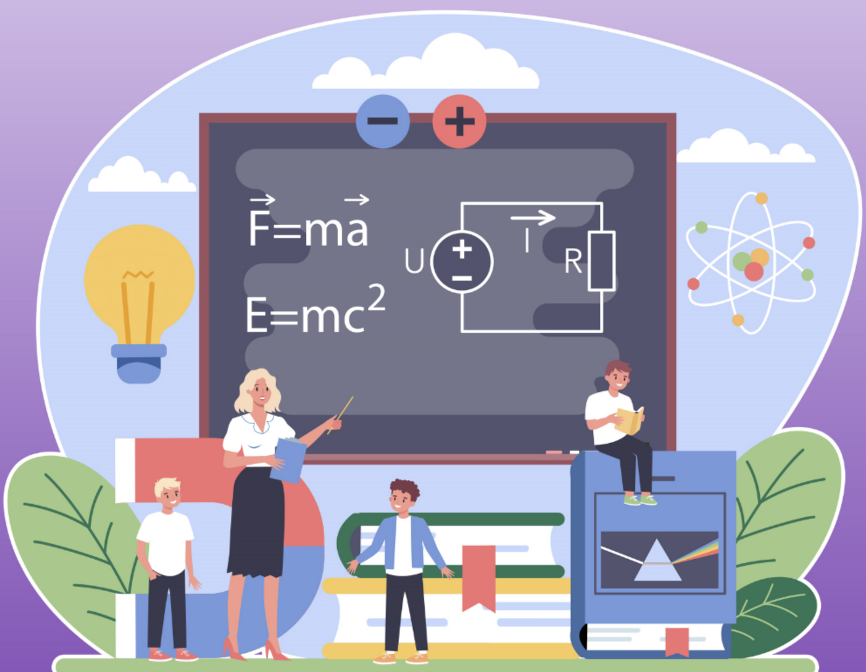
भौतिकी
Physics

कक्षा / Class XI
2025-26

विद्यार्थी सहायक सामग्री
Student Support Material



केन्द्रीय विद्यालय संगठन~Kendriya Vidyalaya Sangathan

संदेश

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना एवं नवाचार द्वारा उच्च – नवीन मानक स्थापित करना केन्द्रीय विद्यालय संगठन की नियमित कार्यप्रणाली का अविभाज्य अंग है। राष्ट्रीय शिक्षा नीति 2020 एवं पी. एम. श्री विद्यालयों के निर्देशों का पालन करते हुए गतिविधि आधारित पठन-पाठन, अनुभवजन्य शिक्षण एवं कौशल विकास को समाहित कर, अपने विद्यालयों को हमने ज्ञान एवं खोज की अद्‌भुत प्रयोगशाला बना दिया है। माध्यमिक स्तर तक पहुँच कर हमारे विद्यार्थी सैद्धांतिक समझ के साथ-साथ, रचनात्मक, विश्लेषणात्मक एवं आलोचनात्मक चिंतन भी विकसित कर लेते हैं। यही कारण है कि वह बोर्ड कक्षाओं के दौरान विभिन्न प्रकार के मूल्यांकनों के लिए सहजता से तैयार रहते हैं। उनकी इस यात्रा में हमारा सतत योगदान एवं सहयोग आवश्यक है - केन्द्रीय विद्यालय संगठन के पांचों आंचलिक शिक्षा एवं प्रशिक्षण संस्थान द्वारा संकलित यह विद्यार्थी सहायक-सामग्री इसी दिशा में एक आवश्यक कदम है । यह सहायक सामाग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है। केन्द्रीय विद्यालय संगठन की विद्यार्थी सहायक- सामग्री अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री संकलन की विशेषज्ञता के लिए जानी जाती है और शिक्षा से जुड़े विभिन्न मंचों पर इसकी सराहना होती रही है। मुझे विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर निरंतर मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुँचाएगी।

शुभाकांक्षा सहित ।

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TABLE OF CONTENT

Student Support Material contains the following:

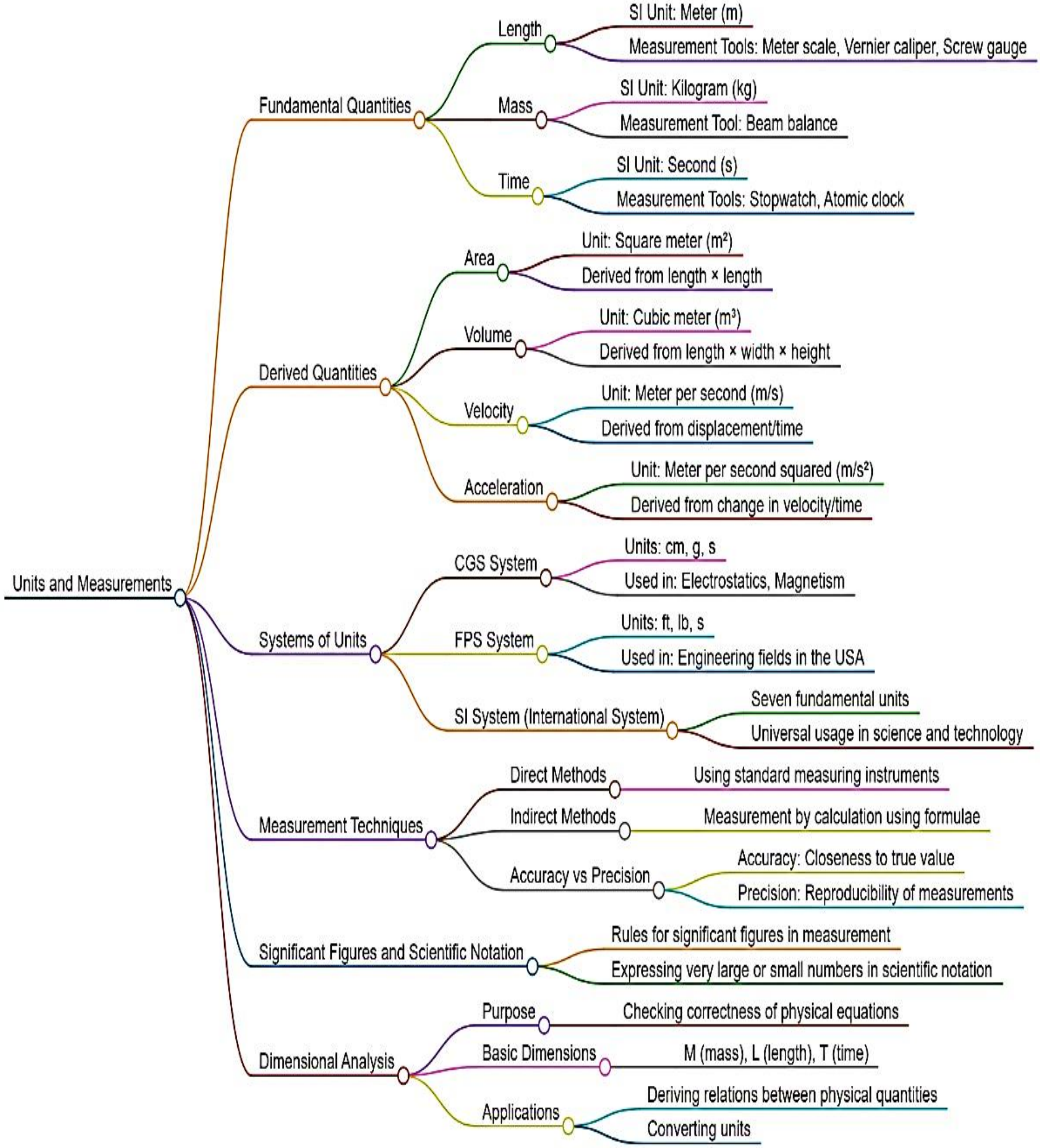
- **MCQs, ASSERTION & REASONING questions, 2 Marks, 3 Marks, Case Based Questions and 5 Marks questions.**
- **Questions are provided with solutions. Exercise questions are having only answers.**
- **Chapter wise syllabus in the beginning of each chapter**
- **The gist of each chapter audio recording QR code.**
- **Chapter wise assessment (google form).**
- **Links of video lessons Prepared by KVS PGT(Physics)**
- **Link for three practice question paper is also included in digital resources**
- **The CBSE curriculum 2025-26 is given in form of QR code**

PHYSICS CURRICULUM 2025-26

UNITS AND MESUREMENTS

Content of the chapter: Units and Measurements, Need for measurement, Units of measurement, systems of units,SI units, fundamental and derived units, significant figures, Determining the uncertainty in result, Dimensions of physical quantities, dimensional analysis and its applications.

MIND MAP



GIST OF THE CHAPTER:

- Every measurement has two parts. The first is a number (n) and the next is a unit(u). Q = n u. Ex : Length of an object = 40 cm.
- The number expressing the magnitude of a physical quantity is inversely proportional to the unit selected.
- If n_1 and n_2 are the numerical values of a physical quantity corresponding to the units u_1 and u_2, then $n_1u_1 = n_2u_2$. Ex : 2.8 m = 280 cm;   6.2 kg = 6200 g
- The quantities that are independent of other quantities are called fundamental quantities. The units that are used to measure these fundamental quantities are called fundamental units.
- There are four systems of units namely C.G.S, M.K.S, F.P.S and SI
- The quantities that are derived using the fundamental quantities are called derived quantities. The units that are used to measure these derived quantities are called derived units.
- The early systems of units :

Fundamental Quantity	**System of units**		
	MKS	CGS	FPS
Length	Meter	Centimeter	Foot
Mass	Kilogram	Gram	pound
Time	Second	Second	Second

Fundamental and supplementary physical quantities in SI system (Systeme Internationale d'units) :

Physical quantity	**Unit**	**Symbol**
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	Ampere	A
Thermodynamic temperature	Kelvin	K
Intensity of light	candela	Cd
Quantity of substance	mole	mol

- Supplementary quantities:

Plane angle	radian	rad
Solid angle	steradian	S rad

- If Q is the unit of a derived quantity represented by $Q = M^aL^bT^c$, then $M^aL^bT^c$ is called dimensional formula and the exponents a ,b and c are called the dimensions.
- Dimensional Constants: The physical quantities which have dimensions and have a fixed value are called dimensional constants. Ex: Gravitational constant (G), Planck's constant (h), Universal gas constant (R), Velocity of light in vacuum (C) etc.
- Dimensionless quantities are those which do not have dimensions but have a fixed value.
- Dimensionless quantities without units. Ex: Pure numbers, π, e, $\sin\theta$, $\cos\theta$,$\tan\theta$ …. etc.,

- Dimensionless quantities with units. Ex : Angular displacement – radian, Joule's constant – joule/calorie, etc.,
- Dimensional variables are those physical quantities which have dimensions and do not have fixed value. Ex: velocity, acceleration, force, work, power… etc.
- Dimensionless variables are those physical quantities which do not have dimensions and do not have fixed value. Ex: Specific gravity, refractive index, coefficient of friction, Poisson's ratio etc.
- Dimensional formulae are used to a) verify the correctness of a physical equation, b) derive relationship between physical quantities and c) to convert the units of a physical quantity from one system to another system.
- Law of homogeneity of dimensions: In any correct equation representing the relation between physical quantities, the dimensions of all the quantities must be the same on both sides of '+' or '–' or '=' sign must have the same dimensions.
- A physical quantity Q has dimensions a, b and c in length (L), mass (M) and time (T) respectively, and n_1 is its numerical value in a system in which the fundamental units are L_1, M_1 and T_1 and n_2 is the numerical value in another system in which the fundamental units are L_2, M_2 and T_2 respectively, then

$$n_2 = n_1\left[\frac{L_1}{L_2}\right]^a\left[\frac{M_1}{M_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

- Fourier laid down the foundations of dimensional analysis.
- Limitations of dimensional analysis :
  a) Dimensionless quantities cannot be determined by this method. Dimensionless Constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
  b) This method is not applicable to trigonometric, logarithmic and exponential functions.
  c) In the case of physical quantities which are dependent upon more than three physical quantities, this method will be difficult.
  d) In some cases, the constant of proportionality also possesses dimensions. In such cases we cannot use this system.
  e) If one side of equation contains addition or subtraction of physical quantities, we cannot use this method to derive the expression.
- **Significant figures** are the digits in a number that express its measured or calculated precision. They include all certain digits plus one uncertain digit.
- **Rules for Counting Significant Figures**:
  a) All non-zero digits are significant. Example: 237 → 3 significant figures.
  b) Any zero between two non-zero digits is significant. Example: 1003 → 4 significant figures.
  c) Leading zeros in decimals numbers with values less than 1 (zeros before the first non-zero digit) are not significant. Example: 0.0052 → 2 significant figures.
  d) Trailing zeros in a number with a decimal point are significant. Example: 4.500 → 4 significant figures.
  e) Trailing zeros in a whole number without a decimal may or may not be significant, depending on context. Example: 5000 → could have 1, 2, 3, or 4 significant figures (needs scientific notation to clarify).

  Rounding off is the process of reducing the number of digits in a number while keeping its value close to the original. It simplifies numbers for easier interpretation or calculation while maintaining reasonable accuracy.

MULTIPLE CHOICE QUESTIONS:

1. Which of the following is a fundamental quantity?
a). Velocity b) Force c) Length d). Pressure
2. Which of the following quantities is derived from fundamental quantities?
a). Mass b).Time c).Temperature d). Acceleration
3. The unit of work is derived from which fundamental quantities?
a). Mass and length b). Force and distance c). Mass, length, and time d). Power and time
4. How many significant figures are in the number 0.004506?
a) 4 b) 3 c) 6 d) 5
5. Which number has 3 significant figures?
a) 0.070 b) 700 c) 7.00×10^2 d) 0.007
6. Round 76.483 to 3 significant figures.
a) 76.5 b) 76.4 c) 76.48 d) 76
7. Which of the following quantities is dimensionless?
a). Pressure b). Strain c) Velocity d). Force
8. The dimensional formula of Planck's constant (h) is:
a). $[ML^2T^{-1}]$ b). $[M L^2 T^{-1}]$ c) .$[ML^2T^{-1}]$ d). $[MLT^{-2}]$
9. Which of the following equations is dimensionally correct?
a) $v=u+at^2v$ b) $v= u + at^2$ c) $v=ut+at^2$ d) $s=ut+1/2at^2$
10. If velocity v, time t, and acceleration a are fundamental quantities, the dimension of distance is:
a). vt b). at c). at^2 d). va

ANSWER WITH SOLUTIONS

1 - Answer: c. Length
Explanation: Length is a fundamental quantity, whereas velocity, force, and pressure are derived quantities.
2 - Answer: d. Acceleration
Explanation: -Acceleration is derived from the fundamental quantities of length and time (Acceleration = velocity/time = (length/time)/time = length/time²)
3 - Answer: c. Mass, length, and time
Explanation: Work = Force × Distance = (mass × acceleration) × distance =$kg\cdot m^2/s^2$
4 - Answer: a. 4
Explanation: Leading zeros are not significant. The digits 4, 5, 0, and 6 are significant.
5 - Answer: c 7.00×10^2
Explanation: All digits (7, 0, 0) in 7.00 are significant because trailing zeros after a decimal are significant.
6 - Answer: a 76.5
Explanation: The third significant figure is 4, and the next digit (8) is ≥5, so we round up.
7 -Answer: b
Explanation: Strain = change in length / original length = (L / L) = dimensionless
8 -Answer: a
Explanation: Energy = hν
$E = [M L^2 T^{-2}]$, $h = [M L^2 T^{-1}]$,$\nu = T^{-1}$ RHS =LHS
9 - Answer: b
Explanation: LHS = RHS
10 - Answer: c
Explanation: From $s=ut+1/2at^2$

ASSERTION REASON BASED QUESTION

In the following questions a statement of assertion(A) is followed by a statement of reason(R) mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion
(b) If both assertion and reason are true but reason is the not correct explanation of assertion
(c) If assertion is true but reason is false
(d) If both assertion and reason are false

1. Assertion (A): The SI unit of length is the meter, and it is defined by the distance travelled by light in a vacuum in 1/299,792,458 seconds.
Reason (R): The definition of the meter was revised in 1983 to make it independent of physical objects like a platinum-iridium rod
2. Assertion (A): The SI unit of mass is the kilogram, and it is defined using a platinum-iridium cylinder kept at the International Bureau of Weights and Measures.
Reason (R): The kilogram is the only SI base unit that still has a physical reference.
3.Assertion (A): In the SI system, the unit of time is the second, which is based on the vibrations of caesium atoms.
Reason (R): The SI second is defined as the time it takes for 9,192,631,770 oscillations of radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom.
4. Assertion (A): In the International System of Units, the unit of electric current is the ampere.
Reason (R): The definition of the ampere is based on the force between two parallel conductors carrying electric currents in a vacuum
5.Assertion (A): The SI unit of temperature is the Kelvin.
Reason (R): The Kelvin scale is based on the absolute zero, which is the lowest possible temperature where all molecular motion ceases
6. Assertion (A): The dimensional formula of force is $[MLT^{-2}]$
Reason (R): The dimensional formula of any physical quantity is derived by equating it to the base quantities (mass, length, time) raised to appropriate powers
7. Assertion (A): The dimensional formula of the gravitational potential energy is $[ML^2T^{-2}]$
Reason (R): Gravitational potential energy is given by $U=mgh$ where m is mass, g is acceleration due to gravity, and h is height. The dimensional formula of g is $[LT^{-2}]$

ANSWERS OF A& R

1. A 2.A 3. A 4. A 5.A 6.A 7.A

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

1. List the seven fundamental units..
2. Using the example of velocity explain about the derived quantities.
3. Is force a fundamental or derived quantity? Justify.
4. How many significant figures are there in 0.004560?
5. Round off the number 76.867 to 3 significant figures.
6. Deduce the dimensional formula for the following physical quantities
a) coefficient of viscosity b) surface tension
7. State the principle of dimensional homogeneity.
8. Check if the equation v=u + at is dimensionally correct.

ANSWER WITH SOLUTIONS

1. : There are seven fundamental quantities in total in the International System of Units (SI), i.e., Length, mass, time, electric current, temperature, amount of substance, and luminous intensity.
2 -: Derived quantities are those physical quantities that are derived from fundamental quantities. Example: Velocity is derived from length and time (Length x Time)
3 -Answer: Force is a derived quantity.
It is given by Newton's second law: F = ma
It depends on mass (fundamental) and acceleration (derived), hence derived.
4 -: There are 4 significant figures: 4, 5, 6, and the trailing 0 (after the decimal and non-zero digits)
5.- : 76.867 rounded to 3 significant figures is 76.9.
6 -: a) $ML^{-1}T^{-1,}$ b) ML^0T^{-2}
7 -Answer: The principle of dimensional homogeneity states that all terms in a physically meaningful equation must have the same dimensions.
8 -Answer: LHS: $[v] = [LT^{-1}]$ RHS: $[u] = [LT^{-1}]$
$[a\cdot t] = [LT^{-2}]\ [T]$ ⇒ Both sides = $[LT^{-1}]=[L\ T^{-1}]+[LT^{-2}]\ [T]$
yes, the equation is dimensionally correct.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1. The Van der Waal equation for 1 mole of a real gas is ($P + a/V^2$) ($V – b$) where P is the pressure, V is the volume, T is the absolute temperature, R is the molar gas constant and a, b are Van dar Waal constants. Find the dimensions of a/b

2. The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. Using the method of dimensional analysis, derive the relation between λ, g, and ρ

3. What are significant figures? State any two rules for determining significant figures. Give one example.

4. Round off the following numbers to 3 significant figures:
(i) 0.004376 (ii) 56.987 (iii) 12345

5. What is the dimensional formula of Planks constant.

6. State the principle of dimensional homogeneity. Use it to check whether the equation v=u+at is dimensionally correct.

7. Derive the dimensional formula of pressure. Write its SI unit.

ANSWER WITH SOLUTIONS:

1. Van der Waals equation: $(P+a/V^2)/(V-b)=RT$
Dimensions of volume $V=[L^3]$ Dimensions of pressure P $[P]=[ML^{-1}T^{-2}]$
Dimensions of a: From $a/V^2= P$ $a/[L^6=[ML^{-1}T^{-2}] \Rightarrow [a]=[ML^5T^{-2}]$
Dimensions of b:
From (V−b) the terms must have the same dimension:
$[b]=[V]=[L^3]$ $[b] = [V] = [L^3]$
Dimensions of a = ML^5T^{-2} Dimensions of b = L^3
Hence, a/b= $ML^5T^{-2} / L^3 \Rightarrow ML^2T^{-2}$

2 - Ans
We express each quantity in terms of the fundamental dimensions [M],[L],[T]:

Quantity	Symbol	Dimensions
Velocity	v	$[LT^{-1}]$
Wavelength	λ	[L]
Density	ρ	$[ML^{-3}]$
Acceleration due to Gravity	g	$[LT^{-2}]$

Assume that:
$v=k\lambda^a\rho^b g^c$
where k is a dimensionless constant, and a,b,c are powers to be determined.
LHS: $[v]=[LT^{-1}]$
RHS: $[\lambda^a\rho^b g^c]=[L]^a[ML^{-3}]^b\,[LT^{-2}]^c=M^bL^{a-3b+c}T^{-2c}$
Set LHS = RHS:
Mass (M): $0=b \Rightarrow b=0$
Length (L): $1=a-3b+c = a + c$
Time (T): $-1=-2c \Rightarrow c=1/2$
Now from $1=a+c$ and $c = ½$
$a=1-1/2 = ½$ so the relation becomes ⇒ $v=k\,\lambda^{½}g^{½}$
now squaring both sides $v^2 = kg\lambda$

3 – Ans Significant figures are the digits in a measurement that are known accurately.
Two Rules for Determining Significant Figures:
All non-zero digits are always significant.
Example: In 456, all three digits are significant ⇒3 significant figures.
Zeros between two non-zero digits are significant.
Example: In 4003, the zeros between 4 and 3 are significant ⇒4 significant figures.
Example Combining the Rules:
Number: 204.05 Digits: 2, 0, 4, 0, 5
All non-zero digits (2, 4, 5) are significant.

Zeros between non-zero digits (the 0s) are also significant.
⇒Number of significant figures = 5
4 – Ans (i) 0.004376 → 0.00438 (ii) 56.987 → 57.0 (iii) 12345 → 12300
Q5 – Ans Planck's constant h appears in the relation for energy of a photon:
E=hν where: E = energy → dimensional formula = $[ML^2T^{-2}]$
ν = frequency → dimensional formula = $[T^{-1}]$
From the equation: h=E/ν
Substitute dimensions: $[h]=[ML^2T^{-2}]/[T^{-1}]=[ML^2T^{-1}]$
This is the dimensional formula of Planck's constant.
6 – Ans Principle of dimensional homogeneity: Every term in a physically meaningful equation must have the same dimensions. Equation: v=u +at
Now,v = final velocity = $[LT^{-1}]$ u = initial velocity = $[LT^{-1}]$
a = acceleration = $[LT^{-2}]$ t = time = [T]
$[LT^{-1}] = [LT^{-1}] + [LT^{-2}]\,[T]$
L.H.S=R.H.S The equation is dimensionally correct.
7 – Ans Pressure = Force / Area
We know Force = $[MLT^{-2}]$ Area = $[L^2]$
Pressure=$[MLT^{-2}][L^{-2}]=[ML^{-1}T^{-2}]$

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1.What are significant figures? Write the rulws of significant fig. Idenfy the total num of significant fig in the following
a)0.035 b)6.020 c) 8000 d)2.3000 e)π
2, How do you round off numbers to a certain number of decimal places?
3.How can we derive the dimensional formula for physical quantities?
Explain the general method for deriving dimensional formulas.
Derive the dimensional formulas for the following physical quantities:
(a) Energy (b) Pressure (c) Power

ANSWER WITH SOLUTIONS:

1 - Ans : Definition:
Significant figures are the digits in a number that express its measured or calculated precision. They include all certain digits plus one uncertain digit.
Rules for Counting Significant Figures:
All non-zero digits are significant. *Example:* 237 → 3 significant figures.
Any zero between two non-zero digits is significant. *Example:* 1003 → 4 significant figures.
Leading zeros (zeros before the first non-zero digit) are not significant.
Example: 0.0052 → 2 significant figures.
Trailing zeros in a number with a decimal point are significant. *Example:* 4.500 → 4 significant figures.
Trailing zeros in a whole number without a decimal may or may not be significant, depending on context.
Example: 5000 → Could have 1, 2, 3, or 4 significant figures (needs scientific notation to clarify).

Examples:

Number	Significant Figures	Explanation
0.035	2	Leading zeros not significant
6.020	4	Includes zeros between digits
8000	1 (unless specified)	No decimal; ambiguous significance
2.3000	5	All digits including trailing zeros
π	infinite	Rational num

2 - Ans: Definition: Rounding off is the process of reducing the number of digits in a number while keeping its value close to the original. It simplifies numbers for easier interpretation or calculation while maintaining reasonable accuracy.

Rules for Rounding Off:1. Identify the digit to round off to.
Count from the first non-zero digit to find the required number of significant figures or decimal places.
2. Check the next digit (the one to the right):If < 5, leave the rounding digit unchanged.
If ≥ 5, increase the rounding digit by 1.
3. Drop all digits after the rounding digit.
Examples:
(i) Round 5.7486 to 3 significant figures: → 5.75
(The fourth digit is 8 → round up)
(ii) Round 0.003478 to 2 significant figures:
First two significant digits are 3 and 4 → next digit is 7 → round up → 0.0035
(iii) Round 189.456 to 2 decimal places:
The third decimal digit is 6 → round up the second decimal digit → 189.46
3 – Ans Definition: A dimensional formula is an expression that shows how a physical quantity depends on the basic dimensions: Mass (M),Length (L), and Time (T).
For example, velocity = $[L\ T^{-1}]$.
Importance of Dimensional Formula:
To check the correctness of physical equations (dimensional homogeneity).
To derive relationships between physical quantities.
To convert units from one system to another.
To understand how quantities scale with others.
1. Dimensional Formula for Energy (Work): Energy=Force×Distance
We know, Force = $[M\ L\ T^{-2}]$ Distance = $[L]$
$\Rightarrow$Energy=$[MLT^{-2}]\cdot[L]=[ML^2T^{-2}]$
Dimensional formula for energy = $[M\ L^2\ T^{-2}]$
2. Dimensional Formula for Pressure:
Pressure=Force/Area We know, Force = $[M\ L\ T^{-2}]$ Area = $[L^2]$
$\Rightarrow$Pressure=$[MLT^{-2}]/[L^2]= [ML^{-1}T^{-2}]$
Dimensional formula for pressure = $[M\ L^{-1}\ T^{-2}]$
3. Dimensional Formula for Power:
Power=Work/time We already have: Energy = $[M\ L^2\ T^{-2}]$ Time = $[T]$
$\Rightarrow$Power= $[ML^2T^{-1}]\ [T^{-1}]$=[ML2T−3]
Dimensional formula for power = $[M\ L^2\ T^{-3}]$
Conclusion: Dimensional formulas are essential tools in physics that help check the validity of equations, derive new formulas, and maintain unit consistency in calculations.

CASE BASED QUESTIONS

1) A student of class 11th during his/her practical class took diameter of a spherical object using vernier calliper. She takes three readings: Reading 1: 12.45 cm Reading 2: 12.50 cm Reading 3: 12.46 cm The true diameter of this object is known to be 12.48 cm. Based on given information answer the following

1) The average spherical object from the given measurements
a)12.48 b)12.43 c)12.45 d)12.00

(2) Discuss the precision of the measurements based on the true length of the rod.
a) minimum b) maximum c) zero d) uncertain

(3) The accuracy of the measurements based on the tru length of the rod will be.
a) 0.01 b) 0.1 c) 0,001 d) 0.0001

(4) find the volume of the spherical object
a)1016.97 b)1058.52 c) 1063.78 d) 1061.65

ANSWER WITH SOLUTIONS

1 – Ans Average length= $\frac{\text{Reading 1+Reading 2+Reading 3}}{3}$

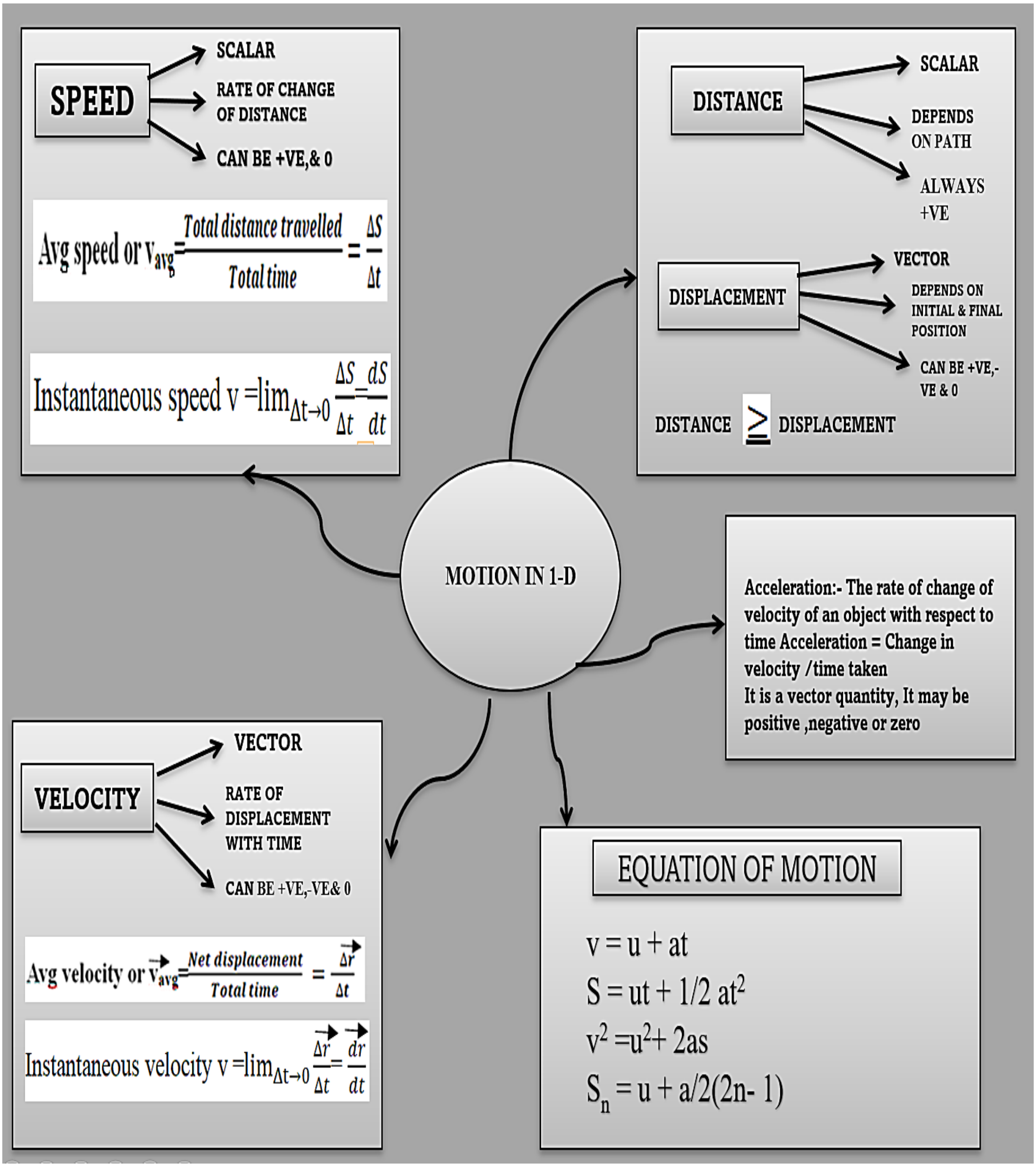
$$= \frac{12.45+12.50+12.46}{3} = \frac{37.41}{3} = 12.47\text{cm}$$

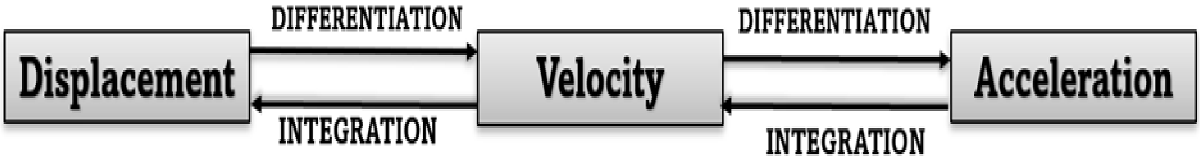
CHAPTER–2: MOTION IN A STRAIGHT LINE

CONTENT OF THE CHAPTER:

Frame of reference, Motion in a straight line, Elementary concepts of differentiation and integration for describing motion, uniform and non- uniform motion, average speed and average velocity and instantaneous velocity, uniformly accelerated motion, velocity - time and position-time graphs. Relations for uniformly accelerated motion.

MIND MAP OF THE CHAPTER





GIST OF THE CHAPTER

What is Motion? (And Why Should You Care)

- Motion = When something changes its position over time.
- Rest = When something stays in the same position (but remember, even a parked car is moving if Earth is your reference!).

For example:-Imagine you're sitting in a moving train. To your friend next to you, you're at rest. But to someone outside, you're in motion. Mind-blowing, right?

2. **Distance vs. Displacement – The Big Difference**

Distance	Displacement
Total path covered (scalar)	Shortest path from start to end (vector)
Always positive (+ve)	Can be +ve, -ve, or zero

For example :-If you walk 5 km East and then 5 km West, your distance = 10 km, but displacement = 0 (because you're back where you started).

3. **Speed vs. Velocity** – Not the Same Thing!

Speed	Velocity
How fast something moves (scalar)	Speed + Direction (vector)
Always positive	Can be +ve or -ve

For example: A car going 60 km/h North has a speed = 60 km/h and velocity = +60 km/h (North). But, If it turns back at 60 km/h South, its velocity = -60 km/h (because direction changed).

4. **Acceleration** – It's Not Just Speeding Up!

Acceleration = Rate of change of velocity (can be +ve or -ve).

Deceleration (Retardation) = Negative acceleration (slowing down).

For example :- When a rocket launches, it accelerates upward (a = +ve).

When a car brakes, it decelerates (a = -ve).

5. **Equations of Motion**

If acceleration is constant, we use:

Final Velocity: $v=u + at$

(Where: u = initial velocity, v = final velocity, a = acceleration, t = time)

Distance Covered: $s=ut+1/2at^2$

No-Time Equation: $v^2=u^2+2as$

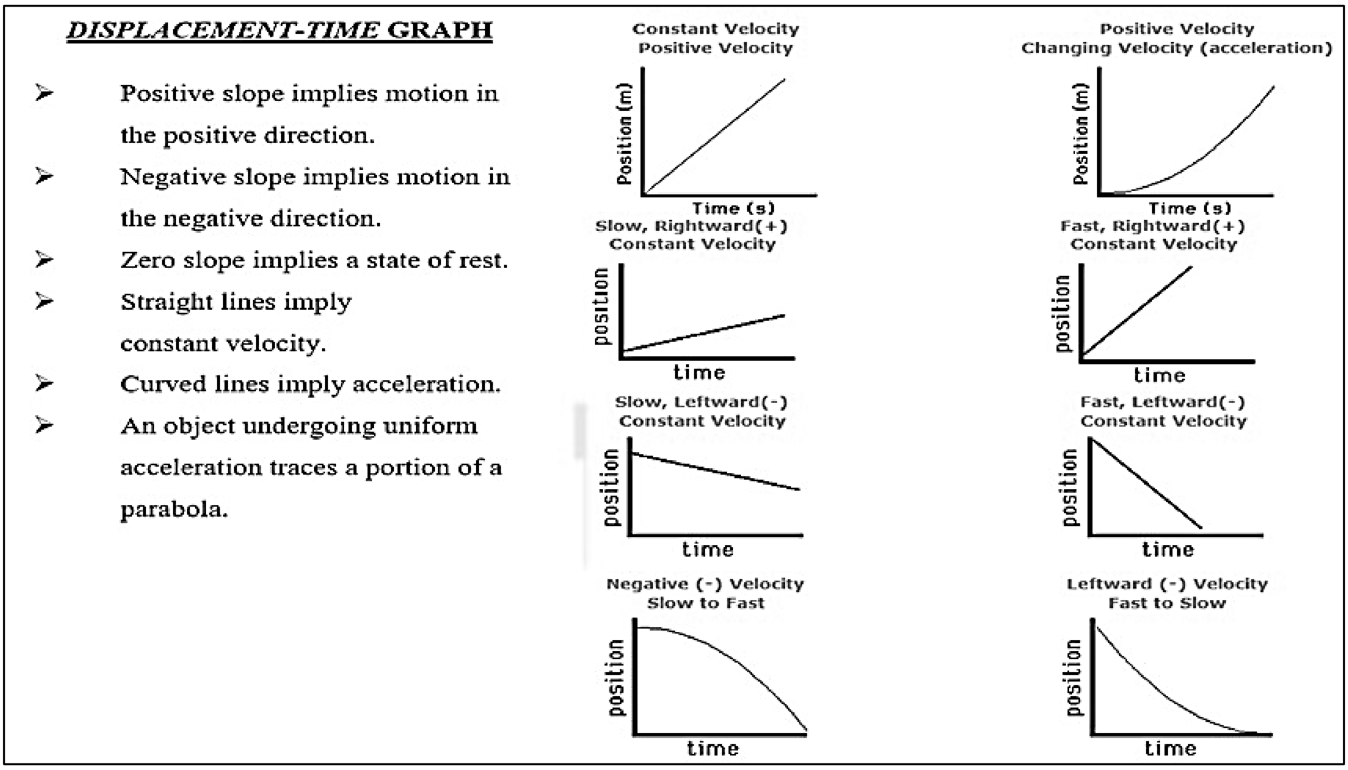
"suvat" Trick (Remember these 5 letters: s=displacement, u=initial velocity, v=final velocity, a=acceleration, t=time).

6. **Graphs** – The Secret to Visualizing Motion

A. **Position-Time Graph**

Slope = Velocity (Steeper slope = Faster speed).

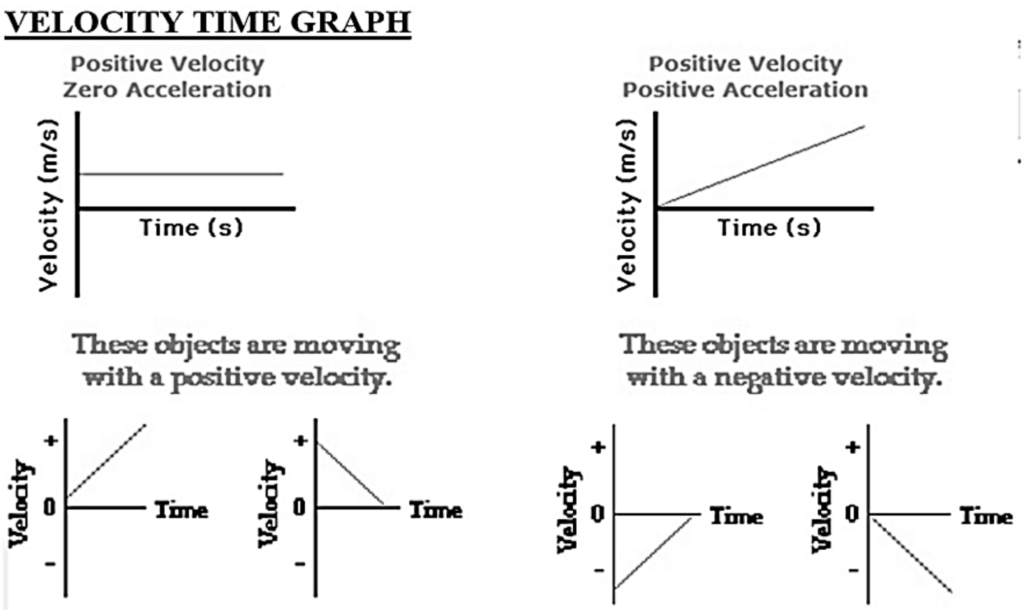
Flat line = No motion (at rest).



B.**Velocity-Time Graph**

Slope = Acceleration

Area under graph = Displacement



For

Example:

A horizontal line means constant speed (zero acceleration).

A straight upward slope means constant acceleration.

For detailed explanation of this chapter

https://youtu.be/yc_uLkZjNo8?si=pBGx5CnJ3bBT-Eqe

https://youtu.be/cbEV1afFOXc?si=mGqimsoDzuLbZzrk

MULTIPLE CHOICE QUESTIONS(MCQ)

1. The slope of velocity time graph gives

(a) distance (b) displacement (c) acceleration (d) speed

2. If the displacement of an object is proportional to square of time, then the object is moving with

(a)constant acceleration (b) uniform motion

(c) constant velocity (d) non uniform motion

3. An athlete finishes a round of circular track of radius R in 40 sec. What is his displacement at the end of 2 min 20 sec?

(a) 2R (b) 2πR (c) 7πR (d) Zero

4. A ball is dropped from height h. The velocity attained by the ball just before reaching the ground is

(a) $\sqrt{2gh}$ (b) $\sqrt{3gh}$ (c) $\sqrt{gh}$ (d)zero

5. A car travels from A to B at a speed of 20 Km/hr and returns at a speed of 30 Km/hr the average speed of the car for the whole journey is

(a) 5 Km/hr (b) 24 Km/hr (c) 25 Km/hr (d) 50 Km/hr

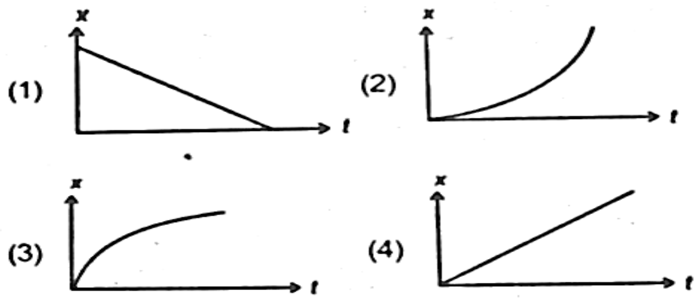
6. A 100 m long train is moving with a uniform velocity of 45 Km/hr.The time taken by the train to cross a bridge of length 1 Km is

(a) 58 second (b) 68 second (c) 78 second (d) 88 second

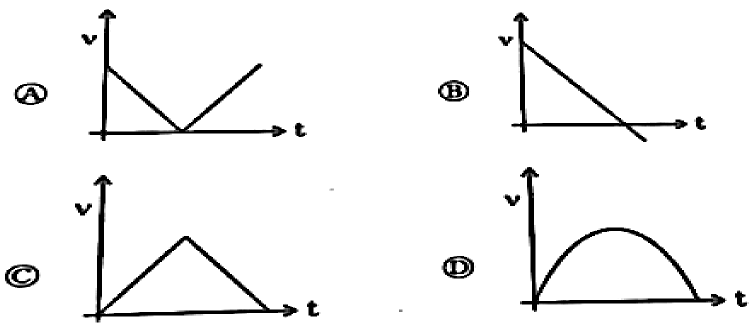
7. If $t=\sqrt{x}+4$, then $\left(\frac{dx}{dt}\right)_{t=4}$ is

(a) 4 (b) zero (c) 8 (d) 16

8. The position-time (x–t) graph for positive acceleration is



9. A ball is thrown vertically upward. Ignoring the air resistance, which one of the following plot represents the velocity-time plot for the period ball remains in air?



SOLUTIONS OF MCQ :-

1. (c) The slope of velocity time graph gives acceleration

2.(a) If the displacement of an object is proportional to square of time, then the object is moving with constant acceleration

3 (a) 2R

4 (a) $\sqrt{2gh}$

5 (b) $v_{avg}=\frac{2\times20\times30}{20+30}$ =24 Km/hr

6 (d) Total Distance=Length of train+ Length of Bridge

Total Distance=(100+1000)=1100 m

Speed 45 km/hr=$45\times\frac{5}{18}=\frac{25}{2}m/s$

Time taken=Total distance/speed= $\frac{1100}{25/2}$=88 sec

7. (b) $x=(t-4)^2=t^2-8t+16$

$\frac{dx}{dt}=2t-8$ At t=4, $\frac{dx}{dt}=0$

8. (b) as the x-t graph bends upwards the slope ($\tan\theta$) increases and so will increases Acceleration is positive for uniform acceleration x-t graph is parabola about position axis

9.(a) During upward motion the magnitude of velocity decreases while during downward motion magnitude of velocity increases hence v-t graph is a straight line with a negative slope

ASSERTION REASON BASED QUESTION

In the following questions a statement of assertion(A) is followed by a statement of reason(R) mark the correct choice as

- **(a) If both assertion and reason are true and reason is the correct explanation of the assertion**
- **(b) If both assertion and reason are true but reason is the not correct explanation of assertion**
- **(c) If assertion is true but reason is false**
- **(d) If both assertion and reason are false**

1. **ASSERTION:** The speed of a body can be negative.
REASON: If the body is moving in the opposite direction of positive motion, then its speed is negative.

2. **ASSERTION:** An object can have constant speed but variable velocity.
REASON: SI unit of speed is m/s.

3. **ASSERTION:** In real-life, in a number of situations, the object is treated as a point object.
REASON: An object is treated as point object, as far as its size is much smaller than the distance, it moves in a reasonable duration of time.

4. **ASSERTION:** The speedometer of an automobile measures the average speed of the automobile.
REASON: Average velocity is equal to total distance divided by total time taken.

5. **ASSERTION**: A body can have acceleration even if its velocity is zero at that instant of time.
REASON: The body will be momentarily at rest when reverses its direction of motion

6. **ASSERTION**: When a body is dropped or thrown horizontally from the same height it would reach the ground at the same time
REASON: Horizontal velocity has no effect on the vertical motion

SOLUTIONS OF A & R :-

1. (d) Speed can never be negative because it is a scalar quantity. So, if a body is moving in negative direction, then also the speed will be positive.

2. (b) Velocity is a vector quantity, so it has both direction and magnitude. Hence, an object can have variable velocity by keeping its magnitude constant, i.e. speed and by changing direction only.

3. (a) The approximation of an object as point object is valid only, when the size of the object is much smaller than the distance it moves in a reasonable duration of time.

4. (d) The assertion that the speedometer measures average speed is incorrect; it measures instantaneous speed. The reason, stating average velocity is total distance divided by total time, is also incorrect. Average velocity is calculated as total displacement divided by total time

5. (a) When a body thrown up vertically will have zero velocity at the highest point but its acceleration will be equal to g.

6. (a) Horizontal velocity has no effect on time of flight. Therefore both assertion and reason are true and the reason is the correct explanation of the assertion

SHORT ANSWER QUESTIONS(2 MARKS)

1. Two balls of different masses are thrown vertically upward with the same initial speed. Which one will rise to a greater height?
2. Can a body have a constant speed and still have a varying velocity?
3. The position coordinate of a moving particle is given by $x=6+18t + 9t^2$(Where x is in meters and t is in seconds) what is the velocity at t=2 second?
4. Derive an equation for the distance travelled by a body moving with uniform acceleration during the nth second of its motion.
5. Is it possible to have a constant rate of change of velocity when velocity changes both in magnitude and election if yes give one example
6. State in the following cases whether the motion is One, two or three dimensional

a kite flying on a windy day

a speeding car on a long straight highway

7. Even when rain is falling vertically downwards the front screen over moving car gets wet while the back screen remains dry why

SOLUTIONS:-

1: Ans: When two balls of different masses are thrown vertically upward with the same initial speed, both of them will rise to a greater height.

2: Yes, a particle in uniform circular motion has a constant speed but varying velocity because of the change its direction of motion at every point

3:Given $x=6+18t + 9t^2$

$v = \frac{dx}{dt} = \frac{d}{dt}(6+18t + 9t^2)=18+18t$

At t=2 sec $v=18+18x2=54ms^{-1}$

4: Prove $S_n = u + a/2(2n- 1)$

5: Yes, in projectile motion a body has uniform acceleration in the downward direction while its velocity changes both in direction and magnitude at every point of its trajectory.

6: (a) Three-Dimensional (b)One-dimensional

7: This is because the rain strikes the car in the direction of relative velocity of rain with respect to car.

SHORT ANSWER QUESTIONS(3 MARKS)

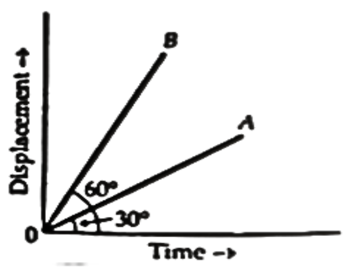
1. A body is moving in a straight line and begins to slow down gradually. In such a situation:

(a) Identify the condition of motion.

(b) State the directions of velocity and acceleration vectors.

(c) Justify your answer with a suitable real-life example

2. Two straight lines drawn on the same displacement time graph makes angle 30° and 60° with time axis respectively as shown in the figure



   (a) which line represents greater velocity
   (b) what is the ratio of the two velocities
   (c) What is the nature of motion represented by each line?

3. Draw following graphs representing motion of an object under freefall neglect air resistance

(a) variation of position with respect to time

(b) variation of velocity with respect to time

(c) variation of acceleration with time

4. A car moving along a straight highway with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

5. A body travels from A to B at 40 ms^{-1}& from B to A at 60 ms^{-1}. Calculate the average speed and average velocity.

6. The position of and object moving along x-axis is given by $x=a+bt^2$, where a=8.5m and b=2.5 ms^{-2} and (t) is measured in seconds. What is the velocity at t=0s and t=2.0s? What is the average velocity between t=2.0s and t=4.0s ?

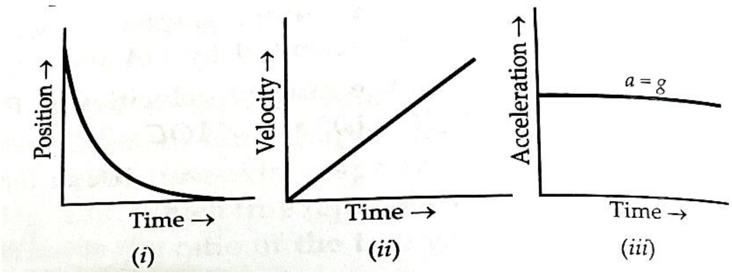
7. The displacement (in metre) of a particle moving along x-axis is given by $x=18t+5t^2$. Calculate
(i) the instantaneous velocity t=2s
(ii) average velocity between t=2s to t=3s
(iii) instantaneous acceleration.

SOLUTIONS :-

Ans 1: (a) Condition of Motion:
The body is undergoing retardation or deceleration—a condition where the speed of the body decreases with time.
(b) Direction of Vectors: The velocity vector is in the direction of motion of the body.
The acceleration vector is opposite to the direction of velocity, as it acts to reduce the body's speed.
(c) Justification with Example:A common example is a car moving forward and gradually coming to a stop when the brakes are applied. While the car continues to move forward for a while, the acceleration due to braking acts in the opposite direction, slowing it down until it stops

Ans 2 .(a) The line with a greater slope (i.e., 60°) represents greater velocity
(b) Ratio of two velocities=$\frac{v_A}{v_B} = \frac{\tan 30}{\tan 60} = \frac{1}{3}$
(c) Nature of Motion : Each straight line on a displacement-time graph represents uniform motion.

3 :



4 : The initial velocity of the car = u ,Final velocity of the car = v
Distance covered by the car before coming to rest = 200 m
Using the equation, v = u + at ⇒ t = (v – u)/a = 11.44 sec.

5: Avg Speed=$\frac{Total\ distance\ travelled}{Total\ time} = \frac{2v_1v_2}{v_1+v_2}$ =48 Km/hr

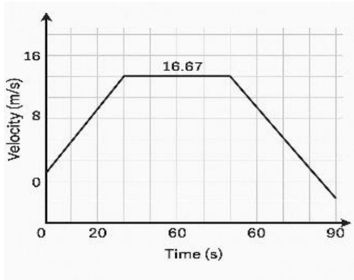
Avg Velocity=$\frac{Total\ displacement\ travelled}{Total\ time}$

As body returns to its initial state Therefore net displacement is zero
Hence Avg velocity=0

6: $v(t)=\frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + 2bt$
At t=0; v=0 & At t=2 s v=2x2.5x2=10 ms^{-1}
At t=2 s x=a+4b & At t=4 s x=a+16b
Avg Velocity=$\frac{x_2-x_1}{t_2-t_1} = \frac{a+16b-a-4b}{4-2}$=6b
Hence, Avg Velocity= 6x2.5=15 ms^{-1}

7 : As,x=18t+5t^2 v(t)=$\frac{dx}{dt} = 18 + 10t$ & a(t)=$\frac{dv}{dt} = 10$ ms^{-2}
velocity at t=2 v (2) =18+10(2) =38ms^{-1}
v (3) =18+10(3) =48ms^{-1}
Since acceleration is constant
V $_{avg}$=$\frac{v(3)+v(2)}{2} = 43$ ms^{-1} & instantaneous acceleration a=10 ms^{-2}

LONG ANSWER TYPE QUESTIONS (5 MARKS)



1. While waiting at a metro station, a student records the motion of a train as it arrives. The velocity-time graph of the train is as follows:
(a) Draw a velocity-time graph for the entire motion.
(b) Calculate the total distance covered by the train.
(c) Find the total time taken and the average speed of the train.
(d) Identify the nature of acceleration in each phase and explain its physical meaning
2. Water drops fall freely from a tap at a height of 4.9 m. If time interval between successive drops is equal and the 4th drop is released when the first lands on the ground, find the separation between the second and third drops
3.A player throws a ball upwards with an initial speed of 29.4ms^{-1}
(a) What is the direction of acceleration during the upward motion of the ball?
(b) What are the velocity and acceleration of the ball at the highest point of its motion?
(c) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.
(d) To what height does the ball rise and after how long does the ball return to the player's hands? (Take g = 9.8 ms^{-2} and neglect air resistance).

SOLUTIONS:

1:(a) Linearly rising from 0 to 16.67 m/s in 20 s Horizontal line at 16.67 m/s for 40 s

Linearly decreasing from 16.67 m/s to 0 in 30 s

(b) **Total Distance Covered**

During acceleration (0 to 20 s): a=$\frac{v-u}{t_1}$=$\frac{16.67-0}{20}$=0.8335 m/s^2 & s$_1$= ut+$\frac{a}{2}t^2$=166.7 ms^{-1}

During constant speed (20 to 60 s): s$_2$=v×t$_2$=16.67×40=666.8 ms^{-2}

During deceleration (60 to 90 s): a=$\frac{v-u}{t_3}$ =−0.556m/s^2 s$_3$= vt_3+$\frac{a}{2}t_3{}^2$=250.05 ms

Total distance: s=s$_1$+s$_2$+s$_3$=166.7+666.8+250.05=1083.55 m

(c) Total time t=20+40+30=90 s = 90 s

Avg speed or v$_{avg}$ = $\frac{\textbf{\textit{Total distance travelled}}}{\textbf{\textit{Total time}}} = \frac{\mathbf{1083.55}}{\mathbf{90}}$ ≈12.04m/s

(d) Nature of acceleration:

First 20 s: Positive uniform acceleration (speed increasing)

Next 40 s: Zero acceleration (constant speed)

Last 30 s: Negative uniform acceleration (deceleration to rest)

2 : As, $h = 4.9$ m

Time for a drop to reach ground $t=\sqrt{\frac{2h}{g}}=\frac{\sqrt{2X9.8}}{9.8}=1$ sec

when the first drop lands on ground, the 4th is ready to come down. Therefore there are 3 equal time gaps of $\left(\frac{1}{3}\right)$between two consecutive drops. Time of travel of 2nd and 3rd drops are $\frac{2}{3}$ & $\frac{1}{3}$ respectively.

Separation between 2nd and 3rd drops$=\frac{g}{2}\left|(\frac{2}{3})^2-(\frac{1}{3})^2\right|$=1.63 m

3.(a) Irrespective of the direction of the motion of the ball, acceleration (which is actually acceleration due to gravity) always acts in the downward direction towards the centre of the Earth.

(b) At maximum height, velocity of the ball becomes zero. Acceleration due to gravity at a given place is constant 9.8 ms^{-2}

(c) During upward motion, the sign of position is positive, sign of velocity is negative, and sign of acceleration is positive. During downward motion, the signs of position, velocity, and acceleration are all positive.

x > 0 for both upward and downward motions, v < 0 for upward and v > 0 for downward motion, a > 0 throughout the motion

Using $v^2 - u^2 = 2as$ ⇒ s= 44.1 m, & Using v=u+ at ⇒ t=3 s

Total time of flight=time of ascent+ time of descent=6 s

CASE STUDY BASED QUESTION

1. A person is standing on the terrace of a building 100 meters high. At time t=0, he throws a ball vertically upward with an initial velocity u. After a time interval Δt (where Δt<2 sec), he throws a second ball vertically upward with an initial velocity equal to half that of the first ball.At time t=2 seconds (measured from the instant the first ball is thrown), the vertical separation between the two balls is observed to be +15 meters, with the first ball being above the second. Assume acceleration due to gravity g=10 m/s^2, and neglect air resistance.

(i) How does the gap between the balls change?
(a) Increases (b) Decreases (c) Becomes zero (d) Remains constant

(ii) Find the initial velocity of ball 1.
(a) 10 m/s (b) 20 m/s (c) 30 m/s (d) 40 m/s

(iii) With what velocity was second ball thrown?
(a) 10 m/s (b) 20 m/s (c) 30 m/s (d) 40 m/s

(iv) Calculate the time interval between their throw.
(a) 0.01 s (b) 0.1 s (c) 1 s (d) 10 s

OR

(iv) The type of motion of balls is
(a) accelerated (b) uniform (c) periodic (d) random

2.Shweta is driving her car on a straight highway from City A to City B, which are 150 km apart. She starts his journey at 9:00 AM. For the first 50 km, she drives at 60 km/h. For the next 50 km, due to road repairs, she drives at 40 km/h. For the last 50 km, the road clears and he speeds up to 75 km/h.
Meanwhile, her friend Aruna starts from City B toward City A at 10:00 AM, driving at a constant speed of 60 km/h on the same road.

(i) What is the total time Shweta takes (including breaks) to reach City B from City A??
(a) 2 h 40 min (b) 2 h 45 min (c) 3 h 30 min (d) 4 h

(ii) What is Shweta's average speed for the entire trip?
(a) 50 km/h (b) 55 km/h (c) 45 km/h (d) 60 km/h

(iii) At what time will Shweta and Aruna cross paths?
(a) 11:45 AM (b) 11:30 AM (c) 11:00 AM (d) 12:15 PM

(iv) What is the relative speed of the two cars when moving toward each other?
(a) 50 km/h (b) 100 km/h (c) 85 km/h (d) 90 km/h

SOLUTIONS

1:(i) (d) The gap between the balls remains constant as their acceleration is the same.

(ii) (b) Let velocity of first ball be u and time gap of second ball be x.

Thus first ball has time (t + x) in upward motion.

$S_1 = u(t + x) + 1/2(-g)(t + x)^2$

Similarly, for second ball, $S_2 = \frac{ut}{2} - \frac{1}{2}\ gt^2$

Taking difference, $S_1 - S_2 = u\left(t + x - \frac{t}{x}\right) - \frac{g}{2}[(t + x)^2 - t^2]$

$= u\left(\frac{t}{2} + x\right) - \frac{g}{2}(2xt + x^2)$

Now, $S_1 - S_2 = 15$ m at t = 2s As x < 2s, x = 1s

$15 = u\left(\frac{2}{2} + 1\right) - \frac{1}{2} \times 10\{2 \times 1 \times 2 + 1(1)2\}$ ⇒ u=20 m/s (velocity of first ball)

(iii) (a) Velocity of second ball = = 10 m/s

(iv) (c) Time interval = x = 1s.

Or

(iv) (a) accelerated

2(i) **(c)** 3 h 5 min

First 50 km: 50 / 60 = 0.83 h = 50 min Next 50 km: 50 / 40 = 1.25 h = 75 min

Last 50 km: 50 / 75 = 0.67 h = 40 min

Driving time = 50 + 75 + 40 = 165 min = 2 h 45 min Breaks = 20 min

Total time = 2 h 45 min + 20 min = 3 h 5 min

(ii) (b) 55 km/h

Average Speed = Total Distance / Total Time = 150 / 2.75 ≈ 54.55 km/h

(c) 11:00 AM (Closest Option)

Shweta drives first 50 km at 60 km/h ⇒ Shweta Time=50/60=0.83 hr

She reaches 50 km mark at 9:50 AM Then she drives next 50 km at 40 km/h in 10 min

Distance=Speed×Time=40×1/6 ≈6.67 km.

Total distance travelled by Shweta till 10 AM=50+6.67=56.67 Km

Aruna started his journey at 10 AM at a constant speed 60 Km/h

Therefore, relative velocity at 10 AM=40+60=100 KM/h

Also, distance between them =150−56.67=93.33 km

Time=distance/relative velocity=93.33/100=0.933 hr≈56 minutes

Hence, they will meet at 10:56 Am

(iv) Option b relative velocity at 10 AM=40+60=100 KM/h

CHAPTER-3 MOTION IN A PLANE

CONTENT OF THE CHAPTER: Scalar and vector quantities; position and displacement vectors, general vectors and their notations; equality of vectors, multiplication of vectors by a real number; addition and subtraction of vectors, Unit vector; resolution of a vector in a plane, rectangular components, Scalar and Vector product of vectors. Motion in a plane, cases of uniform velocity and uniform acceleration, projectile motion, uniform circular motion.

MIND MAP

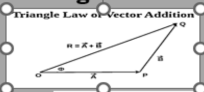
VECTORS

- Unit vector : → It is written as $\hat{A}$

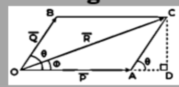
(i) A unit vector whose magnitude is unity & direction is arbitrary $\hat{A} = \left|\frac{\vec{A}}{A}\right|$

VECTOR ADDITION

Triangle law



Parallelogram law



$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Direction is given by

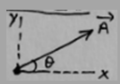
$\phi = \tan^{-1}\left(\frac{\vec{B}\ \sin\theta}{\vec{A} + \vec{B}\ \cos\theta}\right)$

COMPONENTS OF VECTOR

IN 2-D

A_x = A cosθ,

A_y = A sinθ

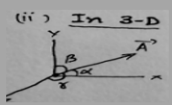


IN 3-D

A_x = A cosα,

A_y = A cosβ

A_y = A cosγ



Dot product or scalar product

$\vec{A}.\vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta$

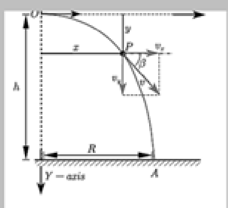
Cross product or vector product

$\vec{A} \times \vec{B} = |\vec{A}|\ |\vec{B}| \sin\theta$

PROJECTILE MOTION

HORIZONTAL PROJECTILE MOTION

When an object or particle (called a projectile) is thrown with some initial velocity along the horizontal and it moves under the action of gravity alone.



The horizontal projection of a projectile

Trajectory of the projectile is a parabola. $y = \frac{gx^2}{2\,u^2}$

Velocity of the projectile at any time $v = \sqrt{u_x^2 + g^2t^2}$

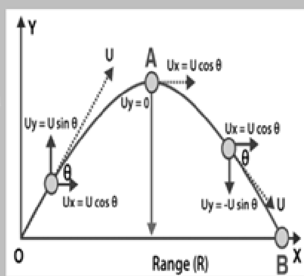
TIME OF FLIGHT $T = \sqrt{\frac{2h}{g}}$

HORIZONTAL RANGE $R = u\sqrt{\frac{2h}{g}}$

IN PROJECTILE MOTION TOTAL ENERGY OF THE PROJECTILE REMAINS CONSTANT K.E+P.E=CONSTANT

ANGULAR PROJECTILE MOTION

When a body is thrown with some initial velocity with an angle and allowed to move under the action of gravity alone



Trajectory of the projectile $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

MAX HEIGHT AT A $H = \frac{u^2\sin^2\theta}{2g}$

TIME OF FLIGHT $T = \frac{2u\sin\theta}{g}$

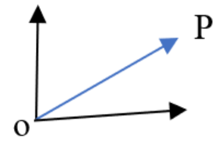
HORIZONTAL RANGE $R = \frac{u^2\sin 2\theta}{g}$

GIST OF THE CHAPTER

- **Scalar And Vector Quantities**

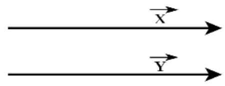
Scalar Quantities	**Vector Quantities**
Those physical quantities which have only magnitude. Examples-distance, speed etc.	Those physical quantities which have both direction and magnitude. Examples- displacement, velocity, 10m displacement towards east, acceleration, force etc.

- **Representation of vector**- A vector quantity is represented a straight line with an arrowhead over it $\vec{A}$
- **Types of vectors**



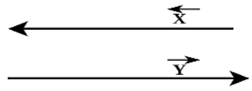
Position vector- A vector which gives position of an object, with reference to origin of a coordinate system is called position vector.

Displacement vector- It is that vector which tells how much and in which direction an object has changed its position in a given time interval.



Equal Vectors

Equal vectors- Two vectors are set to be equal if they have same magnitude and same direction.



Negative Vectors

Negative vector- The negative of a vector is defined as the another vector having the same magnitude but having an opposite direction.

Unit vector : → It is written as $\hat{A}$

(i) A unit vector whose magnitude is unity & direction is arbitrary $\hat{A} = \left|\frac{\vec{A}}{A}\right|$

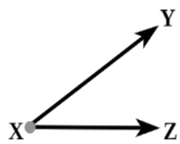
(ii) A unit vector is dimensionless quantity of magnitude equal to unity.

(iii) In cartesian co-ordinates, $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$ are the unit vectors along x, y and z-axes, respectively.

Polar vectors : These are those vectors which have a linear directional effect. For example, force, linear momentum, linear velocity etc.


Collinear Vector

Collinear Vector: A collinear vector can be defined when two or more than two vectors are parallel to one another irrespective of the magnitude or the direction.



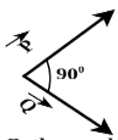
Co-Initial Vector

Co-Initial Vectors: Co-initial vectors refer to the vectors that have a common initial point

Coplanar Vectors :A set of vectors is said to be Coplanar vectors if they lie on the same plane. These vectors can be perpendicular to each other, can be parallel, or can be any random vector on the same plane.

Axial vectors or Rotational vectors : These vectors represent rotational effect. They are always directed along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque, angular momentum etc. are few examples of axial vectors.

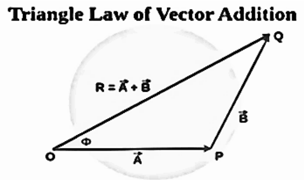
Zero or Null vector-It is a vector which has zero or null magnitude.

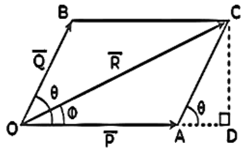


Orthogonal Vectors

Orthogonal Vector - two vectors are said to be orthogonal if they are perpendicular to each other. also in other words two vectors are orthogonal if there dot product is 0

- **Vector addition methods:**

<table>
<tr>
<td><u>Triangle law</u>
If two vectors can be represented both in magnitude and direction by the two sides of triangle taken in the same order then their resultant is represented completely both in magnitude and direction by the third side of the triangle.


$\vec{R} = \vec{A} + \vec{B}$</td>
<td><u>Parallelogram law</u>
if two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram, then their sum (resultant) is represented by the diagonal of that parallelogram, also in magnitude and direction, originating from the same point.

$R = \sqrt{A^2 + B^2 + 2ABcos\theta}$
Direction is given by
$\phi = \left(\frac{\vec{B}\ sin\theta}{\vec{A}+\vec{B}\ cos\theta}\right)$</td>
</tr>
</table>

- **Product of vectors**

Dot product or scalar product

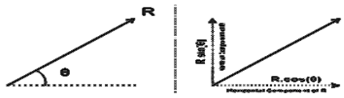
$\vec{A}.\vec{B}=|\overrightarrow{A}|\ \ |\overrightarrow{B}|\ cos\ cos\ \theta$

Cross product or vector product

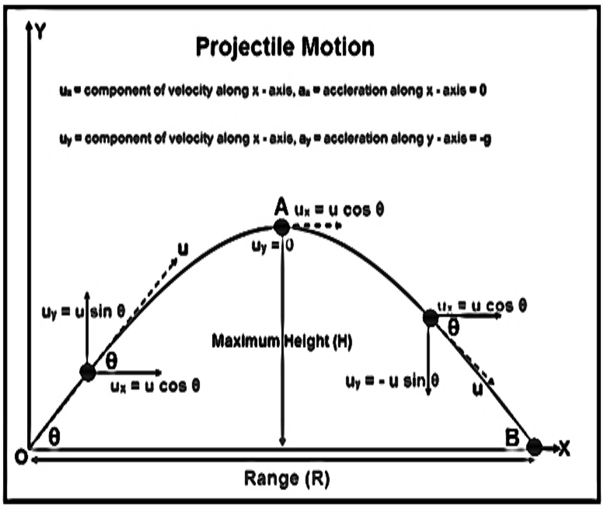
$\vec{A} \times \vec{B}=|\overrightarrow{A}|\ \ |\overrightarrow{B}|\ sin\ sin\ \theta$

- **Resolution of vector**:

R_x=Rcosθ , R_y = R sinθ



Projectile Motion : Projectile is defined as a body thrown with some initial velocity except vertically upward or downward and then allowed to move under the action of gravity alone. The path followed by a projectile is known as its trajectory.



Equation of trajectory of a projectile

y= xtanθ - $\frac{gx^2}{2cos^2{}_{\theta}u^2}$

Maximum height at A H=$\frac{u^2 sin^2\theta}{2g}$**,Time of flight T=** $\frac{2usin\theta}{g}$ **Horizontal Range R=**$\frac{u^2 sin2\theta}{g}$

Centripetal acceleration :(ac) It is the acceleration required to move the body in circular path with a constant angular velocity.

(b) The centripetal acceleration acts on the particle along the radius which is directed towards the centre of circular path **$a_c = \frac{v2}{r} = \omega^2$** **$a_c = 4\pi^2\ v^2 r$**

MULTIPLE CHOICE QUESTIONS(MCQ)

1. Which physical quantity remains unchanged during projectile motion?

(A) Momentum (B) Kinetic energy

(C) Vertical component of velocity (D) Horizontal component of velocity

2. The square of resultant of two equal forces is three times of their dot product, the angle between the forces is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

3.A person can throw a ball up to a maximum range of 100 meter how high above the ground we can pour the same ball ?

(A)5 m (B) 200 m (C) 100 m (D) 50 m

4.If a particle has negative velocity and negative acceleration, its speed

(A) Zero (B) decreases (C) remains same (D) increases

5.Identify the unit vector in the following.

(A) $\hat{\imath}+\hat{\jmath}$ (B) $\frac{\hat{\imath}}{\sqrt{2}}$ (C) $\hat{k}-\frac{\hat{\jmath}}{\sqrt{2}}$ (D) $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

6. If the velocity is $\vec{v}=\mathbf{2}\hat{\imath}+t^2\hat{\jmath}-9\hat{k}$ then the magnitude of acceleration at t = 0.5 s is

(A) $1ms^{-2}$ (B) 2 ms^{-2} (C) zero (D) -1 ms^{-2}

7. Two bullets are fired simultaneously horizontally and with different speeds from the same place which bullet will hit the ground first

(A)The faster one (B) depends on their mass

(C) The slower one (D) both will reach simultaneously

8. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle the motion of the particle takes place in a plane it follows that

(A) Its velocity is constant (B) Its acceleration is constant

(C) Its is kinetic energy is constant (D) It moves along a straight line

8. Which one of the following is the correct expression of centripetal acceleration?

(A) $4\pi^2\upsilon^2R$ (B) $\frac{4\pi^2\upsilon^2}{R}$ (C) $2\,\pi\,\upsilon$ R (D) $\frac{2\,\pi\,\upsilon\,R}{R}$

SOLUTIONS OF MCQ :

1. Option (D) is correct.

3.Option D

4.Option (D)

5. Option (D)

6. Option (A)

7.Option D

8. Ans. Option (A) is correct. As, $a_c = R\,\omega^2$ Putting $\omega = 2\,\pi\,\upsilon$ $a_c = 4\pi^2\upsilon^2R$

ASSERTION REASON BASED QUESTION

In the following questions a statement of assertion(A) is followed by a statement of reason(R) mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of the assertion

(b) If both assertion and reason are true but reason is the not correct explanation of assertion

(c) If assertion is true but reason is false

(d) If both assertion and reason are false

1. ASSERTION: -For a Projectile, the time of flight becomes n times the original value if its speed of projection is made n times.

REASON: For a projectile, due to this the range of the projectile which becomes n times

2. ASSERTION: For uniform circular motion, the displacement and acceleration are directed towards the center along the radius.

REASON: Centripetal acceleration is represented as $a_c = \frac{\omega^2}{R}$

3. ASSERTION: A javelin thrower should try to launch the javelin at an angle 45° with the ground. (Ignore height of the thrower and air resistance)

REASON: The maximum range does not depend upon angle of projection.

4. ASSERTION: If $\vec{P}.\vec{Q}=\vec{P}.\vec{R}$ Then $\vec{P}$ may not always be equal to $\vec{R}$.

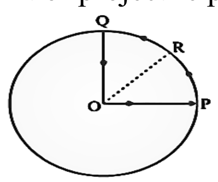
REASON: The dot product of two vectors involves sine of the angle between the two vectors.
5.ASSERTION: Two balls of different masses are thrown vertically upward with same speed, they will pass through their point of projection in the downward direction with the same speed.
REASON: The maximum height and downward velocity attained at the point of Projection are dependent of the mass of the ball.
6. ASSERTION: The path of a projectile on earth is parabolic but for a satellite it is elliptical.
REASON: Acceleration due to gravity decreases with height from earth's surface.
7.ASSERTION: Minimum number of non-equal vectors in a plane required to give zero resultant is three.
REASON: If $\vec{P}+\vec{Q}+\vec{R}=0$, then they must lie in one plane.
8. ASSERTION: If there were no gravitational force, the path of the projected body always is a straight line.
REASON: Gravitational force makes the path of projected body always parabolic.

SOLUTIONS OF A & R:-

1. Option (C)
As $T_f=\frac{2usinsin\,\theta}{g}\Rightarrow T'_f=\frac{2(nu)sinsin\,\theta}{g}=nT_f$
$R=\frac{u^2\theta}{g}\Rightarrow R'=\frac{n^2u^2\theta}{g}\Rightarrow n^2R$,Hence A is true R is false
2. Option D As during uniforms of your motion the acceleration and displacement is along the tangent and in reason the formula is incorrect. Therefore both are incorrect
3. Option C When a body is projected its range depends on
4.Option C As the dot product's dependence on the angle between vectors.
5. Option C As motion is governed by force of gravity and acceleration due to gravity (g) is independent of mass of object
6. Option B As both assertion and reason are correct but reason is the explaining the assertion for both cases.
7. Option (A) is correct. The minimum number of non-equal vectors in a plane required to give a zero resultant is three and they must be coplanar .
8. Option (A) is correct . In the absence of gravity, once a body is projected, no vertical acceleration acts on it. The combination of horizontal constant velocity and vertical acceleration due to gravity results in a parabolic trajectory.

SHORT ANSWER QUESTIONS(2 MARKS)

1. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?
2. Check whether the vectors are orthogonal or not
$\vec{A}=2\,\hat{\imath}+3\hat{\jmath}$ **and** $\vec{B}=4\,\hat{\imath}-5\hat{\jmath}$
3. Two vectors are given as $\vec{r}=2\,\hat{\imath}+3\hat{\jmath}+5\hat{k}$ and $\vec{F}=3\,\hat{\imath}-2\hat{\jmath}+4\hat{k}$. Find the resultant vector $\vec{\tau}=\vec{r}X\vec{F}$
4. A football is kicked into the air vertically upwards. Find its
(a) acceleration (b) velocity at the highest point?
5. A projectile achieves a maximum range that is $\frac{2}{\sqrt{3}}$ times its actual range. What is the angle of projection corresponding to the actual range ?
6. What is the angle between the direction of velocity and acceleration at the highest point of projectile path
7. A cyclist starts from center O of a circular park of radius 1 km and moves along the path OPRQO as shown. If he maintains constant speed of 10 m/s, what is his acceleration at point R in magnitude & direction?



8. What do you mean by rectangular components of a vector? Explain how a vector can be resolved into two rectangular components in a plane ?

SOLUTIONS:-

1. As, $R = \frac{u^2\theta}{g}$ $\Rightarrow$R = R $_{max}$, θ = 45°, i.e., θ = sin 90° = 1

$R_{max} = \frac{u^2}{g}$ Suppose H = height up to which the ball goes when the cricketer throws it with velocity u. Since, the final velocity of the ball, v = 0.

$v^2 - u^2 = 2as$ here, v = 0, a = – g, s = H $\Rightarrow$ $H = \frac{1}{2}\frac{u^2}{g} = 50\ m$

2. Condition for orthogonal vectors ; $\vec{A} \cdot \vec{B} = 0$

$\vec{A} \cdot \vec{B}$ = (2 $\hat{\imath}$+3$\hat{\jmath}$) . (4 $\hat{\imath}$-5$\hat{\jmath}$) = 8($\hat{\imath}$. $\hat{\imath}$)−15 ($\hat{\jmath}$. $\hat{\jmath}$)= $\vec{A} \cdot \vec{B}$ = 8−15 = − 7

As, $\vec{A} \cdot \vec{B} \neq 0$. Hence $\vec{A}$ and $\vec{B}$ are not orthogonal to each other.

3. As $\vec{\tau} = \vec{r} X \vec{F} =$ $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 5 \\ 3 & -2 & 4 \end{vmatrix}$ = 22 $\hat{\imath}$+ 7 $\hat{\jmath}$− 13 $\hat{k}$

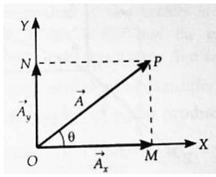
4. (a) Acceleration will always be vertical downward and is called acceleration due to gravity
(b) When football reaches the highest point it is momentarily at rest velocity, v=0. As, it is continuously retarded by acceleration due to gravity (g).

5. $R_{max} = \frac{2}{\sqrt{3}} R \Rightarrow \frac{u^2}{g} = \frac{2}{\sqrt{3}}\frac{u^2\theta}{g} \Rightarrow \theta = \frac{\sqrt{3}}{2} = Sin60 \therefore \theta = 30$

6. 90° . At the highest point the vertical component of velocity becomes zero the projectile has only a horizontal velocity while the acceleration due to gravity acts vertical downwards

7. $Centripetal\ acceleration,\ a = \frac{v^2}{r} = \frac{10^2}{1000} = 0.1 ms^{-2}\ along$ RO

8. When a vector is resolved along 2 mutually perpendicular directions the components so obtained are called a rectangular component of the given vector as shown in the figure



Here $\overrightarrow{A_x} = A_x\hat{\imath}$ & $\overrightarrow{A_y} = A_y\hat{\jmath}$ Also,$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$

$A_x = A\cos\theta$ & $A_y = A\sin\theta \Rightarrow A = \sqrt{A_x{}^2 + A_Y{}^2}$ $Also,$ $\tan\theta = \frac{A_y}{A_x}$

SHORT ANSWER QUESTIONS(3 MARKS)

1. The sum & difference of two vectors are equal in magnitude. Mathematically prove that they are mutually perpendicular to each other.
2. Prove that the horizontal range is same when angle of projection is
(i) greater than 45° by certain value and (ii) less than 45° by the same value.
3. A man moving in rain holds his umbrella inclined to the vertical even though the rain drops are falling vertically downwards . Why ?
4. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r.
5. Consider the x–axis as representing east, the y–axis as north and z–axis as vertically upwards. Give the vector representing each of the following points.
(a) 5 m north east and 2 m up (b) 4 m south east and 3 m up (c) 2 m north west and 4 m up
6. The resultant of two forces whose magnitudes are in the ratio 3:5 is 28 N. If the angle of their inclination is 60°, then find the magnitude of each force
7. At what angle do the two forces (P + Q) and (P – Q) act so that the resultant is $\sqrt{3P^2 + Q^2}$.
8. Find the angle between the vectors $\vec{A} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{B} = -\hat{\imath} + \hat{\jmath} - \hat{k}$?

SOLUTIONS :-

1: Given $\vec{A}+\vec{B}=\vec{A}-\vec{B}$

Taking square on both sides

$(\vec{A}+\vec{B})^2=(\vec{A}-\vec{B})^2 \Rightarrow A^2+B^2+2\vec{A}\vec{B}=A^2+B^2-2\vec{A}\vec{B}$

$4\vec{A}.\vec{B}=0 \ \Rightarrow \vec{A}.\vec{B}=0$

Therefore these two vectors are perpendicular to each other.

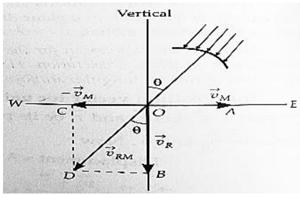
2. (i) When the angle of projection is $\theta=45+\alpha$

R$=\frac{u^2\theta}{g}=\frac{u^2(45+\alpha)}{g}=\frac{u^2 sinsin\,(90+2\alpha)}{g}=\frac{u^2 coscos\,(2\alpha)}{g}$

(ii) When the angle of projection is $\theta=45-\alpha$

R'$=\frac{u^2\theta}{g}=\frac{u^2(45-\alpha)}{g}=\frac{u^2 sinsin\,(90-2\alpha)}{g}=\frac{u^2 coscos\,(2\alpha)}{g}$

Clearly R= R'



3. The man experiences the velocity of rain relative to himself to protect himself from the rain the man should hold umbrella in the direction of relative velocity of rain with respect to the man

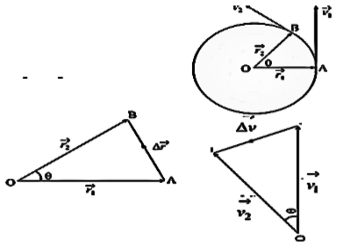
$$\overrightarrow{v_{RM}}=\overrightarrow{v_R}-\overrightarrow{v_M}=\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OD}$$

$\overrightarrow{OD}$ *makes angle* θ with the vertical, then $=\frac{DB}{OB}=\frac{\overrightarrow{v_M}}{\overrightarrow{v_R}}$

So, the man can protect himself from rain by holding umbrella at an angle θ with the vertical in the direct of his motion.

4. Centripetal Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

Magnitude of centripetal acceleration



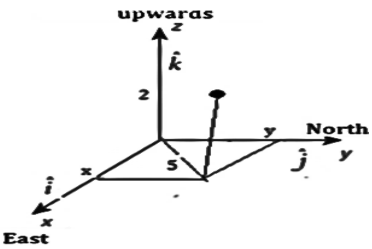
$$|\vec{a}|=\frac{|\Delta\vec{v}|}{\Delta t}=\frac{1}{\Delta t}\frac{v|\Delta\vec{r}|}{r}=\frac{v}{r}\frac{|\Delta\vec{r}|}{\Delta t}$$

$$|\vec{a}|=\frac{v^2}{r}$$

$$\left(\because v=\frac{|\Delta\vec{r}|}{\Delta t}\right)$$

5.(a) From figure x - component $=5\cos45°\hat{\imath}=5\sqrt{2}\,i$

y - component $=5\sin45°\hat{\jmath}=5\sqrt{2}\,\hat{\jmath}$ z - component $=2\,\hat{k}$



Then position vector $\vec{r}=\frac{5}{\sqrt{2}}(\hat{\imath}+\hat{\jmath})+2\hat{k}$

(b) ||ly ,vector $\vec{r}=\frac{4}{\sqrt{2}}(\hat{\imath}-\hat{\jmath})+3\,\hat{k}$

(c) ||ly,vector $\vec{r}=\sqrt{2}\,(\hat{\imath}+\hat{\jmath})+4\,\hat{k}$

6. Let F_1andF_2 be the two forces.

Then $F_1=3x$,$F_2=5x$,R=28N and θ=60°

R$=\sqrt{P^2+Q^2+2PQ\cos\cos\theta}$ $\Rightarrow 28=\sqrt{(3x)^2+(5x)^2+2(3x)(5x)cos60}$°$\Rightarrow$7x=28$\Rightarrow$x=4

$\therefore F_1$=12N, F_2=5×4=20N

7. Let F_1andF_2 be the two forces.

Then F_1=P+Q,F_2=P-Q, R$=\sqrt{3P^2+Q^2}$

$R^2=F_1{}^2+F_2{}^2+2F_1F_2\cos\theta$

$$\Rightarrow 3P^2+Q^2=\sqrt{(P+Q)^2+(P-Q)^2+2(P+Q)(P-Q)cos\,\theta}$$

$\Rightarrow P^2-Q^2=2(P^2-Q^2)cos\,\theta \quad \Rightarrow cos\,\theta \ =\frac{1}{2} \qquad \therefore\ \theta=60°$

8.$|\vec{A}|=\sqrt{1^2+2^2+(-1)^2}=\sqrt{6}$

$|\vec{B}| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$

$\vec{A}.\vec{B} = 1 \times (-1) + 2 \times 1 + (-1) \times (-2) = -1 + 2 + 2 = 3$

$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2} \quad \theta = 60$

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. (a) Define centripetal acceleration. Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r.

(b) A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone

2. The position vector of a particle is given by $\vec{r} = 3t\hat{i} - 2t^2\hat{j} + 4\hat{k}$ m where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle ?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s ?

3. Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an angle θ. Using parallelogram law of vector addition, find the magnitude and direction of their resultant. Calculate the magnitude and gives direction of R when

(a) $\vec{A}$ and $\vec{B}$ act along the same straight line in the same direction.

(b) $\vec{A}$ and $\vec{B}$ act along the same straight line but in opposite directions.

(c) $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

4. (a) Define resolution of a vector with an example ?

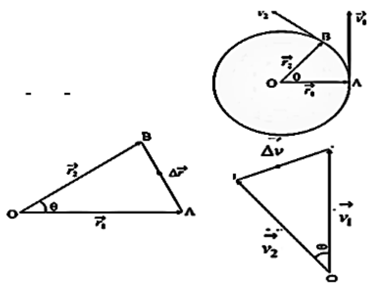
(b) A projectile can have the same range R for two angles of projection. If t_1 & t_2 be the time of flight in the two cases then prove that $t_1 t_2 = \left(\frac{2R}{g}\right)$

SOLUTIONS

1. (a)

Centripetal Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

Magnitude of centripetal acceleration



$$|\vec{a}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{1}{\Delta t}\frac{v|\Delta\vec{r}|}{r} = \frac{v}{r}\frac{|\Delta\vec{r}|}{\Delta t}$$

$$|\vec{a}| = \frac{v^2}{r}$$

$$\left(\therefore v = \frac{|\Delta\vec{r}|}{\Delta t}\right)$$

(b) Radius of the horizontal circle, r = 80 cm = 0.80 m, frequency = f = $\frac{14}{25}$ rps

Angular speed of revolution of the stone is $\omega = 2\pi f \Rightarrow \omega = 2 \times 22.7 \times \frac{14}{25} \Rightarrow \omega = \frac{88}{25}\ rad/s$

Magnitude of acceleration produced in the stone will be equal to the magnitude of centripetal acceleration.

$a_c = r\omega^2 = 0.80 \times (\frac{88}{25})^2 = 9.90\ ms^{-2}$

We know that, the direction of the acceleration is towards the centre of the circle along its radius.

2.(a) As $\vec{r} = 3t\hat{i} - 2t^2\hat{j} + 4\hat{k}$

Also, $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t\hat{i} - 2t^2\hat{j} + 4\hat{k}) = 3\hat{i} - 4t\hat{j}\ ms^{-1}$

Also, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3\hat{i} - 4t\hat{j}) = -4\hat{j}\ ms^{-2}$

(b) velocity of the particle $\vec{v} = 3\hat{i} - 4t\hat{j}\ ms^{-1}$

At t=2 s $\Rightarrow 3\hat{i} - 4 \times 2\hat{j} = 3\hat{i} - 8\hat{j}$

Magnitude $\vec{v} = \sqrt{3^2 + (-8)^2} = \sqrt{9+64} = \sqrt{73}$=8.544 ms^{-1}

Direction $\theta = \left(\frac{v_y}{v_x}\right) = \left(\frac{-8}{3}\right) = -70\ with\ x\ axis$

3. When two vectors A and B are inclined at an angle θ, the magnitude R of their resultant is given by $R = \sqrt{A^2 + B^2 + 2ABcos\theta}$ Direction is given by $\theta = \left(\frac{\vec{B}\ sin\theta}{\vec{A}+\vec{B}\ cos\theta}\right)$

(a) θ = 0° (Vectors in the same direction):

Magnitude: $R = \sqrt{A^2 + B^2 + 2AB}$ Direction: Same as A or B

(a) θ = 180° (Vectors in opposite directions):

Magnitude $R = \sqrt{A^2 + B^2 - 2AB}$ Direction: Along the larger vector

(b) θ = 90° (Perpendicular vectors):

Magnitude: $R = \sqrt{A^2 + B^2}$ Direction $\theta = \left(\frac{\vec{B}}{\vec{A}}\right)$

4. (a) Resolution of a Vector

Resolution of a vector is the process of decomposing a vector into two or more components along specified directions (e.g., horizontal and vertical). $A_x = Acos\ \theta$ & $A_y = Asin\ \theta$

Example: -Flying of bird, Motion of catapult etc.

(b) Projectile Range and Time of Flight Relationship

$\theta_1 = \theta$ & $\theta_2 = 90 - \theta$

$t_1 = \frac{2usin\theta}{g}$, & $t_2 = \frac{2usin\theta_2}{g} = \frac{2usin(90-\theta)}{g} = \frac{2ucos\theta}{g}$

$$t_1t_2 = \frac{2usin\theta}{g} \times \frac{2ucos\theta}{g} = \frac{4u^2sincos\theta}{g^2} = \frac{2}{g}\left(\frac{u^2sin2\theta}{g}\right) \Rightarrow t_1t_2 = \frac{2R}{g}$$

CASE STUDY BASED QUESTIONS

1. **Operation Sindoor Projectile Motion in Combat Scenario**:

During *Operation Sindoor*, the Indian Army's artillery unit must neutralize an enemy bunker located on a hilltop. The bunker is 200 meters horizontally away and 50 meters above the artillery position. A mortar is fired with an initial velocity of 80 m/s at an angle of 30° with the horizontal. Assume g=10 m/s^2. Ignore air resistance.

(i) What is the maximum height attained by the mortar shell?

(a) 80 m (b) 160 m (c) 40 m (d) 320 m

(ii) What would be the horizontal range of the mortar if launched on level ground(same velocity and angle) ?

(a) 3203 m (b) 554.2 m (c) 640 m (d) 160 m

(iii) What is the time of flight for the mortar shell if launched on level ground?

(a) 8 s (b) 16 s (c) 4 s (d) 10 s

(iv) To ensure the mortar shell reaches the elevated bunker (50 m height), which adjustment is most critical?

(a) Increase the launch angle (b) Decrease the launch angle

(c) Increase the launch velocity (d) Both B and C

2. A student is practicing the javelin throw. The javelin is thrown with an initial velocity of 32 m/s at an angle of 35° above the horizontal, from a height of 2 meters above the ground. Assume gravitational acceleration g=9.8 m/s^2 and air resistance is negligible. Take cos(35°)≈0.8192

(i) What is the horizontal component of the initial velocity?

(a) 26.2 m/s (b) 22.6 m/s (c) 18.7 m/s (d) 20.1 m/s

(ii) What is the maximum height (from the ground) reached by the javelin during its flight?

15.7 m (b) 17.2 m (c) 19.3 m (d) 21.6 m

(iii) At the maximum height of the javelin's trajectory, which of the following statements is TRUE?

(a). Both horizontal and vertical components of velocity are zero
(b). Only the vertical component of velocity is zero
(c). Only the horizontal component of velocity is zero
(d). The javelin comes to a complete stop momentarily

(iv) Which factor determines how long the javelin stays in the air (time of flight), assuming no air resistance?

(a) Only the horizontal component of velocity
(b) Only the angle of projection
(c) Only the vertical component of velocity and initial height
(d) Both horizontal and vertical components equally

SOLUTIONS

1(i) Option A H=$\frac{u^2\theta}{2g}$==$\frac{80^2 30}{2\times10}$ =80m.

(ii) Option (b) is correct. R=$\frac{u^2 sin2\theta}{g}$=$\frac{80^2(30)}{10}$=$\frac{6400\sqrt{3}}{10\times2}$=554.2m.

(iii) Option (a) is correct. T=.$\frac{2usin\theta}{g} = \frac{2\times80sin30}{10} = 8s$

(iv) Option (d) is correct

For an elevated target, reducing the angle or increasing the velocity compensates for the additional vertical displacement. A lower angle prioritizes horizontal distance, while higher velocity extends the range.

2 (i) a u_x=ucosθ=32cos(35∘)≈26.2 m

2(ii) c H=$\frac{u^2}{2g}$=$\frac{(18.4)^2}{2\times9.8}$ =≈17.3m

Total Height from Ground=17.3+2=19.3 m

(iii) b Because no external force acts in the horizontal direction

(iv) c Only the vertical component of velocity and initial height

CHAPTER 4: LAWS OF MOTION

SYLLABUS

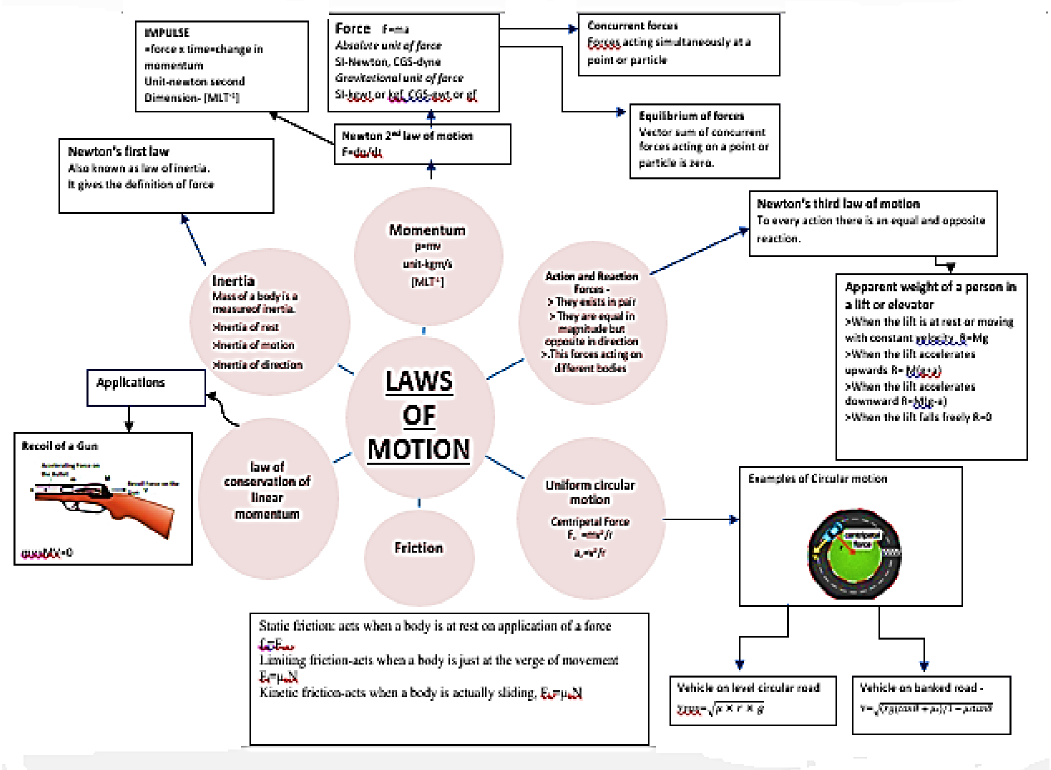
Intuitive concept of force, Inertia, Newton's first law of motion; momentum and Newton's second law of motion; impulse; Newton's third law of motion.

Law of conservation of linear momentum and its applications.

Equilibrium of concurrent forces, Static and kinetic friction, laws of friction, rolling friction, lubrication.

Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on a banked road).

MIND MAP



GIST OF THE CHAPTER: -

- **Inertia:** It is the property of a body by which it continues to be in state of rest or uniform motion along a straight path unless an external unbalanced force acts on the body. Inertia is of three types:
  (a) Inertia of rest (b) Inertia of motion (c) Inertia of direction
  Inertia to linear motion is measured by the mass of the body. Larger the mass; greater is the inertia of the body i.e. it is more difficult to change the state of rest or uniform motion of the body.
- **Linear Momentum**: It is the quantity of motion present in a body. Mathematically, it is measured as product of mass and velocity v of the body. Momentum p = m v
  It is a vector in the direction of velocity. Its SI unit is kg ms^{-1} or Ns. Dimensionally momentum is $[MLT^{-1}]$
- **Newton's First Law of Motion**
  Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.
- **Newton's Second Law of Motion**

The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.

Mathematically, $F = dp/dt = d/dt\,(mv) = m\,dv/dt$ (for a system with constant mass)

$F = ma$ Where, $a = dv/dt$ is Instantaneous acceleration of body.

- **Newton's Third Law of Motion:**

  It states: To very action, there is an equal and opposite reaction. Mathematically,

  $F_{BA} = (-)F_{AB}$, Where F_{BA} is force on B due to A and F_{AB} is force on A due to B.

  The forces of action and reaction do not cancel each other because they act on different bodies.

  Unit of force in SI system is newton. $1N = 1\ kg\ m/s^2$

  The gravitational unit of force in kg wt. or kg f. $1\ kg\ wt = 1\ kg\ f = 9.8\ kg\ m/s^2 = 9.8N$

- **Impulse of a Force**: A large force acting for a small-time interval is said to impart an impulse to the object. It is a vector quantity. Impulse $I = F\,\Delta t$

  The SI unit of impulse is N–s or kg ms^{-1}. Dimensionally impulse is $[MLT^{-1}]$

  Impulse = Force × Time= $p_2 - p_1 = m\,(v-u)$

  Impulse $I = F.\,t$; if force is constant, $I = \int F.dt$; if the force in variable

  Impulse of a force can also be measured as the area under the force time graph.

- **Apparent Weight of a Body in a Lift**

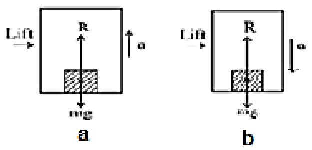
  In a lift at rest or in uniform motion (upwards or downwards) i.e. when a = 0; the weight of a body is its true weight. We have $R = mg$ (fig a.)

  When a lift moves upwards with uniform acceleration 'Q', apparent weight of a body in the lift increase, We have $R - mg = ma$, $R = m\,(g + a) > mg$

  When a lift moves downwards with a uniform acceleration 'a'; apparent weight of a body is less than its $R + mg = ma$, $R = m\,(g + a) < mg$

  In a freely falling lift a=g

  $R=m(g-g)=0$; The body experience weightlessness

  Note-When a person of mass m climbs up a rope suspended from a rigid support with acceleration 'a'; the tension in the rope in $T = m\,(g + a)$

  When the person climbs down the rope with acceleration 'a'; the tension in the rope is $T = m\,(g - a)$.

  If the rope supports the weight of the man (with man neither climbing up nor down); the tension in the rope equals his weight i.e. $T = m\,g$

- **Law of Conservation of Linear Momentum**

  According to this Law, the total linear momentum, of a given system, remains conserved if the net external force acting on this system is zero.

  If $F_{ext}=0$, We have, $p_1+p_2+ .....+p_n =$ a constant

  or $m\,v_1+m_2\,v_2 +.....+ m_nv_n =$ a constant

  For a system of two bodies, undergoing a collision, we have, $m_1\,u_1 + m_2\,u_2 = m_1\,v_1 + m_2\,v_2$

- **RECOILING OF A GUN**

  Let us apply this law to a bullet and the gun system. Here the system is at rest to start with.

  Hence, when a bullet is fired from a gun we have,

  Initial momentum = zero

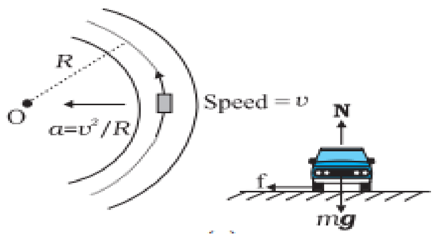
  So, Final momentum of the bullet & gun system must again be zero.

  $mv + MV = 0$ Hence, Recoil velocity of the gun. $V= -\,(m/M)\,v$

- **Concurrent Forces**

The forces acting at the same point of a body are called concurrent forces.
Equilibrium of Concurrent Forces: For a number of concurrent forces acting on a body in equilibrium, the forces can be represented by a closed polygon taken in order or the resultant force is zero. $F_1 + F_2 + F_3 + ... + F_n = 0$

- **Friction:** Whenever a body moves or tends to move over the surface of another body, a force comes into play to opposes their relative motion. This force is known as force of friction. It opposes motion and acts parallel to the surface of contact of bodies.
- **Static Friction:**
  The force of friction which comes into play between two bodies before one object actually begins to move over the other is called static friction (fs). Static friction is a self adjusting force (both in magnitude as well as direction). It is always equal and opposite to the applied force as long as there is no relative motion.
- **Limiting Friction:** The maximum force of static friction which comes into play when a body just starts moving over the surface of another body is called limiting friction or the maximum force of static friction fs max. The force of friction never exceeds fs max.
- **Kinetic Friction:**
  The force of friction which comes into play when a body is in motion over the surface of another body is called kinetic or dynamic friction. It is denoted by f_k and is less than limiting friction.
  Laws of Limiting Friction
  (1) The force of limiting friction depends on the nature of the two surfaces in contact and their state of roughness.
  (2) The force of limiting friction acts tangential to the surfaces in contact and in a direction opposite to that of the applied force.
  (3) The force of limiting friction between any two surfaces is independent of the shape and the area of contact so long as the normal reaction remains unchanged.
  (4) The force of limiting friction between two given surfaces is directly proportional to the normal reaction between the surfaces. f is directly proportional to N or $f = \mu_s N$; where μ_s is
- **Coefficient of limiting friction**.
  coefficient of limiting friction =Limiting Friction f_{smax}/Normal Reaction(N) and coefficient of kinetic friction $\mu_k = f_k / N$
  We have $\mu_k < \mu s$ because f_k is always less than fsmax. .
- **Body on Level Circular Road**:
  Photo source-NCERT
  The friction between the tyres and the road provides the centripetal force.
  $v = \sqrt{\mu r g}$
  which gives maximum safe speed of a vehicle on road for negotiating a curve of radius 'r'. (Without skidding).



- **Banking of Roads (Tracks):**
  Photo source-NCERT
  Curved roads and tracks are banked to reduce the role of friction for providing centripetal force on curves. For a track / road of radius 'r' banked at an angle θ; the maximum safe speed at which a vehicle can negotiate a curve is given by $v = \sqrt{gr \tan\theta}$
  The above max safe speed does not take into account the friction between the tyres and the road.

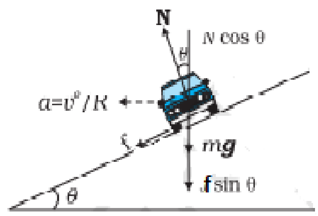
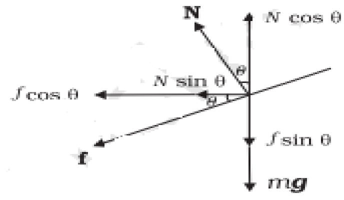


Photo source- NCERT

If the force of friction is also taken into account; we have,

V=√rg (μ+ tanθ)/(1-μtanθ)



- **Bending of a Cyclist:**

  In order to negotiate a circular turn of radius 'r' at a speed v; a cyclist must bend at an angle θ with the vertical such that tanθ =v^2 /rg

MULTIPLE CHOICE QUESTIONS-

1) A body of mass 10 kg is moving with a velocity of 5m/s. The momentum of the body is-

a)10 kgm/s. b)5 kgm/s. c)50 kgm/s. d) 25 kgm/s

2) When the velocity is tripled the banking angle should be

a) 1/3 b) 1/9 c) 3 times d) 9 times

3) A stone of mass 0.5kg tied to the end of the string in whirled round in the circle of radius 0.5m with a speed of 4rps in a horizontal plane. The tension in the string-

a) 2N b) 4N c) 5N d) 20N

4) A body of mass m is moving with an acceleration 'a' under the action of a force 'g/4'. The weight of the body is-

1) g/4a 2) g/4a 3) g^2/2a 4) g^2/4a

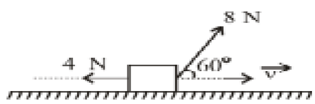
5) Force produces an acceleration of a_1 in a body and the same force produces an acceleration of a_2 which is twice to a_1 in another body. If the two bodies are combined and the same force is applied on the combination, the acceleration produced in it is-

a) 3/2 a_1 b) 2/3 a_1 c) 3/2 a_2 d) 2/3a_2

6) A rubber ball falls from a height h and rebounds to a height h/3. A rubber ball of double the mass falling from the same height h rebounds to a height-

1) h 2) h/3 3) 3h/4 4) 3h

7) Two forces of magnitudes 4 N and 8 N are acting on a box, when the box moves rightward across a frictionless horizontal support. The speed of the box at time t is 1m/ s the change in kinetic energy of the box is-



(a) Zero (b) 80 J (c) 8 J (d) 4

8).An object of mass 5 kg is attached to the hook of a spring balance and the balance is suspended vertically from the roof of a lift. The reading on the spring balance, when the lift is going up with an acceleration of 0.25 ms^{-2}(g= 10m/s^{-2})

(A). 51.25 N (B). 48.75 N (C). 52.75 N (D). 47.25N

SOLUTIONS:-

1)p=mv=10x5=50 kgm/s

2) tanθ= $v^{2/}$rg

When , v'=3v

Therefore, tanθ'=9v^2/rg=9tanθ

3) Here the tension provides necessary centripetal force

v=rω= 0.5x4=2m/s

T = mv^2/r =0.5x$(2)^2$/0.5= 0.5x4/0.5=2/0.5=4N

4)F=ma g/4=ma m=$\frac{g}{4a}$

Weight W = mg= $\frac{g}{4a}$xg= g^2/4a

5) Force applied on one body F=m_1 a_1, same Force applied on second body F=m_2 a_2=2ma_1

Same force applied on combined body=F=(m_1+ m_2)a= (F/a_1+F/a_2)a ⇒a= 2a_1/3

6) Solution-The change in momentum does not depends on the rebound height. Therefore, the rubber ball of double the mass will also rebound to a height "h/2."

7)0

8)m(g+a)= 5(10+0.25)=5x10.25=51.25

ASSERTION AND REASONING-

Directions-Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct and reason both are correct and reason is a correct explanation for assertion.
(b) Assertion and reason both are correct and reason is not a correct explanation for assertion
(c) Assertion is correct but reason is incorrect.
(d) Both Assertion and reason is incorrect.

1) Assertion-The book is at rest on a study table means no net force is acting on the book
Reason-The weight of the book and reaction force are acting in the same direction.
2)Assertion- Due to friction between tyres and road there is wear and tear of tyres
Reason- Friction is a necessary evil
3) Assertion-A bullet fired by a gun with high speed causes more harm than the same bullet fired with moderate speed.
Reason-Change in momentum depends on mass and velocity.
4) Assertion-We know novice player tries to catch the cricket ball almost instantly and this hurts him.
Reason-Force not only depends on the change in momentum but also on how fast the change in momentum is brought about.
5)Assertion-When a fixed force is applied for a certain interval of time on two bodies of different masses initially at rest the lighter body picks up a greater speed than the heavier body
Reason-Same force for the same time causes same change in momentum for different bodies

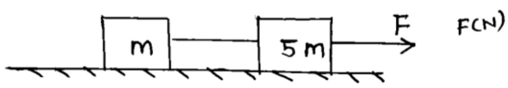
6)Assertion- Driving at optimum speed on a banked road will cause little wear and tear of the tyres.
Reason -Frictional force provides the necessary centripetal force.
7) Assertion-In a system of three forces in equilibrium, the resultant of any two forces obtained by the parallelogram law of forces must be equal and opposite to the third force.
Reason-These forces are non-concurrent forces.
8)Assertion- Frictional force is non conservative force
Reason- frictional force depends on the path not on initial and final position of the object.

ANSWERS

1)(c) because W and R are action and reaction forces and action and reaction acts on different bodies so they never cancels out.
2)(b) Both are contradict to each other..
3)(a)
4)(a) Force is directly to the rate of change of momentum.
5)(a) Same force for the same time causes same change in momentum for different bodies is correct so the lighter body will gain higher velocity and heavier body will lose velocity.
6)(b) Driving at optimum speed on a banked road, the required centripetal force for circular motion is provided by the horizontal component of the normal reaction force.
7)(c) Forces acting at one point are concurrent.
8) (a) Frictional force is a non conservative forces and non conservative forces depends on the path.

VERY SHORT ANSWER TYPE QUESTIONS-

1) A monkey seems to climbs up and another monkey seems to move down at same rope hanging over a pulley with same uniforms acceleration. If the respective masses of monkeys are in the ratio 1:3. Find the common acceleration?
2) Is it possible for a body to have constant speed but variable velocity? Justify your answer with two examples.
3) A particle moves in a circle of radius 20 cm at 120 revolutions per minute. Calculate the linear acceleration of the particle.
4)The tension in the string shown in fig



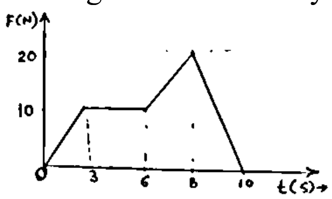
5) A scooter is moving with speed of 10m/s on a circular track of radius 25m.Its speed is increasing at the rate of 3m s^{-2}. Calculate its resultant acceleration.
6)What is a centripetal force. How it is different from centrifugal force. Identify source of the centripetal force acting on respective objects in the following cases?
(i) Electron revolving around the nucleus.
(ii) Earth revolves around the sun.

ANSWERS-

1) Hint: From formula of acceleration we can calculate m_1=m, m_2= 3m , $a=\frac{1}{2}g$
2) Yes when body moves in a circle with constant speed, its velocity is variable. This is because velocity is a vector quantity whose magnitude is speed but its direction changes continuously due to the change of the direction of motion,2 examples
3) $a = \frac{v}{r} = \frac{\omega r}{r}$
$\omega^2 r = (2\pi v)^2$, r=20cm , v = 120/60 = 2rpsa = $(2\pi \times 2)^2 \times 20 = 60\,\pi^2 \times 20 = 320\,\pi^2$ cm s^{-1}
4) Draw free body diagram and get
F-T= 5ma-----------(1) F=ma-----------(2)
From 1 and 2, we get T=F/6
5)v=10ms^{-1},r=25m
Therefore, $a_c=v^2/r= 10^2/25 = 100/25 =4$ Net acceleration $a=\sqrt{16+9}$ =5ms^{-2}
6) Centripetal force-It is the force that keeps an object moving in a circular path, always directed towards the centre of circle.
Difference between centripetal force and centrifugal force
Centripetal force is the force that keeps an object moving in a circular path, always directed towards the centre of circle. While centrifugal force is a pseudo force, an apparent outward force experienced by an object moving in a circular path when viewed from a non-inertial frame.
Centripetal force acts towards the centre of the circle, while centrifugal force acts outwards away from the centre.
(i)Electrostatic force of nucleus on electron. ii) Gravitational force of sun

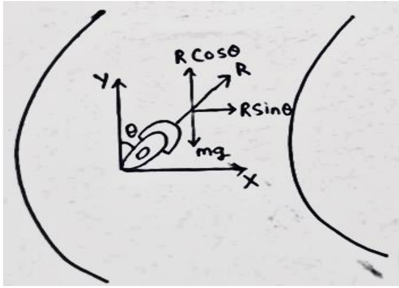
SHORT ANSWER TYPE QUESTIONS

Q1) Why does a cyclist lean inward when moving along a curved path. Determine the angle through which the cyclist bends from the vertical to negotiate a curve.
Q2. What is recoil velocity? Find the expression for the recoil velocity of a gun when a bullet is fired from it.
Q3. An athlete of mass 50 kg climbs on a rope which can withstand a maximum tension of 400 N. The rope will break if the athlete
i) Climbs up with an acceleration of 6 ms $^{-2}$
ii) slips down with acceleration of 5 ms $^{-2}$
iii) climbs up with a uniform speed of 2ms^{-1}. (Given g=10ms^{-2})
Q4 Using an example justify that the friction is a necessary evil give three advantages and disadvantages of friction mostly observed in our daily routine.
Q5.A bomb is thrown in a horizontal direction with a velocity of 50 ms^{-1} .It explodes into two part of masses 5 kg and 03 kg. The heavier fragment continuous to move in the horizontal direction with a velocity twice the initial velocity of lighter particle. If the final velocity of lighter fragment is 60 m/s then calculate the final velocity of the heavier fragment.
Q6 A hunter with a machine gun can fire 20g bullets with a velocity of 800ms^{-1}. A 40 kg tiger springs at him with a velocity of 10ms^{-1}. How many bullets must he fire onto the tiger in order to stop it in his track?
Q7.The force time graph of a particle is mass 2kg which is initially at rest is shown below:



ANSWERS-

1) To produce necessary centripetal force required for turning.
Resolved R into two components-
RCosθ balances the weight= RCosθ=mg--------(1)
RSinθ is directed toward the centre and provides necessary centripetal force
RSinθ= mv^2/r-------------(2)
Divide 2 by 1 we get



$\frac{RSin\theta}{RCos\theta} = \frac{mv2}{r} x \frac{1}{mg}$ ⇒ tanθ= $\frac{v^2}{rg}$ ⇒θ=$tan^{-1}\frac{v^2}{rg}$

2)The velocity with which a gun moves backward after firing a bullet is known as recoil velocity
expression for recoil velocity
Linear momentum of system(gun + bullet) before firing=0
Linear momentum of system(gun + bullet) after firing= mv+ MV
According to the conservation of linear momentum
mv+ MV=0 ⇒ MV= -mv ⇒ V= -mv/M
Negative sign shows that the velocity of gun is in the opposite direction to that of the velocity of the bullet.

3) let T be the tension in the rope.In this case equation of motion is given by
ma=T-mg ⇒ T=mg+ma=m(g+a)
=50(10+6)=50X16=800N The rope will break
ii) In this case mg –T =ma
T=mg-ma=m(g-a) =50(10-5)=50X5=250N The rope will not break
iii) In this case , the boy climbing of with uniforms speed so there is no acceleration and hence.
T=mg=50X10=500N (therefore,a=0) Thus, the rope will break.

4) Advantages-a)Friction enables us to walk
b)Friction between the pen and paper allows us to write.
c)It is essential in braking in vehicles.
Disadvantages-
a)Friction produces energy loss due to heating in various parts of a machinery.
b) Continuous friction can cause wear and tear of tyres and machinery components.
c) Forest fires are caused due to friction between branches of tress.

5) Solution :
m_1=5 kg, m_2= 3 kg v_1=2 u_2
Initial momentum of bomb = mu=8X50=400 kg ms^{-1}.
Final momentum of the fragments=m_1v_1+ m_2v_2
= m_1u_2+ m_2v_2
=5x 2u_2+ 3x60=10u_2+ 180
According to the law of conservation of linear momentum
Initial momentum=final momentum
400=10u_2+ 180 10u_2=400- 180
u_2=220/10=22 m/s
v_1= 2 u_2= 2x22= 44m/s

6 Here , mass of bullet m = 20 g = 0.02 kg ,Velocity of a bullet , v = 800ms^{-1}
Linear momentum of a bullet = mv =0.02×800=16 kg ms^{-1},Mass of a tiger M = 40 kg,
Velocity of tiger V = 10ms^{-1}
Therefore , linear momentum of tiger = MV =40×10=400kg ms^{-1}
Let n be the number of bullets fired from the machine gun. Thus to stop the tiger in its track .
Momentum of n bullets = momentum of tiger
n×40=400 n=400/40=10

7) Impulse = area under F – t graph =1/2×2×10+10x2+$\frac{1}{2}$(10+20)×2+1/2×4×20=100Ns
Velocity = 100/5=50m/s

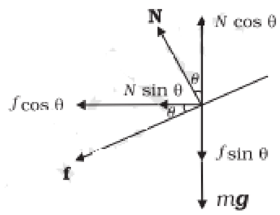
LONG ANSWER TYPE QUESTIONS-

1) What is meant by banking of roads? Explain the need for it. Obtain an expression for the maximum speed with which a vehicle can safely negotiate a curved road banked at angle θ. The coefficient of friction between road and wheels is μ.

2) A body of mass 15g is pushed up on a rough inclined plane making an angle of 60°with a horizontal. The length of the inclined plane is 10m. The body goes up the inclined plane and comes to rest at the top of the inclined plane. From rest position, it again moves down. It is found that the time of ascent is half of the time of descent. Calculate-

a) The coefficient of friction between the body and the plane.
b) The acceleration of the body when it moves up the plane.
c) Time of descent.
d) The acceleration of the body when it moves down the plane
e) The force of friction between the body and the plane (take g=10 m/s^2)

ANSWERS-



1)It means raising the level of the road as we move from the inside to the outside of the curved road

A vehicle cannot remain on flat circular road of its speed exceeds the speed given by v =$\sqrt{\mu g r}$.In such cases the force of friction is insufficient to provide the necessary centripetal force. To overcome this problem the road is banked.

Resolved R into two components-RCosθ and RSinθ

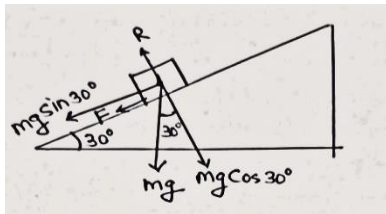
Similarly FCosθ and FSinθ be the horizontal and vertical components of force of friction

For equilibrium of the car, RCosθ=mg+ FSinθ, RCosθ – Fsinθ=mg-----------(1)

(RSinθ – FCosθ) acts towards the centre of the circular banked road and provides necessary centripetal force to the car, RSinθ – FCosθ= $\frac{mv2}{r}$---------(2)

Divide 2 by 1, we get $\frac{\frac{mv2}{r}}{mg}=\frac{RSin\theta - FCos\theta}{RCos\theta - Fsin\theta}=\frac{Sin\theta - F/RCos\theta}{Cos\theta - F/Rsin\theta}=\frac{tan\theta+\mu}{1-\mu tan\theta}$, V=($\frac{(tan\theta+\mu)}{(1-\mu tan\theta)}$rg)$^{1/22}$

i)



When the body moves up, net force acting on body is given by-f=mgsin 30°+F= mgsin 60°+μ mgcos 30°

a= f/m= mgsin 30°+μ mgcos 30°/m= gsin 30°+μ gcos 30°=9.8x1/2+ μ x9.8x$\sqrt{3}$/2=4.9 (1 + μ$\sqrt{3}$)

Time of ascent-,t=$\sqrt{\frac{2S}{a}}$= $\sqrt{2S/4.9(1+\mu\sqrt{3})}$, Upward acceleration,a= 4.9(1 + $\sqrt{3}$μ)

Time of descent t'= $\sqrt{\frac{2S}{a}}$= $\sqrt{2S/4.9(1-\sqrt{3}-\mu)}$, Downward acceleration,a= 4.9(1 − $\sqrt{3}$μ)

According to the question-

t=1/2 t', $\sqrt{2S/4.9(1+\sqrt{3}\mu}$= 1/2x $\sqrt{2S/4.9(1-\sqrt{3}\mu}$, μ= $\sqrt{3}$ /5

Upward acceleration =0.8x9.8 m/s^2 Downward acceleration=0.2x 9.8 m/s^2

Time of ascent=5/7 seconds Force of friction=F= μmg=509.208x 10^{-4}N

CASE BASED STUDY QUESTIONS

Conservation of linear momentum

1.The second and 3rd laws of motion lead to an important consequence the law of conservation of momentum. The law states that In an isolated system mutual forces between pairs of particles in the system can go cost momentum change in individual particles but since the mutual forces for each pair are equal and opposite the momentum changes cancel in pairs and a total momentum remains unchanged.

Take a familiar example. A bullet is fired from the gun. If the force on the bullet by the gun is F, the force on the gun by the bullet is –F, according to the third law. The two forces act for a common interval of time

Δt. According to second law and $F\Delta t$ is the change in momentum of the bullet and $-F\Delta t$ is the change in momentum of the gun. Since initially both are at rest, the change in momentum equals the final Momentum for each. Thus if P_b is the momentum of the bullet after firing and P_g is the recoil of the gun. $P_g = -P_b$ that is a total momentum of the (bullet + gun) system is conserved

i) A bullet of mass 20 g is fired from a gun of mass 1 kg with recoil velocity of gun 5 m/s. The muzzle velocity will be-

(a) 5 m/s (b) 50 m/s (c) 100 m/s (d) 250 m/s

ii) Two masses of M and 5M are moving with equal kinetic energy. The ratio of their linear momenta is-

(a) 1:25 (b) 1:5 (c) 25:1 (d) 5:1

iii) A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the nuzzle speed of the shell is 80 m/s. What is the recoil speed of the gun?

iv) A 1000 kg lorry is moving with a speed of 10m/s strikes a car of mass parked at roadside. After collision, the two move together at 8m/s. What is the mass of the car?

2.Weight in a lift or elevator

The person of 70 kg is standing on a platform scale balance kept on the floor of an elevator Cab is capable of motion in up or down direction with either a uniform velocity or a constant acceleration. The value of velocity as well as acceleration may be adjusted at any suitable value. Newton's 2^{ND} law of motion can be applied for the motion of elevator .$g = 10\ m/s^2$

(i) What would be the readings on the scale, if elevator is moving upwards with a uniform speed of 10m/s?

(ii) What would be the readings on the scale, if elevator is moving downwards with a uniform acceleration of $5 m/s^2$?

(iii) What would be the readings on the scale, if elevator is moving upwards with a uniform acceleration of $5 m/s^2$?

(iv). If the elevator cab suddenly goes out of order and starts falling freely under gravity then observed weight of the person will be?

ANSWERS-

1) Solution- According to the conservation of linear momentum

Initial momentum before firing= final momentum after firing

$0= MV_g+mv_b$, $MV_g= -mv_b$, $v_b = MV_g/m= 1x5x1000/20=250$ m/s

2)$P_1= \sqrt{2ME}$, $P_2=\sqrt{2x5ME}= \sqrt{10ME}$

$\frac{P1}{P2} = \frac{\sqrt{2ME}}{\sqrt{10ME}} = \frac{1}{5}$

3)According to the conservation of linear momentum

Initial momentum before firing= final momentum after firing

$0= Mv_g+mv_b$,$Mv_g= -mv_b$, $v_g= mv_b /M= 0.020x80/100=1.6/100=0.016$ m/s

4)let the mass of lorry and car be M and m respectively.

According to the conservation of linear momentum

Initial momentum before collision= final momentum after collision

$Mu_l+mu_c=(M+m)V=1000x10+mx0=(1000+m)x8,10000= 8000+ 8m$

$8m=10000-8000$, $m=2000/8=250$ kg

2. Answers-(a) Here, m = 70 kg, $g = 10\ m/s^2$

The weighing machine in each case measures the reaction R i.e., the apparent weight.

(a) When the lift moves upwards with a uniform speed, its acceleration is zero.

R = mg = 70 x 10 = 700 N

(b) When the lift moves downwards with $a = 5\ ms^{-2}$

R = m (g – a) = 70 (10 – 5) = 350 N

c) When the lift moves upwards with $a = 5\ ms^{-2}$

R = m (g + a) = 70 (10 + 5) = 1050 N

(d) If the lift were to come down freely under gravity, downward acc. a = g

:. R = m(g -a) = m(g-g) = Zero

CHAPTER– 5: WORK, ENERGY AND POWER

CONTENT OF THE CHAPTER:-Work done by a constant force and a variable force; kinetic energy, work- energy theorem, power. Notion of potential energy, potential energy of a spring, conservative forces: non-conservative forces, motion in a vertical circle; elastic and inelastic collisions in one and two dimensions.

Work

Work is said to be done whenever a force acts on a body and the body moves through some distance

Sw $= \vec{F} \cdot \vec{S}$ = FS Cos θ (Where θ is angle between force applied $\vec{F}$ and displacement vector · $\vec{S}$

Nature Of Work Done

If θ = 0°, W= FS i.e Work done is Maximum

If θ = 90°, W= 0 i.e Work done is Zero

Work Done by a Variable Force

The work done by a variable force in changing the displacement from S1 to S2 is W = $\int_{S_2}^{S_1} \vec{F}.d\vec{S}$ = Area under the force displacement graph

Energy

It is defined as ability of a body to do work. It is measured by the amount of work that a body can do. The unit of energy used at the atomic level is electron volt (eV) and SI unit is J.

Kinetic Energy

It is energy possessed by a body by virtue of its motion. The K.E. of a body of mass m moving with speed v is K = ½ MV² = $\frac{p^2}{2m}$

Potential Energy

It is energy possessed by a body by virtue of its position (in a field) or configuration (shape or size). For conservative fprce in one dimension, the potential energy function U(x) may be defined as $F(x) = \frac{dU(x)}{dx}$

OR $\Delta U = U_f - U_i = \int_{x_i}^{x_f} F(x)dx$

Power

The rate of doing work is power.

Average Power: it is defined as the ratio of small amount of work done W to the time taken t to perform the work. P = $\frac{W}{t}$

The SI Unit of power is Watt (W)

Work Energy Theorem

The work done by the net force acting on a body is equal to change in kinetic energy of body.

W= Change in Kinetic Energy = ½ MV² – ½ mu² => W= Δ K.E.

The work energy theorem may be regarded as the scalar form of Newton's second law of motion.

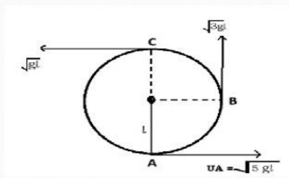
Potential Energy of a Spring

According to Hooke's law, when a spring is stretched through a distance x, the restoring force F is such that F ∝ x (Where k is spring constant or F= -Kx and its unit is Nm⁻¹)

The work done is stored as potential energy U of the spring.

$W = \int_0^x kxdx = \frac{1}{2}kx^2$ => U = $\frac{1}{2}kx^2$

Vertical Circular Motion



Minimum Velocity is required at each point to complete circle

Head-on Collision or One-Dimensional Collision

It is the collision in which the colliding bodies move along the same straight-line path before and after the collision

Velocity of Approach = Velocity of Separation or $U_1 - u_2 = v_2 - v_1$

$$v'_2 = \frac{2m_1}{m_1 + m_2}v_1 - \frac{m_1 - m_2}{m_1 + m_2}v_2$$

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2$$

Collision

A collision between two bodies is said to occur if either they physically collide against each other or the path of motion of one body is influenced by the other.

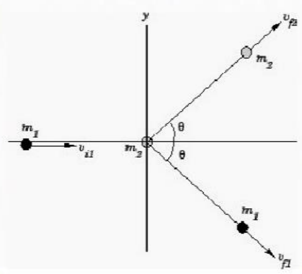
Types of Collisions

Elastic Collision: Both the momentum and kinetic energy of the system remains conserved.

Inelastic Collision: only the momentum of system is conserved but the kinetic energy is not conserved.

Oblique Collision

If the two bodies do not move along with the same straight-line path before and after collision, the collision is said to be oblique collision.



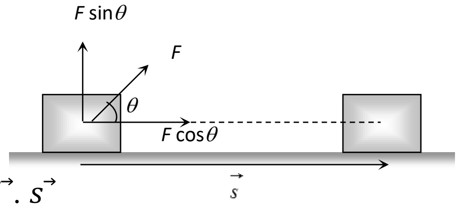
GIST OF THE CHAPTER:-

Introduction:

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

Work Done by a Constant Force:

Let a constant force F be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s. Then



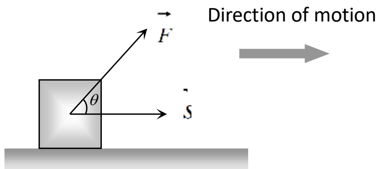
work done by the force in displacing the body

through a distance s is given by:

W = (F cos θ) s = Fs cos θ ⇒W = (F cos θ) s = Fs cos θ $= \vec{F}.\vec{s}$

<u>Nature of Work Done</u>:

Positive work:

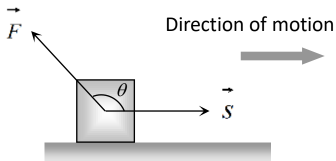


Positive work means that force (or its component)

is parallel to displacement.

Angle between force and displacement (ɵ): (0º ≤ θ < 90º).

Negative Work:



Negative work means that force (or its component)

is opposite to displacement.

Angle between force and displacement (ɵ): (90º < θ ≤ 180º).

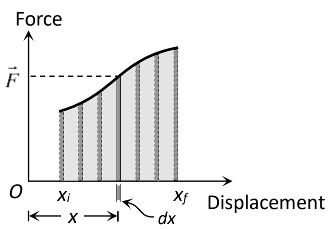
Zero Work:

Zero work means that force is perpendicular to displacement. Angle between force and displacement ɵ= 90º.

Work Done by a Variable Force:

When the magnitude and direction of a force varies with position, the work done by such a force for an infinite small displacement is given by: $dW = \vec{F}.\vec{ds}$

The total work done in going from initial position A to B



$$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F\,dx$$

$$\therefore W = \int_{x_i}^{x_f} (\text{Area of strip of width } dx)$$

$$\therefore W = \text{Area under curve between } x_i \text{ and } x_f$$

Relation between Different Units:

1 joule = 10^7 erg, 1 eV = 1.6×10^{-19} joule

1 kWh = 3.6×10^6 joule, 1 calorie = 4.18 joule

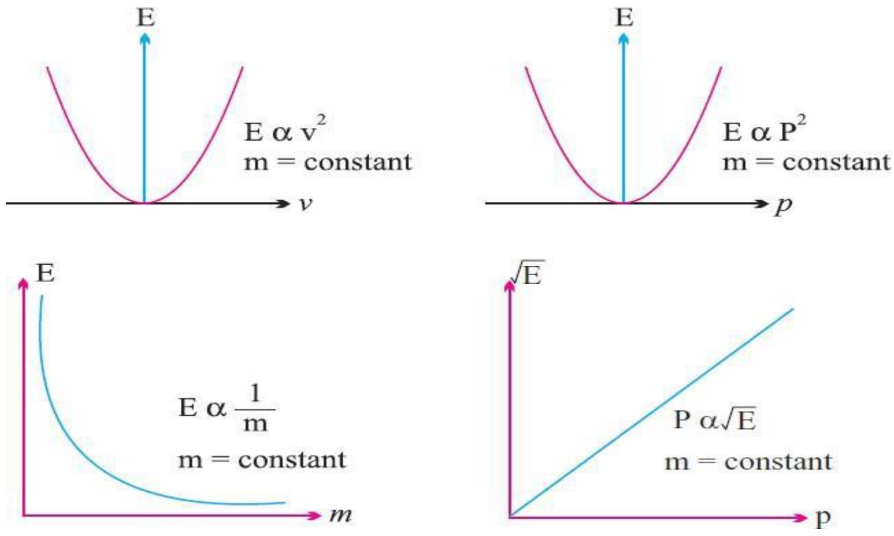
Kinetic Energy: The energy possessed by a body by virtue of its motion is called kinetic energy. Let m = mass of the body, v = velocity of the body then K.E. = $\frac{1}{2}mv^2$

Work-EnergyTheorem: It states that the work done by force acting on a body is equal to the change produced in the kinetic energy of the body. $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Relation of Kinetic Energy (E) with Linear Momentum (p):

$\boldsymbol{E} = \frac{p^2}{2m}$, $\boldsymbol{p} = \sqrt{2mE}$

Various graphs of Kinetic Energy:



Potential Energy: Potential energy is defined only for conservative forces.

In the space occupied by conservative forces, every point is associated with certain energy which is called the energy of position or potential energy.

Electrical Potential Energy

It is the energy associated with state of separation between charged particles that interact via electric force. For two point charge q_1 and q_2, separated by distance r.

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

While for a point charge q at a point in an electric field where the potential is V U = qV

As charge can be positive or negative, electric potential energy can be positive or negative.

Gravitational Potential Energy

It is the usual form of potential energy and this is the energy associated with the state of separation between two bodies that interact via gravitational force.

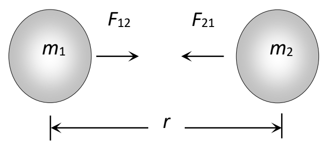


Fig. 6.23

For two particles of masses m1 and m2 separated by a distance r

Gravitational potential energy $U = -\frac{G m_1 m_2}{r}$

1) If a body of mass m at height h relative to surface of earth then

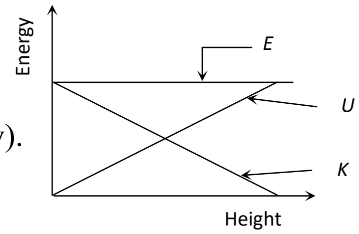
Gravitational potential energy $U = \frac{mgh}{1 + \frac{h}{R}}$ (Where R = radius of earth, g = acc. due to gravity at the surface of earth.)

(2) If h << R then above formula reduces to U = mgh.

(3) If V is the gravitational potential at a point, the potential energy of a particle of mass m at that point will be

U = mV

(4) Energy height graph: When a body projected vertically upward from the ground level with some initial velocity then it possess kinetic energy but its initial potential energy is zero. As the body moves upward its potential energy increases due to increase in height but kinetic energy decreases (due to decrease in velocity). At maximum height its kinetic energy becomes zero and potential Energy maximum but throughout the complete motion, total energy remains constant the figure.



Law of Conservation of Energy: For an isolated system or body in presence of conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depend upon time. This is known as the law of conservation of mechanical energy.

Law of Conservation of Total Energy:

If the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical etc., which is conserved.

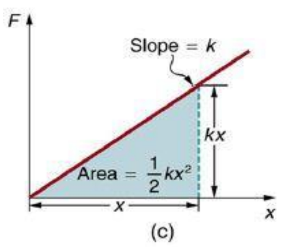
Elastic Potential Energy:

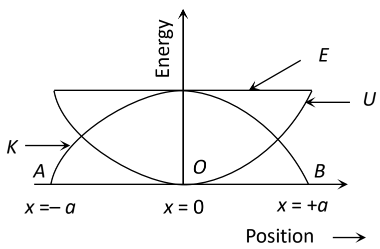
Restoring force and spring constant: When a spring is stretched or compressed from its normal position (x = 0) by a small distance x, a

restoring force is produced in the spring to bring it to the normal position.

According to Hooke's law, this restoring force is proportional to the displacement x and its direction is always opposite to the displacement. F α x, F=Kx

Elastic potential Energy: E= ½Kx^2





If spring is stretched from initial position xi to final position x_f then work done is equal to increase

in elastic potential energy. $\boldsymbol{W} = \Delta \boldsymbol{U} = = \frac{1}{2}k(x_2^2 - x_1^2)$

Power: Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{\text{av.}}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{\text{inst.}}) = \frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$P_{\text{inst}} = \vec{F}.\vec{v}$ [As $\vec{v} = \frac{d\vec{s}}{dt}$]

I.e. power is equal to the scalar product of force with velocity.

Dimensions of Power: [P] = $[ML^2T^{-3}]$.

SI unit of Power: watt (W) or joule/second (J/s).

Other units of Power are kilowatt (kW), Megawatt (MW), horse power (hp).

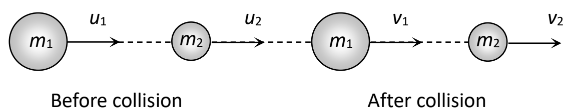
1 kW = 10^3 W, 1 MW = 10^6 W, 1 hp = 746 W, 1 watt = 1 joule/second = 10^7 erg/s

Collision: Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch.

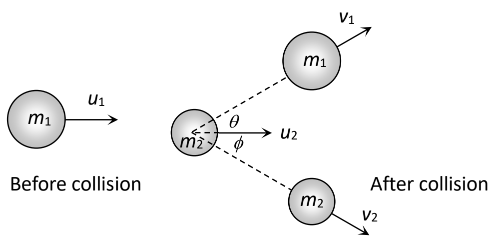
Types of Collision:

Perfectly Elastic Collision:



If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic collision.

The coefficient of restitution e = 1



Perfectly Elastic Oblique Collision

By law of conservation of momentum

Along x-axis, $m_1u_1 + m_2u_2 = m_1v_1\cos\theta + m_2v_2\cos\phi$ ...(i)

Along y-axis, $0 = m_1v_1\sin\theta - m_2v_2\sin\phi$ ...(ii)

By law of conservation of kinetic energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Special condition : If $m_1 = m_2$ and $u_2 = 0$ substituting these values in equation (i), (ii) and (iii) we get

$u_1 = v_1\cos\theta + v_2\cos\phi$ ....(iv)

$0 = v_1\sin\theta - v_2\sin\phi$ ...(v)

and $u_1^2 = v_1^2 + v_2^2$ …(vi)

Squaring (iv) and (v) and adding we get

$u_1^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta+\phi)$ …(vii)

Using (vi) and (vii) we get $\cos(\theta+\phi) = 0$

$\therefore \theta + \phi = \pi/2$

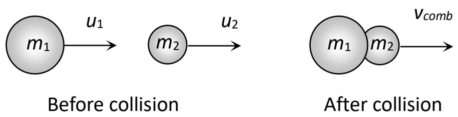
Inelastic collision:

If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to be inelastic.

The coefficient of restitution $0 < e < 1$

Perfectly inelastic Collision:

If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic.



The coefficient of restitution e = 0

*Linear momentum is conserved both in elastic and inelastic collision. Perfectly Elastic Head on Collision:

Let two bodies of masses m_1 and m_2 moving initial velocities u_1 and u_2 in the same direction they collide such that after collision their final velocities are v_1 and v_2 respectively.

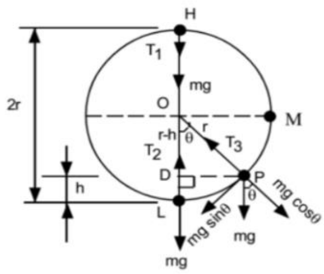
According to law of conservation of momentum and kinetic energy:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Velocity of approach = Velocity of Separation, $u_1 - u_2 = v_2 - v_1$

Motion in Vertical Circle: This is an example of non- uniform circular motion. In this motion, body is under the influence of gravity of earth.



Velocity at any point on vertical loop: If u is the initial velocity imparted to body at lowest point then, velocity of the body at height h is given by:

$$v = \sqrt{u2 - 2gh} = \sqrt{u2 - 2gl\,(1 - \cos\theta)}$$

Tension at any point P on the Vertical Loop:

$$T = mgcos\theta + \frac{mv^2}{R}$$

Tension at lowest point (L) $\quad T_{max} = mg + \frac{mv^2}{R}$

Tension at highest point (H) $\quad T_{min} = \frac{mv^2}{R} - mg$

MULTIPLE CHOICE QUESTIONS

1. A force $F = (5\hat{i} + 3\hat{j})$ newton is applied over a particle which displaces it from its origin to the point $\boldsymbol{r} = (2\hat{\boldsymbol{i}} - 1\hat{\boldsymbol{j}})$ metres. The work done on the particle is

(a)– 7J (b) 13J (c)7J (d)11 J

2. A force acts on a 30 gm particle in such a way that the position of the particle as a function of time is given by $x = 3t - 4t^2 + t^3$, where x is in metres and t is in seconds. The work done during the first 4 seconds is

(a) 5.28 J (b) 450 mJ (c) 490 mJ (d) 530 mJ

3. A body of mass 6kg is under a force which causes displacement in it given by $S = \frac{t^2}{4}$ metres where t is time. The work done by the force in 2 seconds is

(a) 12 J (b) 9 J (c) 6 J (d) 3 J

4. A particle is acted upon by a force of constant magnitude, which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that

(a) Its velocity is constant (b) Its acceleration is constant

(c) Its kinetic energy is constant (d) It moves in a straight line

5. Two springs have their force constant as k_1 and $k_2 (k_1 > k_2)$. When they are stretched by the same force

(a) No work is done in case of both the springs

(b) Equal work is done in case of both the springs

(c) More work is done in case of second spring

(d) More work is done in case of first spring

6. The potential energy between two atoms in a molecule is given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$; where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium when

(a) $x = \sqrt[6]{\frac{11a}{5b}}$ (b) $x = \sqrt[6]{\frac{a}{2b}}$ (c) $x = 0$ (d) $x = \sqrt[6]{\frac{2a}{b}}$

7. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by

(a) 100 J (b) 200 J (c) 300 J (d) 400 J

8. A sphere of mass m, moving with velocity V, enters a hanging bag of sand and stops. If the mass of the bag is M and it is raised by height h, then the velocity of the sphere was

(a) $\frac{M+m}{m}\sqrt{2gh}$ (b) $\frac{M}{m}\sqrt{2gh}$ (c) $\frac{m}{M+m}\sqrt{2gh}$ (d) $\frac{m}{M}\sqrt{2gh}$

SOLUTIONS- MULTIPLE CHOICE QUESTIONS

1. C $W = \vec{F}.\vec{s} = (5\hat{i} + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = 7$ J

2. (a) $v = \frac{dx}{dt} = 3 - 8t + 3t^2$ $\therefore v_0 = 3m/s$ And $v_4 = 19m/s$

$W = \frac{1}{2}m(v_4^2 - v_0^2)$ (According to WE theorem) $= \frac{1}{2} \times 0.03 \times (19^2 - 3^2) = 5.28J$

3. (d) $s = \frac{t^2}{4} \therefore ds = \frac{t}{2}dt$ $F = ma = \frac{md^2s}{dt^2} = \frac{6d^2}{dt^2}\left[\frac{t^2}{4}\right] = 3N$

Now $W = \int_0^2 F\,ds = 3J$

4. (c) When a force of constant magnitude which is perpendicular to the velocity of particle acts on a particle, work done is zero and hence change in kinetic energy is zero.

5. (c) $W = \frac{F^2}{2k}$

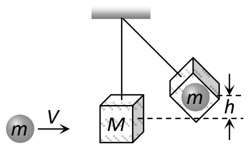
If both springs are stretched by same force then $W \propto \frac{1}{k}$

As $k_1 > k_2$ therefore $W_1 < W_2$ i.e. more work is done in case of second spring.

6.(d) Condition for stable equilibrium $F = -\frac{dU}{dx} = 0$, $x = \sqrt[6]{\frac{2a}{b}}$

7. (c) $100 = \frac{1}{2}kx^2$ $W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}k[(2x)^2 - x^2]$ $= 3 \times \left(\frac{1}{2}kx^2\right) = 3 \times 100 = 300J$

8. (a)



By the conservation of linear momentum

Initial momentum of sphere = Final momentum of system

$mV = (m+M)v_{sys.}$ … (i)

If the system rises up to height h then by the conservation of energy

$\frac{1}{2}(m+M)v_{sys.}^2 = (m+M)gh$ … (ii) $\Rightarrow v_{sys.} = \sqrt{2gh}$

Substituting this value in equation (i) $V = \left(\frac{m+M}{m}\right)\sqrt{2gh}$

ASSERTION AND REASON TYPE QUESTIONS

In the following questions a statement of assertion (A) is followed by a statement of reason(R) mark the correct choice as:

(a) If both assertion and reason are true and reason is the correct explanation of the assertion

(b) If both assertion and reason are true but reason is the not correct explanation of assertion

(c) If assertion is true but reason is false

(d) If both assertion and reason are false

1. **Assertion**: A person working on a horizontal road with a load on his head does no work.

Reason: No work is said to be done, if directions of force and displacement of load are perpendicular to each other.

2. **Assertion**: The work done during a round trip is always zero.

Reason: No force is required to move a body in its round trip.

3. **Assertion**: A light body and heavy body have same momentum. Then they also have same kinetic energy.

Reason: Kinetic energy does not depend on mass of the body.

4. **Assertion**: The instantaneous power of an agent is measured as the dot product of instantaneous velocity and the force acting on it at that instant.

Reason: The unit of instantaneous power is watt.

5. **Assertion**: The change in kinetic energy of a particle is equal to the work done on it by the net force.

Reason: Change in kinetic energy of particle is equal to the work done only in case of a system of one particle.

6. **Assertion**: A spring has potential energy, both when it is compressed or stretched.

Reason: In compressing or stretching, work is done on the spring against the restoring force.

7. **Assertion**: Comets move around the sun in elliptical orbits.

Reason: The gravitational force on the comet due to sun is not normal to the comet's velocity but the work done by the gravitational force over every complete orbit of the comet is zero.

SOLUTIONS- ASSERTION AND REASON

1(a) The work done, $W = \vec{F}.\vec{s} = Fs\cos\theta$, when a person walks on a horizontal road with load on his head then $\theta = 90°$ .Hence $W = Fs\cos 90° = 0$

Thus no work is done by the person.

2 (d) In a round trip work done is zero only when the force is conservative in nature.

Force is always required to move a body in a conservative or non-conservative field

3 (d) When two bodies have same momentum then lighter body possess more kinetic energy because $E = \frac{P^2}{2m}$

$\therefore E \propto \frac{1}{m}$ when P = constant

4 (b) $P = \vec{F}.\vec{v}$ and unit of power is Watt.

5(c) Change in kinetic energy = work done by net force.

This relationship is valid for particle as well as system of particles.

6 (a) the work done on the spring against the restoring force is stored as potential energy in both conditions when it is compressed or stretched.

7 (c) The gravitational force on the comet due to the sun is a conservative force. Since the work done by a conservative force over a closed path is always zero (irrespective of the nature of path), the work done by the gravitational forces over every complete orbit of the comet is zero.

SHORT ANSWER QUESTIONS (2 MARKS)

1. When a conservative force does positive work on a body, then what would happen to potential energy the body?

Solution: Potential energy of the body decreases since W = - ΔU

2. A body of mass 3 kg is under a force, which causes a displacement in it is given by S = $\frac{t^3}{3}$ (in m). Find the work done by the force in first 2 seconds.

Solution: S = $\frac{t^3}{3}$ or $dS = t^2 dt$

$$a = \frac{d^2S}{dt^2} = \frac{d^2}{dt^2}\left[\frac{t^3}{3}\right] = 2t\ m/s^2$$

Now work done by the force $W = \int_0^2 F.dS = \int_0^2 ma.dS$

$$\int_0^2 3 \times 2t \times t^2 dt = \int_0^2 6t^3 dt = 24 \text{ J}$$

3. Power of a water pump is 2 kW. If g = 10 m/s2, then find the amount of water it can raise in on minute to a height of 10 m.

Solution: P = $\frac{mgh}{t}$ m = $\frac{p \times t}{gh} = \frac{2 \times 10^3 \times 60}{10 \times 10} = 1200$ kg

As volume = $\frac{\text{mass}}{\text{density}}$ $v = \frac{1200kg}{10^3\,kg/m^3} = 1.2m^3$ Volume = $1.2m^3 = 1.2 \times 10^3\,litre = 1200\,litre$

4. Two bodies having same mass 40 kg are moving in opposite directions, one with a velocity of 10 m/s and the other with 7 m/s If they collide and move as one body, then find velocity of the combination.

Solution: By the conservation of momentum

$$40 \times 10 + (40) \times (-7) = 80 \times v$$

$$v = 1.5 m/s$$

5. A body of mass 5 kg explodes at rest into three fragments with masses in the ratio 1:1: 3. The fragments with equal masses fly in mutually perpendicular directions with speeds of 21 m/s. Find velocity of the heaviest fragment.

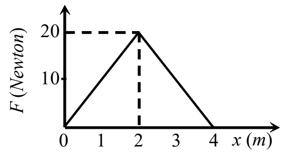
Solution: $P_x = m \times v_x = 1 \times 21 = 21\,kg\ m/s$

$P_y = m \times v_y = 1 \times 21 = 21\,kg\ m/s$ Resultant = $\sqrt{P_x^2 + P_y^2} = 21\sqrt{2}$ kg m/s

The momentum of heavier fragment should be numerically equal to resultant of Px and Py

$3 \times v = \sqrt{P_x^2 + P_y^2} = 21\sqrt{2}$ $v = 7\sqrt{2}$ $= 9.89$ m/s

6. The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 m/s. When the distance covered by the body is 4m, then what will be its kinetic energy?



Solution: Initial K.E. of the body = $\frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4 = 50\ J$

Work done against resistive force= Area between F-x graph $= \frac{1}{2} \times 4 \times 20 = 40J$

Final K.E. = Initial K.E. - Work done against resistive force

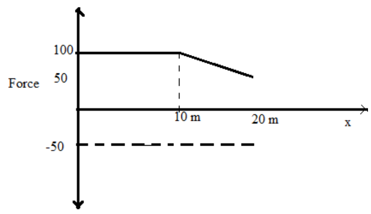
$= 50 - 40 = 10\ J$

SHORT ANSWER QUESTIONS (3 MARKS)

1. A porter pushes a trunk on a railway platform which has a rough surface. He applies a force of 100 N over a distance of 10 m. Thereafter, He gets progressively tired and his applied force reduces linearly with distance to 50 N. The total distance through which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N versus displacement. Calculate the work done by the two forces over 20 m.

Solution:

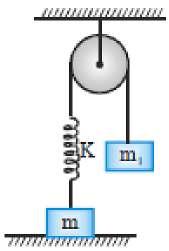
Work done = area under the curve



Work done by porter = 1750 J

Work done by friction = − 1000 J

2. For what minimum value of m1 the block of mass m will just leave the contact with surface?



Solution: let extension in the string is x.

For $m_1 g x = \frac{1}{2}kx^2$ or $2m_1 g = kx$

For block to be lifted $kx \geq mg$ or $2m_1 g \geq mg$ Or $m_1 \geq m/2$

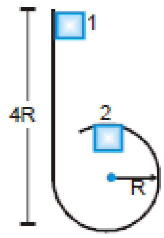
Therefore min value is m/2.

3. For a particle rotating in a vertical circle with uniform speed, the maximum and minimum tension in the string are in the ratio 5: 3. If the radius of vertical circle is 2m, then find the speed of revolving body.

Solution:

$T_{max} = mg + \frac{mv^2}{R}$ And $T_{min} = \frac{mv^2}{R} - mg$

Now $\frac{T_{max}}{T_{min}} = \frac{5}{3}$ Solving we get v =$4\sqrt{5}$ m/s



4. A cube of mass M starts at rest from point 1 at a height 4R, where R is the radius of the circular track. The cube slides down the frictionless track and around the loop.

Find the force which the track exerts on the cube at point 2.

Solution: Applying conservation of energy at point 1 and 2

$K_1 + U_1 = K_2 + U_2$

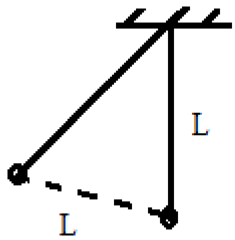
0 + mg (4R) = ½ mv^2 + mg (2R)

Solving v = $\sqrt{4gR}$

Now at point 2 N + mg = mv^2/R OR N = mv^2/R – mg = 3mg

5. (a) The bob of a pendulum is released from a horizontal position. If the length of the pendulum is L, what is the speed with which it arrives at the lower most point, given that it dissipates its 10 % energy against air resistance?

(b) If the same bob is releases as shown in the figure, find the speed at the lowermost point if it dissipates no energy.



Solution:

(a) 90 % of mgL = $\frac{1}{2}mv^2$ Solving v = $\sqrt{3.6\,gL}$

(b) $K_f + U_f = K_i + U_i$

$\frac{1}{2}mv^2 + 0 = 0 + mg(l - lcos\theta)$ (Hereθ=60 degree)

On Solving V = $\sqrt{gL}$

6. A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{\frac{3}{2}}$ where a = 5 unit.

What is the work done by the net force during its displacement from x = 0 m to x = 2 m?

Solution: W = Kf –Ki on Solving W = 50 J

7. If an engine delivers constant to a car starting from rest then show that the square displacement is proportional to cube of time elapsed.

Solution: W = ΔK

Pt = ½ mv2 – 0, $v = \sqrt{\frac{2Pt}{m}}$, $\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$

Rearranging and integrating $x^2 \alpha t^3$

LONG ANSWER QUESTIONS (5 MARKS)

1. (a) Show that in case of one dimensional elastic collision of two bodies, the relative velocity of separation after the collision is equal to the relative velocity of approach before collision.

(b) Hence write expressions for the velocities of two bodies in terms of their initial velocities before collision.

2. A body tied to one end of a string is made to revolve in a vertical plane. Derive the expression for the velocity of the body and tension in the string at any point. Hence find (i) tension at the bottom and top of the circle. (ii) Minimum velocity at the lowest point so that it is just able to loop the vertical loop and (c) the minimum velocity at the top.

3. (A) Draw the graph of equation $F_s = -kx$ where F_s is the spring force and x is the displacement of the block from equilibrium position. Using the graph, show that maximum work done by the spring at x_m is $W_s = -\frac{1}{2}kx_m^2$ (k = spring constant)

(B) What will be the work done by external force if the spring is displaced from x_1 to x_2?

4. (a) Define work done. How can we calculate work done by

(i) Constant force (ii) Variable force and

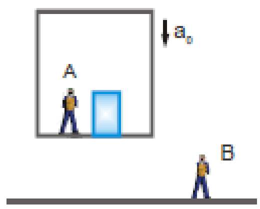
(iii) From graph if an object is displaced along a straight line.

(b) Find work done by a force F = 2x (SI unit) if it displaces an object from origin to a

Point (1, 2,-2)

SOURCE BASED QUESTIONS:

Q 1. A block of mass m is kept in an elevator which starts moving downward with an acceleration a0 as shown in figure. The block is observed by two observers A and B for a time interval t0.



(i) The observer B finds that the work done by gravity is

(a) $\frac{1}{2}mg^2t_0^2$ (b) -$\frac{1}{2}mg^2t_0^2$ (c) $\frac{1}{2}mgat_0^2$ (d) $-\frac{1}{2}mgat_0^2$

(ii) The observer B finds that work done by normal reaction N is

(a) Zero (b) - Nat_0^2 (c) $\frac{Nat^2}{2}$ (d) none of these

(iii) The observer B finds that work done by pseudo force is

(a) Zero (b) $-ma^2t_0$ (c) $+ma^2t_0$ (d) $-mgat_0$

(iv) According to observer B, the net work done on the block is

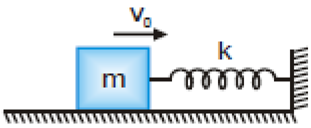
(a) $\frac{1}{2}ma^2t_0^2$ (b) -$\frac{1}{2}ma^2t_0^2$ (c) $\frac{1}{2}mgat_0^2$ (d) $-\frac{1}{2}mgat_0^2$

(v) According to the observer A

(a) The work done by gravity is zero (b) the work done by normal reaction is zero

(c) The work done by pseudo force is zero (d) all the above

2. A block of mass m moving with a velocity v0 on a smooth horizontal surface strikes and compresses a spring of stiffness k till mass comes to rest as shown in the figure. This phenomenon is observed by two observers:



A: Standing on the horizontal surface;

B: Standing on the block

(i) To an observer A, the work done by spring force is-

(a) Negative but nothing can be said about its magnitude (b) $-\frac{1}{2}mv_0^2$

(c) Positive but nothing can be said about its magnitude (d)$+\frac{1}{2}mv_0^2$

(ii) To an observer A, the work done by the normal reaction N between the block and the spring on the block is

(a) Zero (b) $-\frac{1}{2}mv_0^2$ (c)$-\frac{1}{2}mv_0^2$ (d) None of these

(iii) To an observing A, the net work done on the block is

(a) $-mv_0^2$ (b) $+mv_0^2$ (c) $-\frac{1}{2}mv_0^2$ (d) zero

(iv) According to the observer A

(a) The kinetic energy of the block is converted into the potential energy of the spring

(b) The mechanical energy of the spring–mass system is conserved

(c) The block loses its kinetic energy because of the negative work done by the conservative force of spring

(d) All the above

(V) To an observer B, when the block is compressing the spring

(a) Velocity of the block is decreasing (b) retardation of the block is increasing

(c) Kinetic energy of the block is zero (d) all the above

ANSWERS:

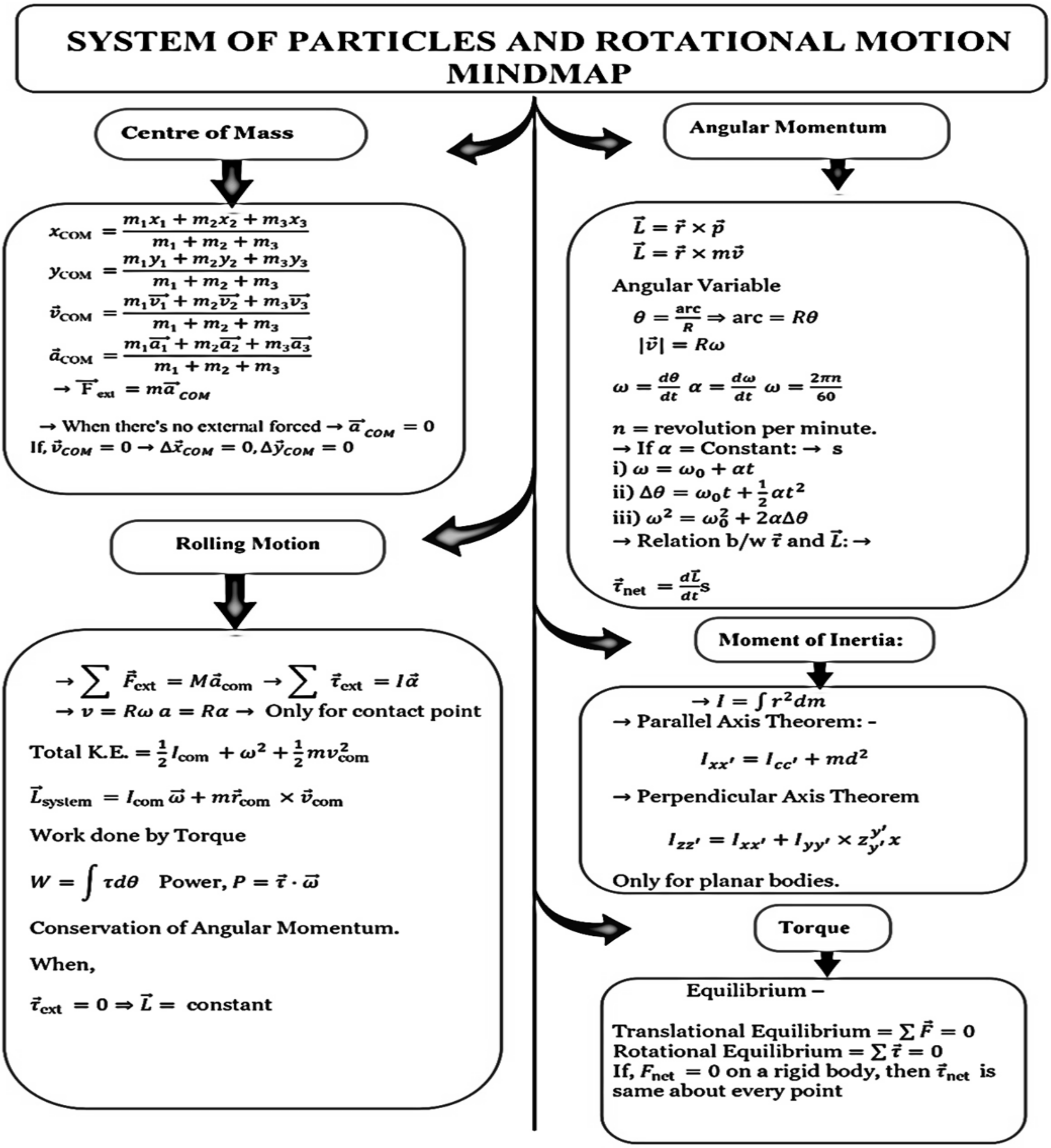
1 (i) c (ii) d (iii) a (iv) b (v) d

2 (i) b (ii) b (iii) c (iv) d (v) c

CHAPTER–6: SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

CONTENT OF CHAPTER:-Centre of mass of a two-particle system, momentum conservation and Centre of mass motion. Centre of mass of a rigid body; centre of mass of a uniform rod. Moment of a force, torque, angular momentum, law of conservation of angular momentum and its applications. Equilibrium of rigid bodies, rigid body rotation and equations of rotational motion, comparison of linear and rotational motions. Moment of inertia, radius of gyration, values of moments of inertia for simple geometrical objects (no derivation).

MIND MAP



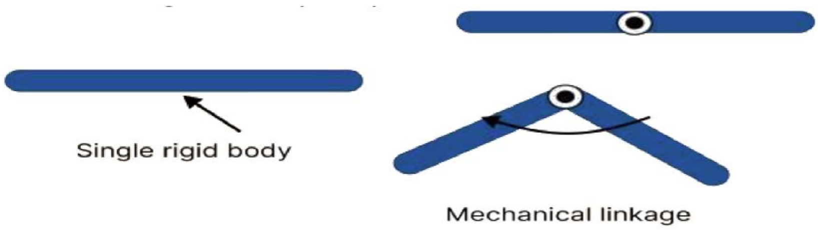
GIST OF THE CHAPTER

A rigid body is one in which the distances between different particles of the body do not change even though forces are acting on them.

Axis of rotation: The line along which the body is fixed is known as the axis of rotation. Examples for rotation about an axis are a ceiling fan and a merry-go-round etc.

In pure rotation of a rigid body

Image: Linkage of a rigid body



about a fixed axis, every particle of the rigid body moves in a circle which lies in a plane perpendicular to the axis and has its centre on the axis. Every point in the rotating rigid body has the same angular velocity at any time.

CENTRE OF MASS:

The centre of mass of a body is a point where the entire mass of the body can be supposed to be concentrated. The nature of motion executed by the body shall remain unaffected if all the forces acting on the body were applied directly at this point.

For a system of two particles of masses m_1 and m_2 having their position vector has $\vec{r}_1$ and $\vec{r}_2$ respectively, with respect to origin of the coordinate system, the position vector $\vec{R}_{CM}$ of the centre of mass is given by $\vec{R}_{CM} = \frac{m_1\vec{r}_1+m_2\vec{r}_2}{m_1+m_2}$

If $m_1 = m_2 = m$, then $\vec{R}_{CM} = \frac{\vec{r}_1+\vec{r}_2}{2}$

Thus, the centre of a mass of two equal masses lies exactly at the centre of the line joining the two masses.

For a system of N-particles of masses $m_1, m_2, m_3 \ldots m_N$ having their position vectors as $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N$ respectively, with respect to the origin of the coordinate system, the position vector $\vec{R}_{CM}$ of the centre of mass is given by

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N}{m_1 + m_2 + \cdots m_N} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{M}$$

The coordinates of centre of mass for a 3-D object is given as:

$$X_{CM} = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i x_i}{M}$$

$$Y_{CM} = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i y_i}{M}$$

$$Z_{CM} = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_j} = \frac{\sum_{i=1}^{N} m_i z_i}{M}$$

For a continuous distribution of mass, the coordinates of centre of mass are given by

$$X_{CM} = \frac{1}{M}\int xdm;\ Y_{CM} = \frac{1}{M}\int ydm;\ Z_{CM} = \frac{1}{M}\int zdm$$

VELOCITY OF CENTRE OF MASS

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_N\vec{v}_N}{m_1 + m_2 + \cdots m_N} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{M}$$

ACCELERATION OF CENTRE OF MASS

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_N\vec{a}_N}{m_1 + m_2 + \cdots m_N} = \frac{\sum_{i=1}^{N} m_i\vec{a}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i\vec{a}_i}{M}$$

Angular velocity and acceleration:

ANGULAR VELOCITY: It is defined as the time rate of change of angular displacement and is given by, $\vec{\omega} = \frac{d\theta}{dt}$

Angular velocity is directed along the axis of rotation. Angular velocity is a vector quantity. Its SI unit is rad/s and its dimensional formula is $[M^0L^0T^{-1}]$.

Relationship between linear velocity and angular velocity

The linear velocity of a particle of a rigid body rotating about a fixed axis is given by,

$\vec{v} = \vec{r} \times \vec{\omega}$ where $\vec{r}$ is the position vector of the particle with respect to an origin along the fixed axis. As in pure translational motion, all body particles have the same linear velocity at any instant. Similarly, in pure rotational motion, all body particles have the same angular velocity at any instant.

ANGULAR ACCELERATION: It is defined as the time rate of change of angular velocity, and it is given by $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

Angular acceleration is a vector quantity. Its SI unit is rads $^{-2}$ and its dimensional formula is $[M^0L^0T^{-2}]$.

EQUATION OF ROTATIONAL MOTION:

After a brief introduction to angular velocity and angular acceleration, let us see how they are related to the kinematic equations.

For a certain initial angular velocity, final angular velocity (ω_0, ω) with time t , the kinematic equations of rotational motion is given as:

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ These equations are valid for uniform angular acceleration.

$\omega^2 - \omega_0^2 = 2\alpha\theta$

MOMENT OF INERTIA: Moment of inertia of a rigid body about a given axis of rotation is defined as the sum of the product of masses of all the particles of the body and the square of their respective perpendicular distances from the axis of rotation. It is denoted by symbol I and is given by,

$$I = \sum_{i=1}^{N} m_i r_i^2$$

Moment of inertia is a scalar quantity. Its SI unit is kgm^2 and its dimensional formula is $[M^1L^2T^0]$. It depends upon Position of the axis of rotation, Orientation of the axis of rotation, Shape of the body, Size of the body, Distribution of mass of the body about the axis of rotation.

RADIUS OF GYRATION: It is defined as the distance from the axis of rotation at which, if the whole mass of the body were concentrated, the moment of inertia of the body would be the same as the actual distribution of the mass of the body. It is denoted by the symbol K.

Radius of gyration of a body about an axis of rotation may also be defined as the root mean square distance of the particles from the axis of rotation.

$$\text{i.e.,}\ K = \sqrt{\frac{r_1^2 + r_2^2 + \cdots + r_N^2}{N}}$$

The moment of inertia of a body about a given axis is equal to the product of the mass of the body and square of its radius of gyration about that axis. i.e., $I = MK^2$.

The SI unit of radius of gyration is metre and its dimensional formula is $[M^0L^1T^0]$.

TORQUE: The torque or moment of force is a measure of the turning effect of force about the axis of rotation. It is denoted by the symbol $\vec{\tau}$.Torque $\vec{\tau} = \vec{r} \times \vec{F}$

In magnitude, $\tau = rF\sin\ \theta$ Where θ is the angle between r and F .

PRINCIPLE OF MOMENTS: According to the principle of moments, a body will be in rotational equilibrium if the algebraic sum of the movements of all forces acting on the body about a fixed point is zero.

CENTRE OF GRAVITY: The center of gravity of a body is that point where the total gravitational torque on the body is zero.

ANGULAR MOMENTUM: Angular momentum of a particle about a given axis is the moment of linear momentum of the particle about that axis. It is denoted by the symbol L .

$$\vec{L} = \vec{r} \times \vec{p}\ \text{i.e.}\ L = rp\sin\ \theta$$

It is a vector quantity. Its SI unit is kgm^2 s^{-1} and dimensions are $[ML^2T^{-1}]$

RELATIONSHIP BETWEEN TORQUE AND ANGULAR MOMENTUM: $\boldsymbol{\vec{\tau} = \frac{d\vec{L}}{dt}}$

LAW OF CONSERVATION OF ANGULAR MOMENTUM: If no external torque acts on a system, then the total angular momentum is said to be conserved.

i.e. $\vec{\tau} = 0$ then $\frac{d\vec{L}}{dt} = 0$ or L = constant

A rigid body is said to be in equilibrium when:

It is in translational equilibrium i.e. the total external force acting on it is zero. i.e. $\sum F_i = 0$.

It is in rotational equilibrium, i.e. the total external torque acting on it is zero. i.e. $\sum \vec{\tau}_i = 0$.

Relation between angular momentum and moment of inertia:

Angular momentum, $L = I\omega$.

Important points:

Kinetic energy of rotational motion, $K_R = \frac{1}{2} I\omega^2$.

For a redistribution of masses in a system: $I_1\omega_1 = I_2\omega_2$

Work energy theorem in rotational motion: $\frac{1}{2}I_2\omega_2^2 - \frac{1}{2}I_1\omega_1^2$=work done

Types of motion include Translational motion, rotational motion, and rolling motion.

Translational motion is also known as kinematics.

ROLLING MOTION is the motion which happens when a body is rolling on a surface or an inclined plane.

Kinetic energy of a rolling body = translational kinetic energy + rotational kinetic energy

i.e. $\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2\left[1 + \frac{K^2}{R^2}\right]$

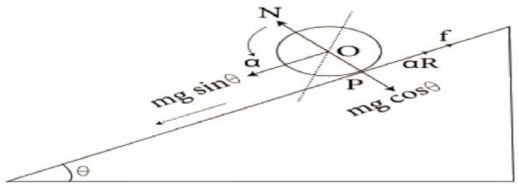


Image: Body rolling down on an inclined plane

When a body rolls down on an inclined plane of inclination without slipping its velocity at the bottom of incline is given by: $v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$.

When a body rolls down on an inclined plane without slipping its acceleration at the bottom of incline is given by: $a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$.

When a body rolls down on an inclined plane without slipping its time at the bottom of incline is given by: $t = \sqrt{\frac{2l\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}}$.

MULTIPLE CHOICE QUESTIONS (MCQ'S):

1. If a body is rotating about an axis passing through its center of mass, the angular momentum of the body is directed along its ——————.
(a) Circumference (b) Radius (c)Axis of rotation (d) None of the options

2. Linear velocities of all the particles of the body in rotational motion is —————-.
(a)1 (b)0 (c)Same (d)Different

3. Moment of inertia depends on
(a) Shape and size of the body (b) Mass
(c) Position of the axis of rotation (d) All of these

4. Which of the following is the mathematical representation of law of conservation of total linear momentum?
(a) dP/dt = 0 (b) dF/dt = 0 (c) dP/dt = F_{internal} (d) dF/dt = P

5. Two bodies of masses 5kg and 15kg are located in the Cartesian plane at (1,0) and (0,1). What is the location of their centre of mass?
(a) ¼, ¼ (b) ¾, ¾ (c) ¾, ¼ (d) ¼, ¾

6. The distance between the centres of carbon and oxygen in the carbon monoxide molecules is 1.13×10^{-10} m. The distance of the centre of mass of the molecule relative to the oxygen atom is
(a) 0.48×10^{-10} m (b) 0.64×10^{-10} m (c) 0.56×10^{-10} m (d) 0.36×10^{-10} m

7. For increasing the angular velocity of an object by 10%, the kinetic energy has to be increased by
(a) 40% (b) 20% (c) 10% (d) 21%

8. A circular platform is mounted on a frictionless vertical axle. Its radius R = 2 m and its moment of inertia about the axle is 200 kg m^2. It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of 1 ms^{-1} relative to the ground.

ANSWERS of MCQ:

1. Option I Axis of rotation,
2. Option (d),
3. Option (d)
4. Option (a)
Hint: The law of conservation of linear momentum is derived from Newton's second law. It states that total linear momentum is constant when external forces add up to zero. Newton's second law: dP/dt = F_{ext}. Here, F_{ext} must be zero for conservation of linear momentum, thus dP/dt = 0.
5. Option (d)
Hint: Sum of masses $=m_1+m_2= 5 + 15 = 20$
x-coordinate :$X_{CM}=(m_1x_1+ m_2x_2)/(m_1+m_2)=(5\times1 + 15\times0)/20 = ¼$
y-coordinate : $Y_{CM}=(m_1y_1+ m_2y_2)/(m_1+m_2)=(5\times0 + 15\times1)/20 = ¾$.
6. Option (a)
Hint: The location of the centre of mass from an object in a two-object system is inversely proportional to the mass of the object.
The atomic mass of oxygen = 16
The atomic mass of carbon = 12
Total atomic mass = 16 + 12 = 28
The location of centre of mass relative to oxygen atom = $(1.13 \times 10^{-10}) \times 12 / 28= 0.48 \times 10^{-10}$ m.
7. Option (d)
Hint: Let initial energy = K , angular velocity = ω after change become K' and ω'
$\omega' =1.1\omega$(increasing of angular velocity 10%)
We know $K=½ I\omega^2$
$K'=½ I(\omega')^2 =½ I (1.1\omega)^2=1.21(½ I\omega^2) =1.21K$
Therefore, kinetic energy should be increased by 21%
8. Option I
Hint: Given,Radius of the platform R=2 m, Moment of inertia of the platform
I=200 kg m^2
Mass of the man m=50 kg, Speed of the man relative to the ground v=1 m/s
system is initially at rest, Initial angular momentum $L_i=0$
From the law of conservation of angular momentum Final angular momentum is the sum of the angular momentum of the man and the platform.
$L_{initial}= L_{final}$ ➔ $0= L_{man} – I\omega$
➔$I\omega$ = mvR ➔ ω=mvR/I ➔ $\omega=(50\times1 \times 2)/200=100/200=1/2$ rad s^{-1}
Time taken by the man relative to the platform T= $2\pi / \omega'$
Here, $\omega' = \omega+ v/R =(½) + (½) =1$ rad s^{-1}
∴ T= 2π / 1 = 2π s

ASSERTION REASON TYPE QUESTIONS:

Directions:
Each of the following questions consists of two statements: an Assertion (A) and a Reason I. Answer them by selecting the correct option:
(a) Both Assertion (A) and Reason I are true, and Reason I is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason I are true, but Reason I is not the correct explanation of Assertion (A).
I Assertion (A) is true, but Reason I is false.
(d) If both, Assertion (A) and Reason I are false.

1. Assertion (A): Moment of inertia depends on the distribution of mass and the axis of rotation.
Reason I: Moment of inertia is a scalar quantity.
2. Assertion (A): The center of mass of a uniform semi-circular ring lies at its geometric center.
Reason I: The center of mass always lies at the center of the figure.
3. Assertion (A): A body can rotate even if its center of mass remains at rest.
Reason I: Torque causes rotational motion, independent of translational motion.
4. Assertion (A): Angular momentum of a system is conserved in the absence of external torque.
Reason I: Torque is the time derivative of angular momentum.
5. Assertion (A): The radius of gyration of a body is equal to the distance of its center of mass from the axis of rotation.
Reason I: Radius of gyration is defined as the distance from axis at which mass may be assumed to be concentrated.
6. Assertion (A): In rolling, all points of a rigid body have the same linear velocity.
Reason I: The rotational motion does not affect the linear velocity.
7. Assertion (A): The velovity of a body at the bottom of an inclined plane of given height is more when it slides down the plane compared to when it is rolling down the same plane.
Reason I: In rolling down, a body acquires both kinetic energy of translation and rotation.
8. Assertion (A): The moment of inertia of a rigid body reduces to its minimum value, when the axis of rotation passes through its centre of gravity.
Reason I: The weight of a rigid body always acts through its centre of gravity.

ANSWERS:

1.(c) 2.(c) 3.(a) 4.(a) 5.(b) 6.(d) 7.(b) 8.(a)

SHORT ANSWER TYPE 1:

1. Two blocks of masses 10 kg and 20 kg are placed on the x-axis. The first mass is moved on the axis by a distance of 2 cm . By what distance should the second mass be moved to keep the position of centre of mass unchanged ?

Ans:- Two masses m_1 and m_2 are placed on the X-axis

$$m_1 = 10 \text{ kg}, \ m_2 = 20 \text{ kg}$$

The first mass is displaced by a distance of 2 cm

$$\therefore \overline{X_{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{10 \times 2 + 20x_2}{30}$$

$$\Rightarrow 0 = \frac{20 + 20x_2}{30}$$

$$\Rightarrow 20 + 20x_2 = 0$$

$$\Rightarrow 20 = -20x_2$$

$$\Rightarrow x_2 = -1cm$$

∴ The 2^{nd} mass should be displaced by a distance 1 cm towards left so as to kept the position of centre of mass unchanged.

2. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Answer: Let F_1 and F_2 be the forces exerted by the level ground on front wheels and back wheels respectively. Considering rotational equilibrium about the front wheels,
F_2 x 1.8 = mg x 1.05
F_2 = (1.05/1.8) x 1800 x 9.8 N =10290 N
Force on each back wheel is =10290/2 N or 5145 N.
Considering rotational equilibrium about the back wheels.
F_1 x 1.8 = mg (1.8 – 1.05) = 0.75 x 1800 x 9.8
F_1=(0.75 x 1800 x 9.8)/1.8 = 7350 N
Force on each front wheel is 7350/2 N or 3675 N.

3. (i) Why do we prefer to use a wrench, having a long arm?

(ii) Under what condition, the torque due applied can be considered to be zero?
Answer: We know, the turning effect of a force is $\tau = r \times F$
Now when the arm of the wrench is long, r is larger. Therefore, a smaller force will be required to produce the same turning effect.
(ii) As we know, $\tau = r\ F\sin\theta$ Now, if $\theta = 0$ or 180,
Or, r = 0, then $\tau = 0$ Now, r = 0 refers to the applied force passing through the axis of rotation.

4. (i) Is there any possibility of coinciding with the geometrical centre and C.M. of a body? Cite example.
(ii) Does the C.M of a body necessarily have to lie inside a body?
Answer: (i) C.M. and geometrical centre may coincide when the body in consideration has a uniform mass density. So, yes, they may coincide. For instance, C.M. and geometrical centre are the same in the case of a cube or a sphere or cylinder etc.
(ii) No, the C.M of the body in consideration does not necessarily be inside the body itself. I also may lie outside the body. In the case of the semi-circular ring, it is at the center which is outside the ring.

5. (i) What are the factors on which the M.I of a body depends?
(ii) When a cat can land on its feet after a fall, which property of Physics is being used by her?
Answer: (i) The M.I. of a body depends on the following factors-
Position of the axis of the rotation
Mass of the body.
Way of distribution of mass about the axis of rotation.
(ii) If the cat can land on its feet after a fall, she is using the principle of conservation of angular momentum.

6. (i) What is a rigid body?
(ii) Why cannot a single force balance the torque?
Answer: (i) A rigid body refers to an object in which the distance between all the constituting particles remains fixed under any influence of external force. Therefore, a rigid body conserves its shape during motion.
(ii) The effect of torque can be seen to produce angular acceleration and its effect is entirely different from that of the force that causes linear acceleration. Therefore, a single force cannot balance the torque.

SHORT ANSWER TYPE 2 :

1. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s-1. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?
Answer: M = 20 kg
Angular speed, ω = 100 rad s^{-1}; R = 0.25 m
Moment of inertia of the cylinder about its axis
$=1/2\ MR^2 = ½ \times 20\ (0.25)2\ kg\ m^2 = 0.625\ kg\ m^2$
Rotational kinetic energy,
$E_r = ½\ I\ \omega^2 = ½ \times 0.625 \times (100)^2\ J = 3125\ J$
Angular momentum,
$L = I\ \omega = 0.625 \times 100\ Js = 62.5\ Js.$

2. Using the expression of power and K.E. of rotational motion, derive Relation $\tau = I\alpha$.
Answer: As we know that power is given by
$P = \tau(\omega)$………………(i) & K.E. = $½\ I\omega^2$………………(ii)
Now, the power of rotational motion is equal to time rate of work done during the rotational motion. Since the work done is stored in the form of kinetic energy,
P= d/dt (K.E. of rotational motion) Using the equations (i) and (ii), we have,
$\tau(\omega)=d/dt(1/2I(\omega)^2)=1/2I\ d/dt(\omega^2)$
$=1/2\times I\times(2\omega d\omega/dt)=I\omega\alpha(\because d\omega dt=\alpha)$……. … (iii)
From (i) and (iii), we get: $\tau\omega=I\omega\alpha \Rightarrow \tau=I\alpha$.

3. A circular ring having a diameter of 40cm and mass of 1 kg is rotating about an axis normal to its plane and passing through the center with a frequency of 10 rotations pers second. Now, calculate the angular momentum about the axis of rotation.

Answer: From the question we can obtain,

R = 40/2 cm = 20 cm = 0.2 m M = 1 kg ,v = 10 rotations / sec

Now, we can calculate the moment of inertia as follows:

$M.I = MR^2 = 1 \times (0.2)^2 = 0.04$ kg sq.m

$\omega = 2\pi \times 10 = 20\pi$ rad / sec

Thus, angular momentum can be calculated as,

$\Rightarrow L = 0.04 \times 20\pi$

$\Rightarrow L = 2.51$ kg sq m / sec

4. Three separate particles of masses of 1g, 2g, and 3g have their center of mass at the precise point (2,2,2). What should be the position of the fourth particle of mass 4g so that the combined center of mass may be at the point (0,0,0).

Answer: Given m_1=1g m_2=2g m_3=3g m4=4g

Centre of mass of three particle system $r_{cm}=(x_{cm}, y_{cm}, z_{cm})=(2,2,2)$

Position of 4th Particle r_4=? If centre of mass of four particle is (0,0,0)

Centre of mass for 3 particle system is

$X_{cm}=(m_1x_1+m_2x_2+m_3x_3)/(m_1+m_2+m_3)= (x_1+2x_2+3x_3)/(1+2+3)$

$2=(x_1+2x_2+3x_3)/6$ $x_1+2x_2+3x_3=12$…………(i)

Similarly $y_1+2y_2+3y_3=12$…………(ii), $z_1+2z_2+3z_3=12$…………(iii)

Centre of mass for 4 particle system is

$X_{cm}=(m_1x_1+m_2x_2+m_3x_3+ m_4x_4)/(m_1+m_2+m_3+ m_4)= (x_1+2x_2+3x_3+4x_4)/(1+2+3+4)$

➔ $0=(12+4x_4)/10$ ➔ $12+4x_4=0$ ➔ $x_4=-3$ similarly $y_4=-3$ $z_4=-3$

Thus the new particle should be at (-3,-3,-3).

5. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minutes. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2.(a) What is his new angular speed? (Neglect friction)(b) Is kinetic energy conserved in the process? If not, from where does the change come about?

Answer: Here, $I_1= 7.6 + 2 \times 5 (0.9)^2 = 15.7$ kg m^2

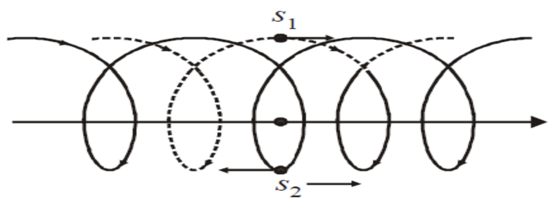
$\omega_1 = 30$ rpm $I_2 = 7.6 + 2 \times 5 (0.2)^2 = 8.0$ kg m^2 ω_2 = ?

According to the principle of conservation of angular momentum,$I_1\omega_1=I_2\omega_2$

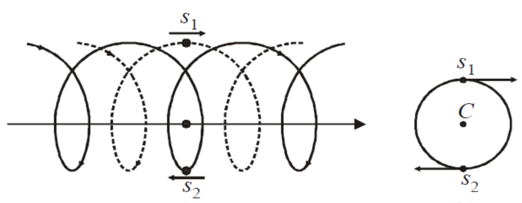
➔$\omega_2= (I_1 \omega_1) /I_2 = (15.7 \times 30)/8.0 = 58.88$ rpm

No, kinetic energy is not conserved in the process. In fact, as moment of inertia decreases, K.E. of rotation increases. This change comes about as work is done by the man in bringing his arms closer to his body.

6. The motion of binary stars, S_1 and S_2 is the combination of *X* and *Y* .What are *X* and *Y* refer to?



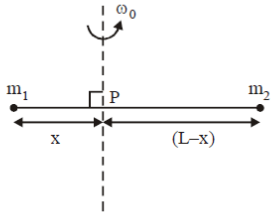
Answer: Here *X* and *Y* refer to motion of the CM and motion about the CM When no external force acts on the binary star, its CM will move like a free particle [Fig. (a)]. From the CM frame, the two stars will seems to move in a circle about the CM with diametrically opposite positions.



(a) (b)

(a) Trajectories of two stars. S1 (dotted line) and S2 (solid line) forming a binary system with their centre of mass C in uniform motion

(b) The same binary system, with the centre of mass C at rest. So, to understand the motion of a complicated system, we can separate the motion of the system into two parts. So, the combination of the motion of the CM and motion about the CM could described the motion of the system.



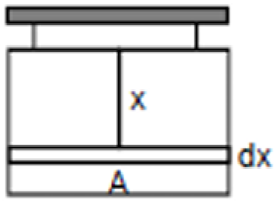
LONG ANSWER TYPE QUESTIONS:

1. (a) What are the differences between the centre of gravity and the centre of mass?

(b) A square plate of mass 120 g and edge 5.0 cm rotates about one of edges. If it has a uniform angular acceleration of $0.2 rad/s^2$, what torque acts on the plate?

Answer: a) (i)The centre of gravity is a point in a body where weight distribution is equal in all directions. It depends on the gravitational field.

(ii)The centre of mass is a point where mass distribution is uniform in all directions. It does not rely on the gravitational field. However, a body's centre of gravity and centre of mass is positioned at the same point in a homogeneous gravitational field.



(b)A square plate of mass 120 gm and edge 5 cm rotates about one of the edge. Let take a small area of the square of width dx and length a which is at a distance x from the axis of rotation.

Therefore mass of that small area

$m/a^2 \times adx$(m = mass of the square ; a = side of the plate

$$I = \int_0^a (m/a^2) \times ax^2 dx = (m/a)(x^3/3)\Big]_0^a$$

$$= ma^2/3$$

Therefore torque produced $= I \times \alpha = (ma^2/3) \times \alpha$

$$= \{(120 \times 10^{-3} \times 5^2 \times 10^{-4})/3\}0.2$$

$$= 0.2 \times 10^{-4} = 2 \times 10^{-5}\ \text{N} - \text{m}.$$

2. (a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value ? Assume that the turntable rotates without friction.

(b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

Ans: (a) given initial moment of inertia = I_1,then final moment of inertia $I_2 = (2/5)\ I_1$

Angular speed n_1= 40 rev/min Angular speed n_2= 40 rev/min

By principle of conservation of angular momentum $I_1\omega_1 = I_2\omega_2$

$I_1 (2\pi n_1) = I_2 (2\pi n_2) \Rightarrow n_2 = (I_1 n_1) / I_2 = (I_1\ 40) / (2/5)\ I_1$=100 rev/min

(b)Final K.E $_{Rotation}$ / initial K.E $_{Rotation}$ $= \frac{1}{2} I_2\omega_2^2 / \frac{1}{2} I_1\omega_1^2 = \frac{1}{2} I_2 (2\pi n_2)^2 / \frac{1}{2} I_1(2\pi n_1)^2$

$= I_2 n_2^2 / I_1 n_1^2 = (2/5)\ I_1 (100)^2 / I_1 (40)^2$

Final K.E $_{Rotation}$ / initial K.E $_{Rotation}$ =2.5

From this we can say final K.E becomes more because the child uses his internal energy when he folds his hands to increase the kinetic energy.

3. (a) A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

(b) Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time.

Answer: (a) Given Force applied F= 30 N , mass m=3kg , radius r= 40 cm =0.4 m

Moment of inertia of the hollow cylinder about its axis.

$I = MR^2 = 3(0.4)^2 = 0.48$ kg m^2

Torque $\tau = r \times F = 0.4\times30 = 12$ Nm

If α is angular acceleration produced, then from $\tau = I\alpha \Rightarrow \alpha = \tau/I = 12/0.48 = 25$ rad s^{-2}

Linear acceleration $a = R\alpha = 0.4\times25 = 10$ ms^{-2}

Let M be the mass and R the radius of the hollow cylinder, and also of the solid sphere. Their moments of inertia about the respective axes are $I_1 = MR^2$ and $I_2 = 2/5\ MR^2$

Let τ be the magnitude of the torque applied to the cylinder and the sphere, producing angular accelerations $\alpha_1$and α_2 respectively. Then $\tau = I_1\ \alpha_1 = I_2\ \alpha_2$

The angular acceleration α_2 produced in the sphere is larger. Hence, the sphere will acquire larger angular speed after a given time.

CASE BASED QUESTIONS:

1. **Centre of Mass**: The centre of mass of a body or a system of bodies is the point which moves as though all of the mass were concentrated there and all external forces were applied to it. Hence, a point at which the entire mass of the body or system of bodies is supposed to be concentrated is known as the centre of mass. If a system consists of more than one particle (or bodies) and net external force on the system in a particular direction is zero with centre of mass at rest. Then, the centre of mass will not move along that direction. Even though some particles of the system may move along that direction.

(i) Two bodies of masses 1 kg and 2 kg are lying in xy-plane at (-1, 2) and (2, 4), respectively. What are the coordinates of the centre of mass?

(a) 1, 10/3 (b) (1,0) (c) (0,1) (d) None of these

(ii) Two balls of same masses start moving towards each other due to gravitational attraction, if the initial distance between them is L. Then, they meet at

(a) L/2 (b) L (c) L/3 (d) L/4

(iii) The centre of mass of a system of two particles divides, the distance between them

(a) in inverse ratio of square of masses of particles

(b) in direct ratio of square of masses of particles

(c) in inverse ratio of masses of particles

(d) in direct ratio of masses of particles

(iv) Two particles A and B initially at rest move towards each other under a mutual force of attraction. At the instant, when the speed of A is v and the speed of B is 2v, the speed of centre of mass of the system is

(a) zero (b) v (c) 1.5 v (d) 3v

OR

All the particles of a body are situated at a distance R from the origin. The distance of centre of mass of the body from the origin is

(a) = R (b) ≤ R (c) > R (d) ≥ R

Answer: (i) a (ii) a (iii) c (iv) a OR b

CHAPTER-7 : GRAVITATION

CONTENT OF CHAPTER:- Kepler's laws of planetary motion, universal law of gravitation. Acceleration due to gravity and its variation with altitude and depth. Gravitational potential energy and gravitational potential, escape speed, orbital velocity of a satellite, energy of an orbiting satellite**.**

Newton's Law of Gravitation

Gravitational force (F) between two bodies is directly proportional to product of masses and inversely proportional to square of distance between them.

$$\overline{F} = \frac{Gm_1 \cdot m_2}{r^2}$$

Acceleration due to Gravity

*For a body falling freely under gravity, the acceleration in body is called acceleration due to gravity.

*Relationship between g and G

$$g = \frac{GMe}{R_e^3} = \frac{4}{3}\Pi G R \rho$$

Where G = gravitational constant

p= density of earth M_e R_e be the mass and radius of earth

Characteristics of Gravitational Force

* It is always attractive
* It is independent of the medium
* It is conservative and central forces
* It holds good over a wide range of distance

Kepler's Law of Planetary Motion

Law of Orbits: Every Planet revolves around the Sun in an elliptical orbit and the Sun is situated at one of its foci.

Law of Areas: The areal Velocity of planet around the sun is constant.

i.e. $\frac{da}{dt} = a\ Constant$

Law of Periods: The Square of the time period of revolution of a planet is directly proportional to the cube of semi major axis of the elliptical orbit. $T^2 \propto a^3$

Variation of Acceleration due to gravity (g)

Due to altitude(h)

$$g_h = g\left(1 - \frac{2h}{Re}\right)$$

Value of g goes on decreasing with height

Due to depth(d)

$$g_d = g\left(1 - \frac{d}{Re}\right)$$

The value of g decreases g with depth

Due to Rotation of Earth

$$g_\lambda = g - Re^{\omega} \cos^2 \lambda$$

At equator $\lambda = 0^0$

$$g_{\lambda min} = g - Re\,\omega^2$$

At poles $\lambda = 90^0$

$$g_{\lambda max = }g_P = g$$

Gravitational Potential Energy

Work done in bringing the given body from infinity to a point in the gravitational field U = GMm/r

Gravitational Potential

Work done in bringing a unit mass from infinity to a point in the gravitational field

$$v = -\frac{GM}{r}$$

Escape Speed

The minimum speed of projection of a body from surface of earth so that it just crosses the gravitational field of earth $v_r = \sqrt{\frac{2GM}{R}}$

Earth's Satellite

Time Period of Satellite

$$T = \frac{2\pi}{R_C}\sqrt{\frac{(Re + h)}{g}}$$

For satellite orbiting close to the earth's surface $T = 2\pi\sqrt{\frac{R_e}{g}}$ = 84.6 min

Energy of Satellite

Kinetic Energy K = $\frac{GM_r m}{2(Re+h)}$

Potential energy U =$-\frac{GM_r m}{Re+h}$

Total Energy

E= K+U=$\frac{GM_r m}{2(Re+h)}$

Orbit Speed of Satellite

The minimum speed required to put the satellite into a given orbit $V_0 = Re\sqrt{\frac{g}{Re+h}}$, For satellite orbiting close to the earth's surface $v_0 = \sqrt{gR_e}$

Types of Satellite

Polar Satellite

Time Period: 100 min, Revolve in polar orbit around the earth

Height: 500 – 800 KM

Uses: Weather forecasting and Military Spying

Geostationary Satellite

Time Period: 24 hours, same angular speed in same direction with earth

Height: 36000 KM, Uses: GPS, Satellite Communication (TV)

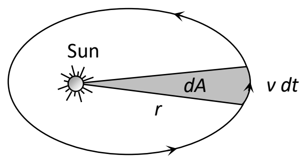
GIST OF THE CHAPTER:-

1-Kepler's law of planetary motion

(a) *Kepler's first law (law of orbit):* Every planet revolves around the sun in an elliptical orbit with the sun situated at one focus of the ellipse.

(b) *Kepler's second law (law of area):* The radius vector drawn from the sun to a planet
sweeps out equal areas in equal intervals of time, i.e., the areal velocity of the planet around the sun is

Areal velocity $= \frac{dA}{dt} = \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv$



$\therefore \quad \frac{dA}{dt} = \frac{L}{2m}$ constant.

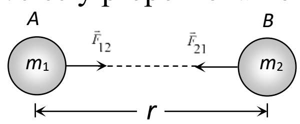
(c) *Kepler's third law (law of period):* The square of the time period of revolution of a
planet around the sun is directly proportional to the cube of the semi-major axis of the elliptical orbit of the planet around the sun.

$$T^2 \propto r^3$$

2-Universal law of Gravitation (Newton's law of gravitation)

Newton's law of gravitation:

It states that the gravitational force of attraction acting between two-point mass bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them, $F \propto \frac{m_1 m_2}{r^2}$



or $F = G\frac{m_1 m_2}{r^2}$

where G is the universal gravitational constant and in SI unit, G =6.67X10^{-11}Nm^{2}kg^{-2}.

Weight of a body:

It is the gravitational force of attraction exerted on a body by the earth towards its centre lying on or near the surface of the earth.

The weight of a body (w) = mass of body × acceleration due to gravity(g) i.e. $W = mg$

The unit of weight of a body is Newton.

3-Acceleration due to gravity (g):

It is an acceleration set up in a body while falling freely under the effect of the gravitational force of the earth alone.

The acceleration due to gravity (g) is given by

$$g = \frac{GM}{R^2}$$

where M and R are the mass and radius of the earth.

The value of g changes with the height, depth, rotation and shape of the earth. The value of g on the surface of the earth is 9.8 ms^{-2} .

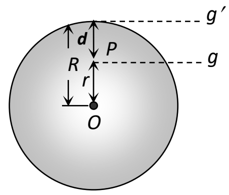
4- Variation in acceleration due to gravity with altitude- Acceleration due to gravity decreases with altitude. Acceleration due to gravity at a height 'h' from the earth's surface is given by

(i) As we go above the surface of the earth, the value of g decreases because $g' \propto \frac{1}{r^2}$.

(ii) If $r = \infty$ then $g' = 0$, *i.e.*, at infinite distance from the earth,
the value of g becomes zero.

(iii) If $h << R$ *i.e.*, height is negligible in comparison to the radius

then from equation (iii) we get

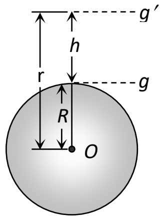


$$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2}$$

$$\approx g\left[1-\frac{2h}{R}\right]$$ [this relation is valid only for $h << R$]

5-Variation in acceleration due to gravity with depth.

Acceleration due to gravity decreases with the increase in depth d from the earth's surface and it becomes zero at the centre of the earth. Acceleration due to gravity at a depth 'd' from the earth's surface is given by



$$g' = g\left[1-\frac{d}{R}\right]$$

6-Gravitational potential energy

The gravitational potential energy of a body, at a point in the gravitational field of the earth, is defined as the amount of work done in bringing the body from infinity to that point without acceleration. The gravitational potential energy of a body is given by

$$U = -\frac{GMm}{r}$$

here M is the mass of the earth and m is the mass of the body. r is the distance between the centre of the body and the centre of the earth.

7-Gravitational potential:

The gravitational potential at a point in a gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Gravitational potential at a point in earth's gravitational field is given by,

$$V = -\frac{W}{m}$$

It is a scalar Quantity and the SI unit is Joule/Kg

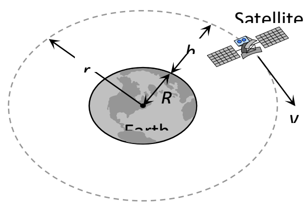
The gravitational potential at a point in the earth's gravitational field-

$$V = -\frac{GM}{r}$$

where M is the mass of the earth and r is the distance of the point from the centre of the earth.

Gravitational potential energy (U) = gravitational potential × mass of body

8- The orbital velocity of a satellite- The orbital speed of the satellite is the minimum velocity required tangentially to the circular orbit to put the satellite into an orbit around earth. The orbital velocity($\boldsymbol{V_O}$) of a satellite revolving around the Earth in circular orbit at a height h from earth's surface is given by

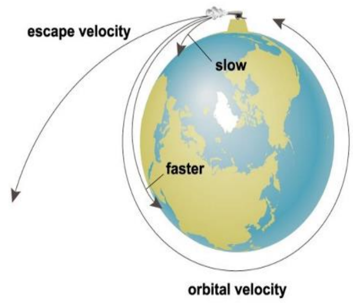


$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$ [As $GM = gR^2$ and $r = R+h$]

where M and R are the mass and radius of the earth and 'h' is height of satellite from earth surface.

9-Escape Velocity: The escape velocity on earth (or any other planet) is defined as the minimum velocity required to project a body vertically upwards from the surface of Earth (or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. The escape velocity $\boldsymbol{Ve}$ on the earth (or any other planet) is



$$v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e = \sqrt{2gR} \qquad [\text{As } GM = gR^2]$$

On the earth surface, g= 9.8 m/s^2 and R = 6.4×10^6 meters, then value of the escape velocity from the earth surface will be 11.2 kms^{-1}.

Relation between Escape velocity and Orbital velocity

For a point close to the earth's surface, the escape velocity and orbital speed are related as $v_e = \sqrt{2}\, v_o$

Energy of Satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).(1) Potential energy :

$$U = mV = \frac{-GMm}{r} \qquad \left[\text{As } V = \frac{-GM}{r}\right]$$

(2) Kinetic energy : $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$ $\qquad \left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$

(3) Total energy : $E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r}$

MULTIPLE CHOICE QUESTIONS

Q1. The gravitational force of attraction between two mass m_1 and m_2 kept at a distance r from each other is F. What will be the gravitational force if m_1 is doubled, m_2 is halved and the distance between them is made thrice?

a. F b. F/9 c. 4F/9 d. None of these

Q2.The mass and radius of earth is reduced to half the acceleration due to gravity would

(a) remains same (b) decreases by half (c) decreases by same amount (d) increases by same amount

Q3. At what point regarding earth the value of Force of gravitational attraction of earth is

a. At the equator b. At the poles

c. At a point in between equator and any pole d. At surface of earth.

Q4. Which of the following relation is correct?

a. GM = g R^2 b. GM = gR^3 c. gM = GR^2 d. GM = gR

Q5. What will be the effect on acceleration due to gravity When the average density of Earth is doubled,

a. Remain constant b. Will become doubled

c. Will become four times d. None of these

Q6. We have a planet whose mass is doubled and radius is halved to that of the earth Then what will be the value of gravity on it's surface If acceleration due to gravity is g on the surface of the Earth,

a. 8g b. 4g c. g d. 6g

Q7. If the law of gravitation, instead of being inverse-square law, becomes an inverse-cube law-

(a) Planets will not have elliptic orbits.

(b) Circular orbits of planets is not possible.

(c) Projectile motion of a stone thrown by hand on the surface of the earth will not be parabolic.

(d) There will be no gravitational force inside a spherical shell of uniform density.

Q8. For earth, Acceleration due to gravity
a. Decreases with depth		b. Increases with depth
c. Remain constant with depth		d. May increase or decrease

ANSWERS:-

1 b. F/9 (Hint: $F = \frac{GMm}{r^2}$)

2 d. (Hint: $g = \frac{GM}{R^2}$ and $g' = \frac{G\frac{M}{2}}{\left(\frac{R}{2}\right)^2} = 2g$, % increase $g = \frac{\Delta g}{g} \times 100$)

3. a. , (Hint: Radius of earth is less at equator $g = \frac{GM}{R^2}$)
4. a. (Hint : GM = g R^2)
5. b (Hint - $g = \frac{4}{3}\pi\rho GR$)
6. a. (Hint - g= GM/R^2)
7. a.
8. a. (Hint - $g' = g\left[1 - \frac{d}{R}\right]$

ASSERSSION – REASON QUESTIONS

Read the questions below and answer
(1) Both (A) & (R) are true and the (R) is the correct explanation of the (A)
(2) Both (A) & (R) are true but the (R) is not the correct explanation of the (A)
(3) (A) is true but (R) is false
(4) Both (A) and (R) are false

Q1 Assertion (A): Two satellites A and B are in the same orbit around the earth, such that A Leads B. Satellite B can overtake satellite A by increasing its speed.
Reason (R): Orbital speeds of two satellites in same orbit may different
Q2Assertion (A): For a system of masses at some finite distance, gravitational field can be zero but gravitational potential can not be zero.
Reason (R): Gravitational field is a scalar quantity while gravitational potential is a vector quantity.
Q3 **Assertion (A):** Period of revolution of satellite in circular orbit around earth is inversely proportional to cube of its orbital speed.
Reason (R): Period of revolution in uniform circular motion is given by T = =2πr/v, where r is radius of orbit and v is speed.
Q4 Assertion (A): Assuming zero potential at infinity, the gravitational potential at a point can never be positive.
Reason (R): The magnitude of gravitational force between two particles has inverse square dependence on the distance between two particles.
Q5 Assertion (A): Gravitational field of a uniform spherical shell outside it is same as that of particle of same mass placed at its centre of mass.
Reason (R): For the calculation of gravitational force between any two uniform spherical shells, they can always be replaced by particles of same mass placed at respective centres.
Q6 **Assertion (A):** The gravitational force between two finite bodies is necessarily along the line joining their centre of mass.
Reason (R): The gravitational force between two particles is not central.
Q7 **Assertion (A):** Moon revolving around earth does not come closer despite earth’s gravitational attraction.
Reason (R): A radially outward force balances earth’s force of attraction during revolution of moon.

ANSWERS :

1 – D   2- C   3- A   4- B   5- C   6- D   7- C

SHORT ANSWER QUESTION(2 MARKS)

Q1. Suppose there existed a planet that went around the sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

Answer

According to Kepler's third law

If planets move faster with double speed then it will take half time period of revolution around sun as that of earth. Te =1 yearTp = 0.5 year r_e = 1A.U. $r_p = r_e(Tp/Te)^{2/3}$ = $1AU(0.5year/1year)^{2/3}$ = 0.63 AU

Q2. The gravitational force on the Earth due to the Sun is greater than that of the moon, However, the tidal effect of the moon's pull is greater than the tidal effect of the Sun. Why?

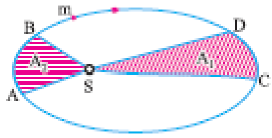
Answer: . The tidal effect depends inversely on the cube of the distance, unlike the force which depends inversely on the square of the distance. The distance of the moon from the ocean water is very small compared to the distance of the sun from the ocean water on earth. Therefore, the tidal effect of the Moon's pull is greater than the tidal effect of the sun.

Q3. Derive the expression for acceleration due to gravity at the earth's surface

Given that M is mass and R is the radius of the earth.

Answer:

Q4. The figure shows elliptical orbit of a planet *m* about the sun S. The shaded area of SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from D to C andt_2, is time to move from A to B, what is the relation between $t_1$and t_2 ?



Answer: Using Kepler's II Law $t_1=2t_2$

Q5. Prove that the weight of a body at the centre of the Earth is zero.

Answer: An atmosphere means the presence of a mixture of a number of gases. The molecules of these gases are in a state of continuous random thermal motion moving at different velocities. As the value of escape velocity on the surface of the moon is small (only 2.5 km s^{-1}), the molecules of gases with velocities greater than the escape velocity moved out of the atmosphere. As time passed, nearly all the molecules escaped from the moon's atmosphere.

Q6. Hydrogen escapes faster from the Earth than oxygen. Why?

Answer.

The thermal speed of hydrogen is much larger than oxygen. Therefore, a large number of hydrogen molecules are able to acquire escape velocity than that of oxygen molecules. Hence hydrogen escapes faster from the Earth than oxygen.

SHORT ANSWER QUESTION (3 MARKS)

Q1. Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC. What is the force acting on a mass 2m placed at the centroid G of the triangle = BG = CG = 1m in the given diagram

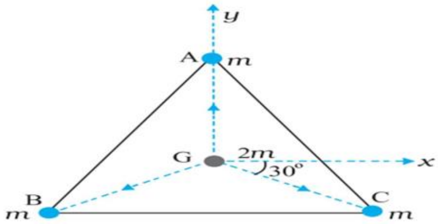
Ans1.

Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC. What is the force acting on a mass 2m placed at the centroid G of the triangle?

$FGA = \frac{Gm(2m)}{1^2}\ \hat{\mathbf{j}}$

$FGB = \frac{Gm(2m)}{1^2}\ (-\mathbf{i}\cos30^0 - \mathbf{j}\sin30^0)$

$FGB = \frac{Gm(2m)}{1^2}\ (\mathbf{i}\cos30^0 - \mathbf{j}\sin30^0)$

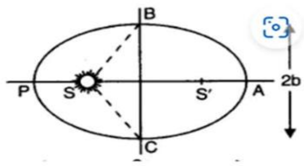


Net force $F = F_{GA} + F_{GB} + F_{GC} = 0$

Q2. State the Kepler laws of the planetary motions.

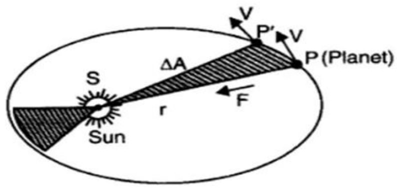
Ans2.Kepler's 1st Law of Orbits:

This law is popularly known as the law of orbits. The orbit of any planet is an ellipse around the Sun with Sun at one of the two foci of an ellipse. In an ellipse, there are two foci and Sun is situated at one of the foci of the ellipse.



Kepler's 2nd Law of Areas:

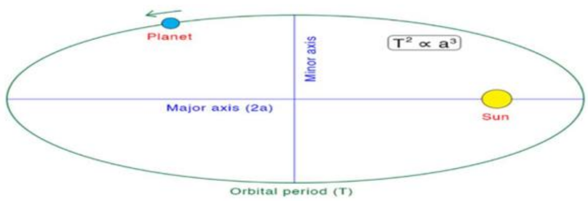
This law is known as the law of areas. The line joining a planet to the Sun sweeps out equal areas in equal interval of time. The rate of change of area with time will be constant.



Areal Velocity = $V = \frac{\Delta A}{\Delta t}$

Areal Velocity = $\frac{L}{2m}$ where L = Angular momentum and m are mass of the planet

Kepler's 3rd Law of Periods: This law is known as the law of Periods. The square of the time period(T) of the planet is directly proportional to the cube of the semimajor axis(a) of its orbit. $T^2 \propto a^3$



Q3.In a two stage launch of a satellite, the first stage brings the satellite to a heightof 150 km and the 2nd stage gives it the necessary critical speed to put it in acircular orbit. Which stage requires more expenditure of fuel ? Given massof earth = 6.0×10^{24} kg, radius of earth = 400 km.

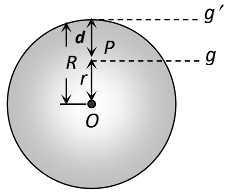
Answer: Work done on satellite in first stage = W1 = PE at 150 km – PE at the surface

Ratio w1/w2 = 3/64 (less than one)

W2 > W1, so second stage requires more energy.

Q4. Derive the expression for the acceleration due to gravity at a depth 'd' from thesurface of the earth.
Ans4. Acceleration due to gravity at the surface of the earth

$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$ ...(i)



Acceleration due to gravity at depth d from the surface of the earth

$g' = \frac{4}{3}\pi\rho G(R-d)$ ...(ii)

From (i) and (ii) $g' = g\left[1 - \frac{d}{R}\right]$

Q5. Three equal masses of each mass m are placed at the three vertices of an equilateral triangle of each side a. Find the gravitational potential at the centroid of the triangle.

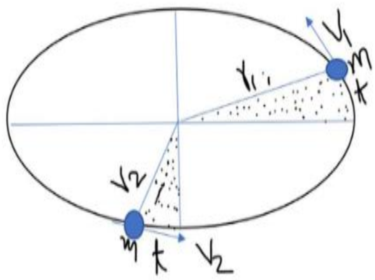
5-MARKS QUESTIONS

Q1. Define the gravitational potential energy of a body on the surface of the Earth. A body of mass m is at a distance of r from the centre of the earth. If M is the mass of the earth and R is the radius of the earth, then find the gravitational potential energy of the body.
Answer : Define the term and then derive the expression
Q2. Define escape velocity. Does the escape velocity of a body from the Earth depend on (a) the mass of the body, (b) the location from where it is projected, (c) the direction of projection, and (d) the height of the location from where the body is launched? Why the atmosphere of Jupiter contains light gases (generally hydrogen) whereas the Earth's atmosphere has little hydrogen gas?
Answer
Escape Velocity is the minimum velocity an object must have to escape a celestial body's gravitational field permanently, or without ever falling back again. Escape velocity of the body is given by

$h \Rightarrow v_e = \sqrt{2gR}$ [As $GM = gR^2$]

Where M is mass of the earth and R is radius of the earth and g is acceleration due to gravity on the earth surface.
(a)Escape velocity does not dependent upon mass of the body to be escaped
(b)Escape velocity does not dependent the location
(c)Escape velocity does not dependent the direction of the projection
(d)Yes, Escape velocity depends on the height of the location.
The escape velocity of Jupiter is much larger than the escape velocity of Earth. So in order to escape from the surface of Jupiter, a very large velocity is required. Since the thermal velocity of hydrogen gas molecules is lesser than the escape velocity of Jupiter, therefore hydrogen can't escape from the surface of Jupiter.
Q3.(i)State Kepler's second law of planetary motion.
(ii) Show that Kepler's Second law is the law of conservation of angular momentum.
(iii) A Saturn year is 29.5 times the Earth year. How far is Saturn from the Sun if the Earth is 1.50 x 10^8 km away from the Sun?

Answer:
Kepler's 2nd Law of Areas: The second law states that the areal velocity is constant i.e. area covered by the radius vector is the same in equal intervals of time. If the velocity and radius at the time, t_1 is v_1 and r_1 while at another place these are v_2 and r_2 in the same time, then the area covered by the planet in these intervals are



(ii) using ½ $(v_1t)\ r_1$ = ½ $(v_2t)\ r_2$ (area of triangle = ½ base×height)
$mv_1 r_1 = mv_2 r_2$
$L_1 = L_2$
(iii)A Saturn year is 29.5 times the Earth year. Earth is 1.50 x 10^8 km away from the Sun

Using Kepler's III law distance = 14.32×10^{11} m.

Q4. Derive the expression for escape velocity for a body projected from the surface of the earth. Calculate escape velocity on the surface of the earth if the radius of the earth is 6400 Km and acceleration due to gravity is 9.8 m/s^2 on the surface of the earth. A body is projected out thrice the escape velocity. What will be the velocity of the body far away from the earth? Ignore the presence of the sun and other planets.

Answer: Escape velocity is the minimum velocity with which a body must be projected vertically upward so that it can just escape the gravitational field of the Earth.

Expression for escape velocity:

$v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e = \sqrt{2gR}$ [As $GM = gR^2$]

Total Kinetic energy by which body is projected

$K_1 = \frac{1}{2} m V_1^2$

Minimum Kinetic energy required by body to overcome earth gravitational field

$K_2 = \frac{1}{2} m V_2^2$

Kinetic Energy of body just after crossing earth gravitational field

$K_3 = \frac{1}{2} m V_3^2$

From conservation of energy

$K_1 = K_2 + K_3$

$\frac{1}{2} m V_1^2 = \frac{1}{2} m V_2^2 + \frac{1}{2} m V_3^2$

$V_3^2 = V_1^2 - V_2^2$

Here V = 11.2 km/s and $V_1 = 3 \times 11.2$ km/s $V_3 = 31.68$ km/s

CASE STUDY-BASED QUESTIONS

Q1. Origin of Gravitation

The **origin of gravity** evolved from Newton's description of a force between masses to Einstein's conceptualization of gravity as the curvature of spacetime. While Newton's laws work well for everyday situations, Einstein's theory provides a deeper understanding of gravity, explaining phenomena that Newton's theory could not, such as the bending of light and the behaviour of objects near black holes. Today, gravity is understood as a result of the interaction between mass and the fabric of spacetime, a concept that has been validated through both theory and experiment.

(i) Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will

(a) move towards each other

(b) move away from each other

(c)will become stationary

(d) keep floating at the same distance between them

(ii) Two spheres of masses m and M are situated in air and the gravitational force between them is 2F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(a) 6F (b) 2F (c) F/6 (d) F/2

(iii)Which of the following best describes gravity according to Newton's Law of Universal Gravitation?

a) It is a force that attracts two objects based on their mass and the distance between them

b) It is a force that repels objects based on their mass and the distance between them

c) It is caused by the curvature of spacetime around objects

d) It is a quantum mechanical effect between subatomic particles

(iv) Why is gravity so much weaker than the other fundamental forces (strong nuclear force, electromagnetic force, and weak nuclear force)

(a) it acts specifically (b)it acts universally

(c)its value decrease with height (d) it obeys inverse square law

(v) **What is one of the main challenges in the study of gravity at quantum scales**?

a) Gravity is too strong to be studied in quantum mechanics

b) Gravity has no effect at small scales
c) Gravity does not interact with subatomic particles
d) Gravity does not fit well with the principles of quantum mechanics

Q2. Case study 'g' IN OUR LIFE

Early in our lives, we become aware of the tendency of all material objects to be attracted to the earth. Anything thrown up falls down towards the earth, going uphill is lot more tiring than going downhill, raindrops from the clouds above fall towards the earth and there are many other such phenomena. Historically it was the Italian Physicist Galileo (1564-1642) who recognized the fact that all bodies, irrespective of their masses, are accelerated towards the earth with a constant acceleration. It is said that he made a public demonstration of this fact. To find the truth, he certainly did experiments with bodies rolling down inclined planes and arrived at a value of the acceleration due to gravity which is close to the more accurate value obtained later.

Q(i).Which of the following factors causes the acceleration due to gravity to be slightly weaker at the equator compared to the poles?

A) Earth's shape (oblate spheroid) B) Earth's rotation
C) Altitude above sea level D) Both A and B

Q(ii). At which location will the acceleration due to gravity be the least on Earth?

a) At the North Pole b) At the South Pole
c) At sea level at the equator d) At the summit of Mount Everes

Q(iii). Which of the following is NOT a factor that directly influences the acceleration due to gravity?

a) The mass of the object b) The altitude above Earth's surface
c) The Earth's radius d) The latitude of the location

Q(iv). Does the gravitational force between two particles depends upon the medium between them?
Q(v). How does the acceleration due to gravity vary vertically downward from the earth's surface?

ANSWER -CASE STUDY BASED QUESTIONS

1. Case Study (ORIGIN OF GRAVITY)

Answer:i – (d) ii – (b) iii –(a) iv – (b) v – (d)

2. Case study

'g' IN OUR LIFE

Answer(i): d)BothAandB

(The Earth's bulging at the equator and its rotation both contribute to a weaker gravitational pull.

Ans(ii). c at sea level at the equator (Because of the centrifugal force from Earth's rotation and the Earth's bulging at the equator.)

Ans(iii).(a) **Ans(iv).** No **Ans(v).** Decreases as depth increases

CHAPTER-8 : MECHANICAL PROPERTIES OF SOLID

SYLLABUS- Elasticity, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity (qualitative idea only), Poisson's ratio; elastic energy. Application of elastic behavior of materials (qualitative idea only).

MIND MAP-

MECHANICAL PROPERTIES OF SOLID

STRESS AND STRAIN

- Elasticity-property of the body due to which it tends to regain its original shape and size.
- Plasticity-property due to which the body does not regain its original shape and size.
- Stress-The restoring force per unit area. **Stress=F/A**
- Change in configuration per unit original configuration

Longitudinal Strain= $\frac{\Delta L}{L}$

Volumetric Strain= $\frac{\Delta V}{V}$

Shearing Strain= $\frac{\Delta x}{L} \approx \theta$

Hooke's Law

- For small deformations Stress α Strain
- Stress = E x Strain
- E= modulus of elasticity

Energy Stored in a Stretched Wire

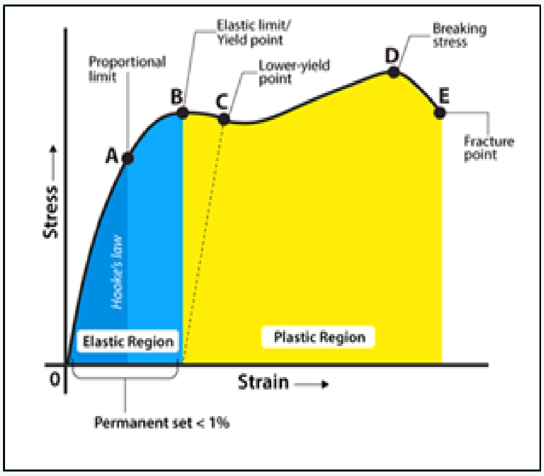
E= $\frac{1}{2}$ x Stress x Strain x Volume

Poisson's Ratio

$\sigma = \frac{Lateral\ strain}{Longitudinal\ strain}$

υ<σ< 0.5

Stress-Strain curve



Elastic Modulus

1.Young's Modulus of Elasticity-

$$Y = \frac{Tensile\ or\ Compressive\ stress}{Longitudinal\ Strain} = \frac{FL}{A\Delta L}$$

2.Shear Modulus of Elasticity-

$$\eta = \frac{Shearing\ stress}{Shearing\ Strain} = \frac{F}{A\theta}$$

2.Bulk Modulus of Elasticity-

$$K = \frac{Hydraulic\ stress}{Hydraulic\ Strain} = \frac{-PV}{\Delta V}$$

GIST OF LESSON

1.Elastic Property of Matter

(1) **Elasticity :** The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

(2) **Plasticity :** It is basically the reciprocal of elasticity means which it does not regain its original shape and size after the removal of deforming force is called plasticity.

(3) **Perfectly elastic body :** If on the removal of deforming forces the body regain its original configuration completely it is said to be perfectly elastic. A quartz fibre and phosphor bronze is the nearest approach to the perfectly elastic body.

(4) **Perfectly plastic body :** If the body does not have any tendency to recover its original configuration, on the removal of deforming force, it is said to be perfectly plastic. Paraffin wax, wet clay are the nearest approach to the perfectly plastic body.

Note-Practically there is no material which is either perfectly elastic or perfectly plastic and the behavior of actual bodies lies between the two extremes.

(5) **Elastic limit :** The maximum deforming force up to which a body retains its property of elasticity is called elastic limit of the material of body.(Elastic limit is the property of a body whereas elasticity is the property of material of the body)

(6) **Elastic after effect :** The time delay in which the substance regains its original condition after the removal of deforming force is called elastic after effect.

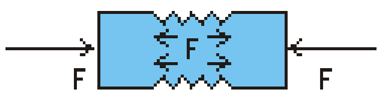
2.Stress

The internal restoring force acting per unit area of cross section of the body is called stress. At equilibrium, restoring force is equal in magnitude to external force. If external force F is applied on the area A of a body then, **Stress =Force/area**

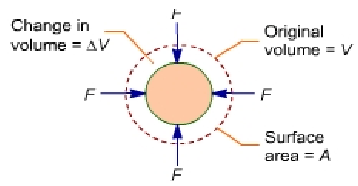
Unit : N/m^2 (S.I.) , $dyne/cm^2$ (C.G.S.) ,Dimension : $[ML^{-1}T^{-2}]$ On this basis there are two types of stresses: Normal and Shear or tangential stress

(1) **Normal stress:** Here the force is applied normal to the surface. It is again of two types:

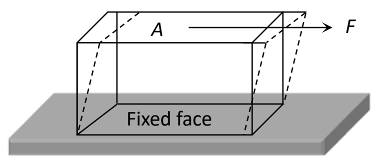
(i) Longitudinal stress -It occurs only in solids. Longitudinal stress produced due to increase in length of a body under a deforming force is called **tensile stress.** Longitudinal stress produced due to decrease in length of a body under a deforming force is called **compressive stress.**



(ii) ***Bulk or Volume stress*** **-**It occurs in solids, liquids or gases. In case of fluids only bulk stress can be found. It produces change in volume and density, shape remaining same. Deforming force is applied normal to surface at all points.



(2) **Shear or tangential stress:** when nearby layers of solid move on each other *i.e.* when there is a relative displacement between various layers of solid. Here deforming force is applied tangential at least one of the faces. It produces change in shape, volume remaining the same.



3.Strain

The ratio of change in configuration to the original configuration is called strain. Being the ratio of two like quantities, it has no dimensions and units. Strain are of three types

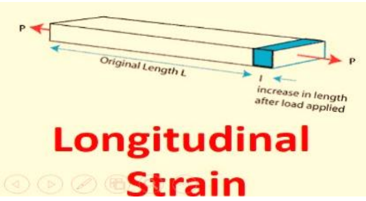
(1) **Longitudinal strain-**It is the ratio of change in length to the original length.

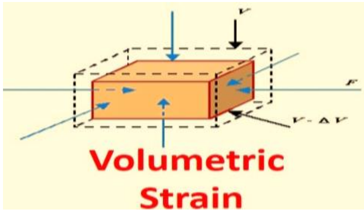
$$\text{Linear strain} = \frac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$$

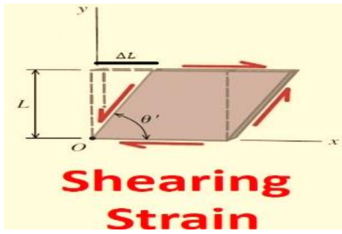
(2) **Volumetric strain-**It is the ratio of change in volume to the original volume.

$$\text{Volumetric Strain} = \frac{change\ in\ volume\ \Delta V}{original\ volume\ V}$$

(3) **Shearing strain-**If the deforming force produces a change in the shape of the body without changing its volume.It is defined as angle in radians through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force φ=x/L







4.Hooke's law and Modulus of Elasticity

According to this law, within the elastic limit, stress is proportional to the strain. i.e. stress $\propto$ strain

or $\frac{\text{stress}}{\text{strain}} = \text{constant} = E$ (The constant E is called modulus of elasticity)

5.Young's Modulus of Elasticity(Y)

It is the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{Al} \Rightarrow Y = \frac{MgL}{\pi r^2 l}$$

6.Bulk Modulus of Elasticity-When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, such that the shape remains the same, then there is a change in volume. Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus. This is denoted by K.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}} \Rightarrow K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

where p = increase in pressure; V = original volume; ΔV = change in volume

The negative sign shows that with increase in pressure p, the volume decreases by ΔV i.e. if p is positive, ΔV is negative. The reciprocal of bulk modulus is called compressibility.

7.Modulus of Rigidity

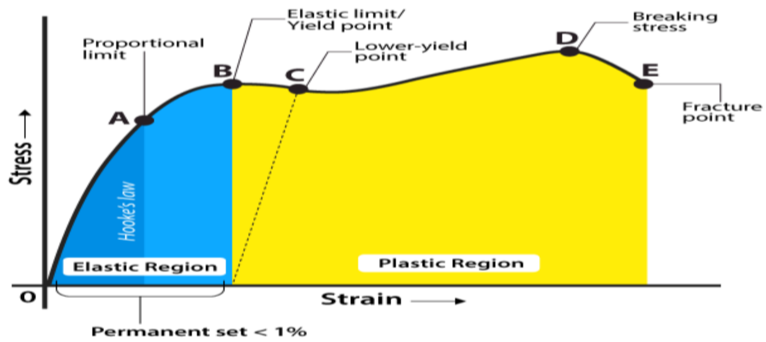
Within limits of proportionality, the ratio of tangential stress to the shearing strain is called modulus of rigidity of the material of the body and is denoted by η

$$\text{i.e. } \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi}$$ (Only solids can exhibit this)

8.Stress-strain Curve-

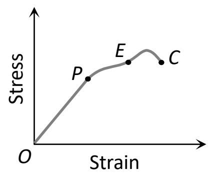
The different regions in the stress-strain diagram are:

(i) Proportional Limit
(ii) Elastic Limit
(iii) Yield Point
(iv) Ultimate Stress Point
(v) Fracture or Breaking Point

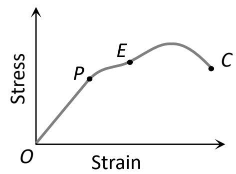


Note-

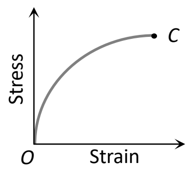
Brittle material- The plastic region between *E* and *C* is small for brittle material
Ductile material- The material of the wire have a good plastic range
Elastomers-Such materials have no plastic range and the breaking point lies very close to elastic limit. Example rubber(here stress strain curve is non linear)



Brittle material



Ductile material

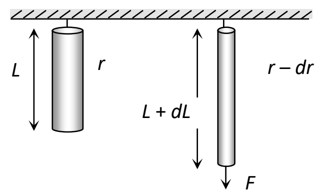


Elastomer

9.Work done in stretching a wire

Work done in stretching a wire W= ½ stress × strain × volume
This is work done in stretching a wire is store in the wire in the form of elastic PE.
Elastic PE = W = ½ stress × strain × volume

10.Poisson's Ratio- Lateral strain : The ratio of change in radius or diameter to the original radius or diameter is called lateral strain.**Longitudinal strain :** The ratio of change in length to the original length is called longitudinal strain.



The ratio of lateral strain to longitudinal strain is called Poisson's ratio (σ).

i.e. $\sigma = \dfrac{\text{Lateral strain}}{\text{Longitudin al strain}}$ or $\sigma = \dfrac{-dr/r}{dL/L}$

Negative sign indicates that the radius of the bar decreases when it is stretched.
Poisson's ratio is a dimensionless and a unit less quantity.
Theoretical Value of σ must be between −1.0 and +0.5 but practical value of σ ranging between 0.0 and 0.5.

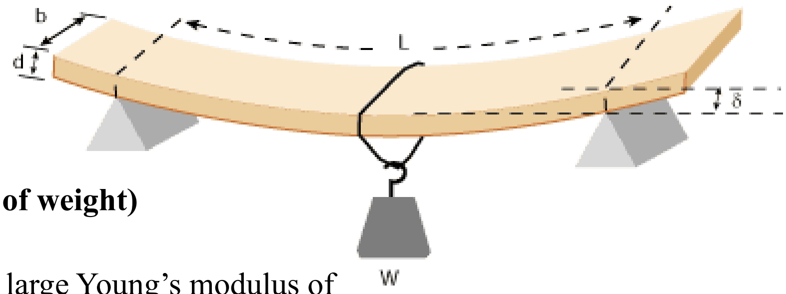
11 Aplications

1.Cranes are used for lifting heavy loads. These cranes have strong metallic ropes. Thickness of ropes is selected so that under the action of heavy load they don't pass the elastic limit.

2. Bridges are designed such that under the load of traffic the force of winds & its own weight it should not bend too much.

3. A hollow shaft is stronger than a solid shaft made of same material

4. Bending of beams $\delta = \dfrac{W L^3}{4Y\ b\ d^3}$.



Minimum bending for a given load

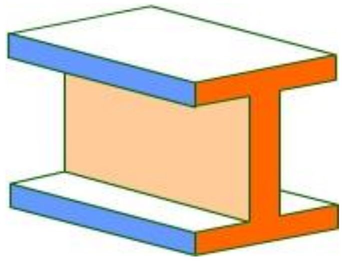
[a] δ α W (One should have small value of weight)

[b] δ α L^3 **(Length must be small)**

[c] δ α 1/Y (One should use material with large Young's modulus of elasticity)

[d] δ α 1/b (**Breadth of the rod must be large)**

[e] δ α 1/d^3 **(Height must be large)**



5.Maximum height of a mountain on earth can be estimated from the elastic behavior of earth. At the base of the mountain, the pressure is given by $P = h\rho g$ and it must be less than elastic limit (K) of earth's supporting material. $K > P > h\rho g \ \therefore\ h < \dfrac{K}{\rho g}$ or $h_{\max} = \dfrac{K}{\rho g}$

MULTIPLE CHOICE QUESTIONS

1. An Indian rubber cord L metre long and area of cross-section A meter^2 is suspended vertically. Density of rubber is D kg/m^3 and Young's modulus of rubber is E N/m^2. If the wire extends by l metre under its own weight, then extension l is

(a) L^2Dg/E (b) $L^2Dg/2E$ (c) $L^2Dg/4E$ (d) L

2. To break a wire, a force of 10^6 N/m^2 is required. If the density of the material is 3x10^3 kg/m^3, then the length of the wire which will break by its own weight will be

(a) 34 m (b) 30 m (c)300 m (d)3 m

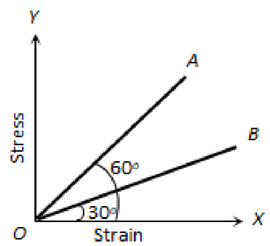
3. There are two wires of same material and same length while the diameter of second wire is 2 times the diameter of first wire, then ratio of extension produced in the wires by applying same load will be

(a) 1 : 1 (b)2 : 1 (c)1 : 2 (d)4 : 1

4.An elastic material of Young's modulus Y is subjected to a stress S. The elastic energy stored per unit volume of the material is

(a) $\dfrac{2Y}{S^2}$ (b) $\dfrac{S^2}{2Y}$ (c) $\dfrac{S}{2Y}$ (d) $\dfrac{S^2}{Y}$

5. The stress versus strain graphs for wires of two materials A and B are as shown in the figure. If Y_A and Y_B are the Young 's moduli of the materials, then



(a) $Y_B = 2Y_A$ (b) $Y_A = Y_B$

(c) $Y_B = 3Y_A$ (d) $Y_A = 3Y_B$

6. The diameter of a brass rod is 4 mm and Young's modulus of brass is $9\times10^{10}\ N/m^2$. The force required to stretch by 0.1% of its length is

(a) $360\pi N$ (b)36 N (c) $144\pi\times10^3\ N$ (d) $36\pi\times10^5\ N$

7. When a spiral spring is stretched by suspending a load on it, the strain produced is called

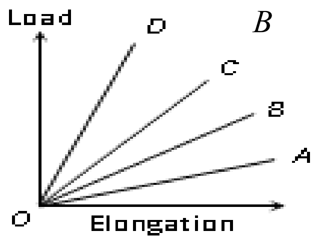
(a) Shearing (b)Longitudinal (c)Volume (d)Transverse

8. Two wires of equal lengths are made of the same material. Wire A has a diameter that is twice as that of wire B. If identical weights are suspended from the ends of these wires, the increase in length is

(a) Four times for wire A as for wire B (b)Twice for wire A as for wire B

(c) Half for wire A as for wire B (d)One-fourth for wire A as for wire B

9. The load versus elongation graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line



(a) OD (b)OC

(c)OB (d)OA

10. K is the force constant of a spring. The work done in increasing its extension from l_1 to l_2 will be

(a) $K(l_2 - l_1)$ (b) $\frac{K}{2}(l_2 + l_1)$ (c) $K(l_2^2 - l_1^2)$ (d) $\frac{K}{2}(l_2^2 - l_1^2)$

ANSWER

1.(b) The weight of the cord is $W=mg=DALg$ The average stress is stress$=F_{avg}/A=(\frac{1}{2}DALg)/A=DLg/2$

so strain=stress/$E=\frac{DLg}{2E}$

Since strain = l/L ➔ l=strain x L➔$(\frac{DLg}{2E})$ L➔ l=$\frac{DL^2g}{2E}$

2. (a)Breaking Stress=P=F/A=mg/A=dALg/A=> $L=\frac{P}{dg}=\frac{10^6}{3\times10^3\times10}=\frac{100}{3}=34m$

3. (d) $l=\frac{FL}{AY}\therefore\ l\propto\frac{1}{r^2}$ (F, L and Y are constant) $\frac{l_1}{l_2}=\left(\frac{r_2}{r_1}\right)^2=(2)^2=4$

4.(b) W=$\frac{1}{2}$ stress x strain x volume ➔work/volume =$\frac{1}{2}$x S x $\frac{S}{Y}=\frac{S^2}{2Y}$

5. (d) $\frac{Y_A}{Y_B}=\frac{\tan\theta_A}{\tan\theta_B}=\frac{\tan 60}{\tan 30}=\frac{\sqrt{3}}{1/\sqrt{3}}=3\Rightarrow Y_A=3Y_B$

6.(a) $F=\frac{YAl}{L}=\frac{9\times10^{10}\times\pi\times4\times10^{-6}\times0.1}{100}=360\ \pi\ N$

7. (a) A small part of the spring bear tangential stress, causing straining strain.

8. (d) $l=\frac{FL}{AY}\Rightarrow l\propto\frac{1}{r^2}$ (F,L and Y are same) $l_B=\frac{l_A}{4}$

9. (a) $l=\frac{FL}{AY}\therefore\ l\propto\frac{1}{r^2}$ (Y, L and F are constant)*i.e.* for the same load, thickest wire will show minimum elongation. So graph D represent the thickest wire.

10. (d) At extension l_1, the stored energy $=\frac{1}{2}Kl_1^2$,At extension l_2, the stored energy $=\frac{1}{2}Kl_2^2$

Work done in increasing its extension from l_1 to $l_2=\frac{1}{2}K(l_2^2-l_1^2)$

ASSERTION & REASONING QUESTIONS

These questions of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses

(a)If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion

(b)If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.(c)If Assertion is true but the Reason is false.(d)If Assertion & Reason Both are False.

1. Assertion : Spring balances show correct readings even after they had been used for a long time interval.

Reason : On using for long time, spring balances enhances its elastic strength.

2. Assertion : Glassy solids have sharp melting point.
Reason : The bonds between the atoms of glassy solids get broken at the same temperature.
3. Assertion : Bulk modulus of elasticity (K) represents incompressibility of the material.
Reason : Bulk modulus of elasticity is proportional to change in pressure.
4. Assertion : The bridges declared unsafe after a long use.
Reason : Elastic strength of bridges losses with time.
5. Assertion : Two identical solid balls, one of ivory and the other of wet-clay are dropped from the same height on the floor. Both the balls will rise to same height after bouncing.
Reason : Ivory and wet-clay have same elasticity.
6. Assertion : The stretching of a coil is determined by its shear modulus.
Reason : Shear modulus change only shape of a body keeping its dimensions unchanged.
7. Assertion : Identical springs of steel and copper are equally stretched. More work will be done on the steel spring.
Reason : Steel is less elastic than copper.
8. Assertion : On elastic bodies, though we apply force and experience elongation, it is a general practice to plot stress versus strain instead of plotting force versus elongation graph.
Reason : Stress versus strain plotting makes the graph independent of the geometry of the simple body and reflects only the material property.

ANSWER

1. (d) When a spring balance has been used for a long time, the spring in the balance fatigued and there is loss of strength of the spring. In such a case, the extension in the spring is more for a given load and hence the balance gives wrong readings.

2. (d) In a glassy solid (*i.e.*, amorphous solid) the various bonds between the atoms or ions or molecules of a solid are not equally strong. Different bonds are broken at different temperatures. Hence there is no sharp melting point for a glassy solid.

3. (a) Bulk modulus of elasticity measures how good the body is to regain its original volume on being compressed. Therefore, it represents incompressibility of the material. $K = \frac{-PV}{\Delta V}$ where P is increase in pressure, ΔV is change in volume.

4. (a) A bridge during its use undergoes alternating strains for a large number of times each day, depending upon the movement of vehicles on it when a bridge is used for long time, it losses its elastic strength. Due to which the amount of strain in the bridge for a given stress will become large and ultimately, the bridge may collapse. This may not happen, if the bridges are declared unsafe after long use.

5. (d) Ivory is more elastic than wet-clay. Hence the ball of ivory will rise to a greater height. In fact the ball of wet-clay will not rise at all, it will be somewhat flattened permanently.

6. (a) Because, the stretching of coil simply changes its shape without any change in the length of the wire used in coil. Due to which shear modulus of elasticity is involved.

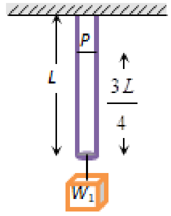
7. (c) Work done $= \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \times Y \times (\text{Strain})^2$.(Since, elasticity of steel is more than copper, hence more work has to be done in order to stretch the steel.)

8. (a) the stress-strain graph provides a more material-independent and universal representation of the material's behavior. A force versus elongation graph is dependent on the sample's dimensions, while stress and strain are independent of the sample's size and shape, allowing for easier comparison of different materials

2-MARK QUESTIONS

1. Steel is more elastic than rubber. Explain.

Answer-Consider two wire, one of steel and another of rubber having equal length L and cross sectional area A . When subjected to same deforming force F, the extension produce in steel is lS and in rubber is lR such that lR>lS. Then ($Y_S = FL/Al_s$ and $Y_r = FL/Al_r$ or $Y_s/Y_r = l_r/l_s$ as $l_s<l_r$)therefore Ys>Yr Hence steel is more elastic.



2. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the what is the stress produce in the wire at a height $3L/4$ from its lower end?

Answer- As the wire is uniform so the weight of wire below point P is $\frac{3W}{4}$

$\therefore$ Total force at $P = W_1 + \frac{3W}{4}$ and area $= S \therefore$ Stress at point $P = \frac{\text{Force}}{\text{Area}} = \frac{W_1 + \frac{3W}{4}}{S} = \frac{W_1}{S} + \frac{3W}{4S}$

3. A wire of length L and radius r is rigidly fixed at one end. On stretching the other end of the wire with a force F, the increase in its length is l. If another wire of same material but of length $2L$ and radius $2r$ is stretched with a force of $2F$, the how much length of wire will be increased.

Answer- $l = \frac{FL}{\pi r^2 Y}$ after solving we get the increment in length will be same.

4. A uniform cube is subjected to volume compression. If each side is decreased by 1%,then Calculate bulk strain produced in cube.

Answer-Volume of cube $V = L^3$ $\therefore$ % in $V = 3 \times$ (% change in L)= $3\,(1\%) = 3\%$

$\therefore \Delta V = 3\%$ of $V \Rightarrow$ Volumetric strain $= \frac{\Delta V}{V} = \frac{3}{100} = 0.03$

5. A ball falling in a lake of depth 200 m shows 0.1% decreases in its volume at the bottom. What is the bulk modulus of the material of the ball?

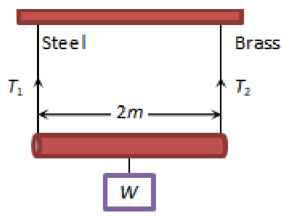
Answer - $K = \frac{P}{\Delta V/V} = \frac{hdg}{\Delta V/V} = \frac{200 \times 10^3 \times 9.8}{0.001} = 19.6 \times 10^8\ N/m^2$

6. What is elastic fatigue? Give an example.

Answer: The property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming force is called elastic fatigue. It is suggested that after a certain time the bridges must be demolished and reconstructed as they might have developed elastic fatigue.

3-MARK QUESTIONS

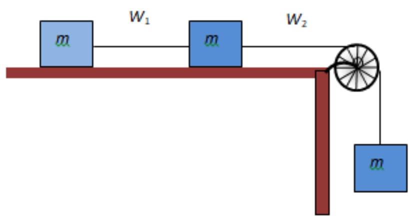
1.A 2m long rod is suspended with the help of two wires of equal length. One wire is of steel and its cross-sectional area is 0.1 cm^2 and another wire is of brass and its cross-sectional area is 0.2 cm^2. If a load W is suspended from the rod and stress produced in both the wires is same then what will be the ratio of tensions produce in steel and brass wire.



Answer- Stress = $\frac{\text{Tension}}{\text{Area of cross-section}}$ = constant

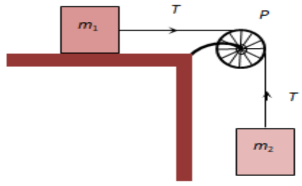
$$\therefore \frac{T_1}{A_1} = \frac{T_2}{A_2} \Rightarrow \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{0.1}{0.2} = \frac{1}{2} = 0.5$$

2. Three blocks, each of same mass m, are connected with wires W_1 and W_2 of same cross-sectional area A and Young's modulus Y. (Neglecting friction).



What is the amount of strain developed in wire W_2 ?

Answer- we know that $T = \dfrac{m_1 m_2}{m_1 + m_2} g$



In the given problem $m_1 = (m+m) = 2m$ and $m_2 = m$

$\therefore$ Tension $= \dfrac{2}{3} mg$ $\therefore$ Stress $= \dfrac{T}{a} = \dfrac{2}{3a} mg$ and

$$\text{Strain} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{2}{3}\frac{mg}{aY}$$

3.What are the factors affecting elasticity?

Answer- i)Effect of hammering and rolling on a material: It causes a decrease in the plasticity of the material due to the break-up of crystal grains into smaller units and thus, the elasticity of the material increases.

ii)**Effect of Annealing on a material:** Annealing results in an increase in the plasticity of the material due to, the formation of large crystal grains. Henceforth, the elasticity of the material decreases.

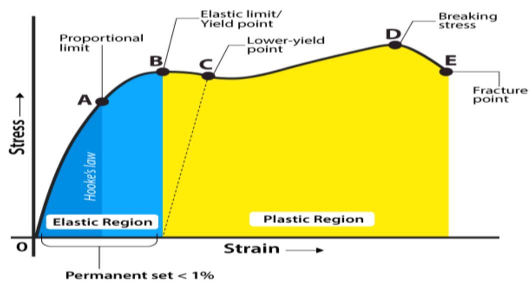
iii)**Effect of the presence of impurities on a material:** The effect of the presence of impurities in a material can be both ways i.e. it can increase as well as decrease the elasticity of the material. The type of effect depends upon the nature of the impurity present in the material.

iv)**Effect of temperature on a material:** The increase in the temperature of the material in most cases causes a decrease in the elasticity of the material. The elasticity does not change with the change in temperature.

5-MARK QUESTIONS

1.Draw stress and strain curve. Explain the various region or points that are necessary to understand this graph (i) Proportional Limit (ii) Elastic Limit (iii) Yield Point (iv) Ultimate Stress Point (v) Fracture or Breaking Point.

Answer-The different regions in the stress-strain diagram are:



(i) Proportional Limit-It is the region in the stress-strain curve that obeys Hooke's Law. In this limit, the stress-strain ratio gives us a proportionality constant known as Young's modulus. The point OA in the graph represents the proportional limit.

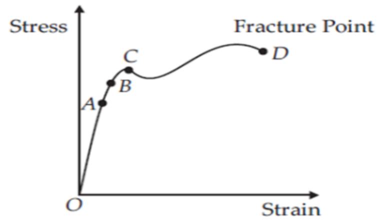
(ii) Elastic Limit-It is the point in the graph up to which the material returns to its original position when the load acting on it is completely removed. Beyond this limit, the material doesn't return to its original position, and a plastic deformation starts to appear in it.

(iii) Yield Point-The yield point is defined as the point at which the material starts to deform plastically. After the yield point is passed, permanent plastic deformation occurs. There are two yield points (i) upper yield point (ii) lower yield point.

(iv) Ultimate Stress Point-It is a point that represents the maximum stress that a material can endure before failure. Beyond this point, failure occurs.

(v) Fracture or Breaking Point-It is the point in the stress-strain curve at which the failure of the material takes place.

CASE STUDY BASED QUESTIONS



1. The graph shown below shows qualitatively the relation between the stress and the strain as the deformation gradually increases. Within Hooke's limit for a certain region stress and strain relation is linear. Beyond that up to a certain value of strain the body is still elastic and if deforming forces are removed the body recovers its original shape.

(i) If deforming forces are removed up to which point the curve will be retraced?

(a) upto *OA* only (b) upto *OB* (c) upto *C* (d) Never retraced its path

(ii) In the above question, during loading and unloading the force exerted by the material are conservative up to

(a) *OA* only (b) *OB* only (c) *OC* only (d) *OD* only

(iii) During unloading beyond *B*, say *C*, the length at zero stress in now equal to

(a) less than original length (b) greater than original length

(c) original length (d) can't be predicted

(iv) The breaking stress for a wire of unit cross-section is called

(a) yield point (b) elastic fatigue (c) tensile strength (d) Young's modulus

Answer-(i)b (ii)b (iii)b (iv)c

2. A metal wire of original length L = 2 m and cross-sectional area A = 1 mm² is stretched by applying a force of 100 N. The student observes that the extension in length is ΔL = 1 mm. She is asked to calculate stress, strain, and Young's modulus using standard formulas.

(i) What is the stress in the wire?

(a) 10^6 Pa (b) 10^7 Pa (c) 10^8 Pa (d) 10^5 Pa

(ii) What is the strain produced in the wire?

(a) 0.0005 (b) 0.001 (c) 0.01 (d) 0.1

(iii) What is the Young's modulus of the material?

(a) 2×10^{11} Pa (b) 1×10^{11} Pa (c) 5×10^{10} Pa (d) 1×10^{10} Pa

(iv) What type of deformation is shown in this case?

(a) Plastic deformation (b) Temporary strain (c) Elastic deformation (d) Shearing

Answer-(i)c (ii)a (iii)a (iv)a

CHAPTER–9: MECHANICAL PROPERTIES OF FLUIDS

SYLLABUS- Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes), effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its simple applications (Torricelli's law and Dynamic lift). Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.

MIND MAP-

Pressure

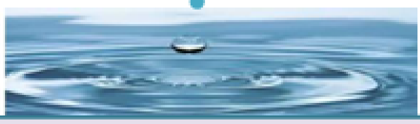
- Fluid:that can flow
- Pressure P=F/A
- Pascal's law-Pressure is same at all points at the same horizontal level for a fluid at rest
- At depth h, absolute pressure = $P_0 + \rho gh$
- Gauge pressure=Absolute pressure-atmospheric pressure

Pressure applied at any point is transmitted equally in all directions

1Pa=1N/m^2
1 bar=10^{5}Pa
1 torr=1mm of Hg
1atm=1.013x10^{5}Pa

Archimedes Principle

Upthrust=weight of liquid displaced be the body=Vρg

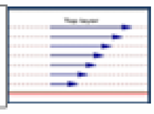


MECHANICAL PROPERTIES OF FLUIDS

Viscosity and Stoke's Law

- Viscosity: a resistance to fluid motion
- Coefficient of viscosity η

Viscous Force (between layers) F=-η A (dv/dx)



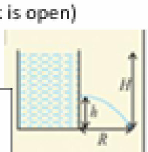
- Stoke's Law-viscous force on a body moving in the fluid is given by F=6πηrv
- Terminal velocity $v_t = \dfrac{2r^2(\rho-\sigma)g}{9\eta}$
- Reynold's Number- $R = \dfrac{\rho v D}{\eta}$
- R<1000- → Streamline flow
- 1000<R<2000 → Transition flow
- R>2000 → Turbulent flow

Equation of continuity and Bernoulli's Theorem

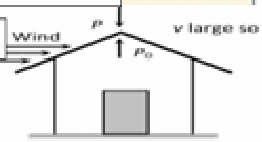
- In stream line flow,velocity of particles passing at any point is same at that point.
- Equation of Continuity (using conservation of mass)(for incompressible fluid) $A_1v_1 = A_2v_2$
- Bernoulli's Principle-For an incompressible and non viscous fluid(using conservation of energy) P+(1/2)ρv^2+ρgh=constan
- Application of Bernoulli's theorem
- Torricelli's Law(when the tank is open)

$V_{efflux} = \sqrt{2g(H-h)}$

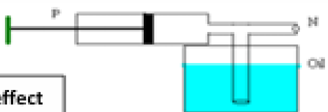
h Range of fluid R=$\sqrt{2h(H-h)}$ (When h=H/2 range is max)



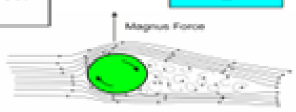
Blowing off Roof



Sprayer

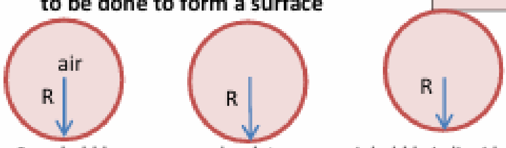


Magnus effect



Surface tension and Capillary

- Surface tension =Force per unit length T=F/l
- Surface energy= amount of work Done to be done to form a surface E=S x A



Soap bubble $\Delta P = \frac{4T}{R}$ | droplet $\Delta P = \frac{2T}{R}$ | air bubble in liquid $\Delta P = \frac{2T}{R}$

- Angle of contact-angle between tangent to the liquid surface at the point of contact and solid surface inside the liquid.
- When angle of contact is obtuse the liquid will fall in capillary tube
- When angle of contact is acute, the liquid will rise in capillary tube
- Capillary rise - $h = \dfrac{2T\cos\theta}{r\rho g}$

GIST OF LESSON-

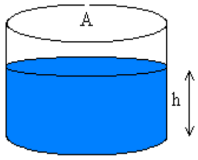
Fluid are those substance which begins to flow when external force is applied on it. Liquids and gases are fluids. The branch of physics which deals with the study of fluids at rest is called **hydrostatics** and the branch which deals with the study of fluids in motion is called **hydrodynamics**.

1.Pressure -At a point force applied per unit area is called pressure. $\boxed{Pressure = \frac{Force}{Area} = \frac{F}{A}}$

- It is scalar quantity. Its SI unit is N/m^2 also known as Pascal.
- Other practical units of pressure are atmosphere, bar and torr (*mm* of *Hg*)

$$1atm = 1.01 \times 10^5\, Pa = 1.01\,\text{bar} = 760\ \text{torr}$$

2.Thrust and Liquid pressure: - **Thrust**- The total normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface. Unit = Newton (S.I) and Dyne (C.G.S.).



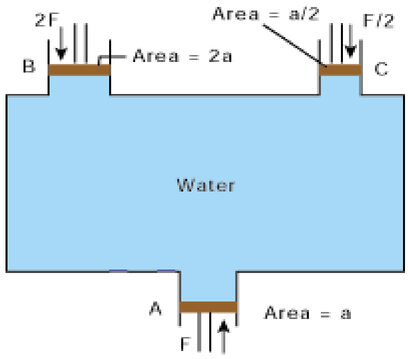
Liquid Pressure:- The normal force (or thrust) exerted by liquid at rest per unit area of the surface in contact with it, is called pressure of liquid or hydrostatic pressure.

$$Pressure\ at\ a\ point\ is\ P = \frac{total\ thrust}{total\ area} \qquad \boxed{P = \frac{A\,h\,\rho\,g}{A} = h\,\rho\,g}$$

3.Pascal's Law- The increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected. $P_1 = P_2 = P_3$ => $F/a = 2F/2a = \frac{F/2}{a/2}$

i.e., F/a in each case.

Example : Hydraulic lift, hydraulic press and hydraulic brakes



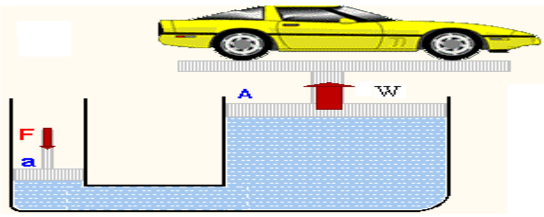


Fig- Hydraulic lift

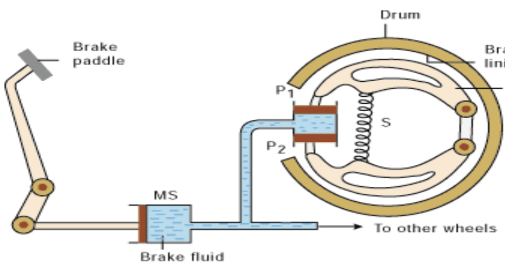
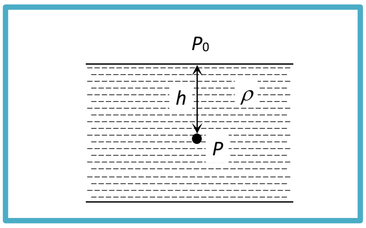


Fig-Hydraulic brakes

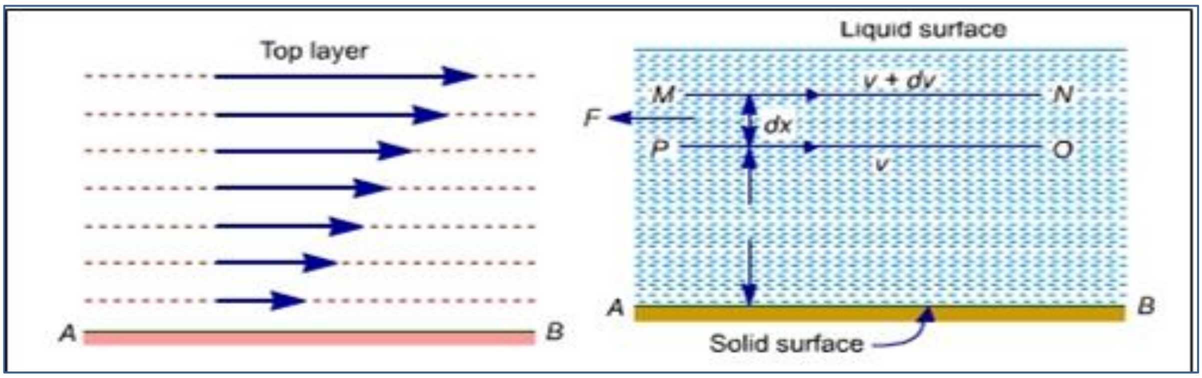
4.Effect of gravity on fluid pressure-

- If P_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ, hydrostatic pressure P is given by $P = P_0 + h\rho g$
- The pressure difference between hydrostatic pressure P and atmospheric pressure P_0 is called gauge pressure. $P - P_0 = h\rho g$



5.Viscosity and Newton's law of Viscous Force:-Property of liquid by virtue of which a liquid opposes the relative motion between the different layers is called viscosity. Property of liquid fluids due to which a backward dragging force or a viscous drag act tangentially on the layers of the fluid in motion and it tries to stop the motion

According to Newton the backward tangential force F on any; layer depends on following factor $F \propto A$ - (1)



$F \propto \frac{dv}{dx}$ - - - - (2)Combined these two equations $\boxed{F = \eta A \frac{dv}{dx}}$, Where $\boxed{\eta = -\frac{F}{A\frac{dv}{dx}}}$ is proportionality constant called coefficient of viscosity and − sign indicate the direction of force is opposite to direction of motion. If A =1 and $\frac{dv}{dx} = 1$ *per* sec then η = – F

- The coefficient of viscosity is defined as the viscous force acting per unit area between two layers moving with unit velocity gradient.
- Dimensional formula [$M^1 L^{-1} T^{-1}$]
- **Units**: -(1) in CGS the unit of η is dyne × sec /m or $gm^1cm^{-1}s^{-1}$ and is called poise (2) In SI, the unit of η is N× s/m or kg/(m× s) and is called 1 decapoise. 1decapoise =10 Poise.
- Viscosity of liquid is much greater (about 100 times more) than that of gases i.e. $\eta_L > \eta_G$
- **Dependence of viscosity:**
  (1) **On temperature of fluid** (2) **On pressure of fluid**

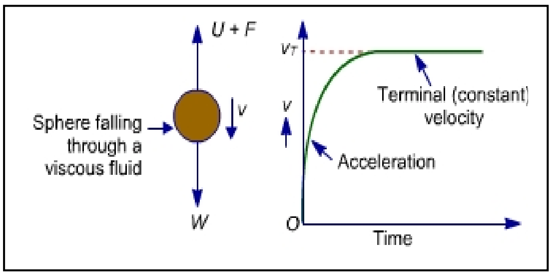
6.Stoke law- If sphere of radius r moving with uniform velocity through a medium of viscosity η then backward dragging (Viscous) force **F = 6 π η r v** this is Stoke's law

Derivation: from dimension analysis

7.Terminal velocity- The constant maximum velocity with which the body moves in a fluid is called terminal velocity.

Then U + F = W - - - (1)

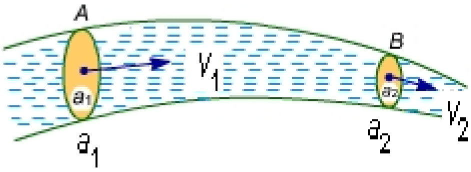
$$\boxed{v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}}$$



From this formula: -1) $v_t \propto r^2$. 2) $v_t \propto 1/\eta$ 3)$v_t \propto (\sigma - \rho)$.

i) If σ > ρ then r. h. s of equation is positive, then direction of terminal velocity is downward. ii) If σ < ρ then r. h. s of equation is negative. Then direction of sphere is upward i.e. rising up. For this reason air bubbles in a liquid are seen to move in upward direction.

8.Equation of continuity- In a time Δt, the volume of liquid entering the tube of flow in a steady flow is $a_1 V_1 \Delta t$. The same volume must flow out as the liquid is incompressible. The volume flowing out in Δt is $aV \Delta t$.



$\boxed{a_1 \times v_1 = a_2 \times v_2}$ This equation is called equation of continuity. In general
$a \times v$ =constant .

- The product ($a \times v$) represents volume flux or flow volume (rate) of flow of liquid in given time.
- $a\, v$ = constant, $v \propto 1/a$

9.Reynolds number-The nature of flow of a liquid through a pipe depends upon four factors.1) Density of the liquid 2) velocity of liquid 3) Diameter of the pipe 4) coefficient of viscosity $v \propto \frac{\eta}{D\,\rho}$ Hence

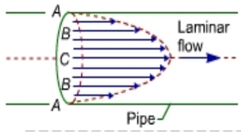
$v = R \frac{\eta}{D\,\rho}$ Here $\boxed{R = \frac{\rho\, v D}{\eta}}$ is called Reynolds number. This is a pure number and it is a dimensionless quantity.It is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid. $R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}}$

10.Streamline,Laminar flow and turbulent flow-

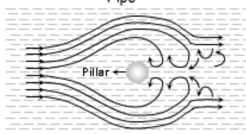
Stream line flow-Stream line flow of a liquid is that flow in which each element of liquid passing through a point travels along the same path and with the same velocity as the preceding element passes through that point.In this flow, the velocity of liquid flow is always less than the critical velocity of the liquid.Value of R is less than 2000 the flow of is said to be laminar or streamline.

streamline flow.

Laminar flow-The laminar flow is generally used synonymously with streamlined flow.When value of R lies between 2000 and 3000 the flow is unstable changing from streamline to turbulent flow.



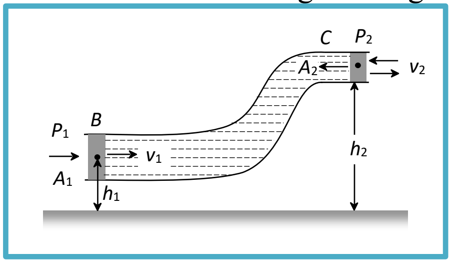
Turbulent flow-. 2.When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid becomes disordered or irregular. Such a flow is called a turbulent flow.when the value of R is more than 3000 then the flow is turbulent.



11.Energy of liquid inn motion-: liquid in motion possess three types of energy.

Kinetic energy	**Potential energy**	**Pressure energy**
The energy due to its motion K.E = ½ mv^2 K. E per unit mass = ½ v^2 K. E per unit volume =½ $\rho\, v^2$ K.E per unit weight ½ (v^2 /g) called velocity head	the energy due to position P.E.= m g h P. E. per unit mass= gh P. E. per unit volume=ρ g h P.E. per unit weight = h potential head	the energy due to pressure exerted on the liquid while it is flowing Pre Energy=PV Pre energy per unit mass=P/ρ Pre energy per unit volume=P Pre energy per unit weight= P/ρg called pressure head

12.Bernoulli's Theorem-According to this theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow, provided there is no source or sink of the fluid along the length of the pipe.



Mathematically for unit volume of liquid flowing through a pipe.

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

13.Application of Bernoulli's Theorem-

- Attraction between two closely parallel moving boats (or buses))
- Working of an aeroplane
- Action of atomiser
- Blowing off roofs by wind storms
- Magnus effect
- Torricelli's theorem:– Velocity of Efflux :–Statement– "The velocity of efflux of a liquid from a hole is equal to the velocity of a body falling freely from height h *i.e.* $V = \sqrt{2gh}$ "

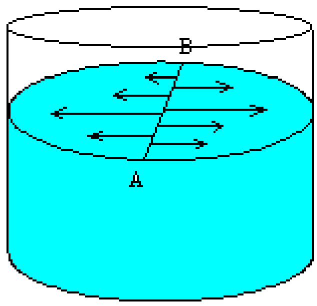
Suggested video link- https://youtu.be/S8iT7ACAl7s?si=4rhoAh-JgQfr4DUW

14.Intermolecular forces-The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic. The intermolecular forces of attraction may be classified into two types.

1.Cohesive force- The force of attraction between molecules of same substance. Ex. (i) Two drops of a liquid coalesce into one when brought in mutual contact.(ii) It is difficult to separate two sticky plates of glass welded with water.

2.Adhesive force- The force of attraction between the molecules of the different. Ex. (i) Adhesive force enables us to write on the blackboard with a chalk.(ii)Water wets the glass surface due to force of adhesion.

15.SURFACE TENSION- The property of a liquid due to which its free surface tries to have minimum surface area and behaves as if a stretched elastic membrane is called surface tension. Surface tension =



$$\boxed{\frac{Force}{Length\ \ of\ \ the line} = \frac{F}{l}}$$

If L = 1 then T = F so,Tangential force acting per unit length on either side of the imaginary line drawn on the liquid surface at rest is called **surface tension**. It is denoted by 'T'. Its S I unit is N/m and C.G.S.Dyne /cm. It's dimension formula is $[M^1L^0T^{-2}]$

16.SURFACE ENERGY-We know that due S.T. liquid has a tendency to contact and occupy minimum surface area .If the area of liquid has to be increased we have to do work against the force of surface tension. This work done is stored in the liquid surface film as its P.E.The potential energy per unit area of the surface film is called surface energy.

Amount of work done in increasing the area of the surface film through unity is called surface energy.

Surface energy = S.T. × Area , S.T. $= \dfrac{Surface\ energy}{Area}$

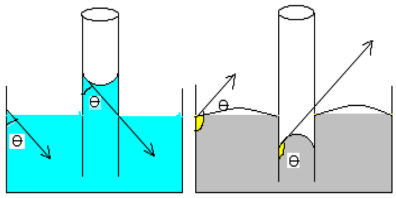
So, surface tension is equal to surface energy per unit area.

17.EXCESS OF PRESSURE INSIDE A LIQUID DROP AND A BUBBLE: The difference in pressure depends upon the S.T of liquid and radius of curvature of the liquid surface. So in equilibrium the pressure inside a bubble or drop is greater than outside and the difference of pressure between two sides of the liquid surface is called excess pressure. In case of a drop, excess pressure is provided by $P_{IN} - P_{OUT} = P = \frac{2T}{R}$

In case of a bubble, excess pressure is provided by $P_{IN} - P_{OUT} = P = \frac{4T}{R}$

So, excess pressure is directly proportional to S.T & inversely proportional to the radius.

18.ANGLE OF CONTACT- -Angle made by the tangent to the curved liquid surface at the point of contact of the liquid surface at the point of contact of the liquid surface with the solid surface inside the liquid is called angle of contact.In the case of water the angle of contact is acute 35°. Where in case of mercury it is obtuse = 130°.For pure water θ = 0°.



The value of angle of contact :– (1) Depends upon the nature of the liquid and solid in contact. (2) Depends upon the medium which exists above the free surface of liquid (3) Is independent of the inclination of the solid to liquid surface.(4) Is fixed for a given pair of solid and liquid and surrounding medium.(5) $\theta = 140^\circ$ for mercury and glass. (6) $\theta = 8^\circ$ for water and glass. (7) $\theta = 0^\circ$ for pure water and glass.

19.CAPILLARITY- A very fine bore is known as the capillary (hair like). The property of rise and depression of a liquid inside a capillary tube is called capillarity. .

Ascent Formula: $\boxed{T = \frac{h\ r\ d\ g}{2\cos\theta}}$ here 'r'-radius of the capillary tube, 'h'-height of the liquid tube,'d'-density of the liquid and Θ –angle of contact.

Tube of Insufficient length: - Since $T = \frac{h\ R\ d\ g}{2} \Rightarrow h\ R = \frac{2T}{d\ g}$

$h\ R$ = Constant . i.e. $hR = LR'$ If capillary tube is small i.e. h is small $\because L < h \therefore R' > R$

So if tube is small then radius of curvature of meniscus increases so that the product of h and R remain constant. Hence the meniscus adjusts its shape and become flatter.

21.FACTORS AFFECTINNG SURFACE TENSION- The surface tension of a liquid depends upon (*i*) its nature (*ii*) contamination and (*iii*) temperature.

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Suggested video link- https://youtu.be/AgbDxs75dHc?si=-rBg52v5vLjHlqm9

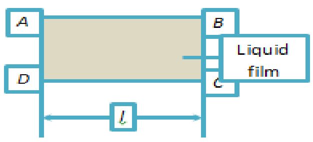
MULTIPLE CHOICE QUESTIONS

1. After terminal velocity is reached, the acceleration of a body falling through a fluid is
(a) equal to g (b) zero (c) less than g (d) greater than g

2. A wooden stick 2*m* long is floating on the surface of water.The surface tension of water 0.07 *N*/*m*. By putting soap solution on one side of the sticks the surface tension is reduced to 0.06 *N*/*m*. The net force on the stick will be
(a)0.07 *N* (b)0.06 *N* (c)0.01 *N* (d)0.02 *N*

3. A 10 *cm* long wire is placed horizontally on the surface of water and is gently pulled up with a force of $2\times10^{-2}N$ to keep the wire in equilibrium. The surface tension in Nm^{-1} of water
(a)0.1 *N/m* (b)0.2 *N/m* (c)0.001 *N/m* (d) 0.002 *N/m*

4. A liquid film is formed over a frame *ABCD* as shown in figure. Wire *CD* can slide without friction. The mass to be hung from *CD* to keep it in equilibrium is



(a) $\frac{Tl}{g}$ (b) $\frac{2Tl}{g}$ (c) $\frac{g}{2Tl}$ (d) $T\times l$

5. Radius of a soap bubble is increased from R to $2R$ work done in this process in terms of surface tension is

(a) $24\pi R^2 S$ (b) $48\pi R^2 S$ (c) $12\pi R^2 S$ (d) $36\pi R^2 S$

6. If pressure at half the depth of a lake is equal to 2/3 pressure at the bottom of the lake then what is the depth of the lake?

(a) 10 *m* (b)20 *m* (c)60 *m* (d)30 *m*

7. Spherical balls of radius 'r' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is

(a)Inversely proportional to 'r' but directly proportional to velocity 'v'

(b)Directly proportional to both radius 'r' and velocity 'v'

(c)Inversely proportional to both radius 'r' and velocity 'v'

(d)Directly proportional to 'r' but inversely proportional to 'v'

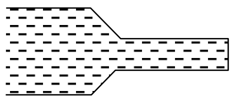
8. A cylindrical tank has a hole of 1 cm^2 in its bottom. If the water is allowed to flow into the tank from a tube above it at the rate of 70 cm^3/sec. then the maximum height up to which water can rise in the tank is

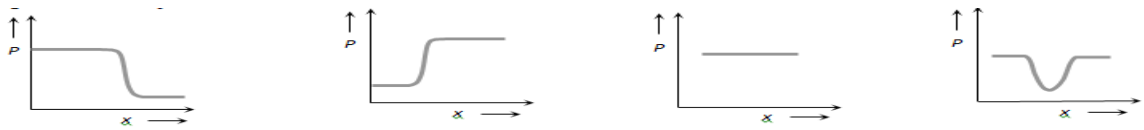
(a)2.5 *cm* (b)5 *cm* (c)10 *cm* (d)0.25 *cm*

9. There is a hole in the bottom of tank having water. If total pressure at bottom is 3 *atm* (1 *atm* = $10^5 N/m^2$) then the velocity of water flowing from hole is **[CPMT 2002]**

(a) $\sqrt{400}\ m/s$ (b) $\sqrt{600}\ m/s$ (c) $\sqrt{60}\ m/s$ (d)None of these

10. Water flows through a frictionless duct with a cross-section varying as shown in fig. Pressure p at points along the axis is represented by





(a) (b) (c) (d)

ANSWERS

1. **(b)** zero (The constant maximum velocity with which the body moves in a fluid is called terminal velocity).

2. **(d)**Force on one side of the stick $F_1 = T_1 \times L$ =0.14N other side of the stick $F_2 = T_2 \times L$ =0.12N So net force on the stick $= F_1 - F_2 = 0.14 - 0.12 = 0.02N$

3. **(a)**Force on wire due to surface tension $F = T \times 2l \Rightarrow \therefore T = \frac{F}{2l} = \frac{2\times 10^{-2}}{2\times 10\times 10^{-2}} = 0.1\ N/m$

4. **(b)**Weight of the body hung from wire (mg) = upward force due to surface tension ($2Tl$) $\Rightarrow m = \frac{2Tl}{g}$

5. **(a)** $W = 8\pi T\left(R_2^2 - R_1^2\right) = 8\pi S[(2R)^2 - (R)^2] = 24\pi R^2 S$

6.**(b)** pressure ay half the depth of a lake $= P_0 + \frac{h}{2}\rho g$

so $P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g) \Rightarrow \frac{1}{3}P_0 = \frac{1}{6}h\rho g \Rightarrow h = \frac{2P_0}{\rho g} = \frac{2\times 10^5}{10^3 \times 10} = 20m$

7. **(b)** $F = 6\pi\eta r v$

8. **(a)**The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

Volume of water flowing out per second = $Av = A\sqrt{2gh}$

and volume of water flowing in per second $= 70\, cm^3/\sec$.

$\therefore\ A\sqrt{2gh} = 70 \Rightarrow 1 \times \sqrt{2gh} = 70 \Rightarrow 1 \times \sqrt{2 \times 980 \times h} = 70\ \ \therefore\ h = \frac{4900}{1960} = 2.5\, cm.$

9. **(b)** Pressure at the bottom of tank $P = h\rho g = 3 \times 10^5 \frac{N}{m^2}$ and velocity of water $v = \sqrt{2gh}$

$\therefore v = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2 \times 3 \times 10^5}{10^3}} = \sqrt{600}\ m/s$

10. **(a)**When cross section of duct decreases the velocity of water increases and in accordance with Bernoulli's theorem the pressure decreases at that place.

A-R TYPE QUESTIONS-

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If the assertion and reason both are false.

1. Assertion : It is easier to spray water in which some soap is dissolved.
Reason : Soap is easier to spread.

2. Assertion : The angle of contact of a liquid decrease with increase in temperature.
Reason : With increase in temperature, the surface tension of liquid increase.

3. Assertion : The impurities always decrease the surface tension of a liquid.
Reason : The change in surface tension of the liquid do not depend upon the degree of contamination of the impurity.

4.Assertion : The water rises higher in a capillary tube of small diameter than in the capillary tube of large diameter.
Reason : Height through which liquid rises in a capillary tube is inversely proportional to the diameter of the capillary tube.

5. Assertion : When height of a tube is less than liquid rise in the capillary tube, the liquid does not overflow.
Reason : Product of radius of meniscus and height of liquid in capillary tube always remains constant.

6. Assertion : The concept of surface tension is held only for liquids.
Reason : Surface tension does not hold for gases.

7. Assertion : At critical temperature, surface tension of a liquid becomes zero.
Reason : At this temperature, intermolecular forces for liquids and gases become equal. Liquid can expand without any restriction.

ANSWERS

1.**(c)** When a liquid is sprayed, the surface area of the liquid increases. Therefore, work has to be done in spraying the liquid, which is directly proportional to the surface tension.
Because on adding soap, surface tension of water decreases, the spraying of water becomes easy.

2. **(c)** With increase in temperature surface tension of the liquid decreases and angle of contact also decreases.

3.(d) The presence of impurities either on the liquid surface or dissolved in it, considerably affect the force of surface tension, depending upon the degree of contamination. A highly soluble substance like sodium chloride when dissolved in water increase the surface tension. But the sparing soluble or substance like phenol when dissolved in water reduces the surface tension of water.

4.**(a)** For smaller r the value of h is higher.

5.**(a)** As h decreases, R increases and the liquid meniscus becomes more and more flat, but the liquid does not overflow.

6. **(b)** We know that the intermolecular distance between the gas molecules is large as compared to that of liquid. Due to it the forces of cohesion in the gas molecules are very small and these are quite large for liquids. Therefore, the concept of surface tension is applicable to liquid but not to gases.
7. **(a)** Zero surface tension means no opposition to expansion.

2-MARKS QUESTIONS

1. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius r = 2.5 × 10^{-5} m. The surface tension of sap is T = 7.28 × 10^{-2} N/m and the angle of contact is 0^o. Does surface tension alone account for the supply of water to the top of all tress?(density103 kg/m^3)

Answer-r = 2.5 × 10^{-5} m, T = 7.28 × 10^{-2} N/m, θ = 0º, ρ = 103 kg/m^3,
The maximum height h is given as: h=$\frac{2T\cos\theta}{r\rho g}$(Substituting the values we get, h = 0.6 m)

2. The surface tension and vapour pressure of water at 20ºC is 7.28 × 10^{-2} N/m and 2.33 × 10^3 Pa, respectively. What is the radius of the smallest spherical water droplet which can form without evaporating at 20ºC?

Answer- T = 7.28 × 10^{-2} N/m, , P = 2.33 × 10^3 Pa

Radius of drop = r, 2T/r is the excess pressure which is greater than the vapour pressure.

Vapour pressure = excess pressure in drop so we can calculate r = 6.25 × 10^{-5} m

3. Water rises in a vertical capillary tube upto a height of 2.0 *cm*. If the tube is inclined at an angle of 60^o with the vertical, then upto what length the water will rise in the tube.

Answer- The height upto which water will rise $l = \frac{h}{\cos\alpha} = \frac{2cm}{\cos 60} = 4cm$. [h = vertical height, α = angle with vertical]

4. What is force on the base of a tank of base area 1.5 m^2 when it is filled with water up to a height of 1m ($\rho_{water} = 10^3\ kgm^{-3}$, P= 1.013 × 10^5 g=10 m s^{-1})

Answer- Absolute pressure at the bottom of the container is

P =$P_o + h\rho g$ =1.01 × 10^5 + 1 × 10^3 × 10 = ′ 1.1× 10^5 Pa

Then force on the base is $F_{base} = PA = 1.1 \times 10^5 \times 1.5$ =1.65x10^5N

5.Write Pascal Law. Name any two applications of Pascal's Law

Answer-Pascal's Law:- According to this law, "If we neglect the effect of gravity, pressure is equally transmitted in all directions "to all other points of the liquid. The two applications of Pascal's Law are :-

a) Hydraulic Lift b) Hydraulic Brake

6. Explain the effect of Temperature on viscosity of (a)Liquid (b)Gases

Answer- Effect of temperature: With increase in temperature, viscosity of liquids decrease and viscosity of gases increase.

7.A rectangular plate of 10cm X 5cm is moving at constant rate of 2 cm/s on a 0.5 mm thick layer of glycerine on a horizontal table at **20**°C. If the coefficient of viscosity of glycerine at **20**°C is $\boldsymbol{830\ mPl}$, find the velocity gradient and force acting on the plate.

Answer- Given : Δv = 2 cms^{-1} , Δx = 0. 5 mm ,velocity gradient or strain rate = Δv / Δx

= 2 cms^{-1}/0. 5 mm = 40s^{-1} , = 830 × 10^{-3} Pl ; $A = (10 \times 10^{-2}) \times (5 \times 10^{-2}) = 5 \times 10^{-3}\ m^2$

,We know $\boldsymbol{F} = -\boldsymbol{\eta A}\ \boldsymbol{\Delta v}\ \boldsymbol{\Delta x}$ => $F = 830 \times 10^{-3} \times 5 \times 10^{-3} \times 40$ N

F= 0.166 N

3-MARKS QUESTIONS

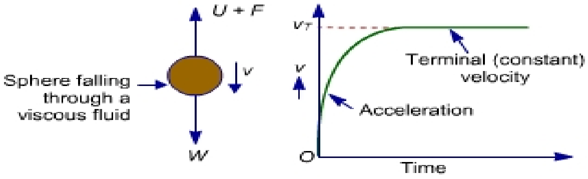
1.Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of 6 cms^{-1} . If they coalesce to form one big drop, what will be the terminal speed of bigger drop? (Neglect the buoyancy of the air)

Answer- We have $8 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ => R = 2r

And we know that terminal velocity $v_T \propto (Radius)^2$ We have

$\frac{V_T}{v_T} = \frac{R^2}{r^2}$ or $V_T = v_T \frac{(2r)^2}{r^2}$ $V_T = 0.24$ m/s

2. Derive the expression for terminal velocity of a rain drop falling in a vessel filled with fluid.



Answer- Terminal velocity- when a body is dropped in a liquid, it, accelerates first due to gravity. In liquid it experience weight (force) 'W' along downward, buoyancy force 'U' along upward. Since downward force is greater so the body moves along downward. As body in motion the layers of air in contact with body also set in motion due to this an upward viscous force 'F' act on the body. As velocity of the solid body increases Value of 'F' increases. But after same time the downward force 'W' become exactly equal to the total upward force (U+F). Now there is no net force acting on the body so from Newton's first law the velocity becomes constant (and maximum).

The constant maximum velocity with which the body moves in a fluid is called terminal velocity.

When the spherical body of radius 'r' moving with constant terminal velocity v_t. Then U + F = W - - - (1)

U = weight of the liquid displaced = mass of the liquid displaces × g = σ V g

W = weight of the solid = mass of the solid sphere × g = ρ V g

From Stoke's law F = 6 π η r v_t (Where ρ is density of ball, σ is density of liquid.)

$\sigma V g + 6\pi\eta r v_t = \rho V g \Rightarrow 6\pi\eta r v_t = \sigma V g - \rho V g \Rightarrow 6\pi\eta r v_t = V g(\sigma - \rho)$

$6\pi\eta r v_t = \frac{4}{3}\pi r^3 g(\sigma - \rho)$ Hence $\boxed{v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}}$ From this formula: -

1) $v_t \propto r^2$. 2) $v_t \propto 1/\eta$ 3) $v_t \propto (\sigma - \rho)$.

i) If σ > ρ then r. h. s of equation is positive, then direction of terminal velocity is downward. ii) If σ < ρ then r. h. s of equation is negative. Then direction of sphere is upward i.e. rising up. For this reason air bubbles in a liquid are seen to move in upward direction.

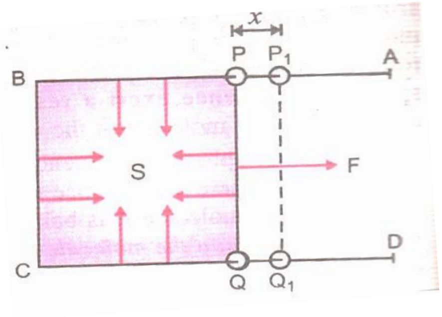
3. A water drop of diameter 2mm is split up into 10^9 identical water drops. Calculate the work done in this process. (The surface tension of water is $7.3 \times 10^{-2}\ Nm^{-1}$)

Answer- Let a water drop of radius R be split up into 10^9 identical water drops each of radius r.(R = D/2 = 2/2 = 1 mm= 1×10^{-3}m ,No. of droplets n = 10^9; T = 7.3×10^{-2}N/m)

W = $4\pi R^2 T\ [n^{1/3} - 1] = 4\pi\ (10^{-3})2 \times 7.3 \times 10^{-2}\ [(10^9)1/3 - 1] = 9.17 \times 10^{-4}$ J

4. Define Surface Energy and derive the expression of relation between surface energy and surface tension.

Answer-SURFACEENERGY: To bring a water molecule from the interior of the liquid to the free surface, some work is required to be done which will be stored as potential energy of the increased area. This potential energy of increased area is called surface energy.



SURFACE ENERGY: Let us consider a rectangular frame of wire PQRS, where arm RS can slide on the arm PR and QS. If this frame is dipped in a soap solution, then a soap film is produced in the frame PQRS.Due to surface tension, the soap film exerts a force on the frame.

Let 'l' be the length of the arm RS, then the force acting on the arm RS towards the film is given by

F=T×2l ……… (1)

Here the length is taken twice due to the two-surface area of soap film. Let the arm RS be displaced to a new position 'AB' through a distance 'x'. Hence work done is W=Force× Displacement or

W=(T×2l) ×x …………(2) W=T×(2l×x)Work E=T×ΔA ……...(3) Which is required expression for surface energy Where ΔA=2(l× x)= increase in the area of the film on both sides T= W /ΔA or T= E/ ΔA

"surface tension of a liquid is numerically equal to the surface energy per unit area of the liquid surface."

5.Prove that excess pressure inside a soap bubble is inversely proportional to radius of bubble.

Answer- we will derive the expression for the excess pressure inside a soap bubble.
The work done by the excess pressure inside a soap bubble in displacing the surface is given as follows.
dW=F×s ⇒dW=p×4πR^2×dR …… (1)
In the above equation, we have substituted the surface area of the sphere. The increase in the potential energy is calculated as follows,
dU=T×A_{inc} dU=T[2{4π$(R + dR)^2$−4πR^2}]
Continue the further calculation.
dU=T[2{4π(R^2+dR^2+2R×dR)−4πR^2}]
dU=T[2{4πR^2+4πdR^2+4π×2R×dR−4πR^2}]
dU=T[2{4πdR^2+4π×2R×dR}]
The first within the bracket can be equated to zero because of the negligible value. Thus, we get,
dU=T[2{0+4π×2R×dR}] or dU=T[2{4π(2RdR)}]
From the equations (1) and (2), we get,

p×4πR^2×dR=T[2{4π(2RdR)}] ⇒ p=4T/R

Where T is the surface tension and R is the radius of the soap bubble.

Here, the surface tension of the soap bubble can be considered to be a constant value.

Therefore, the excess pressure inside a soap bubble is inversely proportional to its radius.
Means critical velocity is the value of velocity of flow of liquid below which flow is streamlined and above which it become turbulent.

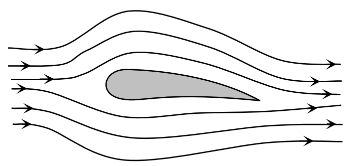
6. A bucket of water is suspended from a spring balance. Does the reading of balance change (a) when a piece of stone suspended from a string is immersed in the water without touching the bucket? (b) when a piece of iron or cork is put in the water in the bucket?

Answer-(a) Yes, the reading of the balance will increase but the increase in weight will be equal to the loss in weight of the stone $(V\sigma g)$ and not the weight of stone $(V\rho g)[> V\sigma g$ as $\rho > \sigma]$.

(b) Yes, the reading of the balance will increase but the increase in weight will be equal to the weight of iron or cork piece.

7.Explain the correct reason behind these phenomena's (i)Special design of wings of an aeroplane.(ii) A person standing near the railway track is pulled toward the train, when fast moving train passes close to him.

Answer-(i) - Wings of an aeroplane are made such that the upper surface is made wing in as compared to the lower surface of the wing is made concave. Due to their shape of wing, the air currents at the top have a large velocity then at the bottom accordingly to Bernoulli their the pressure above the surface of the. This difference of pressure is helpful in giving avertical lift to the plane. But if upper surface is more covered then the airflow does not remain steady and turbulence is set in.



(ii) A person standing near the railway track is pulled toward the train, when fast moving train passes close to him. The fast train large velocity decrees the pressure between the person and the train while pressure on the other side of man is large which pushes the person towards the track.

5-MARKS QUESTIONS

1. (a)Define different units of pressure and also find the height of atmospheric pressure.

(b) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure dp over a differential height dh?

(c) A fish is swimming 10 m below the surface of water in a lake. Estimate the pressure on it.. (1 + 2 + 2)

Answer-(a) Units of pressure are: 1atm=Pressure exerted by 0.76m of mercury column=hρg

$=0.76\times13.6\times10^3\times9.8$

$=1.013\times10^5 \text{N}/m^2$ or$=1.013\times10^5$Pa

Another units (i)1torr=Pressure exerted by 1mm of mercury column or 1mm of Hg=133.28Pa

760 torr=1atm

1bar=10^5Pa

Height of the atmosphere: The atmospheric pressure, Pa =hρg

or h= P a/ ρg = 1.013×10^5/ 1.3×9.8 = 7951m ≈ 8km

(b)Consider a part (packet) of atmosphere of thickness dh, As the pressure at a point in fluid is equal in all directions. So the pressure on upper layer is p acting downward and on lower layer is (p+dp) acting upward. Force due to pressure is balanced by Buoyant force by air (p+dp) A−p.A=−Vρg

p.A+dpA−pA=−A(dh)ρg

dp.A=−ρpg(dh)A

dp/p=−ρg(dh)

Negative shows that pressure decreases as height increases.

(c) $P_a \approx 10^5 Pa$(1 atm), g = 10 ms^{-2} Density of water ρ = 10^3 kgm^{-3})

Absolute Pressure **P** = P_a + h ρ g

Absolute Pressure **P** = $10^5 + 10 \times 10^3 \times 10$

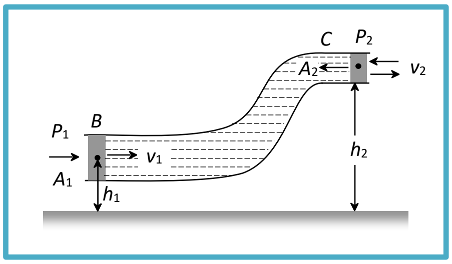
Absolute Pressure **P** = 2×10^5≈ 2 atm.

2. (i)State and prove Bernoulli's theorem. (3 + 2)

(ii)Explain any two applications of Bernoulli's theorem.

Answer- According to this theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow, provided there is no source or sink of the fluid along the length of the pipe.

Mathematically for unit volume of liquid flowing through a pipe. $P+\rho gh+\frac{1}{2}\rho v^2$ = constant



To prove it consider a liquid flowing steadily through a tube of non-uniform area of cross-section as shown in fig. If P_1 and P_2 are the pressures at the two ends of the tube respectively, work done in pushing the volume V of incompressible fluid from point B to C through the tube will be $W=P_1V-P_2V=(P_1-P_2)V$ ......(i)

This work is used by the fluid in two ways.

(i) In changing the potential energy of mass m (in the volume V) from mgh_1 to mgh_2,

i.e., $\Delta U = mg(h_2-h_1)$ ......(ii)

(ii) In changing the kinetic energy from $\frac{1}{2}mv_1^2$ to $\frac{1}{2}mv_2^2$, *i.e.,* $\Delta K=\frac{1}{2}m(v_2^2-v_1^2)$ ......(iii)

Now as the fluid is non-viscous, by conservation of mechanical energy

$$W = \Delta U + \Delta K$$

i.e., $$(P_1 - P_2)V = mg(h_2 - h_1) + \frac{1}{2}m(v_2^2 - v_1^2)$$

or $$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

This equation is the so-called Bernoulli's equation and represents conservation of mechanical energy in case of moving fluids.

(i) Bernoulli's theorem for unit mass of liquid flowing through a pipe can also be written as: $\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$

(ii) Dividing above equation by g we get $\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$

Here $\frac{P}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head. From this equation Bernoulli's theorem can be stated as.

In stream line flow of an ideal liquid, the sum of pressure head, gravitational head and velocity head of every cross section of the liquid is constant.

Explain any two---(1+1)

CASE BASED QUESTIONS

1. The property due to which the free surface of liquid tends to have minimum surface area and behaves like a stretched membrane is called surface tension. It is a force per unit length acting in the plane of interface between the liquid and the bounding surface *i.e.*, S = F/L, where F = force acting on either side of imaginary line on surface and L = length of imaginary line. Surface tension decreases with rise in temperature. Highly soluble impurities increases surface tension and sparingly soluble impurities decreases surface tension.

(i) The excess pressure inside a soap bubble is three times than excess pressure inside a second soap bubble, then the ratio of their surface area is
(a) 9 : 1 (b) 1 : 3 (c) 1 : 9 (d) 3 : 1

(ii). Which of the following statements is not true about surface tension?
(a) A small liquid drop takes spherical shape due to surface tension.
(b) Surface tension is a vector quantity.
(c) Surface tension of liquid is a molecular phenomenon.
(d) Surface tension of liquid depends on length but not on the area.

(iii) Which of the following statement is not true about angle of contact?
(a) The value of angle of contact for pure water and glass is zero.
(b) Angle of contact increases with increase in temperature of liquid.
(c) If the angle of contact of a liquid and a solid surface is less than 90°, then the liquid spreads on the surface of solid
(d) Angle of contact depend upon the inclination of the solid surface to the liquid surface.

(iv)A liquid does not wet the solid surface if the angle of contact is
(a) 0° (b) equal to 45° (c) equal to 90° (d) greater than 90°

Answer
(i). (c) (ii)(b) (iii)(d) (iv) (d)

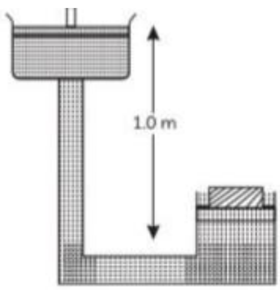
2. A hydraulic lift is a device for moving objects using force created by pressure on a liquid inside a cylinder that moves a piston upward. Incompressible oil is pumped into the cylinder, which forces the piston upward. When a valve opens to release the oil, the piston lowers by gravitational force. The principle for hydraulic lifts is based on Pascal's law for generating force or motion, which states that pressure change on an incompressible liquid in a confined space is passed equally throughout the liquid in all directions. The concept of Pascal's law and its application to hydraulics can be seen in the example below, where a small amount of force is applied to an incompressible liquid on the left to create a large amount of force on the right.

(i) The rectangular vessel's base measures 10 cm x 18 cm. Mixture is pumped into the hole to a depth of 4 cm. What is the base's thrust? [g= 10 m/s².] (density=1000 kg/m^3)
(a) 7.2 N (b) 8.3 N (c) 5.7 N (d) 7.5 N

(ii) A girl weighs 50 kg and balances on a single heel while wearing high heel shoes. The heel is circular, with a 1 cm diameter. How much pressure does the heel put on the horizontal floor?
(a) 8.24 x 10^6 Nm^{-2} (b) 6.24 x 10^6 Nm^{-2} (c) 10.24 x 10^6 Nm^{-2} (d) 2.24 x 10^6 Nm^{-2}

(iii) A hydraulic automobile lift is designed to lift vehicles weighing up to 3000 kg. The piston carrying the load has a cross- sectional area of 425 cm². What is the maximum pressure that the smaller piston must withstand? (a) 6.92 x 10^5 Nm^{-2} (b) 6.02 x 10^4 Nm^{-2} (c) 6.92 x 10^3 Nm^{-2} (d) 6.92 x 10^2 Nm^{-2}

(iv) A hydraulic press with a larger piston of diameter 30 cm at a height of 1.0 m above the smaller piston of diameter 5 cm is shown in the figure. The smaller piston has a mass of 10 kg. What is the force that the larger piston exerts on the load? Oil in the press has a density of 750 kg/m³.
(a) 3000.0 N (b) 3009.6 N (c) 3009.3 N (d) 3009.2 N

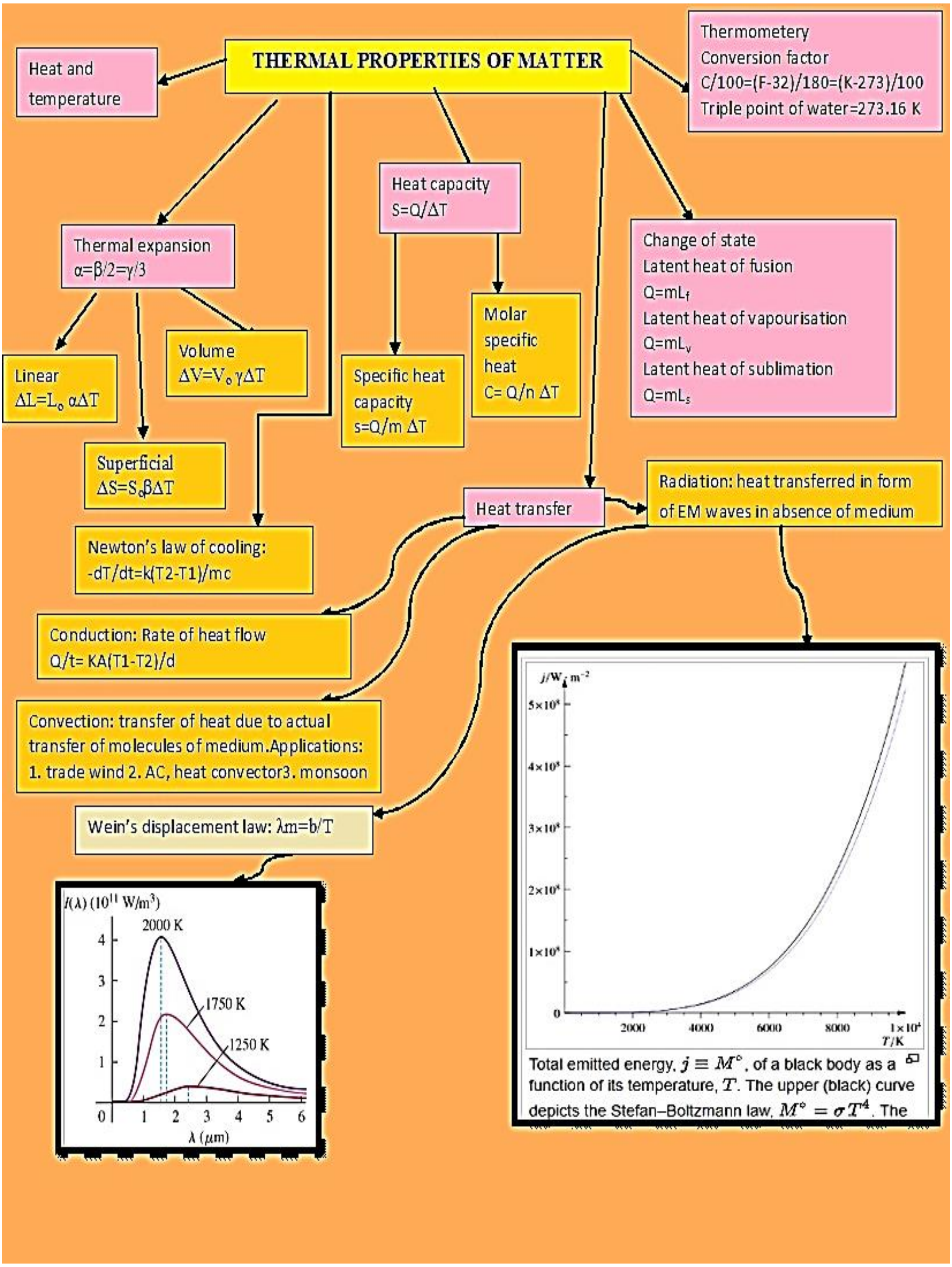


ANSWER
(i). (a) (ii)(b) (iii)(a) (iv) (b)

CHAPTER-10: THERMAL PROPERTIES OF MATTER

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wein's displacement Law, Stefan's law.

MIND MAP



GIST OF THE CHAPTER

Heat vs. Temperature

Heat	Temperature
Heat is energy in transit.	Temperature measures the degree of hotness or coldness of a body.
Heat causes changes,	Temperature is the effect of heat.

Measuring Temperature: Thermometers

Temperature is measured using devices called **thermometers**. The most common type is the **liquid-in-gas thermometer**, which uses the expansion of liquids like mercury or alcohol to measure temperature.

Calibration of Thermometers

To define standard temperature scales, two reference points are commonly used:

1. The freezing point of water
2. The boiling point of water at standard pressure

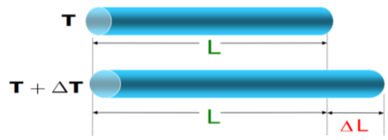
Temperature Scales: Celsius, Fahrenheit, and Kelvin

Conversion Formula: C/100=(F-32)/180

Thermal Expansion: Solids

When a solid's temperature increases, its molecules vibrate more, this, in turn, results in an increase in the dimensions of the body. This increase in the dimension is known as the Thermal Expansion. You can loosen a metal jar lid by running hot water over it.

Types of Thermal Expansion:



1. **Linear Expansion**: Expansion in length.

Coefficient of Linear Expansion: It is the fractional change in length per degree change in temperature. It is represented by

$$\alpha = \frac{\Delta L}{L\Delta T}$$

2. **Area Expansion**: Increase in surface area.

It is represented as:

$$\beta = \frac{\Delta s}{s\Delta T}$$

3. **Volume Expansion**: Increase in the volume of the solid. It is represented by

It is represented as:

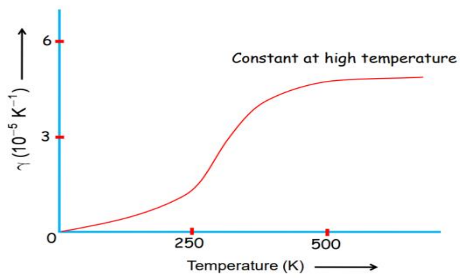
$$\gamma = \frac{\Delta V}{V\Delta T}$$

Thermal Expansion of Solids: Examples

- A **metal jar lid** can be loosened by running hot water over it, causing the metal to expand.

Relationship Between Coefficients:

β=2α and γ=3α



Thermal Stress

When a material is prevented from expanding or contracting due to temperature changes, **thermal stress** develops. For example, a rod that is rigidly fixed at both ends will develop compressive stress if the temperature increases.

Thermal Stress Formula:

Stress=YαΔT Where:

Applications of Thermal Expansion

- **Railway Tracks**: Small gaps are left between rails to allow expansion during hot weather.
- **Steel Bridges**: One end is fixed, while the other rests on rollers to accommodate expansion.

Anomalous Expansion of Water

Water behaves differently from most substances. When cooled below 4°C, instead of continuing to contract, it begins to expand. This unique behavior is why **water pipes burst in winter**.

Thermal Expansion of Gases

Gases expand more than solids and liquids when heated. The coefficient of volume expansion γ\gammaγ for gases is more dependent on temperature.

For an **ideal gas**:

$PV=RT$

Heat Capacity (S) is the amount of heat required to change the temperature of a body. It is measured in **Joules per Kelvin (J/K)**. $S=\Delta Q/\Delta T$

Specific Heat Capacity

The heat capacity per unit mass of a substance. Its SI unit is **J/kg K**.

$s=1/m(\Delta Q/\Delta T)$

Molar Specific Heat Capacity

Defined as the heat capacity per mole of a substance. The SI unit is **J/mol K**.

$C=(\Delta Q/\Delta T)/n=M(\Delta Q/\Delta T)/m=Ms$

At Constant Pressure and Volume:

***Cp**: Molar specific heat capacity at constant pressure.*

Cv: Molar specific heat capacity at constant volume.

Water as a Cooling Agent

- Water, with its high specific heat, is an efficient cooling agent.
- This means that a small amount of water can absorb a large amount of heat with a relatively minor rise in temperature.
- Because of this property, water is widely used in cooling systems for automobiles and engines.
- If a liquid with lower specific heat were used, its temperature would rise significantly for the same heat absorption, making water the preferred choice.

Law of Heat Exchange

The law of heat exchange can be used to calculate specific heat:

- **Heat lost by body A = Heat gained by body B**

Given two bodies, A and B:

- Body A: Mass mA, specific heat SA, temperature TA
- Body B: Mass mB, specific heat sB, temperature TB

Heat will flow from A to B until both reach a common temperature, T. $m1s1(T1-T)=m2s2(T-T2)$

Change of State

Matter exists in three states: solid, liquid, and gas. When transitioning from one state to another, the temperature remains constant. Common state changes include:

During these processes, the substance absorbs or releases heat, but its temperature doesn't change until the entire material has transformed.

Melting Point

The **melting point** is the temperature at which a solid and its liquid state are in thermal equilibrium. At standard atmospheric pressure, this is known as the normal melting point.

Regelation

Regelation refers to the melting of ice under pressure and its re-solidification when the pressure is removed. A common demonstration involves placing a weighted wire on an ice block. The pressure from the wire causes the ice to melt, allowing the wire to pass through without breaking the ice block.

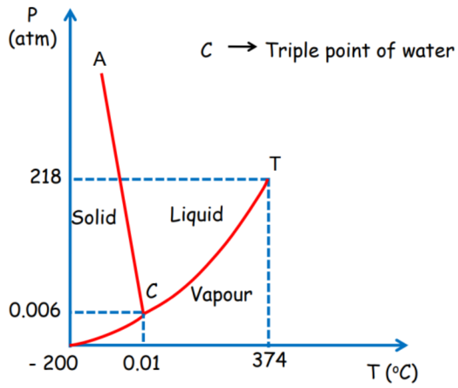
Vaporization and Boiling Point

Sublimation

Sublimation is the direct conversion of a solid into a gas without passing through the liquid phase. Examples include camphor and iodine.

Triple Point

The **triple point** is the specific temperature and pressure where all three states (solid, liquid, and gas) of a substance coexist in equilibrium. For water, the triple point is 273.16 K at 6.11×10^-3 Pa.



Latent Heat

Latent heat is the amount of heat required for a substance to change its state without changing its temperature. It is defined as: L=Q/m

Where Q is the heat energy and m is the mass. There are two types of latent heat:

1. **Latent heat of fusion (Lf)** for solid-liquid transitions.
2. **Latent heat of vaporization (Lv)** for liquid-gas transitions.

Heat Transfer

1. **Conduction**: Transfer of heat through a material by direct contact.
2. **Convection**: Transfer of heat in fluids (liquids or gases) through fluid motion.
3. **Radiation**: Transfer of heat through electromagnetic waves without requiring a medium.

Conduction

In conduction, heat flows through a material due to temperature differences between adjacent parts. A classic example is heating one end of a metal rod; the other end gradually becomes hot due to conduction.

In a **steady state**, no part of the conductor absorbs heat, while in a **variable state**, different sections heat up over time. **Thermal Conductivity**

The rate at which heat flows through a material is determined by its thermal conductivity, K. This is given by:Q=KA (T1-T2)/L

Convection

Convection is the transfer of heat within a fluid (liquids and gases) from regions of higher temperature to lower temperature, aided by the movement of the fluid itself.

Natural Convection in Liquids

When the temperature of the lower region in a fluid is higher than that of the upper region, the density of the lower region decreases.

Buoyancy causes the warmer, less dense fluid to rise while cooler fluid particles from the upper region descend to take their place. This cycle continues until the temperature becomes uniform throughout the fluid.

Natural Convection in Air

- During the day, the ground heats faster than water bodies, causing the air in contact with the ground to heat up, expand, and rise.
- Cooler air replaces this warm air, creating air currents that circulate. At night, this process reverses as the ground cools faster than water.

Sea and Land Breezes

Trade Winds

- Trade winds are steady surface winds that blow from the northeast towards the equator. This happens because the equatorial region receives more solar heat than the polar regions, setting up a convection current.
- Due to Earth's rotation, air descends at about 30° N latitude and returns to the equator.

Forced Convection

In forced convection, the movement of the fluid is driven by external forces such as pumps or fans. For example, the air heating systems used in homes work on the principle of forced convection.

Radiation

Radiation is the transfer of heat through electromagnetic waves, requiring no medium. Solar energy is a prime example of radiation.

- Heat energy travels at the **speed of light** (3×10^8 m/s).
- When thermal radiation falls on an object, part of it is reflected while the rest is absorbed. The **amount of heat absorbed** depends on the color of the body.
- **Black surfaces** absorb and emit radiant energy better than lighter-colored ones. That's why we prefer to wear light-colored clothes in summer and darker ones in winter.

Practical Uses of Radiation

- **Blackened cooking utensils** absorb maximum heat, helping cook food efficiently.
- We see **Newton's law of cooling** apply when objects release heat at a rate proportional to the temperature difference between the object and its surroundings.

MULTIPLE CHOICE QUESTIONS

1. A cup of hot coffee is left on table after some time coffee cools down. What is the mode of transfer of heat for this cooling?
 a) conduction b) convection c) radiation d) all of these

2. A block of ice is heated until it melts completely. What happens to the temperature of ice water mixture during this process?
a) It increases continuously b) it decreases continuously
c) it remains constant d) it increases and then decreases

3. An aluminum piece and a wooden piece have been left in a room for a few hours. When you go and touch them the aluminum seems colder than wood. Select the correct option. Assume room temperature to be 25°.
a) Both have the same temperature greater than 25°
b) Aluminum has a greater temperature than wooden piece
c) Aluminum has a lower temperature than wooden piece
d) Both have the same temperature equal to 25°

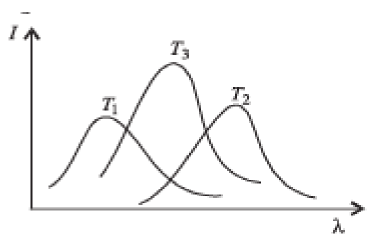
4. An ice cube is kept on the surface of an aluminum block and another ice cube on the surface of a plastic block. Both blocks have the **same temperature**. Select the correct statement.
a) Ice will melt faster on the aluminum block b) Ice will melt faster on the plastic block
c) Ice will melt equally fast on both the blocks d) Ice will not melt on the plastic block

5. What is the slope of line drawn for a graph between temperature in °F & °C?
a) 1.8 b) 5/9 c) slope is variable d) 2

6. A liquid cools from 65°C to 55°C in 5 mins. If the surrounding temperature is 35°C, calculate the time taken for temperature to fall from 45°C to 35°C.
a) 25sec b) 25min c) 30min d) 30sec

7. An aluminium sphere is dipped into water. Which of the following is true?
(a) Buoyancy will be less in water at 0 °C than that in water at 4 °C.
(b) Buoyancy will be more in water at 0 °C than that in water at 4 °C.
(c) Buoyancy in water at 0 °C will be same as that in water at 4 °C.
(d) Buoyancy may be more or less in water at 4 °C depending on the radius of the sphere.

8. The plots of intensity of radiation versus wavelength of three black bodies at temperatures T1, T2 and T3 are shown. Then,



(a) T3 > T2 > T1 (b) T1 > T2 > T3

(c) T2 > T3 > T1 (d) T1 > T3 > T2

SOLUTIONS:

1.Answer d

Explanation: conduction, convection and radiation all contribute to cooling of coffee

2. Answer c

Explanation: during phase change temperature remains constant until phase change is complete

3. Answer: d

Explanation: Both have been left in a room for a few hours, so both will have the same temperature as that of the room. The only reason aluminum feels colder is because it conducts heat away from our body faster than wood.

4. Answer: a

Explanation: Ice will melt faster on the aluminum block because it has a higher thermal conductivity than plastic and hence conducts heat more rapidly than plastic. Which is why ice melts faster on aluminum.

5. Answer: a

Explanation: °F = (9/5)°C + 32. (y=mx+c)

Therefore from this we can say that slope of line drawn for graph between temperature in °F & °C is 9/5 = 1.8.

6. Answer: b

Explanation: Average temperature of liquid in the first case = 60°C.

Rate of fall of temperature = (65-55) / 5 = 2°C /min.

Which is also equal to k (60 – 35) = 25k, where k is a constant.

∴ 25k = 2 Or k = 2/25

Average temperature in the second case = 40°C

Rate of fall of temp = 45 – 35 / t = k (40-35)

∴ t = 25mins.

7. Answer a

Explanation: The density of water is maximum at 4 degrees Celsius

8. Answer: d

Explanation: According to Wien's law if λ is the wavelength at maximum intensity λT = constant. Shorter the wavelength of the peak, greater the temperature for intensity vs λ graph. λmax. is not maximum wavelength but λ at maximum intensity)

ASSERTION AND REASONING TYPE QUESTIONS

1. ASSERTION: Specific heat capacity of a body is always lesser than its heat capacity.
REASON: Heat capacity is heat required to raise temperature of unit mass of substance by unity.

2. ASSERTION: Thermal coefficient of expansion for volume is approximately 3/2 times of superficial coefficient of thermal expansion.
REASON: On heating if change in temperature is same then percentage rise in volume is greater than that in surface area.

3. ASSERTION: If two different substances of equal mass at different temperature are mixed with each other equilibrium temperature is always different from their mean temperature.
REASON: Two substances have different specific heat capacity.

4. ASSERTION: The monsoon is due to convection in fluids.
REASON: Specific heat capacity of water is greater than that of land.
5. ASSERTION: If placed in same refrigerator, iron will always lose heat at higher rate than water at same temperature if their mass is same.
REASON: rate of fall of temperature is directly proportional to temperature difference of system and surrounding only.
6. ASSERTION: A brass disc is just fitted in a hole in a steel plate. The system must be cooled to loosen the disc from the hole.
Reason : The coefficient of linear expansion for brass is greater than the coefficient of linear expansion for steel.
7. ASSERTION: The coefficient of volume expansion has dimension K^{-1}
REASON: The coefficient of volume expansion is thrice of that of coefficient of linear expansion.
8. ASSERTION: The melting point of ice decreases with increase of pressure.
Reason : Ice contracts on melting.

SOLUTION

1. ANSWER: Both assertion and reason are false
Explanation: specific heat capacity of a substance is defined as amount of heat required to raise the temperature of unit mass of that substance by 1°C
Mathematically, s=C/m, if m>1, then s<C but if m<=1 assertion is false.
2 Answer: Assertion and Reason both are true but reason is not correct explanation of assertion
Explanation: since $\alpha=\beta/2=\gamma/3$ => $\gamma=3\beta/2$
$\Delta S=S_o\beta\Delta T$ => $\Delta S/S_o=\beta\Delta T$
Similarly, $\Delta V/V_o=\gamma\Delta T$
Therefore, ratio of percentage rise in volume to percentage rise in surface=$\gamma/\beta=3/2$
Hence percentage rise in volume=3/2 times of percentage rise in surface.
Thus both A and R are true but R is not correct explanation of A.
3. Answer: Assertion and reason both are true and reason is correct explanation of assertion. Explanation: since specific heat capacity is characteristic property of a substance it is different for different substances
Let s1 and s2 be specific heat capacities of two different substances of equal mass at temperature T1 and T2and T is equilibrium temperature then, after mixing by principle of calorimetry.
=> $m_1s_1(T_1-T)=m_2s_2(T-T_2)$ assuming that $T_1>T_2$
=> $s_1T_1-s_1T=s_2T-s_2T_2$ ($m_1=m_2$) => $(s_1+s_2)T=s_1T_1+s_2T_2$
=> T=(s1T1+s2T2)/(s1+s2)
Thus, T is not mean of T1 and T2, until $s_1=s_2$, or $T_1=T_2$
4. Answer: Assertion and Reason both are true and reason is correct explanation of assertion.
Since, specific heat capacity of water is higher than land, ocean/ sea are cooler than land during summer. It creates low pressure area in land and high pressure area in sea. Hence moist air start blowing from sea to land known as monsoon.
5. Answer: assertion is true but reason is false
Explanation: rate of fall in temperature is directly proportional to temperature difference but depends on mass and specific heat capacity as well. Thus reason is not true.
Since specific heat of water is greater than iron, iron will cool down faster than water.
6. Answer: Assertion and Reason both are true but reason is not correct explanation of assertion.
Even if coefficient of linear expansion were equal on cooling inner radius of steel disc will increase due to contraction
7. Answer: Assertion and Reason both are true but reason is not correct explanation of assertion.

Explanation: even if $\gamma=\alpha$, dimension will be K^{-1}
8. Answer: (a) Assertion and Reason both are true and reason is correct explanation of assertion.
With rise in pressure distance between molecules of water decreases thus, melting point of ice can be acquired at lower temperatures.

2 MARKS QUESTION

1. Analyze the situation:
A metal spoon is left in a hot cup of coffee. Why does the handle become hot?
Answer: Conduction is the direct transfer of heat between particles in physical contact.
In this case, the metal spoon is in direct contact with the hot coffee.
Heat energy from the coffee transfers to the spoon handle through conduction, making it hot.
2. Evaluate the statement:
"Water is a good coolant because of its high specific heat capacity."
Is this statement true or false? Explain
Answer: true
* Specific heat capacity is the amount of heat energy required to raise the temperature of a unit mass of substance by 1 degree Celsius.
* Water has a high specific heat capacity (4.186 J/g°C), meaning it can absorb and release large amounts of heat energy without a large change in temperature.
* This makes water an effective coolant, as it can absorb heat from surroundings without rising in temperature quickly.
3. Calculate the heat energy required to raise temperature of 500g water from 20°C to 80°C.
(Specific heat capacity of water = 4.186 J/g°C)
Answer: Given values: Mass of water (m) = 500g = 0.5 kg,Initial temperature (T_1) = 20°C
Final temperature (T_2) = 80°C,Specific heat capacity (c) = 4.186 J/g°C = 4186 J/kg°C
Calculate temperature difference (ΔT) = T_2 - T_1 = 80°C - 20°C = 60°C
Apply formula $Q = mc\Delta T$ Q = 0.5 kg * 4186 J/kg°C * 60°C
=Q = 130940 J or 130.94 kJ
4. Identify the type of heat transfer in a vacuum flask.
Answer: A vacuum flask has minimal air particles between its inner and outer walls.
This minimizes conduction and convection heat transfer. However, heat can still be transferred through:
Radiation: Electromagnetic waves pass through the vacuum, transferring heat. Minimal conduction through the flask's inner and outer wall connection points.
Vacuum flasks reduce heat transfer, keeping liquids hot or cold for longer.
5. Compare latent heat of fusion and vaporization of water.
Answer: Latent Heat of Fusion (LHF) of water: 334 J/g. Energy required to change 1g of ice to water at 0°C.Solid to liquid phase change.
Latent Heat of Vaporization (LHV) of water: 2256 J/g
Energy required to change 1g of water to vapor at 100°C.
Liquid to gas phase change.
Comparison:
LHV is approximately 6.75 times greater than LHF.
More energy is required to change water from liquid to gas than from solid to liquid.
6. Determine the coefficient of linear expansion of a metal rod given ΔL = 0.1m, L_0 = 1m, ΔT = 25°C.
Answer: Formula: $\alpha = \Delta L / (L_0 \ \Delta T)$ given values: Change in length (ΔL) = 0.1 m
Original length (L_0) = 1 m, Change in temperature (ΔT) = 25°C
using values into formula:

$\alpha = 0.1$ m / (1 m X 25°C) =$\alpha = 0.004$ °C^{-1} or 4×10^{-3} °C^{-1} =$\alpha = 4 \times 10^{-3}$ °C^{-1}

3 MARKS QUESTIONS

1. A metal rod 30.0 cm long expands by 0.075 cm when its temperature is raised from 0°C to 100°C.A rod of a different metal and of same length expands by 0.045 cm for the same raise in temperature. A third rod also 30.0 cm long is made up of the pieces of each of the above metals placed and to end. and expends 0.065 cm between 0°C and 100°C. Find the length of each portion of the composite bar.

Answer: Let's denote: α_1=coefficient of linear expansion of metal 1

α_2=coefficient of linear expansion of metal 2

L_1=length of metal 1 in the composite rod

L_2=length of metal 2 in the composite rod

Given: L=30.0cm(initial length of each rod),ΔT=100°C-0°C=100K(temperature change)

For metal 1:ΔL_1=0.075cm $\alpha 1=\Delta L_1/(L\times\Delta T)=0.075/(30\times100)=2.5\times10^{-5}K^{-1}$

For metal 2: ΔL_2=0.045cm $\alpha 2=\Delta L_2/(L\times\Delta T)=0.045/(30\times100)=1.5\times10^{-5}K^{-1}$

For the composite rod:ΔL=0.065cm

$\alpha_{avg}=\Delta L/(L\times\Delta T)=0.065/(30\times100)=2.17\times10^{-5}K^{-1}$

Since the composite rod is made of two metals, we can write:

$\alpha_{avg}=(\alpha_1\times L_1+\alpha_2\times L_2)/L$

Substituting values: $2.17\times10^{-5}=(2.5\times10^{-5}\times L1+1.5\times10^{-5}\times L2)/30$

We know that L1+L2=30cm.Let's solve the equations:

$2.17\times10^{-5}\times30=2.5\times10^{-5}\times L_1+1.5\times10^{-5}\times(30-L_1)$

$6.51\times10^{-4}=2.5\times10^{-5}\times L_1+4.5\times10^{-4}-1.5\times1010^{-5}\times L\_=2.01\times10^{-4}=1\times10^{-5}\times L_1$

L_1=20.1cm , $L_2=30-L_1=30-20.1=9.9$cm

The length of metal 1 in the composite rod is approximately 20.1cm,and the length of metal 2 is approximately 9.9cm.

2. Suppose that a steel hoop could be constructed around the earth's equator, just fitting it at a temperature 20°C. What would be the thickness of space between hoop and the earth if the temperature of the hoop were increased by 1 °C.$\alpha_{steel}= 1.2 \times 10^{-5}$°C^{-1}

Answer: Let's denote:

R = radius of the earth L = circumference of the earth = $2\pi R$ ΔT = 1°C (temp change)

$\alpha = 1.2 \times 10^{-5}$ /°C (coefficient of linear expansion)

The change in circumference of the hoop is: $\Delta L=L\times\alpha\times\Delta T=2\pi R\times1.2\times10^{-5}\times1=2.4\pi R\times10^{-5}$

The change in radius is: $\Delta R=\Delta L/(2\pi) =2.4\pi R\times10^{-5}/(2\pi) =1.2R\times10^{-5}$

The thickness of space between the hoop and the earth would be the difference in radii, which is ΔR.

$\Delta R=1.2\times6.37\times10^{6}\times10^{-5}=7.644$m

3.Derive an expression for the rate of heat flow through a composite slab consisting of two materials with different thermal conductivities and thicknesses. Assume that the temperatures at the outer surfaces of the slab are T1 and T2, and that the interface between the two materials is at temperature T.

Answer

Let K1 and K2 be the thermal conductivities, and d1 and d2 be the thicknesses of the two materials.

The rate of heat flow through the first material is: Q1 = K1 × A × (T1 - T) / d1

The rate of heat flow through the second material is: Q2 = K2 × A × (T - T2) / d2

At steady state, Q1 = Q2 = Q

Equating the two expressions :K1 × A × (T1 - T) / d1 = K2 × A × (T - T2) / d2

Solving for T: $T = (K1 \times d2 \times T1 + K2 \times d1 \times T2) / (K1 \times d2 + K2 \times d1)$
The rate of heat flow is: $Q = K1 \times A \times (T1 - T) / d1$
Substituting the expression for T: $Q = A \times (T1 - T2) / (d1 / K1 + d2 / K2)$

4. A composite wall consists of two layers: a 10 cm thick layer of brick (thermal conductivity = 0.7 W/m·K) and a 5 cm thick layer of insulation (thermal conductivity = 0.2 W/m·K). The inner surface of the brick layer is at 20°C, and the outer surface of the insulation layer is at 0°C. Calculate the heat flux through the composite wall and the temperature at the interface between the two layers.
Answer: Let's denote the temperature at the interface as T.
The heat flux through the brick layer is:$q = K_{brick} \times (T1 - T) / d_{brick}$
The heat flux through the insulation layer is: $q = K_{insulation} \times (T - T2) / d_{insulation}$
At steady state, the heat flux through both layers is the same.
$0.7 \times (20 - T) / 0.1 = 0.2 \times (T - 0) / 0.05$ Solving for T, we get T = 15.56°C
The heat flux is :$q = 0.7 \times (20 - 15.56) / 0.1 = 31.08\ W/m^2$

5. A 50 g aluminum ball (specific heat capacity = 900 J/kg°C) at 100°C is dropped into 100 g of water at 20°C. The final temperature of the water and ball is 24°C. Calculate the heat absorbed by the water and the heat lost by the aluminum ball. Explain why the final temperature is not exactly 25°C (the theoretical average).
Answer: Step 1: Calculate the heat lost by the aluminum ball
The heat lost by the aluminum ball is given by $Q = m \times c \times \Delta T$, where m is the mass of the ball, c is the specific heat capacity, and ΔT is the change in temperature.
$Q_{lost} = 0.05$ kg × 900 J/kg°C × (100°C - 24°C) = 0.05 × 900 × 76 = 3420 J
Step 2: Calculate the heat absorbed by the water
The heat absorbed by the water is given by $Q = m \times c \times \Delta T$, where m is the mass of the water, c is the specific heat capacity of water (approximately 4186 J/kg°C), and ΔT is the change in temperature.
$Q_{absorbed} = 0.1$ kg × 4186 J/kg°C × (24°C - 20°C) = 0.1 × 4186 × 4 = 1674.4 J
Step 3: Explain the difference in heat lost and absorbed
The difference in heat lost by the aluminum ball and heat absorbed by the water can be attributed to heat loss to the surroundings. Some of the heat energy lost by the aluminum ball is absorbed by the container or lost to the environment, rather than being entirely absorbed by the water.
Step 4: Explain why the final temperature is not exactly the theoretical average
The final temperature is not exactly the theoretical average (25°C) due to the difference in specific heat capacities of aluminum and water, as well as heat losses to the surroundings. The calculation of the final temperature would require considering these factors, and in an ideal scenario without heat loss, the final temperature would be closer to the calculated value based on the heat balance equation.
The heat lost by the aluminum ball is 3420 J, and the heat absorbed by the water is 1674.4 J. The difference highlights the role of heat loss and the specific heat capacities in determining the final temperature of the mixture.

CASE STUDY BASED QUESTION

1. Water has a peculiar property, which is different from any other substance it's density decreases with temperature and density of solid state (ice) is lesser than density of liquid(water). If observed carefully the density is maximum at 4ºC. it helps us in many ways and is important for sustenance of life on Earth. Especially aquatic life.
i) which one of the following has highest density

a) ice b) water c) vapour d) different at different temperatures

ii) density of water is higher than ice, it helps in

a) saving life of aquatic animals in cold weather b) formation of top soil

c) both a and b d) none

iii) On compressing the melting point of ice

a) increases b) decreases c) remains same d) decreases than increases

OR

On higher altitudes rice takes more time to cook in open vessel in comparison to that at sea levelBecause at higher altitudes

a) boiling point of water increases c) cooking temperature required by rice increases

b) boiling point of water decreases d) cooking temperature required by rice decrease

iv) the volume occupied by water and ice of equal mass will be

a) equal b) greater for water than ice

c) greater for ice than water d) may not be same sometimes

2. Specific heat capacity is defined in terms of heat required to raise the temperature of unit mass of a substance by unity. While molar specific heat is heat required to raise temperature of one mole of substance by unity. Generally in case of a gas we usually discuss about molar specific heat only but it does not imply that we cannot calculate specific heat of a gas. Generally we take two types of specific heats in gases at constant volume and at constant pressure.

i) How many molar specific heat can be defined for gases?

a) 2 b) 1 c) 3 d) none of the above

ii) Specific heat of substances during fusion would be

a) zero b) equal to that of solid c) infinite d) equal to that of liquid

OR

if specific heat at constant volume of an ideal gas of molar mass 4 is 6R then specific heat at constant pressure would be

a) 5R b) 7R c) 10R d) 3R

iii) Solids and liquids practically have only one specific heat because

a) Volume is independent of temperature b) Pressure is independent of temperature

c) Volume is almost constant d) pressure and volume are independent of each other

iv) Water has highest specific heat capacity it makes it suitable for

a) coolant b) cooking medium c) drinking d) all of the above

3.when any substance is heated it experiences expansion. Thermal expansion can be categorized in three categories

Linear along length, Superficial expansion of surface, Volumetric expansion of volume

There are many applications of thermal expansions in our day-to-day life.

i) what is the ratio of coefficient of volumetric expansion and coefficient of superficial expansion of same substance

a) 1:1 b) 3:2 c) 2:3 d) none of the above

ii) which of the following is application of thermal expansion

a) bimetallic strip b) thermostat c) sagging of cable d) all of the above

iii) in a bimetallic strip Cu and Steel are used because

a) they are good conductor of heat b) they are cheaper than silver and Aluminum

c) the difference of thermal coefficient of expansion d) all of the above

iv) if overhead cables are laid in summer we should take into account

a) linear expansion b) superficial expansion c) volumetric expansion d) none of them

OR

For fixing a hoop around the wheel it is slightly heated and then allowed to cool to use

a) linear expansion b) linear contraction c) superficial expansion d) both a and b

solutions

1. i) (b) water

Water has its highest density at 4°C.

ii) (a) saving life of aquatic animals in cold weather

The fact that ice is less dense than water allows it to float on top of bodies of water, insulating the water below and helping to preserve aquatic life during cold weather.

iii)1. (b) decreases

The melting point of ice decreases under pressure.

2. (b) boiling point of water decreases

At higher altitudes, the atmospheric pressure is lower, causing water to boil at a lower temperature, which can increase cooking time.

iv) (c) greater for ice than water

Since ice is less dense than water, a given mass of ice will occupy a greater volume than the same mass of water.

2. i) d) none of the above

ii) c) infinite

or

c) 10R

iii) c) Volume is almost constant

iv) a) coolant

3. i) (b) 3:2

The ratio of coefficient of volumetric expansion (γ) to coefficient of superficial expansion (β) is 3:2, since $\gamma = 3\alpha$ and $\beta = 2\alpha$, where α is the coefficient of linear expansion.

ii) (d) all of the above

All the options listed are applications or examples of thermal expansion.

iii) (c) the difference of thermal coefficient of expansion

The primary reason for using Cu and Steel in a bimetallic strip is the significant difference in their thermal coefficients of expansion.

iv) (a) linear expansion

When laying overhead cables in summer, it's essential to consider linear expansion, as the cables will expand in length due to the increased temperature.

Or

For fixing a hoop around the wheel: (d) both a and b

The process involves heating the hoop (linear expansion) and then cooling it (linear contraction) to secure it tightly around the wheel.

5 MARKS QUESTION

1.(a) What is thermal expansion? Describe an experiment to demonstrate linear expansion in solids. (1 mark)

(b) Derive the relationship between change in length (ΔL) of a solid and its coefficient of linear expansion (α), initial length (L_0) and change in temperature (ΔT). (2 marks)

(c) A metal wire of length 1.5 m expands to 1.502 m when heated from 25°C to 75°C. Calculate its coefficient of linear expansion. (α in $10^{-5}/°C$) (2 marks)
Solution
(a) Thermal expansion is the change in dimensions of a substance due to change in temperature.
An experiment to demonstrate linear expansion is by heating a metal rod clamped at one end and observing the expansion using a scale.
(b) Let initial length = L_0, final length = $L_0 + \Delta L$, initial temperature = T_1, final temperature = T_2
$\Delta T = T_2 - T_1$ $\Delta L = L_0 \times \alpha \ \Delta T$ or $\alpha = \Delta L / (L_0 \ \Delta T)$
(c) Given: ΔL = 1.502 m - 1.5 m = 0.002 m, L_0 = 1.5 m, ΔT = 75°C - 25°C = 50°C
$\alpha = \Delta L / (L_0 \times \Delta T) = 0.002 / (1.5 \times 50) = 2.67 \times 10^{-5}\ °C^{-1}$
2. (a) What is conduction of heat? Give one example of a good conductor of heat. (1 mark)
(b) State the factors on which the rate of heat conduction depends. (2 marks)
(c) A copper wire of length 10 m and cross-sectional area 0.5 cm² conducts 60 J of heat energy in 1 minute when the temperature difference between its ends is 20°C. Calculate its thermal conductivity. (2 marks)
Solution: a) Conduction of heat is the transfer of heat energy through direct contact between particles of matter.One example of a good conductor of heat is Copper.
(b) The rate of heat conduction depends on:
1. Temperature difference between the ends of the conductor
2. Cross-sectional area of the conductor
3. Length of the conductor
4. Thermal conductivity of the material
(c) Given:
Q = 60 J, t = 1 min = 60 s, ΔT = 20°C, L = 10 m, A = 0.5 cm² = 0.00005 m²
Thermal conductivity (K) = $Q L / (A \Delta T \ t)$
$= 60 \times 10 / (0.00005 \times 20 \times 60)$
= 385 W/mK
3. (a) Explain the term 'apparent expansion' of a liquid. (1 mark)
(b) Describe an experiment to demonstrate the expansion of a liquid on heating. (2 marks)
(c) A flask made of glass (coefficient of volume expansion = $9 \times 10^{-6}/°C$) is filled with a liquid (coefficient of volume expansion = $1.2 \times 10^{-3}/°C$) at 20°C. If the flask is heated to 80°C, calculate the apparent expansion of the liquid. (2 marks)
solution(a)Apparent expansion of a liquid is the net expansion observed when a liquid expands inside a container that also expands with temperature.
It is the difference between the actual expansion of the liquid and the expansion of the container.
(b)Experiment:
- Fill a transparent glass tube with a liquid (like water or oil) up to a mark.
- Heat the tube gently using a Bunsen burner.
- Observe the rise in liquid level against the markings on the tube.
- Record initial and final temperatures and liquid levels.
(c)Given:
- Coefficient of volume expansion of glass = $9 \times 10^{-6}/°C$,
Coefficient of volume expansion of liquid = $1.2 \times 10^{-3}/°C$
- Initial temperature = 20°C ,Final temperature = 80°C
- ΔT = 80 - 20 = 60°C
Apparent expansion = Actual expansion of liquid - Expansion of glass
$= (1.2 \times 10^{-3} - 9 \times 10^{-6}) \times 60$

$= 7.14 \times 10^{-2}$ or approximately *7.14%*

4. A metal coffee mug and a ceramic coffee mug are both filled with hot coffee at the same temperature (80°C).
After 30 minutes, compare the final coffee temperatures and explain the reasons.
(a) Describe the heat loss process in both mugs. (1 mark)
(b) Which mug will have hotter coffee remaining after 30 minutes? (1 mark)
(c) Explain the role of thermal conductivity and specific heat capacity in this scenario. (2 marks)
(d) Calculate the approximate temperature drop in the metal mug assuming its handle and body are made of aluminum (thermal conductivity = 237 W/mK, specific heat capacity = 900 J/kg°C). Initial coffee quantity = 200g. (1 mark)
Want me to provide the solution?
Solution; (a) Description of heat loss process:
Heat loss occurs through convection and conduction.
- Convection: Coffee transfers heat to surrounding air.
- Conduction: Mug material transfers heat from coffee to surroundings.

(b) The ceramic mug will have hotter coffee remaining after 30 minutes.
Reasons:
1. Lower Thermal Conductivity: Ceramic (0.8 W/mK) conducts heat slower than metal (aluminum: 237 W/mK).
2. Less Heat Escape: Ceramic mug loses heat more slowly, keeping coffee warmer.
3. Insulating Properties: Ceramic acts as a thermal insulator, reducing heat transfer.

(c) Explanation of thermal properties:
1. Thermal Conductivity*: Measures ability of materials to conduct heat.
   - Metals (high conductivity) lose heat quickly.
   - Ceramics (low conductivity) lose heat slowly.
2. Specific Heat Capacity: Measures heat energy required to raise temperature of unit mass by 1°C.

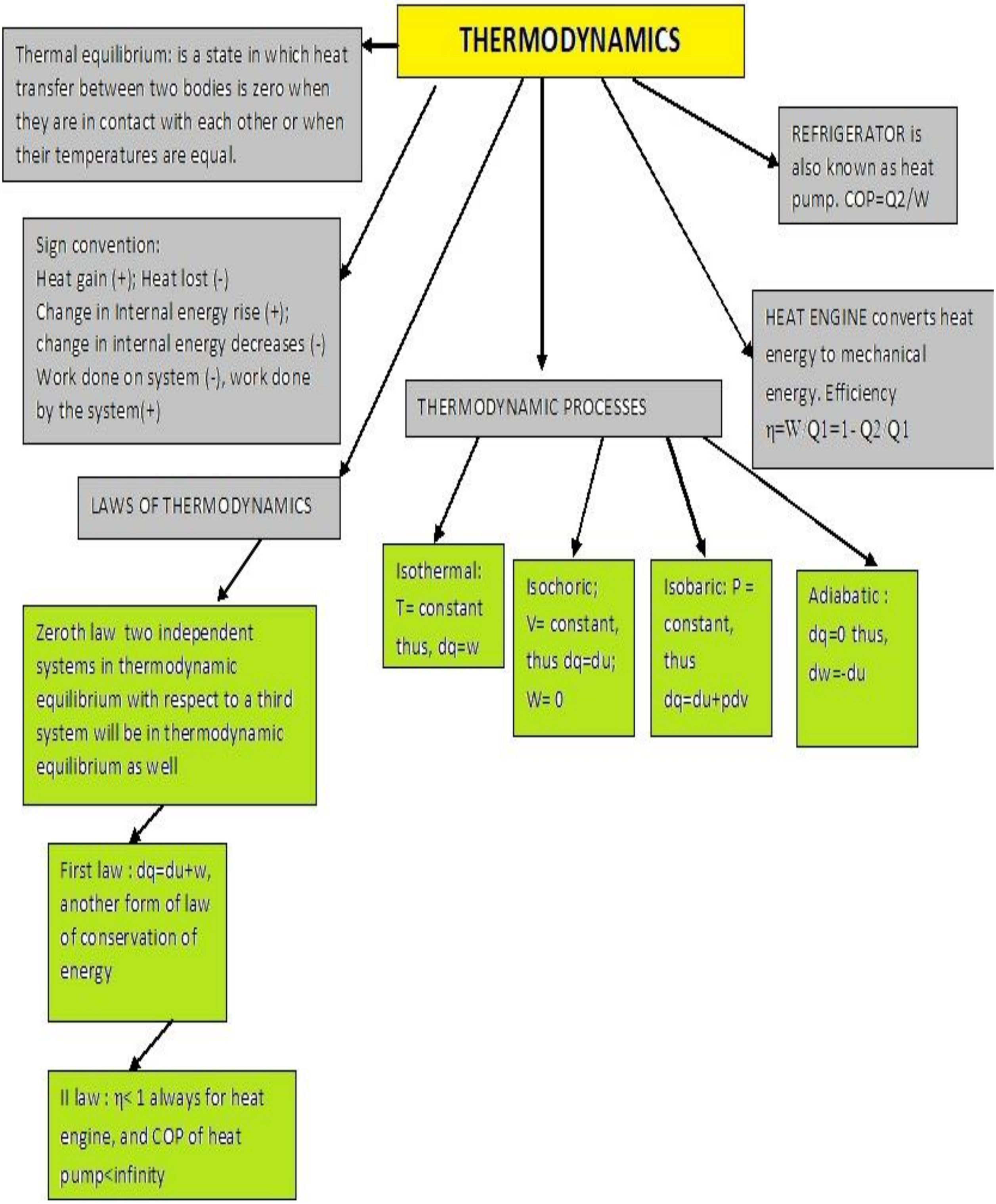
(d) Calculation of temperature drop in metal mug:
Given:
- Initial temperature (T_1) = 80°C,Final time (t) = 30 minutes = 1800 seconds
- Initial coffee quantity (m) = 200g = 0.2 kg, Specific heat capacity of water (c) = 4186 J/kg°C
- Specific heat capacity of aluminum (c_al) = 900 J/kg°C
- Thermal conductivity of aluminum (K) = 237 W/mK
- Density of aluminum (ρ) = 2700 kg/m³
- Mug wall thickness (d) = assumed 2 mm = 0.002 m
- Mug outer surface area (A) = assumed 0.025 m² (approximate)
- Temperature of surroundings (T_2) = assumed 20°C

Formula to calculate heat loss (Q):
$Q = (K\ A\ (T_1 - T_2)\ t) / d$
Plugging values:
$Q = (237 \times 0.025 \times (80 - 20) \times 1800) / 0.002$
$Q = 1018350$ J
Temperature drop (ΔT) formula:
$\Delta T = Q / (m\ c)$
Plugging values:
$\Delta T = 1018350 / (0.2 * 4186)$ $= \Delta T = 12.2$ °C
Therefore, the final coffee temperature = Initial temperature – ΔT = 80°C - 12.2°C= 67.8°C

CHAPTER 11: THERMODYNAMICS

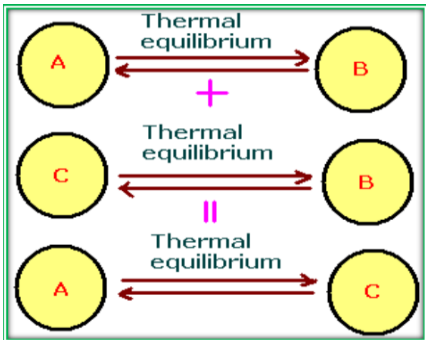
Thermal equilibrium and definition of temperature, zeroth law of thermodynamics, heat, work and internal energy. First law of thermodynamics, Second law of thermodynamics: Thermodynamic state variable and equation of state. Change of condition of gaseous state - isothermal, adiabatic, reversible, irreversible, and cyclic processes.

MIND MAP



GIST OF THE CHAPTER

Thermal Equilibrium



A system is said to be in thermal equilibrium when its macroscopic variables—like pressure, temperature, and volume—remain constant over time. This concept is crucial in understanding the flow of heat and energy between systems.

Zeroth Law of Thermodynamics

The Zeroth Law of Thermodynamics states that if two systems (A and B) are in thermal equilibrium with a third system (C), then A and B are also in thermal equilibrium with each other. This law forms the basis for temperature measurement.

Heat and Internal Energy

- Heat: Heat is a form of energy that naturally flows from a body at a higher temperature to a body at a lower temperature.
- Internal Energy: This is the total energy stored in a system due to molecular motion and configuration. It depends on the system's state and not on how it arrived there.

Work and The First Law of Thermodynamics

Work in thermodynamics is the energy transferred without involving temperature differences. For example, pushing the piston in a gas-filled cylinder constitutes work.

The First Law of Thermodynamics is essentially the law of conservation of energy. It states: $\Delta Q=\Delta U+\Delta W$.

- ΔQ is the heat supplied to the system,
- ΔU is the change in internal energy,
- ΔW is the work done by the system.

Thermodynamic State Variables: State variables are those macroscopic variables that determine the equilibrium state of a thermodynamic system. Examples: pressure, temperature, volume, and mass.

- The connection between state variables is called the equation of state.
- The equation of state for an ideal gas is represented as $PV = \mu RT$
- For a given amount of gas, only two variables are independent, that is either P and V or T and V.

Quasi-static Process: It is an infinitely slow process such that the system remains in thermal and mechanical equilibrium with the surroundings throughout.

Thermodynamic Processes

Thermodynamics involves various processes that help us understand energy transformations:

Isothermal Process In an isothermal process, the system's temperature remains constant. During an isothermal expansion of a gas, the work done is given by:

$$Q = W = \mu RT \ln\left(\frac{V_2}{V_1}\right)$$

This process results in a curve called an isotherm when plotted on a pressure-volume graph.

Adiabatic Process In an adiabatic process, no heat is exchanged between the system and its surroundings. The temperature of the system changes due to internal factors. The relationship for an adiabatic process is:

$$PV^{\gamma} = \text{constant}$$

$$\text{where, } \gamma = \frac{C_P}{C_V}$$

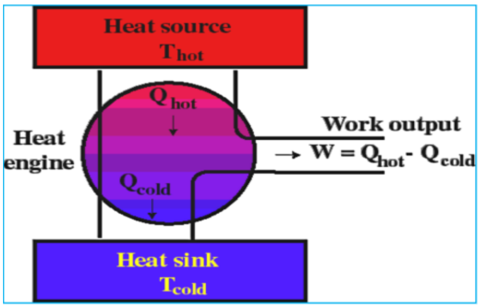
Isochoric Process An isochoric process keeps the volume constant, meaning no work is done. The heat added only changes the internal energy and temperature.

Isobaric Process In an isobaric process, the pressure remains constant. The work done in this process is: $W=P(V2-V1)=\mu R(T2-T1)$

Heat Engines
A heat engine converts heat into mechanical work. It works by transferring heat from a high-temperature source to a low-temperature sink, performing work in the process.

Efficiency of a Heat Engine
The efficiency η of a heat engine is defined as the ratio of the work done to the heat absorbed: $\eta = W/Q1$

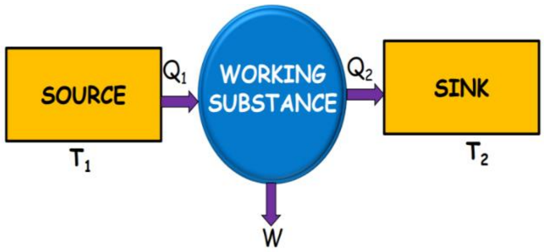
$W = Q1 - Q2$ =>. EFFICIENCY=1- Q2/Q1



Refrigerator
The refrigerator is the reverse of a heat engine. It extracts heat from a cold reservoir, using external work, and releases it to a hotter reservoir. The refrigeration cycle ensures that substances stay cool by constantly moving heat away.

Second Law of Thermodynamics
The Second Law of Thermodynamics introduces the concept of irreversibility and entropy:

- Kelvin-Planck Statement: No heat engine can convert 100% of absorbed heat into work.
- Clausius Statement: It is impossible to transfer heat from a cold body to a hot body without external work.

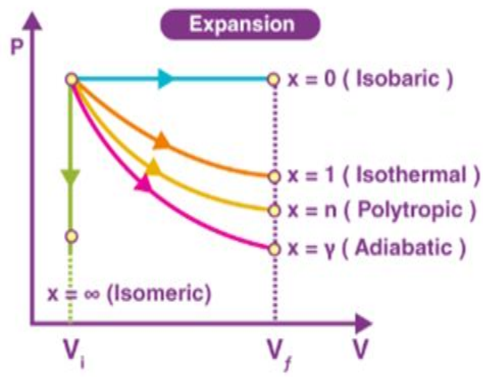


Reversible and Irreversible Processes
- Reversible Processes: These are ideal processes where the system and surroundings can return to their original states with no changes elsewhere.
- Irreversible Processes: These processes involve natural spontaneous changes, such as heat flow from a hot to a cold body or the diffusion of gases.

Carnot Engine
The Carnot engine represents an ideal reversible heat engine that operates between two temperatures. The Carnot cycle includes:

1. Isothermal expansion
2. Adiabatic expansion
3. Isothermal compression
4. Adiabatic compression

The Carnot Theorem states that no engine can be more efficient than a Carnot engine operating between the same two temperatures.



Efficiency of Carnot Engine
The efficiency of a Carnot engine is $\eta = W/Q1$ or $\eta = 1 - (T2/T1)$ Where T1 is the temperature of the heat source and T2 is the temperature of the heat sink.

A pressure-volume (PV) diagram, also known as an indicator diagram, is a graph that visually represents changes in pressure and volume during a thermodynamic process. It's a valuable tool for understanding work done by or on a system, and it can be used to determine a system's efficiency. The area under the curve of a PV diagram represents the work done.

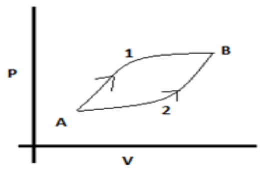
MULTIPLE CHOICE QUESTIONS

1. First law of thermodynamics is based on?

a) Conservation of energy b) Conservation of mass

c) Conservation of momentum d) Conservation of work

2. What is the relation between the internal energy and heat supplied in the process 1 & 2 shown in the diagram? Both paths start at A and end at B.



a) $U_1 > U_2$, $Q_1 > Q_2$ b) $U_1 < U_2$, $Q_1 > Q_2$

c) $U_1 = U_2$, $Q_1 = Q_2$ d) $U_1 = U_2$, $Q_1 > Q_2$

3. Zeroth law of thermodynamics helped in the creation of which scale?

a) Temperature b) Heat energy c) Pressure d) Internal energy

4. Internal energy of a system is defined as?

a) The sum of kinetic energies of all molecules of the system

b) The sum of kinetic and potential energies of all molecules of the system

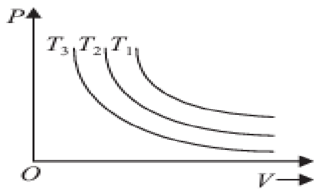
c) The sum of potential energies of the system

d) The average kinetic energy of all molecules

5. If an engine takes 5J of heat energy from a reservoir and rejects 3J to the surroundings, what is its efficiency?

a) 2/5 b) 3/5 c) 1/5 d) 0

6. The isothermal diagram of a gas at three different temperatures T1, T2 and T3, is shown in the given figure. Then



a) T1 < T2 < T3 b) T1 < T2 > T3 c) T1 > T2 > T3 d) T1 > T2 < T3

7. The relation between the slope of isothermal curve and slope of adiabatic curve

a) slope of adiabatic curve = γ times slope of isothermal curve

b) slope of isothermal curve = γ times slope of adiabatic curve

c) slope of adiabatic curve = 1/ γ times slope of isothermal curve

d) slope of isothermal curve = 1/ γ times slope of adiabatic curve

8. In a thermodynamic system working substance is an ideal gas. Its internal energy is in the form of (a) kinetic energy only (b) kinetic and potential energy (c) potential energy (d) electric energy.

SOLUTIONS:

1.Answer: a

Explanation: The first law of thermodynamics is based on the conservation of energy. It deals with work done and heat energy supplied or removed from a system. It basically says that energy supplied to a system is conserved.

2. Answer: d

Explanation: The initial and final states are the same for both processes, so the value of internal energy will be the same ($U_1 = U_2$).

The area under the PV curve gives work done.

In the given diagram, area under 1 is greater than area under 2. So, $W_1 > W_2$.

We know, $\Delta Q = \Delta U + \Delta W$, which implies that $Q_1 > Q_2$ since U is same for both and $W_1 > W_2$.

3. Answer: a

Explanation: Zeroth law of thermodynamics talks about thermal equilibrium or relates bodies having no heat flow. Temperature difference is what leads to heat flow. So, this law helped in making temperature measurable and an universal property of matter.

4. Answer: b

Explanation: The internal energy of a system corresponds to the energy possessed by all molecules. Thus it is the sum of kinetic and potential energies of all molecules in the system considered. Also note that potential energy is frame dependent, so we choose a frame in which the centre of mass is at rest.

5. Answer: a

Explanation: Efficiency is defined as work done/heat input.

Work done = 5-3=2J. ,Heat input = 5J,Thus, efficiency = 2/5.

6. Answer: a

Explanation: The given diagram shows that the curves move away from the origin at higher temperature.

7. Answer: a)

Explanation: For isothermal process, PV = constant, Differentiating both side PdV +VdP =0

or dP/dV=- P/V ...(i)

Again for adiabatic process PV^{γ} = constant

Again differentiating both side dP/dV=- γP/V

slope of adiabatic curve = γ × slope of isothermal curve

8. Answer: b)

ASSERTION AND REASONING TYPE QUESTIONS

1. ASSERTION:Work done in isobaric process per mole is R (T_2-T_1) in terms of temperature.
   REASON: For an isobaric process pressure is constant.
2. ASSERTION: Net heat gain by earth is not zero.
   REASON:Heat always flow from body at higher temperature to body at lower temperature
3. ASSERTION: When a chilled bottle of carbonated drink is opened a slight fog is formed around the opening.
   REASON: Temperature of drink is lesser than that of atmosphere.
4. ASSERTION: When heat is supplied to a system its internal energy may decrease.
   REASON: By first Law of thermodynamics ΔQ=ΔU+ΔW
5. ASSERTION: We cannot change temperature of a body without giving or taking heat.
   REASON: Energy is conserved for every system.
6. ASSERTION: The heat supplied to a system is always equal to the increase in its internal energy.

Reason : When a system changes from one thermal equilibrium to another, some heat is absorbed by it.

7. Assertion : In adiabatic compression, the internal energy and temperature of the system get decreased.

Reason : The adiabatic compression is a slow process

8. Assertion : In isothermal process whole of the heat energy supplied to the body is converted into internal energy.

Reason : According to the first law of thermodynamics ΔQ = ΔU.

.

SOLUTIONS

1. Answer: a) Both A and R are true and R is correct explanation of A

PV=RT ⇒ W=P(V2-V1) ⇒ W=R(T2-T1)

2. ANSWER: BOTH A and R are false.

Heat gain by Earth= heat lost, otherwise slowly the temperature of earth should increase to very high values. Heat may flow from body at lower temperature to higher temperature in forced convection like a.c.

3. Answer: both A and R are true but R is not correct explanation of A.

Explanation; if chilled bottle of carbonated drink is opened formation of slight fog is due to sudden release of carbon di oxide in adiabatic manner hence expansion of gas takes place thus decreasing the internal energy or temperature thus formation of fog takes place.

4. Answer: Both A and R are true and R is correct explanation of A.

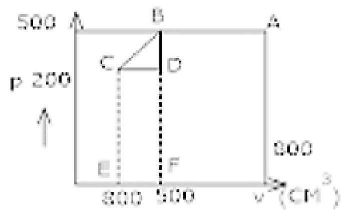
Explanation: Theoretically if ΔQ=ΔU+ΔW, change in internal energy will be negative if heat provided is less than work done by the gas.

5. Answer: Both A and R false

Explanation: We can change temperature of a body by compressing, friction etc.
Energy is conserved for an isolated system only.
6. Answer Both A and R are false
Explanation: the heat supplied to a system is equal to the increase in internal energy only when system is adiabatic
Similarly if system is adiabatic heat absorption is zero.
7. Answer both A and R are false
Explanation: adiabatic process is always a fast process and work is done at the cost of internal energy. $\Delta W = -\Delta U$, thus for compression internal energy will increase
8. Answer both A and R are false
Explanation: if T=constant change in internal energy does not take place and $\Delta Q = \Delta U + W$

2 MARKS QUESTIONS

1. A motor car tyre has a Pressure of four atmosphere at A room temperature of 27° C. If the Tyre suddenly bursts, calculate the temperature of escaping gas?
Ans. Since the tyre suddenly bursts, the change taking place is adiabatic, for adiabatic change:PV^{γ}=CONSTANT⇒ $P^{\gamma-1}/T^{\gamma}$=constant
Using P1=4 atm; P2=1 atm; T1= 300 K; and γ=1.4
$T2=T1(1/4)^{0.4/1/4} \Rightarrow T2= 300x\ (1/4)^{2/7} \Rightarrow T2= 201.8\ K=-71.2^{o}C$
2. Why does absolute zero does not corresponds to zero energy?
Ans. Absolute zero corresponds to no kinetic energy, but total energy consists of kinetic as well as potential energy. Thus even at absolute zero total energy is not zero.
3. A heat engine operates between 527°C and 127°C.
Calculate its maximum possible efficiency using Carnot's equation.
Answer: T_1 = 527°C = 800K T_2 = 127°C = 400K
* Efficiency (η) = 1 - (T_2/T_1) = 1 - (400/800) = 0.5 or 50%
4. A refrigerator consumes 120 J of electrical energy to transfer 480 J of heat energy.
Calculate its coefficient of performance (COP).
Answer: Electrical energy input (W) = 120 J   Heat energy transferred (Q) = 480 J
* Coefficient of Performance (COP) = Q / W = 480 J / 120 J = 4
Explanation: COP measures the efficiency of a refrigerator.
A higher COP indicates better performance.In this case, the refrigerator transfers 4 units of heat energy for every 1 unit of electrical energy consumed.
5.If a drop of water falls on a very hot iron, it takes long time to evaporate. Explain.
Answer: When a drop of water falls on a very hot iron, it gets separated from the iron by a thin layer of water vapour which is a bad conductor of heat. So, heat is conducted from the iron to the water drop very slowly so that it evaporates in a long time.
6. What is the ratio of final volume to initial volume if the air (γ=1.5) is compressed adiabatically till its temperature is doubled?
Ans.Since for an adiabatic Process, PV^{γ}= constant
Since PV= RT for ideal gas
$\Rightarrow P=RT/V \Rightarrow RTV^{\gamma}/V= CONST \Rightarrow (V2/V1)^{\gamma-1}=T1/T2=1/2$
$\Rightarrow V2/V1=(1/2)^{1/(\gamma-1)}=(1/2)^{1/0.5}=(1/2)^{2}=1/4$
7. Calculate the work done by the gas in the P-V graph going from A to B and B to C.



Ans: work done in P-V graph is area under the curve
Thus work done by the gas from A to B
W1= -500X (800-500) x10^{-6}⇒-15x10^{-2} J
Work done by gas from B to C
W2= -(1/2x300x200x10^{-6}+200x200x10^{-6}) =-7x10^{-2} J
Net work done will be W=W1+W2= -22x10^{-2} J

SHORT ANSWER QUESTIONS (3 MARKS)

1. Derive the work done during the isothermal Process?
Ans. Consider an ideal gas expanding isothermally from volume Vi to Vf.
The work done (W) is given by: appropriate graph
W = ∫PdV
For an ideal gas, PV = nRT
At constant temperature (T), P = nRT/V
Substituting P in the work equation:
W = ∫(nRT/V)dV
Evaluating the integral from Vi to Vf:
W = nRT ∫(1/V)dV from Vi to Vf
⇒W = nRT ln(Vf/Vi)
This is the expression for work done in an isothermal process.

2. Five moles of an ideal gas are taken in a Carnot engine working between 100^O C and 30^0C. The useful work done in 1 cycle is 420 J. Calculate the ratio of the volume of the gas at the end and beginning of the isothermal expansion?
Answer: Given:n = 5 moles,T1 = 100°C = 373 K,T2 = 30°C = 303 K, W = 420 J
Efficiency of Carnot engine:
η = 1 - (T2/T1) = 1 - (303/373) = 0.187
η = W / Q1 ⇒ 0.187 = 420 / Q1 ⇒ Q1 = 2246.52 J
Q1 = nRT1 ln(V2/V1)
2246.52 = 5 x 8.314x 373 x ln(V2/V1) ⇒ ln(V2/V1) = 2246.52/ (5x 8.314 x 373)
ln(V2/V1) = 0.145 ⇒ V2/V1 = $e^{0.145}$= 1.156
The ratio of the volume of the gas at the end and beginning of the isothermal expansion is approximately 1.156.

3. One kilomole of a gas at 400K expands isothermally until its volume is doubled. Find the amount of work done and heat produced.
Answer: - Number of moles (n): 1 kilomole = 1000 mole Temperature (T): 400 K ,(Vf/Vi): 2
The work done (W) in an isothermal expansion is given by:
W = nRT ln(Vf/Vi)
Substituting the given values:
W = 1000 x 8.314x 400 x ln(2) = 2305376 J = 2305.38 KJ
Since it's an isothermal process, ΔU = 0, and Q = W.

4. show that Newton's law of cooling is an approximation of Stefan's law.
Answer: Stefan's Law: E = σT^4
Net energy radiated: E_net = $\sigma(T^4 - T_0^4)$
Approximation for small ΔT: $T^4 - T_0^4 = 4T0^3(T - T_0)$
E_net = σ x $4T0^3$(T - T0) ⇒ dQ/dt ∝ $(T - T_0)$
Newton's Law: dQ/dt = $-k(T - T_0)$

5. An electric heater supplies heat to a system at a rate of 100 W. If the system performs work at a rate of 75 joules per second, at what rate is the internal energy increasing?

Answer: - Heat supplied (Q): 100 W = 100 J/s ,Work done (W): 75 J/s

First Law of Thermodynamics: $\Delta U = Q - W$

Rate of Internal Energy Increase: $dU/dt = dQ/dt - dW/dt$= 100 J/s - 75 J/s= 25 J/s OR 25W

6. What amount of heat must be supplied to 2×10^{-2} kg of nitrogen at room temperature to raise its temperature by 45 °C at constant pressure, given that the molecular mass of nitrogen is 28 g and R is 8.3?

Answer: To find the answer, we calculate the number of moles: n = 20 g / 28 g/mol = 0.714 mol. Then, $C_p = 7/2\ R$ = 29.05 J/mol·K.

The heat supplied $Q = n \times C_p \times \Delta T = 0.714 \times 29.05 \times 45 = 933.47$ J. So, approximately 933.47 J of heat is required.

CASE STUDY BASED QUESTIONS

Case Study 1: Heat Engine

A heat engine operates between two temperatures, 500 K and 300 K. It absorbs 1000 J of heat from the source and rejects 600 J of heat to the sink.

1. What is the efficiency of the heat engine?

a) 20% b) 30% c) 40% d) 50%

2. What is the work done by the heat engine?

a) 200 J b) 400 J c) 600 J d) 800 J

3. If the heat engine's efficiency is increased to 50%, what would be the new heat rejected to the sink?

a) 400 J b) 500 J c) 300 J d) 200 J

4. What would happen to the efficiency of the heat engine if the temperature of the source is increased?

a) It would decrease b) It would remain the same

c) It would increase d) It would become unpredictable

5. Which of the following is a characteristic of a Carnot engine?

a) It is a practical engine b) It has the maximum possible efficiency

c) It operates at a single temperature d) It rejects no heat to the sink

Case Study 2: Refrigerator

A refrigerator operates between two temperatures, 250 K and 300 K. It absorbs 500 J of heat from the cold body and rejects 700 J of heat to the hot body.

1. What is the coefficient of performance (COP) of the refrigerator?

a) 1.5 b) 2.5 c) 3.5 d) 4.5

2. What is the work done on the refrigerator?

a) 100 J b) 200 J c) 300 J d) 400 J

3. If the refrigerator's COP is increased to 3, what would be the new heat absorbed from the cold body if the work done remains the same?

a) 400 J b) 600 J c) 800 J d) 1000 J

4. What is the primary function of a refrigerator?

a) To convert heat into work b) To transfer heat from a colder body to a hotter body

c) To increase the temperature of a body d) To decrease the pressure of a gas

5. Which of the following statements is true about a refrigerator?

a) It violates the second law of thermodynamics b) It operates in a cycle

c) It converts all the heat into work d) It has 100% efficiency

SOLUTIONS

Case Study 1: Heat Engine

1. Answer: c) 40%Efficiency = (Work done) / (Heat absorbed) = (1000 J - 600 J) / 1000 J = 400 J / 1000 J = 0.4 or 40%.
2. Answer: b) 400 J Work done = Heat absorbed - Heat rejected = 1000 J - 600 J = 400 J.
3. Answer: b) 500 J Efficiency = 1 - (Heat rejected / Heat absorbed); 0.5 = 1 - (Heat rejected / 1000 J); Heat rejected = 500 J.
4. Answer: c) Efficiency increases with an increase in the source temperature.
5. Answer: b) It has the maximum possible efficiency

Case Study 2: Refrigerator

1. Answer: b) 2.5   COP = Heat absorbed / Work done = 500 J / (700 J - 500 J) = 2.5.
2. Answer: b) 200 J Work done = Heat rejected - Heat absorbed = 700 J - 500 J = 200 J.
3. Answer: b) 600 J COP = Heat absorbed / Work done; 3 = Heat absorbed / 200 J; Heat absorbed = 600 J.
4. Answer: b) A refrigerator transfers heat from a colder body to a hotter body.
5. Answer: b) It operates in a cycle   A refrigerator operates in a cycle.

5 MARKS QUESTIONS

1. (a) Define internal energy of an ideal gas. (1 mark)
(b) Derive an expression for the internal energy of an ideal gas in terms of temperature. (3 marks)
(c) Explain the physical significance of internal energy. (1 mark)

Solution: (a) Internal energy (U) is the total energy possessed by the system due to the motion of its molecules and interactions between them.
U is a state function depending only on the current state of the system.
It includes kinetic energy of molecules, potential energy of molecular interactions, and other forms of energy like vibrational and rotational energy.

(b) $\Delta U = Q_v = nCv\Delta T$ This equation shows that change in internal energy (ΔU) is equal to heat supplied (Q) at constant volume.
$U = nCvT$ This equation shows internal energy (U) is directly proportional to temperature (T) and number of moles (n).
Alternatively, $U = (3/2)nRT$
This equation shows internal energy (U) in terms of gas constant (R) and temperature (T).
(c) Internal energy determines temperature of a system & work output of a heat engine

2. a) Define isothermal and adiabatic processes. (1 mark)
(b) Derive an expression for work done in an isothermal expansion of an ideal gas. (3 marks)
(c) Explain graphical representation of these processes. (1 mark)

Solution: (a) Isothermal process: A constant temperature process where the system absorbs or releases heat without changing its temperature.
This means that the temperature of the system remains the same throughout the process.
Adiabatic process: A process in which no heat exchange occurs between the system and surroundings, resulting in a change in temperature.
This means that the system is completely insulated from its surroundings.

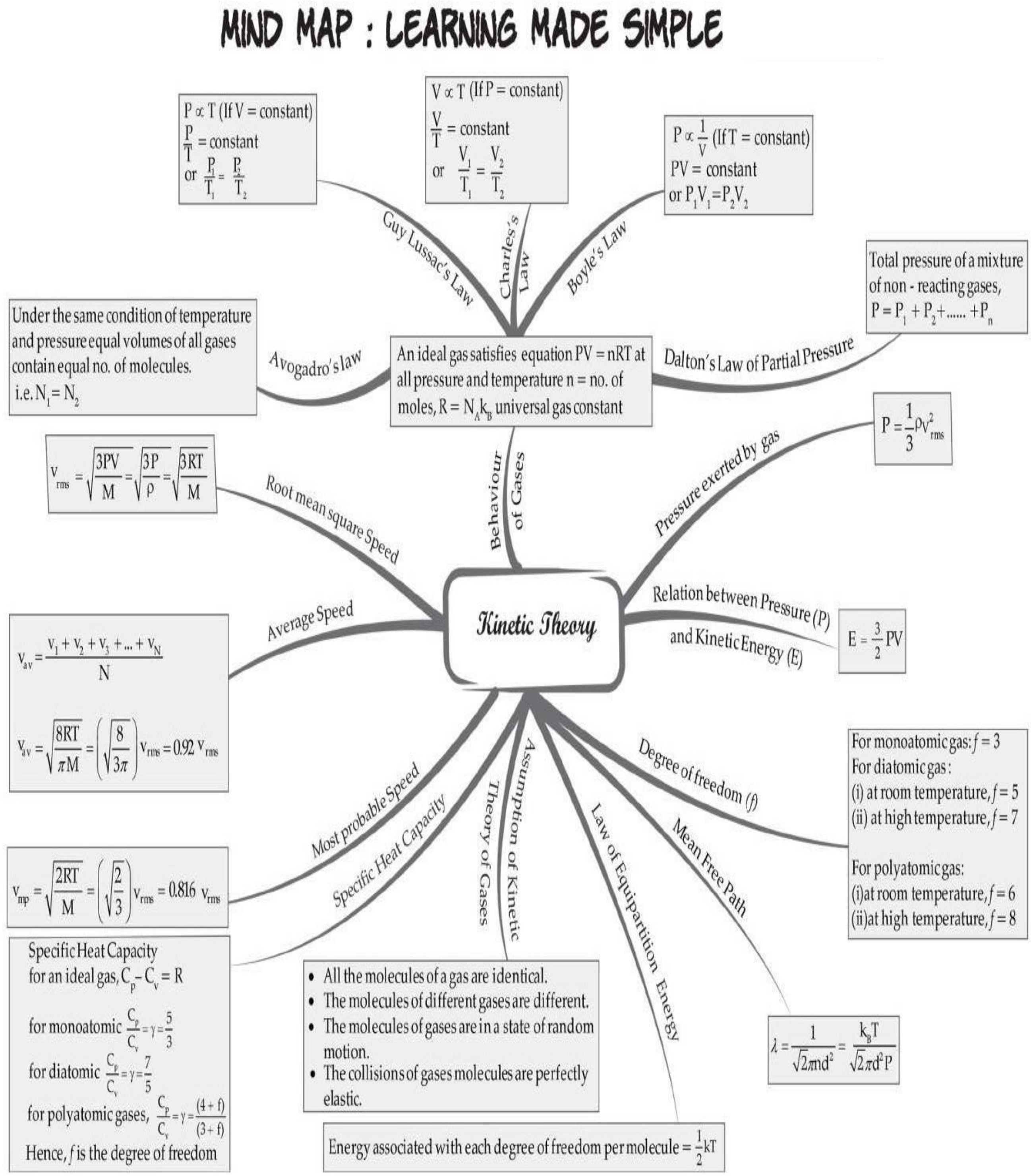
(b) For an isothermal expansion:
$P_1V_1 = P_2V_2$ (Boyle's Law)
This equation shows that at constant temperature; the product of pressure and volume is constant.
Work done (W) = $\int PdV$ from V1 to V2
This is the mathematical expression for work done in an isothermal process.
W = nRT $\int(dV/V)$ from V1 to V2
After integrating, we get:
$W = nRT \ln(V2/V1)$ *Also* ,$W = 2.303\ nRT \log(V2/V1)$

This equation gives the work done in an isothermal expansion of an ideal gas.
(c) Graphical representation:
- Isothermal process: Horizontal line on PV diagram.
This indicates that pressure and volume are inversely proportional at constant temperature.
- Adiabatic process: Steeper slope than isothermal on PV diagram.
This indicates rapid change in pressure with volume due to no heat exchange.
3. (a) Define specific heat capacity at constant volume and constant pressure. (1 mark)
(b) Derive a relation between these two specific heat capacities for an ideal gas. (3 marks)
(c) Explain the significance of this relation. (1 mark)
Solution: (a) Specific heat capacity at constant volume (Cv):
Amount of heat required to raise temperature of unit mass of gas by 1°C at constant volume.
It is a measure of the amount of heat energy required to change the temperature of a unit mass of a gas by one degree Celsius (or Kelvin) while keeping the volume constant.
Specific heat capacity at constant pressure (Cp):
Amount of heat required to raise temperature of unit mass of gas by 1°C at constant pressure.
It is a measure of the amount of heat energy required to change the temperature of a unit mass of a gas by one degree Celsius (or Kelvin) while keeping the pressure constant.
(b) For an ideal gas:
$C_p - C_v = R$ where R is gas constant.
Derivation:
1. $\Delta U = nC_v\Delta T$ (at constant volume) This equation shows the change in internal energy (ΔU) of an ideal gas at constant volume. Where, n is the number of moles of gas, Cv is specific heat capacity at constant volume, and ΔT is change in temperature.
2. $\Delta Q = nC_p\Delta T$ (at constant pressure) This equation shows the change in enthalpy (ΔH) of an ideal gas at constant pressure.Where,n is the number of moles of gas, Cp is specific heat capacity at constant pressure, and ΔT is change in temperature.
3. $\Delta Q - \Delta U = n(C_p-C_v)\Delta T = nR\Delta T$ This equation shows the relation between change in enthalpy and change in internal energy. Therefore, $C_p - C_v = R$
(c) Significance:
This relation helps in calculating one specific heat capacity if the other is known.
It also explains why Cp is always greater than Cv for an ideal gas because some energy is used to do work in expanding the gas at constant pressure.

CHAPTER-12: KINETIC THEORY OF GASES

SYLLABUS:-Equation of state of a perfect gas, work done in compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure. Kinetic interpretation of temperature; rms speed of gas molecules; degrees of freedom, law of equi-partition of energy (statement only) and application to specific heat capacities of gases; concept of mean free path, Avogadro's number.



GIST OF THE CHAPTER:

Kinetic theory of gases relates the three basic features of gases: volume, pressure and temperature. The kinetic theory was developed by Maxwell, Boltzmann and Gibbs in the nineteenth century.

BEHAVIOUR OF GASES

Gases at *low pressures* and *high temperatures* shows *ideal gas behavior*.

Thei deal gas equation can be written as $PV = \mu RT$

Where μ is the number of moles and *R is universal gas constant.* R =8.314J mol^{-1} K^{-1}

Also in terms of Boltzmann's constant ideal gas equation can be written as

$PV = k_B NT$ or $P = k_B nT$

Where n=N/V The value of Boltzmann constant in SI units is 1.38 x 10^{-23}JK^{-1}

KINETIC THEORY OF AN IDEAL GAS

Kinetic theory of gases is a theory, which is based on the concept of molecular motion as is able to explain the behavior of gases.

POSTULATES OF KINETIC THEORY:

- The molecules of a gas are supposed to be point masses, the size of a molecule being negligible compared to the distance between them.
- There is no force of attraction or repulsion between molecules.
- The molecules are in a state of random motion, moving with all possible velocities in all possible directions.
- During their motion, they collide with one another and also with the walls of the container. These collisions are elastic.
- Between successive collisions, the molecules move in straight lines with uniform velocity. The distance travelled between two successive collisions is called free path. Average distance between the successive collisions is called mean free path
- Time for a collision is negligibly small compared to the time taken to traverse mean free path.
- The mean KE of the molecule is a constant at a given temperature and is proportional to absolute temperature.

CONCEPT OF PRESSURE.

The pressure exerted by a gas may be defined as the total momentum imparted to unit area of the walls of the container per second due to molecular impacts (collisions).

ROOT MEAN SQUARE (RMS) VELOCITY OF GAS MOLECULES.

rms velocity of gas molecules is the square root of the mean of thee squares of individual velocities of the molecules.

If$c_1, c_2, \ldots\ldots c_n$ are the velocities of a gas molecules, then mean square velocity,

$$\overline{c^2} = \frac{c_1^2 + c_2^2 + c_3^2 + \ldots\ldots + c_n^2}{n}$$

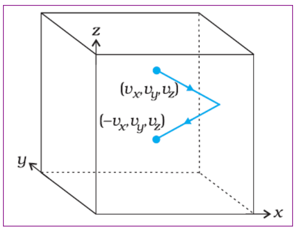
Hence root mean square velocity

$$c_{rms} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + \ldots\ldots + c_n^2}{n}}$$

At a temperature *T=300K,the root mean square speed* of a molecule in nitrogen gas is :

$v_{rms} = 516$m/s

Pressure of an Ideal Gas



A molecule with velocity (v_x, v_y, v_z) *hits the planar wall parallel to yz plane* of area *A (=* l^2*)*.The velocity after collision is($-v_x$,v_y, v_z).

The change in momentum of the molecule is : $-mv_x - (mv_x) = -2mv_x$

By the principle of conservation of momentum, the momentum imparted to the wall in the collision = $2mv_x$. Thus the number of molecules with velocity(v_x,v_y,v_z)*hitting the wall in time Δt is ½A* v_x *Δt n, where n is the number of molecules* per unit volume.

The total momentum transferred to the wall in time Δ*t is :* $Q = (2mv_x)\,(1/2\ nAv_x\Delta t)$

The force on the wall is the rate of momentum transfer *Q/Δt and pressure is force per unit* area : $P = Q/(A\Delta t) = nmv_x^2$

The above equation therefore, stands for pressure due to the group of molecules with speed v_x *in the x-direction and n stands for the* number density of that group of molecules. The total pressure is obtained by summing over the contribution due to all groups:

$P = n\, m\, \bar{v}^2{}_x$

Now the gas is isotropic ,i.e. there is no preferred direction of velocity of the molecules in the vessel. Therefore, by symmetry,

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$
$$= (1/3)\,[\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}\,] = (1/3)\,\overline{v^2}$$

Thus $P = \frac{1}{3}\, n\, m\, \bar{v}^2$

KINETIC INTERPRETATION OF TEMPERATURE

We have $P = \frac{1}{3}\, n\, m\, \bar{v}^2$ We may write $PV = \frac{1}{3}\, n\, V\, m\, v^2$ $PV = \frac{2}{3}\, N \times \frac{1}{2}\, m\, v^2$

Where N(=nV)is the number of molecules in the sample. The quantity in the bracket is the average translational kinetic energy of the molecules in the gas. Since the internal energy E of an ideal gas is purely kinetic, $E = N \times \frac{1}{2}\, m\, \bar{v}^2$

Thus $PV = 2/3\, E$

But we have $PV = k_BNT$ or $P = k_BnT$

Thus $E = 3/2\, k_BNT$ $E/N = ½\, m\, v^2 = 3/2\, k_BT$

Thus the *average kinetic energy of a molecule is proportional to the absolute temperature of the gas*; it is independent of pressure, volume or the nature of the ideal gas.

$P = \frac{1}{3}\,(n_1 m_1 v_1^2 + (n_2 m_2 v_2^2 + \ldots\ldots\ldots)$

MEAN FREE PATH

Mean free path of a molecule in a gas is the average distance travelled by the molecule between two successive collisions.

The mean free path, in gases, is of the order of thousands of angstroms.

Let $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ are the free paths travelled by the molecules in successive collissions, the mean free path is given by $\lambda = (\lambda_1 + \lambda_2 + \lambda_3 + \ldots\ldots\ldots\ldots + \lambda_n)/\, n$

Equation for mean free path

$$\lambda = \frac{1}{\sqrt{2}\, n\, \pi\, d^2}$$

Degrees of freedom:

The total number of independent modes(ways)in which a system can possess energy is called the degree of freedom (f). The degree of freedom is of three types:

(i)Translational degree of freedom (ii)Rotational degree of freedom

(iii)Vibrational degree of freedom

General expression for degree of freedom f=3N–R

where N = Number of independent particles, R = Number of independent restrictions

(1) Monoatomic gas: It can have 3 degrees of freedom (all translational).

(2) Diatomic gas: A diatomic molecule has5 degree of freedom:3 translational and 2 rotational.

(3) Triatomic gas

Law of Equipartition of Energy:
For a dynamic system in thermal equilibrium, the energy of the system is equally distributed amongst the various degrees of freedom and the energy associated with each degree of freedom per molecule is 1/2 kT, where k is Boltzmann constant.
Avogadro's Law: Equal volumes of all gases under S.T.P. contain the same number of molecules equalling 6.023×10^{23}.
Specific Heat of Gases (i) In adiabatic process *i.e.,* ΔQ = 0, ∴ C = $\frac{C\,\Delta Q}{m(\Delta T)}$ = 0 *i.e.,* C = 0
(ii) In isothermal process *i.e.,* ΔT = 0 ∴ C = $\frac{C\,\Delta Q}{m(\Delta T)}$ *i.e.,* C = ∞
Specific heat of gas can have any positive value ranging from zero to infinity.

MULTIPLE CHOICE QUESTIONS (MCQ)

1. The gas having average speed four times as that SO_2 (molecular mass 64) is
(a) He (molecular mass 64) (b) O_2 (molecular mass 4)
(c) H_2 (molecular mass 32) (d) CH_4 (molecular mass 16)
2. The temperature of a given mass is increased from 27^{O}C to 327 ^{O}C. The rms velocity of the molecules increases
(a) $\sqrt{2}$ times (b) 2 times (c) 2 $\sqrt{2}$ times (d) 4 times
3. The ratio of the vapour densities of two gases at a given temperature is 9: 8. The ratio of the rms velocities of their molecules is
(a) 3/2 $\sqrt{2}$ (b) 2$\sqrt{2}$ / 3 (c) 9:8 (d) 8:9
4. A diatomic gas molecule has translational, rotational and vibrational degrees of freedom. The C_P / C_V is
(a) 1.67 (b) 1.4 (c) 1.29 (d) 1.33
5. A gas is filled in a container at pressure P.If the mass of molecules is halved and their rms speed is doubled. The resultant pressure would be
(a) 2P (b) 4P (c)P/4 (d) P/2
6. In kinetic theory of gases ,it is assumed that:
(a) The collisions are not perfectly elastic.
(b) The molecular collisions change the density of the gas.
(c) The molecules don't collide with each other on the well.
(d) Between two collisions the molecules travel with uniform velocity.
7. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C.
(a) –73K (b) 3K (c) 20K (d) 80K
8. The work done by (or on) a gas per mole per kelvin is the gas constant called
(a) Universal gas constant (b) Boltzmann's constant
(c) Gravitational constant (d) Entropy
9. A gas behaves as an ideal gas at
(a) Low pressure and high temperature (b) low pressure and low temperature
(c) High pressure and low temperature (d) high pressure and high temperature
10. The root mean square velocity of gas molecules is 10km/sThe gas is heated till its pressure becomes four times. The velocity of gas molecules will be
(a) 10Km/s (b) 20Km/s (c) 40Km/s (d) 80Km/s

ANSWERS (MCQ)

1. A V rms = $\sqrt{\frac{3RT}{M}}$ $\frac{V_1}{V_2} = \frac{M_1}{M_2}$
2. A V rms α $\sqrt{T}$

3. B V rms α $\sqrt{\frac{1}{\rho}}$
4. B
5. A Using, $P=1/3\rho v^2_{rms}=1/3mNv^2_{rms}/V$ $\therefore P\propto mv^2_{rms}$
6. D – Theory Based If the average speed is halved and the root mean square (rms) speed is doubled, then pressure P becomes twice, i.e., 2P.
7. C – RMS Velocity of a Gas Molecule
   The RMS velocity of a gas molecule is given by: $v=\sqrt{3RT/M}$
   Let the temperature at which the velocity of hydrogen is equal to the velocity of oxygen at 47°C be T. $\sqrt{3RT/2}=\sqrt{3R\times(273+47)/32}$.
   Squaring both sides: $\Rightarrow T=20K$
   A – Gas Constant The gas constant is a fundamental physical constant that relates the energy of a gas to its temperature, volume, and pressure.
   It is given as:R= 8.3145J/mol/K. The work done by or on a gas per mole per Kelvin is called the gas constant R.

9. A – Ideal Gas Behavior
   A gas behaves as an ideal gas at low pressure and high temperature.
   A gas behaves ideally when its molecules are widely spaced, move freely, and do not interact with each other.
   10. A $PV=1/3MV^2rms$ $V^2rms= 3PV/M$ $V^2rms= 3P/\rho$ =10km/s

ASSERTION & REASON QUESTIONS

For Assertion and Reason Questions, two statements are given – one labelled Assertion (A) and other labelledReason (R).
Select the correct answer to these questions from the options as given below.
A. If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
C. If Assertion is true but Reason is false.
D. If both Assertion and Reason are false.

1. Assertion: The total translational kinetic energy of all the molecules of one mole of an ideal gas is 1.5 times the product of its pressure and its volume.
   Reason: The molecules of a gas collide with each other and the velocities of the molecules change due to collision.

1. Assertion: Air pressure in a car tyre increases during driving.
   Reason: Absolute zero temperature is not zero energy temperature.
2. Assertion: The ratio C_P,/C_V, for diatomic gas is more than that for a monoatomic gas.
   Reason: The molecules of a monoatomic gas have more degrees of freedom than those of a diatomic gas.
3. Assertion: For an ideal gas at constant temperature, the product of the pressure and volume is a constant.
   Reason: The mean square velocity of the molecules is inversely proportional to mass.
4. Assertion: The root mean square and most probable speeds of the molecules in a gas are the same.
   Reason: The Maxwell distribution for the speed of molecules in a gas is symmetrical.
5. Assertion: Vibrational energy of a diatomic molecule corresponding to each degree of freedom is k_BT.
   Reason: For every molecule, vibrational degree of freedom is 2.

6. Assertion: The ratio C_P/C_V is more for helium gas than for hydrogen gas.
   Reason: Atomic mass of helium is more than that of hydrogen.
7. Assertion: An undamped spring-mass system is simplest free vibration system.
   Reason: It has three degrees of freedom.

ANSWERS

1. B Assertion is true and reason is false. $E = 1/2k_B T \times N_A = 3/2 RT = 3/2 P$
2. B Both assertion and reason are true but the reason is not a correct explanation of the assertion. Air pressure in a car tyre increases due to rise in temperature during driving.
3. A Both assertion and reason are true. For monoatomic gas, $C_P/C_V = 1+2/f = 1+2/3 = 5/3$For diatomic gas, $C_P/C_V = 1+2/f = 1+2/5 = 7/5$
4. B Both assertion and reason are true but the reason is not a correct explanation of the assertion. According to Boyle's law, PV = constant, at a given temperature.
5. D Both assertion and reason are false. The two speeds are different from each other. Also, the Maxwell distribution for the speed to molecules in a gas is asymmetrical.
6. D Assertion is false and reason is true. Each diatomic molecule has vibrational energy equal to $2 \times 1/2k_B T$ because a vibrational frequency has both kinetic and potential modes and it contributes 2 degrees of freedom.
7. B Both assertion and reason are true but the reason is not a correct explanation of the assertion. Monoatomic helium has 3 degrees of freedom while diatomic hydrogen has 5 degrees of freedom.

SHORT ANSWER QUESTIONS (2 MARKS)

1. A sealed container with a fixed volume is filled with gas. If the temperature of the gas is doubled, what happens to the pressure? Justify your answer using kinetic theory.
2. Why the land has a higher temperature than the ocean during the day but a lower temperature at night.
3. Why does a gas exert pressure on the walls of its container? Explain using kinetic theory.
4. When an automobile travels for a long distance the air pressure in the tyres increases. Why?
5. A gas storage tank has a small leak. The pressure in the tank drop more quickly if the gas is hydrogen than if it is oxygen. Why?
6. Why is the concept of ideal gas an approximation? In what conditions does real gas behave like an ideal gas?
7. If the mass of each gas molecule is doubled but temperature remains constant, what happens to the average kinetic energy?

ANSWERS OF EXERCISE QUESTIONS

1. From the kinetic theory, (at constant volume). So, if temperature is doubled, pressure also doubles. This is because the average kinetic energy of molecules increases, leading to more frequent and forceful collisions with the walls.
2. Specific Heat of water is more than land (earth). Therefore, for given heat change in temp. of land is more than ocean (water).
3. Gas molecules move randomly and collide with the walls of the container. Each collision transfers momentum, and the continuous bombardment of molecules creates a force per unit area, i.e., pressure.
4. Work is done against friction. This work done is converted into heat. Temperature rises. $PV = nRT$, As volume of tyre is const. $P \propto T$.

5. Rate of diffusion of a gas is inversely proportional to the square root of the density. So hydrogen leaked out more rapidly.
6. Ideal gas laws assume no intermolecular forces and negligible volume of molecules. Real gases approximate ideal behaviour at low pressure and high temperature when intermolecular forces are weak and volume occupied is negligible.
7. The average kinetic energy depends only on temperature, not mass. So, it remains unchanged even if the mass doubles.

SHORT ANSWER QUESTIONS (3 MARKS)

1. Demonstrate your understanding of the kinetic theory of gases by listing its fundamental assumptions and explaining the significance of each.
2. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains ne on(monoatomic),the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contains equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is vrms the largest?
3. Analyse the factors affecting the mean free path (λ) of gas molecules and explain how each parameter influences it.
4. Equal masses of oxygen and helium gases are supplied equal amount of heat. Which gas will undergo a greater temperature rise and why?
5. Explain the process of evaporation and analyze how it leads to cooling, using the concept of kinetic energy of molecules.

ANSWERS

1. The following are the assumptions of the kinetic theory of gases:
    a) A gas is made up of a vast number of molecules that should all be elastic spheres and identical.
    b) A gas's molecules are in a constant state of rapid and unpredictable mobility.
    c) Gas molecules are extremely small in comparison to the distance between them.
    d) There is no attraction or repulsion between the molecules.
    e) Molecule collisions with one another and with the vessel's walls are perfectly elastic.

2. (a)Yes, because according to Avogadro's hypothesis, equal volume of all the gases has same number of molecules under the condition of same temperature and pressure.

(b)Using $v_{rms}=\sqrt{3kT/m}$,we get $v_{rms} \propto 1/\sqrt{m}$, i.e. vrms will not be same in the three cases because it depends upon mass of the gas. It will be the largest for neon

3. (i) diameter of molecule (ii) Pressure of gas

4. Heat supplied to oxygen = Heat supplied to Helium $mc_1\Delta T_1 = mc_2\Delta T_2$ $\Delta T_1 > \Delta T_2$

5. During evaporation fast moving molecules escape a liquid surface so the average kinetic energy of the molecules left behind is decreased thus the temperature of the liquid is lowered.

CASE BASED QUUESTIONS:

I. A balloon is filled with helium gas and sealed at room temperature (300 K). It is then placed in sunlight, and the gas inside gets heated. The balloon is flexible and can expand to accommodate the increased pressure. A science student observes that as the temperature increases, the balloon expands and rises higher in the air.The student wants to understand this behavior using the kinetic theory of gases. According to this theory, gas molecules are in continuous random motion, colliding with each other and the container walls. These collisions are responsible for exerting pressure.

1. According to the kinetic theory of gases, what is the main reason for the increase in pressure (or volume) as the temperature of the gas increases?

a) The number of gas molecules increases
b) The gas molecules become heavier
c) The average kinetic energy of gas molecules increases
d) The size of the gas molecules increases

2. If the temperature of the helium gas in the balloon increases from 300 K to 600 K, and the pressure is kept constant, the volume of the balloon will:

a) Remain the same b) Become half c) Become double d) Become four times

3. The kinetic theory of gases assumes that:

a) There are strong attractive forces between gas molecules
b) Gas molecules are in random motion and undergo elastic collisions
c) Gas molecules lose energy during collisions
d) Gas pressure is caused by gravity

4. Which of the following correctly describes what happens to the root mean square speed of the gas molecules when temperature increases?

a) It decreases linearly b) It remains constant
c) It increases linearly with temperature
d) It increases with the square root of temperature

5. As the balloon rises to higher altitudes, the external atmospheric pressure decreases. What is the likely effect on the balloon?

a) The balloon will shrink b) The balloon will remain the same size
c) The balloon will expand further d) The balloon will burst and fall

II. The kinetic theory of gases is a fundamental concept in physics that describes the behavior of gases based on the motion of their constituent particles. According to this theory, gases are composed of a large number of tiny particles, such as atoms or molecules, that are in constant random motion. The pressure exerted by a gas is a result of the collisions between these particles and the walls of the container. The kinetic energy of the gas particles is directly proportional to their temperature, and as the temperature increases, the particles move with higher velocities, resulting in an increase in pressure. The kinetic theory also explains the relationship between temperature and the average kinetic energy of the gas particles. At the same temperature, all gases have an equal average kinetic energy per molecule. This theory has been essential in understanding various gas properties and phenomena, including diffusion, effusion, and the ideal gas law.

1. The kinetic theory of gases describes the behavior of gases based on the motion of their constituent particles, which are:
   (a) Constantly at rest (b) In constant random motion
   (c) Arranged in a regular pattern (d) Composed of protons and electrons
2. The pressure exerted by a gas is a result of the collisions between gas particles and:
   (a) The walls of the container (b) Each other
   (c) The surrounding atmosphere (d) The gravitational force
3. The kinetic energy of gas particles is directly proportional to their:
   (a) Mass (b) Temperature (c) Volume (d) Pressure
4. According to the kinetic theory, at the same temperature, all gases have an equal average kinetic energy per:
   (a) Molecule (b) Atom (c) Gram (d) Litre

III. A gas is enclosed in a frictionless piston-cylinder arrangement. Initially, the gas is at room temperature and atmospheric pressure. The piston is slowly pushed inward, compressing the gas isothermally (temperature remains constant). As the gas is compressed, the volume decreases, and the pressure increases. The student performing this experiment refers to the kinetic theory of gases to understand the molecular explanation behind the increase in pressure. The theory assumes that gas molecules are in constant motion and pressure arises due to their collisions with the walls of the container.

1. What happens to the number of collisions per unit area per second of gas molecules with the container walls when the volume is decreased?
   a) Decreases b) Increases c) Remains constant d) Becomes zero
2. During isothermal compression, which of the following quantities remains unchanged?
   a) Pressure b) Volume c) Temperature d) Number of collisions
3. According to the kinetic theory, pressure of a gas is directly related to:
   a) The shape of the container b) The gravitational field
   c) The rate of collisions of molecules with container walls
   d) The color of gas molecules
4. If the volume of the gas is halved during the isothermal compression, what happens to the pressure?
   a) It becomes one-fourth b) It remains the same
   c) It becomes double d) It becomes zero

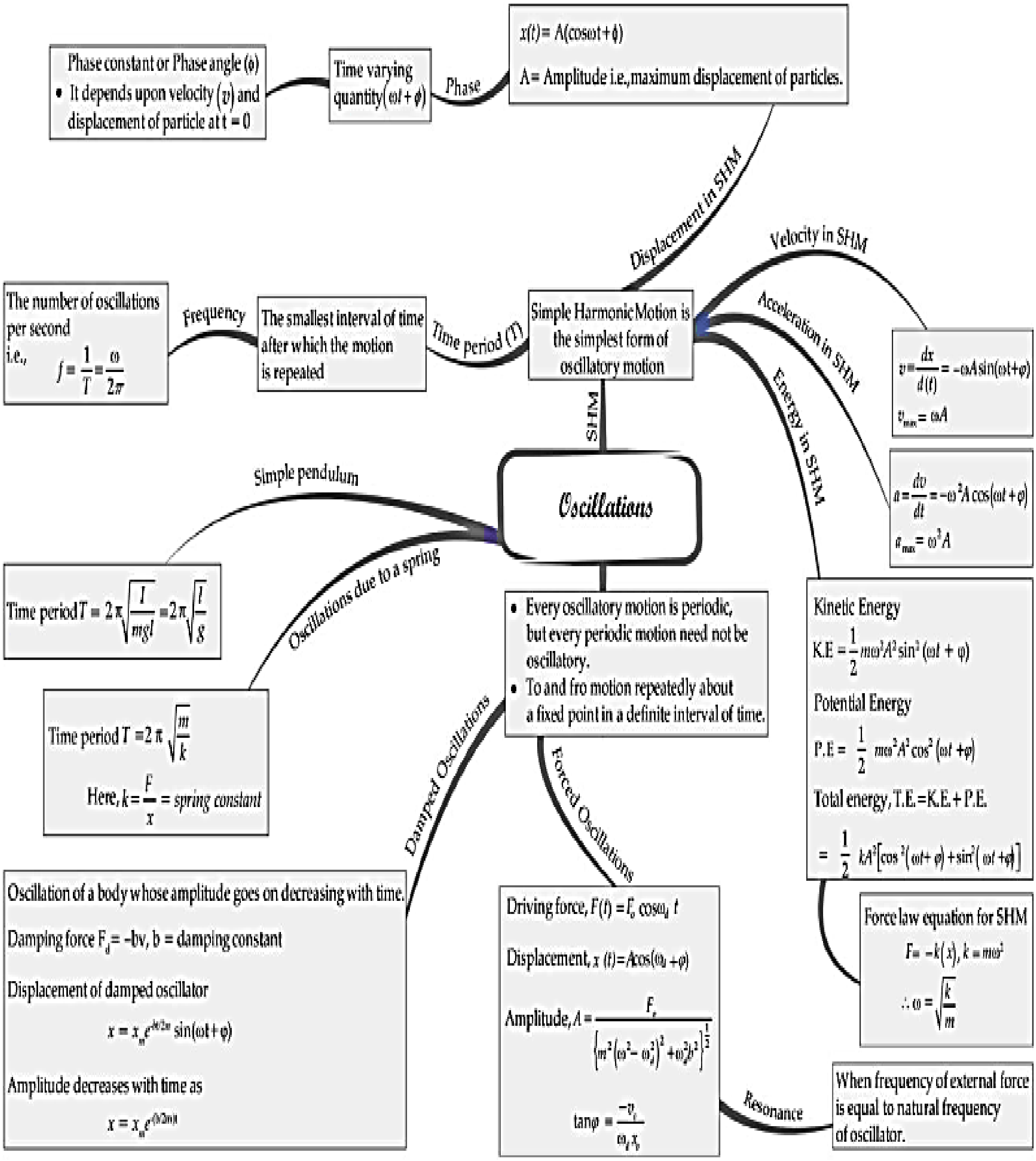
ANSWERS:

I.	1. C	2. C	3. B	4. D	5. C
II.	1. B	2. A	3. B	4. A	
III.	1. B	2. C	3. C	4. C	

CHAPTER–13: OSCILLATIONS

Periodic motion - time period, frequency, displacement as a function of time, periodic functions and their applications. Simple harmonic motion (S.H.M), uniform circular motion and its equations of motion; phase; oscillations of a loaded spring- restoring force and force constant; energy in S.H.M. Kinetic and potential energies; simple pendulum-derivation of expression for its time period.

MIND MAP



GIST OF THE CHAPTER

Periodic Motion

Periodic motion refers to any movement that repeats itself after a fixed interval of time. Common examples include the motion of a pendulum, the rotation of the Earth, and the vibrations of a tuning fork.

Oscillatory Motion

Oscillatory motion is a specific type of periodic motion where an object moves back and forth repeatedly about a central or fixed position.

Simple harmonic motion is a specific type of oscillatory motion, in which

(a) particle moves in one dimension,
(b) particle moves to and fro about a fixed mean position (where Fnet = 0),
(c) net force on the particle is always directed towards means position, and
(d) magnitude of net force is always proportional to the displacement of particle from the mean position at that instant.

So, $F_{net} = -kx$ where, k is known as force constant

$ma = -kx \Rightarrow a = -(k/m)x \Rightarrow a = -\omega^2 x$ where, ω is known as angular frequency.

$d^2x/dt^2 = -\omega^2 x$ This equation is called as the differential equation of S.H.M.

The general expression for x(t) satisfying the above equation is: $x(t) = A\sin(\omega t + \theta)$

Note: Every harmonic motion is periodic, but all periodic motions may not be harmonic.

- When studying SHM (e.g., a **simple pendulum**), the restoring force depends on the angle θ that the pendulum makes with the vertical. The exact motion is not truly simple harmonic unless we **approximate**.

To make it SHM, we use the **small-angle approximation**:
$\sin\theta \approx \theta$(in radians, when θ is small, typically $<10^\circ$)

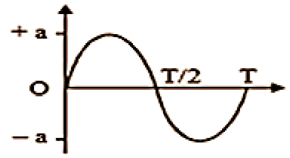
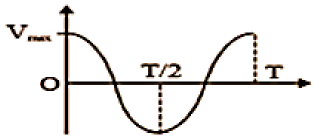
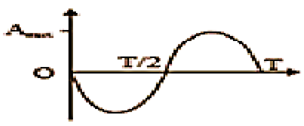
- Amplitude(A): The amplitude of particle executing S.H.M. is its maximum displacement on either side of the mean position.
- Time Period(T): Time period of a particle executing S.H.M. is the time taken to complete one cycle. $T = 2\pi/\omega$ $T = 2\pi\sqrt{\frac{m}{k}}$
- Frequency (ν): Number of oscillations per unit time. $\nu = 1/T$ (Unit: Hz)
- Angular Frequency (ω) $\omega = 2\pi\nu$ (Unit: rad/s)
- Phase (φ): Describes the position and direction of motion of the particle at a given instant. It is measured as argument (angle) of sine in the equation of S.H.M.
- Initial Phase (φ_0) or phase constant: The phase when time t=0.

Periodic Function

A periodic function is a mathematical function that repeats its values at regular intervals of its input. The sine and cosine functions are typical examples, both having a fundamental period T: $f(t)=\sin(\omega t)$ $g(t)=\cos(\omega t)$

These are collectively referred to as **harmonic functions**.

A basic form of a harmonic function is: $f(t)=A\cos(\omega t)$ where $\omega=2\pi/T$
If ωt increases by a multiple of 2π, the function value remains unchanged. Therefore, the function is periodic with a period: $T=2\pi/\omega$

		At mean position	At extreme positions
	**Position** $X(t) = A\sin(\omega t + \varphi)$	$x = 0$	$x = +A, -A$
	**Velocity** $v(t) = A\omega\cos(\omega t + \varphi)$ $v(x) = \pm\,\omega\sqrt{A^2 - x^2}$	$v_{min} = 0$	$v_{max} = \omega A$
	**Acceleration** $a(t) = -\omega^2 A\sin(\omega t + \phi)$ $a(x) = -\omega^2 x$	$a_{min} = 0$	$a_{max} = \omega^2 A$

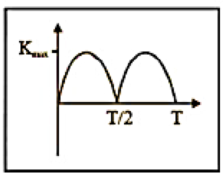
- **ENERGY**

Kinetic energy

$K = ½\, mv^2$

$K = 1/2\, m\omega^2(A^2 - x^2)$

$= ½\, m\omega^2 A^2 \cos^2(\omega t + \phi)$



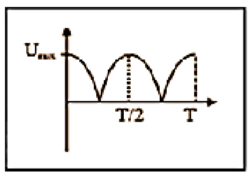
At mean position,
- $K_{max} = 1/2\, m\omega^2 A^2$
  $= 1/2\, kA^2$

At extreme positions,
- $K_{min} = 0$

Potential Energy

$U = 1/2\, kx^2$

$U = ½\, m\omega^2 A^2 \sin^2(\omega t + \phi)$



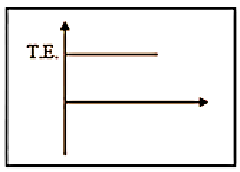
At mean position,
- $U_{min} = 0$

At extreme positions,
- $U_{max} = 1/2\, m\omega^2 A^2 = 1/2\, kA^2$

Total Energy

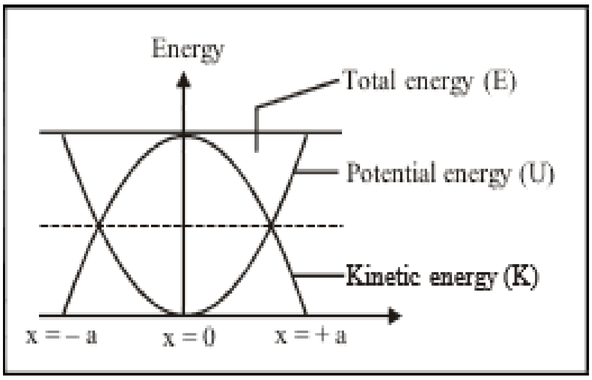
$T.E = 1/2\, kA^2$

$= 1/2\, m\omega^2 A^2$



It is constant at all time instant and at all positions.

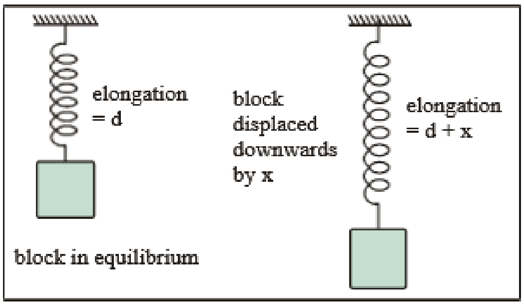
➢ **<u>Energy – position graph</u>**

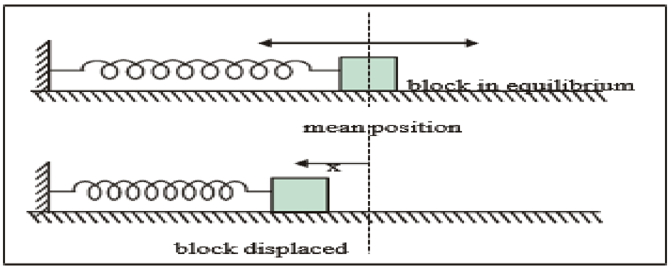


Special Cases

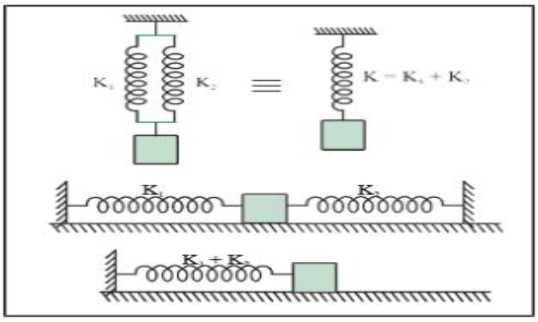
Mass on a Spring (Vertical or Horizontal):

T=2π√m/k

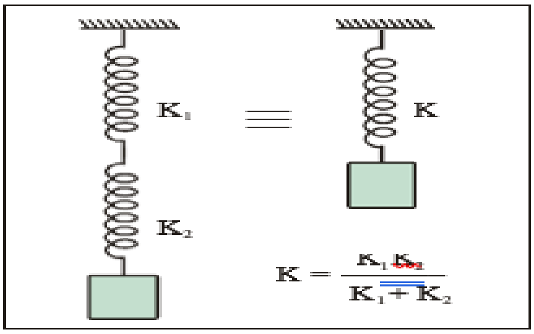




➢ **Two Springs in Parallel:**



Two Springs in Series:



$k_{eff}=k_1+k_2$

$T=2\pi\sqrt{\frac{m}{(k1+k2)}}$

$1/\ k_{eff}=1/k_1+1/k_2$;

$T=2\pi\sqrt{\frac{m(k1+k2)}{k1\ k2}}$

➢ **Simple Pendulum (length l):**

Time period of a simple pendulum, $T=2\pi\sqrt{\frac{l}{g}}$

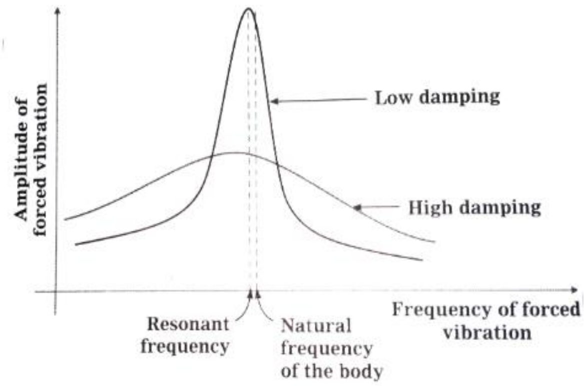
Time period of a simple pendulum in a lift

$T = 2\pi\sqrt{\frac{l}{g+a}}$ (if acceleration of lift is upwards)

$T = 2\pi\sqrt{\frac{l}{g-a}}$ (if acceleration of lift is down wards)

Resonance

Resonance occurs when the frequency of an external force matches the natural frequency of an oscillator. Under resonance, the amplitude of oscillation becomes maximum.



PhET Simulation

- . **Periodic and Oscillatory Motion** **https://phet.colorado.edu/en/simulation/masses-and-springs**
- Simple Harmonic Motion (SHM) https://phet.colorado.edu/en/simulation/pendulum-lab
- **Energy in SHM** (KE & PE exchange) https://phet.colorado.edu/en/simulation/masses-and-springs
- Resonance https://phet.colorado.edu/en/simulation/resonance

MULTIPLE CHOICE QUESTIONS

1) The nature of the acceleration vs displacement graph for a particle is executing S.H.M. is

a) A straight-line b) A circle c) An ellipse d) A hyperbola

2) Two particles execute S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, and each time their displacement is half of their amplitude. The phase difference between them is

a) 30° b) 60° c) 90° d) 120°

3) A particle is performing simple harmonic motion along x-axis with amplitude 4 cm and time period 1.2 sec. The minimum time taken by the particle to move from x =2 cm to x = + 4 cm and back again is given by

a) 0.6 s b) 0.4 s c) 0.3 sec d) 0.2 sec

4) The change (or changes) observed when the period of a simple pendulum is doubled is (or are)

a) Its length is doubled b) The mass of the bob is doubled

c) Its length is made four times

d) The mass of the bob and the length of the pendulum are doubled

5) The period of oscillation of a simple pendulum of constant length at earth's surface is T. Its period inside a mine is

a) Greater than T b) Less than T c) Equal to T d) Cannot be compared

6) The total energy of a particle executing S.H.M. is proportional to

a) Displacement from equilibrium position b) Frequency of oscillation

c) Velocity in equilibrium position d) Square of amplitude of motion

7) A particle executes simple harmonic motion along a straight line with an amplitude A. The potential energy is maximum when the displacement is

a) ±A b) Zero c) ±A/√2 d) ±A/√2

8) A particle is vibrating in a simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position, is its energy half potential and half kinetic

a) 1 cm b) 2/√2 cm c) 3 cm d) 2 √2cm

9) Two equations of two S.H.M. are y=asin (ωt−α) and y=bcos (ωt−α). The phase difference between the two is

a) 0° b) a° c) 90° d)180°

ANSWERS:

1) **A ;** The graph of acceleration vs. displacement for a particle in S.H.M. is a straight line with negative slope — indicating that acceleration is always directed opposite to displacement.

2) D For Particle 1: sin(ωt) =1/2=π/6

For Particle 2: sin (ωt+ϕ) =1/2 The sine function equals ½ at π/6 and 5π/6. Since the particles are moving in opposite directions, the phase difference corresponds to the latter: ωt+ϕ=5π/6

Substituting ωt=π/6, π/6+ϕ=5π/6

ϕ=5π/6−π/6 = 2π/3 radians = 120°

3) B $x_1=4\sin\omega t_1$ $\sin\omega t_1 = x_1/4 = 2/4 = 1/2$ ∴ $\omega t_1 = \pi/6$

$x_2=4\sin\omega t_2$ $\sin\omega t_2 = x_2/4 = 4/4 = 1$ ∴ $\omega t_2 = \pi/2$

$t_2 - t_1 = (\pi/2 - \pi/6)/\ \omega = (\pi/2 - \pi/6) \times T/2\pi = T/6 = 1.2/6 = 0.2$

Therefore, time taken by the particle to move from x = 2cm to x =4cm and back again is 0.2+0.2=0.4s.

4) C The **time period** T of a simple pendulum is given by: $T=2\pi\sqrt{\frac{l}{g}}$ ----------(1)

To make the time period 2T, let the length be changed to l'.Then, $2T = 2\pi\sqrt{\frac{l'}{g}}$ ------------(2)

From (1) and (2),2= √l'/√l (or) l'=4l

5) A ; Inside a mine, the acceleration due to gravity decreases.

T∝√1/ g. If g increases, T decreases. If g decreases (like on the Moon or inside a mine), T increases.

6) D ; The total energy E in S.H.M. is the sum of kinetic and potential energy, and is constant for a given motion: $E=1/2m\omega^2A^2$; $E \propto A^2$

7) A ; The potential energy (P.E.) in S.H.M. at any displacement x is:

P.E.=1/2 kx^2. So, P.E. is maximum when x is maximum, i.e., when x=±A

8) D Kinetic energy, $K=1/2m\omega^2(a^2-x^2)$ Potential energy, $U=1/2m\omega^2x^2$

Given, $KE = PE$ $1/2m\omega^2(a^2-x^2) = 1/2m\omega^2x^2$ x= a/√2 = 4/√2 = 2√2

9) C; y_2=bcos(ωt−α)= bsin((ωt−α)+90°)

ASSERTION REASON TYPE QUESTIONS

a) Both assertion and reason are true and the reason is the correct explanation of the assertion.
b) Both assertion and reason are true but reason is not the correct explanation of the assertion.
c) Assertion is true but reason is false.
d) Assertion and reason both are false.

1) Assertion: In a S.H.M., kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude.
Reason: In SHM, kinetic energy is zero when potential energy is maximum

2) Assertion: Simple harmonic motion is a uniform motion.
Reason: Simple harmonic motion occurs due to gravitational force only.

3) Assertion: Acceleration is proportional to the displacement. This condition is not sufficient for motion in simple harmonic.
Reason: In simple harmonic motion direction of displacement is also considered.

4) Assertion: Sine and cosine functions are periodic functions.
Reason: Sinusoidal functions repeat it values after a definite interval of time.

5) Assertion: The graph between velocity and displacement for a harmonic oscillator is a parabola.
Reason: Velocity remains constant throughout the motion in simple harmonic motion.

6)Assertion: When a simple pendulum is made to oscillate on the surface of moon, its time period increases.
Reason: Moon is much smaller compared to Earth.

7) **Assertion:** Resonance is special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency of external periodic force and the amplitude of forced vibration is maximum.
Reason: The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force.

ANSWERS OF QUESTIONS

1)b Both assertion and reason are true but reason is not the correct explanation of the assertion

2)d. In simple harmonic motion, as y changes, velocity v will also change. So simple harmonic motion is not uniform motion. This reason is false because SHM is not necessarily caused only by gravity.it can result from other restoring forces too, such as spring force (Hooke's Law)

3)a In SHM, the acceleration must be proportional to the negative of displacement. So the sign (direction) matters. Just "proportional" is not enough;it must be opposite in direction.

4)a Their fundamental period is 2π, so they are indeed periodic functions.

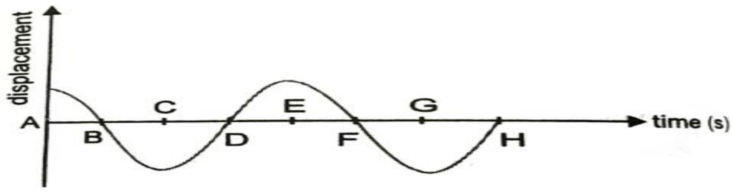
5)c. The reason is false because in SHM, velocity does not remain constant — it changes with displacement

6)b While it's true that the Moon is smaller, the correct reason for the increase in time period is that the gravitational acceleration on the Moon is less — not merely that the Moon is smaller in size.

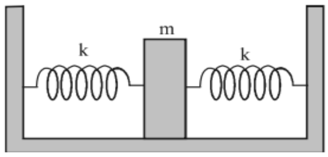
7)c The amplitude does not always increase with frequency. It only increases up to a point — specifically, when the frequency approaches the natural frequency. After that, the amplitude decreases again.

VERY SHORT ANSWER TYPE QUESTIONS

1) Displacement versus time curve for a particle executing S.H.M. is shown in figure. Identify the points marked along the time axis at which (i) velocity of the oscillator is zero, (ii) speed of the oscillator is maximum.



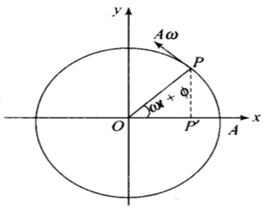
2) Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in figure. When the mass is displaced from equilibrium position by a distance x towards right, find the restoring force.



3) What is the ratio of maximum acceleration to the maximum velocity of a simple harmonic oscillator?

4) What is the ratio between the distance travelled by the oscillator in one time period and amplitude?

5) In the figure, what will be the sign of the velocity of the point P1, which is the projection of the velocity of the reference particle P. P is moving in a circle of radius R in anti-clockwise direction.



6) Show that for a particle executing S.H.M., velocity and displacement have a phase difference is π/2.

ANSWERS

1) We know that in SHM, the velocity is zero when particle is at the extreme position (i.e., displacement is maximum) and velocity is maximum when particle is at the mean position, i.e., displacement is zero. Thus, velocity of oscillator is zero at locations A, C, E, and G. The speed of oscillator is maximum at locations B, D, F and H.

2) In this case, since two springs are connected in series, each spring experiences the same displacement, x, and therefore each spring exerts a restoring force of -kx. Since two springs

are acting on the block, the total restoring force is the sum of the restoring forces from each spring. Therefore: $F = (-kx) + (-kx) = -2kx$.

Since the restoring force acts towards the equilibrium position, the magnitude of the force is 2kx.

3) The maximum acceleration (A_{max}) of a simple harmonic oscillator is given by the formula: $A_{max} = \omega^2 r$ The maximum velocity (V_{max}) is given by the formula: $V_{max} = \omega r$

$A_{max} / V_{max} = \omega^2 r / \omega r = \omega$

4) The total path (distance) travelled by the oscillator in one time period is:

Distance=A+A+A+A=4A (From mean position → to +A; Then from +A → to −A ; Then from −A → back to mean position)

Amplitude = A ⇒Ratio of distance to amplitude = 4A/A = 4

5) As particle of reference is moving from A to P in anticlockwise direction, the projection of the velocity of the reference particle is from A to P′. i.e., towards O.i.e.,along negative x-axis.

Hence sign of the velocity at point P′ is negative.

6) Let the displacement of the particle at time t be: $x(t)=A\sin(\omega t+\phi)$

$v(t)= dx/dt =d[A\sin(\omega t+\phi)]/dt=A\omega\cos(\omega t+\phi) \Rightarrow v(t) = A\omega \sin (\omega t+\phi+\pi/2)$

Therefore, velocity leads displacement by a phase of $\pi/2$

SHORT ANSWER QUESTIONS (3 MARKS)

1) (a) Write the displacement equation representing the following conditions obtained for a simple harmonic motion: Amplitude = 0.01m, Frequency = 600Hz, Initial phase = $\pi/6$

(b) All trigonometric functions are periodic, but only sine or cosine functions are used to define SHM. Why?

2) a) Can an object executing SHM have acceleration without having velocity?

b) Can an ideal simple pendulum be constructed in practice? Give reason too.

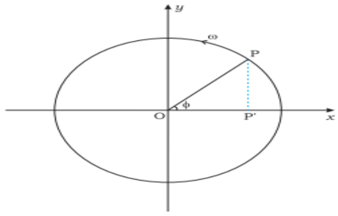
3) The maximum velocity of a particle, executing S.H.M with amplitude of 7mm is 4.4 m/s. What is the period of oscillation? At what displacement from the mean position is the particle's speed exactly half of its maximum speed?

4) A speaker cone in a sound system vibrates back and forth in simple harmonic motion. Its total displacement (stroke length) during vibration is 6 cm, and its angular frequency is 150 rad/min. Calculate the maximum speed of the vibrating cone. (Take $\pi = 3.14$)

5) A block of mass 2 kg is attached to a spring with a spring constant of 100 N/m. It is pulled to a distance of 0.2 m from its equilibrium position and released from rest on a smooth horizontal surface. Calculate:

a) The total energy of the system,
b) The kinetic energy and potential energy when the block is 10 cm away from the equilibrium position.

6) A particle P executes uniform circular motion with angular velocity ω as shown in the figure. Its displacement is given by: $x(t) = A \cos(\omega t + \varphi)$. Obtain the expression for its velocity and acceleration



7) Which among the given functions represent SHM:

a) $\sin 2\omega t$ b) $\sin \omega t + \cos \omega t \sin \omega t$ c) $\sin\omega t+\cos 2\omega t$ Justify your answer.

8) What are the two basic characteristics of a simple harmonic motion? When will the motion of a simple pendulum be simple harmonic?

ANSWERS

1) (a) The standard equation for the displacement Y in simple harmonic motion is given by:

$$Y=A\sin(\omega t+\phi) = A\sin(2\pi f t+\phi)=0.01\sin(2\pi \times 600t+\pi/6) =0.01\sin(1200\pi t+\pi/6)$$

(b) All trigonometric functions are periodic. The sine and cosine functions can take value between -1 to +1 only. So, they can be used to represent a bounded motion like SHM. But the functions such as tangent, cotangent, secant and cosecant can take value between 0 and ∞ (both negative and positive). So, these functions cannot be used to represent bounded motion like SHM.

2) a) Yes, an object executing SHM can have maximum acceleration but zero

velocity when situated at its extreme position.

b) We can never design an ideal simple pendulum because neither the pendulum bob can be an infinite, point mass nor the string be massless and inextensible.

3) $v_{max}=A\omega$; $\omega=4400/7$ rad/s ; $V_{max} = 4.4$ m/s

$T=2\pi/\omega$; $T\approx 0.01$ s $v=\omega\sqrt{A^2 - x^2}$; $v = v_{max}/2$; therefore, $x\approx 6.06\times 10^{-3}$m

4) $v_{max}=\omega A$; amplitude $A=6/2 =$ **0.03** m Angular frequency $\omega=150$ rad/min
Convert to rad/s: 2.5 rad/s ; $v_{max}=\omega A=2.5\times 0.03=0.075$ m/s

5) $\omega = \sqrt{\frac{k}{m}}$; $\omega\approx 7.07$rad/s

6) Expression for velocity: $v(t)= - A\omega\sin(\omega t+ \varphi)$;

Expression for acceleration: $a(t) = -A\omega^2\cos(\omega t+ \varphi)$

7) (a) and (c)

8) Two basic characteristics of SHM; To make it SHM, we use the **small-angle approximation**: $\sin\theta\approx\theta$(in radians, when θ is small, typically $<10^\circ$)

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1) A body of mass 0.5 kg is executing SHM with amplitude 0.1 m and spring constant 200 N/m. Calculate:
(a) Time period of oscillation (b) Maximum velocity (c) Total energy
(d) Kinetic and potential energy when the body is 0.05 m from the mean position

2) A block attached to a spring, moves back and forth on a frictionless surface in SHM. Using equations of motion, derive expressions for the potential and kinetic energies at any instant. Justify why energy is conserved in SHM and how changing the amplitude affects the total energy.

3)A particle of mass m is attached to a spring and performs linear simple harmonic motion with amplitude A and angular frequency ω.

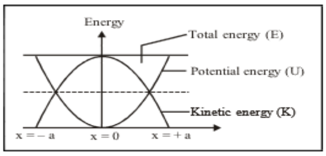
(a) Derive the expression for the total mechanical energy of the particle and state whether it depends on time.
(b) Draw neat, labelled graphs to show the variation of kinetic energy and potential energy of the particle with time during one complete oscillation.
(c) It is observed that both K.E. and P.E. repeat twice in one oscillation. Justify this observation with reference to the frequency of kinetic and potential energies compared to the frequency of the particle.

ANSWERS :

1) (a) Time period of oscillation : $T=2\pi/\omega = 0.314$s (b) vmax = A ω = 2 m/s
(c) Total energy = $\frac{1}{2} kA^2 = 1$J
(d) potential energy= $\frac{1}{2} kAx^2 = 0.25$J Kinetic energy = E- P E=0.75J

2) Correct derivation : potential energy=$1/2\ m\ \omega^2A^2 \sin^2(\omega t+\phi)$ and kinetic energy=$1/2\ m\ \omega^2A^2 \cos^2(\omega t+\phi)$

3) Total mechanical energy = $1/2\ m\ \omega^2A^2$ and it is independent of time. KE and PE complete two full cycles during one oscillation of the particle.



CASE BASED QUESTIONS:

I. A rescue operation is being conducted using a helicopter that lowers a stretcher on a strong but flexible cable. The helicopter hovers steadily as the stretcher, with a person on it, is gently lowered and comes to rest. After a while, the person on the stretcher starts to oscillate **vertically** (like a mass on a spring) due to slight air currents. The system behaves approximately like a **mass-spring system** performing **simple harmonic motion (SHM)**. The effective spring constant of the cable is k=100 N/m and the total mass of the stretcher with the person is m=25 kg. The maximum vertical displacement from the mean position is 10 cm.

1. What is the time period of oscillation of the stretcher?

A. 1 s B. π s C. 2π s D. 0.99s

2.**What is the maximum potential energy stored in the cable during the oscillation?**

A. 0.5 J B. 1.0 J C. 2.5 J D. 5.0 J

3.**What will be the maximum speed of the stretcher during the oscillation?**

A. 0.2 m/s B. 1.0 m/s C. 2.0 m/s D. 5.0 m/s

4.**If the mass of the person increases, what happens to the time period?**

A. It increases B. It decreases C. It remains constant D. It becomes zero

5. **If a sudden gust of wind applies random forces to the stretcher, what type of oscillation does it become?**

A. Free oscillation B. Damped oscillation C. Forced oscillation D. Resonance

II. A group of soldiers is marching in step across a long suspension bridge. The rhythmic marching causes the bridge to begin oscillating slightly. The bridge is modelled as a **large mass-spring system**, with a natural frequency of **2 Hz**. Each soldier exerts a force at a frequency of **2 Hz**, in sync with the natural frequency of the bridge. This creates **resonance**.

The oscillations gradually increase in amplitude until the soldiers are ordered to **break step** and walk randomly to stop the resonance.

1. What kind of oscillation is exhibited by the bridge when soldiers march in step at its natural frequency?

A. Free oscillation B. Damped oscillation C. Forced oscillation D. Resonant oscillation

2. What will happen to the amplitude of the bridge's oscillations if resonance continues unchecked?

A. It decreases B. It remains constant

C. It increases gradually D. It becomes zero

3. Why are soldiers asked to "break step" while crossing bridges?

A. To increase the time period B. To decrease the time period

C. To avoid synchronization with the bridge's natural frequency

D. To synchronize with the bridge's natural frequency

4. If the bridge's mass is doubled, what happens to its natural frequency?

A. It remains the same B. It increases C. It decreases D. It becomes infinite

5. Suppose the bridge has a damping mechanism installed. What effect does this have on resonance?

A. It increases amplitude B. It reduces sharpness of resonance

C. It makes oscillations chaotic D. It causes phase reversal

ANSWERS:

I. **1)B** **2)A** **3)A** **4)A** **5) C**

II. **1)D** In resonance, the system absorbs more energy each cycle, causing the amplitude to increase significantly — possibly to dangerous levels.

2)C

3)C Synchronizing steps can cause **forced oscillations** at the bridge's natural frequency, leading to resonance and possible collapse.

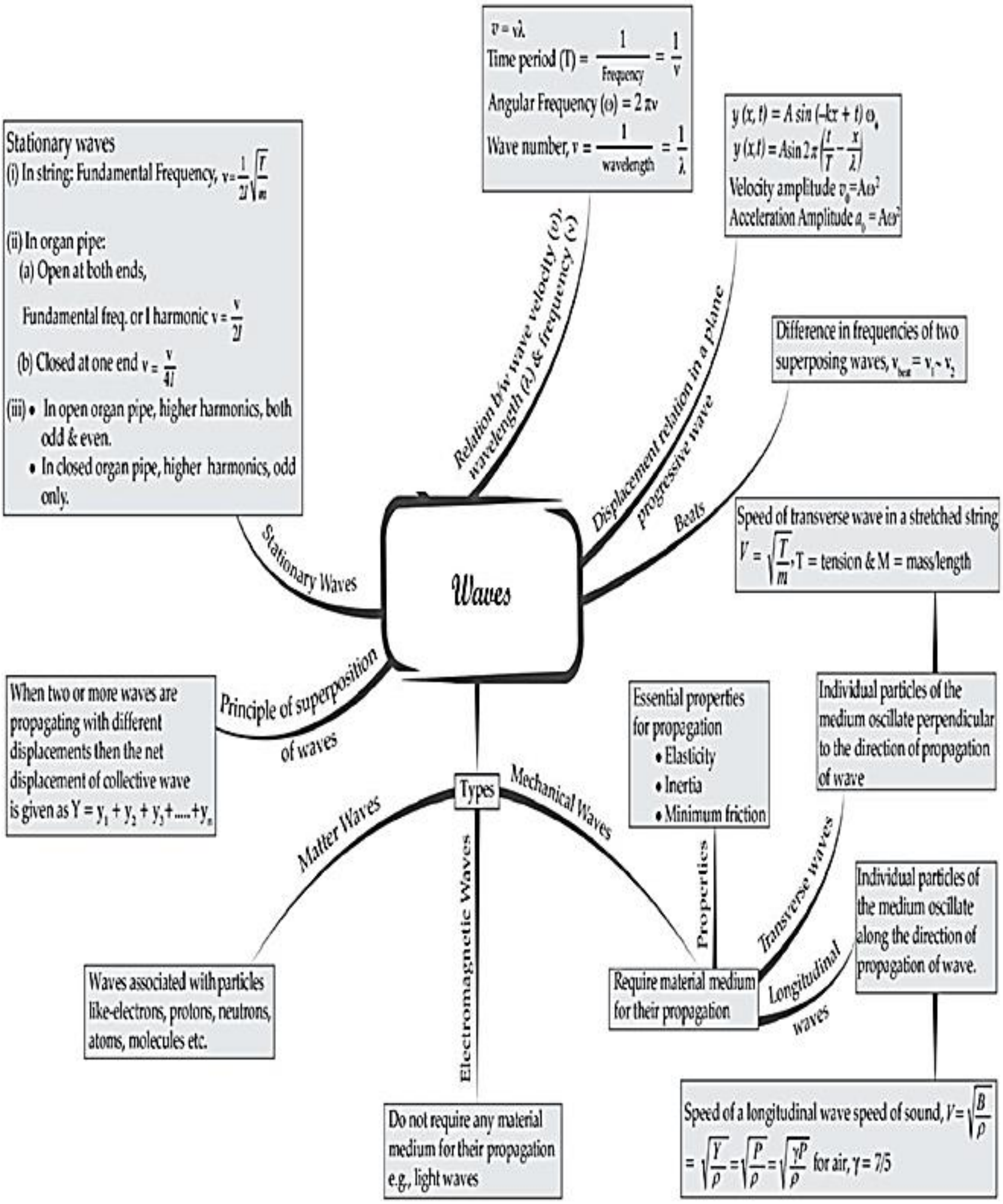
4) C $f=1/2\pi\sqrt{k/m}$ If mass increases, frequency decreases since $f\propto 1/\sqrt{m}$

5) B Damping reduces energy buildup in resonance, so the amplitude doesn't increase too much. This makes resonance **less sharp and less dangerous**.

CHAPTER–14: WAVES

Wave motion: Transverse and longitudinal waves, speed of travelling wave, displacement relation for a progressive wave, principle of superposition of waves, reflection of waves, standing waves in strings and organ pipes, fundamental mode and harmonics, Beats.

MIND MAP



GIST OF THE CHAPTER

● WAVE MOTION

•Waves:-Wave is a form of disturbance which travels through a material medium due to the
repeated f periodic motion of the particles of the medium about their mean positions
without any actual transportation of matter.

• Characteristics of wave

The characteristics of waves are as follows:

(i) The particles of the medium traversed by a wave execute relatively small vibrations about their mean positions but the particles are not permanently displaced in the direction of propagation of the wave.

(ii) Each successive particle of the medium executes a motion quite similar to its predecessors along/perpendicular to the line of travel of the wave.

(iii) During wave motion only transfer of energy takes place but not that of a portion of the medium.

Waves are mainly of three types: (a) mechanical or elastic waves, (b) electromagnetic waves and (c) matter waves.

• **Mechanical waves**

Mechanical waves can be produced or propagated only in a material medium. These waves are governed by Newton's laws of motion. For example, waves on water surface, waves on strings, sound waves etc.

• **Electromagnetic Waves**

These are the waves which require no material medium for their production and propagation, i.e., they can pass through vacuum and any other material medium. Common examples of electromagnetic waves are visible light; ultra-violet light; radio waves, microwaves etc.

• **Matter waves**

These waves are associated with moving particles of matter, like electrons, protons, neutrons etc.

Mechanical waves are of two types:

(i) Transverse wave motion, (ii) Longitudinal wave motion,

• **Transverse wave motion**

In transverse waves the particles of the medium vibrate at right angles to the direction in which the wave propagates.

Waves on strings, surface water waves and electromagnetic waves are transverse waves.

In electromagnetic waves (which include light waves) the disturbance that travels is not a result of vibrations of particles but it is the oscillation of electric and magnetic fields which takes place at right angles to the direction in which the wave travels.

• **Longitudinal wave motion**

In these types of waves, particles of the medium vibrate to and fro about their mean position along the direction of propagation of energy.

These are also called pressure waves.

Sound waves are longitudinal mechanical waves.

• **Wavelength (λ)**

The distance travelled by the disturbance during the time of one vibration by a medium particle is called the wavelength (λ).

In case of a transverse wave the wavelength may also be defined as the distance between two successive crests or troughs.

In case of a longitudinal wave, the wavelength (λ) is equal to distance from centre of one compression (or refraction) to another.

• **Wave Velocity (υ)**

Wave velocity is the time rate of propagation of wave motion in the given medium.
It is different from particle velocity. Wave velocity depends upon the nature of medium.
Wave velocity (υ) = frequency (v) x wavelength (λ)

• **Amplitude**:-The amplitude of a wave is the maximum displacement of the particles of the medium from their mean position.

• **Frequency**:-The number of vibrations made by a particle in one second is called Frequency. It is represented by v. Its unit is hertz (Hz) $v = 1/T$

• **Time Period**:-The time taken by a particle to complete one vibration is called time period. $T = 1/v$, it is expressed in seconds.

• The velocity of transverse waves in a stretched string is given by : $v = \sqrt{\frac{T}{\mu}}$

where T = tension in the string
μ = mass per unit length of the string (linear mass density of the string).
SI unit of μ is kg/m.

• The velocity of the longitudinal wave in an elastic medium is given by: $v = \sqrt{\frac{E}{\rho}}$

where E = modulus of elasticity of the medium
ρ = the density of the medium. (In case of solids, E is Young's modulus of elasticity (Y), then : $v = \sqrt{\frac{Y}{\rho}}$)

• Factors Influencing Velocity of Sound

The velocity of sound in any gaseous medium is affected by a large number of factors like density, pressure, temperature, humidity, wind velocity etc.
(i) The velocity of sound in a gas is inversely proportional to the square root of density of the gas.
(ii) The velocity of sound is independent of the change in pressure of the gas, provided temperature remains constant.
(iii) The velocity of sound in a gas is directly proportional to the square root of its absolute temperature.
(iv) The velocity of sound in moist air is greater than the velocity of sound in dry air.
(v) If wind flows at an angle θ to the direction of propagation of sound, the velocity of sound is $v + w \cos\theta$, where w is the velocity of wind.

• General Equation of Progressive Waves

"A progressive wave is one which travels in a given direction with constant amplitude, i.e., without attenuation."
As in wave motion, the displacement is a function of space as well as time, hence displacement relation is expressed as a combined function of position and time as:
$y(x, t) = A \sin(kx - \omega t + \Phi)$
We may also choose a cosine function instead of sine function. Here A, K, ω and Φ are four constant for a given wave and are known as amplitude, angular wave number, angular frequency and initial phase angle of given wave.

• Relation between phase and path difference

• A wave motion can be reflected from a rigid as well as from a free boundary. A travelling wave, at a rigid boundary or a closed end, is reflected with a phase reversal but the reflection at an open boundary takes place without any phase change.

PRINCIPLE OF SUPERPOSITION OF WAVES

When a number of waves travel through a medium simultaneously, the resultant dispalcement of any particle of the medium at any given time is equal to the algebraic sum of the displacements due to the individual waves.

$$y = y_1 + y_2 + y_3 + \cdots + y_n$$

The superposition of two waves may lead to following three different effects:

Interference – Superposition of two waves having equal frequency and nearly equal amplitude.

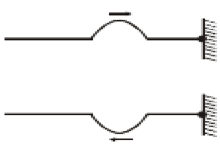
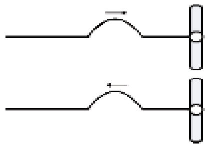
Stationary waves – Superposition of equal waves from opposite direction.

Beats – Superposition of two waves of slightly different frequency in same direction.

REFLECTION OF WAVES

The phenomenon of change in path of mechanical waves on striking a boundary is called reflection.

According to the nature of reflected surface ,reflections are of two types:

Rigid Boundary	Free boundary
	

STATIONARY WAVES

$$y = 2a\cos kx \sin \omega t \;= A \sin \omega t$$

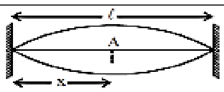
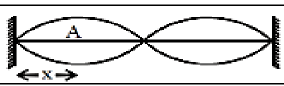
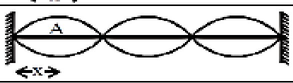
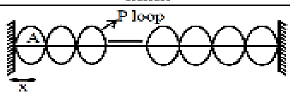
It represents a stationary wave, a wave in which the wave form does not move.

	Rigid Boundary	Free boundary
Incident wave	$y_1 = a\sin(\omega t - kx)$	$y_1 = a\sin(\omega t - kx)$
Reflected wave	$y_2 = -a\sin(\omega t + kx)$	$y_2 = a\sin(\omega t + kx)$
Resultant wave (Superposition principle)	$y = y_1 + y_2 = -2a\ \sin kx \cos \omega t = -A\cos \omega t$	$y = y_1 + y_2 = 2a\cos kx\ \sin \omega t = A\sin \omega t$
Position of nodes (A=0)	$x = 0, \lambda/2, \lambda, 3\lambda/2$......	$x = \lambda/4\ ,3\lambda/4, 5\lambda/4$ ....
Position of antinodes (A=max.)	$x = \lambda/4\ ,3\lambda/4, 5\lambda/4$	$x = 0, \lambda/2, \lambda, 3\lambda/2$......

Types of stationary waves

- Longitudinal Stationary waves [Organ Pipe (Ex. Resonance Tube)]
- TransverseStationary waves [Vibration in stretched string (Ex. Sonometer)]

MODES OF VIBRATIONS OF STRINGS

MODE	DIAGRAM	WAVELENGTH	FREQUENCY	HARMONIC	OVERTONE
Fundamental		$\lambda_1 = 2L$	$\nu_1 = \frac{v}{2L}$	First	Zero
Second		$\lambda_2 = L$	$\nu_2 = 2\nu_1$	Second	First
Third		$\lambda_3 = \frac{2L}{3}$	$\nu_3 = 3\nu_1$	Third	Second
....	....	......	......	......	.....
p^{th}		$\lambda_p = \frac{2L}{p}$	$\nu_p = p\nu_1$	p^{th}	$(p-1)^{th}$

BEATS:- The periodic variations in the intensity of sound due to superposition of two sound waves of slightly different frequencies are called beats.
One rise and one fall of intensity constitute one beat.
The number of beats produced per second is called beat frequency. $\boldsymbol{\nu_{beat} = \nu_1 - \nu_2}$.
Beats may be used to determine the frequency of a tuning fork.

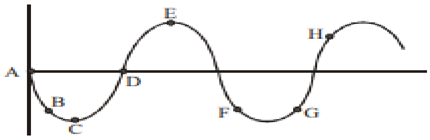
PhET Simulations:

- **https://phet.colorado.edu/en/simulation/wave-on-a-string**
  Explore transverse wave motion, wavelength, amplitude, reflection (fixed/free ends), damping, and standing waves.

- https://phet.colorado.edu/en/simulation/wave-interference
  Superposition Principle & Interference: Simulate 2D water wave interference, varying source frequency, phase and observe constructive/destructive patterns.

MULTIPLE CHOICE QUESTIONS

1) The diagram below shows the propagation of a wave. Which points are in same phase?



(a) F and G (b) C and E (c) B and G (d) B and F

2) Standing waves are produced due to superposition of two identical progressive waves travelling in

a) Same direction b) Opposite direction
c) Perpendicular direction d) none of these

3) If equation of stationary wave is given by $y = 15 \ \sin 5x \cos 8t$ .Calculate the wavelength.

a) $1.26 \ m$ b (1.57m c (1.88 m d) $1.25 \ m$

4) Velocity of sound waves in air is 330 m/s. For a particular sound wave in air, a path difference of 40 cm is equivalent to phase difference of 1.6S. The frequency of this wave is
(a) 165 Hz (b) 150 Hz (c) 660 Hz (d) 330 Hz

5) A string is stretched between two fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this string is:
(a) 205 Hz (b) 10.5 Hz (c) 105 Hz (d) 155 Hz

6) A sonometer wire supports a 4 kg load and vibrates in fundamental mode with a tuning fork of frequency 416 Hz. The length of the wire between the bridges is now doubled. In order to maintain fundamental mode, the load should be changed to
(a) 1 kg (b) 2 kg (c) 4 kg (d) 16 kg

7) A travelling wave reflected at an open boundary undergoes a phase change of

(a) π (b) 0 (c) π /2 (d) π/ 3

8) Four wires of identical lengths, diameters and of the same materials are stretched on a sonometer wire. If the ratio of their tensions is $1:4:9:16$,then the ratio of their fundamental frequencies are:

a) $1:2:3:4$ b) $1:3:5:7$ c) $4:3:2:1$ d) $16:9:4:2$

ANSWERS

1. (d) The displacement of the points B and F are equal in magnitude and sign. So these points are in same phase.

2. (b) Opposite direction 3.(a) $1.26\ m$

4. (c) $\Delta x=(\lambda/2\pi)\Delta\phi$, $\lambda=2\pi\Delta x/\Delta\phi = 2\pi(0.4)/1.6 = 660$Hz

5. (c) In a stretched string all multiples of frequencies can be obtained i.e., if fundamental frequency is n then higher frequencies will be 2n, 3n, 4n ... 75 cm So, the difference between any two successive frequencies will be 'n' According to question, n = 420 – 315 = 105 H.

6. (d) 16kg

7. (b) In case of an open boundary: Incident wave is $y_1 = a \sin(wt - kx)$ and reflected wave is $y_2 = a \sin(wt + kx)$. No phase change occurs.

8. (a) $1:2:3:4$

ASSERTION REASON TYPE QUESTIONS

1.Assertion : Compression and rarefaction involve changes in density and pressure.
Reason : When particles are compressed, density of medium increases and when they are rarefied, density of medium decreases.
2. Assertion : Solids can support both longitudinal and transverse waves but only longitudinal waves can propagate in gases.
Reason : For the propagation of transverse waves, medium must also necessarily have the property of rigidity.
3. **Assertion :** Sound wave is an example of longitudinal wave.
Reason : In longitudinal waves, the constituents of the medium oscillate perpendicular to the direction of wave
propagation.
4. Assertion : Particle velocity and wave velocity both are independent of time.
Reason : For the propagation of wave motion, the medium must have the properties of elasticity and inertia.
5.Assertion : The change in air pressure affect the speed of sound.
Reason : The speed of sound in a gas is proportional to the square root of pressure.
6. Assertion : Two waves moving in a uniform string having uniform tension cannot have different velocities.
Reason : Elastic and inertial properties of string are same for all waves in same string. Moreover speed of wave in a string depends on its elastic and inertial properties only.

ANSWERS:

1) **(a)** Compression → High pressure & high density; Rarefaction → Low pressure & low density
2) **(a)** Gases and liquids do not have rigidity → no transverse wave propagation in them.
3) **(c)** In longitudinal waves, the constituents of the medium oscillate parallel to the direction of wave propagation
4) **(d)** particle velocity is time-dependent.
5) **(d)** In an ideal gas, at constant temperature, speed of sound is independent of pressure.
6) **(d)** With change in pressure, density of medium also changes and so $P/$□ remains constant.

VERY SHORT ANSWER QUESTIONS (2 MARKS)

1. What is the minimum frequency with which a string of length L stretched under tension T can vibrate?
2. On the basis of dimensional analysis, derive the formula for the speed of transverse waves on a stretched string.
3. If a splash is heard 4.23 s after a stone is dropped into a well 78.4 m deep, find the speed of the sound in air.
4. An organ pipe produces a fundamental frequency $128\ Hz$. When blown forcefully , it produces first overtone of $384\ Hz$. Is the pipe open or closed ?
5. The frequency of the fundamental note of a closed organ pipe and that of an open organ pipe are the same. What is the ratio of their lengths.
6. How will the fundamental frequency of a closed organ pipe be affected if instead of air it is filled with a gas heavier than air ?

ANSWERS

1) The minimum frequency is $1/2L\sqrt{\frac{T}{\mu}}$
2) The speed of transverse waves on a stretched string is given by $v=\sqrt{\frac{T}{\mu}}$
3) 340.87 m/s
4) For an open pipe, the first overtone is the second harmonic which is twice the fundamental frequency, $f_1 = 2f_0$. For a closed pipe, the first overtone is the third harmonic which is three times the fundamental frequency, $f_1 = 3f_0$.comparing the two cases, it indicates that the pipe is closed.
5) Ratio is 1:2
6) The fundamental frequency of the closed organ pipe will decrease when filled with a gas heavier than air.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

1) Giving reasons for your selection, select pairs out of the following four waves in a medium which will give rise to

(i) beats (ii) destructive interference (iii) stationary waves:

a) $y_1 = A\cos 2\pi\,(v_1 t + \frac{x}{\lambda_1})$ b) $y_2 = A\cos\left[2\pi\left(v_1 t + \frac{x}{\lambda_1}\right) + \pi\right]$

c) $y_3 = A\cos 2\pi\left(v_2 t + \frac{x}{\lambda_2}\right)$ d) $y_4 = A\cos 2\pi\left(v_2 t - \frac{x}{\lambda_2}\right)$

Given $v_1 - v_2$ is small.

2) Two tunin g forks give6 beats/s when sounded simultaneously. The frequency of one of the forks is480 Hz.When the other fork is loaded with a piece of wax , the beats increase. What is the frequency of the second fork?

3) Two sounds of very close frequencies, 256 Hz and 260 Hz are produced simultaneously. What is the frequency of the resultant sound ? Also write the no of beats heard in one second.

4) Two progressive sound waves each of frequency 170 Hz and travelling in opposite direction in air superimpose to produce stationary waves. The speed of sound in air is 340 m/s. What is the separation between (i) two successive nodes (ii) two successive antinodes and (iii) a node and its nearest antinodes.

5) A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg wt into between two rigid supports 1 meter apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance, when carrying an alternating current of frequency θ. Find the frequency of the alternating source.

6) A wire having a linear mass density of 5.0 x 10-3 kg/m is stretched between two rigid support with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the wire resonates is 490 Hz. Find the length of the wire.

ANSWERS

1)(i) Beats : a) and c) ; Reason-Slightly different frequencies, same direction

(ii) Destructive interference: a) & b); Reason-Same frequency, π phase difference

(iii) Stationary waves: c) & d) ; Reason- Same frequency, opposite directions

2) beat frequency is the absolute difference between the frequencies of the two forks. The frequency of the second fork can be either 480 + 6 = 486 Hz or 480 – 6 = 474 Hz. Loading the second fork with wax decreases its frequency. If the second fork's frequency was 486 Hz, loading it with wax would decrease the frequency and the beat frequency would decrease. Since the beat frequency increases, the second fork's frequency must be 474 Hz.

3) 258 Hz, 4

4) 1m, 1m, 0.5 m

5) 50 Hz

6) 2.14 m

LONG ANSWER QUESTIONS

1) (i) A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?

(ii) A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly exited by a 430 Hz source? Will this same source be in resonance with the pipe if both ends are open? (Speed of sound = 340 m/s).

2) a) Write displacement equation respecting the following condition obtained in SHM. Amplitude = 0.01m, Frequency = 600Hz, Initial phase =π/6

b) The equation of a plane progressive wave is,y=10sin2π(t−0.005x) where y & x are in cm and t in second. Calculate the amplitude, frequency, wavelength & velocity of the wave.

3) A source of sound emits waves of frequency 500 Hz in air. The speed of sound in air is 340 m/s. (a) What is the wavelength of the sound wave?
(b) How much time does the wave take to travel a distance of 170 m?

ANSWERS

1) (i)In the fundamental mode, the wavelength of longitudinal vibration is: $\lambda=2L=2\times1=2m$
Speed of sound in steel:
$v=f\lambda=2530\times2=5060$ m/s
(ii) Resonance in a Pipe

Case 1: Pipe closed at one end
only odd harmonics allowed:
$f_n=nv/4L$; n = 1, 3, 5 and $f_n\approx425n$; 430Hz:$n\approx1.01$
So first harmonic (n = 1) is excited.

Case 2: Pipe open at both ends Allowed harmonics: $f_n=nv/2L$; n = 1, 2, 3…and $f_n\approx835n$; 430Hz: $n\approx0.506$

2) a) $X(t) = 0.01 \sin(1200\pi t+\pi/6)$ m; b) The amplitude is the coefficient of the sine function , A = 10 cm, f = 1 Hz; λ=200cm; $v=f\lambda$ =200cm/s

3) a)$v=f\lambda$ =50 m/s; using s= vt, t = 6s

CASE STUDY BASED QUESTION

I. If you drop a little pebble in a pond of still water, the water surface gets disturbed. The disturbance does not remain confined to one place, but propagates outward along a circle. If you continue dropping pebbles in the pond, you see circles rapidly moving outward from the point where the water surface is disturbed. It gives a feeling as if the water is moving outward from the point of disturbance. If you put some cork pieces on the disturbed surface, it is seen that the cork pieces move up and down but do not move away from the centre of disturbance. This shows that the water mass does not flow outward with the circles, but rather a moving disturbance is created. Similarly, when we speak, the sound moves outward from us, without any flow of air from one part of the medium to another. The disturbances produced in air are much less obvious and only our ears or a microphone can detect them. These patterns, which move without the actual physical transfer or flow of matter as a whole, are called waves

i) The waves transmit
a) mass b) energy c) both mass and energy d) neither mass nor energy

ii) Speed of sound wave in air
a) is independent of temperature b) increases with pressure
c) increases with an increase in humidity d) decreases with an increase in humidity

iii) When a pebble is dropped into still water, the ripples observed are an example of:
a) Movement of water molecules outward b) Transfer of water from center to edges
c) Outward propagation of disturbance d) Rotational motion of water

(iv) The propagation of ripples on water shows that:
a) Water flows from the center outward b) Only water at the center moves
c) Water moves in circular paths
d) Energy is transferred without actual movement of water

II. Beats is an interesting phenomenon arising from interference of waves. When two harmonic Sound waves of slightly different frequencies and comparable amplitude are heard at the same time, we hear a sound of similar frequency (the average of two close frequencies), but we hear something else also. We hear audibly distinct waxing and waning of the intensity of the sound, with a frequency equal to the difference in the two close frequencies Artists use this phenomenon often while tuning their instruments with each other. They go on tuning until their sensitive ears do not detect any beats.

(i)Two tuning forks when sounded together produced 4beats/sec. The frequency of one fork is 256. The number of beats heard increases when the fork of frequency 256 is loaded with wax. The frequency of the other fork is

a)504 b) 520 c) 260 d) 252

ii)Two adjacent piano keys are struck simultaneously. The notes emitted by them have frequencies $n_1$and n_2 . The number of beats heard per second is

a) $n1 \sim n2$ b) $n1 + n2$ c) $n1 \times n2$ d) $n1 - n2$

(iii)Two tuning forks have frequencies 450 Hz and 454 Hz respectively. On sounding these forks together, the time interval between successive maximum intensities will be

a) $1/4sec$ b) $1/2sec$ c) $1sec$ d) $2sec$

(iv) Beats are the result of

(a) diffraction (b) destructive interference

(c) constructive and destructive interference

(d) superposition of two waves of nearly equal frequency

ANSWERS

1.(i) b. (ii) b.

(iii) c. The speed of sound in air increases with an increase in humidity. This is because humid air is less dense than dry air, and as the density of the medium decreases, so does the speed of sound**.**

(iv)c. (v) d.

2**.(i) c.** The beat frequency is $|f2-f1|=4 \Rightarrow f2=256\pm4 \rightarrow$ So f2 could be 252 Hz or 260 Hz

If f2 =252, then after lowering f1, the beat difference $|f1-f2|$ becomes smaller, so beats decrease. If f2 =260, and f1 is lowered, then $|f2-f1|$ becomes more than 4, so beats increase.

(ii) a.

(iii) a $f_{beat}=|f2-f1|= |454-450|=4Hz$; $T = 1/f_{beat} = ¼ = 0.25s$

(iv) d.

DIGITAL RESOURCES

CHAPTER NAME	WORKSHEET	WORKSHEET
CHAPTER–1:	Units and Measurement-Worksheet-1	Units and Measurements-Worksheet-2
CHAPTER–2:	MOTION IN A STRAIGHT LINE WORKSHEET-1	MOTION IN A STRAIGHT LINE WORKSHEET-2
CHAPTER–3:	MOTION IN A PLANE WORKSHEET-1	MOTION IN A PLANE WORKSHEET-2
CHAPTER–4:	Laws of Motion- Worksheet-1	Laws of Motion -Worksheet-2
CHAPTER–5:	Work Energy and Power-Worksheet	Work Energy and Power-Worksheet-2
CHAPTER–6:	System of Particles and Rotational Motion	
CHAPTER–7:	Gravitation -Worksheet-1	Gravitation -Worksheet-2
CHAPTER–8:	Mechanical Properties of of Solids-Worksheet	MECHANICAL PROPERTIES OF SOLID -2
CHAPTER–9:	Mechanical Properties of of Liquids-Worksheet	Mechanical Properties of fluids Worksheet-2
CHAPTER–10:	Thermal Properties of Matter-Worksheet	
CHAPTER–11:	THERMODYNAMICS-Worksheet	Thermodynamics-Worksheet
CHAPTER–12:	Kinetic Theory-Worksheet-1 K SIVADAS	Kinetic Theory-Worksheet-2 K SIVADAS
CHAPTER–13:	Oscillations-Worksheet	OSCILLATIONS-2
CHAPTER–14:	Waves-Worksheet	waves-2
PRACTICE QUESTION PAPERS	11 PHY PP - 5 QP.pdf	11PHY PP - 5 MS.pdf
	11 PHY QP SET 1.pdf	11 PHY MS SET 1.pdf
	11 PHY QP SET 4.pdf	11 PHY MS SET 4.pdf

Video Link

PROJECTILE MOTION MRS.DEEPA.mp4

Motion in Straight Line.mp4

Laws of motion Deepa malviya.mp4

Motion in Straight Line Azad.mp4

Oscillations Varsha.mpeg
Waves Varsha.mpeg