Activity 1

OBJECTIVE

To construct a square-root spiral.

MATERIAL REQUIRED

Coloured threads, adhesive, drawing pins, nails, geometry box, sketch pens, marker, a piece of plywood.

METHOD OF CONSTRUCTION

1. Take a piece of plywood with dimensions 30 cm × 30 cm.
2. Taking 2 cm = 1 unit, draw a line segment AB of length one unit.
3. Construct a perpendicular BX at the line segment AB using set squares (or compasses).
4. From BX, cut off BC = 1 unit. Join AC.
5. Using blue coloured thread (of length equal to AC) and adhesive, fix the thread along AC.
6. With AC as base and using set squares (or compasses), draw CY perpendicular to AC.
7. From CY, cut-off CD = 1 unit and join AD.

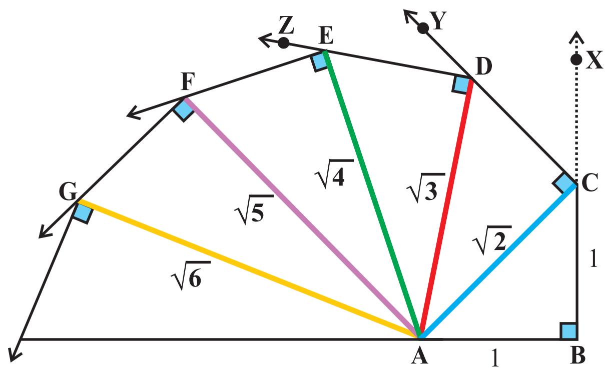

Fig. 1

8. Fix orange coloured thread (of length equal to AD) along AD with adhesive.
9. With AD as base and using set squares (or compasses), draw DZ perpendicular to AD.
10. From DZ, cut off DE = 1 unit and join AE.
11. Fix green coloured thread (of length equal to AE) along AE with adhesive [see Fig. 1].

    Repeat the above process for a sufficient number of times. This is called "a square root spiral".

Demonstration

1. From the figure, $AC^2 = AB^2 + BC^2 = 12 + 12 = 2$ or $AC = \sqrt{2}$.

   $AD^2 = AC^2 + CD^2 = 2 + 1 = 3$ or $AD = \sqrt{3}$.

2. Similarly, we get the other lengths AE, AF, AG, ... as $\sqrt{4}$ or 2, $\sqrt{5}$, $\sqrt{6}$ ....

Observation

On actual measurement

AC = ... , AD = ... , AE = ... , AF = ... , AG = ...

$\sqrt{2}$ = AC = ... (approx.),

$\sqrt{3}$ = AD = ... (approx.),

$\sqrt{4}$ = AE = ... (approx.),

$\sqrt{5}$ = AF = ... (approx.)

Application

Through this activity, existence of irrational numbers can be illustrated.

Activity 3

Objective

To verify the algebraic identity :

$(a + b)^2 = a^2 + 2ab + b^2$

Material Required

Drawing sheet, cardboard, cello-tape, coloured papers, cutter and ruler.

Method of Construction

1. Cut out a square of side length a units from a drawing sheet/cardboard and name it as square ABCD [see Fig. 1].
2. Cut out another square of length b units from a drawing sheet/cardboard and name it as square CHGF [see Fig. 2].

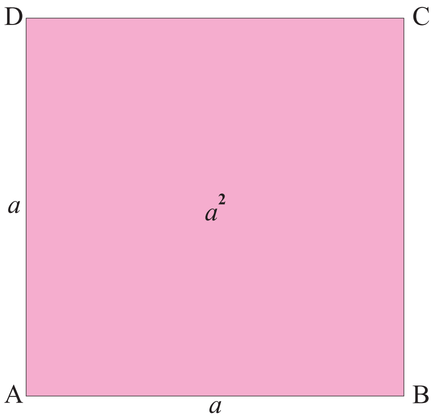

Fig. 1

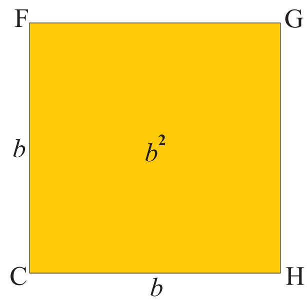

Fig. 2

3. Cut out a rectangle of length a units and breadth b units from a drawing sheet/cardbaord and name it as a rectangle DCFE [see Fig. 3].
4. Cut out another rectangle of length b units and breadth a units from a drawing sheet/cardboard and name it as a rectangle BIHC [see Fig. 4].

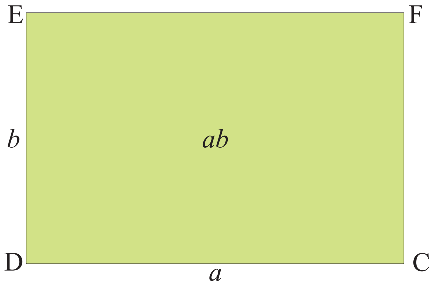

Fig. 3

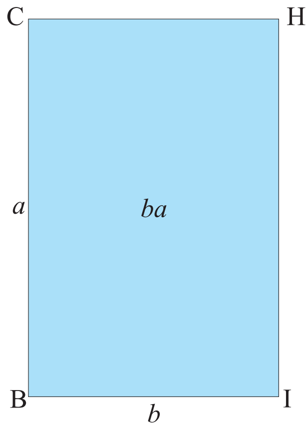

Fig. 4

5. Total area of these four cut-out figures

   = Area of square ABCD + Area of square CHGF + Area of rectangle DCFE + Area of rectangle BIHC

   $= a^2 + b^2 + ab + ba = a^2 + b^2 + 2ab.$

6. Join the four quadrilaterals using cello-tape as shown in Fig. 5.

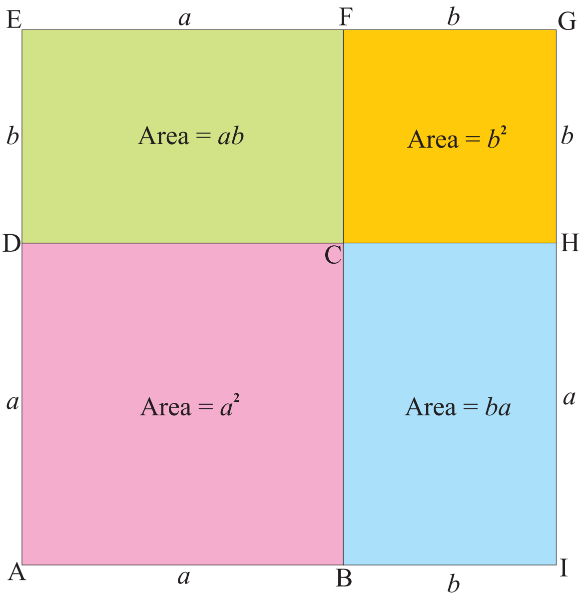

Fig. 5

Clearly, AIGE is a square of side ($a + b$). Therefore, its area is $(a + b)^2$. The combined area of the constituent units = $a^2 + b^2 + ab + ab = a^2 + b^2 + 2ab$.

Hence, the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$

Here, area is in square units.

Observation

On actual measurement:

a = ..., b = ... $(a+b)$ = ...,

So, a^2 = ... b^2 = ..., ab = ...

$(a+b)^2$ = ..., $2ab$ = ...

Therefore, $(a+b)^2 = a^2 + 2ab + b^2$.

The identity may be verified by taking different values of a and b.

Application

The identity may be used for

1. calculating the square of a number expressed as the sum of two convenient numbers.
2. simplifications/factorisation of some algebraic expressions.

Activity 4

Objective

To verify the algebraic identity :

$(a - b)^2 = a^2 - 2ab + b^2$

Material Required

Drawing sheets, cardboard, coloured papers, scissors, ruler and adhesive.

Method of Construction

1. Cut out a square ABCD of side a units from a drawing sheet/cardboard [see Fig. 1].
2. Cut out a square EBHI of side b units ($b < a$) from a drawing sheet/cardboard [see Fig. 2].
3. Cut out a rectangle GDCJ of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 3].
4. Cut out a rectangle IFJH of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 4].

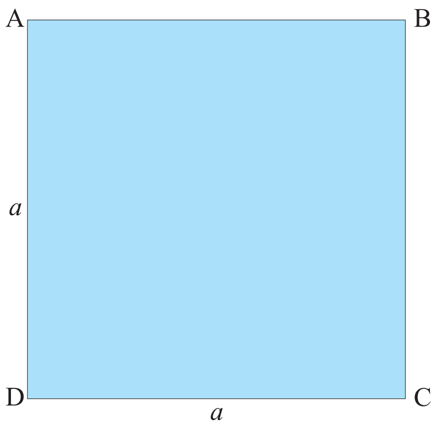

Fig. 1

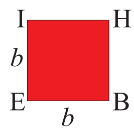

Fig. 2

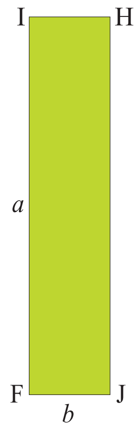

Fig. 4

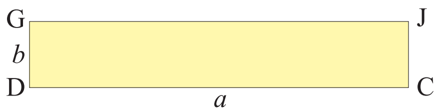

Fig. 3

5. Arrange these cut outs as shown in Fig. 5.

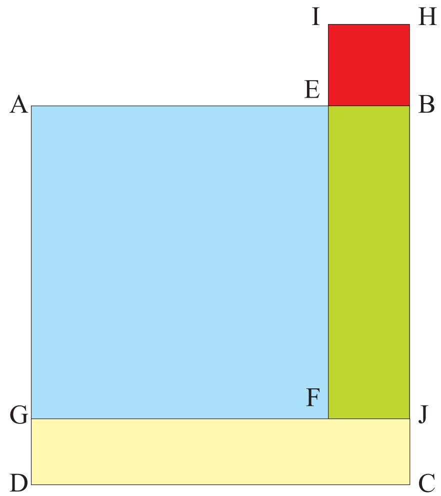

Fig. 5

DEMONSTRATION

According to figure 1, 2, 3, and 4, Area of square ABCD = a^2, Area of square EBHI = b^2

Area of rectangle GDCJ = ab, Area of rectangle IFJH = ab

From Fig. 5, area of square AGFE = AG × GF = $(a - b)(a - b) = (a - b)^2$

Now, area of square AGFE = Area of square ABCD + Area of square EBHI

– Area of rectangle IFJH – Area of rectangle GDCJ

$= a^2 + b^2 - ab - ab$

$= a^2 - 2ab + b^2$

Here, area is in square units.

OBSERVATION

On actual measurement:

a = ..., b = ..., $(a - b)$ = ...,

So, a^2 = ..., b^2 = ..., $(a - b)^2$ = ...,

ab = ..., $2ab$ = ...

Therefore, $(a - b)^2 = a^2 - 2ab + b^2$

APPLICATION

The identity may be used for

1. calculating the square of a number expressed as a difference of two convenient numbers.
2. simplifying/factorisation of some algebraic expressions.

Activity 5

Objective

To verify the algebraic identity :

$a^2 - b^2 = (a + b)(a - b)$

Material Required

Drawing sheets, cardboard, coloured papers, scissors, sketch pen, ruler, transparent sheet and adhesive.

Method of Construction

1. Take a cardboard of a convenient size and paste a coloured paper on it.
2. Cut out one square ABCD of side a units from a drawing sheet [see Fig. 1].
3. Cut out one square AEFG of side b units ($b < a$) from another drawing sheet [see Fig. 2].

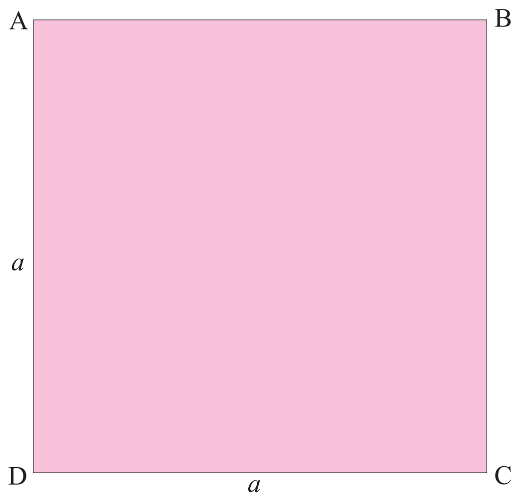

Fig. 1

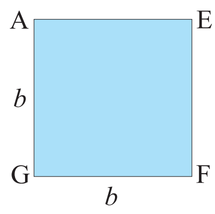

Fig. 2

4. Arrange these squares as shown in Fig. 3.
5. Join F to C using sketch pen. Cut out trapeziums congruent to EBCF and GFCD using a transparent sheet and name them as EBCF and GFCD, respectively [see Fig. 4 and Fig. 5].

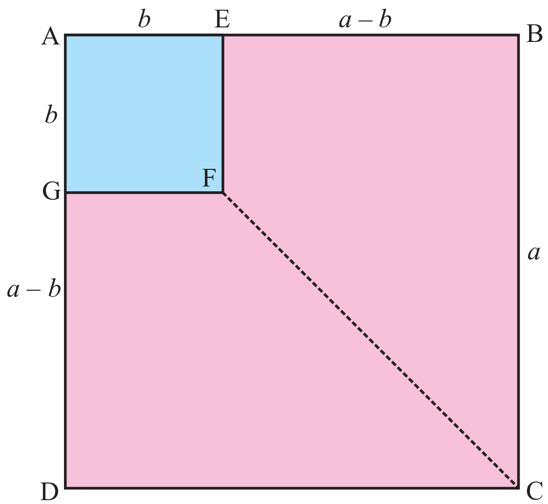

Fig. 3

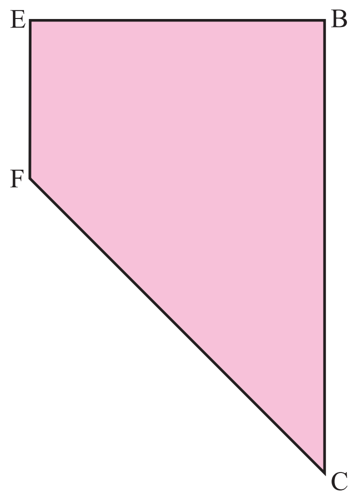

Fig. 4

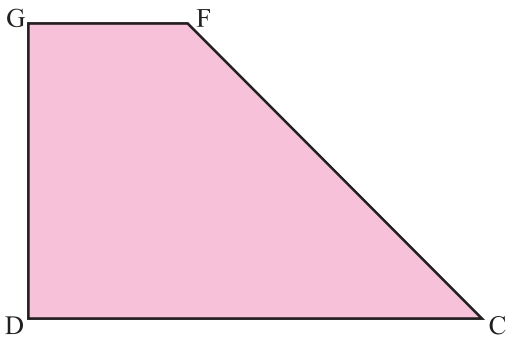

Fig. 5

6. Arrange these trapeziums as shown in Fig. 6.

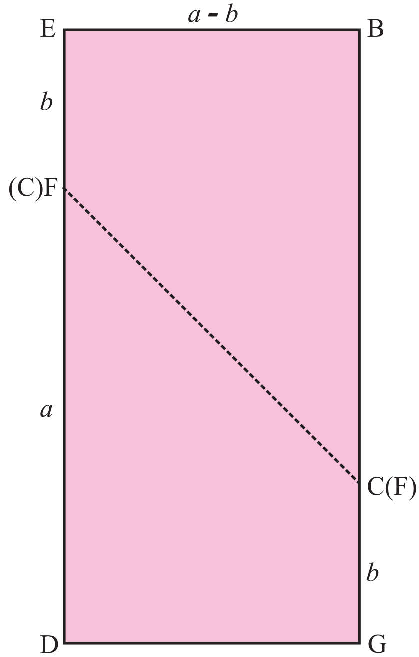

Fig. 6

Demonstration

Area of square ABCD = a^2

Area of square AEFG = b^2

In Fig. 3,

Area of square ABCD – Area of square AEFG

= Area of trapezium EBCF + Area of trapezium GFCD

= Area of rectangle EBGD [Fig. 6].

= ED × DG

Thus, $a^2 - b^2 = (a+b)\ (a-b)$

Here, area is in square units.

Observation

On actual measurement:

a = ..., b = ..., $(a+b)$ = ...,

So, a^2 = ..., b^2 = ..., $(a-b)$ = ...,

$a^2 - b^2$ = ..., $(a+b)\ (a-b)$ = ...,

Therefore, $a^2 - b^2 = (a+b)\ (a-b)$

Application

The identity may be used for

1. difference of two squares
2. some products involving two numbers
3. simplification and factorisation of algebraic expressions.

Activity 6

Objective

To verify the algebraic identity :

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Material Required

Hardboard, adhesive, coloured papers, white paper.

Method of Construction

1. Take a hardboard of a convenient size and paste a white paper on it.
2. Cut out a square of side a units from a coloured paper [see Fig. 1].
3. Cut out a square of side b units from a coloured paper [see Fig. 2].
4. Cut out a square of side c units from a coloured paper [see Fig. 3].
5. Cut out two rectangles of dimensions $a \times b$, two rectangles of dimensions $b \times c$ and two rectangles of dimensions $c \times a$ square units from a coloured paper [see Fig. 4].

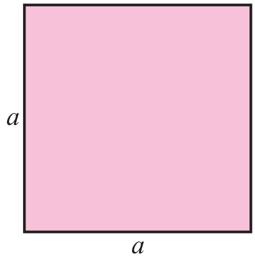

Fig. 1

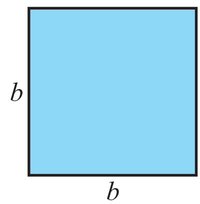

Fig. 2

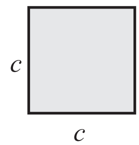

Fig. 3

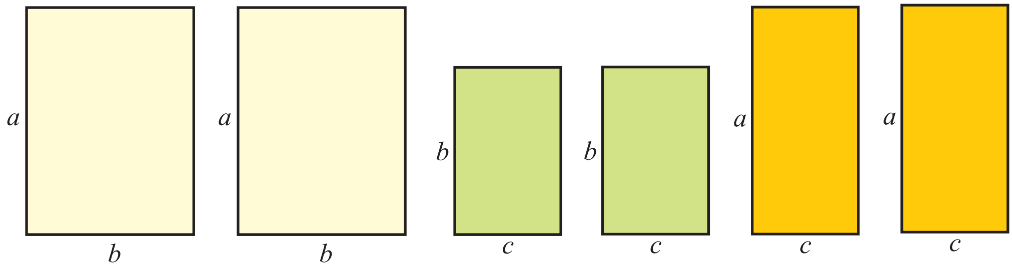

Fig. 4

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6. Arrange the squares and rectangles on the hardboard as shown in Fig. 5.

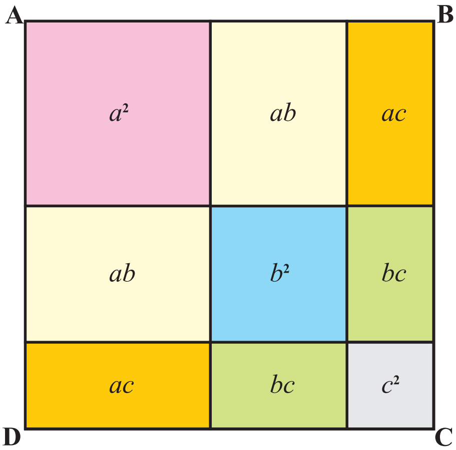

Fig. 5

Demonstration

From the arrangement of squares and rectangles in Fig. 5, a square ABCD is obtained whose side is $(a+b+c)$ units.

Area of square ABCD = $(a+b+c)^2$.

Therefore, $(a+b+c)^2$ = sum of all the squares and rectangles shown in Fig. 1 to Fig. 4.

$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, area is in square units.

Observation

On actual measurement:

a = ..., b = ..., c = ...,

So, a^2 = ..., b^2 = ..., c^2= ...., ab= ....,

bc= ...., ca = ..., $2ab$ = ..., $2bc$ = ...,

$2ca$= ...., $a+b+c$ = ..., $(a+b+c)^2$ = ...,

Therefore, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Application

The identity may be used for

1. simiplification/factorisation of algebraic expressions
2. calculating the square of a number expressed as a sum of three convenient numbers.

Activity 7

Objective

To verify the algebraic identity :

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Material Required

Acrylic sheet, coloured papers, glazed papers, saw, sketch pen, adhesive, Cello-tape.

Method of Construction

1. Make a cube of side a units and one more cube of side b units ($b < a$), using acrylic sheet and cello-tape/adhesive [see Fig. 1 and Fig. 2].
2. Similarly, make three cuboids of dimensions $a \times a \times b$ and three cuboids of dimensions $a \times b \times b$ [see Fig. 3 and Fig. 4].

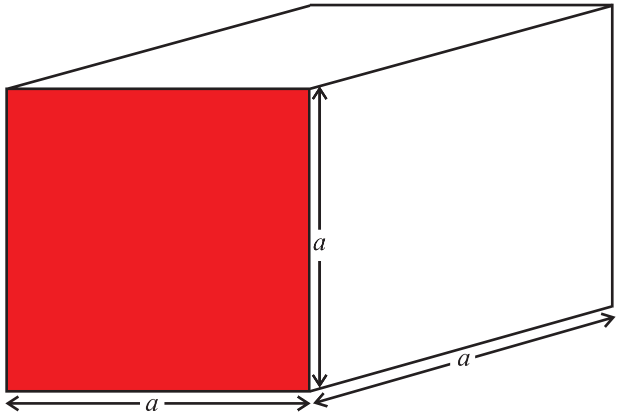

Fig. 1

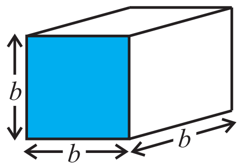

Fig. 2

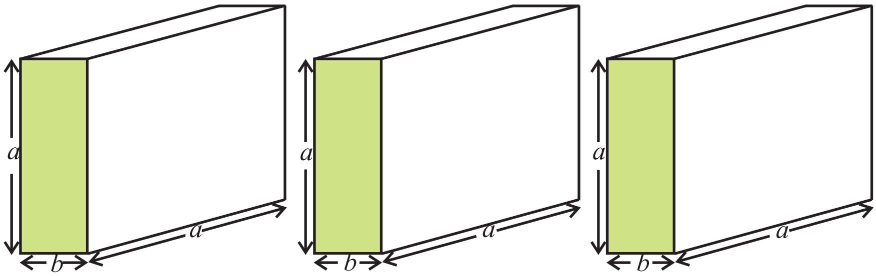

Fig. 3

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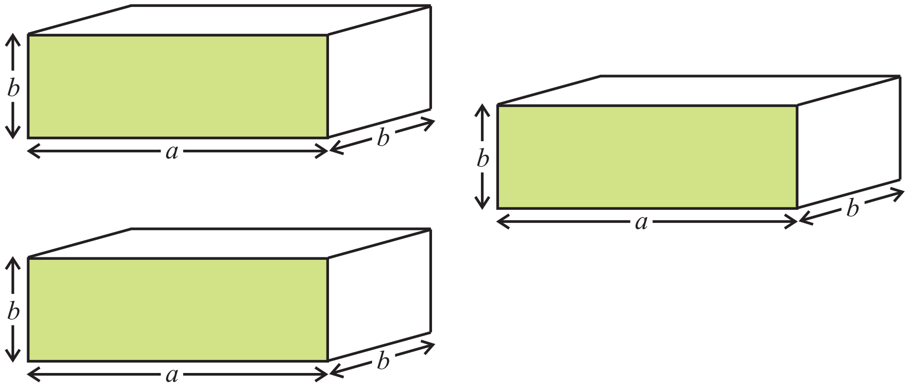

Fig. 4

3. Arrange the cubes and cuboids as shown in Fig. 5.

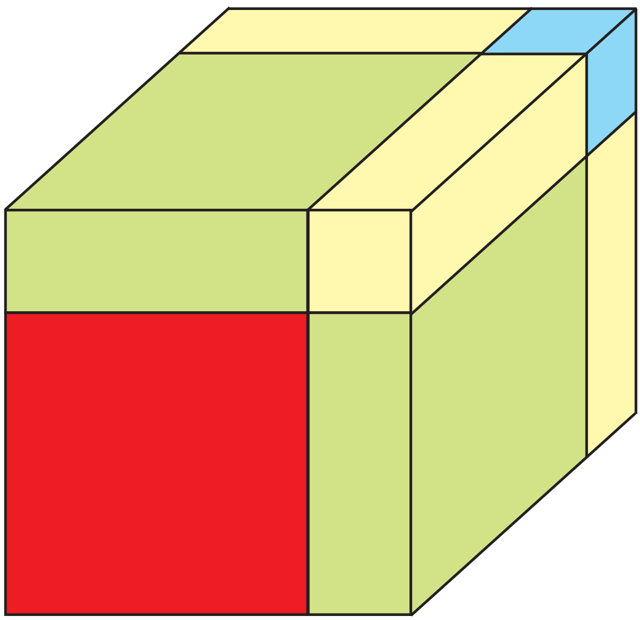

Fig. 5

Demonstration

Volume of the cube of side $a = a \times a \times a = a^3$, volume of the cube of side $b = b^3$

Volume of the cuboid of dimensions $a \times a \times b = a^2b$, volume of three such cuboids $= 3a^2b$

Volume of the cuboid of dimensions $a \times b \times b = ab^2$, volume of three such cuboids $= 3ab^2$

Solid figure obtained in Fig. 5 is a cube of side $(a + b)$

Its volume $= (a + b)^3$

Therefore, $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Here, volume is in cubic units.

Observation

On actual measurement:

a = ..., b = ..., a^3 = ...,

So, a^3 = ..., b^3 = ..., a^2b = ..., $3a^2b$ = ...,

ab^2 = ..., $3ab^2$ = ..., $(a+b)^3$ = ...,

Therefore, $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Application

The identity may be used for

1. calculating cube of a number expressed as the sum of two convenient numbers
2. simplification and factorisation of algebraic expressions.

Activity 8

Objective

To verify the algebraic identity

$(a - b)^3 = a^3 - b^3 - 3(a - b)ab$

Material Required

Acrylic sheet, coloured papers, saw, sketch pens, adhesive, Cello-tape.

Method of Construction

1. Make a cube of side $(a - b)$ units $(a > b)$using acrylic sheet and cellotape/ adhesive [see Fig. 1].
2. Make three cuboids each of dimensions $(a-b) \times a \times b$ and one cube of side b units using acrylic sheet and cellotape [see Fig. 2 and Fig. 3].
3. Arrange the cubes and cuboids as shown in Fig. 4.

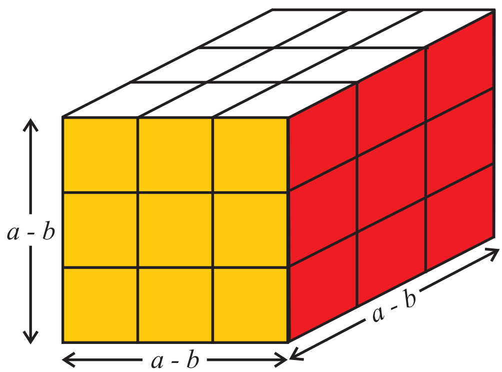

Fig. 1

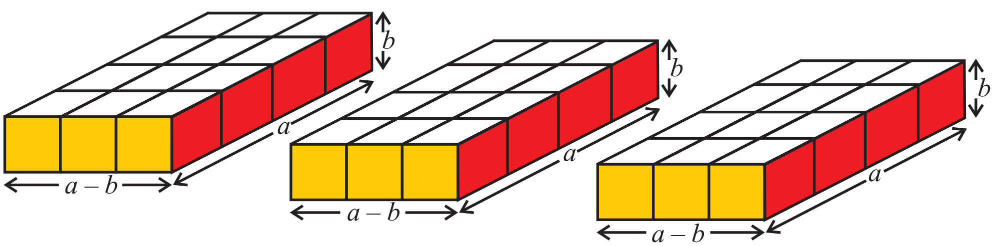

Fig. 2

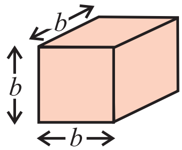

Fig. 3

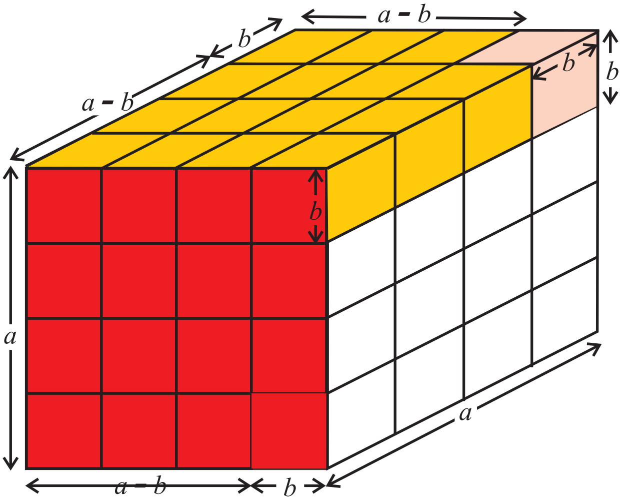

Fig. 4

DEMONSTRATION

Volume of the cube of side $(a - b)$ units in Fig. 1 = $(a- b)^3$

Volume of a cuboid in Fig. 2 = $(a-b)\ ab$

Volume of three cuboids in Fig. 2 = $3\ (a-b)\ ab$

Volume of the cube of side b in Fig. 3 = b^3

Volume of the solid in Fig. 4 = $(a-b)^3 + (a-b)\ ab + (a-b)\ ab + (a - b)\ ab + b^3$

$= (a-b)^3 + 3(a-b)\ ab + b^3$ (1)

Also, the solid obtained in Fig. 4 is a cube of side a

Therefore, its volume = a^3 (2)

From (1) and (2),

$(a-b)^3 + 3(a-b)\ ab + b^3 = a^3$

or $(a-b)^3 = a^3 - b^3 - 3ab\ (a-b)$.

Here, volume is in cubic units.

OBSERVATION

On actual measurement:

a = ..., b = ..., $a–b$ = ...,

So, a^3 = ..., ab = ...,

b^3 = ..., $ab(a–b)$ = ...,

$3ab\ (a–b)$ = ..., $(a–b)^3$ = ...,

Therefore, $(a–b)^3 = a^3 - b^3 - 3ab(a–b)$

APPLICATION

The identity may be used for

1. calculating cube of a number expressed as a difference of two convenient numbers
2. simplification and factorisation of algebraic expressions.

NOTE

This identity can also be expressed as :

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

Activity 9

OBJECTIVE

To verify the algebraic identity :

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

MATERIAL REQUIRED

Acrylic sheet, glazed papers, saw, adhesive, cellotape, coloured papers, sketch pen, etc.

METHOD OF CONSTRUCTION

1. Make a cube of side a units and another cube of side b units as shown in Fig. 1 and Fig. 2 by using acrylic sheet and cellotape/adhesive.
2. Make a cuboid of dimensions $a \times a \times b$ [see Fig. 3].
3. Make a cuboid of dimensions $a \times b \times b$ [see Fig. 4].
4. Arrange these cubes and cuboids as shown in Fig. 5.

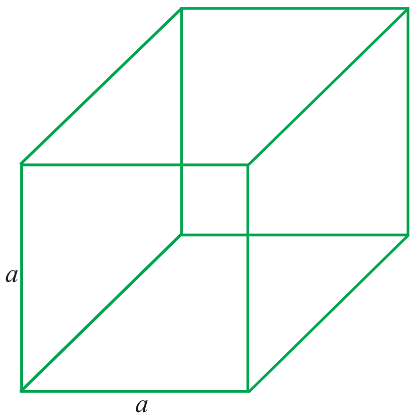

Fig. 1

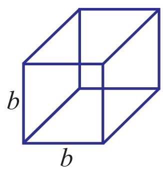

Fig. 2

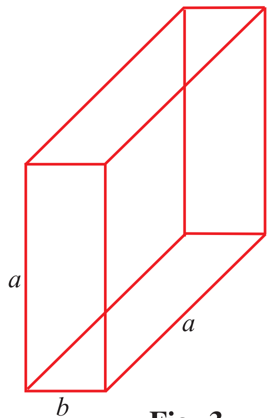

Fig. 3

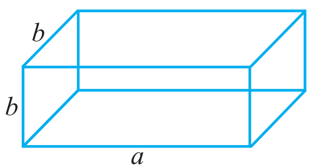

Fig. 4

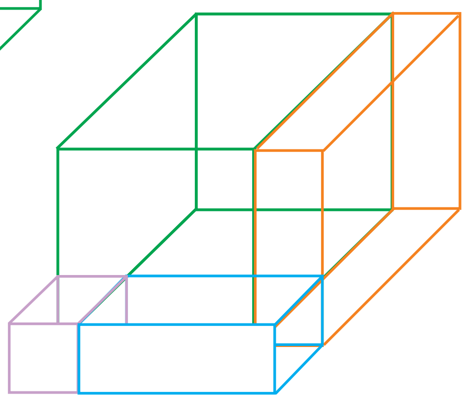

Fig. 5

Demonstration

Volume of cube in Fig. 1 = a^3

Volume of cube in Fig. 2 = b^3

Volume of cuboid in Fig. 3 = a^2b

Volume of cuboid in Fig. 4 = ab^2

Volume of solid in Fig. 5 = $a^3+b^3+a^2b+ab^2$
$= (a+b)(a^2+b^2)$

Removing cuboids of volumes a^2b and ab^2, i.e., $ab\ (a+b)$ from solid obtained in Fig. 5, we get the solid in Fig. 6.

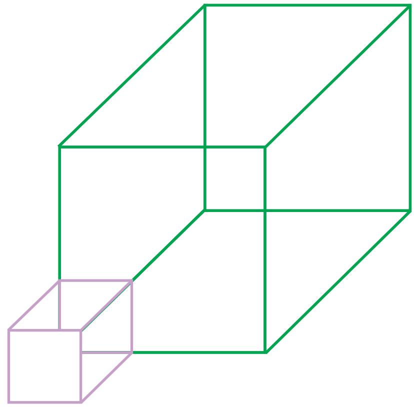

Fig. 6

Volume of solid in Fig. 6 = $a^3 + b^3$.

Therefore, $a^3 + b^3 = (a+b)(a^2+b^2) - ab\,(a+b)$

$= (a+b)(a^2+b^2-ab)$

Here, volumes are in cubic units.

Observation

On actual measurement:

a = ..., b = ...,

So, a^3 = ..., b^3 = ..., $(a+b)$ = ..., $(a+b)a^2$ = ...,

$(a+b)\,b^2$ = ..., a^2b = ..., ab^2 = ...,

$ab\,(a+b)$ = ...,

Therefore, $a^3 + b^3 = (a+b)(a^2+b^2-ab)$.

Application

The identity may be used in simplification and factorisation of algebraic expressions.

14/04/18

Activity 10

Objective

To verify the algebraic identity :

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Material Required

Acrylic sheet, sketch pen, glazed papers, scissors, adhesive, cellotape, coloured papers, cutter.

Method of Construction

1. Make a cuboid of dimensions $(a–b) \times a \times a$ $(b < a)$, using acrylic sheet and cellotape/adhesive as shown in Fig. 1.
2. Make another cuboid of dimensions $(a–b) \times a \times b$, using acrylic sheet and cellotape/adhesive as shown in Fig. 2.
3. Make one more cuboid of dimensions $(a–b) \times b \times b$ as shown in Fig. 3.
4. Make a cube of dimensions $b \times b \times b$ using acrylic sheet as shown in Fig. 4.

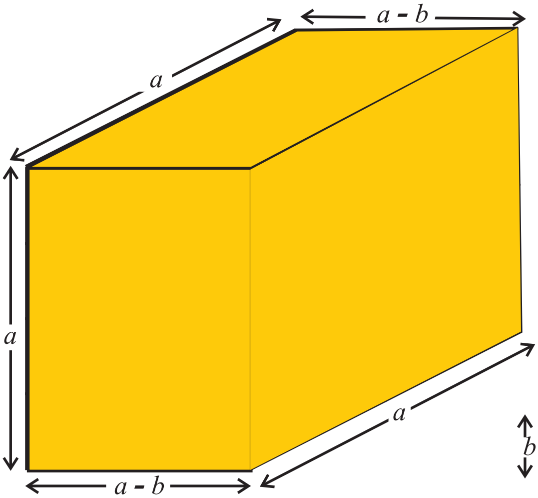

Fig. 1

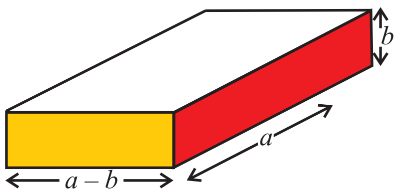

Fig. 2

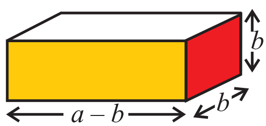

Fig. 3

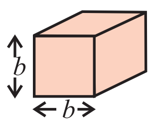

Fig. 4

5. Arrange the cubes and cuboids made above in Steps (1), (2), (3) and (4) to obtain a solid as shown in Fig. 5, which is a cube of volume a^3 cubic units.

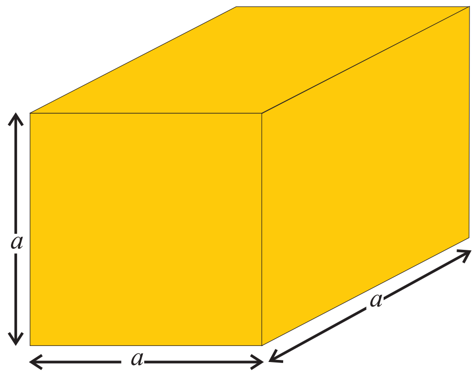

Fig. 5

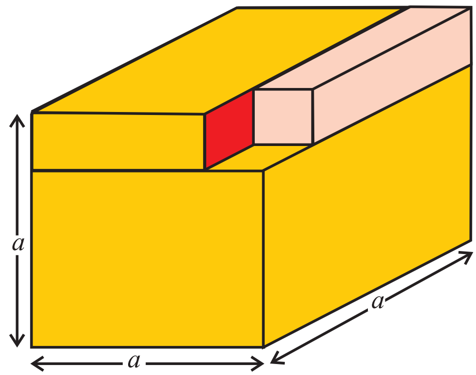

Fig. 6

Demonstration

Volume of cuboid in Fig. 1 = $(a–b) \times a \times a$ cubic units.

Volume of cuboid in Fig. 2 = $(a–b) \times a \times b$ cubic units.

Volume of cuboid in Fig. 3 = $(a–b) \times b \times b$ cubic units.

Volume of cube in Fig. 4 = b^3 cubic units.

Volume of solid in Fig. 5 = a^3 cubic units.

Removing a cube of size b^3 cubic units from the solid in Fig. 5, we obtain a solid as shown in Fig. 6.

Volume of solid in Fig. 6 = $(a–b)\ a^2 + (a–b)\ ab + (a–b)\ b^2$

$$= (a–b)\ (a^2 + ab + b^2)$$

Therefore, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Observation

On actual measurement:

a = ..., b = ...,

So, a^3 = ..., b^3 = ..., $(a-b)$ = ..., ab = ...,

a^2 = ..., b^2 = ...,

Therefore, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Application

The identity may be used in simplification/factorisation of algebraic expressions.