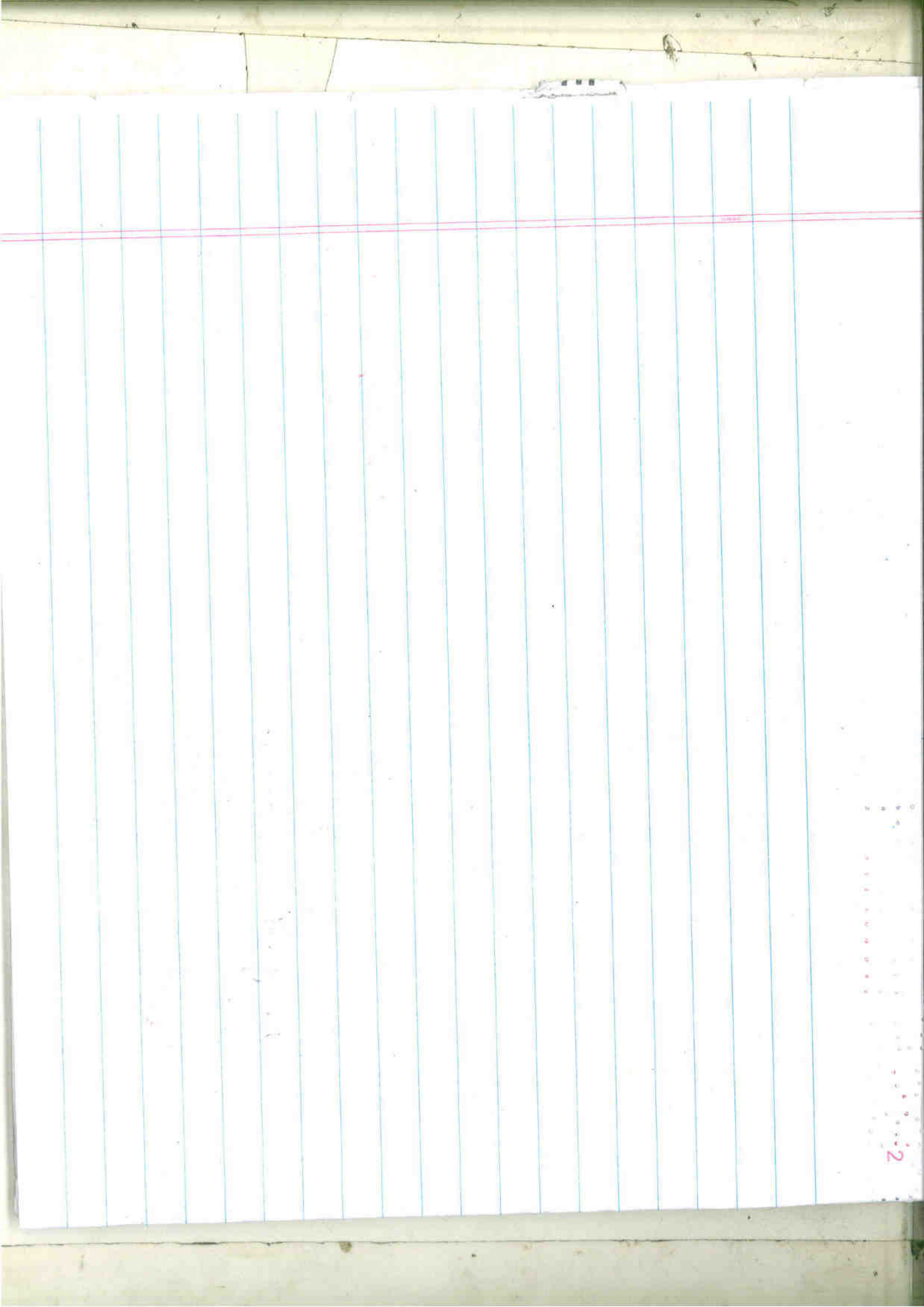




Section D

b) Reflexivity:

Let (a, b) be an arbitrary element belonging to $N \times N$
 So, $(a, b) \in N \times N$.

$$\text{As, } a - a = b - b = 0,$$

$$(a, b) R (a, b) \text{ for all } (a, b) \in N \times N$$

$\therefore R$ is reflexive

• Symmetric:

Let $(a, b), (c, d) \in N \times N$ such that $(a, b) R (c, d)$.
 So,

$$a - c = b - d \quad [\text{Given Relation}]$$

Multiplying by (-1) on both sides,

$$c - a = d - b$$

$$\text{Hence, } (c, d) R (a, b)$$

For $(a, b) R (c, d)$ there is $(c, d) R (a, b)$ for $(a, b), (c, d) \in N \times N$

$\therefore R$ is symmetric

• Transitivity:

Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$

From the two relations,

$$a - c = b - d \quad \text{--- ①}$$

$$c - e = d - f \quad \text{--- ②}$$

① + ②

$$a - c + c - e = b - d + d - f$$

$$\text{So, } a - e = b - f$$

Hence, $(a, b) R (e, f)$

For $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$ for $(a, b), (c, d), (e, f) \in N \times N$

Hence, R is transitive

As R is reflexive, Symmetric and transitive, hence R is an equivalence relation.

As the required line bisects line segment AB, midpoint of line segment AB lies on required line.

Let midpoint of line segment AB be P.

$$P = \left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2} \right) = (3, 4, 6)$$

As required line is perpendicular to two lines,

$$L_1: \frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \text{and} \quad L_2: \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

the direction vector of required line is found by taking cross product of direction vectors of L_1 and L_2 . Let direction vector of required line be $\vec{a}$.

$$\vec{a} = (3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$= \hat{i} (80 - 56) - \hat{j} (-15 - 21) + \hat{k} (24 + 48)$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

- DR's of required line = $\langle 24, 36, 72 \rangle = \langle 2, 3, 6 \rangle$
- Simplified Direction vector = $2\hat{i} + 3\hat{j} + 6\hat{k}$

So, Required line is

$$\vec{r} = \vec{OP} + \lambda (2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = 3\hat{i} + 4\hat{j} + 6\hat{k} + \lambda (2\hat{i} + 3\hat{j} + 6\hat{k}) \quad [\text{Vector equation}]$$

Cartesian equation:

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-6}{6}$$

34)(a) Let $\frac{1}{x} = a$, $\frac{1}{y} = b$, $\frac{1}{z} = c$.

The equations are,

$$2a + 3b + 10c = 4$$

$$4a - 6b + 5c = 1$$

$$6a + 9b - 20c = 2$$

The matrix equation is,

$$\begin{matrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{matrix} \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{matrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{matrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

${}_{3 \times 1} A \quad {}_{3 \times 1} X \quad {}_{3 \times 1} B$

Its of the form $AX = B$

$$\begin{aligned} |A| &= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36) \\ &= 2(75) - 3(-110) + 10(72) \\ &= 150 + 330 + 720 \\ &= 1200 \end{aligned}$$

As $|A| \neq 0$, above equation have an unique solution

$$\text{So, } AX = B$$

Pre-multiplying by A^{-1} ,

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$A = \begin{bmatrix} 2^{(+)} & 3^{(-)} & 10^{(+)} \\ 4^{(-)} & -6^{(+)} & 5^{(-)} \\ 6^{(+)} & 9^{(-)} & -20^{(+)} \end{bmatrix}$$

$$\cdot \operatorname{adj} A = \begin{bmatrix} 75 & 110 & -72 \\ -(-60-90) & -40-60 & -(18-18) \\ 15+60 & -(10-40) & -12-12 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\cdot A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$X = A^{-1}B$$

$$X = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$X = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

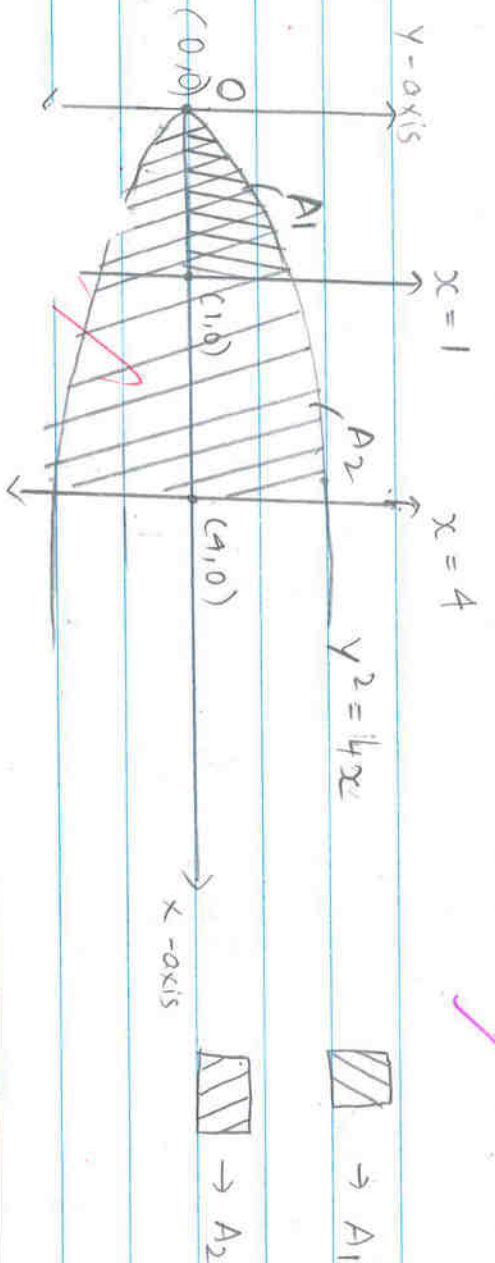
$$X = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

So, $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

Hence, $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = \frac{1}{5}$

Ans: $x = 2$, $y = 3$, $z = 5$

35)



$$y^2 = 4x \Rightarrow y = 2\sqrt{x}$$

$$A_1 = \int_0^1 y dx = 2 \int_0^1 \sqrt{x} dx = 2 \left(\frac{x^{3/2}}{3/2} \right)$$

$$= \frac{4}{3} \left(x^{3/2} \right) \Big|_0^1 = \frac{4}{3} (1 - 0) = \frac{4}{3} \text{ sq. units}$$

$$A_2 = 2 \int_1^4 y dx = 2 \int_1^4 2\sqrt{x} dx = 4 \int_1^4 \sqrt{x} dx = 4 \left(\frac{x^{3/2}}{3/2} \right)$$

$$= \frac{8}{3} \left(x^{3/2} \right) \Big|_1^4 = \frac{8}{3} (4^{3/2} - 1) = \frac{64}{3} \text{ sq. units}$$

Ans) $A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1 : 16$

Section E

$$E = \frac{V^2}{500} - \frac{V}{4} + \frac{1}{14}$$

1) When $V = 40 \text{ km/hr}$,

$$E = \frac{(40)^2}{500} - \frac{40}{4} + 14$$

$$= \frac{1600}{500} - 10 + 14$$

$$= 3.2 + 4$$

$$= 7.2 \text{ (2/100 km)}$$

Ans: $F = 7.2 \text{ (2/100 km)}$

(ii)

$$F = \frac{V^2}{500} - \frac{V}{4} + 14$$

$$\frac{dF}{dV} = \frac{2V}{500} - \frac{1}{4} + 0$$

$$\text{Ans. } \frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}$$

iii) (a) $\frac{d^2F}{dV^2} = \frac{1}{250}$

Now, put $\frac{dF}{dV} = 0$

$$\frac{V}{250} - \frac{1}{4} = 0 \Rightarrow \frac{V}{250} = \frac{1}{4}$$

$$\text{So, } V = 62.5 \text{ (km/h)}$$

As $\frac{d^2F}{dV^2}$ is positive for $V = 62.5 \text{ km/hr}$,

F attains minimum at that point.

Ans: $V = 62.5 \text{ km/hr}$

i)

$$x + 2y \geq 10$$

$$x + y \geq 6$$

$$3x + y \geq 8$$

$$x \geq 0$$

$$y \geq 0$$

The above are the constraints determining the feasible region

ii)

Corner Points

A (10, 0)

B (2, 4)

C (1, 5)

D (0, 8)

$$Z = 16x + 20y$$

$$16 \times 10 + 20 \times 0 = 160$$

$$16 \times 2 + 20 \times 4 = 112$$

$$16 \times 1 + 20 \times 5 = 116$$

$$16 \times 0 + 20 \times 8 = 160$$

The cost is minimum when $x = 2$ and $y = 4$.
Minimum cost = ₹ 112

38) (i) P (airplane will not crash) = $1 - \frac{0.000001}{100}$

$$= 1 - \frac{10^{-5}}{10^2}$$

$$= 1 - 10^{-7}$$

$$= 1 - 0.0000001$$

$$= 0.9999999$$

Ans: 0.9999999

$$i) P(A/E_1) + P(A/E_2) = 0.95 + 1 = 1.95$$

$$ii) \alpha P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$$

$$= (10^{-7}) (0.95) + (1 - 10^{-7}) (1)$$

$$= 0.95 \times 10^{-7} + 1 - 10^{-7}$$

$$= 1 - 0.05 \times 10^{-7}$$

$$= 1 - 5 \times 10^{-9}$$

$$= 1 - 0.000000005$$

$$= 0.999999995$$

$$\text{Ans: } P(A) = 0.999999995$$

Section C

26)(a) $x = e^{\cos 3t}$

Taking log on both sides,

$$\log_e x = \cos 3t \quad \text{--- ①}$$

Differentiating w.r.t t ,

$$\frac{1}{x} \frac{dx}{dt} = -3 \sin 3t$$

So, $\frac{dx}{dt} = x(-3 \sin 3t) \quad \text{--- ②}$

$$y = e^{\sin 3t}$$

Taking log on both sides,

$$\log_e y = \sin 3t \quad \text{--- ③}$$

Differentiating w.r.t t ,

$$\frac{1}{y} \frac{dy}{dt} = 3 \cos 3t$$

$$\frac{dy}{dx} = y(3 \cos 3t) \quad \text{--- (4)}$$

$$\textcircled{4} \div \textcircled{2}$$

$$\frac{dy}{dx} = \frac{y(3 \cos 3t)}{x(-3 \sin 3t)} = -\frac{y}{x} \frac{(\cos 3t)}{(\sin 3t)}$$

As $\log x = \cos 3t$ and $\log y = \sin 3t$ from ① and ②,

$$\frac{dy}{dx} = -\frac{y}{x} \frac{(\log x)}{(\log y)} = -\frac{y \log x}{x \log y}$$

$\therefore$ Hence proved that

$$\frac{dy}{dx} = -\frac{y \log x}{x \log y}$$

27)

$$I = \int_{-2}^2 \sqrt{\frac{2-x}{2+x}} dx$$

Take $x = 2 \cos \theta$
 $dx = -2 \sin \theta d\theta$

The limits change

x	2	-2
θ	0	π

$$\left[\begin{array}{l} 2 = 2 \cos \theta \rightarrow \theta = 0 \\ -2 = 2 \cos \theta \rightarrow \theta = \pi \end{array} \right]$$

So,

$$I = \int_{\pi}^0 \sqrt{\frac{2-2\cos\theta}{2+2\cos\theta}} (-2\sin\theta d\theta)$$

$$I = \int_0^{\pi} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} (-2\sin\theta d\theta)$$

$$I = \int_0^{\pi} \frac{2 \sin^2 \theta/2}{2 \cos^2 \theta/2} (2 \sin \theta d\theta)$$

$$I = \int_0^{\pi} \tan^2 \theta/2 (2 \sin \theta d\theta)$$

Using property $\int_a^b f(x) dx = - \int_b^a f(x) dx$

$$I = \int_0^{\pi} (\tan^2 \theta/2) (2 \sin \theta d\theta) \quad \left[\sin \theta = 2 \sin \theta/2 \cos \theta/2 \right]$$

$$I = \int_0^{\pi} \frac{\sin^2 \theta/2}{\cos^2 \theta/2} (4 \sin \theta/2 \cos \theta/2 d\theta)$$

$$I = 4 \int_0^{\pi} \sin^2 \theta/2 d\theta$$

$$I = 4 \int_0^{\pi} \left(\frac{1 - \cos \theta}{2} \right) d\theta$$

$$I = 2 \int_0^{\pi} (1 - \cos \theta) d\theta$$

$$= 2 (\theta - \sin \theta) \Big|_0^{\pi}$$

$$= 2 ((\pi - \sin \pi) - (0 - \sin 0))$$

$$= 2\pi$$

$$28) (a) \quad 2xy + y^2 - 2x^2 \frac{dy}{dx} = 0.$$

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

$$\frac{dy}{y} = \frac{y}{x} + \frac{1}{2} \frac{y^2}{x^2} \quad - (1)$$

Take $y = vx$.

Differentiating w.r.t x , $\frac{dy}{dx} = v + x \frac{dv}{dx} \quad - (2)$

Sub $\frac{y}{x} = v$ in ① and equate ① and ②

$$v + x \frac{dv}{dx} = v + \frac{1}{2} v^2$$

$$\frac{x dv}{dx} = \frac{v^2}{2}$$

So, on re-arranging,

$$\int \frac{dx}{x} = 2 \int \frac{dv}{v^2}$$

Integrating on both sides,

$$\ln|x| + c = 2 \int v^{-2} dv$$

$$\ln|x| + c = 2 \left[\frac{v^{-2+1}}{-2+1} \right] = \frac{-2}{v}$$

$$\ln|x| + c = \frac{-2}{v} = \frac{-2x}{y} \quad [v = \frac{y}{x}]$$

$$\text{So, } c = -\frac{2x}{y} - \ln|x|$$

$$\text{Given: } y'(1) = 2$$

$$\text{So, } c = -\frac{2(1)}{2} - \ln(1)$$

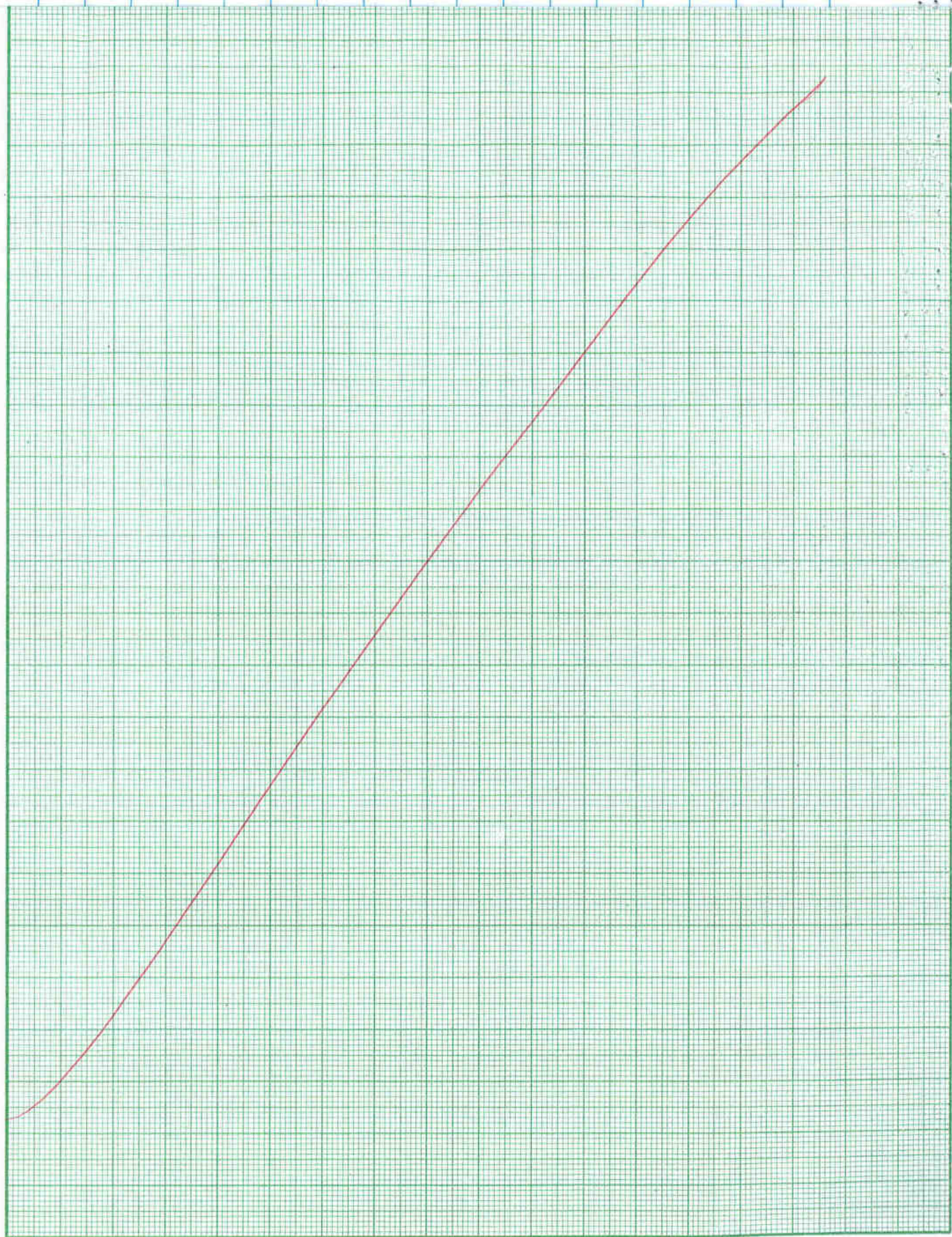
$$c = -1$$

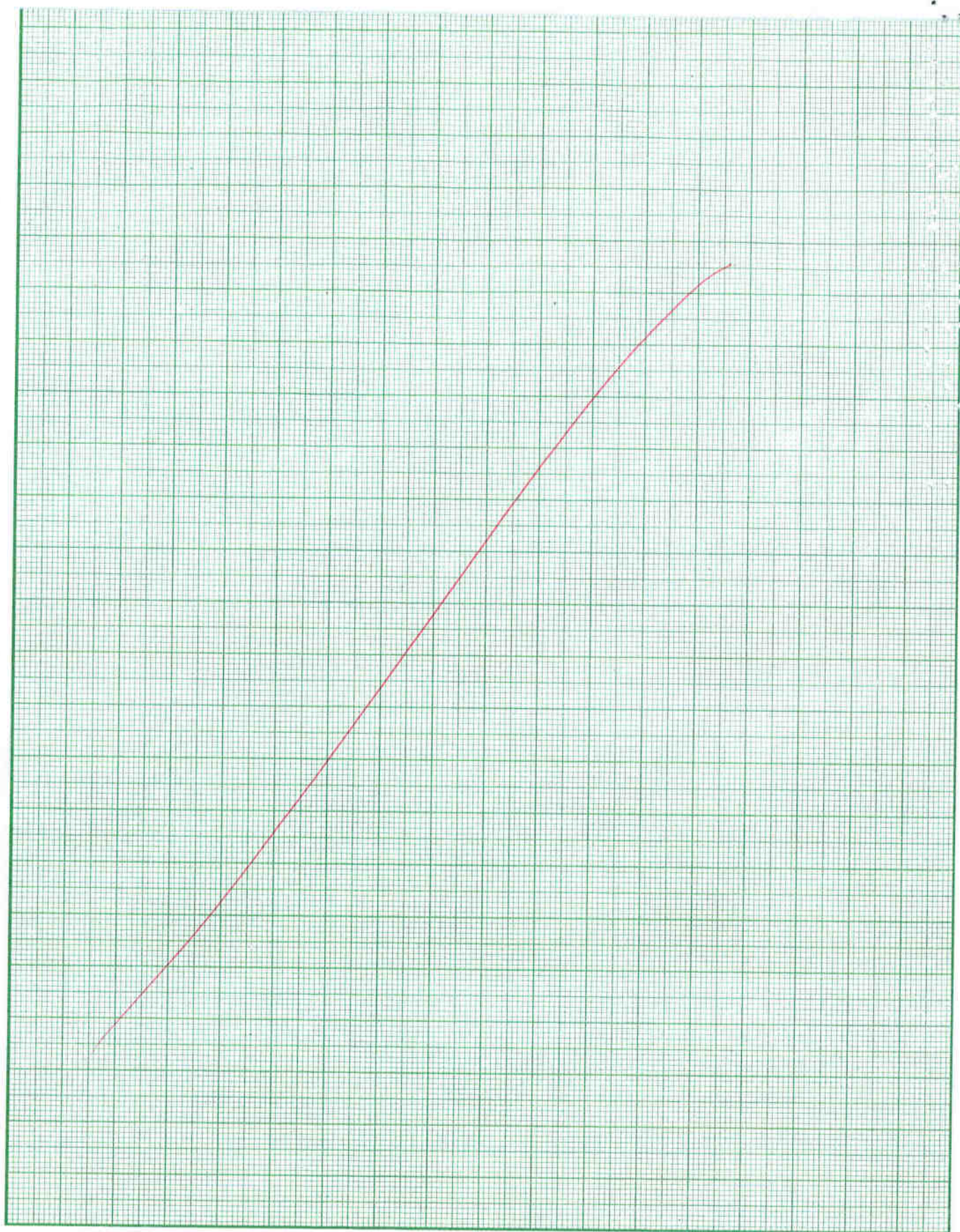
Particular solution is,

$$\ln|x| - 1 = -\frac{2x}{y}$$

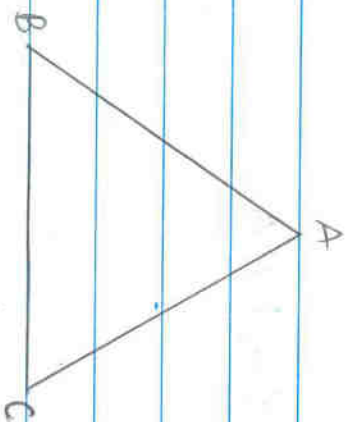
$$y = \frac{-2x}{\ln|x| - 1} = \frac{2x}{1 - \ln|x|}$$

$$\text{Ans: } y = \frac{2x}{1 - \ln|x|}$$





$$\begin{aligned}
 1) \quad \vec{OA} &= 2\hat{i} - \hat{j} + \hat{k} \\
 \vec{OB} &= \hat{i} - 3\hat{j} - 5\hat{k} \\
 \vec{OC} &= 3\hat{i} - 4\hat{j} - 4\hat{k}
 \end{aligned}$$



$$\begin{aligned}
 \vec{AB} &= \vec{OB} - \vec{OA} \\
 &= -\hat{i} - 2\hat{j} - 6\hat{k}
 \end{aligned}$$

$$\begin{aligned}
 \vec{BC} &= \vec{OC} - \vec{OB} \\
 &= 2\hat{i} - \hat{j} + \hat{k}
 \end{aligned}$$

$$\begin{aligned}
 \vec{CA} &= \vec{OA} - \vec{OC} \\
 &= -\hat{i} + 3\hat{j} + 5\hat{k}
 \end{aligned}$$

$$\begin{aligned}
 (i) \quad \cos \angle ABC &= \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|} \\
 &= \frac{(\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})}{(\sqrt{1^2 + 2^2 + 6^2}) (\sqrt{2^2 + 1^2 + 1^2})}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{2 - 2 + 6}{(\sqrt{41}) (\sqrt{6})} \\
 &= \frac{6}{\sqrt{41}}
 \end{aligned}$$

$$\text{So, } \angle ABC = \cos^{-1} \left(\frac{\sqrt{6}}{\sqrt{41}} \right)$$

$$(ii) \cos \angle BAC = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$$

$$= \frac{(-\hat{i} - 2\hat{j} - 6\hat{k}) \cdot (\hat{i} - 3\hat{j} - 5\hat{k})}{(\sqrt{1^2 + 2^2 + 6^2})(\sqrt{1^2 + 3^2 + 5^2})}$$

$$= \frac{-1 + 6 + 30}{(\sqrt{41})(\sqrt{35})} = \sqrt{\frac{35}{41}}$$

$$\angle BAC = \cos^{-1} \left(\sqrt{\frac{35}{41}} \right)$$

$$(iii) \cos \angle ACB = \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}| |\vec{CB}|}$$

$$= \frac{(-\hat{i} + 3\hat{j} + 5\hat{k}) \cdot (-2\hat{i} + \hat{j} - \hat{k})}{(\sqrt{1^2 + 3^2 + 5^2})(\sqrt{2^2 + 1^2 + 1^2})} = \frac{2 + 3 - 5}{(\sqrt{35})(\sqrt{6})}$$

$$\text{So, } \angle ACB = \pi/2$$

$$\text{Ans: } \angle BAC = \cos^{-1} \sqrt{\frac{35}{41}}, \quad \angle ABC = \cos^{-1} \sqrt{\frac{6}{41}}, \quad \angle ACB = \pi/2$$

30) Let X denote the absolute difference of numbers appearing on top of dice

X can take values 0, 1, 2, 3, 4, 5.

• For $X=0 \rightarrow \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$$\hookrightarrow P(X=0) = \frac{6}{36} = \frac{1}{6}$$

• For $X=1 \rightarrow \{(1,2), (2,3), (3,4), (4,5), (5,6), (2,1), (3,2), (4,3), (5,4), (6,5)\}$

$$\hookrightarrow P(X=1) = \frac{10}{36} = \frac{5}{18}$$

• For $x=2 \rightarrow \{(1,3), (2,4), (2,5), (4,6), (3,1), (4,2), (5,3), (6,4)\}$

$$\hookrightarrow P(X=2) = \frac{8}{36} = \frac{2}{9}$$

• For $x=3 \rightarrow \{(1,4), (2,5), (3,6), (4,1), (5,2), (6,3)\}$

$$\hookrightarrow P(X=3) = \frac{6}{36} = \frac{1}{6}$$

• For $x=4 \rightarrow \{(1,5), (2,6), (5,1), (6,2)\}$

$$\hookrightarrow P(X=4) = \frac{4}{36} = \frac{1}{9}$$

• For $x=5 \rightarrow \{(1,6), (6,1)\}$

$$\hookrightarrow P(X=5) = \frac{2}{36} = \frac{1}{18}$$

The Probability distribution table is,

x	0	1	2	3	4	5
$P(x)$	$1/6 = 6/36$	$10/36 = 5/18$	$8/36 = 2/9$	$6/36 = 1/6$	$4/36 = 1/9$	$2/36 = 1/18$

31) $I = \int x^2 \cdot \sin^{-1}(x^{3/2}) dx$

Take $t = x^3$

$dt = 3x^2 dx \rightarrow \frac{dt}{3} = x^2 dx$

$\sqrt{x} = t$

$\frac{1}{2\sqrt{x}} dx = dt$

$\frac{1}{3} \int 2t \sin^{-1} t dt$

$\frac{1}{3} \left[2t \sin^{-1} t - \int \frac{t^2}{\sqrt{1-t^2}} dt \right]$

So, $I = \frac{1}{3} \int (1) \sin^{-1}(x^{3/2}) dx$

Take $u = \sin^{-1}(x^{3/2})$, $dv = dx$

Applying By-Parts,

$3I = x \sin^{-1} x - \int x \left(\frac{1}{\sqrt{1-x}} \right) \cdot \left(\frac{1}{2\sqrt{x}} \right) dx$

$$3I = x \sin^{-1} x - \frac{1}{2} \int \sqrt{\frac{x}{1-x}} dx$$

Take $\int \frac{x}{\sqrt{1-x}} dx = I_1$

Put

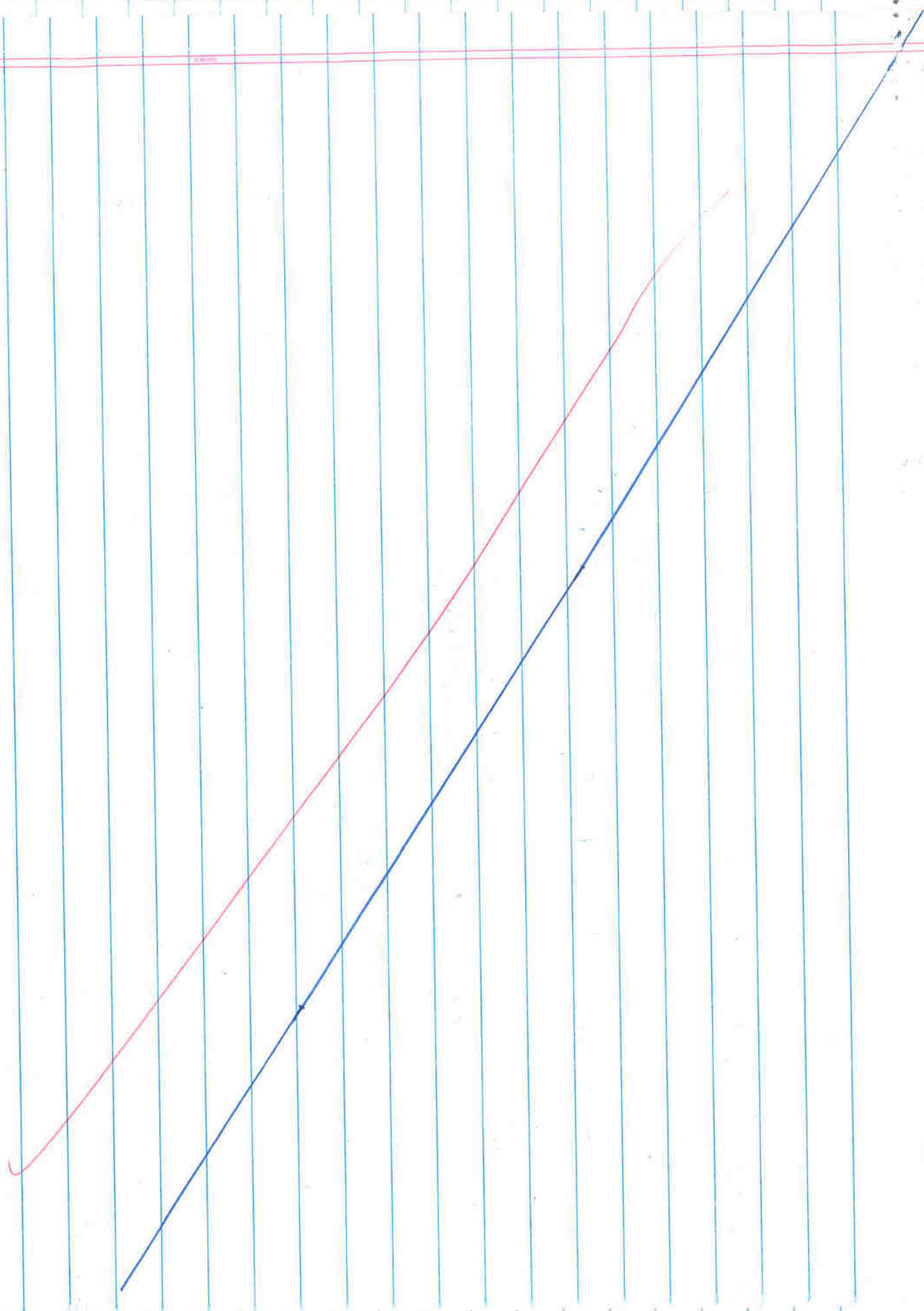
$$1-x = t^2$$

$$\rightarrow x = 1-t^2$$

$$-dx = 2tdt \Rightarrow -dt = +dx$$

$$I_1 = - \int \sqrt{\frac{1-t^2}{t}} (dt)(2t)$$

$$= -2 \int \sqrt{t(1-t^2)} dt$$



Section - B

$$21) (a) \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-1)$$

$$= -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \frac{\pi}{3} - \tan^{-1}(1)$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{3} - \frac{5\pi}{12} = -\frac{\pi}{12}$$

$$\text{Ans: } \frac{-\pi}{12}$$

1) b) $y = \operatorname{cosec}(\cot^{-1}x)$

~~$$\begin{aligned} \frac{dy}{dx} &= -\operatorname{cosec}(\cot^{-1}x) \cdot \cot(\cot^{-1}x) \cdot \left(\frac{-1}{1+x^2}\right) \\ &= -x \operatorname{cosec}(\cot^{-1}x) \\ &= -x \sqrt{1+\cot^2(\cot^{-1}x)} \\ &= -x \sqrt{1+x^2} \end{aligned}$$~~

2) b) $y = \operatorname{cosec}(\cot^{-1}x)$

Diff wr.t x ,

$$\frac{dy}{dx} = -\operatorname{cosec}(\cot^{-1}x) \cdot \cot(\cot^{-1}x) \cdot \left(\frac{-1}{1+x^2}\right) \quad [\cot(\cot^{-1}x) = x]$$

$$\frac{dy}{dx} = \frac{x \operatorname{cosec}(\cot^{-1}x)}{1+x^2}$$

$$[\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}]$$

$$\frac{dy}{dx} = \frac{x \sqrt{1 + \cot^2(\cot^{-1}x)}}{1+x^2}$$

$$\frac{dy}{dx} = \frac{x (\sqrt{1+x^2})}{1+x^2}$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$$

$$\text{So, } (\sqrt{1+x^2}) \frac{dy}{dx} = x$$

$$(\sqrt{1+x^2}) \frac{dy}{dx} - x = 0.$$

• Hence, Proved the required

(13)

$$f(x) = x + \frac{1}{x}$$

Diff wrt x on both sides

$$f'(x) = 1 - \frac{1}{x^2}$$

$$\text{Put } f'(x) = 0$$

$$1 - \frac{1}{x^2} = 0 \Rightarrow 1 = \frac{1}{x^2} \Rightarrow x^2 = 1$$

$$\text{So, } x = \pm 1$$

$$\text{Now, } f''(x) = \frac{2}{x^3}$$

$$f''(1) = \frac{2}{1} > 0.$$

$f(x)$ attains minima at $x = 1$.

$$f''(-1) = \frac{2}{-1} < 0$$

$f(x)$ attains maxima at $x = -1$.

$$S_0, M = 1, m = 1$$

$$M - m = 1 - 1 = 0$$

$$\text{Local maximum value} = M = f(-1) = -1 + \frac{1}{(-1)} = -2$$

$$\text{Local minimum value} = m = f(1) = 1 + \frac{1}{1} = 2$$

$$S_0, M - m = -2 - (2) = -4$$

$$\text{Ans: } -4$$

$$I = \int \frac{(e^{4x} - 1)}{e^{4x} + 1} dx$$

Divide Numerator and Denominator by e^{2x}

So,

$$I = \int \frac{(e^{2x} - e^{-2x})}{e^{2x} + e^{-2x}} dx$$

Take $u = e^{2x} + e^{-2x}$

$$du = (2e^{2x} - 2e^{-2x}) dx$$

So, $\frac{du}{2} = (e^{2x} - e^{-2x}) dx$

Hence, $I = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \log_e |u| + c$

Ans: $I = \frac{1}{2} \log_e |e^{2x} + e^{-2x}| + c$

25) $f(x) = e^x - e^{-x} + x - \tan^{-1}x$

Differentiating w.r.t x ,

$$f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2}$$

$$= e^x + e^{-x} + \frac{1+x^2-1}{1+x^2}$$

$$= e^x + e^{-x} + \frac{x^2}{1+x^2}$$

For any value of x in its domain,
 $e^x > 0$, $e^{-x} > 0$, $\frac{x^2}{1+x^2} > 0$ $\left[\frac{-1}{1+x^2} > -1 \rightarrow 1 - \frac{1}{1+x^2} \right]$

So, $f'(x) = e^x + e^{-x} + \frac{x^2}{1+x^2} > 0$

As $f'(x)$ is always greater than 0, $f(x)$ is a strictly increasing function

Section - A

Product = 0

Ans: (A) 0

$$f(x) = 9x^2 + 6x - 5$$

$$= (3x)^2 + 2(3x)(1) + 1 - 6$$

$$= \frac{(3x+1)^2 - 6}{1}$$

$$= (3x+1)^2 - 6$$

Given: $x \geq 0$

$$3x \geq 0$$

$$3x+1 \geq 1 \rightarrow (3x+1)^2 \geq 1$$

$$\text{So, } (3x+1)^2 - 6 \geq -5$$

Ans: (c) Bijective

3)

$$\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = abc \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= abc [-1(1-1) - 1(-1-1) + 1(1+1)]$$

$$= abc [2+2]$$

$$= 4abc = kabc \quad [k=4]$$

Ans: (D) 4

4)

At $x = -3$:

$$\cdot LHL = |-3| + 3 = 6 = f(-3)$$

$$\cdot RHL = -2(-3) = 6$$

At $x = 3$:

$$\cdot LHL = -2(3) = -6$$

$$\cdot RHL = f(3) = 6(3) + 2 = 20$$

Ans: (B) 1

$$7) f(x) = x^3 - 3x^2 + 12x - 18$$

$$\begin{aligned} f'(x) &= 3x^2 - 6x + 12 \\ &= 3(x^2 - 2x + 4) \\ &= 3(x-2)^2 \end{aligned}$$

Ans: (B) strictly increasing on $\mathbb{R}$.

$$8) \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx$$

Ans: (B) Zero(0)

$$7) (A) \cos x - \sin\left(\frac{\pi}{4} - x\right)$$

$$8) (C) \vec{a} \cdot \vec{b} < |\vec{a}| |\vec{b}|$$

$$1) (D) (0, 0, 0)$$

10,

(c) a feasible region ✓

11)

$$P(S/E) = \frac{P(S \cap E)}{P(E)} = \frac{P(E)}{P(E)} = 1$$

Ans: (C) 1 ✓

12)

$$\bullet a_{11} = 1 - 3 = -2$$

$$\bullet a_{12} = 1 - 6 = -5, \quad a_{21} = 2 - 3 = -1$$

$$\text{So, } a_{12} + a_{21} = -6$$

$$\bullet a_{13} = 1 - 9 = -8, \quad a_{31} = 3 - 3 = 0$$

Ans: (C) $a_{13} > a_{31}$ ✓

13) ✓

$$\frac{d(\tan^{-1}(x^2))}{dx} = \frac{1}{1+x^4} (2x) = \frac{2x}{1+x^4}$$

Ans: (B) $\frac{2x}{1+x^4}$ ✓

$$1) \left(\frac{d^2 y}{dx^2} \right)^2 + \left(\frac{dy}{dx} \right)^3 = x \sin \left(\frac{dy}{dx} \right)$$

Ans: (D) not defined

$$i) (\hat{i} + \hat{k}) \times (\hat{i} - \hat{k}) = \hat{j} + \hat{j} = 2\hat{j} \quad [\text{Unit vector} = \hat{j}]$$

Ans: (B) $\hat{j}$

$$ii) \frac{x-1}{2} = \frac{y}{-1} = \frac{z+1/2}{3}$$

Ans: (D) 2, -1, 3

$$1) [F(x)]^2 = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 2x & -\sin 2x & 0 \\ \sin 2x & \cos 2x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ans: (B) 2

18)

$$\alpha = 30^\circ, \beta = 120^\circ$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 30^\circ + \cos^2 120^\circ + \cos^2 \gamma = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \cos^2 \gamma = 1 + \cos^2 \gamma$$

$$\text{So, } \cos \gamma = 0 \Rightarrow \gamma = 90^\circ$$

$$\text{Ans: (A) } 90^\circ$$

19)

$$(B'AB)^T = B^T A^T B$$
$$= B'AB \quad [A = A']$$

Ans: (D) Assertion (A) is false, but Reason (R) is true

(20) (C) Assertion (A) is true, but Reason (R) is false.

$$I = \int x^2 \sin^{-1}(x^{3/2}) dx$$

$$\text{Let } x^2 = x^3 \rightarrow \frac{2x dx}{3} = x^2 dx$$

So,

$$I = \frac{1}{3} \int \sin^{-1}(x^3) dx$$

$$I = \frac{1}{3} \int 2x \sin^{-1} x dx$$

Now, $dv = 2x dx$ and $v = \sin^{-1} x$

Applying By-parts $[uv - v du]$,

$$3I = x^2 \sin^{-1} x - \int \frac{x^2}{\sqrt{1-x^2}} dx$$

$$= x^2 \sin^{-1} x + \int \frac{(1-x^2-1)}{\sqrt{1-x^2}} dx$$

$$= x^2 \sin^{-1} x + \int (\sqrt{1-x^2}) dx - \int \frac{1}{\sqrt{1-x^2}} dx$$

$$3I = x^2 \sin^{-1} x + \frac{x}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1}(t) - \sin^{-1} t + C$$

$$= \left(t^2 - 1/2 \right) \sin^{-1} t + \frac{t}{2} \sqrt{1-t^2} + C$$

Subbing $t^2 = x^3$,

$$3I = \left(x^3 - \frac{1}{2} \right) \sin^{-1}(x^{3/2}) + \frac{x^{3/2}}{2} \sqrt{1-x^3} + C$$

So,

$$I = \frac{1}{3} \left[\left(x^3 - \frac{1}{2} \right) \sin^{-1}(x^{3/2}) + \frac{x^{3/2}}{2} \sqrt{1-x^3} \right] + C' \quad [C' = C/3]$$