Applied Mathematics

कक्षा / Class XI

2025-26

विद्यार्थी सहायक सामग्री
Student Support Material

<u>संदेश</u>

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना एवं नवाचार द्वारा उच्च - नवीन मानक स्थापित करना केन्द्रीय विद्यालय संगठन की नियमित कार्यप्रणाली का अविभाज्य अंग है। राष्ट्रीय शिक्षा नीति 2020 एवं पी. एम. श्री विद्यालयों के निर्देशों का पालन करते हुए गतिविधि आधारित पठन-पाठन, अनुभवजन्य शिक्षण एवं कौशल विकास को समाहित कर, अपने विद्यालयों को हमने ज्ञान एवं खोज की अद्भुत प्रयोगशाला बना दिया है। माध्यमिक स्तर तक पहुँच कर हमारे विद्यार्थी सैद्धांतिक समझ के साथ-साथ, रचनात्मक, विश्लेषणात्मक एवं आलोचनात्मक चिंतन भी विकसित कर लेते हैं।   यही कारण है कि वह बोर्ड कक्षाओं के दौरान विभिन्न प्रकार के मूल्यांकनों के लिए सहजता से तैयार रहते हैं। उनकी इस यात्रा में हमारा सतत योगदान एवं सहयोग आवश्यक है - केन्द्रीय विद्यालय संगठन के पांचों आंचलिक शिक्षा एवं प्रशिक्षण संस्थान द्वारा संकलित यह विद्यार्थी सहायक- सामग्री इसी दिशा में एक आवश्यक कदम है । यह सहायक सामग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है। केन्द्रीय विद्यालय संगठन की विद्यार्थी सहायक- सामग्री अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री संकलन की विशेषज्ञता के लिए जानी जाती है और शिक्षा से जुड़े विभिन्न मंचों पर इसकी सराहना होती रही है। मुझे विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर निरंतर मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुँचाएगी।

शुभाकांक्षा सहित ।

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INDEX

UNIT-1

NUMBERS, QUANTIFICATION AND NUMERICAL APPLICATIONS

NUMBERS AND QUANTIFICATION

1) Binary Numbers: Definition of number system (decimal and binary), Conversion from decimal to binary system and vice-versa
2) Indices Logarithm and Antilogarithm: Applications of rules of indices Introduction of logarithm and antilogarithm Common and Natural Logarithms
3) Laws and properties of logarithms: Fundamental laws of logarithm
4) Simple applications of logarithm and antilogarithm: Express the problem in the form of an equation and apply logarithm/antilogarithm

BINARY NUMBERS

Computer receives, stores and processes the information (or data) using two digits '0' and '1' called **binary digits or bits**. Any number using two digits 0 and 1 is called binary number.
In computers and other electronic device we use binary number system. It consists of two digits 0 and 1. So the base of this number system is 2.

DECIMAL NUMBER SYSTEM

In this system we have ten digits i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Hence the base of this system is 10.
Let us consider the number 1532.
As the digit 1 occupies thousands place, the place value of the digit $1 = 1 \times 10^3 = 1000$
Similarly, the place value of digit $5 = 5 \times 10^2 = 5 \times 100 = 500$,
The place value of digit $3 = 3 \times 10^1 = 30$
The place value of digit $2 = 2 \times 10^0 = 2$
So, $1532 = 1 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 2 \times 10^0 = 1532$
In decimal system, we represent the number 1532 as $(1532)_{10}$

BINARY NUMBER SYSTEM

In this system we have two digits i.e. 0 and 1. Hence, the base of this system is 2.
Let us consider a binary number 11100 which can be written in decimal system as $1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 16 + 8 + 4 + 0 + 0 = 28$
The binary number 11101 in binary system is represented as $(11100)_2$
So $(11100)_2 = (28)_{10}$

CONVERSION OF DECIMAL NUMBER TO BINARY NUMBER

To convert a given decimal number to a binary number proceed as under.
Step 1. Divide the given decimal number by 2 and write down the remainder.
Step 2. Again, divide the quotient obtained in step 1 by 2 and again write down the remainder.
Step 3. Repeat the step 2 again and again till you get the quotient as 1.
Step 4. Write the last quotient (i.e.1) and remainder in the reverse order (i.e. from bottom to the top).

BINARY ADDITION

Binary addition is done in the same way as in decimal system.
$0 + 0 = 0$
$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 0$ and 1 carry over to next left column.

BINARY SUBTRACTION

Binary subtraction is done in the same way as in decimal system.

$0 - 0 = 0$

$1 - 0 = 1$

$1 - 1 = 0$

$0 - 1 = 1$ with a borrow of 1 from the next left column.

BINARY MULTIPLICATION

Binary multiplication is done in the same way as in decimal system.

$0 \times 0 = 0$

$1 \times 0 = 0$

$1 \times 0 = 0$

$1 \times 1 = 1$

BINARY DIVISION

Binary division is done in the same way as in decimal system. Like decimal system, division by 0 is meaningless in binary system also. So, there are only two possible rules of binary division. These two rules are given below:

$0 \div 1 = 0$

$1 \div 1 = 1$

<div align="center">

INDICES AND LOGARITHMS

</div>

LAWS OF EXPONENTS FOR REAL NUMBERS

Laws of exponents for real numbers are:

If a, b are positive real numbers and m, n are rational numbers, then the following results hold:

(i) $a^m \cdot a^n = a^{m+n}$

(ii) $(a^m)^n = a^{mn}$

(iii) $a^m / a^n = a^{m-n}$

(iv) $a^m \cdot b^m = (ab)^m$

(v) $(a / b)^m = a^m / b^m$

(vi) $a^{-n} = 1 / a^n$

(vii) $a^n = b^n, n \neq 0 \implies a = b$

(viii) $a^m = a^n \implies m = n$ provided $a \neq 1$

(ix) $a^0 = 1, a \neq 0$

(x) $\sqrt[n]{x} = x^{1/n}$

LOGARITHMS

Definition. If a is any positive real number (except 1), n is any rational number and $a^n = b$, then n is called algorithm of b to the base a. It is written as $\log_a b$ (read as log of b to the base of a).

Thus,

$a^n = b$ if and only if $\log_a b = n$

$a^n = b$ is called the exponential form and $\log_a b = n$ is called logarithmic form

For example:

$5^4 = 625$ $\therefore \log_5 625 = 4$

$7^0 = 1,$ $\therefore \log_7 1 = 0$

$(10)^{-2} = 0.01$ $\therefore \log_{10}(0.01) = -2$

Properties of Logarithms

(1) $\log (a.b) = \log a + \log b$

(2) $\log \left(\frac{a}{b}\right) = \log a - \log b$

(3) $\log a^b = b \log a$

Important Points:

(i) $\log_a 1 = 0$ (ii) $\log_a a = 1$

where a is any positive real number (except 1).

- Logarithms to the base 10 are called common logarithms.
- $\log_a x = \log_a y \Rightarrow x = y$
- If no base is given, the base is always taken as 10.
    For example, $\log 2 = \log_{10} 2$.

LOGARITHM TABLES:

The logarithm of a number consists of two parts, the whole part or the integral part is called the characteristic and the decimal part is called the mantissa. While the former can be known by mere inspection, the latter has to be obtained from the logarithm tables.

Characteristic:

The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of digits to the left of the decimal point in the given number. The characteristic of the

logarithm of any number less than one (1) is negative, numerically one more than the number of zeros to the right of the decimal point. If there is no zero, then obviously −1.

The following table will illustrate it:

Number	Standard form	Characteristic
37	3.7×10^1	1
4623	4.623×10^3	3
6.21	6.21×10^0	0
0.8	8×10^{-1}	$-1 = \bar{1}$
0.07	7×10^{-2}	$-2 = \bar{2}$

Mantissa:

The mantissa is the fractional part of the logarithm of a given number.

Number Mantissa Logarithm:

- Log 4594 = (......... 6623) = 3.6623
- Log 459.4 = (......... 6623) = 2.6623
- Log 45.94 = (......... 6623) = 1.6623
- Log 4.594 = (......... 6623) = 0.6623
- Log .4594 = (......... 6623) = $\bar{1}$.6623

ANTILOGARITHMS

If *x* is the logarithm of a given number *n* with a given base, then *n* is called the antilogarithm (antilog) of *x* to that base.

This can be expressed as follows:

- If $\log_a b = n$, then b=antilog (n)

Example:
- If log61720=4.7904   log 61720 = 4.7904, then 61720=antilog 4.7904

NUMERICAL APPLICATIONS

1) Clock: - Number of rotations of minute hand/hour hand of a clock in a day Number of times minute hand and hour hand coincides in a day

2) Calendar: - Definition of odd days, odd days in a year/ century Day corresponding to a given date

3) Time, Work and Distance: - Basic concept of time and work Problems on time taken/distance covered/work done, Establish the relationship between work and time, Compare the work done by the individual/ group w.r.t. time Calculate the time taken/ distance covered/Work done from the given data

4) Seating Arrangement: - Linear Circular seating arrangement, position of person in a seating arrangement

CLOCK

1. Clock has two hands. The shorter hand is the hour hand and the longer hand is the minute hand.
2. The face of the clock called dial is a complete circle having 360°. The dial is divided into 12 equal parts. These parts are marked as 1, 2, 3, ..., 11, 12.
3. The angle between two consecutive numbers is 30°.
4. Angle turned by an hour hand in 1 hour = 30°.
5. Angle turned by an hour hand in 1 minute = 0.5°.
6. Angle turned by minute hand in 1 minute = 6°.
7. The relative angular speed of minute hand with respect to hour hand = 6° - 0.5° = 5.5° per minute.

Relation between time and angle between the hands of clock

1. Angle turned by hour hand in H hours 30° × H (30H)°
2. Angle turned by hour hand in M minutes = (½ M)°
3. Angle turned by hour hand in H hours M minutes = (30 H + ½ M)°
4. Angle turned by minute hand in H hours M minutes = (0×H+6M) ° = 6 M°
5. Angle between hands of clock at H hours M minutes = (30 H - (11/2) M)°
   (Value of A is always taken as positive, negative sign is ignored)

Some Facts about Clock

1. Both the hands of clock meet (coincide) 11 times in 12 hours.
2. Both the hands meet after every (12×60)/11 = 65 5/11 min. The meeting takes place at 12:00, 1:5 5/11, 2:10 10/11 ... and so on.
3. Both the hands of clock are opposite to each other (at 180°) 11 times in 12 hours.
4. Both the hands of clock are at right angles (at 90°) to each other 22 times in 12 hours.
5. Any angle other than 0° and 180° between the two hands is made 22 times in 12 hours.

CALENDAR

A chart showing days, weeks and months of a particular year is called calendar. It has two types of calendar years.
1. Leap year: It has 366 days.
2. Non-leap or common or ordinary year. It has 365 days.
3. A year is divided into twelve months.
4. A month is divided into weeks. A week has 7 days.

5. The international is to start the week on Monday.
6. Numbers of week in a year is 52.

To determine whether a year is a leap year:

1. Any year which is not divisible by 4 is not a leap year e.g. 2001, 2002, 2003, 2005 etc.
2. Any year which is divisible by 4 but not divisible by 100 is a leap year e.g. 1904, 1908, 1912 etc.
3. Any year which is divisible by 400 is a leap year egg 1200, 1600, 2000 etc. These century years are called century leap years.
4. Any year which is divisible by 100 but not divisible by 400 is not a leap year e.g. 1700, 1800, 1900 etc.

Odd Days

1. In a Non leap year there are 1 odd day
2. In a leap year there are 2 odd days.
3. In a century i.e. 100 years, there are 5 odd days.
4. In 200 years, there are 3 odd days.
5. In 300 years, there are 1 odd day.
6. In 400 years, 0 odd days.

TIME AND WORK

1. 1 day's work = 1 / number of days to complete the work
2. Number of days needed to complete the work = 1 / (1 day's work)
3. Time required to do a certain work = $\dfrac{work\ to\ be\ done}{work\ to\ be\ done\,1\ day's\ work}$
4. Remuneration is in proportion of work done.

TIME AND DISTANCE

Speed of a moving body is defined as the distance covered by it in unit time.

1. Distance = Speed × Time
2. Speed = Distance / Time
3. Time = Distance X Speed
4. When distance is constant, then speed $\propto 1/$ time
   i.e., Speed × Time = Constant $\Rightarrow S_1 T_1 = S_2 T_2$
5. When time is constant, then distance $\propto$ speed
   i.e., $D_1 / S_1 = D_2 / S_2$
6. When speed is constant, then distance $\propto$ time
   i.e., $D_1 / T_1 = D_2 / T_2$

SEATING ARRANGEMENT

In seating arrangement, we have to arrange a group of persons fulfilling the given conditions.

1. **Linear arrangement**: In this type of arrangement, we have to arrange the group of persons in a line fulfilling the given conditions.
2. **Circular arrangement**: In this type of arrangement, we have to arrange the group of persons around a circular table fulfilling the given conditions.
3. **Arrangement around a square table**: In this type of arrangement, we have to arrange the group of persons around a square table fulfilling the given conditions.

MULTIPLE CHOICE QUESTIONS(SOLVED)

Q 1) Subtract the binary number 10000 from 10111

(a) 111   (b) 1111   (c) 1110   (d) 10001

Answer (a) 111
Solution:

10111 in decimal = $1\times2^4+0\times2^3+1\times2^2+1\times2^1+1\times2^0=16+0+4+2+1=23$

10000 in decimal = $1\times2^4=16$

So, $23-16=7$, and the binary result should be 00111.

Q 2) If log (325.6) = 2.5127, then log (0.03256) is

(a) $\bar{2}.5127$ (b) – 2.5127   (c) -1.5127   (d) 1.5127

Answer (a) $\bar{2}.5127$

Solution: log (325.6) = 2.5127

        Means log (3.256) = 0.5127

Then log (0.03256) = log 3.256 - 2log10 = 0.5127 $-\bar{2}$ = $\bar{2}.5127$

Q 3) The decimal equivalent of the binary number 11001 is

(a) 26   (b) 25   (c) 28   (d) 30

Answer (b) 25

Solution: 11001 = $1\times2^4+1\times2^3+0\times2^2+0\times2^1+1\times2^0=16+8+0+0+1=25$

Q 4) The Value of $(256)^{0.16} \times (256)^{0.09}$

(a) 3   (b) 4   (c) 5   (d) 6

Answer (b) 4

Solution: $(256)^{0.16+0.09} = (256)^{0.25} = (4)^{4 \times \frac{1}{4}} = 4$

Q 5) The Maximum gap between two successive leap years is.

(a) 2   (b) 4   (c) 8   (d) 12

Answer (c) 8

Solution:

Leap years normally occur every 4 years, but there are exceptions based on the rules of the Gregorian calendar:

Leap Year Rules: A year is a leap year if

         It is divisible by 4, and

         Not divisible by 100, unless it is also divisible by 400

Example of an 8-year gap:

Between 1896 and 1904:

     1900 is not a leap year because it's divisible by 100 but not by 400.

     So, leap years go:

        1896 Leap Year

        1900 not a leap year

        1904 Leap Year

Gap = 1904 - 1896 = 8 years

Q 6) Which Two Months of Year have the Same Calendar?

(a) April, July   (b) oct, nov   (c) jan, March   (d) July, oct

Answer (a) April, July

Solution: Days between 1st April and 30th June = 91 Which is divisible by 7

Q 7) Characteristic of Log (0.0003798) is

(a) 3   (b) 3   (c) 4   (d) 4

Answer (d)

Solution: Log (0.0003798) = log (3.798 x 10^{-4}) = log 3.798 + log 10^{-4} = log (3.798) – 4

So Characteristic of Log (0.0003798) is - 4

Q 8) If log (3x+1) =2 then the value of x is

(a) 1/3   (b) 99   (c) 33   (d) 12

Answer (c)

Solution: $\log(3x+1) = 2 \Rightarrow 3x+1 = 10^2$

$3x+1 = 100$ i. e $x = 99$

Q 9) How many odd days are there in 300 years?

(a) 0   (b) 1   (c) 2   (d) 3

Answer (b) 1

Solution: Count leap years in 300 years
In 300 years:

Leap years = $\lfloor 300/4 \rfloor - \lfloor 300/100 \rfloor = 75 - 3 = 72$

72 Leap years and 228 normal years (300 - 72)

Calculate total odd days

From 228 normal years:
$228 \times 1 = 228$ odd days

From 72 leap years:
$72 \times 2 = 144$ odd days

Total odd days = $228 + 144 = 372$

Now divide by 7 to find the remainder (for 7 days = 0 odd days):

$372 \div 7 = 53$ weeks + 1 odd day

1 odd day in 300 years

Q 10) How many odd days are there in a century?

(a) 2   (b) 3   (c) 4   (d) 5

Answer (D) 5

Solution: Count leap years in 100 years
In 100 years:

Leap years = $\lfloor 100/4 \rfloor - \lfloor 100/100 \rfloor = 25 - 1 + 0 = 24$

Normal years = $100 - 24 = 76$

Count odd days

Each normal year gives 1 odd day $\rightarrow 76 \times 1 = 76$ odd days

Each leap year gives 2 odd days $\rightarrow 24 \times 2 = 48$ odd days

Total odd days = $76 + 48 = 124$

Now divide by 7 to find the remainder (for 7 days = 0 odd days):

$124 \div 7 = 17$ weeks and 5 odd days

Hence 5 odd days in 100 years

Q 11) The total number of days from 26th January 2008 to 15th May 2008 is

(a) 109   (b) 110   (c) 111   (d) 108

Answer (c) 111

Solution: January=6 days, February=29 days, March=31 days, April=30 days May=15 days
Total=6+29+31+30+15=111 days

Q 12) It was Thursday on Feb 12, 2004. What was the day of the week on Feb 12, 2003?

(a) Friday   (b) Wednesday   (c) Saturday   (d) Tuesday

Answer (d) Tuesday

Solution: 2004 is a leap year (divisible by 4 and not by 100) So, 2003 b/w 2004 includes Feb 29, meaning the year has 366 days

Days between Feb 12, 2003 to Feb 12, 2004 is exactly 1 year, and since 2004 is a leap year, that year has 366 days=52 weeks +2 odd days

So, the day moves back by 2 days when going backward in time.

Count backward from Thursday

Feb 12, 2004 = Thursday

Going back 1 day → Wednesday

Going back 2 days → Tuesday

Feb 12, 2003 was a Tuesday.

Q 13) The angle between two hands of a clock at 8:30 a.m. is

(a) 75°   (b) 85°   (c) 45°   (d) 65°

Answer (a)

Solution: Using Formula: Angle=| 30H−5.5M |

Where H = hour, M = minutes, the hour hand moves 0.5^0 per minute (or 30° per hour)

The minute hand moves 6^0 per minute

Put the values at 8:30:

H=8 M=30, Answer =75^0

Q 14) In a day, how many times is a right angle formed between minute hand and hour hand?

(a) 11   (b) 22   (c) 33   (d) 44

Answer (d) 44

Solution: In 12 hours, the hour and minute hands form a right angle (90°) exactly 22 times.
So, in 24 hours = 22 x 2 = 44 times

Q 15) A works twice as fast as B. If B can complete a work in 12 days independently, then number

of days in which A and B together can finish the work is

(a) 18 days   (b) 4 days   (c) 8 days   (d) 6 days

Answer (b) 4 days

Solution: B's one day work = 1 / 12

A works twice as fast, so A's one day work = 2×1 / 12=1 / 6

A + B's one day work = 1 / 12 + 1 / 6 = 1 / 4

So, work will Complete in 4 days

Q 16) A and B together can do a piece of work in 10 days. C can do the same work alone in 15

days. If A, B and C work together, then number of days to finish the work is

(a) 6 days   (b) 5 days   (c) 4 days   (d) 8 days

Answer (a) 6 days

Solution: A + B's one day work = 1 / 10

C's one day work = 1 / 15

(A + B + C)'s one day work =1 / 10 +1 / 15 = 1 / 6

So, work will Complete in 6 days

ASSERTION – REASON QUESTIONS (SOLVED)

Two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below: -

    a. Both A and R are true and R is the correct explanation of the assertion
    b. Both A and R are true but R is not the correct explanation of the assertion
    c. A is true, but R is false
    d. A is false, but R is true

Q 1) Assertion (A): Binary number 1101101 can be written as 109 in decimal system.

Reason (R): $(1101101)_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$.

Answer (a)

Solution: $(1101101)_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$. = 64+32+0+8+4+0+1 = 109.

Assertion (A) is true. Also, Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

Q 2) Assertion (A): Sum of binary numbers 1101 and 1010 is 10111.

Reason (R): Rules for binary addition are 0+0=0; 0+1=1; 1+0=1; 1 + 1 = 2.

Solution:

	1	1	0	1
+	1	0	1	0
1	0	1	1	1

Assertion (A) is true. Reason (R) is false. 1+1=0 with 1 carry over to next left column.
Option (c) is the correct answer.

Q 3) Assertion (A): If $2700 = 2^P \times 3^q \times 5^r$, then p = 2, q = 3, r = 2.

Reason (R): If p and q are different positive prime numbers,

then $p^m . q^n = p^l . q^k \Rightarrow m = 1, n = k$.

Solution: Given, If $2700 = 2^P \times 3^q \times 5^r$
$2700 = 2^2 \times 3^3 \times 5^2$ therefore p=2, q=3, r = 2
Assertion (A) is true. Also, Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).
Option (a) is the correct answer.

Q 4) Assertion (A): If $\log_{(2x-1)} 343 = 3$, then x = 4.

Reason (R): If $\log_a b = x$, then $a^x = b$.

Solution: We know that, $\log_a b = x = a^x = b$
Reason (R) is true.
So, $(2x-1)^3 = 343$, $(2x-1)^3 = 7^3$,   2x - 1 =7,   x=4
Assertion (A) is true and Reason (R) is the correct explanation of Assertion
Option (a) is the correct answer.

Q 5) Assertion (A): The 1500 was not a leap Year.

Reason (R): Any Year which is divisible by 4 is a leap Year.

Solution: Since 1500 is century Year and it is not divisible by 400, So it is not Leap year

Assertion (A) is true. Reason (R) is false.

Option (c) is the correct answer.

Q 6) Assertion (A): Angle between two hands of a clock at 4:50 is 135°.

Reason (R): Angle between hands of clock at H hours M minutes: Angle=| 30H−5.5M |

Solution: Angle between two hands of clock at H hours M minutes is | 30H−5.5M |

Reason (R) is true on Calculating We got A = 155°

Assertion (A) is false.

Option (d) is the correct answer.

Q 7) Assertion (A): Sumit is 20% more efficient than Arif. If Sumit can do a piece of work in 25 days, then Arif can do the same work in 30 days.

Reason (R): Time required to do a certain work = work to be done / one day's work

Solution: Given Sumit can do a piece of work in 25 days

So, Arif can do the same work in (25 + 25 x 20 /100) days = 30 days

So, assertion A is true But Reason R is not Correct explanation of Assertion A

Option (b) is the correct answer.

Q 8) Assertion (A): If walking at 5/6 of usual speed Rishabh is 10 minutes late, then the usual time to cover the distance is 50 minutes.

Reason (R): When distance is constant, then speed inversely proportion to time i.e. $S_1*T_1= S_2*T_2$

Solution: Let the usual speed of Rishabh be x km/h and time taken to cover the distance be t minutes, then 5/6 x*(t + 10) = x*t

5t+50=6t,   t = 50 minutes.

Assertion (A) is true. Also, Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

Option (a) is the correct answer.

Q 9) Assertion (A): If A can do a piece of work in 30 days and B can do the same work in 20 days, then they will finish the work together in 12 days.

Reason (R): If a person can do a piece of work in x days and another person can do the same work in y days, then they together can finish the work in (xy) / (x +y) days.

Solution: A 's one day work = 1 / 30

B's one day work = 1 / 20

A + B's one day work = 1 / 30 + 1 / 20 = 1 / 12

Assertion (A) is true. Also, Reason (R) is true and Reason (R) is the correct explanation of Assertion (A).

Option (a) is the correct answer

Q 10) Assertion (A): $\log_a(b/c) = \log_a b − \log_a c$

Reason (R): $\log_a(b \times c) = \log_a b + \log_a c$

Solution: Both are true but Reason is not the correct explanation for the Assertion.

Option (b) is the correct answer.

Q 11) Assertion (A): log (1 + 2 + 3) = log 1 + log 2 + log 3

Reason (R): $\log_a (a \times b \times c) = \log_a a + \log_a b + \log_a c$

Solution: Both are true but Reason is not the correct explanation for the Assertion.
Option (b) is the correct answer.

VERY SHORT ANSWER TYPE QUESTIONS

Q 1) Evaluate: $\log_3(27) + \log_5(1/25)$

Solution: $\log_3(27) = \log_3(3^3) = 3\log_3(3) = 3 \times 1 = 3$

$\log_5(1/25) = \log_5(5^{-2}) = -2\log_5(5) = -2 \times 1 = -2$

Therefore, $\log_3(27) + \log_5(1/25) = 3 + (-2) = 1$

Q 2) Evaluate: $\log_2(32) - \log_3(1/81)$

Solution: $\log_2(32) = 5$, $\log_3(1/81) = -4$

so, result = $5 - (-4) = 9$.

Q 3) Convert the decimal number 156 to binary.

Solution:

$156 \div 2 = 78$ remainder 0

$78 \div 2 = 39$ remainder 0

$39 \div 2 = 19$ remainder 1

$19 \div 2 = 9$ remainder 1

$9 \div 2 = 4$ remainder 1

$4 \div 2 = 2$ remainder 0

$2 \div 2 = 1$ remainder 0

$1 \div 2 = 0$ remainder 1   Reading remainders from bottom to top: 10011100.

Q 4) Convert the binary number 1101101 to decimal.

Solution: $1101101 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 64 + 32 + 0 + 8 + 4 + 0 + 1 = 109$.

Q 5) Add the binary numbers 10101 and 1110.

Solution:

```
  10101
+ 01110
= 100011
```

Therefore, the sum is 100011 in binary.

Q 6) Subtract the binary number 1011 from 11001.

Solution:

```
  11001
− 01011
= 01110
```

Therefore, the result is 1110 in binary.

Q 7) Multiply binary numbers 110 and 101.

Solution:

```
      110
×     101
_____

      110   (110 × 1)
+     000   (110 × 0)
+ 110       (110 × 1, shifted two places)
```

$$\overline{}$$

100110

Therefore, product = 100110 in binary.

Q 8) Divide binary number 101010 by 10.

Solution: Convert: 101010 (binary) = 42 (decimal)

10 (binary) = 2 (decimal)

$42 \div 2 = 21$

21 in binary = 10101.

Q 9) Find the angle between the hour and the minute hands at 7:20.

Solution: Minute hand = $20 \times 6 = 120°$

Hour hand = $7 \times 30 + (20/60) \times 30 = 210 + 10 = 220°$

Angle = $|220 - 120| = 100°$

Q 10) If 30th March 2020 was Monday, what would be the day after 61 days?

Solution: Given that on 30th March 2020, it was Monday.

Number of days = 61 = 7 x 8+5

Number of odd days = 5.

Since 30th March was Monday and there are 5 odd days in next 61 days i.e. Tuesday, Wednesday, Thursday, Friday and Saturday.

The day after 61 days of 30th March 2020 is Saturday.

Q 11) If the second day of April month is a Friday, then find the last day of the next month.

Solution: Given 2nd April is Friday.

From 3rd April to last day of the next month.

Number of days (April and May) = 28 + 31 = 59

So, the number of odd days = 7 x 8 + 3 i.e. 3 days.

Since 2nd April is Friday and there are 3 odd days up to last day of next month i.e. Saturday, Sunday, Monday. The last day of the next month is Monday.

Q 12) A man covers a distance at 4 km/h in 3 hour 30 minutes. How much time will he take to cover the same distance at 21 km/h?

Solution: Given time = 3 h 30 min = 7/ 2 hour and speed = 4 km/h

Distance = speed x time = 7/2 x 4 = 14 km

Now the time taken to cover 14 km at 21 km/h = 14 /21 hour = 40 minutes

Q 13) Find the angle between two hands of the clock at 7: 20.

Solution: Here, H = 7, M = 20.

So, angle between two hands of the clock at 7: 20 A = (30×7-×20) = (210-110) ° = 100°.

Q 14) Baban and Ramsukh together erect a shed in 12 days. Baban alone can do it in 20 days. much time would Ramsukh take working alone to erect the shed?

Solution: Baban and Ramsukh together take 12 days to erect a shed

one day's work of Baban and Ramsukh together = 1/ 12

Now Baban alone can erect the shed in 20 days,

Baban's one day work = 1/ 20 ...

Ramsukh's one day work = 1/15 – 1/ 20 = 1 / 30

Ramsukh alone can erect the shed in 30 days.

Q 15) A train 120 m long is running at 108 km/h. How long will it take to cross a lamp

Solution: Distance to be covered = 120 m, Speed of train = 108 km/h = 108 x (5 /18) = 30 m/s

Time Required = 120 / 30 Sec = 4 Seconds

Q 17) What day of the week will 15th August 2025 fall on, if 15th August 2024 was a Thursday?

Solution: 2024 is a leap year, so it has 366 days.

Days passed = 366, Remainder when divided by 7 = 2

So, 15th August 2025 = Thursday + 2 days = Saturday

Q 18) $\log_{10}(0.001 \times (\sqrt{3} / 100))$ in terms of $\log_{10}(10)$

Solution: $\log_{10}(10^{(-3)} \times 10^{(2/3)}) = \log_{10}(10^{(-7/3)}) = -7/3$.

Q 19) Convert binary 101.101 to decimal.

Solution:

$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 4 + 0 + 1 + 0.5 + 0 + 0.125 = 5.625$.

SHORT ANSWER TYPE QUESTIONS

Q 1) **Convert decimal number 23.625 to binary.**

Solution:

Integer part: $23 \rightarrow 10111$

Fractional part: $0.625 \times 2 = 1.25 \rightarrow 1$

$0.25 \times 2 = 0.5 \rightarrow 0$

$0.5 \times 2 = 1.0 \rightarrow 1$

So, binary = 10111.101

Q 2) Solve the following equation for x: -

$$(\sqrt[3]{4})^{2x+\frac{1}{2}} = \frac{1}{32}$$

Solution: $2^{3/2(2x+1/2)} = 2^{-5}$

on Comparing

$3 / 2 (2x + 1/ 2) = -5$

$x = -4$

Q 3) A and B together can build a wall in 30 days. If A is twice as good as a workman as B, In how many days will A alone finish the work?

Solution: A one-day work = B 2 days' work

A one-day work + B one day work = 1 / 30

A one-day work + A ½ day work = 1 /30

A 1+ ½ day work = 1 /30

A one-day work = 1 / 45

A can finish work = 45 days

Q 4) If 15 February 2010 was Monday. What was the day on 10 January 2006?

Solution: Total Odd days up to 14 Feb 2010 = 1 +1+2+1 +1=6

15 Feb was Monday So Count 6 Days Backwards

So, Answer is Tuesday

Q 5) A and B together can dig a pond in 20 days. They worked together for 8 days and then B leaves the work. How long will A take to finish the work if A alone can dig the pond in 30 days?

Solution: Since A and B together can dig the pond in 20 days,

one day's work of A and B together = 1 / 20

8 day's work of A and B together = 8 x 1 / 20 = 2 / 5

Remaining work = 1 – 2 / 5 = 3 / 5

Since A alone can dig the pond in 30 days,

A's one day work = 1 / 30

The number of days taken by A to complete the remaining work

work to be done / A's one day work = (3/ 5) / (1 / 30) = 18

A will finish the remaining work in 18 days

Q 6) If 2 men or 3 boys take 40 hours to do a certain piece of work, how long will 4 men and 9 boys working together take to complete the work?

Solution: Since 2 men's work = 3 boy's work,

1 man's work boys' work = 3 / 2 boy's work

4 men's work= 3/ 2 x 4 = 6 boys' work

4 men and 9 boys work = 6+9 i.e. 15 boys' work

Since 3 boys can do the work in 40 hours,

1 boy can do the work in 3 x 40 i.e. 120 hours

15 boys can do the work in 120 15 hours = 8 hours.

Hence, 4 men and 9 boys working together will complete the work in 8 hours.

Q 7) If $2^x = 3^y = 6^{-z}$, Prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$.

Solution: Let $2^x = 3^y = 6^{-z} = k$

Then $2 = k^{1/x}$, $3 = k^{1/y}$ $6 = k^{-1/z}$

Now we take $6 = k^{-1/z}$

$2 \ X \ 3 = k^{-1/z}$

$k^{1/x} \ k^{1/y} = k^{-1/z}$

$k^{\frac{1}{x}+\frac{1}{y}} = k^{-1/z}$

On Comparing we got $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$.

Q 8) If log 7-log 2+ log 16-2 log 3-log 7/45 = 1 + log n, find n.

Solution: Given log 7 - log 2 + log 16-2 log 3-log (7 / 45) = 1 + log n

7 log 7-log 2+ log 16-log (3)^2-log (7 / 45) = log 10+ log n

log (7 x 16) / (2x3^{2}x 7 / 45) = log (10 x n)

log (7x16 x 45) / (2 x 9 x 7) = log 10n

log 40 = log 10n

10n = 40 ⇒ n = 4.

Q 9) At what time between 2 and 3 o'clock are the hands of the clock together?

Solution: Use the formula:

Time=60H / 11

Where H = the hour after which hands coincide

Here, H = 2:

Time=60×2 / 11=12011≈10.91 minutes

So, hands will be together at 2:10:55 (approx.)

Q 10) At what time do the hands of the clock meet between 6:00 to 7:00?

Solution: At 6 o'clock, the hour hand is at 6 and the minute hand is at 12. It means the angle between two hands of clock is 180°.

To catch the hour hand, the minute hand has to cover a relative distance of 180° at a relative speed of 5.5° per minute.

The time required to coincide the hands = = 180 / 5.5 minutes = 32 minutes 44 sec.

Hence, the hands of the clock coincide at 6:32:44.

Q 11) A clock loses 5 minutes every hour and was set right at 10:00 a.m. on a Sunday. When will it show the correct time again?

Solution: For clock to show the correct time again, it should lose 12 hours.

Given that, the clock loses 5 minutes in 1 hour

it loses 60 minutes i.e. 1 hour in 12 hours

it loses 12 hours in 12 x 12 = 144 hours

it loses 12 hours in 144 / 24 = 6 days.

Hence, it will show the correct time again at 10:00a.m. on Saturday.

LONG ANSWER TYPE QUESTIONS

Q 1) Evaluate using Log tables:-

$$\frac{(27.8)^{\frac{1}{2}} \times (58.49)^{\frac{2}{3}}}{(0.0007)^{\frac{1}{4}}}$$

Solution: Let x = $\frac{(27.8)^{1/2} \times (58.49)^{2/3}}{(0.0007)^{1/4}}$

Log x = $\frac{1}{2}$ log 27.8 + $\frac{2}{3}$ log 58.49 – ¼ log 0.0007

Log x = $\frac{1}{2}$ log (2.78 x 10) + $\frac{2}{3}$ log (5.849 x 10) – ¼ log (7.00 x 10⁻⁴)

$\quad$ = $\frac{1}{2}$ (0.4440 + 1) + $\frac{2}{3}$ (0.7676+1) – ¼ (0.8451 - 4)

$\quad$ = $\frac{1}{2}$ (1.4440) + $\frac{2}{3}$ (1.7676) – ¼ (-3.1549)

$\quad$ = 0.7220 + 1.1784 – (−0.7887)

$\quad$ = 2.6891

$\quad\quad$ Antilog (log x) = Antilog (2.6891) since antilog (0.6891) = 4.887

$\quad\quad\quad$ x $\quad$ = 4.887×10²

$\quad\quad\quad$ x $\quad$ = 488.7

Q 2) X can do a piece of work in 60 days, where Y can do the same work in 40 days Both work together, X left 10 days before the completion of work. Find how many days will it take to complete to work?

Solution: X and Y one day work working together = 1/60 + 1 / 40 = 1 /24

Let x days to Complete the Remaining work

Then X and Y together worked for x-10 days and Y worked alone for 10 days

X and Y' s (x-10) days' work = (x-10) /24

Then

(x-10) /24 + ¼ = 1

X = 28

Q 3) 100 persons begin to work together on a project which was expected to be completed in 40 days. But after few days 40 persons left. As a result, the project got delay by 10 days. How many days after the commencement of the project did the 40 persons left?

Solution: Let 40 persons leave after x days of the commencement of the project.

Given 100 persons can do the project in 40 days.

So, 100 person's one day's work = 1 / 40

100 person's x day's work = x / 40

Remaining work = 1- x / 40 = (40-x) / 40

After leaving 40 persons, we are left with 60 persons. These 60 persons finished the work in (40-x+10) days i.e. 50-x days.

60 person's one day's work = 60 / (40x100) = 3 / 200

$\quad$ 60 person's (50-x) day's work = 3(50-x) / 200

3(50-x) / 200 = (40-x) /40

5(40-x) = 3(50 – x)

X=40

40 persons left after 25 days of the commencement of the project.

Q 4) If 4th October 1986 was Saturday, what would be the day on 10th April 1991?

$\quad$ **Solution:** Number of odd days:

$\quad$ From 5th October 1986 to 4th October 1987 = 1

$\quad$ From 5th October 1987 to 4th October 1988 = 2 (1988 is a leap year)

$\quad$ From 5th October 1988 to 4 October 1989-1

$\quad$ From 5th October 1989 to 4 October 1990 = 1

$\quad$ Now from 5th October 1990 to 10th April 1991

Number of days = 27+ 30 + 31 + 31 + 28 + 31 + 10 = 188 days
              Oct   Nov   Dec   Jan   Feb March April
              1990   1990 1990 1991   1991 1991 1991
So, number of odd days = 7 x 26+6 i.e. 6 odd days
Total number of odd days = 1+2+1+1+6=11
11=7x1+44 odd days.
Since 4th October 1986 was Saturday and there are 4 odd days up to 10th April 1991 i.e.
Sunday, Monday, Tuesday, Wednesday
So, April 1991 Wednesday.

CASE- STUDY BASED QUESTIONS

Q 1) Read the following information carefully and answers the questions given below it
Nine persons A, B, C. D, E. F. G, H and I are watching a football match in a stadium, silting
in an Ei at one end of the row. It is seated adjacent to F and 1. C is to the immediate right of
D and at fourth place to right of A. H is immediate left of G. G is at sixth place to the left of
E. F is immediate right of C
(i) Write the seating arrangement according to the given conditions
(ii) Who is sitting at the centre of the row?
(iii) Who is at another end of the row?
                                OR
What is the position of I w.r.t. G.
Solution: (i) It is given that E is one end of the row.
So, we have two cases   ----------E or E ---------
It is given that G is at sixth place to the left of E
So, the second arrangement will be rejected because there is no left place to E
Thus, we have _ _G_ _ _ _ _E
Again, it is given that H is immediate left of G
So, we have   _ H G_ _ _ _ _E
Now, it is given that C is to the immediate right of D and at fourth place to the right of A.
So, these can be arranged like A _ _ D C
It is given that F is immediate right of C. So, A _ _ D C F
It is given that B is seated adjacent to F and 1.
So, we have A _ _ D C F B I
On combining (1) and (2), we have the final arrangement   A H G D C F B I E
(ii) C is sitting at the centre of the row
(iii) A is at the other end of the row
          OR
 I is the fifth place to the right of G

Q 2) Rohit is organizing a birthday party and invites 8 of his friends. They are seated around a
circular table. The host, Rohit, sits at the North position. His friends: A, B, C, D, E, F, G,
and H take the remaining seats. The following information is known:
- B is sitting third to the right of A.
- D is sitting opposite to B.
- C is sitting to the immediate left of E.
- G is sitting second to the left of D.
- F is sitting between A and H.
Based on above information answer the following
1. Who is sitting to the immediate right of A?
2. Who is sitting opposite to E?

3. What is the position of H with respect to C?

<div align="center">OR</div>

If G changes seats with E, who will be sitting to the right of G now?

Solution: 1. The immediate right of A is B, based on the given third right rule and the circular seating.

2. The opposite of E is C, considering relative positions and circular arrangement.

3. H is to the right of C

<div align="center">OR</div>

After G and E switch, C will be to the right of G.

Q 3) A student checks that January 1, 2022, was a Saturday. They are asked to calculate the day of the week for various other dates using the concept of odd days and leap years. Based on above information Answer the following: -

1. What day of the week was January 1, 2023?

2. How many odd days are there in 100 years?

3. How many leap years are there between 1901 and 2000?

OR

What day of the week will it be on 1st January 2100?

Solution:

1. 2022 is not a leap year, so Jan 1, 2023 will be one day ahead → Sunday.

2. In 100 years: 76 normal + 24 leap years → 76*1 + 24*2 = 124 odd days → 124 mod 7 = 5 odd days.

3. Leap years = (2000 - 1901 + 1) // 4 = 24 (excluding 1900 as it is not a leap year).

OR

From 2001 to 2100 → 76 normal + 24 years = 124 odd days → 5 days ahead → Saturday + 5 = Thursday.

<div align="center">

HIGHER ORDER THINKING SKILLS

</div>

Q1) A clock gains 5 seconds in 2 minutes and was set right at 9:00 am. If it shows 2: 30 in the afternoon on the same day. What is the correct time?

Solution: Given that, the clock gains 5 seconds in 2 minutes

it gains 12 x 5 = 60 seconds i.e. 1 minute in 12 x 2 = 24 minutes

when incorrect clock moves 25 minutes, correct clock moves 24 minutes.

Now,

from 9:00 a.m. to 2: 30 p.m. on the same day the time passed by incorrect clock = 5 hours 30 minutes = 330 minutes.

When incorrect clock moves 330 minutes, correct clock moves = (24 x 300) /25 = 316 minutes 48 seconds = 5 hours 16 minutes 48 seconds

Hence, the correct time is 2: 16: 48 p.m.

Q 2) The signal poles on a railroad are placed 100 m a part. How many poles will be passed by a tram in 8 hours if the speed of the train 45 km / hr.

(2) if 7201 poles are to be installed within two stations at equal distance covering distance of 360 km. Find out the distance between two consecutive poles.

Solution: (1) Given speed of the train 43 km/h and time taken 8 hours

So, distance covered = speed X time = (360x1000) m = 360000 m

So, number of poles = 360000/ 100 +1 = 3601

Since, there are 4+ 1 poles in a distance of 400 m

Se 3601 poles will he passed by the train in 8 hours

(2) Distance = 360 km = 360 X 1000 = 360000 m

Let the distance between two consecutive poles be x m, then

7201 = 360000 / x + 1

7200 = 360000 / x

X= 50 m

Hence the distance between two consecutive poles is 50 m

EXCERCISE

MULTIPLE CHOICE QUESTIONS

Q 1) The decimal equivalent of the binary number 10101 is

    (a) 21   (b) 12   (c) 22   (d) 31

Q 2) If log 0.0007392 = -3.1313, then log 73.92 is

    (a) 1.1313   (b) 1.8687   (c) 2.1313   (d) 2.8687

Q 3) The product $\sqrt[3]{2}\ \sqrt[4]{2}\ \sqrt[12]{32}$ is

    (a) $\sqrt{2}$ (b) 2   (c) $\sqrt[12]{2}$ (d) $\sqrt[12]{32}$

Q 4) Tuesday fell on which of the following dates of June 2002?

    (a) 3, 10, 17, 24   (b) 4, 11, 18, 25   (c) 2, 9, 16, 23, 30   (d) 1, 8, 15, 22, 29

Q 5) 22. If it was Friday on 4th May 1956, then what was the day on 6 July 1957?

    (a) Wednesday   (b) Monday   (c) Saturday   (d) Tuesday

Q 6) At what time between 7 0'clock and 8 0'clock will the hands of a clock make an angle of 10°?

    (a) 07:35   (b) 07:40   (c) 07:45   (c) 07:45

Q 7) In a day, how many times is a straight angle formed between minute hand and hour hand?

    (a) 11   (b) 22   (c) 33   (d) 44

Q 8) A car travels for 2 hours at a speed of 40 km/h and then travels at 50 km/h What is the average speed of car?

    (a) 45 km/h   (b) 44.44 km/h   (c) 48 km/h   (d) 47 km/h

Q 9) Anil is twice efficient as Komal. They together can finish a piece of work in 50 days. In how many day Komal can finish the same work alone?

    (a) 75 days   (b) 110 days   (c) 140 days   (d) 150 days

ASSERTION – REASON BASED QUESTIONS

Two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below: -

    a. Both A and R are true and R is the correct explanation of the assertion
    b. Both A and R are true but R is not the correct explanation of the assertion
    c. A is true, but R is false
    d. A is false, but R is true

Q 1) Assertion (A): A: The binary equivalent of the decimal number 10 is 1010.

Reason (R): R: In binary, each digit represents a power of 3.

Q 2) Assertion (A): A: log (1) = 0 in all bases.

Reason (R): R: The logarithm of any number to the base 1 is always 1.

Q 3) Assertion (A): A: The hands of a clock coincide every 65 minutes.

Reason (R): R: The hands coincide 11 times in 12 hours.

Q 4) Assertion (A): A: January 1, 2000 was a Monday.

Reason (R): R: The calendar follows a fixed pattern every 400 years.

Q 5) Assertion (A): A: If A and B together can complete a work in 6 days, and A alone can do it in 10 days, then B can do it alone in 15 days.

Reason (R): R: The combined work rate of A and B is the sum of their individual rates.

VERY SHORT ANSWER TYPE QUESTIONS

Q 1) Multiply the Binary Numbers: 101011 by 1101

Q 2) Convert 569 into decimal number.

Q 3) simplify: $16^{\frac{3}{4}} + 3^{-4} \ 3^6 + \left(\frac{1}{3}\right)^{-2}$.

Q 4) Solve for x such that $\log_2 (\log_3 x) = 4$.

Q 5) Thursday fell on which dates of April 2007.

Q 6) At what time 4:00 and 5:00 will the hands of clock be at right angles?

SHORT ANSWER TYPE QUESTIONS

Q 1) simplify

$$\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times \left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div \left(\frac{5}{2}\right)^{-3} \right]$$

Q 2) If $3 \log \sqrt{m} + 2 \log \sqrt[3]{n} = 1$, find the value of $m^9 n^4$

Q 3) A watch gains 5 seconds in 3 minutes and was set right at 7 a.m. If it shows 4: 15 in the afternoon on the same day. What is the correct time?

Q 4) A can do a work in 6 days and B can do it in 8 days. They worked together for 2 days and then B left the work. How many days will A require to finish the work?

Q 5) Convert the binary number 1101.001 into its decimal (base-10) equivalent.

LONG ANSWER TYPE QUESTIONS

Q 1) 100 persons begin to work together on a project which was expected to be completed in 40 days. But after few days 40 persons left. As a result, the project got delay by 10 days. How many days after the commencement of the project did the 40 persons left?

Q 2) Evaluate using Log tables: -

$$\frac{(2.75)^{\frac{3}{2}} \times (0.567)^{\frac{1}{3}}}{(745.23)^{\frac{1}{4}}}$$

Q 3) If 4th October 1985 was Friday, what would be the day on 13th June 1997?

CASE- STUDY BASED QUESTIONS

Q 1) A and B together can do a piece of work in 24 days, B and C together can do the same piece of work in 40 days. C and A together can do it in 30 days. Based on the above information, answer the following questions:

(a) Find the number of days in which A, B and C working together can finish the work.

(b) Find the number of days in which A alone can finish the work.

(c) Find the number of days in which B and C alone can finish the work.

If A, B and C work together and earn ₹48000, then find A's share, B's share and C's share.

Q 2) Read the following information carefully and answer the questions given below it: -

Eleven friends A, B, C, D, E, F, G, H, I, J and K are watching a movie in a cinema hall sitting in a row. H is immediate left of D and third to the right of I. J is the immediate neighbor of A and B and third to the left of G. A is the second to the right of E who is at one end of the row. F is sitting next to the right of D and D is second to the right of C.

(a) Write the seating arrangement.

(b) Who is sitting at the center of row?

(c) Who are the neighbors of H?

OR

How many persons are sitting between E and F?

HIGHER ORDER THINKING SKILLS

Q 1) A clock losses 5 seconds in 3 minutes and was set right at 6:00 pm. If it shows 3: 30 in the afternoon on the same day. What is the correct time?

Q 2) A person was born on 29th February 2000. How many birthdays will he celebrate on his actual birth date till 2100?

Q 3) How many times between 1st January 2000 and 31st December 2020 did 1st January fall on a Sunday?

ANSWERS

MULTIPLE CHOICE QUESTIONS

Q1 (a) Q2 (b) Q3 (b) Q4 (c) Q5 (c) Q6 (b) Q7 (b) Q8 (a) Q9 (d)

ASSERTION – REASON TYPE QUESTIONS

Q1 (c)   Q2 (d)   Q3 (d)   Q4 (c)   Q5 (a)

VERY SHORT ANSWER TYPE QUESTIONS

Q1) $(10000101111)_2$ Q2) $(1000111001)_2$ Q3) 19   Q4) 3^{16}

Q5) 5, 12, 19, 26   Q6) 4:05:27 and 4: 38: 11

SHORT ANSWER TYPE QUESTIONS

Q1)   1   Q2) 10^6 Q3) 4:00 PM   Q4) 2 and half days   Q5) 13.125

LONG ANSWER TYPE QUESTIONS

Q1)   25 days   Q2) 0.138   Q3) Friday

CASE- STUDY BASED QUESTIONS

Q 1) (a) 20 days (b) 40 days (c) 60 days and 120 days OR Rs 24000, Rs 16000, Rs 8000

Q 2) (a) EKAJBIGCHDF (b) I (c) C and D OR 9

HIGHER ORDER THINKING SKILLS

Q 1) 04:07 PM   Q 2) 24 times Q 3) 3 times

UNIT-2 : ALGEBRA
SETS AND RELATIONS

SETS

The concepts of set, Representations of set, Various type of sets, Subsets of real number, intervals, Venn diagrams, Operation on sets, Union of sets, Intersection of sets, Difference of sets, complement of sets and De Morgan's Laws

DEFINITION AND FORMULAS

Set: Set is the well-defined collection of the objects.

Well Define means if asked a question to the person and reply are same.

Representation of sets: Sets can represent in set builder form and roster/tabular form.

Set Builder form: A = {x: x is a natural number and x=2n, nϵZ}

Roster/Tabular form: B = {a, e, i, o, u}

Types of Sets:

Finite Set: Number of elements are countable.

Example A= {2,4,6,8} there is 4 elements in this set.

Infinite Set: Number of elements are uncountable.

Example B = {1,2,3,4,5...} or B= {x: x is collection of all natural numbers.}

Empty Set or Null Set: Number of elements is Zero or No element in Set.

Example: A = {   }, B = {x: x is even composite number.}

Singleton Set: Number of elements in a Set only one.

Example A= {3}, B= {x: x is even prime number}

<center>Equal Set and Equivalence Set</center>

Equal Set: If the number of elements and elements in two or more sets are the same.

Example: A= {1,3,5,7,9}, B= {x: x is odd number less than 10}

Equivalent Set: In two or more set, if number of elements are same, element may be same or not.

Example: $A = \{a, b, c, d\}, B = \{1,3,7,9\}$

Subset: A set A is said to be a subset of a set B if every element of A is also the element of B, then Set A is a subset of Set B.

$A = \{ 2,5,9\}\ and\ B = \{1,2,4,5,9\}\ here\ A{\subset}B$

Number of subset if n elements in a set is 2^n and number of proper subsets of a set is 2^n -1.

Let $A = \{3,10,25\}$ then number of subsets of set A is 2^3 = 8.

Venn diagrams: Venn diagrams as the pictorial representation of relationship between sets.

Operations on sets: Union of sets, Intersection of sets, Difference of sets, Complement of a set, De Morgan's Laws

Intervals: Open interval, closed interval, semi open interval and semi closed interval

Power of set: Collection of all subsets of a set.

Define Interval

(a, b) Means all real values between a and b.

[a, b] Means all real values between a and b including a and b.

{a, b} Means value of a and b only.

Formulas: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$n(A-B) = n(A) - n(A \cap B)$

$n(B-A) = n(B) - n(A \cap B)$.

DE Morgan's Law : $(A \cup B)' = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$

RELATIONS

Ordered pairs: Explain the significance of specific arrangement of elements in a pair
Cartesian product of two sets
Find the number of elements in a Cartesian product of two sets
Ordered pair, order of elements in an ordered pair and equality of ordered pairs
Cartesian product of two non- empty sets
Relations Express relation as a subset of Cartesian product
Find domain and range of a relation
Definition of Relation, examples pertaining to relations in the real number system
Relations
Definition and Formulae

Ordered Pair: An ordered pair is a pair of objects taken is a specific order and the order in which they appear in the pair is significant. The ordered pair is written as (a, b), where 'a' is the first member of the pair and 'b' is the second member of the pair.

Equality of ordered pairs: Two ordered pairs (a, b) and (c, d) are said to be equal if a = c and b = d, that is, to say, $(a, b) = (c, d) \Rightarrow a = c$ and $b = d$.

Cartesian Product of Two Sets: Let P and Q be two non-empty sets. Then the Cartesian product of P and Q in this order is written as $P \times Q$ and is defined as the set of all ordered pairs (p, q) such that p ϵ P, q ϵ Q that is,

$$P \times Q = \{(p, q): p \,\epsilon\, P, q \,\epsilon\, Q\}$$

Cartesian Product: Consider two non-empty sets, say, A = {a, b, c} and set B = {apple, apricot, banana, custard apple, mango}. Now, the Cartesian product of A and B, i.e., $A \times B$ has 15 elements, listed as $A \times B$ = {(a, apple), (a, apricot), (a, banana) ... (c, mango)}.

$A \times B$ using arrow diagram
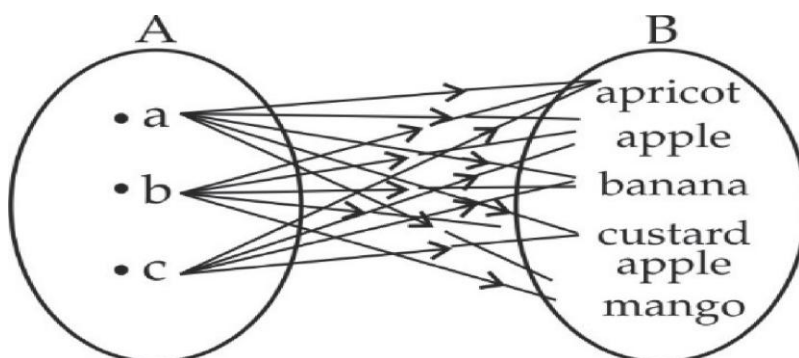
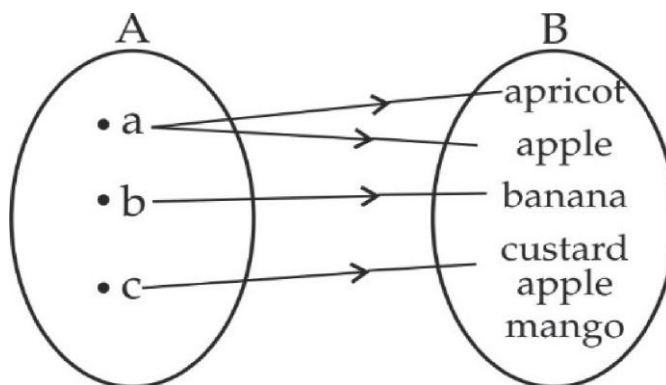**Relation:** Subset of A × B by defining a relation R between the first element belonging to A and second belonging to B of each ordered pair (a, b) as:

R = {(a, b) : a is the first letter of the name of the fruit in B, a ∈ A, b ∈ B} Therefore,

R = {(a, apple), (a, apricot), (b, banana), (c, custard apple)}

Here, R is said to be a 'Relation from the set A to the set B' An arrow diagram of this relation R is as shown in below Figure,

Domain: The set of all the first elements of the ordered pairs in a relation 'R' from set A to a set B is called the domain of the relation R.

Range: The set of all second elements in a relation R from a set A to a set B is called the range of the relation R.

Co-domain: In a relation R from a set A to a set B, B called the co-domain of the relation R.

Note: n (A × B) = n (A) × n (B)

If n(A) = m and n(B) = n then number of relations from A to B = 2^{mn}

MULTIPLE CHOICE QUESTIONS

1. Which bracket is using a Set denoted?

   (a) ( )   (b) {   }   (c) [ ]   (d) None of these.

Answer (b)

Solution: Use of curly bracket.

2. What will be the set of the interval (3, 4]?
(a) {x: 3 < x < 4}   (b) {x: 3 ≤ x ≤ 4}   (c) {x: 3 < x ≤ 4} (d) {x: 3 ≤ x < 4}

Answer (c)

Solution: - Value of x is more than 3 and less than equal to 4

3. Which of the following is not a set of letters of word PRINCIPAL?
a) {P,R,I,N,C,A,L} (b) {C,A,P,I,N,R,L} (c) {P,R,I,N,C,I,P,A,L} (d) {L,N,I,P,C,A,R}

Answer (c)

Solution: - English alphabets of word PRINCIPAL and write repeating letter once.

4. Write the set {x: x is a natural number and $x^2-9=0$} in roster form.
a) {3 ,-3}   (b) {-3}   (c) {3}   (d) {9}

Answer (c)

Solution: - $x^2 - 9 = 0$

$(x + 3)(x - 3) = 0$

Either x = -3 or x = 3 but x is a natural number so that x = 3 is the solution.

5. Which one of the following is not a set?
(a) The collection of all whole numbers less than 200. (b) The collection of all boys in your class.
(c) The collection of talented actors in Hollywood. (d) The collection of all books written by Chetan Bhagat.

Answer (c)

Solution: - Word talented is not well- defined.

6. How many subsets are possible for set A = {1,2,3}.

(a) 3   (b) 6   (c)   1   (d) 8

Answer (d)

Solution: - Use the formula = 2^n, where n = number of elements in a set.

7. Which of the following set is equivalent and equal set.

(a) $A = \{2,5,7\}, B = \{a, b, c\}$

(b) $A = \{2,5,7\}, B = \{5,2,7\}$

(c) $A = \{2,5,7\}, B = \{5,2\}$

(d) Both a and b.

Answer (b)

Solution: - Since, every equal set is also an equivalent set but converse is not true.

8. If Set A is subset of set B then AUB is

(a) A   (b) B   (c) both a and b   (d) None of these.

Answer (b)

Solution: - If A set is subset of B, means all elements of Set A in Set B so that collection of set A and B is set B

9. If $A = \{1,2,3,4,5\}, B = \{2,4,6,8,10\}, C = \{1,3,5,7,9\}$ then universal set for set A, B and C is

(a) All natural numbers. (b) All whole numbers (c) All integers (d) all of these.

Answer (d)

Solution: - Universal set is supper set, Here Set of N, W and Z is supper set of Set A, B and C.

10. If $A = \{2,4,6\}, B = \{1,3,5\} then A - B$

(a) {2,4,6}   (b) {1,1,1}   (c) {1,3,5}   (d){1,2,3}

 Answer (a)

Solution: - A – B means element of A excluding element of B .in set A

11.On real axis if A=[1,5] and B=[3,9],then A-B is

(a) (5,9)   (b) (1,3)   (c) [5,9)   (d) [1,3)

Answer (d)

Solution:- A – B means element of A excluding element of B .in set A

12. For any two set A and B, A∩(AUB) is

 (a)   A   (b) B   (c) ϕ   (d) A∩B

Answer (a)

Solution:- AUB contains the element of set A and B , Common element in A∩(AUB) is A

13 Which of the following is not correct ?

(a) N⊂R   (b) N⊂Q   (c) Q⊂R   (d) N⊂T(Irrational Number).

Answer (d)

Solution:- Irrational numbers do not contain natural numbers.

14 If n(A-B) =10, n(B-A) =23, n(AUB)=50, then n(A∩B) is

(a) 7   (b) 17   (c) 27   (d) 33

Answer (b)

Solution: - Using formula $n(AUB) = n(A-B) + n(B-A) + n(A∩B)$

$$50 = 10 + 23 + n(A∩B)$$

$$n(A∩B) = 50 – 33 = 17$$

15 For any two set X and Y, $X∩(XUY)^c$ is equal to

(a) X   (b) Y   (c) ϕ   (d) X∩Y

Answer (c)

Solution: - Complement of X U Y means no element of Set X and Y, so $X∩(XUY)^c$ is empty set.

16 If A = {a, b} and B= {1,2,3}, then the number of relations from A to B

(a) 4   (b) **8**   (c)   6   (d) 64

Answer (d)

Solution: -Use concepts, number of relations from A to B is 2^{mn}

17 Given R = {(x, y): x, y ϵ Z, y= x – 3}, then which ordered pair belongs to R
(a) (1, 4)   (b) (0 ,3)   (c) (5, 2)   (d) (-4, 1)

Answer (c) ;

Solution: - option c satisfies the given equation.

18 If R = { (x, y) : x, y ϵ W , $x^2 + y^2 = 169$}, then domain of R is
a) {0,5,12,13} (b) {-13, -12,-5,0,5,12,13} (c) {0,1,2,3…13} (d) {0, ±1, ±,2±3,…±13}

Answer (a)

Solution: - $y^2 = 169$ - x^2 after putting the value of x from 0 to 13 we got y = 0 , 5 ,12 ,13

19 Let A = {3,5} and B = {7,11} and R be the relation from A to B defined as R {(a,b): aϵA,bϵB , a-b is odd }, then
a) R = A × B   (b) R = ∅   (c) R ⊂ A × B   (d) R ⊂ B × A

Answer (b)

Solution: - difference two odd numbers always equal to even

20 Let n(A) = m and n(B) = n ,then the number of non -empty relation from A to B.is
(a) m^n (b) $n^m - 1$ (c) $2^{mn} - 1$. (d) 2^{mn}

Answer (c)

Solution: - number of relations of two sets is always equal to 2^{mn} and one is an empty set.

<div align="center">ASSERTION- REASON QUESTIONS</div>

Each of these questions contains two statements: Assertion (A) and Reason (R). Read the given Statement carefully and choose the correct answer from the four options given below.
(a) A is true, R is true and R is the correct explanation of A.
(b) A is true, R is true and R is not the correct explanation of A.
(c) A is true and R is false.
(d) A is false and R is true.

1 Assertion (A) :Two finite sets A and B have m and n elements respectively. If the ratio of cardinal number of power set B to proper set of A is 128:1 and m +n = 15 then m=4 ,n=11
   Reason (R): Number of elements in the power set of A = (Number of elements in set A)^2
Answer (c)
Solution:- $\frac{2^n}{2^m} = \frac{128}{1}$
       $= 2^{n-m} = 2^7$ n-m = 7 (i) and m+n =15(ii) (Given) solving (i) and (ii) m=4 and n=11.
Number of elements in power set A is $2^{\text{No.of elements in set A}}$, Hence Assertion is correct and Reason is false.

2. Let A and B are two finite sets.
 Assertion (A): n(AUB) =n(A- B) +n(B-A) +n(A∩B).
 Reason (R) : n(AUB) = n(A) + n(B) + n(A∩B)
Answer (c)
Solution:- n(A-B) = n(A) – n(A∩B)   (i)
     n(B-A) = n(B) – n(A∩B) …..   (ii)
Adding
    n(A-B) + n(B-A) = n(A) + n(B) - 2n(A∩B)
    n(A-B) + n(B-A) + n(A∩B) = n(A) + n(B) – n(A∩B) = n(AUB)

 Hence, Assertion is correct and Reason is false.

3. Assertion (A): Roster form of set A= {x : x∈ Z , $x^2 < 49$ } is A = { -6,-5,-4,-3,2,-1,0,1,2,3,4,5,6 }.

Reason (R) : Squares of integers 0, ±1, ±2, ±3, ±4, , ±5 and ±6 are less than 49.

Answer (a)

Solution:- Assertion and Reason both are correct and R is the correct explanation of A.

4. Assertion (A): If A = {1,2,3} then the cardinal number of the power set of A is 9.

   Statement II: Number of subsets of a set containing n elements = 2^n

   Answer (d)

Solution:- The Cardinal number in power set A is $2^3 = 8$ and power set is a set of subsets then assertion is false and reason is correct.
5. Assertion (A) I: If A ⊂ B then A ∩ B = A
   Reason (R): If A and B two sets, then A ⊂ B means every element of Set A is also element of B.

Answer (a)

Solution:- Assertion and Reason both are true and R is correct explanation of A

6. Assertion (A) : A – B = A – (A∩B)

Reason (R): A- B = A U B – B

Answer (b)

Solution:- Assertion and Reason both are true and R is not correct explanation of A

7. Assertion (A):   If   A ⊂ B then A – B = ϕ

Reason (R): If A and B are disjoint sets then n (AUB) = n(A) + n(B)

Answer: (b)

Solution:- Since, A ⊂ B means all elements of A in B so that A-B is empty set.

A and B are disjoint sets means there is no common element in A and B so that

n(A∩B) = 0, Hence assertion and reason both are correct but R is not correct explanation of A

8. Assertion (A): The collection of all-natural numbers less than 100 is a set.

Reason (R): A set is a well-defined collection of distinct objects.

Answer (a)

Solution:- Since, collection of all-natural number less than 100 is well defined collection so that Assertion and Reason both are correct and R is correct explanation of A

9.Assertion (A) : If n(A) = 3, n(B) = 6 and A ⊂ B then the number of elements in AUB is 9.

Reason (R): If A and B sets are disjoint then n(AUB) is n(A) + n(B) .

Answer: (d)

Solution:- Since, A ⊂ B then n(AUB) = n(B) =6

A and B is disjoint sets means there is no common elements so that n(AUB) is

n(A) + n(B), Hence Assertion is false and Reason is correct.

10. Assertion (A) : Set of English alphabets is the universal set for the vowels in English alphabets.

Reason (R) : The set of vowels is the subset of consonants in the English alphabet.

Answer (c)

Solution:- Since, the set of vowels is a subset of the set English alphabet so that set of English alphabets is a universal set for the set of vowels.

We know that the set of vowels is not a subset of consonants, hence Assertion is correct but Reason is false.

11.Assertion (A):The cartesian product of A × A has 16 elements, among ordered pairs (1 ,2),(3 ,4) are found.

Set A = {1 ,2 ,3,4}.

Reason (R) : If A and B are two finite sets, then n(A × B) = n(A) × n(B) .

Solution : Option (a)

A has 4 elements.

By definition, the Cartesian product A×A consists of all ordered pairs (a,b) such that a∈ A and b ∈A

So, the total number of ordered pairs is:

n(A×A) =n(A)×n(A)=4×4=16

Ordered pairs like (1,2) and (3,4) are among these 16.

Reason (R): If A and B are two finite sets, then n(A × B)= n(A) × n(B)

This is a fundamental property of Cartesian products of sets

12. Assertion (A): If a relation R on set A ={1,2,3,4,…14,15} is defined by

R = {(x ,y) : y – 2x = 0 ,x ,y ∈ A},then Range of R = { 2,4,6,8,10,12,14}.

Reason (R): If R is a relation from A to B, then domain of R is the set of all first components of ordered pairs which belong to R .

Answer (b),

Solution:- R = {(1,2), (2,4) ,(3,6),(4,8),(5,10),(6,12),(7,14)}

Range of R = { 2,4,6,8,10,12,14}. Assertion is correct

Reason is also correct but it is not the correct explanation of Assertion.

Question 13. Assertion(A) : If (x + 1 ,y – 2) = (3 ,1) ,Then x = 2 and y = 3.

Reason (R) : Two ordered pairs are equal , if their corresponding elements are equal.

Answer. (a)

Solution:- x + 1 = 3 or x = 3 – 1=2 and y-2 = 1 or y =1 + 2 = 3.Assertion is correct.

Fundamental rule ,Reason is correct and R is correct explanation of a.

Question 14. Assertion: (a) If (x ,1),(y,2) and (z ,1) are the elements of A × B and n(A) = 3 ,n(B) = 2,then A = {x ,y ,z} and B = {1, 2}.

Reason (R) : n(A) = 3 and n(B) = 2 ,then n(A×B) = n(A) × n(B).

Answer: (b)

Solution:- A × B = {(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)}.

n(A × B) = n(A) × n(B) = 3×2 = 6.

Question 15. Assertion : The domain of the relation R = {(x + 2 ,x + 4) : x ϵN ,x < 8 } is {3,4,5,6,7,8,9 }.

Reason: The range of the relation R = {(x + 2 ,x + 4) : x ϵN ,x < 8 } is {1,2,3,4,5,6,7 }.

Answer (c)

Solution:- R = {(3,5), (4,6), (5,7),(6,8),(7,9),(8,10),(9,11)}

Domain R = {3,4,5,6,7,8,9} and range = {5,6,7,8,9,10,11}

VERY SHORT ANSWER QUESTIONS

1.Describes the each of following in roster form

(a) {x : x is a positive integer and a divisor of 9}

(b) { x : x is a letter of the word PROPORTION}

Solution 1(a) Divisor of 9 means factors of 9 , i.e 1,3 and 9 {1,3,9}

(b) collection of English alphabets of PROPORTION but repeated element uses once.

{P, R, O, T, I, N}

2.State which of the following sets are finite and which are infinite.

(a) A={x : xϵ Z and x^2 -5x +6=0}

(b) B={x : xϵ Z and x^2 is even }

(c) C={ x : xϵZ and x^2 = 36}

(d) D= { x : xϵZ and x> -10}

Solution : 2(a) x^2 -5x +6=0

x^2 -3x -2x +6=0

(x-3)(x-2)=0

x=3,x=2

Finite set

2(b) Roster form of set B= {2,4, 6…}, Infinite Set

2 (c) x^2 =36 ,x= 6,-6 ,therefore set C is a finite set.

2(d) Roster for of set D= {-9, -8, -7…}, Infinite Set

3.Write the following intervals in the set builder form.

(a) (-9, 0)   (b) [6,12]   (c) [-20,3)   (d) (6,9]

Solution 3(a) $\{x: x \epsilon R \text{ and } -9 < x < 0\}$

3(b) $\{x: x \epsilon R \text{ and } 6 \leq x \leq 12\}$

3(c) $\{x: x \epsilon R \text{ and } -20 \leq x < 3\}$

3(d) $\{x: x \epsilon R \text{ and } 6 < x \leq 9\}$

4.Let A and B two sets such that ; $n(A) = 20$, $n(AUB) = 42$ and $n(A \cap B) = 4$.Find.

(a) n (A-B)   (b) n(B-A).

Solution: 4(a) n(A-B)= n(A) – n(A∩B)

$= 20 – 4 = 16$

4(b) n(AUB)= n(A) + n(B) – n(A∩B)

n(B)= n(AUB) - n(A) + n(A∩B)

= 42 -20 +4

= 26

n(B-A)= n(B) – n(A∩B)

= 26 - 4 = 22

5.Write the all-possible subset of set A={1,2,3}.

Solution: Subset of Set A are $\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$ and $\{ \}$

6.Out of 500 car owners investigated,400 owned car A and 200 owned car B,50 owned both A and B cars. Is this data correct?

Solution: Let U= Car owners investigated

X= Set of persons who owned car A

Y= Set of persons who owned car B

$Then \; n(U) = 500 \;\; , n(X) = 400 \;\; , n(Y) = 200 \; and \; n(X \cap Y) = 50.$

$We \; know \; that \; n(XUY) = n(X) + n(Y) - n(X \cap Y)$

$= 400 + 200 – 50$

$= 550$

$Since \quad XUY \subset U, n(XUY) \leq n(U)$

$550 \leq 500 \; , which \; is \; wrong.$

Hence, the given data is incorrect.

7. $Let \; A = \{2,4,6,8,10\}, B = \{1,2,3,4\} and \; C = \{1,3,5,7\}. find \; (A \cap B)UC.$

$Solution: A \cap B = \{2,4\}, (A \cap B) \; UC = \{1,2,3,4,5,7\}.$

8.Let U= {1,2,3,4,5,6,7,8}, A={1,2,5,7}, B={2,3,5}. find (AUB)^c

Solution: (AUB)={1,2,3,5,7}

(AUB)^c = U – (AUB)

= {4,6,8}.

9.What universal set(s) would you propose for each of the following.

(a) The set of right-angled triangles (b) The set of isosceles triangles.

Solution : 9(a) Set of triangles or set of polygon,

9(b) Set of triangles or set of polygons

10.Let $A = \{1,2,\{5,6\}7\}$ Which of the following are true.

$(a) \{5,6\} \epsilon A \;\; (b) \{5,6\} \subset A \;\; (c) \{1\} \subset A \;\; (d) \{\{5,6\}\} \subset A.$

Solution 10 (a) True, {5,6}is element of set A

10 (b) False , {5,6}is element of set A not Subset of A

10 (c) True, 1 is element of set A and {1} is set which is subset of set A

10(d) True,{ 5 ,6} is an element of set A and) {{5,6}}is a set which is a subset of set A.

11. Let A = {1,2,3}, find A×A

Solution: $A \times A = \{(1,1,(1,2),(1,3),(2,1),(2,2),(2,3),((3,1),(3,2),(3,3)\}.$

12. If $(x + y, 3x – 5) = (10, 4)$. find the value of x and y.

Solution: comparing both sides,

$$x + y = 10 \ldots\ldots\ldots..(i)$$

and $\quad 3x – 5 = 4 \quad \ldots\ldots\ldots\ldots(ii)$

from (ii) $3x = 4 + 5 = 9$

$\quad x = 3$, put in (i) $3 + y = 10$ or $y = 10 -3 = 7$.

13. Let $A = \{ 1,2,3,4,5\}$.Define a relation R from A to A by

$R = \{(x , y) : x > y\}$. write the r in tabular form and also write range.

Solution ; $\{(2,1). (3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(5,4)\}.$

$\quad Range\ of\ R = \{1,2,3,4\}.$

SHORT ANSWER TYPE QUESTIONS

1.Let $U = \{1,2,3,4,5,6\}, A = \{2,3\} and\ B = \{3,4,5\}$. Show that $(AUB)^c=A^c\cap B^c$

Solution: LHS $\qquad\qquad\qquad\qquad$ RHS

$\quad$ AUB = {2,3,4,5} $\qquad\qquad$ A^c = U-A={1,4,5,6}

$\quad (AUB)^c$ =U –(AUB) $\qquad$ B^c = U-B = {1,2,6}

$\qquad$ = {1,6} $\qquad\qquad\qquad$ $A^c\cap B^c$ = {1,6}

$\qquad$ LHS = RHS

2.Let $A = \{x : x\epsilon N\ and\ x2 − x − 6 = 0, \}, B = \{x: x2 − 5x + 6 = 0 , x\epsilon N \}and\ C = \{x: x2 − 4 = 0, x\epsilon Z\}.$

$Find\ (a)\ AUB\ (b)\ (A \cap B)\ UC$

Solution: Roster form of set $\quad$ A = {3}, B = {2,3} and C = {-2,2}

$\quad (a)\ AUB = \{2,3\}\ (b)\ A \cap B = \{3\}, (AUB) \cap C = \{ \}\ or\ \phi$

3.Let U= {x: x ϵN and x≤ 10}, A= {x: x=2n, n ϵN and n< 5}and B= {y: y=2^n, n ϵW and n≤3}. Find $A^c \cap B^c$.

Solution: Roster Form U= {1,2,3,4,5,6,7,8,.9,10}, A = {2,4,6,8}, B = {1,2,4,8}

$\qquad\qquad A^C$ = U – A ={1,3,5,7,9,10}

$\qquad\qquad B^C$ = U- B = {3,5,6,7,9,10}

$\qquad A^c \cap B^c$ = {3,5,7,9,10}

4.In a survey of 700 students in a college,180 were listed as drinking Limca ,275 as drinking Miranda and 95 were listed as both drinking Limca and Miranda. Find how many students were drinking neither Limca nor Miranda.

Solution: Here U = Survey of students in college = n(U) = 700

$\qquad$ A= Set of students who drinking Limca = n(A) = 180

$\qquad$ B= Set of students who drinking Miranda = n(B) = 275

$\quad And\ A \cap B = Set\ of\ students\ who\ drinking\ Limca\ and\ Miranda\ both = n(A \cap B) = 95.$

$\quad n(AUB) = n(A) + n(B) − n(A \cap B)$

$\qquad\qquad$ = 180 + 275 – 95 = 360

n(Number of students who are neither drinking Limca and Miranda) = 700 -360 =340.

5. In a class of 25 students ,12 have taken Mathematics ,8 have taken Mathematics but not Biology. If each student has taken at least one subject. Find the number of students who have taken (a) Biology but not Mathematics (b) Both Mathematics and Biology.

Solution: Let M= set of students who have taken Mathematics

$\qquad$ B = set of students who have taken Biology.

$\quad$ We have

$\quad n(MUB) = 25, n(M) = 12, n(M − B) = 8$

(a) $n(B - M) = n(MUB) - n(M)$
    $= 25 - 12 = 13$

(b) $n (MUB) = n(M - B) + n (B - M) + n (M \cap B)$
    $25 = 8 + 13 + n (M \cap B)$
    $n (M \cap B) = 25 - 21 = 4$

6. A and B are two sets such that $n(A) = n(B)$ and $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n (A \cap B) = x$,

Find: (a) The value of x   (b) n (AUB).

Solution (a) $n(A) = n(A - B) + n (A \cap B) = 14 + x + x = 14 + 2x$
    $n (B) = n (B - A) + n (A \cap B) = 3x + x = 4x$

Since given that n(A) = n(B)
    $14 + 2x = 4x$
    $x = 7$

(b) n (AUB) = n (A-B) + n(B-A) + n(A∩B)
    $= 14 + x + 3x + x = 14 + 5x = 14 + 5 \times 7 = 49$

7. If A = {x: x ∈ N, x is a factor of 12} and B = {x: x ∈ N, x is a factor of 18} find A ∩ B.

   Solution: A = { 1,2,3,4,6,12} and B = { 1,2,3,6,9,18}

   A∩B = { 1,2,3,6}

8. Write each of the following sets in the roster form:

(a) A= Set of all factors of 24.

(b) B= Set of all integers between- $\frac{-3}{2}$ *and* $\frac{11}{2}$.

 (c) C= Set of all letters in the word 'TRIGONOMETRY'

 Solution: (a) A= { 1,2,3,4,6,8,12,24}
        (b) B = { -1,0,1,2,3,4,5}
         (c) C = {T, R, I, G, O, N, M, E, Y}

9. Which of the following sets are empty and which are singleton sets?

(a) {x: x is an even prime number}.

(b) {x: x is a natural number and –1 < x < 1}.

(c) {x: x is an integer and –1 < x < 1}.

 Solution (a) { 2 } = Singleton set
        (b) { } = Empty or Null set.
         (c) { 0 } = Singleton set

10. Which of the following pairs of sets are equal? Give reasons.

(a) A = {–2, 3}, B = {x is a solution, of $x^2 – x – 6 = 0$}

(b) A = {x : x is a letter of the word 'FOLLOW'} B = {y : y is a letter of the word 'WOLF'}

(c) A = {1, 4, 9}; B = {x : x = n^2 where 'n' is a natural number less than 5}

Solution: (a) A = {-2,3} and B = {-2 ,3} = Equal set.
        (b) A = {F,O,L,W} ,B= {W,O,L,F} = Equal Set
         (c) A = {1,4,9} ,B = { 1,4,9,16} = Not Equal Set .

11. Let A = {a, b, c} and B = {1,2}. Find A × B and B × A. Is this equal?

 Solution: A × B = {(a,1), (a,2), (b,1), (b,2), (c,1),(c,2)}.
        B × A = {(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)}.

   No, A × B ≠ B × A.

12. let A = { 1,2,3,4,5} , B = {2,4,6} C = { a ,b ,c}. Find (A ∩ B)×C.

Solution: (A ∩ B) = {2,4}

(A ∩ B)×C = {(2,a),(2,b),(2,c),(4,a),(4,b),(4,c)}.

LONG ANSWER TYPE QUESTIONS

1. If n(U) =40, n(A) = 25 and n(B) = 20 then find

(a) the greatest value of n(AUB) (b) the least value of n (A∩B).

Solution It is clear that every set is a subset of a universal set.

AUB ⊂ U ⇒ n (AUB) ⊂ n(U)

⇒ **n(AUB) ≤ 40**

The Greatest value of n(AUB) =40

(b) We know that n(AUB) = n(A) + n(B) – n(A∩B)

But n(AUB) ≤ 40

n(A) + n(B) – n(A∩B) ≤ 40

25 + 20 – n(A∩B) ≤ 40

45 -40 ≤ n(AUB)

n(AUB) ≥ 5

The least value of n(AUB) = 5

2. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 Study Physics and Chemistry 50 study Mathematics and Chemistry and 20 none of these subjects. Find the number of students who study all three subjects.

Solution: Let M be the set of students who study Mathematics.

 P be the set of students who study Physics and C be the set of students who study Chemistry.

Then ,$n(U)$=200 ,$n(M)$=120 ,$n(P)$=90

$n(C)$=70, $n(M∩P)$=40, $n(P∩C)$=30 ,$n(C∩M)$=50, $n(M'∩P'∩C')$=20

$n(U)−n(M∪P∪C)$=20,

$∵n(M∪P∪C)$=200−20=180

⇒$n(M∪P∪C)$=$n(M)$+$n(P)$+$n(C)$−$n(C∩M)$+$n(M∩P∩C)$

⇒180=120+90+70−40−30−50+$n(M∩P∩C)$

⇒180=160+$n(M∩P∩C)$

⇒$n(M∩P∩C)$=180−160=20

So, the number of students who study all three subjects is 20 .

 3.In an University, out of 100 students 15 offered Mathematics only,12 offered Statistics only,8 offered only Physics ,40 offered Physics and Mathematics ,20 offered Physics and Statistics ,10 offered Mathematics and Statistics ,65 offered Physics . Find the number of students who

(a) offered Mathematics. (b) offered Statistics (c) Did not offer any of the above three subjects.

Solution: Venn diagram
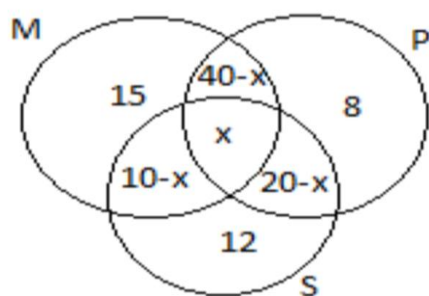
Let M= The Set of students who offered Mathematics

S = The Set of students who offered Statistics

P = The Set of students who offered Physics.

   And x be a set of students who offered all three subjects, then number of members in different regions shown in Venn diagram.

From Venn diagram, we get the number of students who offered Physics

 = (40 - x) + x + (20 – x) 8 = 65

 Or 68 – x = 65

Or x = 3

(a) Number of students who offered Mathematics

= 15 +(10 – x) +x + (40 – x)

 = 65 – x   (Put Value of x)

= 65 – 3

 = 62

(b) Number of students who offered Statistics

 = 12 + (10 – x) +x + (20 – x)

    = 42 – x   (Put Value of x)

    = 42 – 3

  = 39

(c) The number of students who offered any of three subjects

    = 15 +12 + 8 + (10 – x) +(40 – x) + (20 – x) + x

   = 105 – 2x   (Put Value of x)

    = 105 -6

   = 99

Therefore, The number of students who did not offered any of the three subjects

    = 100 – 99

    = 1

4.For sets A ,B and C using property of sets, prove that

(a) (AUB) – B = A- B

(b) A- (A∩B) = A- B

(c) (AUB) – C = (A- C) U (B- C)

Solution: (AUB) – B = (AUB)∩B′   [(X – Y) = X ∩ Y′]

               =(A∩ B′) U (B∩ B′)

                  = (A∩ B′) U Φ = A – B

Solution (b)   A- (A∩B) = A ∩ (A ∩ B) ′

                    = A ∩ (A′ U B′)

                  = (A ∩ A′) U (A ∩ B′)

                  =   Φ U (A ∩ B′)

$$= (A \cap B') = A - B$$

Solution (c) $(A \cup B) - C = (A \cup B) \cap C'$

$$= (A \cap C') \cup (B \cap C')$$

$$= A - C) \cup (B - C)$$

5. If A, B and C are three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, show that B= C

Solution: Given that $A \cup B = A \cup C$ … (i)

$A \cap B = A \cap C$ … (ii)

Let B = B U (B ∩ A)	(Absorption law)
= B U (A∩B)	(Commutative Law)
= B U(A∩C)	(using (ii))
= (B U A) ∩ (B U C)	(Distributive law)
= (A U B) ∩ (B U C)	(commutative law)
= (A U C) ∩ (B U C)	(using (i)
= (A ∩ B) U C	(Distributive law)
= (A ∩ C) U C	(using (ii))
= C U ((A ∩ C)	(Commutative law).
= C	

CASE BASED QUESTIONS

1 . With coronavirus threatening to run riot in India, prevention appears to be the best cure available so far. It is crucial for people to have awareness and knowledge about the virus and need to take proper precautions to discourage its spread. A survey was conducted on 25 people to see if proper precautions were being taken by people and following points were observed: 15 people used face masks, 14 consciously maintained social distancing ,5 used face masks and washed their hand regularly ,9 maintained social distancing and used face masks. 3 were practicing all the three measures. 4 maintained social distancing and washing hands regularly. 4 practised only social distancing norms.

Assuming that everyone took at least one of the precautionary measures, find:
 (i) How many exercised only washing hands as precautionary measure.
 (ii) How many practiced social distancing and washing hands but not wearing masks.
(iii) How many exercised only wearing masks.
 Solution: Venn diagram:
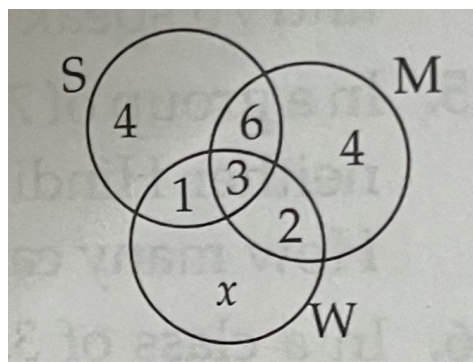
We are given a total of **25** people, all of whom took at least one of three precautionary measures:

Let's define the sets:

Let M= the set of people wearing masks.

S= the set of people maintaining social distancing.

W= the set of people washing hands regularly.

Given:

n(M) = 15

n(S) = 14

n (M ∩ W) = 5

n (M ∩ S) = 9

n (M ∩ S ∩ W) = 3

n (S ∩ W) = 4

People who only followed social distancing = 4

Total people = 25

Everyone took at least one precaution → No one is outside M, S, or W.

We'll use the principle of inclusion-exclusion and Venn diagram logic.

Step 1: Use a Venn diagram model

We want to find:

Let's denote:

a = only M

b = only S

c = only W

d = M ∩ S only (not W)

e = M ∩ W only (not S)

f = S ∩ W only (not M)

g = M ∩ S ∩ W

Total = a + b + c + d + e + f + g = 25

From the given:

People who only did S → b = 4

People who did all three → g = 3

M ∩ W = 5 ⇒ includes those who did all three, so e = 5 - 3 = 2

M ∩ S = 9 ⇒ includes those who did all three, so d = 9 - 3 = 6

S ∩ W = 4 ⇒ includes those who did all three, so f = 4 - 3 = 1

Now use totals to find the rest:

Sum all components:

a + b + c + d + e + f + g = 25

a + 4 + c + 6 + 2 + 1 + 3 = 25

a + c + 16 = 25

a + c = 9 → (Equation 1)

Now let's use the total number in M = 15

M includes: a + d + e + g = 15

a + 6 + 2 + 3 = 15

a = 4

Now from (1): a + c = 9

So, 4 + c = 9 → c = 5

Now answer the questions:

(i) How many people exercise only washing hands as a precautionary measure?

Answer: 5 (value of c)

(ii) How many practiced social distancing and washing hands but not wearing masks?

This is S ∩ H only (not M) = f = 1

(iii) How many exercised only wearing masks?

This is **a = 4**

Question 2. In an examination ,80% students passed in Mathematics ,72% passed in science and 13% failed in both the subjects. If 312 students passed in both the subject, find the

(a) Total number of students appeared in the examination.

(b) Number of students who passed in mathematics.

(c) Number of students failed only in science.

Solution: Number of students failed in Mathematics: 100 -80 = 20%

Number of students failed in science = 100 - 72 = 28%

Number of students failed in both Mathematics and science = 13%

Number of students failed in Mathematic only = 20 -13 = 7%

Number of students failed in science only = 28 – 13 = 15%

Therefore, the students who failed in either of the two subjects or in both subjects = 7% + !5% + 13% = 35%

The students passed in both of two subjects = 100 – 35% = 65%

Thus, if 65 students pass in both subjects, then the total number of student = 100

If 312 students pass in both subjects, then the total number of students : $\frac{100}{65}$ × 312 = 480.

(a) Total number of students appeared in Examination = 480.

(b) Number of students who passed in Mathematics = $\frac{80}{100}$ × 480 = 384

(c) Number of students failed in science only = $\frac{15}{100}$ × 480 = 72 .

Question 3. On the Real axis, if A = [0, 3] and B = [2 ,6] ,then answer the following.

(a) A'   (b) A − B   (c) A∩B

Solution : You have two intervals on the real number line:

A=[0,3]

B=[2,6]

A is from 0 to 3 (inclusive)

B is from 2 to 6 (inclusive)

(a) A′ : Complement of A

The complement of A, denoted A′, means all real numbers not in A.

Since A = [0, 3], the complement is:

A′= (−∞,0) ∪ (3, ∞)

(b) A−B : Set difference A minus B

This means all elements in A excluding those that are also in B.

A = [0, 3]

B = [2, 6]

Their overlap is [2, 3]

So A−B=[0,2)

(c) A∩B : Intersection of A and B

The intersection is the part that both sets share.

A = [0, 3]

B = [2, 6]

So the intersection is:

A∩B=[2,3]

A group of students were studying family relationships. They decided to define a relation R on the set A = {1, 2, 3, 4}, where each number represents a person. The relation R is defined as:

R = {(a, b) | a is the parent of b}

HIGHER ORDER THINKING SKILL QUESTIONS

Question 1. For any two sets A and B prove that : $P(A) = P(B) \Rightarrow A = B$

Solution: Let x be an arbitrary element of A. Then, there exists a subset, say X, of a set A such that $x \in X$.

Now, $X \subset A$

$X \in P(A)$ Since $P(A) = P(B)$

$X \in P(B)$

$X \subset B$

$x \in B \Rightarrow A \subset B$ … (i)

Now, let y be an arbitrary element of set B. Then there exists a subset, say Y, of set B such that $y \in Y$

Now, $Y \subset B$

$Y \in P(B)$ But $P(A) = P(B)$

$Y \in P(A)$

$Y \subset A \Rightarrow y \in A \Rightarrow B \subset A$ …   (ii)

From (i) and (ii) A = B

Question 2.If $a \in N$ such that $aN = \{ax : x \in N \}$. Describe the set $3N \cap 7N$.

Solution: $3N = \{ 3x : x \in N \} = \{ 3,6,9,12,15,18,21…   \}$ and

$7N = \{ 7x : x \in N \} = \{ 7,14,21,28,35,42,…   \}$

Hence, $3N \cap 7N = \{21,42,   \} = \{21x : x \in N\}$

Question 3. For any natural number a, we define $aN = \{ax : x \in N \}$. If $b,c,d \in N$ such that $bN \cap cN = dN$, then prove that d is the L.C.M of b and c.

Solution :

$bN = \{bx : x \in N \}$ = The set of positive integral multiple of b.

$cN   = \{cx : x \in N \}$ = The set of positive integral multiple of c.

$bN \cap cN$ = The set of positive integral multiple of both b and c .

$bN \cap cN = \{kx : x \in N \}$,where k is L.C. M of b and c .

Hence d is L.C.M of b and c.

Question 4. Two finite sets have m and k elements. If the total number of subsets of the first set is 56 more than the total number of subsets of the second set, then the value of m and k.

 Solution: Let the two set A and B ,then $n(A) = m$ and $n(B) = k$, $m > k$

Number of subsets in set $A = 2^m$

Number of subsets in set $B = 2^k$

According to the question,

$$2^m – 2^k = 56$$

Or $2^k (2^{m-k} - 1) = 2^3 \times 7$

Or $2^k = 2^3$ and $2^{m-k} - 1 = 7$

Or $k = 3$ and $2^{m-k} = 8$

Or $k = 3$ and $2^{m-k} = 2^3$

Or $k = 3$ and $m - k = 3$

Put $k = 3$, then $m = 6$

Question 5. Two finite sets A and B have m and k elements respectively. If the ratio of cardinal number of power set of A to cardinal number of powers of set B is 64 : 1 and n(A) + n(B) = 12. Then find the value of m and k .

Solution: Given n(A) = m and n(B) = k

    $n((P(A)) = 2^m$ and $n(P(B)) = 2^k$

A.T.Q

$$2^m : 2^k = 64 : 1$$

Or $2^{m-k} = 2^6$ or $m - k = 6$ …………..(i)

And $m + k = 12$ (Given) ---------------------(ii)

Solving (i) and (ii)

$m = 9$ and $k = 3$

Question 6 . Let A = ∅. Find P(P(A)),Write the cardinal number of P(P(A))

Solution : Given A = ∅

    So , P(A) = {∅ }

   Now , P(P(A)) = { ∅. {∅ } }

Question 7. If A and B are two sets and U is the universal set such that n(U) = 700 and n(A) = 290, n(B) = 240 and n(A∩B) = 110, then find n (A'∩B').

Solution: n (A∪B) = n(A) + n (B) – n(A∩B)

                 = 290 + 240 – 110

                 = 420

     n (A ∪ B) ' = n(U) – n(A ∪ B)

             = 700 – 420 = 280

    Since , n (A'∩B') = n (A ∪ B) ' = 280

       (De morgan's Law)

Question 8. A market research group conducted a survey of 1000 people and reported that 720 people liked product A and 450 people liked product B. What is the least number of people that must have liked both products?

Solution: Number of people liked product A = 720 = n(A)

      Number of people liked product B = 450 = n(B)

     Total number of people in survey = 1000 = n(U)

We know that ,

          n(A∪B) = n(A) + n(B) – n(A∩B)

              = 720 + 450 - n(A∩B)

              = 1170 - n(A∩B)

Since A∪B ⊂ U, n(A∪B) ≤ n(U)

        1170 - n(A∩B) ≤ 1000

       1170 – 1000 ≤ n(A∩B)

      170 ≤ n(A∩B)

     n(A∩B) ≥ 170

Hence, the least number of people who liked both product A and B is 170.

Question 9. In a certain town 25% families own a phone and 15% own a car 65% own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statements in this regard

(1) 10% families own both a car and a phone

(2) 35% families own neither a car or a phone

(3) 40,000 families live in the town

Which one of these statements are correct?

Solution: Let the total number of families be 100.

Define:

P= Number of families owning phones = 25

C= Number of families owning cars = 15

N= Number of families owning neither a phone nor a car = 65

B: Number of families owning both a phone and a Car = 2000

We use the formula for union of two sets:

$n(owning\ at\ least\ one\ a\ phone\ or\ a\ Car) = n(P) + n(C) - n(P \cap C)$

Then, $n(owning\ neither\ a\ Phone\ nor\ a\ Car) = 100 - \{n(P) + n(C) - n(P \cap C)\}$

$65 = 100 - \{25+15 - n(P \cap C)\}$

$n(P \cap C) = 65 - 100 + 40 = 5$

Now, If 5 family having both a phone and a car then total family = 100

If 2000 family having both a phone and a car then total family $= \frac{100}{5} \times 2000 = 40000$.

Now check each statement:

(1) 10% families own both a car and a phone

Incorrect, only 5% own both.

(2) 35% families own neither a car nor a phone

Incorrect, 65% own neither (given).

(3) 40,000 families live in the town

Correct, as calculated.

Question 10. Find all pair of equal set (if any)

$A = \{0\}, B = \{x : x < 5\ and\ x > 15\}\ C = \{x : x - 5 = 0\}, D = \{x : x2 = 25\}$

$E = \{x : x\ is\ a\ positive\ integral\ root\ of\ equation\ x2 - 2x - 15 = 0\}$

Solution: $A = \{0\}$, $B = \{\}$, $C = \{5\}$, $D = \{-5, 5\}$

And E = {5}   (Since $x^2 - 2x - 15 = 0$ or $(x-5)(x+3) = 0$ or $x = 5, -3$)

As per definition of equal set C and E are equal set.

EXERCISE

MULTIPLE CHOICE QUESTIONS

1.Which of the following collection of objects is not a set .

(a) The collection of all integers.

(b) The collection of all months of year beginning with letter J.

(c) The collection of most talented writers of India .

(d) The collection of all prime numbers less than 20.

2. Empty set is a.

(a) Infinite set   (b) Finite set   (c) Unknown set   (d) Universal set

3. The cardinality of the power set of A = {x: x∈ N, x≤10} is

(a) 1024   (b) 1023   (c) 2048   (d) 2043

4.Which of the following is null set .

(a) {x : x ∈ N, 2x -1 = 3 }

(b) {x : x ∈ N , $x^2 < 20$ }

(c){ x : x is an even prime number greater than 2 }

(d) {x : x ϵ Z, 3x + 7 = 1 }

5. If A and B are two finite sets such that n(A) = 25, n(B) = 30, and n(A ∪ B) = 30, then   n(A ∩ B).

   (a)   5   (b) 25   (c) 10   (d) None of these .

6. IfA = { a} and B = {1,2,3}, then the number of relations from B to A is

   (a) 8   (b) 16   (c) 32   (d) 64

7. Let A = {1, 2, 3, 4, 5, 6} and R be the relation defined on A by R = {(x, y): x, y ∈ A, x divides y}, then range of R is

(a) {2, 3, 4, 5, 6}   (b) (1, 2, 3, 4, 5 }   (c) { 2, 4, 6 } (d) {1, 2, 3, 4, 5, 6}

<center>ASSERTION -REASON QUESTIONS</center>

In the following questions a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b)) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

1.Assertion(A) : The collection of all-natural numbers less than 100 is a set.

Reason (R) : A set is a well-defined collection of distinct objects.

2. Assertion(A):The power set of {1, 2} is {∅,{1},{2},{1,2}}

   Reason (R): The power set is the set of all subsets.

3. Assertion(A) : A= {a, b} and   B= {a, b, c} then A is a subset of B.

    Reason (R): All subsets are finite sets.

4. Assertion(A) : If W is the set of whole numbers and N is the set of natural numbers, then

 W – N = { }.

Reason (R):   A – B is the set of elements of A which are not in B.

5. Assertion(A):   If A is the set of letters of the word 'FOLLOW' and B is the set of letters of the word 'WOLF', then A and B are equal sets.

    Reason (R) : Two sets are equal if they have equal number of elements.

6. Assertion(A):Let A and B be two finite sets such that n(A) - n(B) = 1 and number of relations from set A to set B is 64 , n(A) = 3, n(B) = 2.

Reason (R) :   Number of relations from set A to set B = 2nm , when n and m are the number of elements in set A and set B respectively

<center>VERY SHORT ANSWER TYPE QUESTIONS</center>

1 Given that N= {1,2,3, …, 100}. Then write

(a) the subset of N whose elements are even numbers.

(b)) the subset of N whose element are even perfect square numbers.

2. Given, A = {a, b, c, d} and B = {f, b, d, g}, determine the final value of A – B and B – A.

3. Represent the given sets in the Roster Form.

(a) A = {x | x is a positive integer which is less than 10 and 2^x – 1 is an odd number}

(b) B = {x : x^2 + 7x – 8 = 0, x ∈ R}

4. If U = {x : x ∈ N, x ≤ 9}, A = {x : x is an even number, 0 < x < 10}, and B = {2, 3, 5, 7}, what will be the (A U B) ′ ?

5. In a beauty contest, half the number of experts voted for Rajani and two thirds voted for Mahima. 10 voted for both and 6 did not vote for either. How many experts were there in all?

6. Find the linear relation between the components of the ordered pairs of the relation R where

(a) R = ((-1,-1), (0, 2), (1, 5), ...).

(b) R = {(0, 2), (-1, 5), (2, -4), ...).

<center>SHORT ANSWER TYPE QUESTIONS</center>

1. Let A and B be two sets containing 3 and 5 elements respectively. What is the maximum and the minimum number of elements in A ∪ B?

2. If ∪ = {a, e, i, o, u} , A = {a, e, i}, B = {e, o, u}, C = {a, i, u}

Verify the A ∩ (B - C) = (A ∩ B) - (A ∩ C).

3. There are 100 students in a Student Activity Club of which 35 like drawing and 45 like music. 10 students are interested in both drawing and music. Find the number of students that like either of them or neither of them.

4. There are 200 individuals with a dermatological disorder. 120 were exposed to the chemical C_1, 50 to chemical C_2, and 30 to both the chemicals C_1 and C_2. What is the number of individuals exposed to

(a) Chemical C_1 but not chemical C_2?

(b) Chemical C_2 but not chemical C_1?

(c) Chemical C_1 or chemical C_2?

5. According to a survey, 73% of Kashmiris actually like apples, while around 65% of them like oranges. Determine the percentage of Kashmiris who like both apples and oranges.

6. Let R = {(x, y): x, y ∈ Z, y = 2x - 4). If (a, - 2) and (4, b^2) belong to R, find the values of a and b.

<center>LONG ANSWER TYPES QUESTIONS</center>

1. A school awarded 58 medals for Honesty, 20 for Punctuality and 25 for Obedience. If these medals were bagged by a total of 78 students and only 5 students got medals   for all the three values, find the number of students who received medals for exactly two of the three values.

2.Out of 100 students, 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science , 4 in English and Science, 4 in all the three. Find how many passed

(a) in English and Mathematics but not in science.

(b) in Mathematics but not in Science (c) in Mathematics only.

(d) in more than one subject only.

3.If n(A) = 10 and n(B) = 7,then find

(a) the least value of n (A∩B) (b) the greatest value of n (A∩B)

(c) the greatest value of n (A∪B) (d) the least value of n (A∪B)

4.If A = {1,3,5,7…,17} , B = {2,4,6…,18} and N the set of natural numbers is universal set, then show that A′ ∪((A ∪ B) ∩ B′) = N

5.Let A = {2x : x ϵ N and 1 ≤ x < 4}, B = {x + 2 : x ϵ N, and 2 ≤ x < 5} and

C = { x : x ϵ N and 3 < x<}6}. determine the following:

(a) A∩B   (b) A ∪ B   (c) (A ∪ B) ∩ C.

<center>CASE BASED QUESTIONS.</center>

1. In a survey of 40 students, it was found that 21 had taken Mathematics, 16 had taken Physics and 15 had taken Chemistry,7 had taken Mathematics and Chemistry,12 had taken Mathematics and Physics, 5 had taken Physics and Chemistry and 4 had taken all the three subjects.

Based on the above information, answer the following questions :
(a)The number of students who had taken Mathematics only.
(b) The number of students who had taken Physics and Chemistry but not mathematics.
(c) The number of students who had taken exactly one of three subjects.

OR

The number of students who had taken none of the subjects.

2. A survey of 500 television viewers produced the following information, 285 watchfootball,195 watch hockey, 115 watch basketball, 45 watch football and basketball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the three games.

Based on the above information, answer the following questions
(a) How many viewers watch at least one of the three games?
(b) How many viewers watch all of three games?
(c) How many viewers watch football only?

OR

How many viewers watch hockey only?

3.The sum of cardinal numbers of two set A and B is 9. The ratio of cardinal number of power set of A to cardinal number of power set of B is 8 : 1

Based on the above information, answer the following questions:
(a) Find the cardinal number of set A .
(b) Find the cardinal number of set B .
(c) Find the maximum value of n(A∪B)

OR

Find the minimum number value of n(A∪B)

HIGHER ORDER THINKING SKILL QUESTIONS

1. If X = {8^n – 7n -1 : n ϵ N } and Y = {49(n – 1) : n ϵN}. Find X ∪ Y and X ∩ Y .

2.If sets A and B are defined as A = {(x , y) : y = $\frac{1}{x}$, x $\neq$0 and xϵ R } ,B = {(x,y): y = - x , xϵ R} then Find A ∩ B .

3.Let S = {x : x is a positive multiple of 3 less than 100} ,P = {x : x is a prime less than 20} Find n(S) + n(P) .

4.Let A = {x : x ϵ R and x > 4 } and B = {x: x ϵ R, x < 5 }.Find A ∪ B.

5. Which of the following statements are correct. Write the correct form of each of following incorrect statements.

(a) a $\subset$ {a, b , c}   (b) {a} ϵ {a, b, c}   (c) a ϵ {{a}, b}   (d) {a} $\subset$ {{a}, b}

ANSWERS

MULTIPLE CHOICE QUESTIONS ANSWERS

1 (c)   2 (b)   3 (a)   4   (c)   5 (b)   6 (a)   7(d)

ASSERTION-REASON QUESTIONS ANSWERS

1 (a)   2 (a)   3 (c)   4 (d)   5 (c)   6(c)

VERY SHORT QUESTIONS ANSWERS

1 (a) {2,4,6,…100} ,   b) {4,16,,36,64,100}.

2 A – B ={a, c} , B – A = {f, g }.

3 (a) A = {1, 2, 3, 4, 5, 6, 7, 8, 9}. (b) B = {– 8, 1}

4 (A U B)' = {1, 9}

5. 36

6. (a) y= 3x +2 (b) y=2-3x

SHORT ANSWERS TYPE QUESTIONS

1   Maximum element in AUB = 8, Minimum element in AUB = 6.

2 A ∩ (B - C) = (A ∩ B) - (A ∩ C) = {e}

3 30.

4   (a) 90   (b) 20   (c) 140

5 . 38%

6. a= 1 , b= ± 2

LONG QUESTIONS TYPE ANSWERS

1 15 Medals

 2 (a)   2   (b) 3   (c) 3   (d) 9

 3 (a)   0   (b) 7   (c)   17   (d)   10

 4 Verified LHS = RHS

 5 (a) {4 , 6 }   (b)   {2,4,5,6}   (c) {4 ,5}

Case Based Questions Answers :

1. (a) 6   (b)   1   (c)   16 Or (c) 8

 2. (a) 450   (b) 20   (c)   190 Or (c) 90

 3. (a)   6   (b) 3   (c) 9 Or (c) 6

HOT QUESTIONS ANSWERS:

1. Y and X .

 2. A ∩ B = ∅

 3.   41

  4 (4 ,5)

  5(a) incorrect, Correct form is a ∈ {a, b , c}

    (b) Incorrect, correct form is {a} ⊂ {a, b, c}

    (c) Incorrect, correct form is {a} ∈ {{a}, b}

     (d) incorrect, correct form is {a} ∈{{a}, b}.

SEQUENCE AND SERIES

Gist/Summary of the lesson:

Sequence, series, pattern, progression, arithmetic progression, geometric progression, arithmetic mean, geometric mean, relation between AM & GM.

Definitions and Formulae:

Let a be first term d be the common difference then general term

$T_n = a + (n − 1)d$

- Sum of n terms is

$$S_n = \frac{n[2a + (n − 1)d]}{2}$$

$$S_n = \frac{a(r^n − 1)}{r − 1}$$

For any number a and b

$$\text{A.M.} = \frac{a+b}{2} \quad \text{G.M.} = \sqrt{ab}$$

MULTIPLE CHOICE QUESTIONS

1) What is the 10th term of the arithmetic sequence: 3, 7, 11, ...?

   A) 39   B) 40   C) 41   D) 42

   Solution:

   First term, a = 3; Common difference, d = 4

   nth term, $\mathbf{T_n = a + (n − 1)d}$

   $T_{10} = 3 + (10 − 1) × 4 = 3 + 36 = \mathbf{39}$

   Answer: A

2) What is the sum of the first 20 natural numbers?

   A) 200   B) 190   C) 210   D) 220

   Solution:

   Sum of first n natural numbers = $\mathbf{n(n + 1)/2}$

   $S_{20} = 20 × 21 / 2 = \mathbf{210}$

   Answer: C

3) The 5th term of a geometric progression is 81 and the common ratio is 3 What is the first term?

   A) 1   B) 3   C) 9   D) 4

   Solution:

   $T_5 = ar^4 = 81$

   Let a be the first term: $a × 3^4 = 81 \Rightarrow a × 81 = 81 \Rightarrow a = \mathbf{1}$

   Answer: A

4) Find the sum of the first 5 terms of the G.P. 2, 4, 8, ...

   A) 62   B) 60   C) 64   D) 66

   Solution:

   a = 2, r = 2, n = 5

   Sum = $a(r^n − 1)/(r − 1) = 2(2^5 − 1)/(2 − 1) = 2(32 − 1)/1 = 2 × 31 = \mathbf{62}$

   Answer: A

5) In an A.P., if a = 2, d = 5, find the 12th term.

   A) 56   B) 57   C) 58   D) 59

   Solution:

   $T_{12} = a + (n − 1)d = 2 + (12 − 1) × 5 = 2 + 55 = \mathbf{57}$

   Answer: B

6) Which term of the A.P. 7, 13, 19, ... is 151?

   A) 25   B) 26   C) 27   D) 28

Solution:
$T_n = 7 + (n-1)6 = 151$
$(n-1)6 = 144 \Rightarrow n-1 = 24 \Rightarrow n = \mathbf{25}$
Answer: A

7) The sum of the first n terms of an A.P. is $3n^2 + 2n$. Find the first term.
   A) 5   B) 3   C) 2   D) 4
   Solution:
   $S_1 = 3(1)^2 + 2(1) = 3 + 2 = \mathbf{5}$
   Answer: A

8) If $a = 5$, $d = 3$, find the sum of first 10 terms.
   A) 180   B) 185 C) 175   D) 190
   Solution:
   $S_n = n/2[2a + (n-1)d] = 10/2\ [2 \times 5 + 9 \times 3] = 5[10 + 27] = 5 \times 37 = \mathbf{185}$
   Answer: B

9) In a G.P., the 3rd term is 24 and 6th term is 192. Find the common ratio.
   A) 2   B) 3   C) 4   D) 1.5
   Solution:
   $T_6 = ar^5 = 192$
   $T_3 = ar^2 = 24$
   Divide: $ar^5 / ar^2 = r^3 = 192 / 24 = 8 \Rightarrow r = 2$
   Answer: A

10) If the sum of a G.P. is 121 and $a = 1$, $r = 3$, find the number of terms.
    A) 4   B) 5   C) 3   D) 6
    Solution:
    $S_n = a(r^n - 1)/(r - 1) = (3^n - 1)/2 = 121$
    $\Rightarrow 3^n - 1 = 242 \Rightarrow 3^n = 243 \Rightarrow 3^n = 3^5 \Rightarrow n = \mathbf{5}$
    Answer: B

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:
A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
C) Assertion (A) is true but Reason(R) is false.
D) Assertion (A) is false but Reason(R) is true.

1) **Assertion (A):** The sequence 7, 7, 7, 7…… is both arithmetic and geometric.
   Reason(R) : A constant sequence has a common difference and a common ratio (other than zero).
   Answer: A

2) **Assertion (A):** The common difference in an Arithmetic Progression can be zero.
   Reason (R): In an AP, all terms must be different.
   Answer: C
   Solution: An AP can have a common difference of zero (e.g., 5, 5, 5, ...). But R is incorrect because all terms do not have to be different.

3) **Assertion (A):** If the nth term of a sequence is (2n+1), then the sequence is an AP.
Reason (R): The difference between consecutive terms is constant.
Answer: A
Solution: The difference, which is constant. Therefore, it is an AP, and R explains A.

4) **Assertion (A):** The series 2 +4 +8 + 16 +………diverges.
Reason (R): A geometric series is diverges if the common ratio is greater than 1.
Answer: A

5) **Assertion (A):** The sequence 1, 4, 9, 16…… is an AP.
Reason (R): If the difference between the two consecutive terms of a sequence is constant, then it is an AP.
Answer: D
Solution: The sequence is of perfect squares, and the difference between terms is not constant (3, 5, 7, ...). So A is false.

<div align="center">

VERY SHORT ANSWER TYPE QUESTIONS

</div>

1) Find the 10th term of the AP: 3, 7, 11, ...
Solution:
AP: a = 3, d = 4
T_{10} = a + (10 − 1)d = 3 + 9×4 = 39

2) How many terms are there in the AP: 5, 9, 13, ..., 77?
Solution:
a = 5, d = 4, T_n = 77
T_n = a + (n − 1)d ⇒ 77 = 5 + (n − 1)×4
(n − 1) = 18 ⇒ n = 19

3) Find the sum of first 20 terms of AP: 2, 4, 6, ...
Solution:
a = 2, d = 2
S_n = n/2 [2a + (n − 1)d] = 20/2 [4 + 38] = 10×42 = 420

4) What is the 5th term of the GP: 2, 4, 8, ...?
Solution:
a = 2, r = 2
T_5 = a·r^4 = 2×2^4 = 32

5) Find the common ratio of GP: 81, 27, 9, ...
Solution:
r = 27/81 = 1/3

6) If AM between x and y is 10, find x + y.
Solution:
AM = (x + y)/2 = 10 ⇒ x + y = 20

7) If GM between 4 and x is 8, find x.
Solution:
GM = √(4x) = 8 ⇒ 4x = 64 ⇒ x = 16

8) Find the sum of infinite GP: 5, 2.5, 1.25, ...

Solution:
a = 5, r = 0.5 < 1
S∞ = a / (1 – r) = 5 / 0.5 = 10

9) If AM and GM between two numbers are 10 and 8 respectively, find the numbers.
   Solution:
   Let numbers be a and b.
   AM = (a + b)/2 = 10 ⇒ a + b = 20
   GM = √(ab) = 8 ⇒ ab = 64
   Solving: a, b = roots of $x^2 – 20x + 64 = 0$ ⇒
   x = [20 ± √(400 – 256)]/2 = (20 ± √144)/2
   x = (20 ± 12)/2 ⇒ x = 16, 4
   Numbers: 16 and 4

10) Find the 6th term of GP: 1, –2, 4, ...
    Solution:
    a = 1, r = –2
    $T_6 = a·r^5 = 1×(–2)^5 = –32$

SHORT ANSWER TYPE QUESTIONS

Q 1 For the sequence a_n, $a_1 = \frac{1}{2}$, $a_2 = -3$ and $a_n = \frac{a_{n-1}}{a_{n-1}+\frac{1}{3}} - 3$, n>2 find $\frac{a_3}{a_4}$

Solution: n = 3, $a_3 = \frac{a_1}{a_2+\frac{1}{3}} - 3 = \frac{\frac{1}{2}}{-3+\frac{1}{3}} - 3 = -\frac{51}{16}$

n=4, $a_4 = \frac{a_2}{a_3+\frac{1}{3}} - 3 = \frac{-3}{-\frac{51}{16}+\frac{1}{3}} - 3 = -\frac{267}{137}$

$\frac{a_3}{a_4} = \frac{2329}{1424}$

Q 2 Insert 3 Arithmetic mean between 2 and 10.

Solution: Let three A.M. are A_1, A_2 and A_3

Then 2, A_1, A_2, A_3, 10 are in A.P.

2+4d = 10

d = 2

A_1= a+d =2+2=4

$A_2 = A_1$ +d=4+2=6

A_3= A_2+d=6+2=8

Q 3 Find the sum of the product of the corresponding terms of the sequences 2, 4, 8, 16, 32

and 128, 32, 8, 2, $\frac{1}{2}$.

Solution: required sum = 2 x 128 + 4 x 32 + 8 x 8 + 16 x 2 + 32 x $\frac{1}{2}$

= 256 + 128 + 64 + 32 + 16

$= 2^8 + 2^7 + 2^6 + 2^5 + 2^4$

$= \frac{2^8\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\frac{1}{2}} = 496$

Q 4 If the first and the nth term of a G.P. are a and b respectively and p is the product of n terms
, prove that $p^2 = (ab)^n$

Solution: First term = a

n^{th} term = b

$ar^{n-1} = b$

Also p = a × ar × ar^2 × ar^3 × …..n terms

$= a^n r^{1+2+3+4+ …….(n-1) \text{ terms}}$

$$= a^n r^{\frac{n(n-1)}{2}}$$

$$P^2 = a^{2n} r^{n(n-1)}$$

$$P^2 = (aar^{n-1})^n$$

$$P^2 = (ab)^n$$

Q 5 Find three numbers in G.P. whose sum is 7 and product is 8 .

Solution: Let three terms in G.P. are $\frac{a}{r}, a, ar$

$$\frac{a}{r} \times a \times ar = 8 \Longrightarrow a=2$$

$$\frac{a}{r} + a + ar = 7 \qquad (i)$$

Putting the value of a in equation (i) we get an equation

$$2r^2-5r+2 = 0$$

$$R = \frac{1}{2}, 2$$

Required numbers are 4, 2, 1 or 1, 2, 4

LONG ANSWER TYPE QUESTIONS

Q 1 Find the sum of the series: 0.7+0.77+0.777+0.7777+….. ……… to n terms

Solution: S = 7(0.1+0.11+0.111+0.1111+ ……………to n terms)

$$= \frac{7}{9}(0.9 + 0.99 + 0.999 + 0.9999 + \cdots \ldots \ldots to\ n\ terms)$$

$$= \frac{7}{9}\left\{\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \left(1 - \frac{1}{10000}\right) + \cdots \ldots to\ n\ terms \right\}$$

$$= \frac{7}{9}\left\{\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \left(1 - \frac{1}{10^4}\right) + \cdots \ldots to\ n\ terms \right\}$$

$$= \frac{7}{9}\left\{n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \frac{1}{10^4} + \cdots \ldots to\ n\ terms \right)\right\}$$

$$= \frac{7}{9}\left\{n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{\left(1-\frac{1}{10}\right)}\right\}$$

$$= \frac{7}{81}\left(9n - 1 + \frac{1}{10^n}\right)$$

Q 2 What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually.

Solution: We have principal = Rs 500, Rate =10%

Amount at the end of first year = $P\left(1 + \frac{R}{100}\right)$

Amount at the end of second year = $P\left(1 + \frac{R}{100}\right) + P\left(1 + \frac{R}{100}\right)\frac{R}{100}$

$$= P\left(1 + \frac{R}{100}\right)\left(1 + \frac{R}{100}\right)$$

$$= P\left(1 + \frac{R}{100}\right)^2$$

Amount at the end of third year = $P\left(1 + \frac{R}{100}\right)^2 + P\left(1 + \frac{R}{100}\right)^2\frac{R}{100}$

$$= P\left(1 + \frac{R}{100}\right)^3$$

and so on

We find that amounts at the end of various year form a G.P.

Amount at the end of 10th year = 11th term of G.P. = $P\left(1 + \frac{R}{100}\right)^{10}$

$$= 500\left(1 + \frac{10}{100}\right)^{10}$$

$$= 500\left(\frac{11}{10}\right)^{10}$$

Q 3 Find the natural number 'a' for which $\sum_{p=1}^{n} f(a + p) = 16(2^n - 1)$, where the function '$f$' satisfies the relation $f(x + y) = f(x).f(y) \ \forall$ natural numbers x, y and $f(1) = 2$.

Solution: Since $f(x + y) = f(x).f(y)$

Put x = 1, y = 1

$f(1 + 1) = f(1).f(1) = 2.2 = 2^2$

$f(2) = 2^2$

$f(3) = f(2 + 1) = f(2).f(1) = 2^2.2 = 2^3$

$f(3) = 2^3$

Continuing in this way, $f(n) = 2^n \quad \forall \quad n \in N$

Now $\sum_{p=1}^{n} f(a + p) = 16(2^n - 1)$

$f(a + 1) + f(a + 2) + f(a + 3) + \ldots + f(a + n) = 16(2^n - 1)$

$2^{a+1} + 2^{a+2} + 2^{a+3} + \ldots + 2^{a+n} = 16(2^n - 1)$

G.P., first term = 2^{a+1}, r = 2

$\frac{2^{a+1}(2^n - 1)}{2 - 1} = 16(2^n - 1)$

$2^{a+1}(2^n - 1) = 16(2^n - 1)$

$2^{a+1} = 16$

$2^{a+1} = 2^4$

a + 1 = 4

a = 3

CASE STUDY BASED QUESTIONS

1) A bus has rows of seats arranged in AP. The first row has 4 seats, and each successive row has 1 seat more than the previous.
    a. What is the number of seats in the 10th row?
    b. What is the common difference in the sequence?
    c. Find the total number of seats in the first 20 rows.

Solution: (a) no. of seats in 10th row is = a +9d

$= 4 + 9 \times 1 = 13$

(b) common difference in the sequence is 1.

(c) total number of seats in the first 20 rows $= (20/2)\{2 \times 4 + 19 \times 1\} = 10 \times (8 + 19)$
$= 270$ seats

2) A ball is dropped from a height of 100 m. Each time it bounces, it rises to 80% of the previous height.
   a. What is the height after the first bounce?
   b. What is the common ratio of the heights?
   c. Find the total vertical distance travelled by the ball after 3 bounces.

Solution: (a) height after the first bounce is $= 80\%$ of $100 = 80$ m.

(b) common ratio of the heights is $= 80 : 100 = 4 : 5$

(c) total vertical distance travelled by the ball after 3 bounces = Fall $+ 2 \times$ (Sum of bounces) $= 100 + 2 \times (80 + 64 + 51.2) = 100 + 2 \times 195.2 = 100 + 390.4 = 490.4$ m

3) The marks (out of 100) obtained by two students in a test are 36 and 64.
   a. What is the Arithmetic Mean of their marks?
   b. What is the Geometric Mean of their marks?
   c. Show that AM $\geq$ GM for these marks.

Solution:
   a. $(36 + 64)/2 = 50$
   b. $36 \times 64 ------ \sqrt{36 \times 64}$
      $= 6 \times 8 = 48$
   c. It is clear from above
      AM $\geq$ GM

HIGHER ORDER THINKING SKILL QUESTIONS

Question 1:
A person saves ₹50 in the first month, ₹60 in the second month, ₹70 in the third month, and so on. After how many months will his total savings be ₹6000?

Solution:
This forms an **AP** with
- First term (a) = 50
- Common difference (d) = 10
- Total savings (S_n) = 6000

We use the formula for the sum of the first **n** terms of an AP:
$S_n = n/2[2a + (n − 1)d]$
$6000 = n/2[2 \times 50 + (n − 1)10]$
$12000 = n[100 + (n − 1)10]$
$10n^2 + 90n – 12000 = 0$
$n^2 + 9n – 1200 = 0$
$n \sim 55.68$

Rounding down to the nearest whole month: **n = 55 months** (as total savings will cross ₹6000 between 55 and 56 months).

Question 2:
A ball is dropped from a height of 100 meters. It rebounds to 80% of the height from which it falls every time. What is the total vertical distance travelled by the ball before it comes to rest?

Solution:

- Initial fall = 100 m
- First rebound = 80 m

- Next fall = 80 m
- Rebound = 64 m

And so on... (all rebounds form a GP starting from second term onward)

Total distance = first fall + 2 × (sum of infinite GP)

= 100 + 2 × {80/(1-0.8)}= 100 + 2 × 400 = 900 m

EXERCISE

MULTIPLE CHOICE QUESTIONS

1) If a = 10, l = 100, n = 10 in an A.P., what is the sum of the terms?

   A) 500   B) 550   C) 600   D) 650

2) How many terms of the A.P. 5, 11, 17,... are needed for the sum to be 1190?

   A) 10   B) 15   C) 20   D) 17

3) If the 2nd and 5th terms of a G.P. are 6 and 48, find the 4th term.

   A) 24   B) 18   C) 36   D) 30

4) Find the common ratio if the 1st term is 729 and the 6th term is 1.

   A) 1/3   B) 1/2   C) 1/6   D) 1/9

5) Which of the following sequences is an arithmetic progression?

   A) 1, 4, 9, 16   B) 2, 4, 8, 16   C) 5, 10, 15, 20   D) 3, 6, 12, 24

ASSERTION - REASON BASED QUESTIONS

1) Assertion (A): The sequence 1, 3, 6, 10, 15….is not an AP.
   Reason (R): The common difference in the sequence keeps increasing.
2) Assertion (A): The sum of the terms in an arithmetic series can be negative.
   Reason (R): In AP, terms can be negative and decreasing.
3) Assertion (A): In an AP, if the first term is and the common difference is , the nth term is .
   Reason (R): Each term in an AP is formed by adding a fixed number to the previous term.

VERY SHORT ANSWER TYPE QUESTIONS

1) Find the 12th term of AP where a = 7 and d = 3.
2) Find the sum of first 5 terms of GP: 3, 6, 12, ...
3) Which term of AP: 4, 9, 14, ... is 94?
4) Insert 3 AMs between 5 and 17.
5) Insert 2 GMs between 2 and 16.

SHORT ANSWER TYPE QUESTIONS

1) If the AM between two numbers exceeds their GM by 5, and their product is 144, find the numbers.
2) Insert 3 arithmetic means between 4 and 20.
3) Insert 2 geometric means between 3 and 24.
4) Find the 10th term from the end of the AP: 8, 10, 12, ..., 100.
5) If AM and GM of two numbers are 10 and 8 respectively, find the numbers.

LONG ANSWER TYPE QUESTIONS

1. Find the sum of the series: 0.7+0.77+0.777+0.7777+….. …….. to n terms

2. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually.

CASE STUDY BASED QUESTIONS

1) Riya invests ₹10,000 in a scheme that offers 10% interest compounded annually.
   a. What is the amount after 1 year?
   b. What is the common ratio in the sequence of amounts?
   c. Find the total amount after 3 years.
2) A person's starting monthly salary is ₹25,000 and increases by ₹1,500 every year.
   a. What is the salary in the 3rd year?
   b. What is the common difference in the salary sequence?
   c. What is the total salary earned over 5 years?

HIGHER ORDER THINKING SKILL QUESTIONS

Q1. the arithmetic mean of two positive numbers is 10, what is the minimum possible value of their geometric mean?

ANSWERS

MULTIPLE CHOICE QUESTIONS

1. B   2. C   3. A   4. A   5. C

ASSERTION - REASON BASED QUESTIONS

1. A   2. A   3. A

VERY SHORT ANSWER TYPE QUESTIONS

1. 40   2. 93   3. 19th   4. 8, 11, 14   5. 4 and 8

SHORT ANSWER TYPE QUESTIONS

1. 16 and 18   2. 8, 12 and 16   3. 6 and 12   4. 82   5. 4 and 16

LONG ANSWER TYPE QUESTIONS

1. $\frac{7}{81}(9n-1+\frac{1}{10^n})$ 7819n−1+110n

2. 500

CASE STUDY BASED QUESTIONS

1. (a) ₹11,000

   (b) 1.1

   (c) ₹13,310

2. (a) ₹28,000

   (b) ₹1,500

   (c) ₹1,40,000

HIGHER ORDER THINKING SKILL QUESTION

Answer: The minimum GM is **just above 0**, approaching **0** but never exactly 0 as both numbers must be positive.

PERMUTATIONS & COMBINATIONS

GIST/SUMMARY OF THE LESSON(DEFINITION &FORMULAE)

FUNDAMENTAL PRINCIPLE OF COUNTING

The rule of sum: If a first task can be performed in m ways, while a second task can be performed in n ways, and the tasks cannot be performed simultaneously, then performing either one of these tasks can be accomplished in any one of total m+n ways.

The rule of product: If a procedure can be broken down into first and second stages, and if there are m possible outcomes for the first stage and if, for each of these outcomes, there are n possible outcomes for the second stage, then the total procedure can be carried out, in the designated order, in a total of m n ways.

FACTORIAL NOTATION: The number n! (read as "n-factorial") is defined as follows

For any positive integer n; n! = n(n – 1)(n – 2) …….. (3)(2)(1)

For instance, 4! = 4.3.2.1 = 24

Note:

(i)   n! = n(n – 1)(n – 2) ………… 3.2.1
(ii)   n! = n.(n – 1)!
(iii)   0! = 1! = 1
(iv)   (2n)! = 2^n .n![1.3.5.7 ……… (2n – 1)]
(v)   n! = n(n – 1)! = n(n – 1)(n – 2)! = n(n – 1)(n – 2) (n – 3)!
(vi)   n(n – 1) …….. (n – r + 1) = $\frac{n!}{(n-r)!}$

PERMUTATION: Each of the different arrangements, which can be made by taking some or all of a number of objects is called permutation. The number of permutations of n different objects taken r at a time is represented as n_{P_r}

n_{P_r} = n!/ (n -r)! $-$ = n(n – 1)(n – 2) ……… (n – r + 1) (where, 0 ≤ r ≤ n)

Note:

(i)   In permutation, the order of the items plays an important role.
(ii)   The number of all permutations of n distinct objects taken all at a time is n!

***The no. of permutations of n different objects taken r at a time when each object may be repeated any number of times is n^r

Permutation of Alike Objects

This kind of problems involve permutations of different objects in which some of them are similar. The number of permutations of n objects taken all at a time in which, p are alike objects of one kind, q are alike objects of second kind & r are alike objects of a third kind and the rest (n – (p + q + r)) are all different is n! /p!q!r!

Permutation under Restriction

(a) The number of permutations of n different objects, taken r at a time, when a particular object is to be always included in each arrangement, is r. $(n-1)_{p_{(r-1)}}$. The number of permutations of n different objects, taken r at a time, when a particular object is never taken in each arrangement is $(n-1)_{P_r}$

(b) **String method**: The number of permutations of n different objects, taken all at a time, when m specified objects always come together is m! × (n – m + 1)!.

(c) **Gap Method**: The number of permutations when no two given objects occur together.

In order to find the number of permutations when no two given objects occur together.

(a) First of all, put the m objects for which there is no restriction, in a line. These m objects can be arranged in m! ways.

(b) Then count the number of gaps between every two m objects for which there is no restriction, including the end positions. Number of such gaps will be (m + 1).

(c) If m is the number of objects for which there is no restriction and n is the number of objects, two of which are not allowed to occur together,

then the required number of ways = m! $\times (m+1)_{c_n} \times$ n!

The number of permutations when two types of objects are to be arranged alternately

(a) If their numbers differ by 1 put the object whose number is greater in the first, third, fifth.... places, etc. and the other object in the second, fourth, sixth.... places.

(b) If the number of two types of objects is same, consider two cases separately keeping the first type of object in the first, third, fifth places, etc. and the second type of object in the first, third, fifth places.... and then add

COMBINATION

Each of the different groups or selection which can be made by some or all of a number of given objects without reference to the order of the objects in each group is called a combination.

The number of all combinations of n objects, taken r at a time is generally denoted by

C(n, r) or nC_r =n! / r!(n – r)! (0 ≤ r ≤ n)

(a) The number of ways of selecting r objects out of n objects, is the same as the number of ways in which the remaining (n - r) can be selected and rejected.

(b) The combination notation also represents the binomial coefficient. That is, the binomial coefficient n Cr is the combination of n elements chosen r at a time.

(c)   n Cr = n Cn–r

(d) $^n C_r + {^n C_{r-1}} = {^{n+1}C_r}$

(e) n Cx = n Cy ⇒ x = y or x + y = n

(f) If n is even, then the greatest value of n Cr is n Cn/2

(g) If n is odd, then the greatest value of n Cr is $nC_{\frac{n+1}{2}}$

(h) n C$_0$ + n C$_1$ +................+ n C$_n$ = 2^n

Comparison of permutation and combination:

Permutations	Combinations
Different orderings or arrangement of the r objects are different permutations nP_r= n! /(n – r)! **Clue words**: arrangement, schedule, order	Each choice or subset of r object give one combination. Order within the group of r objects does not matter. nC_r = n! /(n – r)!r! **Clue words**: group, committee, sample, selection, subset

Combinations under Restrictions

(a) Number of ways of choosing r objects out of n different objects if p particular objects must be excluded then required number of ways = (n–p)Cr

(b) Number of ways of choosing r objects out of n different objects if p particular objects must be included (p ≤ r) then the required number of ways = (n–p)C_{r-p}

(c) The total number of combinations of n different objects taken one or more at a time = $2^n - 1$.

Combinations of Alike Objects

(a) The number of combinations of n identical objects taking (r ≤ n) at a time is 1.

(b) The number of ways of selecting r objects out of n identical objects is n + 1.

(c) If out of (p + q + r + s) objects, p are alike of one kind, q are alike of a second kind, r are alike of the third kind and s are different, then the total number of combinations is (p + 1)(q + 1)(r + 1)2^S– 1

<h3 style="color:red; text-align:center">MULTIPLE CHOICE QUESTIONS</h3>

1) If   nPr = 3024 then (n, r) is :

   (a) (4, 9)   b) (9, 4)   c) (9, 6)   d) (6, 9)

   ANS:(b) (9,6)

2) If 56P(r + 6) : 54P(r + 3) = 30800 : 1 then 'r' is

   a) 17   b) 38   c) 41   d) 52

   ANS:(c) 41

3) 1 + 1.1! + 2.2! + 3.3! +......+ n.n! is equal to :

   a) n!   b) (n – 1)!   c) (n+1)!   d) n

   ANS;(c) (n+1)!

   Lhs=1!+ 2!- 1!+3!-2!+3!-4!+------------+(n+1)!-n! =(n+1)!

4) The number of arrangements that can be formed out of 'LOGARITHM' so that

   no two vowels come together is

   a) 6! 7p3   b) 6! 7!   c) 6! 3!   d) 7! 3!

   ANS:(a)   6! 7p3

   First arrange consonants then arrange vowels in between. 6! 7p3

5) Everybody in a room shakes hand with everybody else. The total number of handshakes is 66. The total number of persons in the room is
   (a) 11   (b) 12   (c) 13   (d) 14

   ANS:(b) 12

   n C$_2$ =66 then   n(n-1)/2 =66 , n=12

6) If $^{15}C_{3r} = {}^{15}C_{r+3}$, then the value of r is

   a) 3   (b) 4   (c) 5   (d) 8

   ANS:(a) 3

   n Cx = n Cy ⇒ x = y or x + y = n ,

   3r+r+3=15 then r=3

7) $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 =$

   a) $^{47}C_6$ (b) $^{52}C_5$ (c) $^{52}C_4$ (d) None of these

ANS:(c) $^{52}C_4$

$$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

8) $^nC_r + 2{}^nC_{r-1} + {}^nC_{r-2} =$
   a) $^{n+1}C_r$ (b) $^{n+1}C_{r+1}$ (c) $^{n+2}C_r$ (d) $^{n+2}C_{r+1}$

   ANS:(c) $^{n+2}C_r$

$$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

9) If $^8C_r = {}^8C_{r+2}$, then the value of $^rC_2$ is
   (a) 8   (b) 3   (c) 5   (d) 2

   ANS:(b)3

     n Cx = n Cy $\Rightarrow$ x = y or x + y = n

10) In a football championship there were played 153 matches. Every two teams played one match with each other. The number of teams, participating in the championship is

   a) 14   b) 22   c) 18   d) 9

   ANS:(c) 18

   n C_2 =153 then   n(n-1)/2 =153 , n=18

11) If a polygon of n sides has 275 diagonals, then n is equal to

   a) 25   b) 35   c) 20   d) 15

   ANS(a) 25

   n C_2 -n =275 then n(n-3)=25$\times$ 22

   n=25

12). The number of sides of a polygon which has diagonals and its sides same

   a) 6   b) 5   c) 7   d) 3

   ANS:(b) 5

   n C_2 -n=n then n=5 pentagon

13). The H.C.F. of 6!, 8!, 9!, 11!. is

   (a) 6!   (b) 8!   (c) 9!   (d) 11!
   Sol. (a) 6!

14). A polygon has 14 sides then number of its diagonals are:
   (a) 91   (b) 77   (c) 28   (d) none of these
   Sol. (b) 77
   Let the no. of sides is n.
   Then no. of its diagonals is = $^{14}C_2$ – 14= 91 –14 = 77
15). If $^nC_5 = {}^nC_7$, find n.

(a) 12   (b) 15   (c)   14   (d)   18

Sol. (a) 7 +5 = 12

16). How many three digit numbers are there with all distinct digits1,2,3,4,5,6,7,8,9,0 ?

(a) 458   (b) 568   (c) 648   (d) 748

Sol. (c)9.9.8= 648

9	9	8

17). Teacher ask the student, suppose there are 10 boys and 8 girls in a class room, we want

select *either a boy or a girl* to represent the class   in a function. In how many ways the

selection can be made?

(a)10   (b) 8   (c) 18   (d) 80

Ans: (c) 18

Given that 10 boys and 8 girls

We are selecting 1 person, and it can be either a boy or a girl. Since the teacher can select

either a boy or a girl, we add the two results together

Total ways to select: 10 (boys)+8 (girls)=18

So, the selection can be made in $^{18}C_1$ =18 different ways.

18). Meera ask her friend Neeta that if we have a 4 flags of different colours, then how many

different signals can be generated if a signals   requires the   use of 2 flags one below the other?

(a)4   (b) 3   (c) 2   (d) 12

Ans : (d) 12

We are selecting and arranging 2 flags out of 4, where order matters.

This is: $^4P_2$=4×3=12

So, 12 different signals can be generated.

19). In a class room discussion teacher ask the students, tell me about how many *3- digits even numbers*
can be formed from the digits   1,2,3,4,5,6   if *no digit are repeated*?

(a) 60   (b) 85   (c) 108   (d) 98

Ans: (a) 60 =4x5x3

OR

Even digits from {1, 2, 3, 4, 5, 6} are: 2, 4, 6
So the last digit must be 2, 4, or 6 → 3 options

Count valid numbers by fixing the last digit

We will break it into cases based on the choice of the last digit (unit's place).

Case 1: Last digit = 2

Remaining digits to choose from: {1, 3, 4, 5, 6} → 5 digits

Choose and arrange 2 digits for the first two places (hundreds and tens):

Number of ways=P(5,2)=5×4=20

Case 2: Last digit = 4

Remaining digits: {1, 2, 3, 5, 6} → 5 digits

Ways = 5×4=20

Case 3: Last digit = 6

Remaining digits: {1, 2, 3, 4, 5} → 5 digits

Ways = 5×4=20

Total valid numbers=   20+20+20=60

20). In a class room discussion teacher ask the students, tell me about how many *3- digits even numbers* can be formed from the digits 1,2,3,4,5,6 if *digit are repeated*?

      (a)125   (b) 216   (c) 108   (d)98

Ans : (c) 108

  =6x6x3 =108

21). Sunita went to market with her friend Divya, on walking in the market Sunita see the Banner were it is written as 'LOGARITHMS'. Divya asked Sunita can you guess how many words with or without meaning can be formed out of the letters of the word 'LOGARITHMS'. If each letter are used once?

a)   10!   (b) 10!/2   (c) 9!   (d) 9!/2

Ans: (a) 10!   Since all letters are unique, the total number of distinct permutations is simply the factorial of the number of letters=10!

22). Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

(a) 2520   (b) 25200   (c) 252   (d) 25400

Ans: (b) 25200

Step 1: Choose 3 consonants from 7

The number of ways to select 3 consonants from 7 is given by the combination formula: C(7,3)=35

Step 2: Choose 2 vowels from 4

Similarly, the number of ways to select 2 vowels from 4 is: C(4,2)=6

Step 3: Arrange the selected 5 letters

Once we've selected 3 consonants and 2 vowels, we need to arrange these 5 letters. The number of ways to arrange 5 distinct letters is:

5!=5×4×3×2×1=120

Step 4: Calculate the total number of distinct words

Now, we multiply the results from each step: C(7,3)x C(4,2)x5! = 35×6×120=25,200

Therefore, 25,200 distinct words can be formed using 3 consonants and 2 vowels from a set of 7 consonants and 4 vowels, with each letter used exactly once.

23). If $(n + 1)C_3 = 2.nC_2$, then n = ?

   a)   3   (b) 4   (c) 5   (d) 6

Ans: (c) 5

  n=5   then LHS=RHS=10

24). Among 14 players, 5 are bowlers. In how many ways a team of 11 may be formed with *at least 4 bowlers*?

  a) 265   (b) 263   (c) 264   (d) 275

Ans : (c) 264

    We are given:

 Total players: 14 , Bowlers: 5, Non-bowlers: 14 - 5 = 9

 We need to form a team of 11 players with at least 4 bowlers

        Let's break it down by cases:

Case 1: 4 bowlers

   Choose 4 bowlers from 5: C(5,4)=5

 Choose 7 non-bowlers from 9:= C(9,7)=36

 Total no.of ways of selecting 4 bowlers = C(5,4)x C(9,7)=5x36=180

   Case 2: 5 bowlers

  Choose 5 bowlers from 5: C(5,5)=1

  Choose 6 non-bowlers from 9:C (9,6)=84

  Total no.of ways of selecting 5 bowlers = C(5,5)x C(9,6)=1x84=84

  Total number of ways: 180+84=264

25). The number of arrangements of the letters of the word BHARAT taking 3 at a time is

  (a)72   (b) 120   (c) 14   (d) None of these

   Ans :(a) 72

  The words can be formed out of the letters of the word 'BHARAT' taking 3 at a time can be done in 2 different cases : Case - 1 : When all the letters are distinct.

  We have, 5 distinct letters, out of which taking three at a time, the number of words that can be formed = 5P3 = 60

  Case-2 : When 2 A's are selected.

So, we have, 2A's and 1 letter is to selected out of the 4 distinct letters, which can be done in $^4P_1 = 4$ ways.

Now, the 3 letters can be arranged among themselves, but there are 2 A's,

so the number of ways in which arrangement can be done = 3 ! /2 ! = 3

So, in this case, total number of words that can be formed = 4 x 3 = 12.

The number of arrangements of the letters of the word BHARAT taking 3 at a time is = (60 + 12) = 72 ways

ASSERTION - REASON BASED QUESTIONS

Directions:

Each of the following questions consists of two statements: an Assertion (A) and a Reason (R). Answer them by selecting the correct option:

(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

1. Assertion (A): The number of ways to arrange the letters of the word "SCHOOL" is $\frac{6!}{2!}$

Reason (R): The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of kth kind and the rest, if any, are of different kind is $\frac{n!}{p_1!p_2!...p_k!}$.

*Answer: (a) Both A and R are true, and R is the correct explanation of A

2. Assertion (A): The number of 4-digit numbers formed using the digits 1, 2, 3, 4 without repetition is 4^4

Reason (R): Each of the 4 places can be filled by 4 digits.

* Answer: (d) Assertion is false, but Reason is true.

3. Assertion (A): C(n, r)=C(n, n−r) for all 0≤r≤n.

Reason (R): Choosing r objects from n is the same as choosing n−r objects to leave behind.

*Answer: (a) Both A and R are true, and R is the correct explanation of A.

4. Assertion (A): The number of ways of choosing 3 students out of 10 for a group is 120.

Reason (R): In combinations, the order of selection does not matter.

*Answer: (a) Both A and R are true, and R is the correct explanation of A.

5. Assertion (A): The number of permutations of 5 items taken 3 at a time is equal to $^5C_3$.

Reason (R): In permutations, the order of selection is not important.

* Answer: (b) Both A and R are true, but R is not the correct explanation of A.

6. Assertion (A): $12_{C_2} = 12_{C_{10}}$

Reason (R): Selection of the r distinct thing out of n is equal to the rejection of (n-r) thing out of n.

Ans-(a)

Solution:- Attributing to the reason, selection of 2 distinct thing out of 12,is equal to rejection of (12-2=10) thing out of 12, so $12_{C_2} = 12_{C_{10}}$

7. Assertion (A): 3! +4! = 7!

Reason (R): For any positive integer n, n! = 1×2×3…..×n and 0! =1

Ans-(d)

Solution - (R) is the definition of factorial which is true but assertion is false.

As $3! + 4! = (3 \times 2 \times 1) + (4 \times 3 \times 2 \times 1) = 6 + 24 = 30$

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 > 30$$

8. Assertion (A):-For any natural number $n, n_{C_n} = 1$

Reason (R): The number of ways of selecting r objects from a set of n objects, then mathematically it is given by $^nC_r = \frac{n!}{(n-r)!}$.

Ans (c)

Solution: $n_{C_r} = \frac{n!}{(n-r)!r!}$, $n_{C_n} = \frac{n!}{(n-n)!n!} = \frac{n!}{n!} = 1$

9. Assertion (A): $n_{P_r} = n_{C_r} \cdot r!$, $0 < r \leq n$

Reason(R): For each combination of ^nC_r , there are r! permutations.

Ans (a)

Solution: r objects in every combination can be arranged in r! ways. $n_{P_r} = n_{C_r} \cdot r!$, $0 < r \leq n$

10. Assertion (A):-The number of ways of choosing 4 cards from a pack of cards is $^{52}C_4$

Reason (R): permutation is an arrangement in a definite order of a number of objects taken some or all at a time.

Ans (b)

Solution: Selection of 4 card out of 52 can be made in 52_{C_4} ways. And (R) is the definition for permutation

SHORT ANSWER TYPE QUESTIONS

Q.1 How many number of rectangle forming by 5 different horizontal parallel lines and 7 other different vertical parallel line.
Solution: A rectangle forming by two horizontal and two vertical parallel lines
No. of rectangle = $^5C_2 \cdot {}^7C_2 = 10 \cdot 21 = 210$
Q.2 How many arrangement of the letters of the word APPLICATION.
Solution: Total letters 11, A = 2, P=2, I=2
No. of arrangements = $\frac{11!}{2!.\ 2!.\ 2!} = 4989600$
Q.3 There are 12 point on a circle. How many chord can be draw? Find number of intersection of all chord in the circle?
Solution: No. of chord = $^{12}C_2$
No. of intersection of chord = no. of quadrilateral can be made = $^{12}C_4$
Q4. Rashmita is playing with scrambles word cubes. She needs to form a word using all the letters of the word 'ORGANIC' without repetition of the letters.
(i) In how many ways vowels and consonants alternate between each other?

(ii)In how many words the all vowels are together?

Solution: (i)Vowels – O, A, I

Number of words =4! X 3! = 144

(ii)   O A I, R,G, N, C- 5 Places

No of ways to arrange = 5!

O A I , Ways to arrange = 3!

No of words beginning with Vowels = 5! x 3! = 120 x 6 = 720

Q5. Find the sum of the digits in the unit place of all the numbers formed with the help of 2,3,4,5 taken all at a time.

Solution: We are given the digits 2, 3, 4, 5, and

we're to Use all digits at a time to form numbers (so all 4-digit numbers with no repetitions),

To find the sum of the digits in the unit place (i.e., last digit) across all such numbers.

Total no.of numbers using all 4 digits with no repetition =4!

Since all digits are used equally and permutations are evenly distributed.

So each digit {2,3,4,5} will appear in the units place =24/4=6 times

Hence the Sum of digits in the unit place =6×(2+3+4+5)=6×14=84

Q6. Find the number of ways in which a team of eleven players can be selected from 22 players always including 2 of them   and excluding 3 of them.

Solution: Total number of players =22

We have to selected 11 players

We have to exclude 3, so 19 players are available.

Also from these 2 particular players are always included .

Therefore to select 9 players from remaining 17 players in $17c_9$ ways

Q7. Find the number of words which can be formed out of the letters of the word ARTICLE, so that vowels occupy the even place.

Solution: Vowels are A,I,E and consonants are R,T,C,L.

Now vowels occupy three even places (2nd, 4th and 6th) in ways.

In remaining four places four consonants can be arranged in 4! was

so, total number of words = 3! x 4! = 6 x 24 = 144 ways

Q8. How many different words can be formed by using all the letters of the word ALLAHABAD?

Solution: There are 9 letters in given word ,of which 4 are A'&,2 are L'&. So total number of words is the number of arrangements of 9 things of which 4 are similar of one kind , two are similar of one kind ∴

Total number of word = $\frac{9!}{4!2!}$specific.

Q9. There are 10 points on a circle. How many chords can be drawn by using these points?

Solution: One chord can be drawn by using two points. So total number of chord = number of ways of choosing two points out of 10.i.e 10_{C_2}

Q10.Find the total number of five digit numbers that can be formed by using the digits 0,1,2,3, ….,9

Solution: Total number of five digit numbers 9×10×10×10×10=90000

As 0 cannot be used as first place so it has 9 choices and rest all 4 places can be filled in 10 ways.

Q11. At an election, a voter may vote for any number of candidates, not greater than the number to be elected there are 8   candidates and 3 are to be elected. If a voter votes for at least one candidate, then find number of ways in which he can vote?

Solution: The number of ways in which a voter can vote is $^8C_1+^8C_2+^8C_3$ =8+28+56=92

Q12.Find the total number of ways of answering 6 objective type question, each question having 4 choices:

Solution: since each question can be answered in 4 ways ,So total number of ways of answering 6
question is $4×4×4×4×4×4 = 4^6$

Q13. There are 3 candidates for a classical ,4 for a social science and 2 for a natural science
scholarship In how many ways can these scholarship be   awarded?

Solution: Since, classical scholarship can be awarded to any one of three candidates. So awarding
the classical

scholarship. Similarly the social science and natural science scholarship can be awarded in 4 and 2
ways

respectively. So number of   ways 3×4×2=24

SHORT ANSWER TYPES QUESTIONS

1. In India Pakistan world cup match. BCCI decided to choose 11 players from 14 eligible players
in which 5 are bowlers, 4 are batsman. In   how many   ways a team of 11 may be formed with
(a) at least 4 bowlers?   (b) Exactly 4 batsman?   (c)   3 bowlers and 2 batsman?

 Solution: (a)Number of ways chosen at least 4 bowlers

 Given 5 Bowlers 9 other cricketers

 Case I : 4 bowlers & 7 other = $5c_4$ x $9C_7$ =180 ways

 Case II: 5 bowlers & 6 other = $5c_5$ x $9C_6$ =84 ways

Total number of ways = 180+84 =264 ways

 (b)   Exactly 4 batsman?

 Given 4 batsman & 10 others

 Number of ways =$4C_4$ x $10C_7$ =120 ways

 (c) 3 bowlers and 3 batsman?

 Number of ways =$5C_3$ x $4C_3$ x $5C_5$ = 40 ways

2. In a school group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be
selected if the team has
 (i)   No girl?
 (ii) At least one boy and one girl?
 (iii) At least three girls?

(i) **Solution:** No girl.

Number of ways = $7C_5$= 21 ways

(ii)   At least one boy and one girl?

Number of ways =$4C_4$ x $7C_1$ + $4C_3$ x $7C_2$ + $4C_2$x $7C_3$ + $4C_1$ x $7C_4$= 441 ways

(iii)   At least three girls?

Number of ways = $4C_3$ x $7C_2$ + $4C_4$x $7C_1$ = 91 ways

3. From a college of 35 students, 15 students are to be chosen for an excursion party. Ravi, Shyam
and Raju are best friends. They decided that *either we three of them go* or *none of them go for
excursion*. In how many ways can the excursion party be chosen?

Solution: There are two cases

Case I: *when three of them chosen for party*

Rest students chosen by = $^{32}C_{12}$ ways

Case II : *when three of them not chosen for party*

Number of ways 15 students chosen = $^{32}C_{15}$ ways

Total Number of ways =$^{32}C_{12}$ +$^{32}C_{15}$ ways

4.Find the number of different words that can be formed from the letters of the word
INTERMEDIATE such that two vowels never come together ?_.

[Hint: Number of ways of arranging 6 consonants of which two are alike is $\frac{6!}{2!}$ and number of ways of arranging vowels $= 7P_6 \times \frac{1}{3!} \times \frac{1}{2!}$]

Solution: Letters of the word INTERMEDIATE are: Vowels (I, E, E, I, A, E) and consonants (N, T, R, M, D, T)

Now we have to arrange these letters if no two vowels come together.

So, first arrange six consonants in $\frac{6!}{2!}$ ways.

Arrangement of six consonants creates seven gaps.

Six vowels can be arranged in these gaps in $7C_6 \times \frac{6!}{2!3!}$ ways.

So, total number of words $\frac{6!}{2!} \times 7C_6 \times \frac{6!}{2!3!} = 360 \times 7 \times 60 = 151200$

5. Four persons entered the lift cabin on the ground floor of 7 floor house suppose each of them can leave the cabin independently at any floor beginning with the first. What is the total number of ways in which each of the four persons can leave the cabin at any of the six floor?

Solution: Suppose that A, B, C, D are 4 persons .

A can leave the cabin at any of the 6 floors. So A can leave the cabin in 6 ways,

Similarly each of B, C and D can leave the cabin in 6 ways .

Therefore the total number of ways in which each of the 4 persons can leave the cabin at any of the six floor $= 6 \times 6 \times 6 \times 6 = 1296$.

6. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

Solution: Let us denote married couples by S_1, S_2, S_3 where each couple is considered to be a single unit as shown in the following figure:
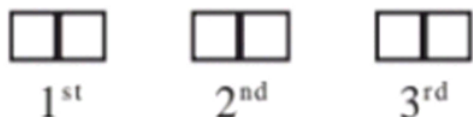
Then the number of ways in which spouses can be seated next to each other is 3! = 6 ways.

Again each couple can be seated in 2! ways.

Thus the total number of seating arrangement so that spouses sit next to each other = 3! × 2! × 2! × 2! = 48.

Again, if three ladies sit together, then necessarily three men must sit together.

Thus, ladies and men can be arranged altogether among themselves in 2! ways.

Therefore, the total number of ways where ladies sit together is 3! ×3! × 2! = 144.

7. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if. (i) 4 letters are used at a time   (ii) all letters are used at a time,

 (iii) all letters are used but first letter is a vowel?

Solution: The total no. of letters in the word MONDAY = 6 = n.

We know that the number of ways of selecting and arranging r different things from n different things is a permutation and we calculate it using the ^nP_r formula.

(i) Number of 4-letter words that can be formed from the letters of the word MONDAY,

 without repetition of letters $= {}^6P_4$

   = 6!/(6 - 4)! = 6!/2! = 360.

(ii) Number of words that can be formed from the letters of the word MONDAY, if all letters are used at a time $= {}^6P_6$ = 6!/(6 - 6)! = 6!/0! = 720.

(iii) Number of words that can be formed from the letters of the word MONDAY, if all letters are used but the first letter is a vowel $= 2 \times 5 \times 4 \times 3 \times 2 \times 1 = 240$

8. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8 books, he does not want to borrow chemistry part-II, unless chemistry part-I is also borrowed. In how many ways can he choose the three books to be borrowed?

Solution: we need to consider the restrictions on borrowing Chemistry Part II. Let's denote the books as follows:

- Let C1 = Chemistry Part I
- Let C2 = Chemistry Part II
- Let B1,B2,B3,B4,B5,B6 = the other 6 books

We have a total of 8 books: C1,C2,B1,B2,B3,B4,B5,B6.

We will break this problem into cases based on whether the boy borrows Chemistry Part I and/or Chemistry Part II.

Case 1: Neither C1 nor C2 is borrowed.

In this case, the boy can choose any 3 books from the 6 remaining books B1,B2,B3,B4,B5,B6.

The number of ways to choose 3 books from 6 is given by the combination formula: $n_{C_r} = \dfrac{n!}{(n-r)!r!}$

So, the number of ways is: $6_{C_3} = \dfrac{6!}{(6-3)!3!} = 20$

Case 2: C1 is borrowed and C2 is not borrowed.

In this case, the boy can choose 2 more books from the 6 remaining books B1,B2,B3,B4,B5,B6.

The number of ways to choose 2 books from 6 is : $6_{C_2} = \dfrac{6!}{(6-2)!2!} = 15$

Case 3: Both C1 and C2 are borrowed.

In this case, the boy can choose 1 more book from the 6 remaining books B1,B2,B3,B4,B5,B6.

The number of ways to choose 1 book from 6 is : $6_{C_1} = \dfrac{6!}{(6-1)!1!} = 6$

Add the number of ways from all cases

Now, we can add the number of ways from all three cases:

Total ways=Case 1+Case 2+Case 3=20+15+6=41

Therefore, the total number of ways the boy can choose 3 books to borrow is 41.

9. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag, if

(i) they can be of any colour

(ii) two must be white and two red and

(iii) they must all be of the same colour.

Solution: Total number of marbles = 6 white +5 red = 11 marbles

(a) If they can be of any colour means we have to select 4 marbles out of 11

∴Required number of ways = $^{11}C_4$

(b) Two white marbles can be selected in $^6C_2$

Two red marbles can be selected in 5_{C_2} ways.

∴Total number of ways = $^6C_2 \times {}^5C_2 = 15 \times 10 = 150$

(c) If they all must be of same colour,

Four white marbles out of 6 can be selected in $^6C_4$ ways. And 4 red marbles out of 5 can be selected in $^5C_4$ ways.

∴Required number of ways = $^6C_4 + {}^5C_4 = 15 + 5 = 20$

10. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected, if the team has (i) no girls   (ii) at least one boy and one girl   (iii) at least three girls

Solution: Number of girls = 4;

Number of boys = 7

We have to select a team of 5 members provided that

(i). Team having no girls

Required number of ways = $^7C_5 = \frac{7 \times 6}{2!} = 21$

(ii) Team having at least one boy and one girl

∴A Required number of ways $= 7C_1 \times 4C_4 + 7C_2 \times 4C_3 + 7C_3 \times 4C_2 + 7C_4 \times 4C_1$

$= 7 \times 1 + 21 \times 4 + 35 \times 6 + 35 \times 4 = 7 + 84 + 210 + 140 = 441$

(iii) Team having at least three girls

∴Required number of ways $= 4C_3 \times 7C_2 + 4C_4 \times 7C_1 = 4 \times 21 + 7 = 84 + 7 = 91$

<u>LONG ANSWER TYPES QUESTIONS</u>

1. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these are

    (i) four cards are of the same suit?

    (ii) four cards belong to four different suits?

    (iii) four cards are face cards?

    (iv) two are red cards and two are black cards?

    (v) cards are of the same colour?

Solution: (i) There are four suits (diamond, spade, club and heart) of 13 cards each.

Therefore, there are $^{13}C_4$ ways of choosing 4 diamond cards, $^{13}C_4$ ways of choosing 4 club cards, $^{13}C_4$ ways of choosing 4 spade cards and $^{13}C_4$ ways of choosing heart cards.

Required number of ways $^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 = 4 \times {}^{13}C_4 = 4 \times \frac{13!}{9! X 4!} = 2860$

(ii) There are 13 cards in each suit. Four cards drawn belong to four different suits means one card is drawn from each suit. Out of 13 diamond cards one card can be drawn in $^{13}C_1$ ways. Similarly, there are $^{13}C_1$ ways of choosing one club card, $^{13}C_1$ ways of choosing one spade card and $^{13}C_1$ ways of choosing   one heart card.

  Number of ways of selecting one card from each suit = $^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

(iii) There are 12 face cards out of which 4 cards can be chosen in $^{12}C_4$ ways.

    Required number of ways = $^{12}C_4 = 495$ ways

(iv) There are 26 red cards and 26 black cards. Therefore, 2 red cards can be chosen in $^{26}C_2$ ways and 2

    black cards can be chosen in $^{26}C_2$ ways. Hence, 2 red and 2 black cards can be chosen in $^{26}C_2 \times {}^{26}C_2 = 105625$ ways.

    (v) Out of 26 red cards, 4 red cards can be chosen in $^{26}C_4$ ways. Similarly, 4 black cards can be chosen in $^{26}C_4$ ways.

    Hence, 4 red or 4 black cards can be chosen in $^{26}C_4 + {}^{26}C_4 = 2 \times {}^{26}C_4 = 29900$ ways

2. Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

    i) do the words start with P   ii) do all the vowels always occur together

iii) do all the vowels never occur together   iv) do the words begin with I and end in P ?

Solution: To solve the problem of finding the number of arrangements of the letters in the word "INDEPENDENCE" and answering the specific questions, we will follow these steps:

Calculate the total number of arrangements of the letters in "INDEPENDENCE".

The word "INDEPENDENCE" consists of 12 letters where:

- I appears 1 time , - N appears 3 times , - D appears 2 times ,- E appears 4 times, - P appears 1 time,   - C appears 1 time

The formula for the total arrangements of letters when there are repetitions is given by:

Total arrangements $= \dfrac{n!}{p_1! p_2! \dots p_k!}$.

Where n is the total number of letters, and $p_1, p_2, \dots$ are the frequencies of the repeated letters.

Thus, we have: Total arrangements $= \dfrac{12!}{3! 2! 4!}$

Calculating this gives: Total arrangements=1663200

(i) Find the number of arrangements that start with P.

If the arrangement starts with P, we fix P in the first position.

The remaining letters are I, N, N, D, E, D, E, N, C, E (11 letters in total).

The number of arrangements of these 11 letters is:

Arrangements$= \dfrac{11!}{3! 2! 4!}$=13860

(ii) Find the number of arrangements where all vowels occur together.

The vowels in "INDEPENDENCE" are I, E, E, E, E. We can treat these vowels as a single entity or block.

Thus, we have the block (IEEEE) and the consonants   N, D, N, D, N, C (7 letters in total).

Now we have 8 entities to arrange (the block and the consonants):

Arrangements $= \dfrac{8!}{3! 2!}$=3360

Now, we also need to arrange the vowels within the block:

Vowel arrangements $= \dfrac{5!}{4!}$=5

Thus, the total arrangements where all vowels are together is: 3360 x 5 = 16800

(iii) Find the number of arrangements where vowels never occur together.

To find this, we can use the total arrangements and subtract the arrangements where vowels are together:

Arrangements where vowels never together = Total arrangements − Arrangements where vowels together

Calculating this gives: 1663200 – 16800 =1646400

(iv) Find the number of arrangements that begin with I and end with P.

If the arrangement starts with I and ends with P, we fix I at the start and P at the end.

The remaining letters are N, N, D, E, D, E, N, C, E (10 letters in total).

The number of arrangements of these 10 letters is:

Arrangements $= \dfrac{10!}{3! 2! 4!}$ = 12600

3. There are 10 professors and 20 students out of whom a committee of 2 professors and 3 students is to be formed. Find the number of ways in which this can be done. Further find in how many of these committees:

   (i)A particular professor is included

   (ii)A particular student is included

   (iii)A Particular student is excluded

Solution: We are given: 10 professors → choose 2

20 students → choose 3

Total number of committees

Ways to choose 2 professors from 10: $^{10}C_2 = 45$

Ways to choose 3 students from 20: $^{20}C_3 = 1140$

Total number of committees: $45 \times 1140 = 51300$

(i) A particular professor is included

Fix 1 specific professor. Now choose 1 more professor from the remaining 9, and 3 students from 20.

Professors: $^9C_1 = 9$

Students: $^{20}C_3 = 1140$

Total: $9 \times 1140 = 10260$

(ii) A particular student is included

Fix 1 specific student. Now choose 2 more students from the remaining 19, and 2 professors from 10.

Students: $^{19}C_2 = 171$

Professors: $^{10}C_2 = 45$

Total: $171 \times 45 = 7695$

(iii) A particular student is excluded

Fix 1 specific student to be not selected. Now choose 3 students from the remaining 19, and

2 professors from 10.

Students: $^{19}C_3 = 969$

Professors: $^{10}C_2 = 45$

Total: $969 \times 45 = 43605$

4. Find the number of permutation of the letters of the word ALLAHABAD. In how many of these permutation

   (i)   All the vowels always occur together

   (ii)   The vowels never occur together

Solution: We are given the word: ALLAHABAD.

Count total letters and repetitions Word: A L L A H A B A D

No of letters = 9

Total Repetitions: A appears 4 times, L appears 2 times ,H, B, D appear once each

Total permutations of ALLAHABAD

When letters repeat, total permutations are given by:

Total permutations = $9!/4! \cdot 2! = 7560$

(i) Vowels always occur together

Vowels: A, A, A, A , Consonants: L, L, H, B, D → 5 consonants

Treat the 4 A's (vowels) as one block. So now we have:1 vowel block & 5 consonants

That's 6 items to permute: vowel-block + 5 consonants (L, L, H, B, D)

Ways to arrange 6 blocks = $6!/2! = 720/2 = 360$ (2! because L repeats twice)

Also, within the vowel block, we have 4 A's → only 1 way to arrange them (all same).

So total permutations with vowels together $= 360 \times 1 = 360$

(ii) Vowels never occur together

This is:Total permutations − Permutations where vowels are together

      $= 7560 - 360 = 7200$

Total permutations − Permutations where vowels are together $= 7560 - 360 = 7200$

5. Arrangement of the letters of the word INSTITUTIONS

  then find (i)Total arrangement.   (ii) If all vowels come together.

(iii) If no vowels come together   (iv) All vowels come together and all consonant come together.

Solution: Total letters = 12

Let's count the frequency of each letter: 3 I's ,2 N's ,2 S's ,3 T's

Vowels = I, U, O → Total vowels = 3 (I) + 1 (U) + 1 (O) = 5 vowels

Consonants = N, S, T → Total consonants = 12 - 5 = 7 consonants

(i) Since some letters repeat, we divide by the factorials of the counts of repeated letters:

Total arrangements$=\dfrac{12!}{2!2!3!3!}$.=3326400

(ii) Treat all vowels (I, I, I, U, O) as a single block.

So we now arrange:1 "vowel block" ,7 consonants: N, N, S, S, T, T, T

Total blocks to arrange = 8 blocks

Step 1: Arrangement of 8 blocks (vowel block + consonants)

Arrangements of blocks$=\dfrac{8!}{2!2!3!}$=1680

Step 2: Arrangements within the vowel block (I, I, I, U, O)= $\dfrac{5!}{3!}$=20

Total arrangements if all vowels occur together = 1680 × 20 =33600

(iii) <u>Number of arrangements if no vowels come together</u>

This is more complex. We'll use inclusion-exclusion:

Total arrangements (from part i) = 3326400

We subtract arrangements where **at least 2 vowels are together**

But here, we want the number where **no vowels are adjacent**.

Instead, it's easier to:

Step 1: Arrange consonants first (7 consonants)

Consonants = N (2), S (2), T (3)

Consonant arrangements$=\dfrac{7!}{2!2!3!}$=210

Now, place vowels in the gaps between consonants so that no vowels are adjacent.

There are 8 possible positions for vowels (i.e., between and around consonants):

_ C _ C _ C _ C _ C _ C _ C _

From these 8 gaps, choose 5 to place the 5 vowels. $^8C_5$=56

Now arrange the vowels (I, I, I, U, O): $\dfrac{5!}{3!}$ =20

Total = 210 × 56 × 20 = 235200

(iv) <u>All vowels come together and all consonants come together</u>

We treat vowels and consonants each as a block.

So we only arrange:[Vowel block][Consonant block] or [Consonant block][Vowel block] → 2 ways

Step 1: Arrange vowels (I, I, I, U, O) = $\dfrac{5!}{3!}$ =20

Step 2: Arrange consonants (N, N, S, S, T, T, T) =$\dfrac{7!}{2!2!3!}$=210

Step 3: Multiply with 2 arrangements of blocks

Total no.of arrangements in which all vowels come together and all consonants come together
 =2×20×210 =8400

CASE STUDY BASED QUESTIONS

1. Republic day is a national holiday of India. It honours the date on which the constitution of India came into effect on 26 January 1950 replacing the Government of India Act (1935) as the governing document of India and thus, turning the nation into a newly formed republic.

Answer the following question, which are based on the word "REPUBLIC"

(i) Find the number of arrangements of the letters of the word 'REPUBLIC"?

(ii) How many arrangements start with a vowel?

(iii) If the number of arrangements of the letters of the word 'REPUBLIC" is abcde , then find the value of (a + b +c +d +e)

OR

(i) If the number of arrangements start with a vowel is abcde, then find the value of (a + b)- (d + e)

Solution: (i) The letters in the word 'REPUBLIC' are all distinct.

There are 8 letters in the given word. So, the number of arrangements are 8! i.e. 40320.

(ii)The vowels in a given word are 'E, I, U'. If we start a word from vowel, we can choose 1 vowel from 3 vowels in $^3C_1$ ways.

Further, remaining 7 letters can be arranged in 7! ways.

Total number of arrangements start with a vowel = $^3C_1$ x 7! = 3 x 5040 = 15120

(iii)Since the number of arrangements are 40320. On comparing, we get a = 4, b = 0, c = 3, d = 2 and e = 0

So, a + b + c + d + e = 4 + 0 + 3 + 2 + 0 = 9

OR

(iv)Since, number of arrangements is 15120 =abcde

On comparing, we get a = 1, b = 5, c = 1 ,d = 2 and e = 0

(a + b) - (d + e) = (1 + 5) - (2 + 0) = 6 - 2 = 4

2. In an examination, a question paper consists of 12 questions divided into two parts i. e Part l and Part **II** containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part.

Based on the information answer the following In how many ways can a student select the questions in such a way that

(i) 3 questions from part I and 5 questions from part II

(ii) 4 questions from part I and 4 questions from part II

(iii) 5 questions from part I and 3 questions from

OR

(iv) In how many ways can a student select the questions

Solution: It is given that the question paper consists of 12 questions divided into two parts - Part I and Part II, containing 5 and 7 questions, respectively. A student has to attempt 8 questions, selecting at least 3 from each part.

This can be done as follows.

i)3 questions from part I and 5 questions from part II can be selected in $^5C_3 \times ^7C_5$ ways.

ii) 4 questions from part I and 4 questions from part II can be selected in
$^5C_4 \times {}^7C_4$ ways.

iii) 5 questions from part I and 3 questions from part II can be selected in
$^5C_5 \times {}^7C_3$ ways

iv) Thus, required number of ways of selecting questions

$$= 5C_3 \times 7C_5 + 5C_4 \times 7C_4 + 5C_5 \times 7C_3$$

$$= \frac{5!}{2!3!} \times \frac{7!}{2!5!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{3!4!} = 210 + 175 + 35 = 420$$

3. A bag contains six white marbles and five red marbles. Four marbles can be drawn from the bag .
Based on the information answer the following .Find the number of ways in which

(i) They can of any colour

(ii) Two must be white and two red

OR

(iii) They must be all of same colour

Solution: (i) $6C_4 + 6C_3 \times 5C_1 + 6C_2 \times 5C_2 + 6C_1 \times 5C_3 + 5C_4 = 330$

(ii) $6C_2 \times 5C_2 = 150$

(iii) $6C_4 + 5C_4 = 20$

<div align="center"><strong style="color:red">HIGHER ORDER THINKING SKILLS</div>

1. Find rank of the word MONDAY in the dictionary wise all arrangements of word MONDAY,
also find the word which will on the rank 400.

Solution: Arrange alphabetically A,D,M,N,O,Y

Starting from A _ _ _ _ _ can be arrange by = 5! =120
Starting from D _ _ _ _ _ can be arrange by = 5! =120
Starting from M A _ _ _ _ can be arrange by = 4! =24
Starting from M D _ _ _ _ can be arrange by = 4! =24
Starting from M N _ _ _ _ can be arrange by = 4! =24
Starting from M O A _ _ _ can be arrange by = 3! = 6
Starting from M O D _ _ _ can be arrange by = 3! = 6
Starting from M O N A _ _ can be arrange by = 2! = 2
Starting from M O N D A Y can be arrange by = 1! = 1
Rank =120+120+24+24+24+6+6+2+1 = 327

		rank
Starting from A _ _ _ _ _ can be arrange by = 5! =120		120
Starting from D _ _ _ _ _ can be arrange by = 5! =120		240
Starting from M _ _ _ _ _ can be arrange by = 5! =120		360
Starting from N A _ _ _ _ can be arrange by = 4! =24		384
Starting from N D A _ _ _ can be arrange by = 3! =6		390
Starting from N D M _ _ _ can be arrange by = 3! =6		396
Starting from N D O A _ _ can be arrange by = 2! =2		398
Starting from N D O M _ _ can be arrange by = 2! =2		**400**

2.Find the sum of all 4 digit numbers formed using the digits 1, 2, 4 and 6.

Solution: Use formula, Sum = (a1 + a2 + a3 + … + an) (n 1)! − (111 …. N times)

Using formula, Sum = (1 + 2 + 4 + 6) 3! (1111) = 13 × 6 × 1111 = 86658

Alternate: Here, the total 4-digit numbers will be 4! = 24.

So, every digit will occur 6 times at every one of the four places.

Since the sum of the given digits = 1 + 2 + 4 + 6 = 13.

So, the sum of all the digits at every place of all the 24 numbers = 13 × 6 = 78.

The sum of the values of all the digits At first place = 78 At the tens place = 780

At the hundreds place = 7800

At the thousands place = 78000 ∴

The required sum 78 + 780 + 7800 + 78000 = 866582

EXERCISE
MULTIPLE CHOICE QUESTIONS

Q.1 Number of words from the letters of the word **BHARAT** in which B and H will never come together is:

   (a) 210   (b) 240   (c)   422   (d) 400

Q.2 here are 10 true-false questions in a examination. If all questions are compulsory then these questions can be answered in:

   (a) 210   (b) 512   (c)   422   (d) 1024

Q.3   If $^{15}C_r : ^{15}C_{r-1} = 11:5$ then r equals

   (a) 15   (b) 11   (c)   5   (d) 4

Q.4 .A polygon has 35 diagonals. Find the number of its sides.

(a) 7   (b) 16   (c)   10   (d)   70

Q.5.Number of rectangles in a chess board

 (a)   1296 (b) 1196 (c) 204   (d) 536

CASE STUDY BASED QUESTIONS

1.In a company, CEO wants to establish a new branch. New branch required a committee of 5 members is to be formed out of 6 gents and 4 ladies.

 In how many ways this can be done, when

i. At least two ladies are included?

ii. ii. At most two ladies are included?

2.Read the following passage and answer the questions given below.

    Every person has Independence thought.

Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,

(i) Do the words start with P

(ii)   Do all the vowels always occur together

(iii) Do the vowels never occur together

OR

Do the words begin with I and end in P?

3.Read the following passage and answer the question given below.

The longest river of North America is ***Mississippi River***

In how many ways can the letters of the word MISSISSIPPI be arranged Such that

(i) All letters are used

(ii)   All I's are together

(iii)   All I's are not together

        OR

All S 's are not together

SHORT ANSWER TYPE QUESTIONS

Q.1 If $^{10}P_r = 5040$, find the value of r

Q.2 If $5.^4C_r = 6.^5C_{r-1}$, find value of r.

Q.3 How many 6-digit number can be formed from the digits 0,1,3,5,7,9 which are divisible by 10 and no digit is repeated?

Q.4 How many words can be formed by using the letters of the word ORIENTAL, so that the vowels always occupy the odd places?

Q.5 How many squares in a chess board?

Q.6 How many palindromes of 5 letters can be made by using letters of the word MATHS?

Q.7 It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Q.8 Given 12 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Q.9 There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?

Q.10 In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

LONG ANSWER TYPE QUESTIONS

Q.1 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?

Q.2 Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 50th word?

Q.3 In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Q.4 How many number of signals that can be sent by 6 flags of different colours taking one or more at a time?

Q.5 A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and at least 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

ANSWERS OF EXERCISE QUESTIONS

MULTIPLE MCHOICE QUESTIONS

Q.1 B   Q.2 D   Q.3 C   Q.4 C Q.5 A

CASE STUDY BASED QUESTIONS

Q.1 (i) 186 (ii) 186

Q.2 (i) 138600   (ii) 16800 (iii) 1663200 – 16800 = 1646400 OR 12600

Q.3 (i) 34650   (ii) 840   (iii) 33810   OR   33810

SHORT ANSWER TYPE QUESTIONS

Q.1 4 Q.2 3   Q.3 120   Q.4 576   Q.5 204

Q.660   Q.7 2880   Q.8 132   Q.9 72   Q.10 63

LONG ANSWER TYPE QUESTIONS

Q.1 (i) 21 (ii) 441 (iii) 91   Q.2 60, NAAIG   Q.3 420   Q.4   1956

Q.5 $2(^{20}C_5 \times {}^{20}C_6)$

UNIT – III

MATHEMATICAL REASONONG

Gist/Summary of the lesson:

Mathematical reasoning in class XI Applied Mathematics pertaining to verbal and non-verbal reasoning. Here logical reasoning involve Coding Decoding, blood relation, odd man out & Syllogism.

Definitions and Formulae:

Definitions:-

1. **Coding decoding:** Coding is a rule for converting the information in the form of letters, word and phrases into the same or other form of representation.

   Decoding is the reverse process of coding. It converts the message sent by a source into the information which can be understood by the receiver.

2. **BLOOD RELATION:** Any relation which is either by birth or by marriage is called blood relation.
3. **Odd Man Out:** All the terms (Alphabets, words, numbers etc) except first, follow a specific rule or pattern.
4. **SYLLOGISM:** Syllogism is a kind of a logical argument in which a conclusion is drawn from two or more given propositions or statements that are assumed to be true.

   Formulae

- Different patterns are here for LETTER coding (1) Repetition (2) Addition-subtraction (3) Multiplication pattern
- Symbolic representation of a family tree
- ODD MAN OUT (Alphabets problems, word problems, Numbers problems)
- Venn diagrams are using for solving SYLLOGISM problems.

MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)

1) In the given diagram, square represents students learning computers, triangle represents students learning swimming and circle represents students preparing for competitions.
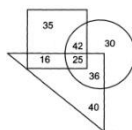
What percentage of students are preparing for competitions only?

a) 30%   b) 13.39%   c) 10%   d) 25%

Solution:

$$\frac{30}{35 + 16 + 42 + 25 + 30 + 36 + 40} \times 100 = \frac{3000}{224} = 13.39\%$$

Answer: b) 13.39%

2) Meenakshi is Kirti's sister. Kaavya is Kirti's mother. Dipesh is Kaavya's father. Esha is Dipesh's mother. Then how is Meenakshi related to Dipesh?

a) Granddaughter   b) Grandfather   c) Daughter   d) Grandmother

Solution:

Meenakshi is Kirti's sister.

Kavya is Kirti's mother and so Meenakshi also Dipesh is Kavya's father, so Meenakshi is grand-daughter of Dipesh.

Answer: a) Granddaughter

3) Radha is daughter of Prem. Mona is sister of Brij who is son of Prem. How Mona is related to Radha?

   a) Aunt   b) Sister   c) Niece   d) Cousin

Solution:
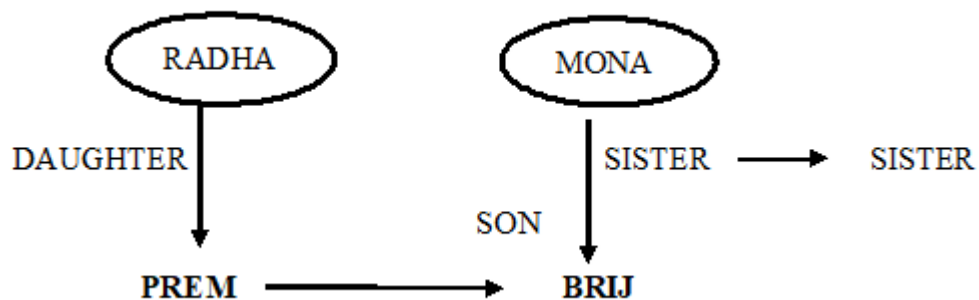
Answer: b) Sister

4) In a certain code language, FOUL is written as HNVO, in the same code language HURT is written as

   a) JTWS   b) ITSW   c) JTSW   d) ITSV

Solution:

We have

$$\begin{array}{cccc} F & O & U & L \\ +2\downarrow & -1\downarrow & +1\downarrow & +3\downarrow \\ H & N & V & O \end{array}$$

Therefore

$$\begin{array}{cccc} H & U & R & T \\ +2\downarrow & -1\downarrow & +1\downarrow & +3\downarrow \\ J & T & S & W \end{array}$$

Answer: c) JTSW

5) If CASUAL represents FXVRDI, then PEOPLE in coded language represents

   a) SHRSQH   b) SBRMPB   c) SBRMOB   d) SHRSOH

Solution:

We have

C	A	S	U	A	L		P	E	O	P	L	E
+3↓	-3↓	+3↓	-3↓	+3↓	-3↓		+3↓	-3↓	+3↓	-3↓	+3↓	-3↓
F	X	V	R	D	I		S	B	R	M	O	B

Answer: c) SBRMOB

6) If +, - , x,÷ are represented by A, B, C, D respectively, then value of 5A7C2B15D5 is

   a) 17   b) 8   c) 16   d) 24

Solution: 5A7C2B15D5 =   5+7 × 2-15÷5 = 5+14-3=16

   Answer: c) 16

7) In a code language **TAPE** is written as 4825, SMART is written as 91834 and **BONE** is written as 7605, then how is **BASERA** written?

   a) 789138   b) 789198   c) 785198   d) 789538

   Solution:

   Here

   T A P E   **S M A R T**   **B O N E**   **B A S E R A**

   ↓ ↓ ↓ ↓   ↓ ↓ ↓ ↓ ↓   ↓ ↓ ↓ ↓   ↓ ↓ ↓ ↓ ↓ ↓

   4 8 2 5   **9 1 8 3 4**   **7 6 0 5**   **7 8 9 5 3 8**

   Answer: d) 789538

8) If **KOLKATA** is coded as 111512111201 then **MUMBAI** will be coded as:

   a) 1523152311   b) 1220121268   c) 1422142210   d) 132113219

   Solution: In the given if we arrange every alphabet according to its number we can solve the given question for e.g. In KOLKATA, K = 11, O = 15, L = 12 etc.

   ∴by this logic by solving Mumbai M = 13 U = 21 M = 13 B = 2 A = 1 I = 9

     which gives us the required solution 132113219

   Answer: d) 132113219

9) Which pair of numbers should come next?

     2, 8; 11, 18; 20, 28; 29, 38

   a) 48, 50   b) 38, 48   c) 38, 42   d) 40, 46

Solution: first numbers are at difference of 9
        Second numbers are at difference of 10
        ∴29 + 9 = 38

     38 + 10 = 48

   Answer: b) 38,48

10) What should be value of x from the given numbers so that x is minimum?

   3, - 4, 7, - 6, x

   a) - 7   b) 7   c) - 12   d) 12

   Solution: as 3 - 4 + 7 - 6 + x = x and minimum is -12

   Answer: c) -12

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).

C) Assertion (A) is true but Reason(R) is false.

D) Assertion (A) is false but Reason(R) is true.

1) **Assertion (A):):** If all balls are bats and some bats are toys, then all balls are toys.
   Reason(R) : All A are B means, A is a subset of B.
   a) A   b) B   c) C   d) D

   Answer: d) D A is false but R is true.

   Solution: For the given Statements two possible Venn diagrams are shown below:
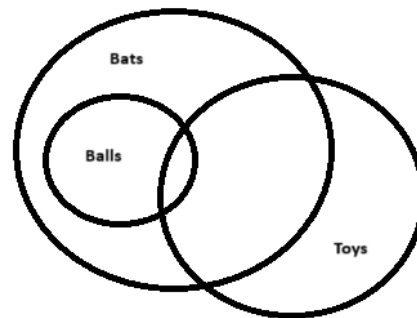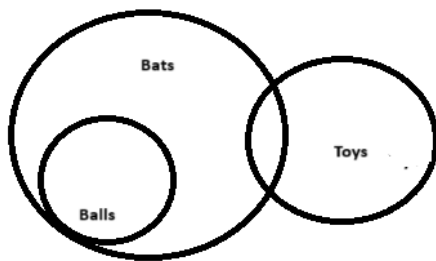
   Since conclusion that all balls are toys cannot be deduced from both the Venn diagrams. So, conclusion is false. ∴ A is false. R is true.

2) **Assertion (A):** In a certain language if **PENCIL** is coded as LICNEP, then **ERASER** is coded as RESARE.

   Reason (R): LICNEP is obtained by writing the letters of the word PENCIL in reverse order.

   a)   A   b) B   c) C   d) D

   Answer: a) A Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

   Solution: If we write the letters of the word PENCIL in reverse order, we get LICNEP.

   ∴ R is true.

So, 'ERASER' can be coded as RESARE.

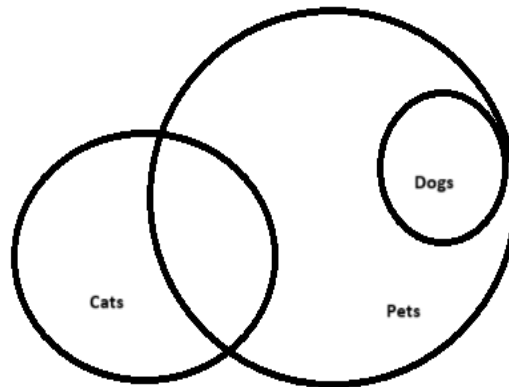∴ A is true and R is the correct explanation of Assertion A.

3) **Assertion (A):** If some cats are dogs and all dogs are pets, then some cats are pets.

Reason (R): Some A are B means A is a subset of B.

a) A   b) B   c) C   d) D

Answer: c) C Assertion (A) is true but Reason(R) is false.

Solution:

4) **Assertion (A):** If in a certain language **MUMBAI** is coded as OWODCK, then **BANGLORE** is coded as DCPINQTG in the same language.

Reason (R) : In a certain language if **ORANGE** is coded as TWFSLJ, then **MANGO** is coded as RFSLT in the same language.

a) A   b) B   c) C   d) D

Answer: b) (B) Both A and R are true but R is not the correct explanation of A.

Solution:: Clearly, each letter of the word MUMBAI is moved two steps forward to obtain the corresponding code

M	U	M	B	A	I		B	A	N	G	L	O	R	E
↓	↓	↓	↓	↓	↓		↓	↓	↓	↓	↓	↓	↓	↓
O	W	O	D	C	K		D	C	P	I	N	Q	T	G

So, 'BANGLORE' is coded as DCPINQTG. A is true.

Now, each letter of the word ORANGE moved 5 steps forward to obtain the corresponding code
So, 'MANGO' is coded as RFSLT.

O	R	A	N	G	E		M	A	N	G	O
↓	↓	↓	↓	↓	↓	⟹	↓	↓	↓	↓	↓
T	W	F	S	L	I		R	F	S	L	T

R is true but R is not the correct explanation of A.

5) **Assertion (A):** If Vikas is the only son of Pulkit's grandfather, then Vikas is the father of Pulkit.

Reason (R): If Saurabh is the brother of Saloni's mother then Saurabh is maternal uncle of Saloni.

   a) A   b) B   c) C   d) D

 Answer: b) (b) Both A and R are true but R is not the correct explanation of A.

 Solution::: Vikas is the only son of Pulkit's grandfather. It means Vikas is the father of Pulkit. ∴ A is true. Since, mother's brother is called the maternal uncle. So, Saurabh is the maternal uncle of Saloni.

∴R is true but R is not the correct explanation of A.

6) **Assertion (A):** Pointing to a boy if Rekha said, his mother, is the only daughter of my father, then Rekha is the mother of that boy.

   Reason (R): Only daughter of Rekha's father is Rekha herself.

   a) A   b) B   c) C   d) D
   Answer: a) (A) Both A and R are true and R is the correct explanation of A.
   Solution:: Only daughter of Rekha's father is Rekha itself. ∴R is true.
   So, Rekha is the mother of that boy.
   ∴A is true and R is the correct explanation of A.

7)   **Assertion (A):** A's father is the only son of B's father. If B is female, then B is aunt of A.

   Reason (R): B is the sister of A's father.

   a) A   b) B   c) C   d) D

   Answer: a) (A) Both A and R are true and R is the correct explanation of A.

   Solution: Given A's father is the only son of B's father and B is female. It means B is the sister of A's father. R is true. ∴ Also, if B is the sister of A's father, then B is aunt of A. A is true. R is the correct explanation of A

8)   **Assertion (A):** A man looks at a picture and say's "Her father's sister's son is my younger brother." How is he related to the person in the   picture?
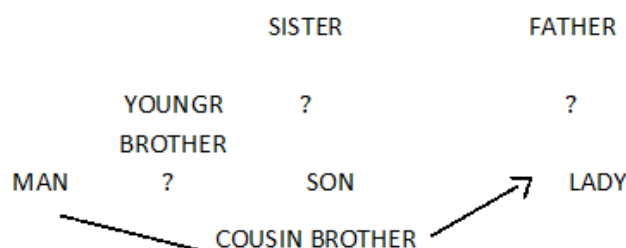
   Reason (R): Man is a cousin brother of lady given picture.

   a)A   b) B   c) C   d) D

   Answer: a) (A) Both A and R are true and R is the correct explanation of A.

   Solution:

   Solution

.R is the correct explanation of A

9) **Assertion (A):** In a certain language if "LUCKNOW" is coded as "NWEMPQY" then "DELHI" is coded as "FGNJK"

   Reason (R): It is LETTER CODING PROBLEM

   a)A   b) B   c) C   d) D

Answer: b)(B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
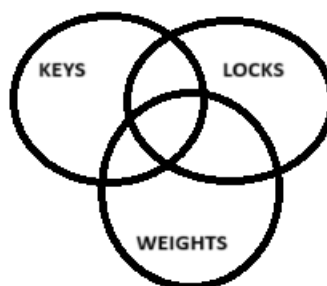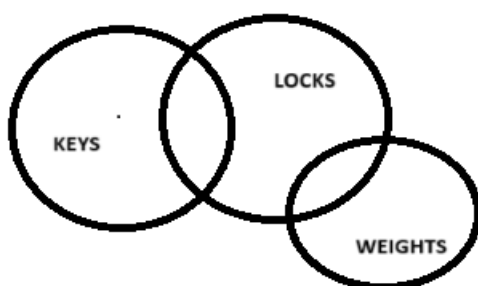
   Solution:

L	U	C	K	N	O	W		D	E	L	H	I
↓+1	↓+1	↓+1	↓+1	↓+1	↓+1	↓+1		↓+1	↓+1	↓+1	↓+1	↓+1
N	W	E	M	P	Q	Y		F	G	N	J	K

10) Assertion (A): If some keys are locks and some locks are weights, then all keys are weights.

   Reason (R): Some A are B means A and B intersect each other.

   Answer: d)(D) Assertion (A) is false but Reason(R) is true.

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

1) Out of the words (i) Advertisement (ii) News (iii) Editor (iv) Paper (v) Date, which is most appropriate to complete the sentence **A newspaper always has ...** .

   Solution:

   All the given words are relevant for a newspaper, but out of these the word 'News' is most appropriate and remaining are remove. So, the given sentence can be completed as 'A newspaper always has news'

2) In a certain language if **CORONA** is coded as 6 and **MALARIA** is coded as 5 then how **CANCER** coded?

   Solution: C = 3, O = 15, R = 18, N = 14, A = 1

   CORONA = 3 + 15 + 18 + 15 + 14 + 1 = 66

   $\frac{66}{11} = 6$

   M = 13, A = 1, L = 12, R = 18, I = 9

   MALARIA = 13 + 1 + 12 + 1 + 18 + 9 + 1 = 55

$$\frac{55}{11} = 5$$

CORONA = 6   MALARIA = 5

CANCER = 3 + 1 + 14 + 3 + 5 + 18 = 44

$$\frac{44}{11} = 4$$

Thus CANCER is coded as '4'

3)   Arrange the following words in a logical and meaningful order.

1.Country 2. Furniture 3. Forest 4. Wood 5. Trees

Solution: From the forest, given words, we can say that country contains forest, for has trees, trees have wood that is used to make furniture,   Hence, the correct order of the given words is 1, 3, 5, 4, 2.

4)   Find the value of the letters and give reason for the steps involved

$$
\begin{array}{ccc}
A & A & B \\
 & \times & B \\
\hline
8 & 8 & A \\
\end{array}
$$

Solution: From the hundred's digit of the answer, it is clear that A x B is single digit number which gives 8 as product. So, it may be 1 x 8, 2 x 4. But B   B x B = A, so, B = 2 and A = 4.

Hence A=4 , B=2

$$
\begin{array}{ccc}
4 & 4 & 2 \\
 & \times & 2 \\
\hline
8 & 8 & 4 \\
\end{array}
$$

5) In a certain language **TABLE** is coded as **NBDXJ** , how **CHAIR** is coded?
Solution Here, the new positions are obtained by multiplying the original position by 2.

So,

C	H	A	I	R
↓	↓	↓	↓	↓
3	8	1	9	18
× 2	× 2	× 2	× 2	× 2
6	16	2	18	36
↓	↓	↓	↓	-26
				10
F	P	B	R	J

Hence CHAIR is coded as 'FPBRJ'

6) Which of the two conclusions is/are true on the basis of given statements:
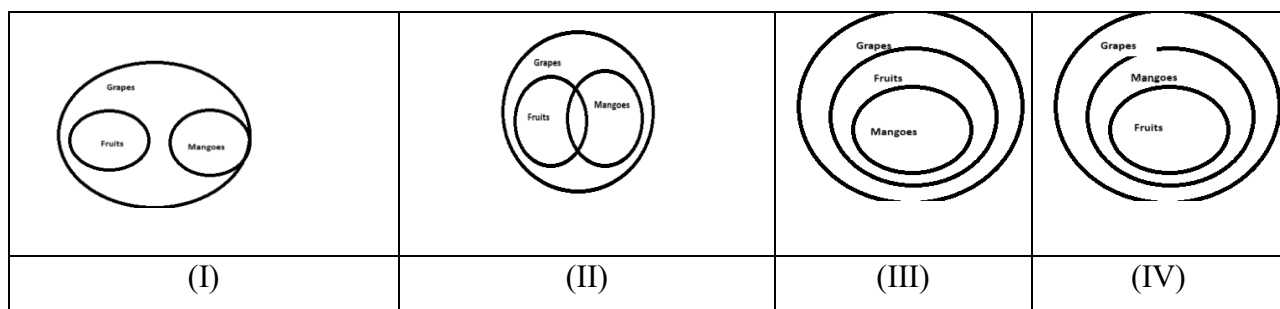   Statements I: All fruits are grapes
                II: All mangoes are grapes
        Conclusions I: Some fruits are mangoes

II: Some grapes are mangoes

Solution: Four possible Venn diagrams are shown below:

(I)	(II)	(III)	(IV)

Conclusion I can be deduced from the Venn diagrams (ii) and (iii) but not by (i) and (iv), so conclusion I is false. Conclusion II can be deduced by   all the four Venn diagrams, so the conclusion is true.

7) Let U = the set of the human beings;

   P = the set of all policemen;

   T = the set of all thieves.

Write the truth value of the following statements from the Venn diagram shown in Fig.

   I. No policemen is a thief.

   II. Thieves are not policemen.

   III. Men who are not policemen are thieves.

   IV. Some policemen are thieves.

Solution: i. It is evident from the Venn diagram that the policemen x is a thief also. So, the given statement is not true. Hence, its truth value is 'F'.

ii. From the Venn-diagram, we find that $P \cap T \neq \emptyset$. So, there are some thieves who are also policemen. Hence, the above statement is not true.

   So, its truth value is 'F'.

iii. It is evident from the Venn-diagram that there are some human beings who are neither policemen nor thieves. So, the above statement is not   true and its truth value is 'F'.

iv. Clearly, policemen x is a thief also. Therefore, the given statement is true and its truth value is 'T'.

8) Which of the two conclusions is/are true on the basis of given statements:
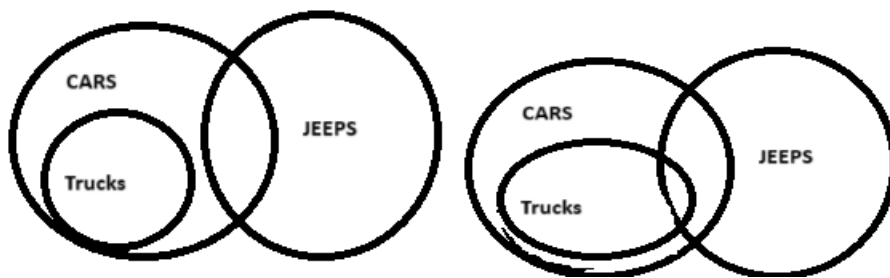   Statements I: Some cars are jeeps
            II: All trucks are cars
Conclusions I: No truck is a jeep
            II: Some jeeps are cars
   Solution: For the given statements two possible Venn diagrams are shown below

Since conclusion I cannot be deduced from the Venn diagram (ii). So the conclusion I is false. Conclusion II can be deduced from both the Venn diagrams, so conclusion II is true.

9) Write the negation of the following statements:

I.   for every real number n, n + 1 > n.

II.   There exists a number which is equal to its square.

Solution: i. Negation of quantifier 'for every' is 'There exists'. Therefore, let p: For every real number n, n + 1 > n. then ∼ p: There exist a real numbers n, n + 1 > n.

ii. Similarly, the negation of quantifier 'There exists' is 'for every/for all'. Therefore, let p: There exists a number which is equal to its square.

∼ p: for every real number x, we have $x \neq x^2$.

or ∼ p: There does not exist a number which is equal to its square.

10) In a certain language If **'SHARPNER'** is coded as **'LPBJFBJJ'** , how **'ERASER'** is coded?

Solution: Clearly, new positions of the letters of word 'SHARPNER' are obtained by multiplying the original position by 2.

Here,

S	**H**	**A**	**R**	**P**	**N**	**E**	**R**
↓	↓	↓	↓	↓	↓	↓	↓
19	**8**	**1**	**18**	**16**	**14**	**5**	**18**
× 2	**× 2**	**× 2**	**× 2**	**× 2**	**× 2**	**× 2**	**× 2**
38	**16**	**2**	**36**	**32**	**28**	**10**	**36**
-26			**-26**	**-26**	**-26**		**-26**
12			**10**	**6**	**2**		**10**
L	**P**	**B**	**J**	**F**	**B**	**J**	**J**

So

E	**R**	**A**	**S**	**E**	**R**
↓	↓	↓	↓	↓	↓
5	**18**	**1**	**19**	**5**	**18**
× 2	**× 2**	**× 2**	**× 2**	**× 2**	**× 2**
10	**36**	**2**	**38**	**10**	**36**
	-26		**-26**		**-26**
	10	**2**	**12**		**10**

| J | J | B | L | J | J |

'ERASER' is coded as 'JJBLJJ'.

SHORT ANSWER TYPE QUESTIONS

1).Write the component statements of the following compound statements and check whether the compound statement is true or false.

I. $2 + 4 = 6$ or $2 + 4 = 7$

II.   A rectangle is a quadrilateral or five - sided polygon

Solution: i. The components of compound statement are:

p: $2 + 4 = 6$

q: $2 + 4 = 7$

We observe, p is true and q is false and both the statements are connected with 'or'.

Hence, the compound statement is true.

ii. The components of compound statement are:

p: A rectangle is a quadrilateral.

q: A rectangle is five-sided polygon.

We observe, p is true and q is false and both the statements are connected with 'or'.

Hence, the compound statement is true

2).Write the negation of the following statements and check whether the resulting statements are true:

I.   The sum of 2 and 5 is 9.

II.   Every natural number is greater than zero.

Solution: i. The negation of the given statement is: "It is false that the sum of 2 and 5 is 9".
We know that, $2 + 5 = 7 \neq 9$. So, negation of given statement is true. ii.
The negation of the given statement is: "It is false that every natural number is greater than 0".
We know that, all natural numbers are greater than 0. So, negation of given statement is true.

3) Check whether the following sentences are statements? Give reason your answer.

1. $3 + x = 5$

2.   Every set is a finite set.

3.   The sun is a star.

4. $x^2 - 3x + 2 = 0$.

Solution: i. This is not a statement. But if numerical value is assigned to x, then it becomes a statement.

For $x = 2$, this is a true statement and for $x = 3$ it is false statement.

ii. This sentence is always false, because there are sets which are not finite. Hence, it

is a statement.

iii. Since, sun is a star (it is a scientific fact).

So, the given sentence is always true.

Hence it is a statement.

iv. This is not a statement. But for x = 1 or x = 2,

this is a true statement.

4) Rohit is the husband of Vanshika. Sumita is the sister of Rohit. Anushka is the sister of Vanshika. How Anushka is related to Rohit?

Solution: Rohit is the husband of Vanshika

Sumita is the sister of Rohit

Anushka is the sister of Vanshika

So Anushka is Rohit's wife's sister

Anushka is the sister-in-law of Rohit.

5) Find the values of the letter and give a reason for the steps involved.

	2	A	B
+	A	B	1
	B	1	8

Solution:

	2	A	B
+	A	B	1
	B	1	8

We have to find the value of A and B.

B + 1 we get 8, i.e., a number whose unit digit is 8.

For this, B must be 7.

So the question has been decoded as

	2	A	7
+	A	B	1

	7	1	8

∵ A + 7 we get 1, i.e., a number whose unit digit is 1.

For this, A must be 4, as 4 + 7 = 11.

So the question has been decoded as,

	2	4	7
+	4	B	1
	7	1	8

Hence A=4 and B = 7.

6) Find A and B in the addition

	1	2	A
+	6	A	B
	A	0	9

Solution: Observe the unit column of the addition.

A + B cannot exceed 18 and the unit digit in the sum is 9, so A + B = 9

This is possible when

A = 1, B = 8; A = 2, B = 7, ..., A = 8, B = 1

Now, observe ten's column As 2 + A cannot exceed 11 and the digit below the sum of ten's column is 0, therefore

2 + A = 10

⇒ ∴ A = 8

A = 8, B = 1

Now check the sum

	1	2	**8**
+	6	**8**	**1**
	8	0	9

Hence, the required digits are A = 8, B = 1

7) Find the values of the letter and give a reason for the step involved

	B	3	4	5
+	C	9	B	A
	8	B	A	2

Solution:

	B	3	4	5
+	C	9	B	A

	8	B	A	2

We have to find the value of A, B, and C.

For this, 5 + A we get 2, a number whose unit digit is 2

Clearly 5 + 7 = 12

So, A= 7, and Question becomes,

```
      B   3   4   5
+     C   9   B   7
      8   B   7   2
```

Now, we have 1 + 4 + B = 7, A number whose unit digit is 7

So, the number should be 2, as 1 + 4 + 2 = 7

∴ B = 2

Now, the question reduced to,

```
      2   3   4   5
+     C   9   2   7
      8   2   7   2
```

Again we have 1 + 2 + C = 8

So, the number should be 5, as 1 + 2 + 5 = 8

∴C = 5

So, the question reduced to,

```
      2   3   4   5
+     5   9   2   7
      8   2   7   2
```

Hence , A = 7; B = 2; C = 5

8) Identify the quantifiers and write negation of the following statements.
    I.   For all even integers, x, x^2 is also even.
    II.   There exists a number which is multiple of 6 and 9.
Solution:
        i.   The quantifier is 'for All'.
        The negation of the statement is:
        "There exists an even integer x such that x^2 is not even".
        ii.   The quantifier is "There exists".
        The negation of the statement is:
        "There does not exist a number which is a multiple of both 6 and 9.

9) Arrange each of the following words in meaningful logical order.

        I.   1. Probation, 2. Interview, 3. Selection, 4. Appointment, 5. Advertisement, 6. Application

        II.   1. Gold, 2. Iron, 3. Sand, 4. Platinum, 5. Diamond

Solution:

    i.   As for a job, a person first see an advertisement, then fill application form and go for interview. If he/she is selected, got appointment letter and final stage is of probation.

    Hence, the Correct arrangement of given words is 5, 6, 2, 3, 4, 1.

    ii. All the given words represent substances which can be arranged in the increasing order of their cost. The least costly is sand after which comes the cost of iron, followed by gold, diamond and costliest among all is platinum. Hence, the Correct arrangement of given words is 3, 2, 1, 5, 4.

LONG ANSWER TYPE QUESTIONS

1).Question: Read the information given below and answer the questions that follow:

A family of Ravi, Rekha, Gaurav, Saurabh, Payal, Mohit, Monika, Swasti and Shaurya consists of 3 generations. Ravi is the husband of Rekha and has 2 sons and one daughter Payal. Mohit is Rekha's son - in - law. Saurabh is Payal's brother and Monika's brother - in - law. Gaurav has two children.

i.How is Gaurav related to Mohit?

ii.How is Saurabh related to Ravi?

iii. How is Ravi related to Shaurya?

iv. How is Mohit related to Payal?

v. How is Mohit related to Swasti?

    Solution:
    Ravi is the husband of Rekha and has 2 sons and one daughter Payal
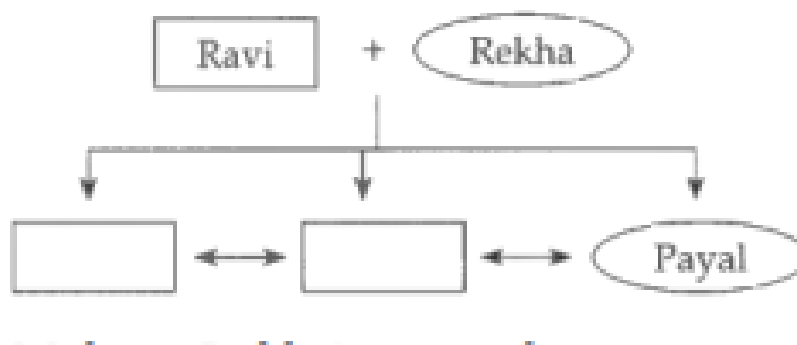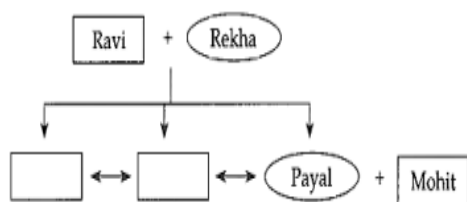
Saurabh is Payal's brother and Monika's brother-in-law
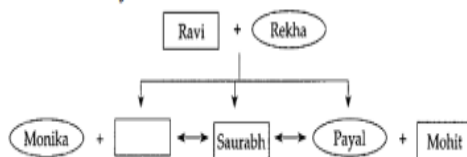
Since it is a family of 3 generations, Gaurav has 2 children so Gaurav has to be married to Monika and Swasti and Shaurya must be the children of Gaurav and Monika.
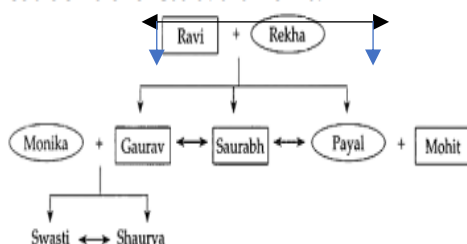
Hence the whole family tree is represented as above

    i. Gaurav is Mohit's brother-in-law.

   ii.   Saurabh is Ravi's son.

   iii. Ravi grandfather of Shaurya.

   iv. Mohit is the husband of Payal.

   v. Mohit is Swasti's uncle.

2).Question: In a family, there are 7 members P, Q, R, S, T, U and V. P the head of the family is a Lawyer. R and T are brothers and both are doctors. S, the wife of T is a social worker. Q is the son of U, who is the daughter - in - law of P. V is the cousin sister of Q. If there are two married couples and a female teacher in the family, answer the following questions:

1) Who is married to R?
2) What is the profession of U?
3) How T is related to V?
4) How P is related to S?

Solution: The given problem can be summarised as:

           P
           ↓
U   ↔   R   . .   T   ↔   S
↓   ↓
Q   V

   I.U is married to R

  ii. Teaching is the profession of U.

 III.T is father of V

 IV.P is father-in-law of S

      At first, we define some symbols.

      Male-A→ A

      Female-A→ A

      A is son of B   B→B

|
A

A is daughter of B→B

|
A

A is sister of B→ A··B

A is brother of B→ A··B

A is wife of B → A - B

A is husband of B→A··B

A IS GRANDSON OF B→ *B*

↑

A

A IS GRANDDAUGHTER OF B→ *B*

↑

A

When sex of A is not defined→ *A*

3).Question: In each question below are two statements followed by two conclusions numbered I and II. You have to take the two given statements   to be variance from commonly known facts and decide which of the given conclusions logically follows the two given statements disregarding commonly known facts.

Give answer: (A) if only conclusion I follows

(B)if only conclusion II follows

(C)if either I or II follows

(D)if neither I nor II follows

(E)if both I and II follow

I. **Statements:**
    a. Some fools are intelligent.
    b. Some intelligent are great.
Conclusions:
    a. Some fools are great.
    b. All great are intelligent.

II. **Statements:**
    a. All stones are water.
    b. Some waters are clean.
Conclusions:
    c. Some stones are clean.
    d. No stone in clean.

V. **Statements:**
    a. All keys are locks.
    b. All locks are screws
Conclusions:
    c. All screws are keys.
    d. Some locks are keys.

   IV. Statements:
    a. Some books are pen.
    b. No pen is pencil.
Conclusions:
    a. Some books are pencil.
    b. No books is a pencil.

Solution:

 I). Conclusion I is mediate inference and conclusion II is immediate inference. Since, middle term is not distributed conclusion I does not follow. Conclusion II is not true as per the rule of immediate inference. Hence, option (D) is correct i.e., neither I nor II follows.

ii. Conclusion I and II are mediate inference since, middle term is not distributed no conclusion follows. But both the conclusions form a complimentary pair. Hence, either I or II follows i.e., option (C).

 III. Conclusion II follows being the converted form of statement (i). Conclusion I is mediate inference and does not follow as the term   screws is not distributed in the statement. Hence, only conclusion II follows i.e., option (B).

 iv. Both the conclusions are mediated inferences and do not follow because the term which is distributed in the conclusion is not   distributed in the statement. Hence, option (D) is correct i.e., neither I nor II follows.

 4).Question: In a family of seven members, three generations are living together. The family has two married couples with two children each.   Dharmesh has two grandchildren. There are two housewives. Dharmesh who is Bhavesh's father, is a Doctor and earns the most Yuvraj is the son of Anuja. Anuja is married to an Engineer. Who is Kamla's son. Sonal is the grand - daughter of one of the housewives and she is a classical dancer. Sarika is the sister of an Engineer and herself is a Teacher.

1.   What is Bhavesh's profession?
2.   How many female members are there in the family?
3.   Who are the childern's of Kamla?
4.   Write the names of two married couples.

   Solution: We first draw family chart to understand the relationship with each other.
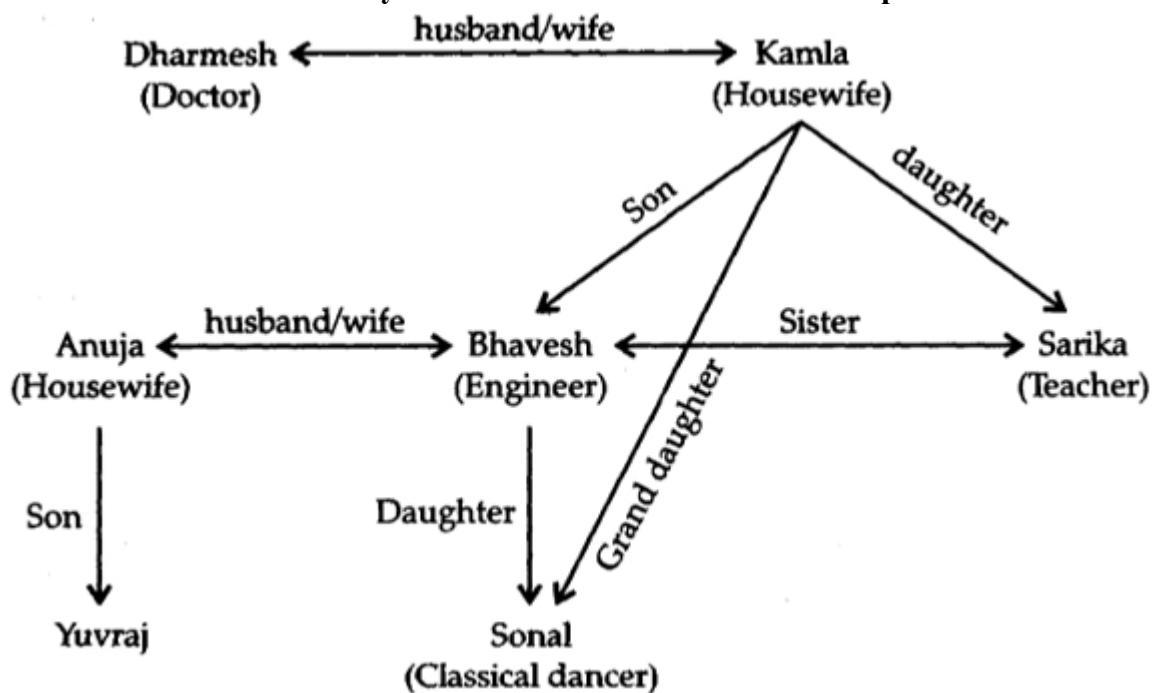
In the above chart, we can see that Dharmesh (Doctor) is the husband of Kamla (housewife). They have two children, one son and one daughter namely Bhavesh (Engineer) and Sarika (Teacher). Bhavesh is married to Anuja (Housewife) and they have two children namely Yuvraj and Sonal.

i.Bhavesh's profession in Engineer.
ii. There are four females in the family.
iii. Bhavesh and Sarika are the children of Kamla.
Married couples in the family are:
Dharmesh - Kamla
and Bhavesh - Anuja

5).Question: Translate the following statements into symbolic form:
i.Rahul passed in Hindi and English.
ii) 2, 3 and 6 are factors of 12.
iii) Either x or x + 1 is an odd integer.
iv) x and y are even integers.
v) A number is divisible by 2 or 3.
vi) Either x = 2 or x = 3 is a root of $3x^2 - x - 10 = 0$

Solution:

i. p: Rahul passed in Hindi.
q: Rahul passed in English.
∴ p ∧ q: Rahul passed in Hindi and English.
ii. p: 2 is a factor of 12
q: 3 is a factor of 12
r: 6 is a factor of 12
∴ p ∧ q ∧ r: 2, 3 and 6 are factors of 12.
iii. p: x is an odd integer
q: x + 1 is an odd integer
∴ p ∨ q: either x or x + 1 is an odd integer
iv. p: x is even integer.
q: y is even integer.
∴ p ∧ q: x and y are even integers.
v. p: A number is divisible by 2
q: A number is divisible by 3
∴ p ∨ q: A number is divisible by 2 or 3.
vi. P: x = 2 is a root of $3X^2 - x - 10 = 0$
q: x = 3 is a root of $3x^2 - x - 10 = 0$
∴ p ∨ q: Either x = 2 or x = 3 is a root of $3x^2 - x - 10 = 0$

CASE STUDY BASED QUESTIONS

1). Question: Read the following text carefully and answer the questions that follow:
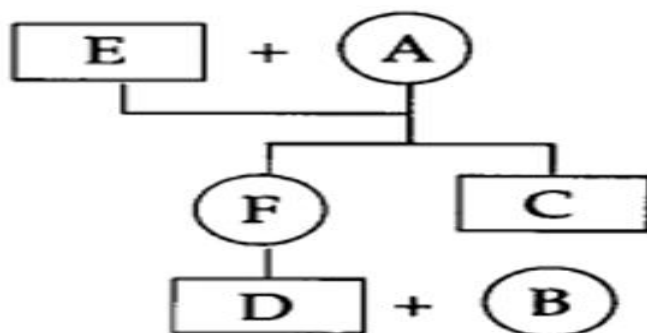
In a family of 6 members A, B,C, D, E and F. B is daughter - in - law of F. D is grandchild of A. C is the only uncle of D. A has two children F   and C, one male and one female. E is the father of C.

I.   Who is the mother - in - law of B? (1)

II.   Who is the grandfather of D? (1)

III.   How D is related to F? (2)

OR

How B is related to D? (2)

A has two children F and C , C is only uncle of D. So, C is male and D is a child of F. B is daughter-in-law of F so, she is wife of D.   Also, E is father of C, so, A is wife of E.

F is mother-in-law of B.

II

A has two children F and C , C is only uncle of D. So, C is male and D is a child of F. B is daughter-in-law of F so, she is wife of D.
Also, E is father of C, so, A is wife of E.
E is grandfather of D.

III.

A has two children F and C , C is only uncle of D. So, C is male and D is a child of F. B is daughter-in-law of F so, she is wife of D.   Also, E is father of C, so, A is wife of E.
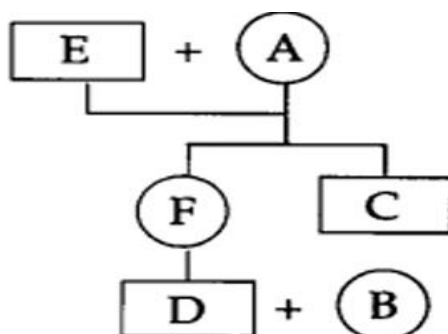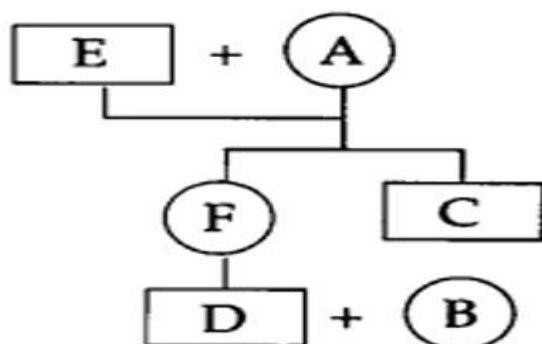
D is son of F.

OR
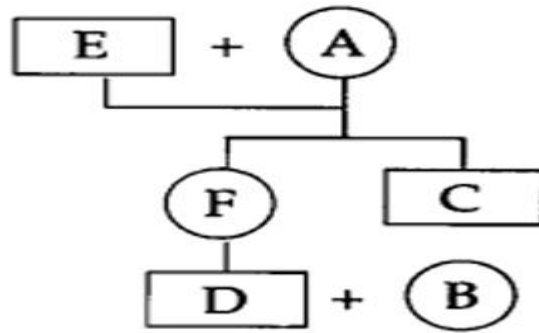
A has two children F and C , C is only uncle of D. So, C is male and D is a child of F. B is daughter-in-law of F so, she is wife of D. Also, E is father of C, so, A is wife of E.
B is wife of D.

2). Question: Read the following text carefully and answer the questions that follow:
 In a family, there are 7 members P, Q, R, S, T, U and V. P the head of the family is a Lawyer. R and T are brothers and both are doctors.   S, the wife of T is a social worker. Q is the son of U, who is daughter - in - law of P. V is the cousin sister of Q. If there are two married couples and a female teacher who is married in the family.
 Who is married to R? (1)
 What is the profession of U? (1)
 How T is related to V? (2)
 OR
 How P is related to S? (2)
 Solution:
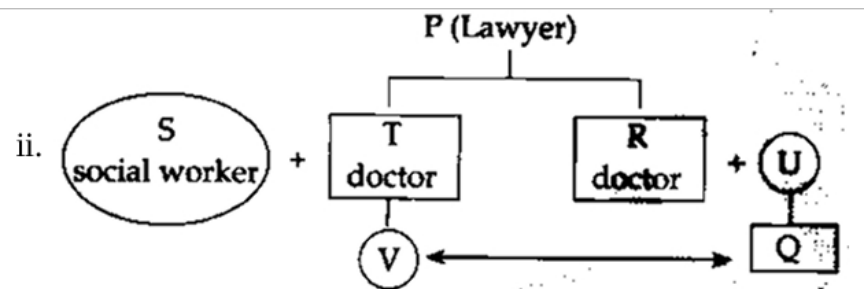
U is a teacher.
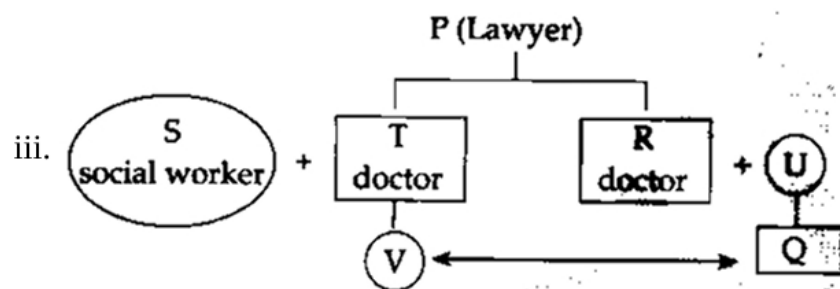
T is father of V.

OR
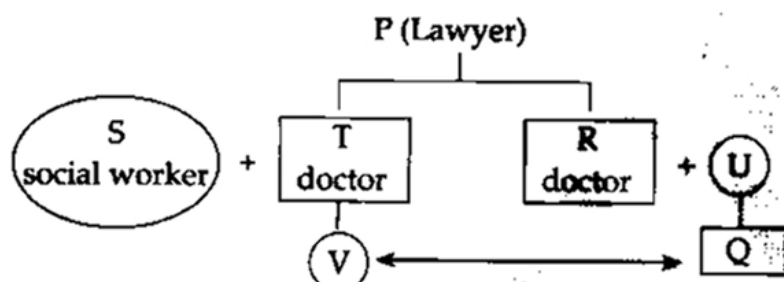
3). Question: **Read the following text carefully and answer the questions that follow:**

In a family there are 6 persons A, B, C, D, E and F. They are Entrepreneur, Lawyer, Teacher, Manager, Doctor and Engineer. The doctor is the grandfather of F, who is an Entrepreneur. The Engineer D is married to A. C the Manager is married to Lawyer. B is the mother of E and F. There are two married couples in the family.

1.   What is the profession of B? (1)

2. What is the profession of E? (1)

3. How A is related to E? (2)

   OR

How D is related to F? (2)

Solution:

1. The doctor is the grandfather of F, who is an entrepreneur:

$$\boxed{\begin{array}{c} ? \\ \textbf{DOCTOR} \end{array}}$$

$$\downarrow$$

$$?$$

$$\downarrow$$

$$P$$

(Entrepreneur)

Engineer D is married to A.

D (Engineer) + A (?)

Manager C is married to Lawyer.

C (Manager) ⇒ + ? (Lawyer)

B is the mother of E and F B is female and of the second generation. Because F is in the third generation.

So, we have

Also, given that there are two married couples. So, we have a married couple of first-generation.

```
  ┌──────────┐         ┌──────────┐
  │    A     │    +    │    D     │
  │ (Doctor) │         │(Engineer)│
  └──────────┘         └──────────┘
        │
        ▼
  ┌──────────┐         ┌──────────┐
  │    C     │    +    │    B     │
  │ (Manager)│         │ (Lawyer) │
  └──────────┘         └──────────┘
        │
   ┌────┴────────────────┐
   ▼                     ▼
   E                     F
(Teacher)          (Entrepreneur)
```
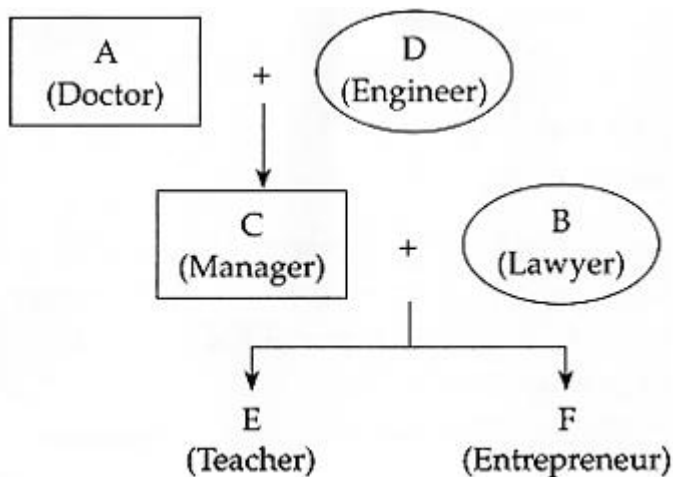
D is grandmother of F

HIGHER ORDER THINKING SKILLS

Que 1: In a family, there are 7 members P, Q, R, S, T, U and V. P the head of the family is a Lawyer. R and T are brothers and both are doctors. S, the wife of T is a social worker. Q is the son of U, who is the daughter - in - law of P. V is the cousin sister of Q. If there are two married couples and a female teacher in the family, answer the following questions:

i).Who is married to R?
ii).What is the profession of U?
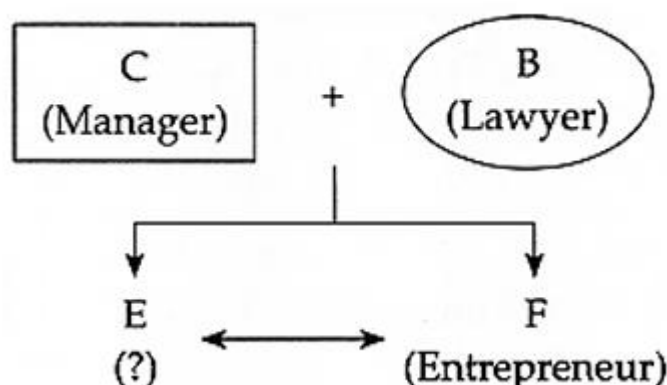iii)How T is related to V?
iv)How P is related to S?
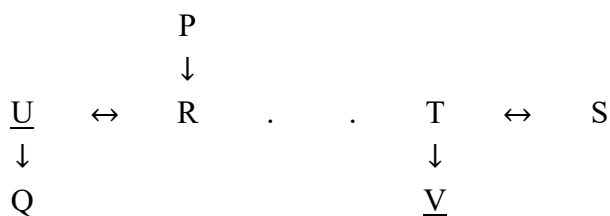
  Solution: The given problem can be summarised as:

```
              P
              ↓
  U    ↔    R   .   .   T   ↔   S
  ↓                     ↓
  Q                     V
```

  I.U is married to R
  ii. Teaching is the profession of U.
  III.T is father of V
  IV.P is father-in-law of S
     At first, we define some symbols.

    Male-A→ A

    Female-A→ A

    A is son of B   B→B

              |

              A

    A is daughter of B→B

              |

<div align="center">

A̲

A is sister of B→ A··B

A is brother of B→ A··B

A is wife of B → A - B

A is husband of B→A··B

A IS GRANDSON OF B→ *B*

↑

A

A IS GRANDDAUGHTER OF B→ *B*

↑

A̲

When sex of A is not defined→ *A*

</div>

Que 2: Find the values of the letter and give a reason for the step involved

```
    B   3   4   5
+   C   9   B   A
  ─────────────────
    8   B   A   2
```

Solution:

```
    B   3   4   5
+   C   9   B   A
  ─────────────────
    8   B   A   2
  ─────────────────
```

We have to find the value of A, B, and C.

For this, 5 + A we get 2, a number whose unit digit is 2

Clearly 5 + 7 = 12

So, A= 7, and Question becomes,

```
    B   3   4   5
+   C   9   B   7
  ─────────────────
    8   B   7   2
  ─────────────────
```

Now, we have 1 + 4 + B = 7, A number whose unit digit is 7

So, the number should be 2, as 1 + 4 + 2 = 7

∴ B = 2

Now, the question reduced to,

```
      2   3   4   5
  +   C   9   2   7
      ───────────────
      8   2   7   2
```

Again we have $1 + 2 + C = 8$

So, the number should be 5, as $1 + 2 + 5 = 8$

$\therefore C = 5$

So, the question reduced to,

```
      2   3   4   5
  +   5   9   2   7
      ───────────────
      8   2   7   2
```

Hence , A = 7; B = 2; C = 5

EXCERCISE

MULTIPLE CHOICE QUESTIONS

1. Find the odd one out:
A) Apple
B) Orange
C) Mango
D) Chair

2. If "HOUSE" is written as "IPVTF," how is "TABLE" written?
A) UBCMF
B) TBCME
C) UBCNE
D) UBCLF

3. All apples are fruits. Some fruits are bananas. Which of the following is correct?
A) All bananas are apples
B) Some fruits are apples
C) No apple is a banana
D) All fruits are bananas

4. Which of the following is not a type of reasoning?
A) Deductive
B) Inductive
C) Analytical
D) Emotional

5. If no squares are circles, and all circles are triangles, what is true?
A) No triangle is a square.
B) Some triangles are squares.

C) All squares are triangles.

D) None of these.

ASSERTION - REASON BASED QUESTIONS

General Instructions:

Options:

- A - Both A and R are true, and R is the correct explanation of A.
- B - Both A and R are true, but R is NOT the correct explanation of A.
- C - A is true, but R is false.
- D - A is false, but R is true.

1. **Assertion (A):** Birds have feathers.

   Reason (R): Feathers help birds to fly.

2. **Assertion (A):** If some kites are lions and some lions are wolves, then all kites are wolves.

   Reason (R): Some A are B means A and B intersect each other.

3. **Assertion (A):** A's father is the only son of B's father. If B is female, then B is aunt of A.

   Reason (R): B is the sister of A's father.

4. **Assertion (A):** If all pens are pencils and some pens are sticks, then all pens are sticks.

   Reason (R): All A are B means, A is a subset of B.

5. **Assertion (A):** MUKESH's father is the only son of KRISHNA's father. If KRISHNA is female,

   then HRISHNA is aunt of MUKESH.

   Reason (R): KRISHNA is the sister of MUKESH's father.

VERY SHORT ANSWER TYPE QUESTIONS

1. Which of the two conclusions is/are true on the basis of given statements:

Statements I: Some cars are jeeps

II: All trucks are cars

Conclusions I: No truck is a jeep

II: Some jeeps are cars

2. Which of the two conclusions is/are true on the basis of given statements:

Statements I: Some cups are plates

II: All plates are pots

Conclusions I: Some plates are cups

II: Some pots are cups

3. If 'men are very busy' means, '1234', 'Busy person need encouragement' means '4567', 'encouragement is very important' means '3589' and 'Important persons are rare' means, '2680', then

I). What is the code for **encouragement.**

II). According to given code, what is the code of **Men need encouragement**?

4. RAM is the husband of SEETA. Sumita is the sister of RAM. MANDAVI is the sister of SEETA. How MANDAVI is related to RAM?

5. Arrange each of the following words in meaningful logical order.

1. Probation, 2. Interview, 3. Selection, 4. Appointment, 5. Advertisement, 6. Application

SHORT ANSWER TYPE QUESTIONS

1. P is the son of Q, who is the husband of R, who is the sister of S, who is the father of T, who is the brother of U, who is the daughter of V.
   - I. How is P related to V?
   - II. How is R related to U?
   - III. How is P related to S?

2. In a family of seven members, three generations are living together. The family has two married couples with two children each. Dharmesh has two grandchildren. There are two housewives. Dharmesh who is Bhavesh's father, is a Doctor and earns the most Yuvraj is the son of Anuja. Anuja is married to an Engineer. Who is Kamla's son. Sonal is the grand - daughter of one of the housewives and she is a classical dancer. Sarika is the sister of an Engineer and herself is a Teacher.
   - I. What is Bhavesh's profession?
   - II. How many female members are there in the family?
   - III. Who are the childern's of Kamla?

3. Find the values of the letter and give a reason for the step involved

```
    B   3   4   5
+   C   9   B   A
  ─────────────────
    8   B   A   2
```

4. . Find the values of the letter and give a reason for the steps involved.

```
      2    A    B
+     A    B    1
  ─────────────────
      B    1    8
```

5.) Find A and B in the addition

```
      1    2    A
+     6    A    B
  ─────────────────
      A    0    9
```

LONG ANSWER TYPE QUESTIONS

1. If A is the son of P, B is P's sister, B has a son Q and a daughter C, R is the maternal uncle of Q, then answer the following questions.
I)How is A related to Q?
II)How is C related to R?
III)What is the relationship of P with C and Q?
IV)How many nephews does R have?
V)How many brothers does B have?

2. Read the information given below and answer the questions that follow:
In a family there are 6 persons A, B, C, D, E and F. They are Entrepreneur, Lawyer, Teacher, Manager, Doctor and Engineer. The doctor is the grandfather of F, who is an Entrepreneur.
Engineer D is married to A. C the Manager is married to Lawyer.
B is the mother of E and F. There are two married couples in the family.
I. What is the profession of B?
II. What is the profession of E?
III. How A is related to E?
IV. How D is related to F?
V. How many female members are there in the family?

3. Study the following statements A, B, C, D and E and answer the following questions:

(A) 1, 5, 9 means : 'you better go'

(B) 1, 6, 7 means : 'better come here'

(C) 5, 6, 7 means : 'you come here'

(D) 1, 5, 6 means : 'better you here'

(E) 3, 7, 9 means : 'come and go'

I. How many minimum number of statements are necessary to find the code no. of 'better'?

II. Which numeral means 'go'?

III. Which numeral means 'you'?

IV. Which numeral means 'and'?

4. In each question below are two statements followed by two conclusions numbered I and II. You have to take the two given statements to be variance from commonly known facts and decide which of the given conclusions logically follows the two given statements disregarding commonly known facts.

Give answer: (A) if only conclusion I follows

(B)if only conclusion II follows

(C)if either I or II follows

(D)if neither I nor II follows

(E)if both I and II follow

I).Statements:

Some fools are intelligent.

Some intelligent are great.

Conclusions:

Some fools are great.

All great are intelligent.

II).Statements:

All stones are water.

Some waters are clean.

Conclusions:

Some stones are clean.

No stone in clean.

III).Statements:

All keys are locks.

All locks are screws

Conclusions:

All screws are keys.

Some locks are keys.

IV)Statements:

Some books are pen.

No pen is pencil.

Conclusions:

Some books are pencil.

No books is a pencil.

5. In a certain code 'Ding Dong Dang' means 'A Hacking the enemy', 'Ping Pond Dong' means 'Enemy is retreating', 'Ding Ping Mong' means 'Attacking and retreating'.

From the above information, answer the following questions -

I. Which code stands for Enemy?

II. Which code stands for Attacking?

III. Which code stands for Retreating?

IV. Which code stands for And?

CASE STUDY BASED QUESTIONS

1. Read the following text carefully and answer the questions that follow:

Reasoning and coded language are beautiful aspects of learning which increases our curiosity as well as looks after safety. In different departments where secrecy is to be preserved coded language is used. As at present we are first beginners let us also try to answer few questions which two friends are exchanging in coded language or trying to reason out.

I).In a certain code if c = 6, m = 26, y = 50, then find the value of B + E + T. (1)

II)Find the odd one out in ACE, FHI, LNP, QSU, UWX. (1)

III)If EROS is coded as 1682, CHAIR as 73456 and PREACH as 961473, then find the code for ARCHSE. (2)

OR

If GOLD is written as IQNF then how YKPF will be written? (2)

2. Teacher asked students about family relationship questions. Teacher states that "In a house there are six members A, B, C, D, E and F. They are related to each other in some manner".

A is B's sister. C is B's mother. D is C's father. E is D's mother. Then, how is A related to D? (1)

A and B are a married couple, A being the male member. D is the only son of C, who is the brother of A. E is the sister of D. B is the daughter - in - law of F, whose husband has died. How is E related to C? (1)

All the six members of a family A, B, C, D, E and F ate travelling together. B is the son of C but C is not the mother of B. A and C are a married couple. E is the brother of C. D is the daughter of A. F is the brother of 8. Who is the mother of B? (2)

   OR

Pointing to a photograph, A tells E, "I am the only daughter of this lady and her son is your maternal uncle." How is the A related to E's father. (2)

3. In a class of few students, teacher give students few code and ask them questions related to those codes. Codes are: # $ @: Dust is harmful; @!%: harmful object kill; #*%: Dust can kill; 123: DOG; 59: For; 547: Lungs.

I.What is the code for Dust in this language? (1)

II.What is the code for kill in this code language? (1)

III.What is the code for Dust kill DOG? (2)

OR

What is the code for "Dust is harmful For Lungs". (2)

HIGHER ORDER THINKING SKILLS

1. If A is the son of P, B is P's sister, B has a son Q and a daughter C, R is the maternal uncle of Q, then answer the following questions.

I. How is A related to Q?

II. How is C related to R?

III. What is the relationship of P with C and Q?

IV. How many nephews does R have?

V. How many brothers does B have?

2. **Assertion (A):** If in a certain language **MUMBAI** is coded as OWODCK, then **BANGLORE** is coded as DCPINQTG in the same language.

Reason (R) : In a certain language if **ORANGE** is coded as TWFSLJ, then **MANGO** is coded as RFSLT in the same language.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

3. Read the following text carefully and answer the questions that follow:

Reasoning and coded language are beautiful aspects of learning which increases our curiosity as well as looks after safety. In different departments where secrecy is to be preserved coded language is used. As at present we are first beginners let us also try to answer few questions which two friends are exchanging in coded language or trying to reason out.

In a certain code if c = 6, m = 26, y = 50, then find the value of B + E + T. (1)

Find the odd one out in ACE, FHI, LNP, QSU, UWX. (1)

If EROS is coded as 1682, CHAIR as 73456 and PREACH as 961473, then find the code for ARCHSE. (2)

OR

If GOLD is written as IQNF then how YKPF will be written? (2)

ANSWERS

MULTIPLE CHOICE QUESTIONS

1. D) Chair   2. A) UBCMF   3. B) Some fruits are apples

4. D) Emotional   5. A) No triangle is a square.

ASSERTION - REASON BASED QUESTIONS

1. (A) - Both A and R are true, and R is the correct explanation of A.

2. (D) Assertion (A) is false but Reason(R) is true.

3. (A) Both A and R are true and R is the correct explanation of A.

4. (D) - A is false, but R is true.

5. a) Both A and R are true and R is the correct explanation of A.

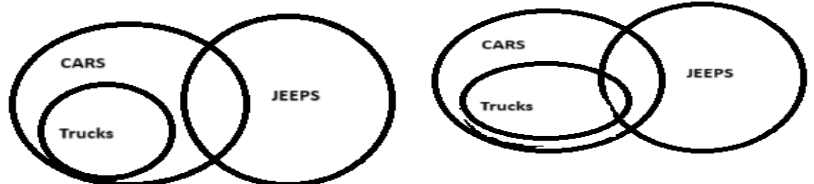### VERY SHORT ANSWER TYPE QUESTIONS

1. **Solution:** For the given statements two possible Venn diagrams are shown below

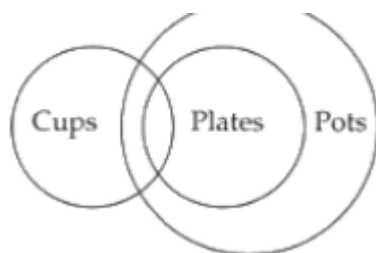Since conclusion I cannot be deduced from the Venn diagram (ii). So the conclusion I is false. Conclusion II can be deduced from both the Venn   diagrams, so conclusion II is true

2. For the given statements, the possible Venn diagram is shown in the adjacent figure.

Since both, the conclusions can be deduced from the Venn diagram, so both the conclusions are true.

3. Given statements and their codes are as follows:

(I) Men are very busy - 1234

(II) Busy persons need encouragement – 4567

(III) Encouragement is very important – 3589

(IV) Important persons are rare – 2680

i. In second (II) and third (III) sentences, common word is 'encouragement' and common code no. is '5'.

Hence, number 5 stands for encouragement.

ii. In first (I) and fourth (IV) sentences, the common word is 'are', hence 'are' stands for '2' from first (I) and third (III) sentences,   'very' stand for '3' from first (I) and second (II) sentences, 'busy' stands for '4'. Hence, we get from first sentence 'Men stand for '1' similarly 'needs' stand for '7'. From this, we conclude that 'Men need encouragement' will be coded as '1 5 7'.

4. Solution: RAM is the husband of SEETA

Sumita is the sister of RAM

MANDAVI is the sister of SEETA

So MANDAVI is RAM's wife's sister

MANDAVI is the sister-in-law of RAM.

5. i. As for a job, a person first see an advertisement, then fill application form and go for interview. If he/she is selected, got appointment letter and final stage is of probation. Hence, the Correct arrangement of given words is 5, 6, 2, 3, 4, 1.

SHORT ANSWER TYPE QUESTIONS

1. P is the son of Q, who is the husband of R, who is the sister of S, who is the father of T, who is the brother of U, who is the daughter of V.

i. P is nephew of V

ii. R is aunt of U

iii. P is nephew of S

2. i. Bhavesh's profession in Engineer. ii. There are four females in the family. iii. Bhavesh and Sarika are the children of Kamla.

3.

Solution:

	B	3	4	5
+	C	9	B	A
	8	B	A	2

We have to find the value of A, B, and C.

For this, 5 + A we get 2, a number whose unit digit is 2

Clearly 5 + 7 = 12

So, A= 7, and Question becomes,

	B	3	4	5
+	C	9	B	7
	8	B	7	2

Now, we have 1 + 4 + B = 7, A number whose unit digit is 7

So, the number should be 2, as 1 + 4 + 2 = 7

∴ B = 2

Now, the question reduced to,

	2	3	4	5
+	C	9	2	7
	8	2	7	2

Again we have 1 + 2 + C = 8

So, the number should be 5, as 1 + 2 + 5 = 8

∴C = 5

So, the question reduced to,

	2	3	4	5
+	5	9	2	7
	8	2	7	2

Hence , A = 7; B = 2; C = 5

4

Solution:

	2	A	B
+	A	B	1
	B	1	8

We have to find the value of A and B.

B + 1 we get 8, i.e., a number whose unit digit is 8.

For this, B must be 7.

So the question has been decoded as

	2	A	7
+	A	B	1
	7	1	8

∵ A + 7 we get 1, i.e., a number whose unit digit is 1.

For this, A must be 4, as 4 + 7 = 11.

So the question has been decoded as,

	2	4	7
+	4	B	1
	7	1	8

Hence A=4 and B = 7.

5. Solution: Observe the unit column of the addition.

A + B cannot exceed 18 and the unit digit in the sum is 9, so A + B = 9

This is possible when

A = 1, B = 8; A = 2, B = 7, ..., A = 8, B = 1

Now, observe ten's column As 2 + A cannot exceed 11 and the digit below the sum of ten's column is 0, therefore

2 + A = 10

⇒ ∴ A = 8

A = 8, B = 1

Now check the sum

	1	2	8
+	6	8	1
	8	0	9

Hence, the required digits are A = 8, B = 1

LONG ANSWER TYPE QUESTIONS

1. i. A is the cousin brother of Q.

   ii. C is the niece of R.

   iii. P is the maternal uncle of C and Q both.

   iv. R has two nephews, A and Q.

   v. B has two brothers, P and R.

2. i. B is a Lawyer.

ii. E is a teacher.

iii. A is the grandfather of E.

iv. D is the grandmother of F.

v. Since the gender of E and F is not known. So, the number of female members cannot be determined.

3. i. In statements A and B, A and D & B and D common word is 'better' and in all the combinations common code is 1. Hence, code no. of better is 1 and minimum number of required statements are 2 (two). ii. In statements A and E common word is 'go' and common code no. is '9'. Hence, numeral '9' means 'go'.

iii. In statements A and C common words is 'you' and common code no. is '5'. Hence, numeral '5' means 'you'.

iv. From statements B, C and E, it is clear that common word is 'come' and its code no. is 7. Now, we know that code no. for go is 9 and code no. for come is 7. Hence, from statement E, it is clear that code no. for 'and' is '3'

4. i. Conclusion I is mediate inference and conclusion II is immediate inference. Since, middle term is not distributed conclusion I does not follow. Conclusion II is not true as per the rule of immediate inference. Hence, option (D) is correct i.e., neither I nor II follows.

ii. Conclusion I and II are mediate inference since, middle term is not distributed no conclusion follows. But both the conclusions form a complimentary pair. Hence, either I or II follows i.e., option (C).

iii. Conclusion II follows being the converted form of statement (i). Conclusion I is mediate inference and does not follow as the term screws is not distributed in the statement. Hence, only conclusion II follows i.e., option (B).

iv. Both the conclusions are mediated inferences and do not follow because the term which is distributed in the conclusion is not distributed in the statement. Hence, option (D) is correct i.e., neither I nor II follows

5.Let

(I) Ding Dong Dang-Attacking the enemy

(II) Ping Pond Dong-Enemy is retreating

(III) Ding Ping Mong-Attacking and retreating

i. In the I and II sentences, common word is 'enemy' and common code is 'Dong'. Hence, 'Dong' stands for 'enemy'.

ii. ii. In the I and III sentences, common word is 'Ding' and common code is 'Attacking'. Hence, code for 'Ding' is 'Attacking'.

iii. iii. From the II and III sentences, we can see that common word is 'Ping' and common code is 'retreating'. Hence, 'Ping' stands for 'retreating'.

iv. iv. From the III sentence, it is clear that 'Mong' stands for 'and

CASE BASED QUESTIONS

1. i. 54   ii. UWX   iii. 467321   OR   WIND

2. i. A is the sister of B and B is the daughter of C. So, A is the daughter of C. Also, D is the father of C. So, A is the granddaughter of D.

ii. A is a male and married to B. So, A is the husband and B is the wife. C is the brother of A. D is the son of C. E who is the sister of D will be the daughter of C. B is the daughter-in-law of F whose husband has died that means F is the mother of A.

iii. B is the son of C but C is not the mother of B means C is the father of B. A is married to C means A is the mother of B. F is brother of B means F is son of A and C. D is daughter of A means D is daughter of A and C. Therefore, A is the mother of B. OR

Clearly, the A's brother is E's maternal uncle. So, A is E's mother or his father's wife.

3.. i. Dust is harmful: # $ @ Dust can kill: # * % So, common code is the code for 'Dust' i.e., #

ii. Harmful object kill% @ ! % Dust can kill% # * % Thus, common code is the code for 'kill' i.e., %

iii. Code for Dust': # Code for 'Kill' % Code for DOG': 123

OR

Dust is harmful% # $ @ harmful object kill: @ ! % So, code for 'harmful: @ and code for 'is': $; code for 'For': 59; code for Lungs: 123.

HIGHER ORDER THINKING SKILLS

1. i. A is the cousin brother of Q.

ii. C is the niece of R.

iii. P is the maternal uncle of C and Q both.

iv. R has two nephews, A and Q.

v. B has two brothers, P and R

2. (b) Both A and R are true but R is not the correct explanation of A.

3. i. 54 ii. UWX iii. 467321 OR WIND

UNIT IV –CALCULUS

Gist /Summary of the lesson:

- Functions: Dependent variable and independent variable, function as a rule or law that defines a relationship between one variable (the independent variable) and another variable (the dependent variable)

- Domain and Range of a function: Domain as a set of all values of independent variable Co-domain as a set of all values of dependent variable. Range of a function as set of all possible resulting values of dependent variable

- Types of functions and their graphical representation: Following types of functions with definitions, characteristics and their graphs. Constant function, Identity function, Polynomial function, Rational function, Composite function, Logarithmic function, Exponential function, Modulus function, Algebraic function.

- Concepts of limits and continuity of a function: Left hand limit, Right hand limit, Limit of a function, Continuity of a function

- Instantaneous rate of change: The ratio $\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$ as instantaneous rate of change, where Δy is change in y and Δx is change in x at any instant.

- Differentiation as a process of finding derivative: Derivatives of functions (non-trigonometric only)

- Derivatives of algebraic functions using Chain Rule: If $y= f(u)$ where $u=g(x)$ then differential coefficient of y w.r.t x is $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Definitions and Formulae

- A function in mathematics is an expression, rule or law that defines relationship between one variable to another variable. A function is a special kind of relation, which is a correspondence between two sets.

- Functions: If A and B are two non-empty sets, then a relation f from a set A to a set B, i.e., a subset of A × B, is said to be a function from A to B, if for each a ∈ A, there exists one and only one b ∈ B such that (a, b) ∈ f

- If A and B are two finite sets having $m \ and \ n$ elements respectively, then

  (i)   Number of relations from A to B is 2^{mn}.
  (ii)   Number of functions from A to B is n^m.

- Algebra of Real Functions Let f : D1 → R and g : D2 → R be two real functions with domain D1 and D2, respectively. Then, algebraic operations as addition, subtraction, multiplication and division of two real functions are given below.

- Addition of two real functions : The sum function (f + g) is defined by (f + g)(x) = f(x) + g(x), ∀ x ∈ D1 ∩ D2. The domain of (f + g) is D1 ∩ D2.

- Subtraction of two real functions : The difference function (f – g) is defined by (f – g)(x) = f(x) – g(x), ∀x ∈ D1 ∩ D2. The domain of (f – g) is D1 ∩ D2.

- Multiplication of two real functions : The product function (fg) is defined by (fg)(x) = f(x).g(x), ∀x ∈ D1 ∩ D2. The domain of (fg) is D1 ∩ D2.

- Quotient of two real functions : The quotient function is defined by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, ∀x ∈ (D1 ∩ D2)– {x : g(x) ≠ 0}.

The domain of $\left(\dfrac{f}{g}\right)$ is D1 $\cap$ D2 $-$ {x: g(x) $\neq$ 0}.

- Multiplication of a real function by a scalar: The scalar multiple function cf is defined by (cf)(x) = c.f(x), $\forall$:x $\in$ D1 where, c is scalar (real number). The domain of cf is D1
- Equal Functions: Two functions f and g are said to be equal iff
  (i)   domain of f = domain of g
  (ii)   codomain of f = codomain of g
  (iii)   f(x)=g(x) $\forall$x $\in$ D1 $\cap$ D2

Various Functions and their Graphs:

- Identity function: Let R be the set of real numbers. A real valued function f defined as
  f: R $\to$ R by y = f(x) = x for each value of x $\in$ R is called the identity function. For the identity function, Domain = R and Range = R. Graph of identity function is given below.
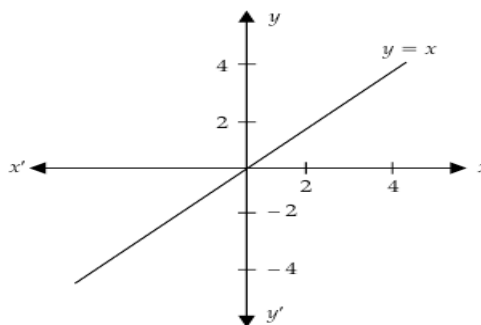
- Constant function: The function f: R $\to$ R defined by f(x) = C for each x $\in$ R is called constant function. (where C is a constant). Its graph is a straight line parallel to
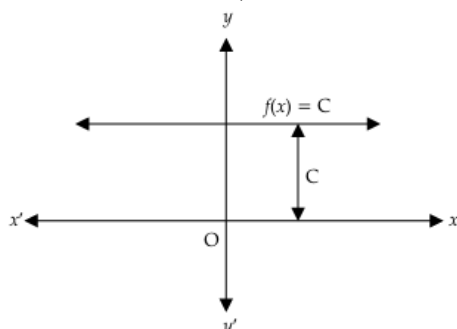
- Modulus function: The function f: R $\to$ R defined by f(x) = |x| for each x $\in$ R is called modulus function or absolute valued function.

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

- Exponential function: If a is any positive real number, then the function f defined by f(x) = a^x; a ≠ 1 is called the exponential function. Here, Domain = R and Range = $(0, \infty)$
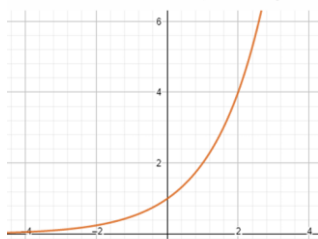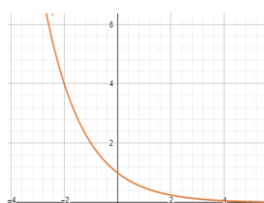
Some properties of exponential functions are:

(i) $a^0 = 1$

(ii) $a^x \cdot a^y = a^{x+y}$ for all x, y ∈R

(iii) $a^x \div a^y = a^{x-y}$ for all x, y ∈R

(iv) $(a^x)^y = a^{xy}$ for all x, y ∈R

(v) $a^{-x} = \dfrac{1}{a^x}$ for all x ∈R

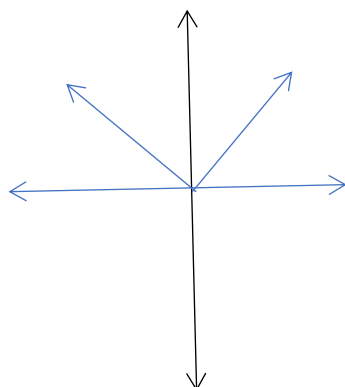- Logarithmic function: If a is any positive real number, a ≠ 1, and if x is any positive real number, then logarithmic function with base a is denoted by symbol $\log_a x$ and is defined as y = f (x) = $\log_a x$ if and only if x = a^y. If the base= e , then $\log_e x$ is called natural logarithm of x

where e= $1 + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \ldots\ldots\ldots$. The value of e lies between 2 and 3.
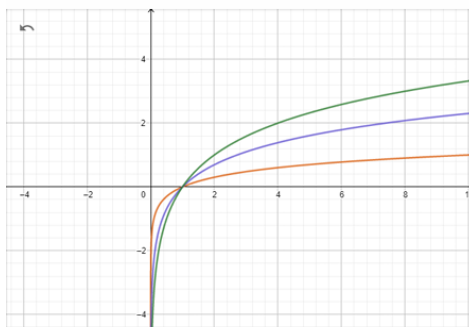
- Polynomial function: The function that can be expressed in the form of a polynomial is called a polynomial function. A polynomial is generally represented as P(x). The highest power of the variable of P(x) is known as its degree. A polynomial function in degree in n can be written as

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_2 x^2 + a_1 x + a_0,$$

- Graphs of some polynomial functions are given below:

Quadratic Polynomial Function: f: R → R, f(x) = x^2

Domain = R and Range = $[0, \infty)$
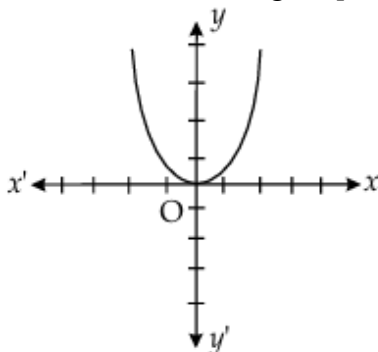
- Cubic Polynomial Function: f: R → R, f(x) = x³, Here Domain= R and Range =R
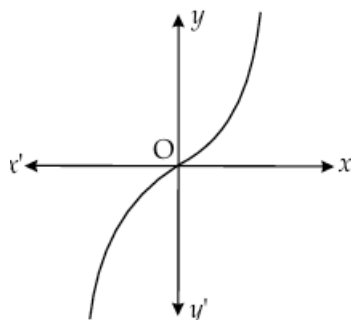
- Limit and Continuity:
  Left hand and right hand limit A real number l_1, is the left hand limit of function f(x) at x = a, if the value of f(x) can be made as close as l_1, at point close to a and on the left of a.
  Symbolically, L.H.L = $\lim_{x \to a^-} f(x)= l_1$
  A real number l_2 is the right hand limit of function f(x) at x = a, if the values of f(x) can be made as close as l_2 at points close to a and on the right of a.
  Symbolically, R.H.L. = $\lim_{x \to a^+} f(x) = l_2$

- Continuity of a function at a point – A real valued function f(x) is said to be continuous at $x = a$ if LHL= RHL= f (a)
  $ie,$ $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) =$ f (a)

- Derivative of a function- The derivative of a function f(x) is defined by

  $$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

- Product Rule-

  If y= u v   then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

- Quotient Rule-

  If y= $\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

- Chain Rule-

  If $y = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$

- **Standard Derivatives**

Sl. No	Function	Derivative
1	x^n	nx^{n-1}
2	K (constant)	0
3	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$

4	e^x	e^x
5	$\log_e x$	$\dfrac{1}{x}$
6	a^x	$a^x \log a$

MULTIPLE CHOICE QUESTIONS

1) The number of elements in set A is 3 and the number of elements in set B is 2. The number of possible relations that can be defined from A to B is

   a) 8   b) 4   c) 64   d) 512

   Solution:

   AXB has 2x3=6 elements and therefore it has $2^6 = 64$ subsets. The number of possible relations from set A to set B is the number of subsets of AXB, as every relation is a subset of AX B. Hence the number of relations is 64.

   Answer: c

2) Let A = {1, 2, 3} and B = {2, 3, 4} and f1, f2 and f3 are three subsets of A × B as given below.

   Which of the following is a function?

   f_1 = {(1, 2), (2, 3), (3, 4)} f_2 ={(1, 2), (1, 4), (2, 3), (3, 4)}

   f_3 ={(1, 2), (3, 3), (2, 3), (3, 4)} f_4 ={(1, 2), (2, 3)}

   a) f_1 b) f_2 c) f_3 d) f_4

   Solution:

   f_2 is not a function as 1 has two images, f_3 is not a function as 3 has two images and f_4 is not a function as 3 has no image.

   Answer: a

3) The domain of the function f(x) = $\frac{x+2}{3-x}$ is

   a) R-{-2}   b) R-{2}   c) R-{-3}   d) R-{3}

   Solution:

   Clearly, f(x) is defined for all x satisfying 3 – x ≠ 0 i.e., x ≠ 3 Hence, D_f = R – {3}

   Answer: d

4) Given that a function, f = {(2, 1), (5, 1), (8, 1), (11, 1), (14, 3), (17, 1)}, what is its range?

   a) {2}   b) {2, 5, 8, 11, 14, 17}   c) {1,3}   d) {3}

   Solution:

   The range of a function is the set of all images. So, Range = {1,3}

   Answer: c

5) The points of discontinuity of the function

$$f(x) = \begin{cases} 2x + 3, & if \ x \le 2 \\ 2x - 3, & if \ x > 2 \end{cases} \text{ are}$$

a) -2 b) 2 c) ± 2 d) (-2,2)

Solution:

At x =2, LHL = 2(2) + 3 =7
RHL = 2(2) - 3 = 1
LHL $\neq$ RHL
So f(x) is not continuous at x=2. x = 2 is the point of discontinuity.

Answer: b

6) If $f(x) = \begin{cases} 3x - 5, & x \le 4 \\ 2k, & x > 4 \end{cases}$ is continuous at x=4 then k is

a) $\frac{7}{2}$ b) $\frac{2}{7}$ c) $\frac{-7}{2}$ d) $\frac{-2}{7}$

Solution:

At x= 4, LHL = 3(4)-5 =7
RHL = 2k
LHL = RHL
7=2k, k= $\frac{7}{2}$

Answer: a

7) If f(x) =x+1 and g(x)= 2x-3 ,then (f.g)(x) is
   a) $2x^2$ -3 b) $2x^2$ –x - 3 c) $2x^2$ + x - 3 d) $2x^2$ –x +3

Solution:

(f.g)(x)= f(x). g(x) = (x+1) (2x-3)= $2x^2$ –x - 3

Answer: b

8) The edge of a cube is changing at the rate of 0.3 cm/s, the rate of change of its surface area when edge is 3 cm is

a) $12.8cm$ b) $10.8 \ cm^2$ c) $10.8 \ cm^2/s$ d) $10.8 \ cm/s$

Solution:

Let x be the edge of the cube . Given $\frac{dx}{dt}$ = 0.3 cm/s .
Surface area, S = $6x^2$
Then $\frac{ds}{dt}$ = 12x $\cdot \frac{dx}{dt}$ = 12x $\times$ 0.3, $\frac{dS}{dt}$ = 3.6x

And $\frac{dS}{dt}$ at x = 3 is 3.6 $\times$ 3 = 10.8 cm^2/s

Answer: c

9) If A = {2, 4, 6, 9} and B = {4, 6, 18}, then the set of ordered pairs (a,b) where a $\in$ A and b $\in$ B, such that a is a factor of b and a < b is
   a) {(2, 4), (2, 6), (2, 18), (6,18),(9,18)} b) {(2, 4), (2, 6), (2, 18), (6,18),(9,18), (4,6)}

   c){(2,2),(2, 4), (2, 6), (2, 18),(4,4), (6,18),(9,18)} d) {(2, 4), (2, 6), (6,18),(9,18)}

10) The value of $(81)^{1/4}$ x $(16)^{3/4}$

a) 18   b)9   c) 27   d) 24

Solution:

$(81)^{1/4}$ x $(16)^{3/4}$ $=(3^4)^{\frac{1}{4}}$ x $(2^4)^{\frac{3}{4}}$ $= 3$ x $(2^3) = 24$

Answer: d

11) The domain of the function f defined by f(x)=$\frac{x^2+2x+1}{x^2-x-6}$ is

a) R   b) R-{3,-2}   c) R-{-3,2}   d) R-{3,2}

Solution:

f(x) is defined only if $x^2 - x - 6 \neq 0$

(x-3) (x+2) $\neq$ 0, ,   x$\neq$ 3 ,   x$\neq -2$

So domain of f = R-{3,-2}

Answer: b

12) The domain and range of the real function f(x) = $\frac{x-2}{2-x}$ is

a) R , {1}   b) R-{2},{-1}   c) R-{-2}, {-1}   d) R-{2}, {1}

Solution:

f(x) is defined only if $2 - x \neq 0$

(x-2) $\neq$ 0,   x$\neq$2

So domain of f = R-{2}

$\frac{x-2}{2-x}$ = -1 for every x $\neq$2

Hence range ={-1}

Answer: b

13) If f(x) =x+1 and g(x)= 2x-3 ,then (f0g)(x) is

a) $2x^2$ -3   b) $2x^2$ –x - 3   c) $2x^2 + x - 3$ d) 2x-2

Solution:

(fog)(x) = f(g(x)) =f(2x-3)= 2x-3+1=2x-2

Answer: d

14)Domain of f(x) = $\sqrt{x + 4}$ is

a) [-4, ∞)   b) [4, ∞)   c) [-4, 4]   d) (-∞, 4])

Solution:

f(x) is defined if x+4≥ 0, ie x ≥ -4

So Domain = [-4, ∞)

Answer: a

15) The derivative of f(x) = 2x² –x – 3 is

a) 4x-x   b)4x-1   c) 4x²-x-3   d) 4x² –x

Solution:

f'(x) = 2.(2x)-1-0 = 4x-1

Answer: b

16) Consider $f(x) = \begin{cases} 2x + 1, if x < 2 \\ k, if x = 2 \\ 3x - 1, if x > 2 \end{cases}$ *t*he value of k, if f(x) is continuous at x = 2.

   a)) $\frac{3}{2}$ b)) $\frac{-5}{2}$ c)) $\frac{7}{2}$ d) $\frac{5}{2}$

Solution:

At x= 2,   LHL = 2(2)+1 = 5

RHL = 3(2) -1= 5,   f(2) = 2k when x =2

LHL = RHL = f(2)

5=2k,   k= $\frac{5}{2}$

Answer: d

17) Range of the function f(x) = lxl + 1 is

   a) R   b) [1, ∞)   c) [0, ∞)   d) R-{0}

Solution:

lxl ≥ 0,   lxl +1 ≥ 1 , f(x) ≥ 1

Range = [1, ∞)

Answer: b

18) If f(x) = x² – 3x + 2, then f '(1) is

   a) -1   b) 1   c) -5   d) 7

Solution:

f '(x) = 2x-3

f '(1) = 2(1)-3 = 2-3 =-1

Answer: a

19) If y = e^{x^2} then $\frac{dy}{dx}$ is

a) e^{x^2} b) e^{2x} c) $2x \, e^{x^2}$ d) $2e^x$

Solution:

y = e^{x^2}

$\frac{dy}{dx}$ = e^{x^2} (2x)   = $2x \, e^{x^2}$

20) If f(x)=x+7 and g(x)=x-7,x∈ R , then (f og)(7) is

   a)-49   b)14   c)0   d)7

Solution:

(f 0g) (7) =f(g(7))= f(7-7)= f(0)=7

Answer: d

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

   (A) Both Assertion (A) and Reason (R) true and Reason (R) is the correct explanation of Assertion (A).
   (B) Both Assertion (A) and Reason (R) true and Reason (R) is not the correct explanation of Assertion (A).
   (C) Assertion (A) is true but Reason (R) is false.
   (D) Assertion (A) is false but Reason (R) is true.

1) ASSERTION (A): The relation R: A→B is defined by R= {(1, x), (2, y), (3, x)},
then its domain is {1, 2, 3} and range is {x, y}.
REASON(R) : The range of a relation R is always the co-domain set
Answer: C
Solution:

Clearly the domain of R is {1, 2, 3} and range is {x, y}. Hence A is true

R is false because range is a subset of codomain, but need not be the codomain.

2) ASSERTION(A): The value of k for which the function $f(x) = \begin{cases} kx^2 & if \quad x \le 1 \\ 4 & if x > 1 \end{cases}$ is

continuous at x=1 is 4
REASON(R): A function is continuous at x=a if LHL= RHL= f(a)

Answer: A
Solution:

At x=1, LHL = k, RHL= 4 and f(1)=4. Hence A and R are true and R is the correct explanation of A.

3) ASSERTION(A): log 2+ log3 = log5

  REASON(R): loga+ logb= log(ab)

Answer: D
Solution:

log 2+ log3 = log (2x3)= log6. Hence A is false. But R is true.

4)ASSERTION(A): $\lim_{x \to 0} \frac{2^x - 1}{x} = \log 2$

  REASON(R): $\lim_{x \to 0} \frac{a^x - 1}{x} = \log a$

Answer: A
Solution:

Here A and R are true and R is the correct explanation of A.

5) ASSERTION(A): The domain of the function $f(x) = \sqrt{x - 4}$ is $[4, \infty)$

REASON(R): The domain of a function is the set of all values for which f(x) is defined.

Answer: A
Solution:

Here A and R are true and R is the correct explanation of A.

6) ASSERTION(A): $\frac{d}{dx}(e^{3\log x}) = 3x^2$
REASON(R): $e^{3\log x} = 3x$

Answer: C
Solution:

$e^{3\log x} = x^3$, $\frac{d}{dx}(e^{3\log x}) = 3x^2$ A is true.

R is false.

7) ASSERTION(A): The relation {(a,1),(b,2)} defined from A to B where A={a,b}, B={1,2,3}

is a function
REASON(R): All relations are functions.

Answer: C

Solution:

As every element in the domain has one and only one image in the codomain, R

is a function. All relations are not functions.Hence A is true and R is false.

8) ASSERTION(A): If A={a ,b}, B={1,2,3} ,the number of relations from A to B is 64

REASON(R): If n(A)= p and n(B)= q, then number of relations from A to B is 2^{pq}.

Answer: A

Solution:

n(AXB) = pq

Number of relations from A to B= Number of subsets of AXB = 2^{pq}.

Hence both A and R are true and R is the correct explanation of A.

9) ASSERTION(A): The range of the relation R= {(a,1), (b,2), (c,3)}is {1,2,3}

REASON(R): The range of a relation from A to B is a subset of B.

Answer: B

Solution:

The range of R= The set of all images= {1,2,3}

Range is a subset of codomain B

Hence both A and R are true and R is not the correct explanation of A.

10) ASSERTION(A): The function f(x) = 2^x is an increasing function.

REASON(R): The function f(x) = a^x is an increasing function for all values of a.

Answer: C

Solution:

For a>1 the function f(x) = a^x is an increasing function. For 0<a<1 the function f(x) = a^x is

a decreasing function. Hence A is true and R is false.

VERY SHORT ANSWER TYPE QUESTIONS

1) Question: Evaluate: $\lim_{x \to 2} \frac{x^2-4}{x-2}$

Solution: $\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \to 2}(x+2) = 4$

2) Question: Find derivative of f(x)=x² with respect to x using first principle.

Solution: $f^1(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2-x^2}{n} = \lim_{h \to 0} \frac{x^2+h^2+2hx-x^2}{h} = \lim_{h \to 0} \frac{h(h+2x)}{h} = 2x$

3) Question: Find the domain and range of the function f(x) = $\frac{x-2}{x-1}$

Solution: f(x) is defined only if $x-1 \neq 0, x \neq 1$

Domain =R-{1}

Let y=$\frac{x-2}{x-1}$, $(x-1)y = x-2$, $xy-y = x-2$

$xy-x = y-2$, $x(y-1) = y-2$

$x = \frac{y-2}{y-1}$, Range= R-{1}

4) Question: Find the range of the function f(x) = x^2 +2, x∈ R

Solution: $x^2 \geq 0, \forall x \in R \Rightarrow x^2 + 2 \geq 2$

Range = [2, ∞)

5) Question: Find the range of f(x) = - $|x|$

Solution: $|x| \geq 0 \, \forall x \in R \Rightarrow -|x| \leq 0, \, \forall x \in R$
⇒ Range = (- ∞, 0]

6) Question: Let f be a function defined by f: x→ $5x^2 + 2$, $x \in R$, find f(3) × f(2)

Solution: $f(3) = 5 \times 3^2 + 2 = 47$

$f(2) = 52^2 + 2 = 22$
$f(3) \times f(2) = 47 \times 22 = 1034$

7) Question: Find the domain of f(x) =$\sqrt{x-1}$

Solution: f(x) is defined only if $x-1 \geq 0$

⇒ $x \geq 1 \Rightarrow Domain = [1, \infty)$

8) Question: A function f is defined by f(x) =$x^2 + 2$, $x \in \{-1,1,0,2\}$. List the elements of f .

Find its range.

Solution: $f(-1) = (-1)^2 + 2 = 3$ $f(1) = (1)^2 + 2 = 3$

$f(0) = (0)^2 + 2 = 2$ $f(2) = (2)^2 + 2 = 6$

$f = \{(-1,3),(1,3),(0,2),(2,6)\}$

$\Rightarrow$ Range= $\{3,2,6\}$

9) Question: Find the domain for which the functions f(x) = 2x^2 -1 and g(x) = 1-3x are equal

Solution: $2x^2 - 1 = 1 - 3x$ $\Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow (2x - 1)(x + 2) = 0 \Rightarrow x = \frac{1}{2}, x = -2$

The domain of the function is $\left\{-2, \frac{1}{2}\right\}$

10) Question: If f(x) = 2x^2 -1 and g(x) = $\sqrt{x}$ find f0g and g0f

Solution: f 0g(x) = f(g(x)) = f($\sqrt{x}$) = 2($\sqrt{x}$)^2- 1 = 2x-1

g0f(x) = g(f(x)) = g(2x^2 -1) = $\sqrt{2x^2 - 1}$

11) Question: Evaluate $\lim\limits_{x \to 2} \frac{3^x - 3^2}{x-2}$

Solution: Let x=2+h , as $x \to 2, h \to 0$

$\lim\limits_{x \to 2} \frac{3^x - 3^2}{x-2} = \lim\limits_{h \to 0} \frac{3^{2+h} - 3^2}{2+h-2} = 9 \lim\limits_{h \to 0} \frac{3^h - 1}{h} = 9 \log 3$

12) Question: Find the value of k if the function

$f(x) = \begin{cases} kx^2 + 2 & if \ x \leq 2 \\ 6 & if \ x > 2 \end{cases}$ is continuous at x=2

Solution: LHL $= \lim\limits_{x \to 2-} kx^2 + 2 = 4k+2$

RHL = 6, f(2) = 4k+2

As the function is continuous at x= 2, LHL= RHL= f(2)

4k+2 = 6 $\Rightarrow 4k = 4 \Rightarrow k = 1$

13) Question: Find the derivative of $f(x) = x^2 \ at \ x = 5 \ using \ first \ principle$

Solution: $f^1(5) = \lim\limits_{h \to 0} \frac{f(5+h) - f(5)}{h} = \lim\limits_{h \to 0} \frac{((5+h)^2) - 5^2}{h} = \lim\limits_{h \to 0} \frac{(25+10h+ h^2) - 25}{h}$

$= \lim\limits_{h \to 0} \frac{h(10+h)}{h} = 10$

14) Question: Find the derivative of f(x) = $x^3 e^x$

Solution: $f^1(x) = x^3 \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x^3)$

$= x^3 e^x + e^x \cdot 3x^2$

15) Question: If y= 2x^3- 5logx +3, then find $\frac{dy}{dx}$

Solution: $\frac{dy}{dx} = 2 \times 3x^2 - 5 \cdot \frac{1}{x} + 0 = 6x^2 - \frac{5}{x}$

16) Question: **If** $y = x^3 \log x$ $find$ $\frac{dy}{dx}$

Solution: $\frac{dy}{dx} = x^3(1/x) + \log x \, 3x^2 = x^2 + 3x^2 \log x$

17) Question: $Find\ the\ range\ of\ f(x) = 3 - 4x, x > 0$
   Solution: $x > 0 \Rightarrow -4x < 0 \Rightarrow 3 - 4x < 3$

   Range = $(-\infty, 3)$

18) **Question:** Find the derivative of $x^3(4 + 5x^{-3})$

   Solution: $Let\ y = x^3(4 + 5x^{-3})$

   $\frac{dy}{dx} = x^3(-15x^{-4}) + (4 + 5x^{-3})3x^2 = \frac{-15}{x} + 12x^2 + \frac{15}{x} = 12x^2$

19) Question: $If\ f(x) = \begin{cases} 1 + x^2 & if\ x \le 1 \\ x + 2 & if x > 1 \end{cases}$, does $\lim\limits_{x \to 1} f(x)$ exist?
   Solution: LHL = 1+1= 2, RHL= 1+2= 3,

   LHL$\ne$ RHL

   Hence $\lim\limits_{x \to 1} f(x)$ does not exist.

20) Question: Evaluate $\lim\limits_{x \to 0} \frac{2x+3}{x^2-1}$

   Solution: $\lim\limits_{x \to 0} \frac{2x+3}{x^2-1} = \frac{2.0+3}{0^2-1} = \frac{3}{-1} = -3$

SHORT ANSWER TYPE QUESTIONS

1) Question: Find all possible values of "a " if $\lim\limits_{x \to a} \frac{x^9 - a^9}{x-a} = \lim\limits_{x \to 5} 4 + x$
   Solution: $9a^8 = 9$, a=± 1
2) Question: If f(x) =$\frac{x+1}{x-1}$ then find f(x^2).$[f(x)]^2$
   Solution:

   f(x^2)=$\frac{x^2+1}{x^2-1}$, $[f(x)]^2 = (\frac{x+1}{x-1})^2$

   Final answer $\frac{(x^2+1)(x+1)}{(x-1)^3}$
3) Question: Evaluate the following limit $\lim\limits_{x \to 3} \frac{x^3-243}{x^3-27}$

   Solution:

   $\lim\limits_{x \to 3} \frac{x^5-3^5}{x^3-3^3} = \lim\limits_{x \to 3} \frac{\frac{x^5-3^5}{x-3}}{\frac{x^3-3^3}{x-3}} = \frac{5.3^4}{3.3^2} = 15$

4) **Question:** If y=f(x) =$\frac{1-x}{1+x}$ then show that x=f(y)

   Solution:

$y=\frac{1-x}{1+x}$

$(1+x)\ y = 1-x$

$x=\frac{1-y}{1+y}$ $=f(y)$

5) Question: Represent the relation R given by

R = {(x, y): $y = x + \frac{6}{x}$ where x, y ∈ N, and x < 4} in roster form. Write its domain and range.
Is R a function?
Solution: R= { (1,7), (2,5), (3,5)}

Domain={1,2,3},Range= {7,5}

R is a function because every element in the domain has one and only one image.

6) Question: Find the derivative of f(x) $= \frac{x^5 - \log x}{e^x}$ with respect to x.

Solution: $f^1_{(x)} = \frac{e^x[5x^{\underline{4}}\frac{1}{x}]-[x^5-\log x]e^x}{(e^x)^2}$

7) Question: Find the derivative of f(x) $= \frac{x^2-1}{x}$

Solution: $f'(x) = 1 + \frac{1}{x^2}$

8) Question: Find the relationship between a and b if

$f(x) = \begin{cases} ax+1 & if\ x \leq 3 \\ bx+3 & if\ x > 3 \end{cases}$ is continuous at x=3

Solution: LHL = 3a+1, RHL = 3b +3, f(3) = 3a+1

LHL= RHL=f(3) ⇒ 3a+1 = 3b +3

⇒ 3a-3b = 2 ⇒ $a - b = \frac{2}{3}$

9) Question: Differentiate $f(x) = 2^x - \frac{1}{3}e^x + \log x$

Solution: f'(x)= $2^x \log 2 - \frac{1}{3}e^x + \frac{1}{x}$

10) Question: Let f={(1,1),(2,3),(0,−1),(−1,−3) ..…} be a function from Z to Z defined

by $f(x) = ax + b$, for some integers a and b, Determine a and b

Solution: Since (1,1) ∈ $f, f(1) = 1 \Rightarrow a + b = 1$
(2,3) ∈ $f, f(2) = 3 \Rightarrow 2a + b = 3$

Solving for a and b, we get a= 2 and b = -1

11) Question: If y= $x + \frac{1}{x}$, prove that $x^2\frac{dy}{dx} - xy + 2 = 0$

Solution: y= $x + \frac{1}{x}$, $\frac{dy}{dx} = 1 - \frac{1}{x^2}$

$x^2\frac{dy}{dx} = x^2 - 1$

⇒ $x^2\frac{dy}{dx} - xy + 2 = x^2 - 1 - (x^2 + 1) + 2 = 0$

12)Question: $If \ f(x) \equiv \begin{cases} \frac{x^2-9}{x-3} & if \ x \neq 3 \\ k & if \ x = 3 \end{cases}$ is continuous at x=3 , find the value of k

Solution: $\lim\limits_{x \to 3} f(x) = \lim\limits_{x \to 3} \frac{x^2-9}{x-3} = \lim\limits_{x \to 3}(x + 3) = 6$

As the function is continuous at x=3 , $\lim\limits_{x \to 3} f(x) = f(3) \Rightarrow k = 6$

LONG ANSWER TYPE QUESTIONS

1) Question: Find the value of k for which the function f(x) is continuous at x=0 , where

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & if -1 \leq x < 0 \\ \frac{2x+1}{x-1} & if \ 0 \leq x < 1 \end{cases}$$

Solution :

$\lim\limits_{x \to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} = \lim\limits_{x \to 0^+} \frac{2x+1}{x-1}$

$\lim\limits_{x \to 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}$ x $\frac{\sqrt{1+kx} + \sqrt{1-kx}}{\sqrt{1+kx} + \sqrt{1-kx}}$ = -1

$\lim\limits_{x \to 0^-} \left(\frac{1+kx - 1 + kx}{x(\sqrt{1+kx} + \sqrt{1-kx})} \right)$ = -1

$\lim\limits_{x \to 0^-} \left(\frac{2kx}{x(\sqrt{1+kx} + \sqrt{1-kx})} \right)$ = -1 , $\frac{2k}{2}$ = -1 $\Rightarrow$ k = -1

2) Question: Find the derivative of y = $2x^2$+ 3x. with respect to x using first principle.

Solution: $f(x) = 2x^2 + 3x$
f(x+h)= $2(x+h)^2 + 3(x+h) = 2(x^2+2xh+h^2) + (3x+3h)$

$= 2x^2+4xh+2h^2+3x+3h$

$\frac{dy}{dx} = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h \to 0} \frac{[2x^2 + 4xh + 2h^2 + 3x + 3h] - [2x^2 + 3x]}{h}$

$= \lim\limits_{h \to 0} \frac{h(4x + 2h + 3)}{h}$

$= 4x + 3$

3) Question: If f (x) = $x^3 - \frac{1}{x^3}$, then find f (x) + f$\left(\frac{1}{x}\right)$ and f (x) - f$\left(\frac{1}{x}\right)$.

Solution: $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$

$$= \frac{1}{x^3} - x^3$$

$$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$$

$$f(x) - f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} - \left(\frac{1}{x^3} - x^3\right) = 2\left(x^3 - \frac{1}{x^3}\right)$$

4) Question: Find the derivative of $f(x) = \frac{1}{x^2}$ using first principle.

Solution: $f^1_{(x)} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\left(\frac{1}{x+h}\right)^2 - \frac{1}{x^2}}{h \cdot} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h\, x^2(x+h)^2} = \lim_{h \to 0} \frac{x^2 - (x^2 + 2hx + h^2)}{h\, x^2(x+h)^2}$

$$= \lim_{h \to 0} \frac{-2hx - h^2}{h\, x^2(x+h)^2} = \lim_{h \to 0} \frac{-2x - h}{x^2(x+h)^2} = \lim_{h \to 0} \frac{-2x}{x^4} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

CASE BASED QUESTIONS

1) On heating, the volume of a metal cube is increasing at a rate of 9 cubic centimetres per Second. When its edge is 10 centimetres, (i) How fast is the edge of the cube increasing? (ii)How fast is the surface area increasing?

Solution:

Let a be the edge of the cube.

$\frac{dv}{dt} = 9cm^3/sec$

$v = a^3 \Rightarrow \frac{dv}{dt} = 3a^2 \frac{da}{dt}$

$9 = 3a^2 \frac{da}{dt} \Rightarrow \frac{da}{dt} = \frac{3}{a^2} = \frac{3}{10^2} = 0.03 cm/sec$

Surface area of cube, $S = 6a^2$

$\frac{ds}{dt} = 12a \frac{da}{at} = 12a \cdot \frac{3}{a^2} = \frac{36}{a} = \frac{36}{10} = 3.6 cm^2/sec$

2) Read the following passage and answer the questions given below:

The relation between the height of the plant ('y" in cm) with respect to its exposure to the sunlight is governed by the following equation y = 4x - $\frac{1}{2}$ x ^{2} , where ' x ' is the number of days the plant is exposed to the sunlight.
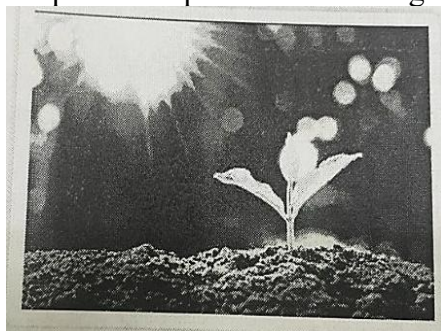
(i)   Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.
(ii)   What will be the height of the plant after 2 days

Solution:

(i)   Rate of growth of plant with respect to number of days = $\frac{dy}{dx} = 4 - x$

(ii)   Height of the plant after 2 days = 4(2) - $\frac{1}{2}$ 2 ^{2} = 8-2 = 6 cm

3) $P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company.

  (i) When the production is 2units what will be the profit of the company?
  (ii) What will be production of the company when the profit is Rs 38250?
  Solution:

(i)   Profit = $-5x^2 + 125x + 37500 = -5(2)^2 + 125(2) + 37500$
               = -20 + 250 + 37500 = Rs 37730

(ii) $38250 = -5x^2 + 125x + 37500$
        $5x^2 - 125x + 750 = 0$
        $x^2 - 25x + 150 = 0$
        $(x-15)(x-10) = 0$
        x = 15 or 10

HIGHER ORDER THINKING SKILLS

1) Question: Find the equations of a line tangent to $y = x^3 - 2x^2 + x - 3$ at the point x=1.
   Solution:

Slope of tangent $= \frac{dy}{dx} = 3x^2 - 4x + 1 = 3 - 4 + 1 = 0$

$x_{1=1}, y_1 = 1 - 2 + 1 - 3 = -3$

Equation of tangent is

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$
$$y - (-3) = 0(x - 1)$$
y+3 = 0

EXERCISE

MULTIPLE CHOICE QUESTIONS

1) If a set A has 3 elements and set B has 2 elements, then the number of functions from A to B is
     a) 9   b) 6   c) 64   d) 8
2) Range of the function f(x) = lxl is
     a) R   b) R-{0}   c) $[0, \infty)$ d)$(- \infty, 0]$

3) If $f(x) = x^2 + 2$ and $g(x) = x^2 - 3$, $x \in R$, then $(f.g)(1)$ is

     a) 6   b)-6   c)2   d)-3

4) If the radius of a circle is increasing at the rate of 0.1 cm/s, then the rate of change of

   its area when radius is 5cm

     a) 5π cm^2 b) 5π cm^2/s c) 0.05π cm^2 d) π cm^2/s

5) $\lim\limits_{x \to 0} \frac{|x|}{x}$ is

a)1   b)-1   c) 0   d) Does not exist

6) The derivative of $\sqrt{x} - \frac{1}{x}$ is

    a) $\frac{\sqrt{x}}{2} + \frac{1}{x^2}$ b) $\frac{1}{2\sqrt{x}} + \frac{1}{x^2}$ c) $\frac{1}{2\sqrt{x}} - \frac{1}{x^2}$ d) $\frac{\sqrt{x}}{2} + \frac{1}{x^2}$

7) $\lim\limits_{x \to 3} \frac{x^2-9}{x-3}$

    a)9   b)0   c)6   d) Doesn't exist

8) The domain of f(x) = $\sqrt{9 - x^2}$

    a) [-3,3]   b) (-3,3)   c) (-3,3]   d) {-3,3}

9) The derivative of log (log x) is

    a) $\frac{1}{x\log x}$ b) $\frac{x}{\log x}$ c) $\frac{\log(\log x)}{\log x}$ d) None of these

10)The range of the function f(x) =$|x| + 3$ is

    a) (-3, ∞)   b) [3, ∞)   c) [-3, ∞)   d) (- ∞,3)

ASSERTION-REASON BASED QUESTIONS

1) ASSERTION(A): The domain of the function f(x)=$\frac{x^2-5x+3}{x-2}$ is R- {2}

   REASON(R): The function f(x) is a polynomial function.

2) ASSERTION(A): There are 9 functions from A to B if n(A)=2 and n(B)=3

   REASON(R): If n(A)=m and n(B)=n, there are n^m functions from A to B.

3) ASSERTION(A): The domain of the function $f(x) = \sqrt{25 - x^2}$ is $[-5,5]$

   REASON(R): The range of the function $f(x) = \sqrt{25 - x^2}$ is $[0,5]$

4) ASSERTION(A): The function f(x)= $|x|$ is a continuous function.

   REASON(R): Every polynomial function is continuous.

5) ASSERTION(A): The function f(x)=$\begin{cases} 2x - 3 & if\ x \leq 2 \\ 5 & if\ x > 2 \end{cases}$ is not continuous at x=2

   REASON(R): For a function to be continuous at x=a, LHL=RHL=f(a)

6) ASSERTION(A): For the function f(x)=$\begin{cases} 5x + 2 & if\ x \leq 2 \\ 7 & if\ x > 2 \end{cases}$ $\lim\limits_{x \to 2-} f(x) = 12$

    and $\lim\limits_{x \to 2+} f(x) = 7$

   REASON(R): $f(x) = \begin{cases} 5x + 2 & if\ x \leq 2 \\ 7 & if\ x > 2 \end{cases}$ is not continuous at x=2

VERY SHORT ANSWER TYPE QUESTIONS

1) Let f be a function defined by f(x) = 4x-3, x∈ R ,Find x such that f(x) =17

2) Find the domain of the function $f(x) = \frac{x^2+3x+2}{x^2-5x+6}$

3) Find the range of the function f(x) = $\sqrt{16 - x^2}$

4) Evaluate $\lim\limits_{x \to 3} \frac{x^2-5x+6}{x-3}$

5) Evaluate $\lim\limits_{x \to 0} \frac{e^{5x}-1}{x}$

6) Find the domain of the function $f(x) = log(2x - 3)$

7) Evaluate $\lim\limits_{x \to 0} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{x}$

8) Evaluate $\lim\limits_{x \to 1} \frac{log\,x}{x-1}$

9) Find the derivative of $\sqrt{5x + 4}$

10) Find the derivative of $(3x^2 + 5)^3$

SHORT ANSWER TYPE QUESTIONS

1) If f and g are two functions defined by $f(x) = x^2 \; and \; g(x) = 2x + 3,$

   find $fog \; and \; gof$

2) Evaluate $\lim\limits_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

3) If y= log(x+$\sqrt{x^2 - a^2}$) , find $\frac{dy}{dx}$

4) Find the derivative of $\sqrt{x + 2}$ using first principle

5) Find the derivative of x^5 logx

6) If f(x) = $\sqrt{4 - x^2}$, find f'(x)

LONG ANSWER TYPE QUESTIONS

1) $f(x) = \begin{cases} a + bx, x < 1 \\ 4, x = 1 \\ b - ax, x > 1 \end{cases}$ is continuous at x=1 what are the possible values of a and b

2) Find the derivative of the following function using first principle.
   $f(x) = \frac{2x+3}{x-2}$

3) Find the derivative of the following function using first principle
   f(x) = (x-1) (x-2)

CASE STUDY BASED QUESTION

1 Let function y= x^2 that measures the area of a metallic square of side x.
   If at any given time the side of the square is 5 cm, and we heat the square
   uniformly increasing the side (i) what is the rate of change of the
   area with respect to the side in that moment?
   (ii) What is the area of the square when x=10cm?

2 The area and perimeter of a circle are functions of its radius. (i) Find the rate of change
   of the area of a circle with respect to its radius when the radius is 4cm
   (ii) Find the rate of change of the perimeter of circle with respect to its radius.

HIGHER ORDER THINKING SKILLS

1) If y= $\frac{e^x + e^{-x}}{e^x - e^{-x}}$, prove that $\frac{dy}{dx} = 1 - y^2$

2) If y= $log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, prove that $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$

ANSWERS

MULTIPLE CHOICE QUESTIONS

(1) d   (2) c   (3) b   (4) d   (5) d   (6) b   (7) c   (8) a   (9) a   (10) b

ASSERTION-REASON BASED QUESTIONS

1) C   2) A   3) B   4) B   5) A   6) B

VERY SHORT ANSWER TYPE QUESTIONS

1) 5   2) R-{2,3}   3)[0,4]   4) 1   5) 5

6) $\left(\left(\frac{3}{2}, \infty\right)\right.$ 7) 2   8) 1   9) $\frac{5}{2\sqrt{5x+4}}$ 10) 18x$(3x^2 + 5)^2$

SHORT ANSWER TYPE QUESTIONS

1) $fog(x) = (2x + 3)^2$ $gof(x) = 2x^2 + 3$ 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{x^2-a^2}}$ 4) $\frac{1}{2\sqrt{x+2}}$

5) $x^4 + 5x^4 \log x$ 6) $\frac{-x}{\sqrt{4-x^2}}$

LONG ANSWER TYPE QUESTIONS

1) a=0, b=4   2) $\frac{-7}{(x-2)^2}$ **3) 2x-3**

CASE STUDY BASED QUESTION

1 (i) 10cm^2 /cm   (ii) 100 cm^2

2 (i) 8π cm^2 /cm (ii) 2π cm /cm

UNIT-5 -PROBABILITY

PROBABILITY
SOME IMPORTANT RESULTS/CONCEPTS

**Random Experiment
An Experiment that can be conducted any number of times under similar conditions and even the possible outcomes are known but exact outcome is unknown is called a Random Experiment.

Sample Space: The set of all possible outcomes of a trial (random experiment) is called its sample space. It is generally denoted by S and each outcome of the trial is said to be a sample point.

EVENT: Every subset of a sample space is called an event.

(a) Simple event: A simple event or an elementary event is an event containing only a single sample point.

(b) Compound events: Compound events or decomposable events are those events that are obtained by combining together two or more elementary events.

(c) Mutually exclusive or disjoint events: Events are said to be mutually exclusive or disjoint or incompatible if the occurrence of any one of them prevents the occurrence of all the other events.

(d) Mutually non-exclusive events: The events which are not mutually exclusive are known as compatible events or mutually nonexclusive events.

(e) Independent events: Events are said to be independent, if the happening (or non-happening) of one event is not affected by the happening (or non-happening) of other events.

(f) Dependent events: Two or more events are said to be dependent, if the happening of one event affects (partially or totally) the other event**.**

(g)Equally likely events: Events which have the same chance of occurring are said to be equally likely events

COMPLEMENT OF EVENTS: The complement of an event 'A' with respect to a sample space S is the set of all elements of 'S' which are not in A. It is usually denoted by A'or A^c

(a) The union $E_1 \cup E_2$ of events E_1 and E_2 is the event of at least one of the events E_1 , E_2 happening.

(b) The intersection $E_1 \cap E_2$ of events E1 and E2 is the event of both the events E_1 , E_2 happening.

ALGEBRA OF EVENTS:

Verbal description of the event	Equivalent set theoretic notation
Not A	A^c
A or B (at least one of A or B)	$A \cup B$
A and B	$A \cap B$
A but not B	$A \cap B^C$
Neither A nor B	$A^C \cap B^C$
At least one of A, B or C	$A \cup B \cup C$

Exactly one of A and B	$(A^C \cap B) U (A \cap B^C)$
All three of A, B and C	$(A \cap B \cap C)$
Exactly two of A, B and C	$(A \cap B \cap C^C) U ((A \cap B^C \cap C) U (A^C \cap B \cap C)$

PROBABILITY If a random experiment results in n mutually exclusive, equally likely and exhaustive outcomes out of which m are favorable to the occurrence of an event A, then the probability of occurrence of A is given by P (A) = m /n

$$P(E) = \frac{\text{Number of outcomes favorable to A}}{\text{Number of total outcomes}}$$

It is obvious that $0 \leq m \leq n$.

If an event A is certain to happen, then m = n, thus P (A) = 1.

If A is impossible to happen, then m = 0 and so P (A) = 0.

Hence we conclude that $0 \leq P (A) \leq 1$

ODDS IN FAVOUR, ODDS AGAINST

(a) The odds in favor of the event E = P(E) P(E') .

(b) The odds against the event E = P(E') P(E)

(c) If odds in favor of the event E = a: b then P(E) = a/(a+ b)

(d) If odds against the event E = a: b then P(E) = b/ (a +b)

For any event $E, 0 \leq P(E) \leq 1$ or $P(E) \in [0.1]$

$* P(\varnothing) = 0, P(S) = 1$

* If E_1, E_2 are mutually exclusive then $E_1 \cap E_2 = \varnothing \Rightarrow P(E_1 \cap E_2) = 0$

* If E_1, E_2 are exhaustive then $E_1 \cup E_2 = S \Rightarrow P(E_1 U E_2) = 1$

Addition Theorem on Probability

If A,B are any two events of a random experiment and P is a probability function then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Note :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) \quad -P(C \cap A) + P(A \cap B \cap C)$$

Conditional Event:

Suppose A and B are two events of random experiment. IF the event B occurs after the occurrence of the event A is called the conditional event B given A. It is denoted by $P\left(\dfrac{B}{A}\right)$

Conditional Probability :

If A and B are two events of a sample space S and $P(A) \neq 0$, then the probability of B after the event A has occurred is called the conditional probability of B given A. It is denoted by $P\left(\dfrac{B}{A}\right)$ and

$P\left(\dfrac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)}$ where P(A) not equal to zero

Note : $P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)}$ where P(B) not equal to zero

Multiplication Theorem on Probability

If A and B are two events of a random experiment with $P(A) > 0, P(B) > 0$ then

$$P(A \cap B) = P(A)P\left(\dfrac{B}{A}\right) = P(B)P\left(\dfrac{A}{B}\right)$$

(i) $P(A \cap B) = P(A).P(B).$ If A and B are independent events.

(ii) $P(A \cap B \cap C) = P(A).P(B).P(C),$ if A,B,C are independent events.

(iii) $P(A \cap B) = 0.$ if A, B are mutually exclusive events.

(i) $P(A \cup B) = P(A) + P(B).$ If A,B are mutually exclusive events.

(ii) $P(A \cup B \cup C) = P(A) + P(B) + P(C),$ If A,B ,C are mutually exclusive events.

(iii) $P(A \cup B) = 1 - P(\overline{A}).P(\overline{B}).$ if A,B are independent events

(iv) $P(A \cup B \cup C) = 1 - P(\overline{A}).P(\overline{B}).P(\overline{C}),$ if A,B,C are independent events

(i) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

(ii) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

Theorem on total probability:

If E_i(i=1,2,3….n) be a partition of sample space and all Ei have non-zero probability. A be any event associated with the sample space, which occurs with E_1 or E_2 or E_3 …..En then

P(A) = P(E_1)P(A| E_1)+ P(E_2)P(A| E_2)+ P(E_3)P(A| E_3)+……………..+ P(E_n)P(A| E_n)

Baye's Theorem :

Suppose $E_1, E_2 ...... E_n$ are n mutually exclusive and exhaustive events of a random experiment with $P(E) \neq 0$ for $i = 1, 2, 3 ...., n$. For any event A of the random experiment with $P(A) \neq 0$ then

$$P(E_i / A) = \frac{P(E_i) P(A / E_i)}{\sum\limits_{i=1}^{n} P(E_i) P(A / E_i)}, k = 1, 2, 3, \ldots, n$$

MULTIPLE CHOICE QUESTIONS

1.If P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4, what is the value of P (A ∩ B)?
a. 0.32   b. 0.25   c. 0.1   d. 0.5

Answer: A. 0.32
SOLUTION: Given, P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4
By conditional probability, we have;
P (B|A) = P(A ∩ B)/P(A)   ⇒ P (A ∩ B) = P(B|A). P(A) = 0.4 x 0.8 = 0.32
Answer: a

2.If P (A) = 6/11, P (B) = 5/11 and P (A ∪ B) = 7/11, what is the value of P(B|A)?
a. ⅓   b. ⅔   c. 1   d. none of the above

Answer: b. ⅔
Explanation: By definition of conditional probability we know;
P(B|A) = P(A ∩ B)/P(A) …(i)
Also,
P(A ∩ B) = P(A) + P(B) – P(A U B)
= 6/11 + 5/11 – 7/11= 4/11
Now putting the value of P(A ∩ B) in eq.(i), we get;
P(B|A) = (4/11)/(6/11) = 4/6 = ⅔

3.Find P(E|F), where E: no tail appears, F: no head appears, when two coins are tossed in the air.
a. 0   b. ½   c. 1   d. none of the above

Answer: a. 0
SOLUTION: Given,
E: no tail appears
And F: no head appears
⇒ E = {HH} and F = {TT}
⇒ E ∩ F = ϕ
As we know, two coins were tossed;
P(E) = ¼
P(F) = ¼
P(E ∩ F) = 0/4 = 0
Thus, by conditional probability, we know that;
P(E|F) = P(E ∩ F)/P(F)
= 0/(¼)
= 0

4.If P(A ∩ B) = 70% and P(B) = 85%, then P(A/B) is equal to:
a. 17/14   b. 14/17   c. ⅞   d. ⅛

Answer: b. 14/17

SOLUTION: By conditional probability, we know;
P(A|B) = P(A ∩ B)/P(B)
= (70/100) x (100/85)

= 14/17

5. The probability that at least one of the events A & B occurs is 3/5. If A & B occur simultaneously with probability 1/5.

then $P(\overline{A}) + P(\overline{B}) =$

(a) $\dfrac{2}{5}$
(b) $\dfrac{4}{5}$
(c) $\dfrac{6}{5}$
(d) none

Ans:c

SOLUTION: $P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B) = 2 - (P(AUB) + P(A \cap B))$
P(AUB)=P(A)+P(B)-P(A∩ B)and P($\overline{A}$)= 1-P(A)

6. A speaks truth in 80% cases and B in 60% cases. In what percentage of case are they likely to contradict each other in starting the same fact :

(a) 44%
(b) 56%
(c) 42%
(d) 48%

ANS:a

P(E)=P(A∩ $\bar{B}) + P(\bar{A} \cap B)=\dfrac{80}{100} \times \dfrac{40}{100}+\dfrac{20}{100} \times \dfrac{60}{100}=\dfrac{44}{100}$

7. One of the two events must occur. If the chance of one is 2/3 of the other, then odds in favour of other are :

(A) 2 : 3
(B) 1 : 3
(C) 3 : 1
(D) 3 : 2

ANS:D
P(A)+$\dfrac{2}{3}$P(A)=1,P(A)=$\dfrac{3}{5}$ Odds in favour P(A):P(A^i)= 3:2

8. A problem of mathematics is given to three students whose chances of solving the problem are 1/3, 1/4 and 1/5 respectively. The probability that the question will be solved is

(A) 2/3
(B) ¾
(C) 4/5
(D) 3/5

ANS:D

P(E)=1-$\dfrac{2}{3} \times \dfrac{3}{4} \times \dfrac{4}{5}$=3/5

9. Two persons A and B throw a die alternatively. Person who first throws 6 is the winner. If A starts throwing. Then the probability of A's winning is

(A) 5/11
(B) 6/11
(C) 2/11
(D) 9/11

ANS:B

P(A)= $\dfrac{6}{11}$

10. If E and F be mutually exclusive events in a sample space such that P(E) = 0.3 and P(F) = 0.6, then P($\overline{E} \cap \overline{F}$) equals

(A) 0.3
(B) 0.9
(C) 0.1
(D) 0.7

ANS:C

$P(\overline{E} \cap \overline{F}) = 1 - (P(E) + P(F)) = 0.1$

11. For any two events A and B, P(A')=1/2, P(B')=2/3 and $P(A \cap B) = 1/4$, then $P\left(\dfrac{A'}{B'}\right)$ equals:

    (a) 8/9 (b) 5/8 (c) 1/8 (d) ¼

Ans: (b)

$P(A) = 1/2,\ P(B) = 1/3$

$P(AUB) = 1/2 + 1/3 - 1/4 = 7/12$

$P(A'/B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{1 - P(AUB)}{P(B')} = \dfrac{1 - 7/12}{2/3} = \dfrac{5/12}{2/3} = 5/8$

12. For any two events A and B, $P(A) = 4/5\ and\ P(A \cap B) = 7/10$, then $P(B/A)$ is

    (a) 1/10 (b) 1/8 (c) 17/20 (d) 7/8

Ans: (d)

$P(B/A) = \dfrac{7/10}{4/5} = 7/8.$

13. If A and B are two independent events such that P(A)=1/2 and P(B)=1/4, then P(B'/A) is

    (a) 1/4 (b) 3/4 (c) 1/8 (d) 1

Ans: (b)

$P(B'/A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{P(B')P(A)}{P(A)} = P(B') = 3/4$

14. If two events A and B, P(A-B)=1/5 and P(A)= 3/5, then P(B/A) is equal to

    a. ½ (b) 3/5 (c) 2/5 (d) 2/3

Ans: (d)

$P(A - B) = 1/5,\ P(A) = 3/5$

$P(A \cap B) = 3/5 - 1/5 = 2/5$

$P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{2/5}{3/5} = 2/3.$

15. If P(A/B)=0.3, P(A)= 0.4 and P(B)=0.8, then P(B/A) is equal to

    (a) 0.6 (b) 0.3 (c) 0.06 (d) 0.4

Ans: (a)

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A \cap B)}{0.8} = 0.3$

$P(A \cap B) = 0.24$

$P(B/A) = 0.24/0.4 = 0.6$

16. If A and B are two events such that P(A/B)=2.P(B/A) and P(A)+P(B)=2/3, then P(B) is

    (a) 2/9 (b) 7/9 (c) 4/9 (d0 5/9.

Ans: (a)

$\dfrac{P(A \cap B)}{P(B)} = 2\dfrac{P(A \cap B)}{P(A)}$

$P(A) = 2P(B)$

$3P(B) = 2/3$

$P(B) = 2/9$

17. If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is

a.   1/9   (b)   4/9   (c) 1/18   (d)   1/2

Ans: (d)

$A = \text{Sum } 9 = \{(3,6),(4,5),(5,4),(6,3)\}$

$B = \text{one die shows } 4 = \{(1,4),(2,4),(3,4),(5,4),(6,4),(4,1),(4,2),(4,3),(4,5),(4,6)\}$

$P(A) = 4/36 \quad P(B) = 10/36$

$P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{2/36}{4/36} = 1/2$

18. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is

a.   27/32   (b) 5/32   (c) 31/32   (d) 1/32

Ans: (c)

$P(\text{atleast one H}) = 1 - P(\text{none is H})$

$= 1 - 1/32 = 31/32.$

19. Ramesh can hit a target 2 out of 3 times. He tried to hit the target twice. The probability that the she missed the target exactly once is

a.   2/3   (b)   1/3   (c) 4/9   (d) 1/9

Ans: (c)

$P(A) = 2/3 \quad P(A') = 1/3 \quad (A - \text{hit}, A' - \text{not hit})$

$P(\text{only once hit}) = \dfrac{2}{3} \cdot \dfrac{1}{3} + \dfrac{1}{3} \cdot \dfrac{2}{3} = 4/9.$

20. Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is

a. 27/32   (b) 5/32   (c) 31/32   (d) 1/32

Ans: (c)

$P(\text{atleast one H}) = 1 - P(\text{none is H})$

$= 1 - 1/32 = 31/32.$

ASSERTION-REASONING QUESTIONS-SOLVED

Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is **not** the correct explanation of the Assertion (A)

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

1   **Assertion (A):** If A and B are two independent events with P(A)=1/5 and P(B)=1/5, then P(A'/B) is 1/5.

$\text{Reason } (R) \; : \; P(A'/B) = \dfrac{P(A' \cap B)}{P(B)}$

Ans: (d)

$P(A'/B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(A').P(B)}{P(B)} = P(A')$

$= 1 - P(A) = 4/5$

So A is false and R is True.

2.   **Assertion (A) :** Let A and B be two events such that P (A) =1/5 and P (A or B) =1/2 then P(B)=3/8 for A and B are independent events.

Reason (R) : For independent events P (A or B) = P (A) + P(B) - P(A).P(B).

Ans: (a)   As A is true and R is the correct explanation for A.

For Assertion :

$P(A \cup B) = 1/2$

$P(A) + P(B) - P(A).P(B) = 1/2$

$P(B) = 3/8 \quad (\because P(A) = 1 - 1/5 = 4/5)$

R is correct exxplanation.

3. **Assertion (A):** Two coins are tossed simultaneously. The probability of getting two heads, if it is know that at least one head comes up, is 1/3.

   Reason (R) : Let E and F be two events with a random experiment, then

   $$P(F/E) = \frac{P(E \cap F)}{P(E)}.$$

   Ans: (a)

   For Assertion

   $F = \{HH\} \quad E = \{HH, HT, TH\}$

   $P(F/E) = \dfrac{1/4}{3/4} = 1/3$

4. **Assertion (A) :** If A and B are mutually exclusive events with P (A') =5/6 and P (B)=1/3. Then P (A/B')=1/4.

   Reason (R) : If A and B are two events such that P (A)=0.2, P(B)=0.6 and P(A/B)=0.2 then the value of P(A/B') is 0.2.

   Ans: (b) as A is true and R is not the correct explanation for A.

   For Assertion :

   $P(A) = 1/6$

   $P(A/B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{P(A) - P(A \cap B)}{1 - P(B)} = \dfrac{1/6 - 0}{1 - 1/3} = 1/4$

   Reason :

   $P(A \cap B) = 0.12$

   $P(A/B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{P(A) - P(A \cap B)}{1 - P(B)} = \dfrac{0.08}{0.4} = 0.2$

5. Let A and B be two events associated with an experiment such that
   $P(A \cap B) = P(A).P(B)$

   Assertion (A): P(A/B)=P(A) and P(B/A)=P(B)

   Reason (R): $P(A \cup B) = P(A) + P(B)$

   Ans: (c) as A is correct but R is false.

6. Let A and B be two independent events.
   Assertion (A): If P(A)=0.3 and $P(A \cup \bar{B})=0.8$ then P(B) is 2/7

   Reason (R) : $P(\bar{E}) = 1 - P(E)$, for any event E.

   Ans : (a) as

   $P(A \cup B') = P(A) + P(B') - P(A).P(B')$

   $P(B') = 5/7$

   $P(B) = 2/7$

   Hence A is true and R is the correct explanation for A.

7. **Assertion (A):** Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely.
If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then P(E/F)=1/4 and P(F/E)=1/13.
Reason (R): E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.
Ans: (b) As A is correct but R is not the correct explanation of A.

8. Consider that following statements:
Assertion (A): Let A and B be two independent events. Then P(A and B)= P(A)+P(B)
Reason (R): Three events A,B and C are said to be independent if
$$P(A \cap B \cap C) = P(A).P(B).P(C)$$

Ans: (d) as $P(A \text{ and } B) = P(A) + P(B) - P(A).P(B)$, hence A is false and R is True.

9. **Assertion (A):** In rolling a die, event A= {1, 3, 5} and event B = {2, 4} are mutually exclusive events.
Reason (R): in a sample space two events are mutually exclusive if they do not occur at the same time.

Ans: (a) A is true as $P(A \cap B) = \phi$ and R is the correct explanation of A.

10. For any two events A and B. P(A)=p and P(B)=q
Assertion (A): The probability that exactly one of the events A and B occurs is p+q-2pq
Reason (R): P(A $\cup$ B)=P(A)+P(B)-P(A $\cap$ B)

Ans: (b)
A and R are true and R is the not correct explanation of A.

<div align="center">

VERY SHORT ANSWER QUESTIONS

</div>

1. A coin is tossed twice. If E – both head and tail have occurred F- at most one tail is occurred find $P(E), P(F), P(E/F), P(F/E)$

Sol : $S = \{HH, TH, TH, TT\}$, $E = \{HT, TH\}$, $F = \{HH, HT, TH\}$

$E \cap F = \{HT, TH\}$

$P(E) = \dfrac{2}{4} = \dfrac{1}{2}$ $P(F) = \dfrac{3}{4}$; $P(E \cap F) = \dfrac{2}{4} = \dfrac{1}{2}$

$P(E/F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{1/2}{3/4} = \dfrac{2}{3}$

$P(F/E) = \dfrac{P(E \cap F)}{P(E)} = \dfrac{1/2}{1/2} = 1$

2. If $P(E) = 0.40, P(F) = 0.35, F(E \cup F) = 0.55$ then find $P(E/F)$

Sol : $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$0.55 = 0.40 + 0.35 - P(E \cap F)$

$$P(E \cap F) = 0.20$$

$$P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{0.20}{0.35} = \frac{4}{7}$$

3. Two events A and B are such that $P(A) = 0.5, P(B) = 0.4 \, and \, P(A/B) = 0.3$.

    (i) State whether the events are independent

    (ii) Find the value of $P(A \cap B)$

Sol: (i) P(A/B) ≠ $P(A)$ $\Rightarrow$ the events A and B are not independent

   (ii) $P(A \cap B) = P(B).P\left(\frac{A}{B}\right) = 0.4 \times 0.3 = 0.12$

4. A problem is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of the problem being solved if both of them try independently

Sol: Let $P(E_1) = \frac{1}{3}. \ P(E_2) = \frac{1}{4}$

   Probability of the problem being solved

   $$P(E_1 \cup E_2) = 1 - P\left(\overline{E_1 \cup E_2}\right)$$

   $$= 1 - P\left(\overline{E_1} \cap \overline{E_2}\right)$$

   $$= 1 - P\left(\overline{E_1}\right) P\left(\overline{E_2}\right) = 1 - \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) = 1 - \frac{1}{2}$$

   $$= \frac{1}{2}$$

5. A number X is drawn from {1,2,3,.......100} find the probability that $x + \frac{100}{x} > 29$.

Sol: $S = \{1, 2, ...100\}$, $n(S) = 100$

   $x + \frac{100}{x} > 29$ $\Rightarrow x^2 - 29x + 100 > 0$, $(x-4)(x-25) > 0 \Rightarrow x < 4 \ (or) \ x > 25$

   $A = \{1, 2, 3, 26, 27, ...100\} \Rightarrow n(A) = 78$

   $$P(A) = \frac{n(A)}{n(S)} = \frac{78}{100}$$
   Required probability

6. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, then what is the conditional probability that both are girls? Given that
    (i) the youngest is a girl?
    (ii) at least one is a girl?

Answer:

Let B and b represent elder and younger boy child. Also, G and g represent elder and younger girl child. If a family has two children, then all possible cases are

S = {Bb, Bg, Gg, Gb}

∴ n(S) = 4

Let us define event A : Both children are girls, then A = {Gg} ⇒ n(A) = 1

(i) Let E1 : The event that youngest child is a girl.

Then, E1 = {Bg, Gg} and n(E1) = 2

so $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$

and $A \cap E_1 = \{Gg\} \Rightarrow n(A \cap E_1) = 1$

so $P(A \cap E_1) = \dfrac{n(A \cap E_1)}{n(S)} = \dfrac{1}{4}$

Now, $P\left(\dfrac{A}{E_1}\right) = \dfrac{P(A \cap E_1)}{P(E_1)} = \dfrac{1/4}{1/2} = \dfrac{1}{2}$

∴ Required probability $= \dfrac{1}{2}$

(ii) Let E2: The event that at least one is girl.

Then, E2 = {Eg, Gg, Gb} ⇒ n(E2) = 3,

7 .If P(not A) = 0.7, P(B) = 0.7 and P(B/A) = 0.5, then find P(A/B).

Answer:

Given, $P(A') = 0.7$, $P(B) = 0.7$ and $P\left(\dfrac{B}{A}\right) = 0.5$

Clearly, $P(A) = 1 - P(A') = 1 - 0.7 = 0.3$

Now, $P\left(\dfrac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)}$

⇒ $0.5 = \dfrac{P(A \cap B)}{0.3}$

⇒ $P(A \cap B) = 0.15$

∴ $P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.15}{0.7}$ ⇒ $P\left(\dfrac{A}{B}\right) = \dfrac{3}{14}$

8. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'number is even' and B be the event 'number is marked red'. Find whether the events A and B are independent or not.

Answer:

When a die is thrown, the sample space is

S = {1, 2, 3, 4, 5, 6}

⇒ n(S) = 6

Also, A: number is even and B: number is red.

∴ A = {2, 4, 6} and B = {1, 2, 3} and A ∩ B = {2}

⇒ n(A) = 3, n(B) = 3 and n(A ∩ B) = 1

Now, $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

$P(B) = \dfrac{n(B)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

and $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{1}{6}$

Now, $P(A) \times P(B) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4} \neq \dfrac{1}{6} = P(A \cap B)$

$\therefore \quad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events.

9. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Answer:

Let us denote the numbers on black die by B1, B2, ……., B6 and the numbers on red die by R1, R2, ….., R6.

Then, we get the following sample space.

s = {(B1, R1) ,(B1, R2), ……., (B1, R6), (B2, R2), ………, (B6, B1), (B6, B2), ……., (B6,R6)

Clearly, n(S) = 36

Now, let A be the event that sum of number obtained on the die is 8 and B be the event that red die shows a number less than 4.

Then, A = {(B2, R6), (B6,R2), (B3,R5), (B5,R3), (B4,R4)}

and B = {(B1, R1), (B1,R2), (B1,R3), (B2,R1), (B2,R2), (B2,R3) ,…….., (B6,R1), (B6,R2), (B6,R3)}

⇒ A ∩ B = {(B6, R2), (B5, R3)}

Now, required probability,

p(A/B) = P(A∩B)/P(B)=2/36/18/36=2/18=1/9

10. Evaluate P(A ∪ B), if 2P (A) = P(B) = 5/13 and P(A/ B) = 25.

Answer:

We have, $2P(A) = P(B) = \dfrac{5}{13}$

$\Rightarrow P(A) = \dfrac{5}{26}$ and $P(B) = \dfrac{5}{13}$ and $P(A/B) = \dfrac{2}{5}$

$\because P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)}$

$\therefore \quad \dfrac{2}{5} = \dfrac{P(A \cap B)}{5/13}$

$\Rightarrow \quad P(A \cap B) = \dfrac{2}{5} \times \dfrac{5}{13} = \dfrac{2}{13}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{5}{26} + \dfrac{5}{13} - \dfrac{2}{13}$

$= \dfrac{5+10-4}{26} = \dfrac{11}{26}$

SHORT ANSWER TYPE QUESTIONS

1. A pair of dice is rolled what is the probability that they sum to 7 given that neither shows a 2

Sol: $n(s) = 6^2 = 36$ A – event that sum is 7

B – neither die shows a 2

$$A = \{(1,6), (2,5), (3,4), (5,2), (6,1), (4,3)\}$$

$$B = \{(1,1), (1,3), (1,4), (1,5), (1,6)(3,1), (3,3)(3,4), (3,5), (3,6), (4,1), (4,3), (4,5), (4,6),$$

$$(5,1), (5,3), (5,4), (5,5), (5,6), (6,1), (6,3), (6,5), (6,4), (6,6)\}$$

$$A \cap B = \{(1,6), (3,4), (4,3), (6,1)\}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)} = \frac{4/36}{25/36} = \frac{4}{25}$$

2. The probability that Australia wins a match against India in a cricket game is given to be $\frac{1}{3}$. If India and Australia play 3 matches what is the probability that Australia will loose all the 3 matches.

Sol: A Event that Australia wins match against India

$$P(A) = \frac{1}{3}. \Rightarrow P(\overline{A}) = \frac{2}{3}$$

Probability that Australia loose all the 3 matches

$$= P(\overline{A}). P(\overline{A})P(\overline{A}) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

3. A, B, C are three horses in a race The probability A to win the race is twice that of B, the probability of B is twice that of C what are B the probabilities of A, B, C to win the race.

Sol : $P(A) = 2P(B), P(B) = 2P(C), P(A) = 4P(C)$

$$P(A) + P(B) + P(C) = 1$$

$$4P(C) + 2P(C) + P(C) = 1 \Rightarrow 7P(C) = 1 \Rightarrow P(C) = \frac{1}{7}$$

$$P(B) = \frac{2}{7}, P(A) = \frac{4}{7}$$

4. Find the probability that a non leap year will have

(a) 53 Sundays(b) 52 Sundays

Sol: $365 = 52(7) + 1(\text{odddays})$

S=$\{su, m, tu, w, th, fr, sa\}$

A= Event of getting 53 Sundays

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{7}$$

B= Event of getting 52 Sundays =$\{m,t,w,th,f,sa\}$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{7}$$

5. If two numbers are selected at random from 20 consecutive natural numbers. Find the probability that the sum of the two numbers is an odd numbers

Sol : $n(s) = 20_{C_2} = 190$

A= Event that sum is odd (one – even and one – odd)

$n(A) = 10C_1 \times 10C_1 = 10 \times 10 = 100$

$P(A) = \frac{n(A)}{n(S)} = \frac{100}{190} = \frac{10}{19}$

6. If a and B are events $P(A) = 0.5, P(B) = 0.4, P(A \cap B) = 0.3$ Probability that neither A nor B occurs

$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

=1-(0.5+0.4-0.3)

$= 1 - 0.6$

$= 0.3$

7. A Problem is given to amir and Ramu their respective probability of solving are $\frac{1}{2}$ and $\frac{1}{3}$. Find the probability that Exactly one will solve the problem.

Sol : Amir solving the problem $\Rightarrow P(A) = \frac{1}{2}$

Ramu solving the problem $\Rightarrow P(B) = \frac{1}{3}$

P (Exactly one solve the problem) $= P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$= P(A)P(\overline{B}) + P(\overline{A})P(B)$

$= \frac{1}{2}\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

$= \frac{1}{2}.$

8. Find the probability that a leap year will have (a) 53 Mondays, (b) 52 Mondays

Sol : $366 = 52(7) + 2(\text{odd days})$

$S = \{(S, M), (M, T), (T, W), (W, Th), (Th, F), (F, Sa), (Sa, S)\}$

A : Event of getting 53 Mondays

$A = \{(S, M), (M, T)\} P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$

B : Event of getting 52 Mondays

B={(T,W),(WA,TH),(TH,F)(F,SA),(SA,SU)}=5

P(B)=$\frac{n(B)}{n(S)} = \frac{5}{7}$

9. Ten Cards 1 to 10 are placed in a box A card is drawn from the box at random. If the number on card drawn is more than 5, what is the Probability that it is an odd number.

Sol : A – Event that number on the card drawn n odd

B – Event that number on the card is more than 5.

$$S = \{1,2,3.....10\} \quad A = \{1,3,5,7,9\} \quad B = \{6,7,8,9,10\}$$

$$A \cap B = \{7,9\}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/10}{5/10} = \frac{2}{5}.$$

10. If A and B are independent events with $P(A) = 0.2, P(B) = 0.5$ then find $P(A/B), P(B/A), P(A \cap B), P(A \cup B)$

Sol: $P(A \cap B) = P(A)P(B) = (0.2)(0.5) = 0.1$

$P(A/B) = P(A) = 0.2$

$P(B/A) = P(B) = 0.5$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.5 - 0.1 = 0.6$

11. If A and B are independent events with $P(A) = 0.6, P(B) = 0.7$ compute $P(A \cap B), P(A \cup B), P(B/A), P(\overline{A} \cap \overline{B})$

Sol : $P(A \cap B) = P(A)P(B) = 0.42$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.7 - 0.42 = 0.88$

$P(B/A) = P(B) = 0.7$

$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.88 = 0.12$

12. A and B are two events then show that $P(A/B)P(B) + P\left(\frac{A}{\overline{B}}\right)P(\overline{B}) = P(A)$

Sol : $P(A/B)P(B) + P\left(\frac{A}{\overline{B}}\right)P(\overline{B})$

$\frac{P(A \cap B)}{P(B)}.P(B) + \frac{P(A \cap \overline{B})}{P(\overline{B})}.P(\overline{B})$

$P(A \cap B) + P(A) - P(A \cap B) = P(A)$

LONG ANSWER TYPE QUESTIONS

1.. Two persons A and B are rolling a die on the condition to the person who gets 3 first will win the game If A starts the game then find the probabilities of A and B respectively to win the game

Sol : P be probabilities of getting 3 (success) $= \frac{1}{6}$

q be probability of not getting 3 (failure) $= \frac{5}{6}$

Probability of A to the game

$P(A) = p + qqp + qqqqp + \ldots..$

$= P(1 + q^2 + q^4 + \ldots.)$

$= P\left(\frac{1}{1-q^2}\right) = \frac{P}{1-q^2} = \frac{1/6}{1-\frac{25}{36}} = \frac{1/6}{11/36} = \frac{6}{11}$

Probability of B to win the game $P(B) = 1 - \frac{6}{11} = \frac{5}{11}$

2. A number is selected at random from $\{1, 2, \ldots\ldots.100\}$ Given that the number selected is divisible by 2, find the probability that is divisible by 3 or 5

Sol: $A_2 =$ event that the number is divisible by 2

$A_3 =$ event that the number is divisible by 3

$A_5 =$ event that the number is divisible by 5

P (Required probability

$P(A_3 \cup A_5 / A_2) = \frac{P((A_3 \cup A_5) \cap A_2)}{P(A_2)}$

$= \frac{P((A_3 \cap A_2) \cup (A_5 \cap A_2))}{P(A_2)}$

$= \frac{P(A_3 \cap A_2) + P(A_5 \cap A_2) - P(A_3 \cap A_5 \cap A_2)}{P(A_2)}$

$= \frac{\frac{16}{100} + \frac{10}{100} - \frac{3}{100}}{\frac{50}{100}} = \frac{23}{50} = 0.46$

3. If A, B, C are three independent events of an experiment such that

$P(A \cap \overline{B} \cap \overline{C}) = \frac{1}{4}, P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{8}, P(\overline{A} \cap \overline{B} \cap \overline{C}) = \frac{1}{4}$ then find. $P(A), P(B), P(C)$.

Sol :
$P(A \cap \overline{B} \cap \overline{C}) = \frac{1}{4} \Rightarrow P(A)P(\overline{B})P(\overline{C}) = \frac{1}{4}$ ………………… (1)

$P(\overline{A} \cap \overline{B} \cap \overline{C}) = \frac{1}{8} \Rightarrow P(\overline{A})P(\overline{B})P(\overline{C}) = \frac{1}{8}$ ………………….. (2)

$P(\overline{A} \cap \overline{B} \cap \overline{C}) = \frac{1}{4} \Rightarrow P(\overline{A})P(\overline{B})P(\overline{C}) = \frac{1}{4}$ ……………………(3)

$\frac{2}{3} \Rightarrow \frac{P(B)}{P(A)} = \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

$\frac{1}{3} \Rightarrow \frac{P(A)}{P(A)} = 1 \Rightarrow P(A) = \frac{1}{2}$ From (3)

$\frac{1}{2}\left(\frac{2}{3}\right)(1 - P(C)) = \frac{1}{4} \Rightarrow P(C) = \frac{1}{4}$

HIGHER ORDER THINKING SKILLS

1. The probability of 3 events A,B,C are such that $P(A) = 0.3, P(B) = 0.4, P(C) = 0.8,$ $P(A \cap B) = 0.09, P(A \cap C) = 0.28, P(A \cap B \cap C) = 0.08$ and $P(A \cup B \cup C) \geq 0.75.$ Show that $P(B \cap C)$ lies in the interval $[0.21, 0.46]$

Sol:$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - (C \cap A) + P(A \cap B \cap C)$

$$= 0.3 + 0.5 + 0.8 - 0.09 - P(B \cap C) - 0.28 + 0.08$$

$$= 1.21 - P(B \cap C)$$

$$0.75 \le P(A \cup B \cup C) \le 1$$

$$0.75 \le 1.21 - P(B \cap C) \le 1$$

$$0.21 \le P(B \cap C) \le 0.46 \implies P(B \cap C) \in [0.21, 0.46]$$

2. Bag A contains 5 white and 3 block balls and Bag B contains 3 white and 5 block ball. Two balls are transferred from A to B and then 2 balls are drawn from bag B find the probability that they are one white and one black

Sol: E – Event of transferring 2 white balls from A to B

F- Event of transferring 1 white and 1 black ball from A to B

G – Event of transferring 2 black balls from A to B

A – Event of drawing 1 black and 1 white ball from B

$$P(E) = \frac{5_{C_2}}{8_{C_2}} = \frac{5}{14}, P(F) = \frac{5_{C_1} \times 3_{C_1}}{8_{C_2}} = \frac{15}{28}, P(G) = \frac{3_{C_2}}{8_{C_2}} = \frac{3}{28}$$

$$P(A) = P(E)P\left(\frac{A}{E}\right) + P(F)P\left(\frac{A}{F}\right) + P(G)P\left(\frac{A}{G}\right)$$

$$= \frac{5}{14} \cdot \frac{5_{C_1} \times 5_{C_1}}{10_{C_2}} + \frac{15}{28} \cdot \frac{4_{C_1} \times 6_{C_1}}{10_{C_2}} + \frac{3}{28} \cdot \frac{3_{C_1} \times 7_{C_1}}{10_{C_2}}$$

$$= \frac{1}{45}\left[\frac{125}{14} + \frac{90}{7} + \frac{9}{4}\right] = \frac{673}{1260}$$

CASE STUDY BASED QUESTIONS

1. In a shooting test probability of A, B, C hitting the targets are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ respectively. If all of them fire at the same target.

(i) Find the probability that only one of them hits the target.

(ii) If all of them fire at the same target. Find the probability that only one of them hits the target.

Sol : (i) $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$

A, B , C are independent events

Probability that only one hits the target

$$= P(A \cap \overline{B} \cap \overline{C}) + P(\overline{A} \cap \overline{B} \cap C) + P(\overline{A} \cap B \cap \overline{C})$$

$$= P(A)P(\overline{B})P(\overline{C}) + P(\overline{A})P(\overline{B})P(C) + P(\overline{A})P(B)P(\overline{C})$$

$$= \frac{1}{4}$$

(ii) $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(C) = \frac{1}{5}$

A, B, C are independent events

Probability that at least one hits the target

$$P(A \cup B \cup C) = 1 - P(\overline{A \cup B \cup C})$$

$$= 1 - P(\overline{A} \cap \overline{B} \cap \overline{C})$$

$$= 1 - P(\overline{A}) P(\overline{B}) P(\overline{C}) = 1 - \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right) = 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

2. During July in Bengaluru, the probability that it will rain on a randomly chosen day is $\frac{4}{5}$. Pragna either walks to school or gets a ride with her parents in their car. If it rains, the probability that Pragna' parents will take her to school by car is $\frac{5}{6}$. If it does not rain, the probability that Pragna' parents will take her to school by car is $\frac{1}{12}$.
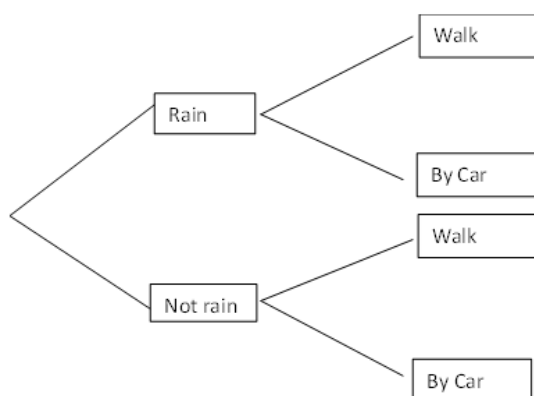
(i) Represent the above information in a tree diagram. On your diagram show all the respective probabilities.

(ii) What is the probability that it is a rainy day and Pragna walks to school?

(iii) What is the probability that Pragna' parents take her to school by car?

Sol:

(ii)P(school by walk on a non-rainy day)$= \frac{4}{5} \times \frac{1}{6}$ $=$ $\frac{4}{30}$

(iii)P(school by car) $= \frac{4}{5} \times \frac{5}{6} + \frac{1}{5} \times \frac{1}{12} =$ 41/60

3. There are two antiaircraft guns, name as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at

the same time.

Based on the above information, solve the following

Q1. What is the probability that the shell fired from exactly one of them hit the plane?

Q2. If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

SOLUTION:

i)P (Shell fired from exactly one of them hits the plane)= P [(Shell from A hits the plane and shell from B does not hit the plane) or (shell from A does not hit the plane and shell from B hits the plane)]=$0.3 \times 0.8 + 0.7 \times 0.2 = 0.24 + 0.14 = 0.38$.

(ii) P (Shell fired from B hit the plane/Exactly one of them hit the plane)

$$= \frac{P(shell\ fired\ from\ B\ hit\ the\ plane \cap Exactly\ one\ of\ them\ hit\ the\ plane)}{P(Exactly\ one\ of\ them\ hit\ the\ plane)}$$

$$= \frac{P(Shell\ from\ only\ B\ hit\ the\ plane)}{P(Exactly\ one\ of\ them\ hit\ the\ plane)}$$

$= \frac{0.14}{0.38} = \frac{7}{19}$ (Since,$P(\bar{A} \cap B) = P(\bar{A}) \times P(B) = (1 - 0.3) \times 0.2 = 0.14$)

EXERCISEFOR PRACTICE
MULTIPLE CHOICE QUESTIONS

1.A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the probability of getting exactly one red ball is

(a)45/196   (b)135/392   (c)15/56   (d)15/29

2.A flashlight has 8 batteries out of which 3 are dead. If two batteries are selected without replacement and tested, the probability that both are dead is

(a)33/56   (b)9/64   (c)1/14   (d) 3/28

3. Probability that A speaks truth is 4/5.A coin is tossed .A reports that a head appears. The probability that actually there was a head is

(a)4/5   (b)1/2   (c)1/5   (d) 2/5

4. A and B are two students. Their chances of solving a problem correctly are 1/3 and ¼ respectively. If the probability of their making a common error is 1/20 and they obtain the same number , then the probability of their answer to be correct is

(a)1/12   (b)1/40   (c)13/120   (d) 10/13

ASSERTION – REASON QUESTIONS

1.Assertion: 20 persons are sitting in a row. Two of these persons are selected at random. The probability that the two selected persons are not together is 0.9.

Reason: If $\underline{A}$ denotes the negation of an event A, then $P(\bar{A}) = 1 – P(A)$.

SHORT ANSWER TYPE QUESTIONS

1.In shop A, 30 tin pure ghee and 40 tin adulterated ghee are kept for sale while in shop B, 50 tin pure ghee and 60 tin adulterated ghee are there. One tin of ghee is purchased from one of the shops randomly and it is found to be adulterated. Find the probability (i) Getting adulterated ghee   (ii) it is getting from shop B.

2.Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six. Do you also agree that the value of truthfulness leads to more respect in the society ?

3.. In a game of Archery, each ring of the Archery target is valued. The centre most ring is worth 10 point and rest of the rings are allotted points 9 to 1 in sequential order moving outwards. Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely to earn 10 points with a probability of 0.9. Based on the above information answer the following questions. If both of them hit the Archery target , then find the probability that

(i) exactly one of them earns 10 points (ii) both of them earn 10 points.

4. If $P(A) = 4/7$, $P(B) = 0$, then find $P(A/B)$.

5. Write the probability of an even prime number on each die, when a pair of dice is rolled.

6. Two independent events A and B are given such that $P(A) = 0.3$ and $P(B) = 0.6$ find P(A and not B).

7. A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event 3 on the die. Check whether A and B are independent events or not.

8. Let A and B be two events. If $P(A)=0.2$, $P(B)=0.4$,$P(A∩B) = 0.6$, then find $P(A∪B)$.

9. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

10. A problem in mathematics is given to 3 students whose chances of solving it are $1/2,1/3$ and $1/4$,. What is the probability that The (i) problem is solved (ii) exactly one of them will solve it?

11. X is taking up subjects, Mathematics, Physics and Chemistry in the examination. His probabilities of getting Grade A in these subjects are $0.2,0.3$ and 0.5 respectively.

Find the probability that he gets (i) Grade A in all subjects. (ii) Grade A in no subject

(iii) Grade A in two subjects.

12. A speak truth in 60% of the case, while B in 90% of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

13. There are three urns A, B and C. Urn A contains 4 white balls and 5 blue balls. Urn B contains 4 white balls and 3 blue balls. Urn C contains 2 white balls and 4 blue balls. One ball is drawn from each of these urns. What is the probability that out of these three balls drawn, two are white balls and one is a blue ball?

14.(a) 12 cards numbered 1 to 12, are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number.

(b) 12 cards, numbered 1 to 12, are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 5, find the probability that it is an odd number.

15. 10%of the bulbs produced in a factory are of red colour and 2%are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.

16. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10,1/5 ,1/10 and 2/5. The probabilities that he will be late are 1/4, 1/3, and 1/12, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

17. Suppose that the reliability of a HIV test is specified as follows: of people having HIV, 90% of the test detects the disease but 10% go undetected. Of the people free of HIV, 99% of the tests are judged HIV-ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ve. What is the probability that the person actually has HIV?

18. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letters came from TATA NAGAR?

ANSWERS

1.(c) 2.(d) 3.(a) 4.(d) 5. (a)

SHORT ANSWERTYPE QUESTIONS

1. (i)43/77 (ii) 21/43   2. 4/9   3.(i)0.26 (ii) 0.72 , 4.does not exist. 5. 5/18 6.0.12 7. Yes 8. 0

9. 3   10.(i) ¾ (ii) 11/24   11. (i) 0.03 (ii) 0.28 (iii) 0.22 12. 42%, yes 13. 64/189

14. (a) 5/9 (b) 3/7 15. 1/5 16.1/2 17. 0.083 approx.   18. 7/11 .

PRACTICE QUESTIONS

1. Senior students tend to stay up all night and therefore are not able to wake up on time in morning. Not only this but their dependence on tuitions further leads to absenteeism in school. Of the students in class XII, it is known that 30% of the students have 100% attendance. Previous year results report that 80% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the class XII. Using above information answer the following:

(i). Find the conditional probability that a student attains A grade given that he is not 100 % regular

student.

(ii) Find the probability of attaining A grade by the students of class XII .

(iii) Find the probability that student is 100% regular given that he attains A grade

ANSWER: (i)1/10   (ii)31/100   (iii) 24/31   (or) 7/31

2.A school conducted a survey of their school staff to find their beverage preferences. Each of them picked either tea or coffee as their first preference and then with sugar or without sugar as their second preference as shown in the below tree diagram.
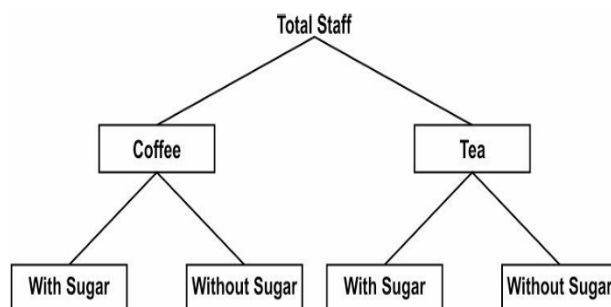
Some of the insights from the survey are given below.
♦ 60% percent of the staff prefer coffee.
♦ 90% of those who prefer coffee prefer it with sugar
♦ 20% of those who prefer tea prefer it without sugar.
i)   What is the probability that a person selected randomly from the staff prefers a beverage with sugar? Show your steps.
ii)   What is the probability that a person from the staff selected at random prefers coffee given that it is without sugar? Show your steps. 2
ANSWERS: (i)43/50   (ii)3/7

3. According to the recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights. Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55% , 37% and 17% due to severe, moderate and light turbulence respectively.

On the basis of the above information, answer the following questions:
(i)   Find the probability that an airplane reached its destination late. [2 marks]
(ii)   If the airplane reached its destination late, find the probability that it was due to moderate turbulence. [2 marks]

SOLUTION
Let E_1 = severe, E_2 = moderate, E_3 = Light Turbulence
(i) $P(E_1) = P(E_2) = P(E_3) = 1/3$
        P(airplane reached destination late) = $P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$
        $= 1/3(55/100) + 1/3(37/100) + 1/3(17/100) = 109/300$
(ii)   P(If the airplane reached its destination late, find the probability that it was due to moderate turbulence) $P(E_3/A) = P(E_3).P(A/E_3)/P(A) = 17/109$

UNIT- VI: DESCRIPTIVE STATISTICS

Gist/Summary of the lesson:
- Measure of central tendency, range, quartile,
- measures of dispersion, mean deviation, variance and standard deviation,
- Understand meaning of dispersion in a data set
- Range, quartile deviation, mean deviation and standard deviation

Definitions and Formulae:
- Dispersion indicates the extent to which the individual measures differ from an average.
- Mean deviation is the average of absolute values of deviations about mean or median.
- The mean of the squared deviations is called variance and denoted by var (x) or σ^2
- Variance = $[\sum fd^2 / N - (\sum fd / N)^2] \times h$
- Standard deviation is the square root of variance.

MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)

1) If the range of 14, 12, 17, 18, 16, x is 20 and x>0, then the value of x is

A) 42 B) 28 C) 32 D) cannot be determined

Solution: we have, range = 20

So, greatest value – smallest value = 20

x-12 = 20

x = 32

Answer: C

2) If the range of 15, 14, x, 25, 30, 35 is 23, then the least possible value of x is

A) 13 B) 12 C) 13   D) 11

Solution: Clearly, least possible value of x = Greatest value – Range = 35 -23 =12.

Answer: B

3.   If the range of 18, 13, 14, 42, 22, 26, x is 44 and x>0, then the sum of digits of x is

A) 16 B) 14   C) 12   D) cannot be determined

Solution: Given that range is 44. Therefore, greatest value is more than 44. Thus, x is the greatest value.

Greatest value – Least value =44

Greatest value = 13 + 44 = 57

Hence sum of digits = 5+7 = 12

Answer: C

4.   The highest score of a certain data exceeds to lowest score by 16 and coefficient of range is 1/3. The sum of the highest score and the lowest score is

A) 36 B) 48 C) 24   D) 18

Solution: Let the lowest and highest score be a and b respectively.

b – a = 16 and (b – a)/(b + a) = 1/3

b + a = 48

Answer: B

5.   The mean deviation from the median is

A.   to that measured from another value

B.   Maximum if all observations are positive.

C.   Greater than that measured from any other value.

D.   Less than that measured from any other value.

Answer: D

6.   Which of the following is **not** a measure of central tendency?

A) Mean

B) Median

C) Mode

D) Standard Deviation

Solution: Standard deviation is a measure of dispersion, not central tendency.

Answer: D

7.   The arithmetic means of the numbers 4, 8, 6, 5, and 3 is:

A) 5

B) 6

C) 4

D) 5.2

Solution: Mean = (4 + 8 + 6 + 5 + 3) / 5 = 26 / 5 = 5.2

Answer: D

8.   Which measure of central tendency is most affected by extreme values?

A) Median

B) Mode

C) Mean

D) None of the above

Solution: The mean is sensitive to outliers, unlike the median and mode.

   Answer: C

9.   If all values in a data set are increased by 5, what happens to the standard deviation?

A) Increases by 5

B) Decreases by 5

C) Remains the same

D) Becomes zero

Solution: Standard deviation is based on the spread of data, not the data's location.

Answer: C

10.   The range of the data set {3, 7, 8, 5, 10} is:

A) 3

B) 5

C) 7

D) 8

Solution: Range = Maximum - Minimum = 10 - 2 = 8

Answer: D

11.   What is the formula for quartile deviation (Q.D.)?

A. $(Q_3 + Q_1)/2$

B. $(Q_3 - Q_1)/2$

C. $Q_3 - Q_1$

D. $(Q_1 - Q_2)/2$

Answer: B

Solution: Q.D. = $(Q_3 - Q_1) / 2$

Answer: B

12.   Quartile deviation is also known as:

A. Standard deviation

B. Interquartile range

C. Semi-interquartile range

D. Coefficient of variation

Solution: Q.D. = Semi-interquartile range = $(Q_3 - Q_1) / 2$

Answer: C

13.   Quartile deviation is best used when:

A. Data is skewed

B. Data is symmetrical

C. Data has outliers

D. Both A and C

Solution: Quartile deviation is robust to outliers and skewed data.

Answer: D

14. If Q1 = 20 and Q3 = 60, what is the quartile deviation?
A. 40
B. 20
C. 30
D. 10
Solution: Q.D. = (Q3 − Q1) / 2 = (60 − 20)/2 = 20
Answer: B
15. Coefficient of quartile deviation is given by:
A. (Q3 − Q1)/(Q3 + Q1)
B. (Q3 + Q1)/(Q3 − Q1)
C. Q3/Q1
D. Q1/Q3
Solution: Coefficient of Q.D. = (Q3 − Q1) / (Q3 + Q1)
Answer: A
16. Mean deviation is also known as:
A. Average deviation
B. Quartile deviation
C. Range
D. Standard deviation
Solution: Mean deviation = Average deviation from mean/median/mode.
Answer: A
17. Mean deviation is the average of:
A. Squared deviations
B. Absolute deviations
C. Percentage deviations
D. None
Solution: M.D. = Mean of absolute deviations from a central point.
Answer: B
18. M.D. about the mean is computed as:
A. Σ|X − Mean| / N
B. Σ (X − Mean) ² / N
C. Σ (X − Mean) / N
D. Σ|X − Median| / N
Solution: M.D. (Mean) = Σ|X − Mean| / N
     Answer: A
19. Which of the following is not affected by extreme values?
A. Mean deviation from mean
B. Standard deviation
C. Quartile deviation
D. None
Solution: Quartile deviation is not sensitive to extreme values.
     Answer: C
20. If the values are 3, 6, 9, 12, 15, what is the mean deviation from the mean?
A. 3
B. 4
C. 2.4
D. 6
Solution:
Mean = (3+6+9+12+15)/5 = 9
M.D. = [|3−9| + |6−9| + |9−9| + |12−9| + |15−9|]/5
= (6+3+0+3+6)/5 = 18/5 = 3.6
Answer: C

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

A.   Both Assertion (A) and Reason(R) are true, and Reason(R) is the correct explanation of Assertion (A).

B.   Both Assertion (A) and Reason(R) are true, but Reason(R) is NOT the correct explanation of Assertion (A).

C.   Assertion (A) is true, but Reason(R) is false.

D.   Assertion (A) is false, but Reason(R) is true.

1. **Assertion (A):** The mean is affected by extreme values.

   Reason(R): Mean considers all values in the dataset.

   Answer: A

Solution: Since the mean includes every data point, an extremely high or low value will affect the overall average.

2. **Assertion (A)**: The median is not affected by outliers.

   Reason (R): Median is the middle value and does not depend on magnitude.

   Answer: A

Solution: Median depends on the position of data when sorted, not their actual values.

3. **Assertion (A)**: Standard deviation can never be negative.

   Reason (R): Standard deviation is the square root of variance.

   Answer: A

 Solution: Variance is a squared quantity and is always non-negative. Hence, its square root (standard deviation) is also non-negative.

4. **Assertion (A)**: Mode is the most frequently occurring value in a dataset.

   Reason (R): Mode is not useful for continuous data.

Answer: C

Solution: Mode is the most frequent value, but it can be used in continuous data, especially when grouped into class intervals.

5. **Assertion (A):** In a symmetrical distribution, mean = median = mode.

   Reason (R): In symmetric distributions, the tail lengths are equal on both sides.

   Answer: A

Solution: In a perfectly symmetrical distribution like the normal curve, all three central tendencies coincide.

6. **Assertion (A):** Range is a good measure of dispersion for small data sets.

   Reason (R): Range only depends on the highest and lowest values.

   Answer: A

Solution: For small datasets, range can give a quick estimate of spread, but for larger data, it can be misleading due to ignoring other values.

7. **Assertion (A):** Variance can be zero.

   Reason (R): If all values are the same in a dataset, variance is zero.

   Answer: A

Solution: When there is no variability in data (all values are equal), there is no spread, hence variance = 0.

8. **Assertion (A):** Mean is always greater than the median.

   Reason (R): Mean is pulled in the direction of the skew.

   Answer: D

Solution: Mean can be greater than, less than, or equal to the median depending on the skewness of the data. However, mean is pulled in the direction of skewness.

9. **Assertion (A):** Quartile deviation is based on the middle 50% of the data.
Reason (R): It is calculated using Q1 and Q3.

<center>**Answer: A**</center>

Solution: Quartile Deviation = (Q3 - Q1) / 2, so it focuses on the interquartile range.

10. **Assertion (A):** Coefficient of variation is useful for comparing variability between datasets with different units.
Reason (R): It is a dimensionless number expressed as a percentage.

<center>**Answer: A**</center>

Solution: Coefficient of variation = (Standard Deviation / Mean) × 100% and allows comparison regardless of unit.

<center>**VERY SHORT ANSWER TYPE QUESTIONS**</center>

1. **Question:** The arithmetic means of 5, 10, 15, 20, and 25.

Solution: Mean = (5 + 10 + 15 + 20 + 25)/5 = 75/5 = **15**

2. **Question:** The mean of 4 numbers is 20. If three numbers are 15, 22, and 18, find the fourth number.

Solution: Total = 20 × 4 = 80
Fourth number = 80 - (15 + 22 + 18) = 80 - 55 = **25**

3. **Question:** Find the median of the data: 8, 12, 5, 10, 15.

Solution: Arrange: 5, 8, 10, 12, 15
Median = middle value = **10**

4. **Question:** Find the mode of the data: 2, 3, 4, 4, 5, 4, 6.

Solution: Mode = most frequent value = **4**

5. **Question:** The median of 7 observations is 25. If six values are 15, 18, 22, 30, 35, and 40, find the 7th value.
Solution:: Median is the 4th value after arranging.
Arrange: 15, 18, 22,?, 30, 35, 40
Median = 25 → So, 4th value must be 25 → **25.**

6. **Question:** Calculate the arithmetic mean of the first 10 natural numbers.
Solution:: Mean = (1 + 2 + ... + 10)/10 = (10×11)/2 ÷ 10 = 55/10 = **5.5**

7. **Question:** The mean of 5 numbers is 16. One number is changed from 20 to 30. Find the new mean.
Solution: Total increase = 30 - 20 = 10
New total = 5 × 16 + 10 = 80 + 10 = 90
New means = 90/5 = **18**

8. **Question:** If the mode of a data is 25 and the mean is 20, find the median using the empirical relation.
Solution: Mode = 3 × Median – 2 × Mean
25 = 3M – 2×20
25 = 3M – 40 → 3M = 65 → M = **21.67**

9. **Question:** Find the median of the data: 40, 35, 50, 55, 45.
Solution: Arrange: 35, 40, 45, 50, 55 → Median = **45**

10. **Question:** The mode of a dataset is 18, and the median is 20. Find the approximate mean.
Solution: Mode = 3 × Median – 2 × Mean
18 = 3×20 – 2×Mean → 18 = 60 – 2M → 2M = 42 → Mean = **21**

11. **Question:** Find the range of the data: 15, 22, 30, 10, 25.
Solution: Range = Max – Min = 30 – 10 = **20**

12. **Question:** Find the mean deviation about the mean for 2, 4, 6.
Solution::
Mean = (2+4+6)/3 = 4
M.D. = [|2–4| + |4–4| + |6–4|]/3 = (2+0+2)/3 = **1.33**

13. **Question:** Find the standard deviation of the data: 4, 4, 4, 4.
Solution:
Mean = 4, All deviations = 0
Standard deviation = **0**
14. **Question:** Find the variance of 3, 5, 7.
Solution: Mean = 5
Variance = $[(3–5)^2 + (5–5)^2 + (7–5)^2]/3 = (4 + 0 + 4)/3 = $ **2.67**
15. **Question:** The standard deviation of a dataset is 5. Find its variance.
Solution: Variance = (Standard Deviation)^2 = 5^2 = **25**
16. **Question:** Find the range and interquartile range (IQR) of the data: 5, 10, 15, 20, 25.
Solution: Range = 25 – 5 = 20
Q1 = 10, Q3 = 20 → IQR = 20 – 10 = **10**
17. **Question:** Find the mean deviation about the median for 1, 2, 3, 4, 5.
Solution: Median = 3
M.D. = (2+1+0+1+2)/5 = 6/5 = **1.2**
18. **Question:** What is the standard deviation of first 3 odd natural numbers?
Solution: Data: 1, 3, 5 → Mean = 3
SD = $\sqrt{[(4+0+4)/3]} = \sqrt{(8/3)} = $ **1.63**
19. **Question:** If all values of a dataset are increased by 5, how does it affect standard deviation?
Solution:
No change in standard deviation (it measures spread, not position)
20. **Question:** If each value of a dataset is multiplied by 3, how is the standard deviation effected?
Solution: Standard deviation is also multiplied by 3 → New SD = 3 × Original SD

SHORT ANSWER TYPE QUESTIONS

1.Question: Find the standard deviation for the data: 6, 8, 10, 12, 14
Solution: Given data set is 6, 8, 10, 12, 14
Mean (μ) = (6+8+10+12+14)/5 = 10

Variance (σ^2) = $[(6−10)^2 + (8−10)^2 + (10−10)^2 + (12−10)^2 + (14−10)^2]/5 = 8$
Standard Deviation (σ) = $\sqrt{8}$
2.Question: Find the mean of the data: 5, 7, 9, 12, 15.
Solution: Mean = (5 + 7 + 9 + 12 + 15) / 5 = 48 / 5 = 9.6
3.Question: Find the median of the data: 10, 15, 20, 25, 30, 35, 40.
Solution: Number of terms = 7 (odd), Median = Middle term = 4th term = 25
4.Q**estion**: Find the mode of the data: 12, 15, 12, 18, 20, 15, 15, 12.
Solution: 12 appears 3 times, 15 appears 3 times, 18 and 20 appear once.
Bimodal: 12 and 15
5.Question: Calculate the mean for the data: 6, 8, 10, 12, 14 using assumed mean method (assume 10).
Solution: Let A = 10.
Deviations (d) = x – A → -4, -2, 0, 2, 4
Sum of deviations = 0
Mean = A + (Σd/n) = 10 + (0/5) = 10
6.**Question**: Find the variance and standard deviation of the data: 3, 4, 5, 6, 7.
Solution: Mean = (3+4+5+6+7)/5 = 25/5 = 5
Σ(x – mean)^2 = (4+1+0+1+4) = 10
Variance = 10/5 = 2
SD = $\sqrt{2}$ = 1.41
7.Question: The mean of five observations is 18. Four of them are 16, 20, 18, and 22. Find the fifth observation.

Solution: Sum of 5 observations = 5 × 18 = 90

Sum of 4 = 16 + 20 + 18 + 22 = 76

Fifth observation = 90 – 76 = 14

8.Question: Find the combined mean of two groups having means 60 and 70, and sizes 30 and 20 respectively.

Solution: Combined Mean = (60×30 + 70×20) / (30 + 20)

= (1800 + 1400)/50 = 3200/50 = 64

9.Question: Calculate the median for the data: 4, 8, 15, 21, 27, 30.

Solution: Even number of terms = 6

Median = (3rd + 4th)/2 = (15 + 21)/2 = 18

10.Question: Find the mean and standard deviation for data: 2, 4, 6, 8, 10.

Solution: Mean = (2+4+6+8+10)/5 = 30/5 = 6

Variance = [(16+4+0+4+16)/5] = 40/5 = 8

SD = $\sqrt{8}$ = 2.83

11.Question: If the variance of a data is 16, find the standard deviation.

Solution: SD = $\sqrt{\text{Variance}}$ = $\sqrt{16}$ = 4

12.Question: The mean of 6 numbers is 12. One number is removed, and the new mean is 11. Find the removed number.

Solution: Sum of 6 = 6×12 = 72

Sum of 5 = 5×11 = 55

Removed number = 72 – 55 = 17

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1.Question: Find the variance and standard deviation for the following data:

10, 12, 14, 16, 18

Solution: Mean ($\bar{x}$) = (10 + 12 + 14 + 16 + 18)/5 = 70/5 = 14

Variance (σ^2) = [(10−14)² + (12−14)² + (14−14)² + (16−14)² + (18−14)²] / 5

= [16 + 4 + 0 + 4 + 16] / 5 = 40 / 5 = 8

Standard deviation (σ) = $\sqrt{8}$ ≈ **2.83**

2) Question: Find the standard deviation of the first 5 natural numbers.

Solution: First 5 natural numbers: 1, 2, 3, 4, 5

Mean = (1 + 2 + 3 + 4 + 5) / 5 = 15 / 5 = 3

Variance = [(1−3)² + (2−3)² + (3−3)² + (4−3)² + (5−3)²] / 5

= [4 + 1 + 0 + 1 + 4] / 5 = 10 / 5 = 2

Standard deviation = $\sqrt{2}$ ≈ **1.41**

3) Question: The marks of 5 students are 45, 50, 55, 60, and 65. Find the variance and standard deviation.

Solution: Mean = (45 + 50 + 55 + 60 + 65)/5 = 275 / 5 = 55

Variance = [(45−55)² + (50−55)² + (55−55)² + (60−55)² + (65−55)²]/5

= [100 + 25 + 0 + 25 + 100]/5 = 250 / 5 = 50

Standard deviation = $\sqrt{50}$ ≈ **7.07**

4) Question: Find the variance and standard deviation for the data: 3, 7, 7, 19, 21.

Solution: Mean = (3 + 7 + 7 + 19 + 21) / 5 = 57 / 5 = 11.4

Variance = [(3−11.4)² + (7−11.4)² + (7−11.4)² + (19−11.4)² + (21−11.4)²] / 5

= [70.56 + 19.36 + 19.36 + 57.76 + 92.16] / 5 = 259.2 / 5 = 51.84
Standard deviation = √51.84 ≈ **7.2**

5) Question: The ages (in years) of 6 workers are: 22, 25, 27, 30, 35, 40. Calculate the standard deviation.

Solution: Mean = (22 + 25 + 27 + 30 + 35 + 40) / 6 = 179 / 6 ≈ 29.83
Now calculate deviations and squares:
(22−29.83)² = 61.39
(25−29.83)² = 23.33
(27−29.83)² = 8.00
(30−29.83)² = 0.03
(35−29.83)² = 26.66
(40−29.83)² = 103.36
Sum = 222.77
Variance = 222.77 / 6 ≈ 37.13
Standard deviation = √37.13 ≈ 6.09

CASE STUDY BASED QUESTIONS

A class of students recorded the number of hours they studied in a week. The data is presented below:

Hours Studied	0–10	10–20	20–30	30–40	40–50
Number of Students (f)	5	8	12	6	4

Questions:
  i. What is the class width of the given data?
  ii. Find the mid-point of the class 20–30.
  iii. Calculate the variance and standard deviation of the data.

Solution:
  i. width = 10
  ii. mid-point of 20–30 = (20 + 30)/2 = 25
  iii.

Class	f	x (Mid)	fx	x²	fx²
0–10	5	5	25	25	125
10–20	8	15	120	225	1800
20–30	12	25	300	625	7500
30–40	6	35	210	1225	7350
40–50	4	45	180	2025	8100

Σf = 35, Σfx = 835, Σfx² = 24875
Mean = Σfx / Σf = 835 / 35 ≈ 23.86
Variance = [Σfx² / Σf] – (mean)² = 24875/35 – (23.86)² ≈ 710.71 – 569.48 ≈ 141.23
Standard Deviation = √141.23 ≈ 11.88

The following table shows the monthly electricity consumption (in kWh) of households in a locality:

Consumption (kWh)	100–150	150–200	200–250	250–300	300–350
Number of Houses (f)	6	11	15	10	8

Questions:
        i. What is the class mark of the class 150–200?

ii. How many households consume 250–300 kWh?

iii.Find the standard deviation.

Solution:

1. Class mark = (150 + 200)/2 = 175
2. 10 households
3.

Class	f	x	fx	x²	fx²
100–150	6	125	750	15625	93750
150–200	11	175	1925	30625	336875
200–250	15	225	3375	50625	759375
250–300	10	275	2750	75625	756250
300–350	8	325	2600	105625	845000

Σf = 50, Σfx = 11400, Σfx^2 = 2,794,250

Mean = 11400 / 50 = 228

Variance = $[\Sigma fx^2 / \Sigma f]$ – (mean)² = 2794250/50 – 228² = 55885 – 51984 = 3901

Standard Deviation = $\sqrt{3901} \approx 62.45$

3. teacher recorded the marks out of 100 scored by students in an exam:

Marks	0–20	20–40	40–60	60–80	80–100
No. of Students (f)	4	6	10	5	5

Questions:

1. What is the total number of students?
2. What is the class mark of the class 60–80?
3. Find the variance and standard deviation.

Solution:

1. Total = 4 + 6 + 10 + 5 + 5 = 30
2. Class mark = (60 + 80)/2 = 70
3.

Class	**f**	**x**	**fx**	**x²**	**fx²**
0–20	4	10	40	100	400
20–40	6	30	180	900	5400
40–60	10	50	500	2500	25000
60–80	5	70	350	4900	24500
80–100	5	90	450	8100	40500

Σf = 30, Σfx = 1520, Σfx^2 = 95700

Mean = 1520 / 30 = 50.67

Variance = 95700/30 – (50.67)² $\approx$ 3190 – 2567.45 = 622.55

Standard Deviation = $\sqrt{622.55} \approx 24.94$

HIGHER ORDER THINKING SKILLS

Q1. A company has 3 departments with the following salaries (in ₹) of employees:

- Dept A: 25,000, 28,000, 30,000
- Dept B: 40,000, 42,000, 45,000
- Dept C: 20,000, 22,000, 25,000

Which department has the highest income inequality? Use standard deviation to justify your answer.

Solution: Calculate Mean and SD for each department.

- Dept A:
  - Mean = (25000 + 28000 + 30000)/3 = 27667
  - SD = $\sqrt{[(25000-27667)^2 + (28000-27667)^2 + (30000-27667)^2)/3]}$
    = $\sqrt{[(711111 + 111111 + 544444)/3]}$ = $\sqrt{(455555)} \approx 675$
- Dept B: Mean = 42333, SD $\approx$ 2041
- Dept C: Mean = 22333, SD $\approx$ 2041

Conclusion: Dept B and C have the same standard deviation, but relative to their means, Dept C shows higher relative variability, so Dept C has the highest income inequality.

Q2. In a survey of test scores in two different classes:
- Class A: Mean = 70, SD = 5
- Class B: Mean = 72, SD = 10

Which class shows more consistent performance, and why?

Solution:

Consistency is measured by Coefficient of Variation (CV):
- Class A: CV = (5 / 70) × 100 = 7.14%
- Class B: CV = (10 / 72) × 100 = 13.89%

Conclusion: Class A has a lower CV, so it shows more consistent performance.

Q3. A dataset has 10 observations with mean = 50 and variance = 25. If each observation is increased by 5, what will be the new mean and standard deviation?

Solution:
- New Mean = 50 + 5 = 55
- New Variance = unchanged = 25
- New Standard Deviation = $\sqrt{25}$ = 5

Concept: Adding a constant shifts the mean but does not affect variance or SD.

Q4. Two students calculated the mean of the same dataset:
- Student A used all values and found the mean to be 60.
- Student B excluded the highest value (100) and got 55.

What can you infer about the influence of outliers on mean?

Solution: Excluding the outlier (100) reduced the mean from 60 to 55.

Inference: The mean is sensitive to extreme values, i.e., outliers can significantly affect the average.

Q5. A teacher has to select the best performing student from two students based on 3 test scores:

Student	Test 1	Test 2	Test 3
A	80	90	100
B	85	85	85

Who should the teacher choose if they value both performance and consistency?

Solution:
- Student A: Mean = 90, SD = $\sqrt{[(100-90)^2 + (90-90)^2 + (80-90)^2]/3}$ = $\sqrt{[100 + 0 + 100]/3}$ $\approx$ 8.16
- Student B: Mean = 85, SD = 0

Conclusion:
- Student A has higher average, but Student B is perfectly consistent.
- If the teacher values both, B may be better for reliability, but A is better for potential.

Answer depends on the weightage of consistency vs. performance.

EXCERCISE
MULTIPLE CHOICE QUESTIONS

1. The correlation coefficient lies between:
A. 0 and 1
B. −1 and 1
C. 0 and ∞
D. −1 and 0

2. r = +1, the relationship is:
A. No correlation
B. Perfect positive correlation
C. Weak correlation
D. Perfect negative correlation

3. Which of the following implies a strong negative correlation?
A. r = −0.90
B. r = +0.85
C. r = 0
D. r = −0.10

4. If two variables move in opposite directions, the correlation is:
A. Zero
B. Positive
C. Negative
D. Undefined

ASSERTION - REASON BASED QUESTIONS

1. Assertion (A): Quartile deviation is based on the middle 50% of the data.
Reason (R): It is calculated using Q1 and Q3.
Answer: Both A and R are true, and R is the correct explanation of A.

2. Assertion (A): Mean deviation can be negative.
Reason (R): Mean deviation is the average of the deviations from the mean.
Answer: A is false, but R is true.
(Mean deviation is always positive as absolute deviations are taken.)

3. Assertion (A): Quartile deviation is a more reliable measure of dispersion than standard deviation.
Reason (R): Quartile deviation ignores extreme values.
Answer: A is false, but R is true.
(Standard deviation is generally considered more reliable, though Q.D. is less affected by outliers.)

4. Assertion (A): Correlation coefficient (r) is always between -1 and 1.
Reason (R): The correlation coefficient measures the direction and strength of a linear relationship between two variables.
Answer: Both A and R are true, and R is the correct explanation of A.

5. Assertion (A): Mean deviation about the mean is always less than standard deviation.
Reason (R): Standard deviation gives more weight to larger deviations due to squaring.
Answer: Both A and R are true, and R is the correct explanation of A.

VERY SHORT ANSWER TYPE QUESTIONS

1. What is Quartile Deviation?
2. Write the formula for Quartile Deviation.
3. What is Mean Deviation?
4. Write the formula for Mean Deviation about the mean.

5. What is the range of the correlation coefficient (r)?
6. What does a correlation coefficient of 0 indicate?

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SHORT ANSWER TYPE QUESTIONS
</div>

1. Calculate the mean of the following frequency distribution:

x	2	4	6
f	3	5	2

2. For the data 5, 7, 9, 11, 13, compute the range and standard deviation.

3. Find the mean and variance of first 5 natural numbers.

4. A student scored 40, 45, 35, 50, and 30 in five tests. Calculate mean and standard deviation.

LONG ANSWER TYPE QUESTIONS

1. **Question:** The ages (in years) of 6 workers are: 22, 25, 27, 30, 35, 40. Calculate the standard deviation.
2. **Question:** The weights (in kg) of 4 children are: 18, 20, 22, and 24. Find the variance and standard deviation.

CASE BASED QUESTIONS

1. The weights (in kg) of a group of athletes are recorded as:

Weight (kg)	50–55	55–60	60–65	65–70	70–75
No. of Athletes (f)	3	7	10	8	2

Answer the following questions
1. What is the mid-value of the class 65–70?
2. What is the modal class?
3. Find the variance and standard deviation.

2.
The ages of employees in a firm are given below:

Age (years)	20–30	30–40	40–50	50–60	60–70
No. of Employees (f)	2	6	12	5	3

Answer the following questions:
1. What is the class size?
2. What is the total number of employees?
3. Calculate the standard deviation.

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HIGHER ORDER THINKING SKILLS
</div>

1. A student records the number of hours studied over 7 days as follows: 2, 3, 5, 7, 5, 3, 2. Find the mean, variance, and standard deviation.
2. Design a dataset of 5 numbers where the mean is 20, and standard deviation is exactly 0. What does this tell you about the dataset?

ANSWERS

MULTIPLE CHOICE QUESTIONS

1. B
2. B
3. A
4. D

ASSERTION - REASON BASED QUESTIONS

1. A
2. D
3. D
4. A
5. A

VERY SHORT ANSWER TYPE QUESTIONS

1. Quartile Deviation is half the difference between the third quartile (Q3) and the first quartile (Q1). It measures the spread of the middle 50% of a dataset.

2. Quartile Deviation = (Q3 – Q1) / 2

3. Mean Deviation is the average of the absolute differences between each data value and the mean (or median) of the dataset.

4. Mean Deviation = $(\Sigma|x – \bar{x}|) / n$, where $\bar{x}$ is the mean and n is the number of observations.

5. The correlation coefficient (r) ranges from –1 to +1.

SHORT ANSWER TYPE QUESTIONS

1. Mean = 3.8
2. Range = 13 – 5 = 8
SD = $\sqrt{8}$ = 2.83
3. Mean = 3
Variance = 2
4. Mean = 40
SD = $\sqrt{50}$ = 7.07

LONG ANSWER TYPE QUESTIONS

1. Standard deviation = $\sqrt{37.13} \approx 6.09$

2. Variance = 5
Standard deviation = $\sqrt{5} \approx 2.24$

CASE BASED QUESTIONS

1.   (i)Mid- value= 67.5

      (ii)Modal class = 60–65 (highest frequency = 10)
     (iii)Mean = 62.33   Variance = 28.75

Standard Deviation = $\sqrt{28.75} \approx 5.36$

2.   (I)   size = 10
    ii.Total no. of employees = 2 + 6 + 12 + 5 + 3 = 28

     (iii)   Mean = 1270 / 28 ≈ 45.36
     Variance = $60600/28 – (45.36)^2 \approx 2164.29 – 2057.73 = 106.56$
     Standard Deviation = $\sqrt{106.56} \approx 10.32$

HIGHER ORDER THINKING SKILLS

1. Mean = 3.857 hours, Variance = 2.976, Standard Deviation ≈ 1.726

2. The dataset has no variation; all values are identical and equal to the mean.

UNIT VII: - BASICS OF FINANCIAL MATHEMATICS

DEFINATIONS AND FORMULAE

Compound interest

Principal (P): - The money borrowed (or the money lent) is called Principal.

Interest(I): - The additional money paid by a borrower to the lender for having used his(her) money is called interest.

Amount(A): - The sum of principal and interest is called Amount.

Thus,   Amount = principal + interest

Rate of Interest(R): - It is the interest paid on Rs 100 for a specific period.

Types of interest: - there are two types of interest: -

1. **Simple interest:** - it is the interest calculated on the original money (principal) for any given time and rate.

    Simple Interest (S.I) $= \frac{P \, X \, R \, X \, T}{100}$

2. **Compound Interest: -** At the end of the first year (or any other fixed period), if the interest accrued is not paid to the money lender but is added to the principal, then this amount becomes the principal for next year (or any other fixed period) and so on. This process is repeated until amount for the whole time is found. The time period after which interest added each time is called the conversion period.

    Compound Interest (C.I) = compound Amount – Principal.

    Compound Amount (A) = P $(1 + \frac{r}{100})^n$

    A = Final amount

    r = rate of interest per conversion period

    n= is the number of conversion period

In solving problems on compound Interest, remember the following:

*   When the rate of interest for the successive fixed periods is r1%, r2%, r3%........, then the amount is given by

    A = P$(1 + \frac{r1}{100})$ $(1 + \frac{r2}{100})$ $(1 + \frac{r3}{100})$………

*   S.I and C.I is same for first conversion period on the same sum and same rate.
*   When the total time is not a complete number of conversion periods, we consider simple interest for the last partial period. For example, if time is 2 years 5 months and interest rate is r% p.a. compounded annually then

    A = P$(1 + \frac{r}{100})^2(1 + \frac{5r}{12 \, x \, 100})$

    Equivalent rates of interest: - Two annual rate of interest with different conversion period if they yield same compound at the end of the year.

    For example, consider an amount of 10000 invested at 4% per annum compounded quarterly. So, the amount at the end of one year = $1000(1.01)^4$= 10406. This is equivalent to interest of 4.06% compounded annually because S.I = 10000 × 4.06 × 1/100 = 406 and amount = 10000 + 406 = 10406.

    Nominal and effective rate of interest: - When the interest compounded more than once in a year, the given annual rate is called nominal or annual nominal rate. The rate of actually earned is called effective rate. In the above example, 4% is nominal rate of interest while 4.06% is effective rate of interest.

    Effective rate of interest = $(1 + \frac{r}{100p})^p$ - 1

    r = rate of interest per annum

p = number of times compounded annually.

Real interest: - the real interest rate is the interest rate that takes inflation into consideration. It is also called inflation adjusted interest rate.

Real rate of interest = nominal interest – rate of inflation.

Annuity: - An annuity is a sequence of equal payments made at equal interval of time.

Example: Monthly insurance premiums, pension payments, EMIs.

Types of Annuities

Ordinary annuity: - Payment made at the end of each period.

Annuity due: - Payment made at the beginning of each period

Deferred annuity: - Payment start after a certain period.

FUTURE VALUE OF ORDINARY ANNUITY

$A = R \frac{[(1+i)^n - 1]}{i} = S_{\bar{n}/i}$

$i = \frac{r}{100}$ R = periodic payment.

n = no. of periods

PRESENT VALUE OF ORDINARY ANNUITY

$P = R \frac{[1 - (1+i)^{-n}]}{i} = R a_{\bar{n}/i}$

Future value of annuity due

$A = R(1+i) \frac{[(1+i)^n - 1]}{i} = R(S_{\overline{n+1}/i} - 1)$

$i = \frac{r}{100},$ R = periodic payment.

n = no. of periods

present value of annuity due

$P = R(1+i) \frac{[1 - (1+i)^{-n}]}{i} = R(1 + a_{\overline{n-1}/i})$

Future value of deferred annuity

$P = R \frac{[1 - (1+i)^{-n}]}{i} = R a_{\bar{n}/i}$

$i = \frac{r}{100},$ R = periodic payment.

n = no. of periods

TAXATION

 Meaning of Taxation

- **Taxation** is the process by which a government collects money from citizens and businesses to fund public services.
- It is **compulsory** and **legally imposed**.

  Types of Taxes

➤ **Direct Taxes**

- Paid **directly** to the government by the individual or organization.
- Examples: **Income Tax**, **Corporate Tax**, **Wealth Tax**.

➤ **Indirect Taxes**

- Collected by intermediaries (like shopkeepers) and passed to the government.
- Examples: **GST (Goods and Services Tax)**, **Sales Tax**, **Excise Duty**.

  Key Terms in Taxation

Term	Meaning
Income	Total earnings (salary, rent, interest, etc.)

Term	Meaning
Gross Income	Total income before any deductions
Net Income (Taxable Income)	Gross income – Deductions (like under Section 80C)
Tax Rate	Percentage of income to be paid as tax
Tax Slab	Income range and corresponding tax rate
Rebate	A relief or refund given under certain conditions
TDS (Tax Deducted at Source)	Tax deducted before payment is made to the recipient

Income Tax Slabs (Example Format)

(These vary by year and country – check current data for actual figures)

Income Range	Tax Rate
Up to ₹2,50,000	Nil
₹2,50,001 – ₹5,00,000	5%
₹5,00,001 – ₹10,00,000	20%
Above ₹10,00,000	30%

Calculation of Income Tax

Steps:

1. Calculate **Gross Total Income** (sum of all incomes).
2. Subtract **Deductions** (under sections like 80C, 80D, etc.).
3. Result is Net Taxable Income.

UTILITY BILLS

Electricity Bill Terms

1. Unit/Consumption – The amount of electricity used, measured in kilowatt-hours (kWh).
2. It is a price assigned to different utilities determined by the government authorities. It consists of two parts. Fixed charge and variable charge.
3. Fixed Charge – A base fee charged regardless of the consumption.
4. Variable: it depends on consumption.
5. Energy Charge – The cost based on actual units consumed (e.g., ₹ per kWh).
6. Meter Reading – Measurement of electricity used, usually from an electricity meter.
7. Slab Rate – Tiered pricing where cost per unit increases with more consumption.
8. Surcharge – An extra charge (e.g., for late payment or higher usage).
9. Subsidy – A government-provided discount for specific consumers.
10. Total Bill Amount – Sum of fixed charge, energy charge, and applicable taxes/surcharges.

MULTIPLE CHOICE QUESTIONS

1. A bank offers 6% interest compounded quarterly. What will be the effective annual rate (EAR)?

   (a) 6%   (b) 6.09%   (c) 6.14%   (d) 6.20%

   Solution: EAR = $(1 + \frac{6}{100})^4$ - 1 = $(1.015)^4$ - 1 = 1.0614 - 1 = 0.0614 → 6.14%

2. Which investment gives higher return over 1 year for same rate and principal?

   (a) 10% p.a. compounded annually   (b) 10% p.a. compounded half-yearly

   (c)10% p.a. compounded quarterly   (d). All are equal

   Answer: (c)

   Explanation: More frequent compounding means more interest accumulation

3. Meena invests ₹2,000 at the end of every year in an annuity that pays 10% p.a. compounded Annually for 3 years. What is the total value of her investment at the end of 3 years?

   (a) ₹6,200   (b) ₹6,620   (c) ₹6,820   (d) ₹7,200

Answer: (b)

Explanation:

Given:

(Use: $A = \dfrac{P \times [(1+r)^n - 1]}{r}$

P=₹2000, r=10%=0.10, n=3

$A = 2000 \dfrac{\times[(1+0.10)3-1]}{0.1} = 2000 \dfrac{\times[1.331-1]}{0.1} = 2000 \times 3.31 = ₹6620$

4. Why does an annuity due accumulate more value over time compared to an ordinary annuity (Assuming same terms)

  (a) It has a higher interest rate

  (b) It adds more money to the investment

  (c) Payments are made at the beginning of each period, allowing more time to earn interest

  (d) It compounds annually instead of monthly

Answer: (c)

Explanation: In an annuity due, each payment gets an extra period to earn interest compared to an Ordinary annuity.

5. The difference between simple and compound interest compounded annually on a certain sum of Money for 2 years at 4% per annum is ₹1. Then the sum is

  (a) ₹625   (b) ₹630   (c) ₹640   (d) ₹650

Answer = (a)

Solution: - $C.I = P\left(1 + \dfrac{r}{100}\right)^n - P$

$C.I = P\left(1 + \dfrac{4}{100}\right)^2 - P$

$C.I = P \times 0.0816 = \dfrac{816P}{10000}$

$S.I = \dfrac{8 \times P}{100} = \dfrac{800P}{10000}$

$C.I - S.I = 1,$

$\dfrac{816P}{10000} - \dfrac{800P}{10000} = 1$

$P = \dfrac{10000}{16} = ₹625$

6. Which section of the Income Tax Act allows deductions on investments like PPF, LIC, ELSS, etc.?

  (a) Section 10   (b) Section 24   (c) Section 80C   (d) Section 87A

  Answer: (c)

  Explanation: Section 80C allows deductions on investments up to ₹1.5 lakh in specified.

7. The income tax act was passed in the year of

  (a) 1951   (b) 1961   (c) 1963   (d) 1971

  Answer: (b)

8. If the nominal rate is 5% compounded monthly, find the corresponding effective rate. Given $(1.004167)^{12} = 1.05116$

  (a) 4.908%   (b) 5.116%   (c) 5.001%   (d) 5.06%

Answer: - (b) 5.116%

Solution   Effective rate of interest $= \left(1 + \dfrac{r}{100p}\right)^p - 1$

Effective rate of interest $= \left(1 + \dfrac{5}{100 \times 12}\right)^{12} - 1 = (1.004167)^{12} - 1 = 0.05116 = 5.11\%$

9. If the compound interest on ₹8,000 in 2 years is ₹1,632, what is the rate of interest per annum?

   (a) 10%   (b) 12%   (c) 8%   (d) 9%

   Answer: (b) 10%

   Solution: A=8000+1632=9632

   $A=P(1+r)^2 \Rightarrow 9632=8000(1+r)^2 \Rightarrow (1+r)^2=1.204 \Rightarrow r=0.10=10\%$

10. Section 87A refers to

   (a) Tax rebate   (b) Interest on home loan

   (c) Investment in NSC   (d) Investment in ELSS

   Answer: - (a)

11. What equal payment made at the beginning of each year for 1 0 years pay for a piece of land priced at Rs. 400000, if the money worth is 7% per annum compounded annually? $(1.07)^{-10} = 0.5082$

   (a). Rs. 53225.46   (b) Rs 53209.07   (c) Rs 45000.26   (d) Rs 53202.64

   Answer (b)

   Solution: - $P = \dfrac{R(1+i)[1-(1+i)^{-n}]}{i}$

$$400000 = R(1+0.07)\dfrac{[1-(1+0.07)^{-10}}{0.07}$$

$$400000 = R \times 1.07 \times \dfrac{[1-0.5082]}{0.07}$$

$$400000 = \dfrac{R \times 1.07 \times 0.4918}{0.07}$$

$$R = \dfrac{400000 \times 0.07}{(1.07 \times 0.4918)}$$

$$R = Rs. 53209.07$$

12. Deduction of interest on home loan is allowed under section

   (a) 80C   (b) 80TTA   (c) 24   (d). 80E

   Solution: (c)

13. A retailer purchases a fan for ₹ 1500 from a wholesaler and sells it to a consumer at 10% profit. If the sales are intra-state and the rate of GST is 12%, the tax (under GST) paid by the wholesaler to the Central Government is:

   (a) ₹ 9   (b) ₹ 180   (c) ₹ 90   (d). ₹ 99

   Solution: (b)

14. Health and Education cess are payable on:

   (a) Taxable income   (b) Education loan   (c) Gross income   (d) Income tax

   Solution: income tax (c)

15. A shopkeeper bought a TV from a distributor at a discount of 25% of the listed price of ₹ 32000. The shopkeeper sells the TV to a consumer at the listed price. If the distributor to the State Government is the sales are intra-state and the rate of CST is 18%, The tax (under GST) paid by:

   (a) ₹ 2880   (b) ₹ 4320   () ₹ 720   (d) ₹ 2160

   Answer (a)

   Solution: Listed price = ₹ 32000, discount = 25% of ₹ 32000 = ₹ 8000

        S.P. of distributor = ₹ 32000 - ₹ 8000 = ₹ 24000

        CGST = 9% of ₹ 24000 = ₹ 2160

        SGST = 9% of ₹ 24000 = ₹ 2160

        S.P. including tax = ₹ 24000 + ₹ 2160 + ₹ 2160 = ₹ 28320

16. A retailer purchases a fan for ₹ 1500 from a wholesaler and sells it to a consumer at 10% profit. If the sales are intra-state and the rate of GST is 12%, The cost of the fan to the consumer inclusive of tax is

   (a) ₹ 1650   (b) ₹ 1830   c. ₹ 1848   d. ₹ 1800

   Answer: (a)

   Solution: If a retailer purchases a fan for ₹ 1500 from a wholesaler and sells it to a consumer at 10% profit and the rate of GST is 12%

   then, including tax (under GST) the selling price would be $= \frac{1500 \, x \, 110}{100} = 1650$

17. Section 80E refers to the deduction of:

   (a) Interest on fixed deposit   (b) interest on savings account

   (c) interest on education loan   (d) Interest on a home loan

   Answer: interest on education loan (c)

18. compute the amount on ₹10000 compounded annually for $2\frac{1}{2}$ years at 4 % per annum:

   (a) ₹ 11032.32   (b) ₹ 21032.20   (c) ₹ 12032.32   (d) ₹ 11023.23

   Answer: (a)

   Solution: amount at the end of $2\frac{1}{2}$ years = ₹10000 $(1 + \frac{4}{100})^2(1 + \frac{2}{100})$

                            = ₹11032.32

19. Deduction of tuition fees of two children is allowed under the section:

   (a) 80C   (b) 80D   (c) 80E   (d) 80TTA

   Answer: a. 80C

20. Deduction of medical insurance premium is allowed under the section:

   (a) 80C   (b) 80D   (c) 80E   (d) 80TTA

   Answer: (b) 80D

ASSERATION AND REASON QUESTIONS

 These questions are Assertion and Reason based question. Two statements are given, one labelled Assertion (A) and other labelled Reason ®. Select the correct answer from the codes (a), (b), (c) and (d) as given below:

 (a) Both A and R are true and R is the correct explanation of A.

 (b) Both A and R are true but R is not the correct explanation of A.

 (c) A is true but R is false.

 (d) A is false but R is true.

1. Assertion (A): Deduction of tuition fee of two children is allowed under section 80E of income tax.

    Reason (R): A person who donate to Prime Minister's National Relief fund can avail deduction under section 80G of income tax.

    Answer: (d) A is false but R is true.

    Explanation: A is false. As deduction of tuition fee of two children is allowed under section 80C but R is true

2. Assertion (A): For the same principal, time, and rate, compound interest is always less than simple interest.

    Reason (R): Compound interest considers only the original principal for interest calculation.

    Answer: (d)

Explanation: A is false. As deduction of tuition fee of two children is allowed under section 80C but R is true

3.   A shopkeeper in U.P. buys an article for ₹ 10000 from a wholesaler in Delhi. Shopkeeper sells   the article to a consumer in M.P. at a profit of 20%. Rate of GST is 12%.   Assertion (A): IGST paid by the consumer is ₹1440.
Reason (R): IGST paid by the wholesaler to the Central Government is ₹120.
Answer: (c) A is true but R is false.
Explanation:
Selling price of the article by the shopkeeper = ₹ (10000 + 10000 x 20/100) = ₹ 12000.
IGST paid by the consumer = 12% of ₹ 12000 = ₹ 1440.
A is true.
IGST paid by the wholesaler to the Central Government = 12% of ₹ 10000 = ₹ 1200. R is false.

4. Assertion (A): Deduction of principal of home loan amount is allowed under section 80C of income tax.
Reason (R): Deduction of interest of home loan amount is allowed under section 80TTA of income tax.
Answer: (c) A is true but R is false. Explanation: A is true but R is false.

5. Manufacturer A sells a T.V. to a dealer B for and ₹ 8000 dealer B sells it to consumer C at a profit of 25%. Sales are intra-state and rate of GST is 15%.
Assertion (A): CGST paid by the dealer B to the Central Government is ₹ 150.
Reason (R): CGST paid by the dealer B to the Central Government = CGST collected from consumer C - CGST paid to manufacturer A.
Answer: (a) Both A and R are true and R is the correct explanation of A.
Explanation: As the sales are intra-state and the rate of GST is 15%, so GST comprises of 7.5% as CGST and 7.5% as SGST. Since dealer B sells the T.V. to consumer C at a profit of 25%,
So, selling price of T.V. by dealer B = ₹ 8000 + 25% of ₹ 8000 = ₹ 10000.
Amount of CGST paid by dealer B to manufacturer A = 7.5% of ₹ 8000 = ₹ 600
Amount of CGST collected by dealer B from consumer C = 7.5% of ₹ 10000 = ₹ 750. So, CGST paid by dealer B to the Central Government = CGST collected from consumer - CGST paid to manufacturer = ₹ 750 - ₹ 600 = ₹ 150 A is true.
Also, R is true and R is the correct explanation of A.

6. Assertion (A): In compound interest, the interest is calculated on the principal only.
Reason (R): Compound interest is calculated annually or semi-annually.
Answer: (d)
(In compound interest, interest is calculated on principal + accumulated interest. The reason is correct but not related to assertion.)

7. Assertion (A): The formula for compound interest is $A = P \left(1 + \frac{r}{100} \right)$.
Reason (R): Compound interest accumulates as interest is added to the principal at regular intervals.

Answer: (a)

8. Assertion (A): The formula for the future value of an ordinary annuity is

$$A = R \times \frac{(1+r)^n - 1}{r}$$

Reason (R): The payment in an ordinary annuity grows linearly over time
Answer: (c)
(The formula is correct, but the Reason is incorrect. The payments are equal, but the value grows exponentially due to compounding.)

9. Assertion (A): An annuity is a series of equal payments made at regular intervals.
Reason (R): The amount of an annuity depends only on the number of payments made.
Answer: (c)
(An annuity depends not only on the number of payments but also on the rate of interest and the timing of payments.)

10. Assertion (A): A salaried individual can reduce taxable income by claiming deductions under Section 80C.
Reason (R): Section 80C allows deductions up to ₹1.5 lakh for investments in specified financial instruments.
Answer**:** (a) Both A and R are true, and R is the correct explanation of A

VERY SHORT ANSWER TYPE QUESTIONS

1. Two persons A and B went to a restaurant and ordered for pizza and garlic bread. The rates for pizza and garlic bread are ₹ 250 and ₹ 150 respectively. GST rate on these food items is 5%. Calculate the net amount payable by A and B.
   Also, find CGST and SGST.
   Solution:

   GST paid on pizza = ₹ 250 X $\frac{5}{100}$ = ₹12.50

   Amount paid on pizza = 250 + 12.50 = ₹262.50

   GST paid on garlic bread = ₹ 150 X $\frac{5}{100}$ = ₹ 7.50

   Amount paid on garlic bread = 150 + 7.50 = ₹157.50
   Total amount paid = 262.50 + 157.50 =₹ 420
   Total GST = 12.50 + 7.50 = ₹ 20
   CGST = ₹ 10
   SDST = ₹ 10

2. The marked price of an article is ₹ 5000 and sold at ₹ 5900 including GST. Find the rate of GST, also find CGST and SGST.
   Solution: Marked price = ₹ 5000
   Let rate of GST = x%
   GST = 5000 x $\frac{x}{100}$ = ₹ 50x
   5000 + 50x = 5900,   50x = 900,   x = 18
   Rate of GST = 18%
   GST = ₹ 5900 - ₹ 5000 = ₹ 900
   CGST = ₹ 450,   SGST = ₹ 450

3. A person's annual income is ₹ 8,00,000, how much he will save in taxes if rate of income tax is reduced from 15% to 12%.
   Solution: Annual Income = ₹ 8,00,000
   Income tax at 15% = ₹ 8,00,000 = ₹ 1,20,000
   Income tax at 12% = ₹ 8,00,000 = ₹ 96,000
   Tax saved = ₹ (1,20,000 - 96,000) = ₹ 24,000

4. A dealer in Bhopal (M.P.). He supplies products and services worth ₹ 5,000 to another dealer in Kanpur (UP). If the rate of GST is 28%, find the tax levied under CGST.
   Solution: Since, the given case in of inter-state transaction as the products/services is supplied from one state to another. CGST = 0.

5. A product is sold from Mandi (Himachal Pradesh) to Haridwar (Uttarakhand) for ₹8,000 and then from Haridwar to Dehradun (Uttarakhand). If the rate of tax under GST system is 18% and the profit made by the dealer in Haridwar is ₹3,000, find net GST payable by the dealer in Haridwar.
   Solution: Cost price in Gwalior = ₹ 8,000
   And profit = ₹ 3,000
   Selling price in Gwalior = ₹ (8000 + 3000)
   $$= ₹ 11,000$$
   CGST = ₹ 11,000 = ₹ 990
   SGST = ₹ 11,000 = ₹ 990
   Net GST paid by the dealer in Gwalior

6. Find the amount if Rs. 10,000 is invested at 10% p.a. for 2 years when compounded annually?
   Solution: We know $A = P(1 + \frac{R}{100})^n$
   From given data P = 10,000
   R = 10%
   n = 2 years
   A =10000(1+ 10/100)²
   A = 10000 x 1.21= Rs. 12100.

7. The CI on a sum of Rs 1000 in 2 years is Rs 440. Find the rate of interest?
   Solution: Given P = 1000
   n = 2 years
   CI = 150
   We know CI = A – P
   440 = A – 1000
   A = 1440
   R = ?
   $1440 = 1000(1 + \frac{R}{100})^2$
   $\frac{1440}{1000} = (1 + \frac{R}{100})^2$
   $\frac{12}{10} = 1 + \frac{R}{100}$
   $\frac{12}{10} - 1 = \frac{R}{100}$
   (12-10)/10 = R/100
   2/10 = R/100
   Rearranging we get the Rate of Interest as 20%.

8. The difference between SI and CI for 2 years at 10% per annum is Rs 15. What is the principal?

Solution: We know the formula Difference = $P(\frac{R}{100})^2$

$15 = P(\frac{10}{100})^2$

$15 = P(\frac{100}{10000})$

$P = 100 \times 15$

Therefore, Principal = Rs 1500

9. A certain sum amounts to $ 7200 in 2 years at 6% per annum compound interest, compounded annually. Find the sum?

Solution: Given Data A = $7200

n = 2 years

R = 6%

Formula to Calculate the Amount $A = P(1 + \frac{R}{100})^n$

$7200 = P(1 + \frac{6}{100})^2$

$7200 = P(\frac{106}{100})^2$

$7200 = P(1.1236)$

$P = \frac{7200}{1.1236}$

= $ 6407

Therefore, the Sum is $6407

10. Find the amount of ordinary annuity of ₹10000payable at the end of each year for 3 years at 10% per year compounded annually.

Solution: R= ₹10000, n = 3, i = 10/100

$$A = R\frac{[(1+i)^n - 1]}{i}$$

$$A = 10000\frac{[(1 + 0.1)3 - 1]}{0.1}$$

$$A = 10000 \times 3.31 = 33100$$

Hence the amount of given annuity is ₹ 33100.

11. Find the present value of ₹ 5000 per annum for 12 years , the interest being 4 % per annum compounded annually. [given = 1.04^{-12} = 0.652

Solution: R = ₹ 5000, n = 12, i = 4/100 = 0.04

$$P = R\frac{[1 - (1 + i)^{-n}]}{i}$$

$$P = 5000\frac{[1 - (1 + 0.04)^{-12}}{0.04}$$

$$P = 5000\frac{[1 - 0.652]}{0.04} = 46850$$

12. A consumer in North Dehli consumes 587 units of electricity in a month. He has a connection of 4 kw. calculate the electricity bill of consumer for that month if the surcharge is ₹ 0.40 per unit for that month, fixed charge is ₹50 / kw and energy tax is nil.

Electricity charge:

Unit	Price
0 – 200	₹ 3
201 – 400	₹ 4.5
401 – 600	₹ 6.5

Solution: number of unit consumed = 587

Connection load = 4 kw

Unit	Price	Amount
0 – 200	₹ 3	600
201 – 400	₹ 4.5	900
401 – 587	₹ 6.5	1215.50
Total		2715.50

Fixed charge = 50 x 4 = 200

Surcharge = 0.40 x 587 = ₹ 234.80

Electricity bill = 2715.50 + 200 + 234.80 = ₹ 3150.30

13. Ravi invests ₹1,000 at the end of every year for 3 years in an annuity that earns 10% per annum compound interest. What is the total amount he will have at the end of 3 years?

Solution: P=1000

r=10%=0.1

$$A=\left(\frac{1000\times[(1+0.10)^3-1]}{0.1}\right)$$

A = 1000 x 0.331/0.1=1000×3.31=₹3,310

14. A shopkeeper buys an article whose printed price is ₹ 4000 from a whole seller at a discount of 20% and sell it to a consumer at the printed price. If the sales are intra-state and the rate of GST is 12%. Then find the price of the article inclusive of GST at which the shopkeeper bought it.

Solution: printed price = ₹ 4000

Discount on printed price =4000 x 20/100=₹ 800

Price of the article for shopkeeper = 4000 – 800 = ₹3200

GST paid by shopkeeper = 3200 x 12/100 = ₹384

Total amount paid by shopkeeper to wholesaler = 3200 + 384 = ₹ 3584.

15. Rahul's annual income is ₹6,00,000. He gets a deduction of ₹50,000 under Section 80C. If the income tax rate is 5% for income between ₹2,50,001 and ₹5,00,000, and 10% for income between ₹5,00,001 and ₹10,00,000, calculate his total income tax.

Solution: Taxable Income=₹6,00,000−₹50,000=₹5,50,000

First ₹2,50,000 → **No tax**

Next ₹2,50,000 (₹2,50,001 to ₹5,00,000) → 5% of ₹2,50,000 = ₹12,500

Remaining ₹50,000 (₹5,00,001 to ₹5,50,000) → 10% of ₹50,000 = ₹5,000

Total Tax=₹12,500+₹5,000=₹17,500

16. Riya earns ₹4,80,000 annually. She claims no deductions. Calculate her income tax if the slab rates are:

₹0 – ₹2,50,000: No tax

₹2,50,001 – ₹5,00,000: 5%

Solution: Taxable income = ₹4,80,000

First ₹2,50,000 → No tax

Next ₹2,30,000 (₹2,50,001 to ₹4,80,000) → 5% of ₹2,30,000 = ₹11,500

Answer: ₹11,500

17. Aman's gross income is ₹7,20,000. He claims ₹1,20,000 under Section 80C. Calculate his income tax using these slabs:

₹0 – ₹2,50,000: No tax

₹2,50,001 – ₹5,00,000: 5%

₹5,00,001 – ₹10,00,000: 10%

Solution: taxable income = ₹7,20,000 − ₹1,20,000 = ₹6,00,000

First ₹2,50,000 → No tax

Next ₹2,50,000 → 5% of ₹2,50,000 = ₹12,500
Remaining ₹1,00,000 → 10% of ₹1,00,000 = ₹10,000
Answer: ₹22,500

18. family consumes 350 units of electricity in a month. The electricity charges are as follows:

₹5 per unit for the first 100 units

₹6 per unit for the next 200 units

₹8 per unit for units above 300

Fixed charge = ₹50 per month

Calculate the total electricity bill for the month.

Solution: Units consumed = 350

Breakdown:

First 100 units → ₹5 × 100 = ₹500

Next 200 units (101 to 300) → ₹6 × 200 = ₹1,200

Remaining 50 units (301 to 350) → ₹8 × 50 = ₹400

Energy Charges = ₹500 + ₹1,200 + ₹400 = ₹2,100

Fixed charge = ₹50

Total Bill=₹2,100+₹50=₹2,150.

19. A household used 25 kilolitres (kL) of water in a month. The water rates are:

₹8 per kL for the first 15 kL

₹10 per kL for the next 10 kL

Fixed charge = ₹100 per month

Calculate the total water bill.

Solution: First 15 kL → ₹8 × 15 = ₹120

Next 10 kL → ₹10 × 10 = ₹100

Water Charges = ₹120 + ₹100 = ₹220

Fixed charge = ₹100

Total Bill=₹220+₹100=₹320

20. A household used 18 LPG gas units in a quarter. The gas company charges:

₹45 per unit for the first 12 units

₹50 per unit for the next 6 units

Fixed quarterly charge = ₹60

Calculate the total gas bill for the quarter.

Solution: First 12 units → ₹45 × 12 = ₹540

Next 6 units → ₹50 × 6 = ₹300

Gas Charges = ₹540 + ₹300 = ₹840

Fixed charge = ₹60

Total Bill=₹840+₹60=₹900

SHORT ANSWER TYPE QUESTIONS

1. A manufacturer in a firm manufactures a machine and marks it at ₹ 80,000. He sells the machine to a wholesaler (in Gorakhpur) at a discount of 20%. The wholesaler sells the machine to a dealer (in Mathura) at a discount of 15% on the marked price. If the rate of GST 28%, find tax paid by the whole saler to central Government.

Solution: Given, Marked price = ₹ 80,000

and discount = ₹ 80,000 x $\frac{20}{100}$

= ₹ 16000

Selling price by the manufacturer = ₹(80,000 - 16,000)

= ₹ 64,000

Cost price of a machine by the wholesale = 64,000

and now selling price by the wholesale = ₹$(80000 - 80000 \times \frac{15}{100})$

$$= ₹ (80,000 - 12,000)$$
$$= ₹ 68,000$$

Tax paid by the wholesaler to the central government

Tax on S.P. - Tax on C.P.= 68,000 x $\frac{28}{100}$ - 64,000 x $\frac{28}{100}$)

$$= (68,000 - 64,000) \times \frac{28}{100}$$
$$= 4,000 \times \frac{28}{100}$$
$$= ₹ 1120.$$

2. A shopkeeper sells an article at the listed price of ₹1500. The rate of GST on the article is 18%. If the sales are intra-state and the shopkeeper pays a tax (under GST) of ₹27 to the Central Government, find the amount inclusive of tax at which the shopkeeper purchased the article from the wholesaler.

Solution: Sales are intra-state and the rate of GST is 18%, so GST comprises CGST at 9% and   SGST at 9%

Let the shopkeeper buy the article from the wholesaler for ₹x (excluding tax)

Amount of GST paid by the shopkeeper to wholesaler:

CGST = 9% of ₹x = ₹ $\frac{9x}{100}$,

SGST = 9% of ₹x = ₹ $\frac{9x}{100}$

Input CGST of the shopkeeper = ₹ $\frac{9x}{100}$

The shopkeeper sells the article at the list price of ₹1500

Amount of GST collected by the shopkeeper:

CGST = 9% of ₹1500 = ₹ (1500 x 9/100) = ₹135,

SGST = 9% of ₹1500 = ₹135

Output CGST of the shopkeeper = ₹135

The amount of tax (under GST) paid by the shopkeeper to the Central Government

Output CGST - input CGST = ₹135 - ₹ $\frac{9x}{100}$

According to given, $135 - \frac{9x}{100} = 27$

$\frac{9x}{100} = 108$

x = 1200

Thus, the shopkeeper purchased the article from the wholesaler for ₹1200

CGST = 9% of ₹1200 = ₹108,

SGST = 9% of ₹1200 = ₹108

The amount inclusive of tax (under GST) at which the shopkeeper purchased the article from the wholesaler

Cost of the article to the shopkeeper + GST paid by the shopkeeper

$$= ₹1200 + ₹108 + ₹108 = ₹1416$$

3. A shopkeeper in Haryana buys an article from a wholesaler in Delhi at printed price of ₹ 60,000. The shopkeeper in Haryana sells the article to a consumer in Haryana at a profit of 20% on the basic cost price. If the rate of GST is 18%, find
    i. the amount paid by the shopkeeper to the wholesaler.

ii. the GST paid by the shopkeeper to the wholesaler.

iii. the amount paid by the consumer.

Solution: i. As the shopkeeper in Haryana buys an article for ₹ 60,000 from a wholesaler in Delhi, so the sale is inter-state sale on which

IGST is levied at 18%

IGST collected by the wholesaler = 18% of ₹ 60,000

=60000 x 18/100 = ₹ 10,800

Amount paid by the shopkeeper to the wholesaler = ₹ 60,000 + ₹ 10,800

= ₹ 70,800

ii. Input GST of the shopkeeper = ₹ 10,800

The shopkeeper sells the article to the consumer in Haryana at a profit of 20% on the basic cost price.

Selling price of the shopkeeper = ₹ $(60,000 + 60,000 \times \frac{20}{100})$ = ₹ 72,000

This sale is intra-state sale and the rate of GST is 18%

GST paid by the consumer = 18% of ₹ 72,000 = ₹ 72,000 x $\frac{18}{100}$ = ₹ 12,960

Output GST of the shopkeeper = ₹ 12,960.

GST paid by the shopkeeper = Output GST - Input GST

= ₹ 12,960 - ₹ 10,800 = ₹ 2160

iii. Amount paid by the consumer = Selling price of the shopkeeper + GST

= ₹ 72,000 + ₹ 12,960 = ₹ 84,960

4. A shopkeeper buys an article whose list price is ₹4500 at some rate of discount from a wholesaler. He sells the article to a consumer at the list price and charges GST at the rate of 12%. If the sales are intra-state and the shopkeeper has to pay tax (under GST) of ₹27, to the State Government, find the rate of discount at which he bought the article from the wholesaler.

Solution: The sales are intra-state and the rate of GST is 12%, so GST comprises of CGST at 6% and SGST at 6%

Let the amount of discount be ₹x

Selling price of the article (excluding tax) by the wholesaler

= listed price - discount = ₹(4500 - x)

Amount of GST collected by the wholesaler from the shopkeeper:

CGST = 6% of ₹(4500 - x) = ₹$\frac{6}{100}$x (4500 - x)

= ₹$(270 - \frac{3x}{50})$,

SGST = 6% of ₹(4500 - x) = ₹$(270 - \frac{3x}{50})$

Input GST of shopkeeper:

Input CGST = ₹$(270 - -\frac{3x}{50})$, input SGST = ₹$(270 - -\frac{3x}{50})$

The shopkeeper sells the article to a consumer at the list price i.e. for ₹4500

Amount of GST collected by the shopkeeper from the consumer:

CGST = 6% of ₹4500 = ₹$(\frac{6}{100} \times 4500)$ = ₹270,

SGST = 6% of ₹4500 = ₹270

Output GST of shopkeeper:

Output CGST = ₹270, output SGST = ₹270

The amount of tax (under GST) paid by the shopkeeper to the State Government

Output SGST - input SGST

₹270 - ₹(270 - $\frac{3x}{50}$) = ₹ $\frac{3x}{50}$

According to given, $\frac{3x}{50}$ = 27

x = 450

The amount of discount = ₹450

Thus, the shopkeeper gets a discount of ₹450 from the wholesaler

Rate of discount = ($\frac{450 \times 100}{4500}$) = 10%

$\qquad$ = 10%

5. If income tax is increased by 9%, the net income is reduced by 1%. Find the rate of income tax.

Solution: Income tax is increased by 9% net income is reduced by 1%

$$\frac{income\ tax\ x\ 9}{100} = \frac{net\ income\ x\ 1}{100}$$

9 income tax = Net income = x say

Also, income = Net income + income tax

= x + $\frac{x}{9}$ = $\frac{10x}{9}$

Rate of income tax: ($\frac{x}{9} \div \frac{10x}{9}$)x 100 = 10%

6. Arjun wants to buy a bike costing ₹60,000. He decides to save by depositing a fixed amount annually into a fund that gives 8% interest compounded annually. He plans to buy the bike after 4 years. How much should he deposit every year to accumulate ₹60,000 at the end of 4 years?

Solution: Given: $\quad$ A = ₹60,000, $\quad$ i = $\frac{8}{100}$ = 0.08, $\quad$ n = 4 years

$$A = P\frac{[(1+i)^n - 1]}{i}$$

$$60000 = P\ \frac{([(1+0.08)^4 - 1])}{0.08}$$

$$P = \frac{60000 \times 0.08}{(1.08)^4 - 1)}$$

$$P = \frac{4800}{1.3605 - 1}$$

$$P = ₹13,312.72$$

7. A company offers a retirement pension of ₹25,000 per year for 6 years after an employee retires. The interest rate is 6% per annum compounded annually. What is the present value of this pension plan (i.e., how much does thae company need to set aside today)? Given = $(1.06)^{-6}$ = 0.7050

Solution: R=25,000, i=0.06, n= 6

$$PV = R\frac{\times[1-(1+i)^{-n}]}{i}$$

$$PV = \frac{25000 \times [1-(1.06)^{-6}]}{0.06}$$

$$PV = \frac{25000 \times (1-0.7050)}{0.06}$$

$$PV = \frac{25000 \times 0.2950}{0.06} = 25000 \times 4.9167 = ₹1,22,917.50$$

8. What equal payment made at the beginning of each year for 10 years will pay for a piece of land priced at ₹ 400000, if money worth is 7% per annum compounded annually.

Solution: Given, $i = \frac{7}{100} = 0.07$, n = 10,   P= ₹400000

We have to calculate the amount of each instalment R

For annuity due, present value

$P = R (1 + i)\frac{[1 - (1 + i)^{-n}]}{i}$

$400000 = R(1 + 0.07)\frac{[1 - 1.07^{-10}]}{0.07}$ ....(i)

Let $x = 1.07^{-10}$

Taking log both sides, we get

Log x = -10log 1.07 = -10 x 0.0294 = -0.294

x = antilog (-0.294) = antilog($\bar{1}$.7060)

x = 0.5082

substituting the value of x in (i)

$400000 = R \times 1.07 \times \frac{[1 - 0.5082]}{0.07}$

R = 400000 x 0.07 /(1.07 x 0.4918) = 53209.07

9. A sum of ₹3254 is borrowed from a money lender at 6% per annum compounded annually. If this amount is to be paid in 4 equal instalments, find the annual instalment.

Solution: here loan amount = ₹3254

Solution: Let the yearly instalment be ₹ x

$3254 = x [\frac{100}{100+6} + (\frac{100}{100+6})^2 + (\frac{100}{100+6})^3 + (\frac{100}{100+6})^4]$

$x = \frac{3254 \ x \ 53^4}{(50 \ x \ 103 \ x \ 5309)}$

x = ₹1017

10. A sum of ₹ 25000 invested at 8% p.a compounded semiannually amounts to ₹ 28121.60. compute the time period of investment.

Solution: P = ₹25000, n = ?, A = 28121.60 , r = rate of interest for conversion period

r = 4%

$28121.60 = 25000(1 + \frac{4}{100})^4$

$\frac{28121.60}{25000} = (\frac{26}{25})^n = \frac{17576}{15625} = (\frac{26}{25})^3$ n = 3

LONG ANSWER TYPE QUESTIONS

1. A shopkeeper buys an article whose list price is ₹8000 at some rate of discount from a wholesaler. He sells the article to a consumer at the list price. The sales are intra-state and the rate of GST is 18%. If the shopkeeper pay a tax (under GST) of ₹72 to the State Government, find the rate of discount at which he bought the article from the wholesaler.

Solution: Given:

List of prices of an article = ₹8000

Let the rate of discount given by wholesaler = x%

So,

Discount = x% of ₹8000

       $= \frac{x}{100}$ x ₹8000

     = ₹80x

CP of an article for shopkeeper = ₹8000 – ₹80x

It is given that, CP of article for consumer = ₹8000

Since the sales are intra-state, rate of GST = 18%

CGST = SGST = 9%

Amount of GST paid by shopkeeper to wholesaler,

SGST = CGST = 9% of [₹8000 – ₹80x]

$$= \frac{9}{100} [₹8000 - ₹80x]$$

Amount of GST paid by consumer to shopkeeper,

CGST = SGST = 9% of ₹8000

$$= \frac{9}{100} \times ₹8000 = ₹720$$

$$= (\frac{9}{100}) ₹8000 = ₹720$$

So, the tax paid by shopkeeper to state government = ₹720 - $\frac{9}{100}$ x [₹8000 – ₹80x]

Also, tax paid by shopkeeper to state government = ₹72

₹72 = 720 - (9 x $\frac{80}{100}$) (100 - x)

720 - 72 = $\frac{720}{100}$ (100 - x)

648 = $\frac{720}{100}$ (100 - x)

100 - x = 90

x = 100 - 90 = 10

Hence, the required rate of discount = 10%

2. In financial year 2019-20, the annual income of Mr. Bhasin (age 48 years) was ₹ 9,50,000 (exclusive of HRA). He deposited ₹ 9200 per month in NPS and ₹ 29000 as LIC premium. He paid ₹ 42000 as tuition fee of his two children. He purchased a Mediclaim policy of ₹ 18400. He paid ₹ 1,09,000 as interest and ₹ 18,000 as principal on home loan. Calculate the income tax paid by Mr. Bhasin at the end of the financial year.

Solution: Gross income = ₹ 9,50,000

Less standard deduction = - ₹ 50,000

Balance = ₹ 9,00,000

Deductions under sec. 80 C

Contribution in NPS = ₹ 9200 x 12 = 110400

Contribution in LIC = 29000

Tuition fees = 42000

Principal on home loan = 18000

Total = 1,99400

But maximum admissible is ₹ 1,50,000, therefore = 900000 - ₹ 1,50,000

Balance = ₹ 7,50,000

Deductions under section 24

Interest on home loan = ₹ 1,09,000

So rebate = ₹ 7,50,000 - ₹ 1,09,000

Balance = ₹ 6,41,000

Deductions under section 80D

Mediclaim policy premium ₹ 18400

Balance ₹ 6,22,600

Taxable income ₹ 6,22,600

From income tax slab,

income tax = ₹ 12500 + 20% of ₹ 122600

= ₹ 12500 + ₹ 24520 = ₹ 37020

Health and Education cess = 4% of ₹ 37020 = ₹ 1481

Net income tax = ₹ 37020 + ₹ 1481 = ₹ 38501

3. A manufacturer sells a T.V. to a dealer for ₹ 18000 and the dealer sells it to a consumer at a profit of ₹ 1500. If the sales are intra-state and the rate of GST is 12%, find:

i. the amount of GST paid by the dealer to the State Government.

ii. the amount of GST received by the Central Government.

iii. the amount of GST received by the State Government.

iv. the amount that the consumer pays for the T.V.

solution: It is a case of intra-state transaction of goods and services

$$SGST = CGST = \frac{GST}{2}$$

Given:

Manufacturer sells T.V. to a dealer = ₹ 18000

Amount of GST collected by manufacturer from dealer

CGST - SGST = 6% of 18000

$$= \frac{6}{100} \times 18000$$

$$= ₹ 1080$$

So, Manufacturer will pay ₹ 1080 as CGST and ₹1080 as SGST

CP of a TV for dealer = ₹ 18000

Profit = ₹ 1500

SP of a TV for dealer to customer - CP + Profit = ₹ 18000 + ₹ 1500 = ₹ 19500

Amount of GST collected by dealer from customer,

CGST = SGST = 6% of ₹ 19500

$$= \frac{6}{100} \times 19500$$

$$= ₹ 1170$$

i. Amount of GST paid by the dealer to the State Government

₹ 1170 - ₹ 1080 = ₹ 90

ii. Amount of GST received by the Central Government

CGST paid by manufacturer + CGST paid by dealer = ₹ 1080 + ₹ 90 = ₹ 1170

iii. Amount of GST received by the State Government

SGST paid by manufacturer + SGST paid by dealer = ₹ 1080 + ₹ 90 = ₹ 1170

iv. Amount that the consumer pays for the T.V.

CP of TV + CGST paid by customer + SGST paid by customer

= ₹ 19500 + ₹ 1170 + ₹ 1170 = ₹ 21840

4. A person deposits ₹50,000 in a bank that offers compound interest compounded annually at the rate of **8% per annum**. He does not withdraw any amount from the account for **4 years**. After 4 years, he decides to deposit an additional ₹20,000 into the account (without withdrawing the original amount), and the interest rate is revised to **10% per annum**, compounded annually. He leaves the entire amount (₹50,000 + ₹20,000 plus accumulated interest) in the bank for **3 more years**.

Calculate:

i. The amount in the account at the end of the first 4 years.

ii.   The total amount in the account at the end of 7 years (i.e., after the next 3 years).

iii.   The total compound interest earned over the entire 7-year period

Solution: Part 1: Amount after the first 4 years

Principal (**P₁**) = ₹50,000

Rate (R₁) = 8% per annum

Time (T₁) = 4 years

We use the compound interest formula:

$$A=P(1+\frac{R}{100})^T$$

$$A1=50000(1+\frac{8}{100})^4=50000\times(1.08)^4$$

First, calculate $(1.08)^4$

$(1.08)^4=1.08\times1.08\times1.08\times1.08=1.3605$ (approx)

$A1=50000\times1.3605=₹68,025$

Amount after 4 years = ₹68,025

Part 2: Amount after next 3 years (with additional ₹20,000)

Now:

Initial amount = ₹68,025

Additional deposit = ₹20,000

New principal for next 3 years = ₹68,025 + ₹20,000 = ₹88,025

New rate (R₂) = 10%

Time (T₂) = 3 years

$$A_2=88025(1+\frac{10}{100})^3=88025\times(1.10)^3$$

$A_2=88025\times1.331=₹117,231.28$

Amount after 7 years = ₹117,231.28

5. Ritika decides to save for her future studies by depositing ₹5,000 at the end of every year into a savings account. The bank offers an interest rate of 6% per annum, compounded annually. She continues making these annual deposits for 10 years. After 10 years, she stops depositing further, but leaves the money in the account for another 5 years, during which the interest continues to be compounded at the same rate of 6% per annum.

You are required to calculate:

i.   The amount accumulated in the account at the end of 10 years.

ii.   The total amount in the account after the next 5 years (i.e., at the end of 15 years).

iii.   The total interest earned over the entire 15-year period.

Solution: Ritika deposits a fixed amount at the end of each year, which makes this an ordinary annuity.

Given:

Annual payment (PPP) = ₹5,000

Interest rate (r) = 6% = 0.06

Number of years (n) = 10

$$A =P\frac{[(1+r)^n-1]}{i}/r = 5000\times[(1+0.06)^{10}-1]/0.06$$

$(1.06)10=1.7908$ (approx)

$A =5000\times(1.7908-1)/0.06=5000\times0.7908/0.06$

$A= 5000\times13.18=₹65,900$

Amount in account after 10 years = ₹65,900

Now, this amount is left untouched for 5 more years, and it earns compound interest at 6% per annum.

Formula for Compound Amount**:**
$A = P \times (1+r)^t$
$A = 65900 \times (1.06)^5$
$(1.06)5 = 1.3382$ (approx)
$A = 65900 \times 1.3382 = ₹88,183.34$
Amount after 15 years = ₹88,183.34

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CASE BASED TYPE QUESTIONS
Read the following taxt carefully and answer the questions that follow:
</div>

1. In financial year 2019-20 Mr Narendra Kumar Mani's (age 55 years) income from salary was ₹ 100000 (exclusive of HRA) and income from interest on saving account was ₹ 18600. He deposited ₹ 15000 per month in GPF and paid ₹ 57000 as LIC premium. He donated ₹ 100000 in Prime Minister's National relief fund. He paid ₹ 31600 as interest on education loan for higher studies of his daughter. He also paid ₹ 212500 as interest on home loan and ₹ 63450 as principal of home loan.

    Income Tax Slab for Financial Year 2019-20
    (For individual tax payers below the age of 60 years)
    Taxable Income Income Tax
    Up to ₹ 250000 NIL
    ₹ 250001 to ₹5000000 5% of taxable income exceeding ₹ 250000
    ₹ 500001 to ₹ 1000000 ₹ 12500 + 20% of taxable income exceeding ₹ 500000
    Above ₹ 1000000 ₹ 1,12500 + 30% of taxable income exceeding ₹ 1000000.

    i. Find the total amount invested or paid by Mr. Mani under Section 80C.   (1)
    ii. Find the taxable income of Mr. Mani.   (1)
    iii. Find the income tax paid by Mr. Mani for F.Y. 2019-20.   (2)

    OR

    If Mr. Mani purchased a health insurance policy for himself and his family worth   (2)
    ₹ 30000, then find the income tax paid by Mr. Mani for F.Y. 2019-20.

    Read the following text carefully and answer the questions that follow:
    Solution:

(i) GPF = ₹ 15000 x 12 = ₹ 180000
    LIC   = ₹ 57000
        Principal of home loan   = ₹ 63450
            Total   = ₹ 300450

(ii) Gross income = ₹ 1500000 + ₹ 18600 = ₹ 1518600
    Less standard deduction   = - ₹ 50000
        Balance   = ₹ 1468600
    Deductions under 80C   = - ₹ 150000
    Balance   =   ₹ 1318600
    Deductions under section 24   =   - ₹ 10000
                         =   ₹ 1308600
    Deductions under sections 80G   =   - ₹ 200000
                         =   ₹ 11,08,600
    Deduction under section 80E   =   - ₹ 31,600
                         =   ₹ 977000

Taxable income is ₹ 977000.

(iii)   Income tax = ₹ 12500 + 20% of ₹ 477000 = ₹ 12500 + ₹ 95400 = ₹ 107900. Health and Education cess = 4% of ₹ 107900 = ₹ 4316.

Net income tax = ₹ 107900 + ₹ 4316 = ₹ 112216.

<center>OR</center>

Mr. Mani had purchased a health insurance policy for himself of ₹ 30,000. But admissible deduction against it is of ₹ 25000.

Taxable income becomes = ₹ 977000 - ₹ 25000 = ₹ 952000.

Income tax = ₹ 12500 + 20% of ₹ 4,52,000 = ₹ 12500 + ₹ 90400 = ₹ 102900. Health and Education cess = 4% of ₹ 102900 = ₹ 4116.

Net income tax = ₹ 102900 + ₹ 4116 = ₹ 107016.

2. In XI standard, teacher was giving lecture on GST topic. Following points were discussed on this topic.

Goods and Services Tax (GST) GST is known as the Goods and Services Tax. It is an indirect tax which replaced many indirect taxes in India such as the excise duty, VAT, services tax, etc. The Goods and Services Tax Act was passed in the Parliament on 29 March 2017 and came into effect on 1 July 2017. In other words, Goods and Services Tax (GST) is levied on the supply of goods and services. Goods and Services Tax Law in India is a comprehensive, multi-stage, destination-based tax that is levied on every value addition. GST is a single domestic indirect tax law for the entire country. In order to address the complex system in India, the Government introduced 4 types of GST which are given below.

i. CGST (Central Goods and Services Tax): levied and collected by Central Government.

ii. SGST (State Goods and Services Tax): levied and collected by State Governments/Union Territories with Legislatures.

iii. UTGST (Union Territory Goods and Services Tax): levied and collected by Union Territories without Legislatures, on intra-state supplies of taxable goods and/or services.

iv. IGST (Integrated Goods and Services Tax): Inter-state supplies of taxable goods and/or services are subject to Integrated Goods and Services Tax (IGST). IGST is the total sum of CGST and SGST/ UTGST and is levied by Centre on all inter-state supplies.

Intra-state means: Supply within the same state. In case of intra-state sale of goods/services, or both If GST rate is 18%, then

CGST = 9% of Sale price

SGST = 9% of Sale price

IGST = 0

Inter-state means: Supply from one state to another state.

In case of inter-state of goods or services or both If GST rate is 18%, then

IGST = 18% of Sale price

Discount is never allowed on amount including GST.

i. When Integrated Goods and Services Tax is applicable?

ii. When SGST is applicable?

iii. Tax levied on sales of goods by business man B in M.P. only, It will come under which GST?

<center>OR</center>

If business man A runs his business and sells his product in Maharashtra then which tax will be applicable on the product.

solution:

(i) Integrated Goods and Services Tax is applicable when there is inter state supply.

(ii) State Goods and Services Tax is applicable when goods are sold within a state.

(iii) Since business B sell his product in Madhya Pradesh only, therefore SGST is applicable.

OR

Business man A operates in Maharashtra and if he sells his product in Maharashtra only, then SGST is applicable.

3. Amara is planning for her retirement. She wants to ensure she has enough money to live comfortably once she stops working at age 60. She plans to invest in a retirement annuity where she will deposit $500 at the end of every month into a fund that earns an interest rate of 6% compounded monthly.

She plans to invest in this annuity for 25 years, starting at age 35 and continuing until she turns 60.

Using the case study above, answer the following:

i) Identify the type of annuity Amara is using and explain why it fits that type.

ii) Calculate how much money Amara will have in her annuity account at the time of retirement.   Given $(1.005)^{300} = 4.467744$

iii) Explain how increasing the interest rate or duration would impact her retirement savings.

Solution

i) Amara is using an ordinary annuity because she makes regular payments at the end of each month. In an ordinary annuity, payments are made at the end of each compounding period (in this case, monthly), and interest is applied accordingly.
Ordinary Annuity formula:

ii) $A = P \times [(1+r)^n - 1]/r)$
Where:
$n = 25 \times 12 = 300$(total number of payments
$P = 500$, $r = 0.06/12 = 0.005$ (monthly interest rate),
$A = 500 \times [(1+0.005)^{300} - 1]/0.005$
$A = 500 \times (4.467744 - 1)/0.005$
$= 500 \times 3.467744/0.005$
$A \approx 346{,}774.40$

Answer: At retirement, Amara will have approximately **$346,774.40** in her annuity account.

iii) Impact of Increasing Interest Rate or Duration
Answer:

Increasing the interest rate: If the interest rate is higher, the money earns more interest each month. This leads to greater compounding, so Amara's final amount would be significantly higher.

Increasing the duration: If Amara starts saving earlier or retires later (e.g. saving for 30 years instead of 25), she would make more contributions and benefit from more compounding periods, increasing the final amount as well.

1. In what time will be a sum of ₹1562.50 produce ₹195.10 at rate 4% per annum compounded interest
   (a) $1\frac{1}{2}$ years (b) 2 years (c) $2\frac{1}{2}$ years (d) 3years

2. At what rate % per annum will a sum of ₹1200 become ₹13230 in 2 years?
   (a) 5% (b) 5.5% (c) 6% (d) 6.5%

3. If ₹8000 is invested at 5% p.a. simple interest for 4 years, what will be the total amount received at the end?
   (a) ₹9600 (b) ₹8800 (c) ₹9600 (d) ₹10400

4 The compound interest on a sum for 2 years is ₹264. The simple interest on the same sum for the same time at the same rate is ₹240. What is the principal
   (a) ₹1200 (b) ₹1500 (c) ₹2000 (d) ₹2400

5. at what rate percent per annum will ₹5000 amount to ₹6050 in 2 years when the interest is compounded annually?
   (a) 8% (b) 10% (c) 12% (d) 9%

6. A sum doubles itself in 5 years at compound interest. In how many years will it become 4 times itself?
   (a) 10 years (b) 7.5 years (c) 15 years (d) 20 years

7. What is the future value of an ordinary annuity of ₹2000 per year for 3 years at 10% p.a. compounded annually?
   a) ₹6000 b) ₹6620 c) ₹7260 d) ₹6600

8. If you deposit ₹500 every year for 4 years in an ordinary annuity at 8% p.a., what will be the future value?
   a) ₹2160 b) ₹2249.28 c) ₹2020 d) ₹2500

9. A person deposits ₹3000 every year for 5 years in an annuity due at 12% p.a. What is the future value ?
   ` a) ₹19,025.57 b) ₹20,948.63 c) ₹21,000.00 d) ₹18,900.1

10. A shopkeeper sells a bag for ₹1,000. If GST is 18%, what is the price the customer pays?
   a) ₹1,118 b) ₹1,080 c) ₹1,180 d) ₹1,800

ANSWERS

1. Answer: d   **2.** Answer: a   3. Answer: d   4. Answer: c   **5.** Answer: b
6. Answer: a   7. Answer: a   8. Answer: b   9. Answer: b   10. Answer: c

ASSERATION AND REASON QUESTIONS

These questions are Assertion and Reason based question. Two statements are given, one labelled Assertion (A) and other labelled Reason ®. Select the correct answer from the codes (A), (B), (C) and (D) as given below:
 (a) Both A and R are true, and R is the correct explanation of A
(b) Both A and R are true, but R is not the correct explanation of A
(c) A is false, but R is true
(d) Both A and R are false

1. **Assertion (A):** Interest earned on savings bank accounts is fully exempt from income tax.
   Reason (R): Section 80TTA allows deduction up to ₹10,000 on interest from savings accounts.

2. **Assertion (A):** A person can claim both HRA exemption and deduction on home loan interest under Section 24.

Reason (R): The Income Tax Act allows both benefits if the taxpayer lives in one city and owns a house in another.

3. **Assertion (A):** The consumer number mentioned in the utility bill helps uniquely identify the user.

   Reason (R): Every customer is assigned a unique consumer ID by the utility provider.

4. **Assertion (A):** The total amount in a utility bill may include fixed charges, usage charges, and taxes.

   Reason (R): Utility providers use a structured billing system to itemize the charges clearly.

5. **Assertion (A):** The amount after 2 years on ₹5,000 at 10% p.a. compounded yearly is ₹6,050.

   Reason (R): Compound interest for 2 years at 10% is ₹1,050.

<div align="center">

ANSWERS

1. Answer: C   2. Answer: A   3. Answer: A   4. Answer: A   **5.** Answer: A

VERY SHORT ANSWER TYPE QUESTIONS

</div>

1. An investment of ₹ 25000earns investment at 9%, compounded annually. What will be the value of investment after 7 years?
2. Money invested in Kisan Vikas Patra double in 6 years. Find the rate of investment.
3. What sum must be invested at the end of each year to provide the fund for the replacement of machine costing ₹ 8000 at the end of 3 years, if the money worth is 5%. Given $(1.05)^3$ =1.1576
4. A shopkeeper sells a shirt for ₹1,000. If the GST rate is 12%, find the amount of GST and the final price paid by the customer.
5. A laptop is sold for ₹40,000 with a GST of 18%. Find (a) GST amount (b) Final price
6. A trader sells a product for ₹5,000 and collects GST of ₹600.
7. Riya deposits ₹2,000 at the end of every year for 3 years in a scheme that gives 5% annual interest, compounded annually. What is the total value of the annuity at the end of 3 years?
8. A person has a taxable income of ₹6,00,000. The income tax slab is: No tax up to ₹2,50,000 5% on next ₹2,50,000 10% on the remaining income. Calculate the total income tax payable.
9. A shopkeeper offers a 10% discount on a product marked ₹5,000. GST is 18%. Should the GST be calculated on the marked price or the discounted price? Also, find the final price paid.
10. A person invests ₹10,000 at 10% per annum for 2 years. Which will earn more: simple interest or compound interest? Calculate the difference.

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ANSWERS

</div>

1 Answer the value of investment after 7 years ₹45680 approximately.

2 Answer 12.3%   3. answer = ₹ 2538.07   4. Answer ₹1,120
5. Answer: ₹7,200 and ₹47,200   6. Answer 12%   7. Answer ₹6,305

8. Answer Income Tax Payable = ₹22,500   9. Answer = ₹5,310
10. Answer =CI earns **₹100 more** than SI over 2 years.

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SHORT ANSWER TYPE QUESTIONS

</div>

1. A man borrows₹ 15000 at 14% per annum compound interest. If he repays ₹ 4300 at the end of the first year and ₹ 5220 at the end of the second year, find the amount of loan outstanding at the beginning of the third year.

2. Find the present value of ₹ 25,000 due 10 years hence when the interest of 8% is compounded:
    i) annually
    ii) semi - annually

3. A sum of money doubles itself in 7 years compound interest. In how many years it becomes four times?

4. A family in Jaipur, Rajasthan consumes 2950 units of electricity in a month. It has a connected load of 11 kW. Calculate the electricity bill for the month if no surcharge is applicable. The energy tax is NIL and the energy duty is ₹ 0.40 per unit. The tariff plan is as given below:

No. of Unit/month:	0-50	51- 150	151-300	301-500	>500
Price	₹4.75	₹ 6.50	₹ 7.35	₹ 7.65	₹ 7.95
Fixed Charges per month:	₹ 230	₹ 230	₹ 275	₹ 345	₹ 400

5. Sheela purchased a land Paying ₹ 200000 and agree to pay an equal amount at the end of each year for 3 years. If the money is worth 8%, then find the amount of each instalment.

6. A customer buys a refrigerator listed at ₹30,000. The shopkeeper offers a discount of 10% on the listed price and then applies 18% GST on the discounted price.
   Critically analyse whether the GST charged benefits the customer or the government more, and calculate the final amount the customer has to pay. Justify your reasoning.

ANSWERS

1. ₹ 9372   2. (i) ₹11,579   (ii) ₹11,409.67

3. In 14 year money will become four times.

4 Electricity bill = Fixed charged + Energy charges + Energy duty
          = ₹400 + ₹22997.50 + ₹1180 = ₹ 24577.50

5. ₹ 77594.56

6. While the customer benefits from the discount, GST is still calculated on the post-discount price, ensuring the government earns tax on the actual transaction. Hence, both benefit, but the government consistently receives tax revenue.
Final amount 31,860

LONG ANSWER TYPE QUESTIONS

1. Mehul deposits ₹ 5000 at the end of each year into an account earning 6% interest compounded annually. How much he will get at the end of 3 years? [Use $(1.06)^3 = 1.191$]

2. A sum of ₹ 32,800 is borrowed at 5% p.a compounded annually to be paid back in 2 years in two equal installments. Find the annual payment. [Use $(1.05)^{-2} = 0.90702$]

3. Riya invests ₹20,000 in a fixed deposit that offers an interest rate of 8% per annum compounded annually.
   (a) Calculate the amount and compound interest she will receive after 3 years.
   (b) If the interest was compounded half-yearly, what would be the amount and compound interest after 3 years?

ANSWERS

1. ₹ 15,916   2. ₹ 17,639   3. (a) A≈₹25,194.24   C.I =₹5,194.24
3(b) A≈₹25,306.38,   C.I = ₹5,306.38

CASE STUDY BASED QUESTIONS

1. These days when we get bills of purchase we notice an entry of GST, some friends didn't know much about GST but from different sources they learnt a lot and try to solve a situation given as:

   A manufacturer listed the price of his articles at ₹ 3200 per article. He allowed a discount of 25% to the wholesaler, who in turn allowed a discount of 20% to a retailer. The retailer sells to the consumer at a discount of 5%. If all the sells are inter - state with GST rate 5%.

   (I) The price per article inclusive of GST which the wholesaler pays will be?
   (II) The price per article inclusive of GST which is how much retailer pays?
   (III) The price which consumer pays for the article will be?

   OR

   Find the SGST paid by whole seller?

2. In XI standard, teacher was discussing the concept of Future Value of Annuity Regular. During his class, he discussed the following few points on this: ₹ 500 was invested in annuity (future value regular) for 10 years at the rate of 14% compounded annually.

   If C.F. be the periodic payments, (cash flow in each period), the future value given by:

   Future value (F.V) of an Annuity= $C.F\left[\frac{(1+i)^n-1}{i}\right]$

   i) What will be the future value of annuity at the end of two years? Given that $(1.14)^2 = 1.2996$
   ii) What will be the future value of the annuity after 3 years? Given that $(1.14)^3 = 1.4815$
   iii) What is the approximate difference between the future value of the annuity for five years and three years? Given that $(1.14)^5 = 1.925$

   OR

   What is the difference between the profit earned by future value of the annuity for two years and the ₹ 1000 invested compounded for two years with the same interest rate?

ANSWERS

1 (i) ₹ 2520   (ii) ₹ 2688   (iii) ₹ 3192   Or ₹ 4

2 (i) ₹ 1070   (ii) ₹ 1719.64   (iii) ₹1585.93 Or ₹ 229.6

UNIT – VIII : CO-ORDINATE GEOMETRY

8.1	Straight line	• Find the slope and equation of line in various form • Find angle between the two lines • Find the perpendicular distance of a given points from a line • Find the distance between two parallel lines	• Gradient of a line • Equation of line: Parallel to axes, point slope form, two points form, slope intercept form, intercept form • Application of the straight line in demand curve related to economics problems
8.2	Circle	• Define a circle • Find different form of equations of a circle • Solve problems based on applications of circle	• Circle as locus of a point in a plane • Equation of a circle in standard form, central form, diameter form and general form.
8.3	Parabola	• Define parabola and related terms	• Parabola as a locus of a point in a plane. • Equation of a parabola in standard form

STRAIGHT LINES

1. **Distance formula**: Distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

   PQ= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally, in the ratio m:n is $(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n})$

3. The coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) externally, in the ratio m:n is $(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n})$.

4. The coordinates of the mid point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2})$.

5. Area of the triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

6. The angle θ made by the line 'l' with positive direction of x-axis and measured anticlockwise is called the inclination of the line. Obviously $0° \leq θ \leq 180°$.

7. If θ is the inclination of a line l, then tan θ is called the slope or gradient of the line l.. The slope of a line is denoted by m. Thus, m = tan θ, θ ≠ 90°. The slope of a line whose inclination is 90° is not defined

8. The slope of the x-axis is zero and slope of the y-axis is not defined.

9. The slope m, of the line through the points (x_1, y_1) and (x_2, y_2) is given by m = $\frac{y_2-y_1}{x_2-x_1}$

10. The acute angle θ between two lines with slopes m_1 and m_2 is tan θ = $\left|\frac{m_2-m_1}{1+m_1 m_2}\right|$, $m_1 m_2 \neq 0$.

11. Two lines are parallel if and only if their slopes are equal.

12. Two lines are perpendicular if and only if product of their slopes $= -1$.

13. Equation of the horizontal line having distance 'a' from x axis is either y=a or y =−a.

14. Equation of the vertical line having distance' b' from y axis is either x= b or x =−b.

    The equation of a straight line passing through the fixed point (x_1, y_1) and having slope m is $(y - y_1) = m(x - x_1)$.

15. The equation of straight line passing through origin and having slope m is y = mx

16. Equation of the line with slope 'm' and y- intercept, 'c' is y = mx +c

17. Equation of the line with slope' m' and x- intercept 'd'. is y = m(x-d)

18. Equation of the line through the points (x_1, y_1) and (x_2, y_2) is given by

    $(y - y_1) = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$.

19. Equation of a line making intercepts a and b on the x -axis and y -axis respectively, is

$$\frac{x}{a} + \frac{y}{b} = 1$$

20. The general equation of the line is Ax +By +C = 0 , where A, B, C are real numbers and at least one of A and B is non zero.

21. The perpendicular distance 'd' of a point $P(x_1, y_1)$ from the line ax+by+c = 0 is

    d = $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

22. The distance between two parallel lines ax+by+c_1=0 and ax+by+c_2=0 is $\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$.

CIRCLE:

23. A circle is the set of all points in a plane, each of which is at a constant distance from a fixed point in the plane, the fixed point is called the centre and the constant distance is called radius.

24. The equation of circle with centre (0,0) and radius r is $x^2 + y^2 = r^2$

25. Equation of circle with centre (h,k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$

26. Equation of the circle with $A(x_1, y_1)$ and $B(x_2, y_2)$ as extremities of a diameter is

    $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

27. The General form of equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0, g^2 + f^2 - c > 0$

    Its centre is (-g,-f) and radius =$\sqrt{g^2 + f^2 - c}$

PARABOLA

28. A parabola is the set of all points in plane which are equidistance from a fixed line and fixed point (not on the line) in the plane.

29. The fixed line is called the directrix of the parabola and the fixed point is called the focus of the parabola.

30. The line passing through the focus and perpendicular to the directrix is called the axis of parabola. The point of intersection of parabola with its axis is called vertex of the parabola.

31. Latus rectum of parabola is the line segment passing through the focus and perpendicular to the axis whose end points lie on the parabola

Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Graph				
Vertex	(0,0)	(0,0)	(0,0)	(0,0)
Focus	(a,0)	(-a,0)	(0,a)	(0,-a)
Directrix	x+a=0	x-a=0	y +a=0	y-a=0
Axis	y= 0	y=0	x=0	x=0
Latus rectum	4a	4a	4a	4a

MULTIPLE CHOICE QUESTIONS

1. The slope of the line joining the points (a+b, a-2b) and (2a-3b,3a-10b) is

   a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) 1   d) 2

   Ans (d)

   Slope $= m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3a - 10b - a + 2b}{2a - 3b - a - b} = 2$

2. The line with slope $\frac{2}{3}$ and passing through the point (1,3) also passes through
   a) (4,5)   b) (-2,0)   c) (-1,1)   d) (2,4)

   Ans (a)

   Equation of line is $(y - y_1) = m(x - x_1)$.

   $(y - 3) = \frac{2}{3}(x - 1)$. Point (4,5) satisfies this equation

3. If four times the y-intercept of a line is equal to thrice the x intercept, then the slope of the line is

a) $\frac{3}{4}$ b) $\frac{4}{3}$ c) $-\frac{3}{4}$ d) $-\frac{4}{3}$

Ans (c)

Equation of line is bx+ay= ab ,given 4b=3a

slope $=\frac{-b}{a}=-\frac{3}{4}$

4. The distance between the lines x+y+2=0 and 3x+3y-7=0 is

a) $\frac{13}{3}\sqrt{2}$ b) $\frac{13}{3\sqrt{2}}$ c) $\frac{13}{3}$ d) $\frac{13}{\sqrt{3}}$

Ans (b)

$c_1 = 2, c_2 = \frac{-7}{3}$,distance $=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}} = \frac{\left|2+\frac{7}{3}\right|}{\sqrt{2^2+\left(\frac{-7}{3}\right)^2}}$

5. Find the value of k ,such that the slope of the line passing through the points (2,4) and (-1,k) is 3

a)2   b) -5   c)3   d)4

Ans (b)

Slope $=\frac{k-4}{-1-2}= 3$, k= -5

6. The equation of the line parallel to the y-axis and at a distance of 4 units to its right is
   a) x= 4   b) y = 4   c) x = - 4   d) y = - 4

Ans (a) , x= a = 4

7. The intercepts of a line are the roots of the equation $x^2 - 5x + 6 = 0$, then the line is

a) $\frac{x}{3}+\frac{y}{2}= 1$ b) $\frac{x}{2}+\frac{y}{3}= 1$ c) $\frac{x}{5}+ y = 1$ d) either(a) or (b)

Ans (d)

Solution ,The roots are 2 and 3 ,so a = 2, b = 3 or a = 3, b= 2

8. The area of the circle with centre at (1,2) and passing through (4,6)

a)5π b)10π c)25π d) π

Ans (c)

Radius $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}=\sqrt{(6 - 2)^2 + (4 - 1)^2}= 5$

Area $= \pi r^2 = 25\pi$

9. If $x^2 + y^2 + px + qy = 0$ is the equation of a circle with centre (1,2) and radius $\sqrt{5}$, then the values of p and q are

a) p = 1 ,q =1   b) p = 2, q = 4   c) p = -2, q = -4   d) p=2, q=0

Ans (c)

The centre of the circle is $\left(\frac{-p}{2},\frac{-q}{2}\right) = (1,2)$ so p = -2,q = -4

10. The distance of the point P(1,-3) from the line $2y - 3x = 4$ is

a)13   b) $\frac{7}{13}\sqrt{3}$ c) $\sqrt{13}$ d) $\frac{7}{13}$

Ans (c)

Eqn of line is 2y-3x-4=0

Distance $= d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{|-3\times1+2(-3)-4|}{\sqrt{(-3)^2+2^2}} = \frac{13}{\sqrt{13}} = \sqrt{13}$

11. Coordinate of focus of parabola $y^2 = 3x$ are

    a)(-3,0)   b) (0.75,0)   c) (0,0.75)   d) (-1,0)

Ans (b)
4a= 3 ,a= 0.75

12. The equation of the parabola whose axis is along y-axis vertex at orign and passing through (-2, 5) is

    a) $x^2 = -5y$ b) $5y^2 = 4x$ c)$5x^2 = 4y$ d) $y^2 = -5x$

Ans (c)

Solution $x^2 = 4ay$ passes through (-2,5)

So 4 =20a $\Rightarrow a = \frac{1}{5}$,eqn of parabola is $5y^2 = 4y$

13. The equation of the parabola with focus at (3,0) and directrix x+3=0

    a) $x^2 = 12y$ b) $y^2 = -12x$ c) $y^2 = 12x$ d) $x^2 = -12y$

Ans (c)

Focus (a,0)=(3,0) ,so a=3, Equation of parabola $y^2 = 4ax = 12x$

14. If the parabola $y^2 = 4ax$ passes through the point(3,2) then the length of its latus rectum is
    a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{1}{3}$ d) 16

Ans (b)

$y^2 = 4ax$ passes through (3,2) we get 4= 12a $\Rightarrow$ a $= \frac{1}{3}$

So length of latus rectum = 4a $= \frac{4}{3}$

15. Find the radius of the circle $x^2 + y^2 + 2x - 4y = 0$

    a) 2   b)$\sqrt{5}$ c)$\sqrt{6}$ d)1

Ans (b)

G = 1, f = -2 , c = 0, r $= \sqrt{1^2 + (-2)^2 - 0} = \sqrt{5}$

16. Slope of a given line is $-\frac{1}{5}$. Slope of a line perpendicular to the given line is
    a) $-\frac{1}{5}$ b)$\frac{1}{5}$ c)5   d)-5

Ans (c)

Slope $= \frac{-1}{m} = 5$

17. The equation of the line passing through (2, -3) and parallel to the x-axis is

    a)   y = 3   b) x = 3   c) x = -3   d) y= -3

   Ans(d)   y = k =-5

18. Slope of the line $3x - 2y + 5 = 0$ is

    a) $\frac{-2}{3}$ b) $\frac{-3}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$

   Ans (d) ,   a= 3 , b= -2 , slope $= \frac{-a}{b} = \frac{-3}{-2} = \frac{3}{2}$

19. Centre of the circle $(x + 1)^2 + (y - 3)^2 - 5 = 0$ is

    a)(-1,3)   b) (1,-3)   c) (-1,3)   d) (-1,-3)

   Ans (a) $[x - (-1)]^2 + [y - 3]^2 = \left(\sqrt{5}\right)^2$,   h = -1,k =3 , centre =(h,k)= (-1,3)

20. The equation of parabola whose vertex is at origin and focus is (0,3)

    a) $x^2 = -12y$ b) $x^2 = 12y$ c) $y^2 = 12x$ d) $y^2 = -12x$

   Ans (b)

   Focus (0,a) = (0,3) so a=3

   Equation of parabola is $x^2 = 4ay = 12y$

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ASSERTION AND REASON QUESTIONS

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In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1. Assertion (A): Slope of line 3y - 2x + 1 = 0 is $\frac{3}{2}$

   Reason (R): Slope of line ax + by + c = 0 is $-\frac{a}{b}$

Ans: (d)

2. Assertion (A): Distance of point (1, -1) from line 3x + 4y - 7 = 0 is $\frac{8}{5}$ units.

Reason (R): Distance of point (x_1, y_1) from line ax + by + c = 0 is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$

Ans: (a)

3. Assertion (A): Lines 2x - y + 1 = 0 and x + 2y = 5 are perpendicular to each other.

Reason (R): Angle between lines y = $m_1x + c_1$ and $y = m_2x + c_2$ is given by,

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$.

Ans: (b)

4. Assertion (A): For the line 2x + y - 5 = 0, x-intercept is $\frac{5}{2}$

Reason (R): For the line x + by + c = 0 , x-intercept is $-\frac{c}{a}$.

Ans: (a)

5. Assertion (A): Distance between lines ax + by + c_1 = 0 and bx - cy + c_2 = 0 is $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

Reason (R): If two lines are perpendicular, then product of their slope is (-1).

Ans: (d)

6. Assertion (A): Radius of circle $(x - 1)^2 + (y + 2)^2 = 7$ is $\sqrt{7}$

Reason (R): For the circle $(x - h)^2 + (y - k)^2 = r^2$ centre is (h, k)

Ans: (b)

7. Assertion (A): Centre of the circle $(x + 2)^2 + y^2 = 7$ lies on x axis.

Reason (R): Coordinates of any point on x-axis is (a, o), where a ∈ R.

Ans: (a)

8. Assertion (A): Focus of parabola y^2 = 3x lies on x-axis.

Reason (R): Axis of parabola y^2 = 4ax is x-axis, vertex (0, 0) and focus lies on x-axis.

Ans: (a)

9. Assertion (A): Length of latus rectum of the parabola x^2 = -7y is -7.

Reason (R): Distance/length is always positive

Ans: (d)

10. Assertion (A):Lines 2x+3y-6= 0and 3x-2y+6 =0 are perpendicular

   Reason(R) : Two lines are perpendicular then slope of one line is reciprocal of the other

 Ans:(c)

<h2 style="text-align:center;color:red;">VERY SHORT ANSWER QUESTIONS</h2>

   1) What is the value of y so that the line through (3, y) and (2, 7) is parallel to the line through (-1, 4) and (0, 6)?

Sol:

m_1 = slope of the line through $(3, y)$ and $(2,7)$ = $\frac{7-y}{2-3}$ = $\frac{7-y}{-1}$.

m_2= slope of the line through (-1, 4) and (0, 6) = $\frac{6-4}{0-(-1)}$ = $\frac{2}{1}$ = 2.

since the given lines are parallel ∴ $m_1 = m_2$

$\Rightarrow \frac{7-y}{-1} = 2 \Rightarrow 7 - y = -2 \Rightarrow$ y = 7+2 = 9

2) Find the equation of a line joining the points (-1, 3) and (4,-2).

Sol:
We have the equation of the line passing through (x_1, y_2) and (x_2, y_2) is
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

∴ The equation of the required line is $y - 3 = \frac{-2-3}{4-(-1)}(x - (-1))$

$\Rightarrow y - 3 = \frac{-5}{5}(x + 1)$ $\Rightarrow y - 3 = -(x + 1) \Rightarrow x + y - 2 = 0.$

3) Find the equation of a straight line which cuts off an intercept 4 on the positive direction of x-axis and the intercept 3 on the negative direction of y-axis.

Sol: The equation of the straight line is $\frac{x}{a} + \frac{y}{b} = 1$.

Here, a = 4 , b = - 3

∴ The required equation of the line is $\frac{x}{4} + \frac{y}{-3} = 1$.

$\Rightarrow 3x - 4y = 12$

$\Rightarrow 3x - 4y - 12 = 0$

4) Find the slope of the line which passes through the origin and the midpoint of the line segment joining the points (0, -4) and (8, 0).

Sol: The midpoint the line segment joining (0, - 4) and (8, 0) is $\left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)$.

The slope of the required line is $\frac{-2-0}{4-0} = -\frac{1}{2}$

5) Line through the points (-2, 6) and (4, 8) is perpendicular to the line through the points

(8, 12) and (x, 24). Find the value of x.

Sol:

Slope of the line through (-2, 6) and (4, 8) is $\frac{1}{3}$ and the slope of the line through (8, 12) and (x, 24)

is $\frac{12}{x-8}$ Since the two lines are perpendicular,

$\frac{1}{3}$ X $\frac{12}{x-8} = -1$

$\Rightarrow -3x + 24 = 12$

$\Rightarrow x = 4.$

6) Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

Sol: The acute angle between two lines is given by $\tan \theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

Here, m_1 = slope of line (1) = $-\sqrt{3}$

m_2 = slope of line (2) = $-\frac{1}{\sqrt{3}}$

$$\therefore tan\,\theta = \left| \frac{-\sqrt{3} - \left(-\frac{1}{\sqrt{3}}\right)}{1 + (-\sqrt{3})\left(-\frac{1}{\sqrt{3}}\right)} \right| = \frac{|-3 + 1|}{\sqrt{3} + \sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$$

The acute angle between the lines is 30°.

7) Find the equation of the straight line that passes through the point (3, 4) and perpendicular to the line 3x + 2y + 5 = 0.

Sol: Given equation is 2y = -3x-5

$$y = \frac{-3}{2}x - \frac{5}{2} \text{, so slope} = \frac{-3}{2}$$

slope of the perpendicular line $= \frac{-1}{m} = \frac{2}{3}$

The equation of the line perpendicular to the line 3x + 2y + 5 =0 and passes through (3, 4).is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{3}(x - 3)$$

we get the equation of the line perpendicular to the given Line as 2x-3y- 6= 0.

8) What are the points on x-axis whose perpendicular distance from the line 4x + 3y = 12 is 4?

Sol: Any point on x-axis is of the form $(\alpha, 0)$ Given, the distance of $(\alpha, 0)$ from 4x + 3y -12 = 0 is 4.

$$\Rightarrow \frac{|4\alpha + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}} = 4 \Rightarrow \frac{|4\alpha - 12|}{5} = 4$$

$\Rightarrow |$ 4α - 12 1 = 20 $\Rightarrow$ 4α - 12 = $\pm$ 20 $\Rightarrow$ α - 3 = ±5

$\Rightarrow \alpha$ -3=5 or α -3 = -5

$\Rightarrow$ α = 8 or α = -5 + 3 = -2.

Hence, the required points are (8, 0) and (-2, 0).

9) Find the equation of a circle whose centre is (-2, 3) and radius is 4.

Sol. Since the centre of the circle is (-2, 3) and its radius is 4, therefore, the equation of the circle is

$$\left(x - (-2)\right)^2 + (y - 3)^2 = 4^2 \qquad\qquad \text{(central form)}$$

$$\Rightarrow (x + 2)^2 + (y - 3)^2 = 16$$

$$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$\Rightarrow x^2 + y^2 + 4x - 6y - 3 = 0.$$

10) Find the equation of the circle with centre (2, 2) and which passes through the point (4, 5).

Sol. The centre of the circle is C(2,2) and it passes through the point P(4,5)

Radius of circle = CP

$$= \sqrt{(4-2)^2 + (5-2)^2}$$

$$= \sqrt{4+9} = \sqrt{13}.$$

∴ The equation of the circle $(x-2)^2 + (y-2)^2 = (\sqrt{13})^2$ (central form)

i.e. $x^2 + y^2 - 4x - 4y = 5$.

11) Find the equation of a circle having (1, -2) as its centre and passing through the intersection of the lines 3x + y = 14 and 2x + 5y = 18

Sol. Given lines are 3x + y = 14 ...(i)   and 2x + 5y = 18 …(ii)

Solving (1) and (ii) simultaneously, we get x = 4, y = 2 .

The point of intersection, say P, of the given lines is (4, 2). Since the centre of the circle is C(1,- 2) and it passes through the point P(4,2) its radius = CP = $\sqrt{(4-1)^2 + (2-(-2)^2} = \sqrt{9+16} = 5$.

∴ The equation of the circle is

$(x-1)^2 + (y+2)^2 = 5^2$ (central form)

Or $x^2 + y^2 - 2x + 4y - 20 = 0$.

12) Find the equation of the circle when the end points of a diameter are A(- 2, 3) and B(3, - 5)

Sol. Using diameter form, the equation of the circle having A(-2, 3) and B (3, -5) as the end points of a diameter is

$$(x - (-2))(x - 3) + (y - 3)(y - (-5)) = 0$$

Or $(x + 2)(x - 3) + (y - 3)(y + 5) = 0$

Or $x^2 - x - 6 + y^2 + 2y - 15 = 0$

Or $x^2 + y^2 - x + 2y - 21 = 0$.

13) Find the coordinates of focus, the equation of axis, the equation of the directrix and the length of latus-rectum of the parabola represented by the equation $3y^2 = 8x$.

Sol. The given equation is $3y^2 = 8x$ …(i)

i.e. $y^2 = \frac{8}{3}x$ which is comparable with $y^2 = 4ax$ so (i) represents a standard (right hand) parabola, and its axis lies along the taxis. Hence, x-axis i.e y = 0 is the axis of the given parabola

Also therefore, focus is (a, 0) i.e. $\left(\frac{2}{3}, 0\right)$ and the equation of directrix is | x+a = 0

x + $\frac{2}{3}$ = 0   i.e. 3x + 2 =0.

Length of latus-rectum = $4a = \frac{8}{3}$

14) Find the coordinates of focus, the equation of directrix and the length of latus-rectum of the conic represented by the equation $5x^2 = - 12y$

Sol. The given equation is $5x^2 = - 12$

$i.e.\ x^2 = -\dfrac{12}{5}y$ which is comparable with $x^2 = - 4ay$ so (i) represents a parabola of the fourth standard form

Here $4a = \dfrac{12}{5} \Rightarrow a = \dfrac{3}{5}$.

The focus is (0, - a) $ie\ \left(0, -\dfrac{3}{5}\right)$ and the equation of the directrix is $y - \dfrac{3}{5} = 0$

ie. 5y - 3 = 0 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ | y - a = 0

The length of latus-rectum = 4a = $\dfrac{12}{5}$

15) Find the equation of the parabola with focus (6,0) and directrix x = - 6 Also find the length of latus-rectum.

Sol. Given the focus of the parabola is F (6, 0) which lies on x-axis and directrix is the vertical x= - 6 i .e x + 6 = 0 so the x-axis itself is the axis of the parabola.

Therefore, the parabola is of first standard form and its equation is $y^2 = 4ax$ with a = 6 Hence, the required equation of the parabola is $y^2 = 4ax$ ie. $y^2 = 24x$

Length of latus-rectum = 4a = 24 .

16) Find the equation of a line passing through the points (0, 3) and (-1, -4).

Sol. Slope of the line joining the points A(0, 3) and B(- 1, - 4) is

$m = \dfrac{-4-3}{-1+0} = -\dfrac{7}{-1} = 7$.

∴ equation of line AB is y - 3 = 7(x + 0)

⇒7x - y + 3 = 0 is required equation.

17) Find the equation of a line whose y-intercept is - 3 and which is perpendicular to the line joining the points (0,5) and (- 1, 3).

Sol. Slope of line through (0, - 5) and (- 1, 3) is

$\dfrac{3+5}{-1-0}$ = - 8.

∴Slope of perpendicular line is $\dfrac{1}{8}$; y-intercept = - 3, i.e. c = -3

∴ Equation of a line is y = mx+c

Equation is y = $\dfrac{1}{8}$x-3 $\qquad$ (or) x - 8y - 24 = 0

18) Find the equation of circle whose centre is (1, 1) and touches lines 3x + 4y = 32

Sol: Centre is (1, 1), distance of centre from line = radius

$=\left|\dfrac{3+4-32}{\sqrt{9+16}}\right| = \left|\dfrac{-25}{5}\right| = 5$ units

$\therefore$ Circle is $(x - 1)^2 + (y - 1)^2 = 5^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 2y + 1 = 25$

$\Rightarrow x^2 + y^2 - 2x - 2y - 23 = 0$

19) If one end of the diameter of a circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (8,4), show that coordinates of the other end are (-4, 2).

Sol: Let the other end be (α, β)

Centre is (2, 3), centre is mid-point of diameter.

$\frac{8+\alpha}{2} = 2; \frac{4+\beta}{2} = 3$

$\Rightarrow \alpha = -4, \beta = 2,$ i.e. (-4, 2)

Hence, coordinates of the other end are (-4, 2).

20) Find the equation of parabola whose centre is at origin, axis along the x-axis and passes through the point (2, 3).

Sol. Parabola is $y^2 = 4ax$ ….(i)

It passes through the point (2, 3)

$\therefore 9 = 4a \times 2 \Rightarrow a = \frac{9}{8}$

Substituting in (i), parabola is

$y^2 = 4 \times \frac{9}{8} \times x \Rightarrow 2y^2 = 9x$

SHORT ANSWER TYPE QUESTIONS

1) Find the point on the x-axis which is equidistant from the points (7, 6) and (3, 4).

Sol. Let P(x, 0) be any point on the x-axis.

Let the given points be A(7, 6) and B(3, 4).

According to given, AP = BP

$\Rightarrow \sqrt{(x - 7)^2 + (0 - 6)^2} = \sqrt{(x - 3)^2 + (0 - 4)^2}$

$\Rightarrow (x - 7)^2 + 36 = (x - 3)^2 + 16$

$\Rightarrow x^2 - 14x + 49 + 36 = x^2 - 6x + 9 + 16$

$\Rightarrow -8x = -60 \Rightarrow x = \frac{15}{2}$

Hence, the required point is $\left(\frac{15}{2}, 0\right)$

2) The centre of a circle is C(-2, 5) and one end of a diameter is A(3, -7), find the coordinates of the other end.

Sol. Let the other end of the diameter of the given circle be B(α, β) whose one end is A(3, -7).

Mid-point of AB is $\left(\frac{\alpha+3}{2}, \frac{\beta-7}{2}\right)$.

Since the centre C(-2, 5) of the circle is the mid-point of AB, $\frac{\sigma+3}{2} = -2$ and $\frac{\beta-7}{2} = 5$

$\Rightarrow \sigma = -7$ and $\beta = 17$

∴The coordinates of the other end of the diameter are (-7, 17).

3) If the vertices of a triangle are (1, k), (4, -3) and (-9, 7) and its area is 15 sq. units, find the value(s) of k.

Sol. Area of triangle formed by the given points

$= \frac{1}{2}|1.(-3-7) + 4.(7-k) + (-9).(k+3)| = 15$ (given)

$\Rightarrow |-13k - 9| = 30 \Rightarrow |13k + 9| = 30$

$$\Rightarrow 13k + 9 = \pm 30$$

$$\Rightarrow 13k = -39, 21$$

$\Rightarrow k = -3, \frac{21}{13}$.

4) Find the equation of a straight line whose y-intercept is-3 and which is

(i) parallel to the line joining the points (2, 3) and (4,-5)

(ii) perpendicular to the line joining the points (0,-5) and (-1, 3)

Sol. Here c= y-intercept= - 3

(i) Let m be the slope of the required line.

Since the required line is parallel to the line joining the points A(- 2, 3) and B(4, - 5)

∴m = slope of AB = $\frac{-5-3}{4-2} = \frac{-8}{6} = -\frac{4}{3}$

The equation of the required line is $y = -\frac{4}{3}x + (-3)$

$\Rightarrow$ 4x + 3y + 9 =0 |y = mx + c

(ii) Let m be the slope of the required line.

Slope of the line joining the points A(0, - 5) and B(- 1, 3)

$= \frac{3-(-5)}{-1-0} = \frac{8}{-1} = -8$.

Since the required line is perpendicular to AB, its slope = $\frac{1}{8}$ $\left| m_2 = -\frac{1}{m_1} \right.$

The equation of the required line is y = $\frac{1}{8}$ x + (- 3) | y =mx+c

$\Rightarrow$ x - 8y - 24 = 0

5) Find the value of k so that the line 2x + ky - 9 = 0 may be

(i) parallel to 3x - 4y + 7 = 0
(ii) perpendicular to 3y + 2x - 1 = 0
Solution. Given line is $2x + ky - 9 = 0$ ....(1)

Slope of line (1)$= -\frac{coeff.\ of\ x}{coeff.\ of\ y} = -\frac{2}{k}$

(i) Given line is 3x - 4y + 7 = 0   ....(2)

Slope of line (2) $= -\frac{3}{-4} = \frac{3}{4}$.

As the lines (1) and (2) are parallel, $-\frac{2}{k} = \frac{3}{4}$ $(\because m_1 = m_2)$

$\Rightarrow k = -\frac{8}{3}$

(ii) Given line is 3y+2x-1=0 i. e .2x + 3y - 1 = 0

Slope of line (3) $= -\frac{2}{3}$ .....(3)

As the lines (1) and (3) are perpendicular, $\left(-\frac{2}{k}\right)\left(-\frac{2}{3}\right) = -1$ $|m_1 m_2 = -1$

$\Rightarrow 4 = -3k \Rightarrow k = -\frac{4}{3}$.

6) The line through the points (h, 3) and (4, 1) intersects the line 7x - 9y - 19 = 0 at right angles.
Find the value of h.
Sol. The slope of the line joining the points (h, 3) and (4, 1)

$= \frac{1-3}{4-h} = -\frac{2}{4-h}$.

The slope of the line $7x - 9y - 19 = 0\ is\ -\frac{7}{-9} = \frac{7}{9}$

As these lines are e perpendicular, $\left(-\frac{2}{4-h}\right) * \frac{7}{9} = -1$

$\Rightarrow 14 = 36 - 9h \Rightarrow 9h = 22 \Rightarrow h = \frac{22}{9}$.

7) Find the equation of the line perpendicular to the line x - 7y + 5 = 0 and having x-intercept 3.
Sol. Given line is $x - 7y + 5 = 0$

Its slope $= -\frac{1}{-7} = \frac{1}{7}$ ...(i)

.:. Slope of a line perpendicular to (i)= -7 . $\left|m_2 = -\frac{1}{m_1}\right.$

The equation of the line with x-intercept 3 and perpendicular to (i) is

y = - 7(x - 3) $|y = m(x - d)$

∴7x + y - 21 = 0 .

8) Find the equation of a circle having (1,-2) as its centre and passing through the intersection of the
lines 3x + y = 14 and 2x + 5y = 18.
Sol. Given lines are x + y = 14   ...(i) and x + 5y = 18   .....(ii)
Solving (i) and (ii) simultaneously, we get x = 4, y = 2.
∴The point of intersection, say P, of the given lines is (4, 2).
Since the centre of the circle is C(1, -2) and it passes through the point P (4, 2),

its radius = CP = $\sqrt{(4-2)^2 + (2 - (-2)^2} = \sqrt{9 + 16} = 5$

Equation of circle is $(x - h)^2 + (y - k)^2 = r^2$

$(x - 4)^2 + (y - 2)^2 = 5^2$.

9) If the equations of the two diameters of a circle are x - y= 5 and 2x + y = 4 and the radius of the
circle is 5, find the equation of the circle
Sol: To get the centre of the circle we solve the equations of the diameters of the circle

x - y = 5   ……(1)

and 2x + y = 4   ……(2)

Solving (1) and (2), we get x = 3 and y = -2. So the centre of the circle is (3, -2). Also given radius of the circle is 5.

Hence, the equation of the circle is $(x - 3)^2 + (y + 2)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 6x + 4y - 12 = 0$ is the required equation of the circle.

10) Find the centre and the radius of the circle $x^2 + y^2 + 6x - 4y + 4 = 0$.

Sol: Given equation of the circle is $x^2 + y^2 + 6x - 4y + 4 = 0$.

$\Rightarrow (x^2 + 6x) + (y^2 - 4y) + 4 = 0$

$\Rightarrow (x + 3)^2 - 9 + (y - 2)^2 - 4 + 4 = 0$

$\Rightarrow (x + 3)^2 + (y - 2)^2 = 9$

$\Rightarrow (x - (-3))^2 + (y - 2)^2 = 3^2$

Hence, the centre of the circle is (-3,2) and radius 3.

11) The equation of the circle with (-1, 2) and (4, -3) as the end points of the diameter.

Sol: The equation of the circle is of the form $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

$\Rightarrow (x + 1)(x - 4) + (y - 2)(y + 3) = 0$

$\Rightarrow x^2 - 3x - 4 + y^2 + y - 6 = 0$

$\Rightarrow x^2 + y^2 - 3x + y - 10 = 0.$

12) Consider the parabola y^2 = -8x. The line 2x + y + k = 0 passes through focus of the parabola. Then find the value of k.

Sol: The parabola y^2 = -8x is of the form y^2 = -4ax where 4a = 8 $\Rightarrow a = 2$

Focus is (- a, 0) = (-2,0) .

The line x + y + k = 0 passes through (-2, 0).

$\therefore \ 2(-2) + 0 + k = 0 \Rightarrow -4 + k = 0 \Rightarrow k = 4.$

<p align="center" style="color:red">**LONG ANSWER TYPE QUESTIONS**</p>

1) Determine, the equation of the perpendicular bisector of the line segment joining the points A (2, 3) and B (6, -5).

Sol: Let M be the mid-point of AB.

$M = \left(\frac{2 + 6}{2}, \frac{3 + (-5)}{2}\right) = (4, -1)$

Slope of AB $= \frac{-5 - 3}{6 - 2} = \frac{-8}{4} = -2$

Now, the slope of the perpendicular bisector

$= \frac{-1}{slope \ of \ AB} = \frac{-1}{-2} = \frac{1}{2}.$

So the required perpendicular bisector passes through (4,-1) and having slope $\frac{1}{2}$.

Equation of the perpendicular bisector is $y - y_1 = m(x - x_1)$

$\Rightarrow y - (-1) = \frac{1}{2}(x - 4)$

$\Rightarrow 2(y + 1) = x - 4 \Rightarrow x - 2y - 6 = 0$

2) Find the equation of the line which passes through the point (3, 4) and the sum of the intercepts on the axes is 14.

Sol: Let the intercept be a and b

Given that a + b = 14 ⟹ b = 14 − a

The equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{a} + \frac{y}{14-a} = 1$ ….(1)

Also given that, it passes through (3, 4).

From (1), we get $\frac{3}{a} + \frac{4}{14-a} = 1$

$\Rightarrow 3(14-a) + 4a = a(14-a)$

$\Rightarrow 42 - 3a + 4a = 14a - a^2$

$\Rightarrow a^2 - 13a + 42 = 0$

$\Rightarrow (a-6)(a-7) = 0$

$\Rightarrow a = 6 \text{ or } a = 7$

When, a = 6 b = 14 - 6 = 8 .

Then the equation of the straight line is $\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 8x + 6y - 48 = 0$

$\Rightarrow 4x + 3y - 24 = 0$

$When, a = 7, b = 14 - 7 = 7$

Then the equation of the straight line is $\frac{x}{7} + \frac{y}{7} = 1$

$\Rightarrow x + y = 7$

Hence, the required equation of straight line is 4x + 3y- 24 = 0 or x + y -7 = 0.

3) Find the centre and radius of the circle $2x^2 + 2y^2 - 3x + 5y - 7 = 0$.

Sol: The given equation can be written in the general form as

$x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0.$

Comparing with the general form of the circle we get $g = -\frac{3}{4}, f = \frac{5}{4} \text{ and } c = -\frac{7}{2}.$

So, the centre of the circle is (-g, f) = $\left(\frac{3}{4}, -\frac{5}{4}\right)$

And radius = $\sqrt{g^2 + f^2 - c} = \sqrt{\frac{9}{16} + \frac{25}{16} - \frac{7}{2}} = \sqrt{\frac{9}{16} + \frac{25}{16} + \frac{7}{2}} = \sqrt{\frac{90}{16}} = \frac{3\sqrt{10}}{4}.$

4) Consider the following population and year graph. Find the slope of the line AB and using it, find what will be the population in the year 2020?
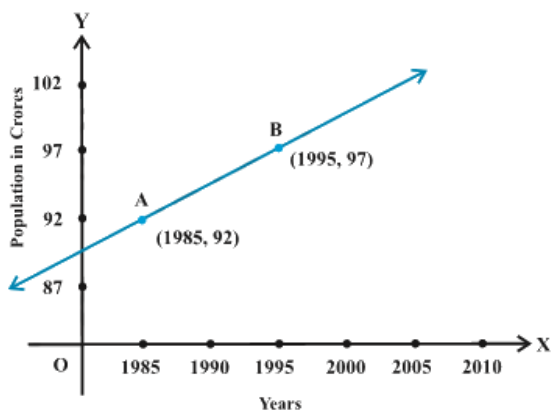
Sol. Slope of the line $AB = \dfrac{97-92}{1995-1985} = \dfrac{5}{10} = \dfrac{1}{2}$

Let P(x, p) be any point on the line where x denotes the years and p denotes the population.

Therefore, the points A(1985, 92), B(1995, 97) and P(x, p) are collinear

$\Rightarrow$ slope of AB = slope of BP

$\Rightarrow \dfrac{1}{2} = \dfrac{p-97}{x-1995} \Rightarrow p - 97 = \dfrac{1}{2}(x - 1995)$

$\Rightarrow p = 97 + \dfrac{1}{2}(x + 1995)$

Now, when $x = 2020, p = 97 + \dfrac{1}{2}(2020 - 1995)$

$\Rightarrow p = 97 + \dfrac{25}{2} = 109.5$.

Hence, the population in the year 2020 will be 109.5 crores.

<div align="center">

HIGHER ORDER THINKING SKILLS

</div>

1) Find the equation of the circle with radius 5 units whose centre lies on x-axis and passes through the point (2, 3).

sol. As the centre of the circle lies on x-axis, let its centre be C(h, 0).

Since the circle passes through the point A(2, 3) and has radius 5,

$$CA = 5\ (2-h)^2 + (3-0)^2 = 5^2 \Rightarrow (2-h)^2 = 16$$

$$\Rightarrow 2 - h = 4, -4 \Rightarrow h = -2, 6.$$

$\therefore$ The centre of the circle is (-2, 0) or (6, 0).

The equation of the circle is $(x + 2)^2 + (y - 0)^2 = 5^2 \text{ or } (x - 6)^2 + (y - 0)^2 = 5^2$

$i.e.\ x^2 + y^2 + 4x - 21 = 0 \text{ or } x^2 + y^2 - 12x + 11 = 0.$

There are two circles satisfying the given conditions.

2) Find the equation of a parabola with focus at (-1,-2) and directrix x - 2y + 3 = 0
Sol. The focus of the parabola is F(- 1, - 2) and directrix is the line x - 2y + 3 = 0.
Let P(x,y) be any point in the plane of focus and directrix, and MP be perpendicular distance from P to the directrix, then P lies on the parabola iff FP = MP

$$\Rightarrow \sqrt{(x+1)^2 + (y+2)^2} = \frac{|x-2y+3|}{\sqrt{1^2+(-2)^2}}$$

$$\Rightarrow 5[(x+1)^2 + (y+2)^2] = (x-2y+3)^2 \text{n}$$

$$\Rightarrow 5(x^2 + 2x + 1 + y^2 + 4y + 4) = x^2 + 4y^2 + 9 - 4xy + 6x - 12y$$

$$\Rightarrow 4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0, \text{ which is the required equation of the parabola.}$$

3) Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Sol: As the centre lies on the x-axis, we can take it as (h, 0). Also, the point (2, 3) lies the circle, therefore, distance between (2, 3) and (h, 0) = radius.

$$\Rightarrow \sqrt{(h-2)^2 + (0-3)^2} = 5$$

$$\Rightarrow (h-2)^2 = 16 \Rightarrow h - 2 = \pm 4$$

$$\Rightarrow h = 2 \pm 4 \Rightarrow h = 6, -2.$$

So the centre of the circle is (-2, 0) or (6, 0).

The equation of the circle with centre (-2, 0) is $(x+2)^2 + y^2 = 25$

$$\Rightarrow x^2 + y^2 + 4x + 4 = 25 \Rightarrow x^2 + y^2 + 4x - 21 = 0.$$

The equation of the circle with centre (6,0) is $(x-6)^2 + y^2 = 25$

$$\Rightarrow x^2 + y^2 - 12x + 36 = 25 \Rightarrow x^2 + y^2 - 12x + 11 = 0.$$

4) Two lines passing through the point (2,3) intersect each other at an angle of 60. If the slope of one line is 2, find the equation of other line.

Sol: Let the slope of the required line be m. Since angle between two lines is 60.

$$\tan\theta = \left|\frac{m-2}{1+m.2}\right| \Rightarrow \sqrt{3} = \left|\frac{m-2}{1+2m}\right|$$

$$\Rightarrow \left|\frac{m-2}{1+2m}\right| = \pm\sqrt{3} \Rightarrow \frac{m-2}{1+2m} = \sqrt{3} \text{ or } \frac{m-2}{1+2m} = -\sqrt{3}$$

$$\Rightarrow \sqrt{3} + 2\sqrt{3}m = m - 2 \text{ } 0r \text{ } m - 2 = -\sqrt{3} - 2\sqrt{3}m$$

$$\Rightarrow (2\sqrt{3} - 1)m = -(\sqrt{3} + 2) \text{ or } (2\sqrt{3} + 1)m = 2 - \sqrt{3}$$

$$\Rightarrow m = -\frac{\sqrt{3}+2}{2\sqrt{3}-1} \text{ or } m = \frac{2-\sqrt{3}}{2\sqrt{3}+1}$$

$\therefore$ The equation of the other line is

$$y - 3 = -\frac{\sqrt{3}+2}{2\sqrt{3}-1}(x+2) \text{ or } y - 3 = \frac{2-\sqrt{3}}{2\sqrt{3}-1}(x-2).$$

CASE STUDY BASED QUESTIONS

1) A triangular park has two of its vertices as B(-4, 1) and C(2, 11). The third vertex A is a point dividing the line joining the points (3, 1) and (6, 4) in the ratio 2: 1.

Based on the above information, answer the following questions:

(i) The coordinates of third vertex A are

(a) (5,3)   (b) (3,5)   (c) (- 5, 3)   (d) (5,-3)

(ii) The equation of passing through B and C is

(a) 5x - 3y - 23 = 0   (b) 5x - 3y + 23 = 0   (c) 3x + 5y - 23 = 0   (d) 5x + 3y - 23 = 0

(iii) The equation of line passing through A and parallel to BC is

(a) 5x - 3y + 16 = 0   (b) 5x - 3y + 34 = 0   (c) 5x - 3y - 16 = 0   (d) 5x + 3y - 16 = 0

(iv) The equation of the line passing through A and perpendicular to BC is

(a) 3x + 5y - 30 = 0   (b) x + 5y + 30 = 0   (c) 3x - 5y + 30 = 0   (d) 3x - 5y = 0

Sol:

(i) Coordinates of $A = \left(\frac{2 \text{ X } 6 + 1 \text{ X } 3}{2 + 1}, \frac{2 \text{ X } 4 + 1 \text{ X } 1}{2 + 1}\right)$. i.e. (5,3)

∴Option (a) is the correct answer

(ii) Equation of line through B(- 4, 1) and C(2,11) is

$$y - 1 = \frac{11 - 1}{2 + 4} (x + 4)$$

$$\Rightarrow y - 1 = \frac{5}{3} (x + 4) \Rightarrow 3y - 3 = 5x + 20$$

$$\Rightarrow 5x - 3y + 23 = 0$$

∴Option (b) is the correct answer.

(ii) Slope of line BC $= \frac{5}{3}$

∴Slope of line parallel to BC $= \frac{5}{3}$

Equation of line through A(5, 3) and parallel to BC is

$$y - 3 = \frac{5}{3} (x - 5) \Rightarrow 3y - 9 = 5x - 25$$

$$\Rightarrow 5x - 3y - 16 = 0$$

∴Option (c) is the correct answer.

(iv) Slope of line perpendicular to BC = -3

Equation of line through A(5, 3) and perpendicular to BC is

$$y - 3 = -\frac{3}{5} (x - 5) \Rightarrow 5y - 15 = -3x + 1$$

$$\Rightarrow 3x + 5y - 30 = 0$$

∴Option (a) is the correct answer.

2) Equation of a straight-line path is 2x + y-12 = 0. A man is standing at a point (2, 3). He wants to reach the straight-line path in least possible time.

Based on the above information, answer the following questions:

(i) The slope of the path followed by man is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2   (d) -2

(ii) Equation of the path followed by man is

(a) 2x+y-4=0   (b) 2x-y+4= 0   (c) x-2y+4=0   (d) x+2y+4=0

(iii) Coordinates of point where path followed by man and given straight line path meet is

(a) (2, 4)   (b) (4,4)   (c) (-2, 4)   (d) (4,-4)

(iv) The distance covered by man in reaching the straight line path is

(a) $\sqrt{5}$ units   (b) $\sqrt{6}$ units   (c) 2 units   (d) 3 units

Sol: (i) y= -2x +12 so slope = -2, slope of perpendicular path $=\frac{-1}{-2} = \frac{1}{2}$

Ans (a)

(ii) Equation of path y- 3 $= \frac{1}{2} (x - 2)$

2y-6 = x-2 (or) x- 2y+4=0

Ans (c)

(iii) Solving 2x+y-12 =0 and x-2y+4= 0 we get (4,4)

Ans (b)

(iv) Distance $=\sqrt{(4 - 2)^2 + (4 - 3)^2} =\sqrt{5}$

Ans (a)

3) Students of a school are taken to a railway museum to learn about railways heritage and its history .

An exhibit in the museum depicted many rail lines on the track near the railway station .

On the basis of above information , answer the following questions :

(i) If the line l_1 passes through the points $(1,2)$ and $(3,4)$, find the equation of another line perpendicular to l_1 and passing through $(3,5)$.

(ii) If the line l_3 passes through $(1,3.5), (2,3)$, then find the equation of l_3.

Sol: (i) Slope of $l_2 = -\left(\frac{1}{slope\ of\ l_1}\right) = -\frac{1}{1} = -1$

Equation of l_2 is $y - 5 = -1(x - 3) \Rightarrow x + y - 8 = 0$

$$(ii)\ y - 3 = -\frac{1}{2}(x - 2) \Rightarrow x + 2y - 8 = 0$$

EXERCISE

MULTIPLE CHOICE QUESTIONS

1) The acute angle between the lines x - y = 0 and y = 0 is

(a) $\frac{n}{6}$ 　　　　　(b) $\frac{\pi}{4}$ 　　　　　(c) $\frac{x}{3}$ 　　　　　(d) $\frac{5\pi}{6}$

2) The point on line 3x - y - 1 = 0 whose ordinate is 5, is

(a) (0,4) 　　　　(b) (2,5) 　　　　(c) (5, 2) 　　　　(d) (-2, 5).

3) The lines $x + (k - 1)y + 1 = 0$ and $2x + k^2 y - 1 = 0$ are at right angles if

(a) k = 1 　　　　(b) k > 1 　　　　(c) k = -1 　　　　(d) |k| = 2

4) The distance between the lines y = mx $+A_1$ and y = $mx + A_2$ is

(a) $A_2 - A_1$ 　　　(b) 0 　　　(c) $\frac{A_1-A_2}{\sqrt{m}}$ 　　　(d) $\frac{|A_2-A_1|}{\sqrt{1+m^2}}$

5) Slope of right bisector of line segment joining the points (1, 2) and (2, 3) is :

(a) 1 　　　　(b) – 1 　　　　(c) -3 　　　　(d) 0

6) The radius of the circle $x^2 + y^2 + 4x - 3y - 1 = 0$ is :

(a) $\sqrt{\frac{29}{2}}$ 　　　　(b) $\frac{\sqrt{29}}{2}$ 　　　　(c) $\sqrt{29}$ 　　　　(d) none of these.

7) Equation of circle with centre (0, 0) and radius 5 is :

(a) $x^2 + y^2 + 25 = 0$ 　　(b) $x^2 + y^2 = 0$ 　　(c) $x^2 + y^2 = 25$ 　　(d) none of these

8) Equation of parabola whose vertex is at origin and focus is (0,3) is

(a) $x^2 = -12y$ 　　(b) $x^2 = 12y$ 　(c) $y^2 = 12x$ 　　(d) $y^2 = -12x$

9) Focus of the parabola $x^2 = - 4ay$ is :

(a) (a, 0) 　　　　(b) (-a, 0) 　　　　(c) (0, a) 　　　　(d) (0, - a).

10) The y -intercept of the line 3x-4y +1=0

(a) $\frac{-1}{4}$ 　　　　(b) $\frac{1}{4}$ 　　　(c) $\frac{4}{3}$ 　　　(d) $.\frac{-3}{4}$

ASSERTION – RESION QUESTIONS

1) ASSERTION A: Equation of line with slope 5 and y-intercept 3 is y = 5x + 3

REASON: Equation of line with slope m and x intercept d is y=m(x-d)

2) ASSERTION: The lines 3x - 4y+6=0 and 3x - 4y - 6=0 are parallel.

REASSON: The Lines are parallel if their slopes are equal

3) ASSERTION. The centre of circle $x^2 + y^2 = 5$ is (0,0)

REASON: The Centre of circle $x^2 + y^2 = r^2$ is (0,0)

4) ASSERTION: The axis of parabola $x^2 = 8y$ is y-axis

REASON: The axis of parabola $x^2 = 4by$ is x-axis

5) ASSERTION: The distance between the points (0,0) and (4,3) is 5.

REASON. The distance between two points is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

6) ASSERTION: For the line 3x+2y – 4 = 0 , x- intercept is $\frac{4}{3}$

REASON: For the line ax + by + c = 0, x-intercept is $-\frac{c}{a}$

VERY SHORT ANSWER TYPE QUESTIONS

1) Find the equation of the line passing through the point (1, 2) and perpendicular to the line x + y + 1 = 0
2) Find the distance between the lines $9x + 8y + 7 = 0 \ and \ 9x + 8y = 10$
3) Find the equation of circle with centre $(2, -3) \ and \ passes \ through \ origin.$
4) Find the equation of a Circle whose Centre (2,2) passes through the point (4,5)
5) Find the equation of parabola with focus (0,-2) and directrix y = 2
6) Find the focus, directrix of the 5 $y^2 = 2x$

SHORT ANSWER TYPE QUESTIONS

1) Find the equation of a line which is perpendicular to the line 4x - y + 8 = 0 and passes through the mid-point of the line segment joining (1, 5) and (3, 11).
2) Find the equation of the line passing through the mid-point of the line segment joining the points (1, 3) and (2, -1) and parallel to the line 3x - y = 7
3) Find the value of k, if the straight line 2x + 3y + 4 + k (6x - y + 12) = 0 is perpendicular to the line 7x + 5y - 4 = 0.
4) Find the centre and radius of the circle $x^2 + y^2$ - 8x + 10y - 12 = 0
5) Show that the line 3x + 4y + 7 = 0 touches the circle $x^2 + y^2$ - 4x - 6y – 12 = 0.
6) Find the equation of the parabola having focus at (-1,-2) and the directrix x - 2y + 3 = 0

LONG ANSWER TYPE QUESTIONS

1) Find the points on the x-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

2) Find the equation of the line passing through the point (1, 3) and making an angle of 45° with the line x - 3y + 4 = 0

3) A circle has radius 3 units and its centre lies on the line y = x - 1 .If it passes through the point (7, 3). find the equations of the circle.

CASE STUDY BASED QUESTIONS

1) A man is moving on curved road and notes as that his distance from a flag post and from a straight wall always remain same. If flag post is at the point (2, 0) and wall is represented by x + 2 = 0 then
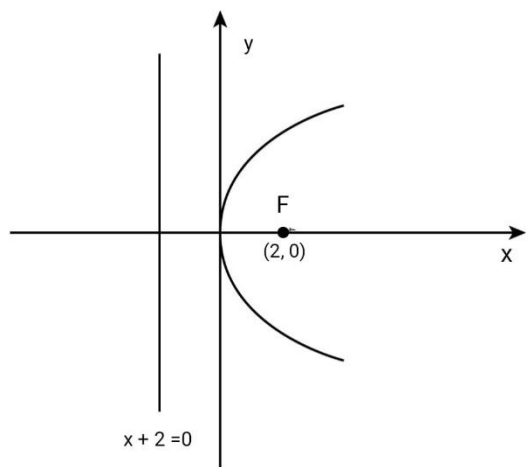
(i) Find the equation of the road.
(ii) Find the length of latus rectum of the curve.
(iii) Find the eccentricity of the path.

OR

(iii) Find the coordinates of the vertices of path.

2) A man visited India gate and moving on a path, he observed that his distance from India gate is always 500 metres. Taking India gate as origin. Answer the following:

(i) Find the equation of path of the man.

(ii) If he saw a balloon seller is at point (600, 800) then check whether the man has to move towards India gate or away from it.

(iii) If man is considered to be at point (400, 300), then find the distance between balloon seller and man.

OR

(iii) Find the area of triangle formed by joining the India gate, the position of man and the position of balloon seller.

HIGHER ORDER THINKING SKILLS

1) A line is such that its segment between the lines 5x - y + 4 = 0 and 3x + 4y - 4 = 0 is bisected at the point (1, 5). Obtain its equation

2) The vertices of the triangle are A(2, 3) B(4, - 1) and C(1,2) Find the length and equation of the perpendicular drawn from the point A on side BC.

3) Find the equation of the circle whose centre lies on the line 2x-y =3 and which passes through (3,-2) and (-2 ,0)

ANSWERS

MULTIPLE CHOICE QUESTIONS

1) (b) 2) (b) 3) (c) 4)(d) 5)(b) 6)(b) 7)(c) 8)(c) 9)(b) 10 (b)

ASSERTION - REASON QUESTIONS

1) (b) 2) (a) 3) (b) 4)(c) 5)(a) 6)(a)

VERY SHORT ANSWER TYPE QUESTIONS

1) $y - x - 1 = 0$ 2) $\frac{17}{\sqrt{145}}$ 3) $x^2 + y^2 - 4x + 6y = 0$

4) $x^2 + y^2 - 4x - 4y - 5 = 0$ 5) $x^2 = -8y$ 6) focus $\left(\frac{1}{10}, 0\right)$ and directrix $x = -\frac{1}{10}$

SHORT ANSWER TYPE QUESTIONS

1) Equation of line is x + 4y - 34 = 0   2) $6x - 2y = 7$.

3) $k = -\frac{29}{37}$ 4) Centre (4, -5); radius $= \sqrt{53}$

6) $4x^2 + y^2 + 4x + 4xy + 32y + 16 = 0$

LONG ANSWER TYPE QUESTIONS

1) (8, 0) and (-2, 0).   2) $x + 2y - 7 = 0$

3) $x^2 + y^2 - 8x - 6y + 16 = 0$, $x^2 + y^2 - 14x - 12y + 76 = 0$

CASE BASED QUESTIONS

1)Ans: (i) y² = 8x

(ii) 8 units

(iii) 1 OR (iii) (0, 0

2) Ans:

(i) $x^2 + y^2$ = 250000

(ii) Man has to move away from India gate

(iii) $100\sqrt{29}$ m OR (iii) Area = 70000 m^2

HIGHER ORDER THINKING SKILLS

1) 107x -3y - 92 =0   2) Length of AD = $\sqrt{2}$, Equation of BC = x + y – 3 = 0

Equation of AD= x – y + 1 = 0.

3) **Ans** $x^2 + y^2 + 3x + 12y + 2 = 0$

SESSION: 2025-26
CLASS XI
APPLIED MATHEMATICS

Time Allowed: 3 Hours   **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains -five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion -Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 mark each.
4. Section C has 6 Short Answer (SA)-type questions of 3 mark each.
5. Section D has 4 Long Answer (LA)-type questions of 5 mark each.
6. Section E has 3 sources based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

Q.1   Subtract the binary number 10000 from 10111
    (A) 111   (B) 1111   (C) 1110   (D) 10001

Q.2   If $\log (325.6) = 2.5127$, then $\log (0.03256)$ is
    (A) $\overline{2}.5127$ (B) $- 2.5127$ (C) -1.5127   (D) $\overline{1}.5127$

Q.3   If a Cone, a hemi sphere and a Cylinder have equal bases and have same height, then the ratio of their volumes is
    (A) $2:1:3$ (B) $1:3:2$ (C) $1:2:3$ (D) $3:2:1$.

Q.4   Pointing to a woman in a photograph , Ramesh said " She is the daughter of the father of the sister of my brother " . How is that woman related to Ramesh ?
    (A) Wife   (B) Daughter   (C) Sister   (D) Mother

Q.5   The Domain of the function f defined by $f(x) = \frac{x^2+2x+1}{x^2-x-6}$
    (A) $R - \{-3, 2\}$ (B) $R - \{3, -2\}$ (C) $R - (3, -2)$ (D) $R - [-3, -2]$

Q.6   If $y = x^{1/2} + x^{-1/2}$ then dy/dx at $x = 1$ is
    (A) 1   (B) -1 (C) 2   (D) 0

Q.7 $\lim_{x \to > \frac{3}{2}}[x] = ?$
    (A) 1   (B) 1.5   (C) 2   (D) 3

Q.8   The Probability that a Leap Year contains 53 Tuesday is
    (A) 1/7   (B) 2/7   (C) 3/7   (D) 4/7

Q.9   The least number of complete years in which a sum of money put out at 20 % compound interest will be more than doubled is
    (A) 2 years   (B) 4 years   (C) 5 years   (D) 6 years

Q.10   Variance is independent of change of
    (A)   origin only   (B)   scale only
    (C) origin and scale both   (D) None of these

Q.11   If r is the correlation coefficient, then
    (A) $|r| \leq 1$ (B) $r \leq 1$ (C) $|r| \geq 1$ (D) $r \geq 1$

Q.12   If for a distribution $\sum x_i^2 = 2400$ and $\sum x_i = 250$ and the total number of observation is 50, then standard deviation is:

(A) $\sqrt{20}$ (B) $\sqrt{21}$ (C) $\sqrt{22}$ (D) $\sqrt{23}$

Q.13 The equation of a line with slope -2 and which intersects x-axis at 3-unit on the left of origin is

(A) 2x-y-6=0   (B) 2x+y-6=0   (C) 2x+y+6=0   (D) 2x-y+6=0

Q.14 Which of the following tax is subsumed in GST ?

(A) VAT   (B) Entry Tax   C) Entertainment Tax   D) All A,B,C

Q.15 Health and Education cess is payable on.

(A) Gross Income (B) Taxable income (C) Income Tax (D) Education loan

Q.16 The slope of a line which is parallel to y- axis is

(A) 0   (B) 1   (C) Not defined   (D) $\frac{1}{2}$

Q.17 Which Two Months of Year have the Same Calender ?

(A) April , july   (B) oct , nov   (C) jan , March   (D) July , oct

Q.18 The Number of Two digits numbers divisible by 6 are :-

(A) 12   (B) 15   (C) 16   (D) 17

For questions 19 and 20, two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

(a) Both A and R are true and R is the correct explanation of the assertion

(b) Both A and R are true but R is not the correct explanation of the assertion

(c) A is true, but R is false

(d) A is false, but R is true

Q.19 <u>Assertion</u> (A) : The year 1500 was not a leap Year .

<u>Reason</u> (R) : Any Year which is divisible by 4 is a leap Year .

(A) (i)   (B) (ii)   (C) (iii)   (D) (iv)

Q.20 <u>Assertion</u> (A) : If n^{th} term of an A.P. is $a_n = np +q$, then common difference of AP is p

<u>Reason</u> (R) : Common difference of an A.P. is $d = a_n - a_{n-1}$.

(A) (i)   (B) (ii)   (C) (iii)   (D) (iv)

SECTION-B

Q.21 Find the Value of n If : n x $10c_4 = 10P_4$

OR

Find the Value (s) of r If $9C_r - 8c_3 = 8c_2$

Q.22 Find the derivative of $(x^3 + x^2 + 3) (x – 5)$ with respect to x.

Q.23 If $f(x) = \frac{3x-2}{2x-3}$, Find the Value of (fof) (x) , x ≠ 3/2

Q.24 Suppose an investor wants to have 1 lakh to retire 5 years from now . How much would he have to invest today with an annual rate of return equal to 15 percent ?

Q.25 Two statement followed by two conclusions are given below . You have to decide which of the given conclusions logically follows from the given two statements disregarding commonly known facts.

Statements : (i) Some vegetables are fruits.

(ii) No fruit is black.

Concluions : (I) Some fruits are vegetables.

(II) No vegetables is black.

SECTION-C

Q.26 Find the Sum of n terms of the Series

$0.6 + 0.66 + 0.666 + 0.6666 + \ldots\ldots\ldots$

OR

Evaluate $7^{1/2}$ x $7^{1/4}$ x $7^{1/8}$ x $\ldots\ldots\ldots\ldots$ to infinite terms .

Q.27 Probability of solving specific problem independently by A and B are $\frac{1}{2}$ *and* $\frac{1}{3}$ respectively.

If both try to solve the problem independently , find the probability that

(i) the problem is solved

(ii) exactly one of them solves the problem.

Q.28 If y log x = x – y , then Prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Q.29 If 'GOMTI' is coded as 74521 and 'YAMUNA' is coded as 835963, then how is 'GANGA' coded?

Q.30 Find the mean and variance for the following data: 6, 7, 10, 12, 13, 4, 8, 12.

Q.31 Find the equation of circle concentric with circle $x^2 + y^2 – 3x – 4y – \frac{21}{4} = 0$ and half its

area.

SECTION-D

Q.32 Evaluate using Log tables: $\frac{(27.8)^{\frac{1}{2}} \times (58.49)^{\frac{2}{3}}}{(0.0007)^{\frac{1}{4}}}$

OR

Q 32) (i) X can do a piece of work in 60 days , where Y can do the same work in 40 days Both work together, X left, 10 days before the completion of work. Find how many days will it take to complete to work ?

Q 32) (ii) If 15 February 2010 was Monday, What was the day on 10 January 2006?

Q.33 Find the mean, variance and standard deviation of following data:

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Q.34 In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study

Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 Study Physics and

Chemistry 50 study Mathematics and Chemistry and 20 none of these subjects. Find the number of

students who study all three subjects.

OR

For sets A ,B and C using property of sets, prove that

(a) (AUB) – B = A- B

(b) A- (A∩B) = A- B

(c) (AUB) – C = (A- C) U (B- C)

Q.35 Mr. Abhijit from Bangalore. The reading of electric meter of his house is

found to be 5678 units. If the previous month's reading was 4803 units and

connection load is 4 kw, calculate his electricity bill for that month. Tariff plan is

given below:

Energy charges

Number of units	Price per unit (in Rs.)
0-150	Rs. 5.5
151-300	Rs. 6
301-500	Rs. 6.5
> 500	Rs. 7

Fixed charges Rs.110 per kw/Month. Energy tax is 5% of tariff rates. Surcharge is Rs. 0.26 per unit.

SECTION-E

Q.36   CASE STUDY – 1

With coronavirus threatening to run riot in India, prevention appears to be the best cure available so far. It is crucial for people to have awareness and knowledge about the virus and need to take proper precautions to discourage its spread. A survey was conducted on 25 persons to see if proper precautions were being taken by people and following points were observed.

(a) 15 persons used face masks.

(b) 14 consciously maintained social distancing.

(c) 5 used face masks and washed their hands regularly.

(d) 9 maintained social distancing and used face masks.

(e) 3 were practicing all the three measures.

(f) 4 maintained social distancing and washing hands regularly.

(g) 4 practised only social distancing norms.

Assuming that everyone took at least one of the precautionary measures,

 Find

(i) how many exercised only washing hand as precautionary measure?

(ii) how many practised social distancing and washing hand but not wearing masks?

(iii) how many exercised only one activity measures / social distance / masks all ?

 OR

how many exercised only two activities measures / social distance / masks all ?

Q.37   CASE STUDY – 2

Roshan invested Rs 50000 in bank at the rate of 10 % per annum compounded annually .

On the basis of this information , answer the following questions :

(i) What will be the interest for the first year ?

(ii) What will be the interest for the second year ?

(iii) After three years , how much amount Roshan will get on the given rat e of interest compounded annually ?

 OR

Find the difference between the simple interest and compound interest for first two years .

Q. 38   CASE STUDY – 3

A doctor is visit to a patient. From the past experience, it is known that he will come by a cab, metro, bike or by other means of transport are respectively 0.3,0.2,0.1 and 0.4.The probability that he will be late are 0.25,0.3,0.35 and 0.1 if he comes by cab,metro,bike and other means of transportation respectively.

Based on above information Answer the following questions .

(i)   When the doctor arrives late ,what is the probability that he Comes by metro?

(ii)   What is the probability that doctor is late by all means?

Q 1   Answer (A)
       Solution
       10111 in decimal = $1\times2^4+0\times2^3+1\times2^2+1\times2^1+1\times2^0=16+0+4+2+1=23$
       10000 in decimal = $1\times2^4=16$
       So, 23−16=7, and the binary result should be 00111

Q 2   Answer (A)
       Solution :- log (325.6) = 2.5127
               Means log (3.256) = 0.5127
       Then log (0.03256) = log 3.256 - 2log10 = $0.512\overline{7}$ - 2 = $\overline{2}.5127$

Q 3   Answer (C)
       Solution :-
       The ratio is Vof cone : V_of hemi sphere : V_cylinder
       Substitute the calculated volumes: $(1/3)\pi r^3 : (2/3)\pi r^3 : \pi r^3$
                   $\pi r^3 : 2\pi r^3 : 3\pi r^3$ Now, divide each term by πr^3:
                   1 : 2 : 3

       Q 4Answer (C)

Solution :- Since Ramesh is pointing to a *woman* in the photograph, we are looking for a female relative who is the daughter of Ramesh's father. This makes the woman Ramesh's **sister**

Q 5   Answer (B)
Solution :- In Denominator $(x-3)(x+2)$ not equal to 0
The function $f(x)$ is undefined at $x=3$ and $x=-2$. Therefore, the domain includes all real numbers except $x=3$ and $x=-2$

Q 6   Answer (D)
Solution :- $\frac{dy}{dx} = \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{-3/2}$
On putting the value of x=1 we got $\frac{dy}{dx} = 0$

Q 7   Answer (A)
Solution :- Understand the behavior of the greatest integer function around $x=3/2=1.5$
       $\left[\frac{3}{2}\right] = [1.5] = 1$ therefore $\lim\limits_{x\to1.5} [x] = 1$

Q 8   Answer (B)
Solution :- Number of weeks = 366 days / 7 days = 52 weeks and 2 days.
This means a leap year consists of 52 complete weeks plus 2 extra days. For a leap year to have 53 Tuesdays, one of these two extra days must be a Tuesday.
This can be arranged MT , TW , WTh , Th F , F Sa , SaSu , SuM
From the analysis above, there are only 2 favorable cases out of the 7 possible cases where the leap year starts on a Monday or a Tuesday, resulting in 53 Tuesdays in the year.
The probability that a leap year contains 53 Tuesdays is 2/7.

Q 9   Answer (D)
Solution :- Let the principal sum be *P*. The interest rate is 20% per year, which is 0.20 as a decimal.
 The money is deposited at compound interest, so the amount *A* after *n* years is given by the formula:
  $A=P(1+r)^n$ where *r* is the interest rate and *n* is the number of years.

We want to find the least number of complete years *n* such that the amount *A* is more than double the principal *P*.

This means we need to find the smallest integer *n* for which:

A>2P

Substituting the formula for

$P(1+0.20)^n>2P$

We can divide both sides by

$(1.20)^n>2$ on taking log both sides we got least possible value is n= 4

Q 10   Answer (A)

Solution :- The correct statement is that variance is independent of **change of origin**.

Q 11   Answer (A)

Solution :- *r* always lies between **-1 and +1**

Q 12   Answer (A)

Solution :- using formula standardard deviation = $\frac{1}{n}\sqrt{n\sum x^2 - (\sum x)^2}$

On putting the value of we get andard deviation = $\sqrt{23}$

Q 13   Answer (C)

Solution :- the point-slope form of a line equation:

$y-y_1=m(x-x_1)$

Using the slope *m*=−2 and the point (-3, 0):

$y-0=-2(x-(-3))$, $y=-2(x+3)$ answer is $y=-2x-6$

Q 14   Answer (D) ,

Solution :- VAT , Entry Tax   , Entertainment All are value added tax .

Q 15   Answer (C)

Solution :- Health and Education cess is payable only on Income Tax

Q 16   Answer (C)

Solution :- The slope of a line parallel to the y-axis is **undefined**.

A line parallel to the y-axis is a vertical line . The slope of a line is defined as the ratio of the change in the y-coordinate to the change in the x-coordinate between any two points on the line: $m= (y_2 - y_1) / (x_2 - x_1)$

For any two distinct points on a vertical line, the x-coordinates will always be the same ($x_2 = x_1$).

Therefore, the denominator (x_2-x_1) becomes zero.

Division by zero is undefined in mathematics.

Thus, the slope of a line parallel to the y-axis is undefined.

Q 17   Answer (A)

Solution :-   Total Days Between the Start of April and the Start of July: To find out if April and July have the same calendar, we need to calculate the total number of days from the start of April to the start of July:

Days remaining in April (after the 1st) = 30 days

Days in May = 31 days

Days in June = 30 days

Total days = 30 + 31 + 30 = 91 days

Division by 7: Now, we divide the total number of days (91) by the number of days in a week (7):

91÷7=13 with a remainder of 0 therefore both have same calendar in a year

Q 18   Answer (B)

Solution :- Smallest two-digit number divisible by 6 is 12

And the largest two digit number is 96

The two-digit numbers divisible by 6 form an arithmetic progression: 12, 18, 24, ..., 96. The first term (a) is 12. The common difference (d) is 6. The last term (l) is 96.

We can use the formula for the n-th term of an arithmetic progression: l=a+(n−1)d

Substitute the values: 96=12+(n−1)6

Now, solve for n: 96−12=(n−1)6 , 84=(n−1)6 , 84/6 =n−1 , 14=n−1 , n=14+1 n=15

Therefore, there are **15** two-digit numbers divisible by 6.

Q 19   Answer (C)

Solution :- Since the year 1500 is century Year and it is not divisible by 400 , So it is not Leap year

Assertion (A) is true. Reason (R) is false.

Option (c) is the correct answer.

Q 20   Answer (A)

Solution :- Both are True with correct Expanation

The common difference (d) of an A.P. is the difference between any two consecutive terms. So, we can find the common difference by subtracting the first term from the second term: $d=a_2−a_1$

d = (2p+q)−(p+q) , d = 2p+q−p−q   , d = 2p−p+q−qd=p

Q 21   n×C(10,4)=P(10,4)

C(10,4)= ,   10! / 4!(10−4)! = 210

P(10,4)= 10! / (10−4)! = 5040

n×210=5040

  n = 24

OR

by using as above , Correct formula for Combination

  We got Answer   r = 3 or 6

Q 22 Derivative is $(x^3 + x^2 + 3) + (3x^2 + 2x) (x − 5)$

Q 23   f(f(x))=f $(\frac{2x−3}{3x−2})$ = $\frac{2\frac{2x−3}{3x−2} −3}{3\frac{2x−3}{3x−2}−2}$ after solving this we will get , (fof) (x) = x

Q 24   We know that Present value = $\frac{A_n}{(1+i)^n}$

Here $A_n = 100000$, i = 15 % =0.15 , n = 5 years

 Present value = $\frac{100000}{(1+0.15)^n}$= Rs 49719.08

Q 25   Conclusion I is immediate inference and follows from statement (i).

Conclusion II does not follow as vegetable is distributed in conclusion, but not in the statement, Hence, only conclusion I follows.

Q 26   Taking common 6 and Multiply by 9

Form the GP and use the formula

Answer = 2/27 (9n -1 + 1/10^n)

Or

For 7 $^{½ + ¼ +1/8 + ……..}$

using formula sum upto infinite

  For Correct calculation and Ans = 7

Q 27   Problem is Solved = 2/3

  Exactly one of them solved the problem = 1/2

Q 28   use the chain rule when differentiating terms involving y.

Differentiating both sides of ylogx=x−y with respect to x:

We will got $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

  Q 29

G	O	M	T		I
7	4	5	2		1

Y	A	M	U	N	A
8	3	5	9	6	3

Then

  GANGA coded as 73673

Q 30   Mean x = $\frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

Σ(xi−x⁻)² = 9 + 4 + 1 + 9 + 16 + 25 + 1 + 9 = 74

∴ Variance = $\frac{Σ(xi−x⁻)^2}{Σf_i} = \frac{9 + 4 + 1 + 9 + 16 + 25 + 1 + 9}{8} = \frac{74}{8} = 9.25$

Q 31 Centre = (3/2, 2) and radius, r = sqr root (23 / 2)

Find Radius of New Circle = sqr root (23) /2

Equation

$$\left(x - \frac{3}{2} \right)^2 + (y - 2)^2 = \frac{23}{4}$$

  Q 32   Let x = $\frac{(27.8)^{1/2} \times (58.49)^{2/3}}{(0.0007)^{1/4}}$

Log x = ½ log 27.8 + 2/3 log 58.49 – ¼ log 0.0007

Log x = ½ log (2.78 x 10) + 2/3 log (5.849 x 10) – ¼ log (7.00 x 10^{-4})

      = ½ (0.4440 + 1) + 2/3 (0.7676+1) – ¼ (0.8451 - 4)

      = ½ (1.4440) + 2/3 (1.7676) – ¼ (-3.1549)

      = 0.7220 + 1.1784 – (−0.7887)

      = 2.6891

OR

   1) X and Y one day work working together = 1/60 + 1 / 40 = 1 /24

Let x days to Complete the Remaining work

Then X and Y together worked for x-10 days and Y worked alone for 10 days

X and Y' s (x-10) days work = (x-10) /24

Then

(x-10) /24 + ¼ = 1

X = 28

2) Total Odd days Upto 14 Feb 2010 = 1 +1+2+1 +1=6

15 Feb was Monday So Count 6 Days Backwards

So Answer is Tuesday

Q 33 **Let Assumed Mean =65**

Classes	Mid-points	f_i	$d_i=x_i -65$	d_i^2	$f_i\,d_i$	$f_i\,d^2_i$
30-40	35	3	-30	900	-90	2700
40-50	45	7	-20	400	-140	2800
50-60	55	12	-10	100	-120	1200
60-70	65	15	0	0	0	0
70-80	75	8	10	100	80	800
80-90	85	3	20	400	60	1200
90-100	95	2	30	900	60	1800
Total		50			-150	10500

Mean = 60 , Variance = 201, S.D = $\sqrt{201}$ =14.18

Hence, $$r_s = 1-\frac{6[\Sigma d^2+\Sigma\frac{1}{12}(m^3-m)]}{n(n^2-1)}$$

$$=1-\frac{6[56+\frac{1}{12}(2^3-2)+\frac{1}{12}(3^3-3)+\frac{1}{12}(2^3-2)+\frac{1}{12}(2^3-2)+\frac{1}{12}(2^3-2)+\frac{1}{12}(2^3-2)]}{10(10^2-1)}$$

$$= 1-\frac{6(60.5)}{990} = 1-0.367 = 0.633.$$

Q 34   Solution: Let M be the set of students who study Mathematics.

P be the set of students who study Physics and C be the set of students who study Chemistry.

Then ,$n(U)$=200 ,$n(M)$=120 ,$n(P)$=90

$n(C)$=70, $n(M\cap P)$=40, $n(P\cap C)$=30 ,$n(C\cap M)$=50, $n(M'\cap P'\cap C')$=20

$n(U)-n(M\cup P\cup C)$=20,

∵$n(M\cup P\cup C)$=200−20=180

⇒$n(M\cup P\cup C)$=$n(M)$+$n(P)$+$n(C)$−$n(C\cap M)$+$n(M\cap P\cap C)$

⇒180=120+90+70−40−30−50+$n(M\cap P\cap C)$

⇒180=160+$n(M\cap P\cap C)$

⇒$n(M\cap P\cap C)$=180−160=20

So, the number of students who study all the three subjects is 20 .

OR

Solution: (AUB) – B = (AUB)∩B′   [(X – Y) = X ∩ Y′]

=(A∩ B′) U (B∩ B′)

$$= (A \cap B') \, U \, \Phi = A - B$$

Solution (b) $A - (A \cap B) = A \cap (A \cap B)'$

$$= A \cap (A' \, U \, B')$$

$$= (A \cap A') \, U \, (A \cap B')$$

$$= \Phi \, U \, (A \cap B')$$

$$= (A \cap B') = A - B$$

Solution (c) $(A U B) - C = (A U B) \cap C'$

$$= (A \cap C') \, U \, (B \cap C')$$

$$= A - C) \, U \, (B - C$$

Q 35   Number of unit consume = 5678 - 4803 = Rs. 875

Energy charge = (150 x 5.5) + (150 x 6)+ (200x6.5) +(375 x7)

= 825+ 900+ 1300+ 2625 = Rs. 5650

Fixed charge = 110 x 4 = 440

Surcharge = 875 x 0.26 = 227.50

Energy tax = 5 % of (5650+ 440) = 304.50

Total electricity bill = 5650 + 440 + 227.50 + 304.50 = Rs. 6622

Q 36)   (i) 5

(ii) 1

(iii) 5+4+4=13   or   6+2+1

Q37) (i)   Interest for the first year is given by SI= $\frac{PRT}{10}$

$=\frac{50000 \times 10 \times 1}{100}$=Rs. 5000

(ii) Interest for the first year = Rs. 5000

Principal for the second year will be 55000.

Therefore, interest for the second year=$\frac{55000 \times 10 \times 1}{100}$=Rs. 5500

(iii) Interest for the second year = Rs. 5500

Principal for the second year will be 60500.

Therefore, interest for the second year=$\frac{60500 \times 10 \times 1}{100}$=Rs. 6050

Therefore, total interest after three years

= 5000 + 5500 + 6050

=Rs. 16550

Total amount = 50000 + 16550 = ` Rs. 66500

OR

Simple interest for two years=$\frac{50000 \times 10 \times 2}{100}$=Rs. 10000

Compound interest for two years

= 5000 + 5500 = 10500

Therefore, difference = 10500 – 10000 = 500

Q 38)   (i) Prob that doctor arriving late and he comes by metro =2/7

(ii) Req prob= 2/7 +5/14 +1/6 +4/21   =1

SAMPLE QUESTION PAPER – 2 (2025 – 26)
CLASS 11 - APPLIED MATHEMATICS

Time Allowed: 3 hours   **Maximum Marks :** 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

I. This Question paper contains 38 questions. All questions are compulsory.

II. This Question paper is divided into five Sections - A, B, C, D and E.

III. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion - Reason based questions of 1 mark each.

IV. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) - type questions, carrying 2 marks each.

V. In Section C, Questions no. 26 to 31 are Short Answer (SA) - type questions, carrying 3 marks each.

VI.   In Section D, Questions no. 32 to 35 are Long Answer (LA) - type questions, carrying 5 marks each.

VII   In Section E, Questions no. 36 to 38 are case study - based questions carrying 4 marks each.

VIII. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub - part each in 2 questions of Section E.

IX. Use of calculators is not allowed.

SECTION A

1   If a single letter is selected at random from the word **PROBABILITY**, then the probability of vowels is   **[1]**

   a) $\frac{2}{11}$

   b) $\frac{1}{3}$

   c) $\frac{4}{11}$

   d) $\frac{3}{11}$

2   The coefficient of correlation is   **[1]**

   a) the square root of the coefficient of determination

   b) the square of the coefficient of determination

   c) greater than the coefficient of determination

   d) equal to the coefficient of determination

3   The income tax act was passed in the year   **[1]**

    a) 1961

    b) 1947

    c) 1991

    d) 1960

4   If $3^{5x} = \frac{1}{81}$, then x is   **[1]**

    a) - 4

    b) 4

    c) $\frac{-4}{5}$

    d) $\frac{4}{5}$

5   If R = {(x, y) : x, y∈ **W** , $x^2 + y^2$ = 169}, then domain of R is   **[1]**

    a) { - 13, - 12, - 5, 0, 5, 12, 13}

    b) {0, 5, 12, 13}

    c) {0, 1, 2, 3,... 13}

    d) {0,± 1, ± 2,... ± 13}

6   2^4 = 16 in logarithmic form is   **[1]**

    a) $\log_4 16 = 2$

    b) $\log_2 16 = 4$

    c) $\log_4 2 = 16$

    d) 4 log 2 = log 16

7   If A and B are two events such that P(B) = $\frac{3}{5}$, P($\frac{A}{B}$) = $\frac{1}{2}$ and P(A ∪ B) = $\frac{4}{5}$, thenP(A)   **[1]**
    equals.

    a) $\frac{3}{5}$

    b) $\frac{1}{2}$

    c) $\frac{1}{5}$

    d) $\frac{3}{10}$

8 The equation of circle which passes through the origin and whose centre is (3, 4) will be: **[1]**

a) $x^2 + y^2 - 3x - 4y = 0$

b) $x^2 + y^2 + 6x + 8y = 0$

c) $x^2 + y^2 - 6x - 8y = 0$

d) $x^2 + y^2 + 3x + 4y = 0$

9 Danger leads to _____. **[1]**

a) Help

b) Fear

c) Attack

d) Enemy

10 Mean deviation of n observations x_1 , x_2 , ... x_n from their mean $\bar{x}$ is **[1]**

a) $\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$

b) $\sum_{i=1}^{n}(x_i - \bar{x})$

c) $\sum_{i=1}^{n}(x_i - \bar{x})^2$

d) $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

11 If log 50 = log 2 $+\frac{x}{2}$ log 5, then x is **[1]**

a) 3

b) 1

c) 4

d) 2

12 The amount of money today which is equal to a series of payments in the future is: **[1]**

I.   nominal value of annuity

II.   sinking value of annuity

III.   present value of annuity

IV.   future value of annuity

a) ii and iii

b) iv and i

c) only iii

d) i and ii

13 A shopkeeper bought a TV from a distributor at a discount of 25% of the listed price of ₹ 32000. The shopkeeper sells the TV to a consumer at the listed price. If the sales are intra - state and the rate of GST is 18%, the tax (under GST) paid by the distributor to the State Government is: **[1]**

a) ₹ 2160

b) ₹ 2880

c) ₹ 720

d) ₹ 4320

14 If A and B are two events such that P(A) = 0.2, P(B) = 0.4 and P(A∪ B) = 0.6, then P($\frac{A}{B}$) is equal to: **[1]**

a) 0.5

b) 0.3

c) 0

d) 0.8

15 If A and B are two independent events such that 0 < P(A) < 1 and 0 < P(B) < 1, then which of the following is not correct? **[1]**

a) A' and B' are independent

b) A and B' are independent

c) A' and B are independent

d) A and B are mutually exclusive

16 Relationship between annual nominal rate of interest and annual effective rate of interest, if frequency of compounding is greater than one: **[1]**

a) Effective rate > Nominal rate

b) Effective rate <= Nominal rate

c) Effective rate < Nominal rate

d) Effective rate = Nominal rate

17 The straight lines l_1, l_2, l_3 are parallel and lie in the same plane. A total number of m points are taken on l_1; n points on l_2; k points on l_3, The maximum number of triangles formed with vertices at these points are   **[1]**

a) $^{(m+n+k)}C_3$

b) $^{(m+n+k)}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$

c) $^mC_3 \times {}^nC_3 \times {}^kC_3$

d) $^mC_3 + {}^nC_3 + {}^kC_3$

18 If n(A) = 3, n(B) = 2, then number of non-empty relations from set A to set B are   **[1]**

a) 8

b) 63

c) 4

d) 64

19 **Assertion(A):** If the variance of a mesokurtic curve is 7, then fourth central moment is 147.   **[1]**

Reason(R): For mesokurtic curve, β_2 = 3.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20 **Assertion (A):** If A.M. and G.M. between two positive numbers are 20 and 16 respectively, then numbers are 32 and 8.   **[1]**

Reason (R): If A and G are A.M. and G.M. respectively of two positive numbers, then numbers are $A \pm \sqrt{A^2 - G^2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

SECTION B

21 A can do a piece of work in 80 days. He works at it for 10 days and then B alone finishes the remaining work in 42 days. In how much time will A and B, working together, finish the work?   **[2]**

22 Find the values of the letter and give a reason for the steps involved.   **[2]**

$$\begin{array}{r} B\ A \\ \times\ B\ 3 \\ \hline 5\ 7\ A \end{array}$$

OR

Find the values of the letter and give a reason for the steps involved.

$$\begin{array}{r} A\ B \\ \times\quad 5 \\ \hline C\,A\,B \end{array}$$

23 A school follows the following criterion for grading a student for annual result.   **[2]**

Assessment	Weightage
Homework	25%
Quiz	30%
Test	10%
Final Exam	35%
	100%

A student Scored 88 marks in Homework, 71 marks in Quiz, 97 marks in Test and 90 marks in final exam for mathematics. Find the students annual result in mathematics.

24 Differentiate the function $\frac{1}{x}$ with respect to x.   **[2]**

OR

Find the derivative of the given function: $\log\left(\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\right)$

25 Convert the decimal number13 to the equivalent binary number.   **[2]**

SECTION C

26 If AM and GM are between two positive numbers x and y are 13 and 12 respectively, find the numbers.   **[3]**

OR

Find three numbers in G.P. whose product is 216 and the sum of their products in pairs is 156.

27 Find the values of the letter and give a reason for the steps involved.   **[3]**

$$\begin{array}{r} 2\,A\,B \\ +\,A\,B\,1 \\ \hline B\,1\,8 \end{array}$$

28 Find domain and range of the real function f(x) = $\frac{1}{1-x^2}$.   **[3]**

29 The simple interest on a certain sum of money for 3 years at 5% per annum is₹ 1200. Find the amount due and the compound interest on this sum of money at the same rate after 3 years, interest is reckoned annually.   **[3]**

30 For an industrial connection monthly consumption of water is 40 Kl, calculate the Water bill. Tariff rates can be considered as the table given below: **[3]**

Monthly Consumption (in Kilolitre)	Service Charge (in ₹)	Volumetric Charge (Per Kl in ₹)
Upto 20	146.41	5.27
20-30	219.62	*26.36
> 30	292.82	43.93
Plus Sewer Maintenance Charges: 60% of water volumetric charge		

31 In the adjacent Venn diagram, if n(ξ) = 80, n(P) = 40, n(Q) = 28, n(P ∩ Q) = 12 and n(P ∩ R) = 10,   [3]
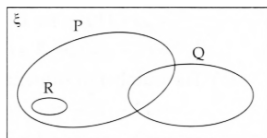
    I.   mark the number of elements in each region.

    II.   determine the value of n (P∪ Q) and n ((Q ∪ R)').

SECTION D

32 Find n, if, (n + 2)! = 60 . (n - 1)!   [5]

OR

If there are 6 periods on each working day of a school, in how many ways can one arrange, 5 subjects such that each subject is allowed at least one period?

33 Evaluate: $\lim_{x \to 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)}$ [5]

34 Find the mean, variance and standard deviation of first 10 multiples of 4.   [5]

OR

Calculate the mean deviation from the mean of the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	5	8	15	16	6

35 Mr. Saxena from Bhopal, M.R has an electricity connection of 5 kW. He consumed 1264 units in one month. Calculate his electricity bill for that month. Tariff plan is given below: [5]

No. of Units (in kWh)	0 - 50	51 -100	101- 300	> 300
Price per unit (in ₹)	4.05	4.95	6.30	6.50

Fixed charge =₹ 250 per kW per month

Surcharge =Nil, Energy duty = ₹ 0.63 per unit

SECTION E

36 **Read the following text carefully and answer the questions that follow:**   [4]

A market is in the form of a triangle whose vertices are B(- 2, 0), C(1, 12). The third vertex A of this trianagle lies on the mid point of the line joining the points (2, 1) and (4, 13).

    I.   What will be the coordinates of A? (1)

II. Find the slope of the line joining the points B and C? (1)

III. Equation of the line joining the points B and C? (2)

OR

Does point A lies on the line BC? (2)

37 **Read the following text carefully and answer the questions that follow:** **[4]**

Different organizations collect the data and analyse it quantitatively. During one such analysis some mistake crept in. The result given was that mean and variance of 100 observations as 40 and 5.1 but later on rechecking it was found that one observation was mistakenly taken as 50 instead of 40.

I. What is incorrect sum of variates? (1)

II. What is correct sum of observations? (1)
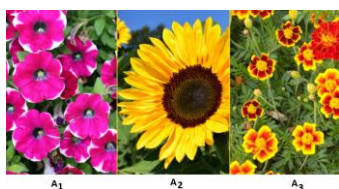
III. What is incorrect Σx^2 ? (2)

OR

What is corrected variance? (2)

38 **Read the following text carefully and answer the questions that follow:** **[4]**

A shopkeeper sells three types of flower seeds A $_1$, A $_2$, and A $_3$. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%.

I. What is the probability of a randomly chosen seed to germinate? (1)

II. What is the probability that the seed will not germinate given that the seed is of type A_3 ? (1)

III. What is the probability that the seed is of the type A_2 given that a randomly chosen seed does not germinate? (2)

OR

Find the probability that it is of the type A $_1$ given that a randomly chosen seed does not germinate? (2)

SECTION A

1. (c) $\frac{4}{11}$

Explanation:

Total number of alphabets in the word 'PROBABILITY = 11

Number of vowels $= 4$

$\therefore$ Required probability $= \frac{4}{11}$

2. (a) the square root of the coefficient of determination

Explanation:

The coefficient of Determination is the square of the Coefficient of Correlation. R square or coeff. of determination shows percentage variation in y which is explained by all the x variables together.

3. (a) 1961

Explanation:

The income tax act was passed in the year 1961.

4.

(c) $\frac{-4}{5}$

Explanation:

$$3^{5x} = \frac{1}{81}$$
$$3^{5x} = \frac{1}{3^4}$$
$$3^{5x} = 3^{-4}$$

Now base is same, so power same

$$\therefore 5x = -4$$
$$x = \frac{-4}{5}$$

5. (b) {0,5,12,13}

Explanation:

Given, R $= \{(x, y): x, y \in \mathbf{W}, x^2 + y^2 = 169\}$

$$\therefore R = \{(0,13), (5,12), (12,5), (13,0)\}$$

$\therefore$ domain of $R = \{0,5,12,13\}$

6. (b) $\log_2 16 = 4$

Explanation:

$2^4 = 16$ in logarithmic form.

As we know that

if $a^y = x$

then $\log_a x = y$

$\therefore \log_2 16 = 4$

7. (b) $\frac{1}{2}$

Explanation:

Here, $P(B) = \frac{3}{5}, P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$

$$\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{1}{2} = \frac{P(A \cap B)}{\frac{3}{5}}$$

$$\Rightarrow P(A \cap B) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} \text{ and } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{4}{5} = P(A) + \frac{3}{5} - \frac{3}{10}$$

$$\therefore P(A) = \frac{4}{5} - \frac{3}{5} + \frac{3}{10} = \frac{8 - 6 + 3}{10} = \frac{1}{2}$$
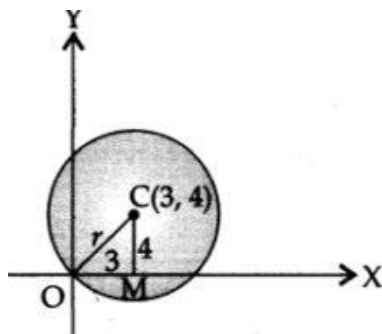
8. (c) $x^2 + y^2 - 6x - 8y = 0$

Explanation:

If the origin be on the circumference of the circle and coordinates of the centre C be (3,4), then from the adjoining figure, it is clear that

$$OC^2 = OM^2 + CM^2$$

i.e., $r^2 = 3^2 + 4^2 = 9 + 16 = 25$

Thus, the equation of required circle with centre (3,4) and radius 25 is given by

$$(x - 3)^2 + (y - 4)^2 = 25$$
$$\Rightarrow (x^2 - 6x + 9) + (y^2 - 8y + 16) = 25$$
$$\Rightarrow x^2 + y^2 - 6x - 8y = 0$$

9. (b) Fear

Explanation:

On the basis of the four options given above it looks as if all the four can be filled in the place of blank space. But we have t select one that is most appropriate and indispensable. We have experienced in our life that danger always leads fear. All the other options are remote but fear is most proximate option.

10. (a) $\frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$

Explanation:

Mean Deviation, MD $= \frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$

where, $\bar{x}$ is mean n is number of observations

11.

(c) 4

Explanation: $\log 50 = \log 2 + \frac{x}{2}\log 5$

$$\log 50 = \log 2 + \log 5^{\frac{x}{2}}$$

$$\log 50 = \log \left(2 \times 5^{\frac{x}{2}}\right)$$

$$\therefore 50 = 2 \times 5^{\frac{x}{2}}$$

$$2 \times 5^{\frac{x}{2}} = 50$$

$$5^{\frac{x}{2}} = 25$$
$$5^{\frac{x}{2}} = 5^2$$
$$\therefore \frac{x}{2} = 2$$
$$x = 4$$

12. (c) only iii

Explanation:

present value of annuity

13. (a) ₹ 2160

Explanation:

₹ 2160

14. (c) 0

Explanation:

Given $P(A) = 0.2, P(B) = 0.4, P(A \cup B) = 0.6$, then

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.2 + 0.4 - 0.6 = 0$$
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = 0$$

15.   (d) A and B are mutually exclusive

Explanation:

hey cannot be mutually exclusive because

$$P(A \cap B) = P(A) \cdot P(B) = \neq = 0 (\because \text{ A and B are independent}$$

16. (a) Effective rate > Nominal rate

Explanation:

If interest is compounded more than once a year the effective interest rate for a year exceeds the per annum nominal interest rate i.e., effective rate > nominal rate

17. (b) $^{(m+n+k)}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$

Explanation:

Here, total number of points are $(m + n + k)$ which must give $^{(m+n+k)}C_3$ number of triangles but m points on line l_1 taking 3 points at a time gives $^mC_3$ combinations which produce no triangle. Similarly, $^nC_3$ and $^kC_3$ number of triangles cannot be formed.

Therefore, the required number of triangles is

$$(m + n + k)C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3.$$

18. (b) 63

Explanation:

as $n(A \times B) = 6$

$\therefore$ Total relations $= 2^6 = 64$

Total non-empty relations $= 64 - 1 = 63$

19. (a) Both A and R are true and R is the correct explanation of A .

Explanation:

Given variance $(\mu_2) = 7$

We know that for mesokurtic curve, $\beta_2 = 3$.

∴ R is true.

Now, $\beta_2 = \frac{\mu_4}{\mu_2^2}$

$$\Rightarrow 3 = \frac{\mu_4}{7^2} \Rightarrow \mu_4 = 3 \times 49 = 147$$

∴ A is true and R is the correct explanation of A .

20. (a) Both A and R are true and R is the correct explanation of A .

Explanation:

Let a and b be two positive numbers, then

$A = \frac{a+b}{2}$ and $G^2 = ab$

$\Rightarrow a + b = 2A$ and $ab = G^2$.

Now, the quadratic equation whose roots are a and b is

$$x^2 - (a + b)x + ab = 0$$
$$\Rightarrow x^2 - 2Ax + G^2 = 0.$$

So, roots of the above quadratic equation are

$$x = \frac{-(-2A) \pm \sqrt{(-2A)^2 - 4 \times 1 \times G^2}}{2 \times 1}$$

$$\Rightarrow x = A \pm \sqrt{A^2 - G^2}$$

∴ R is true.

Now, given $A = 20, G = 16$.

So, numbers are $20 \pm \sqrt{(20)^2 - 16^2} = 20 \pm \sqrt{400 - 256} = 20 \pm 12 = 32$ and 8 .

∴ A is true and R is the correct explanation of A .

SECTION B

21. Work done by A in 10 days $= \frac{1}{80} \times 10 = \frac{1}{8}$

Remaining work $= \left(1 - \frac{1}{8}\right) = \frac{7}{8}$

Now, $\frac{7}{8}$ work is done by B in 42 days

Whole work will be done by B in $\left(42 \times \frac{8}{7}\right) = 48$ days

∴ A's 1 day's work $= \frac{1}{80}$ and B's 1 days' work $= \frac{1}{48}$

∴ (A + B) 's 1 days' work $= \frac{1}{80} + \frac{1}{48} = \frac{8}{240} = \frac{1}{30}$

Hence, both will finish the work in 30 days.

22.
```
   BA
 × B3
  57 A
```
$A \times 3 = A$

$\Rightarrow A = 5[\because 3 \times 5 = 15]$

```
       B5
     × B3
   ────────
   (3 B+1)5
   B²(5 B) ×
   ────────
     5 7 5
```

$$\Rightarrow 3 B + 1 + 5 B = 7 \dots (1)$$

if $B = 2$ i.e.

$3 \times 2 + 1 + 5 \times 2 = 17$

$$\Rightarrow B = 2$$

$$25$$
$$\times\ 23$$
$$75$$
$$50\ \times$$
$$\overline{575}$$
$$A = 5, B = 2$$
$$2$$
$$AB$$
$$\times\ 5$$
$$CAB$$
$$5 \times B = B$$
$$\Rightarrow\ B = 5[\because 5 \times 5 = 25]$$
$$\text{Now, } 5 \times A + 2 = A$$
$$\Rightarrow\ A = 7[\because 5 \times 7 + 2 = 37]$$

Thus

$$75$$
$$\times\ 5$$
$$\overline{}\ 375$$
$$A = 7, B = 5, C = 3$$
$$A = 3$$

23. Weighted mean $= \dfrac{\sum_{i=1}^{n} x_i w_i}{\sum_{i=1}^{n} w_i} = \dfrac{25 \times 88 + 30 \times 71 + 10 \times 97 + 35 \times 90}{100}$

$$= \frac{8450}{100} = 84.5$$

$\therefore$ student obtained 84.5%

24. diff. $\dfrac{1}{x}$ w.r.t. x

$$y = \frac{1}{x}$$
$$\frac{dy}{dx} = \frac{d}{dx}x^{-1}$$
$$= -1x^{-1-1}$$
$$= -x^{-2}$$
$$= -\frac{1}{x^2}$$

<div align="center">OR</div>

Let $y = \log\left(\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\right) = \log\left(\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} \times \dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\right)$

$= \log\left(\dfrac{(x+1)+(x-1)+2\sqrt{x+1}\sqrt{x-1}}{(x+1)-(x-1)}\right) = \log\left(\dfrac{2x + 2\sqrt{x^2-1}}{2}\right)$

$= \log\left(x + \sqrt{x^2-1}\right)$, differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2-1}} \cdot \left[1 + \frac{1}{2}(x^2-1)^{\frac{-1}{2}} \cdot 2x\right]$$

$$= \frac{1}{x+\sqrt{x^2-1}} \cdot \left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{1}{x+\sqrt{x^2-1}} \cdot \frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}} = \frac{1}{\sqrt{x^2-1}}$$

25. Step I: Divide the number by 2 to get the quotient. Keep the whole part for the next step and set the remainder aside.

Step II: Divide the whole part of the quotient from step I by 2. Again, keep the whole part for the next step and set the remainder aside.

Step III: Repeat step 2 until the whole part is 0.

The given decimal number is 13

$$
\begin{array}{c|c|c}
2 & 13 & \\
\hline
2 & 6 & 1 \\
\hline
2 & 3 & 0 \\
\hline
2 & 1 & 1 \\
\hline
2 & 0 & 1 \\
\end{array}
$$

Put the remainder together in reverse order.
So the required binary number is 1101 .

SECTION C

26. Given: $\frac{x+y}{2} = 13 \Rightarrow x + y = 26$

and $\sqrt{xy} = 12 \Rightarrow xy = 144$

(4) Mathpix Snipping

From (i) and (ii)

$$
\begin{aligned}
& x(26 - x) = 144 \Rightarrow 26x - x^2 = 144 \\
& \Rightarrow x^2 - 26x + 144 = 0 \\
& \Rightarrow (x - 18)(x - 8) = 0 \\
& \Rightarrow x - 18 = 0 \text{ or } x - 8 = 0 \\
& \Rightarrow x = 18 \text{ or } 8
\end{aligned}
$$

$\therefore$ Numbers are 18, 8 or 8, 18.

OR

Let three numbers in G.P. be $\frac{a}{r}$, a, ar

$\therefore$ Their product $= \frac{a}{r} \cdot a \cdot ar = 216$ (given)

$$
\Rightarrow a^3 = 216 = (6)^3 \Rightarrow a = 6
$$

Also sum of their products in pairs $= 156$ (given)

$$
\begin{aligned}
& \Rightarrow \frac{a}{r} \cdot a + a \cdot ar + ar \cdot \frac{a}{r} = 156 \\
& \Rightarrow a^2 \left(\frac{1}{r} + r + 1\right) = 156 \\
& \Rightarrow 6^2 \cdot \frac{1 + r^2 + r}{r} = 156 \\
& \Rightarrow 3 \cdot \frac{r^2 + r + 1}{r} = 13 \\
& \Rightarrow 3r^2 + 3r + 3 = 13r \\
& \Rightarrow 3r^2 - 10r + 3 = 0 \\
& \Rightarrow (r - 3)\left(r - \frac{1}{3}\right) = 0 \Rightarrow r = 3, \frac{1}{3}
\end{aligned}
$$

When $r = 3$, numbers are $2, 6, 18$ and when $r = \frac{1}{3}$, numbers are $18, 6, 2$

27.
$$
\begin{array}{r}
2\,AB \\
+AB\,1 \\
\hline
B18
\end{array}
$$

We have to find the value of A and B .

∵ B + 1 we get 8 , i.e., a number whose unit digit is 8 .
For this, B must be 7 .
So the question has been decoded as

$$
\begin{array}{r}
2\,A7 \\
+A71 \\
\hline
718
\end{array}
$$

∵ A + 7 we get 1 , i.e., a number whose unit digit is 1 .
For this, A must be 4 , as 4 + 7 = 11.
So the question has been decoded as,

$$
\begin{array}{r}
1 \\
247 \\
+471 \\
\hline
718
\end{array}
$$

Hence A = 4 and B = 7.

28. For domain: $1 - x^2 \neq 0\,x \neq \pm 1$

$$\therefore \text{ Domain } = R - \{-1,1\}$$

For range: $y = \frac{1}{1-x^2} \Rightarrow y - yx^2 = 1 \Rightarrow yx^2 = y - 1$

$$x = \pm \sqrt{\frac{y-1}{y}}$$

$$\frac{y-1}{y} > 0, y \neq 0 \Rightarrow y^2 - y > 0$$

$$\Rightarrow \left(y - \frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^2 \Rightarrow y - \frac{1}{2} > \frac{1}{2} \text{ or } y - \frac{1}{2} < -\frac{1}{2}$$

$$\Rightarrow y > 1 \text{ or } y < 0$$

$$\text{Range } = (-\infty, 0) \cup (1, \infty)$$

29. Given simple interest for 3 years = ₹1200

∴ Simple interest for one year $= \frac{1}{3}$ of ₹ 1200 = ₹400

$$\text{S.I. } = \frac{P \times R \times T}{100} \Rightarrow ₹400 = \frac{P \times 5 \times 1}{100}$$

$$\Rightarrow P = ₹\frac{400 \times 100}{5 \times 1} = ₹8000$$

Amount after one year = ₹8000 + ₹400 = ₹8400
Principal for the second year = ₹8400
Interest for the second year $= ₹\frac{8400 \times 5 \times 1}{100} = ₹420$
Amount after 2 years = ₹8400 + ₹420 = ₹8820

Interest for the third year $= ₹\frac{8820 \times 5 \times 1}{100} = ₹441$

Amount due after 3 years $= ₹8820 + ₹441 = ₹9261$

Compound interest for 3 years $= ₹9261 - ₹8000 = ₹1261$

30. Volumetric Charge for consumption upto 20kl $= ₹20 \times 5.27 = ₹105.4$

Volumetric Charge for consumption between $20 - 30kl = ₹10 \times 26.36 = ₹263.6$

Volumetric Charge for consumption between $30 - 40kl = ₹10 \times 43.93 = ₹439.3$

Total volumetric Charge for consumption of 40kl $= ₹(105.4 + 263.6 + 439.3) = ₹808.3$

Service Charge $= ₹292.82$

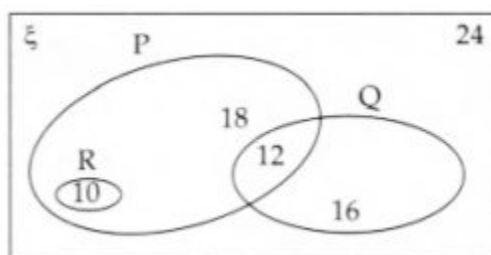Sewage Charges $= 60\%$ of Volumetric Charges

$$= 808.3 \times 60\% = ₹484.98$$

Amount of water bill for the given month $= ₹(808.3 + 292.82 + 484.98) = ₹1586.1$

Thus, amount of domestic water bill is ₹ 1586

31. i. The number of elements in different regions are shown in the adjoining figure.

ii. From the Venn diagram, we get

$$n(P \cup Q) = 10 + 18 + 12 + 16 = 56$$
$$n(Q \cup R) = 16 + 12 + 10 = 38$$
$$\Rightarrow n((Q \cup R)') = 80 - 38 = 42$$

Section D

32. $(n + 2)! = 60(n - 1)!$

$$\Rightarrow (n + 2)(n + 1)n(n - 1)! = 60(n - 1)!$$
$$\Rightarrow (n + 2)(n + 1)n = 5 \times 4 \times 3$$
$$\Rightarrow n = 3$$

OR

Out of six periods 5 subjects can be arranged in $^6P_5$ ways and for remaining period any subject can be taken. Total arrangements =

$$^6P_5 \times {}^5P_1 = 720 \times 5 = 3600$$

33. We have, $\lim\limits_{x \to 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)} = \lim\limits_{x \to 0} \frac{\left(2 \times \sin \frac{3x+5x}{2} \times \cos \frac{3x-5x}{2}\right)}{\left(2 \times \cos \frac{6x+4x}{2} \sin \frac{6x-4x}{2}\right)}$

$= \lim\limits_{x \to 0} \frac{\sin 4x \cos x}{\cos 5x \sin x}$

$$= \lim_{x \to 0} \frac{\sin 4x}{\cos 5x \times \frac{\sin x}{\cos x}} \times \frac{4x}{4x}$$

$$= 4 \times \lim_{x \to 0} \frac{\sin 4x}{4x} \times \frac{1}{\cos 5x} \times \frac{x}{\tan x} \left[\because \lim_{x \to 0} \frac{\sin \theta}{\theta} = 1 \text{ and } \lim_{x \to 0} \frac{\theta}{\tan \theta} = 1 \right] = 4$$

$$\therefore \lim_{x \to 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)} = 4$$

34. Here, the variables are

$$4,8,12,16,20,24,28,32,36,40.$$

Here, N = 10,

So, Mean, $\bar{x} = \frac{4+8+\cdots+40}{10}$

$$= \frac{4(1+2+\cdots 10)}{10}$$
$$= \frac{4}{10} \cdot \frac{10(10+1)}{2}$$
$$= 2 \times 11$$
$$= 22$$

x_i	$X_{i-\bar{x}}$	$(x_i - \bar{x})^2$
4	-18	324
8	-14	196
12	-10	100
16	-6	36
20	-2	4
24	2	4
28	6	36
32	10	100
36	14	196
40	18	324
Total	$\sum(x_i - \bar{x}) = \mathbf{0}$	$\sum(x_i - \bar{x})^2 = 1320$

We have n = 10, $\sum(x_i - \bar{x})^2 = 1320$

$$\therefore \text{ Variance, } \sigma^2 = \frac{1}{n} \sum (x_i - \bar{x})^2 = \frac{1}{10} \times 1{,}320 = 132$$

and $\sigma = \sqrt{132} = 11.49$

$\therefore$ Mean = 22, Variance = 132 (Approx.) and Standard deviation = 11.49 (Approx.).

OR

We make the table from the given data.

Class	Mid value (x_i)	f_i	$u_i = \dfrac{x_i - 25}{10}$	$f_i u_i$	$|x_i - \bar{x}|$	$f_i|x_i - \bar{x}|$
0-10	5	5	-2	-10	22	110
10-20	15	8	-1	-8	12	96
20-30	25	15	0	0	2	30
30-40	35	16	1	16	8	128
40-50	45	6	2	12	18	108
Total		$\sum f_i = 50$		$\sum f_i u_i = 10$		$\sum f_i|x_i - \bar{x}| 472$

Here, $\sum f_i = 50$, $a = 25$, $\sum f_i u_i = 10$, $h = 10$ and $\sum f_i|x_i - \bar{x}| = 472$

$$\therefore \text{ Mean, } \bar{x} = 25 + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 25 + \frac{10}{50} \times 10 = 27 \left[\because \bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h \right]$$

$$\therefore \text{ Mean deviation from mean } = \frac{\sum f_i|x_i - \bar{x}|}{\sum f_i} = \frac{472}{50} = 9.44$$

35. Electricity consumed = 1264 unit (Kwh)

Unit Range	Unit (KWh)	Rate (₹/Kwh)	Charge (₹)
0-50	50	4.05	202.5
51-100	50	4.95	247.5
101-300	200	6.30	1260.0
300-1264	964	6.50	6266.0
Total Electricity Charges (₹)	1264		7976.0

Fixed Charges (₹ 250/KW) = $250 \times 5 = 1250$

Energy duty (₹ 63/ Unit) = $0.63 \times 1264 = 796.32$

$\therefore$ Electricity bill = ₹7976.0 + ₹1250.0 + ₹796.32 = ₹10022.32

SECTION E

36. i. Midpoint of (2,1) and (4,13) is given by (3,7).

ii. Slope $= \frac{12-0}{1-(-2)} = \frac{12}{3} = 4$

iii. Equation of BC: $= (y - 0) = 4(x + 2)$

$= y - 2 = 4x$

OR

Equation of BC is $y - 2 - 4x = 0$. Putting coordinates of A(3,7) in this equation, we get: 7-2-12 $\neq$ 0, therefore we ca that point A will not lie on the line BC .

37. i. Mean $= 40$, n $= 100$, sum $= 100 \times 40 = 4000$

ii. Corrected sum $= 4000 - 50 + 40 = 3990$

iii. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$$\Rightarrow (5.1)^2 = \frac{\Sigma x^2}{100} - (40)^2$$

$$\Rightarrow (26.01 + 1600)100 = \Sigma x^2$$

OR

Corrected $\Sigma x^2 = 162601 - (50)^2 + (40)^2$

$= 162601 - 2500 + 1600 = 161701$

Corrected $\sigma^2 = \frac{161701}{100} - (39.9)^2$

$= 1617.01 - 1592.01 = 25$

38. i. The probability of a randomly chosen seed to germinate is 0.49 .

ii. $\frac{65}{100}$

iii. $\frac{16}{51}$

BLUE PRINT

S.N	UNIT	Section A (1 mark)	Section B (2Marks)	Section C (3Marks)	Section D (5Marks)	Section E (4Marks)	Total
1	Numbers, Quantification &Numerical Application	4(4)	-------	-------	1(5)	----	5(9)
2	Algebra	1(1)	1(2)	1(3)	1(5)	1(4)	5(15)
3	Mathematical Reasoning	1(1)	1(2)	1(3)	--------	-------	3(6)
4	Calculus	3(3)	2(4)	1(3)	------	-------	6(10)
5	Probability	1(1)	------	1(3)	------	1 (4)	3(8)
6	Descriptive Statistics	4(4)	-----	1(3)	1(5)	-------	6(12)
7	Basics of Financial Mathematics	4(4)	1(2)	---------	1(5)	1 (4)	7(15)
8	Co-ordinate Geometry	2(2)	-------	1(3)	--------	-------	3(5)
		20(20)	5(10)	6(18)	4(20)	3(12)	38(80)

CLASS: XI   Max. Marks:80
 Subject: Applied Mathematics   Time: 3 Hours

General Instructions:

This question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.

Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.

Section – A: It comprises of 20 MCQs of 1 mark each.

Section – B: It comprises of 5 VSA type questions of 2 marks each.

Section – C:It comprises of 6 SA type of questions of 3 marks each.

Section – D: It comprises of 4 LA type of questions of 5 marks each.

Section – E: It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks.

SECTION-A

1. The value of radix in binary number system is: -

(a) 1   (b)   2   (c)   8   (d)   10

2. How many odd days are in 300 years : -

(a) 0   (b)   1   (c) 2   (d) 3

3.Characteristic of log(48.75) is:-

(a) 1   (b)   2   (c)   3   (d) 4

4.The angle between hour hand and minute hand at 8:30 A.M

(a) 75^0 (b) 80^0 (c) 85^0 (d) 45^0

5.If A = {1,2} and B = {4,5,6}, then number of relations from B to A is:-

(a) 8   (b)   16   (c)   32   (d)   64

6. $\lim\limits_{x\to 0} \frac{|x|}{x}$ is equal to

(a) 1   (b)   -1   (c)   2   (d) does not exist.

7.If f(x) =px + q, where p and q are integers f(-1) = 1 and f(2) = 13 ,then p and q are :-

(a) p =4,q=5 (b) p= -4 ,q=5   (c) p= -4 ,q = -5 (d) p= 4,q = -5

8.Derivative of $\frac{1}{x}$ with respect to x is:-

(a) - x—2 (b) 0 (c) x—2 (d) None of these

9. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, then find $P(A \cap B)$. If A and B are independence events.

(a) $\frac{4}{5}$ (b $\frac{3}{25}$ (c) $\frac{1}{25}$ (d) $\frac{3}{5}$

10. The measure central tendency of a statistical data which takes into account all the data is .

(a) Mean (b) Median (c) Mode (d) Range.

11. The size of each payment of an annuity is called -------.

(a) Principal (b) payment period (c) periodic payment (d) Term

12. The variance of first 5 natural number is:-

(a) 1 (b) 3 (c) 2 (d) none of these

13. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A fails or B fails is: -

(a) 0.5 (b) 0.06 (c) 0.56 (d) 0.44

14. :-Full form of GST.

(a) Goods and sale tax. (b) Goods and sell tax

(c) Goods and services tax (d) Goods and services transports.

15. Quartiles divide the data set into --------- equal parts.

(a) 3 (b) 4 (c) 5 (d) 2

16. For what value of k are the points (1,5),(k,1) and (4,11) collinear?

(a) 1 (b) - 1 (c) 2 (d) - 2

17. If equation of circle $x^2 + (y-1)^2 = 1$,then point lie on the circle is:

(a) (0,0) (b) (1,0) (c) (0,1) (d) (1,1)

18. The score of a MCQ test of 10 students are given below:

13,52,42,22,44,105,45,88,88,76

The percentile rank of score 88 is:-

(a) 75 (b) 85 (c) 80 (d) 90

Each of these questions contains two statements Assertion (A) and Reason (R). Read the given Statement carefully and choose the correct answer from the four options given below.

(a) A is true, R is true and R is correct explanation of A.

(b) A is true, R is true and R is not correct explanation of A.

(c) A is true and R is false.

(d) A is false and R is true.

19.Assertion(A) If A and B are mutually exclusive events associated with a random experiment, then

P(A∪B) = P(A) + P(B).

 Reason (R) : For exhaustive events A and B ,P(A∪B) =1 .

20. Assertion(A): Assertion (A) I: If A ⊂ B then A ∩ B = A

 Reason (R): If A and B two sets, then A ⊂ B means every element of Set A is also element of B.

SECTION: B

21.Let A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, find A × (B ∩ C).

22.In certain language if "INDIA" is coded as "KMFHC",how "AMERICA" is coded?

OR

Find the value of x if $\log\log_{10} x - \log_{10}(2x - 1) = 1$

23. If $\lim_{x \to 2} \frac{x^n - 16}{x-2} = 32$ .Find the value of n.

24. Find derivative of $\frac{x+1}{x-1}$ w.r.to x .

25.In single throw of two dice, what is probability of getting

(a) A total of 9?   (b) Doublets?

OR

If an integer from 1 to 1000 is chosen at random, find the probability that the integer is a multiple of 2 and 9.

SECTION: C

26.In a G.P ,the third term is 24 and 6th term is 192.find the 10th term.

OR

If A_1 and A_2 are two arithmetic mean between a and b, then prove that $A_1 + A_2$ = a + b .

27. A and B are brothers, C and D are sisters. A's son is D's brother. How B is related to C.

28 . If y= $x + \frac{1}{x}$, prove that $x^2 \frac{dy}{dx} - xy + 2 = 0$

OR

Find the value of k if the function

$$f(x) = \begin{cases} kx^2 + 2 & if \ x \leq 2 \\ 6 & if \ x > 2 \end{cases} \quad \text{is continuous at x=2}$$

29. Find the probability that a leap year will have (a) 53 Mondays, (b) 52 Mondays.

OR

Ten Cards 1 to 10 are placed in a box A card is drawn from the box at random. If the number on card drawn card is more than 5, what is the Probability that it is an odd number.

30. Find the equation of the straight line that passes through the point (3, 4) and perpendicular to the line $3x + 2y + 5 = 0$.

31.   Find the mean deviation about the median of following data

x	5	7	9	10	12	15
f	8	6	2	2	2	6

SECTION :D

32. If 4th October 1986 was Saturday, what would be the day on 10th April 1991?

OR

100 persons begin to work together on a project which was expected to be completed in 40 days. But after few days 40 persons left. As a result, the project got delay by 10 days. How many days after the commencement of the project did the 40 persons left?

33. How many words can be made from the letter in the word MONDAY, assuming that no letter is repeated if

(a) 4 letters are used at a time?

(b) all letters are used at a time?

(c) all vowels are togethers?

34. Find the mean, variance and standard deviation of following data:

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

OR

For the following frequency distribution, compute the percentile rank corresponding to the score 66.

C.I	48-52	53-57	58-62	63-67	68-72	73-77	78-82	83-87	88-92	93-97
Frequency	4	5	10	7	6	3	8	5	7	4

35 A shopkeeper buys an article whose list price is ₹8000 at some rate of discount from a wholesaler. He sells the article to a consumer at the list price. The sales are intra-state and the rate of GST is 18%. If the shopkeeper pays a tax (under GST) of ₹72 to the State Government, find the rate of discount at which he bought the article from the wholesaler.

SECTION: E

36. In a University, out of 100 students 15 offered Mathematics only,12 offered Statistics only,8 offered only Physics ,40 offered Physics and Mathematics ,20 offered Physics and Statistics ,10 offered Mathematics and Statistics ,65 offered Physics.

On the basis of above information answer the following questions.

(a) How many students offered Mathematics?

(b) How many students offered Statistics?

(c) How many students did not offer any of the above subject?

37.A manufacturer listed the price of his goods at Rs 1600 per articles. He allowed a discount of 25% to a wholesaler, who in turn allowed a discount of 20% on the listed price to a retailer. The retailer sells one article to a consumer at a discount of 5% on listed price. All sales are intra-state and rate of GST is 5%.

  Based on the above information, answer the following questions

(a) The price per article inclusive of GST, which the retailer is pays.

(b) The amount which the consumer is pays for a article.

 (c) The GST paid by the wholesaler to the state government for a article.

<div align="center">OR</div>

The GST paid by the retailer to the Central government for a article.

38. Amara is planning for her retirement. She wants to ensure she has enough money to live comfortably once she stops working at age 60. She plans to invest in a retirement annuity where she will deposit Rs500 at the end of every month into a fund that earns an interest rate of 6% compounded monthly.

She plans to invest in this annuity for 25 years, starting at age 35 and continuing until she turns 60.
Using the case study above, answer the following:

(a) Identify the type of annuity Amara is using and explain why it fits that type.

(b) Calculate how much money Amara will have in her annuity account at the time of retirement.   Given $(1.005)^{300} = 4.467744$

 (c) Explain how increasing the interest rate or duration would impact her retirement savings.

SECTION: A

1   (b)

Solution: Use two digits 0 and 1.

2   (b)

Solution: 100 years has 5 odd days ,200 years has 10 odd days and 300 years has 15 odd days. $7 \times 2 + 1$ i.e 1 odd day.

3   (a)

Solution: Standard form 4.875×10^1

4   (a)

Solution Formula for angle between hour and minute hands

Angle=|30H−5.5M|

Where:

- H = hour
- M = minutes

At **8:30**:

- H=8H
- M=30M

Angle=|30×8−5.5×30|=|240−165|=75^0

5   (d)

Solution: Number of relations $2^{mn} = 2^6 = 64$

6   (d)

 Solution: LHL = -1 and RHL =1 .

7   (a)

Solution: Now we have a system of two linear equations with two variables p and q: Equation 1: −p+q=1 Equation 2: 2p+q=13 . The final answer is p=4,q=5 .

8   (a)

Solution: derivative is $\frac{-1}{x^2}$

9   (b)

Solution: P(A∩B) = P(A) P(B) = $\frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$.

10   (a)

 Solution: The mean is calculated by summing all the values in a dataset and dividing by the total number of values. This inherently means that every single data point contributes to the final calculated value of the mean.

11   (c)

Solution: The size of each payment of an annuity is typically called an installment or **a** periodic payment**.**

12   (c)

 Solution : Variance of first n natural number =$\frac{n^2-1}{12} = \frac{25-1}{12}= 2$

13   (d)

Solution: Since A and B failing are independent events, the probability of both A and B failing P(A∩B)) is the product of their individual probabilities:

P(A∩B)=P(A)×P(B)

P(A∩B)=0.2×0.3=0.06

Now, substitute the values into the union formula:

P(A∪B)=0.2+0.3−0.06 =0.5−0.06

P(A∪B)=0.44

14 (c)

Solution: Good Service Tax.

15 (b)

Solution: Quartiles divide the data set into **four** equal parts

16 (b)

Solution 0=2⅟₂|1(1−11)+k(11−5)+4(5−1)| 0=2⅟₂|1(−10)+k(6)+4(4)| 0=2⅟₂|−10+6k+16| 0=2⅟₂|6k+6|

Multiply both sides by 2: 0=|6k+6|

This means: 6k+6=0 6k=−6 k=−1

17 (d)

Solution: Equation of the circle is given as $x^2+(y−1)^2=1$, satisfy the point (1,1).

18 (c)

Solution: Arranging the data in ascending order, the apply ,$P_R = \frac{L+0.5 \times E}{n} \times 100$, L = 7,E=2 and n =10

19 (b)

Solution: since, A∩B = 0 , P(A∪B) = P(A) + P(B) is correct .R is also correct but no relation between A and R

20 (a)

Solution: Assertion and Reason both are true and R is correct explanation of A.

SECTION B

21. B ∩ C = { 4 } and A × (B ∩ C)= { (1,4),(2,4),(3,4)}.

22. Letters of word INDIA are moved 2 steps forward and one step backward alternatively. Therefore AMERICA is coded as CLGQKBC.

OR

Use Property of logarithm, then $\frac{x}{2x-1} = 10$, x = $\frac{10}{19}$

23. $\lim_{x \to 2} \frac{x^n -16}{x-2} = \lim_{x \to 2} \frac{x^n -2^4}{x-2} = 4 \times 2^{4-1} = 32$, n =4

24   of $\frac{dy}{dx} = \frac{(x-1)\times1-(x+1)\times1}{(x-1)^2} = \frac{-2}{(x-1)^2}$.

25. n(S) = 36, (a) n(Sum of 9) = 4 , P(a total of 9) = $\frac{1}{9}$

  (b) P(Doublets) =$\frac{6}{36} = \frac{1}{6}$

Or

18,36, 54 ….990. (E) = 55 and n(S) = 1000 ,P(Divisible by 2 and 9) = $\frac{55}{1000} = \frac{11}{200}$

26 $ar^2 = 24$, $ar^5 = 192$, solving r = 2 and a = 6 ,Therefore 10 term = 3024.

OR

a, A_1, A_2, b are in A.P . b = a +3d , d $= \frac{b-a}{3}$

A_1 = a + d = a $+ \frac{b-a}{3}$

A_2 = a +2 d = a $+ \frac{2(b-a)}{3}$ or $A_1 + A_2$ = a+ b

27. A's son is D's brother: This means A's son, D, and C (since C and D are sisters) all share the same father.

A is the father of D and C's brother: Therefore, A is the father of C and D.

A and B are brothers: Since A is C's father, and B is A's brother, B is C's paternal uncle.

So, B is C's uncle.

28. y= $x + \frac{1}{x}$, $\frac{dy}{dx} = 1 - \frac{1}{x^2}$

Or $x^2 \frac{dy}{dx} = x^2 - 1$

L.H.S $= x^2 \frac{dy}{dx} - xy + 2 = x^2 - 1 - (x^2 - 1) + 2 = 0$ RHS

OR

L.H.L. $= \lim\limits_{x \to 2^-} kx^2 + 2$ = 4k+2

R.H.L. = 6, f(2) = 4k+2

As the function is continuous at x= 2,s L.H.L= R.H.L= f(2)

4k+2 = 6 ⇒ $4k = 4$ ⇒ $k = 1$

29. 366 = 52× 7 + 2 (odd days)

$S = \{(S, M), (M, T), (T, W), (W, Th), (Th, F), (F, Sa), (Sa, S)\}$

A : Event of getting 53 Mondays

$A = \{(S, M), (M, T)\}$ $P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$

B : Event of getting 52 Mondays

$B = \{(T, W), (W, Th), (Th, F), (F, Sa), (Sa, S)\}$ $P(A) = \frac{n(B)}{n(S)} = \frac{5}{7}$

OR

A – Event that number on the card drawn n odd

B – Event that number on the card is more than 5.

$S = \{1, 2, 3 ..... 10\}$ $A = \{1, 3, 5, 7, 9\}$ $B = \{6, 7, 8, 9, 10\}$

$A \cap B = \{7, 9\}$

$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{2/10}{5/10} = \frac{2}{5}$

30. Given equation is: 2y = -3x-5

$$y = \frac{-3}{2}x - \frac{5}{2}, \text{ so slope} = \frac{-3}{2}$$

slope of the perpendicular line $= \frac{-1}{m} = \frac{2}{3}$

The equation of the line perpendicular to the line 3x + 2y + 5 =0 and passes through (3, 4).is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{3}(x - 3)$$

Required Equation of line 2x-3y- 6= 0.

31. Median = 7

x	5	7	9	10	12	15	
f	8	6	2	2	2	6	Sum =26
c.f	8	14	16	18	20	26	
I x -M I	2	0	2	3	5	8	
f Ix-MI	16	0	4	6	10	48	Sum =84

$$M.D = \frac{1}{n}\sum f_{i\,|x-M|} = \frac{84}{26} = 3.23(\text{Appr.})$$

32. Number of odd days:

From 5th October 1986 to 4th October 1987 = 1

From 5th October 1987 to 4th October 1988 = 2 (1988 is a leap year)

From 5th October 1988 to 4 October 1989-1

From 5th October 1989 to 4 October 1990 = 1

Now from 5th October 1990 to 10th April 1991

Number of days = 27 + 30 + 31 + 31 + 28 + 31 + 10 = 188 days
                  Oct   Nov   Dec   Jan   Feb   March April
                  1990   1990   1990 1991   1991   1991   1991

So, number of odd days = 7 x 26+6 ie. 6 odd days
Total number of odd days = 1+2+1+1+6=11

11=7 ×1+44 odd days.
Since 4th October 1986 was Saturday and there are 4 odd days upto 10th April 1991 ie. Sunday, Monday, Tuesday, Wednesday

So , April 1991 Wednesday.

<center>OR</center>

Let 40 persons leave after x days of the commencement of the project.

Given 100 persons can do the project in 40 days.

So, 100 person's one day's work = 1 / 40

100 person's x day's work = x / 40

Remaining work = 1- x / 40 = (40-x) / 40

After leaving 40 persons, we are left with 60 persons. These 60 persons finished the work in
    (40-x+10) days i.e. 50-x days.

60 person's one day's work = 60 / (40 × 100) = 3 / 200

60 person's (50-x) day's work = 3(50-x) / 200

3(50-x) / 200 = (40-x) /40

5(40-x) = 3(50 – x)
X=40

40 persons left after 25 days of the commencement of the project.
33.The formula for permutations is: $P(n ,r)=n!/(n−r)!$

So, for (a): $P(6,4)=6!/(6−4)!=6!/2!=(6×5×4×3×2×1)/(2×1)$ $P(6,4)=720/2=360$

Answer (a): 360 words

(b) all letters are used at a time?

This means we are arranging all 6 distinct letters. This is a permutation of n distinct items taken n at
a time, which is n!.

So, for (b): $P(6,6)=6!=6×5×4×3×2×1=720$

Answer (b): 720 words

(c) all vowels are together?

First, identify the vowels in "MONDAY": O, A. Treat the group of vowels (OA) as a single unit.

Now, the "items" to arrange are: (OA), M, N, D, Y. There are 5 such items.

The number of ways to arrange these 5 items is 5!. $5!=5×4×3×2×1=120$

Next, consider the internal arrangement of the vowels within their group (OA). The vowels are O
and A, and they can be arranged in 2! ways. $2!=2×1=2$

Total words = (Arrangement of blocks) × (Arrangement of vowels) Total words =
$5!×2!=120×2=240$

Answer (c): 240 words

34.Let Assumed Mean =65

Classes	Mid-points	f_i	$d_i=x_i$ -65	d_i^2	$f_i d_i$	$f_i d^2_i$
30-40	35	3	-30	900	-90	2700
40-50	45	7	-20	400	-140	2800

50-60	55	12	-10	100	-120	1200
60-70	65	15	0	0	0	0
70-80	75	8	10	100	80	800
80-90	85	3	20	400	60	1200
90-100	95	2	30	900	60	1800
Total		50			-150	10500

Mean = 60 , Variance = 201, S.D = $\sqrt{201}$ =14.18

OR

C.I	Frequency	cf
47.5-52.5	4	4
52.5-57.5	5	9
57.5-62.5	10	19=C
62.5-67.5	7 =f	26
67.5-72.5	6	32
72.5-77.5	3	35
77.5-82.5	8	43
82.5-87.5	5	48
87.5-92.5	7	55
92.5-97.5	4	59
	n =59	

Here x = 66-62.5=3.5,C =19 ,f=7,h=5 and n=59

$$P_R = \left(\left(C + \frac{x}{h} \times f\right) \frac{100}{n}\right) = 40.50$$

35. List of prices of an article = ₹8000

   Let the rate of discount given by wholesaler = x%

   So,

   Discount = x% of ₹8000

   $\qquad = \frac{x}{100}$ x ₹8000

   $\qquad = ₹80x$

   CP of an article for shopkeeper = ₹8000 – ₹80x

   It is given that, CP of article for consumer = ₹8000

   Since the sales are intra-state, rate of GST = 18%

   CGST = SGST = 9%

   Amount of GST paid by shopkeeper to wholesaler,

   SGST = CGST = 9% of [₹8000 – ₹80x]

   $\qquad = \frac{9}{100}$ [₹8000 - ₹80x]

   Amount of GST paid by consumer to shopkeeper,

   CGST = SGST = 9% of ₹8000

   $\qquad = \frac{9}{100}$ x ₹8000 = ₹720

   $\qquad = \left(\frac{9}{100}\right)$ ₹8000 = ₹720

   So, the tax paid by shopkeeper to state government = ₹720 - $\frac{9}{100}$ x [₹8000 –

   ₹80x]

Also, tax paid by shopkeeper to state government = ₹72

₹72 = 720 - (9 x $\frac{80}{100}$) (100 - x)
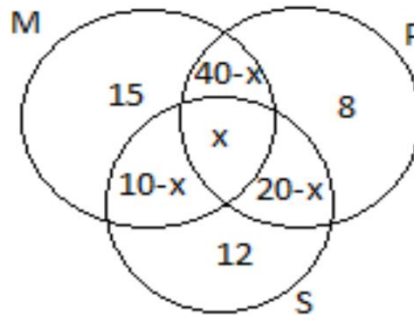
720 - 72 = $\frac{720}{100}$ (100 - x)

648 = $\frac{720}{100}$ (100 - x)

100 - x = 90

x = 100 - 90 = 10

Hence, the required rate of discount = 10%

36. Venn diagram

Let M= The Set of students who offered Mathematics

    S = The Set of students who offered Statistics

    P = The Set of students who offered Physics.

   And x be set of students who offered all three subjects, then number of members in different region shown in Venn diagram.

From Venn diagram, we get the number of students who offered Physics

  = (40 - x) + x + (20 – x) 8 = 65

 Or 68 – x = 65

Or x = 3

(a) Number of students who offered Mathematics

= 15 +(10 – x) +x + (40 – x)

 = 65 – x   (Put Value of x)

= 65 – 3

 = 62

(b) Number of students who offered Statistics

  = 12 + (10 – x) +x + (20 – x)

    = 42 – x   (Put Value of x)

    = 42 – 3

   = 39

(c) The number of students who offered any of three subjects

    = 15 +12 + 8 + (10 – x) +(40 – x) + (20 – x) + x

   = 105 – 2x   (Put Value of x)

    = 105 -6

   = 99

Therefore, The number of students who did not offered any of the three subjects

    = 100 – 99

    = 1

37. Let the listed price of the article be P=Rs 1600. The rate of GST is 5%.

(a) The price per article inclusive of GST, which the retailer pays.

First, let's find the price at which the wholesaler buys the article from the manufacturer after a 25% discount: Discount for wholesaler =25% of 1600=Rs 400 Price for wholesaler =1600−400=Rs 1200

Next, the wholesaler allows a 20% discount on the listed price to the retailer: Discount for retailer =20% of 1600=Rs 320 Price before GST for retailer =1600−320=Rs 1280

Now, we need to calculate the GST at 5% on the price the retailer pays: GST for retailer =5% of 1280 =Rs 64

The price per article inclusive of GST, which the retailer pays, is the price before GST plus the GST:

 Price for retailer inclusive of GST =1280+64=Rs 1344

(b) The amount which the consumer pays for an article.

The retailer sells the article to the consumer at a discount of 5% on the listed price: Discount for consumer =5% of 1600=Rs 80 Price before GST for consumer =1600−80=Rs 1520

Now, we need to calculate the GST at 5% on the price the consumer pays: GST for consumer =5% of 1520=Rs 76

The amount which the consumer pays for an article is the price before GST plus the GST: Price for consumer inclusive of GST =1520+76=Rs 1596

(c) The GST paid by the wholesaler to the state government for an article.

The wholesaler bought the article for Rs 1200 and sold it to the retailer for Rs 1280 (before GST). GST paid by the wholesaler on purchase =5% of 1200=Rs 60 (Input Tax Credit) GST collected by the wholesaler on sale =5% of 1280=Rs 64 (Output Tax)

The GST paid by the wholesaler to the state government Output Tax−Input Tax Credit=64−60=Rs 4

38. (a) Amara is using an ordinary annuity because she makes regular payments at the end of each month. In an ordinary annuity, payments are made at the end of each compounding period (in this case, monthly), and interest is applied accordingly.

(b) Ordinary Annuity formula:

$A = P \times [(1+r)^n - 1]/r$

 Where:

 $n = 25 \times 12 = 300$ (total number of payments

 $P = 500,$ $r = 0.06/12 = 0.005$ (monthly interest rate),

 $A = 500 \times [(1+0.005)^{300} - 1]/0.005$

 $A = 500 \times (4.467744 - 1)/0.005$

 $= 500 \times 3.467744/0.005$

 $A \approx 346,774.40$

Answer: At retirement, Amara will have approximately Rs 346,774.40 in her annuity account.

(c) Impact of Increasing Interest Rate or Duration

Increasing the interest rate: If the interest rate is higher, the money earns more interest each month.

USEFUL LINKS

CBSE CURRICULUM https://cbseacademic.nic.in/curriculum_2026.html
CBSE STUDENT SUPPORT MATERIAL https://cbseacademic.nic.in/appliedmaths-supportmaterial_XII.html
CBSE QUESTION PAPER 2025 https://www.cbse.gov.in/cbsenew/question-paper.html
CBSE MARKING SCHEME 2025 https://www.cbse.gov.in/cbsenew/marking-scheme.html
CBSE SAMPLE PAPER 2024-25 https://cbseacademic.nic.in/SQP_CLASSXii_2024-25.html
NCERT YOUTUBE VIDEOS https://www.youtube.com/@NCERTOFFICIAL/search?query=CLASS%2011%20APPLIED%20MATHEMATICS