 and ∞ (both negative and positive). So these functions cannot be used to represent bounded motion like SHM.	3M 2M 2M 1M 2M

तत् त्वं पूषन् अपावृणु
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