तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

गणित
Mathematics

कक्षा / Class XI
2025-26

विद्यार्थी सहायक सामग्री
Student Support Material

केन्द्रीय विद्यालय संगठन~Kendriya Vidyalaya Sangathan

<u>संदेश</u>

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना एवं नवाचार द्वारा उच्च - नवीन मानक स्थापित करना केन्द्रीय विद्यालय संगठन की नियमित कार्यप्रणाली का अविभाज्य अंग है। राष्ट्रीय शिक्षा नीति 2020 एवं पी. एम. श्री विद्यालयों के निर्देशों का पालन करते हुए गतिविधि आधारित पठन-पाठन, अनुभवजन्य शिक्षण एवं कौशल विकास को समाहित कर, अपने विद्यालयों को हमने ज्ञान एवं खोज की अद्‌भुत प्रयोगशाला बना दिया है। माध्यमिक स्तर तक पहुँच कर हमारे विद्यार्थी सैद्धांतिक समझ के साथ-साथ, रचनात्मक, विश्लेषणात्मक एवं आलोचनात्मक चिंतन भी विकसित कर लेते हैं। यही कारण है कि वह बोर्ड कक्षाओं के दौरान विभिन्न प्रकार के मूल्यांकनों के लिए सहजता से तैयार रहते हैं। उनकी इस यात्रा में हमारा सतत योगदान एवं सहयोग आवश्यक है - केन्द्रीय विद्यालय संगठन के पांचों आंचलिक शिक्षा एवं प्रशिक्षण संस्थान द्वारा संकलित यह विद्यार्थी सहायक- सामग्री इसी दिशा में एक आवश्यक कदम है । यह सहायक सामाग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है। केन्द्रीय विद्यालय संगठन की विद्यार्थी सहायक- सामग्री अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री संकलन की विशेषज्ञता के लिए जानी जाती है और शिक्षा से जुड़े विभिन्न मंचों पर इसकी सराहना होती रही है। मुझे विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर निरंतर मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुँचाएगी।

शुभाकांक्षा सहित ।

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CHAPTER – 1 SETS

Definitions and Formulae

Set: A Set is a well-defined collection of distinct objects.

Empty Set: A set which does not contain any element is called the empty set or the null set or the void set. The empty set is denoted by the symbol φ or { }.

Finite and Infinite Sets: A set which is empty or consists of a definite number of elements is called finite otherwise, the set is called infinite.

Equal Sets: Two sets A and B are said to be equal if they have exactly the same elements and we write A = B. Otherwise, the sets are said to be unequal and we write $A \neq B$.

Equivalent sets: Two sets A and B are said to be equivalent if they have same number of elements.

Subsets: A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, $A \subset B$ if whenever $a \in A$, then $a \in B$. Thus $A \subset B$ if $a \in A \Rightarrow a \in B$

If A is not a subset of B, we write $A \not\subset B$.

- Every set A is a subset of itself, i.e., $A \subseteq A$.
- φ is a subset of every set.
- If $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.
- If a set A has only one element, we call it a singleton set. Thus $\{a\}$ is a singleton set.

Closed Interval : $[a , b] = \{x \in R : a \leq x \leq b\}$

Open Interval : $(a , b) = \{ x \in R : a < x < b\}$

Closed open Interval : $[a , b) = \{ x \in R : a \leq x < b\}$

Open closed Interval : $(a , b] = \{ x \in R : a < x \leq b \}$

If A is a set with n(A) = m, then no. of subsets of set A = 2^m.

Universal Set: The largest set under consideration is called Universal set.

UNION OF SETS: The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both).

INTERSECTION OF SETS:

The intersection of two sets A and B is the set of all those elements which belong to both A and B.

Disjoint sets: If A and B are two sets such that $A \cap B = \varphi$, then A and B are called disjoint sets.

DIFFERENCE OF SETS:

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

COMPLEMENT OF A SET:

Let U be the universal set and A be a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A.

Practical Problems on Union and Intersection of Two Sets:

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$, if $A \cap B = \varphi$.

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

MULTIPLE CHOICE QUESTIONS WITH SOLUTIONS

1. Which one of the following sets is infinite?

(A) The set of whole numbers less than 10 (B) The set of prime numbers less than 10

(C) The set of integers less than 10 (D) The set of factors of 10

Answer: (C) The set of integers less than 10

Explanation: The set of whole numbers less than 10 = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} is finite

The set of prime numbers less than 10 = {2, 3, 5, 7} is finite

The set of integers less than 10 = {..., -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9} is infinite since the negative integers go on for ever.

The set of factors of 10 = {1, 2, 5, 10} is finite

2. Which one of the following is the null set?

(A) The set of subsets of null set (B) The set of even prime numbers

(C) The set of factors of 7 (D) The set of rational expressions for π

Answer: (D) The set of rational expressions for π

Explanation: The null set is a subset of itself, so the set of subsets of the null set has one element which is the null set itself.
The set of even prime numbers = {2} is not the null set.
The set of factors of 7 = {1, 7} is not the null set. π is an irrational number, so cannot be expressed as a rational number. Although we often approximate $to\ \frac{22}{7}$, this is not an exact value. So the set of rational expressions for π is the null set.

3. The set builder form of the set {1, 4, 9, 16, 25, …} is

(A) X = {x: x is a set of prime numbers} (B) X = {x: x is a set of whole numbers}
(C) X = {x: x is a set of natural numbers} (D) X = {x: x is a set of square numbers}

Answer: (D) X = {x: x is a set of square numbers}
Explanation: Given X = {1, 4, 9, 16, 25,…}= $\{1^2, 2^2, 3^2, 4^2, 5^2, \ldots\}$
Therefore, X = {x: x is a set of square numbers}

4. If the set S = {x ∈ Z | x ≥ -2} ∪ { x ∈ Z | x ≤ 4}, then what is another name for S?

(A) $S = \{x \in \mathbf{Z}\colon -2 \le x \le 4\}$ (B) $S = \{x \in \mathbf{Z}\colon -2 < x < 4\}$
(C) $S = \{-2, -1, 0, 1, 2, 3, 4\}$ (D) **Z**

Answer: (D) Z
Explanation: ∪ means 'union', so let us see what happens when we join the sets:
{x ∈ Z | x ≥ -2} = {-2, -1, 0, 1, 2, 3, ...} {x ∈ Z | x ≤ 4} = {..., -1, 0, 1, 2, 3, 4}
So together they are {..., -2, -1, 0, 1, 2, 3, 4, ...} = Z, the set of Integers.
So {x ∈ Z | x ≥ -2} ∪ { x ∈ Z | x ≤ 4} = {..., -2, -1, 0, 1, 2, 3, 4, ...} = Z

5. The set builder notation of set S = $\{\ldots, -2, -1, 0, 1, 2, 3, 4, 5\}$ is

(A) $S = \{x \in \mathbf{Z}\colon -5 \le x \le 5\}$ (B) $S = \{x \in \mathbf{Z}\colon x \le 5\}$
(C) $S = \{x \in \mathbf{Z}\colon x < 5\}$ (D) $S = \{x \in \mathbf{R}\colon x \le 5\}$

Answer: (B) $S = \{x \in \mathbf{Z}\colon x \le 5\}$
Explanation: The numbers listed in S are all integers, so x ∈ Z is correct.
The integers listed finish at 5, but the "..." shows that the list continues indefinitely to -∞
Therefore x ≤ 5. Putting x ∈ Z and x ≤ 5 together gives us answer B in set builder notation.

6. A, B and C are three sets such that A is a subset of B and B is a subset of C. Which one of the following statements must always be true?

(A) B is a subset of A (B) C is a subset of B
(C) C is a subset of A (D) A is a subset of C

Answer: (D) A is a subset of C
Explanation: A is a subset of B means every element of A is in B and B is a subset of C means every element of B is in C. Therefore, if both statements are true, it follows that every element of A is in C ⇒ A is a subset of C is always true

7. Let A and B be two sets in the same universal set. Then A-B equals to

(A) A∩B (B) A′ ∩ B (C) A∩ B′ (D) A′ ∩ B′

Answer: (C) A∩B′
Explanation: The venn diagram represents A-B which is equal to A∩B′

8. Which of the following real values lie outside the interval (1, 8)?

(A) 1.5 (B) 7 (C) 4 (D) 1and 8

Answer: (D) 1 and 8
Explanation: 1 and 8 lies outside the set as (a , b) = {x : a < x < b}

9. How do we write the set {x: x ∈ R, -8 ≤ x < -5} in interval notation?

(A) (-8, -5) (B) (-8, -5] (C) [-8, -5) (D) [-8, -5]

Answer: (C) [-8, -5)
Explanation: (a , b) = {x : a < x < b} and [a , b] = {x : a ≤ x ≤ b}. So the interval is closed at -8 as -8 is included but it is open at -5 as it is not included in the set.

10. Which of the following statement regarding the set A = {{-2, 2},{-1, 1}, 0} **is true?**

(A) $\{\{0\}\} \subset A$ (B) $\{\{-1, 1\}\} \subset A$ (C) $\{-2, 2\} \subset A$ (D) $\{-1, 0, 1\} \subset A$

Answer: (B) $\{\{-1,1\}\}\subset A$

Explanation: $\{\{0\}\}\subset A$ is false as $\{0\}\subset A$, $\{-2,2\}$ $\subset A$is false as it is a member of set so it belongs to A

$\{-1,0,1\}\subset A$is false as $\{-1,1\}\in A$ *but* -1 *and* 1 *doesn't belongs to A*

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

B) Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion (A).

C) Assertion (A) is true but Reason(R) is false.

D) Assertion (A) is false but Reason(R) is true.

1. **Assertion:** The set A={2,4,6,8,10} can be written in set-builder form as A={x : x is an even natural number less than or equal to 10}
   Reason: The set-builder form lists all elements while roster form describes the property of elements.
   Answer: C)-Assertion (A) is true but Reason(R) is false
   Solution: The roster form lists all elements while set-builder form describes the property of elements.
2. **Assertion:** The set $\{x \in R: 3< x \leq 7\}$ is represented in interval notation as (3,7).
   Reason: Open interval indicates exclusion and closed interval indicates inclusion of an endpoint in an interval.
   Answer: D)-Assertion (A) is false but Reason(R) is true.
   Solution: The set $\{x \in R: 3< x \leq 7\}$ is represented in interval notation as (3, 7].
3. **Assertion:** The empty set is a subset of every set.
   Reason: A set with no element is called a null set.
   Answer: A)-Both A and R are true and R is the correct explanation for A.
   Solution: The empty set is defined as a set with no elements, which is also known as a null set. Since the empty set has no elements, it is trivially a subset of any other set because every element of the empty set is an element of the other set.
4. **Assertion:** If $A \subseteq B$ and $B \subseteq A$, then A = B.
   Reason: Two sets are equal if and only if each element of one set is also an element of the other set.
   Answer: A)-Both A and R are true and R is the correct explanation for A.
   Solution: The assertion is true because if every element of A is in B ($A \subseteq B$) and every element of B is in A ($B \subseteq A$), then the two sets must contain the same elements, meaning they are equal.
5. **Assertion:** If W is the set of whole numbers and N is the set of Natural numbers then W-N is an empty set
   Reason: A-B is the set of elements of A which are not in B.
   Answer: D)-Assertion (A) is false but Reason(R) is true
   Solution: W-N equals to {0}. So it is not an empty set
6. **Assertion:** The union of two sets is the set of elements that are in either set.
   Reason: The intersection of two sets is the set of elements that are in both sets.
   Answer: B)-Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion (A).
   Solution: The assertion is correct (the union is the set of elements in either set), but the reason is incorrect. The reason defines the intersection, not the union.
7. **Assertion:** If $A\subseteq B$, then the Venn diagram will show that the circle representing A is completely inside the circle representing B.
   Reason: A set is always a subset of itself.
   Answer: B)-Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion (A).
   Solution: A set is a subset of itself doesn't explain why A is inside B in the Venn diagram.
8. **Assertion:** If $A\cap B = A$, then $A \subseteq B$.
   Reason: The intersection of two sets is always equal to the smaller set.

Answer: C) Assertion (A) is true but Reason(R) is false.
Solution: If the intersection of A and B gives back A which means every element of A is also in B. So, A is a subset of B. Reason is not true as the intersection of two sets is the set of elements common to both.

9. **Assertion:** If A∪ B = B, then A ⊆ B.
Reason: The union of two sets contains all elements that belongs to either set.
Answer: A) Both Assertion (A) and Reason(R) are true but Reason(R) is the correct explanation of Assertion (A).
Solution: If A∪B=B, this means adding all elements of A to B does not change B. So all elements of A must already be in B. So, A is a subset of B. Reason is true as that's the definition of union of sets. (R) is the correct explanation of A since the reason explains why A∪B=B.

10. **Assertion:** If A is the set of letters of the word 'FOLLOW' and B is the set of letters of the word 'WOLF', then A and B are equal sets.
Reason: Two sets are equal if they have same number of elements.
Answer: C) Assertion (A) is true but Reason(R) is false
Solution: Two sets are equal if they have same elements. Sets having same number of elements are known as equivalent sets.

VERY SHORT ANSWER TYPE QUESTIONS WITH SOLUTIONS

1. **Write the solution set of the equation $x^2 + 5x + 6 = 0$ in roster form.**
Solution: $x^2 + 5x + 6 = 0 \Rightarrow (x + 2)(x + 3) = 0 \Rightarrow x = -2$ or $x = -3$
Therefore, the solution set of the given equation can be written in roster form as {-3, –2}.

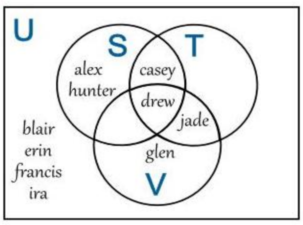
2. **List all the subsets of the set {–1, 0, 1 }.**

Solution: Let A = { –1, 0, 1 }. The subset of A having no element is the empty set φ. The subsets of A having one element are { –1 }, { 0 }, { 1 }. The subsets of A having two elements are {–1, 0}, {–1, 1} ,{0, 1}. The subset of A having three elements of A is A itself.
So, all the subsets of A are φ, {–1}, {0}, {1}, {–1, 0}, {–1, 1}, {0, 1} and {–1, 0, 1}.

3. **Show that the set of letters needed to spell " CATARACT " and the set of letters needed to spell " TRACT" are equal.**
Solution: Let X be the set of letters in "CATARACT". Then X = { C, A, T, R }
Let Y be the set of letters in " TRACT". Then Y = { T, R, A, C }
Since every element in X is in Y and every element in Y is in X. It follows that X = Y.

4. **Let A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8}. Find A – B and B – A.**
Solution: A – B = {1, 3, 5}, since the elements 1, 3, 5 belong to A but not to B
B – A = {8}, since the element 8 belongs to B and not to A.

5. **From the above Venn diagram, what is the set (S ∩ T) ∪ V?**



Solution: ∪ means "Union" (in either set)
∩ means "Intersection" (must be in both sets)
S ∩ T = {casey, drew}
V = {drew, jade, glen}
(S ∩ T) ∪ V = {casey, drew} ∪ {drew, jade, glen}
= {casey, drew, jade, glen}

6. **Find the complement of A if the Universal Set U = {x ∈Z | -4 ≤ x < 4} and A = {0}.**
Solution: U = { x ∈Z | -4 ≤ x < 4} = {-4, -3, -2, -1, 0, 1, 2, 3} A = {0}
Thus A' = U − A = {-4, -3, -2, -1, 1, 2, 3}.

7. **Let A, B and C be three sets. If A ∈ B and B ⊂ C, is it true that A ⊂ C? If not, give an example.**
Solution: No. Let A = {1}, B = {{1}, 2} and C = {{1}, 2, 3}.
Here A ∈ B as A = {1} and B ⊂ C.
But A ⊄ C as 1 ∈ A and 1 ∉ C.

8. **Find the set (A U B)' if U = {x: x ∈ N, x ≤ 9}; A = {x: x is an even number, 0 < x < 10} and B = {2, 3, 5, 7}.**
Solution: U = {x: x ∈ **N**, x ≤ 9} ⇒ U = {1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {x: x is an even number, 0 < x < 10} ⇒ A = {2, 4, 6, 8}

B = {2, 3, 5, 7}⇒ A U B = {2, 3, 4, 5, 6, 7, 8} ⇒ (A U B)' = U- (A U B) = {1, 9}

9. Let X = {1, 2, 3, 4, 5, 6}. If n represent any member of X, express the values of n as sets.

(i) n + 5 = 8

(ii) n is greater than 4

Solution: (i) Let B = {x | x ∈ X and x + 5 = 8}

Here, B = {3} as x = 3 ∈ X and 3 + 5 = 8 and there is no other element belonging to X such that x + 5 = 8.

(i) Let C = {x | x ∈ X, x > 4}⇒ C = {5, 6}

10. Express set A = {x | x is a positive integer less than 10 and $2^x – 1$ is an odd number} in roster form.

Solution: $2^x – 1$ is always an odd number for all positive integral values of x since 2^x is an even number. In particular, $2^x – 1$ is an odd number for x = 1, 2, … , 9.

⇒ A = {1, 3, 5, 7, 9}

SHORT ANSWER TYPE QUESTION WITH SOLUTIONS

1. **Use the properties of sets to prove that for all the sets A and B, A – (A ∩ B) = A – B**

Solution: A – (A ∩ B) = A ∩ (A ∩ B)′ (since A – B = A ∩ B′)

= A ∩ (A′ ∪ B′) [by De Morgan's law]

= (A∩A′) ∪ (A∩ B′) [by distributive law]

= φ ∪ (A ∩ B′)

= A ∩ B′ = A – B

2. **Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}. Find A′, B′, A′ ∩ B′, A ∪ B and hence show that (A∪B)′ = A′∩ B′.**

Solution: Given, U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}

⇒A′ = {1, 4, 5, 6}

⇒B′ = {1, 2, 6}.

Hence, A′ ∩ B′ = {1, 6}

A ∪ B = {2, 3, 4, 5} ⇒ (A ∪ B)′ = {1, 6}

Therefore, (A∪ B)′ = {1, 6} = A′ ∩ B′

3. **In a survey of 600 students in a school, 150 students were found to be drinking Tea and 225 drinking Coffee, 100 were drinking both Tea and Coffee. Find how many students were drinking neither Tea nor Coffee. (Not for Summative Assessment)**

Solution: Total number of students = 600

Number of students who were drinking Tea = n(T) = 150

Number of students who were drinking Coffee = n(C) = 225

Number of students who were drinking both Tea and Coffee = n(T ∩ C) = 100

n(T U C) = n(T) + n(C) – n(T ∩ C)

= 150 + 225 -100

= 375 – 100 = 275

Hence, the number of students who are drinking neither Tea nor Coffee = 600 – 275 = 325

4. **Using properties of sets, prove that:**

(i) A ∪ (A ∩ B) = A **(ii) A ∩ (A ∪ B) = A**

Solution: (i) We know the union is distributive over the intersection.

A ∪ (A ∩ B) = (A ∪ A) ∩ (A ∪ B) [Since, A ∪ A = A]

= A ∩ (A ∪ B) = A

(ii) We know the union is distributive over the intersection.

A ∩ (A ∪ B) = (A ∩ A) ∪ (A ∩ B)

= A ∪ (A ∩ B) [Since, A ∩ A = A]

= A

5. **Let A, B, C be the sets such that A ∪ B = A ∪ C and A ∩ B = A ∩ C. Show that B = C.**

Solution: Let us have b ∈ B ⇒ b ∈ A ∪ B

Also it is given A∪ B = A∪ C

Therefore b ∈ AUC

Hence we get b ∈ A or b ∈ C

If b ∈ A then b ∈ A∩ B which is equals to A ∩ C ⇒ b ∈ C

In both cases B is subset of C.

Similarly Let us have c ∈ C ⇒ c ∈ A ∪ C

Also it is given AU B = AU C

Therefore b ∈ AUB

Hence we get b ∈ A or b ∈ B

If b ∈ A then b ∈ A∩ C which is equals to A ∩ B ⇒ b ∈ B

In both cases C is subset of B.

Therefore **B** = **C**

6. **If L = {1, 2, 3, 4}, M = {3, 4, 5, 6} and N = {1, 3, 5} then verify that**

L – (M ∪ N) = (L – M) ∩ (L – N)

Solution: L = {1, 2, 3, 4}, M = {3, 4, 5, 6} and N = {1, 3, 5}

⇒ (M ∪ N) = {1, 3, 4, 5, 6}, (L – M) = {1, 2}, (L – N) = {2, 4}

⇒ L – (M ∪ N) = {2} and (L – M) ∩ (L – N) = {2}

⇒ L – (M ∪ N) = (L – M) ∩ (L – N)

7. **If P is the set of whole numbers less than 5, Q is the set of even numbers greater than 3 but less than 9 and R is the set of factors of 6. Then what is (P ∩ Q) ∪ (Q ∩ R)?**

Solution: P = {0, 1, 2, 3, 4} Q = {4, 6, 8} R = {1, 2, 3, 6}

P ∩ Q = {4} Q ∩ R = {6}

So (P ∩ Q) ∪ (Q ∩ R) = {4} ∪ {6} = {4, 6}

8. **Let A = {a, b, c} and B = {1, 2}. Find A ∪ B and B ∪ A. If C = {4, 6} then prove that A ∪ (B∩C) = (A ∪B) ∩ (AUC)**

Solution: AU B = {a, b, c, 1, 2} B ∪ A = {1, 2, a, b, c}.

A ∪ (B ∩C) =**{a, b, c}** . **A ∪B** = {a, b, c, 1, 2} **AUC** = {a, b, c, 4, 6}

Now **(A ∪B)** ∩ **(AUC) = {a, b, c}**

Thus A ∪ (B∩C) = (A ∪B) ∩ (AUC)

9. **Let A, B, and C be sets such that A ⊆ B and B ⊆ C. Prove that AU B = B and AU C = C**

Solution: Let x ∈ A ∪ B

Then x ∈ A **or** x ∈ B.

If x ∈ A and since A ⊆ B, then x ∈ B

If x ∈ B, then obviously x ∈ B

So in both cases, x ∈B ⇒AU B⊆ B

Also we have B⊆ A ∪ B ⇒AU B= B

Let x ∈ A ∪ C

Then x ∈ A **or** x ∈ C.

If x ∈ A and since A ⊆ C, then x ∈ C

If x ∈ C, then obviously x ∈ C

So in both cases, x ∈C ⇒AU C ⊆ C

Also we have C ⊆ A ∪ C ⇒AU C= C

10. **Using properties of sets, prove that (AUB) – B = A- B**

Solution: (AUB) – B = (AUB) ∩ B′ [(X – Y) = X ∩ Y′]

= (A∩ B′) U (B∩ B′)

= (A∩ B′) U ∅

= A – B

LONG ANSWER TYPE QUESTIONS WITH SOLUTIONS

1. **Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n.**

Solution: No. of elements in A = m

No. of elements in B = n

According the question, No. of subsets of set A – No. of subsets off set B = 112

$$2^m - 2^n = 112 \Rightarrow 2^n(2^{m-n} - 1) = \mathbf{112} \quad \textbf{because } \mathbf{m} > n$$
$$2^n(2^{m-n} - 1) = 16 \times 7 \Rightarrow 2^n(2^{m-n} - 1) = 2^4 \times 7$$

By comparing to both sides we get $2^n = 2^4$ **and** $2^{m-n} - 1 = 7 \Rightarrow$ **n = 4 and** $2^{m-n} = 8$

$$2^{m-n} = 8 \Rightarrow m - n = 3 \Rightarrow m = n + 3 \Rightarrow m = 4 + 3 = 7$$

3. Let A and B be two sets such that: n (A) = 20, n (A ∪ B) = 42 and n (A ∩ B) = 4. Find

(i) n (B) **(ii) n (A – B)** **(iii) n (B – A)**

Solution: (i) n (A ∪ B) = n (A) + n (B) – n (A ∩ B)

42 = 20 + n (B) – 4

∴ n (B) = 26

(ii) n (A – B) = n (A ∪ B) – n (B) = 42 – 26 = 16

∴ n (A – B) = 16

(iii) n (B – A) = n (B) – n (A ∩ B)

n (B – A) = 26 – 4 = 22

∴ n (B – A) = 22

4.**Let X = the set of all letters in the word 'NEW DELHI' and Y = the set of all letters in the word 'CHANDIGARH'. Find (i) X∪Y (ii) X ∩Y (iii) X-Y.**

Also verify that (a) n(X∪Y)= n(X) + n(Y) –n(X ∩Y)

(b) n(X-Y)= n(X∪Y) - n(Y)

Solution: X = {N, E, W, D, L, H, I} Y= { C, H, A, N, D, I, G, R}

X∪Y = {N, E, W, D, L, H, I, C, A, G, R}

X ∩Y = {N, D, H, I} X –Y = { E, W, L}

n(X∪Y) = 11 n(X) = 7 n(Y) = 8 n(X ∩Y) = 4 n(X –Y) =3

n(X) + n(Y) –n(X ∩Y) = 7+8-4=11 ⇒ n(X∪Y)= n(X) + n(Y) –n(X ∩Y)

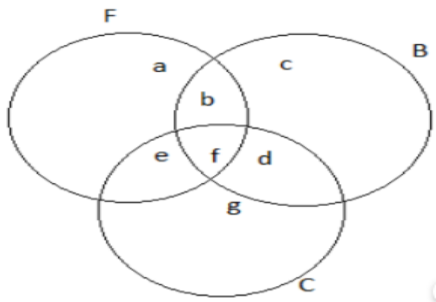
n(X∪Y) - n(Y) = 11- 8 = 3 ⇒ n(X-Y)= n(X∪Y) - n(Y)

5. **A college awarded** 38 **medals in football,** 15 **in basketball and** 20 **in cricket. If these medals went to a total of** 50 **men and only five men got medals in all the three sports, how many received medals in exactly two of the three sports? (Not for Summative Assessment)**

Solution: Let F denotes medals in Football

B denotes medals in Basketball

C denotes medals in Cricket



According to the question we get,

f = 5 ………. (i)

a+ b + e + f = 38

$\Rightarrow a + b + e = 33$ ……. …. (ii)

b + c + d + f = 15 ……… (iii)

$\Rightarrow b + c + d = 10$

e + f + d + g = 20 …………. (iv)

$\Rightarrow e + d + g = 15$

a + b + c + d + e + f + g = 50 ………..(v)

From (iv) and (v) we get a + b + c = 30 ………(vi)

Now from equation (ii) and (vi) we get e – 3 = c …….(vii)

put value of c in the equation (v) as shown

a + e + g + b + e + d = 50−5+3

Also from equation (iii) and (v) we get a + e + g = 35

Therefore the medals received in exactly 2 of three sports is b + d + e = 13

CASE STUDY BASED QUESTIONS WITH SOLUTIONS

1. In a school of 500 students:
   - 300 participate in Drama (D)
   - 250 in Music (M)
   - 200 in Art (A)
   - 150 participate in both Drama and Music
   - 100 in both Music and Art
   - 130 in both Drama and Art
   - 80 participate in all three activities

   Based on the above information, answer the following: (Not for Summative Assessment)

   (i) How many students are involved in exactly one activity?
   (ii) How many are involved in only Drama?
   (iii) (a) How many are involved in none of the three?

   OR

   (b) How many are involved in at least two activities?

   Solution: Total students = 500
   Students in Drama (D) = 300
   Students in Music (M) = 250
   Students in Art (A) = 200
   Students in Drama and Music (D ∩ M) = 150
   Students in Music and Art (M ∩ A) = 100
   Students in Drama and Art (D ∩ A) = 130
   Students in all three (D ∩ M ∩ A) = 80
   Let's use these symbols: Only D = x   Only M = y   Only A = z   D ∩ M only (not A) = a
   M ∩ A only (not D) = b   D ∩ A only (not M) = c   D ∩ M ∩ A = d = 80

1. D ∩ M = 150 ⇒ (a + d) = 150 ⇒ a = 70
2. M ∩ A = 100 ⇒ (b + d) = 100 ⇒ b = 20
3. D ∩ A = 130 ⇒ (c + d) = 130 ⇒ c = 50

   Now calculate:

- Only D = x = D - (a + c + d) = 300 - (70 + 50 + 80) = 100
- Only M = y = M - (a + b + d) = 250 - (70 + 20 + 80) = 80
- Only A = z = A - (b + c + d) = 200 - (20 + 50 + 80) = 50

(i) Only D + Only M + Only A =100+80+50=230= 100 + 80 + 50 = 230
(ii) Only Drama = 100 students
(iii) (a) First, compute total students involved in at least one activity = 100+80+50+70+20+50+80=450
Total students = 500 ⇒ students involved in none of the three = 500−450 = 50
(b) D ∩ M only = 70   D ∩ A only = 50   M ∩ A only = 20   All three = 80
Students involved in at least two activities =70+50+20+80=220

2. In class XI of one international school in Hyderabad, there are 200 students out of which 80 have taken Mathematics, 120 have taken Economics and 90 have taken Physical Education. If 50 have taken Mathematics and Economics, 60 have taken Economics and Physical Education, 40 have taken Mathematics and Economics. If 20 students have taken all three subjects then on the basis of above information answer the following:

(i) Find the number of students who have taken at least one of the subjects.
(ii) Find the number of students who have taken at most one of the subjects. (Not for Summative Assessment)

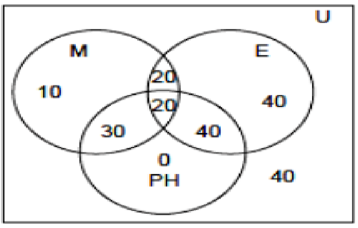
Solution: Let 'M' represents the set of students who have taken Maths. 'E' represents the set of students who have taken Economics. 'PH' represents the set of students who have taken Physical Education.

(i) at least one of the subjects
= M ∪ E ∪ PH

= 10 + 40 + 20 + 40 + 30 + 20

= 160 (at least one subject means one subject or two subjects or three subjects)

(ii) at most one of the subjects = one subject or none of the subjects = 10 + 40 + 40 + 0 = 90



3. In a sports club:
   - Set F: Members who play Football = {Mike, John, David, Ravi}
   - Set B: Members who play Basketball = {Ravi, Arjun, John}
   - Set T: Members who play Tennis = {Arjun, Mike}

   Based on the above information, answer the following: **(Not for Summative Assessment)**

   (i) Identify members who play **both** Football and Basketball **but not Tennis**.

   (ii) Is $B \subseteq F \cup T$? Explain.

   (iii) (a) If a new set $G = F \cap T \cap B$, what type of set is G?

   OR

   (b) Find $(F \cup B \cup T) - (F \cap B)$.

   Solution: (i) Ravi, John (ii) Yes, All elements of set B are also in $F \cup T$

   (iii) (a) G is an empty set (iii) (b) {Mike, Arjun, David }

4. For a scholarship program:
   - Set X: Students aged 18 years or older but not more than 25
   - Set Y: Students aged more than 22 years up to and including 30

   Based on the above information, answer the following:

   (i) Represent Sets Y in **interval notation**.

   (ii) Write a set-builder form for Set X.

   (iii) (a) Find the intersection of X and Y and represent it as an interval.

   OR

   (b) Are Sets X and Y disjoint ? Justify.

   Solution: (i) Y = (22,30) (ii) $X=\{x \in R: 18 \leq x \leq 25\}$

   (iii) (a) (22,25] (iii) (b) **No**, because they share the interval (22,25]

5. On the real axis, if A = [0, 3] and B = [2 ,6] ,then answer the following questions.

   (a) Find A' (b) Find A – B (c) Find $A \cap B$

   Solution: You have two intervals on the real number line:

   A = [0,3] and B = [2,6]

   A is from 0 to 3 (inclusive) and B is from 2 to 6 (inclusive)

   (a) A′: Complement of A

   The complement of A, denoted A′, means all real numbers not in A.

   Since A = [0, 3], the complement is:

   $A' = (-\infty, 0) \cup (3, \infty)$

   (b) A−B: Set difference A minus B

   This means all elements in A excluding those that are also in B.

   A = [0, 3] B = [2, 6]

   Their overlap is [2, 3]

   So A−B = [0,2)

   (c) $A \cap B$: Intersection of A and B

   The intersection is the part that both sets share.

   A = [0, 3] B = [2, 6]

   So the intersection is: $A \cap B = [2,3]$

CHAPTER-2: RELATIONS & FUNCTIONS

Definition and Formulae

Cartesian Products of Sets:

Given two non-empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, i.e., P × Q = {(p , q) : p ∈ P, q ∈ Q}

- ❖ A × A × A = {(a, b, c) : a, b, c ∈ A}. Here (a, b, c) is called an ordered triplet.

RELATION :

A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.

- ❖ The second element is called the image of the first element.
- ❖ The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.
- ❖ The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the co-domain of the relation R.
- ❖ RANGE ⊆ CO-DOMAIN.
- ❖ The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A) = p and n(B) = q, then n (A × B) = pq and the total number of relations is 2^{pq}

FUNCTION:

A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B. In other words, a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in 'f ' have the same first element.

- ❖ If f is a function from A to B and (x, y) ∈ f, then f (x) = y, where y is called the image of a under f and x is called the pre-image of y under f.
- ❖ A function which has either R or one of its subsets as its range is called a real valued function. Further, if its domain is also either R or a subset of R, it is called a real function.

SOME STANDARD FUNCTIONS

IDENTITY FUNCTION

Let **R** be the set of real numbers. Define the real valued function f : **R** →**R** by y = f(x) = x for each x ∈ **R**. Such a function is called the identity function.

Here the domain and range of f are **R**.

CONSTANT FUNCTION:

Define the function f: **R** → **R** by y = f (x) = c, x ∈ **R** where c is a constant and each x ∈ **R**.

Here domain of f is **R** and its range is {c}.

POLYNOMIAL FUNCTION:

A function f: **R**→**R** is said to be polynomial function if for each x in **R**,

y = f (x) = $a_0 + a_1x + a_2x^2 + \dots\dots\dots + a_nx^n$, where n is a non-negative integer and a_0 , $a_1, a_2, \dots\dots, a_n$ ∈ **R**.

RATIONAL FUNCTIONS: They are functions of the type $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomial functions of x defined in a domain, where g(x) ≠ 0.

MODULUS FUNCTION: The function f: **R**→**R** defined by f(x) = |x| for each x ∈ **R** is called modulus function. For each non-negative value of x, f(x) is equal to x. But for negative values of x, the value of f(x) is the negative of the value of x, i.e., $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$.

SIGNUM FUNCTION: The function f : **R**→**R** defined by $f(x) = \begin{cases} 1, if x > 0 \\ 0, if x = 0 \\ -1, if x < 0 \end{cases}$ is called the signum function. The domain of the signum function is **R** and the range is the set {–1, 0, 1}.

GREATEST INTEGER FUNCTION:

The function f: **R** → **R** defined by f(x) = [x], x ∈ **R** assumes the value of the greatest integer, less than or equal to x.

Such a function is called the greatest integer function.

[x] = –1 for –1 ≤ x < 0

[x] = 0 for 0 ≤ x < 1

[x] = 1 for 1 ≤ x < 2

[x] = 2 for 2 ≤ x < 3 and so on.

MULTIPLE CHOICE QUESTIONS WITH SOLUTIONS

1. The function f is defined on the real numbers by $f(x) = 2 + x - x^2$. What is the value of f(-3)?

(A) - 10 (B) -4 (C) 8 (D) 14

Answer: (A) - 10

Solution: $f(x) = 2 + x - x^2$. So $f(-3) = 2 + (-3) - (-3)^2 = 2 - 3 - (+9) = -10$

2. Find the Cartesian product of P = {2,5} and Q = {7,8}

(A) {(2, 7), (2, 8), (5, 7), (5, 8)} **(B)** {14, 16, 35, 40}

(C) {(7, 2), (7, 5), (8, 2), (8, 5)} (D) {(2, 5), (7, 8)}

Answer: (A) **{(2, 7), (2, 8), (5, 7), (5, 8)}**

3. Let A and B be any two sets such that n(B) = p, n(A) = q then the total number of functions f : A → B equals to

(A) pq (B) q^p (C) p^q (D) Cannot be determined

Answer: (C) p^q

Any element of set A, say x_i can be connected with the element of set B in p ways. Hence, there are exactly p^q functions.

4. Find the set T if T×M equals to {(2,3), (2, 8), (3, 3), (3, 8)}

(A) {(2, 3)} **(B)** {2, 3, 8} **(C)** {(2, 3), (2, 8)} (D) {(3, 8)}

Answer: (A) **{(2, 3)}**

5. If n(A) and n(B) is 1 and 2 respectively and the ordered pair (9, -3) is in B × A. Then find the set A.

(A) **{9}** (B) **{9, -3}** (C) **{-3}** **(D) cannot be determined**

Answer: (C) **{-3}**

6. A relation R from A to B is given by $R = \{(x, y) : y = 2x + 1, x \in A, y \in B\}$ where A ={ 1, 2, 3 } and B ={ 2, 3, 4, 5, 6 }then find the roster form of this relation.

(A) R = {(1, 3), (2, 5), (3, 7)} **(B) R =** {(3, 1), (5, 2)}

(C) R = {(1, 3), (2, 5)} (D) $R = \left\{\left(\frac{1}{2}, 2\right), (1, 3), \left(\frac{3}{2}, 4\right), (2, 5), \left(\frac{5}{2}, 6\right)\right\}$

Answer: (A) R = **{(1, 3), (2, 5), (3, 7)}**

Explanation: y = 2(1)+1=3 y = 2(2)+1=5 y = 2(3)+1=7

7. Find the range of the following relation R from a set of natural numbers N to itself.

R = {(x, y): y = 2x+1, x, y∈ N}.

(A) Set of all natural numbers (B) set of all odd natural numbers (C) Set of all odd natural numbers except 1 **(D) Set of all even natural numbers**

Answer: Set of all odd natural numbers except 1

Explanation: y = 2(1)+1=3 y = 2(2)+1=5 y = 2(3)+1=7 and so on

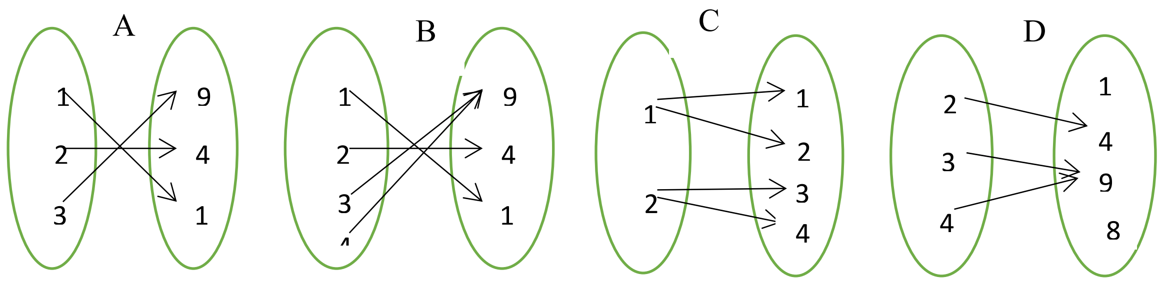
8. Write an equation to represent the function from the following table of values:

x	**-2**	**-1**	**0**	**1**	**2**
y	**-4**	**-2**	**0**	**2**	**4**

(A) $y = -2x$ (B) $y = 2x$ (C) $y = x + 1$ **(D) $y = x + 2$**

Answer: (B) $y = 2x$
Explanation: The y value is always 2 times the x value. So the equation is y = 2x.

9. Which one of the following relations is not a function?



(A) A (B) B (C) C **(D) D**

Answer: (C) C
Explanation: In C the numbers 1 and 2 are both related to more than one number, so this cannot be a function.

10. Which relation is not a function?

(A) $\mathbf{f(x) = \sqrt{x}}$ **(B)** $\mathbf{f(x) = -\sqrt{x}}$ **(C)** $\mathbf{f(x) = \pm\sqrt{x}}$ **(D)** $\mathbf{f(x) = \sqrt{x} - 1}$

Answer: (C) $\mathbf{f(x) = \pm\sqrt{x}}$
Explanation: A, B and D all represent functions because, for each value of x, there is just one value of f(x).

ASSERTION - REASON BASED QUESTIONS WITH SOLUTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
B) Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion (A).
C) Assertion (A) is true but Reason(R) is false.
D) Assertion (A) is false but Reason(R) is true.

1. **Assertion (A):** Every function is a relation.
   Reason (R): A function is a set of ordered pairs in which no two ordered pairs have the same first element.
   Answer: A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
   Solution: By definition, every function is a special kind of relation f from a set A to a set B where every element of set A has one and only one image in set B.
2. **Assertion (A):** The domain of the function f(x) = $\sqrt{4 - x^2}$ is [−2, 2].
   Reason (R): The square root function is only defined for non-negative real numbers.
   Answer: C) Assertion (A) is true and Reason(R) is false.
   Solution: Inside the square root, $4 - x^2 \geq 0$ implies x∈ [−2, 2].
3. **Assertion (A):** The function f(x) = $\frac{1}{x}$ is defined for all real numbers.
   Reason (R): The domain of a function includes all real numbers except where the function is undefined.
   Answer: D) Assertion (A) is false but Reason(R) is true.
   Solution: The function f(x) = $\frac{1}{x}$ is not defined for x = 0
4. **Assertion (A):** The Cartesian product A × A of set A= {1, 2, 3, 4} has 16 elements, among which ordered pairs (1, 2), (3, 4) are found.
   Reason (R): If A and B are two finite sets, then n(A × B) = n(A).n(B)
   Answer: A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
   Solution: n(A) = 4 so number of elements in A × A is 16
5. **Assertion (A):** If a relation R on a set A = {1, 2, 3,……, 14, 15} is defined by R={(x, y): y-2x = 0, x, y ∈ A}, then range of R={2, 4, 6, 8, 10, 12, 14}.

Reason (R): Let a relation R: A→ B, then domain of R is the set of all first components of ordered pairs which belong to R

Answer: B) Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion (A).

Solution: Assertion deals with range.

6. **Assertion(A):** The domain of the function f(x) = $\frac{1}{x-1}$ is R - {1}.

   Reason(R): The function f(x) is undefined at x = 1, so it can never have value at x = 1.

   Answer: A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

   Solution: The denominator cannot be zero, so the function is undefined at x = 1.

7. **Assertion(A):** If A = {1, 2, 3}, B = {2, 4}, then the number of relations from A to B is equal to 2^6 .

   Reason(R): The total number of relations from set A to set B is equal to $\{2^{n(A)n(B)}\}$.

   Answer: (A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

   Solution: By the property of relation, the total number of relations from set A to set B is $\{2^{n(A)n(B)}\}$.

8. **Assertion (A):** The relation R = $\{(x, y): x^2 + y^2 = 1\}$ is a function.

   Reason (R): For a given value of x, there can be more than one value of y=f(x)..

   Answer: D) Assertion (A) is false but Reason(R) is true.

   Explanation: The given relation defines a **circle**, and for some value of x, there are two values of y, so it's not a function.

9. **Assertion (A):** The modulus function is defined for all real numbers.

   Reason (R): |x| ≥ 0 for all x∈ R.

   Answer: B) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

   Solution: The domain is all real numbers (definition based), and the range is [0,∞) due to the non-negative nature, which the Reason states.

10. **Assertion (A):** The graph of a constant function is a straight line parallel to the x-axis.

    Reason (R): A constant function has the form f(x) = c, where c is a fixed real number..

    Answer: A) Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.

    Solution: Since f(x) is always constant, the line is horizontal (parallel to x-axis)..

VERY SHORT ANSWER TYPE QUESTIONS WITH SOLUTION

1. **Find x and y, if (x + 3, 5) = (6, 2x + y)**

   Solution: (x + 3, 5) = (6 , 2x+y) ⇒ x+3 = 6 ⇒ x=3

   5 = 2x+y ⇒ 5 = 6 + y ⇒ y = -1

2. **If the ordered pairs (x, –1) and (5, y) belong to the set {(a, b): b = 2a –3}, find the values of x and y.**

   Solution: b = 2a – 3 ⇒ -1= 2x – 3 ⇒ 2 = 2x ⇒ x =1, Similarly y= 2(5) - 3⇒ y = 10 – 3 =7

3. **If P = {1, 2}, form the set P×P×P and write down the number of elements in P × P × P**

   Solution: P × P × P = {(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)}.

   No. of elements = 8

4. **Find the domain of function $f(x) = \frac{1}{\sqrt{x}}$.**

   Solution: Because the denominator of the fraction is a square root, it is undefined for negative values of x. So x ≥ 0. However, 1/0 is also undefined, so x cannot equal zero.

   Therefore the domain = {x ∈ R | x > 0}

5. **Find the range of function $f(x) = \sqrt{x} - 2$.**

   Solution: $\sqrt{x}$ can only be zero or positive i.e. √(x) ≥ 0

   Then, $\sqrt{x} - 2$ must be greater than or equal to -2.

   So, the range = {y ∈ R | y ≥ -2}

6. **Find the range of function $f(x) = \frac{|x-4|}{x-4}$.**

Solution: $f(x) = \frac{|x-4|}{x-4} = \begin{cases} \frac{x-4}{x-4} = 1 \\ \frac{-(x-4)}{x-4} = -1 \end{cases}$

Thus the range of $f(x) = \frac{|x-4|}{x-4}$ *is* $\{\mathbf{1}, -\mathbf{1}\}$

7. **Find the domain of function** $\boldsymbol{f(x) = \frac{x}{x^2+3x+2}}$.

   Solution: f is a rational function of the form $\frac{g(x)}{h(x)}$, where g (x)= x and h (x) = $x^2 + 3x + 2$.

   Now h (x) ≠ 0 ⇒ $x^2 + 3x + 2 \neq 0$ ⇒ (x + 1) (x + 2) ≠ 0

   ⇒ Domain of the given function is R – {– 1, – 2}.

8. **Find the domain of f + g , if f and g are two functions given by**
   f = {(2, 4), (5, 6), (8, –1), (10, –3)} and g ={(2, 5), (7, 1), (8, 4), (10, 13), (11, –5)}

   Solution: Since Domain of f = D_f = {2, 5, 8, 10} and Domain of g = D_g = {2, 7, 8, 10, 11}

   ∴ Domain of (f + g) = {x | x ∈ $D_f \cap D_g$ } = {2, 8, 10}

SHORT ANSWER TYPE QUESTION WITH SOLUTION

1. **Find x and y if: (i) (4x + 3, y) = (3x + 5, – 2) (ii) (x – y, x + y) = (6, 10)**

   Solution: (i) (4x + 3, y) = (3x + 5, – 2)

   4x + 3 = 3x + 5 ⇒ x = 2 and y = – 2

   (ii) x – y = 6   x + y = 10

   ∴ 2x = 16 ⇒ x = 8

   ⇒ 8 – y = 6 ⇒ y = 2

2. **If** $\mathbf{f(x) = x^2 + x - 6}$ **and** $\mathbf{g(x) = \frac{1}{x+3}}$ **then what is** $\mathbf{(f.g)(x)}$ **and what is its domain?**

   Solution: $(f.g)(x) = f(x)g(x) = (x^2 + x - 6) \times \frac{1}{x+3} = (x-2)(x+3) \times \frac{1}{x+3}$

   $$(f.g)(x) = x - 2 \text{ if } x + 3 \neq 0$$

   The domain of (f.g) depends on the domains of f and g. D_f = R and D_g = {x ∈ **R**: x ≠ -3}

   So Dom(f.g) = {x ∈ **R**: x ≠ -3}

3. **If** $\mathbf{f(x) = (x-3)^2}$ **and** $\mathbf{g(x) = (5-x)^2}$ **then what is** $\mathbf{(f-g)(x)}$ **and what is its domain and range?**

   Solution: $(f - g)(x) = f(x) - g(x) = (x-3)^2 - (5-x)^2$

   $= x^2 - 6x + 9 - 25 - x^2 + 10x = 4x - 16$

   This is the equation of a straight line, so Domain = R and Range = R

4. **If** $\mathbf{f(x) = \sqrt{x-3}}$ **and** $\mathbf{g(x) = \sqrt{4-x}}$ **then what is the domain of** $\mathbf{(f+g)}$?

   Solution: $(f+g)(x) = f(x) + g(x) = \boldsymbol{\sqrt{x-3} + \sqrt{4-x}}$

   f(x) is only defined for values of x such that x - 3 ≥ 0 ⇒ x ≥ 3

   Therefore D_f = {x ∈ R | x ≥ 3}

   g(x) is only defined for values of x such that 4 - x ≥ 0 ⇒ x ≤ 4

   Therefore D_g = {x ∈ R | x ≤ 4}

   The domain of f + g is the set of all Real Numbers that are in the domain of f and in the domain of g. Therefore D_{f+g} = {x ∈ R | x ≥ 3 and x ≤ 4} = {x ∈ R | 3 ≤ x ≤ 4}

5. **Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a linear function from Z into Z. Find f(x).**

   Solution: f is a linear function ⇒ f (x) = mx + c.

   Also, (1, 1) ⇒ f (1) = m + c ⇒ m + c = 1

   (0, – 1) ∈ R, f (0) = c ⇒ -1 = c

   ⇒ m - 1 = 1.

   This gives m = 2 and f(x) = 2x – 1.

6. **Let A={1, 2, 3} and B={x, y}.**

   (a) Find the Cartesian product A×B.

   (b) Explain why A×B ≠ B×A

   (c) Find A×A×B

   Solution: (a) A×B = {(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)}

(b) A×B = {(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)}
B×A = {(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)}
⇒ A×B ≠ B×A as order in each pair matters.

(c) {(1, 1, x),(1, 2, x),(1, 3, x), (2,1, x),(2, 2, x),(2, 3, x), (3, 1, x),(3, 2, x),(3, 3, x), (1,1, y), (1, 2, y), (1, 3, y), (2, 1, y), (2, 2, y), (2, 3, y), (3,1,y), (3, 2, y), (3, 3, y)}

7. **Let P ={x∈ Z: 0< x < 5} and Q={y ∈ Z: y is a prime number and y<10}**
(a) Find the Cartesian product P×Q
(b) Find a subset S⊆ P×Q such that the sum of elements in each ordered pair is a multiple of 3. List all such pairs.
Solution: (a) P = {1, 2, 3, 4}   Q = {2, 3, 5, 7}
P × Q = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7)}
(b) S = {(1, 2), (1, 5), (2, 7), (3, 3), (4, 2), (4, 5)}

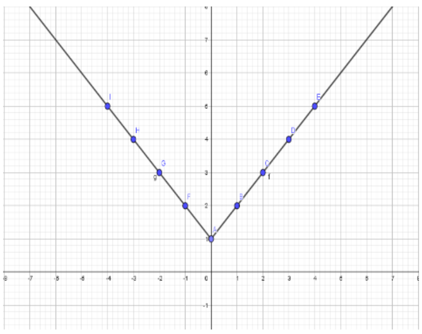
8. **Let A= {1, 2, 3, 4, 5}, B ={2, 4, 6} and C ={a, b, c}. Find (A∩B)× C and (A×C)∩(B×C). Are they equal?**
Solution: (A ∩ B) = {2, 4}
$(A \cap B) \times C = \{(2,a),(2,b),(2,c),(4,a),(4,b),(4,c)\}$.
$A \times C = \{(1,a),(2,a),(3,a),(4,a),(5,a),(1,b),(2,b),(3,b),(4,b),(5,b),(1,c),(2,c),(3,c),(4,c),(5,c)\}$
$B \times C = \{(2,a),(2,b),(2,c),(4,a),(4,b),(4,c),(6,a),(6,b),(6,c)\}$
$(A \times C) \cap (B \times C) = \{(2,a),(2,b),(2,c),(4,a),(4,b),(4,c)\}$
Yes, they are equal.

LONG ANSWER TYPE QUESTIONS WITH SOLUTIONS

1. **Determine the domain and range of the following relation**
R = {(x, y): x ∈ N, y ∈N and x + y = 10}.
Solution: Given set N = {1, 2, 3,…}
A relation R in N is defined as R = {(x, y): x, y ∈ N, x + y = 10} ∀ x, y ∈ N
So, x R y ⇔ x + y = 10 ⇔ y = 10 – x when x = 1, then y = 10 – 1 = 9 ∈ N then (1, 9) ∈ R
when x = 2, then y = 10 – 2 = 8 ∈ N then (2, 8) ∈ N
when x = 3, then y = 10 – 3 = 7 ∈ N then (3, 7) ∈ R
when x = 4, then y = 10 – 4 = 6 ∈ N then (4, 6) ∈ R
Similarly, (5, 5) ∈ R, (6, 4) ∈ R, (7, 3) ∈ R, (8, 2) ∈ R, (9, 1) ∈ R
R = {(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)}
Domain of R = {1, 2, 3, 4, 5, 6, 7, 8, 9}   Range of R = {9, 8, 7, 6, 5, 4, 3, 2, 1}

2. **Draw the graph of following function and write it's domain and range.**

$$\mathbf{f(x) = \begin{cases} 1 - x, & \text{if } x < 0 \\ 1, & \text{if } x = 0 \\ x + 1, & \text{if } x > 0 \end{cases}}$$



Solution: f(x) = 1 – x, x < 0, this gives
f(– 4) = 1 – (– 4)= 5;
f(– 3) =1 – (– 3) = 4,
f(– 2) = 1 – (– 2)= 3
f(–1) = 1 – (–1) = 2; etc,
f(1) = 2,
f (2) = 3,
f (3) = 4,
f(4) = 5 and so on for f(x) = x + 1, x > 0.
Domain: All real numbers.   Range: [1, ∞]

3. **Assume that A = {1, 2, 3,…,14}. Define a relation R from A to A as follows-**
R = {(x, y): 3x – y = 0, such that x, y ∈ A}.
Determine and write down its range, domain, and co domain.

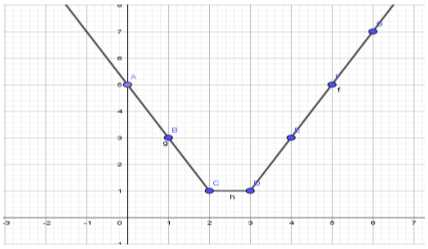
Solution: R = {(x, y): 3x – y = 0, where x, y ∈ A}.
It means that R = {(x, y): 3x = y, where x, y ∈ A}
Hence, R = {(1, 3), (2, 6), (3, 9), (4, 12)}
Since the domain of R is defined as the set of all first elements of the ordered pairs in th given relation. Hence, the domain of R = {1, 2, 3, 4}
To determine the co domain, we know that the entire set A is the co-domain of the relation R.
Therefore, the co-domain of R = A = {1, 2, 3,…,14}
As it is known that, the range of R is defined as the set of all second elements in the relation ordered pair. Hence, the range of R is given is {3, 6, 9, 12}

4. **Draw the graph of function f(x) = |x – 2| + |x - 3|**

Solution: f(x) = | x – 2 | + | x – 3 |

$$\text{When } x < 2, f(x) = -(x-2) - (x-3) = -x + 2 - x + 3 = -2x + 5$$
$$\text{When } 2 \le x \le 3, \quad f(x) = (x-2) - (x-3) = x - 2 - x + 3 = 1$$
$$\text{When } x > 3, \; f(x) = (x-2) + (x-3) = x - 2 + x - 3 = 2x - 5$$

x	0	1	2	3	4	5	6	7
f(x)	5	3	1	1	3	5	7	9



CASE STUDY BASED QUESTIONS WITH SOLUTIONS

1. Maths teacher started the lesson Relations and Functions in Class XI. He explained the following topics:
Ordered Pairs: The ordered pair of two elements a and b is denoted by (a, b) : a is first element (or first component) and b is second element (or second component).
Two ordered pairs are equal if their corresponding elements are equal. i.e., (a, b) = (c, d) ⇒ a = c and b = d
Cartesian Product of Two Sets: For two non-empty sets A and B, the cartesian product A × B is the set of all ordered pairs of elements from sets A and B.
In symbolic form, it can be written as A × B= {(a, b): a ∈ A, b ∈ B}
Based on the above topics, answer the following questions.
(i) If (a – 3, b + 7) = (3, 7), then find the value of a and b
(ii) If (x + 6, y – 2) = (0, 6), then find the value of x and y
(iii) If (x + 2, 4) = (5, 2x + y), then find the value of x and y
(iv) Find x and y, if (x + 3, 5) = (6, 2x + y).
Solution: (i) We know that, two ordered pairs are equal, if their corresponding elements are equal.
So (a – 3, b + 7) = (3, 7)
⇒ a – 3 = 3 and b + 7 = 7 [equating corresponding elements]
⇒ a = 3 + 3 and b = 7 – 7 ⇒ a= 6 and b = 0
(ii) (x + 6, y – 2) = (0, 6)
⇒ x + 6 = 0 ⇒ x = -6 and y – 2 = 6 ⇒ y = 6 + 2 = 8
(iii) (x + 2, 4) = (5, 2x + y)
⇒ x + 2 =5 ⇒ x = 5 – 2 = 3 and 4 = 2x + y ⇒ 4 = 2 × 3 + y ⇒ y = 4 – 6 = -2
(iv) x + 3 = 6, 2x + y = 5 ⇒ x = 3, y = 1

2. Let **A = {1, 2, 3, 4}** and **B = {a, b, c, d}**. Consider the following relations from A to B:
Relation R_1: {(1, a), (2, b), (3, c), (4, d)}

Relation R_2: {(1, a), (2, b), (2, c), (3, d)}
Relation R_3: {(1, a), (2, a), (3, a), (4, a)}

Answer the following questions using the information given above.

(a) Which of the relations is a function?
(b) Which of the represents many to one function?
(c) If we define a new relation R_4 = {(1, a), (2, b), (3, c)}, Is it a function from A to B? Justify your answer.

Solution: (a) R_1 and R_3

(b) R_3

(c) No, as R_4 doesn't assign any value to **4**, which is in A. So it **violates** the rule that **every element of the domain must have an image.**

3. You are given a table that shows a relation from **set A = {1, 2, 3, 4}** to set B.

X	1	2	3	4
f(x)	5	7	5	8

Answer the following questions using the information given above.

(a) Is this relation a function? Justify it with proper explanation.
(b) Which of the following would **invalidate** this table as a function? Explain it.

A) Adding (2, 9) B) Adding (5, 9)
C) Adding (4, 8) D) Replacing (3, 5) with (3, 7)

Solution: (a) Yes because for each value of x, we get different values of f(x).
(c) By adding (2, 9) we get two different values of function for x=2.

4. You're given the following description of a graph of a function f(x)

The graph is a continuous curve passing through the points:
(–3, 4), (–2, 2), (0, 0), (2, –2) and (3, –1)
It is defined for $x \in [-3, 3]$
The curve decreases from x = –3 to x = 2, then increases from x = 2 to x = 3.
Answer the following questions using the information given above.

(a) What is the **domain** of the function?
(b) What is the **range** of the function?
(c) On which interval is the function **decreasing**? What is the **minimum value** of the function?

Solution: (a) [−3, 3] (b) [−2, 4]

(c) The curve goes downhill (function values drop) from x = –3 to x = 2. The lowest point on the graph is (2, –2), so minimum value is –2.

5. You're given the function: f(x) = | x – 2 |

This is a **V-shaped graph**, and its key properties are:
It shifts the standard |x| graph **2 units to the right.**
The vertex (point of minimum) is at (2, 0)
The function is piecewise defined as

$$\boldsymbol{f(x) = \begin{cases} 2 - x, & if\ x < 2 \\ x - 2, & if\ x \geq 2 \end{cases}}$$

Answer the following questions using the information given above.

(a) What is the value of f(2)
(b) Check whether (1, 3) lies **on** the graph or not?
(c) What is the **domain** and range of f(x) = | x – 2 |?

Solution: (a) f(2) = | 2 – 2 | = 0 (b) f(3)=|3−2|=1 ⇒ (3, 1) is on the graph
(c) Domain = (−∞, ∞) as it is defined for all real numbers.
Range = [0, ∞) as Absolute value is **never negative**. Smallest value is at x=2x where f(2)=0

CHAPTER – 3: TRIGONOMETRIC FUNCTIONS

Definitions and Formulae:

Degree measure: If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is said to have a measure of one degree, written as 1°.

Radian measure: There is another unit for measurement of an angle, called the radian measure. Angle subtended at the centre by an arc of length 1 unit in a unit circle (circle of radius 1 unit) is said to have a measure of 1 radian.

Domain and Range

Function	Domain	Range
$sin\ x$	R	[-1, 1]
$cos\ x$	R	[-1, 1]
$tan\ x$	R- {x=(2n+1)π/2, $n \in Z$ }	R
$cot\ x$	R-{x=n π, $n \in Z$ }	R
$sec\ x$	R- {x=(2n+1)π/2, $n \in Z$}	$(-\infty, -1] \cup [1, \infty)$
$cosec\ x$	R-{x=n π, $n \in Z$ }	$(-\infty, -1] \cup [1, \infty)$

Signs in Quadrants (ASTC Rule)

- **1st Quadrant**: All positive
- **2nd Quadrant**: sine & cosecant is positive
- **3rd Quadrant**: tangent & cotangent is positive
- **4th Quadrant**: cosine & secant is positive

(A mnemonic: **All Students Take Coffee**)

Important Formulae:

- If in a circle of radius r, an arc of length l subtends an angle of θ radians, then $l = r\,\theta$
- cos (2n π + x) = cosx , sin (2n π + x) = sinx
- $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$, $\cot(-x) = -\cot x$,
- $\text{cosec}(-x) = \text{cosec } x$, $\cos(-x) = \cos x$, $\sec(-x) = \sec x$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\cos\left(\frac{\pi}{2} - x\right) = sinx$, $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- $\cos\left(\frac{\pi}{2} + x\right) = -sinx$, $\sin\left(\frac{\pi}{2} + x\right) = cosx$
- $\cos(\pi - x) = -cosx$, $\sin(\pi - x) = \sin x$
- $\cos(\pi + x) = -cosx$, $\sin(\pi + x) = -\sin x$
- $\cos(2\pi - x) = cosx$, $\sin(2\pi - x) = -\sin x$
- **If none of the angles x, y and (x ± y) is an odd multiple of $\frac{\pi}{2}$, then**

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}, \quad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

- **If none of the angles x, y and (x ± y) is a multiple of π , then**

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}, \quad \cot(x - y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - tan^2 x}{1 + tan^2 x}$

- $\sin 2x = 2\sin x \cos x = \frac{2tanx}{1+tan^2x}$
- $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\tan 3x = \frac{3tanx - tan^3x}{1-3tan^2x}$
- $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$
- $\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$
- $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$
- $2\cos x\cos y = \cos(x+y) + \cos(x-y)$
- $-2\sin x\sin y = \cos(x+y) - \cos(x-y)$
- $2\sin x\cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x\sin y = \sin(x+y) - \sin(x-y)$
- $\sin(A+B)\sin(A-B) = sin^2A - sin^2B = cos^2B - cos^2A$
- $\cos(A+B)\cos(A-B) = cos^2A - sin^2B = cos^2B - sin^2A$

MULTIPLE CHOICE QUESTIONS

1) The large hand of a clock is 42 cm long. How much distance does its extremity move in 20 minutes?
   (a) 88 cm (b) 80 cm (c) 75 cm (d) 77 cm

   Ans. b) 88 cm

   Solution : Angle sweeps by the minute hand in 20 minutes= $6° \times 20 = 120° = \frac{2\pi}{3}$ radians . Thus , the distance does its extremity move in 20 minutes = arc of length l = rθ

   $= 42 \text{ x } \frac{2\pi}{3} = 88\ cm$

2) The value of $\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 89°\ is$

   (a) 1 (b) 0 (c) $\frac{1}{2}$ (d) not defined

   Ans. (a) 1

   Solution: $\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 88° \tan 89°$

   $= (\tan 1° \tan 89°)(\tan 2° \tan 88°) \ldots\ldots (\tan 44° \tan 46°) \tan 45°$

   $= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots \ldots (\tan 44° \cot 44°) \times 1$

   $= 1 \times 1 \times 1 \times \ldots\ldots \times 1 = 1$

3) If $\sin x + \cos x = \frac{1}{5}$ then tan 2x is

   (a) $\frac{25}{17}$ (b) $\frac{7}{25}$ (c) $\frac{25}{7}$ (d) $\frac{24}{7}$

   Ans. d)

   Solution :$(\sin x + \cos x)^2 = \frac{1}{25} \Rightarrow 1 + \sin 2x = \frac{1}{25}$

   $\Rightarrow \sin 2x = -\frac{24}{25}$ & $\cos 2x = -\frac{7}{25} \Rightarrow \tan 2x = \frac{24}{7}$

4) Value of sin 10° + sin 20° + sin 30° +......+ sin360° is

   (a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

   Ans. b) 0

   Solution : $\sin(360° - x) = -\sin x \Rightarrow$ sin 10° + sin 20° + sin 30° +......+ sin360° = 0

5) If sin A = $\frac{3}{5}$, 0< $A < \frac{\pi}{2}$ and cos B = $-\frac{12}{13}$, $\pi < B < \frac{3\pi}{2}$, then the value of sin (A −B) is

(a) $-\frac{13}{82}$ (b) $-\frac{15}{65}$ (c) $-\frac{13}{65}$ (d) $-\frac{16}{65}$

Ans . (d) $-\frac{16}{65}$

Solution : sin A = $\frac{3}{5}$ ⟹ cos A = $\frac{4}{5}$, cos B = $-\frac{12}{13}$ ⟹ $\sin B = -\frac{5}{13}$

∴ $\sin(A-B) = \frac{3}{5} \times \left(-\frac{12}{13}\right) - \frac{4}{5} \times \left(-\frac{5}{13}\right) = -\frac{36}{65} + \frac{20}{65} = -\frac{16}{65}$

6) 1 + cos 2x + cos 4x + cos 6x =

(a) 2 cos x cos 2x cos 3x (b) 4 sin x cos 2x cos 3x

(c) 4 cos x cos 2x cos 3x (d) None of these

Ans. **c)4cosx cos2x cos3x**

Solution : 1 + cos 4x + cos 2x + cos 6x = $2\cos^2 2x$ + 2cos4x cos2x

⟹ 2cos2x(cos2x+ cos4x) ⟹ 2cos2x .2cos3x cosx ⟹4cosx cos2x cos3x

7) If in two circles, arcs of the same length subtend angles 45° and 60° at Centre, then the ratio of their radii is

(a) 2:3 (b) 2:5 (c) 3:4 (d) 4:3

Ans. (d) 4:3

Solution : ATQ, we have $l_1 = l_2$ ⟹ $r_1 \theta_1 = r_2\theta_2$

⟹ $r_1 : r_2 = \frac{\pi}{3} : \frac{\pi}{4} = 4:3$

8) The value of $\sin(\pi - \theta)\sin(\pi + \theta)\ cosec^2\,\theta$ =

(a) 0 (b) 1 (c) -1 (d) $\frac{1}{2}$

Ans. (c) -1

Solution: $\sin(\pi - \theta)\sin(\pi + \theta)\ cosec^2\,\theta = \sin\theta\,(-\sin\theta) \times \frac{1}{sin^2\theta} = -1$

9) The value of tan 3A – tan 2A – tan A is equal to

(a) tan 3A tan 2A tan A (b) –tan 3A tan 2A tan A

(c) tan A tan 2A – tan 2A tan 3A – tan 3A tan A (d) None of these

Ans. (a) tan 3A tan 2A tan A

Solution :tan 3A = tan (2A + A) = $\frac{\tan 2A + \tan A}{1 - \tan 2A \tan A}$

⟹ tan3A – tan A tan 2A tan 3A = tan2A + tan A

⟹ tan 3A − tan2A − tanA = tanA tan2A tan 3A

10) If tan A = $\frac{b}{a}$, then value of a cos 2A+b sin 2A=

(a) –b (b) a (c) $-a$ (d) b

Ans. (b) a

Solution : a cos 2A + b sin 2A= $a\left(\frac{1-tan^2A}{1+tan^2A}\right) + b\left(\frac{2\tan A}{1+tan^2A}\right)$

$$\Rightarrow a\left(\frac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}}\right) + b\left(\frac{2 \times \frac{b}{a}}{1+\frac{b^2}{a^2}}\right) = a$$

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

- **A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).**
- **B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).**
- **C) Assertion (A) is true but Reason(R) is false.**
- **D) Assertion (A) is false but Reason(R) is true.**

1) **Assertion (A):** The maximum value of sin x + cos x is 2.

Reason(R) : The maximum value of sin x is 1 and maximum value cos x is 1

Answer : D

Solution: The maximum value of sin x + cos x is $\sqrt{2}$ at x = $\frac{\pi}{4}$. So the Assertion (A) is false and reason (R) is true.

2) **Assertion (A) :** tan 5A – tan 3A – tan 2A = tan 5A tan 3A tan 2A

Reason(R) : x = y + z ⇒ tan x – tan y – tan z = tan x tan y tan z.

Answer: A

Solution : If tan A = tan (B+ C) ⇒ tan A = $\frac{\tan B + \tan C}{1 - \tan B \tan C}$ which imply the result

tan A – tan B – tan C = tan A tan B tan C.

Hence, Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

3) **Assertion (A):** $\cos^3\alpha + \cos^3\left(\alpha + \frac{2\pi}{3}\right) + \cos^3(\alpha + \frac{4\pi}{3}) = 3\cos\alpha\cos\left(\alpha + \frac{2\pi}{3}\right)\cos(\alpha + \frac{4\pi}{3})$

Reason(R) : If a + b + c = 0 then $a^3 + b^3 + c^3 = 3abc$

Answer: A

Solution: Assertion (A): $\cos\alpha + \cos\left(\alpha + \frac{2\pi}{3}\right) + \cos\left(\alpha + \frac{4\pi}{3}\right) = 0$

$\Rightarrow \cos^3\alpha + \cos^3\left(\alpha + \frac{2\pi}{3}\right) + \cos^3(\alpha + \frac{4\pi}{3}) = 3\cos\alpha\ \cos\left(\alpha + \frac{2\pi}{3}\right)\ \cos(\alpha + \frac{4\pi}{3})$ (true)

Hence, Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

4) **Assertion (A):** The ratio of the radii of two circles at the centres of which two equal arcs subtend angles of 30° and 70° is 21: 10.

Reason(R): Number of radians in an angle subtended at the centre of a circle by an arc is equal to the ratio of the length of the arc to the radius of the circle.

Answer: D

Solution: Assertion (A): $r_1 : r_2 = \theta_2 : \theta_1 \Rightarrow r_1 : r_2 = 7\frac{\pi}{18} : \frac{\pi}{6} = 7 : 3$ (false)

Reason(R) : $\theta = \frac{l}{r}$ (True)

5) **Assertion (A) :** $(\sin\theta + \cos\theta)^2 = \sin 2\theta$

Reason(R) : $\sin^2\theta + \cos^2\theta = 1$

Answer: D

Solution: $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$, So assertion is false and reason is true.

6) **Assertion (A):** $\cos(\pi - \theta) = -\cos\theta$

Reason(R): Cosine is an even function.

Answer: B

Solution: The assertion is true, but the reason is not the correct explanation. $\cos(\pi - \theta) = -\cos\theta$ is due to angle transformation, not the even nature of cosine. Cosine being even means cos(−x) = cos(x).

7) **Assertion (A) :** Value of $\tan\left(-\frac{11\pi}{4}\right)$ is 1.

Reason (R) : $\sin(3\pi + \theta) = \sin\theta$

Answer: C)

Solution: $\tan\left(-\frac{11\pi}{4}\right) = -\tan\left(\frac{11\pi}{4}\right) = -\tan\left(3\pi - \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

And $\sin(3\pi + \theta) = -\sin\theta$

8) **Assertion (A):** the value of $\cot\frac{A}{2} - \tan\frac{A}{2}$ is 2 tanA

Reason(R) : cot A is reciprocal of tan A

Answer: D

Solution: $\cot\frac{A}{2} - \tan\frac{A}{2} = \frac{1-\tan^2\frac{A}{2}}{\tan\frac{A}{2}} \Rightarrow 2\left(\frac{1-\tan^2\frac{A}{2}}{2\tan\frac{A}{2}}\right) = 2 \times \frac{1}{\tan A} \Rightarrow 2\cot A$

9)Assertion (A): The range of 4 + 5 cos x is [−1, 9]

Reason(R): The value of cos x lies from −1 to 1.

Answer: A

Solution: $-1 \le \cos x \le 1 \Rightarrow -5 \le 5\cos x \le 5$

$\Rightarrow -5+4 \le 4+5\cos x \le 5+4 \Rightarrow -1 \le 4+5\cos x \le 9$

⟹ both Assertion (A) and Reason (R) is correct and Reason(R) is the correct explanation of Assertion (A).

10)Assertion (A): The domain of tan x is **R− { (2n+1)** $\frac{\pi}{2}$, **n ∈ ℤ }**

Reason(R) : tan x is undefined where cos x = 0.

Answer: A

Solution: cos x = 0 at x = (2n+1) $\frac{\pi}{2}$, $n \in Z$

⟹ Hence, Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

VERY SHORT ANSWER TYPE QUESTIONS

1) **Find the value of** $\cos\left(\frac{3\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} + x\right)$

Solution: $\cos(\frac{3\pi}{4} - x) - \cos(\frac{3\pi}{4} + x) =$

$= \cos\frac{3\pi}{4}\cos x + \sin\frac{3\pi}{4}\sin x - \cos\frac{3\pi}{4}\cos x + \sin\frac{3\pi}{4}\sin x$

$= 2\sin\frac{3\pi}{4}\sin x = 2 \times \frac{1}{\sqrt{2}}\sin x = \sqrt{2}\sin x$

2) **The angles of a triangle are in A.P., then, find the angles of the triangle in radians if greatest angle is three times the smallest angle.**

Solution: let A, B and C be the angles of triangle. Since, A, B and C are in A.P.

So, 2B = A + C

$\because \; A + B + C = \pi \Rightarrow 2B + B = \pi$

$\Rightarrow B = \frac{\pi}{3}$ and $C = 3A$

So, $A + \frac{\pi}{3} + 3A = \pi \Rightarrow A = \frac{\pi}{6}$ and $C = \frac{\pi}{2}$

3) **Find the value of** $\cot\left(\frac{-15\pi}{4}\right)$.

Solution: $\cot(\frac{-15\pi}{4}) = -\cot(\frac{15\pi}{4})$

$= -\cot(2\pi + \frac{7\pi}{4}) = -\cot(\frac{7\pi}{4})$

$= -\cot\left(2\pi - \frac{\pi}{4}\right) = -(-\cot\frac{\pi}{4}) = 1$

4) **Prove that :** $2\sin^2\frac{\pi}{6} + \mathrm{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

Solution: L.H.S : $2\sin^2\frac{\pi}{6} + \mathrm{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = 2(\frac{1}{2})^2 + \mathrm{cosec}^2\left(\pi + \frac{\pi}{6}\right)(\frac{1}{2})^2$

$= \frac{2}{4} + (-\mathrm{cosec}\,\frac{\pi}{6})^2 \times \frac{1}{4} = \frac{1}{2} + 4 \times \frac{1}{4} = \frac{3}{2}$

5) **Show that :** $\cos^2\left(\frac{\pi}{4} + x\right) - \sin^2\left(\frac{\pi}{4} - x\right)$ **is independent of x.**

Solution: we have , $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

$\Rightarrow \cos^2\left(\frac{\pi}{4} + x\right) - \sin^2\left(\frac{\pi}{4} - x\right) = \cos\left(\frac{\pi}{4} + x + \frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} + x - \frac{\pi}{4} + x\right)$

$= \cos(\frac{\pi}{2})\cos 2x = 0 \times \cos 2x = 0$

Thus, the given expression is independent of x.

6) **If** $\sin x = \frac{1}{6}$, **find the value of sin 3x .**

Solution: we have, $\sin 3x = 3\sin x - 4\sin^3 x$

$= 3\left(\frac{1}{6}\right) - 4\left(\frac{1}{6}\right)^3 = \frac{3}{6} - \frac{4}{216} \quad = \frac{13}{27}$

7) **Show that :** $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8x}}} = 2\cos x$

Solution: L.H.S : $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8x}}} = \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8x)}}}$

$=\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 4x}}} = \sqrt{2+\sqrt{2+2\cos 4x}}$

$= \sqrt{2+\sqrt{2\times 2\cos^2 2x}} = \sqrt{2+2\cos 2x}$

$= \sqrt{2(1+\cos 2x)} = \sqrt{2\times 2\cos^2 x} = 2\cos x$ = R.H.S

8) **If cot x = $-\frac{5}{12}$, x lies in second quadrant, find the values of sec x and cosec x.**

Solution: Since, $\cot x = -\frac{5}{12} \implies \tan x = -\frac{12}{5}$

Now, $1 + \tan^2 x = \sec^2 x \implies \sec^2 x = 1 + \left(-\frac{12}{5}\right)^2 = 1 + \frac{144}{25} = \frac{169}{25}$

$\text{Sec } x = \pm\frac{13}{5} \implies \sec x = -\frac{13}{5}$, because x lies in second quadrant

Similarly, $\text{cosec}^2 x = 1 + \frac{25}{144} = \frac{169}{144} \implies \text{cosec } x = \pm\frac{13}{12}$

$\text{cosec} x = \frac{13}{12}$, as x lies in second quadrant.

9) **A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?**

Solution:

Number of revolutions made in 60 seconds = 360

∴ Number of revolutions made in one seceond= $\frac{360}{60} = 6$

when the wheel makes one revolution, it turns through 2π radian.

∴ Number radians turned by the wheel in one seceond = 6 × 2π = 12 π radians

10): Prove that : $\sin(45^\circ + A)\sin(45^\circ - A) = \frac{1}{2}\cos 2A$

Solution:

We have , $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

$\implies \sin(45^\circ + A)\sin(45^\circ - A) = \sin^2 45^\circ - \sin^2 A$

$= \frac{1}{2} - \sin^2 A \implies \frac{1-2\sin^2 A}{2} = \frac{\cos 2A}{2}$

SHORT ANSWER TYPE QUESTIONS

1) **Prove that: cot x cot 2x – cot 2x cot 3x – cot 3x cot x=1**

Solution: We have 3x = x + 2x

cot 3x = cot (x + 2x)

$\cot 3x = \frac{\cot x \cot 2x - 1}{\cot x + \cot 2x}$

$\cot x \cot 3x + \cot 2x \cot 3x = \cot x \cot 2x - 1$

⟹ cot x cot 2x – cot 2x cot 3x – cot 3x cot x=1

2) **Prove that :** $\sin(-420^\circ)(\cos 390^\circ) + \cos(-660^\circ)(\sin 330^\circ) = -1$

Solution:

L.H.S : $\sin(-420^\circ)(\cos 390^\circ) + \cos(-660^\circ)(\sin 330^\circ)$

$= -\sin\frac{7\pi}{3}\cos\frac{13\pi}{6} + \cos\frac{11\pi}{3}\sin\frac{11\pi}{6}$

$= -\sin\left(2\pi + \frac{\pi}{3}\right)\cos\left(2\pi + \frac{\pi}{6}\right) + \cos\left(4\pi - \frac{\pi}{3}\right)\sin\left(2\pi - \frac{\pi}{6}\right)$

$= -\sin\frac{\pi}{3}\ \cos\frac{\pi}{6} - \cos\frac{\pi}{3}\ \sin\frac{\pi}{6}$

$= -\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right) = -\sin\frac{\pi}{2} = -1 = \text{R.H.S.}$

3) **Show that : $\tan 70° = \tan 20° + 2\tan 50°$**

Solution: We have, $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$\Rightarrow \tan A - \tan B = \tan(A-B)(1+\tan A\tan B)$

Put A = 70° and B = 20°, we get

$\tan 70° - \tan 20° = \tan 50°(1+\tan 70° \tan 20°)$

$\Rightarrow \tan 70° - \tan 20° = \tan 50°(1+\tan 70° \tan(90°-70°))\}$

$\Rightarrow \tan 70° - \tan 20° = \tan 50°(1+\tan 70° \cot 70°)$

$\Rightarrow \tan 70° - \tan 20° = 2\tan 50°$

4) **What must be the radius of a circular running path, round which an athlete must run 5 times in order to describe 1760 metres?**

Solution: Since, it is a circular path. So, $\theta = \frac{l}{r}$

During one rotation, angle is subtended at the centre $= 2\pi$

$\therefore\quad 2\pi = \frac{l}{r} \quad \Rightarrow l = 2\pi r = \text{perimeter}$

$\Rightarrow 5 \times 2\pi r = 1760 \text{ m} \qquad \Rightarrow \pi r = 176 \text{ m}$

$\Rightarrow r = 176 \times \frac{7}{22} = 8 \times 7 \text{ m} = 56 \text{ m}$

5) **Prove that : $\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A} = \tan A$**

Solution: L.H.S.

$\frac{\sin A+\sin 2A}{1+\cos A+\cos 2A} = \frac{\sin A+2\sin A\cos A}{1+\cos A+2\cos^2 A-1}$

$\frac{\sin A(1+2\cos A)}{\cos A(1+2\cos A)} = \frac{\sin A}{\cos A} = \tan A = \text{R.H.S.}$

6) **In any cyclic quadrilateral ABCD, prove that: cos A + cos B + cos C + cos D=0**

Solution: A + C=π (Opposite angles of cyclic quadrilateral are supplementary) .

$A = \pi - C$.

$\cos A = \cos(\pi - C)$.

$\cos A = -\cos C$

$\cos A + \cos C = 0$ ------------------------(1)

Similarly , $\cos B + \cos D = 0$ ----------------(2)

By adding equation (1) and (2), we get

$\cos A + \cos B + \cos C + \cos D = 0$

7) **If 3 tan A tan B = 1, prove that 2 cos (A + B) = cos (A - B)**

Solution: We have

$3\tan A\tan B = 1 \Rightarrow \frac{3\sin A\sin B}{\cos A\cos B} = 1$

$\Rightarrow \frac{\cos A\cos B}{\sin A\sin B} = \frac{3}{1}$

$\Rightarrow \frac{\cos A\cos B + \sin A\sin B}{\cos A\cos B - \sin A\sin B} = \frac{3+1}{3-1}$ (Applying componendo & dividendo)

$\Rightarrow \frac{\cos(A-B)}{\cos(A+B)} = \frac{2}{1} \quad \Rightarrow 2\cos(A+B) = \cos(A-B)$

8) **Show that : $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right) = \frac{1}{8}$**

Solution: : L.H.S : $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right) = \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$= sin^2\frac{\pi}{8}\ sin^2\frac{3\pi}{8} = \frac{1}{4}\left(1 - cos\frac{\pi}{4}\right)(1 - cos\frac{3\pi}{4}) = \frac{1}{4}\ (1 - \frac{1}{\sqrt{2}})\ (1 + \frac{1}{\sqrt{2}})$

$= \frac{1}{4}\left(1 - \frac{1}{2}\right) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

LONG ANSWER TYPE QUESTIONS

1) If $\sin x = \frac{3}{5}$, where $0 < x < \frac{\pi}{2}$, find the values of sin2x , cos 2x , tan2x and sin4x

Solution : we have , $\sin x = \frac{3}{5}$, where $0 < x < \frac{\pi}{2}$,

$\cos x = \sqrt{1 - \sin^2 x} \implies \cos x = \sqrt{1 - \frac{9}{25}} \implies \cos x = \frac{4}{5}$

$\therefore\ \tan x = \frac{\sin x}{\cos x} = \frac{3}{4}$

Thus, we obtain

$\sin 2x = 2\sin x \cos x = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

$\cos 2x = 1 - 2\sin^2 x = 1 - 2 \times (\frac{3}{5})^2 = \frac{7}{25}$

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{24}{7}$

and , $\sin 4x = 2\sin 2x \cos 2x = 2 \times \frac{24}{25} \times \frac{7}{25} = \frac{336}{625}$

2) Prove that : $cos^2 x + cos^2\left(x + \frac{2\pi}{3}\right) + cos^2\left(x - \frac{2\pi}{3}\right) = \frac{3}{2}$

Solution : L.H.S. $cos^2 x + cos^2\left(x + \frac{2\pi}{3}\right) + cos^2\left(x - \frac{2\pi}{3}\right)$

$= \frac{1}{2}\{2\ cos^2 x + 2\ cos^2\left(x + \frac{2\pi}{3}\right) + 2cos^2\left(x - \frac{2\pi}{3}\right)\}$

$= \frac{1}{2}\ [1 + \cos 2x + \{1 + cos\ 2\left(x + \frac{2\pi}{3}\right) + 1 + cos\ 2\left(x - \frac{2\pi}{3}\right)\}]$

$= \frac{1}{2}\ [\ 3 + \cos 2x + \{\ cos\ \left(2x + \frac{4\pi}{3}\right) + cos\ \left(2x - \frac{4\pi}{3}\right)\}]$

$= \frac{1}{2}\ [\ 3 + \cos 2x + \{\ 2cos\ 2x\ cos\frac{4\pi}{3}\}]$

$= \frac{1}{2}\ [\ 3 + \cos 2x + \{\ 2cos\ 2x \times (-\frac{1}{2})\}] = \frac{1}{2}\ \{3 + \cos 2x - \cos 2x\ \} = \frac{3}{2}$

3) If $\tan x = -\frac{4}{3}$, x lies in quadrant III , Find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

Solution: we have , $\tan x = -\frac{4}{3}$, $\pi < x < \frac{3\pi}{2}$

$\implies \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \implies \frac{x}{2}$ lies in quadrant II

Now, $\cos x = \frac{-1}{\sqrt{1+\tan^2 x}} = \frac{-1}{\sqrt{1+\frac{16}{9}}} = -\frac{3}{5}$

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \frac{2}{\sqrt{5}}$

$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-3/5}{2}} = \frac{-1}{\sqrt{5}}$

$\implies \tan\frac{x}{2} = -2$

4) Find the value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

Solution: We have

$= \tan 9° + \tan 81° - \tan 27° - \tan 63°$

$= \tan 9° + \tan(90° - 9°) - \tan 27° - \tan(90° - 27°)$

$= \tan 9° + \cot 9° - (\tan 27° + \cot 27°)$

Also, $\tan 9° + \cot 9° = \frac{1}{\sin 9° \cos 9°} = \frac{2}{2\sin 9° \cos 9°} = \frac{2}{\sin 18°}$

Similarly, $\tan 27° + \cot 27° = \frac{2}{\sin 54°} = \frac{2}{\sin(90° - 36°)} = \frac{2}{\cos 36°}$

$\therefore$ tan 9° + cot 9° − (tan 27° + cot 27°) $= \frac{2}{\sin 18°} - \frac{2}{\cos 36°}$

$= \frac{2\times4}{\sqrt{5}-1} - \frac{2\times4}{\sqrt{5}+1} = 4$

5) **Prove that : $\cos\frac{\pi}{7}\ \cos\frac{2\pi}{7}\ \cos\frac{4\pi}{7} = -\frac{1}{8}$**

Solution: L.H.S.

$\cos\frac{\pi}{7}\ \cos\frac{2\pi}{7}\ \cos\frac{4\pi}{7} = \frac{1}{2\sin\frac{\pi}{7}}\left\{2\sin\frac{\pi}{7}\ \cos\frac{\pi}{7}\ \cos\frac{2\pi}{7}\ \cos\frac{4\pi}{7}\right\}$

$= \frac{1}{2\sin\frac{\pi}{7}}\left\{\sin\frac{2\pi}{7}\ \cos\frac{2\pi}{7}\ \cos\frac{4\pi}{7}\right\}$

$= \frac{1}{4\sin\frac{\pi}{7}}\left\{2\sin\frac{2\pi}{7}\ \cos\frac{2\pi}{7}\ \cos\frac{4\pi}{7}\right\} = \frac{1}{4\sin\frac{\pi}{7}}\left\{\sin\frac{4\pi}{7}\ \cos\frac{4\pi}{7}\right\}$

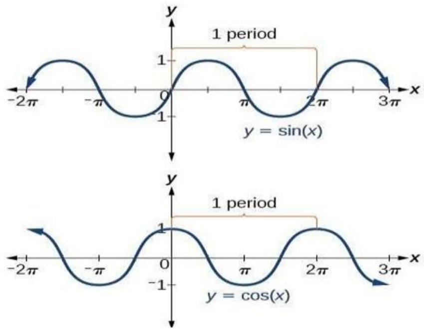
$= \frac{1}{8\sin\frac{\pi}{7}}\left\{2\sin\frac{4\pi}{7}\ \cos\frac{4\pi}{7}\right\} = \frac{1}{8\sin\frac{\pi}{7}}\left\{\sin\frac{8\pi}{7}\right\}$

$= \frac{1}{8\sin\frac{\pi}{7}}\left\{\sin(\pi + \frac{\pi}{7})\right\} = \frac{1}{8\sin\frac{\pi}{7}}\left\{-\sin\frac{\pi}{7}\right\}$

$= -\frac{1}{8}$ = R.H.S

CASE STUDY BASED QUESTIONS

1) Sine and cosine functions can be used to model many real life scenarios-radio waves, tides, musical tones, electrical signals. Following are shown the graphs of sine and cosine curves. Looking to these graphs, answer the following questions



(i) If we draw line y= $\frac{1}{2}$, at how many points the graph of sine and this line intersects between interval [$-\pi$, π].

(ii) At how many points the graph of cosine cuts on X axis between (0, 2π) ?

(iii) If both sine and cosine curve are drawn on the same graph and on the same interval [-2 π, 2π] , then at how many points their graph will intersects ?

OR

At how many points will be sin x = √2 in interval [$-\pi$, π]

Solution: (i) From the above graph. The line y= $\frac{1}{2}$ intersects the sine graph at **two** points.

Solution(ii) clearly from the graph, the graph of cosine cuts on X axis between (0, 2π) at **two** points

Solution:(iii) If we will draw the graph of sine and cosine on the same interval [-2 π, 2π], then the graph of both function will intersect at **four** points.

OR

Solution: The value of sin x lies from -1 to 1 only, so sin x =√2 is not possible in interval [$-\pi$, π]

2) **Given $\cos x = -\frac{4}{5}$ and $\sin y = \frac{5}{13}$, x & y both lie in second quadrant.**

Based on the following information's, answer the following: -

(i) Find the value of sin (x − y).

(ii) Find the value of cos (x + y).

(iii) The value of tan (x + y) =

OR

The value of cot (x - y) =

Explanation(i) sin (x − y) = sin x cosy − cos x sin y

$= \frac{3}{5} \times \left(-\frac{12}{13}\right) - \left(-\frac{4}{5}\right) \times \frac{5}{13} = -\frac{16}{65}$

Explanation:(ii) cos (x + y) = cos x cosy – sin x sin y

$= \left(-\frac{4}{5}\right) \times \left(-\frac{12}{13}\right) - \frac{3}{5} \times \frac{5}{13} = \frac{33}{65}$

Explanation: (iii) tan x = -3/4 , tan y = - 5/12

$\therefore \tan(x + y) = \frac{tanx+tany}{1-tanx\ tany} = \frac{-\frac{3}{4}-\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{-56}{33}$

OR

Solution: cot x = $-\frac{4}{3}$ and cot y = $-\frac{12}{5}$

$\therefore \cot(x - y) = \frac{\cot x \cot y + 1}{coty - cotx} = \frac{\frac{4}{3}\times\frac{12}{5}+1}{-\frac{12}{5}+\frac{4}{3}} = \frac{63}{-16}$

3) **Trigonometry is the combination of 2 words – 'Trigon' means triangle and 'metron' means measure. It is a branch of geometry that studies relationship between lengths and angles of a triangle. Degree and radian units of measurement of angles are used, also called Indian system of measurement of triangles. In this system π *radian* = 180°; 1°=60 minute and 1minute=60 seconds; 1 right angle =90°.The length of arc l is given by $l = r\,\theta$.**

On the basis of above information answer the following questions:

(i) convert $\frac{11}{36}$ radians into degree minutes and seconds.

(ii) convert $\frac{7\pi}{18}$ into degree measure

(iii) Find the length of arc made by minute's hand of a clock in 5 minutes having radius 7 cm.

OR

If the arcs of the same length in two circles subtend angles 65° and 80° at the centre, then the ratio of their radii.

Solution:(i) $\frac{11}{36} \times \frac{180}{\pi}$ degree $= \frac{11\times5\times7}{22}$ degree $= \frac{35}{2}$ degree $= 17°30'$

Solution: (ii) $\frac{7\pi}{18} \times \frac{180}{\pi}$ degree $= 7 \times 10$ degree $= 70°$

Solution: (iii)Angle turns by minute hand in 1 minute $= \frac{360°}{60} = 6°$

∴ Angle turns by minute hand in 5 minutes $= 5 \times 6° = 30° = \frac{\pi}{6}$ radian

∴ length of an arc made by minute hand in 5 minutes $l = r\theta = 7 \times \frac{\pi}{6} = \frac{7\pi}{6}$

OR

Solution: we have , $\theta_1 = 65° = 65 \times \frac{\pi}{180} = \frac{13\pi}{36}$ & $\theta_2 = 80 \times \frac{\pi}{180} = \frac{4\pi}{9}$

∴ ATQ , $r_1\theta_1 = r_2\theta_2 \Rightarrow \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{4\pi}{9} \times \frac{36}{13\pi} \Rightarrow \frac{r_1}{r_2} = \frac{16}{13}$

CHAPTER 4
COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Definitions and Formulae:

- A number of the form (a + ib) where a , b ∈ R, the set of real numbers, and i =$\sqrt{-1}$(iota) is called a complex number.
- It is denoted by z, z = a + ib. "a" is called the real part of complex number z and "b" is called the imaginary part of complex number z.
- Two complex numbers are said to be equal i.e. $z_1 = z_2$.

  (a + ib) = (c + id)⇒ a = c and b = d
- A complex number z is said to be purely real if Im(z) = 0 and is said to be purely imaginary if Re(z) = 0.
- Algebra of Complex Numbers:
- Addition of two complex numbers:

  Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the sum $z_1 + z_2$ is defined as follows: $z_1 + z_2 = (a + c) + i (b + d)$, which is again a complex number.
- Difference of two complex numbers:

  Given any two complex numbers z_1 and z_2, the difference $z_1 - z_2$ is defined as follows: $z_1 - z_2 = z_1 + (-z_2)$.
- Multiplication of two complex numbers: Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the product $z_1 z_2$ is defined as follows: $z_1 z_2 = (ac - bd) + i(ad + bc)$
- The multiplication of complex numbers possesses the following properties:

  i) The existence of multiplicative inverse:

  For every non-zero complex number z = a + ib or a + bi (a ≠ 0, b ≠ 0), we have the complex number $\frac{1}{z} = z^{-1} = \frac{a-ib}{a^2+b^2}$, called the multiplicative inverse of z, such that $z.\frac{1}{z} = 1$

  ii) The distributive law for any three complex numbers z_1, z_2, z_3,

  (a) $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$ (b) $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$
- Division of two complex numbers : Given any two complex numbers z_1 and z_2, where $z_2 \neq 0$, the $\frac{z_1}{z_2} = z_1 \frac{1}{z_2}$.
- Modulus a Complex Number : Let z = a + ib be a complex number. Then, the modulus of z, is denoted by | z |, is defined to be the non-negative real number $\sqrt{a^2 + b^2}$,
- A complex number with modulus 1 is called uni-modular complex number
- Properties of Modulus :

  If z, z_1, z_2 are three complex numbers then

  i) |z| = 0 => z = 0 i.e., real part and imaginary part are zeroes.

  ii) |z| = | z | = |– z |

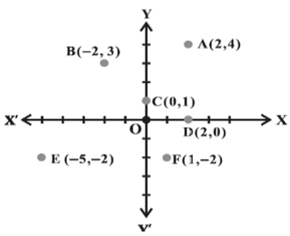
  iii) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1.z_2)$

  iv) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2Re(z_1.z_2)$

  v) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
- Conjugate of a Complex Number :

  Let z = a + ib then its conjugate is denoted by $\bar{z} = a - ib$
- Modulus using conjugate:$|z|^2 = z \cdot \bar{z}$
- Argand Plane :

  Some complex numbers such as 2 + 4i, – 2 + 3i, 0 + 1i, 2 + 0i, – 5 –2i and 1 – 2i which correspond to the ordered pairs (2, 4), (– 2, 3), (0, 1), (2, 0), (–5, –2), and (1, – 2),



respectively, have been represented geometrically by the points A, B, C, D, E, and F, respectively.

The plane having a complex number assigned to each of its point is called the complex plane or the Argand plane.

- The square roots of a negative real number:
  - $\sqrt{a}\times\sqrt{b} = \sqrt{ab}$ *if both a and b are non negitive*
  - $\sqrt{a}\times\sqrt{b} = -\sqrt{ab}$ if both are negative
  - $\sqrt{a}\times\sqrt{b} = \sqrt{ab}\, i$ *if one of them is negitive*

MULTIPLE CHOICE QUESTIONS

1) Multiplicative inverse of complex number (1+i) is …….

A) $\frac{1}{2}(1-i)$ B) $\frac{-1}{2}(1-i)$ C) $\frac{1}{2}(1+i)$ D) $\frac{1}{2}(-1-i)$

Solution: Let $z = 1 + i$. Then the inverse is: $\frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{2} = \frac{1}{2}(1-i)$

Answer: A

2) If $\left[\frac{(1-i)}{(1+i)}\right]^{100} = a + ib$,then (a, b) lie in

A) 1st quadrant B) real axis C) imaginary axis D) origin

Solution: $\frac{1-i}{1+i} = \frac{(1-i)^2}{(1+i)(1-i)} = \frac{1-2i+i^2}{1-i^2} = \frac{1-2i-1}{1+1} = \frac{-2i}{2} = -i$

Then $(-i)^{100} = [(-i)^4]^{25} = 1^{25} = 1$, which lies on the real axis.

Answer: B

3) What is the conjugate of the complex number z = 3 + 4i

A) -3 + 4i B)3 - 4i C) -3 - 4i D) 4+3i

Solution: the conjugate of the complex number a + bi is a - bi

Answer: B

4) The value of $i^{29} + \frac{1}{i^{29}}$

A) 1 B) 0 C)-1 D) I

Solution: $i^{29} + \frac{1}{i^{29}}$ Since $i^4 = 1$, $i^{29} = i^{(4\cdot7+1)} = i^1 = i, \frac{1}{i} = -i$

$i + (-i) = 0$

Answer: B

5) $z_1 = 1 + 2i$ and $z_2 = 2 + 3i$. Then z_1z_2=

A) 2+6i B)3+2i C) -4+7i D)4-7i

Solution: $(1 + 2i)(2 + 3i) = 1\cdot2 + 1\cdot3i + 2i\cdot2 + 2i\cdot3i$

$= 2 + 3i + 4i + 6i^2 = 2 + 7i - 6 = -4 + 7i$

Answer: C

6) The value of $i^{19} + i^9$ is

A) -1 B) 1 C) 0 D) 2

Solution: $i^{19} + i^9 = i^3 + i^1 = -i + i = 0$

Answer: C

7) The conjugate of $\frac{2-i}{(1-2i)^2}$ is

A) $\frac{2+11i}{25}$ B) $\frac{2-11i}{25}$ C) $\frac{-2-11i}{25}$ D) $\frac{-2+11i}{25}$

Solution: Conjugate of $\frac{2-i}{(1-2i)^2}$

First compute conjugate of numerator and denominator separately:
Conjugate of numerator: $2+i$
Conjugate of denominator: $(1+2i)^2 = (1+2i)^2$

$$= 1 + 4i + 4i^2 = 1 + 4i - 4 = -3 + 4i$$

$$\frac{2+i}{(-3+4i)} \Rightarrow \text{Among options, that corresponds to} - \frac{2+11i}{25}$$

Answer: C

8) If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

A) 1 B) 2 C) 3 D) 4

Solution: If $\left(\frac{1+i}{1-i}\right)^m = 1$, find least positive integer m

$\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+2i+i^2}{1+1} = \frac{1+2i-1}{2} = \frac{2i}{2} = i \Rightarrow i^m = 1$

$\Rightarrow m = 4$

Answer: D

9) The modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ is

A) 3 B) 2 C) 5 D) 1

Solution: First simplify both terms:

$\frac{1+i}{1-i} = i, \frac{1-i}{1+i} = -i \quad \Rightarrow i - (-i) = 2i \Rightarrow |2i| = 2$

Answer: B

10) Let $z_1 = 2 - i$, $z_2 = -2+i$ Find $\text{Re}\left(\frac{z_1 z_2}{\bar{z_1}}\right)$

A) $\frac{2}{5}$ B) $\frac{-2}{5}$ C) $\frac{2}{3}$ D) $\frac{-2}{3}$

Solution: First compute $z_1 z_2 = -(-2+i)^2 = -4 + 4i - i^2 = -3 + 4i$

$$\left(\frac{z_1 z_2}{\bar{z_1}}\right) = \frac{-3+4i}{2+i} \cdot \frac{2-i}{2-i}$$

$$= \frac{-2+11i}{5}$$

$$\Rightarrow \text{Re} = \frac{-2}{5}$$

Answer: B

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
C) Assertion (A) is true but Reason(R) is false.
D)Assertion (A) is false but Reason(R) is true

1) Assertion (A) : The real value of a for which $3i^3 - 2ai^2 + (1-a)i + 5$ is real is -2.
Reason (R) : A complex number $z = x + iy$ is purely imaginary if $x \neq 0$ and $y = 0$.
Answer: C
Solution: $3i^3 - 2ai^2 + (1-a)i + 5$= -3i =$-3i + 2a + (1-a)i + 5$ is real ∴ coefficient of i = 0∴ $-3 + 1 - a = 0 \;\therefore a = \; -2$ which is true. Reason is false

2) Assertion (A) : Conjugate of $2 + 3i$ lies in the fourth quadrant in complex plane.
Reason (R) : $x + iy$ lies in the fourth quadrant in complex plane if $x < 0$ and $y > 0$.
Answer: C
Solution: Conjugate of $2 + 3i$ *is* $2 - 3i$ the corresponding point is (2, -3) which is in fourth quadrant. A is true, clearly R is false

3) Assertion (A) : The complex number $\frac{(1-i)^3}{1-i^3}$ is equal to - 2 .

Reason (R) : $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0 \forall n \in \mathbb{N}$.

Answer: C

Solution: $\frac{(1-i)^3}{1-i^3} = \frac{(1-i)^3}{1+i} \times \frac{1-i}{1-i} = \frac{(1-i)^4}{2}$

$= \frac{(1+i^2-2i)^2}{2} = \frac{(-2i)^2}{2} = \frac{-4}{2} = -2$ A is true

R is also true but it is the not correct explanation of A.

4) Assertion (A) : If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then $\frac{z_1}{z_2}$ is pure imaginary

Reason (R):if z is pure imaginary tha z + $\bar{z}$ = 0

Answer: A

Solution$|z_1 + z_2|^2 = (z_1 + z_2)(\bar{z_1} + \bar{z_2}) = z_1\bar{z_1} + z_1\bar{z_2} + z_2\bar{z_1} + z_2\bar{z_2}$

$|z_1|^2 + |z_2|^2 + z_1\bar{z_2} + z_2\bar{z_1}$ but this = $|z_1|^2 + |z_2|^2 \therefore z_1\bar{z_2} + z_2\bar{z_1} = 0$

divide by $z_2\bar{z_2}$ then $\frac{z_1}{z_2} + \frac{\bar{z_1}}{\bar{z_2}} = 0 \Rightarrow \frac{z_1}{z_2}$ is pure imaginary

if z is pure imaginary then $\bar{z}$ =-z then z+$\bar{z}$ =0 , R is true and it is the correct explanation of A

5) Assertion (A) : if z_1, z_2 *are two unimodular* $(|z_1| = |z_2| = 1)$ *complex numbers then* $|z_1 + z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right|$

Reason (R):if z is unimodular complex number then $\bar{z} = \frac{1}{z}$

Answer: A

Solution: z is unimodular the $|z| = 1 \Rightarrow z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$ R is True

Since z_1, z_2 *are two unimodular* , $\bar{z_1} = \frac{1}{z_1}$ and $\bar{z_2} = \frac{1}{z_2}$ ∴ A is also True and R is the correct explanation of A.

VERY SHORT ANSWER TYPE QUESTIONS

1) Find the reciprocal of 3- 4i in a+ib form.

Solution: reciprocal of x+iy is $\frac{x-iy}{x^2+y^2}$

reciprocal of 3- 4i is $\frac{3+4i}{25}$ i.e/ $\left(\frac{3}{25} + \frac{4}{25}i\right)$

2) Show that the value of $1 + i^{10} + i^{20} + i^{30}$ is a real number

Solution: $1 + i^{10} + i^{20} + i^{30} = 1 + i^{20} + i^{18}.i^2 + i^{28}.i^2$ but $i^{4\,multipe}$=1 andi^2= -1

=1+1+1(-1)+1$(-1) = 0$ which is a real number

3) For Complex numbers $z_1 = -1 + i$, $z_2 = 3 - 2i$ show that,

Im (z_1z_2) = Re (z_1) Im(z_2) + Im (z_1) Re (z_2)

Solution: Step 1: Compute z_1z_2 :

$(-1 + i)(3 - 2i) = -3 + 2i + 3i - 2i^2 = -3 + 5i + 2 = -1 + 5i \Rightarrow \text{Im}(z_1z_2) = 5$

Step 2: RHS:

$\text{Re}(z_1) = -1, \text{Im}(z_1) = 1, \text{Re}(z_2) = 3, \text{Im}(z_2) = -2$

$\Rightarrow -1 \cdot (-2) + 1 \cdot 3 = 2 + 3 = 5$

4) Find the conjugate of $(5+\sqrt{2}i)^3$ in a + ib form

Solution: (i) $(5 + \sqrt{2}i)^3$

Let $z = 5 + \sqrt{2}i$, then $\overline{z^3} = (\bar{z})^3 = (5 - \sqrt{2}i)^3$ = 125 -75$\sqrt{2}$ i -30 + 2$\sqrt{2}$ i

$= (125 - 30)$ - 73$\sqrt{2}$ I = (95) - 73$\sqrt{2}$ i

5) Find ihe value $\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right)$ in a + ib form

Solution: = $\left(\frac{1}{3} + 4 + \frac{4}{3}\right) + \left(\frac{7}{3} + \frac{1}{3} - 1\right)i = \frac{17}{3} + \frac{5}{3}i$

6) Evaluate the following

i) $\sqrt{36} \times \sqrt{-225}$ ii) $\sqrt{-36} \times \sqrt{-225}$

Solution: i) $\sqrt{36} \times \sqrt{-225} = \sqrt{36i^2} \times \sqrt{225} = 6i \times 15 = -90i$

$[\sqrt{a}\times\sqrt{b} = \sqrt{ab}\, i \text{ if one of them is negitive}]$

ii) $\sqrt{-36} \times \sqrt{-225} = -\sqrt{36} \times \sqrt{225} = -90$

$[\sqrt{a}\times\sqrt{b} = -\sqrt{ab}$ if botha and b are negativ]

7) If $(x - iy)(3 + 5i)$= -6 +24 i then find the values of x and y

Solution: $(x - iy)(3 + 5i) = 3x + 5ix - 3iy - 5yi^2 = (3x + 5y) + i(5x - 3y)$

Equating:

- Real: $3x + 5y = -6$
- Imag: $5x - 3y = 24$

Solve:

Multiply 1st by 3: $9x + 15y = -18$

Multiply 2nd by 5: $25x - 15y = 120$

Add: $34x = 102 \Rightarrow x = 3$

Then $3x + 5y = -6 \Rightarrow 9 + 5y = -6 \Rightarrow y = -3$

Answer: $x = 3, y = -3$

8) if n is any integer , then the value of $(1 - i)^n \left(1 - \frac{1}{i}\right)^n$

Solution: We know that $\frac{1}{i}$ = -i

Given expression becomes $(1 - i)^n(1 + i)^n = (1 - i^2)^n = 2^n$

9) Find the non-zero integral solutions of the equaton $|1 - i|^x = 2^x$

Solution: $|1 - i|^x = 2^x \Rightarrow \left(\sqrt{2}\right)^{x..} = 2^x \Rightarrow 2^{\frac{x}{2}} = 2^x \Rightarrow \frac{x}{2} = x \Rightarrow x = 0$

∴ it has no non-zero solutions i.e. there are zero non-zero solutions are there.

10) Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

Solution: $\frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2-(1-i)^2}{2} = \frac{4i}{2} = 2i$

magnitude of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ $=|2i| = 2$

SHORT ANSWER TYPE QUESTIONS

1) Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form

Solution: Reduce:$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right) \cdot \left(\frac{3-4i}{5+i}\right)$= :$\left(\frac{1}{1-4i}\frac{1+4i}{1+4i} - \frac{2}{1+i}\frac{1-i}{1-i}\right) \cdot \left(\frac{3-4i}{5+i}\right)\frac{5-i}{5-i}$

:$\left(\frac{1+4i}{17} - \frac{1-i}{1}\right) \cdot \left(\frac{11-23i}{26}\right) = \left(\frac{-16+21i}{17}\right) \cdot \left(\frac{11-23i}{26}\right) = \frac{307+599i}{442}$

2) If z_1= 2 – i , z_2= 1 + i, find $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$

Solution: Numerator: $z_1 + z_2 + 1 = (2 - i) + (1 + i) + 1 = 4$

Denominator: $z_1 - z_2 + 1 = (2 - i) - (1 + i) + 1 = (1 - 2i + 1) = 2 - 2i$

$$\left|\frac{4}{2 - 2i}\right| = \left|\frac{4(2 + 2i)}{(2 - 2i)(2 + 2i)}\right| = \left|\frac{8 + 8i}{8}\right| = |1 + i| = \sqrt{2}$$

3) Evalute $\left(i^{18} + \frac{1}{i^{25}}\right)^3$

$i^{18} = i^2 = -1,\qquad \frac{1}{i^{25}} = \frac{1}{i} = -i$

solution: $\therefore (-1 - i)^3 = (a + bi)^3 = a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3$

$a = -1, b = -1 \Rightarrow (-1)^3 + 3(-1)^2(-1)i + 3(-1)(1)(-1) + (-1)^3(-i)$

$=-1 - 3i + 3 + i = 2 - 2i$

4) Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

solution: Let's compute numerator and denominator:

Numerator: $(3-2i)(2+3i) = 6+9i-4i-6i^2 = 6+5i+6 = 12+5i$

Denominator: $(1+2i)(2-i) = 2-i+4i-2i^2 = 2+3i+2 = 4+3i$

Now the expression is $\frac{12+5i}{4+3i} = \frac{12+5i}{4+3i} \cdot \frac{4-3i}{4-3i} = \frac{(12+5i)(4-3i)}{25}$

$$= \frac{63-16i}{25}$$

5) If (a + ib)(c + id)(e + if)(g + ih) = A + iB then prove that $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Solution: $(a+ib)(c+id)(e+if)(g+ih) = A+iB$

$\Rightarrow |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$\Rightarrow |(a+ib)||(c+id)||(e+if)||(g+ih)| = |A+iB|$

$\Rightarrow \sqrt{(a^2+b^2)}\sqrt{(c^2+d^2)}\sqrt{(e^2+f^2)}\sqrt{(g^2+h^2)} = \sqrt{(A^2+B^2)}$

squaring on both sides $\Rightarrow (a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

6) If $(x+iy)^3$ = u+ iv then show that $\frac{u}{x}+\frac{v}{y} = 4(x^2 - y^2)$.

Solution

$(x+iy)^3 = x^3 + 3x^2iy + 3x(iy)^2 + (iy)^3$

$= x^3 + 3x^2iy - 3xy^2 - iy^3$

$= (x^3 - 3xy^2) + i(3x^2y - y^3)$

$So, u = x^3 - 3xy^2 \text{ and } v = 3x^2y - y^3$

$Now\ compute\ \frac{u}{x} + \frac{v}{y} = x^2 - 3y^2 + 3x^2 - y^2 = 4(x^2 - y^2)$

7) Find the real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real

Solution:

Let $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

For z to be purely real, its imaginary part must be 0.

Multiply numerator and denominator by the conjugate of the denominator:

$z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{1+2i\sin\theta}{1+2i\sin\theta} = \frac{3(1+2i\sin\theta)+2i\sin\theta(1+2i\sin\theta)}{1+4\sin^2\theta} = \frac{3+6i\sin\theta+2i\sin\theta-4\sin^2\theta}{1+4\sin^2\theta}$

$\frac{(3-4\sin^2\theta)+i(8\sin\theta)}{1+4\sin^2\theta}$ is pure real so Imaginary part = 0

So, imaginary part of z =0 $\Rightarrow$ 8 $\sin\theta$ =0$\Rightarrow$

Therefore, $\theta = n\pi$ where $n \epsilon Z$

8) Evaluate $\sum_{n=1}^{13}(i^n + i^{n+1}),\ where\ n\varepsilon N$

Solution:

$\sum_{n=1}^{13}(i^n + i^{n+1}) = \sum_{n=1}^{13} i^n(1+i) = (1+i)\sum_{n=1}^{13} i^n = (1+i)(i+i^2+i^3+i^4+\cdots..+i^{13})$

$= (1+i)([i+i^2+i^3+i^4]+[i^5+\cdots+i^8]\ldots.+[i^{10}+\cdots.+i^{12}+i^{13}])$

$= (1+i)(0+0+\cdots..+0+i) = (1+i)i = i+i^2 = i-1 = -1+i$

LONGANSWER TYPE QUESTIONS

1) If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$

Solution: $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = \left|\frac{\beta-\alpha}{\beta\bar{\beta}-\bar{\alpha}\beta}\right| = \left|\frac{\beta-\alpha}{\beta(\bar{\beta}-\bar{\alpha})}\right|$ $\qquad \beta\bar{\beta} = 1$

$\left|\frac{\beta-\alpha}{\beta(\overline{\beta-\alpha})}\right| = \frac{|\beta-\alpha|}{|\beta||(\overline{\beta-\alpha})|}$ $\qquad |z_1z_2| = |z_1||z_2|$ and $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$\frac{|\beta-\alpha|}{|\beta||(\beta-\alpha)|}$ $\qquad |\bar{z}| = |z| \qquad \frac{1}{|\beta|} = 1$

2) if |a + ib| = 1 then prove that $\frac{(1+b+ia)}{(1+b-ia}$ = b + ia

Solution: |a + ib| = 1 ⇨ $a^2+b^2=1$

LHS = $\frac{(1+b+ia)}{(1+b-ia}=\frac{(1+b+ia)}{(1+b-ia}\times\frac{(1+b+ia)}{(1+b+ia}=\frac{(1+b+ia)^2}{(1+b)^2+a^2}=\frac{(1+b)^2+2(1+b)ai-a^2}{1+b^2+2b+a^2}$ but $a^2+b^2=1$

$\frac{1+b^2+2b+2ai+2bai-a^2}{1+b^2+2b+a^2}=\frac{2b^2+2b+(1+b)2ai}{2(1+b)}$ = b + ia

3) If z = x+iy and the real part of $\frac{\bar{z}+2}{\bar{z}-1}$ is4, show that $x^2+y^2-3x+2=0$

Solution: $\frac{\bar{z}+2}{\bar{z}-1}=\frac{x+2-iy}{x-1-iy}=\frac{x+2-iy}{x-1-iy}\times\frac{x-1+iy}{x-1+iy}=\frac{(x+2)(x-1)+y^2+(x+2)-(x-1)iy}{(x-1)^2+y^2}$

But real part is 4 ⇨ $\frac{(x+2)(x-1)+y^2}{(x-1)^2+y^2}=4$ ⇨ $x^2+x-2+y^2=4x^2+4-8x+4y^2$

⇨$3x^2+3y^2-9x+6=0$ ⇨ $x^2+y^2-3x+2=0$

4) show that $\left|\frac{z-2}{z-3}\right|=2$ represents a circle. Find its centre and radius'

Solution: put z= x+iy where x and y ϵR

$$\left|\frac{x+iy-2}{x+iy-3}\right|=2 ⇨ \sqrt{\frac{(x-2)^2+y^2}{(x-3)^2+y^2}}=2 ⇨ (x-2)^2+y^2=4[(x-3)^2+y^2]$$

⇨$x^2+y^2-\frac{20}{3}x+\frac{32}{3}=0$ *which represent a circle whose centre is* $(-g,-f)=$ $\left(\frac{10}{3},0\right)$ *and radius* $=\sqrt{g^2+f^2-c}=\frac{2}{3}$*units*

5) If $|z^2-1|=|z|^2+1$ the show that z lies on Y-axis

Solution: we have to show that z lies on Y axis i.e. z is pure imaginary

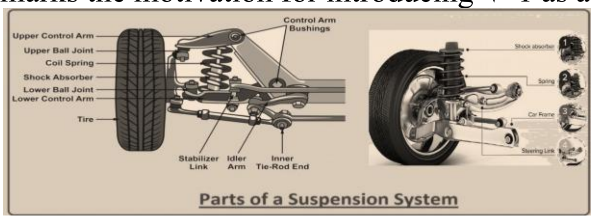
put z = x+iy then$|(x+iy)^2-1|$=$|(x+iy)|^2+1$

$\sqrt{(x^2-y^2-1)^2+(2xy)^2}$=$x^2+y^2+1$

⇨-$4x^2=0$⇨x=0 and z= iy pure imaginary∴ z lies on y axis.

CASE STUDY BASED QUESTIONS

1) During the study of oscillations in a new car model, an equation like $x^2+4=0$ arises. This equation has no real solution, making it impossible to proceed with real numbers alone. To handle such situations, engineers introduce complex numbers to find meaningful solutions and analyze system behavior. This need marks the motivation for introducing √−1 as a new number.



Basing on the data answer the following questions.

(1) Find the roots of the quadratic equation $x^2+4=0$ and write the roots in the form a + bi.

(2) If the two solutions are denoted by z_1 and z_2, verify $z_1 \cdot z_2$and z_1+z_2

(3) Find the conjugate and modulus of the roots.

OR

write the position of z_1 and z_2 in Argan plane.

Solution :

(1) The equation $x^2+4=0$ gives $x^2=-4$, so x = ±√(-4) = ±2i.

(2) Let z_1 = 2i and z_2 = -2i.

Then $z_1\cdot z_2$ = (2i)(-2i) = $-4i^2$ = -4(-1) = 4

And z_1+z_2 = 2i - 2i = 0

3) Conjugate of 2i is -2i, and modulus of 2i is

|2i| = $\sqrt{(0^2+2^2)}$ = 2

Conjugate of -2i is 2i, and modulus of- 2i is

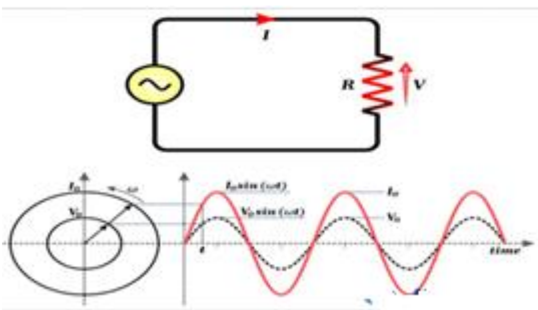
|-2i| = $\sqrt{(0^2+2^2)}$ = 2

OR

$z_1=(0,2)$ lies on positive imaginary axis

$z_2 = (0, -2)$ lies on negative imaginary axis

2) Electrical Engineering and Circuit Analysis



In AC (alternating current) electrical circuits, voltage and current do not always vary in sync. To represent both magnitude and phase, electrical engineers use complex numbers. Suppose an engineer records the voltage across a circuit as V = 3 + 4i volts and the current flowing as I = 1 − 2i amperes. These complex quantities allow calculations involving impedance, power, and energy distribution. Complex numbers help visualize and compute phase differences effectively, which is not possible with real numbers alone.

(1) Calculate the complex power S = V · I.
(2) Find the modulus of voltage and current. What do they physically represent?
(3) Find the conjugate of the current I.

OR

write the quadrants of the Argand plane in which by z_1 and z_2 lies

Solutions: (1) V = 3 + 4i, I = 1 - 2i

S = V · I = (3 + 4i)(1 - 2i) = 3 - 6i + 4i - 8i² = 3 - 2i + 8 = 11 - 2i

(2)$|V| = \sqrt{3^2 + 4^2} = 5\ volts$

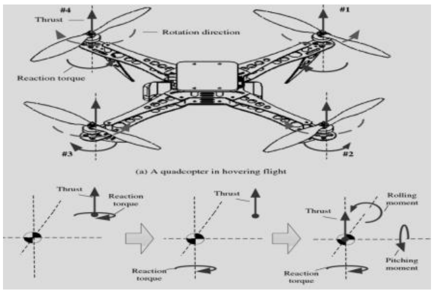
)$|I| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ amperes|

These represent the magnitudes of voltage and current respectively.

(3) Conjugate of I = 1 - 2i is 1 + 2i

(4) V (3, 4) lies in the first quadrant and I (1, -2) lies
In the fourth quadrant of the Argand plane.

3) Drone Stabilization System



In advanced drone technology, computer algorithms simulate and control movement in a 2D space. Complex numbers are used to represent positions and rotations efficiently. Suppose at one instant, the drone is at position z_1 = 2 + 2i, and after a simulated rotation and translation, its new position becomes z_2 = -1 + 3i. Using complex addition and multiplication, programmers can determine the resultant path, orientation, and adjust controls accordingly. Plotting these points on an Argand plane helps in visualizing movements as geometric transformations.

based on the information answer the following questions
(1) Compute the resultant position z = $z_1 + z_2$.
(2) Find $z_1 \cdot z_2$ and interpret whether the output is real or complex.
(3) Find the modulus and conjugate of z
(4) Plot z_1, z_2, and z on the Argand plane.

Solutions : (1) z = $z_1 + z_2$ = (2 + 2i) + (-1 + 3i) = 1 + 5i

(2) $z_1 \cdot z_2$ = (2 + 2i)(-1 + 3i) = -2 + 6i -2i + 6i² = -2 + 4i -6 = -8 + 4i

→ It is a complex number

(3) Conjugate of z = 1 + 5i is 1 - 5i

Modulus of z = √(1² + 5²) = √26

(4) z_1 = (2, 2), z_2 = (-1, 3), and z = (1, 5) can be plotted as points in the Argand plane representing initial, intermediate, and final positions respectively.

CHAPTER - 5: LINEAR INEQUALITIES

Definitions and Formulae:

- Definition 1: Two real numbers or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an inequality.
- Definition 2 : the inequality not containing any variable is numerical inequality.
- Definition 3: the inequality containing variables is literal inequalities
- Definition 4: The inequality contains the symbol '<' or '>' is called strict inequality.
- Definition 5: The inequality contains the symbol '≤' or '≥' is called slack inequality.
- Definition 6: solution of an inequality in one variable is a value of the variable which makes it a true statement.
- Definition 7: If the degree of an inequality is one then it is called Linear inequality
- Definition 8: If the degree of an inequality is two then it is called quadratic inequality

MULTIPLE CHOICE QUESTIONS SOLVED

1. Which of the following is a solution of the inequality: $2x + 3 < 7$?

A) $x = 3$ B) $x = 2$ C) $x = 1$ D) $x = 4$

Answer : C

Solution: $2x + 3 < 7 \Rightarrow 2x < 7-3 \Rightarrow 2x < 4 \Rightarrow x<2 \Rightarrow x = 1< 2$

2. The solution set of the inequality $3x - 5 \geq 1$ is:

A) $x \geq 2$ B) $x \leq 2$ C) $x \geq 1$ D) $x \leq 1$

Answer : A

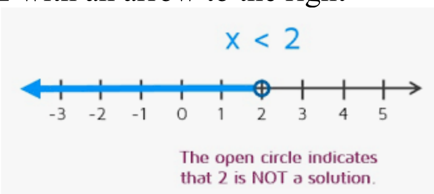
Solution: $3x - 5 \geq 1 \Rightarrow 3x \geq 6 \Rightarrow x \geq 2$

3. Which of the following is the correct representation of the solution of: $x < -2$ on the number line?

A) A filled circle at -2 with an arrow to the right
B) An open circle at -2 with an arrow to the left
C) A filled circle at -2 with an arrow to the left
D) An open circle at -2 with an arrow to the right

Answer : B

Solution:



4. Solve the inequality: $-4x > 8$.

A) $x < -2$ B) $x > -2$ C) $x > 2$ D) $x < 2$

Answer : A

Solution: $-4x > 8$ divide by -4 both sides $\Rightarrow x<-2$ [when both sides are multiplied (or divided) by a negative number, then the inequality is reversed.]

5. What is the solution of the inequality: $5 - 2x \leq 9$?

A) $x \geq -2$ B) $x \leq -2$ C) $x \geq 2$ D) $x \leq 2$

Answer : B

Solution: $5 - 2x \leq 9 \Rightarrow - 2x \leq 4$divide by -2 both sides $\Rightarrow x\geq -2$ [when both sides are multiplied (or divided) by a negative number, then the inequality is reversed.]

6. Which one of the following represents a linear inequality in one variable?

A) $x^2 + 1 < 3$ B) $2x + 3 < 7$ C) $x + y > 2$ D) $x^3 < 5$

Answer : B

7. If the inequality $(2x - 5)/(x + 1) \leq 0$ holds, then the solution set is:

A) $x \leq -1$ or $x \geq$ B) $-1 < x \leq \frac{5}{2}$

C) $x \geq -1$ and $x \leq 3$ D) $x < -1$ or $x \geq 3$

Answer : B

Solution: if $\frac{p}{q} \leq 0$, then case I) $p \leq 0$ and $q > 0 \Rightarrow (2x - 5) \leq 0$ and $(x + 1) > 0 \Rightarrow x \leq \frac{5}{2}$ and $x > -1$

case II) $P \geq 0$ and $q < 0 \Rightarrow (2x - 5) \geq 0$ and $(x + 1) < 0 \Rightarrow x \geq \frac{5}{2}$ and $x < -1$ which is not possible

8. If the inequality $(2x - 5)/(x + 1) \leq 1$ holds, then the solution set is:

A) $x \in (-1, 6]$ B) $x \in (-1, 6)$ C) $x \in [-1, 6]$ D) $x \in [-1, 6)$

Answer : A

Solution: $(2x - 5)/(x + 1) \leq 1 \Rightarrow$: $\frac{(2x-5)}{(x+1)} - 1 \leq 0 \Rightarrow$ on simplification $\frac{x-6}{x+1} \leq 0$

if $\frac{p}{q} \leq 0$, then case I) $p \leq 0$ and $q > 0 \Rightarrow x - 6 \leq 0$ and $(x + 1) > 0 \Rightarrow x \leq 6$ and $x > -1 \Rightarrow x \in (-1, 6]$

case II) $P \geq 0$ and $q < 0 \Rightarrow x - 6 \geq 0$ and $(x + 1) < 0 \Rightarrow x \geq 6$ and $x < -1$ which is not possible

9. If $2x < 10$ and $x > 1$, what is the range of x?

A) $1 < x < 5$ B) $x < 1$ or $x > 5$ C) $x > 10$ D) $x < 5$

Answer : A

10 Let $x \in R$ be such that $3 < 2x + 1 < 7$. Then the solution set for x is:

A) $x \in (1, 3)$ B) $x \in (2, 4)$ C) $x \in (1, 2)$ D) $x \in (1, 4)$

Answer : A

Solution : $3 < 2x + 1 \Rightarrow 1 < x$ and $2x + 1 < 7 \Rightarrow x < 3 \Rightarrow 1 < x < 3$

ASSERTION - REASON BASED QUESTIONS SOLVED

1. Assertion (A) : If $a < b, c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

   Reason (R) : If both sides of the inequality are divided by the same negative number, then the inequality is reversed.

   Answer: D

   Solution: here R is True and A is not true If $a < b, c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

2. Assertion (A) : $|3x - 5| > 9 \Rightarrow x \in \left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$

   Reason (R) : The region containing all the solutions of an inequality is called the solution region.

   Answer : A

3. Assertion (A) : $|3x - 5| > 9 \Rightarrow x \in \left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{14}{3}, \infty\right)$

   Reason (R) : If $|x| > K$ the $x < -k$ and $x > K$

   Answer: A

   Solution: $|3x - 5| > 9 \Rightarrow -9 > 3x - 5$ *and* $3x - 5 > 9 \Rightarrow x < -\frac{4}{3}$ and $x < \frac{14}{3}$ A is true R is true and it is the correct explanation of A

4. Assertion (A) : If $|x - 2| < 5$, then $x \in (-3,7)$.

   Reason (R) : $|x - a| < r \Rightarrow a - r < x < a + r$

   Answer: A

   Solution: $|x - 2| < 5 \Rightarrow -5 < x - 2 < 5 \Rightarrow -3 < x < 7$ A is true

R is also true and correct explanation of A

5. Assertion (A) : If $\frac{x+3}{|x+3|} \geq 0$, then $x \in (-3, \infty)$.

Reason (R) : If $\frac{p}{q} \geq 0 \Rightarrow both\ p\ and\ q\ should\ have\ same\ sign$

Answer: A

Solution: $\frac{x+3}{|x+3|} \geq 0 \ \Rightarrow both\ x+3\ and$

$|x+3| should\ have\ same\ signbut\ \ |x+3|$is $>\ \ 0 \Rightarrow x+3\ \geq 0$

$\Rightarrow x \geq -3$ Ais true

R is true and it is the correct explanation of A

6. Assertion (A) : If $11x - 9 \leq 68$, then $x \in (-\infty, 7)$.

Reason (R) : If both the sides of an inequation are either positive of negative, then on taking their reciprocals, the order of the inequation is reversed.

Answer: D

Solution: We have. $11x - 9 \leq 68 \Rightarrow 11x \leq 77 \Rightarrow x \leq 7$

$\therefore x \in (-\infty, 7)$

So, Assettion (A) is false but Reasen (R) is true.

$\therefore$ The answer is option (d).

VERY SHORT ANSWER TYPE QUESTIONS SOLVED

1. Solve the inequality,4x – 5<x+13,when x is a natural number

Solution: 4x – 5<x+13

⇨3x<18⇨x<6⇨ x= 1, 2, 3, 4, 5

2. Solve the inequality,4x – 5<x+13,when x is a whole number

Solution: 4x – 5<x+13 ⇨3x<18⇨x<6⇨ x= 0, 1, 2, 3, 4, 5

3. Solve the inequality, 4x – 5<x+13,when x is a real number

Solution: 4x – 5<x+13⇨3x<18⇨x<6⇨ $x\varepsilon\ (6, \infty)$

4. Solve the inequality $|x+3| \leq 10$.

Solution: we know that $|x| \leq k$ ⇨ $-K \leq X \leq K$

$-10 \leq x+3 \leq 10$⇨ $-13 \leq x \leq 7$

5. Solve the inequality $|x+3| \geq 10$.

Solution: we know that $|x| \geq k$ ⇨ $x \geq k\ , x \leq -k$

$x+3 \geq 10, x+3 \leq -10$⇨$x \geq 7, x \leq -13$

6. Solve $1 \leq |x-2| \leq 3$

Solution: $1 \leq |x-2|$⇨$x-2 \geq 1, x-2 \leq -1$ ⇨ $x \leq 1\ or\ x\ \geq 3\ ----1$

$|x-2| \leq 3$ ⇨ $-3 \leq x-2 \leq 3$ ⇨ $-1 \leq x \leq 5 ----------------2$

$from\ 1\ and\ 2\ x\epsilon[-1,1] \cup [3,5]$

7. A solution is to be kept between 40^0c and 45^0c.What is the range of temperature in degree Fahrenheit, if the conversion formula is F=$\frac{9}{5}$ C+32?

Solution: let the temperature of the solution be $x^0C\ or\ y^0F$ ⇨ $\mathrm{y} = \frac{9}{5}\ \mathrm{x} + 32$

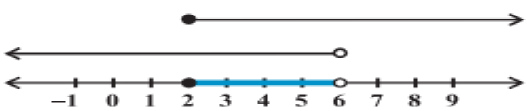
given that $40 < x < 45$ ⇨ $\frac{9}{5} \times 40 < \frac{9}{5} \times x < \frac{9}{5} \times 45 \ \ (multiplied\ by\ \frac{9}{5}\ through\ out)$

⇨$72 < \frac{9}{5} \times x < 81$ ⇨ $72 + 32 < \frac{9}{5} \times x + 32 < 81 + 32$ ($adding$ 32)

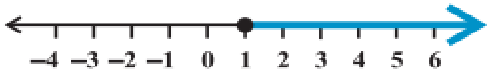
⇨104<y<114 $\therefore$ $the\ rage\ of$ temperature in degree Fahrenheit $is\ 104^0\ F\ to\ 114^0F$

8. Represent $2 \leq x \leq 6)$ on numberline.

Solution:



9. Represent x≥ 1 on numberline.
Solution:



10. Solve the inequality $\left|x-\frac{5}{2}\right| \leq \frac{31}{2}$

Solution: $\left|x-\frac{5}{2}\right| \leq \frac{31}{2}$ => $-\frac{31}{2} \leq x-\frac{5}{2} \leq \frac{31}{2}$

Add $\frac{5}{2}$ to all parts

$$-\frac{31}{2}+\frac{5}{2} \leq x \leq \frac{31}{2}+\frac{5}{2}$$
$$-\frac{26}{2} \leq x \leq \frac{36}{2}$$
$$-13 \leq x \leq 18$$

Answer: $x \in [-13{,}18]$

SHORT ANSWER TYPE QUESTIONS SOLVED

1. Ravi obtained 70 and 75 marks in first two-unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Solution: Let third test marks be x.

(70 + 75 + x)/3 ≥ 60

145 + x ≥ 180 ⇒ x ≥ 35

Answer: Minimum marks = 35

2. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.

Solution: Let fifth test marks be x.

(87 + 92 + 94 + 95 + x)/5 ≥ 90

368 + x ≥ 450 ⇒ x ≥ 82

Answer: Minimum marks = 82

3. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Solution: let the consecutive positive odd inters be n, n+2

by given condition 1 n+2 <10 i.e. n <8

by given condition 2: n+ n+2 >11 ⇨2n>9 and n>4.5 therefore 4.5<n<8 and odd

n = 5, 7,

Pairs with sum > 11: (5,7), , (7,9)

4. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23 Solution: let the consecutive positive odd inters be n, n+2

by given condition 1 n >5 by given condition 2: n+ n+2 <23 ⇨2n<21 and n<10.5 therefore 5<n<10.5 and even n = 6, 8, 10

Pairs with given conditions: (6,8), , (8,10)(10, 12)

5. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side.

Solution : Sides: x, 3x, 3x-2

$x + 3x + 3x - 2 \geq 61 \Rightarrow 7x - 2 \geq 61 \Rightarrow x \geq 9$

Answer: Minimum shortest side = 9 cm

6. A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

Solution : Sides: x, x+3, 2x

$x + x+3 + 2x \leq 91 \Rightarrow 4x + 3 \leq 91 \Rightarrow x \leq 22$

Also, $2x \geq x+3+5 \Rightarrow x \geq 8$

Answer: $8 \leq x \leq 22$

7. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm , then find the minimum length of the shortest side.

Solution : Let the length of shortest side be x cm .

According to the given information,

Longest side $= 2 \times$ shortest side $= 2x$cm

and third side $= 2 +$ shortest side $= (2 + x)$cm

perimeter of triangle $= x + 2x + (x + 2) = 4x + 2$

According to the question, perimeter > 166cm

$$\Rightarrow 4x + 2 > 166 \Rightarrow 4x > 166 - 2 \Rightarrow 4x > 164$$
$$\Rightarrow x > 164/4$$
$$= 41\text{cm}$$

8. Solve for x: $|x + 1| + |x| > 3$

Solution : LHS $= |x + 1| + |x|$

As both the terms contain modulus by equating the expression within modulus to zero, We get $x = -1,0$ as critical points.

These critical points divide the line in three parts as $(-\infty, -1), [-1,0), [0, \infty)$.

Case I: when $-\infty < x < -1$

$$|x + 1| + |x| > 3$$
$$\Rightarrow -x - 1 - x > 3 \Rightarrow -2x > 4 \Rightarrow x < -2$$

Case II: when $-1 \leq x < 0$

$$|x + 1| + |x| > 3$$

$\Rightarrow x + 1 - x > 3 \Rightarrow 1 > 3$ (not possible)

Case III: when $0 \leq x < \infty$

$$|x + 1| + |x| > 3$$
$$\Rightarrow x + 1 + x > 3 \Rightarrow 2x > 2 \Rightarrow x > 1$$

Combining the results of cases, we get $x \in (-\infty, -2) \cup (1, \infty)$

9.Solve the system of inequalities:

$$5(2x - 7) - 3(2x + 3) \leq 0, 2x + 19 \leq 6x + 47$$

Solution:

We have $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 47$

$\Rightarrow 10x - 35 - 6x - 9 \leq 0$ and $-4x \leq 28$

$\Rightarrow 4x - 44 \leq 0$ and $x \geq -7$

$\Rightarrow 4x \leq 44$ and $x \geq -7$

$\Rightarrow x \leq 11$ and $x \geq -7 \Rightarrow x \in [-7,11]$

Hence, Solution set is $[-7,11]$.

10 . A plumber can be paid under two schemes as given below.

- Rs 600 and Rs 50 per hr.
- Rs 170 per hr.

If the job takes n hours for what values of n does the scheme I give the plumber the better wages?
solution : payment under 1st schemeP1 =600 +50n
payment under 1st schemeP2 =170n
P1 > P2 ⇨600 + 50n >170n ⇨ 120 n <600⇨n<5
no. of hours should be less than 5 hours for scheme 1 pavement to be better

LONG ANSWER TYPE QUESTIONS SOLVED

1. Solve the in equation $|7x - 2| \leq 11$
Solution: We know: |A| ≤ B ⇒ -B ≤ A ≤ B
-11 ≤ 7x - 2 ≤ 11 ⇒ -9 ≤ 7x ≤ 13 ⇒ -9/7 ≤ x ≤ 13/7
Answer: x ∈ [-9/7, 13/7]
2. Solve for real number x, $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$and represent it on the number line.
Solution: Simplify RHS:
= (25x - 10 - 21x + 9)/15 = (4x - 1)/15
Inequality becomes:
x/4 < (4x - 1)/15 ⇒ 15x < 16x – 4 ⇒ -x < -4 ⇒ x > 4
Answer: x > 4
3. A milk of 80% concentration is diluted at home by the seller by adding some water to it so that milk concentration is reduced between 65% to 87% If 640 liters of milk of 80% concentration is available, how much water has been added? Which value system the seller is lacking?
Solution : Let x be the litres of water added to 640 litres of 80% milk.
Milk content = 512 litres
We want 65% ≤ (512 / (640 + x)) ≤ 87%
⇒ 0.65 < 512 / (640 + x) < 0.87
Solve: Left: 512 > 0.65(640 + x) ⇒ x < 147.69
Right: 512 < 0.87(640 + x) ⇒ x > -51.49
Since x ≥ 0, final answer: 0 < x < 147.69
Maximum water that can be added ≈ 147.69 litres
4. In drilling world's deepest hole it was found that the temperature T in degree Celsius, X km below the earth's surface was given by
T=30+25(x-3),3≤ $x \leq 15.3$ At what depth will the temperature be between 155^0c and 205^0c.
Solution:
T = 30 + 25(x - 3), where 3 ≤ x ≤ 15.3
We want: 155 ≤ T ≤ 205 ⇒ 155 ≤ 30 + 25(x - 3) ≤ 205
⇒ 125 ≤ 25(x - 3) ≤ 175 ⇒ 5 ≤ x - 3 ≤ 7
⇒ 8 ≤ x ≤ 10
Answer: Depth is between 8 km and 10 km
5. A solution of 9% acid is to be diluted by adding 3%acid solution to it .The resulting mixture is to be more than 5% but less than 7%acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?
Solution: Let x = litres of 3% acid solution added to 460 litres of 9% acid.
Acid content: 41.4 + 0.03x
Total volume = 460 + x
by given conditions 0.05 < (41.4 + 0.03x)/(460 + x) < 0.07
Solve:
Left: 41.4 + 0.03x > 0.05(460 + x) ⇒ x < 920

Right: $41.4 + 0.03x < 0.07(460 + x) \Rightarrow x > 230$
Answer: $x \in (230, 920)$

CASE STUDY BASED QUESTIONS SOLVED

1. Aditya's mother gave him Rs. 200 to buy some packets of rice and maggie from the market. The cost of one packet of rice is Rs. 30 and that of one packet of maggie is Rs. 20. Let x denotes the number of packet of rice and y denotes the number of packets of maggie.

(a) Find the inequality that represents the given situation.
(b) If he buys 4 packets of rice and spends entire amount of Rs. 200, then find the maximum number of packets of maggie that he can buy. (c) Solve the following inequality for real x : $4x + 3 < 5x + 7$ (2)
Ans. (a) Total amount = Rs. 200
Cost of one packet of rice = Rs. 30
And cost of one packet of maggie = Rs. 20
Here, x and y denote the number of packets of rice and maggie respectively, Total amount spent by Aditya is $30x + 20y$.
∴ Required inequality is $30x + 20y \leq 200$
(b) If he spends his entire amount, then
We have, $30x + 20y = 200$ ......(i)
Since, number of packet of rice $= 4$
∴ At $x = 4$, equation (i) becomes $30 \times 4 + 20y = 200$
$\Rightarrow 120 + 20y = 200$
$\Rightarrow 20y = 200 - 120$
$\Rightarrow 20y = 80$
∴ Maximum number of packets of maggie that he can buy is 4 .
(c) Given that, $4x + 3 < 5x + 7$ Now by subtracting 7 from both the sides, we get
$4x + 3 - 7 < 5x + 7 - 7$
The above inequality becomes, $4x - 4 < 5x$
Again, by subtracting 4x from both the sides,
$4x - 4 - 4x < 5x - 4x$
$\Rightarrow x > -4$
∴ The solutions of the given inequality are defined by all the real numbers greater than -4 .
The required solution set is $(-4, \infty)$.

2. An intelligence quotient (IQ) is a total score derived from a set of standardized tests or subtests designed to assess human intelligence. The abbreviation "IQ" was coined by the psychologist William Stern for the German term Intelligence quotient, his term for a scoring method for intelligence tests at University of Breslau he advocated in a 1912 book.

Manjula is a Psychology student and nowadays she is learning about Intelligence Quotient (IQ). She calculates the result as follows:
Intelligence Quotient $= \frac{\text{Mental Age}}{\text{Chronological Age}} \times 100$
(a) What could be the range if mental age if a group of children with chronological age of 15 years have the IQ range as $90 \leq \text{IQ} \leq 150$?
(b) What could be the range of IQ if a group of children with age of 12 years have the mental age

range as $9 \leq MA \leq 15$?

OR

(b) What could be the range of IQ if a group of children with mental age of 18 years have the mental age range as $12 \leq CA \leq 15$?

Ans. (a) $90 \leq IQ \leq 150 \Rightarrow 90 \leq \frac{MA}{15} \times 100 \leq 150$

$$\Rightarrow \frac{90}{100} \leq \frac{MA}{15} \leq \frac{150}{100} \Rightarrow \frac{90}{100} \times 15 \leq MA \leq \frac{150}{100} \times 15$$

$$\Rightarrow \frac{27}{2} \leq MA \leq \frac{45}{2} \Rightarrow 13.5 \leq MA \leq 22.5$$

(b) $IQ = \frac{MA}{CA} \times 100 \Rightarrow MA = \frac{IQ \times CA}{100}$ As, $9 \leq MA \leq 15$

$$\Rightarrow 9 \leq \frac{IQ \times CA}{100} \leq 15 \Rightarrow 9 \leq \frac{IQ \times 12}{100} \leq 15 \Rightarrow 900 \leq IQ \times 12 \leq 1500$$

$$\Rightarrow \frac{900}{12} \leq IQ \leq \frac{1500}{12} \Rightarrow 75 \leq IQ \leq 125$$

OR

(b) $IQ = \frac{MA}{CA} \times 100 \Rightarrow CA = \frac{MA}{IQ} \times 100$

As $12 \leq CA \leq 15$,

$$\Rightarrow 12 \leq \frac{MA}{IQ} \times 100 \leq 15$$

$$\Rightarrow 12 \leq \frac{18}{IQ} \times 100 \leq 15 \Rightarrow \frac{12}{100} \leq \frac{18}{IQ} \leq \frac{15}{100}$$

$$\Rightarrow \frac{12}{1800} \leq \frac{1}{IQ} \leq \frac{15}{1800} \Rightarrow \frac{1800}{12} \geq IQ \geq \frac{1800}{15}$$

$$\Rightarrow 150 \geq IQ \geq 120$$

3. A beaker contains 600 litres of 12% solution of boric acid. This is to be diluted by adding 3% solution of boric acid to it. Based on this information answer the following questions.

1. If x litres of 2% boric acid solution is added to the beaker, find the quantity of acid in the resulting solution.
2. Find the initial quantity of water in the beaker.
3. How many litres of water will have to be added to the beaker that the resulting mixture will contain more than 5% but less than 8% acid content?

Solution: 1) $600 \times \frac{12}{100} + x \times \frac{3}{100} = 72 + \frac{3x}{100}$

2) $600 \times \frac{88}{100} = 528\ lt$

3) $5 < \frac{600 \times \frac{12}{100} + x \times \frac{3}{100}}{600 + x} \times 100 < 8$⇨5(600+x)<7200+3x<8(00+x)

480<x <2100

CHAPTER – 6: PERMUTATIONS AND COMBINATIONS

Definitions And Formulae

Fundamental principles of counting:

Fundamental principle of multiplication –

- "If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurrences of the events in the given order is m × n."

➢ If there are three different events such that one event occurs in **m** different ways, second event happens in **n** different ways and the third event occurs in **p** different ways ,then all three Events simultaneously will happen in **m × n × p** ways.

➢ **Fundamental principle of addition**–If there are two jobs such that first work can be performed independently in **m** number of ways and the second work independently can be done in **n** number of ways ,then either of the two jobs can be performed in **(m +n)** ways.

➢ **Factorial notation**

The notation **n!** represents the product of first **n** natural numbers, i.e., the product 1 × 2 × 3 × . . . × (n – 1) × n

is denoted as **n!.**

We read this symbol as 'n factorial'.

Thus, 1 × 2 × 3 × 4 . . . × (n – 1) × n = n !

➢ n ! = n (n – 1) !

= n (n – 1) (n – 2) ! [provided (n ≥ 2)]

= n (n – 1) (n – 2) (n – 3) ! [provided (n ≥ 3)]

➢ Permutation and combination are two fundamental concepts in mathematics that deal with the arrangement and selection of elements from a set.

➢ Permutation refers to the arrangement of elements in a specific order, while combination refers to the selection of elements without regard to order.

What is Permutation?

➢ Permutations A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

➢ The number of permutations of n different objects taken r at a time, where 0 < r ≤ n and the objects do not repeat is n (n – 1) (n – 2). . .(n – r + 1),

which is denoted by P (n , r) OR $nP_r = \frac{n!}{(n-r)!}, 0 \le r \le n$

$nP_0 = 1 = nP_n$

➢ The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.

➢ PERMUTATION OF A LIKE OBJECTS

The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k

are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1!p_2!\ldots p_k!}$.

➢ The number of ways in which (m + n) different things can be divided into two groups containing m and n things is $\frac{(m+n)!}{m!\ n!}$.

➢ The no. of permutations of n different objects taken r at a time:

❖ when a particular object to be always included in each arrangement is ${}^{n-1}P_{r-1}$

❖ when a particular object is never taken in each arrangement in ${}^{n-1}P_r$

Combinations – If we have to select combinations of items from a given set of items such that the order or arrangement doesn't matter, then we use combinations . Such that to find the number of ways of selecting **r** objects from a set of **n** objects, then mathematically it is given by $^nC_r = \frac{n!}{r!(n-r)!}$.

- (i) $n_{C_0} = 1 = ^n C_n$ (ii) $n_{C_1} = n = ^n C_{n-1}$
- (iii) $n_{C_1} = n = ^n C_{n-1}$ (iv) $n_{C_2} = \frac{n(n-1)}{2!} = n_{C_{n-2}}$
- (v) $n_{C_3} = \frac{n(n-1)(n-3)}{3!} = ^n C_{n-3}$ (vi) $nC_r = ^n C_s \Rightarrow r + s = n \ \ or \ \ r = s$
- (vii) $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

- RELATIONS *BETWEEN ^nP_r & ^nC_r*

$$^nP_r = ^n C_r . r! , \quad 0 \le r \le n$$

- The no. of arrangements of n distinct objects taken r at a time , so that k particular objects are
- Always included : $^{n-k}C_{r-k} .r!$
- Never included : $^{n-k}C_r .\ r!$
- $^nC_1 + {^nC_2} + {^nC_3} + {^nC_4} + ... + {^nC_n} = 2^n - 1$

MULTIPLE CHOICE QUESTIONS SOLVED

1) Teacher ask the student, suppose there are 10 boys and 8 girls in a class room, we want to select *either a boy or a girl* to represent the class in a function. In how many ways the selection can be made?
(a)10 (b) 8 (c) 18 (d) 80
Ans: (c) 18
Given that 10 boys and 8 girls We are selecting 1 person, and it can be either a boy or a girl. Since the teacher can select either a boy or a girl, we add the two results together
Total ways to select: 10 (boys)+8 (girls)=18
So, the selection can be made in $^{18}C_1$ =18 different ways.

2) Meera ask her friend Neeta that if we have 4 flags of different colours, then how many different signals can be generated if a signal requires the use of 2 flags one below the other?
(a)4 (b) 3 (c) 2 (d) 12
Ans : (d) 12
We are selecting and arranging 2 flags out of 4, where order matters.
This is: $^4P_2 = 4 \times 3 = 12$
So, 12 different signals can be generated.

3) In a class room discussion teacher ask the students, tell me about how many *3- digits even numbers* can be formed from the digits 1,2,3,4,5,6 if *no digit are repeated*?
(a) 60 (b) 85 (c) 108 (d) 98
Ans: (a) 60
=4x5x3

4) In a class room discussion teacher ask the students, tell me about how many *3- digits even numbers* can be formed from the digits 1,2,3,4,5,6 if *digit are repeated*?
(a)125 (b) 216 (c) 108 (d)98
Ans : (c) 108
=6x6x3 =108

5) A mint prepares metallic calendars specifying months, dates and days in the form of monthly sheets (one plate for each month). How many types of February calendars should it prepare to serve for all the possibilities in future years?
(a) 11 (b) 12 (c) 13 (d) 14
Ans : (d) 14
Step 1: Identify the Types of February
February can have two types of configurations based on whether it is a leap year or a non-leap year:
- Leap Year: February has 29 days.
- Non-Leap Year: February has 28 days.
Step 2: Determine the Starting Day of February

February can start on any day of the week. The days of the week are: - Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday

This gives us a total of 7 possible starting days for February.

Step 3: Calculate Total Calendar Types

For each type of February (28 days or 29 days), there are 7 different starting days. Therefore, we can calculate the total number of different February calendars as follows:

- For 28 days: 7 starting days
- For 29 days: 7 starting days

So, the total number of February calendars is:

Total Calendars=(Number of Days in February)×(Number of Starting Days)=2×7=14

6) Sunita went to market with her friend Divya, on walking in the market Sunita see the Banner where it is written as 'LOGARITHMS'. Divya asked Sunita can you guess how many words with or without meaning can be formed out of the letters of the word 'LOGARITHMS'. If each letter is used once?

(a) 10! (b) 10!/2 (c) 9! (d) 9!/2

Ans: (a) 10!

Since all letters are unique, the total number of distinct permutations is simply the factorial of the number of letters=10!

7) Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

(a) 2520 (b) 25200 (c) 252 (d) 25400

Ans: (b) 25200

Step 1: Choose 3 consonants from 7

The number of ways to select 3 consonants from 7 is given by the combination formula: C(7,3)=35

Step 2: Choose 2 vowels from 4

Similarly, the number of ways to select 2 vowels from 4 is: C(4,2)=6

Step 3: Arrange the selected 5 letters

Once we've selected 3 consonants and 2 vowels, we need to arrange these 5 letters. The number of ways to arrange 5 distinct letters is:

$5!=5\times4\times3\times2\times1=120$

Step 4: Calculate the total number of distinct words

Now, we multiply the results from each step: C(7,3)x C(4,2)x5! = 35×6×120=25,200

Therefore, 25,200 distinct words can be formed using 3 consonants and 2 vowels from a set of 7 consonants and 4 vowels, with each letter used exactly once.

8) If ${}^{n+1}C_3 = 2.{}^{n}C_2$ then n = ?

(a) 3 (b) 4 (c) 5 (d) 6

Ans: (c) 5

9) Among 14 players, 5 are bowlers. In how many ways a team of 11 may be formed with *at least 4 bowlers*?

(a) 265 (b) 263 (c) 264 (d) 275

Ans : (c) 264

We are given: Total players: 14 , Bowlers: 5, Non-bowlers: 14 - 5 = 9

We need to form a team of 11 players with at least 4 bowlers

Let's break it down by cases:

Case 1: 4 bowlers

Choose 4 bowlers from 5: C(5,4)=5

Choose 7 non-bowlers from 9:= C(9,7)=36

Total no.of ways of selecting 4 bowlers = C(5,4)x C(9,7)=5x36=180

Case 2: 5 bowlers

Choose 5 bowlers from 5: C(5,5)=1

Choose 6 non-bowlers from 9:C (9,6)=84

Total no. of ways of selecting 5 bowlers = C(5,5)x C(9,6)=1x84=84

Total number of ways: 180+84=264

10) The number of arrangements of the letters of the word BHARAT taking 3 at a time is

(a) 72 (b) 120 (c) 14 (d) None of these

Ans :(a) 72

The words can be formed out of the letters of the word 'BHARAT' taking 3 at a time can be done in 2 different cases : Case - 1 : When all the letters are distinct.

We have, 5 distinct letters, out of which taking three at a time, the number of words that can be formed $= {}^5P_3 = 60$

Case-2 : When 2 A's are selected.

So, we have, 2A's and 1 letter is to selected out of the 4 distinct letters, which can be done in ${}^4P_1 = 4$ ways.

Now, the 3 letters can be arranged among themselves, but there are 2 A's, so the number of ways in which arrangement can be done = 3 ! /2 ! = 3

So, in this case, total number of words that can be formed = 4 x 3 = 12.

The number of arrangements of the letters of the word BHARAT taking 3 at a time

= (60 + 12) = 72 ways

ASSERTION - REASON BASED QUESTIONS SOLVED

Directions:

Each of the following questions consists of two statements: an Assertion (A) and a Reason (R). Answer them by selecting the correct option:

(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

1) Assertion (A): The number of ways to arrange the letters of the word "SCHOOL" is $\frac{6!}{2!}$

Reason (R): The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\frac{n!}{p_1!p_2!...p_k!}$.

Answer: (a) R explains A

2) Assertion (A): The number of 4-digit numbers formed using the digits 1, 2, 3, 4 without repetition is 4^4

Reason (R): Each of the 4 places can be filled by 4 digits.

Answer: (d) Without repetitions choices are reduced gradually in A.

3) Assertion (A): C(n, r)=C(n, n−r) for all 0≤r≤n.

Reason (R): Choosing r objects from n is the same as choosing n−r objects to leave behind.

Answer: (a) R explains A

4) Assertion (A): The number of ways of choosing 3 students out of 10 for a group is 120.

Reason (R): In combinations, the order of selection does not matter.

Answer: (a) R explains A.

5) Assertion (A): The number of permutations of 5 items taken 3 at a time is equal to ${}^5C_3$.

Reason (R): In permutations, the order of selection is not important.

Answer: (d) as Permutation matters in arrangement.

6) Assertion (A):$12_{C_2} = 12_{C_{10}}$

Reason (R): Selection of the r distinct thing out of n is equal to the rejection of (n-r) thing out of n.

Answer: (a)

Since attributing to the reason, selection of 2 distinct things out of 12,is equal to rejection of (12-2=10) things out of 12

$\therefore 12_{C_2} = 12_{C_{10}}$

7) Assertion (A): 3! +4! = 7!

Reason (R): For any positive integer n, n! = 1×2×3…..×n and 0! =1

Answer: d)

Since (R) is the definition of factorial which is true but assertion is false.

As $3! + 4! = (3 \times 2 \times 1) + (4 \times 3 \times 2 \times 1) = 6 + 24 = 30$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 > 30$

8) Assertion (A):-For any natural number $n, n_{C_n} = 1$

Reason (R): The number of ways of selecting r objects from a set of n objects, then mathematically it is given by ${}^{n}C_r = \frac{n!}{(n-r)!}$.

Answer: (c)

Solution: $n_{C_r} = \frac{n!}{(n-r)!r!} \therefore n_{C_n} = \frac{n!}{(n-n)!n!} = \frac{n!}{n!} = 1$

9) Assertion (A): $n_{P_r} = n_{C_r} \cdot r!\ , 0 < r \leq n$

Reason(R): For each combination of ${}^{n}C_r$, there are r! permutations.

Answer: (a)

Solution: r objects in every combination can be arranged in r! ways. $n_{P_r} = n_{C_r} \cdot r!\ , 0 < r \leq n$

10) Assertion (A):-The number of ways of choosing 4 cards from a pack of cards is ${}^{52}C_4$

Reason (R): Permutation is an arrangement in a definite order of a number of objects taken some or all at A time.

Answer: (b)

Solution: Selection of 4 card out of 52 can be made in 52_{C_4} ways. And (R) is the definition of permutation.

VERY SHORT ANSWER TYPES QUESTIONS SOLVED

1) Rashmita is playing with scrambles word cubes. She needs to form a word using all the letters of the word 'ORGANIC' without repetition of the letters.

(i) In how many ways vowels and consonants alternate between each other?

(ii)In how many words the all vowels are together?

Ans: (i)Vowels – O, A, I , Number of words =4! X 3! = 144

(ii) No of words in which vowels are together = $2! \times 4! \times 3! = 288$

2) Find the sum of the digits in the unit place of all the numbers formed with the help of 2,3,4,5 taken all at a time.

Ans: We are given the digits 2, 3, 4, 5, and

we're to Use all digits at a time to form numbers (so all 4-digit numbers with no repetitions),

To find the sum of the digits in the unit place (i.e., last digit) across all such numbers.

Total no.of numbers using all 4 digits with no repetition =4!

Since all digits are used equally and permutations are evenly distributed.

So each digit {2,3,4,5} will appear in the units place =24/4=6 times

Hence the Sum of digits in the unit place =6×(2+3+4+5) = 6×14=84

3) Find the number of ways in which a team of eleven players can be selected from 22 players always including 2 of them and excluding 3 of them.

Ans: Total number of players =22

We have to selected 11 players

We have to exclude 3, so 19 players are available.

Also from these 2 particular players are always included .

Therefore to select 9 players from remaining 17 players in $17c_9$ ways

4) Find the number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place.

Ans: Vowels are A,I,E and consonants are R,T,C,L.

Now vowels occupy three even places (2nd, 4th and 6th) in ways.

In remaining four places four consonants can be arranged in 4Į was

so, total number of words = 3! x 4! = 6 x 24 = 144 ways

5) How many different words can be formed by using all the letters of the word ALLAHABAD?

Ans: There are 9 letters in given word ,of which 4 are A'&,2 are L'&. So total number of words is the number of arrangements of 9 things of which 4 are similar of one kind , two are similar of one kind ∴

Total number of word = $\frac{9!}{4!2!}$ specific.

6) There are 10 points on a circle. How many chords can be drawn by using these points?
Ans: One chord can be drawn by using two points. So total number of chord = number of ways of choosing two points out of 10.i.e 10_{C_2}

7) Find the total number of five digit numbers that can be formed by using the digits 0,1,2,3, ….,9
Ans: Total number of five digit numbers $9\times10\times10\times10\times10=90000$
As 0 cannot be used as first place so it has 9 choices and rest all 4 places can be filled in 10 ways.

8) At an election, a voter may vote for any number of candidates, not greater than the number to be electedthere are 8 candidates and 3 are to be elected. If a voter votes for at least one candidate, then find number of ways in which he can vote?
Ans: The number of ways in which a voter can vote is ${}^8C_1+{}^8C_2+{}^8C_3=8+28+56=92$

9) Find the total number of ways of answering 6 objective type question, each question having 4 choices:
Sol: -since each question can be answered in 4 ways ,So total number of ways of answering 6 question is $4\times4\times4\times4\times4\times4 = 4^6$

10) There are 3 candidates for a classical ,4 for a social science and 2 for a natural science scholarship In how many ways can these scholarships be awarded?
Ans: Since, classical scholarship can be awarded to any one of three candidates. So awarding the classical scholarship. Similarly, the social science and natural science scholarship can be awarded in 4 and 2 ways respectively. So number of ways $3\times4\times2=24$.

SHORT ANSWER TYPES QUESTIONS SOLVED

1) In India Pakistan world cup match. BCCI decided to choose 11 players from 14 eligible players in which 5 are bowlers, 4 are batsman. In how many ways a team of 11 may be formed with
(a) at least 4 bowlers? (b) Exactly 4 batsmen? (c) 3 bowlers and 2 batsmen?
Ans: (a)Number of ways chosen at least 4 bowlers
Case I : 4 bowlers & 7 others = ${}^5C_4 \text{ x } {}^9C_7$ =180 ways
Case II: 5 bowlers & 6 others =${}^5C_5 \text{ x } {}^9C_6$ =84 ways
Total number of ways = 180+84 =264 ways
(b) Exactly 4 batsmen?
Number of ways = ${}^4C_4 \text{ x } {}^{10}C_7$ =120 ways
(c) 3 bowlers and 3 batsmen?
Number of ways =${}^5C_3 \text{ x } {}^4C_3 \text{ x } {}^5C_5$ = 40 ways

2) In a school group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has
(i) No girl? (ii) At least one boy and one girl? (iii)At least three girls?
Ans:
(i) No girl.
Number of ways = ${}^7C_5$ = 21 ways
(ii) At least one boy and one girl?
Number of ways =${}^4C_4 \text{ x } {}^7C_1+{}^4C_3 \text{ x } {}^7C_2+{}^4C_2 \text{ x } {}^7C_3+{}^4C_1 \text{ x } {}^7C_4$ = 441 ways
(iii) At least three girls?
Number of ways=${}^4C_4 \text{ x } {}^7C_1+{}^4C_3 \text{ x } {}^7C_2$ = 91 ways

3) From a college of 35 students, 15 students are to be chosen for an excursion party. Ravi, Shyam and Raju are best friends. They decided that *either we three of them go* or *none of them go for excursion*. In how many ways can the excursion party be chosen?
Solution: There are two cases
Case I: *when three of them chosen for party*

Rest students chosen by = $^{32}C_{12}$ ways

Case II : *when three of them not chosen for party*

Number of ways 15 students chosen = $^{32}C_{15}$ ways

Total Number of ways $= ^{32}C_{12} + ^{32}C_{15}$ ways

4) Find the number of different words that can be formed from the letters of the word INTERMEDIATE such that two vowels never come together.

Solution. Letters of the word INTERMEDIATE are: Vowels (I, E, E, I, A, E) and consonants (N, T, R, M, D, T)

Now we have to arrange these letters if no two vowels come together.

So, first arrange six consonants in $\frac{6!}{2!}$ways.

Arrangement of six consonants creates seven gaps.

Six vowels can be arranged in these gaps in $7C_6 \times \frac{6!}{2!3!}$ ways.

So, total number of words $\frac{6!}{2!} \times 7C_6 \times \frac{6!}{2!3!} = 360 \times 7 \times 60 = 151200$

5) Four persons entered the lift cabin on the ground floor of 7 floor house suppose each of them can leave the cabin independently at any floor beginning with the first. What is. The total number of ways in which each of the four persons can leave the cabin at any of the six floors?

Solution: Suppose that A, B, C, D are 4 persons.

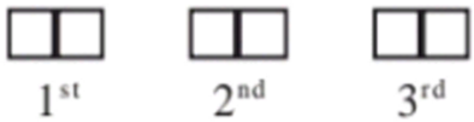
A can leave the cabin at any of the 6 floors. so, A can leave the cabin in 6 ways,

similarly, each of B, C and D can leave the cabin in 6 ways.

therefore, the total number of ways in which each of the 4 persons can leave the cabin at any of the six floors $= 6 \times 6 \times 6 \times 6 = 1296$.

6) Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

Solution: Let us denote married couples by S_1, S_2, S_3 where each couple is considered to be a single unit as shown in the following figure:



Then the number of ways in which spouses can be seated next to each other is 3! = 6 ways. Again, each couple can be seated in 2! ways.

Thus, the total number of seating arrangement so that spouses sit next to each other = $3! \times 2! \times 2! \times 2! = 48$.

Again, if three ladies sit together, they can be arranged in 3! ways

And the ladies and 3men can be arranged altogether among themselves in 4! ways.

Therefore, the total number of ways where ladies sit together is $3! \times 4! = 144$.

7) How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if. (i) 4 letters are used at a time (ii) all letters are used at a time, (iii) all letters are used but first letter is a vowel?

Solution: The total no. of letters in the word MONDAY = 6 = n.

We know that the number of ways of selecting and arranging r different things from n different things is a permutation and we calculate it using the ^nP_r formula.

(i) Number of 4-letter words that can be formed from the letters of the word MONDAY, without repetition of letters $= {}^6P_4 = 6!/(6 - 4)! = 6!/2! = 360$.

(ii) Number of words that can be formed from the letters of the word MONDAY, if all letters are used at a time

$= {}^6P_6 = 6!/(6 - 6)! = 6!/0! = 720.$

(iii) Number of words that can be formed from the letters of the word MONDAY, if all letters are used but the first letter is a vowel $= 2 \times 5 \times 4 \times 3 \times 2 \times 1 = 240$

8) A boy has 3 library tickets and 8 books of his interest in the library. Of these 8 books, he does not want to borro chemistry part-II, unless chemistry part-I is also borrowed. In how many ways can he choose the three books to be borrowed?

Answer: we need to consider the restrictions on borrowing Chemistry Part II. Let's denote the books as follows:

- Let C1 = Chemistry Part I
- Let C2 = Chemistry Part II
- Let B1, B2, B3, B4, B5, B6 = the other 6 books

We have a total of 8 books: C1, C2, B1, B2, B3, B4, B5, B6.

We will break this problem into cases based on whether the boy borrows Chemistry Part I and/or Chemistry Part II.

Case 1: Neither C1 nor C2 is borrowed.

In this case, the boy can choose any 3 books from the 6 remaining books B1,B2,B3,B4,B5,B6.

The number of ways to choose 3 books from 6 is given by the combination formula: $:n_{C_r} = \frac{n!}{(n-r)!r!}$

So, the number of ways is: $6_{C_3} = \frac{6!}{(6-3)!3!}$ =20

Case 2: C1 is borrowed and C2 is not borrowed.

In this case, the boy can choose 2 more books from the 6 remaining books B1, B2, B3, B4, B5, B6.

The number of ways to choose 2 books from 6 is $:6_{C_2} = \frac{6!}{(6-2)!2!}$ =15

Case 3: Both C1 and C2 are borrowed.

In this case, the boy can choose 1 more book from the 6 remaining books B1, B2,B3,B4,B5,B6.

The number of ways to choose 1 book from 6 is $:6_{C_1} = \frac{6!}{(6-1)!1!}$ =6

Add the number of ways from all cases

Now, we can add the number of ways from all three cases:

Total ways=Case 1+Case 2+Case 3=20+15+6=41

Therefore, the total number of ways the boy can choose 3 books to borrow is 41.

9) A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag, if

(i) they can be of any colour (ii) two must be white and two red and

(iii) they must all be of the same colour.

Answer: Total number of marbles = 6 white + 5 red =11 marbles

(a) If they can be of any colour means we have to select 4 marbles out of 11

∴Required number of ways = ${}^{11}C_4$

(b) Two white marbles can be selected in ${}^6C_2$

Two red marbles can be selected in ${}^5C_2$ ways.

∴Total number of ways = ${}^6C_2 \times {}^5C_2 = 15 \times 10 = 150$

(c) If they all must be of same colour,

Four white marbles out of 6 can be selected in ${}^6C_4$ ways. And 4 red marbles out of 5 can be selected in ${}^5C_4$ ways.

∴Required number of ways = ${}^6C_4 + {}^5C_4 = 15 + 5 = 20$

10) A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected, if the team has

(i) no girls (ii) at least one boy and one girl (iii) at least three girls

Answer : Number of girls = 4;

Number of boys = 7

We have to select a team of 5 members provided that

(i). Team having no girls

Required number of ways = $^7C_5 = \frac{7\times 6}{2!} = 21$

(ii) Team having at least one boy and one girl

∴A Required number of ways $= 7C_1 \times 4C_4 + 7C_2 \times 4C_3 + 7C_3 \times 4C_2 + 7C_4 \times 4C_1$

$= 7 \times 1 + 21 \times 4 + 35 \times 6 + 35 \times 4 = 7 + 84 + 210 + 140 = 441$

(iii) Team having at least three girls

∴Required number of ways $= 4C_3 \times 7C_2 + 4C_4 \times 7C_1 = 4 \times 21 + 7 = 84 + 7 = 91$

LONG ANSWER TYPES QUESTIONS SOLVED

1. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these are

(i) four cards are of the same suit? (ii) four cards belong to four different suits?

(iii) four cards are face cards? (iv) two are red cards and two are black cards?

(v) cards are of the same colour?

Solution: (i) There are four suits (diamond, spade, club and heart) of 13 cards each. Therefore, there are $^{13}C_4$ ways of choosing 4 diamond cards, $^{13}C_4$ ways of choosing 4 club cards, $^{13}C_4$ ways of choosing 4 spade cards and $^{13}C_4$ ways of choosing 4 heart cards.

Required number of ways $^{13}C_4 + ^{13}C_4 + ^{13}C_4 + ^{13}C_4 = 4x\ ^{13}C_4 = 4x\ \frac{13!}{9!X4!} = 2860$

(ii) Number of ways of selecting one card from each suit = $^{13}C_1$ x $^{13}C_1$x $^{13}C_1$ x $^{13}C_1$ $=13^4$

(iii) There are 12 face cards out of which 4 cards can be chosen in $^{12}C_4$ ways.

Required number of ways = $^{12}C_4 = 495$ ways

(iv) There are 26 red cards and 26 black cards. Therefore, 2 red cards can be chosen in $^{26}C_2$ ways and 2 black cards can be chosen in $^{26}C_2$ ways. Hence, 2 red and 2 black cards can be chosen in $^{26}C_2$ x $^{26}C_2 = 105625$ ways.

(v) Out of 26 red cards, 4 red cards can be chosen in $^{26}C_4$ ways. Similarly, 4 black cards can be chosen in $^{26}C_4$ ways.

Hence, 4 red or 4 black cards can be chosen in $^{26}C_4 + ^{26}C_4 = 2$ x $^{26}C_4 = 29900$ ways

2. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

i) do the words start with P ii) do all the vowels always occur together

iii) do all the vowels never occur together iv) do the words begin with I and end in P?

Solution: To solve the problem of finding the number of arrangements of the letters in the word "INDEPENDENCE" and answering the specific questions, we will follow these steps:

Calculate the total number of arrangements of the letters in "INDEPENDENCE".

The word "INDEPENDENCE" consists of 12 letters where:

- I appears 1 time , - N appears 3 times , - D appears 2 times ,- E appears 4 times, - P appears 1 time, - C appears 1 time

The formula for the total arrangements of letters when there are repetitions is given by:

Total arrangements $=\frac{n!}{p_1!p_2!...p_k!}$.

Where n is the total number of letters, and $p_1, p_2, \ldots$ are the frequencies of the repeated letters.

Thus, we have: Total arrangements $=\frac{12!}{3!2!4!}$

Calculating this gives: Total arrangements=1663200

(i) Find the number of arrangements that start with P.

If the arrangement starts with P, we fix P in the first position.

The remaining letters are I, N, N, D, E, D, E, N, C, E (11 letters in total).

The number of arrangements of these 11 letters is:

Arrangements= $\frac{11!}{3!2!4!}$=13860

(ii) Find the number of arrangements where all vowels occur together.

The vowels in "INDEPENDENCE" are I, E, E, E, E. We can treat these vowels as a single entity or block.

Thus, we have the block (IEEEE) and the consonants N, D, N, D, N, C (7 letters in total).

Now we have 8 entities to arrange (the block and the consonants):

Arrangements $=\frac{8!}{3!2!}$=3360

Now, we also need to arrange the vowels within the block:

Vowel arrangements $=\frac{5!}{4!}$=5

Thus, the total arrangements where all vowels are together is: 3360 x 5 = 16800

(iii) Find the number of arrangements where vowels never occur together.

To find this, we can use the total arrangements and subtract the arrangements where vowels are together:

Arrangements where vowels never together = Total arrangements − Arrangements where vowels together

Calculating this gives: 1663200 – 16800 =1646400

(iv) Find the number of arrangements that begin with I and end with P.

If the arrangement starts with I and ends with P, we fix I at the start and P at the end.

The remaining letters are N, N, D, E, D, E, N, C, E (10 letters in total).

The number of arrangements of these 10 letters is:

Arrangements $=\frac{10!}{3!2!4!}$ = 12600

3. There are 10 professors and 20 students out of whom a committee of 2 professors and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees:

(i)A particular professor is included (ii)A particular student is included

(iii)A Particular student is excluded

Solution: We are given: 10 professors → choose 2

20 students → choose 3

Total number of committees

Ways to choose 2 professors from 10: $^{10}C_2$ =45

Ways to choose 3 students from 20: $^{20}C_3$=1140

Total number of committees: 45×1140 =51300

(i)A particular professor is included

Fix 1 specific professor. Now choose 1 more professor from the remaining 9, and 3 students from 20. Professors: $^{9}C_1$= 9

Students: $^{20}C_3$= 1140

Total: 9×1140=10260

(ii)A particular student is included

Fix 1 specific student. Now choose 2 more students from the remaining 19, and 2 professors from 10. Students: ${}^{19}C_2$=171

Professors: ${}^{10}C_2$= 45

Total: 171×45= 7695

(iii)A particular student is excluded

Fix 1 specific student to be not selected. Now choose 3 students from the remaining 19, and 2 professors from 10.

Students: ${}^{19}C_3$=969

Professors: ${}^{10}C_2$ =45

Total: 969×45=43605

4. Find the number of permutation of the letters of the word ALLAHABAD. In how many of these permutation

(i) All the vowels always occur together (ii) The vowels never occur together

Solution: We are given the word: ALLAHABAD.

Count total letters and repetitions Word: A L L A H A B A D

No of letters =9

Total Repetitions: A appears 4 times, L appears 2 times, H, B, D appear once each

Total permutations of ALLAHABAD

When letters repeat, total permutations are given by:

Total permutations=9!/4!·2! = 7560

(i) Vowels always occur together

Vowels: A, A, A, A , Consonants: L, L, H, B, D → 5 consonants

Treat the 4 A's (vowels) as one block. So now we have:1 vowel block & 5 consonants

That's 6 items to permute: vowel-block + 5 consonants (L, L, H, B, D)

Ways to arrange 6 blocks=6!/2!=720/2=360 (2! because L repeats twice)

Also, within the vowel block, we have 4 A's → only 1 way to arrange them (all same).

So total permutations with vowels together =360×1=360

(ii) Vowels never occur together

This is:Total permutations−Permutations where vowels are together

=7560−360=7200

Total permutations−Permutations where vowels are together=7560−360= 7200

5. Arrangement of the letters of the word INSTITUTIONS

then find (i)Total arrangement. (ii) If all vowels come together.

(iii) If no vowels come together (iv) All vowels come together and all consonant come together.

Solution: Total letters = 12

Let's count the frequency of each letter: 3 I's ,2 N's ,2 S's ,3 T's

Vowels = I, U, O → Total vowels = 3 (I) + 1 (U) + 1 (O) = 5 vowels

Consonants = N, S, T → Total consonants = 12 - 5 = 7 consonants

(i) Since some letters repeat, we divide by the factorials of the counts of repeated letters:

Total arrangements=$\frac{12!}{2!2!3!3!}$.=3326400

(ii) Treat all vowels (I, I, I, U, O) as a single block.

So we now arrange:1 "vowel block" ,7 consonants: N, N, S, S, T, T, T

Total blocks to arrange = 8 blocks

Step 1: Arrangement of 8 blocks (vowel block + consonants)

Arrangements of blocks = $\frac{8!}{2!2!3!}$=1680

Step 2: Arrangements within the vowel block (I, I, I, U, O) = $\frac{5!}{3!}$=20

Total arrangements if all vowels occur together = 1680 × 20 =33600

(iii) Number of arrangements if no vowels come together

This is more complex. We'll use inclusion-exclusion:

Total arrangements (from part i) = 3326400

We subtract arrangements where **at least 2 vowels are together**

But here, we want the number where **no vowels are adjacent**.

Instead, it's easier to:

Step 1: Arrange consonants first (7 consonants)

Consonants = N (2), S (2), T (3)

Consonant arrangements=$\frac{7!}{2!2!3!}$=210

Now, place vowels in the gaps between consonants so that no vowels are adjacent.

There are 8 possible positions for vowels (i.e., between and around consonants):

_ C _ C _ C _ C _ C _ C _ C _

From these 8 gaps, choose 5 to place the 5 vowels. $^8C_5$=56

Now arrange the vowels (I, I, I, U, O): $\frac{5!}{3!}$ =20

Total = 210 × 56 × 20 = 235200

(iv) All vowels come together and all consonants come together

We treat vowels and consonants each as a block.

So we only arrange:[Vowel block][Consonant block] or [Consonant block][Vowel block] → 2 ways

Step 1: Arrange vowels (I, I, I, U, O)= $\frac{5!}{3!}$ =20

Step 2: Arrange consonants (N, N, S, S, T, T, T) =$\frac{7!}{2!2!3!}$=210

Step 3: Multiply with 2 arrangements of blocks

Total no. of arrangements in which all vowels come together and all consonants come Together =2×20×210 =8400

CASE STUDY BASED QUESTIONS SOLVED

1. Republic day is a national holiday of India. It honours the date on which the constitution of India came into effect on 26 January 1950 replacing the Government of India Act (1935) as the governing document of India and thus, turning the nation into a newly formed republic.

Answer the following question, which are based on the word "REPUBLIC"

(i) Find the number of arrangements of the letters of the word 'REPUBLIC" ?

(ii) How many arrangements start with a vowel?

(iii) If the number of arrangements of the letters of the word 'REPUBLIC" is abcde , then find the value of (a + b +c +d +e)

OR

(iv) If the number of arrangements start with a vowel is abcde, then find the value of (a + b)- (d + e)

Solutions: (i) The letters in the word 'REPUBLIC' are all distinct.

There are 8 letters in the given word. So, the number of arrangements are 8! i.e. 40320.

(ii)The vowels in a given word are 'E, I, U'. If we start a word from vowel, we can choose 1 vowel from 3 vowels in $^3C_1$ ways.

Further, remaining 7 letters can be arranged in 7! ways.

Total number of arrangements start with a vowel = $^3C_1$ x 7! = 3 x 5040 = 15120

(iii)Since the number of arrangements are 40320. On comparing, we get a = 4, b = 0, c = 3, d = 2 and e = 0 So, a + b + c + d + e = 4 + 0 + 3 + 2 + 0 = 9

OR

(iv)Since, number of arrangements is 15120 =abcde

On comparing, we get a = 1, b = 5, c = 1 ,d = 2 and e = 0

(a + b) - (d + e) = (1 + 5) - (2 + 0) = 6 - 2 = 4

2 :In an examination, a question paper consists of 12 questions divided into two parts i. e Part land Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part

Based on the information answer the following

In how many ways can a student select the questions in such a way that

(i) 3 questions from part I and 5 questions from part II

(ii) 4 questions from part I and 4 questions from part II

(iii) 5 questions from part I and 3 questions from part II

OR

(iv)In how many ways can a student select the questions

Solution: It is given that the question paper consists of 12 questions divided into two parts - Part I and Part II, containing 5 and 7 questions, respectively. A student has to attempt 8 questions, selecting at least 3 from each part.

This can be done as follows.

i)3 questions from part I and 5 questions from part II can be selected in ${}^5C_3 \times {}^7C_5$ ways.

ii) 4 questions from part I and 4 questions from part II can be selected in ${}^5C_4 \times {}^7C_4$ ways.

iii) 5 questions from part I and 3 questions from part II can be selected in ${}^5C_5 \times {}^7C_3$ ways.

OR

(iv) Thus, required number of ways of selecting questions

$$= 5C_3 \times 7C_5 + 5C_4 \times 7C_4 + 5C_5 \times 7C_3$$

$$= \frac{5!}{2!3!} \times \frac{7!}{2!5!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{3!4!} = 210 + 175 + 35 = 420$$

3. During the maths class, a teacher clears the concept of permutation and combination to the 11th standard students. At the end of the class he asked the students some questions.

(i) How many numbers between 99 and 1000 (both excluding) can be formed such that every digit is either 3 or 7.

(ii) How many numbers between 99 and 1000 (both excluding) can be formed such that there is no restriction

(iii) The number of numbers lying between 99 and 1000 which can be formed using the digits 2, 3, 7, 0, 6, 8 when no digit is being repeated

OR

(iv) How many numbers are there between 99 and 1000 which have exactly one of their digits as 7?

Solution: We're only looking at 3-digit numbers between 99 and 1000, excluding 99 and 1000.

So we're dealing with the numbers from 100 to 999, inclusive.

That gives us: Total 3-digit numbers = 999 − 100 + 1 = 900 numbers

(i) How many numbers between 99 and 1000 (both excluding) can be formed such that every digit is either 3 or 7?

We're forming 3-digit numbers using only digits 3 and 7.

At each digit place (hundreds, tens, units), we can use either 3 or 7.

So total combinations = 2 choices (hundreds) × 2 choices (tens) × 2 choices (units)= 2 × 2 × 2 = 8 numbers

(ii) How many numbers between 99 and 1000 (both excluding) can be formed such that there is no restriction?

As calculated earlier, all 3-digit numbers from 100 to 999: = 900

(iii) The number of numbers lying between 99 and 1000 which can be formed using the digits 2, 3, 7, 0, 6, 8 when no
digit is being repeated
Available digits: {2, 3, 7, 0, 6, 8} → Total 6 digits
We want to form 3-digit numbers with no repetition.
Important: A 3-digit number cannot start with 0
Step-by-step:
Hundreds digit (first digit): Cannot be 0 → 5 choices (from 2, 3, 7, 6, 8)
Tens digit: Pick from remaining 5 digits
Units digit: Pick from remaining 4 digits
So total = $5 \times 5 \times 4 = 100$

OR

How many numbers are there between 99 and 1000 which have exactly one of their digits as 7?
We want 3-digit numbers (100 to 999) with exactly one of the digits being 7.
Go through cases based on the position of 7.
Case 1: 7 is in hundredth place
Number = 7 _ _
We want exactly one 7 → other two digits ≠ 7
Tens digit: 0–9 except 7 → 9 options
Units digit: 0–9 except 7 → 9 options
So: $1 \times 9 \times 9 = 81$
Case 2: 7 is in tens place
Number = _ 7 _
Hundreds digit: can't be 0 or 7 → 1–9 except 7 → 8 options
Units digit: ≠ 7 → 9 options
So: $8 \times 1 \times 9 = 72$
Case 3: 7 is in units place
Number = _ _ 7
Hundreds digit: 1–9 except 7 → 8 options
Tens digit: ≠ 7 → 9 options
So: $8 \times 9 \times 1 = 72$,
Total = $81 + 72 + 72 = 225$

CHAPTER 7 – BINOMIAL THEOREM

DEFINITIONS AND FORMULAE

Binomial theorem for any positive integer n:

$(a+b)^n = C(n.0)a^n + C(n,1)a^{n-1}b + C(n,2)a^{n-2}b^2 + C(n,3)a^{n-3}b^3 + \cdots + C(n,n)b^n$

Some important conclusions from the Binomial Theorem:

- $(x+a)^n = \sum_{r=0}^{n} C(n,r)x^{n-r}a^r$

Or $(x+a)^n = C(n.0)x^n + C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 + C(n,3)x^{n-3}a^3 + \cdots + C(n,n)a^n$

- The sum of indices of x and a in each term is n
- Since $C(n,r) = C(n,n-r), for\ r = 0,1,2,3,\ldots,n$

  $C(n,0) = C(n,n); C(n,1) = C(n,n-1); C(n,2) = C(n,n-2);..$

- Replacing $a\ by - a$

$(x-a)^n = C(n.0)x^n - C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 - C(n,3)x^{n-3}a^3 + \cdots + (-1)^n C(n,n)a^n$

i.e $(x-a)^n = \sum_{r=0}^{n}(-1)^n C(n,r)x^{n-r}a^r$

- putting $x = 1\ and\ a = x$ in the expansion, we get

  $(1+x)^n = C(n.0) + C(n,1)x + C(n,2)x^2 + C(n,3)x^3 + \cdots + C(n,n)x^n$

  $(1+x)^n = \sum_{r=0}^{n} C(n,r)x^r$

- putting $x = 1\ and\ a = -x$ in the expansion, we get

  $(1-x)^n = C(n.0) - C(n,1)x + C(n,2)x^2 - C(n,3)x^3 + \cdots + (-1)^n C(n,n)x^n$

  $(1-x)^n = \sum_{r=0}^{n}(-1)^n C(n,r)x^r$

- *the coefficient of (r+1)*th *term in the expansion of* $(1+x)^n$ *is* $C(n,r)$
- *the coefficient of* x^r *in the expansion of* $(1+x)^n$ *is* $C(n,r)$
- *if n is odd then* $((x+a)^n + (x-a)^n)$ *and* $((x+a)^n - (x-a)^n)$ *both have same number of terms equal to* $\left(\frac{n+1}{2}\right)$

  if n is even, than $((x+a)^n + (x-a)^n)$ has $\left(\frac{n}{2}+1\right)$ terms and $((x+a)^n - (x-a)^n)$ has $\left(\frac{n}{2}\right)$ terms

- The number of terms in the expansion of $(x_1+x_2+\ldots+x_k)^n$ is given by the formula $^{n+k-1}C_{k-1}$

MULTIPLE CHOICE QUESTION SOLVED

1:**Which of the following represents the binomial theorem for positive integral indices?**

a) $(a+b)^n = a^n + b^n$ b) $(a+b)^n = \sum_{r=0}^{n} C(n,r)a^{n-r}b^r$

c) $(a+b)^n = a^{n+1} + b^{n+1}$ d) none of these

Answer: b (Statement of Binomial theorem)

2: Which of the following is the correct binomial coefficient?

a) $C(n,r) = \frac{n!}{r!(n-r)!}$ b) $(n,r) = \frac{(n-r)!!}{n!r!}$ c) $(n,r) = \frac{r!(n-r)!}{n!}$ d) $(n,r) = \frac{n!}{r!}$

Answer: a

Since $C(n,r) = \frac{n!}{r!(n-r)!}$

3: What is the value of $C(6,2)$ **?**

a) 10 b)15 c) 20 d) 30

Answer: b .using $C(n,r) = \frac{n!}{r!(n-r)!}$

4: Which of the following corresponds to Pascal's triangles coefficients of the expansion $(a+b)^4$

a) 1,3,3,1 b) 1,4,6,4,1 c) 1,5,10,10,5,1 d) 1,2,1

Answer: b ,

For n=4 The Coefficients are C(4,0), C(4,1),C(4,2),C(4,3),C(4,4)

5: The expansion $((x+a)^{51}+(x-a)^{51})$ has__________ terms after simplification.

a) 50 b) 52 c) 26 d) none of these

Answer : c

Using properties *if n is odd then* $((x+a)^n+(x-a)^n)$ *and* $((x+a)^n-(x-a)^n)$ *both have same number of terms equal to* $\left(\frac{n+1}{2}\right)$

6: The total number of terms in the expansion $((x+a)^{100}+(x-a)^{100})$ after simplification is____________

a)202 b) 51 c) 50 d) none of these

Answer: b

Using the properties if n is even , then $((x+a)^n+(x-a)^n)$ has $\left(\frac{n}{2}+1\right)$ terms and $((x+a)^n-(x-a)^n)$ has $\left(\frac{n}{2}\right)$ terms

7: Number of terms in the expansion $(1-2x+x^2)^7$ are _______

a) 14 b) 15 c) 16 d) 17

Answer : b

$(1-2x+x^2)^7$= **$((1-x)^2)^7=(1-x)^{14}$,**

So the no. of terms =15

8. The coefficient of x^5 in $(x+3)^9$ is ___________

a) $C(9,4)3^2$ b) $C(9,4)3^3$ c) $C(9,4)3^4$ d) none of these

Answer: c

The binomial expansion of

$(x+3)^9 = x^9+27x^8+324x^7+2268x^6+10206x^5+30618x^4+61236x^3+78732x^2+59049x+19683$

9. $(1.2)^{10000}$ is _____1000

(a)greater than (b)smaller than (c) equal to (d)none

Answer: a

Let,$(1.1)^{10000}=(1+0.1)^{10000}$

$={}^{10000}C_0+{}^{10000}C_1(1.1)$+other positive terms

=1+10000×1.1+other positive terms

=1+11000+ other positive terms

>1000

Hence, (1.1)10000>1000

10. The approximation of $(0.99)^5$ is

(a) 0.951 b) 0.195 (c)0.591 (d)0.519

Answer: a

$0.99)^5 = (1 - 0.01)^5$

$= {}^5C_0(1)^5 - {}^5C_1 (1)^4 (0.01) + {}^5C_2(1)^3 (0.01)^2$

Here, we can calculate the binomial coefficients ${}^5C_0$, ${}^5C_1$, ... using the nCr formula.

= 1 - 5(0.01) + 10(0.0001)

= 0.951

ASSERTION-REASON BASED QUESTIONS - SOLVED

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

a). Both A and R are true and R is correct explanation of A.

b). Both A and R are true but R is not the correct explanation of A.

c). A is true but R is false.

d). A is false but R is true

1: Assertion(A) : The sum of the coefficients in the expansion $(x+y)^n$ is 2^n

Reason (R): The binomial coefficients in the expansion of $(x+y)^n$ represent the elements in the nth row of Pascal's triangle.

Answer: (B)

2: Assertion (A): in the expansion $(x+y)^n$, the coefficient of $x^{n-k}y^k$ is given by $C(n,k)$

Reason(R): The binomial coefficient $C(n,k)$ is equal to $\frac{n!}{k!(n-k)!}$

Answer: B

3: ASSERTION: The value of $\sum_{r=0}^{n} n_{C_r} 5^r = 6^n$

REASON: The value of $\sum_{r=0}^{n} n_{C_r} x^r = (1+x)^n$

Answer: a

4: Assertion(A) : the number of terms in the expansion of $\{(3x+y)^8-(3x-y)^8\}$ is 4

Reason (R) : if n is even, then $((x+a)^n-(x-a)^n)$ has $\left(\frac{n}{2}\right)$ terms

Answer: a

5: Assertion (A) : The coefficient of the expansions of $(a+b)^n$ are arranged in an array. This array is called Pascal's triangle.

Reason (R): There are 11 terms in the expansion $\{(3x+4y)^{10}+(3x-4y)^{10}\}$

Answer: b

6. ASSERTION: The total number of terms in expansion of $(x^5+y^5)^5$ is 6.

REASON: The total number of terms in expansion of $(x+y)^n$ is n+1.

Answer: A

7 ASSERTION: If n is an odd prime, then integral part of $(5^{1/2}+2)^n$ is divisible by 20n.

REASON:: If n is prime, then ${}^n c_1, {}^n c_2, {}^n c_3 \ldots\ldots {}^n c_{n-1}$ must be divisible by n.

Answer :a

8. ASSERTION: In the expansion of $(1+x)^{30}$, greatest binomial coefficient is ${}^{30}C_{15}$

REASON: In the expansion of $(1+x)^{30}$, the binomial coefficients of equidistant terms from end & beginning are equal.

Answer :b

9. Let n be a positive integer.

ASSERTION: 3^{2n+2} -8n-9 is divisible by 64.

REASON: $3^{2n+2}-8n-9=(1+8)^{n+1}-8n-9$ and in the binomial expansion of $(1+8)^{n+1}$, sum of first two terms is 8n + 9 and after that each term is a multiple of 8^2..

Answer :a

10. ASSERTION: The binomial theorem provides an expansion for the expression $(a+b)^n$.where a, b, n ∈ R.

REASON : All coefficients in a binomial expansion may be obtained by Pascal's triangle.

Answer :d

VERY SHORT ANSWER TYPE QUESTIONS SOLVED

1: Expand $(2x-3y)^4$ by Binomial theorem

Answer: using

$$(x-a)^n = C(n.0)x^n - C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 - C(n,3)x^{n-3}a^3 + \cdots + (-1)^n C(n,n)a^n$$

$$=16x^4-96x^3y+216x^2y^2-216xy^3+81y^4$$

2: Expand $(x^2+2a)^5$ by Binomial theorem.

Answer: using

$$(x+a)^n = C(n.0)x^n + C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 + C(n,3)x^{n-3}a^3 + \cdots + C(n,n)a^n$$

$$=x^{10}+10x^8a+40x^6a^2+80x^4a^3+80x^2a^4+32a^5$$

3: Which is larger $(1.01)^{100000}$ or 10000

Answer : $(1.01)^{100000} = (1+0.01)^{100000}$

$$= C(100000{,}0) + C(100000.1)(0.01) + \textit{other positive terms}$$

$=1+100000(0.01)+ other\ positive\ terms = 1+10000+ other\ positive\ terms > 10000$

So, $(1.01)^{100000} > 10000$

4: Using binomial theorem prove that $6^n - 5n$ always leaves reminder 1 when divided by 25

Answer : $6^n = (1+5)^n = 1 + C(n,1)5 + C(n,2)5^2 + C(n,3)5^3 + \cdots + 5^n$

$= 1 + 5n + 5^2(C(n,2) + C(n,3)5 + \cdots + 5^{n-2})$

$6^n - 5n - 1 = 25k(where\ k = C(n,2) + C(n,3)5 + \cdots + 5^{n-2})$

Hence proved.

5. Expanding $\left(x - \frac{1}{x}\right)^6$,find the term which lies exactly at the middle of the expansion.

Answer : $\left(x - \frac{1}{x}\right)^6 = x^6 - 6x^4 + 15x^2 - 20 + 15\frac{1}{x^2} - 6\frac{1}{x^4} + \frac{1}{x^6}$

So,the middle term is -20

6. Find an approximation of $(0.99)^5$ using the first three terms of its expansion

Answer: 0.99 can be written as

0.99 = 1 – 0.01

Now by applying the binomial theorem, we get

$(o.99)^5 = (1 – 0.01)^5 = {}^5C_0 (1)^5 – {}^5C_1 (1)^4 (0.01) + {}^5C_2 (1)^3 (0.01)^2$

$= 1 – 5 (0.01) + 10 (0.01)^2 = 1 – 0.05 + 0.001 = 0.951$

7. Compute $(96)^4$ using binomial theorem

Answer : $(96)^4 = (100-4)^4$

$= 4_{C_0}100^4 4^0 - 4_{C_1}100^3 4^1 + 4_{C_2}100^2 4^2 - 4_{C_3}100^1 4^3 + 4_{C_4}100^0 4^4$

$= 100000000 - 16000000 + 960000 - 25600 + 256$

$= 84934656$

8. Using binomial theorem, find 11^5 .

Answer : $(10+1)^5 = 10^5 + 5.10^4 + 10.10^3 + 10.10^2 + 5.10^1 + 1$=161051

9. Prove that: $\sum_{k=0}^{n} C(n,r) \quad 3^r = 4^n$

Answer : $(1+3)^n = \sum_{r=0}^{n} C(n,r)3^r$

10. Evaluate : $(\sqrt{2} + 1)^5 + (\sqrt{2} – 1)^5$

Answer: Here, we can see $(x + y)^5 + (x – y)^5 = 2[{}^5C_0\ x^5 + {}^5C_2\ x^3 y^2 + {}^5C_4 xy^4]$

$= 2(x^5 + 10 x^3 y^2 + 5xy^4)$

Thus, $(\sqrt{2} + 1)^5 + (\sqrt{2} – 1)^5 = 2[(\sqrt{2})^5 + 10(\sqrt{2})^3(1)^2 + 5(\sqrt{2})(1)^4]$

$= 58\sqrt{2}$

SHORT ANSWER TYPE QUESTIONS SOLVED

1. Find m such that the coefficient of x^2 in expansion of $\left(mx + \frac{m}{x}\right)^6$ is 960

Answer : $\left(mx + \frac{m}{x}\right)^6 = m^6\left(x + \frac{1}{x}\right)^6$

Then, coefficient of x^2 is $m^6.6_{C_2}$

$m^6.6_{C_2} = 960$

$m^6 = \frac{960}{15} = 64$

$m = 2$

2. Show that $9^{n+1} - 8n - 9$ is divisible by 64 , whenever n is a positive integer

Answer : $(1+a)^n = \sum_{r=0}^{n} C(n,r)a^r$

putting a=8 and m = n+1 we get

$(1+8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$(9)^{n+1} = 9+8n+64({}^{n+1}C_2 + \ldots + {}^{n+1}C_{n+1}(8)^{n-1})$

So, $9^{n+1} - 8n - 9 = 64k$ .implies the expression is divisible by 64

3. If the expansion of $(1-ax)^n = 1 - 21x + 189x^2 - 945x^3 + \cdots$, then find a and n

Answer : $(1-ax)^n = 1 - nax + \frac{n(n-1)}{2}a^2x^2 - \frac{n(n-1)(n-2)}{6}a^3x^3 \ldots,$

Comparing $na = 21$, $\frac{n(n-1)}{2}a^2 = 189$

$n(n-1)a^2 = 378$

Thus, $(n-1)a = 18$

Also $\frac{n}{n-1} = \frac{7}{6}$

$n = 7$ and $a = 3$

4. Find the coefficient of x^5 in binomial expansion of $(1 + 2x)^6(1 - x)^7$

Answer : Using binomial theorem we will expand both the terms.

We know that,

$(x+y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + {}^nC_3x^{n-3}y^3 + \ldots + {}^nC_rx^{n-r}y^r + \ldots + {}^nC_nx^0y^n$

Applying the formula we get,

$(1 + 2x)^6 (1 - x)^7$

$= (1 + {}^6C_1(2x) + {}^6C_2 (2x)^2 + {}^6C_3 (2x)^3 + {}^6C_4 (2x)^4 + {}^6C_5 (2x)^5 + {}^6C_6 (2x)^6)$ *

$(1 - {}^7C_1x + {}^7C_2 (x)^2 - {}^7C_3 (x)^3 + {}^7C_4 (x)^4 - {}^7C_5 (x)^5 + {}^7C_6 (x)^6 - {}^7C_7 (x)^7) =$

$(1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6) (1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 +$

$7x^6 - x^7)$

Clearly, it can determined that the coefficient of x^5 is

⇒ 1 * (-21) + 12 * 35 + 60(-35) + 160 * 21 + 240 * (-7) + 192 * 1

⇒ 171

Therefore, the coefficient of x^5 in $(1 + 2x)^6 . (1 - x)^7$ is 171.

5. Find the last two digits of the number $(13)^{10}$

Sol:

$(13)^{10} = (169)^5 = (170 - 1)^5$

$= 5C_0 (170)^5 - 5C_1 (170)^4 + 5C_2 (170)^3 - 5C_3 (170)^2 + 5C_4 (170) - 5C_5$

$= 5C_0 (170)^5 - 5C_1 (170)^4 + 5C_2 (170)^3 - 5C_3 (170)^2 + 5(170) - 1$

A multiple of 100 + 5(170) – 1 = 100K + 849

∴ The last two digits are 49.

6. Show that $2^{3n}-7n-1$ is divisible by 49, where n is a positive integer.

Answer : Given $2^{3n}-7n-1 = 8^n-7n-1$

$=(1+7)^n-7n-1$

$={}^nC_0+{}^nC_1\,7+{}^nC_27^2+\ldots+{}^nC_n7^n -7n-1$

$=1+7n+{}^nC_27^2+\ldots+{}^nC_n7^n -7n-1$

$={}^nC_27^2+\ldots+{}^nC_n.7^n$

$=49({}^nC_2+\ldots+{}^nC_n.7^{n-2})$

Which is divisible by 49

7. If a and b are distinct integers, prove that $a^n - b^n$ is divisible by a- b whenever $n \in N$

Answer : $a^n - b^n = \{(a-b)+b\}^n$

$$a^n - b^n = (a-b)\left\{(a-b)^{n-1} + \overset{n}{\underset{1}{c}}(a-b)^{n-2}b + \dots\dots\dots + \overset{n}{\underset{n-1}{c}}\, b^{n-1}\right\}$$

Clearly RHS is divisible by a – b

8. Show that $2^{4n} - 15n - 1$ is divisible by 225.

Ans : $2^{4n} = (2^4)^n = (16)^n = (1 + 15)^n$

$\therefore 2^{4n} = 1 + {}^nC_1\, 15 + {}^nC_2\, 15^2 + \dots\dots + {}^nC_n\, 15^n$

$\therefore 2^{4n} - 1 - 15n = 15^2\, [{}^nC_2 + {}^nC_3.15 + \dots\dots\dots + {}^nC_n\, 15^{n-2}] = 225\, k.$

Where k is an integer.

Hence $2^{4n} - 15n - 1$ is divisible by 225.

9. Find the number of terms in the expansion of the following : $(1 + 5\sqrt{2}\,x)^9 + (1 - 5\sqrt{2}\,x)^9$

Answer : Number of terms = (9 +1)/ 2 = 5

10. Prove that $12^n - 11n - 1$ is divisible by 121 For all n∈N by using binomial theorem

Answer :

$$12^n = (1+11)^n = n_{C_0}11^0 + n_{C_1}11^1 + n_{C_2}11^2 + \dots + n_{C_{n-1}}11^{n-1} + n_{C_n}11^n$$

$$= 1 + n11 + n_{C_2}11^2 + \dots + n_{C_{n-1}}11^{n-1} + 11^n$$

$12^n - n11 - 1 = 11^2\left(n_{C_2} + n_{C_3}11 \dots + n_{C_{n-1}}11^{n-3} + 11^{n-2}\right)$ divisible by 11^2 $i.e.$ 121.

LONG ANSWER TYPES QUESTIONS SOLVED

1. The sum of coefficients of the first three terms of the expansion of $\left(x - \frac{3}{x^2}\right)^m$ is 559, where m is a natural number. Find the number of terms in the expansion.

Answer: Here, coefficient of 1st three terms is $C(m,0), -3C(m,1), 9C(m,2)$+…

According to the question, $C(m,0) - 3C(m,1) + 9C(m,2) = 559$

$$1 - 3m + \frac{9m(m-1)}{2} = 559$$

On solving we get m=12

So, number of terms in the expansion are 13

2. If P be the sum of odd terms and Q that of even terms in the expansion $(a+b)^n$

Prove that

i. $P^2 - Q^2 = (x^2 - a^2)^n$

ii. $4PQ = (x+a)^{2n} - (x-a)^{2n}$

Answer: $(x+a)^n = C(n.0)x^n + C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 + C(n,3)x^{n-3}a^3 + \dots + C(n,n)a^n$

$$= t_1 + t_2 + t_3 + \dots + t_{n+1}$$

$$= (t_1 + t_3 + t_5 + \dots) + (t_2 + t_4 + t_6 + \dots)$$

$$(x+a)^n = P + Q - - - - - (i)$$

$$(x-a)^n = C(n.0)x^n - C(n,1)x^{n-1}a + C(n,2)x^{n-2}a^2 - C(n,3)x^{n-3}a^3 + \dots + (-1)^n C(n,n)a^n$$

$$= t_1 - t_2 + t_3 - t_4 + \dots$$

$= (t_1 + t_3 + t_5 + \dots) - (t_2 + t_4 + t_6 + \dots)$

$$(x-a)^n = P - Q - - - - - (ii)$$

Multiply (i) with (ii), we get

$$P^2 - Q^2 = (x^2 - a^2)^n$$

by squaring (i) and (ii) and subtract, we get

$4PQ = (x+a)^{2n} - (x-a)^{2n}$

3. Find the remainder when 3^{3n}-26n is divided by 676, where n∈ N

Solution: Using Binomial theorem:

$$(1+x)^n = C(n.0) + C(n,1)x + C(n,2)x^2 + C(n,3)x^3 + \dots + C(n,n)x^n$$

. $3^{3n}-26n=27^n-26n$

$(1+26)^n = {}^nC_0 + {}^nC_1\,(26) + {}^nC_2(26)^2 + \dots\dots + {}^nC_n(26)^n - 26n$

$= 1+26n+(26)^2 \{ {}^nC_2(26)+........+{}^nC_n(26)^{n-2} \}-26n$

$3^{3n}-26n = 1+676\{ {}^nC_2(26)+........+{}^nC_n(26)^{n-2} \}$

There fore, when $3^{3n}-26n$ is divided by 676 ,we get remainder is 1

4. Show that $2^{4n+4} - 15n - 16$, where n ∈ N is divisible by 225.

Answer: Use the binomial theorem to expand $2^{4n+4} = (2^4)^{n+1}, = (16)^{n+1} = (1+15)^{n+1}$

Now $(1+15)^{n+1}-15n-16$

$= {}^{n+1}C_0+{}^{n+1}C_1(15)+{}^{n+1}C_2(15)^2+........+{}^{n+1}C_{n+1}(15)^{n+1} -15n-16$

$= 1+15(n+1)+(15)^2 \{ {}^{n+1}C_2(15)+........+{}^{n+1}C_{n+1}(15)^{n-2} \}-15n-16$

$=225 \{ {}^{n+1}C_2(15)+........+{}^{n+1}C_{n+1}(15)^{n-2} \}$ which is divisible by 225

CASE STUDY BASED QUESTIONS SOLVED

1. A bakery sells cookies in packs. Each pack contains either chocolates chips or oatmeal cookies. The owner wants to find out the different combinations of chocolate chip and oatmeal cookies in a pack of 5 cookies

(i) How many different combinations of chocolate chip and oatmeal cookies can be made in a pack of 5 cookies?

(ii) What is the probability of having exactly 3 chocolate chip cookies in a pack of 5 cookies?

Answer:

(i)The number of combinations corresponds to the coefficients in the expansion $(x + y)^5$, where x represents chocolate chips and y represents oatmeal cookies. The number of different combinations is given by the 5th row of Pascal's triangle 1,5,10,10,5,1. Therefore there are 6 different combinations.

(ii) The coefficient of x^3y^2 in the expansion $(x + y)^5$ gives the number of ways to have 3 chocolate chip and 2 oatmeal cookies. The coefficient $C(5,3) = 10$.

The total number of combinations is $2^5 = 32$.

Therefore, the probability is $\frac{10}{32}$

2. Four friends applied the knowledge of Binomial Theorem while playing a game to make the equations by observing some conditions they make some equations.

(a) Expand, $(1 – x + x^2)^4$

(b) Expand the expression, $(1 – 3x)^7$.

OR

Show that $11^9 + 9^{11}$ is divisible by 10.

Ans. (a) We have, $(1 – x + x^2)^4 = [(1 – x) + x^2]^4$

$= {}^4C_0(1 – x)^4 + {}^4C_1(1 – x)^3 (x^2) + {}^4C_2(1 – x)^2 (x^2)^2 + {}^4C_3(1 – x)(x^2)^3 + {}^4C_4(x^2)^4$

$= (1 – x)^4 + 4x^2(1 – x)^3 + 6x^4(1 – x)^2 + 4x^6(1 – x) + 1.x^8$

$= (1 – 4x + 6x^2 – 4x^3 + x^4) + 4x^2(1 – 3x + 3x^2 – x^3) + 6x^4(1 – 2x + x^2) + 4(1 – x)x^6 + x^8$

$= 1 – 4x + 6x^2 – 4x^3 + x^4 + 4x^2 – 12x^3 + 12x^4 – 4x^5 + 6x^4 – 12x^5 + 6x^6 + 4x^6 – 4x^7 + x^8$

$= 1 – 4x + 10x^2 – 16x^3 + 19x^4 – 16x^5 + 10x^6 – 4x^7 + x^8$

(b) Here, a = 1, b = 3x, and n = 7

Given, $(1 – 3x)^7$

$= {}^7C_0(1)7 – {}^7C_1(1)^6(3x)^1 + {}^7C_2(1)^5(3x)^2 – {}^7C_3(1)^4(3x)^3 + {}^7C_4(1)^3(3x)^4 – {}^7C_5(1)^2(3x)^5 + {}^7C_6(1)^1(3x)^6 – {}^7C_7(1)^0(3x)^7$

$= 1 – 21x + 189x^2 – 945x^3 + 2835x^4 – 5103x^5 + 5103x^6 – 2187x^7$.

OR

$11^9 + 9^{11} = (10 + 1)^9 + (10 – 1)^{11} = ({}^9C_0.10^9 + {}^9C_1.10^8 + ... {}^9C_9) + ({}^{11}C_0.10^{11} – {}^{11}C_1.10^{10} + ... – {}^{11}C_{11})$

$= {}^9C_0.10^9 + {}^9C_1.10^8 + ... + {}^9C_8.10 + 1 + 10^{11} – {}^{11}C_1.10^{10} + ... + {}^{11}C_{10}.10 – 1$

$= 10[{}^9C_0.10^8 + {}^9C_1.10^7 + ... + {}^9C_8 + {}^{11}C_0.10^{10} – {}^{11}C_1.10^9 + ... + {}^{11}C_{10}]$

= 10 K, which is divisible by 10.

CHAPTER - 8: SEQUENCES AND SERIES

DEFINITIONS AND FORMULAE

Sequence: A sequence is a succession of numbers arranged in a definite order and formed according to a definite law. The elements of a sequence are called its terms.

Functional definition: A sequence is a special class of function whose domain is set N of all natural numbers and range is any set of numbers.

Finite Sequence: Sequence containing finite number of terms or whose terms are countable is called a finite sequence

Series: If the terms of a sequence are combined writing + (plus) sign between them, they form a series.

Progression: When a sequence or a series is written with special conditions or laws, it forms a progression. The progressions are classified into Arithmetic Progression (A.P.), Geometric Progression (G.P.) and Harmonic progression (H.P.) etc.

Arithmetic Progression: Such as sequences or series in which each term exceeds its previous term by a constant number is called Arithmetic Progression.

Arithmetic Mean: In a arithmetic progression, all the terms lying between any two of its terms are known as arithmetic mean between these terms.

If A be the A.M. of a and b, then a, A and b will be in A.P. and A.M. = $\frac{a+b}{2}$

Geometric Progression: Geometric progression is that in which each term except the 1st can be obtained by multiplying its preceding term by a definite number this definite number is called common ratio.

$a, ar, ar^2, ar^3, ar^4, ar^5, \ldots\ldots\ldots \; a r^{n-1}$

Geometric Mean: When three numbers are in G.P. the middle term is the geometric mean of the other two. If G is the G.M. of a and b, then G = $\sqrt{ab}$

Formulae

Sum of n terms of G.P. $\quad S_n = \frac{a(r^n-1)}{r-1}$ if $r > 1$

$S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$

Sum of terms of an infinite G.P. $\quad S_n = \frac{a}{1-r}$ provided $|r| < 1$

MULTIPLE CHOICE QUESTIONS SOLVED

Q 1 Every term of a G.P. is a positive and also every term is the sum of two preceding terms. Then the common ratio of the G.P. is

(a) $\frac{1-\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{2}$ (c) $\frac{-1+\sqrt{5}}{2}$ (d) 1

Ans: (b)

Solution: Let a be the first term and r be the common ratio of the G.P. Then, a > 0 and r > 0.

Now, $a_n = a_{n-1} + a_{n-2}$, $n > 2$

$ar^{n-1} = ar^{n-2} + ar^{n-3} \Rightarrow r^2 = r + 1$

$r^2 - r - 1 = 0 \Rightarrow r = \frac{1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{1+\sqrt{5}}{2} \quad [r > 0]$

Q 2 For the sequence a_n general term is $a_n = \frac{(n-1)(n-2)}{(n+5)}$, then its term at places greater than 2 are all

(a) positive (b) zeroes (c) natural numbers (d) negative

Ans: (a)

Solution: As n − 1 > 0 and n −2 > 0 for n > 2

Q 3 Third term of the sequence a_n whose n^{th} term is $a_n = \frac{n+\frac{1}{2}}{n}$ is

(a) 3 (b) $\frac{5}{4}$ (c) $\frac{9}{8}$ (d) $\frac{7}{6}$

Ans: (d)

Solution: $a_3 = \frac{3+\frac{1}{2}}{3} = \frac{7}{6}$

Q 4 Four arithmetic means inserted between two given numbers, then third arithmetic mean is at

(a) 2^{nd} place (b) 3^{rd} place (c) 4^{th} place (d) 5^{th} place

Ans: (c)

Solution: suppose a and b are two given numbers, then 4 A.M. between them are a A_1, A_2, A_3, A_4 b. A_3 is at 4th place

Q 5 If arithmetic means between a and b, p and q are equal then

(a) ab = pq (b) aq = bp (c) a+p = b+q (d) a-q = p-b

Solution: (d)

Solution: $\frac{a+b}{2} = \frac{p+q}{2}$

Q 6 If x, 2y, 3z are in A.P. , where the distinct numbers x, y, z are in G.P., then common ratio of the G.P. is

(a) 3 (b) $\frac{1}{3}$ (c) 2 (d) $\frac{1}{2}$

Ans: (b)

Solution: Since x, 2y, 3z are in AP, so 4y = x+3z (i)

Also x, y, z are in GP y = x r and z = y r (ii)

From (i) and (ii) we get $3r^2-4r+1 = 0$

Solving this equation we get $r = \frac{1}{3}$ and r = 1 (not possible)

Q 7 If x, y, z are distinct positive integers then the value of (x+y) (y+z) (z+x) is

(a) =8xyz (b) < 8xyz (c) >8xyz (d) = 4xyz

Ans: (c)

Solution: $\frac{x+y}{2} > \sqrt{xy}$, $\frac{z+y}{2} > \sqrt{zy}$, $\frac{x+z}{2} > \sqrt{xz}$

$(x+y)(y+z)(z+x) > 2\sqrt{xy}\ 2\sqrt{zy}\ 2\sqrt{xz} \implies (x+y)(y+z)(z+x) > 8xyz$

Q 8 The length of three unequal edges of a rectangular solid block are in G.P. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. The length of the longest side is

(a) 12 cm (b) 6 cm (c) 18cm (d) 3 cm

Ans: (a)

Solution: Let the lengths be $\frac{a}{r}$, a, ar

Volume of block is $\frac{a}{r} \times a \times ar = 216 \implies a^3 = 6^3 \implies a = 6$ cm.

Given that total surface is 252 cm^2.

$2\left(\frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r}\right) = 252 \implies r = 2, \frac{1}{2}$

Hence the lengths are 3, 6 and 12 cm.

Q 9 In a G.P. of even number of terms, the sum of all the terms is 5 times the sum of odd terms, the common ration of the G.P. is.

(a) $-\frac{4}{5}$ (b) $\frac{1}{5}$ (c) 4 (d) none of these

Ans: (c)

Solution: Let $a, ar, ar^2, \ldots\ldots, ar^{2n-1}$ be a G.P. with 2n terms. Let the sum of all its terms and odd terms be S_1 and S_2 respectively.

It is given that $S_1 = 5\,S_2 \implies (a + ar + ar^2 + \dots + ar^{2n-1}) = 5\,(a + ar^2 + ar^4 \dots + ar^{2n-2})$

Using sum of n terms formula and solving we get $r = 4$

Q 10 The minimum value of the expression $3^x + 3^{1-x}$, $x \in R$, is

(a) 0 (b) $\frac{1}{3}$ (c) 3 (d) $2\sqrt{3}$

Ans: (d)

Solution: Using AM ≥ GM, we obtain

$$\frac{3^x + 3^{1-x}}{2} \geq \sqrt{3^x \times 3^{1-x}} \implies \frac{3^x + 3^{1-x}}{2} \geq \sqrt{3}$$

Hence, after solving this equation, the minimum value of $3^x + 3^{1-x}$ is $2\sqrt{3}$.

ASSERTION – REASON BASED QUESTIONS SOLVED

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
B) Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion (A).
C) Assertion (A) is true but Reason(R) is false.
D) Assertion (A) is false but Reason(R) is true.

Q 1 Assertion (A): The minimum value of $4^x + 4^{1-x}, x \in R$ is 4.

Reason (R): A.M.≥ G.M.

Ans: A)

Solution: $\frac{4^x + 4^{1-x}}{2} \geq \sqrt{4^x \times 4^{1-x}}$

$$\frac{4^x + 4^{1-x}}{2} \geq 2 \implies 4^x + 4^{1-x} \geq 4$$

So Assertion is true & Reason is correct explanation.

Q 2 Assertion (A): If for any real x, $2^{1+x} + 2^{1-x}, \lambda$ and $3^x + 3^{-x}$ are three equidistant terms of an A.P., then $\lambda \geq 3$

Reason (R): A.M.≥ G.M.

Ans: A)

Solution: Assertion;

$2^{1+x} + 2^{1-x}, \lambda$ and $3^x + 3^{-x}$ are in A.P. $\implies 2\lambda = 2^{1+x} + 2^{1-x} + 3^x + 3^{-x}$

$= 2(2^x + 2^{-x}) + 3^x + 3^{-x}$

A.M. ≥ G.M.

$2^x + 2^{-x} \geq 2$, $3^x + 3^{-x} \geq 2$

$2\lambda \geq 4+2 \implies \lambda \geq 3$

Q 3 Assertion (A): If x > 1, the sum to infinite series

$$1+3\left(1-\frac{1}{x}\right)+5\left(1-\frac{1}{x}\right)^2+7\left(1-\frac{1}{x}\right)^3+\dots\dots\dots \text{ is } x^2 - x$$

Reason (R): If $0 < y < 1$, the sum of the series $1+3y+5y^2+7y^3+\dots\dots$ Is $\frac{1+y}{(1-y)^2}$.

Ans: A)

Solution:

$S = 1+3y+5y^2+7y^3+\dots\dots$

$y\,S = y+3y^2+5y^3+\dots\dots\dots\dots$

$S(1-y) = 1+2y+2y^2+2y^3+\dots\dots$

$S(1-y) = 1+\frac{2y}{1-y} \implies S = \frac{1+y}{(1-y)^2}$

So, Reason is true

Putting $y = 1-\frac{1}{x}$ in the above sum, we get $S = x^2 - x$

Assertion is also true and Reason(R) is correct explanation of (A)

Q 4 Assertion (A): A sequence is said to be finite if it has finite numbers of terms.

Reason (R): The n^{th} term of the sequence: 2, 2, $\frac{8}{3}$, 4 ………… Is $\frac{2^n}{n}$.

Ans: B)

Q 5 Assertion (A): The sum of infinite terms of G.P. is given by $S_\infty = \frac{a}{1-r}$, provided |r | < 1.

Reason (R): The sum of n terms of G.P. is $S_n = \frac{a(r^n-1)}{r-1}$

Ans: B)

Q 6 Assertion (A): The value of k for which $\frac{3}{4}$, k , $\frac{4}{3}$ are in G.P. is 1 only.

Reason (R): If x, y, z are in G.P., then $y^2 = x z$

Ans: D)

Solution: $\frac{3}{4}$, k , $\frac{4}{3}$ are in G.P.

$$k^2 = \frac{3}{4} \times \frac{4}{3} \implies k = \pm 1$$

The assertion that the value of k is only 1 is incorrect.

Q 7 Assertion (A): The sequence 8, 4, 2, 1, 0.5, ... is a Geometric Progression.

Reason (R): A sequence is said to be a GP if the ratio of any term to it's immediately preceding term is constant throughout the sequence.

Ans: A)

Solution: The sequence has terms 8, 4, 2, 1, ... The ratio $\frac{4}{8} = \frac{1}{2}, \frac{2}{4} = \frac{1}{2} = \frac{1}{2}$. Since the ratio between consecutive terms is constant ($r = \frac{1}{2}$), the sequence is a GP. The reason R correctly states the definition of a GP, which justifies the Assertion (A).

Q 8 Assertion (A): The sum of the infinite geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ is $\frac{3}{2}$.

Reason (R): The sum of an infinite geometric series exists if the absolute value of the common ratio |r| is less than 1, and the sum is given by $S = \frac{a}{1-r}$

Ans: (A)

Solution: Here, the first term a = 1 and the common ratio $r = \frac{1}{3}$. Since $|r| = |\frac{1}{3}| = \frac{1}{3} < 1$, the sum exists. Using the formula $S = \frac{a}{1-r}$ the sum is $S = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$. Assertion (A) is true.

Reason (R) correctly states the condition for the existence of the sum of an infinite GP and provides the correct formula, which explains A.

Q 9 Assertion (A): The geometric mean between 5 and 125 is 25.

Reason (R): The geometric mean (GM) between two positive numbers a and b is given by $\sqrt{ab}$

Ans: A)

Solution: GM = $\sqrt{5 \times 125} = 25 \implies$ GM = $\sqrt{ab}$

Q 10 Assertion (A): For any two positive numbers arithmetic mean is greater than or equal to Its geometric mean.

Reason (R): For the given numbers a and b , $(a-b)^2 \geq 0$

Ans: A)

Solution: $(a-b)^2 \geq 0 \implies a^2 - 2ab + b^2 \geq 0$

$a^2 + 2ab + b^2 \geq 4ab \implies (a+b)^2 \geq 4ab$

$(a+b) \geq 2\sqrt{ab} \implies \frac{(a+b)}{2} \geq \sqrt{ab}$

VERY SHORT ANSWER TYPE QUESTIONS SOLVED

Q 1 Find the first two terms of the sequence whose n^{th} term is given by $a_n = \begin{cases} \frac{1}{n}, if\ n\ is\ even \\ \frac{2}{n}, if\ n\ is\ odd \end{cases}$

Solution: n is odd then n=1 $\Rightarrow a_1 = \frac{2}{1}$

n is even then n=2 $\Rightarrow a_2 = \frac{1}{2}$

Q 2 n AM's are inserted between 3 and 17 .The last mean is three times the first mean. Find n.

Solution: Let n AM's $A_1, A_2, A_3, \ldots\ldots A_n$ are inserted

Also $A_n = 3A_1 \Rightarrow 17 - d = 3(3+d) \Rightarrow d = 2$

$a_n = a + (n-1)d \Rightarrow 17 = 3 + (n+2-1)2$

n = 6

Q 3 Find the value of K for which $\frac{-2}{7}, K, \frac{-7}{2}$ are in G.P.

Solution: $\frac{-2}{7}, K, \frac{-7}{2}$ are in G.P

Then $K^2 = \frac{-2}{7} \times \frac{-7}{2} \Rightarrow K = \pm 1$

Q 4 Show that the sum of 500 AM's between $\frac{3}{2}$ and $\frac{7}{2}$ is 1250.

Solution: Since sum of n AM's between a and b = n $\left(\frac{a+b}{2}\right) = 500\left(\frac{\frac{3}{2}+\frac{7}{2}}{2}\right) = 1250$

Q 5 Find 8[th] term of G.P. 0.3, 0.06, 0.012, ……….

Solution: here $r = \frac{0.06}{0.3} = 0.2$

$a_8 = ar^7 \Rightarrow a_8 = 0.3\,(0.2)^7$

Q 6 Which term of the G.P. 2, $2\sqrt{2}$, 4, ……… Is 8?

Solution: $a_n = ar^{n-1} \Rightarrow 8 = 2(\sqrt{2})^{n-1}$

$4 = (\sqrt{2})^{n-1} \Rightarrow (\sqrt{2})^4 = (\sqrt{2})^{n-1}$

$\Rightarrow$ n = 5

Q 7 Find the sum $\sqrt{2} + 2 + 2\sqrt{2} + \ldots\ldots\ldots$ up to 8 terms.

Solution: $S_n = a\left(\frac{r^n - 1}{r-1}\right)$

$= \frac{\sqrt{2}\left((\sqrt{2})^8 - 1\right)}{\sqrt{2}-1} = 15(2+\sqrt{2})$

Q 8 The 4[th], 8[th] and 11[th] terms of a G.P. are p q and s respectively. Show that $q^2 = ps$

Solution: $ar^4 = p$, $ar^7 = q$ & $ar^{10} = s$

$\therefore\ q^2 = (ar^7)^2 = ar^4 \times ar^{10} \Rightarrow q^2 = p\,s$

Q 9 If A.M. and G.M. of roots of a quadratic equation are 8 and 5 respectively then obtain the quadratic equation.

Solution: Let a and b are the roots of required quadratic equation

Equation having roots a and b is $x^2 - (a+b)x + ab = 0$

a+b = 16 and ab = 25

required equation is $x^2 - 16x + 25 = 0$

Q 10 The first term of a G.P. is 2 and the sum of infinite terms is 6. Find the common ratio.

Solution: $s_\infty = \frac{a}{1-r}$

$6 = \frac{2}{1-r} \Rightarrow r = \frac{2}{3}$

SHORT ANSWER TYPE QUESTIONS SOLVED

Q 1 For the sequence a_n, $a_1 = \frac{1}{2}$, $a_2 = -3$ and $a_n = \frac{a_{n-1}}{a_{n-1}+\frac{1}{3}} - 3$, n>2 find $\frac{a_3}{a_4}$

Solution: n = 3, $a_3 = \frac{a_1}{a_2+\frac{1}{3}} - 3 = \frac{\frac{1}{2}}{-3+\frac{1}{3}} - 3 = -\frac{51}{16}$

n=4, $a_4 = \frac{a_2}{a_3+\frac{1}{3}} - 3 = \frac{-3}{-\frac{51}{16}+\frac{1}{3}} - 3 = -\frac{267}{137}$

$\frac{a_3}{a_4} = \frac{2329}{1424}$

Q 2 Insert 3 Arithmetic mean between 2 and 10.

Solution: Let three A.M. are A_1, A_2 and A_3

Then 2, A_1, A_2, A_3, 10 are in A.P.

$\Longrightarrow$ 2+ 4d = 10 $\Longrightarrow$ d = 2

Now, A_1= a+d =2+2=4

$A_2 = A_1$ +d=4+2=6

A_3= A_2+d=6+2=8

Q 3 Find the sum of the product of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Solution: required sum = $2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$

= 256 + 128 + 64 + 32 + 16

= $2^8 + 2^7 + 2^6 + 2^5 + 2^4$

$= \frac{2^8\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\frac{1}{2}} = 496$

Q 4 If the first and the n^{th} term of a G.P. are a and b respectively and p is the product of n terms , prove that $p^2 = (ab)^n$

Solution: First term = a

n^{th} term = b $\Longrightarrow$ $a r^{n-1} = b$

Also $p = a \times ar \times ar^2 \times ar^3 \times$ …..n terms

$= a^n r^{1+2+3+4+ \ldots\ldots (n-1) \text{ terms}} \Longrightarrow p = a^n r^{\frac{n(n-1)}{2}}$

$P^2 = a^{2n} r^{n(n-1)} \Longrightarrow P^2 = (aar^{n-1})^n$

$P^2 = (ab)^n$

Q 5 Find three numbers in G.P. whose sum is 7 and product is 8.

Solution: Let three terms in G.P. are $\frac{a}{r}, a, ar$

$\frac{a}{r} \times a \times ar = 8 \Longrightarrow a=2$

$\frac{a}{r} + a + ar = 7$ (i)

Putting the value of a in equation (i) we get an equation

$2r^2-5r+2 = 0 \Longrightarrow r = \frac{1}{2}, 2$

Required numbers are 4, 2, 1 or 1, 2, 4

Q 6 A.M. between two positive numbers is 34 and their G.M. is 16 .Find the numbers.

Solution: Let the numbers be a and b

A.M. = $\frac{a+b}{2} = 34 \Longrightarrow a + b = 68$ (i)

G.M. = $\sqrt{ab} = 16 \Longrightarrow ab = 256$ (ii)

$(a-b)^2 = (a+b)^2 - 4ab$

$= (68)^2 - 4 \times 256$

$(a-b) = 60$ (iii)

Solving equation (i) and (ii) we get $a = 64$ and $b = 4$

Two numbers are 64, 4

Q 7 Find the value of $0.4\overline{37}$

Solution: $0.4\overline{37} = 0.437373737\ldots\ldots\ldots\ldots\ldots\ldots\infty$

$= 0.4 + 0.037 + 0.00037 + \ldots\ldots\ldots\ldots\infty$

$= \frac{4}{10} + \frac{37}{1000} + \frac{37}{100000} + \ldots\ldots\ldots\ldots\infty$

$= \frac{4}{10} + 37(\frac{1}{10^3} + \frac{1}{10^5} + \cdots \ldots \ldots . \infty)$

$= \frac{4}{10} + 37\left(\frac{\frac{1}{1000}}{1-\frac{1}{100}}\right)$

$= \frac{4}{10} + \frac{37}{1000} \times \frac{100}{99} = \frac{433}{990}$

Q 8 Determine the number of terms of a G.P. if $a_1 = 3$, $a_n = 96$, $S_n = 189$

Solution: Let r be the common ratio

$a_n = 96$, $S_n = 189$

$a\, r^{n-1} = 96$, $Sn = \frac{a\,(r^n - 1)}{r-1} = 189$

$3r^{n-1} = 96$, $\frac{3\,(r^n-1)}{r-1} = 189$

$r^{n-1} = 32$ (i) $\frac{(r^n-1)}{r-1} = 63$ (ii)

From equation (i) and (ii) r = 2

Putting the value of r in equation (i) we get n = 6

Q 9 Show that the ratio of the sum of first n terms of a G.P. to the sum of terms $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Solution: Let a, r be the first term and common ratio of G.P.

$$S_n = \frac{a(1-r^n)}{1-r}$$

Sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term,

$$S'_n = ar^n + ar^{n+1} + ar^{n+2} + \ldots\ldots\ldots + ar^{2n-1}$$

$$= \frac{ar^n(1-r^n)}{1-r}$$

$$\frac{S_n}{S'_n} = \frac{\frac{a(1-r^n)}{1-r}}{\frac{ar^n(1-r^n)}{1-r}} = \frac{1}{r^n} \quad \text{Hence Proved}$$

Q 10 Find the sum of the G.P. 3+6+12+24+………+1536.

Solution: a=3, r= 6÷3=2

$a_n = ar^{n-1}$

$1536 = 3 \times 2^{n-1} \Rightarrow 2^{10} = 2^n \Rightarrow n = 10$

$S_n = \frac{a(r^n-1)}{r-1} \Rightarrow S_n = \frac{3(2^{10}-1)}{2-1} = 3069$

LONG ANSWER TYPE QUESTIONS SOLVED

Q 1 Find the sum of the series: 0.7+0.77+0.777+0.7777+….. …….. to n terms

Solution: S = 7(0.1+0.11+0.111+0.1111+ …………….to n terms)

$= \frac{7}{9}(0.9 + 0.99 + 0.999 + 0.9999 + \cdots \ldots \ldots \text{to } n \text{ terms})$

$= \frac{7}{9}\left\{\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \left(1 - \frac{1}{10000}\right) + \cdots \text{..to } n \text{ terms}\right\}$

$= \frac{7}{9}\left\{\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \left(1 - \frac{1}{10^4}\right) + \cdots \text{..to } n \text{ terms}\right\}$

$= \frac{7}{9}\left\{n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \frac{1}{10^4} + \cdots \ldots \text{to } n \text{ terms}\right)\right\}$

$= \frac{7}{9}\left\{n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{\left(1-\frac{1}{10}\right)}\right\}$

$= \frac{7}{81}\left(9n - 1 + \frac{1}{10^n}\right)$

Q 2 What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually.

Solution: We have principal = Rs 500, Rate =10%

Amount at the end of first year = $P\left(1+\frac{R}{100}\right)$

Amount at the end of second year = $P\left(1+\frac{R}{100}\right)+P\left(1+\frac{R}{100}\right)\frac{R}{100}$

$= P\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$

$= P\left(1+\frac{R}{100}\right)^2$

Amount at the end of third year = $P\left(1+\frac{R}{100}\right)^2+P\left(1+\frac{R}{100}\right)^2\frac{R}{100}$

$= P\left(1+\frac{R}{100}\right)^3$

and so on

We find that amounts at the end of various year form a G.P.

Amount at the end of 10^{th} year = 11^{th} term of G.P. = $P\left(1+\frac{R}{100}\right)^{10}$

$= 500\left(1+\frac{10}{100}\right)^{10}$

$= 500\left(\frac{11}{10}\right)^{10}$

Q 3 Find the natural number 'a' for which $\sum_{p=1}^{n} f\,(a+p) = 16\,(2^n-1)$, where the function '$f$' satisfies the relation $f(x+y) = f(x).f(y)$ ∀ natural numbers x, y and $f(1) = 2$.

Solution: Since $f(x+y) = f(x)\,.\,f(y)$

Put x = 1, y =1

$f(1+1) = f(1)\,.\,f(1) = 2\,.\,2 = 2^2$

$f(2) = 2^2$

$f(3) = f(2+1) = f(2)\,.\,f(1) = 2^2\,.\,2 = 2^3$

$f(3) = 2^3$

Continuing in this way, $f(n) = 2^n \quad \forall \quad n \in N$

Now $\sum_{p=1}^{n} f\,(a+p) = 16\,(2^n-1)$

$f(a+1) + f(a+2) + f(a+3) + \ldots\ldots\ldots + f(a+n) = 16\,(2^n-1)$

$2^{a+1} + 2^{a+2} + 2^{a+3} + \ldots\ldots\ldots\ldots + 2^{a+n} = 16\,(2^n-1)$

G.P., first term = 2^{a+1}, r = 2

$\frac{2^{a+1}\,(2^n-1)}{2-1} = 16\,(2^n-1)$

$2^{a+1}\,(2^n-1) = 16\,(2^n-1) \implies 2^{a+1} = 16$

$2^{a+1} = 2^4 \implies a+1 = 4 \implies a = 3$

Q 4 If A and G be the A.M. and G.M. respectively between two positive numbers prove that the Numbers are $\pm\sqrt{(A+G)(A-G)}$.

Solution: Let a, b be two positive numbers such that $A = \frac{a+b}{2}$ and G^2 =ab

We know that quadratic equation having roots as a and b can be given as

$x^2 - (a+b)\,x + ab = 0$

$\implies \quad x^2 - 2Ax + G^2 = 0$

$x = \frac{2A \pm \sqrt{4A^2-4G^2}}{2}$

$x = A \pm \sqrt{A^2-G^2}$ or $A \pm \sqrt{A(+G)(A-G)}$

Q 5 The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Solution: Let the numbers in G.P. be a, ar, ar^2. It is given that the sum of these numbers is 56.

∴ $a + ar + ar^2 = 56$ ... (i)

It is also given that

$a-1, ar-7$ and ar^2 - 21 are in A.P.

$2(ar-7)=(a-1)+(ar^2-21) \Rightarrow 2ar = a+ar^2-8 \Rightarrow a+ar^2 = 2ar+8$ ... (ii)

From (i), we obtain

$$a+ar^2 = 56-ar \text{ ... (iii)}$$

Substituting $a+ar^2 = 56-ar$ on the LHS of (ii), we get

$$2ar+8 = 56-ar \Rightarrow 3ar = 48 \Rightarrow ar = 16 \Rightarrow r = \frac{16}{a}$$

Putting $r = \frac{16}{a}$ in (i), we get

$$a+16+\frac{256}{a} = 56 \Rightarrow a^2+16a+256 = 56a \Rightarrow a^2-40a+256 = 0 \Rightarrow (a-32)(a-8) = 0$$

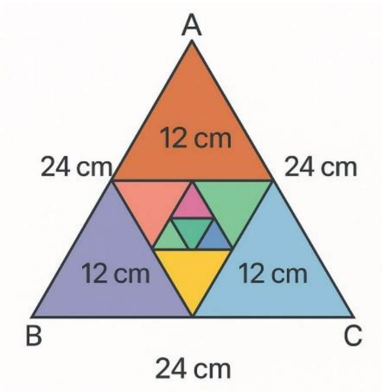
$$\Rightarrow a = 8, 32$$

Putting $a = 8, in\, r = \frac{16}{a}$ we get: $r = \frac{16}{8} = 2.$

Putting $a = 32, in\, r = \frac{16}{a}$ we get: $r = \frac{16}{32} = \frac{1}{2}$

CASE STUDY BASED QUESTIONS SOLVED

Q 1 In Rangoli competition in school, Renu made Rangoli in the equilateral shape. Each side of an equilateral triangle is 24 cm. The mid-point of its sides are joined to form another triangle. This process is going continuously infinite.



Based on above information, answer the following questions.

(i) Find the side of the 5th triangle (in cm)

(ii) Find the sum of perimeter of first 6 triangles (in cm)

(iii) Find the sum of areas of all the triangles (in sq. cm).

OR

(iii) Find the sum of perimeter of all triangles (in cm).

Solution:

(i) Side of first triangle = 24

Side of second triangle = 12

side of third triangle = 6

a = 24, r = $\frac{1}{2}$

Side of the fifth triangle= $a_5 = ar^4 = 24 \times \left(\frac{1}{2}\right)^4 = \frac{24}{16} = 1.5$ cm

(ii) Perimeter of first triangle = 72

Perimeter of second triangle = 36

perimeter of third triangle = 18

$\therefore$ a = 72 r = $\frac{36}{72} = \frac{1}{2}$

∴ Sum of perimeter of first 6 triangle, $S_6 = \frac{a(1-r^6)}{1-r} = \frac{72\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}} = \frac{72\times63\times2}{2^6} = \frac{567}{4}$ cm

(iii) Area of first triangle $= \frac{\sqrt{3}}{4}(24)^2$

Area of second triangle $= \frac{\sqrt{3}}{4}(12)^2$

Area of third triangle $= \frac{\sqrt{3}}{4}(6)^2$

$$a = \frac{\sqrt{3}}{4}(24)^2,\ r = \frac{1}{4}$$

Sum of the areas of all triangles $= \frac{a}{1-r} = \frac{\frac{\sqrt{3}}{4}(24)^2}{1-\frac{1}{4}} = \frac{\frac{\sqrt{3}}{4}(24)^2}{\frac{3}{4}} = \frac{\sqrt{3}}{3}(24)^2$

$= 192\sqrt{3}$ cm

OR

(iii) The sum of perimeter of all triangle = $3(24 + 12 + 6 + \cdots)$

$$S = 3\left(\frac{a}{1-r}\right) = 3\left(\frac{24}{1-\frac{1}{2}}\right) = 144 \text{ cm}$$

Q 2 After striking a floor a certain ball rebounds $\left(\frac{4}{5}\right)^{th}$ of the height from it has fallen. The ball is gently dropped from a height of 100 metres.

Based on this information, answer the following questions:
(i) Find the height to which the ball rebounds in 3rd rebound.
(ii) Find the height to which the ball rebounds in 8th rebound.
(iii) Find the total distance travelled by the ball in first five rebounds.

OR

(iii) Find the total distance travelled by the ball before coming to rest.

Solution:
Initially the ball falls from a height of 100 meters.
After striking the floor it rebounds, and goes to a height of $100 \times \frac{4}{5}$ meters.
Now, it falls from a height of $100 \times \frac{4}{5}$ metres and
after rebounding it goes to a height of $\left(100 \times \frac{4}{5}\right) \times \frac{4}{5} = 100\left(\frac{4}{5}\right)^2$ metres.
This process is continued till the ball comes to rest.
(i) in third rebound it goes up to a height of $100\left(\frac{4}{5}\right)^3 = 51.4$ m.
(i) in 8th rebound it goes up to a height $= 100\left(\frac{4}{5}\right)^8$
(iii) Total distance travelled by the ball in first five rebounds
$= 100 + 2\left[100 \times \frac{4}{5} + 100 \times \left(\frac{4}{5}\right)^2 + \cdots + 100 \times \left(\frac{4}{5}\right)^5\right]$

$$= 100 + 2\left[100 \times \frac{4}{5} \frac{\left\{1 - \left(\frac{4}{5}\right)^5\right\}}{1 - \frac{4}{5}}\right]$$

$$= 100 + 800\left\{1 - \left(\frac{4}{5}\right)^5\right\} = 900 - 800 \times \left(\frac{4}{5}\right)^5 = 900 - 262.14 = 637.86 \text{ metres.}$$

OR

(iii) Total distance travelled by the ball before coming to rest

$$= 100 + 2\left\{100 \times \frac{4}{5} + 100 \times \left(\frac{4}{5}\right)^2 + 100 \times \left(\frac{4}{5}\right)^3 + \cdots\right\}$$

$$= 100 + 2\left\{\frac{100 \times \frac{4}{5}}{1 - \frac{4}{5}}\right\} = 100 + 800 = 900 \text{ metres.}$$

Q 3 A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant. Rohan, being a plant lover, decides to open a nursery and he bought few plants and pots. He wants to place pots in such a way that the number of pots in the first row is 2, in second row is 4 and in the third row is 8 and so on…..

(i) Find the constant multiple by which the number of pots is increasing in every row.
(ii) Find the number of pots in 8th row.
(iii) Find the difference in number of pots placed in 7th row and 5th row.

OR

(iii) If Rohan wants to place 510 pots in total, then find the total number of rows formed in this arrangement.

Solution:

(i) The number of pots in each row is 2, 4, 8, ...
This forms a geometric progression,
Where $a = 2, r = \frac{4}{2} = 2$
Hence, the constant multiple by which the number of pots is increasing in every row is 2.

(ii) Number of pots in 8th row = $a_8 \Rightarrow a_8 = ar^{8-1} = 2(2)^7 = 2^8 = 256$

(iii) Number of pots in 7th row, $a_7 = 2(2)^{7-1} = 2 \cdot 2^6 = 2^7 = 128$
Number of pots in 5th row, $a_5 = 2(2)^{5-1} = 2 \cdot 2^4 = 2^5 = 32$
required answer = $128 - 32 = 96$

OR

(iii) Let there be n number of rows.

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right) \Rightarrow 510 = 2\left(\frac{2^n - 1}{2 - 1}\right) \Rightarrow \frac{510}{2} = 2^n - 1$$

$$\Rightarrow 255 = 2^n - 1 \Rightarrow 256 = 2^n \Rightarrow 2^8 = 2^n \Rightarrow n = 8$$

CHAPTER- 9: STRAIGHT LINES

Definitions and Formulae:

1. Distance formula Distance between the points P (x_1, y_1) and Q (x_2, y_2)

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio m: n.

i) Internal division: $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

ii) External division: $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

3. Mid-point formula:

$$\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$$

4. Area of a triangle, whose vertices are $(x_1, y1)$ $(x_2, y2)$ and (x_3, y_3) is

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

5. Slope of a line: A line in a coordinate plane forms two angles with the x-axis, which are supplementary. The angle θ made by the line l with positive direction of x-axis and measured anticlockwise is called the inclination of the line. Obviously $0° \le \theta \le 180°$

Definition 1: If θ is the inclination of a line l then tan θ is called the slope or gradient of the line l. The slope of a line whose inclination is 90° is not defined. The slope of a line is denoted by m. Thus, m = tan θ, θ ≠ 90° It may be observed that the slope of the x-axis is zero and slope of the y-axis is not defined.

6. Slope of a line when coordinates of any two points on the line are given.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on non-vertical line l whose inclination is θ. Obviously $x_1 \neq x_2$, otherwise the line will become perpendicular to x-axis and its slope will not be defined. The inclination of the line l may be acute or obtuse.

Therefore slope of line is m = $\frac{y_2-y_1}{x_2-x_1}$

7. Conditions for parallelism and perpendicularity of lines in terms of their slopes in a coordinate plane: a) For parallel lines : $m_1 = m_2$, i.e. their slopes are equal.

b) For perpendicular lines $m_1m_2 = -1$ i.e. product of the slope of two lines is -1.

8. Angle between two lines.

The acute angle (say θ) between lines L_1 and L_2 with slopes m_1 and m_2, respectively, is given by tan θ=$\left|\frac{m_2-m_1}{1+m_1m_2}\right|$ as $m_1m_2 + 1 \neq 0$

9. Various Forms of the Equation of a Line

i) Horizontal and vertical line:

a) x=±a which is an equation of line parallel to y-axis

b) $y=\pm a$ which is an equation of line parallel to x-axis

ii. Point slope form: Suppose that $P(x_1,y_1)$ is a fixed point on a non-vertical line L. Whose slope is m. Let A (x, y) be an arbitrary point on L. Then by the definition, the slope of L is given by $m=\frac{y-y_1}{x-x_1}$ and equation of line is $y-y_1=m(x-x_1)$

iii. Two-point form: Let the line L passes through two given points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

iv. Slope-intercept form: Sometimes a line is known to us with its slope and an intercept on one of the axes.

Case I: Suppose a line L with slope m cuts the y-axis at a distance c from the origin. The distance c is called the y-intercept of the line L. obviously, coordinates of the point where the line meet the y-axis are (0, c). Thus, L has slope m and passes through a fixed point (0, c). Therefore, by point-slope form, the equation of L is

$y-c = m(x-0)$ or $y = mx + c$

Thus, the point (x, y) on the line with slope m and y-intercept c lies on the line if and only if

$y = mx + c$

Note that the value of c will be positive or negative according as the intercept is made on the positive or negative side of the y-axis, respectively.

Case II: Suppose line L with slope m makes x-intercept d. Then equation of L is

$y = m(x - d)$

v) Intercept – form: Suppose a line L makes x-intercept a and y-intercept b on the axes. The equation of the line making intercepts a and b on x-and y-axis, respectively, is

$$\frac{x}{a}+\frac{y}{b}=1$$

10) Distance of a Point from a Line.

The distance of a point from a line is the length of the perpendicular drawn from the point to the line.

Let L: $Ax + By + C = 0$ be a line, whose distance from the point $P(x_1,y_1)$ is d.

$$d=\left|\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$$

11) Distance between two parallel lines: We know that slopes of two parallel lines are equal. Therefore, two parallel lines can be taken in the form $y = mx + c_1$ ... (1) and $y = mx + c_2$ -(2)

$$d=\left|\frac{c_1-c_2}{\sqrt{m^2+1}}\right|$$

MULTIPLE CHOICE QUESTIONS SOLVED

Q.1. Find the value of x so that the line through (3, x) and (2, 7) is parallel to the line through (-1, 4) and (0, 6).

a) 3 b) 6 c)9 d)-9

Solution: We know that for two lines to be parallel, their slope must be the same. The given points are A(3,x),B(2,7) and C(-1,4),D(0,6). Slope=$(\frac{6-4}{0+1})$ = $(\frac{7-x}{2-3})$, therefore $\frac{2}{1} = \left(\frac{7-x}{-1}\right)$ ≫-2=7-x therefore x=9

So Answer: (c)

Q.2. Slope of a line which cuts off intercepts of equal lengths on the axes is

a) -1 b) 0 c)2 d) 1

Solution: Given that equation of a line in intercept form is$\frac{x}{a}+\frac{y}{b}$=1

Since a=b therefore x+y=a, Now slope of the line is -1

So Answer: **(a)**

Q. 3. The inclination of the line $x - y + 3 = 0$ with the positive direction of x-axis is

a) 135° b) 35° c) 45° d) -45°

Solution: The equation of the line $x - y + 3 = 0$ can be rewritten as $y = x + 3$. So slope of the line m =1. It means $tan\theta = 1$ and hence θ = 45°

So Answer: **(c)**

Q. 4. An ant is going through a line which is parallel to the f the line 2x+3y= 6 and the ant is also passing through the point (1,1). The equation of the line on which the ant is going on is

a) 2x+3y=5 b) 2x+3y= 10 c) 3x+2y= 6 d) x+y= 2

Solution: Equation of the line parallel to 2x+3y= 6 is given by 2x+3y= c. Since the ant is passing through (1, 1) it means (1,1) lies on the line 2x+3y= c , putting (1,1) in this equation we get 2(1) + 3(1) = c. So c = 5 and hence the required equation is 2x+3y =5

So Answer: **(a)**

Q. 5. The line ax+by+c= 0 will be at a distance of one unit from the origin if:

a) a = b b) a = -b c) $c^2 = a^2 + b^2$ d) c =0

Solution: Using the formula for the distance between a point and a line we get distance, 1 = $\frac{|c|}{\sqrt{a^2+b^2}}$ it means |c|= $\sqrt{a^2+b^2}$, so , $c^2 = a^2 + b^2$

So Answer: ©

Q. 6. If the slope of the line joining the points A(x, 2) and B (6, -8) is – 5/4 , find the value of x.

a)2 b) 3 c) -2 d)-3

Solution: If a line passing through (x_1 ,y_1) and (x_2, y_2) then slope of the line is given by :

Slope = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

Given points are A (x, 2) and B (6,-8) and the slope is -5/4 and $\frac{-8-2}{6-x}$ = -$\frac{5}{4}$

$x = -2$

So Answer: ©

Q. 7. The equation of a line passing through (1,2) and perpendicular to the line x+y+1=0 is

a) y-x+1=0 b) y-x-1=0 c) y-x+2=0 d) x + y=3

Solution: Given that equation of line is x+y+1=0 now equation of line perpendicular to this line is y-x + k=0, since given line passing through the point (1,2), therefore 1-2+ k=0, therefore k=1 now equation of line is y-x+1=0

So Answer: **(a)**

Q. 8. If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is (3, 2), then the equation of the line will be

(a) 2x + 3y = 12 (b) 3x + 2y = 12 (c) 4x – 3y = 6 (d) 5x – 2y = 10

Solution: Let the equation of the required line is $\frac{x}{a}$ + $\frac{y}{b}$ =1 which intersects the x-axis at (a, 0) and y-axis at (0, b) . Now the co-ordinates of mid-point of (a,0) and (0, b) is (3,2) it means $\frac{a+0}{2}$ =3 and $\frac{0+b}{2}$ = 2 (Using mid-point formula), it gives a=6 and b=4 so the equation of the line is $\frac{x}{6}$ + $\frac{y}{4}$ =1 , it gives $2x + 3y = 12$

So Answer: **(a)**

Q. 9. What is the distance between the parallel lines y = 2x + 1 and y = 2x – 4?

(a) 2√5 (b) √5 (c) 4√5 (d) 5√5.

Solution: Using the formula of the distance between parallel lines y= mx+c_1 and y= mx+c_2 that is d=$\left|\frac{c_1-c_2}{\sqrt{m^2+1}}\right|$, we get d=$\left|\frac{1-(-4)}{\sqrt{2^2+1}}\right|$ = $\frac{5}{\sqrt{5}}$ = $\sqrt{5}$

So Answer: (b)

Q. 10. Locus means the path traced by point under given conditions. Locus is simply the equation of a curve may be line or any other curve. The locus of a point, whose abscissa and ordinate are always equal, is

a) x-y = 0 b) x+y =1 c) x+y+1 = 0 d) None of the above

Solution: Let the abscissa and ordinate of a point "P" be(x, y). Given condition: Abscissa = Ordinate, it means x = y. Hence, the locus of a point is x-y = 0.

So Answer: **(a)**

ASSERTION – REASON BASED QUESTIONS SOLVED

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R).Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).

C) Assertion (A) is true but Reason(R) is false.
D) Assertion (A) is false but Reason(R) is true.

1) ASSERTION (A): slope of x-axis is 0.

REASON(R): x-axis makes zero angles with the positive direction of x-axis.

Answer: A
Solution: since $\theta = 0$ therefore m=0 and x-axis are parallel to positive direction of x-axis

2) ASSERTION (A): Two parallel lines have same slope.

REASON(R): Any line parallel to x axis have slope 0.
Answer: B
Solution: Two parallel lines make equal angle with positive direction of x-axis in anticlockwise direction. It means their slope is equal. So Assertion is correct. Reason is also correct but is specific to lines which are parallel to x-axis, so it is not correct explanation of assertion.

3) ASSERTION (A): Product of slope of two perpendicular lines is -1.

REASON (R): Product of slope of x-axis and y-axis is -1.

Answer: B
Solution: Here assertion (A) is correct and Reason (R) is also correct, but Reason is specific to x-axis and y-axis not for general conditions.

4) ASSERTION (A): Parallel lines have same slope.

REASON(R): Line P: 2x+ 3y=5 and Q: 6x+ 9y=20 are parallel to each other.

Answer: A
Solution: Slope of line P= -2/3 and slope of line Q= - 2/3, since the slope of both the lines are equal therefore the two lines are parallel.

5) ASSERTION (A): Equation of line Passing through origin is y = mx.

REASON (R): Line y = mx have intercept on the x-axis is 0 and y-axis is 0.

Answer: B

Solution: Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).

6) ASSERTION (A): The tangent of the angle between the lines x/a + y/b =1 and x/a – y/b = 1 is $2ab/a^2-b^2$

REASON(R) : The tangent of the angle between the lines with clops m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1 + m_2}{1 - m_1 m_2}\right|$$

Answer: C

Solution: Here the assertion is true but the reason the angle between the two lines is not true.

7) Assertion (A): The point (2, 3) lies on the line y = 3x - 7.
Reason (R): Substituting the coordinates of the point in the equation of the line satisfies the equation.
Answer: d) Assertion (A) is false but Reason(R) is true.

Solution: Substituting x = 2 and y = 3 in the equation y = 3x - 7, we get 3 = 3(2) - 7, which simplifies to 3 = 6 - 7, or 3 = -1. This is not true, so the point (2, 3) does not lie on the line. The assertion is false, and the reason is true

8) Assertion (A): The lines 2x + 3y + 5 = 0 and 3x - 2y + 1 = 0 are perpendicular to each other.

Reason (R): Two lines are perpendicular if the product of their slopes is -1.
Answer: (a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Solution: The slope of the first line 2x + 3y + 5 = 0 is -2/3, and the slope of the second line 3x - 2y + 1 = 0 is 3/2. The product of these slopes (-2/3) x (3/2) = -1. Therefore, the lines are perpendicular, and the reason is the correct explanation of the assertion.

9) Assertion(A): Two lines with the same slope are always parallel.
Reason(R): Parallel lines have equal slopes
Answer: (a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A)
Solution: Both are true, the reason correctly explains the assertion, and Answer

10. Assertion (A): Equation of the horizontal line having distance" a "from the x-axis is either y=a or y= -a.
Reason(R): Equation of vertical line having distance "b" from the y-axis is either x=b or x= -b.
Answer: (b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
Solution: Here both are correct statement but R is not giving the reason of A. These are two independent correct statements.

VERY SHORT ANSWER TYPE QUESTIONS SOLVED

1) Question: Show that the line through the points (5, 6) and (2, 3) is parallel to the line through the points (9, -2) and (6, -5).

Solution: We know that for two lines to be parallel, their slope must be the same. Given points are A(5,6),B(2,3) and C(9,-2),D(6,-5)

Slope = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$, since $\left(\frac{3-6}{2-5}\right) = \left(\frac{-5+2}{6-9}\right)$. Hence proved.

2) Question: If a point P(x, y) is equidistant from the points A(6, -1) and B(2, 3), find the relation between x and y.

Solution: Given: Point P(x, y) is equidistant from points A(6, -1) and B(2, 3) i.e., distance of P from A = distance of P from B Squaring both sides,

$\sqrt{(x-6)^2+(y+1)^2}=\sqrt{(x-2)^2+(y-3)^2}$

$(x-6)^2 + (y-1)^2 = (x-2)^2 + (y-3)^2$

$x^2-12x+36+y^2-2y-1 = x^2-4x+4+y^2-6y+9$

$-12x +36+2y +1 = -4x +4 -6y +9$

$x-y = 3$

Therefore, x – y = 3 is the required relation.

3. Find a point on the x-axis which is equidistant from the points A(7, 6) and B(- 3, 4).

Solution: Let the point on x-axis be P(x, 0). Given: Point P(x, 0) is equidistant from points A(7, 6) and B(-3, 4) i.e., distance of P from A = distance of P from B

$\sqrt{(x-7)^2+36} = \sqrt{(x+3)^2+16}$

Squaring both sides,

$(x-7)^2+36 = (x+3)^2 + 16$

$x^2 - 14x +49+36 = x^2 + 6x+ 9 +16$

-20x = -60

x = 3 Therefore, the point on the x-axis is (3, 0).

4. Show that the points A(-5, 1), B(5, 5) and C(10, 7) are collinear.

   Solution: Given: The points are A(-5, 1), B(5, 5) and C(10, 7). Note: Three points are collinear if the sum of lengths of any sides is equal to the length of the third side.

   AB = $\sqrt{(5+5)^2 + (5-1)^2}$ = $\sqrt{100+16}$= $2\sqrt{29} units$

   BC = $\sqrt{(10-5)^2 + (7-5)^2}$ = $\sqrt{25+4}$ = $\sqrt{29} units$

   AC = $\sqrt{(10+5)^2 + (7-1)^2}$ = $\sqrt{225+4}$ = $3\sqrt{29} units$

   From equations 1,2,3 we have AB+BC=AC

   Therefore, the three points are collinear.

5. If the three points A (h, k), B(x_1, y_1) and C(x_2, y_2) lie on a line then show that ($h - x_1$)($y_2 - y_1$) = ($k - y_1$)($x_2 - x$).

   Solution: For the lines to be in a line, the slope of the adjacent lines should be the same. Given points are A(h,k),B(x_1,y_1) and C(x_2,y_2) So slope of AB = BC = CA

   Slope = ($\frac{y_2-y_1}{x_2-x_1}$)

   Slope of AB = $\frac{y_1-k}{x_1-h}$

   Slope of BC = $\frac{y_1-k}{x_1-h}$

   Slope of AC = $\frac{y_2-k}{x_2-h} = \frac{y_1-k}{x_1-h} = \frac{y_1-k}{x_1-h} = \frac{y_2-k}{x_2-h}$

   Now Cross multiplying the first two equality,

   (y_1-k)(x_2-x_1) = (x_1-h)(y_2-y_1)

   (h-x_1)(y_2-y_1) = (k-y_1)(x_2-x_1)

   Hence proved.

6. Find the slope of the line which makes an angle of 30° with the positive direction of the y-axis, measured anticlockwise.

   Solution: According to the given figure, the angle made by the line from X-axis is 90+30 = 120°

   Slope =(y_2-y_1/x_2-x_1)

   We also know that slope of a line is equal to tanθ, Where θ = 120° tan (120°) = tan (90°+30°) = -cot (30°) = -√3 Therefor the slope of the given line is -√3.

7. Find the angle between the lines whose slopes are, $\sqrt{3} and \frac{1}{\sqrt{3}}$

   Solution To find out the angle between two lines, the angle is equal to the difference in θ.

   The slope of a line = tanθ =($\frac{y_2-y_1}{x_2-x_1}$)

   So slope of the first line = $\sqrt{3}$ = tan$\theta \Longrightarrow \theta_1 = 60^0$

   The slope of the second line is = 1/$\sqrt{3} \Longrightarrow \theta_2 = 30^0$

   Now the difference between the two lines is θ1-θ2 = 60°-30° = 30°

8. Find the equation of a line which is equidistant from the lines $x = -2$ and $x = 6$.

   Solution: For the equation of line equidistant from both lines, we will find point through which line passes and is equidistant from both line. As any point lying on x = - 2 line is (- 2, 0) and on x = 6 is (6, 0), so mid - point is

   (x, y) = ($\frac{-2+6}{2}$, $\frac{0+0}{2}$)

   (x, y) = (2,0)

   So, equation of line is x = 2.

9. Find the equation of a line passing through the origin and making an angle of 120° with the positive direction of the x - axis.

   Solution As angle is given so we have to find slope first give by m = tanθ m = tan120°

m = tan(180^0-60^0) = - tan60^0 = -$\sqrt{3}$

(tan (180° - θ) is in II quadrant, tanx is negative) Now equation of line passing through origin is given as y = mx

y=-$\sqrt{3}x$

y+$\sqrt{3}x = 0$ so the required equation of line is y+$\sqrt{3}x = 0$

10. Find the slope and the equation of the line passing through the points(3, - 2) and (- 5, - 7).

Solution: Slope of equation can be calculated using

m = $\frac{y_2 - y_1}{x_2 - x_1}$ m = 5/8

Now using two point formula of the equation of a line

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

y – (-2) = 5/8 (x-3)⇒ 8 (y+ 2) = 5 (x-3) ⇒ 5x- 8y -31 = 0

So the required equation of the line is 5x- 8y -31 = 0

SHORT ANSWER TYPE QUESTIONS SOLVED

Q.1. Prove that A (4, 3), B (6, 4), C (5, 6) and D (3, 5) are the angular points of a square.

Solution: Clearly, AB =$\sqrt{(6-4)^2 + (4-3)^2} = \sqrt{5}$, BC =$\sqrt{(6-4)^2 + (5-4)^2} = \sqrt{5}$

CD = $\sqrt{(5-6)^2 + (3-5)^2} = \sqrt{5}$ DA =$\sqrt{(5-3)^2 + (3-4)^2}$ =$\sqrt{5}$

AB = BC = CD = DA

NOW, $m_1 = slope\ of\ AB = \frac{4-3}{6-4} = \frac{1}{2}$

m_2 = slope of BC = $\frac{6-4}{5-6}$ =-2

$m_{3=} Slope\ of CD = \frac{5-6}{3-5} = \frac{1}{2}$

Clearly, $m_1 m_2$ = (1/2) (-2) = -1

and $m_{1=}\ m_2$

Therefore, AB is perpendicular to CD. Thus, AB =BC=CA=AD, AB is perpendicular to BC and AB is parallel to CD.

Hence, ABCD is a square.

Q. 2. Find the points on the x-axis whose distances from the line x/3 + y/4 = 1 are 4 units.

Solution: Given:

The equation of the line is $\frac{x}{3} + \frac{y}{4} = 1$

4x + 3y = 12

4x + 3y – 12 = 0 …. (1)

Now, compare equation (1) with the general equation of line Ax + By + C = 0, where A = 4, B = 3, and C = -12

Let (a, 0) be the point on the x-axis whose distance from the given line is 4 units.

So, the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$4 = \frac{|4a - 12|}{\sqrt{16 + 9}} = \frac{|4a - 12|}{5}$$

|4a – 12| = 4 × 5 ⇒± (4a – 12) = 20

⇒ a = 8 or a = -2

∴ The required points on the x-axis are (-2, 0) and (8, 0)

Q.3. Find the equation of the line parallel to y-axis and drawn through the point of intersection of the lines x - 7y + 5 and 3x + y = 0.

Solution: On the solving equations x-7y+5=0 and 3x+y=0, we get x= $\frac{-5}{22}$, y = $\frac{15}{22}$

So, the given lines intersect at the point whose coordinates are (- 5/22, 15/22).

We know that, the equation of a line parallel to y-axis is of the form x=constant. So, let the equation of the required line be x = λ → (i)

It passes through (-5/22, 15/22). So, $\frac{-5}{22} = \lambda$

Putting the values of λ in (i), we get x= - 5/22 or, 22x + 5 = 0 as the equation of the required line.

Q.4. If p is the length of the perpendicular from the origin on the line whose intercepts on the axes a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Solution: The equation of the line which makes intercept a,b on the axes is

$\frac{x}{a} + \frac{y}{b} = 1$ i.e. $\frac{x}{a} + \frac{y}{b} - 1 = 0$ ….(i)

Since perpendicular distance of a point (x_1, y_1) from the line

$ax + by + c = 0$ is given by $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

According to question

$$p = \left|\frac{\frac{0}{a} + \frac{0}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right|$$

Implies that $p^2 = \frac{1}{\frac{1}{a^2}+\frac{1}{b^2}} \Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Q.5. Find equation of the line passing through the point (3,4) and cutting of intercepts on the axes whose sum is 14.

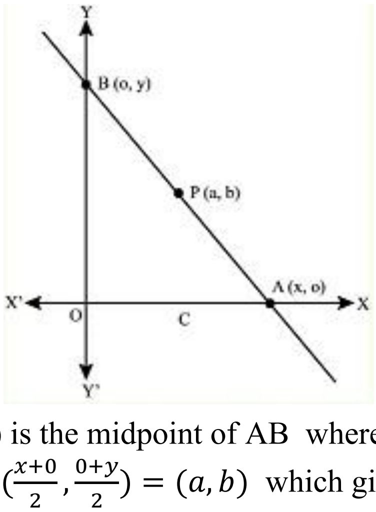
Solution: Let the equation of the line in intercept form be $\frac{x}{a} + \frac{y}{b} = 1$.

Give that a+b =14 and the line passes through the point (3,4)

$\therefore \frac{3}{a} + \frac{4}{14-a} = 1$

Solving above quadratic equation we get a= 7,6 and b= 7,8

Q.6. Let P (a, b) is the mid-point of a line segment between axes. Show that the equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$.



Solution:

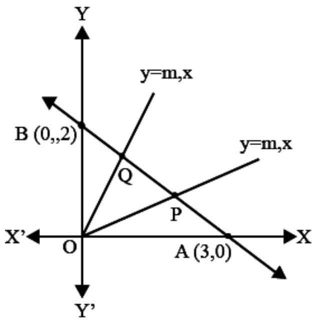
Let P(a, b) is the midpoint of AB where A(x,0) and B(0,y)

Therefore $\left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (a, b)$ which gives x = 2 a and y = 2 b

Equation of line passing through (2a,0) and (0,2b) is

$y - 0 = \frac{2b-0}{0-2a}(x - 2a)$ which gives $\frac{x}{a} + \frac{y}{b} = \mathbf{2}$

Q.7. Find the equations of the straight lines which pass through the origin and trisect the portion of the straight line $2x + 3y = 6$ which intercepted between the axes.



Solution:
A (3,0) and B (0, 2)
The line $2x + 3y = 6$ trisect by points P and Q
Therefore, AP=PQ=QB
P (2,2/3) and Q (1,4/3)
Clearly, P and Q lie on line $y = m_1 x$ and $y = m_2 x$ respectively.
$y = \frac{1}{3}x$ and $y = \frac{4}{3}x$
So, $x - 3y = 0 \; and \; 4x - 3y = 0$

Q.8. Points A and B have coordinates $(3,2), (7,6)$ respectively. Find
(i) The equation of right bisector of the segment AB. (ii) The value of p if $(-2, p)$ lies on it.
Solution: (I) the midpoint of AB=(5,4)
Line passes through mid-point of AB is perpendicular to AB
So, (slope of AB) (slope of line passes through mid-point of AB) $=-1$
slope of AB=1, slope of line passes through mid-point of AB=-1
equation of right bisector of AB is $y - 4 = -1(x - 5)$
$$x + y = 9$$
(ii) $(-2, p)$ lies on $x + y = 9$
There fore $p = 11$

Q.9 Without **using the distance formula, show that points (– 2, – 1), (4, 0), (3, 3) and (–3, 2) are the vertices of a parallelogram.**

Solution: Let the given point be A (-2, -1) , B (4, 0) , C (3, 3) and D (-3, 2)

So now, the slope of AB = (0+1)/(4+2) = 1/6

The slope of CD = (3-2)/(3+3) = 1/6

Hence, the Slope of AB = Slope of CD ∴ AB ∥ CD

Now,mThe slope of BC = (3-0)/(3-4) = 3/-1 = -3

The slope of AD = (2+1)/(-3+2) = 3/-1 = -3

Hence, the Slope of BC = Slope of AD ∴ BC ∥ AD

Thus, the pair of opposite sides are quadrilateral are parallel, so we can say that ABCD is a parallelogram.

Hence, the given vertices, A (-2, -1), B (4, 0), C(3, 3) and D(-3, 2) are vertices of a parallelogram.

Q. 10**. Find the coordinates of the foot of the perpendicular from the point (–1, 3) to the line 3x – 4y – 16 = 0.**
Solution: Let us consider the coordinates of the foot of the perpendicular from (-1, 3) to the line 3x – 4y – 16 = 0 be (a, b)
So, let the slope of the line joining (-1, 3) and (a, b) be m_1
m_1 = (b-3)/(a+1)
And let the slope of the line 3x – 4y – 16 = 0 be m_2
$y = \frac{3}{4}x - 4 \Rightarrow m_2 = 3/4$
Since these two lines are perpendicular, $m_1 \times m_2 = -1$
(b-3)/(a+1) × (3/4) = -1 ⇒ (3b-9)/(4a+4) = -1

3b – 9 = -4a – 4 ⇒ 4a + 3b = 5 …….(1)
Point (a, b) lies on the line 3x – 4y = 16
3a – 4b = 16 ……..(2)
Solving equations (1) and (2), we get
a = 68/25 and b = -49/25
∴ The coordinates of the foot of perpendicular are (68/25, -49/25)

LONG ANSWER TYPE QUESTIONS WITH SOLUTIONS

Q.1. The mid points of the sides of a triangle are (2, 1), (-5, 7) and (-5, -5). Find the equations of the sides of the triangle.

Solution: Let D (2, 1), E (-5, 7) and F (-5, -5) be the mid points of sides BC, CA and AB respectively of Δ ABC.

Slope of AB = Slope of DE
Slope of BC = slope of EF, and Slope of AC = Slope of DF (Mid Point Theorem)
Let m_1, m_2, m_3 be the slopes of AB, BC and CA respectively.
Then
m_1 = Slope of AB = Slope of DE = $\frac{7-1}{-5-2}$ = - 6/7
m_2 = Slope of BC = Slope of EF = $\frac{7+5}{-5+5}$ =(undefined)
m_3 = Slope of CA =Slope of DF = $\frac{1+5}{2+5}$ = 6/7
Side AB passes through F (-5, -5) and has slope $m_1 = \frac{-6}{7}$. So, its equation is
y + 5 = $\frac{-6}{7}$ (x +5) or 6x + 7y + 65 = 0
Side BC is parallel to Y-axis and passes through D (2, 1). So, its equation is x = k. As it passes through (2, 1).
2 = k
Hence equation of BC is x = 2
Side CA passes through E(-5, 7) and has slope m_3=6/7. So, its equation is
y – 7 = $\frac{6}{7}$(x +5)
6x -7y + 79 = 0

Q. 2. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1, 2) in the line $x - 3y + 4 = 0$.

Solution: Let Q (h, k) is the image of the point P (1, 2) in the line
$x - 3y + 4 = 0$ ... (1)
Therefore, the line (1) is the perpendicular bisector of line segment PQ.
Hence slope of line PQ = $\frac{-1}{Slope of line x-3y+4=0}$
So that $\frac{k-2}{h-1} = \frac{-1}{\frac{1}{3}}$ $or\ 3h + k = 5$ ... (2)
And the mid-point of PQ, i.e., point$\left(\frac{h+1}{2}, \frac{k+2}{2}\right)$ will satisfy the equation (1) so that
$\frac{h+1}{2} - 3\left(\frac{k+2}{2}\right) + 4 = 0$ or h-3k = -3 … (3)
Solving (2) and (3) we get, $h = \frac{6}{5} and k = \frac{7}{5}$
Hence, the image of point (1, 2) in the line (1) is, $\left(\frac{6}{5}, \frac{7}{5}\right)$

Q.3. Find the distance of the line $4x - y = 0$ from the point P (4, 1) measured along the line making an angle of 135° with the positive x-axis.

Solution: Given line is $4x - y = 0$ ... (1).
In order to find the distance of the line (1) from the point P (4, 1) along another line, we have to find the point of intersection of both the lines.
For this purpose, we will first find the equation of the second line Slope of second line is tan 135° = –1.
Equation of the line with slope – 1 through the point P (4, 1) is $y - 1 = - 1 (x - 4)$ or $x + y - 5 = 0$ ... (2)
Solving (1) and (2), we get $x = 1$ and $y = 4$

so that point of intersection of the two lines is Q (1, 4).
Now, distance of line (1) from the point P (4, 1) along the line (2) the distance between the points P (4, 1) and Q (1, 4).

PQ = $\sqrt{(1-4)^2+(4-1)^2}$ = $\sqrt{(-3)^2+(3)^2}$ = $\sqrt{9+9}$ = $3\sqrt{2}$

Q. 4. **A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1: n. Find the equation of the line.**

Solution: We know that the coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\left(\frac{1(2)+n(1)}{1+n}, \frac{1(3)+n(0)}{1+n}\right) = \left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

We know that slope, m = $(y_2 - y_1)/(x_2 - x_1)$ = (3 – 0)/(2 – 1) = 3

We know that two non-vertical lines are perpendicular to each other only if their slopes are negative reciprocals of each other.

Then, m = (-1/m) = -1/3

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), only if its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

Here, the point is $\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$

Equation of line is $\left(y - \frac{3}{1+n}\right) = -\frac{1}{3}\left(x - \frac{2+n}{1+n}\right)$

∴ The equation of the line is (1 + n) x + 3(1 + n) y – n – 11 = 0

Q. 5. **A variable line passes through a fixed-point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P.**

Solution: Let the equation of the line be $ax + by = 1$

Now using the formula of the distance between a point and a line we get:

$$\left|\frac{2a-1}{\sqrt{a^2+b^2}}\right| + \left|\frac{2b-1}{\sqrt{a^2+b^2}}\right| + \left|\frac{a+b-1}{\sqrt{a^2+b^2}}\right| = 0$$

⇒ 2a – 1 + 2b – 1 + a + b – 1 = 0 ⇒ 3a + 3b – 3 = 0

⇒ a + b – 1 = 0 ⇒ a + b = 1

So, the equation ax + by = 1 represents a family of straight lines passing through a fixed point.

Comparing the equation ax + by = 1 and a + b = 1, we get

x = 1 and y = 1

So, the coordinates of fixed point is (1, 1)

CASE STUDY BASED QUESTIONS SOLVED

Q.1. A parking lot in a Pharmacy company is triangular shaped with two of its vertices at B(2,0) and C(1,12). The third vertex A is at the midpoint of line joining the points (1,1) and (3,11) .

i) Find the coordinates of A.

ii) Find the equation of the line that passes through the points B(2,0) and C(1,12).

iii) Find the equation of the line parallel to BC and passing through vertex A.

Solution: i) Given that,

The third vertex A is at the midpoint of line joining the points (1,1) and (3,11) .

so, A coordinates (x, y) will be :

→ x = (1 + 3)/2 = 4/2 = 2 .

→ y = (1 + 11)/2 = 12/2 = 6 .

therefore, coordinates of A will be **(2,6).**

ii) The equation of the line that passes through the points B(–2,0) and C(1,12) :

→ slope of line = $\frac{y_2-y_1}{x_2-x_1} = \frac{12-0}{1-2} = \frac{12}{-1} = -12$

then,

→ Equation of line $y - y_1 = m(x - x_1)$

taking (x_1, y_1) as (2,0) we get,

→ (y – 0) = (–12)(x – 2)

→ y = –12x+24

→ **12x+y–24 = 0** which is the required equation of line .

iii) here $m_{BC} = 12$

→ Slope of line parallel to BC = Slope of BC = 12

then, Equation of the line parallel to BC and passing through vertex A(2,6) :-

→ $y - y_1 = m(x - x_1)$

→ y – 6 = 12(x – 2)

→ y – 6 = 12x –24

→ **12x–y–18 = 0,** which is the required equation of line .

Q. 2. Four friends Richa, Sunita, Viki and Raju are sitting on vertices of a rectangle whose coordinates are Richa (2, 6) , Raju (6, 4), Sunita (3, 5) and Viki (5, 3)

Based on the above information answer the following question

a) What is the equation of line formed by Sunita and Raju?
b) What is the equation of the line formed by Richa and Viki?
c) What is the slope of equation of the line formed Richa and Raju?

Solution:

a) We have the positions Sunita (3, 5) and Raju (6, 4)

Slope $m_1 = \frac{4-5}{6-3} = \frac{-1}{3}$

Equation form by Sunita and Raju is x + 3y = 18

b) We have the positions Richa (2, 6) and Viki (5, 3)

Slope, $m_2 = \frac{3-6}{5-2}$ = - 1

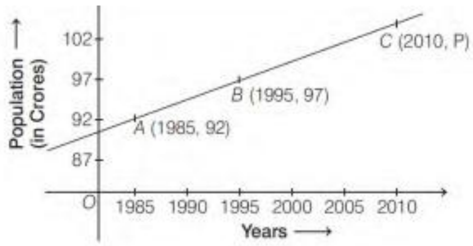
Equation form by Richa and Viki is y -y_1= -1 (x -x_1)

Or, x + y =8

c) We have the positions Richa (2, 6) and Raju (6, 4)

Slope, $m_3 = \frac{4-6}{6-2} = \frac{-1}{2}$

Q. 3. Population Vs Year graph given below.



Based on the above information answer the following questions.

i) What is the slope of the line AB?
ii) Find the equation of line AB?
iii) What is the equation of line perpendicular to AB and passing through (1995, 97)?

Solution: (i) m = $\frac{1}{2}$ (ii) Using two point form: x–2y=1801

(iii) 2x+y=4087

CHAPTER – 10: CONIC SECTION

Definitions and Formulae:

CIRCLES:

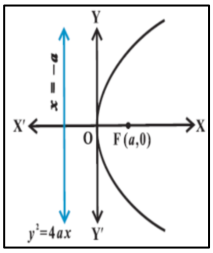
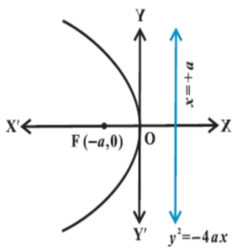
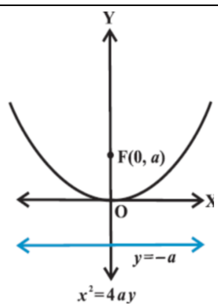
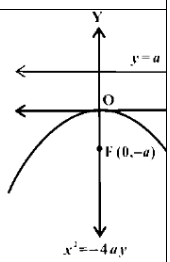
- A circle is a locus of a point which moves in a plane such that its distance from a fixed point in that plane is always constant. The fixed point is said to be the centre of the circle and the constant distance is said to be the radius of the circle.
- The equation of a circle with centre (h, k) and the radius r is given by $(x - h)^2 + (y - k)^2 = r^2$.
- The equation of a circle with centre (0, 0) and the radius r is given by $x^2 + y^2 = r^2$.
- General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ whose centre is (–g, –f) and radius is $\sqrt{g^2 + f^2 - c}$

CONICS:

- Conic Section or a conic is the locus of a point which moves in such a way that its distance from a fixed point bears a constant ratio to its distance from a fixed line.
- The fixed point is called the **focus**, the straight line is called the **directrix** and the constant ratio denoted by **e** is called the **eccentricity of the conics**.
- If $e = 1$, then conic is a parabola.
- If $e < 1$, then conic is an ellipse.
- If $e > 1$, then conic is a hyperbola.
- If $e = 0$, then conic is a circle. (Special case of Ellipse)

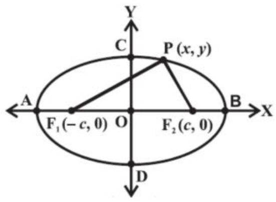
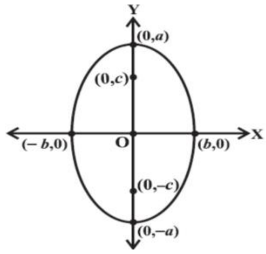
PARABOLA:

- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane.

S. No		$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
					
1.	Vertex	(0,0)	(0,0)	(0,0)	(0,0)
2.	Focus	(a,0)	(-a,0)	(0, a)	(0, -a)

3	Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
4.	Equation of Axis	(x-axis), $y = 0$	(x-axis), $y = 0$	(y-axis), $x = 0$	(y-axis), $x = 0$
5.	Length of Latus Rectum	4a	4a	4a	4a

- Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola

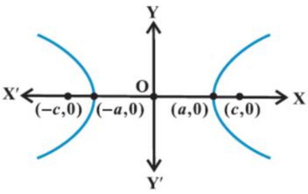
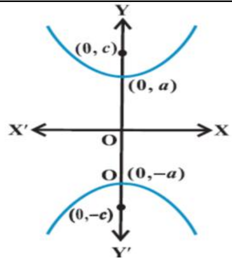
S. No.			
1.	Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, $a > b$
2.	Centre	(0 , 0)	(0 , 0)
3.	Vertices	(± a , 0)	(0 , ± a)
4.	Foci	(± ae, 0) or (± c , 0) where $c^2 = a^2 - b^2$	(0 ± ae) or (0 ± c) where $c^2 = a^2 - b^2$
5.	Eccentricity	e = c/a	e = c/a
6.	Length of major axis	2a	2a
7.	Length of minor axis	2b	2b
8.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

ELLIPSE:

- An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

HYPERBOLA

- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

S. N			
1	Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
2.	Vertices	$(\pm a, 0)$	$(0, \pm a)$
3.	Foci	$(\pm ae, 0)$ or $(\pm c, 0)$ where $c^2 = a^2 + b^2$	$(0 \pm ae)$ or $(0 \pm ae)$ where $c^2 = a^2 + b^2$
4.	Eccentricity	$e = c/a$	$e = c/a$
5.	Centre	(0, 0)	(0, 0)
6.	Length of Transverse axis	2a	2a
7.	Length of conjugate axis	2b	2b
8.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
9.	Equation of Transverse axis	$y = 0$	$x = 0$
10.	Equation of conjugate axis	$x = 0$	$y = 0$

MULTIPLE CHOICE QUESTIONS SOLVED

Q. 1) The number of tangents that can be drawn from (1,2) to $x^2 + y^2 = 5$ is

a) 0 b) 1 c) 2 d) more than 2

Solution: The point (1,2) lies in the interior of the curve $x^2 + y^2 = 5$ which is a circle. So the number of tangents drawn is o.

Answer: a) 0

Q. 2) The length of the latus rectum of the ellipse $3x^2 + y^2 = 12$ is

a) 3 b) $\frac{4}{\sqrt{3}}$ c) 4 d) 8

Solution: Comparing with standard equation $a = 2\sqrt{3}$ and $b = 2$, using the formula of latus rectum $\frac{2b^2}{a}$ we get $l = \frac{4}{\sqrt{3}}$

Answer: b) $\frac{4}{\sqrt{3}}$

Q.3) The line 2x - y + 4 = 0 cuts the parabola $y^2 = 8x$ in P and Q. The mid - point of PQ is

a) (1, - 2) b) (1, 2) c) (- 1, - 2) d) (- 1, 2)

Solution: Solving both the equations 2x - y + 4 = 0 and $y^2 = 8x$. Using the concepts sum of roots.

Answer: d) (- 1, 2)

Q.4) If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latus rectum is

a) 4 b) $\frac{2}{3}$ c) $\frac{4}{3}$ d) $\frac{1}{3}$

Solution: Putting the value (3,2) in equation of parabola 4a = 4/3

Answer: c) $\frac{4}{3}$

Q.5) At what point on the parabola $y^2 = 4x$ the normal makes equal angles with the axes?

a) (4, 4) b) (9, 6) c) (1, - 2) d) (4, - 4)

Solution: c) (1, - 2)

Answer: Normal makes equal angle means slope is 1 or -1. Equating with derivative of $y^2 = 4x$ that is $\frac{dy}{dx}$ it gives co-ordinates of point (1,-2).

Q.6) The length of latus rectum of an ellipse is one - third of its major axis. Its eccentricity would be

a) $\frac{2}{3}$ b) $\sqrt{\frac{2}{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{\sqrt{2}}$

Solution: *Solving* $\frac{2b^2}{a} = \frac{2a}{3}$ we get $a^2 = 3b^2$, $c = \sqrt{2}\,b$ so $e = \frac{c}{a} = \frac{\sqrt{2}\,b}{\sqrt{3}\,b} = \sqrt{\frac{2}{3}}$

Answer: b) $\sqrt{\frac{2}{3}}$

Q.7) Equation of the hyperbola with eccentricity $\frac{3}{2}$ and foci at (± 2, 0) is

a) $\frac{x^2}{9} - \frac{y^2}{9} = \frac{4}{9}$ b) $\frac{x^2}{16} - \frac{y^2}{27} = 1$ c) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ d) $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$

Solution: Here a =± 2 and using the formula of e we get c= 3 so b = $\sqrt{5}$

Answer: d) $\frac{x^2}{4} - \frac{y^2}{5} = \frac{4}{9}$

Q.8) If (2, 4) and (10, 10) are the ends of a latus rectum of an ellipse with eccentricity $\frac{1}{2}$, then the length of semi - major axis is

Solution: Using the distance formula length of latus rectum = 10 units. Comparing with the formula of latus rectum $b^2 = 5a$ and using formula of eccentricity a= 2c. Now using the relation between a, b and c we get a= 0 or a = $\frac{20}{3}$

Answer: d) $\frac{20}{3}$

Q.9) The slope of the tangent at the point (h, h) of the circle $x^2 + y^2 = a^2$ is

a) - 2 b) – 1 c) 0 d) Depends upon h

Solution: Slope of the tangent = negative of reciprocal of slope of the line joining origin and (h, h) = -1

Answer: b) - 1

Q.10) The equation $x^2 + y^2 + 2x - 4y + 5 = 0$ represents.

a) A circle of non - zero radius b) A circle of zero radius

c) A pair of straight lines d) A point

Solution: Here comparing with standard equation r = o so it's a circle having radius that is a point
Answer: d) A point (Best possible answer)

ASSERTION - REASON BASED QUESTIONS SOLVED

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R).Pick the correct option:

a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
c)Assertion (A) is true but Reason(R) is false.
d)Assertion (A) is false but Reason(R) is true.

Q.1 Assertion (A): A line through the focus and perpendicular to the directrix is called the x-axis of the parabola.
Reason (R): The point of intersection of parabola with the axis is called the vertex of the parabola.
Answer: (d) A is false but R is true

Solution: A line through the focus and perpendicular to the directrix is called the axis of the parabola. The point of intersection of parabola with the axis is called the vertex of the parabola.
Q2. Parabola is symmetric with respect to the axis of the parabola.
Assertion(A): If the equation has a term y^2, then the axis of the symmetry is along the $x-$ axis.
Reason(R): If the equation has a term x^2, then the axis of symmetry is along the x -axis.
Answer: (c) Assertion (A) is true but Reason(R) is false.
Solution: If the equation has a term x^2 its axis is y-axis.

Q.3. Assertion(A): The area of the ellipse $2x^2 + 3y^2 = 6$ is more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$
Reason (R): The length of the semi major axis of an ellipse is more than the radius of the circle.

Answer: b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
Solution: After finding the value of a, b and r it can be verified that area of ellipse is more than area of circle here. So, Assertion(A) is true and Reason is also correct but not a valid reason of Assertion.

Q. 4. Assertion(A): The equation $x^2 + y^2 = 4$ represent a circle with centre (0,0) and radius =2 units.
Reason(R): The general equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$.
Answer: (b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
Solution: Here the assertion is correct as its centre is (0,0) and radius is 2 units but the reason (R) do not clearly explaining the process to get centre and radius.

Q.5. Assertion(A): The eccentricity of an ellipse is always less than 1.
Reason(R): The eccentricity of a conic is defined as ratio of c to a ($e = \frac{c}{a}$)
Answer: b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).

Solution: Here A and R both are correct but Reason do not give the correct explanation of A.

Q.6. Assertion (A): A line through the focus and perpendicular to the directrix is called the x-axis of the parabola.
Reason (R): The point of intersection of parabola with axis is called the vertex of the parabola.
Answer: d) Assertion (A) is false but Reason(R) is true.
Solution: A line through the focus and perpendicular to the directrix is called the axis of the parabola. So, A is false and Reason is correct statement here.

Q. 7. Assertion (A): The foci of the hyperbola 9 x^2 -16 y^2 = 144 is (+5, 0).
Reason (R): The formula to find the foci of a parabola is $c^2 = a^2+b^2$
Answer: (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
Solution: After finding c we can easily find the foci.
Q.8. Assertion (A): The eccentricity of a parabola is 1.
Reason (R): The eccentricity of a circle is greater than 1.
Answer: (c) (A) is true but (R) is false.
Solution: The eccentricity of a parabola is exactly 1. The eccentricity of a circle is 0.
Q. 9. Assertion (A): The eccentricity of a hyperbola is always greater than 1.

Reason (R): The distance between the foci of a hyperbola is always greater than the length of its transverse axis.
Answer: (a) Both A and R are true, and R is the correct explanation of A
Solution: In a hyperbola, the eccentricity (e) is defined as the ratio of the distance between the foci to the length of the transverse axis. Since the distance between the foci is always greater than the transverse axis, the eccentricity is always greater than 1.
Q. 10. Assertion(A): One of the farthest points from the centre, lying on the circle $x^2 + y^2 - 25 = 0$ is (3,4).
Reason(R): The point on the circle satisfy the equation of circle and are farthest from the centre of the circle.
Answer: a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
Solution: The farthest point from the centre is a point on the circle. Here (3,4) satisfy the equation of circle so it lie on the circle.

VERY SHORT ANSWER TYPE QUESTIONS SOLVED

1) Question: Find the equation of the circle with radius 4 and Centre (-2, 3).
Solution: Given that:
Radius, r = 4, and center (h, k) = (-2, 3).
We know that the equation of a circle with centre (h, k) and radius r is given as
$(x – h)^2 + (y – k)^2 = r^2$ ….(1)
Now, substitute the radius and center values in (1), we get
Therefore, the equation of the circle is
$(x + 2)^2 + (y – 3)^2 = (4)^2$
$x^2 + 4x + 4 + y^2 – 6y + 9 = 16$
Now, simplify the above equation, we get:
$x^2 + y^2 + 4x – 6y – 3 = 0$
Thus, the equation of a circle with center (-2, 3) and radius 4 is $x^2 + y^2 + 4x – 6y – 3 = 0$
2) Question: What is co-ordinate of the centre and the length of the radius of the circle $2x^2 + 2y^2 – x = 0$.

Solution: Given that, the circle equation is $2x^2 + 2y^2 – x = 0$

This can be written as:

$\Rightarrow (2x^2 - x) + y^2 = 0$

$\Rightarrow 2\{[x^2 – (x/2)] + y^2\} = 0$

$\Rightarrow \{ x^2 – 2x(¼) + (¼)^2\} + y^2 – (¼)^2 = 0$ (Using the completing square method)

Now, simplify the above form, we get

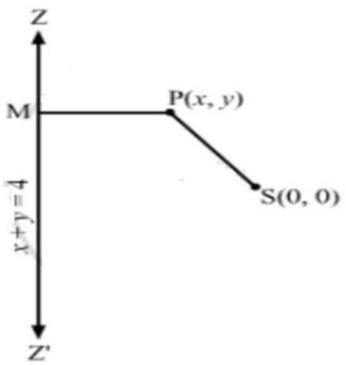
$\Rightarrow (x - (¼))^2 + (y-0)^2 = (¼)^2$

The above equation is of the form $(x – h)^2 + (y – k)^2 = r^2$

Therefore, by comparing the general form and the equation obtained, we can say

h= ¼ , k = 0, and r = ¼. So the centre $(\frac{1}{4}, 0)$ and radius is $\frac{1}{4}$ unit.

3) Question: Determine the coordinates of the focus, the axis of the parabola, the equation of the directrix and the length of latus rectum of the parabola $y^2 = -8x$.
Solution: Given that, the parabola equation is $y^2= -8x$.
It is noted that the coefficient of x is negative.
Therefore, the parabola opens towards the left.
Now, compare the equation with $y^2= -4ax$, we obtain
$-4a= -8$
$\Rightarrow a = 2$
Thus, the value of a is 2.
Therefore, the coordinates of the focus = (-a, 0) = (-2, 0)
Since the given equation involves y^2, the axis of the parabola is the x-axis.
Equation of directrix, x= a i.e., x = 2
We know the formula to find the length of a latus rectum
Latus rectum length= 4a
Now, substitute a = 2, we get
Length of latus rectum = 8 units
4) Question: Write the equation of the parabola with focus (0, 0) and directrix x + y - 4 = 0.
Solution : Let $P(x, y)$ be any point on the parabola whose focus is $S(0,0)$ and the directrix is $x + y = 4$



Now draw PM perpendicular to $x + y = 4$

Therefore, we have: SP = PM

$$\Rightarrow SP^2 = PM^2$$

$$\Rightarrow (x-0)^2 + (y-0)^2 = \left(\frac{x+y-4}{\sqrt{1+1}}\right)^2$$

$$\Rightarrow x^2 + y^2 = \left(\frac{x+y-4}{\sqrt{2}}\right)^2$$

$$\Rightarrow 2x^2 + 2y^2 = x^2 + y^2 + 16 + 2xy - 8y - 8x$$

$$\Rightarrow x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$$

This is the required equation of parabola.
5) Question: Find the distance between the directrices of the hyperbola $x^2 - y^2 = 8$

Solution: Given equation of the hyperbola is $x^2 - y^2 = 8$.

$$\Rightarrow \frac{x^2}{(2\sqrt{2})^2} - \frac{y^2}{(2\sqrt{2})^2} = 1, a = 2\sqrt{2}, b = 2\sqrt{2}$$

Eccentricity, $e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{16}}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

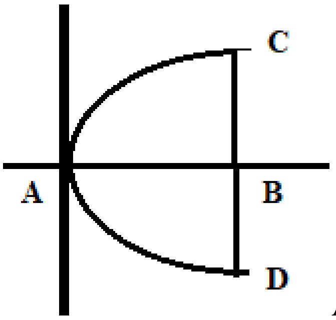
$\therefore$ Distance between the directrices $= \frac{2a}{e} = \frac{2.2\sqrt{2}}{\sqrt{2}} = 4$ unis

6) Question: Aarav is standing at a point whose coordinate is (3, 4). Ayansh wants to move in a path such that its distance from Aarav is always 3 units. Find the equation of path followed by Ayansh.

Solution: Equation of path is $(x - 3)^2 + (x - 4)^2 = 3$

7) Question: Aarini wants to construct a solar parabolic reflector which is 20 cm, wide and 5 cm deep. Find the coordinate of the point at which maximum concentration of sunlight will occur (assuming sunlight travel parallel to the axis of parabolic reflector).

Solution:



AB = 5, CD= 20, CORDINATE OF C(5,10)

Equation of parabola is $y^2=4ax$, $(5)^2=4a(10) \Rightarrow a = \frac{5}{4}$

8) Question: A boy made a model in which Earth moves on a path whose equation is $4x^2 + 9y^2 =36$. Find the possible coordinate of the sun in the model.

Solution: Sun will be at one of the focus

Equation of ellipse $4x^2 + 9y^2 =36$

Standard form $\frac{x^2}{9} + \frac{y^2}{4} = 1, a^2 = 9\ b^2 = 4, a^2 = b^2 + c^2$

$c^2 = 9 - 4 = 5 \Rightarrow c = \pm\sqrt{5}$

So possible co-ordinate of sun is $(\pm\sqrt{5}, 0)$

9) Question: Find foci, vertices , eccentricity and length of latus rectum for the hyperbola $x^2/16 - y^2/9 =1$

Solution: Comparing with standard equation foci $(\pm 5 , 0)$, vertices $(\pm 4 , 0)$ e = 5/4 latus rectum = 9/2

10) Question: **Find the equation of the circle which touches x-axis and whose centre is (1,2).**

Solution: Given that, circle with centre (1,2) touches x-axis.

Radius of the circle is, r = 2

So, the equation of the required circle is:

$(x - 1)^2 + (y - 2)^2 = 2^2$

$\Rightarrow x^2-2x + 1 + y^2-4y + 4 = 4$

$\Rightarrow x^2 + y^2 - 2x-4y + 1 = 0$

SHORT ANSWER TYPE QUESTIONS SOLVED

1)Question: Determine the foci coordinates, the vertices, the length of the major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $(x^2/49) + (y^2/36) = 1$

Solution: On comparing the equation with standard equation, we will get

a= 7 and b = 6

Therefore, $c = \sqrt{(a^2 - b^2)}$

Now, substitute the value of a and b c = $\sqrt{13}$

Hence, the foci coordinates are $(\pm\sqrt{13}, 0)$

Eccentricity, e = c/a = √13/ 7

Length of the major axis = 2a = 2(7) = 14

Length of the minor axis = 2b = 2(6) =12

The coordinates of the vertices are (± 7, 0)

Latus rectum Length= $2b^2/a = 2(6)^2/7 = 2(36)/7 = 72/7$

2) Question: Determine the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), the major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Solution:

Centre = (0, 0), and major axis that passes through the points (3, 2) and (1, 6).

We know that the equation of the ellipse will be of the form when the centre is at (0, 0) and the major axis is on the y-axis,

$(x^2/b^2) + (y^2/a^2) = 1$ …. (1)

Here, a is the semi-major axis.

It is given that, the ellipse passes through the points (3, 2) and (1, 6).

Hence, equation (1) becomes

$(9/b^2) + (4/a^2) = 1$ …(2)

$(1/b^2) + (36/a^2) = 1$ …(3)

Solving equation (2) and (3), we get

$b^2 = 10$ and $a^2 = 40$

Therefore, the equation of the ellipse becomes: $(x^2/10) + (y^2/40) = 1$

3) Question: Determine the equation of the hyperbola which satisfies the given conditions: Foci (0, ±13), the conjugate axis is of length 24.

Solution:

Given that: Foci (0, ±13), Conjugate axis length = 24

It is noted that the foci are on the y-axis.

Therefore, the equation of the hyperbola is of the form:

$(y^2/a^2) - (x^2/b^2) = 1$ …(1)

Since the foci are (0, ±13), we can get

C = 13

It is given that, the length of the conjugate axis is 24,

It becomes 2b = 24

b= 24/2

b= 12

And, we know that $a^2 + b^2 = c^2$

To find a, substitute the value of b and c in the above equation:

$a^2 + 12^2 = 13^2$

$a^2 = 169-144$

$a^2 = 25$.

4) Question: Show that the set of all points such that the difference of their distances from (4, 0) and (–4, 0) is always equal to 2 represent a hyperbola.

Solution:

$$\sqrt{(x+4)^2+y^2} - \sqrt{(x-4)^2+y^2} = 2$$

Solving we will get $\frac{x^2}{1} - \frac{y^2}{4^2-1^2} = 1$ which represent the equation of a hyperbola.

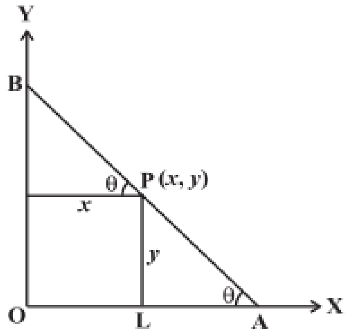
5) Question: If the latus rectum of an ellipse with axis along x-axis and centre at origin is 10, distance between foci = length of minor axis, then find the equation of the ellipse.

Solution:

$\frac{2b^2}{a} = 10, 2c = 2b \Rightarrow c = b, a^2 = b^2 + c^2 \Rightarrow a^2 = 2b^2$

$$\frac{a^2}{a} = 10 \Rightarrow a = 10, b^2 = 50, so\ equation\ of\ ellipse\ is\ \frac{x^2}{100} + \frac{y^2}{50} = 1$$

6) Question: A bar of given length moves with its extremities on two fixed straight lines at right angles. Prove that any point of the bar describes an ellipse.



Solution: Here y = PA sinθ and x= PB cosθ

This gives $\frac{y}{PA}$ = sinθ and $\frac{x}{PB}$ = cosθ

Simplifying we get standard equation of ellipse.

7) Question: If the eccentricity of an ellipse is 5/8 and the distance between its foci is 10, then find latus rectum of the ellipse.

Solution: Here a=8 and b= $\sqrt{39}$ so using the formula of latus rectum we get $\frac{39}{4}$

8) Question: ABCD is a square whose side is a; taking AB and AD as axes, prove that the equation of the circle circumscribing the square is $x^2 + y^2 - a(x + y) = 0$

Solution: We are given that $ABCD$ is a square with side 'a' units.

Let AB and AD represent the x-axis and the y-axis, respectively.

Thus, the coordinates of B and D are $(a, 0)$ and (0, a), respectively.

The endpoints of the diameter of the circle circumscribing the square are B and D.

Therefore, equation of the circle circumscribing the square is $(x - a)(x - 0) + (y - 0)(y - a) = 0$

or, $x^2 + y^2 - a(x + y) = 0$

9) Question: Find the equation of the ellipse the ends of whose major and minor axes are $(\pm 4,0)$ and $(0, \pm 3)$ respectively.

Solution: Given that:

End Points of Major Axis $= (\pm 4,0)$ and End Points of Minor Axis $= (0, \pm 3)$

∴ The Equation of Ellipse is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where, a is the semi-major axis and b is the semi-minor axis.

So, $a = 4$ and $b = 3$

Putting the value of a and b in Eq. (i), we get

$$\frac{x^2}{4^2}+\frac{y^2}{3^2}=1 \Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$$

10) Question: The circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is rolled along the positive direction of x - axis and makes one complete roll. Find its equation in new - position.

Solution: It is told that the circle is rolled along the positive direction of the x-axis and makes one complete roll.

We know that the complete roll of a circle covers the distance $2\pi r$, where r is the radius of the circle.

The centre of the circle moves $2\pi rn$ the positive direction of the x-axis.

Let the d be the distance moved by the centre on completion of one roll.

$$d = 2\pi r(1)$$

$$\Rightarrow d = 2\pi$$

The new position of the centre is $(1 + d, 1) = \text{Centre} = (1 + 2\pi, 1)$

We know that the equation of the circle with centre(p, q) and having radius ' r ' is given by:

$$(x-p)^2 + (y-q)^2 = r^2$$

Now, Equation of circle with centre$(1 + 2\pi, 1)$ and having radius 1 units.

$$\Rightarrow (x - (1 + 2\pi)^2 + (y-1)^2 = 12$$

$$\Rightarrow (x - 1 - 2\pi)^2 + (y-1)^2 = 1$$

$$\Rightarrow x^2 - (2 + 4\pi)x + (1 + 2\pi)^2 + y^2 - 2y + 1 = 1$$

$$\Rightarrow x^2 + y^2 - (2 + 4\pi)x - 2y - (1 + 2\pi)^2 = 0$$

The equations of the circles is$x^2 + y^2 - (2 + 4\pi)x - 2y - (1 + 2\pi)^2 = 0$

LONGANSWER TYPE QUESTIONS SOLVED

Q.1. Find the equation of the ellipse whose foci are (4, 0) and (-4, 0), eccentricity = 1/3.

Solution:

Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

The coordinate of foci are (+ ae, 0) and (-ae, 0).

$\therefore$ ae $= 4[\because$foci $(\pm 4,0)]$

$$\Rightarrow a \times \frac{1}{3} = 4\left[\because e = \frac{1}{3}\right]$$

$$\Rightarrow a = 12$$

$$\Rightarrow a^2 = 144$$

Now, $b^2 = a^2(1 - e^2)$

$$\Rightarrow b^2 = 144\left[1 - \left(\frac{1}{3}\right)^2\right]$$

$$\Rightarrow b^2 = 144\left[1 - \frac{1}{9}\right]$$

$$\Rightarrow b^2 = 144 \times \frac{8}{9}$$

$$\Rightarrow b^2 = 16 \times 8 = 128$$

Substituting, $a^2 = 144$ and $b^2 = 128$ in the above equation of ellipse, we get

$$= \frac{x^2}{144} + \frac{y^2}{128} = 1$$

$$\Rightarrow \frac{1}{16}\left[\frac{x^2}{9} + \frac{y^2}{8}\right] = 1$$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{8} = 16$$

This is the required equation of the ellipse.

Q.2. Find the equation of a circle passing through the points (2, -6), (6, 4) and (-3, 1).

Solution:

Let the equation of the circle passing through the given points be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ……………. (i)

Since, circle passes through the point$(2, -6)$.

So, put $x = 2, y = -6$ in Eq. (i), we get

$$4 + 36 + 4g - 12f + c = 0$$

$\Rightarrow 4g - 12f + c = -40$ .....(ii)

Also, circle passes through the point$(6,4)$.

So, put $x = 6, y = 4$ in Eq. (i), we get

$$36 + 16 + 12g + 8f + c = 0$$

$\Rightarrow 12g + 8f + c = -52$...(iii)

Also, circle passes through the point $(-3,1)$.

So, put $x = -3$ and $y = 1$ in Eq. (i), we get

$$9 + 1 - 6g + 2f + c = 0$$

$\Rightarrow -6g + 2f + c = -10$...(iv)

On subtracting Eq. (iii) from Eq. (ii), we get

$-8g - 20f = 12$ ... (v)

On subtracting Eq. (iv) from Eq. (iii), we get

$18g + 6f = -42$ ... (vi)

On solving Eqs. (v) and (vi) for g and f, we get

$$g = -\frac{32}{13}, f = \frac{5}{13}$$

On putting the values of g and f in Eq. (ii), we get

$$c = -\frac{332}{13}$$

Now, on putting $g = -\frac{32}{13}, f = \frac{5}{13}$ and $c = -\frac{332}{13}$ in Eq. (i), we get

$$x^2 + y^2 - \frac{64}{13}x + \frac{10}{13}y - -\frac{332}{13} = 0$$

$$\Rightarrow 13x^2 + 13y^2 - 64x + 10y - 332 = 0$$

which is the required equation of circle.

Q.3. Find the equation of the hyperbola, the length of whose latus rectum is 4 and the eccentricity is 3.

Solution:

Given: The length of latus rectum is 4 units , and the eccentricity is 3

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The length of the latus rectum is 4 units.

$\Rightarrow$ length of the latus rectum $= \frac{2b^2}{a} = 4$

$\Rightarrow \frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$ ... (i)

And also given, the eccentricity, $e = 3$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = 3$$

$\Rightarrow 1 + \frac{b^2}{a^2} = 9$ [Squaring both sides]

$$\Rightarrow \frac{b^2}{a^2} = 8$$

$$\Rightarrow b^2 = 8a^2$$

$\Rightarrow 2a = 8a^2$ [From (i)]

$$\Rightarrow a = \frac{1}{4} \Rightarrow a^2 = \frac{1}{16}$$

From (i) $\Rightarrow b^2 = 2a = 2 \times \frac{1}{4} = \frac{1}{2} \Rightarrow b^2 = \frac{1}{2}$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{1}{2}} = 1$$

$$\Rightarrow 16x^2 - 2y^2 = 1$$

Q.4. Find the equation of a circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point (5, 4).

Solution: Here, the equation of circle is $x^2 + y^2 + 4x + 6y + 11 = 0$

$$\Rightarrow (x^2 + 4x) + (y^2 + 6y) = -11$$

On adding 4 and 9 both sides to make perfect squares, we get

$$(x^2 + 4x + 4) + (y^2 + 6y + 9) = -11 + 4 + 9$$

$\Rightarrow (x + 2)^2 + (y + 3)^2 = (\sqrt{2})^2$ ... (i)

Its centre is $(-2, -3)$

The required circle is concentric with circle 1, therefore its centre is $(-2, -3)$. Since, it passes through $(5,4)$, therefore radius is $r = CP = \sqrt{(5 + 2)^2 + (4 + 3)^2}$[∵ distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$] $= \sqrt{49 + 49} = 7\sqrt{2}$

Hence, the equation of required circle having centre$(-2, -3)$ and radius $7\sqrt{2}$ is,

$$(x + 2)^2 + (y + 3)^2 = (7\sqrt{2})^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 + 6y + 9 = 98$$

$$\Rightarrow x^2 + 4x + y^2 + 6y - 85 = 0$$

Q.5. Find the (i) length of major and minor axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity, and (v) length of the latus rectum of ellipse:

$\frac{x^2}{25} + \frac{y^2}{9} = 1.$

Solution: Above equation is of the form.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Comparing eq. (i) and (ii), we get

$$a^2 = 25 \text{ } and \text{ } b^2 = 9 \Rightarrow a = 5 \text{ } and \text{ } b = 3$$

i). To find: Length of major axes

Clearly, $a > b$, therefore the major axes of the ellipse are along x-axes.

∴ Length of major axes $= 2a$

Length of major axes $= 2 \times 5$

ii). To find: Coordinates of the Vertices

Clearly, $a > b$

∴ Coordinate of vertices $= (a, 0)$ and $(-a, 0)$

Coordinate of vertices $= (5,0)$ and $(-5,0)$

iii). To find: Coordinates of the foci

We know that,

Coordinates of foci $= (\pm c, 0)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c: $c^2 = a^2 - b^2 = 25 - 9$, $c^2 = 16$

So foci is $(\pm 4,0)$

iv) e $= \frac{c}{a} = \frac{4}{5}$

v) The length of the latus rectum is $\frac{2b^2}{a} = \frac{18}{5}$ units

CASE STUDY BASED QUESTIONS SOLVED

1.) Question: Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8m.

1. Path traced by Arun represents which type of curve. Find the length of major axis?
2. Find the equation of the curve traced by Arun?
3. Find the eccentricity of path traced by Arun?

Solution: Arun is running in a racecourse such that the sum of the distances from the two flag posts from him is always $10m$ and the distance between the flag posts is $8m$.

(i) An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. Hence path traced by Arun is ellipse.

Sum of the distances of the point moving point to the foci is equal to length of major axis $= 10m$

(ii) Given $2a = 10 \& 2c = 8$

$$\Rightarrow a = 5 \& c = 4$$

$$c^2 = a^2 + b^2$$

$$\Rightarrow 16 = 25 + b^2$$

$$\Rightarrow b^2 = 25 - 16 = 9$$

Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

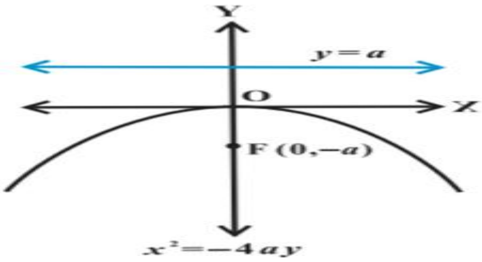
Required equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

(iii) equation is of given curve is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$a = 5, b = 3$ and given $2c = 8$ hence $c = 4$

Eccentricity $= \frac{c}{a} = \frac{4}{5}$

2.) Question: Indian track and field athlete Neeraj Chopra, who competes in the Javelin throw, won a gold medal at Tokyo Olympics. He is the first track and field athlete to win a gold medal for India at the Olympics.



1. Name the shape of path followed by a javelin. If equation of such a curve is given by $x^2 = -16y$, then find the coordinates of foci.

2. Find the equation of directrix and length of latus rectum of parabola $x^2 = -16y$.

3. Find the equation of parabola with Vertex (0,0), passing through (5,2) and symmetric with respect to y - axis and also find equation of directrix.

Solution:

(i) The path traced by Javelin is parabola. A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

compare $x^2 = -16y$ with $x^2 = -4ay$

$$\Rightarrow -4a = -16$$

$$\Rightarrow a = 4$$

coordinates of focus for parabola $x^2 = -4ay$ is $(0, -a)$

$\Rightarrow$ coordinates of focus for given parabola is $(0, -4)$

(ii) compare $x^2 = -16y$ with $x^2 = -4ay$

$$\Rightarrow -4a = -16 \ \Rightarrow a = 4$$

Equation of directrix for parabola $x^2 = -4ay$ is $y = a$

$\Rightarrow$ Equation of directrix for parabola$x^2 = -16y$ is $y = 4$

Length of latus rectum is $4a = 4 \times 4 = 16$

(iii)Equation of parabola with axis along y - axis

$$x^2 = 4ay$$

which passes through $(5,2)$

$$\Rightarrow 25 = 4a \times 2$$

$$\Rightarrow 4a = \frac{25}{2}$$

hence required equation of parabola is

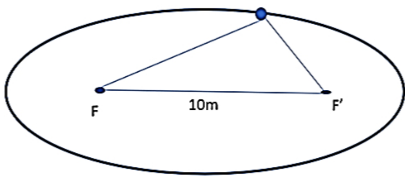
$$x^2 = \frac{25}{2}y$$

$$\Rightarrow 2x^2 = 25y$$

Equation of directrix is $y = -a$. Hence required equation of directrix is $8y + 25 = 0$.

3.) Question: A farmer wishes to install 2 hand pumps in his field for watering. The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.

Name the curve traced by farmer and hence find the foci of curve.



1. Find the equation of curve traced by farmer.
2. Find the length of major axis, minor axis and eccentricity of curve along which farmer moves.

Solution: (i) The curve traced by farmer is ellipse. Because An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Two positions of hand pumps are foci Distance between two foci $= 2c = 10$ Hence $c = 5$ Here foci lie on x axis & coordinates of foci $= (\pm c, 0)$

Hence coordinates of foci $= (\pm 5,0)$

(ii) $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Sum of distances from the foci $= 2a$

Sum of distances between the farmer and each hand pump is $= 26 = 2a$

$$\Rightarrow 2a = 26 \Rightarrow a = 13m$$

Distance between the hand pump $= 10m = 2c$

$$\Rightarrow c = 5m$$

$$c^2 = a^2 - b^2$$

$$\Rightarrow 25 = 169 - b^2$$

$$\Rightarrow b^2 = 144$$

Equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

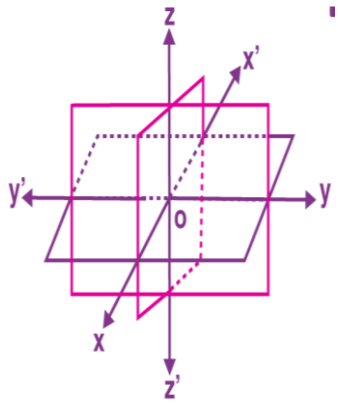
(iii)Equation of ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ comparing with standard equation of ellipse $a = 13, b = 12$ and $c = 5$ (given)

Length of major axis $= 2a = 2 \times 13 = 26$

Length of minor axis $= 2b = 2 \times 12 = 24$

Eccentricity $e = \frac{c}{a} = \frac{5}{13}$

CHAPTER -11: INTRODUCTION TO THREE-DIMENSIONAL GEOMETRY



Definitions and Formulae:

❖ **Coordinate Axes and Planes:**

Three mutually perpendicular axes: x-axis, y-axis and z-axis.

The point where they intersect is called the origin (0, 0, 0).

The three **coordinate planes** are:

XY-plane (z = 0)

YZ-plane (x = 0)

ZX-plane (y = 0)

❖ **Coordinates of a Point:**

A point in 3D space is represented as **(x, y, z)**.

These are the distances from the **YZ, ZX, and XY planes** respectively.

❖ An arbitrary point P in three-dimensional space is assigned coordinates (x_0,y_0,z_0) provided that

(1) The plane through P parallel to the yz-plane intersects the x-axis at $(x_0,0, 0)$;

(2) The plane through P parallel to the xz-plane intersects the y-axis at $(0,y_0,0)$;

(3) The plane through P parallel to the xy-plane intersects the z-axis at $(0,0,z_0)$.

The space coordinates (x_0,y_0,z_0) are called the Cartesian coordinates of P or simply the rectangular coordinates of P.

❖ **Distance Formula in 3D:**

Distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$AB=\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}}$

❖ The three co-ordinate planes divide the whole space into eight compartments, known as **octants**.

Octant→ Co-Ordinates ↓	**I** **XOYZ**	**II** **X'OYZ**	**III** **X'OY'Z**	**IV** **XOY'Z**	**V** **XOYZ'**	**VI** **X'OYZ'**	**VII** **X'OY'Z'**	**VIII** **XOY'Z'**
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

❖ **Coordinate Axes**

➢ **Definition:** The three mutually perpendicular lines (x-axis, y-axis, and z-axis) used to define the position of a point in 3D space.

❖ **Coordinate Planes**

➢ **Definition:** The planes formed by any two of the three coordinate axes.

XY-plane

YZ-plane

ZX-plane

❖ **Origin**

➢ Definition: The point of intersection of the x-, y-, and z-axes is called Origin ; it is denoted by O(0, 0, 0).

❖ **Coordinates of a Point in 3D**

➢ **Definition:** A point in 3D space is represented as **(x, y, z)**, where:

x = distance from the YZ-plane

y = distance from the XZ-plane

z = distance from the XY-plane

MULTIPLE CHOICE QUESTIONS SOLVED

1. What is the distance between the points A(1, 2, 3) and B(4, 6, 3)?
   a. 5 units b. $\sqrt{25}$ units c. $\sqrt{9}$ units d. $\sqrt{10}$ units
   Answer: a. Using distance formula $=\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}}$
   Distance = 5 units
2. The coordinates of a point in 3D are represented as:
   a. (x, y) b. (x, y, z) c. (x, y, t) d. (x, z)
   Answer: b. (x, y, z)
3. Which of the following is the distance formula between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$?
   a. $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ b. $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
   c. $(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$ d. $\sqrt{[(x_1 + x_2)^2 + (y_1 + y_2)^2 + (z_1 + z_2)^2]}$
   Answer: b. $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
4. The origin in 3D geometry is represented as:
   a. (0, 0) b. (0, 0, 0) c. (1, 1, 1) d . none of these
   Answer: B. (0, 0, 0)
5. Which axis is added in 3D geometry that is not present in 2D geometry?
   a. x-axis b. y-axis c. z-axis d. t-axis
   Answer: c. z-axis
6. If point A is at (2, 3, 4), then the x-coordinate of A is:
   a. 2 b. 3 c. 4 d. 0
   Answer: a. 2
7. The coordinates of a point lie in the xz-plane if:
   a. $y = 0$ b. $x = 0$ c. $z = 0$ d. $x = y$
   Answer: a. $y = 0$
8. Which plane does the point (0, 4, 3) lie on?
   a. xy-plane b. yz-plane c. xz-plane d. none of these
   Answer: b. yz-plane
9. In 3D space, the distance of point P(x, y, z) from the origin is:
   a. $x + y + z$ b. $x^2 + y^2 + z^2$ c. $\sqrt{(x^2 + y^2 + z^2)}$ d. $(x + y + z)^2$
   Answer: c. $\sqrt{(x^2 + y^2 + z^2)}$
10. The point (3, 0, 0) lies on the:
    a. y-axis b. z-axis c. x-axis d. origin
    Answer: c. x-axis

ASSERTION-REASON BASED QUESTIONS SOLVED

In the following questions , a statement of Assertion (A) is followed by a statement of Reason (R) . Pick the correct option.

Options:
a). Both A and R are true, and R is the correct explanation of A.
b). Both A and R are true, but R is not the correct explanation of A.
c). A is true, but R is false.
d). A is false, but R is true.

1) **Assertion (A):** The coordinates of a point in space are represented as (x, y, z).
   Reason (R): Three mutually perpendicular axes are required to locate a point uniquely in 3D space.
   Answer: a. With the distance from axes in +ve & -ve sense coordinates are determined.
2) **Assertion (A):** The distance between two points in 3D space is given by
   $\sqrt{\{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\}}$
   Reason (R): This formula is derived using the Pythagoras Theorem.
   Answer: a. Using Pythagoras Thm. i.e. $p^2+b^2=h^2$ we have the formula of distance.
3) **Assertion (A)**: The coordinates of the origin in 3D space are (0, 0, 0).
   Reason (R): Origin is the point where the x-axis and y-axis intersect.
   Answer: c , Origin is the point where the 3 axes meet.
   Explanation: The origin in 3D is where x, y, and z axes intersect, not just x and y.
4) **ASSERTION(A)** – The point (-2,3, -3) lies in the seventh octant.

REASON(R) – Any arbitrary point of seventh octant have all the coordinate negative.
ANSWER b.
All the coordinates of 7^{th} octant are negative so given point will not lie in 7^{th} octant. Hence A is false.

5) **Assertion (A):** Any point on the x-axis in 3D has coordinates of the form (x, 0, 0).
Reason (R): On the x-axis, the y and z coordinates remain constant and equal to 0.
Answer:a ., on each axes the rest 2 components are 0.

6) **Assertion (A):** Point $(a+1, a-1, a^2 -1)$ lies on y-axis, if a = -1.
Reason (R): For any point (x, y, z) on y-axis, x = 0, z =0.
Answer: a , on each axes the rest 2 components are 0.

7) **Assertion (A):** The equation $y^2-3y+2=0$ represents planes parallel to xz-plane in three dimensional space.
Reason (R): The equation $y^2-3y+2=0$ represents points (0, 1, 0) and (0, 2, 0) in three dimensional space.
Answer: c , (0, 1, 0) and (0, 2, 0) are the points not planes

8) **Assertion (A):** The point (3, -4,5) is equidistant from xy-plane and z-axis.
Reason (R): The point (3, –4, 5) is at a distance of 5 units from xy-plane.
Answer: b , Distance from xy-plane=|5|=5 and Distance from z-axis=$\sqrt{9+16}=5$ but R is not an explanation.

9) **Assertion (A):** If $a^2 = b^2 + c^2$, then the point P (a, b, c) is equidistant from yz plane and x-axis.
Reason (R): The sum of the distances of point P (a, b, c) from the coordinate planes is a + b + c.
Answer: c ,
Distance from yz plane is $\sqrt{(b^2 + c^2)}$ & distance from x-axis a. But R is $\sqrt{(a^2+b^2 + c^2)}$ not a+b+c

10) **Assertion (A):**Vertices of triangle are A (2,0,1), B (1,-1,1), C (2,1,-1). Then side AB is $(2)^{1/2.}$

Reason (R): The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) and is given by:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Answer: a.

Using distance formula we are able to find side AB so it is correct explanation

VERY SHORT ANSWER QUESTIONS SOLVED

1.Prove by distance formula, that the points A (1, -1,3), B (2, -4,5) and C (5, -13,11) are collinear.
ANSWER-Using Distance formula $\sqrt{\{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\}}$
$AB=\sqrt{14}, BC = 3\sqrt{14}$, $AC = 4\sqrt{14}$ AB+BC=AC, Therefore collinear.

2. Show that the triangle ABC with vertices A (0,4,1), B (2,3, -1) and C (4,5,0) is right angled.
ANSWER- Using Distance formula $\sqrt{\{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\}}$
$AB=3$, $BC = 3$, $AC = \sqrt{18}$
$\Rightarrow AB^2+BC^2=AC^2$, Therefore right angled.

3. Show that the points A (5, -1,1), B (7, -4,7), C (1, -6,10) and D (-1, -3,4) are vertices of a Rhombus.
ANSWER-By Using Distance formula $\sqrt{\{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\}}$
$AB=BC = CD = AD = 7$, $AC = \sqrt{122}$ & $BD = \sqrt{74}$ i.e. $AC \neq BD$.
Therefore it's a rhombus.

4. Find the locus of a point whose sum of the distances from the points A (2,0,0) and B (-2,0,0) is 10.
ANSWER- Let P(x,y,z) such that PA+PB=10
Using distance formula we have
$\sqrt{\{(x-2))^2 + y^2 + z^2\}} + \sqrt{\{(x+2))^2 + y^2 + z^2\}} = 10$
……$\Rightarrow 21x^2+25y^2+25z^2=525$

5.Show that the points A (2, -1,3), B (1, -3,1) and C (0,1,2) are vertices of an isosceles triangle.

ANSWER-Using distance formula AB=3, BC=$3\sqrt{2}$ & AC=3

i.e. AB=AC

Hence the triangle is isosceles.

6.Show that the points A (2,3,5), B (-4,7, -7), C (-2,1, -10) and D (4, -3,2) are vertices of a rectangle.

ANSWER- Using distance formula AB=CD=14, BC=DA=7 & AC=BD=$\sqrt{245}$.

Hence a rectangle.

7.Find the locus of a point whose each point is equidistant from A (2,3, -4) and B (-1,2,3).

ANSWER-Let the point be P(x,y,z) such that PA=PB, using distance formula

$\sqrt{\{(x-2)^2+(y-3)^2+(z+4)^2\}}=\sqrt{\{(x+1)^2+(y-2)^2+(z-3)^2\}}$. *Squaring*

.....⇒ $6x+2y-14z-15=0$

8. Prove by distance formula, that the points A (1,2,3), B (-1, -1, -1) and C (3,5,7) are collinear.

ANSWER- Using distance formula AB=$\sqrt{29}$, BC=$2\sqrt{29}$ & $AC=\sqrt{29}$

⇒AB+AC=BC.

Therefore, points are collinear.

9. What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincide with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?

ANSWER- The coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin are:

(2,0,0) (2,2,0) (0,2,0) (0,2,2) (0,0,2) (2,0,2) (0,0,0) (2,2,2)

10. If the distance between the points (a,2,1) and (1, -1,1) is 5, then find the values of a?

ANSWER-By distance formula $\sqrt{\{(a-1)^2+9\}}$=5…⇒ $a=5\ or -3$

SHORT ANSWER QUESTIONS SOLVED

1. Determine the point in xy plane equidistant from the points A (1, -1,0), B (2,1,2) and C (3,2, -1).

   ANSWER- Using distance formula

   PA=PB ⇒2a+4b-7=0,

   PB=PC ⇒2a+2b-5=0

   ⇒a=3/2 & b=1 .

2. Prove that the triangle formed by joining the three points with coordinates A(1,2,3) B(2,3,1) and C(3,1,2) is an equilateral triangle.

   ANSWER-Using distance formula AB=BC=CA=$\sqrt{6}$.

3. Show that the points (0,7,10), (-1,6,6) and (-4,9,6) are the vertices of an isosceles right angled triangle.

   ANSWER-Using distance formula AB=BC=$3\sqrt{2}$, AC=6

   $AB^2+BC^2=18+18=36=AC^2$

   ⇒ Triangle is isosceles & right angled.

4. Prove that the points A (1,3,0), B (-5,5,2), C (-9, -1,2) and D (-3, -3,0) taken in order are the vertices of a parallelogram. Also show that ABCD is not a rectangle.

   ANSWER-Using distance formula AB=CD=$\sqrt{44}$, AD=BC =$\sqrt{52}$ ⇒ a parallelogram.

   But AC=$\sqrt{120}$ & $BD=\sqrt{72}$. Therefore it is not a rectangle.

5. Show that the tetrahedron with vertices O (0,0,0), A (0,1,1), B (1,0,1) and C (1,1,0) is a regular one.

   ANSWER-Using distance formula OA=OB=OC=AB=BC=CA=$\sqrt{2}$.

   Therefore it's a regular tetrahedron.

6. Find the coordinates of a point which is equidistant from the four points O (0,0,0), A (2,0,0), B (0,3,0) and C (0,0,8).

   ANSWER-Let point P is equidistant from points O, A , B and C

   ⇒ $x^2+y^2+z^2=(x-2)^2+y^2+z^2=x^2+(y-3)^2+z^2=x^2+y^2+(z-8)^2$

⇒ $x=1, y=3/2$ and $z=4$. Hence Point P(1,3/2,4)

7. If A (-2,2,3), B (13, -3,13) are two points, find the locus of a point P which moves in such a way that 3PA=2PB.

   ANSWER- $x^2+y^2+z^2+28x+12y+10z=119$

8. Determine the point in yz plane equidistant from the points A (1, -1,0), B (2,1,2) and C (3,2, -1).

   ANSWER-Let the point on yz plane is P (0, y, z) is equidistant from A, B & C.

   ⇒ $PA^2=PB^2=PC^2$

   Using distance formula y=2 and z=1

   Point P(0,2,1)

LONG ANSWER QUESTIONS SOLVED

1. Show that the points P (0,1,2), Q (2, -1,3) and R (1, -3,1) are the vertices of an isosceles right-angled triangle.

   ANSWER- PQ=QR=3 & PR=$\sqrt{18}$

   And $PQ^2+QR^2=PR^2$

   ⇒ Triangle PQR is a right-angled isosceles triangle.

2. Find the locus of the point which is equidistant from the points A (0,2,3) and B (2, -2,1).

   ANSWER-Let P (x, y, z) be equidistant from points A and B.

   ⇒$PA^2=PB^2$

   ⇒Using distance formula ……

   $x+2y+z=2$

3. Find the coordinates of a point equidistant from the four points O (0,0,0), A(a,0,0), B (0, b,0) and C (0,0, c).

   ANSWER- Let point P is equidistant from O, A , B & C .

   Using distance formula ⇒$OP^2= AP^2=BP^2=CP^2$

   ⇒ ……………

   ⇒ $x=a/2, y=b/2$ and $z=c/2$

   Hence the point is P (a/2, b/2 , c/2)

4. Find the locus of a point whose distance from y-axis is equal to its distance from the point (2,1, -1).

   ANSWER-Let point P (x,y,z) be equidistant from y-axis and the point (2,1,-1)

   Using distance formula $x^2+z^2=(x-2)^2+(y-1)^2+(z+1)^2$

   The **locus** of the point is given by:

   $4x+2y-y^2-2z=6$

5. Find the coordinates of a point in yz plane which is equidistant from three points A (2,0,3), B (0,3,2) and C (0,0,1).

   ANSWER-Let the point in yz plane is P(0,y,z) such that

   $PA^2=PB^2=PC^2$

   ⇒$6y=10z$ and $6y+2z=12$

   By solving $y=5/3$ and $z=1$

   Therefore the point is P(0,5/3,1)

CASE STUDY QUESTIONS SOLVED

1. A drone delivery company uses a coordinate system to map delivery locations. They represent locations in 3D using the Cartesian system. One of the delivery drones is at point A (2, 3, 5), and the delivery destination is at point B (6, 7, 9).

   Using the above informations, answer the following:

   (i) Find the distance the drone has to travel to reach point B from point A.

   (ii) Find the LOCUS of a target (i.e. collection of points) equidistant from A and B.

   (iii). Find the LOCUS of a point whose sum of distances from P(2,0,0) and Q(-2,0,0) is 50 units.

   OR

   (iv) Find the LOCUS of a point whose difference of distances from P(2,0,0) and Q(-2,0,0) is 5 units.

Answer:

(i) Distance AB=$\sqrt{4^2+4^2+4^2}$ =$\sqrt{48}=4\sqrt{3}$

(ii) $Let\ the\ coordinates\ of\ the\ TARGET\ be\ P(x,y,z) such\ that\ PA=PB$

$\Rightarrow\sqrt{(x-2)^2+(y-3)^2+(z-5)^2}=\sqrt{(x-6)^2+(y-7)^2+(z-9)^2}$

Simplifying x+ y+ z =16

(iii). $Let\ the\ coordinates\ of\ the\ point\ be\ P(x,y,z) such\ that\ PA+PB=50$

$\Rightarrow \sqrt{(x-2)^2+(y)^2+(z)^2}+\sqrt{(x+2)^2+(y)^2+(z)^2}=50$

On simplifying

$$\frac{x^2}{625}+\frac{y^2}{621}+\frac{z^2}{621}=1$$

OR

(iv) $Let\ the\ coordinates\ of\ the\ point\ be\ P(x,y,z) such\ that\ PA-PB=5$

$\Rightarrow \sqrt{(x-2)^2+(y)^2+(z)^2}-\sqrt{(x+2)^2+(y)^2+(z)^2}=5$

On simplifying $x^2-4x-25/4=0$

2.An engineer is constructing a structure that has its base along the XY-plane and a vertical support that reaches up to point P (4, 6, 10). The base support is fixed at point O(0, 0, 0).

Using the above information, answer the following:

(i). Find the length of the vertical support (distance OP).

(ii) Two separate ropes of same length being tied to both the ends are held by a sky diver. Find the locus of curve being described by the diver.

OR

(iii) If another support is placed along the line passing for which sum of distances O and P is 10 units , write the locus of the point.

Answer:

(i) Length of the vertical support = $\sqrt{(4-0)^2+(6-0)^2+(10-0)^2}=2\sqrt{38}$

(ii) $Let\ the\ coordinates\ of\ the\ TARGET\ be\ A(x,y,z)\ such\ that\ PA=OA$

$\Rightarrow \sqrt{(x-4)^2+(y-6)^2+(z-10)^2}=\sqrt{(x)^2+(y)^2+(z)^2}$

On simplifying we get

2x+3y+5z=38

(iii) $\sqrt{(x-4)^2+(y-6)^2+(z-10)^2}-\sqrt{(x)^2+(y)^2+(z)^2}=10$

$\Rightarrow \sqrt{(x-4)^2+(y-6)^2+(z-10)^2}=+\sqrt{(x)^2+(y)^2+(z)^2}+10$

On squaring both sides and simplifying we get the answer.

3.A surveillance camera is mounted at point C(0, -3,0) in a room and it must cover an object placed at P (0,3, 0). The cable connecting the camera runs along the line joining these two points.

Using the above information, answer the following:

Questions:

(i). Find the distance between camera and the object (that is the distance OP).

(ii). Write the locus of the point along which the cable runs such that the point is equidistant from C & P.

OR

(iii). Write the locus of the point along which the cable runs such that the sum of distances from C & P is always 10.

ANSWER

(i). Distance between the camera & the object by distance formula

$\sqrt{0^2+(3-(-3))^2+(0)^2}=6$

(ii). Let the point be A(x,y,z) such that AO=PO

$\sqrt{(x-0)^2+(y+3)^2+(z-0)^2}=\sqrt{(x-0)^2+(y-3)^2+(z-0)^2}$

On simplifying Equation is y=0

OR

(iii)$Let\ the\ coordinates\ of\ the\ TARGET\ be\ A(x,y,z)\ such\ that\ PA+CA=10$

$\Rightarrow \sqrt{(x-0)^2+(y+3)^2+(z)^2}+\sqrt{(x)^2+(y-3)^2+(z)^2}=10$

On simplifying we get $\frac{x^2}{16}+\frac{y^2}{25}+\frac{z^2}{16}=1$

CHAPTER 12 – LIMITS AND DERIVATIVES

DEFINITIONS AND FORMULAE

- **Limits:**

A limit describes the value a function "approaches" as its input gets arbitrarily close to a specific point, regardless of whether the function actually has that value at that point. Understanding limits is essential for defining continuity and derivatives.

Let y = f(x) be a function of x. If at x = a, f(x) takes indeterminate form, then we consider the values of the function which is very near to a. If these values tend to a definite unique number as x tends to a, then the unique number so obtained is called the limit of f(x) at x = a and we write it as $\lim_{x\to a} f(x) = f(a)$.

- **Derivatives:**

A derivative represents the instantaneous rate of change of a function at a specific point. It can be visualized as the slope of the tangent line to the function's graph at that point. Derivatives are used to analyze how functions change and are fundamental in many scientific and engineering applications.

Suppose f is a real-valued function, then

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ is called the derivatives f at x iff $\frac{f(x+h)-f(x)}{h}$ exists finitely.

FORMULAS

Limit

Left Hand and Right-Hand Limits

If values of the function at the point which are very near to a on the left tends to a definite unique number as x tends to a, then the unique number so obtained is called the left-hand limit of f(x) at x = a, we write it as

$$LHL: \lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h)$$

Similarly, right hand limit is

$$RHL: \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h)$$

Existence of Limit

Limit of $f(x)$ exists, if $\boldsymbol{LHL = RHL}$

Some Properties of Limits

Let f and g be two functions such that both $f(x)$ and $g(x)$ exists, then

(i) $\lim_{x\to a}\{(f+g)(x)\} = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

(ii) $\lim_{x\to a}(kf(x)) = k \lim_{x\to a} f(x)$

(iii) $\lim_{x\to a}[f(x) \cdot g(x)] = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x)$

(iv) $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, where $g(x) \neq 0$

Some Standard Limits

(i) $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

(ii) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

(iii) $\lim_{x\to 0} \frac{\tan x}{x} = 1$

(iv) $\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$

(v) $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

(vi) $\lim_{x\to a} \frac{\log(1+x)}{x} = 1$

Derivatives

Suppose f is a real-valued function, then

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ is called the derivatives f at x iff $\frac{f(x+h)-f(x)}{h}$ exists finitely.

Fundamental Derivative Rules of Function

(i) $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

(ii) $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$

(iii) $\frac{d}{dx}[f(x).g(x)] = \left[\frac{d}{dx} f(x)\right].g(x) + f(x)\left[\frac{d}{dx} g(x)\right]$

(iv) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\left[\frac{d}{dx}f(x)\right].g(x)-f(x)\left[\frac{d}{dx}g(x)\right]}{[g(x)]^2}$

Some Standard Derivatives

(i) $\frac{d}{dx}(x^n) = nx^{n-1}$ (ii) $\frac{d}{dx}(sinx) = cosx$

(iii) $\frac{d}{dx}(\cos x) = -sinx$ (iv) $\frac{d}{dx}(\tan x) = sec^2x$

(v) $\frac{d}{dx}(\cot x) = -cosec^2x$ (vi) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(vii) $\frac{d}{dx}(cosec\ x) = -cosec\ x \cot x$ (viii) $\frac{d}{dx}(a^x) = a^x \ln a$

(ix) $\frac{d}{dx}(e^x) = e^x$ (x) $\frac{d}{dx}(logx) = \frac{1}{x}$

MULTIPLE CHOICE QUESTIONS SOLVED

1. $\lim_{x\to 0}\frac{1-\cos x}{\sin x}$ is equal to:

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) 1

ANSWER- (a) $\lim_{x\to 0}\frac{sin^2x}{sinx.(1+cosx)} = \lim_{x\to 0}\frac{sinx}{.(1+cosx)}=0$

2. Find the derivative of sin ^{n} x with respect to x.

(a) n sin $^{n-1}$x. sin x (b) n cos $^{n-1}$x. sin x

(c) (n-1) sin ^{n} x. cos x (d) n sin $^{n-1}$x. cos x

ANSWER-(d) n sin $^{n-1}$x. cos x

3. Evaluate: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

(a) 2 (b) 5 (c) 0 (d) 1

ANSWER-(b) $\lim_{x\to 0}\frac{(x+1)^5-1}{(x+1)-1}=5$

4. Evaluate: $\lim_{x\to a}\frac{sinax}{bx}$

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) - $\frac{a}{b}$ (d)) 0

ANSWER- (a) $\lim_{x\to a}\frac{sinax}{ax}\times\frac{a}{b}$ = a/b

5. Evaluate: $\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$

(a) π (b) $\frac{1}{\pi}$ (c) 0 (d) 1

ANSWER-(b) $1\times\frac{1}{\pi} = \frac{1}{\pi}$

6. Evaluate: $\lim_{x\to\frac{\pi}{2}}\frac{tan2\left(\frac{\pi}{2}-x\right)x}{\left(x-\frac{\pi}{2}\right)}$

(a) - 2 (b) 5 (c) 0 (d) π

ANSWER-(a) $\lim_{x\to\pi/2}\frac{Tan2(\frac{\pi}{2}-x)}{-2(\frac{\pi}{2}-x)}\times 2= - 2$

7. Find the derivative of (sin x. cos x) with respect to x.

a) cos 2x (b) cos x (c) sin 2x (d) sin x

ANSWER-(a) $\frac{dy}{dx}= cosx.cosx - sinx.sinx = cos2x$

8. Find the derivative of [x^{-4}. (3 – 4 x^{-7})] with respect to x.

(a) 12 x^{-5} + 44 x^{-12} (b) -12 x^{-5} + 44 x^{12}

(c) 12 x^{-5} + 44 x^{12} (d) -12 x^{-5} + 44 x^{-12}

ANSWER- (d) $\frac{dy}{dx}$= -12 x^{-5} + 44 x^{-12}

9. Find the derivative of (tan x-sec x) with respect to x.

(a) Sec^2x + sec x tan x (b) Sec^2x-sec x cot x

(c) Sec^2x-sec x tan x (d) Sec x-sec x tan x

ANSWER- 9(c) $\frac{dy}{dx}$=Sec^{2}x-Sec x Tan x

10. Find the derivative of (5 sin x-7 sec x) with respect to x.

a) 5 cos x-7 sec x (b) 7 cos x-5 sec x

(c) 5 cos x +7 sec x tan x (d) 5 cos x-7 sec x tan x

ANSWER-10 (d) $\frac{dy}{dx}$ = 5cos x-7 sec x tan x

ASSERTION-REASON BASED QUESTIONS SOLVED

In the following questions , a statement of Assertion (A) is followed by a statement of Reason (R) . Pick the correct option.

Options:

a. Both A and R are true, and R is the correct explanation of A.

b. Both A and R are true, but R is not the correct explanation of A.

c. A is true, but R is false.

d. A is false, but R is true.

1.Assertion(A): $f(x) = sin^2x + \frac{1}{2}cos2x + cot\alpha, then\ f\text{'}(x) = 0$

Reason(R): Derivative of a constant function is always zero.

ANSWER-(d) f(x) = $cot\alpha + 1/2$, f '(x)=0

2.**Assertion(A)**: The derivative of $y = 2x - \frac{3}{4}$ w. r.t. x is 2.

Reason(R): The derivative of y=cx w.r.t. x is c.

ANSWER-(a) $\boldsymbol{\frac{dy}{dx} = c}$

3. **Assertion(A)**: The derivative of f(x) = $x^3\ w.r.t.\ x$ is $3x^2$.

Reason(R): The derivative of f(x) = $x^n\ w.r.t.\ x$ is x^{n-1}.

ANSWER-(c) As $\frac{d}{dx}(x^n) = nx^{n-1}$

4. **Assertion(A)**:: $\lim_{x\to0}\frac{sinax}{bx}$ is $equal\ to\ \frac{a}{b}$.

Reason(R): $\lim_{x\to0}\frac{sinx}{x} = 1$.

ANSWER- (a) $\lim_{x\to0}\frac{sinax}{ax}\times\frac{a}{b}$

5. **Assertion(A)** : $\lim_{x\to0}\frac{cos2x-1}{cosx-1}$ is $equal\ to$ 4.

Reason(R): $\lim_{x\to0}\frac{tanx}{x} = 1$

ANSWER-(b) **,** A takes help of $\lim_{x\to0}\frac{sinx}{x} = 1$

6. **Assertion(A)** : $\lim_{x\to0}\frac{tanx^0}{x}$ = 1.

Reason(R): $\lim_{x\to0}\frac{tanx}{x} = 1$

ANSWER-(d) tanx$\neq tanx°$

7. **Assertion(A)** : $\lim_{x\to0}\frac{\text{Tan}x}{x}$ =1.

Reason(R): $\lim_{x\to0}\frac{Sinx}{x} = 1$

ANSWER-(a) $\lim_{x\to0}\frac{\tan x}{x} = \lim_{x\to0}\frac{\sin x}{x}\times\frac{1}{cosx}$

8. **Assertion(A)** : $\lim_{x\to0}\frac{\sqrt{1-cos2x}}{x}$ =$\sqrt{2}$sinx

Reason(R): $\lim_{x\to0}\frac{Sinx}{x} = 1$

ANSWER-(a) $\sqrt{1 - cos2x} = \sqrt{2}$sinx

9. **Assertion(A)** $\lim_{x\to0}\frac{\ln(1+|x|)}{x}$=1

Reason(R): R:$\lim_{x\to0}\frac{|x|}{x}$ = doesn't exist

ANSWER-(d) as |x|=$\pm x$

10. **Assertion(A)** : $\lim_{x\to 0} xSin\frac{1}{x} = 0$.

Reason(R): $\lim_{x\to 1} xSin\frac{1}{x} = 1$

ANSWER-(c) as stands for $\lim_{x\to 1} xSin\frac{1}{x}$ = 1×sin1≠1

VERY SHORT ANSWER QUESTIONS SOLVED

1. Find the derivative of $\frac{1}{x}$ with respect to x.

ANSWER - $\frac{d}{dx}$ (1/x) = $\frac{d}{dx}$(x^{-1}) = $-x^{-2}$= $-1/x^2$

2. Find the derivative of $x^3 - 3$ w. r.t. x at x = 10.

ANSWER- $\frac{dy}{dx}=3x^2$, at $x = 10$, $\frac{dy}{dx} = 300$

3. Find the derivative of (5 sec x + 7cos x) with respect to x.

ANSWER- $\frac{dy}{dx}$=5secx.tanx-7sinx

4. Find $f'(x)$, where f(x) = $|x| - 5$, $x \neq 0$

ANSWER- f(x) $\pm x - 5$, f '(x)=± 1

5. Evaluate : $\lim_{x\to\infty} \frac{ax+b}{cx+1}$

ANSWER- $\lim_{x\to\infty} \frac{a+b/x}{c+1/x}$ = a/c

6. If the function f(x) satisfies $\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$. Evaluate: $\lim_{x\to 1} f(x)$

ANSWER-As limit exists & Dr is 0 at x→ 1 , so Nr is 0 at x→ 1 , that is f(x)=2

7. Find the derivative of $(x^3 + x^2 + 3)(x - 5)$ with respect to x.

ANSWER- Derivative is $(x^3 + x^2 + 3) + (3x^2 + 2x)(x - 5)$

8. For the function f(x) = $\frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots\ldots\ldots + \frac{x^2}{2} + x + 1$. Prove that: f '(1) = 100 f '(0).

ANSWER- f '(0)=1 & f '(1)=100 = 100 f '(0) .

9.Evaluate $\lim_{x\to 0} \frac{cos2x-1}{cosx-1}$

ANSWER- $\lim_{x\to 0} \frac{2(cos^2x-1)}{cosx-1}$ = 4

10. Evaluate $\lim_{x\to 0} \frac{sinax}{tanbx}$

ANSWER - $\lim_{x\to 0} \frac{sinax}{tanbx} = \lim_{x\to 0} \frac{sinax}{ax}$ X$\frac{bx}{tanbx}$X$\frac{a}{b}$ = a/b

SHORT ANSWER QUESTIONS SOLVED

1.Evaluate : $\lim_{x\to 0} \frac{(1-cos2x)Sin5x}{x^2sin3x}$

ANSWER-$\lim_{x\to 0} \frac{(2sin^2x)Sin5x}{x^2sin3x}$ =2X5/3 = 10/3

2.Evaluate : $\lim_{x\to 1} \frac{sin\pi x}{x-1}$

ANSWER: $\lim_{x\to 1} \frac{\sin(\pi-\pi x)}{x-1} = \lim_{x\to 1} \frac{\sin(\pi-\pi x)}{x-1} = \lim_{x\to 1} -\pi \frac{\sin-\pi(x-1)}{-\pi(x-1)} = -\pi$

3. Evaluate : $\lim_{x\to\infty} \frac{(x+2)!+(x+1)!}{(x+2)!-(x+1)!}$

ANSWER- : $\lim_{x\to\infty} \frac{(x+3)(x+1)!}{(x+1)(x+1)!} = \lim_{x\to\infty} \frac{(x+3)}{(x+1)}$ =1

4. Evaluate : $\lim_{n\to\infty} \{\frac{1+2+3+\cdots+n}{n+2} - \frac{n}{2}\}$

ANSWER- $\lim_{n\to\infty} \{\frac{n(n+1)}{2(n+2)} - \frac{n}{2}\} = \lim_{n\to\infty} - \frac{n}{(n+2)}$ =-1

5. Evaluate : $\lim_{x\to\pi/2} \frac{1-sinx}{(\frac{\pi}{2}-x)cosx}$

ANSWER- $\lim_{x\to\pi/2} \frac{1-\cos(\pi/2-x)}{(\frac{\pi}{2}-x)\sin(\frac{\pi}{2}-x)}$ =1/2

6. If f(x) = $x^{100}+x^{99}+\ldots+x+1$, then f '(1)

ANSWER- f '(x)= $100x^{100}+99x^{99}+\ldots+2x+1$, f '(1) = 100+99+….+2+1=100 X 101/2

7. If f(x) = $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$ then prove that f '(x) = f (x).

ANSWER – f '(x) = $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$=f(x)

8. If f(x) = $x^{100}-x^{99}+\ldots-x+1$, then f '(1)

ANSWER-f '(1)=100-99+98-97….

9. If y= $\frac{sinx+cosx}{sinx-cosx}$, *then find* $\frac{dy}{dx}$ at x = 0.

ANSWER- $\frac{dy}{dx}=\frac{-(sinx-cosx)^2-(sinx+cosx)^2}{(sinx-cosx)^2}$, At x =0 , dy/dx= …=-2

10. If y = $\sqrt{x}+1/\sqrt{x}$, then find dy/dx at x = 1

ANSWER- dy/dx = $\frac{1}{2\sqrt{x}}$ - $\frac{1}{2}x^{\frac{-3}{2}}$,at x=1, dy/dx = ½ -1/2 = 0

LONG ANSWER QUESTIONS SOLVED

1.Find derivative of f(x) = $\frac{x+1}{x+2}$, Using 1ST principle.

ANSWER-f '(x)=$\lim_{h\to0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to0}\frac{\frac{x+h+1}{x+h+2}-\frac{x+1}{x+2}}{h}$

$=\lim_{h\to0}\frac{h}{(x+h+2)(x+2)h}$ =-1/(x+2)^2

2. Evaluate : $\lim_{h\to0}\frac{(x+h))^2\sin(x+h)-x^2sinx}{h}$

ANSWER-$\lim_{h\to0}\frac{(x+h))^2\sin(x+h)-x^2sinx}{h}$

$=\lim_{h\to0}\frac{x^2\sin(x+h)-x^2sinx}{h}+\frac{(2xh+h^2)sin(x+h)}{h}$

$=\lim_{h\to0}\frac{x^2(\sin(x+h)-sinx)}{h}+\frac{h(2x+h)sin(x+h)}{h}$

$=x^2cosx+2xsinx$

3. Evaluate $\lim_{y\to0}\frac{(x+y)\sec(x+y)-xsecx}{h}$

ANSWER- $\lim_{y\to0}\frac{(x+y)\sec(x+y)-xsecx}{h}$

$=\lim_{y\to0}\frac{x\sec(x+y)-xsecx}{h}+\frac{ysec(x+y)}{y}$

$=\lim_{y\to0}\frac{x(\sec(x+y)-secx)}{h}$

$=\lim_{y\to0}\frac{x(\frac{1}{\cos(x+y)}-\frac{1}{cosx})}{h}$ + sec(x+y)

$=\lim_{y\to0}\frac{x(cosx-\cos(x+y))}{ycosx.\cos(x+y)}$ + sec(x+y)

= x secx.tanx+ secx

4. Find derivative of f(x) = xlogx , Using 1ST principle.

ANSWER- $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{(x+h)\log(x+h)-x\log x}{h}$

$=\lim_{h\to 0}\frac{(x)\log(x+h)-x\log x}{h}+\frac{h\log(x+h)}{h}$

$=\lim_{h\to 0}\frac{x(\log(x+h)-\log x)}{h}$ + log(x+h) = x.$\frac{1}{x}$ +logx =1+logx

5. Differentiate f(x)= (x sinx+ cosx)(x cosx- sinx) with respect to x

ANSWER-

$f'(x) = (x\cos x - \sin x)\frac{d}{dx}(x\sin x + \cos x) + (x \sin x+\cos x)\frac{d}{dx}(x\cos x - \sin x)$

= (xcosx-sinx).(xcosx+sinx-sinx) +(xsinx+cosx)(cosx-xsinx-cosx)

$=x^2(\cos^2x - \sin^2x) - x(2\sin x.\cos x)$

$=x^2\cos 2x - x\sin 2x$

CASE BASED QUESTIONS SOLVED

Question.1

Mahesh was learning limit of a polynomial function from his teacher Mr.Nanda.

Now let $f(x)=a_0+a_1x+a_2x^2+\ldots+a_{n-1}x^{n-1}+a_nx^n$ be a polynomial function , where all a_i's are real numbers and $a_n\neq 0$.

Then limit of the polynomial function

$\lim_{x\to a} f(x)$

$=\lim_{x\to a} a_0 + a_1x + a_2x^2 + \cdots . a_{n-1}x^{n-1}+a_nx^n$

$= a_0+a_1a+a_2a^2+\ldots\ldots\ldots a_{n-1}a^{n-1} + a_n a^n$

= f(a) .

Based on above information evaluate the following

(i) $\lim_{x\to -1} 1 + x + x^2+x^3+\cdots+x^8+x^9$

(ii) $\lim_{x\to 5} x^2$(x-1)

(iii) $\lim_{x\to 2}(x^3+x^2+x-1)$

(iv) $\lim_{x\to 1}\frac{x^4-1}{x-1}$

OR $\lim_{X\to -1}\frac{x^3+1}{x+1}$

ANSWER:-

(i) $1 + (-1) + (-1)^2+(-1)^3+\cdots+(-1)^8+(-1)^9$ =1-1+1-1+…-1=0

(ii) 5^2(5-1)=20

(iii)$2^3+2^2+2-1 = 13$

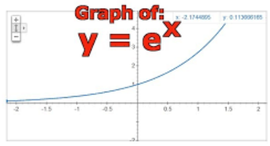
(iv) $\lim_{x\to 1}\frac{(x^2+1)(x+1)(x-1)}{x-1}$ = (1+1)(1+1)=4

OR

(iv) $\lim_{x\to-1} \frac{x^3-(-1)^3}{x-(-1)} = \lim_{x\to-1} \frac{x^3+(1)^3}{x+(1)} = \lim_{x\to-1} \frac{(x^2-x+1)(x+1)}{(x+1)}$ = 1+1+1 = 3

Question.2

The great Swiss mathematician Leonard Euler introduced the number 'e' whose value lies between 2 and 3. This number is useful in defining exponential function, a function of the form f(x)= e^x is called exponential function. The graph of the function is given here.



Domain $(-\infty, \infty)$

Range $(0, \infty)$

To find the limit of function involving exponential function we use the following Theorm :

Theorm: $\lim_{x\to0} \frac{e^x-1}{x} = 1$.

Based on above information find the limits:

(i) $\lim_{x\to4} \frac{e^x-e^4}{x-4}$

(ii) $\lim_{x\to0} \frac{e^{kx}-1}{x}$

(iii) $\lim_{x\to0} \frac{e^{-x}-1}{x}$

(iv) $\lim_{x\to0} \frac{e^{5x}-e^{4x}}{x}$

OR

$\lim_{x\to2} \frac{e^x-e^2}{x^2-4}$

ANSWER:

2(i) $\lim_{x\to4} \frac{e^4(e^{x-4}-1)}{x-4} = \lim_{h\to0} \frac{e^4(e^h-1)}{h} = e^4$

(ii) $\lim_{x\to0} \frac{(e^{kx}-1)}{kx} \times k = k$

(iii) $\lim_{x\to0} \frac{(e^{-x}-1)}{-x}$ ×(-1)= -1

(iv) $\lim_{x\to0} \frac{e^{4x}(e^x-1)}{x} = e^0$.1=1

OR

$\lim_{x\to2} \frac{e^2(e^{x-2}-1)}{(x-2)(x+2)} = \lim_{h\to0} \frac{e^2(e^h-1)}{h} \times \frac{1}{4} = \frac{e^2}{4}$

.

Question. 3 | Bridge Cable Shape

The sag y(x) (in m) of a suspension bridge cable at horizontal distance x (m) from one tower is modelled near mid-span as

$y(x)=0.002x^2-0.3x+10$, $0\le x\le150$

(i) Find the slope of the cable 40 m from the tower.

(ii) Lowest Point

Answer: i) $y'(x)=0.004x-0.3$

$\Rightarrow y'(40) = 0.004(40) - 0.3 = 0.16 - 0.3 = -0.14$

So the cable is descending at 0.14 m per horizontal metre at x=40 m.

(ii) $y'(x)=0.004x-0.3=0$: x=75.

Thus the cable's vertex is 75 m from the tower, consistent with symmetry

Question 4 | Population Growth Check

A town's population (in thousands) after year 2000 is

$P(t)=50(1+1/t)^t$, t>0

(i) Estimate the long-term population limit as $t\to\infty$

(ii) Find the instantaneous change of Q(t) = log P(t) after 1 year.

Answer:

(i) $\lim_{t\to\infty}(1+\frac{1}{t})^t = e$. Therefore

$\lim_{t\to\infty} P(t) = 50e = 135.9$ thousand approx..

(ii) $Q(t) = \log P(t) = t\log(\frac{t+1}{t}) + \log 50$

$Q'(t)= \log(\frac{t+1}{t}) - \frac{1}{t+1}$

$Q'(1)=\log 2-\frac{1}{2}$

5. A drone's vertical position (metres) after t seconds is $s(t)=5t^2-40t+120$,

(i) Find the instantaneous speed when time = t sec.

(ii) The package must be released when the drone's speed is zero. At what time does that happen?

(iii) Also find the acceleration at t =5 and check whether it is accelerating or retarding?

Answer:

1.(i) $s'(t)=10t-40$

(ii) $s'(t)=10t-40=0$, then t=4

(iii) $s''(t)=10$, then $s''(5) = 10 > 0$, which means the drone is accelerating.

Takeaway: Instantaneous speed is the derivative of position; a zero derivative marks a rest point. positive double derivative means acceleration.

CHAPTER 13 - STATISTICS

DEFINITIONS AND FORMULAE

** Median $=\left(\frac{n+1}{2}\right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$=$ mean of $\left(\frac{n}{2}\right)^{th}$ & $\left(\frac{n}{2}+1\right)^{th}$ observations (arranged in ascending or descending order) & the number of observations is odd.

$= l+\left(\frac{\frac{n}{2}-cf}{f}\right)\times h$ where, l = lower limit of median class, n = number of observations,

cf = cumulative frequency of class preceding the median class, f = frequency of median class, h = class size.

**Mode $= l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$, where l = lower limit of the modal class, h = size of the class interval,

f_1 = frequency of the modal class, f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

** **Measures of Dispersion**: The dispersion or scatter in a data is measured on the basis of the observations and the types of the measure of central tendency, used there. There are following measures of dispersion:

(i) Range, (ii) Quartile deviation, (iii) Mean deviation, (iv) Standard deviation.

** Range: Range of a series = Maximum value – Minimum value.

** Mean Deviation : The mean deviation about a central value ‘a’ is the mean of t
he absolute values of the deviations of the observations from ‘a’. The mean deviation from ‘a’ is denoted as M.D. (a).

(i) For ungrouped data

$$\text{M.D.}(\bar{x})=\frac{1}{n}\sum_{i=1}^{n}|x_i-\bar{x}|, \text{ where } \bar{x}=\text{Mean} \qquad \text{M.D.}(M)=\frac{1}{n}\sum_{i=1}^{n}|x_i-\bar{x}|, \text{ where } M=\text{Median}$$

(ii) For grouped data

(a) Discrete frequency distribution

$$\text{M.D.}(\bar{x})=\frac{\sum_{i=1}^{n}f_i|x_i-\bar{x}|}{\sum_{i=1}^{n}f_i}=\frac{1}{N}\sum_{i=1}^{n}f_i|x_i-\bar{x}| \qquad \text{M.D.}(M)=\frac{1}{N}\sum_{i=1}^{n}|x_i-M|$$

(b) Continuous frequency distribution

$$\text{M.D.}(\bar{x})=\frac{1}{N}\sum_{i=1}^{n}f_i|x_i-\bar{x}|, \text{ using } \bar{x}=a+\left(\frac{\sum f_iu_i}{\sum f_i}\right)h$$

$$\text{M.D.}(M)=\frac{1}{N}\sum_{i=1}^{n}|x_i-M|, \text{ using } M=l+\left(\frac{\frac{n}{2}-cf}{f}\right)\times h$$

** Variance: It is the mean of sum of squares of the deviations from the arithmetic mean of the deviation.

(a) For ungrouped data : Var$(X) = \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - (\frac{1}{n}\sum_{i=1}^{n} x_i)^2$

(b) For grouped data: $\sigma^2 = = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2 = \frac{1}{N}\sum_{i=1}^{n} f_i x_i{}^2 - (\frac{1}{N}\sum_{i=1}^{n} f_i x_i)^2$

(c) Short- cut Method: $\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2{}_i = \frac{h^2}{N^2}\sum_{i=1}^{n}[N f_i y_i{}^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2]$ where

$y_i = \frac{x_i - \bar{x}}{h}$

** Standard Deviation: It is the positive square root of variance

S. D. = $\sqrt{Variance} = \sigma$

MULTIPLE CHOICE QUESTIONS (SOLVED)

Q.1 The range for the following data is

Marks less than	10	20	30	40	50	60	70
No of students	0	4	7	10	14	18	20

(a) 50 marks (b) 40 marks (c) 20 marks (d) 60 marks

Solution: (b) 70-10 = 60 marks

Q.2 In a college of 300 students every student reads 5 newspapers and every newspaper read by 60 students. The number of news paper is

(a) at least 30 (b) at most 20 (c) exactly 25 (d) None of these

Solution: Let the number of news papers which are read be n.

Now according to given question ,

60n = 300x5

n = 25 so correct answer is (c)

Q.3 The mean and median of the following ten numbers in increasing order

10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively then $\frac{y}{x}$ is equal to

(a) $\frac{7}{3}$ (b) $\frac{7}{2}$ (c) $\frac{8}{3}$ (d) $\frac{9}{4}$

Solution: (a) median = 35 $\Longrightarrow \frac{34+x}{2} = 35 \Rightarrow$ x=36 and

Mean = 42$\Rightarrow \frac{300+x+y}{10} = 42 \Rightarrow x + y = 120 \Rightarrow y = 84$, Therfore $\frac{y}{x} = \frac{7}{3}$

Q.4 Mean deviation from mean of the following data ; 4,7,8,9,10,12,13,17 is

(a) 2 (b) 3 (c) 4 (d) 5

Solution: (b) Explanation :mean=$\frac{80}{8} = 10 \Rightarrow MD = \frac{6+3+2+1+0+2+3+7}{8} \Rightarrow MD = \frac{24}{8}$=3

Q.5 If the mean of 1,3,4,5,7 is m ,the numbers 3,2,2,4,3,3,p have mean $(m - 1)$ and median q then $(p + q)$ is

(a) 4 (b) 5 (c) 6 (d) 7

Solution: (d) Explanation : $m = \frac{20}{5} \Rightarrow m = 4$

And $\frac{17+p}{7} = (4 - 1) \Rightarrow p = 4$ & $median\ of$ 2,2,3,3,3,3,4 $is\ q = 3\ thus\ p + q = 7$

Q.6.consider any set of 201 observations $x_1 < x_2 < x_3 < x_4 < - - - - - - - - - < x_{201}$. Then the mean deviation of this set of observations about a point K is minimum, when K equals

(a) $\frac{x_1+x_2+x_3+x_4+---------+x_{201}}{201}$. (b) x_1 (c) x_{101} (d) x_{201}

Solution: (c) Explanation-: Given that, $x_1 < x_2 < x_3 < x_4 < - - - - - - - - - < x_{200} < x_{201}$.

$Median = \frac{(201+1)}{2} = 101^{th}\ term \Rightarrow md = x_{101}$, as we know that deviation will be minimum from the median so K=x_{101}

Q7. The standard deviation of 17 numbers is zero. Then

(a) The numbers in geometric progression with common ratio not equal one.

(b) eight numbers are positive, eight numbers are negative and one is zero.
(c) either (a) or (b) (d) None of these.
Solution: (d) Explanation: If S,D. is zero then all 17 numbers must be same. So (a) and (b) can not be true.
Q8.The variance of 2,4,6 8,10 is
a) 8 (b)$\sqrt{8}$ (c) 6 (d) $None\ of\ these$
Solution: $(a)\ Explanation\ mean = \frac{30}{5} \Rightarrow mean = 6$

There fore $Variance = \frac{16+4+0+4+16}{5} \Rightarrow \sigma^2 = 8$
Q9.If each observation of a distribution whose S.D. is σ, is increased by μ then the variance of the new observation is
(a) σ (b) $\sigma + \mu$ (c)σ^2 (d) $\sigma^2 + \mu$
Solution: (c) Explanation: Mean of distribution is also increased by σ therefore SD remains unchanged,
Q10. If each observation of a distribution whose variance is σ^2, is multiplied by K then the S.D. of new observation is
(a) σ (b) K σ (c) $|K|\sigma$ (d) $K^2\sigma$
Solution: (c) Explanation: Mean is also multiplied by K then $\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(Kx_i - K\bar{x})^2$

$\Rightarrow SD = |K|\sqrt{\frac{1}{N}\sum_{i=1}^{n}(Kx_i - K\bar{x})^2} \quad \Rightarrow SD = |k|\sigma$

ASSERTION REASON QUESTIONS SOLVED

DIRECTION : In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as
(a) Both assertion and reason are true and reason is the correct explanation of assertion.
(b) Both assertion and reason are true but reason is not the correct explanation of assertion.
(c) Assertion is true but reason is false.
(d) Assertion is false but reason is true.
Q.1 Assertion (A) The mean deviation about mean for the data 4,7,8,9,10,12,13,14,4.is 9.33
Reason (R) The mean deviation about the mean $\bar{x}$ is $(MD) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$.
Solution::- (a) Explanation: (A)Mean= $\frac{81}{9} \Rightarrow Mean = 9.33$
(R) is correct.
Q.2 Consider the following data

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Then, the mean deviation about the mean is 6.32.
Reason (R) Consider the following data

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Then, the mean deviation about the mean is 15 .
Solution: :- (c) Explanation: By calculating A is true and R is false.
Q.3 Assertion (A) - The mean deviation about median calculated for series, where variability is very high, cannot be fully relied.
Reason (R) – The median is not a representative of central tendency for the series where degree of variability is very high.

Solution: (a) A is true and R is also true.
Explanation: Median is the middle term of the arranged ascending/descending order of data
Q.4. Assertion (A)- The Variance of 20 observations is 5 if each observation is multiplied by 2 ,then new variance of the resulting observations will be 20.
Reason (R) –If each observation is multiplied by a constant K, the variance of the resulting observations becomes K^2 times the original variance.
Solution: (a) A is true and R is also true.
Explanation: variance of new observations = $2^2 X 5 = 20$.
reason is the correct explanation of assertion.
Q.5. Assertion (A)- The Variance of 30 observations is 17 if each observation is increased by 5 ,then new variance of the resulting observations will be 17.
Reason (R) –If each observation is increased by a constant K, the variance of the resulting observations remains unchanged.
Solution: (a) A is true and R is also true.
Explanation: variance of new observations = 17.
reason is the correct explanation of assertion.
Q.6. Assertion (A)- The Variance of 25 observations is 13 if each observation is decreased by 5 ,then new variance of the resulting observations will be 8.
Reason (R) –If each observation is decreased by a constant K, the variance of the resulting observations remains unchanged.
Solution: (d) A is False and R is true.
Explanation:-New variance of the resulting observations will be 13. As we know that If each observation is decreased by a constant K, the variance of the resulting observations remains unchanged.
Q,7. Assertion (A)- The Variance of first n natural numbers is $\frac{n^2-1}{6}$.
Reason (R) – The sum of first n natural numbers $\frac{n(n+1)}{2}$ and the sum of square of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.
Solution: (d) A is False and R is true.
Explanation:- Variance of first n natural numbers is $\frac{n^2-1}{12}$
The sum of first n natural numbers $\frac{n(n+1)}{2}$ and the sum of square of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

Q,8. Assertion (A)- Mean and variance of first ten multiples of 3 are 16.5 and 74.25 respectively.

Reason (R) The mean deviation about the mean $\bar{x}$ is $(MD) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$. Where mean($\bar{x}$) =$\sum_{i=1}^{n}\frac{x_i}{n}$

Solution: (a) A is true and R is also true.

Explanation: By calculation M=16.5 and Variance = 74.25 and

The mean deviation about the mean $\bar{x}$ is $(MD) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$. Where mean($\bar{x}$) =$\sum_{i=1}^{n}\frac{x_i}{n}$

Q,9. If $\sum_{i=1}^{n}(x_i - 5) = 3$ and $\sum_{i=1}^{n}(x_i - 5)^2 = 43$ and total number of terms is 18.

Assertion (A)- Mean of the distribution is 4.1666.

Reason (R)- Standard deviation of the distribution is 1.54.

Solution: (d) A is false and R is true.

Explanation: By calculation mean M=5.166 and standard deviation= 1.54.

Q,10. Assertion (A)- If the mean of n observations $1^2, 2^2, 3^2, - - - - - - - -, n^2$.is $\frac{46n}{11}$,then $n = 11$.

Reason (R)- Mean of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

Solution: (c) A is true and R is false.

.Explanation: $\frac{n(n+1)(2n+1)}{6n} = \frac{46n}{11} \Rightarrow n = 11$.

Mean=$\frac{n(n+1)(2n+1)}{6n} \Rightarrow Mean = \frac{(n+1)(2n+1)}{6}$

VERY SHORT ANSWER TYPE QUESTIONS

Question 1. Find the mean deviation about the mean.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

x_i	f_i	$f_i x_i$	$\lvert x_i - 14 \rvert$	$f_i \lvert x_i - 14 \rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

$$\text{Mean}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{25} \times 350 = 14$$

$$\therefore \quad \text{Mean deviation about mean} = \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - \bar{x} \rvert$$

$$= \frac{1}{25} \times 158 = 6.32$$

Find the mean deviation about the median for the following data .

Question 2.

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Solution :-

x_i	f_i	$c.f.$	$\lvert x_i - 30 \rvert$	$f_i \lvert x_i - 30 \rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	29			148

Here, $\frac{N}{2} = \frac{29}{2} = 14.5$

The $c.f.$ just greater than 14.5 is 21 and the corresponding value of x is 30.

$\therefore$ Median $(M) = 30$

$$\text{M.D. about median} = \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - M \rvert$$

$$= \frac{1}{29} \times 148 = 5.1$$

Question 3. Find the Mean deviation about mean.

Income per day	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800
Number of persons	4	8	9	10	7	5	4	3

Solution:-

Income per day	Mid values x_i	f_i	$f_i x_i$	$\|x_i - 358\|$	$f_i \|x_i - 358\|$
0 – 100	50	4	200	308	1232
100 – 200	150	8	1200	208	1664
200 – 300	250	9	2250	108	972
300 – 400	350	10	3500	8	80
400 – 500	450	7	3150	92	644
500 – 600	550	5	2750	192	960
600 – 700	650	4	2600	292	1168
700 – 800	750	3	2250	392	1176
		50	17900		7896

$$\text{Mean } (\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{50} \times 17900 = 358$$

Mean deviation about mean

$$= \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \bar{x}| = \frac{1}{50} \times 7896 = 157.92$$

Question 4. Find the mean deviation about mean.

Heights (in cm)	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

Solution :-

Height (in cms)	Mid values x_i	f_i	$f_i x_i$	$\|x_i - 125.3\|$	$f_i \|x_i - 125.3\|$
95 – 105	100	9	900	25.3	227.7
105 – 115	110	13	1430	15.3	198.9
115 – 125	120	26	3120	5.3	137.8
125 – 135	130	30	3900	4.7	141
135 – 145	140	12	1680	14.7	176.4
145 – 155	150	10	1500	24.7	247
		100	12530		1128.8

$$\text{Mean } (\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

Mean deviation about mean

$$= \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \bar{x}| = \frac{1}{100} \times 1128.8 = 11.28$$

Question 5. Find the Variance of the following data.

6, 8, 10, 12, 14, 16, 18, 20, 22, 24

Solution: $Mean = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{150}{10}$=15

Variance (σ^2) = $=\frac{330}{10} = 33.$

Question 6.Find the mean and variance of 6,7,10,12,13,4,8,12

Solution: $Mean(M) = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{72}{8} = 9.$

Variance $(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{74}{8} = 9.25$

Question 7. Find the mean and variance of first n natural numbers.

Solution: $Mean(M) = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{n(n+1)}{2n} = \frac{(n+1)}{2}$

Variance $(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - (\frac{1}{n}\sum_{i=1}^{n} x_i)^2$

Variance $(\sigma^2) = \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}$

Question 8. Find the Mean and Standard Deviation of first 10 multiples of 3

Solution--:$Mean(M) = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{165}{10} = 16.5$

Variance $(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n}\sum_{i=1}^{n} x_i{}^2 - (\frac{1}{n}\sum_{i=1}^{n} x_i)^2$

Variance (σ^2) =$346.5 - 272.25 = 74.25$

Standard Deviation=$\sqrt{74.25} = 8.62.$

Question 9. The mean and S.D. of six observations are 8 and 4 respectively. If each observation is multiplied by 3,find the new mean and new standard deviation

Solution: New Mean= 3x8=24

New Standard deviation=3x4=12.

SHORT ANSWER TYPE QUESTIONS SOLVED

Question 1

Find the mean and standard deviation using short-cut method

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution :-

x_i	f_i	$u_i = x_i - 64$	$f_i u_i$	$f_i u_i^2$
60	2	−4	−8	32
61	1	−3	−3	9
62	12	−2	−24	48
63	29	−1	−29	29
64	25	0	0	0
65	12	1	12	12
66	10	2	20	40
67	4	3	12	36
68	5	4	20	80
	100		0	286

Let assumed mean $(A) = 64$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} = 64 + \frac{0}{100} = 64$$

$$\text{S.D.} (\sigma) = \frac{1}{N}\sqrt{N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2}$$

$$= \frac{1}{100}\sqrt{100 \times 286 - (0)^2}$$

$$= \frac{1}{100}\sqrt{28600} = \frac{1}{100} \times 169.1 = 1.69$$

Question 2:
Find the mean and variance for the following frequency distributions

Classes	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequencies	2	3	5	10	3	5	2

Solution :-

Classes	Mid values x_i	f_i	$u_i = \frac{x_i - 105}{30}$	$f_i u_i$	$f_i u_i^2$
0 – 30	15	2	–3	–6	18
30 – 60	45	3	–2	–6	12
60 – 90	75	5	–1	–5	5
90 – 120	105	10	0	0	0
120 – 150	135	3	1	3	3
150 – 180	165	5	2	10	20
180 – 210	195	2	3	6	18
		30		2	76

Let assumed mean $(A) = 105$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 105 + \frac{2}{30} \times 30 = 107$$

$$\text{Variance } (\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{900}{900}[30 \times 76 - 4] = [2280 - 4] = 2276$$

Question 3.Find the mean and variance of given observation:

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

Solution:

Classes	Mid values x_i	f_i	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$	$f_i {u_i}^2$
0 – 10	5	5	–2	–10	20
10 – 20	15	8	–1	–8	8
20 – 30	25	15	0	0	0
30 – 40	35	16	1	16	16
40 – 50	45	6	2	12	24
		50		10	68

Let assumed mean $(A) = 25$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 25 + \frac{10}{50} \times 10$$

$$= 25 + 2 = 27$$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{(10)^2}{(50)^2}[50 \times 68 - (10)^2] = \frac{100}{2500}[3400 - 100]$$

$$= \frac{1}{25} \times 3300 = 132$$

Question 4.
Find the mean, variance and standard deviation using short-cut method.

Heights in cms	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Solution :-

Height in cms	Mid values x_i	f_i	$u_i = \frac{x_i - 92.5}{5}$	$f_i u_i$	$f_i u_i^2$
70 – 75	72.5	3	–4	–12	48
75 – 80	77.5	4	–3	–12	36
80 – 85	82.5	7	–2	–14	28
85 – 90	87.5	7	–1	–7	7
90 – 95	92.5	15	0	0	0
95 – 100	97.5	9	1	9	9
100 – 105	102.5	6	2	12	24
105 – 110	107.5	6	3	18	54
110 – 115	112.5	3	4	12	48
		60		6	254

Let assumed mean $(A) = 92.5$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 92.5 + \frac{6}{60} \times 5$$

$$= 92.5 + 0.5 = 93$$

$$\text{Variance}(\sigma^2) = \frac{h^2}{N^2}[N\Sigma f_i u_i^2 - (\Sigma f_i u_i)^2]$$

$$= \frac{(5)^2}{(60)^2}[60 \times 254 - (6)^2]$$

$$= \frac{25}{3600} \times 15204 = 105.58$$

Standard deviation $(\sigma) = \sqrt{105.58} = 10.27$

Question:-5

The mean and standard deviation of some data for the time taken to complete a test are calculated with the following results:

Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25 seconds. Further, another set of 15 observations $x_1, x_2, ..., x_{15}$, also in seconds, is now available and we have $\sum_{i=1}^{15} x_i = 279 \;\; and \;\; \sum_{i=1}^{15} {x_i}^2 = 5524$. Calculate the standard derivation based on all 40observations.

Solution-As per the given criteria ,In first set, We have Number of observations, n=25 Mean=18.2 ,and standard deviation =3.25

And In second set, We have Number of observations, n=15 and $\sum_{i=1}^{n} x_i = 279 \;\; and \;\; \sum_{i=1}^{n} {x_i}^2 = 5524$ For the first set we have

$$\sum_{i=1}^{25} x_i = 455 \;\; and \; \sum_{i=1}^{25} {x_i}^2 = 14069.0625$$

Therefore for all 40 observations

$$\sum_{i=1}^{40} x_i = 455 + 279 = 734 \;\; and \; \sum_{i=1}^{40} {x_i}^2 = 5524 + 8545.0625 = 14069.0625$$

Therefore

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} {x_i}^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = \frac{14069.0625}{40} - \left(\frac{455}{40}\right)^2$$

=15.004

$$\sigma = \sqrt{15.004} = 3.87$$

Question:6-

If M is the mean and is σ^2 the variance of n observations.and each observation is multiplied by K then prove that the mean and variance of the observations KM and $K^2\sigma^2$, respectively (where, K is a non zero constant). **[3]**

Solution: $New\ mean = \frac{1}{n}\sum_{i=1}^{n} Kx_i = K\frac{1}{n}\sum_{i=1}^{n} x_i = KM$ **&**

New variance $=\sum_{i=1}^{n} Kx_i^{\ 2} - \left(\frac{1}{n}\sum_{i=1}^{n} Kx_i\right)^2 = K^2\sigma^2$

Question:7

The lengths (in cm) of 10 rods in a shop are: 40.0, 52.3, 55.2, 72.9, 52.8, 79.0, 32.5, 15.2, 27.9,30.2 Find mean deviation from the mean.

Solution: mean$=\frac{1}{n}\sum_{i=1}^{n} x_i = \frac{459.8}{10} = 45.98$ & MD$=\frac{164.6}{10} = 16.46$

Question:8 Find the mean and variance of 100 multiple of 5.

Solution : use concepts directly

Question :9 Calculate the mean deviation from the mean of set of first n natural numbers where n is an odd natural number.

Solution: Use concepts directly.

LONG ANSWER TYPE QUESTION SOLVED

Question 1.

The diameters of circles (in mm) drawn in a design are given below:

Diameters	33-36	37-40	41-44	45-48	49-52
No. of Circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

Diameters	Modified Classes	Mid values x_i	f_i	$u_i = \frac{x_i - 42.5}{4}$	$f_i u_i$	$f_i u_i^2$
33 – 36	32.5 – 36.5	34.5	15	–2	–30	60
37 – 40	36.5 – 40.5	38.5	17	–1	–17	17
41 – 44	40.5 – 44.5	42.5	21	0	0	0
45 – 48	44.5 – 48.5	46.5	22	1	22	22
49 – 52	48.5 – 52.5	50.5	25	2	50	100
			100		25	199

Let assumed mean $(A) = 42.5$

$$\text{Mean } (\bar{x}) = A + \frac{\Sigma f_i u_i}{N} \times h = 42.5 + \frac{25}{100} \times 4$$

$$= 42.5 + 1 = 43.5$$

$$\text{Standard deviation}(\sigma) = \frac{h}{N}\sqrt{N\Sigma f_i u_i^{\ 2} - (\Sigma f_i u_i)^2}$$

$$= \frac{4}{100}\sqrt{100 \times 199 - (25)^2} = \frac{1}{25}\sqrt{19275}$$

$$= \frac{1}{25} \times 138.83 = 5.55$$

Question 2: The mean and variance of 8 observations are 9 and 9.25 respectively. If six of the observations are 6 , 7 , 10, 12 , 12 and 13 . Find the remaining two observations.

Solution :-

Let the remaining two observations be x and y.

Therefore, the observations are 6, 7, 10, 12, 12, 13, x, y.

$$\text{Mean, } \bar{x} = \frac{6+7+10+12+12+13+x+y}{8} = 9$$
$$\Rightarrow 60+x+y=72$$
$$\Rightarrow x+y=12 \quad \ldots(1)$$

$$\text{Variance} = 9.25 = \frac{1}{n}\sum_{i=1}^{8}\left(x_i-\bar{x}\right)^2$$
$$9.25 = \frac{1}{8}\left[(-3)^2+(-2)^2+(1)^2+(3)^2+(3)^2+(4)^2+x^2+y^2-2\times 9(x+y)+2\times(9)^2\right]$$
$$9.25 = \frac{1}{8}\left[9+4+1+9+9+16+x^2+y^2-18(12)+162\right]$$
$$\ldots\text{[Using (1)]}$$
$$9.25 = \frac{1}{8}\left[48+x^2+y^2-216+162\right]$$
$$9.25 = \frac{1}{8}\left[x^2+y^2-6\right]$$
$$\Rightarrow x^2+y^2=80 \quad \ldots(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 144$ ...(3)

From (2) and (3), we obtain

$2xy = 64$ ... (4)

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 80 - 64 = 16$

$x - y = \pm 4$ ... (5)

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 4$, when $x - y = 4$

$x = 4$ and $y = 8$, when $x - y = -4$

Thus, the remaining observations are 4 and 8.

Question 3: The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2 , 4 , 10 , 12 , and 14 . Find the remaining two observations.

Solution :-

Let the remaining two observations be x and y.

The observations are 2, 4, 10, 12, 14, x, y.

$$\text{Mean, } \bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$
$$\Rightarrow 56 = 42+x+y$$
$$\Rightarrow x+y=14 \quad \ldots(1)$$
$$\text{Variance} = 16 = \frac{1}{n}\sum_{i=1}^{7}\left(x_i-\bar{x}\right)^2$$
$$16 = \frac{1}{7}\left[(-6)^2+(-4)^2+(2)^2+(4)^2+(6)^2+x^2+y^2-2\times 8(x+y)+2\times(8)^2\right]$$
$$16 = \frac{1}{7}\left[36+16+4+16+36+x^2+y^2-16(14)+2(64)\right]$$
$$\ldots\text{[Using (1)]}$$
$$16 = \frac{1}{7}\left[108+x^2+y^2-224+128\right]$$
$$16 = \frac{1}{7}\left[12+x^2+y^2\right]$$
$$\Rightarrow x^2+y^2=112-12=100$$
$$x^2+y^2=100 \quad \ldots(2)$$

From (1), we obtain

$x^2 + y^2 + 2xy = 196 \ldots (3)$

From (2) and (3), we obtain

$2xy = 196 - 100$

$2xy = 96 \ldots (4)$

Subtracting (4) from (2), we obtain

$x^2 + y^2 - 2xy = 100 - 96$

$(x - y)^2 = 4$

$x - y = \pm 2 \ldots (5)$

Therefore, from (1) and (5), we obtain

$x = 8$ and $y = 6$ when $x - y = 2$

$x = 6$ and $y = 8$ when $x - y = -2$

Thus, the remaining observations are 6 and 8.

CASE STUDY BASED QUESTION SOLVED

Question 1.An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results:

	Firm A:	Firm B:
Number of workers:	1000	1200
Average monthly wages	Rs 2800	Rs2800
Variance of distribution	100	169

Based on the above information, find

(i) Total monthly wages paid by firm A

(ii) Total monthly wages paid by firm B

(iii)Standard Deviation of firm A and B.

Solution:+ (i) Total monthly wages paid by firm A =$1000X2800 = 2800000$

(ii) Total monthly wages paid by firm B=$1200X2800 = 3{,}360{,}000$

(iii) Variance of distribution A=$\sqrt{100} = 10$

Variance of distribution B=$\sqrt{169} = 13$

Question 2. The mean and standard deviation of some data for the time taken to complete a test are calculated with the following results:

Number of observations =25, Mean=18.2 seconds, Standard Deviation = 3.25 seconds

Further another set of 15 observations $x_1, x_2, x_3, \ldots \ldots \ldots \ldots \ldots x_{15}$ also in seconds is now available and we have $\sum_{i=1}^{15} x_i = 279$ *and* $\sum_{i=1}^{15} x_i{}^2 = 5524$.

Based on the above information, answer the following questions:

(i) What is the sum of all 40 observations?

(ii) What is the mean of all 40 observations?

(iii)What is the standard deviation of all 40 observations?

Solution:-(i) $Sum\ of\ all\ 40\ observations = 25x18.2 + 279 = 455 + 279 = 734.$

(ii) Mean= $\frac{734}{40} = 18.35.$

(iii) $S.D. = 3.87$

Question 3. The sum and the sum of squares corresponding to length $x(in\ cm)$ and weight $y(in\ gm)$ of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212,\ \ \sum_{i=1}^{50} {x_i}^2 = 902.8,\ \ \sum_{i=1}^{50} y_i = 261,\ \ \sum_{i=1}^{50} {y_i}^2 = 1457.6$$

Based on the above information, answer the following questions:

(i)Find the mean length of the plant products.

(ii)Find the mean weight of the plant products.

(iii)Find the variance of the length of the plant products.

(iv) Find the S.D. of the length of the plant products.

Solution: (i) $mean = \frac{212}{50} = 4.24$

(ii) mean weight=$\frac{261}{50}$=5.24

(iii) $variance = \frac{902.8}{50} - (4.24)^2 = 18.056 - 17.9776 = 0.0784$

(iv) Standard Deviation=$\sqrt{0.0784} = 0.28.$

CHAPTER 14 - PROBABILITY

DEFINITIONS AND FORMULAE

**Random Experiments : An experiment is called random experiment if it satisfies the following conditions:

(i) It has more than one possible outcome.

(ii) It is not possible to predict the outcome in advance.

** Outcomes and sample space : A possible result of a random experiment is called its outcome.

The set of all possible outcomes of a random experiment is called the sample space associated with the experiment. Each element of the sample space is called a sample point. Any subset E of a sample space S is called an event.

** Impossible and Sure Events : The empty set φ and the sample space S describe events. φ is called an impossible event and S, i.e., the whole sample space is called the sure event.

** Compound Event : If an event has more than one sample point, it is called a Compound event.

** Complementary Event : For every event A, there corresponds another event A' called the complementary event to A. It is also called the event 'not A'.

** The Event 'A or B' : When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'.

** The Event 'A and B' : If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'.

** The Event 'A but not B' : the set A – B denotes the event 'A but not B'. $A - B = A \cap B'$

** Mutually exclusive events : two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In this case the sets A and B are disjoint i.e. $A \cap B = \varphi$.

** Axiomatic Approach to Probability : Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0,1] satisfying the following axioms

(i) For any event E, $P(E) \geq 0$

(ii) $P(S) = 1$

(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

From the axiomatic definition of probability it follows that

(i) $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$

(ii) $P(\omega_1) + P(\omega_2) + ... + P(\omega_n) = 1$

** Equally likely outcomes: All outcomes with equal probability.

** Probability of an event: For a finite sample space with equally likely outcomes

Probability of an event $P(A) = \dfrac{n(A)}{n(S)}$, where n(A) = number of elements in the set A,

n(S) = number of elements in the set S.

** Probability of the event 'A or B' : $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

For mutually exclusive events A and B, we have $P(A \cup B) = P(A) + P(B)$

** Probability of event 'not A' = $P(A') = P(\text{not } A) = 1 - P(A)$.

MULTIPLE CHOICE QUESTIONS SOLVED.

1.Set which contains all possible outcomes is

a) event b) empty set c) sample space d) probability

Solution: (c) Sample Space

Explanation- Set of all possible outcomes is called Sample space

2. What is a complementary event for "At least one head appears" if two coins are tossed simultaneously?

a) Exactly one head appears   b) At least one tail appears
c) At most one tail appears   d) None head appears

Solution:d) None head appears

Explanation--n(S)={HT,TH,TT,HH}, n(E)={HT,TH,HH},$n(\underline{E}) = \{TT\}$

3. Six text books numbered 1,2,3,4,5 and 6 are arranged at random. What is the probability that the text books 2 and 3 will occupy consecutive places ?

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{6}$

Solution: (b) $\frac{1}{3}$,

Explanation –$n(S) = 6! = 720$ and $n(E) = 5!X2! = 240$

Therefore $P(E) = \frac{240}{720} = \frac{1}{3}$

4. Given $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$. What is $P(\bar{A})$?

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{6}$

Solution:-(b)$\frac{1}{3}$

Explanation-$P(AUB) = P(A) + P(B) - P(A \cap B)$

Thus $\frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3}$

$P(A) = \frac{2}{3} \Longrightarrow P(\overline{A}) = 1 - \frac{2}{3} = \frac{1}{3}$

6. From a deck of 52 cards, the probability of drawing a face card is

a) $\frac{4}{13}$ b) $\frac{3}{13}$ c) $\frac{1}{13}$ d) $\frac{1}{4}$

Solution : (c) $= \frac{3}{13}$

Explanation: Since there are 12 face cards in a pack of 52 cards. Therefore, Required probability = $\mathbf{\frac{12}{52}}$

7. If two squares are chosen at random on a chess board, the probability that they have a side common is

(a) $\frac{2}{7}$ (b) $\frac{1}{18}$ (c) $\frac{1}{9}$ (d) $\frac{4}{9}$

Solution: – (b) $\frac{1}{18}$

Explanation - Total number of squares = 64

Two squares can be selected in $^{64}C_2$ ways.

In each column, there are 7 pairs of adjacent squares where each pair share 1 side in common. Total such pairs = 8 × 7 = 56.

In each row, there are 7 pairs of adjacent squares where each pair share 1 side in common.

Total such pairs = 8 × 7 = 56.

Therefore, favourable cases = 56 + 56 = 112.

Required probability =$\frac{112}{64_{C2}} = \frac{112}{2016} = \frac{1}{18}$

8. An unbiased dice is rolled four times. The probability that the minimum number on any toss is not less than 3 is

a) $\frac{16}{81}$ b) $\frac{1}{81}$ c) $\frac{65}{81}$ d) $\frac{80}{81}$

Solution:- (a) $\frac{16}{81}$

Explanation - The probability that the outcome of a single throw of a die is any one of 4, 3, 4, 5 and 5 is equal to $\frac{4}{6} = \frac{2}{3}$ of the die is rolled four times, the required probability is equal to $\left(\frac{2}{3}\right)^4$

9. If three events A, B and C are mutually exclusive, then which one of the following is correct?

a) $P(A \cap B \cap C) = 0$ b) $P(A \cup B \cup C) = 0$ c) $P(A \cap B \cap C) = 1$ d) $P(A \cup B \cup C) = 1$

Solution: - a) $P(A \cap B \cap C) = 0$

Explanation - $P(A \cap B \cap C) = 0$

10. Two dice are thrown together. The probability that neither they show equal digits nor the sum of their digits is 9 will be

a) $\frac{13}{15}$ b) $\frac{13}{18}$ c) $\frac{8}{9}$ d) $\frac{1}{9}$

Solution : b) $\frac{13}{18}$

Explanation – n(s)= 6x6 = 36

Favourable outcomes = 36-6-4=26

Therefore, required probability = $\frac{26}{36} = \frac{13}{18}$

ASSERTION-REASON BASED QUESTIONS SOLVED

In the following questions a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(A) Both (A) and (R) are individually true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are individually true, but (R) is not the correct explanation of(A).
(C) (A) is true, but (R) is false.
(D) (A) is false, but (R) is true.

1. Assertion (A) : $P(A) = \frac{3}{4}$ and $P(B) = \frac{3}{8}$, then $P(A \cup B) \geq \frac{3}{4}$

Reason (R) : $P(A) \leq P(A \cup B)$ and $P(B) \leq P(A \cup B)$; hence $P(A \cup B) \geq \max.\{P(A), P(B)\}$

Answer - (A) Both (A) and (R) are individually true and (R) is the correct explanation of (A).

Expalnation - $P(AUB) = P(A) + P(B) - P(A \cap B)$

Therefore $P(A) \leq P(AUB) and\ P(B) \leq P(AUB)$

2. Assertion (A) : The probability of drawing either an ace or a king from a deck of playing cards in a single draw is $\frac{2}{13}$

Reason (R) : For two events E_1 and E_2 , which are not mutually exclusive probability is given by $P(E_1 \cup E_2)$
$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$.

Solution:- (C) (A) is true, but (R) is false.

Explanation – n(s) = 52

Favourable Outcomes = 8

Probability = $\frac{8}{52} = \frac{2}{13}$

Hence assertion is correct

But reason is false since the events E1 and E2 are mutually exclusive according to given condition in reason.

3. Assertion (A) : Two dice are thrown simultaneously. There are 11 possible outcomes and each of them has a probability $\frac{1}{11}$.

Reason (R) : Probability of an event E is defined as P(E) =Number of favourable outcomes/total number of outcomes

Solution:- (D) (A) is false, but (R) is true

Explanation – since total outcomes = 36

4. Assertion (A) : If A,B,C are three mutually exclusive and exhaustive events of an experiment such that P(A) = 2P(B) = 3P(C), then P(A) = $\frac{6}{11}$.

Reason (R) : If A,B,C are three mutually exclusive and exhaustive events of an experiment then P(A) +P(B)+P(C) = 1.

Solution:- (A) Both (A) and (R) are individually true and (R) is the correct explanation of (A).

Explanation – P(AUBUC) = P(A)+P(B)+P(C)= $\frac{66}{66} = 1$

Hence Both (A) and (R) are individually true and (R) is the correct explanation of (A).

5. Assertion (A): Two cards are drawn from a well shuffled pack of 52 playing cards with replacement.

Reason (R): The probability that both are jack cards is $\left(\frac{1}{13}\right)^2$.

Solution:- (C) (A) is true, but (R) is false.

Explanation - $\frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13} = \left(\frac{1}{13}\right)^2$

6. A can solve 80% of the problems given in a book and B can solve 60%.

Assertion (A): The probability that at least one of them will solve a problem is 0.92.

Reason (R): The probability that none of them will solve problem is0.08.

Solution:- (b) Both A and R are true but R is not the correct explanation of A.

Explanation - Here, P (A) = 0.80 and P (B) = 0.60

Therefore, P(at least one of them will solve a problem)

$= 1 - [P(A) \times P(B)]$

= 1 - [(1 - 0.80) × (1 - 0.60)]

= 1 - (0.20 × 0.40)

= 1 - 0.080

= 0.92

Also, P(none of them solve a problem) = 1 - P(at least one of them will solve a problem)

= 1 - 0.92

= 0.08

7.**Assertion(A):** The probability of a sure event is 1.

Reason (R): The total number of possible outcomes in a sample space is always greater than or equal to 1.

Solution: - (a) Both A and R are true, and R is the correct explanation of A.

8. Assertion (A): The probability of any event is always less than zero.

Reason (R): Probability lies between 0 and 1.

Solution:: (d) Assertion is false, but Reason is true.
9. Assertion (A): If an event has only one favorable outcome, then its probability is 1.
Reason (R): Probability is defined as the ratio of favorable outcomes to total outcomes.
Solution:: (c) Assertion is false, but Reason is true.
10. Assertion (A): If two events are equally likely, then their probabilities are equal.
Reason (R): Equally likely events occur with the same frequency in repeated trials.
Solution:: (b) Both A and R are true, but R is not the correct explanation of A.

VERY SHORT ANSWER TYPE QUESTIONS SOLVED

1. 20 cards are numbered from 1 to 20. One card is drawn at random. What is the probability that the number in the cards is a multiple of 4?
Solution:
We have to find the probability that the number on the cards is a multiple of 4
Given: 20 cards numbered from 1 – 20
Formula: $P(E) = \frac{n(E)}{n(S)}$
one card is drawn at random therefore total possible outcomes are $^{20}C_1$
therefore $n(S) = {}^{20}C_1 = 20$
Let E be the event that the number on the drawn card is a multiple of 4
E= {4, 8, 12, 16, 20}
$n(E) = {}^{5}C_1 = 5$
$P(E) = \frac{1}{4}$
2. In a hand at Whist, what is the probability that four kings are held by a specified player?
Solution:
We have to find the probability that four kings are held by a specified player
Given: game of whist is being played
Formula: P(E) =
we have to find the probability that all four kings are held by a specific player since in one hand at whist, a player has 13 cards
total possible outcomes are $^{52}C_{13}$
therefore $n(S) = {}^{52}C_{13}$
let E be the event that a player has 4 kings
$n(E) = {}^{48}C_9 \, {}^{4}C_4$
probability of occurrence of this event is $P(E) = \frac{11}{4165}$
3. Three unbiased coins are tossed once. What is the probability of getting two heads?
Solution: In tossing three coins, then the sample space of event is given by
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.
And, therefore, n(S) = 8.
Let E = event of getting 2 heads.
Then,E = {HHT, HTH, THH}
and, therefore, n(E) = 3.
P(getting 2 heads) = $P(E) = \frac{3}{8}$

4. If $\frac{5}{14}$ is the probability of occurrence of an event, find

i. the odds in favour of its occurrence.

ii. the odds against its occurrence.

Solution:- i. We know that,
If odds in favor of the occurrence an event are a:b, then the probability of an event to occur is $\frac{a}{a+b}$
Given, probability = $\frac{5}{11}$
We know, probability of an event to occur = $\frac{a}{a+b}$
Here,a = 5 and a + b = 14 i.e. b = 9
So, odds in favor of its occurrence = a : b = 5 : 9
Conclusion: Odds in favor of its occurrence is 5: 9
ii. As we solved in part (i), a = 5 and b = 9
Also, we know, odds against its occurrence are b: a = 9: 5
Conclusion: Odds against its occurrence is 9 : 5

5. Two coins are tossed once. Find the probability of getting at least 1 head.

Solution: When two coins are tossed once, then the sample space of event is given by
S = {HH, HT, TH, TT}
and, therefore,n(S) = 4.
Let E = = event of getting at least 1 head.
Then,
E = {HT, TH, HH]
and, therefore, n(E) = 3.
P(getting at least 1 head) = P(E) = $\frac{3}{4}$

6. P and Q are two candidates seeking admission in I.I.T. The probability that P is selected is 0.5 and the probability that both P an d Q are selected is at most 0.3. Prove that the probability of Q being selected is at most 0.8.

Solution: According to the question,
we can write,
Let A_1 and A_2 be two events defined as follows:
A_1 = P is selected, A_2 = Q is selected.
We have, P (A_1) = 05 and P $(A_1 \cap A_2) \leq 0.3$
Now,$P(A_1 \cup A_2) \leq 1$
$P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1 \Rightarrow 05 + P(A_2) - P(A_1 \cap A_2) \leq 1$
$\Rightarrow P(A_2) \leq 05 + P(A_1 \cap A_2) \Rightarrow P(A_2) \leq 0.5 + 0.3 \Rightarrow P(A_2) \leq 0.8$

7. Two balls are drawn at random from a bag containing 2 white, 3 red, 5 green and 4 black balls, one by one without, replacement. Find the probability that both the balls are of different colours.

Solution: We have to find the probability that both the balls are of different colours.
Given: 2 white, 3 red, 5 green, 4 black
Formula:
two balls are drawn one by one, we have to find the probability that they are of different colours
total possible outcomes are ${}^{14}C_2$
therefore n(S) = ${}^{14}C_2$= 91
let E be the event that all balls are of same colour
E = {WW, RR, GG, BB}

$n(E) = {}^2C_2 + {}^3C_2 + {}^5C_2 + {}^4C_2 = 20$
probability of occurrence is $p(E) = \frac{20}{91}$
Therefore, the probability of non-occurrence of the event (all balls are different) is
$p(E') = 1 - P(E)$
$p(E') = \frac{71}{91}$
8. A coin is tossed. If head comes up, a die is thrown but if tail comes up, the coin is tossed again. Find the probability of obtaining head and an even number.
Solution: We have to find the probability of obtaining a head and an even number
Head and an even number can be obtained in any one of the following ways:
(H, 2),(H, 4), (H, 6).
Favourable number of elementary events = 3
Hence, the required probability = $\frac{3}{8}$
9. A box contains 10 bulbs, of which just three are defective. If a random sample of five bulbs is drawn, find the probabilities that the sample contains exactly two defective bulbs.
Solution: We have to find the probabilities that the sample contains exactly two defective bulbs.
Out of 10 bulbs 5 can be chosen in ${}^{10}C_5$ ways.
So, the total number of elementary events = ${}^{10}C_5$
The number of ways of selecting 2 defective bulbs out of 3 defective bulbs and 3 non-defective bulbs out of 7 non-defective bulbs is ${}^3C_2 \times {}^7C_3$
Favourable number of elementary events = ${}^3C_2 \times {}^7C_3$
So, required probability = $\frac{5}{12}$
10. In a single throw of three dice, determine the probability of getting a total of 5.
Solution:
We have to find the probability of getting a total of 5
Total number of elementary events associated with the random experiment of throwing three dice simultaneously is 6X6X6 = 216.
A total of 5 can be obtain in one of the following ways:
(1, 1, 3), (3, 1, 1), (1, 3, 1), (2, 2, 1), (1, 2, 2), (2, 1, 2)
Favourable number of elementary events = 6
Hence, Required probability = $\frac{6}{216} = \frac{1}{36}$

SHORT ANSWER TYPE QUESTIONS SOLVED

1. Three coins are tossed. Describe two events A and B which are mutually exclusive.
Solution: When three coins are tossed, then the sample space of the event is given by
S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Now, The subparts are:
The two events which are mutually exclusive are when,
A: getting no tails
B: getting no heads
Then, A = {HHH} and B = {TTT}
So, The intersection of this set will be null set.
Or, The sets are disjoint.
2. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12.
Solution: The coin with 1 marked on one face and 6 on the other face.

The coin and die are tossed together.
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
n(S) = 12.
(i) Let A be the event having sum of numbers is 3
A = {(1, 2)}

n (A) = 1

Thus P(A) = $\frac{1}{12}$

(ii) Let B be the event having sum of number is 12

B = {(6, 6)}

n (B) = 1

Thus P(B) = $\frac{1}{12}$

3. Let A and B be two mutually exclusive events of a random experiment such that P(not A) = 0.65 and P(A or B) = 0.65, find P(B).

Solution: It is given that A and B are mutually exclusive events
P(not A) = 0.65 , P(A or B) = 0.65
To find : P(B)
Formula used : P(A) = 1 – P(not A)
P(A or B) = P(A) + P(B) - P(A and B)
For mutually exclusive events A and B, P(A and B) = 0
P(A) = 1 - P(not A)
P(A) = 1 - 0.65
P(A) = 0.35
Substituting the values in the above formula we get,
0.65 = 0.35 + P(B)
P(B) = 0.65 - 0.35
P(B) = 0.30

4. Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have nonadjacent desks? [Hint: First find the probability that the couple has adjacent desks, and then subtract it from 1.]

Solution: Given that total new employees = 6
So, they can be arranged in 6! ways.
n(S) =total number of outcomes = 6! = 6 x5x 4x 3x 2x 1 = 720
Two adjacent desks for married couple can be selected in 5 ways i.e. (1, 2), (2, 3), (3, 4), (4, 5), (5, 6)
Married couple can be arranged in the two desks in 2! Ways
Other four persons can be arranged in 4! Ways
So, number of ways in which married couple occupy adjacent desks= 5x 2!x 4! = 5x 2x 1x 4x 3x 2x 1 = 240
Let E be the event that the couple occupy adjacent desks=240
n(E)= 240
P(E)=P(they occupy adjacent desks) = $\frac{1}{3}$
Required probability= P(they occupy non adjacent desks)P(not E)= 1- P(E)= $\frac{2}{3}$

5. In a certain lottery 10,000 tickets are sold and, ten equal prizes are awarded. What is the probability of not getting a prize if you buy two tickets?

Solution: Given, total number of tickets = 10,000
Number of prize bearing tickets = 10
So number of tickets not bearing prize = 10,000-10 = 9990
Let B be the event that two bought tickets are not prize bearing tickets.
Now, out of 10,000 tickets one can buy 2 ticket in ${}^{10000}C_2$ ways i.e. total number of outcomes, and out of 9990 tickets not bearing any prize, one can buy 2 tickets in ${}^{9990}C_2$ ways i.e. total number of favourable outcomes.
Hence, probability of not getting a prize if two tickets are bought = ${}^{9990}C_2/{}^{10000}C_2$

6. What is the probability that a leap year has 53 Sundays?

Solution: A leap has 366 days i.e. 52 weeks + 2 days.
So, there will be 52 Sundays. (because every week has one Sunday)
So, we want another Sunday from the remaining two days.
The two days maybe, Sunday, Monday or Monday, Tuesday or Tuesday, Wednesday or Wednesday, Thursday or Thursday, Friday or Friday, Saturday or Saturday, Sunday
So, total outcomes are 7 and the desired outcomes are 2 (Sunday, Monday or Saturday, Sunday)
Therefore, the probability of getting 53 Sundays in a leap year = $\frac{2}{7}$

7. Find the probability of getting 2 or 3 tails when a coin is tossed four times.

Solution: We have to find the probability of getting 2 or 3 tails when a coin is tossed four times.
Let S be the sample space associated with the experiment that a coin is tossed four times.
Then n(S) = 2^4 = 16
Consider the following events:
A: Event of getting 2 tails
B : Event of getting 3 tails
Then A = {HHTT, HTHT, HTTH, THTH, TTHH, THHT}
n(A) = 6
P(A) = $\frac{6}{16}$
B = {HTTT, THTT, TTHT, TTTH}
n(B) = 4
P(B) = $\frac{4}{16}$
Since events A and B are mutually exclusive, we have:
P(A and B) = 0
By addition theorem, we have:
P(A or B) = P(A) + P(B) – P(A and B) = $\frac{5}{18}$

8. In an entrance test that is graded on the basis of two examination, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Solution: Let A be the event that the student passes the first examination
and B be the event that the student passes the second examination.
Given, P(A) = 0.8, P(B) = 0.7, and
Probability of passing at least A or B means P(A or B)
Given P(A or B) = 0.95
So = P(A and B) =P(A) + P(B) - P(A or B)=0.8+0.7-0.95=1.5 - 0.95 = 0.55.

Hence, the probability of passing both examination is 0.55.

9. Three coins are tossed once. Let A denotes the event three heads show, B denotes the event two heads and one tail show, C denotes the event three tails show and D denotes the event a head shows on the first coin. Find that which are,

(i) mutually exclusive events.

(ii) simple events.

(iii) compound events.

Solution: When three coins are tossed, then the sample space is

S = {HHH,HHT,HTT,TTT,TTH,THH,THT,HTH}

Given, event A = Three heads show = {HHH}

event B = Two heads and one tail show = {HHT,HTH,THH}

event C = Three tails show = {TTT}

events D = A head shows on the first coin = {HHH,HHT,HTH,HTT }

A ∩ B= {HHH} ∩ {HHT,HTH, THH}= { }

A ∩ C = {HHH} ∩ {TTT}= { }

B∩ C = {HHT,HTH, THH} ∩ {TTT}= { }

C ∩D= {TTT} ∩ {HHH,HHT,HTH,HTT}= { }

And A ∩ B∩ C= {HHH} ∩ {HHT,HTH, THH} ∩ {TTT}={ }

So, A and B; A and C; B and C; C and D; A,B and C are mutually exclusive events.

ii. A and C are simple events, since both have only one sample point.

iii. B and D are compound event, since they have more than one sample point.

10. Two dice are rolled. A is the event that the sum of the numbers shown on the two dice is 5, and B is the event that at least one of the dice shows up a 3. Are the two events

1. mutually exclusive,
2. exhaustive? Give arguments in support of your answer.

Solution: When two dice are rolled, we have n(S) = 6x6 = 36.

Given events are;

A : Sum of the numbers shown on the two dice is 5.

Thus, A = { (1, 4), (2, 3), (4, 1), (3, 2) },And B : at least one of the dice shows up a 3.

Thus, B = { (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) (1, 3), (2, 3), (4, 3), (5, 3), (6, 3) }.

A∩ B = {(2, 3), (3, 2)}does not equal to null set .

Hence, A and B are not mutually exclusive.

Also, A U B does not equal to S.

A and B are not exhaustive events.

LONG ANSWER TYPE QUESTION SOLVED

1. A number is chosen from the numbers 1 to 100. Find the probability of its being divisible by 4 or 6.

Solution: We have to find the probability of its being divisible by 4 or 6.

Let A denote the event that the number is divisible by 4 and B denote the event that the number is divisible by 4.

To find : Probability that the number is both divisible by 4 or 6 = P(A or B)

Probability = P(A or B) = P(A) + P(B) - P(A and B)

Numbers from 1 to 100 divisible by 4 are {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100}.

There are 25 numbers from 1 to 100 divisible by 4

Favourable number of outcomes = 25

Total number of outcomes = 100 as there are 100 numbers from 1 to 100

P(A) = $\frac{25}{100}$
Numbers from 1 to 100 divisible by 6 are{6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96}
There are 16 numbers from 1 to 100 divisible by 6
Favourable number of outcomes = 16
Total number of outcomes = 100 as there are 100 numbers from 1 to 100
P(B) = $\frac{16}{100}$
Numbers from 1 to 100 divisible by both 4 and 6 are{12, 24, 36, 48, 60, 72, 84, 96}
There are 8 numbers from 1 to 100 divisible by both 4 and 6
Favourable number of outcomes = 8
P(A and B) = $\frac{8}{100}$
P(A or B) = P(A) + P(B) - P(A and B)
P(A or B) = $\frac{33}{100}$
The probability that the number is both divisible by 4 or 6 = P(A or B) = $\frac{33}{100}$
2. A fair coin is tossed four times, and a person win Rs.1 for each head and lose Rs. 1.50 for each tail that Turns up.Form the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.
Solution: - Here a coin is tossed four times.
So number of elements in the sample space n(S) will be 2^4 = 16. n(S) = 16.

The sample space,S = {HHHH, HHHT, HHTH, HTHH, HTTH, HTHT, HHTT, HTTT, THHH, THHT, THTH, TTHH, TTTH, TTHT, THTT, TTTT}
Amounts:
i. When 4 heads turns up = Rs(1 + 1+ 1 + 1)= Rs. 4. i.e., Person wins Rs. 4
ii.When 3 heads and 1 tail turns up = Rs(1+ 1+1 –1.50 =Rs. 1.50. i.e.,Person wins Rs. 1.50
iii.When2 heads and 2 tails turns up = Rs(1 + 1– 1.50– 1.50) =– Rs. 1. i.e., Person loses Rs. 1
iv. When1 head and 3 tails turns up = Rs(1– 1.50– 1.50– 1.50)=– Rs 3.50. i.e., Person losesRs. 3.50
v. When4 tails turns up= Rs(– 1.50– 1.50– 1.50– 1.50) =– Rs 6. i.e., Person loses Rs. 6
Let the events for which the person wins Rs 4, wins Rs 1.50, loses Re1, loses Rs 3.50 and loses Rs 6be denoted by E1, E2, E3, E4 and E5.i.e.,
E1 = {HHHH}, E2 = {HHHT, HHTH, HTHH, THHH} E3 = {HHTT, HTHT, HTTH, THTH, THHT, TTHH}E4 = {HTTT, TTTH, THTT, TTHT}, E5 = {TTTT}
Here, n(E1) = 1, n(E2) = 4, n(E3) = 6, n(E4) = 4 and n(E5) = 1.
Hence, P(E1) = $\frac{1}{16}$, P(E2) = $\frac{1}{4}$, P(E3) = $\frac{3}{8}$, P(E4) = $\frac{1}{4}$, P(E5) = $\frac{1}{16}$
3. Two dice are thrown. The events A, B, C, D, E and F are described as follows:
A = Getting an even number on the first die.
B = Getting an odd number on the first die.
C = Getting at most 5 as a sum of the numbers on the two dice.
D = Getting the sum of the numbers on the dice greater than 5 but less than 10.
E = Getting at least 10 as the sum of the numbers on the dice.
F = Getting an odd number on one of the dice.
Describe the following events: A and B, B or C, B and C, A and E, A or F, A and F.
Answer - A = Getting an even number on the first die.
A = { (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

B = Getting an odd number on the first die.
B = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) }
C = Getting at most 5 as sum of the numbers on the two dice.
C = { (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1) }
D = Getting the sum of the numbers on the dice >5 but <10.
D = { (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4, 2), (4, 3), (4, 4), (4, 5), (5, 1), (5, 2), (5, 4), (6, 1), (6, 2), (6, 2), (6, 3) }
E = Getting at least 10 as the sum of the numbers on the dice.
E = { (4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6) }
F = Getting an odd number on one of the dice.
F = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), (3, 4)(3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5) }
A and B are mutually exclusive events, thus A ∩ B = null set
B∪C={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),(5,4), (5,5),(5,6),(2,1),(2,2),(2,3),(4,1)}
B ∩C = {(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)}
A ∩ E = {(4,6),(6,4),(6,5),(6,6)}
A or F = {(2,1),(2,2),(2,3),(2,4),(2,5),(2,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (5,1),(5,2),(5,3),(5,4),(5,5),(5,6)}
A and F={(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),(6,1),(6,3),(6,5)}

Question 3. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that all the three balls are white.

Solution: There are 13 balls in the bag and three balls are to be taken out,
So the number of ways of doing this = n(S) = number of total outcomes = ${}^{13}C_3$
Now, we have to find the probability that all the three balls to be taken out from the bag are white
Let A be the event that all the three balls are white
As there are 5 white balls, so the number of ways in which this can be done= n(A) = number of favourable outcomes= ${}^{5}C_3$
We know that, P(A) = P (all the three balls are white) = ${}^{5}C_3/{}^{13}C_3$ =5/143

Question 4. A die is thrown.
Describe the following events:

(i) A: a number less than 7.
(ii) B: a number greater than 7.
(iii) C: a multiple of 3.
(iv) D: a number less than 4.
(v) E: an even number greater than 4.
(vi) F: a number not less than 3.

Also, find A∪ B, A ∩ B, B ∪ C, E ∩ F, D ∩ E, A - C, D - E, F' and E ∩ F'.

Solution: When a die is thrown, then sample space
S = {1, 2, 3, 4, 5, 6 }
i. A: a number less than 7 = {1, 2, 3, 4, 5, 6 }
ii. B: a number greater than 7 = { } = Null set.
iii. C: a multiple of 3 = {3, 6 }
iv. D: a number less than 4 = {1, 2, 3}
v. E: an even number greater than 4 = {6}

vi. F: a number not less than 3 = {3, 4, 5, 6 }
Now, AU B = The elements which are in A or B or both
= {1, 2, 3, 4, 5, 6 }U {} = {1, 2, 3, 4, 5, 6 }
The elements which are common in both A = B ={1, 2, 3, 4, 5, 6 } and {} = {1, 2, 3, 4, 5, 6 }
B U C = The elements which are in B or C or both
= { }U {3,6} = {3,6}
The elements which are common in both E and F= {6}and {3, 4, 5, 6 } = {6}
The elements which are common in both D and E= {1, 2, 3}and {6} = null set
The elements which are in A but not in C= {1, 2, 3, 4, 5, 6 } - {3,6} = {1, 2, 4, 5 }
The elements which are in D but not in E= {1, 2, 3} - {6} = {1,2,3}
Question 5. Two dice are thrown. The events A, B and C are as follows:
A: getting an even number on the first die
B: getting on odd number on the first die
C: getting the sum of the numbers on the dice ≤ 5
Describe the events
(i) A' (ii) not B (iii) A or B (iv) A and B (v) A but not C (vi) B or C (vii) B and C
(viii) $A \cap B' \cap C'$
Solution: When two dice are thrown then
S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, l), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
A: getting an even number on the first die.= {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2,6),(4,1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
B: getting an odd number on the first die{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}
C: getting the sum of the number on the dice = {(1, 1),(1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}
(i) A' = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)} = B
(ii) not B = {(2, l), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = A
(iii) A or B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)(2, l), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)(4, l), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = S
(iv) A and B = null set
(v) A but not C = A - C = {(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
(vi) B or C = {(1,1), (1,2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (5,1), (5, 2),(5, 3), (5, 4), (5, 5), (5, 6)}
(vii) B and C = {(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)}
(viii) $A \cap B' \cap C'$ = {(2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

CASE STUDY BASED QUESTION

Question 1. Read the following text carefully and answer the questions that follow:
On her vacation, Priyanka visits four cities. Delhi, Lucknow, Agra, Meerut in a random order.

1. What is the probability that she visits Delhi before Lucknow?
2. What is the probability she visit Delhi before Lucknow and Lucknow before Agra?

3. What is the probability she visits Delhi first and Lucknow last?
OR
What is the probability she visits Delhi either first or second?

Solution: 1. Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24 n(S) = 24
Clearly, sample space for this experiment is
S = {ABCD,ABDC,ACBD,ACDB,ADBC,ADCB
BACD,BADC,BCAD,BCDA,BDAC,BDCA
CABD,CADB,CBAD,CBDA,CDAB,CDBA,
DABC,DACB,DCAB,DCBA,DBAC,DBCA}
Let E1 be the event that Priyanka visits A before B.
Then,
E1 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, CABD, CADB, CDAB, DABC, DACB, DCAB}
n(E1) = 12
P(she visits A before B) = P(E1) = $\frac{1}{2}$

2. Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D.
Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24
n(S) = 24
Clearly, sample space for this experiment is
S = {ABCD,ABDC,ACBD,ACDB,ADBC,ADCB
BACD,BADC,BCAD,BCDA,BDAC,BDCA
CABD,CADB,CBAD,CBDA,CDAB,CDBA,
DABC,DACB,DCAB,DCBA,DBAC,DBCA}
E1 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, CABD, CADB, CDAB, DABC, DACB, DCAB}
n(E1) = 12
P(she visits A before B) = $\frac{1}{2}$

3. Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D.
Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24
n(S) = 24
Clearly, sample space for this experiment is
S = {ABCD,ABDC,ACBD,ACDB,ADBC,ADCB
BACD,BADC,BCAD,BCDA,BDAC,BDCA
CABD,CADB,CBAD,CBDA,CDAB,CDBA,
DABC,DACB,DCAB,DCBA,DBAC,DBCA}
Let E3 be the event that she visits A first and B last.
Then,
E3 = {ACDB, ADCB}
n(E3) = 2
P(she visits A first and B last) = P(E3) = $\frac{1}{12}$
OR
Let the Priyanka visits four cities Delhi, Lucknow, Agra, Meerut are respectively A, B, C and D. Number of way's in which Priyanka can visit four cities A, B, C and D is 4! i.e. 24

n(S) = 24
Clearly, sample space for this experiment is
S = {ABCD,ABDC,ACBD,ACDB,ADBC,ADCB
BACD,BADC,BCAD,BCDA,BDAC,BDCA
CABD,CADB,CBAD,CBDA,CDAB,CDBA,
DABC,DACB,DCAB,DCBA,DBAC,DBCA}
Let E4 be the event that she visits A either first or second.
Then,
E4 = {ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, CABD, CADB, DABC, DACB}
n(E4) = 12
Hence, P(she visits A either first or second) = $\frac{1}{2}$

Question 2.. Read the following text carefully and answer the questions that follow:
There are 4 red, 5 blue and 3 green marbles in a basket.

1. If two marbles are picked at randomly, find the probability that both red marbles.
2. If three marbles are picked at randomly, find the probability that all green marbles.
3. If two marbles are picked at randomly then find the probability that both are not blue marbles.

OR
If three marbles are picked at randomly, then find the probability that at least one of them is blue.

Solution: (1) n(S) =12 $P(E) = \frac{4X3}{12X11} = \frac{1}{11}$

(2) $P(E) = \frac{3X2}{12X11X10} = \frac{1}{220}$

(3) $P(E) = \frac{7X6}{12X11} = \frac{7}{22}$

OR

$P(E) = 1 - P(Not\ Blue) = 1 - \frac{7}{44} = \frac{37}{44}$

Question3. Read the following text carefully and answer the questions that follow:
Four friends Dinesh, Yuvraj, Sonu, and Rajeev are playing cards. Dinesh, shuffling a cards and told to Rajeev choose any four cards.

1. What is the probability that Rajeev getting all face card.
2. What is the probability that Rajeev getting two red cards and two black card.
3. What is the probability that Rajeev getting one card from each suit.

OR
What is the probability that Rajeev getting two king and two Jack cards.
Answer – 1. Total number of possible outcomes = ${}^{52}C_4$
We know that there are 12 face cards
Number of favourable outcomes = ${}^{12}C_4$
Required probability = ${}^{12}C_4/{}^{52}C_4$
2.Total number of possible outcomes = ${}^{52}C_4$
We know that there are 26 red and 26 black cards.
Number of favourable outcomes = ${}^{26}C_2 x {}^{26}C_2$
Required probability = ${}^{26}C_{2x}{}^{26}C_2/{}^{52}C_4$

3. Total number of possible outcomes = ${}^{52}C_4$
Number of favourable outcomes = ${}^{13}C_1$

Required probability = $(13C1)^4/52C4$

OR

Total number of possible outcomes = ${}^{52}C_4$

In playing cards there are 4 king and 4 jack cards.

Number of favourable outcomes = = 6 x 6 = 36

Required probability = 36 / ${}^{52}C_4$

Question 4.Two candidates Anil and Surabhi appeared in a written test for a job position in a company.

The probability that Anil will qualify the test is 0.05 and that Surabhi will qualify the test is 0.10, The probability that both will qualify the test is 0.02.

Based on the given information, answer the following questions.

(i) Probability that Anil and Surabhi will not qualify the examination?

(ii) Probability that at least one of them will not qualify the examination?

(iii) Probability that only one of them will qualify the examination?

Solution- (i) $P(AUB) = P(A) + P(B) - P(A \cap B)$

$P(AUB) = 0.05 + 0.10 - 0.02 = 0.13$ AS we know that $P(AUB) + P(\overline{AUB})$ =1

$P(\bar{A} \cap \bar{B}) = 1 - P(AUB) = 1 - 0.13 = 0.87$

(ii) $P(AUB) = P(A) + P(B) - P(A \cap B)$

$P(AUB) = 0.05 + 0.10 - 0.02 = 0.13$

(iii) $P(A \cap \overline{B}) + P(\overline{A} \cap B) = 0.03 + 0.08 = 0.11$

Question 5.Aditya visit has dentist. The probability he will have a tooth extracted is 0.05. the probability that he will have cavity filled is 0.2 and the probability that he will have tooth extracted as well as cavity filled is 0.02. based on the above information answer the following questions.

(i) What are the chances that he will have his tooth extracted or cavity filled ?

(ii) What are the chances that he will have his tooth extracted only ?

(iii) What are the chances that he will have his cavity filled only?

Solution:(i) $P(AUB) = P(A) + P(B) - P(A \cap B)$

$P(AUB) = 0.05 + 0.2 - 0.02 = 0.23$

(ii) $P(A\ Only) = P(A) - P(A \cap B) = 0.05 - 0.02 = .03$

(iii) $P(\ B\ Only) = P(B) - P(A \cap B) = 0.2 - 0.02 = 0.18$

Question 6. A team of medical students doing their internship have to assist during surgeries at a city hospital. The probability of surgeries rated as very complex, routine , simple and very simple are 0.15, 0.20, 0.31 , 0.26 , 0.08 respectively. Based on the above information find the [probability of

(i) complex or very complex.

(ii) Neither very complex nor very simple.

(iii) routine or complex.

Solution- Consider the following events for surgery A=Very complex, B= Complex, C= Routine, D =Simple, E =Very simple.

P(A)=0.15 , P(B)=0.20 , P(C)=.0.31,P(D)=0.26, P(E)=0.08 Clearly A,B,C,D and E are mutually exclusive events.

(i) $P(AUB) = P(A) + P(B) = 0.15 + 0.20 = 0.35$

(ii) $P(\overline{A} \cap \overline{E}) = P(\overline{AUE}) = 1 - P(AUE) = 1 - \{P(A) + P(E)\} = P(\overline{A} \cap \overline{E}) = 1 - \{0.15 + 0.08\}$

$P(\overline{A} \cap \overline{E}) = 1 - 0.23 = 0.77$

(iii) $P(CUB) = P(C) + P(B) = 0.31 + 0.20 = 0.51.$

EXERCISES AND ANSWERS (ALL CHAPTERS)

PRACTICE QUESTION PAPERS (2025-26)

USEFUL LINKS

CBSE CURRICULUM – CLASS XI MATHEMATICS 2025 -26
https://cbseacademic.nic.in/curriculum_2026.html
CLASS 11 MATHEMATICS – NCERT TEXT BOOK
https://ncert.nic.in/textbook.php?kemh1=0-16
CLASS 11 MATHEMATICS LAB MANNUAL- NCERT
https://ncert.nic.in/science-laboratory-manual.php
CLASS 11 MATHEMATICS – VIDEO LESSONS(NCERT)
https://www.youtube.com/@NCERTOFFICIAL/search?query=CLASS%2011%20MATHEMATICS
ANCIENT INDIAN MATHEMATICIANS
https://www.youtube.com/@NCERTOFFICIAL/search?query=ANCIENT%20INDIAN%20MATHEMATICIANS