KENDRIYA VIDYALAYA SANGATHAN
REGIONAL OFFICE, DEHRADUN

COPETENCY BASED QUESTION BANK

CLASS- XII

SUBJECT- MATHEMATICS

1. RELATION AND FUNCTION

MCQ

1. The relation R in the set of natural numbers N defined as R ={ (x,y) : x > y} is
(a) An equivalence relation
(b) Reflexive, symmetric but not transitive
(c) Symmetric, transitive but not reflexive
(d) Transitive but neither symmetric nor reflexive.

2. Let A = {a, b, c}. Then the number of equivalence relations in A containing (a, c) is
(a)1 (b) 2 (c) 3 (d) 4

3. Consider the set A contains n elements. Then the total number of injective functions from A onto itself is
(a) 2^{n^2} (b)$n!$ (c) 2^n –n (d)None of these

4. The number of onto functions from a set A containing 5 elements to set B containing 2 elements is
(a) 10 (b) 32 (c) 30 (d) 0

5. Let A = {x : -1 ≤ x ≤ 1} and f : A → A is a function defined by f(x) = x |x| then f is
(a) a bijective (b) injective but not surjective
(c) surjective but not injective (d) neither injective nor surjective

6. Which of the following functions from Z into Z are bijective?
(a) f(x) = x^3 (b) f(x) = x + 2 (c) f(x) = 2x + 1 (d) f(x) = x^2 + 1

7. The number of bijective functions from set A to itself when A contains 106 elements is
(a) 106 (b) $(106)^2$ (c) 106! (d) 2^{106}

ASSERTION AND REASONING QUESTION

8. **Assertion A**: The number of bijective functions from the set containing 10 elements to itself is 2^{10}

Reason R: The total number of bijections from a set containing *n* elements to itself is *n*!

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

a. Both **A** and **R** are correct and **R** is the correct explanation of **A**
b. Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
c. **A** is correct but **R** is not correct
d. **A** is not correct but **R** is correct

9. **Assertion A:** A relation R = {(1, 1), (1, 3), (3, 1), (3, 3), (3, 5)} defined on the set A={1,3,5} is reflexive.

Reason R: A relation R on the set A is said to be transitive if for. (a, b) ∈R and (b, c)∈R , we have (a,c)∈R.

a) Both A and R are individually true and R is the correct explanation of A.
b) Both A and R are individually true and R is not the correct explanation of A.
c) 'A' is true but 'R' is false
d) 'A' is false but 'R' is true

10. **Assertion A**: A relation R={(x,y):|x−y|=0} defined on the set A = {3, 5, 7} is symmetric.

Reason R: A relation R on the set A is said to be symmetric if for (a,b)∈R, we have (b,a)∈R

a) Both A and R are individually true and R is the correct explanation of A.
b) Both A and R are individually true and R is not the correct explanation of A.
c) 'A' is true but 'R' is false
d) 'A' is false but 'R' is true.

SHORT AND LONG ANSWER QUESTION

11. State whether the function is one – one, onto or bijective f: R → R defined by f(x) = 1+ x^2
12. Checkwhether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a+1} is reflexive, symmetric or transitive.
13. Check whether the relation R in the set R of real numbers, defined by $R = \{(a, b) : 1 + ab > 0)\}$, is reflexive, symmetric or transitive.
14. Define the relation R in the set $N \times N$ as follows:

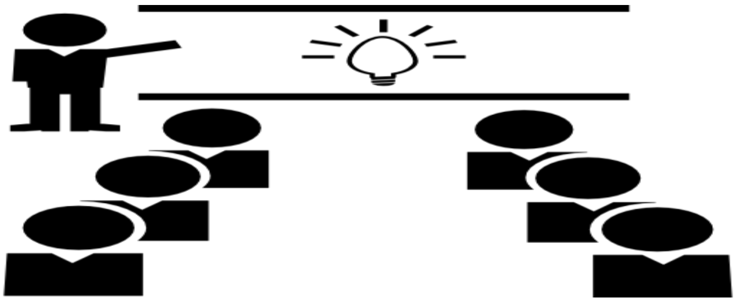
For (a, b), (c, d) $\in N \times N$, (a, b) R (c, d) iff ad (b +c) = b c (a +d). Prove that R is an equivalence relation in $N \times N$.

CASE STUDY BASED QUESTIONS

15. Mr. Pradeep take a lecture in class – XII, in this class students divided into two groups A and B group.

A has 22 Students and group B has 23 students. The relation R: A → B between students are defined

as $R = \{(x, y): x \in A and y \in B\}$



Based of the above information answer the following questions-

(i) Find the number of elements in co-domain of R: A → B.

(ii) The set of all identity elements

(iii) Find the total number of relations that can be formed from set A to B.

(iv) Suppose the relation defined from set A to set B is $R_1 = \{(a_1, b_1), (a_2, b_2), (a_3, b_3)\}$ then find the inverse relation.

16. Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be set of all possible outcomes. A = {S, D}, B = {1, 2, 3, 4, 5, 6}

Based on the above information answer the following:

(i) Let R : B → B be defined by R = {(x, y) : y is divisible by x} is

(a) Reflexive and transitive but not symmetric (b) Reflexive and symmetric but not transitive

(c) Not reflexive but symmetric and transitive (d) Equivalence

(ii) Raji wants to know the number of functions from A to B. How much number of functions are possible?

(a) 6^2 (b) 2^6 (c) 6! (d) 2^{12}

(iii) Let R be a relation on B defined by R = {(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)}. Then R is

(a) Symmetric (b) Reflexive (c) Transitive (d) None of these three

(iv) Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

(a) 6^2 (b) 2^6 (c) 6! (d) 2^{12}

17. In two different societies, there are some school going students – including girl as well as boys. Satish forms two sets with these students, as his college project.

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ where a_i's and b_i's are the school going students of first and second society respectively. Satish decides to explore these sets for various types of relations and functions.

Using the above information given above, answer the following

(i) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

(a) 0, (b) 2^5 (c) 2^{11} (d) 2^{20}

(ii) Let : $A \rightarrow A, R = \{(x, y): x and y are students of same sex\}$. Then relation R is

(a) Reflexive only
(b) reflexive and symmetric but not transitive
(c) reflexive and transitive but not symmetric
(d) an equivalence relation

(iii) Satish and his friend Rajat are interested to know the number of symmetric relations defined on the both the set A and B, separately. Satish decides to find the symmetric relation on A. while Rajat decides to find the symmetric relation on set B. What is difference between their results?

(a) 1024 (b) $2^{10}(15)$ (c) $2^{10}(31)$ (d) $2^{10}(63)$

(iv) Let $R: A \rightarrow B, R = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$, then R is

(a) Neither one-one nor onto
(b) one-one but not onto
(c) only onto but not one-one
(d) one-one and onto both

1. RELATION AND FUNCTION

ANSWERS

QUESTION NO.	ANSWER
1	(d) Transitive but neither symmetric nor reflexive.
2	(b) 2
3	(b) $n!$
4	(c)30
5	(a) a bijective
6	(b) f(x) = x + 2
7	(c) 106!
8	(d) A is not correct but R is correct
9	d) 'A' is false but 'R' is true
10	a) Both A and R are individually true and R is the correct explanation of A.
11	neither one-one, not onto
12	not reflexive, not symmetric, not transitive
13	reflexive, symmetric but not transitive
14	Show Reflexive, symmetric & Transitive to be true
15	i) 23 (ii) $\emptyset$ (iii) 2^{506} (iv) $\{(b_1, a_1), (b_2, a_2), (b_3, a_3)\}$
16	(i) (a) (ii) (a) (iii) (d) (iv) (d) (v) (b)
17	(i) d (ii) d (iii) c (iv) a (v) b

2. INVERSE TRIGONOMETRIC FUNCTIONS

MCQ'S

1. One branch of $\cos^{-1}$ other than the principal value branch corresponds to
   (a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (c) $(0, \pi)$ (d) $[2\pi, 3\pi]$

2. The domain of $y = \cos^{-1}(x^2 - 4)$ is
   (a) $[3, 5]$ (b) $[0, \pi]$ (c) $\left[-\sqrt{5}, -\sqrt{3}\right] \cap \left[-\sqrt{5}, \sqrt{3}\right]$ (d) $\left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$

3. $Let\ \theta = \sin^{-1}\{\sin(-600^0)\}, then\ value\ of\ \theta\ is$
   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $-\frac{2\pi}{3}$

4. If $\sin^{-1} x > \cos^{-1} x\ , then\ x\ should\ be\ in\ the\ interval$
   (a) $(-1, -\frac{1}{\sqrt{2}})$ (b) $(0, \frac{1}{\sqrt{2}})$ (c) $(\frac{1}{\sqrt{2}}, 1)$ (d) $(-\frac{1}{\sqrt{2}}, 0)$

5. The domain of the function $\cos^{-1}(2x - 1)$ is ....
   (a) $[0, 1]$ (b) $[-1, 1]$ (c) $[-2, 1]$ (d) $[0, \pi]$

6. The domain of the function defined by $f(x) = \sin^{-1}x + \cos x$ is
   (a) $[-1, 1]$ (b) $[-1, \pi + 1]$ (c) $(-\infty, \infty)$ (d) φ

7.The value of $\tan^{-1}(tan\frac{3\pi}{4})$ is
a) $-\frac{3\pi}{4}$ b) $-\frac{\pi}{4}$ c) $\frac{\pi}{4}$ d) $\frac{3\pi}{4}$

8. The value of $\sin^{-1}(sin\frac{3\pi}{5})$ is
a) $-\frac{2\pi}{5}$ b) $-\frac{\pi}{5}$ c) $\frac{\pi}{5}$ d) $\frac{2\pi}{5}$

9. $Sin[\frac{\pi}{3} + \sin^{-1}\frac{1}{2}]\ is\ equal\ to$
a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

10. If $4\cos^{-1} x + \sin^{-1} x = \pi, then\ the\ value\ of\ x\ is$
a) $\frac{3}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{2}{\sqrt{3}}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true

11. **Assertion(A)** :$\cos^{-1} x + \sin^{-1} x = 0\ then\ x = \frac{1}{\sqrt{2}}$
Reason (R) :$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$

12. **Assertion(A)** Rangeof $[\sin^{-1} x + 2\cos^{-1} x]\ is\ [0, \pi]$.
Reason (R): Principal branch value of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

13**. Assertion(A):** The domain of the function $\sec^{-1} 2x\ is\ \left(-\infty, -\frac{1}{2}\right] \cup [\frac{1}{2}, \infty)$
Reason (R): $\sec^{-1}(-2) = -\frac{\pi}{4}$

SHORT AND LONG QUESTIONS

14. Find the value of $sin^2 \cos^{-1}(\frac{3}{5}) + tan^2 \sec^{-1}(3)$

15. Find the value of $cot^2[\csc^{-1}(3)] + sin^2[\cos^{-1}(\frac{1}{3})]$

16. Evaluate (i) $\tan^{-1}(2Cos\frac{2\pi}{3})$ (ii) $\tan^{-1}(tan\frac{3\pi}{4})$

17. Evaluate $\sin^{-1}\{Cos(\sin^{-1}\frac{\sqrt{3}}{2})\}$

18. Prove that $Cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$

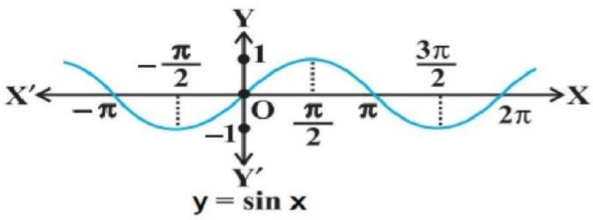
19. Find the value of $\tan^{-1}\left\{2sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

20. Write the principal value of $\cos^{-1} cos\frac{2\pi}{3} + \sin^{-1} sin\frac{2\pi}{3}$

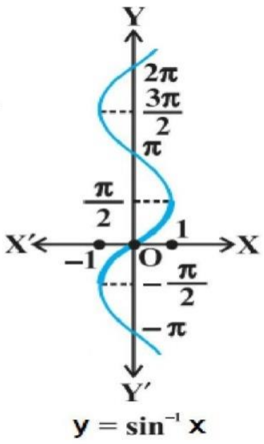
CASE STUDY BASED QUESTIONS

21. Two brothers are plying their bicycles over two curved pathways. Their teacher tells the m that one of the brothers is following the path of curve y= sin x while the other one follows the path of $y = \sin^{-1} x$

Refer the graphs of sine function and its inverse function, given below



Flip about y = x



Based on the above information, answer the following:

(i) The domain and range of Sine function respectively is

(a) [-1,1], R (b) R, [-1,1] (c) R, R (d) None of these

(ii) Once we restrict the domain of Sine function to $[-\pi/2, \pi/2]$ then it becomes

(a) Onto only (b) One-One only (c) One-one onto both (d) None of these

(iii) Considering the graph, write the range of $\sin^{-1} x$ other than the principal branch.

(a) $[0, -\pi/2]$ (b) $[\pi/2, 3\pi/2]$ (c) $[0, \pi]$ (d) None of these

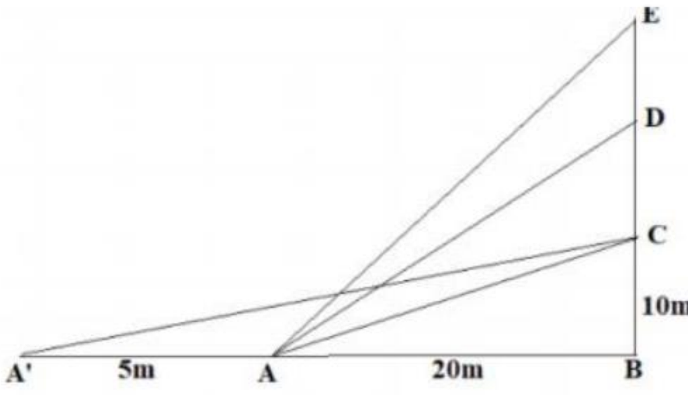
(iv) What will be the principal value for $\sin^{-1}\frac{\sqrt{2}}{2}$

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{3}$

(v) The domain of $\sin^{-1} 3x$ is

(a) [-1, 1] (b) $[-\frac{1}{2}, \frac{1}{2}]$ (c) $[-\frac{1}{3}, \frac{1}{3}]$ (d) None of these

22. The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID -19 protocol. Ram , Robert and Rahim are the three engineers who are working on this project . If " A " is considered to be a person viewing the hoarding board 20 metres away from the building , standing at the edge of a pathway nearby . Ram ,Robert and Rahim suggested to the film to place the hoarding board at three different locations namely C,D and E . "C " is at the height of 10 metres from the ground level. For the viewer A , the angle of elevation of " D "is double the angle of elevation of "C" .The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer .Look at the figure given and based on the above information answer the following .



(i) Measure of $\angle CAB =$

a) $\tan^{-1} 2$ b) $\tan^{-1}\frac{1}{2}$ c) $\tan^{-1} 1$ d) $\tan^{-1} 3$

(ii) Measure of $\angle DAB =$

a) $\tan^{-1}\frac{3}{4}$ b) $\tan^{-1}\frac{4}{3}$ c) $\tan^{-1} 3$ d) $\tan^{-1} 4$

(iii) Measure of $\angle EAB =$

a) $\tan^{-1} 11$ b) $\tan^{-1} 3$ c) $\tan^{-1}\frac{2}{11}$ d) $\tan^{-1}\frac{11}{2}$

(iv) If A' is another viewer standing on the same line of observation across the road. If the width of the road is 5 metre, then the $\angle CA'B =$

a) $\tan^{-1}\frac{1}{2}$ b) $\tan^{-1}\frac{1}{8}$ c) $\tan^{-1}\frac{2}{5}$ d) $\tan^{-1}\frac{11}{21}$

2. INVERSE TRIGONOMETRIC FUNCTIONS

ANSWERS

MCQ'S

1(d)	2(d)	3(a)	4(c)	5(a)
6(a)	7(b)	8(d)	9(a)	10(c)

ASSERTION-REASON BASED QUESTIONS

11.(d) 12. (a) 13. (c)

SHORT AND LONG QUESTIONS

14. Ans.$\frac{216}{25}$

$\cos^{-1}\frac{3}{5} = \sin^{-1}\frac{4}{5}$ and$\sec^{-1} 3 = \tan^{-1} 2\sqrt{2}$

$$sin^2 \sin^{-1}\frac{4}{5} + tan^2 \tan^{-1} 2\sqrt{2} = \frac{16}{25} + 8 = \frac{216}{25}$$

15. Ans. $\frac{80}{9}$

16. (i) Ans. $cos\frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = -cos\frac{\pi}{3} = -\frac{1}{2}$

$$\tan^{-1}\{2\left(-\frac{1}{2}\right\} = \tan^{-1}(-1) = -\frac{\pi}{4}$$

(ii) $-\frac{\pi}{4}$

17. Ans. $\sin^{-1}\{Cos(\sin^{-1}\frac{\sqrt{3}}{2})\}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ $\cos\frac{\pi}{3} = \frac{1}{2}$ $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

19. $\tan^{-1}\left\{2sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

$$= \tan^{-1}\left\{2sin\left(2 \times \frac{\pi}{6}\right)\right\}$$

$$= \tan^{-1}\left\{2sin\left(\frac{\pi}{3}\right)\right\}$$

$$= \tan^{-1} 2 \times \frac{\sqrt{3}}{2} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

20. Ans. $\text{Cos}^{-1} cos\frac{2\pi}{3} + \sin^{-1} sin\frac{2\pi}{3}$

$$= \frac{2\pi}{3} + \frac{\pi}{3} = \pi$$

CASE STUDY BASED QUESTIONS

21. (i) (b) R, [-1,1] (ii)(c) One-one onto both (iii) (b) $[\pi/2, 3\pi/2]$
(iv) (a) $\frac{\pi}{4}$ (v) $[-\frac{1}{3}, \frac{1}{3}]$

22. (i) b) $\tan^{-1}\frac{1}{2}$ (ii) b) $\tan^{-1}\frac{4}{3}$ (iii) d) $\tan^{-1}\frac{11}{2}$
(iv) c) $\tan^{-1}\frac{2}{5}$

3 ,4. MATRICES & DETERMINANTS

MULTIPLE CHOICE QUESTIONS (MCQ)

1. If A and B are square matrices of the same order, then (A+B) (A +B) is equal to:
   (a) A (b) B (c) $A^2 - B^2$ (d) $A^2 + BA + AB + B^2$

2. If A and B are two matrices of the order 3× m and 3× n, respectively, and m= n, then write the order of matrix (5A – 2B) :
   (a) 3x 3 (b) m x n (c) 3 x m (d) none of these

3. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of a, b and c are :
   (a) a=1 , b=2 ,c=3 (b) a = -1 b = 2 , c= -3
   (c) a= -2 , b=0 , c= -3 (d) none of these

4. A matrix which is both symmetric and skew symmetric is :
   (a) Row matrix (b) column matrix (c) null matrix (d) None of these

5. For a 2 X 2 matrix A=$[a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$ then the value of a_{12} is :
   (a) 1/2 (b) 2 (c) 1/3 (d) None of these

6. The value of α for which the matrix A= $\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ is an identity matrix will be :
   (a) $\frac{\pi}{2}$ (b) 0 (c) $\frac{\pi}{3}$ (d) π

7. If $\begin{bmatrix} 2 & 3 \\ 7 & x \end{bmatrix} = \begin{bmatrix} y & z \\ 7 & -5 \end{bmatrix}$, then the value of (x-y+z) is:
   (a) 0 (b) -4 (c) 4 (d) 3

8. The matrix $A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & -4 \\ 1 & 4 & 0 \end{bmatrix}$ is :
   (a) Symmetric (b) Skew Symmetric
   (c) Both Symmetric and Skew Symmetric (d) None of these

9. The number of all possible matrices of order 2x2 with each entry 5, 8 or 9 is:
   (a) 9 (b) 6 (c) 81 (d) 27

10. If $[4 \quad -5 \quad -1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = A$, then the order of A is
    (a) 1× 1 (b) 3× 1 (c) 1× 3 (d) 3× 3

ASSERTION AND REASON BASED QUESTIONS

11. Assertion (A) : Let A and B are matrices of order 3 X2 and 2 X4 respectively, then the order of matrix AB is 3 X 4 .

Reason (R): If A= $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A=A' then x=y .

a) Both A and R are true and R is correct explanation of A
b) Both A and R are true and R is not correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Select correct option .

12. Assertion (A) : if A is a skew symmetric matrix, then A^2 is a symmetric matrix.

Reason (R): if a matrix A is both symmetric and skew-symmetric then matrix A is a zero matrix of order n X n .

a) Both A and R are true and R is correct explanation of A
b) Both A and R are true and R is not correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

13. Assertion (A) : If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A = I$

Reason (R) : AI = IA = A

a) Both A and R are true and R is correct explanation of A
b) Both A and R are true and R is not correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

14. Assertion (A) : $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a scalar matrix

Reason (R) : If all the elements of the principal diagonal are equal , it is called Scalar matrix.

a) Both A and R are true and R is correct explanation of A
b) Both A and R are true and R is not correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

15. Assertion (A) : If A is a Symmetric matrix , then B'AB is also Symmetric.

Reason (R) : (ABC)' = C'B'A'

a) Both A and R are true and R is correct explanation of A
b) Both A and R are true and R is not correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

SHORT ANSWER QUESTION

16. Find the value of x + y from the following equation :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

17. If A is a square matrix such that $A^2 = I$, then find the simplified value of

$(A - I)^3 + (A + I)^3$ - 7A.

18. Matrix A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric , find the values of a and b

LONG ANSWER QUESTION

19. Express the matrix $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix and verify your result.

20. Let = $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$. Using this result , calculate A^{-1}.

21. Find A^{-1} of the matrix , A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and hence solve the system of linear equations

$x - y + z = 4$; $2x + y - 3z = 0$; $x + y + z = 2$

22. Find A^{-1} of the matrix , A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and hence solve the system of linear equations

$x + 2y + z = 4$; $-x + y + z = 0$; $x - 3y + z = 2$

CASE BASED QUESTION

23. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs.100 and Rs.50 each. The number of articles sold by schools A, B and C are given below.

↓ Article School	→ A	B	C
Fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the above information, answer the following questions:

(i) Write a matrix P be a 3× 3 matrix represent the sale of handmade fans, mats and plates by schools.

(ii) Write a matrix Q of 3× 1 represents the sale price of given products.

(iii) Find the collected funds by all the three schools for all products.

24. On the occasion of children's day, class teacher of class XII Shri Singh, decided to donate some money to students of class XII.

If there were 8 students less everyone would have got Rs. 10 more. However, if there were 16 students more, everyone would have got Rs. 10 less.

i) If number of students in class be x and Shri Singh has decided to donate Rs. Y to each student, then write the above information as a system of linear equation.

ii) Write the value of $\begin{vmatrix} 5 & -4 \\ 5 & -8 \end{vmatrix}$.

iii) If A = $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}$ then find A^{-1}.

OR

Find the number of students in class XII and the amount distributed by Shri Singh respectively.

25. A manufacture produces three stationery products Pencil , Eraser and Sharpener which he sells in two markets. Annual sales are indicated below :

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10,000	2,000	18,000
B	6,000	20,000	8,000

If the unit Sale price of Pencil , Eraser and Sharpener are Rs. 2.50 , Rs. 1.50 and Rs. 1.00 respectively , and unit cost of the above three commodities are Rs. 2.00 , Rs. 1.00 and Rs. 0.50 respectively, then based on the above information answer the following:

i) Find total revenue of market A

ii) Find total revenue of market B

iii) Find the cost incurred in market A

OR

Find gross profit in both market

26. Consider two matrices of order 3 x 3

$$A = \begin{bmatrix} 2+x & z+6 & 8 \\ 5 & 8+y & c \\ 4 & a & b \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 & 6 & 8 \\ 5 & 5 & 3c+2 \\ 4 & 1+2a & 2 \end{bmatrix}$$

If these two matrices A and B are equal then

i) Find the value of a, b and c

ii) Find the value of x , y and z

iii) Find the value of 2c + 8a + 6b

OR

Find the value of 7x + 2y

27. On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got Rs.10 more. However, if there were 16 children more, everyone would have got Rs. 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in Rs.)

Based on the information given above, answer the following questions:

i) Write matrix equations to represent the information given above.

ii) Find the number of children who were given some money by Seema.

iii) How much amount is given to each child by Seema?

OR

How much amount Seema spends in distributing the money to all the students of the Orphanage?

3 ,4. MATRICES & DETERMINANTS
ANSWER
(MCQ)

1. (d) $A^2 + BA + AB + B^2$ 2. (c) 3 x m 3. (c) a= -2 , b=0 , c= -3

4. (c) null matrix 5. (a) 1/2 6. (b) 0 7. (b) -4

8. (b) Skew Symmetric 9. (c) 81 10. (a) 1× 1

(ASSERTION & REASON)

11. (b) Both A and R are true and R is not correct explanation of A
12. (b) Both A and R are true and R is not correct explanation of A
13. **(a)** Both A and R are true and R is correct explanation of A
14. **(a)** Both A and R are true and R is correct explanation of A
15. **(a)** Both A and R are true and R is correct explanation of A

(SHORT ANSWER)

16. Given :2 $\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 14 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Equating the corresponding elements , we get

2x + 3 = 7 and 2y – 4 = 14

⇒ x = 2 and y = 9

⇒ x + y = 11 (Answer)

17. We have A^2 = I

Now ,$(A - I)^3$ + $(A + I)^3$ - 7A

= (A^3 - 3 A^2I + 3A I^2 - I^3) + (A^3 + 3 A^2 I + 3A I^2 + I^3) – 7A

= $2A^3$ + 6A I^2 – 7A

= $2A^3$ + 6A I – 7A [$I^2 = I$]

= $2A^2 . A$ + 6A – 7A [AI = A]

= 2 I .A + 6 A – 7A

= 2 A + 6 A – 7A

= A (Answer)

18. We have A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

Since A is symmetric matrix

⇒ A^T = A

$\Rightarrow \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

Equating the corresponding elements on both sides, we get

2b = 3 and 3a = - 2

⇒ b = $\frac{3}{2}$ and a = - $\frac{2}{3}$ (Answer)

(LONG ANSWER)

19. Solution: Let A $= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

A can be expressed as

A = $\frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ .... (I)

Where $(A + A^T)$ and $(A - A^T)$ are symmetric and skew symmetric matrices respectively.

Now, $(A + A^T) = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

$(A - A^T) = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$

Putting these values in (I) , we get

$A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$

Verification :

$\Rightarrow \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} & -\frac{3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$ (Hence verified)

20. Soln. : $A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Also ,

Since, $A^2 - 4A + 7I = 0$

Multiplying on both sides by A^{-1} on its left

$A - 4I + 7A^{-1} = 0$

$$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + 7A^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{-3}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}$

21. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

x = 2 , y = - 1 , Z = 1

22. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

x = $\frac{9}{5}$, y = $\frac{2}{5}$, z = $\frac{7}{5}$

(CASE BASED)

23. : (i) $\begin{bmatrix} 40 & 25 & 35 \\ 50 & 40 & 50 \\ 20 & 30 & 40 \end{bmatrix}$ (ii) $\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$ (iii) $\begin{bmatrix} 5250 \\ 7750 \\ 5500 \end{bmatrix}$

24 : i) 5x – 4y = 40 ; 5x – 8y = - 80 ii) – 20 iii) $\frac{-1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$ OR 32 , Rs 30

25 : i) Total revenue of market A will be

$$[\,A\,] = [\,10000 \quad 2000 \quad 18000\,]\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

⇒ A = 46,000

ii) Total revenue of market B will be

$$[\,B\,] = [\,6000 \quad 20000 \quad 8000\,]\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

⇒ B = 53,000

iii) Total Cost incurred in market A will be

$$[\,A\,] = [\,10000 \quad 2000 \quad 18000\,]\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

⇒ A = 31,000

OR

The cost incurred in market B can be calculated as

$$[B] = [6000 \quad 20000 \quad 8000\,]\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

⇒ B = 36,000

Now , Profit = Revenue – cost

Profit in market A = Rs. 46,000 – Rs. 31,000 = Rs. 15,000

Profit in market B = Rs. 53,000 – Rs. 36,000 = Rs. 17,000

Gross profit in both markets = Rs. 15,000 + Rs. 17,000 = Rs. 32,000

26 : i) a = - 1 , b = 2 , c = -1 ii) x = 6 , y = - 3 , z = 0 iii) 2 OR 36

27 : i) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$ ii) 32 iii) Rs. 30 OR Rs. 960

5. CONTINUITY AND DIFFERENTIABILITY OF FUNCTIONS

MULTIPLE CHOICE QUESTIONS (1 MARK EACH)

Q1 $The\ function\ f(x) = |x-1| + |2x+1| + |3x-2|\ is\ not\ differentiable\ at$

(a) $x = 0$ (b) $x = -\frac{1}{2}\ and\ x = 3$ (c) $x = 1, x = -\frac{1}{2}, x = \frac{2}{3}$ (d) R-{ 1, -1/2}

Q2 The function $f(x) = \begin{cases} \frac{sin^2 4x}{x^2}, x \neq 0 \\ \frac{\lambda}{2} \quad ,x = 0 \end{cases}$ is continuous at $x = 0$, then λ is equal to

a) 3 b) 16 c) 32 d) 18

Q3 The value of k for which f(x)= $\begin{cases} 5x-4\ ,\ 0 < x \leq 1 \\ 4x^2 + 3kx\ ,\ 1 < x < 2 \end{cases}$ is continuous at x=1

(a)-1 (b) -2 (c) 2 (d) 1

Q4 Derivative of $\cos(e^x)$ at x=1 is

(a) -esin e (b) sin e (c) -(1+e) sin e (d) (1+e) sin e

Q5. The derivative of $2^x\ w.r.t.\ 3^x$

(a) $\left(\frac{3}{2}\right)^x \frac{log2}{\log 3}$ (b) $\left(\frac{2}{3}\right)^x \frac{log3}{\log 2}$ (c) $\left(\frac{2}{3}\right)^x \frac{log2}{\log 3}$ (d) $\left(\frac{3}{2}\right)^x \frac{log3}{\log 2}$

Q6. $If\ y = \cos^{-1}(e^x)$, then $\frac{dy}{dx}$ is

(a) $\frac{1}{\sqrt{e^{-2x}+1}}$ (b) $\frac{-1}{\sqrt{e^{-2x}+1}}$ (c) $\frac{1}{\sqrt{e^{-2x}-1}}$ (d) $\frac{-1}{\sqrt{e^{-2x}-1}}$

Q7 If y = A cos x + B sin x, then $\frac{d^2y}{dx^2}$ + my= 0 . The value of (m^2 + 1) is

(a) 1 (b) 0 (c) -1 (d) 2

Q8 The value of $\frac{dy}{dx}$ if $y = e^{log 5x^2}$

a) 1/x b) 5 c) 5x d) 10x

Q9 The value of $\frac{dy}{dx}$ if x = $2at^2$, y = at^4

a) t b) t^2 c) t^3 d) t^4

Q10 For what value of $\boldsymbol{\lambda}$, the function f(x) = $\left\{\begin{matrix} \frac{1-cos4x}{8x^2} & ,x \neq 0 \\ \lambda & ,x = 0 \end{matrix}\right\}$ is continuous at x = 0.

a) 1 b) 2 c) 4 d) -1

ASSERTION –REASON BASED QUESTIONS (1 MARK EACH)

In the following questions a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

a) **Both (A) and (R) are True and R is the correct Explanation of (A)**
b) **Both (A) and (R) are True and R is not the correct Explanation of (A)**
c) **(A) is True but (R) is False**
d) **(A) is False but (R) is True**

Q11.Assertion (A): Let $f(x) = |x+4|\ .f(x)\ is\ not\ differentiable\ at\ x = -4$
Reason (R): All continuous functions are not necessarily differentiable.

Q12. Assertion (A): Let x= sin2t and y = cos2t then $y' = -tan2t$
Reason (R): Let $y = f(x)\ and\ x = h(t), y = g(t)\ where\ t\ is\ a\ parameter\ .Then \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

Q13. Assertion (A): $Let\ \ y = f(x) = \tan^{-1} 2x\,, then\ y' = \frac{-2}{1+x^2}$

Reason (R): $\frac{d}{dx}\tan^{-1}\sqrt{\frac{1+cosx}{1-cosx}} = -\frac{1}{2}$

Q14. Assertion (A): $f(x) = e^{2x} + sin^2x \quad is\ \ continuous\ for\ all\ real\ values\ of\ x.$

Reason (R): The sum of two continuous functions is always a continuous function.

Q15. Assertion (A): The derivative of $sin^2x\ \ w.r.t. cos^2x\ \ is\ equal\ to\ \ -1$

Reason (R): The derivative of f(x) w.r.t. g(x) is given by $\frac{df(x)}{dg(x)}$

VERY SHORT ANSWER QUESTIONS (VSA) (2 MARK EACH)

Q16 If $y = \cos^{-1} x,\ show\ that\ \frac{d^2y}{dx^2} = -coty\ cosec^2y$

Q17.Check the differentiability of $f(x) = |cosx|$ at x=$\frac{\pi}{2}$

Q18 If $x = e^{x/y}\ ,show\ that\ \ \frac{dy}{dx} = \frac{logx-1}{(logx)^2}$

SHORT ANSWER QUESTIONS (SA) (3 MARK EACH)

Q19 Show that the greatest integer function f(x) = [x] is not differentiable at x=2

Q20. If $y = (logx)^2\ \ prove\ that\ \ x^2y_2 + xy_1 - 2 = 0$

Q21. $If\ y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$ Show that $(x^2+1)y' + xy + 1 = 0$

LONG ANSWER QUESTIONS(LA) (5 MARK EACH)

Q22. $If\ y = \tan^{-1}\left(\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right)\ \ then\ show\ that\ y'' = 0$

Q23. $If\ \ x = 2sin2t\ (1 + cos2t)\ \ and\ y = 4cos2t\ (1 - cos2t)$

$then\ show\ that\ \frac{dy}{dx} = 2\ at\ t = \frac{\pi}{4}$

Q24. $If\ y = (\tan^{-1} x)^2\ prove\ that\ \ (1+x^2)^2y_2 + 2x(1+x^2)y_1 - 2 = 0$

CASE BASED QUESTIONS (4 MARK EACH)

Q25. Read the following passage and answer the questions given below:

The absolute value function is defined as f(x) =$|x| = \begin{cases} x, & if\ x > 0 \\ 0, & if\ \ x = 0 \\ -x, & if\ x < 0 \end{cases}$

The function defined above is a continuous function for all real values of x but it is not derivable at x=0. The function $p(x) = |ax - b|\ is\ not\ differentiable\ at\ \ x = \frac{b}{a}$ .Now consider the following modulus functions:

$g(x) = |x-1|\ and\ \ h(x) = |x+1|.$

Answer the following questions.

(a) Check the continuity of the function r(x) = 2g(x) +3 h(x) at x= 0.

(b) Find the right-hand derivative of the function $q(x) = 100 + x\ h(x)\ \ at\ x = 0$

Q26. Read the following passage and answer the questions given below:

An equation of the form y=g(x) is said to define y explicitly as a function of x because the variable y appears alone on one side of the equation and does not appear at all on the other side. For example the expression $sinx + xy = 0\ is\ an\ explicit\ expression\ in\ y.$ If on the other hand it is not possible to express y directly in terms of x then the relation between y and x is said to be expressed implicitly.

For example: $xsinxy + yx^2 = 0\ is\ an\ implicit\ function.$

Answer the following questions.

(a) Find the derivative of $y^{100}\ \ w.r.t.x.$

(b) Find $\frac{dy}{dx}$ $if\ x^3 + y^3 = 3xy$

Q27. Read the following passage and answer the questions given below:
Consider a particle moving along a curve C in the xy- plane in such a way that its x- and y- coordinates as functions of time t, are $given\ by\ \ x = f(t)\ and\ y = g(t).$ $These\ equations\ are\ known\ as\ the\ parametric\ equations\ of\ motion$ $for\ the\ particle\ and\ the\ curve\ C\ is\ called\ as\ the\ trajectory\ of\ the\ particle.$
Consider a particle whose equations of motion are given as $x = t^2, y = t^3\ , t \neq 0$

$$-\infty < t < \infty$$

This curve is known as semi cubical parabola.
Answer the following questions.

(a) Find the rate of change of y w.r.t. x.

(b) Find $\frac{dy}{dx}\ at\ t = 6$

(c) Find the second derivative of y w.r.t. x.

Q28 Read the following passage and answer the questions given below:

$Let\ \ y = f(x)\ be\ a\ differentiable\ function\ such\ that$
$z = f'(x)\ is\ also\ differentiable. Then\ the\ second\ derivative\ of$
$y = f(x) is\ denoted\ by\ y_2\ or\ y''\ or\ \frac{d^2y}{dx^2}\ and\ is\ defined\ by$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dz}{dx}.$$

$Similarly\ we\ can\ define\ the\ higher\ order\ derivtives\ as$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \ f'''(x) = y_3\ and\ so\ on. f^n(x) is\ known$$

$as\ the\ nth\ derivative\ of\ y.$
$Many\ functions\ like\ \ polynomial, trigonometric, exponential,$
$logarithmic\ \ functions\ are\ differentiable\ \ at\ each\ point\ of\ their\ domain.$ Answer the following questions.

(a) If $(sinx)(cosy) = \frac{1}{2}\ \ , then\ \ \ find\ \ \frac{d^2y}{dx^2}\ \ at\ \ x = \frac{\pi}{4}\ , y = \frac{\pi}{4}$

(b) Let = e^{2x} . Then find the value of the product $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$.

Q29. Read the following passage and answer the questions given below:

$The\ function\ f(x) is\ said\ to\ be\ non-differentiable\ at\ x = a\ if$
(1) $f(x)$ is discontinuous at the point x = a
(2) f (x) is continuous function at x = a
and both left hand derivative and right hand derivative
exist but not equal to each other.
(3) f(x) is continuous function at x = a and either or both left hand derivative and right hand derivative are not finite .
All continuous functions are not necessarily differentiable.
$Consider\ the\ function\ \ \ \ h(x) = 2x|x|\ ,$ where |x| is absolute value function.
Answer the following questions:

(1) Find the right hand derivative of $h(x) at\ x = 0$

(2) Find the left hand derivative of $h(x) at\ x = 0$

(3) Check whether h(x) is differentiable at x = 0?

5. CONTINUITY AND DIFFERENTIABILITY OF FUNCTIONS

Answers (MCQ)

1	C	6	D
2	C	7	D
3	A	8	D
4	A	9	B
5	C	10	C

Answers (assertion-reason)

11	12	13	14	15
A	A	D	A	A

Answers: (vsa)

17. not differentiable

Answers (case Based)

Q25	Q26	Q27	Q28	Q29
(a) Continuous (b) 1	(a) $100 y^{99} y'$ (b) $\frac{y-x^2}{y^2-x}$	(a) $\frac{3t}{2}$ (b) 9 (c) $\frac{3}{4t}$	(a) – 4 (b) $-2e^{-2x}$	(a) 0 (b) 0 (c) differentiable

6. APPLICATION OF DERIVATIVES

MULTIPLE CHOICE QUESTIONS

1.The value of b for which the function f(x) = x + cos x + b is strictly decreasing over R is
(a) $b \geq 1$ (c) for no values of b
(b) $b \leq 1$ (d) b=1

2. The area of a trapezium is defined by a function which is given by
$f(x) = (10 + x)\sqrt{100 - x^2}$, then the area when it is maximized is
(a)75 cm^2 (c)7 $\sqrt{3}$ cm^2
(b)75$\sqrt{3}$ cm^2 (d)5 cm^2

3. The maximum value of $[\, x(x-1) + 1]^{1/3}$, $0 \leq x \leq 1$ is
(a) $(1/3)^{1/3}$ (b) 1^2 (c) 1 (d) 0

4. The value of x for which $(x - x^2)$ is maximum is
(a) 3/4 (c) 1/3
(b) 1/2 (d) 1/4

5. Function $f(x) = 2x^3 - 6x + 5$, is an increasing function in the interval
(a) $(-\infty, -1/2) \cup (1/2, \infty)$ (b) $(-1,1)$ c)$(-1, -1/2)$ (d) $(-\infty, -1) \cup (1, \infty)$

6. A company makes closed water storage tank. The water tank is cylindrical in shape.
Let S be the given surface area ,r be the radius of base and h be height of the tank .
Based on the information provided answer the following:
Relation between S,r and h is:
(a) $S = 2\pi rh + 2\pi r^2$
(b) $S = 2\pi rh + \pi r^2$
(c) $S = \pi r^2 h + \pi r^2$
(d) $S = \pi r^2 h + 2\pi r^2$

7. The function $f(x) = x^x$ has stationary point at
a) x = e b) x = 1 c) x = 1/e d) $x = \sqrt{e}$

8. The total revenue in rupees received from the sale of x units of a product is given by R(x) = $3x^2 + 36x + 5$.
The marginal revenue, when x = 15 is
a) 116 b) 96 c) 90 d) 126

9. The side of an equilateral triangle is increasing at the rate of 2 cm/sec. The rate at which area increases
when the side is 10 is :
A) 10 cm^2/sec b) $\sqrt{3}$ cm^2/sec c) 10 $\sqrt{3}$ cm^2/sec d) 10 /3 cm^2/sec

10. If f(x)= a(x - cos x) is strictly decreasing in R, then 'a' belongs to :
A) {0} b) $(0,\infty)$ c) $(-\infty,0)$ d) $(-\infty,\infty)$

ASSERTION –REASON BASED QUESTIONS (1 MARK EACH)

Questions consists of two statements− **Assertion (A) and Reason (R).**
Answer these questions selecting the appropriate option given below –
a) Both A and R are true and R is the correct explanation for A
b) Both A and R are true and R is not the correct explanation for A
c) A is true but R is false
d) A is false but R is true

11. Assertion A : The function $f(x) = x^3 + 3x^2 + 3x + 7$ is increasing for all real values of x.
Reason (R) : For any function y = f(x) to be increasing, $dy/dx > 0$

12. Assertion A : The difference of the greatest & smallest value of $f(x)=\pi/4 - \tan^{-1}x$ on [0,4] is $\pi/4$.
Reason (R) : If a function f decreases on [a,b] then the greatest value of f is f(a) & least value is f(b).

13. Assertion A: The minimum value of $f(x) = x^2 + 2bx + c$ is $c - b^2$.
Reason (R) : $f'(-b) = 0$.

LONG ANSWER & SHORT ANSWER QUESTIONS

14. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

15. A spherical snowball is melting in such a way that its radius decreasing at a rate of 0.1 cm/min. At what rate is the volume of the snowball is decreasing when the radius is 5 cm?

16. A particle is moving along the curve represented by the polynomial $f(x) = (x-2)^2(x-1)$. Find the interval where f(x) is strictly increasing.

17. Prove that the volume of the largest cone that can be inscribed in a sphere of radius ‘a’ is 8/27 of the volume of the sphere.

CASE BASED QUESTIONS

18. The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation
$y=4x-\frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

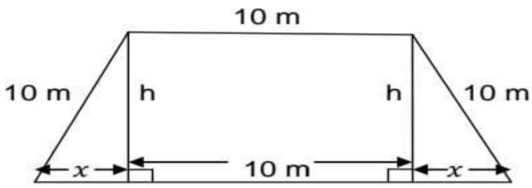
i. The rate of growth of the plant with respect to sunlight is_.

ii. The number of days it will take for the plant to grow to the maximum height are__

iii. Find the maximum height of the plant.

iv. Height of plant after 2 days will be______ .

19. The front gate of a building is in the shape of a trapezium as shown below. Its three sides other than base are10m each. The height of the gate is h meter. On the basis of this information and figure given below, answer the following questions:



i. The area A of the gate expressed as a function of x is______

ii.Find the value of $\frac{dA}{dx}$.

iii. Find the value of x, for which $\frac{dA}{dx} = 0$

iv. Find the maximum area of trapezium.

v. If area of trapezium is maximum, then find $\frac{d^2y}{dx^2}$.

6. APPLICATION OF DERIVATIVES

ANSWERS

QUESTION NO.	ANSWER
1	B
2	C
3	C
4	B
5	D
6	B
7	C
8	D
9	C
10	C
11	A
12	D
13	B
14	(0,1) and (2,∞)
15	10π
16	$\frac{4}{3} < X < 2$
18	i) 4-x ii) 4 iii) 8 iv) 6
19	i) (10+x)$\sqrt{100 - x2}$ ii) $\frac{-2(x+10)(x-5)}{\sqrt{100-x2}}$ iii) 5,-10 iv) 15$\sqrt{75}$ v) $\frac{-30}{\sqrt{75}}$

7. INDEFINITE AND DEFINITE INTEGRALS

MULTIPLE CHOICE QUESTIONS

Q1 : $\int e^{5\log x}$ dx is equal to

(A): $\frac{x^5}{5}$ + C (B): $\frac{x^6}{6}$ + C (C): $5x^4$ + C (D): $6x^5$ + C

Q2: If $\int_0^a 3x^2$ dx = 8 , then the value of a is

(A): 2 (B): 4 (C): $\frac{8}{3}$ (D): 3

Q3: The value of $\int_0^{\pi/6} sec^2\left(\frac{\pi}{6} - x\right)$dx is

(A): $-\sqrt{3}$ (B): $-\frac{1}{\sqrt{3}}$ (C): $\sqrt{3}$ (D): $\frac{1}{\sqrt{3}}$

Q4: The value of $\int(cos^2 2x - sin^2 2x)$dx is

(A): $-\frac{sin4x}{4}$ + C (B): $\frac{cos4x}{4}$ + C

(C): $\frac{sin4x}{4}$ + C (D): $-\frac{cos4x}{4}$ + C

Q5: The value of $\int \frac{3x}{3x-1}$ dx is

(A): $\frac{1}{3}\log|3x-1|$ + C (B): x + $\log|3x-1|$ + C

(C): x + $\frac{1}{3}\log|3x-1|$ + C (D): None of above

Q6: The value of $\int \frac{1}{x-\sqrt{x}}$ dx is

(A): 2 log $(\sqrt{x}-1)$ + C (B): x + $\frac{2}{3}x^{\frac{3}{2}}$ + C

(C): $\frac{1}{2}$log $(\sqrt{x}-1)$ + C (D): None of above

Q7: If f(x) = $\int_0^x tsint\,dt$, then $f'(x)$ is equal to

(A): cosx + x sinx (B): x sin x

(C): x cosx (D): x cosx + sinx

Q8: The value of $\int_0^{\pi/2} log\left(\frac{4+3\,sinx}{4+3\,cosx}\right)$ dx is

(A): 2 (B): 3/4 (C): 1 (D): 0

Q9: $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}$ dx is equal to

(A): 1/2 (B): 3/2 (C): 5/2 (D): 0

ASSERTION- REASON BASED QUESTIONS

Each of the following questions contains two statements: Assertion (A) and Reason (R) .Each of the questions has four alternative choice, only one of which is correct statement

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q10: Assertion (A): $\int_{-1}^{1}(x^3 + sinx + 2)$ dx = 0

Reason (R): $\int_{-a}^{a} f(x) = \begin{cases} 2\int_0^a f(x)dx & if\ f(-x) = f(x) \\ 0 & if\ f(-x) = -f(x) \end{cases}$

Q11: Assertion (A): $\int \frac{1}{x^2+2x+3}$ dx = $\frac{1}{\sqrt{2}} tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)$ + C

Reason (R): $\int \frac{1}{x^2+a^2}$ dx = $\frac{1}{a} tan^{-1}\left(\frac{x}{a}\right)$ + C

Q12: Assertion (A): $\int e^x \left[\frac{1}{x} - \frac{1}{x^2}\right]$ dx = $\frac{e^x}{x}$ + C

Reason (R): $\int e^x\ [\, f(x) + f'(x)]$ dx = $e^x\ f(x)$ +C

VERY SHORT ANSWER TYPE QUESTIONS

Q13: Evaluate $\int \frac{\sin x}{\sin(\,x+a)}$ dx

Q14: Evaluate $\int x^2 \log x$ dx

Q15: Evaluate $\int_0^1 x\,(1-x)^n$ dx

SHORT ANSWER TYPE QUESTIONS

Q16: Evaluate $\int \frac{1}{x^{1/2} + x^{1/3}}$ dx

Q17: Evaluate $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$ dx

Q18: Evaluate $\int \frac{x^2}{x^2+6x+12}$ dx

LONG ANSWER TYPE QUESTIONS

Q19: Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$ dx

Q20. Evaluate dx $\int\left[\sqrt{tanx} + \sqrt{cotx}\ \right]$ dx

Q21: Prove that $\int_0^a f(x)dx\ = \int_0^a f(a-x)dx,$ hence evaluate $\int_0^{\pi} \frac{x\tan x}{secx+tanx}$ dx

CASE BASED QUESTIONS

Q22: For any function f(x)

$\int_a^b f(x)$ dx = $\int_a^{c_1} f(x)$ dx + $\int_{c_1}^{c_2} f(x)$ dx +$\int_{c_2}^{c_3} f(x)$ dx ....+ $\int_{c_n}^{b} f(x)$ dx

where a < c_1 < c_2 < c_3<...........c_n< b

Based on above information, evaluate following integrals

(a) $\int_{-2}^{3}|2x-3|$ dx

(b) $\int_0^3 [x]$ dx where $[x]$ is greatest integer function

Q23: A function is called even function if f$(-x)$ = f(x) and a function is known as odd function if

f$(-x)$ = $-$ f(x).

Following property of definite integrals is very useful to evaluate value of $\int_{-a}^{a} f(x)$ dx

$$\int_{-a}^{a} f(x) = \begin{cases} 2\int_{0}^{a} f(x)dx & if\ \ f(x) is\ an\ even\ function \\ 0 & if\ \ f(x) is\ an\ odd\ function \end{cases}$$

Based on above information, evaluate following integrals

(a) $\int_{-\pi/2}^{\pi/2} (x^3 + sin^3 x)$ dx

(b) $\int_{-\pi/2}^{\pi/2} sin^2 x$ dx

Q24. A Mathematics teacher discuss following property of definite integral with the students of class XII.

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)\,dx\,,\ \ f(x) is\ defined\ on\ [a\,,b]$$

After that he has given some problems based on the above property to the students of his class.

Use the above property and answer the following questions:

(i) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{cot\,x}}$

(ii) $\int_{1}^{4} \frac{\sqrt{5-x}}{\sqrt{5-x}\ +\sqrt{x}}\ dx$

7. INDEFINITE AND DEFINITE INTEGRALS

ANSWER KEY

MULTIPLE CHOICE QUESTIONS

Ans1 (B): $\frac{x^6}{6}$ + C

Ans2 (A): 2

Ans3 (D): $\frac{1}{\sqrt{3}}$

Ans4 (C): $\frac{sin4x}{4}$ + C

Ans5 (C): x + $\frac{1}{3}$ log$|3x-1|$ + C

Ans6 (A): 2 log $(\sqrt{x}-1)$ + C

Ans7 (B): x sin x

Ans8 (D): 0

Ans9. (C): 5/2

ASSERTION- REASON BASED QUESTIONS

Ans10. D

Ans11. A

Ans12. A

VERY SHORT ANSWER TYPE QUESTIONS

Ans13. xcos a + sina log$|\sin(x-a)|$ + C

Ans14. $\frac{x^3}{3}$ log x $-$ $\frac{x^3}{9}$ + C

Ans15. $\frac{1}{(n+1)(n+2)}$

SHORT ANSWER TYPE QUESTIONS

Ans16: $2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\log\left|x^{1/6}+1\right|$ + C

Ans17: $sin^{-1}\left(\frac{e^x+2}{3}\right)$ + C

Ans18: $x - 3\,log|x^2+6x+12| + 2\sqrt{3}\,tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right)$ + C

LONG ANSWER TYPE QUESTIONS

Ans19. $-2\sqrt{1-x} + cos^{-1}\sqrt{x} + \sqrt{x(1-x)}$ + C

Ans20. $\sqrt{2}\,sin^{-1}(sinx - cosx) + c$

Ans21. $\frac{3\,\pi}{\pi^2}$

CASE BASED QUESTIONS

Ans22 (a). 29/2

(b). 3

Ans23 (a). 0

(b). $\frac{\pi}{2}$

Ans. 24.(i) $\frac{\pi}{12}$

(iii) 3/2

8.AREA UNDER CURVE
MULTIPLE CHOICE QUESTIONS (MCQ)

1.Area of the region in the first quadrant enclosed by the $x - axis$, $the\ line\ y = x$ and the circle $x^2 + y^2 = 32$.
(a) $16\,\pi\ sq\ units$ (b) $4\,\pi\ sq\ units$ (c) $32\,\pi\ sq\ units$ (d) $24\,\pi\ sq\ units$

2. Area of the region bounded by the curve $y = x|x|$ $between\ x = -1\ and\ x = 1\ is$
(a) 0 $(b)\frac{1}{3}$ $(c)\frac{2}{3}$ $(d)\frac{4}{3}$

3. For the area bounded by the curve $y = ax$, the line $x = 2\ and\ x - axis$ to be 2 sq. units, the value of a must be equal
(a) 1 (b) 2 (c) 3 (d) 4

4. Area between the curve $y = x\ and\ y = x^3\ is$

$$(a)\frac{\sqrt{3}}{2}\ (b)\frac{1}{2}\ (c)\ 2\sqrt{2}\ (d)\frac{1}{4}$$

5. Area between the curve $y = cos^2x$, $x - axis\ and\ ordinates\ x = 0\ and\ x = \pi\ in\ the\ interval\ (0, \pi)$is
$(a)\frac{2\pi}{3}$ $(b)\ 2\pi$ $(c)\ \pi$ $(d)\frac{\pi}{2}$

6. The area bounded by the curve $y^2 = 16x\ and\ y = mx\ is\ \frac{2}{3}$, $then\ m\ is\ equal\ to$
(a) 1 (b) 2 (c) 3 (d) 4

7. Area of the parabola $y^2 = 4ax\ bounded\ by\ its\ latus\ rectum\ is$
$(a)\frac{2a^2}{3}\ sq\ units$ $(b)\frac{4a^2}{3}\ sq\ units$ $(c)\frac{8a^2}{3}\ sq\ units$ $(d)a^2$

ASSERTION AND REASON TYPE

DIRECTION: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R) . Choose the correct answer out of the following choices.

(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A.(c) A is true and R is false. **(d) A is false but R is true.**

8. Assertion: The area bounded by the curve $y = sinx\ and\ y = -sinx\ from\ 0\ to\ \pi\ is\ 3\ sq\ units.$
Reason : The area bounded by the curve is symmetric about $x - axis.$

9. Assertion: The area bounded by the circle $x^2 + y^2 = a^2$in the first quadrant is given by $\int_0^a \sqrt{a^2 - x^2}\, dx.$
Reason : The same area can also be found by $\int_0^a \sqrt{a^2 - y^2}\, dy.$

10. Assertion: The area bounded by the curve $y = cosx$ in I quadrant with the coordinate axes is 1 sq unit.
Reason:$\int_0^{\frac{\pi}{2}} cosx\, dx = 1.$

SHORT ANSWER TYPE

11. Find the area enclosed by the curve $x = 3\,cost\,,\ y = 2\,sint\,.$

12. Find the areaof the region bounded by the curve $x = at^2\ and\ y = 2at$ between the ordinates corresponding to $t = 1\ \ and\ \ t = 2.$

13. Consider the curve $x^2 + y^2 = 16\ and\ the\ line\ \ y = x$ in the first quadrant. Based on above information, answer the following questions.
(i) Find the point of intersection of both the curves.
(ii) Find the area bounded between the curves.

14. Consider the following equations of curves $y = cosx\ , y = x + 1\ and\ y = 0$ **.** On the basis of above information, answer the following questions.
(i) The curves $y = cosx\ and\ \ y = x + 1$ meets at
$(a)(1,0)(b)(0,1)(c)(1,1)(d)(0,0)$
(ii) Find the area bounded by the given curves. `
15. Find the area of the region bounded by the parabola $y = \ x^2\ and\ \ y = |x|.$

LONG ANSWER TYPE

16. Find the area of the region$\{(\boldsymbol{x}, \boldsymbol{y}): \mathbf{9}\boldsymbol{x}^{\mathbf{2}} + \boldsymbol{y}^{\mathbf{2}} \leq \mathbf{36}\boldsymbol{and}\ \mathbf{3}\boldsymbol{x} + \boldsymbol{y} \geq \mathbf{6}\}$ **.**

17. Sketch the graph $y = \ |x + 1|\ .Evaluate\ \int_{-3}^{1} |x + 1|\ dx$. What does this value represent on the graph?

18. Using integration, find the area of the region: $\{(x, y): 0 \leq 2y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 3\}$

CASE BASED QUESTIONS

19. Amirror in the shape of an ellipse represented by $\frac{x^2}{9} + \frac{y^2}{4} = 1$was hanging on the wall. Arun and his sister were playing with ball inside the house , all of a sudden ball hits the mirror and got a scratch in the shape of line represented by $\frac{x}{3} + \frac{y}{2} = 1$.

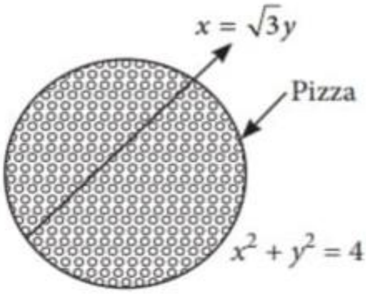
Based on the above information answer the following questions

(i) Find the point of intersection of ellipse and the scratch.

(ii) Represent the area of the smaller region bounded by ellipse and the scratch in terms of integrals.

(iii) Find the area of the smaller region bounded by ellipse and the scratch.

20. A child cuts a pizza with a knife. Pizza is circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edges of the knife represents a straight line given by $x = \sqrt{3}\ y.$



Based on the above information answer the following questions

(i) Find the point of intersection of line and pizza as shown in the figure. **(ii)** Write the expression for the area bounded by the circular pizza and the edge of the knife in the first quadrant.

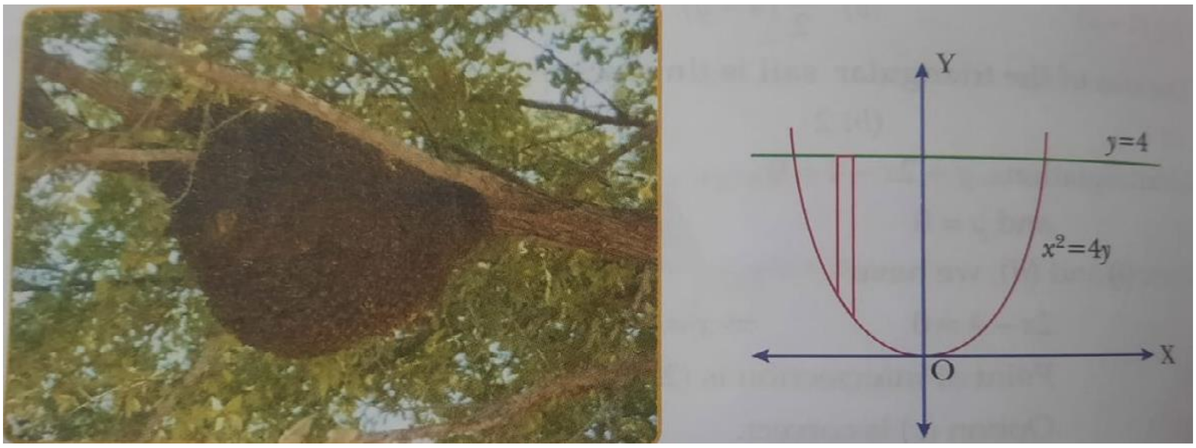
(iii) Find the area of the region bounded by the circular pizza and the edge of the knife in the first quadrant.

21. Garvita a class XII student visited Science Park at Dehradun with her father who is a mathematics teacher . She observes a elliptical carrom board with a very peculiar property that we put two strikers at two of its focus and we hit one of the strike at any point in its boundary it will definitely hit the other striker. Her father give the following information about the carrom board.
(a) Length of major axis and minor axis are 6 and 4 respectively .

Based on the above information answer the following questions
(i) Find the equation of the ellipse .
(ii) Write the expression for the area of ellipse using integrals .
(iii) Using integration find the area of the ellipse.

22. Readthe following and answer the questions that follows

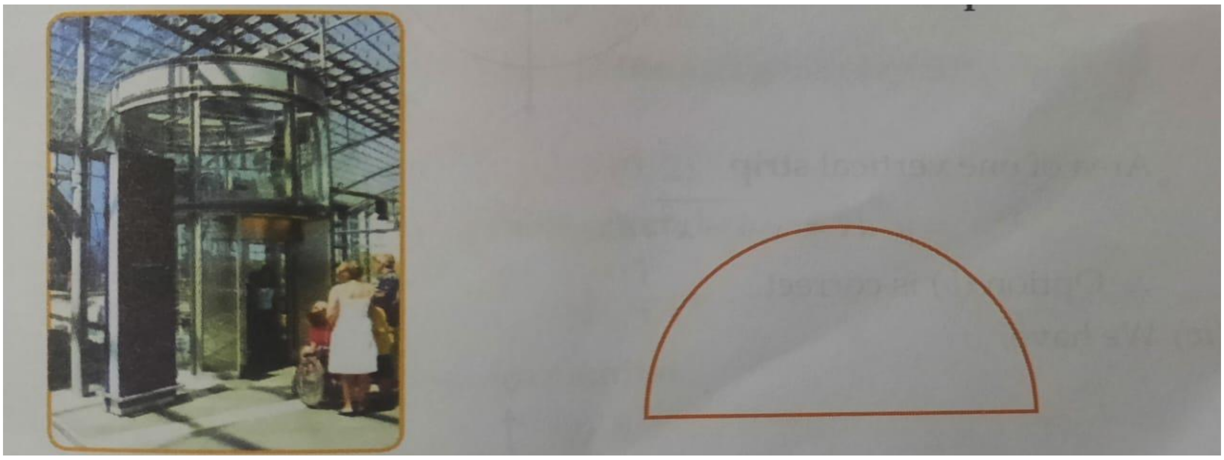
A student designs an open air Honeybee nest on the branch of a tree , whose plane figure is parabolic and the branch of tree is given by a straight line .



(i) Find the point of intersection of the parabola and straight line.
(ii) Find the length of each vertical strip in terms of x .
(iii) Find the area of the region bounded by parabola $x^2 = 4y$ *and line* $y = 4$.

23. Read the following and answer the questions that follows

An architect designs a building whose lift (elevator) is from outside of the building attached to the walls . The floor(base) of the lift (elevator) is in semi-circular shape .



The floor of the elevator whose circular edge is given by the equation $x^2 + y^2 = 4$and the straight edge(line) is given $y = 0$.

(i) Find the point of intersection of the region bounded by the circular edge and the straight edge.
(ii) Write the expression to find the area of the region of the floor of the lift in the building.
(iii) Find the area of the region of the floor of the lift in the building.

8.AREA UNDER CURVE

ANSWER KEY

MULTIPLE CHOICE QUESTIONS (MCQ)

1. $(b)\ 4\pi\ sq\ units$

2. $(c)\frac{2}{3}$

3. $(a)\ 1$

4. $(b)\frac{1}{2}$

5. $(d)\frac{\pi}{2}$

6. $(d)\ 4$

7. $(c)\frac{8a^2}{3}\ sq\ units$

ASSERTION AND REASON TYPE

8. (d) A is false but R is true.

9. **(**b) Both A and R are true, but R is not the correct explanation of A.

10. **(**a) Both A and R are true, and R is the correct explanation of A**.**

SHORT ANSWER TYPE

11.6π sq units .

12. $\frac{\mathbf{56}}{\mathbf{3}}\boldsymbol{a^2}$sq units.

13. (i) $\left(2\sqrt{2}\ ,\ 2\sqrt{2}\right)$ (ii)$\boldsymbol{2\pi}$ sq units

14. (i) $(c)(1,1)$ (ii) $\frac{3}{2}\ sq\ units$.

15. $\frac{\mathbf{1}}{\mathbf{3}}$sq units .

16. $\mathbf{3}\ (\boldsymbol{\pi}-\mathbf{2})$sq units**.**

17. $\int_{-3}^{1}|x+1|\ dx=4$ **.**This represents area bounded between $x=-3\ ,x=1\ and\ \ x-axis$.

18. $\frac{23}{6}\ sq\ units$.

19. $(\boldsymbol{i})(\mathbf{0},\mathbf{2})\ \boldsymbol{and}\ (\mathbf{3},\mathbf{0})(\boldsymbol{ii})\frac{\mathbf{2}}{\mathbf{3}}\int_{\mathbf{0}}^{\mathbf{2}}\sqrt{\mathbf{9}-\boldsymbol{x^2}}\ \boldsymbol{dx}-\mathbf{2}\int_{\mathbf{0}}^{\mathbf{3}}\left(\mathbf{1}-\frac{\boldsymbol{x}}{\mathbf{3}}\right)\boldsymbol{dx}$ $(\boldsymbol{iii})\ \mathbf{3}\left(\frac{\boldsymbol{\pi}}{\mathbf{2}}-\mathbf{1}\right)$

20. (i) $\left(\sqrt{3}\ ,1\right),(-\sqrt{3},-1)$

(ii) $\int_0^{\sqrt{3}}\frac{x}{\sqrt{3}}\ dx\ +\ \int_{\sqrt{3}}^{2}\sqrt{4-x^2}\ dx$

(iii) $\frac{\pi}{3}$ $sq\ units$

21. (i) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(ii) $4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\ dx$

(iii) $6\pi\ \ sq\ units$

22. $(\boldsymbol{i})(-4,4)\ and\ \ (4,4)$

(ii) $\frac{1}{4}(16 - x^2)$

(iii) $\frac{64}{3}\ sq\ units$.

23. $(\boldsymbol{i})(2,0)\ and\ \ (-2,0)$

(ii) $2\int_0^2 \sqrt{4 - x^2}\ dx$

(iii) $2\pi\ \ sq\ units$

9 CHAPTER DIFFERENTIAL EQUTIONS

MCQ (1marks questions)

1. The degree of differential equation $x^2\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^3$ is

(a) 1 (b) 2 (c)3 (d) 6

2. The order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2=\frac{d^2y}{dx^2}$ respectively, are

(a) 1,2 (b) 2,2 (c) 2,1 (d) 4,2

3. The integrating factor of the differential equation $\frac{dy}{dx}(xlogx)+y=2logx$ is

(a) e^x (b) log x (c) log(log x) (d) x

4. The degree of differential equation $sinx+cos\left(\frac{dy}{dx}\right)=y^2$ is

(a) 2 (b) 1 (c) not defined (d) 0

5. The solution of the degree of differential equation $\frac{dy}{dx}+\frac{2y}{x}=0$ with y(1) is given by

(a) $y=\frac{1}{x^2}$ (b) $x=\frac{1}{y^2}$ (c) $x=\frac{1}{y}$

(d) $y=\frac{1}{x}$

6. The product of the order and degree of the differential equation $\frac{d^2y}{dx^2}siny+\left(\frac{dy}{dx}\right)^3cosy=\sqrt{y}$ is

(a) 3 (b) 2 (c) 6 (d) not defined

7. The sum of the order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=siny$ is

(a) 5 (b) 2 (c)3 (d) 4

8. The general solution of the degree of differential equation $xdy-(1+x^2)dx=dx$ is

(a) $y=2x+\frac{x^3}{3}+c$ (b) $y=2logx+\frac{x^3}{3}+c$ (c) $y=\frac{x^2}{2}+c$ (d) $y=2logx+\frac{x^2}{2}+c$

9. The integrating factor of the differential equation $(1-y^2)\frac{dx}{dy}+yx=ay,(-1<y<1)$ is

(a) $\frac{1}{y^2-1}$ (b) $\frac{1}{\sqrt{y^2-1}}$ (c) $\frac{1}{1-y^2}$

(d) $\frac{1}{\sqrt{1-y^2}}$

ASSERTION-REASON QUESTIONS

Each of the following questions contains two statements: Assertion (A) and Reason (R) .Each of the questions has four alternative choice, only one of which is correct statement

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

10. Assertion (A): The degree of differential equation $\frac{d^2y}{dx^2}=1+\sqrt{\frac{dy}{dx}}$ is 2.

Reason (R): The degree of a differential equation is the degree of the highest order derivative occurring in the equation, when differential co-efficient are made free from radicals, fractions and it is written as a polynomial in differential coefficient.

11. Assertion (A): Solution of differential equation ? $(1+x^2)\frac{dy}{dx}+y=tan^{-1}x$ is

$$ye^{tan^{-1}x}=(tan^{-1}x-1)e^{tan^{-1}x}+c$$

Reason (R): The differential equation of the form $\frac{dy}{dx}+Py=Q,$ $where\ P\ ,Q\ be\ the\ functions\ of\ x$ or constant, is a linear type differential equation.

12. Assertion (A): The integrating factor of the differential equation $\frac{dx}{dy}+(tanx).x=sec^2y$ is $secy.$

Reason (R): Linear differential equation of the form $\frac{dy}{dx}+Py=Q,$ where P, Q=f(x) or constant has integrating factor , $IF=e^{\int P\,dy}$

VERY SHORT ANWER QUESTIONS (2 MARKS)

13. Solve the differential equation: $(y+3x^2)\frac{dx}{dy}=x$.

14. Solve the differential equation: $\frac{dy}{dx}=ytanx\,; y=1\ when\ x=0$

15. Find general solution of the differential equation : $log\left(\frac{dy}{dx}\right)=ax+by$

Short Answer Questions (3 marks)

16. Find particular solution of the differential equation $x\frac{dy}{dx}+xcos^2\left(\frac{y}{x}\right)=y$; given that when $x=1, y=\frac{\pi}{4}$.

17. Solve the differential equation

$$(1+x^2)\frac{dy}{dx}+2xy-4x^2=0\ , subject\ to\ the\ initial\ condition\ y(0)=0$$

18. Find general solution of the differential equation: $\frac{d}{dx}(xy^2)=2y(1+x^2)$

LONG ANWER QUESTIONS (5 MARKS)

19. Solve the following differential equation:
$3x^2tanydx+(2-e^x)sec^2ydy=0, given\ that\ when\ x=0, y=\frac{\pi}{4}$

20. Find particular solution of the differential equation $\frac{dy}{dx}=\frac{x(2logx+1)}{siny+ycosy}$ given that $x=1, y=\frac{\pi}{2}$
.

21.Show that the differential equation $\left[xsin^2\left(\frac{y}{x}\right)-y\right]dx+xdy=0$ is homogeneous. Find the particular solution of this differential equation, given that $y=\frac{\pi}{4}$ when x=1.

CASE STUDY QUESTIONS

22.Read the following passage and answer the following questions:
Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx}=k(50-y)$ where x denotes the number of weeks and y the number of children who have been given the drops.

(i) (a) Find the solution of dirrerential equation $\frac{dy}{dx}=k(50-y)$.

(b) Find the value of C in the particular solution given that y(0)=0 and k= 0.049.

(ii) Find the solution that may be used to find the number of children who have been given the polio drops.

23. Read the following passage and answer the following questions:

If an equation is in the form $\frac{dy}{dx}+Py=Q,$
$Where\ P\ , Q\ are\ the\ functions\ of\ x\ then\ such\ equation\ is\ known\ as\ linear\ differential$ eqution . Its solution is given by

$$y\times IF=\int Q\times IF\,dx+C$$

Where $IF=e^{\int Pdx}$

Now suppose we have equation. $\frac{dy}{dx}+\frac{y}{x}=x^2$

(i) Write the value of P.

(ii) Write the value of Q.

(iii) (a) Find the general solution of the differential equation.
OR
(b) If the value of Q replace by sin x , find the solution.

24. Read the following passage and answer the following questions:
An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation. A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make the substitution $y = vx$ and then separate the variables.
Based on the above, answer the following questions:
(I) Show that $\left(x^2 - y^2\right) dx + 2xydy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.
(II) Solve the above equation to find its general solution.

9 CHAPTER DIFFERENTIAL EQUTIONS

Answers

MCQ

1. (a)
2. (c)
3. (b)
4. (c)
5. (a)
6. (b)
7. (c)
8. (d)
9. (d)

Assertion-Reason Questions

10. (a)
11. (b)
12. (a)

Very Short Anwer Questions

13. $y - 3x^2 + cx = 0$
14. $y = secx$
15. $\frac{e^{ax}}{a} + \frac{e^{-bx}}{a} = e$

Short Anwer Questions (3 marks)

16. $tan\frac{y}{x} = -log|x| + 1$
17. $3y(1 + x^2) = 4x^3$
18. $y = 2 + \frac{2}{5}x^2 + C$

Long Anwer Questions (5 marks)

19. $tany = (2 - e^x)^3$
20. $ysiny = x^2logx + \frac{\pi}{2}$.
21. $logx - cot\frac{y}{x} + 1 = 0$

Case Study Questions

22. (i) (a) $-\log|50 - y| = kx + C$

    (b) $C = \log\frac{1}{50}$

    (ii) $y = 50(1 - e^{-kx})$ This is the required solution to find the number of children who have been given the polio drops.

23. (i) $P = \frac{1}{x}$

    $(ii)Q = x^2$

    (iii)(a)$y = \frac{x^3}{4} + \frac{C}{x}$

    (iii)(b)$y = -cosx + \frac{sinx}{x} + \frac{C}{x}$

24. (i) Which is of the form $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$

    (ii)$x^2 + y^2 = Cx$

10. VECTOR ALGEBRA

MCQ (01 Mark Each)

Q1- If $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$ are two equal vectors , then the value of $x + y + z$ is

(a) 0 (b) 1 (c) 2 (d) 3

Q2- Angle between vectors $\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ is

(a) $\theta = \frac{\pi}{4}$ (b) $\theta = \frac{\pi}{3}$ (c) $\theta = \frac{\pi}{2}$ (d) $\theta = \frac{2\pi}{3}$

Q3- The value of p when the projection of $\vec{a} = p\,\hat{\imath} + \hat{\jmath} + 4\,\hat{k}$ on $\vec{b} = 2\,\hat{\imath} + 6\hat{\jmath} + 3\,\hat{k}$ is 4 units

(a) 4 (b) 5 (c) 3 (d) 6

Q4- **(** $\vec{a} + \vec{b}$) X ($\vec{a} - \vec{b}$) is

(a) $|\vec{a}|^2 - |\vec{b}|^2$ (b) 2 ($\vec{a}$ x $\vec{b}$) (c) 2 ($\vec{b}$ x $\vec{a}$) (d) None of these

Q5- If a + b =i and a= 2i – 2j + 2k , then |b| equals

(a) √14 (b) 3 (c) √12 (d) √17

Q6- The value of $|\vec{a} - \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$ is

(a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{3}$ (d) $\sqrt{7}$

Q7- The position vector of midpoint of $\overrightarrow{AB}$ is 3i + 2j -3k .If point B is (2,3,-4) then position vector of $\vec{A}$ is

(a) 5i/2 + 5j/2 + 7k/2 (b) 4i +j – 2k (c) 5i + 5j -7k (d) i/2 –j/2 + k/2

Q8- If θ is the angle between any two vectors $\vec{a}$ *and* $\vec{b}$, then $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to

(a) 0 (b) π / 4 (c) π/2 (d) π

Q9- The vector in the direction of a vector I – 2j + 2k with magnitude 9 is

(a) i – 2j + 2k (b)(i – 2j + 2k) /3 (c) 9(i – 2j + 2k) (d) 3(i – 2j + 2k)

Q10. For what value of a vectors 2i – 3j + 4k and ai + 6j – 8k are collinear or parallel.

(a) -4 (b) 2 (c) 25 (d) none of these

ASSERTION-REASON TYPE QUESTIONS (1 MARK EACH)

Each of the following questions contains two statements : Assertion (A) and Reason (R) .Each of the questions has four alternative choice, only one of which is correct statement

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q11- **Assertion (A)**: $|\vec{a}| = |\vec{b}|$ does not imply $\vec{a} = \vec{b}$.

Reason (R) : If $\vec{a} = \vec{b}$, then $\vec{a}.\vec{b} = |\vec{a}|^2 = |\vec{b}|^2$.

Q12- **Assertion (A):** If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$.

Reason (R) : $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c}.\vec{a})$

Q13- **Assertion(A)** :The direction cosines of vector i+j+k is 1/√3 , 1/√3, 1/√3 .

Reason(R): If vector r = ai +bj+ck its direction ratios are a/|r|, b/|r| ,c/|r| where r= $\sqrt{a^2+b^2+c^2}$

Q14- **Assertion(A)**: If $|\vec{a} \times \vec{b}| = 1$ and $|\vec{a} \cdot \vec{b}| = \sqrt{3}$ then the angle between $\vec{a}$ *and* $\vec{b}$ is π/6

Reason(R) : | $\vec{a}$ X $\vec{b}$ |=| $\vec{a}$ || $\vec{b}$ |sin□ and | $\vec{a}$. $\vec{b}$ |= | $\vec{a}$ || $\vec{b}$ |cos□

Q15- **Assertion(A):** If ($\vec{a} - \vec{b}$).($\vec{a} + \vec{b}$)= 0 then $\vec{a}$ *and* $\vec{b}$ are perpendicular.

Reason (R): the projection of i +3j+k on 3i-3j+6k is -1/7

S A (2 marks questions)

Q16. For what value of p the projection of a = pi +j + 4k on b = 2i + 6j + 3k is 4 units.

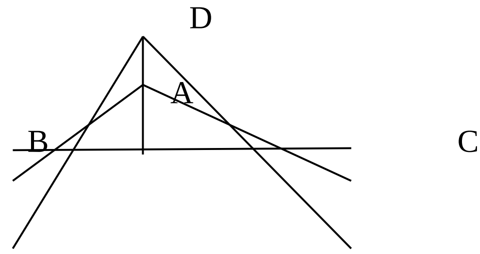
Q17. Find p if (2i +6j + 14k) X (i – p j + 7k) = $\vec{0}$
Q18. What are the direction cosines of a line, which makes equal angles with coordinate axis?

LONG ANWER QUESTIONS

Q19If a = i+j+k, b=4i-2j+3k and c= i-2j+k, find a vector of magnitude 6 units which is parallel to the vectors 2a-b+3c.
Q20. Let a=i+4j+2k, b=3i-2j+7k and c=2i-j+4k. Fin a vector d which is perpendicular to both a and b and c.d=18
Q21. Let a =i – j, b = 3i –k and c = 7i –k. Find a vector d which is perpendicular to both a and b and c.d= 1.
Q22: Dot product of a vector with i+j-3k, i+3j-2k and 2i+j+4k are 0, 5 and 8 respectively. Find the vector.

CASE BASED QUESTIONS

Each of following Questions carries 4 marks. (1+1+2)
(23)- Ambrish purchased a plant holder which is in a shape of tetrahedron ABCD
The coordinates of vertices are A(1,2,3), B(3,2,1), C(2,1,2), D(3,4,3)



(a) Find vector $\overrightarrow{AB}$
(b) Find vector $\overrightarrow{CD}$
(c) Find unit vector $\overrightarrow{BC}$
or
Find the area of Δ BCD

Q(24) – A student appearing for a competitive examination was asked to attempt the following questions : Let $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non zero vectors.

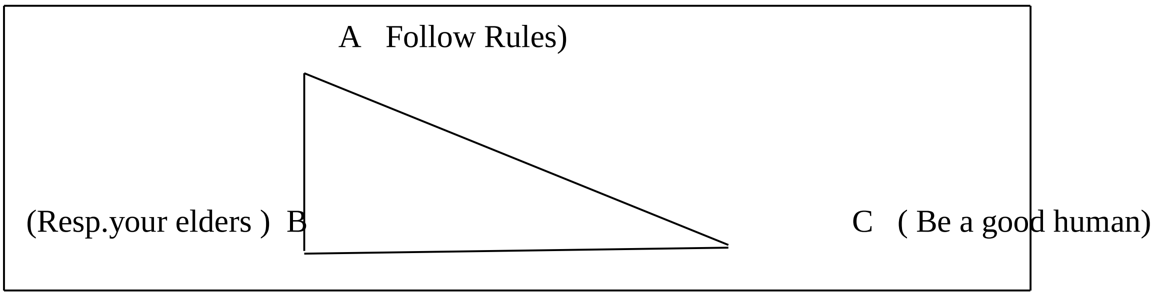
(i) If $\vec{a}$, $\vec{b}$ are such that $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$then find the relation between $\vec{a}$ & $\vec{b}$
(ii) if $\vec{a} = i - 2j$, $\vec{b}$ =2i+j+3k then find the value of $(2\vec{a}+\vec{b}).\{(\overrightarrow{2a}+\vec{b})X(\vec{a}-2\vec{b}\}$,
(iii) Find the area of parallelogram whose diagonals are $\vec{a}$ & $\vec{b}$
Or Find unit vector $\vec{a} - \vec{b}$

Q25. Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P (6,8,4) , Q (21,8,4), R (21,16,10) and S= (6,16,10)

(i). What are the components to the two edge vectors defined by $\vec{a} = \overrightarrow{PQ}$ & $\vec{b} = \overrightarrow{PS}$
(ii). Find the magnitudes of $\vec{a}$ & $\vec{b}$

(iii) What are the components to the vector $\vec{N}$, Which is perpendicular to both $\underset{a}{\rightarrow}$ & $\underset{b}{\rightarrow}$?

Q26- The slogan on chart papers are to be placed in a school bulletin board at the points A, B,& C displaying A(follow Rules) , B (Respect your elders) & C(Be a good human). The coordinates of these three points are A(1,4,2) B(3,-3,-2) & (-2,2,6)



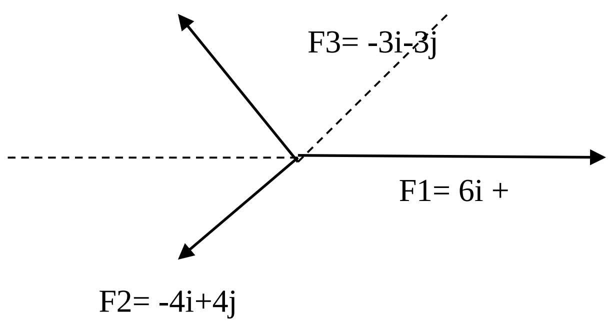
(i) If $\vec{a}$, $\vec{b}$ & $\vec{c}$ are the position vectors of the points A, B & C respectively then find

$$|\vec{a}, +\vec{b} + \vec{c}|$$

(ii)If $\vec{a} = 4i + 6j + 12k$ then find unit vector along $\vec{a}$

(iii) Find area of Δ**ABC**

Q27. Teams A, B ,C went for playing a tug of war game . Teams A,B,C have attached a rope to a metal ring & is trying to pull the ring into their own area . C



Team A pulls with force F1 = 6i + oj N

Team A pulls with force F2 = -4i +4j N

Team A pulls with force F3 = -3i -3j N
0j

(i)What is magnitude of the force team A ?

(ii)Which team will win the game?

(iii)Find the magnitude of the resultants force exerted by the teams

Or In What direction is the ring getting pulled?

10 . VECTOR ALGEBRA

Answers MCQs

1- (a) 0

2- (c) $\theta = \frac{\pi}{2}$

3- (b) p= 5

4- (c) 2 ($\vec{b}$ x $\vec{a}$)

5- (b)

6- (a) $\sqrt{5}$) 7- (b) 8- (b) 9- (d) 10- a = -4,

ANSWERS ASSERTION-REASON QUESTIONS:

(11) b (12) c (13) a (14) a (15) d

Answers of 2 marks questions

16- p = 5, 17- p = - 3 18- $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

LONG ANSWERS TYPE QUESTIONS:

19- 2i-4j+4k 20- 64i-2j-28k 21- $\frac{1}{4}(i + j + k)$ 22- i+2j+k 5- p=1,

ANSWERS CASE-BASED QUESTIONS:

23- Ans (a) 2i -2k (b) i+3j+k (c) –i-j+k/√3 or √6 sq units

24- Ans: (i) a is perpendicular to b (ii) 0 (iii) √70 /2 or -I -3j -3k/√19

25- Ans (i) 15,0,0 & 0 ,8 ,6 (ii) 15 , 10 (iii) 0, -90 ,120

26- Ans: (i) √29 (ii) 2i+3j+6k/7 (iii) √1937 /2 sq.nits

27- Ans: (i) 6 N (ii) Team A (iii) √2 N or □= 3π/4

11. THREE-DIMENSIONAL GEOMETRY

MCQ (01 Mark each)

1- If O is the origin and OP = 3 with direction ratios - 1 , 2 and – 2 , then coordinates of P are

(a) (- 1 , 2 , - 2) (b) ($\frac{-1}{3}$, $\frac{2}{3}$, $\frac{-2}{3}$

(c) (-3 , 6 , 9) (d) (1 , 2 , 2)

2- The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular, if the value of k

is:

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) -2 (d) 2

3- The coordinates of a point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $\frac{6}{\sqrt{2}}$ from the point (1 , 2 , 3)

is :

(a) (56, 43, 111) (b) ($\frac{56}{17}$, $\frac{43}{17}$, $\frac{111}{17}$) (c) (2 , 1 , 3) (d) (-2 , -1 , -3)

4- The angle between the lines2 x = 3 y = $-$ z and6 x =$-$ y = $-$ 4z is

(a) 0^0 (b)30^0 (c) 45^0 (d) 90^0

5- Direction Cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are

(a) $\frac{2}{7}$, $\frac{3}{7}$, $\frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}$, $-\frac{3}{\sqrt{157}}$, $\frac{12}{\sqrt{157}}$ (c) $\frac{2}{7}$, $-\frac{3}{7}$, $-\frac{6}{7}$ (d) $\frac{2}{7}$, $-\frac{3}{7}$, $\frac{6}{7}$

6- The point (x , y , 0) on the XY – plane divides the line segment joining the points (1, 2, 3) and

(3 , 2 , 1) in the ratio:

(a)1 : 2 internally (b) 2 : 1 internally (c) 3 : 1 internally (d) 3 : 1 externally

ASSERTION – REASON BASED QUESTIONS (01 MARK EACH)

In the questions given below are two statements labeled as Assertion (A) and Reason (R) . in the context of the two statements , which one of the following is correct ?

(a) Both A and R are correct: R is the correct explanation of A.

(b) Both A and R are correct: R is not the correct explanation of A.

(c) A is correct , R is incorrect

(d) R is correct , A is incorrect

7- Assertion (A) The points (1, 2, 3) , (- 2 , 3 , 4) and (7 , 0 , 1) are collinear.

Reason (R) If a line makes angles $\frac{\pi}{2}$, $\frac{3\pi}{4}$ and $\frac{\pi}{4}$ with X, Y and Z – axes respectively, then its direction cosines are 0,$-\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

8- Assertion (A) The Cartesian equation of the line which passes through the point (-2,4, -5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{x+3}{-2} = \frac{y-4}{4} = \frac{z+8}{-5}$.

Reason (R) If the Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$, then its vector form is $\vec{r} = 5\,\hat{\imath} - 4\,\hat{\jmath} + 6\,\hat{k} + \mu\,(3\,\hat{\imath} + 7\,\hat{\jmath} + 2\,\hat{k})$

9- Assertion (A) The lines $\vec{r} = \overrightarrow{a_1} + \mu\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \rho\,\overrightarrow{b_2}$ are perpendicular , when $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$.

Reason (R) The angle θ between the lines $\vec{r} = \overrightarrow{a_1} + \mu\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \rho\,\overrightarrow{b_2}$ is given by $\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$.

SHORT ANSWER TYPE QUESTIONS (03 MARK EACH)

10- A line passing through the point A with position vector $\vec{a} = 4\,\hat{\imath} + 2\,\hat{\jmath} + 2\,\hat{k}$ is parallel to the vector $\vec{b} = 2\,\hat{\imath} + 3\,\hat{\jmath} + 6\,\hat{k}$. Find the length of the perpendicular drawn on this line from a point P with position vector $\vec{r}=\hat{\imath} + 2\,\hat{\jmath} + 3\hat{k}$.

11- Find the vector and Cartesian equations of line passing through the point (1,2, - 4) and perpendicular to two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

12- Find the shortest distance between the lines

$\vec{r} = (1 - t)\hat{\imath} + (t - 2)\hat{\jmath} + (3 - 2t)\hat{k}$

and $\vec{r} = (1 + s)\,\hat{\imath} + (2s - 1)\,\hat{\jmath} - (2s + 1)\,\hat{k}$.

13- Show that the lines $\vec{r} = (3\,\hat{\imath} + 2\,\hat{\jmath} - 4\hat{k}) + \lambda\,(\hat{\imath} + 2\,\hat{\jmath} + 2\hat{k})$ and $\vec{r} = (5\,\hat{\imath} - 2\,\hat{\jmath}) + \mu\,(3\,\hat{\imath} + 2\,\hat{\jmath} + 6\hat{k})$ are intersecting. Also, find their point of intersection.

LONG ANSWER TYPE QUESTIONS (05 MARK EACH)

14- Find the coordinates of foot of perpendicular drawn from the point (0, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$

15- Find the coordinates of the foot of perpendicular drawn from a point A (1,8,4) to the line joining the points B (0 , -1 , 3) and C (2, - 3 , -1).

16- Write the vector equations of following lines and hence find the distance between them

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}, \quad \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

17- If the direction cosines of the lines are given by l + m + n = 0 and 3lm – 5mn + 2nl , then show that these lines are mutually perpendicular.

CASE STUDY / SOURCE BASED QUESTIONS

18- Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines :

$\vec{r} = \lambda\,(\hat{\imath} + 2\,\hat{\jmath} - \hat{k}\,)$

And $\vec{r} = (3\,\hat{\imath} + 3\,\hat{\jmath}) + \mu\,(2\hat{\imath} + \hat{\jmath} + \hat{k}\,)$ respectively.

Based on the above information, answer the following questions:

(a) Find the shortest distance between the given lines (02)

(b) Find the point at which the motor cycle may collide. (02)

19- The equation of motion of a missile are x = 3t, y = - 4t, z= t, where the time 't' is given in seconds, and the distance is measured in Kilometers.

Based on the above information, answer the following questions:

1- What is the path of the missile?

(a) Straight line (b) Parabola (c) Circle (d) Ellipse

(01)

2- Which of the following points lie on the path of the missile?

(a) (6,8,2) (b) (6, -8 , -2) (c) (6,-8,2) (d) (-6,-8,2)

(01)

3- At what distance will the missile be from the starting point (0, 0,0) in 5 seconds?

(a) $\sqrt{550}$ km (b) $\sqrt{650}$ km (c)$\sqrt{450}$ km (d) $\sqrt{750}$ km

(01)

4- If the position of missile at a certain instant of time is (5,-8, 10), then what will be the height of the missile from the ground? (The ground is considered as xy- plane)

(a) 12 km (b) 11 km (c) 20 km (d) 10 km

(01)

20- The shortest distance between the skew-lines $\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\,\overrightarrow{b_2}$, is given by

$$d = \left|\frac{(\overrightarrow{b_1}\times\overrightarrow{b_2}).(\overrightarrow{a_2}-\overrightarrow{a_1})}{\overrightarrow{b_1}\times\overrightarrow{b_2}}\right|.$$

Based on the above information, answer the following questions:

(1) The shortest distance between the lines :

$\vec{r} = (\hat{\imath} + \hat{\jmath}) + \lambda\,(2\hat{\imath} - \hat{\jmath} + \hat{k}\,)$and

$\vec{r} = \left(2\,\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \mu\,(3\,\hat{\imath} - 5\,\hat{\jmath} + 2\hat{k}\,)$ is

(a) $\frac{10}{\sqrt{57}}$ (b)$\frac{9}{\sqrt{57}}$ (c) $\frac{10}{\sqrt{59}}$ (d) $\frac{8}{\sqrt{59}}$

(2) The shortest distance between the lines:

$\vec{r} = \left(6\,\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right) + \lambda\,(\hat{\imath} - 2\,\hat{\jmath} + 2\hat{k})$ and

$\vec{r} = \left(-4\,\hat{\imath} - \hat{k}\right) + \mu\,(3\,\hat{\imath} - 2\,\hat{\jmath} - 2\hat{k})$ is

(a) 7 (b) 8 (c) 9 (d) 10

(3) The shortest distance between the lines :

$\vec{r} = (\lambda - 1)\,\hat{\imath} + (\lambda - 1)\,\hat{\jmath} - (1 + \lambda)\,\hat{k}$

$and\ \vec{r} = (1 - \mu)\,\hat{\imath} + (2\mu - 1)\,\hat{\jmath} - (\mu + 2)\,\hat{k}$ is:

$(a) 5$ $(b) \frac{1}{14}$ $(c)\ 5\sqrt{2}$ $(d) \frac{6}{\sqrt{2}}$

(4) The shortest distance between the lines:

$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ $\quad and \quad$ $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+4}{2}$ is:

$(a) \frac{1}{\sqrt{2}}$ $(b) \frac{1}{\sqrt{3}}$ $(c) \frac{1}{2}$ (d) $\frac{1}{\sqrt{5}}$

11. THREE DIMENSIONAL GEOMETRY

ANSWER

1- Ans : (a)

2- Ans: (a)

3- Ans: (b)

4- Ans : (d)

5- Ans: (d)

6- Ans : (d)

7- Ans: (b)

8- Ans : (d)

9- Ans : (a)

10- Ans: $\sqrt{10}$ units

11- Ans : $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$; $\vec{r} = \left(\hat{\imath} + 2\,\hat{\jmath} - 4\hat{k}\right) + \lambda\,(2\,\hat{\imath} + 3\,\hat{\jmath} + 6\hat{k}\,)$

12- Ans : $\frac{8}{\sqrt{29}}$ units

13- Ans : (-1 , - 6 , -12)

14- Ans: (2 , 3, -1)

15- Ans:($-\frac{5}{3}$, $\frac{2}{3}$, $\frac{19}{3}$)

16- $\vec{r} = \left(\hat{\imath} + 2\,\hat{\jmath} - 4\hat{k}\right) + \lambda\,(\,2\hat{\imath} + 3\,\hat{\jmath} + 6\hat{k}\,)$ and $\vec{r} = \left(3\,\hat{\imath} + 3\,\hat{\jmath} - 5\hat{k}\right) +$ μ $(4\,\hat{\imath} + 6\,\hat{\jmath} + 12\hat{k}\,)$;
$\frac{\sqrt{293}}{7}\ units$

17- Proof

18- (a) 0 unit (b) (1, 2, - 1)

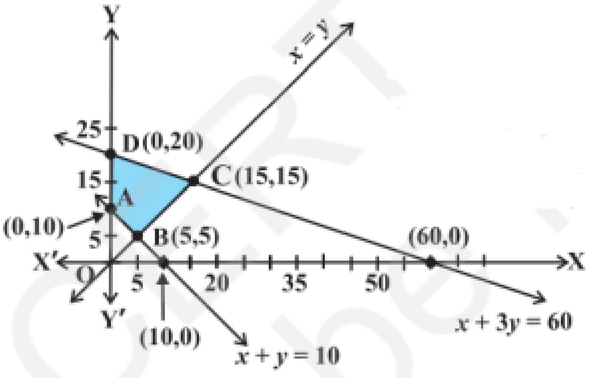
19- (1) (a) Straight line (2) (c) (6,-8,2) (3) (b) $\sqrt{650}$ km (4)(d) 10 km

20- (1) (c) $\frac{10}{\sqrt{59}}$ (2) (c) 9 (3) $(b)\,\frac{1}{14}$ (4) (b) $\frac{1}{\sqrt{3}}$

12 LINEAR PROGRAMMING PROBLEM

MCQ

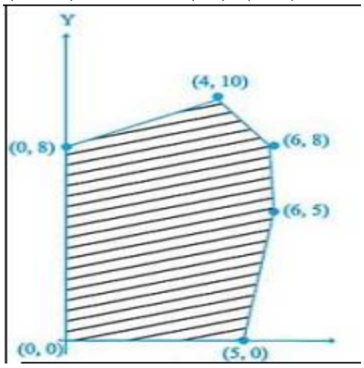
1. Based on following region in the graph at which point Is the objective function Z= 3x+9y



maximum?

(i) Point B (ii) point c (iii) Point D (iv) every point on the line segment CD

2. In the given graph the feasible region of a LPPis shaded. The objective function Z=2x-3y will be minimum at
   (i) (4,10) (iii) (6,8) (iii)(0,8) (iv) (6,5)



3. A linear programming problem is as follows : minimize Z= 30 x+50 y
   Subjected to constraints 3x + 5y ≥ 15 $, 2x + 3y \leq 18 , x \geq 0, y \geq 0$
   In the feasible region ,the minimum value of Z occurs at
   (i) A unique point (ii) no point (iii) infinitely many points (iv) two points
4. For an objective function Z= ax + by where a,b > 0 ,The corner points of the feasible region is determined by a set of constraints are (0,20) ,(10,10) ,(30,30) and (0,40). The condition on a and b such that the maximum Z occurs at both points (30,30) and (0,40) is
   (I) b – 3a=0 (ii) a=3b (iii) a+2b=0 (iv) 2a-b=0
5. In a linear programming problem , the constraints on the decision variables x and y are $x - 3y \geq 0 , y \geq 0 , 0 \leq x \leq 3$. The feasible region
   (i) $is not in the first quadrant$ $(iii) is bounded in the first quadrant$
   (ii) Is unbounded in the first quadrant (iv) does not exist
6. Which of the following point lies in the region $40\ x + 30\ y\ \geq 240$
   (i) (0,0) (ii) (3,5) (iii) (2,3) (iv) (0,5)
7. The optimum value of an objective function occurs at
   (i) All points in feasible region
   (ii) At any vertex point of feasible region
   (iii) At point which is at maximum distance from origin
   (iv) At point which is at minimum distance from origin
8. A lpp can not have
   (i) A unique solution (ii) no solution (iii) 4 solution (iv) infinitely many sol

9. If objective function Z is having its maximum value at two points A and B then its solution will be
   (i) Point A (ii) point B
   (ii) (iii) every point on line segment A and B (iv) all ponts on line joining origin and A
10. If the feasible region for a LPP is ………, then the optimal value of the objective function Z = ax+by may or may not exist.
   (i) bounded (ii) unbounded (iii) in circled form (iv) in squared form

ASSERTION REASON BASED QUESTIONS

11. Maximize /Minimize objective function Z= $5x + 10y$
    Subjected to constraints x+2y≤ 120 ,x+y $\geq 60, x - 2y \geq 0$; $x, y \geq 0$.
    ASSERTION- Z has maximum value at any point on line segment joining (120,0) ,(60,30)
    REASON-If an objective function attains optimum value at two points then all points on line segment joining the two points are feasible solutions
    (i) Both Assertion and Reason are true and Reason is correct explanation of assertion
    (ii) Both Assertion and Reason are true and Reason is not correct explanation of assertion
    (iii) Assertion is true and Reason is false
    (iv) Assertion is false and Reason is true

12. Maximize Z= $x + y$
    Subjected to constraints x-y$\leq -1, -x + y \leq 0; x, y \geq 0$
    ASSERTION- Z has no maximum value
    REASON – Z has bounded feasible region
    (i) Both Assertion and Reason are true and Reason is correct explanation of assertion
    (ii) Both Assertion and Reason are true and Reason is not correct explanation of assertion
    (iii) Assertion is true and Reason is false
    (iv) Assertion is false and Reason is true

13. ASSERTION- A LPP can have only unique solution or infinitely many solutions
    REASON- If the feasible region of a LPP is unbounded or there does not exist any common feasible region then solution may or may not exist
    (i) Both Assertion and Reason are true and Reason is correct explanation of assertion
    (ii) Both Assertion and Reason are true and Reason is not correct explanation of assertion
    (iii) Assertion is true and Reason is false
    (iv) Assertion is false and Reason is true

LONG QUESTIONS

14. Minimize and maximize Z= $5x + 10y$
    Subject to constraints: $x + 2y \leq 120$, $x + y \geq 60, x - 2y0; x, y \geq 0$

15. Maximize Z= $-x + 2y$
    Subjected to: $x \geq 3, x + y \geq 5, x + 2y \geq 6; x, y \geq 0$

16. Maximize Z= $8x + 9y$
    subjected to: $2x + 3y \leq 6, 3x - 2y \leq 6, y \leq 1; x, y \geq 0$

17. Minimize Z=$5x + 7y$
    Subjected to: $2x + y \geq 8, x + 2y \geq 10; x, y \geq 0$

18. Maximize Z=$x + y$

Subjected to: $x - y \leq -1, -x + y \geq 0; x, y \geq 0$

CASE STUDY BASED QUESTIONS

19. A cake shop or baery makes two kind of cakes. First kind of cake requires 200 g of flour and 25g of cream and 2^{nd} type of cake requires 100g of flour and 50g of cream

(i) If bakery makes x cakes of first type and y cakes of 2^{nd} type and it can use maximum 5kg flour then which of the following constraint is correct

(a) $2x + y \geq 50$ (b)$x + y \leq 50$ (c)$2x + y \leq 50$ (d)$2x + y = 50$

(ii) If Bakery can use maximum 1 kg of cream, then which of the following is correct

(a) $x + 2y \geq 40$ (b)$x + y \geq 40$ (c) $x + 2y = 40$ (d)$x + 2y \leq 40$

(iii) If total number of cakes made by bakery is represented by Z then Z is equal to

(a) $Z = x + y$ $(b) z = 2x + y$ $(c) Z = x + 2y$ $(d) Z = 2x + 3y$

(iv) Using above constraints, the maximum number of total cakes which can be made by bakery assuming that there is no shortage of ingredients used in making the cakes is

(a) 40 (b) 30 (c) 20 (d)25

20. A dealer Rahul wants to set up a business of fans. He wishes to purchase a number of ceilings fans and table fans. A ceiling fan costs him Rs 360 and table fan costs Rs 240

(i) If Rahul purchases x ceiling fans ,y table fans .HE has space in his store for at most 20 items then which of the following is correct

(a) $x + y = 20 (b) x + y > 20 (c) x + y < 20 (d) x + y \leq 20$

(ii) If Rahul has only Rs 5760 to invest on both type of fans,then which of the following is correct

(a) $x + y \leq 5760$ $(b)\ 360x + 240y \leq 5760$

$(c) 360x + 240y \geq 5760 (d) 3x + 2y \leq 48$

(iii) If he expects to sell ceiling fan at a profit of Rs 22 and table fan for a profit of Rs 18 then what is the total profit Z also find solution using graphical method

12 LINEAR PROGRAMMING PROBLEM

ANSWER

MCQ

1.iv
2. iii
3. iii
4.i
5.ii
6.ii
7.ii
8.iii
9.iii
10.ii

ASSERTION REASON QUESTIONS

11.i
12.iii
13.iv

LONG QUESTIONS

14.
$max_{z=}600\ at\ every\ point\ on\ line\ segment\ joining\ (120,0),(60,30)\quad and\ min_z = 300\ at\ (60,0)$
15.No max value exist
16.max Z=22.6 at $\left(\frac{30}{13},\frac{6}{13}\right)$
17.min Z=38 at point (2,4)

18.No max value exist

CASE STUDY QUESTION

19. (i) –c
(ii) -d
(iii)-a
(iv)-b
20. (i) -d
(ii)-d
(iii) Z=22x+18y and optimum value of Z=392 at point (8,12)

13 PROBABILITIES

MCQ

1. Probability that Raman speaks truth is $\frac{3}{5}$. A die is rolled. Raman reports that an even number appears. The probability that there is an even number.

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) None of these

2. A rocket has 8 engines out of which 3 are not working. If the two engines are selected without replacement and tested, the probability that both are not working.

(a) $\frac{33}{56}$ (b) $\frac{9}{64}$ (c) $\frac{1}{14}$ (d) $\frac{3}{28}$

3. In a boy's college, 30% students play Cricket, 25% play Football and 10% students play both Cricket and Football. One student is selected at random. The probability that he likes Cricket if he also like Football is-

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$ (c) $\frac{9}{20}$ (d) $\frac{1}{3}$

4. A box contains 6 pens and 10 pencils. Half of the pens and half of the pencils are of blue colour. If one of the items is chosen at random, the probability that it is of blue colour or is a pen is-

(a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{11}{16}$ (d) $\frac{14}{16}$

5. A bag contains 10 good and 6 bad mangoes. One of the mangoes is selected. The probability that it is either good or bad-

(a) $\frac{64}{64}$ (b) $\frac{49}{64}$ (c) $\frac{40}{64}$ (d) $\frac{24}{64}$

6. Two dice are thrown once. If it is known that the sum of the numbers on the dice was less than 6 the probability of getting a sum 3 is-

(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

7. In a box, there are 8 orange, 7 white, and 6 blue balls. If a ball is picked up randomly, what is the probability that it is neither orange nor blue?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{21}$ (d) $\frac{5}{21}$

8. A husband and his wife appear for an interview for two posts. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that only one of them is selected?

(a) $\frac{2}{7}$ (b) 0 (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

ASSERTION-REASONING BASED QUESTION

9. **ASSERTION(A) : If A and B two events, then $P(A \cap B) = P(A).P(B/A)$.**

REASONING(R) : Two events are said to be exhaustive if probability of the one of the events is zero.

(a) A is true, R is true and R is correct explanation for A.
(b) A is true, R is true and R is not correct explanation for A.
(c) A is true and R is false
(d) A is false and R is true.

10. **ASSERTION(A)** : The probability of getting either a king or an ace from a pack of 52 playing cards is 2/13 .

REASONING(R) : For any two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(a) A is true, R is true and R is correct explanation for A.
(b) A is true, R is true and R is not correct explanation for A.
(c) A is true and R is false
(d) A is false and R is true

SHORT AND LONG ANSWER QUESTIONS

11. The probability that Abraham hits the target is 1/3 and the probability that Bhavesh hits it, is 2/5 . If both try to hit the target independently, find the probability that target is hit.

12. Ramesh is going to play a game of chess against one of four opponents in an inter school sports competition. Each opponent is equally likely to be paired against him. The table below 5 shows the chances of Ramesh losing, where paired against each opponent.

Opponent	Chance of losing
Opponent 1	12%
Opponent 2	60%
Opponent 3	x%
Opponent4	84%

If the probability that Ramesh loses the game that day is 1/2 , find the probability for Ramesh to be losing when paired against opponent 3.

13. A bag I contains 5 red and 4 white balls and a bag II contains 3 red and 3 white balls. Two balls are transferred from the bag I to the bag II and then one ball is drawn from the bag II. If the ball drawn from the bag II is red, then find the probability that one red and one white ball are transferred from the bag I to the bag II.

CASE STUDY BASED QUESTIONS

14. Read the following text and answer the following question on the basis of the same: A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.

(i) Let the target is hit by A,B,C. Then the probability that A, B and C all will hit is

(a) 4/5 (b) 3/5 (c) 2/5 (d) 1/5

(ii) What is the probability that B,C will hit and A will lose?

(a) 1/10 (b) 3/10 (c) 7/10 (d) 4/10

(iii) What is the probability that any two of A,B and C will hit?

(a) 1/30 (b) 11/30 (c) 17/30 (d) 13/30

(iv) What is the probability that none of them will hit the target?

(a) 1/30 (b) 1/60 (c) 1/15 (d) 2/15

15. In an office three employees Aman, Aryan and Biswajit process incoming copies of a certain form. Aman processes 50% of the forms, Aryan processes 20% and Biswajit the remaining 30% of the forms. Aman has an error rate of 0.06, Aryan has an error rate of 0.04 and Biswajit has an error rate of 0.03. Based on the above information answer the following:

(i) The conditional probability that an error is committed in processing given that Aryan processed the form is:

(A) 0.0210 (B) 0.04 (C) 0.47 (D) 0.06

(ii) The probability that Aryan processed the form and committed an error is:

(A) 0.005 (B) 0.006 (C) 0.008 (D) 0.68

(iii) The total probability of committing an error in processing the form is:

(A) 0 (B) 0.047 (C) 0.234 (D) 1

(iv) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Aman is:

(A) 1 (B) 30/47 (C) 20/47 (D) 17/4

13 PROBABILITIES

Q.NO.	ANSWERS
1	A
2	D
3	B
4	C
5	A
6	C
7	A
8	A
9	C
10	A
11	$P(A) = P$(A hits target)= 1/3 $P(B) = P$(B hits target)= 2/5 Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ = 1/3 + 2/5 – 1/3 × 1/5 = 3/5
12	Note that $P(E1) = P(E2) = P(E3) = P(E4) = 1/4$ Given $P(A)$ = ½ , $P(A/E1)$ = 12% = 12/100 $P(A/E2)$ = 60% = 60/100 $P(A/E3) = x\% = x/100$ $P(A/E4)$ = 84% = 84/100 Using total probability theorem, we have $P(A) = P(E1)P(A/E1) + P(E2)P(A/E2) + P(E3).P(A/E3) + P(E4)P(A/E4)$ ⇒ 1/2 = 1/4 × 12/100 + 1/4 × 60/100 + 1/4 × x/100 + 1/4 × 84/100 ⇒ $x = 44$ $P(A/E3)$ = 44%
13	$Let\ E\ 1, E\ 2, E\ 3$ and A are event such that E 1 = Both transferred balls from bag I to bag II are red E 2 = Both transferred balls from bag I to bag II are white E 3 = Out of two transferred balls one is red and other is white A = Drawing a red ball from bag II P (E 1) = 5 C 2 /9 C 2 = 5 × 4/ 9 × 8 = 20/72 = 5/18 P (E 2) = 4 C 2 /9 C 2 = 4 × 3 /9 × 8 = 12/72 = 3/18 P (E 3) = 5 C 1 × 4 C 1/ 9 C 2 = 5 × 4 × 2 /9 × 8 = 40/72 = 10/18 P (A/E 1) = 5/8 , P (A/E 2) = 3/8 , P (A/E 3) = 4/8 P (E3/A) = [P(E3) × P(A/E3)]/[P(E1) × P(A/E1) + P(E2) × P(A/E2) + P(E3) × P(A/E3)] = [10/18 × 4/8]/ [5/18 × 5/8 + 3/18 × 3/8 + 10/18 × 4/8] = 20/37
14	I) C II) A III) D IV) B
15	I) B II) C III) B IV) D