Class 12th Science

Summer Holiday Homework

English

1)PROJECT WORK

2) Reading Comprehension: Descriptive /Factual/ Literary Passages-05

3)Reading Comprehension: Case based passages: 05

4) Writing Assignment: Notice Writing, Article, Letter to Editor, Application for Job, Report Writing

5)Revise the following lessons/poems:

The Last Lesson

Lost Spring: Stories of Stolen Childhood

Deep Water

My Mother at Sixty Six

Keeping Quiet

The Third Level

Tiger King

Hindi

1.भक्तिन पाठ में भक्तिन और लेखिका के बीच के संबंध का वर्णन किया गया है। इस संदर्भ में भक्तिन और लेखिक के बीच के संबंध को स्पष्ट करें।

2. शीतल वाणी में आग से कवि का क्या आशय है?पाठ के आधार पर स्पष्ट कीजिए।

3.दिन ढलने के बाद लौट रहे कवि के कदम शिथिल हो जाते है । पर पंथी जल्दी जल्दी घर लौटना चाहता है। पाठ 'एक गीत' के आधार पर बताइये ,ऐसा क्यो?|

4."पतंग ' कविता में 'पतंगों के साथ-साथ वे भी उड़ रहे है ' कथन का आशय स्पष्ट करते हुए बताइए कि 'वे' किसके लिए प्रयुक्त हुआ है |

5.महादेवी ने भक्तिन के जीवन का दूसरा परिच्छेद किसे कहा है और उस परिच्छेद में भक्तिन का अनुभव कैसा था ?

6.बाज़ार दर्शन पाठ के आधार पर बताइए कि भगत जी का व्यक्तित्व बाज़ार में कैसे सार्थकता देता है ?

7.अच्छे फीचर की विशेषताओं का उल्लेख कीजिए।

8.समाचार लेखन के समय प्रायः किन प्रश्नों का जवाब देने की कोशिश की जाती है ? इन्हें किस रूप में जाना जाता है ? समाचार के मुखड़े, बॉडी और समापन में किन प्रश्नों को आधार बनाया जाता है।

9.विशेष लेखन की भाषा और शैली कैसी होनी चाहिए?

10. उल्टा पिरामिड शैली क्या है? इसके तीन भागो को अलग कीजिए।

11.पत्रकारीय लेखन क्या है?पत्रकारिता और सृजनात्मक लेखन में अंतर स्पष्ट कीजिए।

निम्नलिखित पठित पद्यांश को ध्यानपूर्वक पढ़कर उस पर आधारित दिए गए प्रश्नों के सर्वाधिक उपयुक्त उत्तर वाले विकल्प को चुनकर लिखिए-

मैं निज उर के उद्‌गार लिए फिरता हूँ

मैं निज उर के उपहार लिए फिरता हूँ

है यह अपूर्ण संसार

न मुझको भाता

मैं स्वप्नों का संसार लिए फिरता हूँ।

मैं जला हृदय में अग्नि दहा करता हूँ

सुख-दुःख दोनों में मग्न रहा करता हूँ

जग भव-सागर तरने को नाव बनाए,

मैं भव-मौजों पर मस्त बहा करता हूँ

(i)कवि के स्वप्नों का संसार है-

(क) यथार्थ

(ख) आदर्श

(ग) स्वप्निल

(घ) सुखी

(ii) निम्नलिखित कथनों पर विचार करते हुए पद्यांश के अनुसार सही कथन को चयनित कर लिखिए-(क) कवि स्वप्नों का संसार लिए फिरता हैं ।

ख) कवि जला हृदय में अग्नि दहा करता हैं

ग) कवि सुख-दुःख दोनों में मग्न रहा करता है

(घ) उपरोक्त सभी

iii) निम्नलिखित कथन और कारण को ध्यानपूर्वक पढिए और सही विकल्प चुनकर लिखिए

कथन (A) यथार्थ को स्वीकार नहीं करने से कवि के स्वप्न अधूरे रह गए हैं।

कारण (R) कल्पना और वास्तविकता में सामंजस्य की कमी होने से कवि को संसार अधूरा महसूस होता है।

कूट

(क) कथन (A) सही है, परंतु कारण (R) गलत है।

(ख) कथन (A) सही नहीं है, परंतु कारण (R) सही है।

(ग) कथन (A) तथा कारण (R) दोनों सही हैं, किंतु कारण (R) कथन (A) की सही व्याख्या नहीं करता है।

(घ) कथन (A) तथा कारण (R) दोनों सही हैं, किंतु कारण (R) कथन (A) की सही व्याख्या करता है।

iv) भव-मौजों पर मस्त बहने से तात्पर्य क्या है?

(क) संसारिक सुख रूपी लहरें

(ख) मौज-मस्ती करना

(ग) स्वप्न में उपजी सुख-दु:ख की लहरें

(घ) नाव जैसी लहने वाली भावुकता

v) काव्यांश में 'उर' से तात्पर्य है-

(क) हृदय

(ख) इच्छा

(ग) स्मृति

(घ) आवेश

IP

Chapter 1: Social Impacts

Section A – Short Answer Questions

1. Define the term Digital Footprint. Give two examples.
2. What is plagiarism? How can it be avoided?
3. Differentiate between Free Software, Open Source Software, and Proprietary Software.
4. Explain the term Cybercrime with two real-life examples.
5. What is Digital Society? Mention any two benefits and two risks.

Section B – Long Answer Questions

6. Write a short note on:

- E-waste Management

Explain the impact of IT on:

- Education
- Healthcare
- Business
- Environment

Activity Section

7. Case Study Analysis:
Read a news article about a cybercrime (e.g., identity theft, phishing, or hacking).
Write a summary (150-200 words) and explain how it could have been prevented.

Chapter 2: Networking and Internet

Section A – Short Answer Questions

1. Define the following terms:
    - MAC Address
    - Modem
2. What is the difference between LAN, MAN, and WAN?
3. Name and explain any three network devices with diagrams.

Section B – Practical/Diagram Based

5. Draw a neat labeled diagram showing different types of network topologies (Bus, Star, Ring).

Physics

PART A:- INVESTIGATORY PROJECT

S.No	Name of Project	Roll no.
1	To study various factors on which the internal resistance/EMF of a cell depends.	1,8,15,22
2	To study the variations in current flowing in a circuit containing an LDR because of a variation in (a) the power of the incandescent lamp, used to 'illuminate' the LDR (keeping all the lamps at a fixed distance). (b) the distance of a incandescent lamp (of fixed power) used to 'illuminate' the LDR	2,9,16, 23
3	To find the refractive indices of (a) water (b) oil (transparent) using a plane mirror, an equiconvex lens (made from a glass of known refractive index) and an adjustable object needle	3,10,17,24
4	To investigate the relation between the ratio of (i) output and input voltage and (ii) number of turns in the secondary coil and primary coil of a self-designed transformer.	4,11,18,25
5	To investigate the dependence of the angle of deviation on the angle of incidence using a hollow prism filled one by one, with different transparent fluids.	5,12,19,26
6	To estimate the charge induced on each one of the two identical Styrofoam (or pith) balls suspended in a vertical plane by making use of Coulomb's law	6,13,20,27
7	To study the factor on which the self-inductance of a coil depends by observing the effect of this coil, when put in series with a resistor/(bulb) in a circuit fed up by an A.C. source of adjustable frequency.	7,14,21,28

PART B: Easy and Average Level Questions

Revise all the notes from your class notebook and practice examples of NCERT along with the question from homework notebook.
Complete your notes upto the chapter 3.

PART C: Competency based questions: Electrostatic, current electricity units)

PDFs of the questions attached here with.
Thank you and have a safe and happy summer vacation.

Chemistry

PART A:- INVESTIGATORY PROJECT

S.No	Name of Project	Roll no.
1	Corrosion	1,8,15,22
2	Study of quantity of casein present in different samples of milk	2,9,16, 23
3	To study the oxalate ion content in Guava fruit.	3,10,17,24
4	Preparation of Soyabean milk and it's comparison with the natural milk	4,11,18,25
5	Comparative study and qualitative analysis of different brands of cold drinks	5,12,19,26
6	Determination of caffeine in tea sample	6,13,20,27
7	Study of adulterants in food stuffs	7,14,21,28

PART B: Easy and Average Level Questions

Revise NCERT (Examples, In text and Exercise Questions) and CBSE questions (2023,2024,2025) of Solution, Electrochemistry, Chemical Kinetics & d and f block elements.
Write and learn all the formulas of chapter Solution, Electrochemistry & Chemical Kinetics

PART C: Competency Based Questions (SOLUTIONS, ELECTROCHEMISTRY,CHEMICAL KINETICS & D and F BLOCK ELEMENTS)

Solve the competency based questions of these chapters which are already shared with you.

Mathematics

RELATION AND FUCNTIONS

Relations

Ordered Pair: A pair of objects listed in a specific order separated by comma and enclosed in parenthesis is called an *ordered pair i.e. (a, b)* is an order pair with 'a' as first element and *'b*' as second element.

Two order pairs are equal if the corresponding elements are equal i.e. *(a, b) = (x, y) if and only if a = x and b = y.*

Cartesian product of sets: Let A and B be two non-empty sets then the set of all ordered pairs *(a, b) for* $a \in A, b \in B$ is called the Cartesian product of set A with set B. it is denoted as $A \times B$

Symbolically, $A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$

Note that $A \times B \neq B \times A$

For $A = \{1,2,3\}$, $B = \{3,4\}$; $A \times B = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$

Q1. (i) For set $A = \{1,2,3\}, B = \{2,3\}$ find $A \times B$.

(ii) $A = \{0,1,2,3\}$ find $A \times A$

(iii) $A = \{-1,0,1,2\}, B = \{0,2,3\}$ find $B \times A$

(iv) $B = \{0,2,3\}$ find $B \times B$

(v) $A = \{-1,0,1,2\}, B = \{\ \}$ find $A \times B$

Relation: A relation *R* from a non-empty set *A* to non-empty set *B* is a subset of the Cartesian product $A \times B$. Thus R is a relation from *A* to *B* $\Leftrightarrow R \subseteq A \times B$.

If R is a relation from *A* to *B* and if $(x, y) \in A \times B$ then we write xRy and say that *x* is related to *y* .

The total number of relations that can be defined from set A to set B is the number of possible subset of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = p.q$ and total number of relations from *A* to *B* will be 2^{pq} .

Domain: The set of all first components of the ordered pairs in a relation *R* from set *A* to set *B* is called the domain of the relation *R*.

Thus, domain of $R = \{x : x \in A, (x, y) \in R, for\ some\ y \in B\}$

Range: The set of all second components of the ordered pairs in a relation *R* from set *A* to set *B* is called the range of *R*.

Thus, range of $R = \{x : x \in A, (x, y) \in R, for\ some\ y \in B\}$

Co-domain: If *R* is a relation from set *A* to set *B*, then *B* is called the co-domain of *R*.

Note that $Range \subseteq Co - domain$

Example: for $A = \{2,3,5,8,9\}$ and $B = \{4,6,7,15,16,20\}$

And relation *R* from set *A* to set *B* defined as "*is divisor of*"

$R = \{(2,4),(2,6),(2,16),(2,20),(3,6),(3,15),(5,15),(5,20),(8,16)\}$

Now Domain of $R = \{2,3,5,8\}$; Range of $R = \{4,6,15,16,20\}$;

Co-domain of $R = \{4,6,7,15,16,20\} = B$ Hence $Range \subseteq Co - domain$

Inverse of a relation: If *R* be a relation from set *A* to set *B* then the inverse relation of *R* denoted by R^{-1} is a relation from *B* to *A* defined by

$$R^{-1} = \{(y, x) : (x, y) \in R\}$$

Let *A* be any non-empty set, then any subset of $A \times A$ is called a **relation** of *A.*

Q2. Write the relation R, its domain, range and inverse for the following

(i) $R = \{(a,b) : a + b\ is\ odd\}$ on $A = \{1,2,3,4\}$

(ii) $R = \{(a,b) : a + b\ is\ even\}$ on $A = \{2,4,6\}$

(iii) $R = \{(a,b) : a - b = 0\}$ on $A = \{1,2,3,4,5,6,7\}$

(iv) $R = \{(a,b) : a\ divides\ b\}$ on $A = \{1,2,3,4,5\}$

(v) $R = \{(a,b) : both\ a\ and\ b\ are\ either\ even\ or\ odd\}$ on $A = \{1,2,3,4,5\}$

(vi) $R = \{(x, y) : 2x - y = 0\}$ on set of $A = \{1,2,3,4,5,6,7, - - - - - -, 19\}$

(vii) $R = \{(x, y) : 2x + y \le 7\}$ on set of $A = \{1,2,3\}$

(viii) $R = \{(x, y) : |x - y|\ is\ multiple\ of\ 4\}$ on set of $A = \{1,2,3,4,5,6,7,8,9,10\}$

(ix) $R = \{(x, y) : |x - y|\ is\ an\ \text{integer}\}$ on set of set of integers.

(x) $R = \{(x, y) : x + 2y = 10,\ x, y \in N\}$

(xi) $R = \{(x, y) : |x - y| \le 5,\ x, y \in Z\}$

(xii) Let $A = \{1,,2,3,4,5,6,7,8,9\}$. Let R_1 be a relation on A defined as

$R_1 = \{(x, y) : x - y\ is\ divisible\ by\ 3\}$ and R_2 be another relation on A defined as

$R_2 = \{(x, y) : (x, y) \subset \{1,4,7\}\ or\ (x, y) \subset \{2,5,8\}\ or\ (x, y) \subset \{3,6,9\}\}$

Show that $R_1 = R_2$

Types of Relations

Reflexive relation: Let *R* be a relation on set *A*, then *R* is called a reflexive relation if every element of *A* is related to itself i.e. *R* is reflexive if $(x, x) \in R$ for all $x \in A$.

Example: 1. Let $A = \{1,2,3\}$, then $R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ is reflexive.

2. Let $A = \{2,3,4\}$ and relation R on set A defined as *aRb iff a* divides *b* is reflexive.

Q3. Show that following relations are not reflexive:

(i) $R=\{(1,2),(2,1)\}$ on set of $A=\{1,2,3\}$.

(ii) $R=\{(1,1),(2,1)\}$ on set of $A=\{1,2,3\}$.

(iii) $R=\{(1,1),(2,2)\}$ on set of $A=\{1,2,3\}$.

(iv) $R=\{(1,1),(2,2),(1,2)\}$ on set of $A=\{1,2,3\}$.

(v) $R=\{(x,y):2x-y=0\}$ on set of $A=\{1,2,3\}$.

(vi) $R=\{(x,y):x+y\ is\ odd\}$ on set of $A=\{0,1,2,3,4\}$.

(vii) $R=\{(x,y):x+y<6\}$ on set of $A=\{1,2,3\}$.

(viii) $R=\{(x,y):2x+y\le 7\}$ on set of $A=\{1,2,3\}$.

(ix) $R=\{(x,y):x+1=y\}$ on set of $A=\{1,2,3,4,5,6\}$.

(x) $R=\{(x,y):y=x+5,x<4,x\in N\}$.

Q4. Show that following relations are reflexive

(i) $R=\{(1,1),(2,2),(3,3)\}$ on set of $A=\{1,2,3\}$.

(ii) $R=\{(a,b):a=b\}$ on set of $A=\{1,2,3,4,5,6,7,8,9,10\}$.

(iii) $R=\{(T_1,T_2):T_1\sim T_2\ \ with\ \ T_1,T_2\in A\}$ where A is set of triangle in a plane.

(iv) $R=\{(l_1,l_2):l_1\ \ is\ \ parallel\ \ to\ l_2\}$ on set of lines in a plane.

(v) $R=\{(x,y):|x-y|\ is\ divisible\ by\ 3\}$ on set of integers .

(vi) $R=\{(x,y):x+y\ is\ even\}$ on set of $A=\{1,2,3,4,5,6,7,8,9,10\}$.

(vii) $R=\{(x,y):x+y>1\}$ on set of natural numbers.

(viii) $R=\{(x,y):2x+y\le 11\}$ on set of $A=\{1,2,3\}$.

(ix) $R=\{(x,y):\ y\ is\ divisible\ by\ x\}$ on set of $A=\{1,2,3,4,5,6\}$.

(x) $R=\{(x,y):x\le y^2\}$ on set of natural numbers .

Symmetric Relation: A relation on a set A is said to be symmetric *iff*

$$(x,y)\in R\ \Rightarrow (y,x)\in R\ \ ,\forall x,y\in A$$

Note: If $R=R^{-1}$ then R is symmetric.

Example: 1. Let $A=\{1,2,3\}$, then $R=\{(1,2),(2,1)\}$ is symmetric.

2. Let $A=\{a,b,c,d\}$ and let $R_1=\{(a,a),(b,b),(c,c),(a,d),(d,a)\}$ and

$R_2=\{(a,a),(b,b),(c,c),(d,d),(d,a)\}$ are relation on se A then R_1 is symmetric but

R_2 is not symmetric.

Q5. Show that following relations are not symmetric

(i) $R=\{(a,b):a\le b^2,a,b\ are\ real\ numbers\}$

(ii) $R=\{(1,1),(2,2),(1,2)\}$ on set of $A=\{1,2,3\}$

(iii) $R = \{(x, y) : 2x - y = 0\}$ on set of $A = \{1, 2, 3\}$

(iv) $R = \{(x, y) : 2x + y \leq 7\}$ on set of $A = \{1, 2, 3\}$

(v) $R = \{(x, y) : x + 1 = y\}$ on set of $A = \{1, 2, 3, 4, 5, 6\}$

(vi) $R = \{(x, y) : y = x + 5, x < 4, x \in N\}$

(vii) $R = \{(x, y) : x + 2y = 10,\ x, y \in N\}$.

(viii) $R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (2,1), (3,2)\}$ on $A = \{1, 2, 3\}$.

(ix) $R = \{(x, y) : x - y \ \ is\ \ positive\}$ on set of $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$.

(x) $R = \{(a, b) : a\ \ divides\ \ b\}$ on $A = \{1, 2, 3, 4, 5\}$

(xi) $R = \{(a, b) : a > b\ \ where\ a, b\ \ are\ real\ \ numbers\}$

(xii) $R = \{(A, B) : set\ \ A\ is\ subset\ of\ set\ B\}$ on the sets of all possible subsets of

$U = \{1, 2, 3, 4, 5, 6\}$.

Q6. Show that following relations are symmetric

(i) $R = \{(x, y) : 1 + xy > 0\}$ on $A = \{-2, -1, 0, 1, 2, 3\}$.

(ii) $R = \{(T_1, T_2) : T_1 \sim T_2 \ \ with\ \ T_1, T_2 \in A\}$ in set of all triangles in a plane.

(iii) $R = \{(x, y) : x + y\ \ is\ \ odd\}$ on set of $A = \{1, 2, 3\}$

(iv) $R = \{(P_1, P_2) : P_1\ \ and\ \ P_2\ \ have\ \ same\ number\ of\ \ sides\}$ on set of all polygons.

(v) $R = \{(l_1, l_2) : l_1\ \ and\ \ l_2\ \ are\ \ parallel\}$ on set of all lines in a plane.

(vi) $R = \{(x, y) : x\ \ and\ y\ have\ same\ number\ of\ \ pages\}$ on set of all books.

(vii) $R = \{(x, y) : x + y\ \ is\ \ even\}$ on set of $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(viii) $R = \{(x, y) : |x - y|\ \ is\ divisible\ by\ \ 3\}$ on set of $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(ix) $R = \{(T_1, T_2) : T_1 \cong T_2 \ \ with\ \ T_1, T_2\ \ are\ triangles\ in\ a\ plane\}$

(x) $R = \{(T_1, T_2) :\ T_1, T_2\ \ are\ triangles\ with\ equal\ \ area\ \ in\ a\ plane\}$

Transitive Relation: A relation on set *A* is said to be transitive *iff*

$(x, y) \in R\ \&\ (y, z) \in R\ \Rightarrow (x, z) \in R\ , \forall x, y, z \in A$

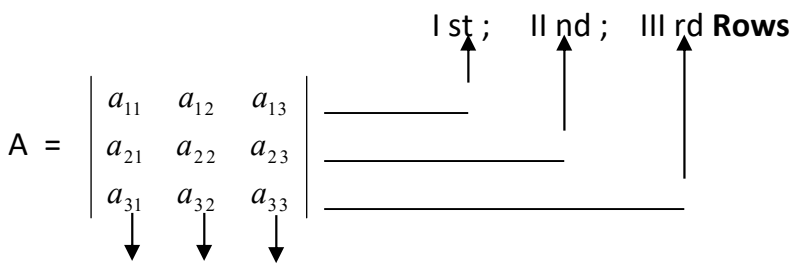
i.e. $xRy\ \ \&\ \ yRz\ \ \Rightarrow\ xRz$

D E T E R M I N A N T S

Every square matrix can be associated to a number, which is known as its determinant. If A = (a_{ij}) is a square matrix having n number of rows and columns, then determinant of A is denoted by $|A|$, e.g.

A = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is a determinant of order 3.

Rows and columns of the determinant: For the determinant A



Position of a point: a_{ij} represent element in 'i' th row and 'j' th column, e.g.

a_{23} is in 2nd row and 3rd column

a_{32} is in 3rd row and 2nd column.

Order of a determinant: Order 1 $A = |a_{11}|$ (one row and one column)

Order 2 $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}.a_{22} - a_{21}.a_{12}$ (Two rows and two columns)

Order 3 $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}C_{11} + a_{12}.C_{12} + a_{12}.C_{13}$ (Three rows and three columns)

Value of the determinant of order 3 can be obtained by calculating the algebric sum of products of any one row / column with corresponding co-factors.

Minor of an element: Minor of a_{ij} in matrix A is denoted by M_{ij} and is the determinant of order (n – 1) obtained by leaving the i th row and j th column of A. For example

In matrix $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$; $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ and $M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$

Co-Factor Of An Element: Co-factor of a_{ij} in matrix A is denoted by C_{ij} and its value is given by $C_{ij} = (-1)^{i+j} M_{ij}$. For example in A = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, $C_{11} = (-1)^2 M_{11} = d$ and $C_{21} = (-1)^3 M_{21} = -b$

Important Results : (i) If A is square matrix of order n and A = k B then $|A| = k^n |B|$

(ii) If A is non singular matrix of order n and then $|adj\ A| = |A|^{n-1}$

Applications of determinants

1. **Area of triangle**: For a triangle having co-ordinates of vertices as $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3).

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Cor. Points (x_1, y_1) (x_2, y_2) and (x_3, y_3).are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

2. **<u>Important results</u>:** 1. Square matrix A is singular then $|A| = 0$.

2. If A is a square matrix of order n, then $|kA| = k^n |A|$

3. If A is a non- singular matrix of order n, then $|Adj\ A| = |A|^{n-1}$

4. If A is a square matrix , then $|A^{-1}| = \frac{1}{|A|}$

5. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ then $A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$ 6. If $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ then $A^n = \begin{bmatrix} a^n & 0 \\ 0 & a^n \end{bmatrix}$

7. If A is a skew-symmetric matrix of order n , then $(A^n)^T = (-1)^n A^n$

3. **Adjoint of a square Matrix** is transpose of matrix obtained by replacing each element of it by its cofactors and is denoted by adj(A)

<u>Property</u> of Adjoint: (Adj A). A = A. (Adj A) = $|A|$ I

4. **Singular Matrix** is the square matrix whose determinant is zero otherwise it is called non-singular matrix.

Note: Inverse of only nonsingular matrices exists.

5. **Inverse of a square matrix:** A non-zero square matrix A of order n is called invertible matrix if there exists a square matrix B of order n such that AB = BA = I and B is called the inverse of A and vice versa. The inverse of A is denoted by A^{-1}.

<u>Properties</u>:(1) $A.A^{-1} = A^{-1}.A = I$ (2) $(AB)^{-1} = B^{-1}A^{-1}$ (3) $(A')^{-1} = (A^{-1})'$ (4) $Adj(A^{-1}) = (AdjA)^{-1}$

6. **Solution of equations:** For the equations

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

Equations in matrix form can be expressed as AX = B with

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

(1) If $|A| \neq 0$, then the system of equations is consistent and has a unique solution given by $X = A^{-1} B$

(2) If $|A|$ = 0 and also (adj A) .B = 0, then the system of equations is consistent and has infinite many solutions.

(3) If $|A|$ = 0 and also (adj A) .B $\neq$ 0, then the system of equations is inconsistent and has no solution.

ASSIGNMENT (DETERMINANTS)

Q 1. Evaluate each of the determinant

(1) $\begin{vmatrix} 3 & 2 \\ 1 & 7 \end{vmatrix}$ (2) $\begin{vmatrix} -3 & 5 \\ 1 & 2 \end{vmatrix}$ (3) $\begin{vmatrix} Sin\,A & Cos\,A \\ -Cos\,A & Sin\,A \end{vmatrix}$

(4) $\begin{vmatrix} -i & 1 \\ -1 & i \end{vmatrix}$ (5) $\begin{vmatrix} 2 & 4 \\ 6 & 10 \end{vmatrix}$ (6) $\begin{vmatrix} 1 & 3 \\ -2 & -5 \end{vmatrix}$

(7) $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ (8) $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Q 2. Find the cofactors of all elements and value of determinants

(1) $\begin{vmatrix} -1 & 3 & 2 \\ 0 & 4 & 5 \\ 1 & -6 & 7 \end{vmatrix}$ (2) $\begin{vmatrix} 2 & -5 & 7 \\ 3 & 2 & 1 \\ 5 & 0 & -4 \end{vmatrix}$ (3) $\begin{vmatrix} 3 & 2 & 9 \\ -7 & 1 & -5 \\ 2 & -4 & 6 \end{vmatrix}$

(4) $\begin{vmatrix} 9 & -2 & 5 \\ 7 & 0 & 1 \\ 9 & -1 & 4 \end{vmatrix}$ (5) $\begin{vmatrix} 1 & 3 & 7 \\ -2 & 5 & 0 \\ 1 & 9 & -6 \end{vmatrix}$ (6) $\begin{vmatrix} 0 & 1 & 2 \\ 7 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$

Q 3. Find the area of the triangle whose vertices are

(1) (4, -2), (-6, 2), (4,5) (2) (1, 3),(4, 9),(-2, -3)

(3) (-3, 5), (3, -6), (7, 2) (4) (1,-3),(-4,2), (1,5)

(5) (3, 8), (-4, 2), (5, -1) (6) (0, 0), (6, 0), (4, 3)

Q 4. If the given points are collinear

(1) (a, 0),(0, b),(1, 1) show that a + b = a. b

(2) $(a,b),(a',b'),(a-a',b-b')$ show that $a.b' = a'.b$

(3) (a, 0),(0, b),(3.a, c) show that c = -2.b (4) (a, 0),(0, b),(p, q) show that $\frac{p}{a}+\frac{q}{b}=1$

Q 5. Find k so that given points are collinear

(1) (5, 5),(k,1),(11,7) (2) (1,-5),(-4,5),(k,7) (3) (3,-2),(k, 2),(8,8)

Q 6. Find k so that triangle with vertices

(1) (k, 4),(2, -6),(5,4) has area 70 units. (2) (2,3), (1, 1), (0, k) has area 2 units.

Q 7. Show that following points lie on a line

(1) (a, b + c),(b, c + a),(c, a + b) (2) $(a, a^2 - b.c),(b, b^2 - c.a),(c, c^2 - a.b)$

Q 8. Solve using the matrix method

(1) $4x - 3y = 11$, $3x + 7y = -1$

(2) $x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$

(3) $x + y = 5$, $y + z = 3$, $z + x = 4$

(4) $\frac{1}{x} + \frac{2}{y} = 2$, $\frac{3}{x} - \frac{8}{y} = -1$

Q 9. (1) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$. Find AC and hence solve the equations

$x - 2y = 10$, $2x + y + 3z = 8$, $-2y + z = 7$.

(2)If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$.Verify that AB = 6 I. Hence solve

$x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$

(3)Find the product of matrices $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$ and $\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$ and use it to solve the equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

(4) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the equations

$x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$

(5) Find A^{-1} for $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the equations

$x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$

(6) If $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \\ 1 & 2 & 0 \end{bmatrix}$, find A^{-1}. Using A^{-1}solve the equations

$2x - y + z = 2$, $3x - z = 2$, $x + 2y = 3$

(7) If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 2 & 0 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}solve the equations

$x - 2y + z = 0$, $-y + z = -2$, $2x - 3z = 10$

(8) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$., find A^{-1} and hence solve

$x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$.

Q 10. (1) The sum of three numbers is 6. If 3rd number is multiplied by 2 and first number is added, we get 7 and by adding second and third number to 3 times the first number we get 12. Use matrix method to find the numbers.

(2) The perimeter of a triangle is 90 cm. The longest side exceeds the shortest side by 16 cm and the sum of the lengths of the longest and shortest side is twice the length of the other side. Use the matrix method to find the sides of the triangle.

(3) Find the three numbers using matrix method such that sum of first and second is 5, sum of second and third is 3 and that of third and first is 4.

Q 11. (1) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ Verify that $A^2 - 4A + I = 0$. Hence find A^{-1}.

(2) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ Verify that $A^2 - 4A + 7 = 0$. Hence find A^{-1}.

(3) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Verify that $A^2 - 4A - 5I = 0$. Hence find A^{-1}.

(4) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Verify that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.

(5) If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ Verify that $A^2 - 7A + 10 = 0$. Hence find A^{-1}.

Q 12. Verify that $(AB)^{-1} = B^{-1}A^{-1}$

(1) $A = \begin{bmatrix} 7 & 5 \\ -9 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ (2) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

Q 13. Find $(AB)^{-1}$

(1) If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -7 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ (2) If $A^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

(3) If $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 5 & 0 \\ 2 & 3 \end{bmatrix}$ (4) If $A^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Q 14. (1) Find A^{-1} for $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$. Hence verify that $A^2 - 4A - I = 0$.

(2) Find A^{-1} for $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$. Hence verify that $A^2 - 4A - 5 = 0$.

(3) Find A^{-1} for $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$. Hence verify that $A^2 + 4A - 42 = 0$

(4) Find A^{-1} for $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$. Hence verify that $A^3 - A^2 - 3A - I = 0$.

(5) Find A^{-1} for $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$. Hence verify that $A^2 - 3A - 7I = 0$

(6) Find A^{-1} for $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$. Hence verify that $A^2 + 4A - 5I = 0$

Q 15. Find A (adj A) without finding (adj A) for

(1) $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ (2) $A = \begin{bmatrix} 1 & 2 & 4 \\ 5 & 3 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ (3) $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

(4) $A = \begin{bmatrix} 1 & 2 \\ 6 & 8 \end{bmatrix}$ (5) $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ (6) $A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$

ANSWERS

Q1. (1) 19, (2) –11, (3) 1, (4) 2, (5) –4, (6) 1 (7) 1

Q2. (1) -51, (2) -179, (3) 256, (4) 12 (5) –157, (6) 4

Q7. (1) $\frac{2}{3}, \frac{11}{3}$, (2) 1,2,-3 (3) 4, (4) -1, 2, -10 (5) 4 , (6) 3a,0

(7) 0, - (a + b + c) (9) 8 (10) 1, $-\frac{7}{4}$ (11) $\pm\sqrt{3}$ (12) 2 (13) $\pm$ 6

Q8. (1) 35, (2) 0, (3) 46, (4) 20, (5) 37.5 (6) 9

Q10. (1) -7, (2) -5, (3) 5

Q11. (1) –9, 19 (2) - 5, 3

Q13. (1) x = 2, y = -1 (2) x = 3, y = -2, z = 1 (3) x = 3, y = 2, z = 1 (4) x = 1, y = 2

Q14. (1) AC = 11 I, x = 4, y = -3, z = 1 (2) x = 2, y = -1, z = 4

(3) AC = 1200 I, x = 2, y = 3, z = 5 (4) AB = 8 I, x = 3, y = -2, z = -1

(5) $A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$, x = 2, y = 3, z = 5 (6) $A^{-1} = \frac{1}{11}\begin{bmatrix} 2 & 2 & 1 \\ -1 & -1 & 5 \\ 6 & -5 & 3 \end{bmatrix}$, x = 1, y = 1, z = 1

(7) $A^{-1} = \begin{bmatrix} 3 & -6 & -1 \\ 3 & -5 & 1 \\ 3 & -4 & -1 \end{bmatrix}$, x = 2, y = 0, z = -2 (8) $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$, x = 9/5, y = 2/5, z =7/5

Q15. (1) x = 3, y = 2, z = 1 (2) x = 22, y = 30, z = 38 (3) x = 3, y = 2, z = 1

Q16. (1) $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ (2) $\frac{1}{7}\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$ (3) $\frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$ (4) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ (5) $\frac{1}{10}\begin{bmatrix} 4 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Q17.

Q18. (1) $\begin{bmatrix} 6 & -33 & 5 \\ -2 & -9 & 0 \\ 1 & -25 & 2 \end{bmatrix}$ (2) $\frac{1}{4}\begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$ (3) $\frac{1}{14}\begin{bmatrix} 5 & 15 \\ 14 & 0 \end{bmatrix}$ (4) $\frac{1}{3}\begin{bmatrix} 5 & 4 \\ -4 & 1 \end{bmatrix}$

Q19.

Q20. (1) -14 I (2) 23 I (3) I (4) - 4 I (5) 4 I (6) I

ONE MARK PROBLEMS

1. Find k so that $\begin{bmatrix} 2 & 3 \\ 4 & k \end{bmatrix}$ is singular.

2. If A = $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ find $|adj\ A|$

3. If A .(Adj A) = $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$,find $|A|$

4. If A is a square matrix of order 3 and $|A|$ = 3, find $|2A|$

5. If A is a non- singular matrix of order 3 and $|A|$ = 5 ,evaluate $|Adj\ A|$

6. If A is a non- singular matrix of order 3 and $|A|$ = 10, Evaluate $|A^{-1}|$

7. If A = $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ then A^5

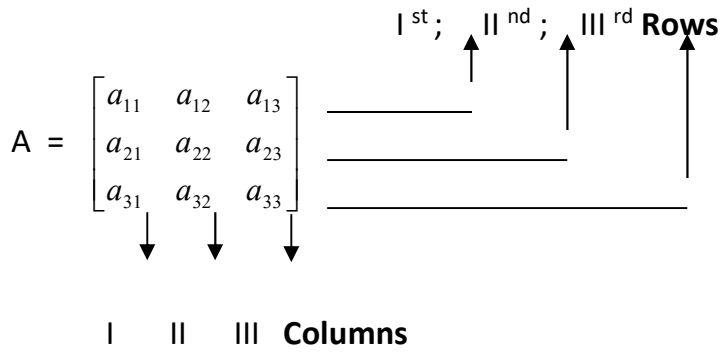
8. If A = $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ then A^4

M A T R I C E S

Matrix is a rectangular array of numbers, real or complex kept inside braces [ ] or ( ) .

The numbers are called elements or members of matrix. Matrices are denoted by capital letters e.g. A, B, C etc. In compact form, matrix A is denoted as $A = [a_{ij}]_{m \times n}$

Rows and columns of a matrix: For the matrix A



Order of a matrix: A matrix having "m" number of rows and "n" number of columns is of order "m $\times$ n". e.g.

Matrix having 2 rows and 3 columns is of order " 2×3"

Matrix having 4 rows and 2 columns is of order " 4×2"

Position of an element in a matrix: The element "a_{ij}" in a matrix A is located at "i^{th}" row and "j^{th}" column. e.g. in matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 6 & 5 \end{bmatrix}$ $a_{12} = 3$, $a_{22} = 6$, $a_{23} = 5$

Types of matrices:

1. **Row matrix:** matrix having only one row is called row-matrix and is of order "$1 \times n$".

   e. g . $A = \begin{bmatrix} 2 & -1 & 8 \end{bmatrix}_{1 \times 3}$

2. **Column matrix:** Matrix having only one column is called column-matrix and is of order "$m \times 1$".

   e. g. $B = \begin{pmatrix} 2 \\ 4 \end{pmatrix}_{2 \times 1}$

3. **Zero matrix:** Matrix having all elements 0 is called zero matrix and is denoted by "O".

   e. g. $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2 \times 3}$, $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2 \times 2}$

4. **Square matrix:** Matrix having same number of rows and columns say "n" is called square matrix of order "n". e. g.

$A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ is a square matrix of order "2" $B = \begin{bmatrix} 2 & 5 & 9 \\ 7 & 5 & 1 \\ 6 & 7 & 8 \end{bmatrix}$ is a square matrix of order "3"

5. **Diagonal matrix:** A square matrix $A = [a_{ij}]$ of order "n" is called diagonal matrix if all the elements except those in leading diagonal are zero i. e. $a_{ij} = 0$ for $i \neq j$ and $a_{ij} \neq 0$ for $i = j$

   e. g. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is a diagonal matrix of order 3.

   Also diagonal matrix can be expressed as A = diag (2,5,-1)

6. **Scalar matrix:** Square matrix $A = [a_{ij}]_m$ of order "m" is called scalar if $a_{ij} = 0$, $i \neq j$ & $a_{ij} = k$, $i = j$ e. g. $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is a scalar matrix of order 2.

7. **Identity or unit matrix:** Square matrix $A = [a_{ij}]_m$ of order "m" is called identity or unit matrix if $a_{ij} = 0, i \neq j$ & $a_{ij} = 1, i = j$. It is denoted by I e. g.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a identity matrix of order 2 and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is identity matrix of order 3.

Properties of Identity Matrices: (1) $A.I = I.A = A$ (2) $A.A^{-1} = A^{-1}.A = I$

Operations on matrices:

8. **Equality of matrices:** Matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if
   (i) both are of same order.
   (ii) $a_{ij} = b_{ij}$ for all i, j i.e. corresponding elements of both matrices are same.

   If A , B are equal, it is denoted by A = B other wise $A \neq B$

9. **Addition of matrices:** Only two matrices of same order can be added or subtracted by adding or subtracting corresponding elements of matrices.

For matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$, C = A + B with $C = [c_{ij}]_{m \times n}$ and $c_{ij} = a_{ij} + b_{ij}$ for all i, j.

Properties (1) A + B = B + A (2) A + (B + C) = (A + B) + C

(3) A + O = O + A = A (4) A + (- A) = (- A) + A = 0

10. **Multiplication of matrix with scalar:** For $A = [a_{ij}]_{m \times n}$ and scalar "k" then multiplication of A with k is denoted by "kA" and $kA = [k.a_{ij}]_{m \times n}$ e. g.

For $A = \begin{bmatrix} -2 & 6 & 7 \\ 4 & 3 & 1 \end{bmatrix}$, $2A = \begin{bmatrix} -4 & 12 & 14 \\ 8 & 6 & 2 \end{bmatrix}$ and $-3A = \begin{bmatrix} 6 & -18 & -21 \\ -12 & -9 & -3 \end{bmatrix}$

11. **Multiplication of matrices:** Matrix A can be multiplied to matrix B if number of columns of A is same as number of rows of B. If $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times p}$ are two matrices then $A.B = [c_{ik}]_{m \times p}$ and $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

Properties: (1) $A.B \neq B.A$ (2) A.(B.C) = (A.B).C

(3) A.(adjA) = (adjA).A = $|A|.I$ (4) A.I = I.A = A for square matrix A.

12. **Positive integral power:** For square matrix $A = [a_{ij}]$ of order "n", $A^1 = A$ & $A^{n+1} = A^n$. A i. e. $A^2 = A.A$ and $A^3 = A^2.A$

13. Transpose of a matrix: For matrix A = $[a_{ij}]_{m\times n}$, transpose of A is denoted by A^T or A' and $A^T=[a_{ji}]_{m\times n}$ e.g.

$A = \begin{bmatrix} 1 & 2 \\ 9 & 4 \\ 2 & 3 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 9 & 2 \\ 2 & 4 & 3 \end{bmatrix}$

Properties of transpose of matrix:

(1) $(A')' = A$, (2) $(A \pm B)' = A' \pm B'$

(3) $(k A)' = k.A'$ (4) $(A.B)' = B'\ A'$

14. Symmetric matrix: Square matrix A = $[a_{ij}]$ is symmetric if $A' = A$ i.e. $a_{ij} = a_{ji}$ e.g. A =

$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & 4 \\ 3 & -4 & 5 \end{bmatrix}$

15. Skew-symmetric matrix: Square matrix A = $[a_{ij}]$ is skew-symmetric if $A' = -A$ i.e.

$a_{ij} = -a_{ji}$ for all i, j e. g. A = $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

16. Square matrix as sum of two matrices: For a square matrix A, A = P + Q where P is symmetric and Q is skew- symmetric.

Here P = $\frac{1}{2}(A + A')$ and Q = $\frac{1}{2}(A - A')$

ASSIGNMENT (MATICES)

Q 1. Write all the possible orders of matrices having

(1) 12 elements (2) 7 elements (3) 9 elements

(4) 1 element (5) 11 elements (6) 6 elements

Q 2. Construct 2×3 matrix A = $[a_{ij}]$ whose elements a_{ij} is given by

(1) $a_{ij} = \frac{(2i+j)^2}{3}$ (2) $a_{ij} = \frac{|-3i+j|}{2}$ (3) $a_{ij} = -4(i-j)$

(4) $a_{ij} = \begin{cases} i+j, & i>j \\ 2i, & i=j \\ 3j, & i<j \end{cases}$ (5) $a_{ij} = \frac{i}{2} + j$ (6) $a_{ij} = \frac{i+2j}{3}$

Q 3. Find the value of variables in each of the cases

(1) $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ (2) $\begin{bmatrix} x-y & z & -2 \\ 4 & x & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$

(3) $\begin{bmatrix} 2a+b & a-b \\ a-c & a+b+c \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 2 & 8 \end{bmatrix}$ (4) $\begin{pmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 11 & 24 \end{pmatrix}$

Q4. Find X and Y if

(1) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$, $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$ (2) $Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$, $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

(3) $2X+3Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$, $3X+2Y=\begin{bmatrix}-2 & 2\\ 1 & -5\end{bmatrix}$

(4) $2X+Y=\begin{bmatrix}4 & 4 & 7\\ 7 & 3 & 4\end{bmatrix}$, $X-2Y=\begin{bmatrix}-3 & 2 & 1\\ 1 & -1 & 2\end{bmatrix}$

Q5. Find X so that for $A=\begin{bmatrix}1 & 2 & 0\\ -7 & 2 & 6\end{bmatrix}$ and $B=\begin{bmatrix}4 & 3 & 2\\ 0 & 1 & 3\end{bmatrix}$

(1) $A+B+X=0$ (2) $A-B-X=0$ (3) $A+2B=3X$

(4) $2A+B-4X=0$ (5) $2A+3X=5B$ (6) $A-2B+X=0$

Q6. (1) If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ show that $F(x).F(y)=F(x+y)$

(2) If $A=\begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}$ show that $I+A=(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Q7. Find A. B if defined when

(1) $A=\begin{bmatrix}1\\ 2\end{bmatrix}$, $B=\begin{bmatrix}2 & 5\end{bmatrix}$ (2) $A=\begin{bmatrix}1 & 5 & 7\\ 0 & 6 & 8\end{bmatrix}$, $B=\begin{bmatrix}7 & 8\\ 9 & 8\\ 6 & 5\end{bmatrix}$

(3) $A=\begin{bmatrix}4\\ 0\\ 6\end{bmatrix}$, $B=\begin{bmatrix}1 & 3 & 4\end{bmatrix}$ (4) $A=\begin{bmatrix}i & 0\\ 0 & -i\end{bmatrix}$, $B=\begin{bmatrix}0 & i\\ i & 0\end{bmatrix}$ (5) $A=\begin{bmatrix}2 & 3 & 6\end{bmatrix}$, $B=\begin{bmatrix}1\\ 2\\ 6\end{bmatrix}$

Q8. Find the square of the following matrices:

(1) $A=\begin{bmatrix}0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}$ (2) $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ (3) $A=\begin{bmatrix}-1 & 3 & 5\\ 1 & -3 & -5\\ -1 & 3 & 5\end{bmatrix}$

(4) $A=\begin{bmatrix}i & 0\\ 0 & -i\end{bmatrix}$ (5) $A=\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$ (6) $A=\begin{bmatrix}1^2 & 2^2\\ 3^2 & 4^2\end{bmatrix}$

Q9. For $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\ 1 & 0\end{bmatrix}$ Verify that

(1) $(A + B)^2 \neq A^2 + B^2 + 2\,A.B$ (2) $(A + B).(A - B) \neq A^2 - B^2$

(3) $A.B \neq B.A$ (4) Evaluate $3A^2-2B$ (5) Evaluate $2A^2 + B$

Q10. (1) If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ Verify that $A^2 - 7A + 10 = 0$

(2) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ Verify that $A^2 - 4A + 7 = 0$.

(3) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ Verify that $A^2 - 4A - 5I = 0$.

(4) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ Verify that $A^2 - 5A + 7I = 0$.

Q11. (1) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ find f (A) where $f(x) = x^2 - 5x + 7I$

(2) If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ find f (A) where $f(x) = x^3 + 3x^2 - 4x$

(3) If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ find f (A) where $f(x) = x^2 - 5x + 6I$

Q12. (1) Find "k" if $A = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$ such that $A^2 = kA + 4I$.

(2) Find x and y if $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ such that $A^2 - xA + yI = 0$.

(3) Find "k" if $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ such that $A^2 = kA - 2I$.

Q13. Find x so that

(1) $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -x \end{bmatrix} = 0$ (2) $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$

(3) $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$ (4) $\begin{bmatrix} 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & x \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ 1 \end{bmatrix} = 0$

Q14. Simplify (1) $\left(\begin{bmatrix} 4 \\ 5 \end{bmatrix} \begin{bmatrix} 7 & 9 \end{bmatrix} \right) + \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix}$ (2) $\left(\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \right) \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

(3) $\begin{bmatrix} 3 & 2 & 5 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 5 \end{bmatrix} - \begin{bmatrix} 7 & -8 \\ 5 & 9 \end{bmatrix}$ (4) $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Q15. Find A in each of the cases

(1) $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$ (2) $A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

(3) $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} A = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$ (4) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} A \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q16. Using PMI,

(1) If A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ Show that $A^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

(2) If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ Show that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

(3) If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ Show that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$

(4) If A = $\begin{bmatrix} \alpha & 1 \\ 0 & \alpha \end{bmatrix}$ Show that $A^n = \begin{bmatrix} \alpha^n & n\alpha^{n-1} \\ 0 & \alpha^n \end{bmatrix}$

Q17. (1) Show that matrix $B'AB$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.

(2) If A and B are symmetric matrices, show that AB is symmetric iff AB = BA.

(3) If A and B are symmetric matrices, show that AB + BA is symmetric.

(4) If A and B are symmetric matrices, show that AB – BA is skew-symmetric.

(5) If A is any square matrix, show that AA' is symmetric.

Q 18. Using elementary transformation find the inverse of following matrices

(1) A = $\begin{bmatrix} 7 & 5 \\ -9 & 1 \end{bmatrix}$ (2) B = $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ (3) A = $\begin{bmatrix} -1 & 5 \\ 2 & 3 \end{bmatrix}$ (4) B = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -7 & 2 \end{bmatrix}$

(5) A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (6) $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$ (7) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ (8) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Q 19. Express each of the matrixes as sum of symmetric and skew-symmetric matrices

(1) $\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 2 & 3 \\ -2 & 4 & 5 \end{bmatrix}$

(4) $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ (5) $\begin{bmatrix} 1 & 2 \\ 6 & 8 \end{bmatrix}$ (6) $\begin{bmatrix} 5 & 1 \\ 6 & 7 \end{bmatrix}$

Q 20. (1) Show that every square matrix can be expressed as sum of symmetric and skew-symmetric matrices.

(2) Show that the diagonal elements of a skew-symmetric matrix are zero.

ANSWERS

Q1. (1) $1\times12, 2\times6, 3\times4, 4\times3, 6\times2, 12\times1$ (2) $1\times7, 7\times1$ (3) $1\times9, 3\times3, 9\times1$

(4) 1×1 (5) $1\times11, 11\times1$ (6) $1\times6, 2\times3, 3\times2, 6\times1$

Q2. (1) $\begin{bmatrix} 3 & 16/3 & 25/3 \\ 25/3 & 12 & 49/3 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 1/2 & 0 \\ 5/2 & 2 & 3/2 \end{bmatrix}$ (3) $\begin{bmatrix} 0 & 4 & 8 \\ -4 & 0 & 4 \end{bmatrix}$

(4) $\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \end{bmatrix}$ (5) $\begin{bmatrix} 3/2 & 5/2 & 7/2 \\ 2 & 3 & 4 \end{bmatrix}$ (6) $\begin{bmatrix} 1 & 5/3 & 7/3 \\ 4/3 & 2 & 8/3 \end{bmatrix}$

Q3. (1) a = 2, b = 4 OR a = 4, b = 2 (2) x = 3/2, y = -3/2, z = 2

(3) a = 3, b = 4, c = 1 (4) a = 1, b = 2, c = 3, d = 4

Q4. (1) X = $\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$, Y = $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ (2) X = $\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

(3) X = $\frac{1}{5}\begin{bmatrix} -12 & 2 \\ 1 & -23 \end{bmatrix}$, Y = $\frac{1}{5}\begin{bmatrix} 13 & 2 \\ 1 & 22 \end{bmatrix}$ (4) X = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$, Y = $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Q5. (1) $\begin{bmatrix} -5 & -5 & -2 \\ 7 & -3 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -3 & -1 & -2 \\ -7 & 1 & 3 \end{bmatrix}$ (3) $\frac{1}{3}\begin{bmatrix} 9 & 8 & 4 \\ -7 & 4 & 12 \end{bmatrix}$

(4) $\frac{1}{4}\begin{bmatrix} 6 & 7 & 2 \\ -14 & 5 & 15 \end{bmatrix}$ (5) $\frac{1}{3}\begin{bmatrix} 18 & 11 & 10 \\ 14 & 1 & 3 \end{bmatrix}$ (6) $\begin{bmatrix} 7 & 4 & 4 \\ 7 & 0 & 0 \end{bmatrix}$

Q7. (1) $\begin{bmatrix} 2 & 5 \\ 4 & 10 \end{bmatrix}$ (2) $\begin{bmatrix} 94 & 83 \\ 102 & 88 \end{bmatrix}$ (3) $\begin{bmatrix} 4 & 12 & 16 \\ 0 & 0 & 0 \\ 6 & 18 & 24 \end{bmatrix}$

(4) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (5) $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$ (6) [44]

Q8. (1) I_3 (2) I_2 (3) A (4) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (5) $\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$ (6) $\begin{bmatrix} 37 & 68 \\ 153 & 292 \end{bmatrix}$

Q9. (4) $\begin{bmatrix} -7 & -14 \\ 22 & 21 \end{bmatrix}$ (5) $\begin{bmatrix} 0 & -7 \\ 17 & 14 \end{bmatrix}$

Q11. (1) $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ (2) $\frac{1}{7}\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$ (3) $\frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

Q12. (1) k = 3 (2) x = 4, y = 1 (3) k = 1

Q13. (1) x = 3 (2) x = -1, -2 (3) x = -3 (4) x = 2

Q14. (1) $\begin{bmatrix} 32 & 37 \\ 37 & 43 \end{bmatrix}$ (2) $\begin{bmatrix} 32 \\ 39 \end{bmatrix}$ (3) $\begin{bmatrix} 15 & 37 \\ -11 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$

Q15. (1) $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$ (4) $\begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$

Q18.

Q29. (1) P = $\begin{bmatrix} 1 & -1 & 1/2 \\ -1 & -1 & 3 \\ 1/2 & 3 & 1 \end{bmatrix}$, Q = $\begin{bmatrix} 0 & 1 & -5/2 \\ -1 & 0 & -1 \\ 5/2 & 1 & 0 \end{bmatrix}$ (2)P = $\begin{bmatrix} 1 & 1/2 & -1 \\ 1/2 & 3 & -1 \\ -1 & -1 & 1 \end{bmatrix}$,Q = $\begin{bmatrix} 0 & 3/2 & -1 \\ -3/2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

(3) P = $\begin{bmatrix} 1 & -3/2 & 0 \\ -3/2 & 2 & 7/2 \\ 0 & 7/2 & 5 \end{bmatrix}$,Q = $\begin{bmatrix} 0 & 1/2 & 2 \\ -1/2 & 0 & -1/2 \\ -2 & 1/2 & 0 \end{bmatrix}$ (4) P = $\begin{bmatrix} 2 & 7/2 \\ 7/2 & -1 \end{bmatrix}$,Q = $\begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix}$

(5) P = $\begin{bmatrix} 1 & 4 \\ 4 & 8 \end{bmatrix}$,Q = $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ (6)) P = $\begin{bmatrix} 5 & 7/2 \\ 7/2 & 7 \end{bmatrix}$,Q = $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$

One Marker Problems

(1) I (2) I (3) $\begin{bmatrix} 14 \end{bmatrix}$ (4) 0 (5) 2

(6) 8 (7) Skew – Symmetric (8) $\begin{bmatrix} a^{16} & 0 \\ 0 & 0 \end{bmatrix}$ (9) $\begin{bmatrix} a^2 + b^2 + c^2 + 2ab \end{bmatrix}$ (10) 81 I

(11) -2 (12) – 1 (13) I (14) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (15) 1

(16) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (17) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (18) $a = -2, b = 0, c = -3$ (19) $x = -1, 6$ (20) 11

(21) $a = 1, b = -2, c = -2$ (22) $x = -2, y = 0$ (23) $x = y$ (24) No diagonal elements

(25) $\begin{bmatrix} 32 & 0 & 0 \\ 0 & 32 & 0 \\ 0 & 0 & 32 \end{bmatrix}$ (26) $\begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$ (27) $a = -3, b = -2$ (28) 512 (29) B

(30) A + B

MATRICES (One Marker Problems)

1. For square matrix *A*, If $A^2 = A$ then evaluate $(I + A)^3 - 7A$

2. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ evaluate A^6

3. If $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$, evaluate $A.A^T$

4. If $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is a skew symmetric matrix then evaluate $\sum_i a_{ij}$.

5. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ satisfy the equation $A^2 = kA$, then find k .

6. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfy the equation $A^4 = kA$, then find k

7. If B is a skew-symmetric matrix, write whether the matrix ABA^T is symmetric or skew-symmetric?

8. If $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$ find A^{16}

9. Evaluate $\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} b \\ a \end{bmatrix} + \begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

10. If $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$ find A^4

11. If $\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$ find x.

12. If $\begin{pmatrix} 2x-1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ x+y \end{pmatrix}$ find value of $x - y$

13. If $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find A^{16}

14. Write square matrix of order 2, which is both symmetric and skew-symmetric..

15. If $A = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$ then find $a_{22} + b_{21}$

16. If A = $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$ find A ^{16}

17. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then find $A^T - B^T$

18. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew – symmetric find values of a, b and c.

19 If the matrix $\begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, find x.

20. In $A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & -2 & -3 \end{bmatrix}$ evaluate $a_{21} + 3\,a_{13}$

21. If A = $\frac{1}{3}\begin{bmatrix} a & 2 & 2 \\ 2 & 1 & b \\ 2 & c & 1 \end{bmatrix}$ obeys the law $A.A^T = I$, find a, b and c

22. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ obeys $A.A^T = 9I$, find x and y

23. If A = $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A^T = A, find x and y.

24. Write all diagonal elements in $B = \begin{bmatrix} -3 & 2 & 1 \\ 2 & 4 & -3 \end{bmatrix}$

25. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ find A^5

26. If A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ find A^5.

27. If A = $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, B = $\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find a and b.

28. Find the number of possible matrices of order 3×3 using 0 or 1.

29. If A and B are square matrices such that AB = A and BA = B then B^2 is_____

30. If A and B are square matrices such that AB = A and BA = B then $A^2 + B^2$ is_____.

INVERSE TRIGONOMETRIC FUNCTION

$\sin^{-1} x$: For x ∈ [-1, 1], $\sin^{-1} x$ is defined as an angle θ whose $\sin\theta$ is x. i. e. $\sin^{-1} x = \theta \Rightarrow \sin\theta = x$.

If $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ then θ is uniquely defined and it is called principle value of $\sin^{-1} x$ e.g. $\sin^{-1}\left(\frac{1}{2}\right)$ $= \frac{\pi}{6}$.

FUNCTION	DOMAIN	RANGE (P. Value)
Sin^{-1} x	[-1,1]	[-π/2, π/2]
Cos^{-1} x	[-1,1]	[0, π]
$\tan^{-1}$ x	R	(-π/2, π/2)

Note: $\text{Sin}^{-1} x \neq \frac{1}{\sin x}$; $\left(\sin^{-1} x\right) \neq \left(\sin x\right)^{-1}$

PROPERTIES:

(I) (i) $\text{Sin}^{-1}(\sin x) = x$ (ii) $\text{Cos}^{-1}(\cos x) = x$ (iii) $\tan^{-1}(\tan x) = x$

(II) (i) $\text{Sin}^{-1}(-x) = -\text{Sin}^{-1}(x)$ (ii) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$ (iii) $\tan^{-1}(-x) = -\tan^{-1}(x)$

(iv) $\text{Cos}^{-1}(-x) = \pi - \cos^{-1}(x)$ (v) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ (vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x$

(III) (i) $\text{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$ (ii) $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$ (iii) $\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$

(iv) $\sin^{-1}(x) = \text{cosec}^{-1}\left(\frac{1}{x}\right)$ (ii) $\cos^{-1}(x) = \sec^{-1}\left(\frac{1}{x}\right)$ (iii) $\tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right)$

(IV) (i) $\text{Sin}^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ (ii) $\text{cosec}^{-1}(x) + \sec^{-1}(x) = \frac{\pi}{2}$ (iii) $\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$.

(V) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}$, if $xy < 1$; $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\frac{x+y}{1-xy}$, if $xy > 1$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}$;

(VI) $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2})$

$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}(x\,y \mp \sqrt{1-x^2}\,\sqrt{1-y^2})$

(VII) $2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$

(VIII) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$, $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$, $2\sin^{-1} x = \cos^{-1}(1 - 2x^2)$

(IX) $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$, $3\tan^{-1} x = \tan^{-1}\frac{3x-x^3}{1-3x^2}$

(X) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$, $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2}$, $\sin^{-1} x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$, $\cos^{-1} x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$

Important Substitutions:

1. For $\sqrt{a^2-x^2}$ Put x = a Sin θ , $\sqrt{1-x^2}$ Put x = Sin θ

2. For $\sqrt{a^2+x^2}$, Put x = a tan θ , $\sqrt{1+x^2}$, Put x = tan θ

3. For $\sqrt{x^2-a^2}$ Put x = a Sec θ, $\sqrt{x^2-1}$ Put x = Sec θ

4. For $\sqrt{a-x}/\sqrt{a+x}$ Put x = a Cos θ, $\sqrt{1-x}/\sqrt{1+x}$ Put x = Cos θ or x = sin θ

Important Trigonometric Formulas:

(i) $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

(ii) $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 2\cos^2\frac{x}{2} - 1 = 1 - 2\sin^2\frac{x}{2} = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

(iii) Sin 3x = 3 Sin x – 4 Sin ^{3}x; Cos 3x = 4 Cos ^{3}x – 3 Cos x; $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

(iv) 1 + Cos x = 2 Cos $^2\frac{x}{2}$; 1 – Cos x = 2 Sin $^2\frac{x}{2}$

(v) $\sqrt{1+\sin x} = \sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}$; $\sqrt{1-\sin x} = \sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}$

(vi) $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x} = \frac{\cos x - \sin x}{\cos x + \sin x}$; $\tan\left(\frac{\pi}{4}-x\right) = \frac{1-\tan x}{1+\tan x} = \frac{\cos x - \sin x}{\cos x + \sin x}$

ASSIGNMENT

1) Find the principal values of each

(i) $\sin^{-1}\left(\frac{1}{2}\right)$ (ii) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (iii) $\tan^{-1}(\sqrt{3})$ (iv) $\cos^{-1}\left(\frac{1}{2}\right)$ (v) $\text{cosec}^{-1}(\sqrt{2})$ (vi) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (vii) $\cos ec^{-1}(2)$ (viii) $\tan^{-1}\left(-\sqrt{3}\right)$ (ix) $\cos^{-1}\left(-\frac{1}{2}\right)$ (x) $\tan^{-1}(-1)$ (xi) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (xii) $\cot^{-1}\left(\sqrt{3}\right)$ (xiii) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (xiv) $\cos ec^{-1}(-2)$

2) Evaluate

(i) $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ (ii) $\tan^{-1}\left(\sqrt{3}\right) - \cot^{-1}\left(-\sqrt{3}\right)$ (iii) $\cos\left(\tan^{-1}(1)\right)$

(iv) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ (v) $\sec^2\left(\tan^{-1}2\right) + \cos ec^2\left(\cot^{-1}3\right)$

(vi) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ (vii) $\tan^{-1}\left(\sqrt{3}\right) + \sec^{-1}(-2)$ (viii) $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

3) Evaluate

(i) $\sec^{-1}(\sec(\frac{13\pi}{6}))$ (ii) $\tan^{-1}(\tan\frac{3\pi}{4})$ (iii) $\sin^{-1}(\sin\frac{4\pi}{3})$ (iv) $\text{cosec}^{-1}(\text{cosec}(-\frac{\pi}{4}))$ (v)

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ (vi) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ (vii) $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ (viii) $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ (ix)

$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ (x) $\cot^{-1}\left(\cot\left(-\frac{\pi}{4}\right)\right)$ (xi) $\tan^{-1}\left(\tan\left(-\frac{3\pi}{4}\right)\right)$

4) Write in simplest form

(i) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ (ii) $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$ (iii) $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$ (iv) $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$ (v)

$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ (vi) $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ (vii) $\cos^{-1}\left(\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}\right)^{\frac{1}{2}}$

5) Show in each

(i) $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$ (ii) $\tan^{-1}\left[\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right] = \tan^{-1}\left(\frac{a}{b}\right)-x$

(iii) $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \frac{x}{2}$ (iv) $\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right] = \frac{\pi}{4}-x$ (v) $\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right] = \frac{\pi}{4}+\frac{x}{2}$ (vi)

$\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right] = \frac{\pi}{4}-\frac{x}{2}$ (vii) $\tan^{-1}\left[\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right] = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$

6) Evaluate

(i) $\tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)$ (ii) $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$ (iii) $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$

((vi) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

ANSWERS

1) (i) $\frac{\pi}{6}$ (ii) $\frac{5\pi}{6}$ (iii) $\frac{\pi}{3}$ (iv) $\frac{\pi}{3}$ (v) $\frac{\pi}{4}$ (vi) $\frac{\pi}{4}$ (vii) $\frac{\pi}{6}$ (viii) $-\frac{\pi}{3}$
(ix) $\frac{2\pi}{3}$ (x) $-\frac{\pi}{4}$ (xi) $\frac{\pi}{6}$ (xii) $\frac{\pi}{6}$ (xiii) $\frac{3\pi}{4}$ (xiv) $-\frac{\pi}{6}$

2) (i) 1 (ii) $-\frac{\pi}{2}$ (iii) $\frac{1}{\sqrt{2}}$ (iv) $\frac{3\pi}{4}$ (v) 15 (vi) $\frac{2\pi}{3}$ (vii) π (viii) -1

3) (i) $\frac{\pi}{6}$ (ii) $-\frac{\pi}{4}$ (iii) $-\frac{\pi}{3}$ (iv) $-\frac{\pi}{4}$ (v) $\frac{\pi}{3}$ (vi) $\frac{5\pi}{6}$ (vii) $\frac{2\pi}{5}$ (viii) $\frac{\pi}{6}$ (ix) $\frac{\pi}{6}$ (x) $\frac{3\pi}{4}$ (xi) $-\frac{\pi}{4}$

4) (i) $\frac{1}{2}\tan^{-1}x$ (ii) $\cos ec^{-1}x$ (iii) $\sec^{-1}x$ (iv) $\sin^{-1}\left(\frac{x}{a}\right)$ (v) $\sin^{-1}x$ (vi) 3 $\tan^{-1}\left(\frac{x}{a}\right)$ (vii) $\frac{1}{2}\tan^{-1}x$

6) (i) $\frac{\pi}{4}$ (ii) 0 (iii) $\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ (iv) $\frac{1}{2}\tan^{-1}x$ (v) $\frac{\pi}{4}$ (vi) $\frac{17}{6}$

9) (i) $\frac{\pi}{4}$ (ii) 0 (iii)x = $\frac{1}{\sqrt{3}}$ (iv) x = $\frac{1}{6}$ (v) $\frac{3}{5}$

List of important questions and examples (NCERT)

EXAMPLES: 2, 3, 5, 6,7, 9, 10, 11, 12, 13

Ex.2.1: 5, 10, 11, 14

Ex. 2.2: 2,4,5,6,7,8,9,10,11,13,14,15,17,18,21

MISC: 1, 4,8,9,10,11,12,14,16,17

Answer one mark problems

(1) 5 (2) $\frac{\pi}{6}$ (3) $\frac{3\pi}{2}$ (4) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (5) $[-1,1]$

(6) 2 (7) 0 (8) $\frac{\pi}{2}$ (9) $\frac{\pi}{2}-2\tan^{-1}x$ (10) $\tan^{-1}\left(\frac{a}{b}\right)+x$

(11) $\tan^{-1}3x-\tan^{-1}2x$ (12) $\sin^{-1}x$ (13) $\cos^{-1}x$ (14) $x-\cos^{-1}\left(\frac{3}{5}\right)$ (15) $\frac{\pi}{4}$

(16) $\sin^{-1}x-\sin^{-1}\sqrt{x}$ (17) 0 (18) $\frac{4}{5}$ (19) 0 (20) $\frac{\pi}{4}$

(21) $\frac{\pi}{4}$ (22) $\frac{24}{25}$ (23) $\frac{\pi}{3}$ (24) $\sin^{-1}x$ (25) $\tan^{-1}x$

(26) $\tan^{-1}x$ (27) $\sin^{-1}x$ (28) $\cos^{-1}x$ (29) $\cos^{-1}x$ (30) $\cos^{-1}x$

(31) $\cos^{-1}x$ (32) 1 (33) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ (34) $\frac{3}{5}$ (35) $\frac{12}{5}$

(36) $x-\frac{\pi}{4}$ (37) $\frac{24}{25}$ (38) $\frac{1}{\sqrt{1+x^2}}$ (39) $1+\frac{y^2}{4}$ (40) $\frac{69}{125}$

Biology

1. WRITE ALL PRACTICALS IN YOUR PRACTICAL NOTEBOOK
2. PREPARE INVESTIGATORY PROJECT FILE ON A DISEASE.
3. WRITE NOTES OF UNIT-2
4. LEARN UNIT-1 AND READY FOR TEST ON LAST WEEK OF JUNE 2025