

Kendriya Vidyalaya Sangathan, Ahmedabad Region

द्वितीय प्री-बोर्ड परीक्षा (2025-26)

Pre-board II Examination (2025-26)

Subject: Mathematics (041)

M.M:80

Class: XII

Time: 3 hrs.

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question Paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

- 1 The integrating factor of the differential equation $(1 - x^2) \frac{dy}{dx} + xy = ax$ is
 (A) $\frac{1}{x^2-1}$ (B) $\frac{1}{\sqrt{x^2-1}}$ (C) $\frac{1}{1-x^2}$ (D) $\frac{1}{\sqrt{1-x^2}}$
- 2 If $\vec{a}$, $\vec{b}$ and $\vec{a} + \vec{b}$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is
 (A) $\frac{2\pi}{3}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$
- 3 The projection of vector $\hat{i}$ on the vectore $\hat{i} + \hat{j} + 2\hat{k}$ is :
 (A) $\frac{1}{\sqrt{6}}$ (B) $\sqrt{6}$ (C) $\frac{2}{\sqrt{6}}$ (D) $\frac{3}{\sqrt{6}}$

- 4 The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :
- (A) $2\hat{j}$ (B) $\hat{j}$ (C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$
- 5 If a line makes angles of 90° , 135° and 45° with the x, y and z axes respectively, then its direction cosines are
- (A) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$
 (C) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (D) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
- 6 If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to
- (A) A (B) I - A (C) I + A (D) 3A
- 7 If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is:
- (A) -2 (B) 2 (C) 3 (D) -3
- 8 A is a square matrix of order 3 and $|A| = 7$, then $|2A A'|$ is equal to
- (A) 98 (B) 392 (C) 14 (D) None of these
- 9 If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, and $A^{-1} = A'$, then value of α is
- (A) π (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) Any real number
- 10 If $A \cdot (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to
- (A) 12 (B) 9 (C) 3 (D) 27
- 11 If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to
- (A) $\frac{-y}{x}$ (B) $\frac{y}{x}$ (C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\frac{y}{x}$
- 12 The function $f(x) = kx - \sin x$ is strictly increasing for
- (A) $k > 1$ (B) $k < 1$ (C) $k > -1$ (D) $k < -1$

13 Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :

- (A) $\sin x e^{\sin^2 x}$ (B) $\cos x e^{\sin^2 x}$
 (C) $-2\cos x e^{\sin^2 x}$ (D) $-2\sin^2 x \cos x e^{\sin^2 x}$

14 $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to

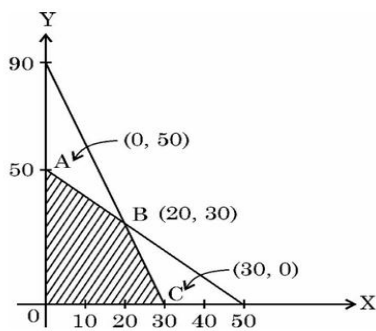
- (A) $\tan x - \cot x + c$ (B) $-\cot x + \tan x + c$
 (C) $\cot x + \tan x + c$ (D) $-\tan x - \cot x + c$

15 The difference of the order and the degree of the differential equation

$$\left(\frac{d^2 y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0 \text{ is:}$$

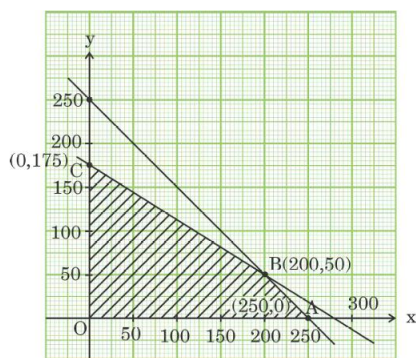
- (A) 1 (B) 2 (C) -1 (D) 0

16 The maximum value of $Z=4x+y$ for a L.P.P. whose feasible region is given below is



- (A) 50 (B) 120 (C) 110 (D) 170

17 The corner points of the bounded feasible region of an LPP are $O(0, 0)$, $A(250, 0)$, $B(200, 50)$ and $C(0, 175)$. If the maximum value of the objective function $Z = 2ax + by$ occurs at the points $A(250, 0)$ and $B(200, 50)$, then the relation between a and b is :



- (A) $a = b$ (B) $2a = 3b$ (C) $2a = b$ (D) $a = 2b$

- 18 If for any two events A and B, $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to
- (A) $\frac{1}{10}$ (B) $\frac{1}{8}$ (C) $\frac{7}{8}$ (D) $\frac{17}{20}$

ASSERTION - REASON BASED QUESTIONS

In the following questions(19 and 20), a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is correct explanation for A
 (b) Both A and R are true but R is not correct explanation for A
 (c) A is true but R is false.
 (d) A is false but R is true
- 19 **Assertion (A):** The range of the function $f(x) = 2 \sin^{-1} x + \frac{3\pi}{2}$, where

$$x \in [-1, 1], \text{ is } \left[\frac{\pi}{2}, \frac{5\pi}{2} \right].$$

Reason (R): The range of the principal value branch of $\sin^{-1} x$ is $[0, \pi]$.

- 20 **Assertion (A):** Equation of the line passing through the points (1,2,3) and (3, -1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R): Equation of a line passing through points (x_1, y_1, z_1) , (x_2, y_2, z_2) , (x_3, y_3, z_3) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

- 21 Write $\tan^{-1} \left[\frac{\sqrt{1+x^2}-1}{x} \right]$ in simplest form.

OR

Evaluate

$$\sin^{-1} \left(\sin \frac{3\pi}{4} \right) + \cos^{-1} \left(\cos \frac{3\pi}{4} \right) + \tan^{-1}(1)$$

- 22 Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices.

- 23 Find the value(s) of 'k', if the function

$$f(x) = \begin{cases} \frac{\sin^2 kx}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases} \text{ is continuous at } x = 0$$

- 24 (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

- 25 If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$ and $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

- 26 Solve the following linear programming problem graphically:

Maximise : $Z = 500x + 300y$,

$$x + 2y \leq 12, \quad 2x + y \leq 12, \quad 4x + 5y \geq 20$$

$$x \geq 0, y \geq 0$$

- 27 Evaluate: $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$

- 28 Evaluate : $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} - e^{-\cos x}} dx$

OR

Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

- 29 Evaluate: $\int \frac{2x+1}{(x+1)^2 (x-1)} dx$

- 30 A and B throw a die alternately till one of them gets a "6" and wins the game. Find their respective probabilities of winning, if A starts the game first.

OR

Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing

a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

- 31** Find the general solution of the differential equation:

$$(xy - x^2) dy = y^2 dx$$

OR

Find the general solution of the differential equation:

$$(x^2 + 1) \frac{dy}{dx} = 2xy = \sqrt{x^2 + 4}$$

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

- 32** Let set $A = \{1, 2, 3, \dots, 10\}$ and R be a relation in $A \times A$, defined by $(a, b) R (c, d)$ iff $a + d = b + c$ for all (a, b) and $(c, d) \in A \times A$. Prove that R is an equivalence relation. Also find equivalence class of $(5, 6)$

OR

Let $f : R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

- 33** If $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that A satisfies the equation $A^3 - 4A^2 - 3A + 11I = O$ and using this result find A^{-1} .

OR

Given $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find AB and use this to solve the equations $x - 2y = 3$, $2x - y - z = 2$ and $-2y + z = 3$

- 34** Using integration, find the area of the region bounded by the line $y = \sqrt{3}x$, the curve $y = \sqrt{4 - x^2}$ and y -axis in the first quadrant.
- 35** Find the equation of the line which bisects the line segment joining points $A(2, 3, 4)$ and $B(4, 5, 8)$ and is perpendicular to the lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

SECTION E

This section comprises 3 case study-based questions of 4 marks each.

- 36** Engine displacement is the measure of the cylinder volume swept by all the pistons of piston engine. The piston moves inside the cylinder bore.

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi \text{ cm}^2$.

(i) If the radius of the cylinder is r cm and height h cm, then write the volume V of cylinder in terms of r .

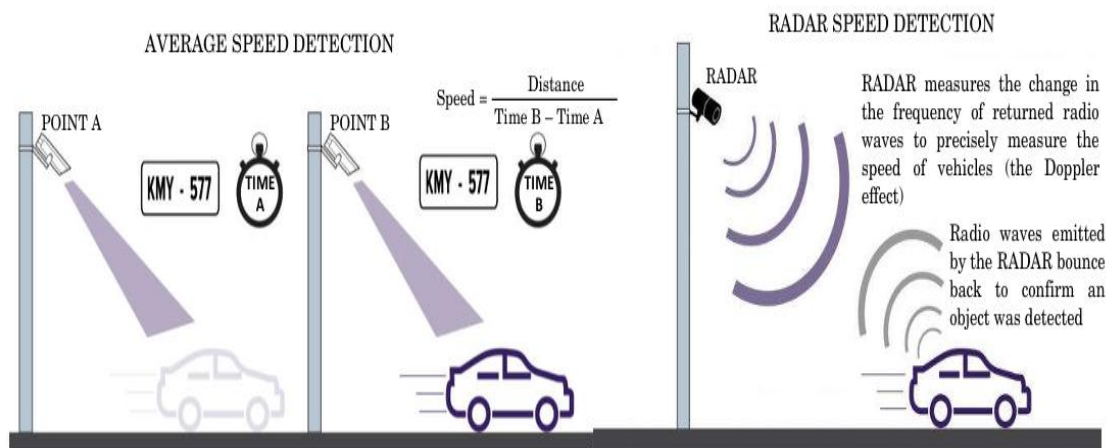
(ii) Find $\frac{dV}{dr}$

(iii) Find radius of the cylinder when its volume is maximum.

OR

(iii) Volume is maximum for $h > r$. State true or false and justify your answer.

- 37** The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ .

Based on the above information, answer the following questions:

- (i) Express q in terms of height of the camera installed on the pole and x .

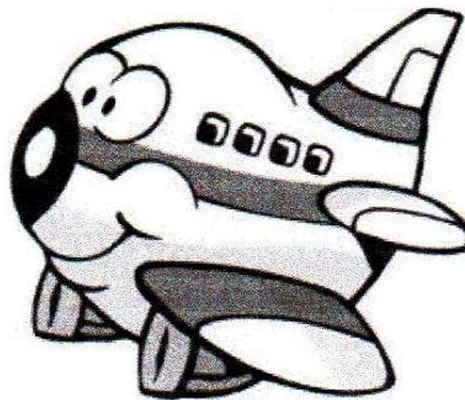
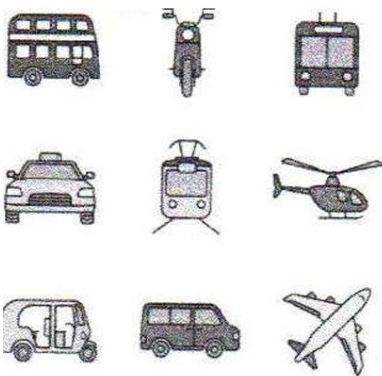
(ii) Find $\frac{d\theta}{dx}$

(iii) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

OR

(iii) If the rate of change of angle of elevation with respect to time of another car at 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car

38 Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.



Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travelers survive. Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

Based on the above information, answer the following questions:

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A|E_1) + P(A|E_2)$.

(iii) Find $P(A)$.

OR

(iii) Find $P(E_2|A)$.