Kendriya Vidyalaya Sangathan Mumbai Region

Subject: Mathematics (041)

Class: XII 2ND PRE-BOARD EXAM

Time Allowed: 3 Hours **SET-1** **Maximum Marks: 80**

General Instructions:

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.*

Section –A (Multiple Choice Questions) Each question carries 1 mark

1.	The area enclosed by $y = cosx, x = -\pi, x = \pi$ and x axis is A) 4 B) 2 C) 1 D) 0
2.	The projection of vector $\hat{i} + \hat{j} - 2\hat{k}$ on vector $3\hat{i} - 2\hat{j} - 6\hat{k}$ is A) 15/7 B) 13/7 C) 2 D) 11/7
3.	If $(2\hat{i} + 3\hat{j} - \hat{k})$ & $(-4\hat{i} + a\hat{j} + b\hat{k})$ are parallel vectors then a is equal to A) 6 B) – 6 C) 2 D) – 2
4.	For an objective function $Z = ax + by$, where $a, b > 0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0, 20), (10, 10), (30, 30) and (0, 40). The condition on a and b such that the maximum Z occurs at both the points (30, 30) and (0, 40) is: A) $b - 3a = 0$ B) $a = 3b$ C) $a + 2b = 0$ D) $2a - b = 0$
5.	If A and B are independent events with P(A) = 0.5 and P(B) = 0.2, then P(AUB) is A) 0.3 B) 0.7 D) 0.5 D) 0.6
6.	The degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = \log y$ A) not defined B) 2 C) 1 D) 3
7.	Solution of the differential equation $\frac{dy}{dx} = \tan x \cot y$ is A) C(cosx)=cosy B) cosx cosy = C C) cosx + cosy = C D) cosx = cosy + C

8.	The maximum value of $y = 4 + 3cos2x$ is A) 10 B) 3 C) 7 D) 4
9.	Value of $\cos^{-1}(-1) - \sin^{-1}(-1)$ = A) π B) 3π/2 C) 2π D) π/2
10.	In a linear programming problem, the constraints on the decision variables x and y are $x - 3y \geq 0$, $y \geq 0$, $0 \leq x \leq 3$. The feasible region A) is not in the first quadrant B) is bounded in the first quadrant C) is unbounded in the first quadrant D) does not exist
11.	If inverse of the matrix $A = \begin{bmatrix} 4 & -1 & a \\ 2 & -3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ does not exist, then a is equal to A) 2 B) – 2 C) 4 D) – 4
12.	$\int_{-1}^{1} \frac{1}{1+x^2} dx$ = A) π/2 B) π/3 C) π/4 D) π
13.	If A is a square matrix of order 3 such that $\|A\| = -2$ then |adj A| = (A) 8 (B) −2 (C) −8 (D) 4
14.	If A is any square matrix, then $A - A^T$ is A) null matrix B) identity matrix C) symmetric matrix D) skew symmetric matrix
15.	If $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, then X = (A) $\begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 0 \\ 1 & 5 \end{bmatrix}$ (C) $\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 7 & 1 \\ 0 & 4 \end{bmatrix}$
16.	If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, and $A + A^T = 0$, then the value of x is A) π B) π/2 C) π/6 D) π/3
17.	The function $f(x) = \frac{x(x-1)}{x(x-1)(x-2)}$ *is*: (A) discontinuous at 3 points (B) discontinuous at 1 point (C) discontinuous at 2 points (D) discontinuous at infinite point

18.	$\int e^x(\sin x - \cos x)dx =$ A) $e^x \sin x + C$ B) $-e^x \sin x + C$ C) $e^x \cos x + C$ D) $-e^x \cos x + C$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

19.	**Assertion:** The relation R defined on the set A = {a, b, c} given by R= {(a,a), (a,b), (b,a)} is symmetric but neither reflexive nor transitive. **Reason:** A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.
20.	**Assertion**: f(x) = x|x| is a continuous and differentiable function at x = 0. **Reason:** All differentiable function are continuous and all continuous functions are differentiable.

Section –B [This section comprises of very short answer type questions (VSA) of 2 marks each]

21.	Find a if $f(x) = \begin{cases} \frac{\sqrt{x+5}-3}{x-4}, & x \neq 4 \\ a, & x = 4 \end{cases}$ is continuous at x = 4.
22.	Express $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $\frac{-\pi}{4} < x < \frac{3\pi}{4}$ in the simplest form.
23.	If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that they are mutually perpendicular to each other, then find $\left\|\vec{a} + \vec{b} + \vec{c}\right\|$. **OR** Find a unit vector in the direction of $\vec{a} + \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + 3\hat{k}$ & $\vec{b} = 2\hat{i} - 2\hat{j} - 5\hat{k}$.
24.	Find the position vector of point C which divides the line segment joining the points A and B having position vectors $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively in the ratio 4:1 externally.
25.	The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area of this triangle increases, when side is 10cm. **OR** Let $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{logx-1}{(logx)^2}$.

.

Section – C [This section comprises of short answer type questions (SA) of 3 marks each]

26.	Evaluate $\int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx$ **OR** Evaluate $\int \frac{5x+3}{(x+5)(x+3)} dx$
27.	Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$ **OR** Find the particular solution of the differential equation $\frac{dy}{dx} + \frac{2y}{x} = x^3$, when y = 1, x = 2.
28.	Find shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ & $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + s(-3\hat{i} + 2\hat{j} + \hat{k})$
29.	Solve the following problem graphically: Maximize Z = x + 6y Subject to constraints: x + y ≥ 10, x + 6y ≤ 60 , x ≤ 30, x ≥ 0, y ≥ 0
30	For the probability distribution given as under find the value of k and the mean. X | 0 | 1 | 2 | 3 P(x) | k | $2k^2$ | k^2 | k **OR** A coin is tossed three times. Find the probability distribution of number of heads obtained in three tosses.
31.	Find the intervals in which the following function is strictly increasing & strictly decreasing $f(x) = (x + 1)^3(x - 3)^3$

Section –D [This section comprises of long answer type questions (LA) of 5 marks each]

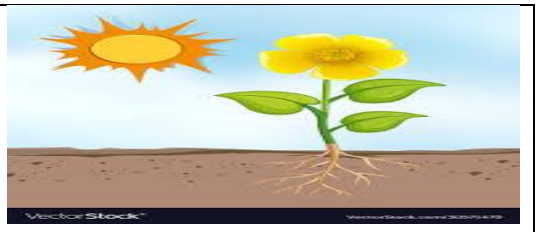
32.	Using integration, find the area of the region enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and –2 ≤ x ≤ 2 .

33.	If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k. Hence write the vector equation of the line perpendicular to these two lines and passing through the point (3, –4, 7). **OR** Find the coordinates of the foot of perpendicular drawn from P(5, 4, 2) to the line $\vec{r}=(-\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}-\hat{k})$. Also find the image of P in this line.
34.	If y = $\frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$ **OR** If $y = e^{acos^{-1}x}$, for $x \in [-1,1]$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$
35.	Find the product of the matrices $\begin{bmatrix}3 & -2 & 3\\ 2 & 1 & -1\\ 4 & -3 & 2\end{bmatrix}\begin{bmatrix}1 & 5 & 1\\ 8 & 6 & -9\\ 10 & -1 & -7\end{bmatrix}$ Hence solve the equations: 3x – 2y + 3z = 8 , 2x + y – z = 1 & 4x – 3y + 2z = 4

Section E

[This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts]

36.	 A boy after passing class X, wants to pursue Science, Commerce or Humanities with chances of selection as 45%, 30% and 25% respectively. He wants to score more than 90% marks in his class XII board. The chances of scoring more than 90% marks provided he has pursued his 10+2 in Science, Commerce or Humanities are 0.4, 0.6, 0.8 respectively. i) Find the probability that the boy secured more than 90% marks in class XII? [2] ii) If the boy secured more than 90% marks in his boards, find the probability that he studied Commerce? [2]

37	The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight. 	
	i) Find the number of days it will take for the plant to grow to the maximum height? [1] ii) Find the maximum height of the plant? [1] iii) a) Find the height of the plant after 2 days? [2] **OR** iii) b) If the height of the plant is $\frac{7}{2}$ cm, then Find the number of days it has been exposed to the sunlight? [2]	
38	The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone. 	
	A relation R is defined on the set $U = \{all\ the\ people\ on\ the\ Earth\}$, such that R = {$(x, y)$: the time difference between the time zones x and y reside in is 6 hours}. (i) Check if the relation is reflexive? [1] (ii) Check if the relation is symmetric? [1] (iii) a) Check if the relation is Transitive? [2] **OR** (iii) b) If R = {(x, y): x and y reside in the same time zone}, check if this relation will be reflexive and symmetric? [2]	

Kendriya Vidyalaya Sangathan Mumbai Region

Subject: Mathematics (041)

Class: XII 2ND PRE-BOARD EXAM

Time Allowed: 3 Hours **SET-2** **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

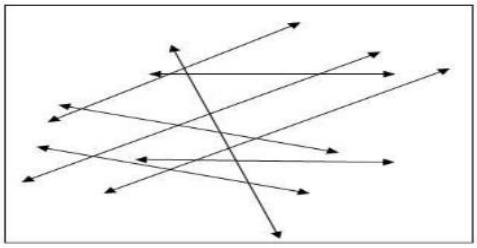
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

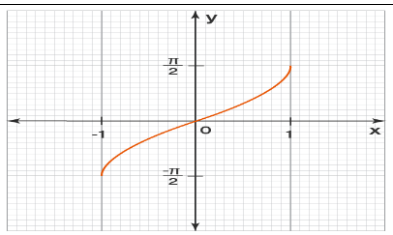
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

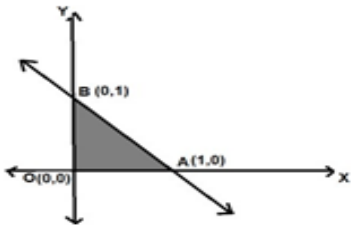
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section A (Multiple Choice Questions) Each question carries 1 mark

1.	Let L be the set of all lines in XY-plane and R be the relation in L defined as R = {L_1, L_2) : L_1 is parallel to L_2}. Which of the following lines related to the line $3x-y-1=0$.  A) $6x-y+1=0$ B) $6x-2y-3=0$ C) $3x+y+1=0$ D) $6x+9y+1=0$
2.	$\int_0^{2\pi} cosec^7 x\, dx$ (A) π (B) 1 (C) 0 (D) 2π
3.	The unit vector perpendicular to both the vectors $\hat{\imath}+2\hat{\jmath}-2\hat{k}$ and $-\hat{\imath}+2\hat{\jmath}+2\hat{k}$ (A)$\frac{1}{\sqrt{5}}(2\hat{\imath}-\hat{k})$ (B) $\frac{1}{\sqrt{5}}(-2\hat{\imath}-\hat{k})$ (C) $\frac{1}{\sqrt{5}}(2\hat{\imath}+\hat{\jmath}+\hat{k})$ (D) $\frac{1}{\sqrt{5}}(2\hat{\imath}+\hat{k})$
4.	If $f(x)=\begin{cases}\frac{2^{x+2}-16}{4^x-16}, & if\ x\neq 2\\ k, & if\ x=2\end{cases}$ is continuous at x = 2, find k. (A) $\frac{1}{2}$ (B) 2 (C) $\frac{1}{4}$ (D) 4
5.	The integrating factor of the differential equation $x\frac{dy}{dx}-y=x^4-3x$ is (A) x (B) $-x$ (C) x^{-1} (D) $log\,(x^{-1})$
6.	If $A=\begin{bmatrix}a & c & -1\\ b & 0 & 5\\ 1 & -5 & 0\end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a-(b+c)$ is (A) 0 (B) 1 (C) -10 (D) 10

7.	The graph shown depicts the function A) $y = \text{cosec}^{-1}x$ B) $y = \cos^{-1}x$ C) $y = \sin^{-1}x$ D) $y = \cot^{-1}x$ 
8.	The adj$A$ for the matrix $A = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$ is equal to (A) $\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & -2 \\ 1 & -3 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 2 \\ -1 & -4 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix}$
9.	Order and degree of differential equation: $\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{4}}$ (A) 4 and 2 (B) 1 and 2 (C) 1 and 4 (D) 2 and 4
10.	$If\ P(A \cap B) = 70\%, P(B) = 85\%, then\ P\left(\frac{A}{B}\right) is\ equal\ to$ $(A)\frac{14}{17}$ (B) $\frac{17}{20}$ (C) $\frac{7}{8}$ (D) $\frac{1}{8}$
11.	If for a square matrix A, $\boldsymbol{A}.(\boldsymbol{adjA}) = \begin{bmatrix} 225 & 0 & 0 \\ 0 & 225 & 0 \\ 0 & 0 & 225 \end{bmatrix}$, then the value of |A|+ |adj A| is equal to: (A) 1 (B) 225 + 1 (C) 225^2+ 15 (D) 225 + 225^2
12.	Number of matrices that are possible having 24 elements is $(A)4$ $(B)6$ $(C)8$ $(D)2$
13.	The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called (A) feasible solutions (B) constraints (C) optimal solutions (D) infeasible solutions
14.	The area of the parallelogram whose diagonals are $\hat{k} + \hat{j}$ and $\hat{k} + \hat{\imath}$ is (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{2}$ (C) 3 (D)$\sqrt{3}$
15	A student observes an open-air Honeybee nest on the branch of a tree, whose plane figure is a parabolic shape given by $x^2 = 4y$. Then the area(in square units) of the region bounded by the parabola $x^2 = 4y$, and the line $y = 4$ is (A) $\frac{32}{3}$ (B) $\frac{64}{3}$ (C) $\frac{128}{3}$ (D) $\frac{256}{3}$

16.	For the objective function Z =3x + 4y, Subject to the constraints x+y $\leq$ 1, x,y ≥0, the shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0,0),(1,0) and (0,1), respectively. The maximum value of Z is  A) 0 B) 3 C) 4 D) 7
17.	If A and B are two skew symmetric matrices then (AB+BA) is (A) Skew-symmetric (B) Null matrix (C) Identity matrix (D) Symmetric matrix
18.	Two positive numbers x and y whose sum is 35 and product is x^2y^5 is maximum are (A) 11, 24 (B)10, 25 (C) 0, 35 (D)17,18

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

19.	**ASSERTION (A)**: If two positive numbers are such that their sum is 16 and sum of their cubes is minimum, then numbers are 8,8. **REASON (R):** If f be a function defined on an interval I and c belongs to I and let f be twice differentiable at c then x = c is a point of local minima if f ' (c) = 0 and f" (c)>0 and f(c) is local minimum value of f(x).
20.	**Assertion(A):** $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{tanx}}\, dx$ is $\frac{\pi}{4}$ **Reason(R):** $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Section B [This section comprises of very short answer type questions (VSA) of 2 marks each]

21.	Express $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $\frac{-\pi}{4} < x < \frac{3\pi}{4}$ in the simplest form.
22.	Find the position vector of point C which divides the line segment joining the points A and B having position vectors $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively in the ratio 4:1 externally.
23.	The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area of this triangle increases, when side is 10cm. **OR** Let $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{logx-1}{(logx)^2}$

24.	If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that they are mutually perpendicular to each other, then find $\left\|\vec{a}+\vec{b}+\vec{c}\right\|$ **OR** Find a unit vector in the direction of $\vec{a}+\vec{b}$ where $\vec{a}=\hat{i}+\hat{j}+3\hat{k}$ & $\vec{b}=2\hat{i}-2\hat{j}-5\hat{k}$ . .
25.	Find a if $f(x)=\begin{cases}\dfrac{\sqrt{x+5}-3}{x-4}, & x\neq 4\\ a, & x=4\end{cases}$ is continuous at x = 4.

Section C [This section comprises of short answer type questions (SA) of 3 marks each]

26.	For the probability distribution find k and mean. X | 0 | 1 | 2 | 3 P(x) | k | $2k^2$ | k^2 | k **OR** A coin is tossed three times. Find the probability distribution of number of heads obtained in three tosses.
27.	Solve the following problem graphically: Maximize Z = x + 6y Subject to constraints: x + y ≥ 10, x + 6y ≤ 60 , x ≤ 30, x ≥ 0, y ≥ 0
28.	Find the intervals in which the following function is strictly increasing & strictly decreasing $f(x) = (x + 1)^3(x – 3)^3$
29.	Find the general solution of the differential equation $\frac{dy}{dx}=\frac{x^3+y^3}{xy^2}$ **OR** Find the particular solution of the differential equation $\frac{dy}{dx}+\frac{2y}{x}=x^3$, when y = 1, x = 2.
30	Evaluate $\int_0^{\pi/2}\frac{\sin^3 x}{\cos^3 x+\sin^3 x}dx$ **OR** Evaluate $\int\frac{5x+3}{(x+5)(x+3)}dx$

31.	Find shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ & $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + s(-3\hat{i} + 2\hat{j} + \hat{k})$

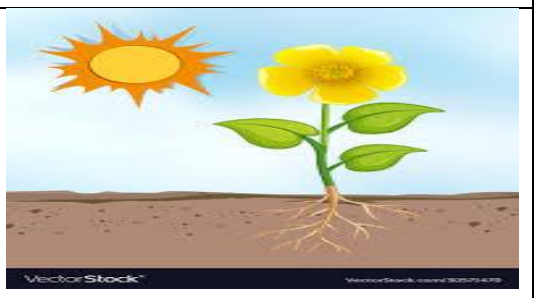
Section D [This section comprises of long answer type questions (LA) of 5 marks each]

32.	Using integration, find the area of the region enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and $-2 \le x \le 2$.
33.	Find the product of the matrices $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -1 & -7 \end{bmatrix}$. Hence solve the equations: 3x – 2y + 3z = 8 , 2x + y – z = 1 & 4x – 3y + 2z = 4
34.	If $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$ **OR** If $y = e^{a\cos^{-1}x}$, for $x \in [-1,1]$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$
35.	If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find value of k. Hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7). **OR** Find the coordinates of the foot of perpendicular drawn from P(5,4,2) to the line $\vec{r} = (-\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$. Also find the image of P in this line.

Section E

[This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts]

36.	A boy after passing class X, wants to pursue Science, Commerce or Humanities with chances of selection as 45%, 30% and 25% respectively. He wants to score more than 90% marks in his class XII board. The chances of scoring more than 90% marks provided he has pursued his 10+2 in Science, Commerce or Humanities are 0.4, 0.6, 0.8 respectively. 

	i) Find the probability that the boy secured more than 90% marks in class XII? [2] ii) If the boy secured more than 90% marks in his boards, find the probability that he studied Commerce? [2]
37	The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone.  A relation R is defined on the set $U = \{all\ the\ people\ on\ the\ Earth\}$, such that R = {$(x, y)$: the time difference between the time zones x and y reside in is 6 hours}. (i) Check if the relation is reflexive? [1] (ii) Check if the relation is symmetric? [1] (iii) a) Check if the relation is Transitive? [2] **OR** (iii) b) If R = {(x, y): x and y reside in the same time zone}, check if this relation will be reflexive and symmetric? [2]
38	The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.  i) Find the number of days it will take for the plant to grow to the maximum height? [1] ii) Find the maximum height of the plant? [1] iii) a) Find the height of the plant after 2 days? [2] **OR** iii) b) If the height of the plant is $\frac{7}{2}$ cm, then Find the number of days it has been exposed to the sunlight? [2]

Kendriya Vidyalaya Sangathan Mumbai Region

Subject: Mathematics (041)

Class: XII 2ND PRE-BOARD EXAM

Time Allowed: 3 Hours **SET-3** **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section –A (Multiple Choice Questions) Each question carries 1 mark

1.	The function f:N $\rightarrow$ N , defined as $f(x) = \|x\|$ is A) neither one – one nor onto B) one – one but not onto C) not one – one but onto D) both one – one and onto
2.	$\int \frac{sin^2x - cos^2x}{sin^2x cos^2x}\, dx$ is equal to (A) tanx + cotx + C (B) tanx + cosecx + C (C) -tanx + cotx + C (D) tanx + secx + C
3.	$\int_0^2 \sqrt{4 - x^2} dx =$ (A) 2log2 (B) -2log2 (C) $\frac{\pi}{2}$ (D) π
4.	The Integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is (A) e^{-x} (B) e^{-y} (C) $\frac{1}{x}$ (D) x
5.	Degree of the differential equation sinx + cos$\left(\frac{dy}{dx}\right)$ =y^2 (A)2 (B)1 (C) not defined (D) 0
6.	The value of p for which the vectors $\vec{a} = 2\hat{\imath} + p\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ are perpendicular to each other (A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{7}{2}$ (D)10

7.	Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$ (A) $\frac{60}{\sqrt{59}}$ (B) $\frac{60}{\sqrt{117}}$ (C) $\frac{60}{\sqrt{114}}$ (D) None of these
8.	The number of feasible solutions of the linear programming problem given as Maximize Z=15x+30y subject to constraints: 3x+y≤ 12, x+2y≤ 10 , x≥ 0 , y≥ 0 is (A)1 (B) 2 (C) 3 (D) infinite
9.	The objective function $Z = ax + by$ of a linear programming problem has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true? (A) a=9,b=1 (B) a=9,b=2 (A) a=3,b=5 (D) a=5,b=3
10.	The value of sin{$\frac{\pi}{3} - sin^{-1}(\frac{-1}{2})$} is (A)$\frac{1}{2}$ (B) 0 (C) 1 (D) – 1
11.	If A is a square matrix of order 3 X 3 such that $\|A\| = 8$,then $\|3A\|$= (A)8 (B) 216 (C)72 (D) 144
12.	A square matrix A = $[a_{ij}]_{n\times n}$ is called a diagonal matrix if a_{ij} = 0 for (A) i = j (B) i < j (C) i > j (D) i ≠ j
13.	The number of all possible matrices of order 3x3 with each entry -1 or 1 is (A) 512 (B) 81 (C) 27 (D) 18
14.	If $A = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ then $A^2 + 2A$ is equal to (A) 4A (B) 3A (C) 2A (D) A
15.	Area (in square units) lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4\ and\ the\ lines\ x = 0\ and\ x = 2\ is$ (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) π
16.	If $f(x) = \begin{cases}\frac{2^{x+2}-16}{4^x-16}, & if\ x \neq 2\\ k & ,if\ x = 2\end{cases}$ is continuous at x = 2, find k. $(A)\frac{1}{2}$ (B) 2 (C) $\frac{1}{4}$ (D) 4
17.	Three persons A, B & C fire a target in turn. Their probabilities of hitting the target are 0.2,0.3 and 0.5 respectively. The probability that the target is hit, is (A)0.993 (B)0.72 (C) 0.82 (D) 0.92

18.	If $y = \frac{cosx-sinx}{cosx+sinx}$, then $\frac{dy}{dx}$ is (A) $sec^2\,(\frac{\pi}{4}-x)$ (B) $-sec^2\,(\frac{\pi}{4}-x)$ (C) log $\left\lvert \sec(-\frac{\pi}{4}-x) \right\rvert$ (D) - log $\left\lvert \sec(-\frac{\pi}{4}-x) \right\rvert$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

19.	**Assertion(A)**:The intervals in which f(x) = log sin x , $0 \le x \le \pi$ is increasing is $\left(0, \frac{\pi}{2}\right)$. **Reason(R)** : A function is increasing in (a,b) if f/ (x) > 0 for each x$\epsilon(a,b)$.
20.	**Assertion(A)**:If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x=± 6 **Reason(R)** : If A is a skew symmetric matrix of odd order, then$\lvert A \rvert$=0.

Section –B [This section comprises of very short answer type questions (VSA) of 2 marks each]

21.	Find the position vector of point C which divides the line segment joining the points A and B having position vectors $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively in the ratio 4:1 externally.
22.	If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that they are mutually perpendicular to each other, then find $\left\lvert \vec{a} + \vec{b} + \vec{c} \right\rvert$. **OR** Find a unit vector in the direction of $\vec{a} + \vec{b}$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + 3\hat{k}$ & $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$.
23.	Express $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $\frac{-\pi}{4} < x < \frac{3\pi}{4}$ in the simplest form.
24.	Find a if $f(x) = \begin{cases} \frac{\sqrt{x+5}-3}{x-4}, & x \neq 4 \\ a, & x = 4 \end{cases}$ is continuous at x = 4.
25.	The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases, when side is 10cm. **OR** Let $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{logx-1}{(logx)^2}$

Section – C [This section comprises of short answer type questions (SA) of 3 marks each]

26. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$$

OR

Find the particular solution of the differential equation

$\frac{dy}{dx} + \frac{2y}{x} = x^3$, when y = 1, x = 2.

27. Evaluate $\int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx$

OR

Evaluate $\int \frac{5x+3}{(x+5)(x+3)} dx$

28. Find the intervals of strictly increasing & strictly decreasing for the function

$f(x) = (x + 1)^3(x - 3)^3$

29. Solve the following problem graphically:

Maximize Z = x + 6y

Subject to constraints:

$x + y \geq 10$, $x + 6y \leq 60$, $x \leq 30$, $x \geq 0$, $y \geq 0$

30 For the probability distribution find k and mean.

X	0	1	2	3
P(x)	k	$2k^2$	k^2	k

OR

A coin is tossed three times. Find the probability distribution of number of heads obtained in three tosses.

31. Find shortest distance between the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ &

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + s(-3\hat{i} + 2\hat{j} + \hat{k})$

Section –D [This section comprises of long answer type questions (LA) of 5 marks each]

32. If $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1 - x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$

OR

If $y = e^{acos^{-1}x}$, for $x \in [-1,1]$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

33.	If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find value of k. Hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7). **OR** Find the coordinates of the foot of perpendicular drawn from P(5,4,2) to the line $\vec{r}=(-\hat{i}+3\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}-\hat{k})$. Also find the image of P in this line.
34.	Using integration, find the area of the region enclosed by the curve $\frac{x^2}{16}+\frac{y^2}{25}=1$ and –2 ≤ x ≤ 2 .
35.	Find the product of the matrices $\begin{bmatrix}3 & -2 & 3\\ 2 & 1 & -1\\ 4 & -3 & 2\end{bmatrix}\begin{bmatrix}1 & 5 & 1\\ 8 & 6 & -9\\ 10 & -1 & -7\end{bmatrix}$ Hence solve the equations: 3x – 2y + 3z = 8 , 2x + y – z = 1 & 4x – 3y + 2z = 4

Section E

[This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts]

36. The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone.



A relation R is defined on the set $U=\{all\ the\ people\ on\ the\ Earth\}$, such that R = $\{(x,y)$: the time difference between the time zones x and y reside in is 6 hours$\}$.

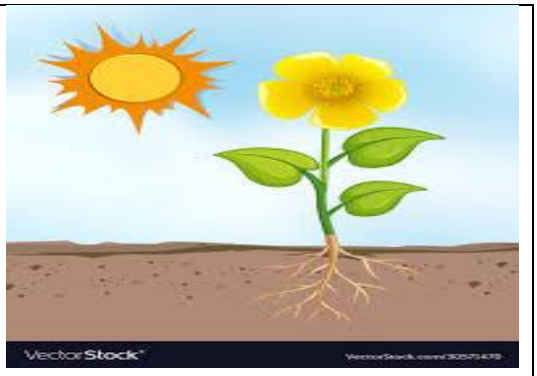
(i) Check if the relation is reflexive? [1]

(ii) Check if the relation is symmetric? [1]

(iii) a) Check if the relation is Transitive? [2]

OR

(iii) b) If R = $\{(x,y)$: x and y reside in the same time zone$\}$, check if this relation will be reflexive and symmetric? [2]

37	The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight. 	
	i) Find the number of days it will take for the plant to grow to the maximum height? [1] ii) Find the maximum height of the plant? [1] iii) a) Find the height of the plant after 2 days? [2] **OR** iii) b) If the height of the plant is $\frac{7}{2}$ cm, then Find the number of days it has been exposed to the sunlight? [2]	
38	A boy after passing class X, wants to pursue Science, Commerce or Humanities with chances of selection as 45%, 30% and 25% respectively. He wants to score more than 90% marks in his class XII board. The chances of scoring more than 90% marks provided he has pursued his 10+2 in Science, Commerce or Humanities are 0.4, 0.6, 0.8 respectively.	
	i) Find the probability that the boy secured more than 90% marks in class XII? [2] ii) If the boy secured more than 90% marks in his boards, find the probability that he studied Commerce? [2]	