तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

गणित
Mathematics

कक्षा / Class IX
2025-26

विद्यार्थी सहायक सामग्री
Student Support Material

केन्द्रीय विद्यालय संगठन~Kendriya Vidyalaya Sangathan

संदेश

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना एवं नवाचार द्वारा उच्च - नवीन मानक स्थापित करना केन्द्रीय विद्यालय संगठन की नियमित कार्यप्रणाली का अविभाज्य अंग है। राष्ट्रीय शिक्षा नीति 2020 एवं पी. एम. श्री विद्यालयों के निर्देशों का पालन करते हुए गतिविधि आधारित पठन-पाठन, अनुभवजन्य शिक्षण एवं कौशल विकास को समाहित कर, अपने विद्यालयों को हमने ज्ञान एवं खोज की अद्भुत प्रयोगशाला बना दिया है। माध्यमिक स्तर तक पहुँच कर हमारे विद्यार्थी सैद्धांतिक समझ के साथ-साथ, रचनात्मक, विश्लेषणात्मक एवं आलोचनात्मक चिंतन भी विकसित कर लेते हैं। यही कारण है कि वह बोर्ड कक्षाओं के दौरान विभिन्न प्रकार के मूल्यांकनों के लिए सहजता से तैयार रहते हैं। उनकी इस यात्रा में हमारा सतत योगदान एवं सहयोग आवश्यक है - केन्द्रीय विद्यालय संगठन के पांचों आंचलिक शिक्षा एवं प्रशिक्षण संस्थान द्वारा संकलित यह विद्यार्थी सहायक- सामग्री इसी दिशा में एक आवश्यक कदम है । यह सहायक सामाग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है। केन्द्रीय विद्यालय संगठन की विद्यार्थी सहायक- सामग्री अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री संकलन की विशेषज्ञता के लिए जानी जाती है और शिक्षा से जुड़े विभिन्न मंचों पर इसकी सराहना होती रही है। मुझे विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर निरंतर मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुँचाएगी।

शुभाकांक्षा सहित ।

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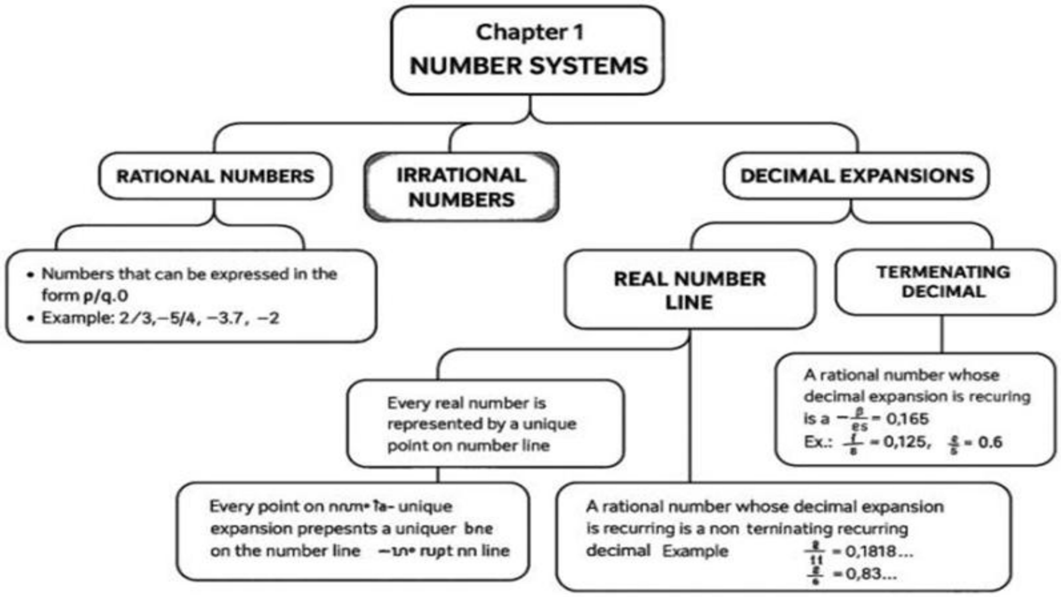
CHAPTER – 1
NUMBER SYSTEMS

Mind Mapping



Gist/Summary Of The Lesson:

The chapter introduces different types of numbers and extends the concept of the number system to include irrational numbers and the real number line. Students learn how to represent numbers on the number line and understand operations such as rationalization and laws of exponents for real numbers.

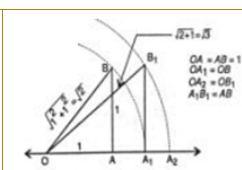
Definitions And Formulae:

- Natural Numbers (N): Counting numbers starting from 1. N = {1, 2, 3, 4, ...}
- Whole Numbers (W): Natural numbers including 0. W = {0, 1, 2, 3, ...}
- Integers (Z): All positive and negative whole numbers including 0. Z = {..., -3, -2, -1, 0, 1, 2, 3, ...}
- Rational Numbers (Q): Numbers that can be expressed in the form p/q, where p, q ∈ Z and q ≠ 0. E.g., 1/2, -3/4, 5
- Irrational Numbers: Numbers that cannot be expressed as p/q. Decimal expansion is non-terminating and non-repeating. E.g., $\sqrt{2}$, $\sqrt{3}$, π
- Real Numbers (R): All rational and irrational numbers together form the real numbers. R = Q ∪ Irrational Numbers
- Terminating Decimal: A decimal that ends after a certain number of digits. E.g., 0.25 = 1/4
- Non-Terminating Recurring Decimal: Decimal expansion that goes on forever but has a repeating pattern. E.g., 0.666... = 2/3
- Non-Terminating Non-Recurring Decimal: Infinite decimals with no repeating pattern; they are irrational. E.g., π = 3.141592...

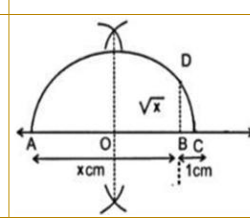
Key Concepts And Formulae

1 Representation of Real Numbers on the Number Line

Rational numbers can be easily plotted using the number line. Irrational numbers like $\sqrt{2}$, $\sqrt{3}$ can be represented using the Pythagoras Theorem:
To locate $\sqrt{2}$: Draw a right triangle with legs of 1 unit each. Hypotenuse = $\sqrt{2}$.
For every positive real number x, $\sqrt{x}$ can be represented by a point on the number line by using the following steps:



(1) Obtain the positive real number x (say)
(ii) Draw a line and mark a point A on it.
(iii) Mark a point B on the line such that AB = x units.
(iv) From point B mark a distance of 1 unit and mark the new point as C.
(v) Find the midpoint of AC and mark the point as O.



(vi) Draw a circle with centre O and radius OC.
(vii) Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Length BD is equal to $\sqrt{x}$.

2 Operations With Real Numbers

All operations (+, −, ×, ÷) are defined on real numbers. Real numbers are closed under addition, subtraction, and multiplication. If R is rational and S is irrational, then R-S, R+S, R×S and R/S are irrational numbers, R≠0, S≠0

3 Laws Of Exponents For Real Numbers

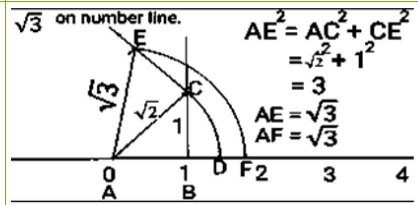
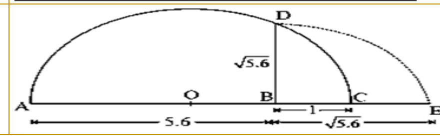
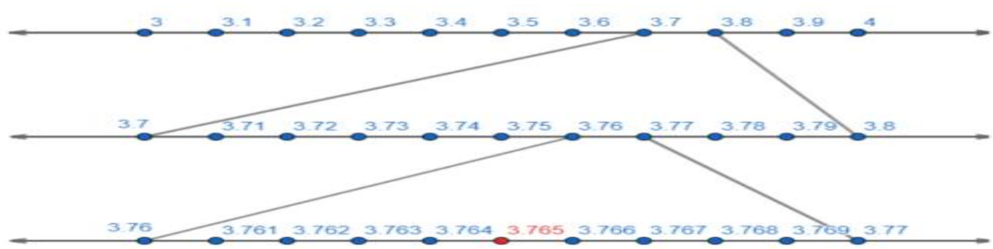
Law	Formula
Product Of Powers	$a^m \times a^n = a^{m+n}$
Quotient Of Powers	$a^m \div a^n = a^{m-n}$
Power Of A Power	$(a^m)^n = a^{m \times n}$
Power Of A Product	$(ab)^m = a^m \times b^m$
Power Of A Quotient	$(a/b)^m = a^m/b^m$
Zero Exponent	$a^0 = 1 \ (a \neq 1)$
Negative Exponent	$a^{-1} = \frac{1}{a}$

Multiple Choice Questions (01 Mark Questions)

	Question	Solution
1	What is the simplest form of the decimal 0.272727...? (a) $\frac{3}{11}$ (b) $\frac{27}{98}$ (c) $\frac{27}{100}$ (d) $\frac{3}{10}$	**Solution:** (a) Let x = 0.272727... 100x = 27.272727... Subtracting, 99x = 27 $x = \frac{27}{99} = \frac{3}{11}$ Answer: A) $\frac{3}{11}$
2	Which of the following is an irrational number? (a) $\sqrt{4}$ (b) $\sqrt{9}$ (c) $\sqrt{2} + \sqrt{3}$ (d) $\sqrt{(4/9)}$	**Solution:** (c) $\sqrt{2} + \sqrt{3}$ is an irrational number since the sum of two irrational numbers is irrational
3	The decimal representation of 1/17 is: (a) Terminating (b) Non-terminating and recurring (c) Non-terminating and non-recurring (d) None of the above	**Solution:** (b) 1/17 = 0.0588235294117647... (non-terminating and recurring)
4	What is the value of $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$? (a) $4 + \sqrt{15}$ (b) $4 - \sqrt{15}$ (c) $(\sqrt{5} + \sqrt{3})/2$ (d) $(\sqrt{5} - \sqrt{3})/2$	**Solution:** (a) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{5+3+2\sqrt{15}}{5-3} = 4 + \sqrt{15}$
5	Which of the following is a rational number? (a) $\sqrt{2} + \sqrt{3}$ (b) $\sqrt{4} + \sqrt{9}$ (c) $\sqrt{2} \times \sqrt{3}$ (d) $\sqrt{2}/\sqrt{3}$	**Solution:** (b) $\sqrt{4} + \sqrt{9} = 2 + 3 = 5$ (rational number)
6	The simplest form of the expression $\frac{\sqrt{2}+1}{\sqrt{2}-1}$ is: (a) $3 + 2\sqrt{2}$ (b) $3 - 2\sqrt{2}$ (c) $\sqrt{2} + 1$ (d) $\sqrt{2} - 1$	**Solution:** (a) $\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2+1+2\sqrt{2}}{2-1}$ $= 3 + 2\sqrt{2}$
7	The value of $(\sqrt{3} + \sqrt{2})^2$ is: (a) $5 + 2\sqrt{6}$ (b) $5 - 2\sqrt{6}$ (c) $3 + 2\sqrt{6}$ (d) $3 - 2\sqrt{6}$	**Solution:** (a) $(\sqrt{3} + \sqrt{2})^2 = 3 + 2 + 2\sqrt{6}$ $= 5 + 2\sqrt{6}$
8	What is the simplest form of the expression $(2 + \sqrt{3})/(2 - \sqrt{3})$? (a) $7 + 4\sqrt{3}$ (b) $7 - 4\sqrt{3}$ (c) $2 + \sqrt{3}$ (d) $2 - \sqrt{3}$	**Solution:** (a) $(2 + \sqrt{3})/(2 - \sqrt{3}) \times (2 + \sqrt{3})/(2 + \sqrt{3}) = (4 + 3 + 4\sqrt{3})/(4 - 3) = 7 + 4\sqrt{3}$
9	Two irrational numbers are added to get a rational number. Which of the following must be true? (a) This is impossible.	**Solution:** C. The numbers are additive inverses.

	(b)The two numbers must be equal. (c)The numbers are additive inverses. (d)One number is a perfect square		
10	What is the p/q form of 0.090909...? (a) $\frac{9}{10}$ (b) $\frac{1}{11}$ (c) $\frac{9}{90}$ (d) $\frac{10}{11}$	**SOLUTION**: (b)$\frac{1}{11}$	
	ASSERTION - REASON BASED QUESTIONS		
	In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option: (a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A). (c) Assertion (A) is true but Reason(R) is false. (d) Assertion (A) is false but Reason(R) is true.		
1	Assertion: Every rational number has a terminating decimal representation. Reason: Rational numbers can be expressed in the form p/q, where p and q are integers and q ≠ 0.	**SOLUTION**:(d) Not every rational number has a terminating decimal representation (e.g., 1/3 = 0.333...). The Reason is true but does not explain the Assertion correctly.	
2	Assertion: √2 is an irrational number. Reason: The decimal representation of √2 is non-terminating and non-recurring.	**SOLUTION**:(a)Both Assertion and Reason are true. The Reason correctly explains why √2 is irrational.	
3	Assertion: Every integer is a rational number. Reason: Integers can be expressed in the form p/q, where p is the integer and q = 1.	**SOLUTION**:(a)Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.	
4	Assertion: The sum of two irrational numbers is always irrational. Reason: The sum of √2 and -√2 is 0, which is rational.	**SOLUTION**:(d)The Assertion is false because the sum of two irrational numbers can be rational (e.g., √2 + (-√2) = 0). The Reason is true and serves as a counter example.	
5	Assertion: The decimal representation of 1/4 is terminating Reason: 1/4 = 0.25, which has a finite number of digits after the decimal point.	**SOLUTION:**(a)Both Assertion and Reason are true. The Reason correctly explains why the decimal representation of 1/4 is terminating.	
6	Assertion: Every rational number can be expressed as a terminating or repeating decimal. Reason: Rational numbers have a finite or repeating decimal expansion due to their fractional form.	**SOLUTION**: (a) Both assertion and reason are true, and the reason is the correct explanation.	
7	Assertion: The decimal expansion of √2 is non-terminating and non-repeating. Reason: √2 is an irrational number, and irrational numbers have non-terminating and non-repeating decimal expansions.	**SOLUTION**: (a)Both assertion and reason are true, and the reason is the correct explanation.	
8	Assertion: The sum of two irrational numbers may or may not be irrational. Reason: The sum of two non-terminating and non-Repeating decimal expansions is always non-Terminating and non-repeating.	**SOLUTION**: (c) Assertion (A) is true but Reason(R) is false	
9	Assertion: Every integer is a rational number. Reason: Integers can be expressed in the form p/q, where q = 1.	**SOLUTION**: (a)Both assertion and reason are true, and the reason is the correct explanation.	
10	Assertion: The number 0.101001000100001... is an irrational number.	**SOLUTION**: (a)Both assertion and reason are true, and the reason is the correct explanation.	

	Reason: The decimal expansion is non-terminating and non-repeating.
	VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)
1	If $2^{x+1} = 4^{x-1}$, find the value of x. **SOLUTION:** $2^{x+1} = (2^2)^{x-1}$; $2^{x+1} = 2^{2x-2}$; x + 1 = 2x - 2 ; x = 3
2	If a = $2^{\frac{1}{3}}$and b = $2^{\frac{2}{3}}$, find the value of a^3b^3. **SOLUTION:**$a^3b^3 = (2^{\frac{1}{3}})^3 \times (2^{\frac{2}{3}})^3 = 2 \times 2^2 = 2^3 = 8$
3	Rationalise the denominator: (√2 + 1) / (√2 - 1). **SOLUTION:** (√2 + 1) / (√2 - 1) × (√2 + 1) / (√2 + 1)= (2 + 1 + 2√2) / (2 - 1) = 3 + 2√2
4	Simplify: (√5 + √3) / (√5 - √3) - (√5 - √3) / (√5 + √3). **SOLUTION:** (√5 + √3) / (√5 - √3) × (√5 + √3) / (√5 + √3) = (5 + 3 + 2√15) / (5 - 3)= 4 + √15 (√5 - √3) / (√5 + √3) × (√5 - √3) / (√5 - √3) = (5 + 3 - 2√15) / (5 - 3)= 4 - √15 Subtracting both expressions, 4 + √15 - (4 - √15) = 2√15
5	Convert 0.456456... to p/q form **SOLUTION**: Let x = 0.456456... ; Multiplying both sides by 1000, we get: 1000x = 456.456456... Subtracting the original equation from this, we get: 999x = 456 ; x = 456/999 ; x = 152/333 So, 0.456456... = 152/333
6	Write 4 irrational numbers between 5/8 and ½. **SOLUTION**: To find four irrational numbers between 5/8 and 1/2, we can start by converting these fractions to decimals. 5/8 = 0.625 ; 1/2 = 0.5 ; Now, we need to find four irrational numbers between 0.5 and 0.625. Here are four examples: 0.54321……., 0.56789……, 0.59123... …., 0.61234... (a non-repeating, non-terminating decimal)
7	What is the value of $256^{0.16} \times 256^{0.09}$? **SOLUTION**:$256^{0.16} \times 256^{0.09} = 256^{0.16+0.09} = 256^{0.25} = 256^{\frac{25}{100}} = 256^{\frac{1}{4}} = (4^4)^{\frac{1}{4}} = 4$
8	Find the value of (√6 + √5) / (√6 - √5) - (√6 - √5) / (√6 + √5). **SOLUTION**: (√6 + √5) / (√6 - √5) × (√6 + √5) / (√6 + √5) = (6 + 5 + 2√30) / (6 - 5) = 11 + 2√30 (√6 - √5) / (√6 + √5) × (√6 - √5) / (√6 - √5) = (6 + 5 - 2√30) / (6 - 5) = 11 - 2√30 Subtracting both expressions, 11 + 2√30 - (11 - 2√30) = 4√30
9	Simplify $\left(\frac{243}{32}\right)^{-\frac{4}{5}}$ **SOLUTION**: $\left(\frac{3^5}{2^5}\right)^{\frac{-4}{5}} = \left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^{4} = \frac{16}{81}$
10	Find two irrational numbers between $\sqrt{2}$ *and* $\sqrt{3}$. **SOLUTION**: $\sqrt{2} = 1.414$ *and* $\sqrt{3} = 1.732$ two irrational numbers between them are $\sqrt{2.5}$ *and* 1.6121121112…………..
	SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)
1	If x = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$and y = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find the value of $x^2 + y^2$. **SOLUTION**: x = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ = 5 + 2√6 ; y = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$= 5 - 2√6 ; $x^2 + y^2 = (5 + 2\sqrt{6})^2 + (5 - 2\sqrt{6})^2$ = 2(25 + 24) = 98
2	If $x = 2 + \sqrt{3}$. Find the value of $x^2 - 4x + 1$ **SOLUTION**: Step 1: Calculate x^2 ; $x^2 = (2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$ Step 2: Calculate 4x ; 4x = 4(2 + √3) = 8 + 4√3 ; Step 3: Calculate $x^2 - 4x + 1$ $x^2 - 4x + 1 = (7 + 4\sqrt{3}) - (8 + 4\sqrt{3}) + 1 = 7 + 4\sqrt{3} - 8 - 4\sqrt{3} + 1 = 0$ So, the value of $x^2 - 4x + 1$ is 0.
3	3. If x = (√2 + 1) / (√2 - 1), find the value of $x^2 + 1/x^2$. **SOLUTION**: x = (√2 + 1) / (√2 - 1) × (√2 + 1) / (√2 + 1) = (2 + 1 + 2√2) / (2 - 1) = 3 + 2√2 1/x = (√2 - 1) / (√2 + 1) × (√2 - 1) / (√2 - 1) = (2 + 1 - 2√2) / (2 - 1) = 3 - 2√2 $x^2 + 1/x^2 = (3 + 2\sqrt{2})^2 + (3 - 2\sqrt{2})^2$; 2(9 + 8) = 34

4	Find 5 rational numbers between ¾ and ⅘ **SOLUTION**: Step 1: Convert both fractions to equivalent fractions with a common denominator, The least common multiple (LCM) of 4 and 5 is 20. ; ¾ = 15/20 ; ⅘ = 16/20 ; ¾ = 15/20 × 6/6 = 90/120 = 16/20 × 6/6 = 96/120 , 91/120, 92/120, 93/120, 94/120, 95/120 You can simplify them if needed: 91/120, 23/30, 93/120, 47/60, 19/24	
5	Represent $\sqrt{3}$ on the real number line. **SOLUTION**:	
6	Represent $\sqrt{5.6}$ on real number line. **SOLUTION**:	
7	If $1617 = 3^a \times 7^b \times 11^c$. Find the value of a,b and c. **SOLUTION**: prime factorisation of $1617 = 3^1 \times 7^2 \times 11^1$on comparing, a=1, b=2, c=1	
8	If $a = 2 + \sqrt{3}\ and\ b = 2 - \sqrt{3}, find\ a^2 + b^2$. **SOLUTION**: $\left(2+\sqrt{3}\right)^2 + \left(2-\sqrt{3}\right)^2 = 4 + 3 + 4 + 3 = 14$	
9	Visualise 3.765 on the number line using successive magnification. **SOLUTION**: 3.765 can be visualised as in the following steps First, we draw a number line and mark points on it after that we will divide the number line between points 3 and 4. And then we will divide the points between 3.7 and 3.8 as the number is between both of them. 	
10	Express the repeating decimal 2.234234... in p/q form. **SOLUTION**: Let x = 2.234234... (1) ; Multiply both sides by 1000 (since the repeating block has 3 digits): 1000x = 2234.234234... (2) ; Subtract (1) from (2): ; 999x = 2232 ; x = 2232 / 999 x = 744 / 333 ; x = 248 / 111 ; So, the p/q form of 2.234234... is 248/111	
	LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)	
1	Simplify: $\left(\frac{x^a}{x^{-b}}\right)^{a-b} \times \left(\frac{x^b}{x^{-c}}\right)^{b-c} \times \left(\frac{x^c}{x^{-a}}\right)^{c-a}$ **SOLUTION**: $(x^{a+b})^{a-b} \times (x^{b+c})^{b-c} \times (x^{c+a})^{c-a} = x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} = 1$	
2	Find the value of a and b in the following: $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}\sqrt{5}b$ **SOLUTION**: $\frac{54+14\sqrt{5}}{44} - (\frac{54+14\sqrt{5}}{44}) = a + \frac{7}{11}\sqrt{5}b$→a= 0 and b = 1.	
3	Prove that: $\frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} = 1$ **SOLUTION**: Prove using rationalisation.	
4	Express $0.6 + 0.\underline{7} + 0.4\underline{7}$ in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. **SOLUTION**: $\frac{167}{90}$	
5	Find the value of $\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{2}{(243)^{\frac{-1}{5}}}$	

SOLUTION: $\frac{4}{(6^3)^{\frac{-2}{3}}}+\frac{1}{(4^4)^{\frac{-3}{4}}}+\frac{2}{(3^5)^{\frac{-1}{5}}}=\frac{4}{6^{-2}}+\frac{1}{4^{-3}}+\frac{2}{3^{-1}}=214$

CASE BASED QUESTIONS (04 MARKS QUESTIONS)

1 Exponents in Science

In a science exhibition, the value of the speed of light is given as $\mathbf{3\times10^{8}}$ m/s and the size of a virus is approximately $\mathbf{2\ \times 10^{-9}}$m. Students are taught to use exponents to simplify and compare such numbers.

(1) Express the product of speed of light and time taken as $3\times10^{8}\times2\ \times10^{-9}$ Simplify using laws of exponents.

(2) Find $(2\ \times10^{-9})^{2}$ using the laws of exponents.

(3) Evaluate $\frac{2^3}{2^5}$ and write your answer in exponent form.

(4) What is the value of $(a^m)^n$ when a = 3, m = 2, and n = 3?

SOLUTION:

(1) $3\times10^{8}\times2\ \times10^{-9}\ =6\times10^{-1}$; (2) $(2\ \times10^{-9})^{2}=2^{2}\times(10^{-9})^{2}=4\times10^{-18}$; (3) $\frac{2^3}{2^5}=2^{3-5}=2^{-2}=(\frac{1}{2})^{2}=\frac{1}{4}$; (4) $(a^m)^n=(3^2)^3=3^6=729$

2 Case Study 2: Rocket Launch Timing

A team of scientists are working on calculating the exact timing of a rocket launch. The rocket's countdown timer works on powers of 10. The current time displayed is $\mathbf{3\times10^{4}}$ seconds before launch.

Questions:

(1) Express 3×10^{4} in standard numerical form.

(2) If the countdown continues and reduces by a factor of 10^2, what is the new time remaining?

(3) Use the laws of exponents to simplify: $(2\times10^{3})\times(4\times10^{2})$

(4) What rule of exponents is applied in the expression $(a^m)^n=a^{mn}$. Apply for a=2, m=3, n=2

SOLUTIONS:

(1) $3\times10^{4}=30000$;

(2) new time= $\frac{3\times10^{4}}{10^{2}}=300$;

(3) $(2\times10^{3})\times(4\times10^{2})=8\times10^{5}$

(4) $(a^m)^n=a^{mn}=2^{3\times2}=2^6=64$

3 Case Study 3: Number Game

Two friends were given a project to tag the numbers given to them as R, IR which are short form for rational number, irrational number respectively. The numbers are: 1.707007000….., 1/5, √49, √2 and √35.020020002….., ¾, $\sqrt{7},\frac{\sqrt{54}}{\sqrt{6}},\frac{1}{2\sqrt{3}-\sqrt{11}}$

Questions:

(1) Identify rational and irrational numbers from the given numbers.

(2) Find two rational numbers between √2 and √3.

(3) Find two irrational numbers between √2 and √3.

(4) Rationalise $\frac{1}{2\sqrt{3}-\sqrt{11}}$

SOLUTIONS:

(1) Rational numbers: ⅕, √49 = 7 (since √49 is a perfect square), ¾, 7, 546, 123 - 11 = 112

Irrational numbers: 1.707007000… (non-terminating and non-repeating), √2, √3,

- 5.020020002… (non-terminating and non-repeating)

(2) √2 ≈ 1.414 ; √3 ≈ 1.732

Two rational numbers between √2 and √3: 1.5 and 1.6

(3) Two irrational numbers between √2 and √3: √2.1 and √2.5

(4) $\frac{1}{2\sqrt{3}-\sqrt{11}} = \frac{2\sqrt{3}+\sqrt{11}}{(2\sqrt{3})^2-(\sqrt{11})^2} = \frac{2\sqrt{3}+\sqrt{11}}{12-11} = 2\sqrt{3}+\sqrt{11}$

HOTS

1 If $a = 2 + \sqrt{3} + \sqrt{5}$ and $b = 2 + \sqrt{3} - \sqrt{5}$, find the value of $(a^2 + b^2) / (a + b)$

SOLUTION: $a^2 = (2 + \sqrt{3} + \sqrt{5})^2 = 4 + 4\sqrt{3} + 4\sqrt{5} + 3 + 2\sqrt{15} + 5 = 12 + 4\sqrt{3} + 4\sqrt{5} + 2\sqrt{15}$.

$b^2 = (2 + \sqrt{3} - \sqrt{5})^2 = 4 + 4\sqrt{3} - 4\sqrt{5} + 3 - 2\sqrt{15} + 5 = 12 + 4\sqrt{3} - 4\sqrt{5} - 2\sqrt{15}$.

$a^2 + b^2 = (12 + 4\sqrt{3} + 4\sqrt{5} + 2\sqrt{15}) + (12 + 4\sqrt{3} - 4\sqrt{5} - 2\sqrt{15}) = 24 + 8\sqrt{3}$.

$a + b = (2 + \sqrt{3} + \sqrt{5}) + (2 + \sqrt{3} - \sqrt{5}) = 4 + 2\sqrt{3}$.

Calculate $(a^2 + b^2) / (a + b)$; $(a^2 + b^2) / (a + b) = (24 + 8\sqrt{3}) / (4 + 2\sqrt{3})$.

Let's simplify by dividing numerator and denominator by 2: $(12 + 4\sqrt{3}) / (2 + \sqrt{3})$.

Multiply by the conjugate $((12 + 4\sqrt{3})(2 - \sqrt{3})) / ((2 + \sqrt{3})(2 - \sqrt{3})) = (24 - 12\sqrt{3} + 8\sqrt{3} - 12) / (4 - 3)$

$= (12 - 4\sqrt{3}) / 1 = 12 - 4\sqrt{3}$. The final answer is: $12 - 4\sqrt{3}$.

2 Prove that $\sqrt{(11 + 2\sqrt{30})} - \sqrt{(7 + 2\sqrt{10})} = \sqrt{2}$.

SOLUTION: $\sqrt{(11 + 2\sqrt{30})} = \sqrt{(6 + 5 + 2\sqrt{30})} = \sqrt{6} + \sqrt{5}$.

$\sqrt{(7 + 2\sqrt{10})} = \sqrt{(5 + 2 + 2\sqrt{10})} = \sqrt{5} + \sqrt{2}$.

$(\sqrt{6} + \sqrt{5}) - (\sqrt{5} + \sqrt{2}) = \sqrt{6} - \sqrt{2}$.

3 If $x = (\sqrt{(a + 2)} + \sqrt{(a - 2)}) / (\sqrt{(a + 2)} - \sqrt{(a - 2)})$, find the value of $x^2 - ax + 1$.

SOLUTION: $x = (\sqrt{(a + 2)} + \sqrt{(a - 2)}) / (\sqrt{(a + 2)} - \sqrt{(a - 2)}) = (a + \sqrt{(a^2 - 4)}) / 2$.

Given the form of x, let's directly evaluate $x^2 - ax + 1 = ((a + \sqrt{(a^2 - 4)}) / 2)^2 - a((a + \sqrt{(a^2 - 4)}) / 2) + 1$.

$x^2 - ax + 1 = (a^2 + 2a\sqrt{(a^2 - 4)} + a^2 - 4) / 4 - (a^2 + a\sqrt{(a^2 - 4)}) / 2 + 1$.

This simplifies to $x^2 - ax + 1 = 0$ after combining like terms and simplifying

The final answer is: 0

4 If $x = (\sqrt{3} + \sqrt{2}) / (\sqrt{3} - \sqrt{2})$ and $y = (\sqrt{3} - \sqrt{2}) / (\sqrt{3} + \sqrt{2})$, find the value of $x^4 + y^4$.

SOLUTION: $x = (\sqrt{3} + \sqrt{2}) / (\sqrt{3} - \sqrt{2}) = 5 + 2\sqrt{6}$, $y = (\sqrt{3} - \sqrt{2}) / (\sqrt{3} + \sqrt{2}) = 5 - 2\sqrt{6}$.

$x^2 = (5 + 2\sqrt{6})^2 = 49 + 20\sqrt{6}$, $y^2 = (5 - 2\sqrt{6})^2 = 49 - 20\sqrt{6}$.

$x^4 + y^4 = (x^2)^2 + (y^2)^2 = (49 + 20\sqrt{6})^2 + (49 - 20\sqrt{6})^2 = 2(49^2 + (20\sqrt{6})^2) = 2(2401 + 2400) = 2 * 4801 = 9602$.

The final answer is: 9602

5 If $x = (\sqrt{(a + b)} + \sqrt{(a - b)}) / (\sqrt{(a + b)} - \sqrt{(a - b)})$, find the value of $x^2 - 2ax + b^2$.

SOLUTION: $x = (\sqrt{(a + b)} + \sqrt{(a - b)}) / (\sqrt{(a + b)} - \sqrt{(a - b)}) = (a + \sqrt{(a^2 - b^2)}) / b$.

Given $x = (a + \sqrt{(a^2 - b^2)}) / b$, let's directly evaluate $x^2 - 2ax + b^2$.

The final answer is: 0

EXERCISE

MULTIPLE CHOICE QUESTIONS

1 Express 1.1666..... in p/q form:

A) 116/99 B) 129/11 C) 105/90 D) 58/50

2 Simplify: $(2^3 \times 3^2)^2$

A) $2^6 \times 3^4$ B) $2^9 \times 3^6$ C) $2^5 \times 3^3$ D) $2^2 \times 3^2$

3 If $x^{-2} = \frac{1}{9}$,then what is the value of x?

A) 3 B) -3 C) ⅓ D) None of these

4 Which of the following numbers is an irrational number?

A) $\sqrt{9}$ B) $\sqrt{16}$ C) $\sqrt{25}$ D) $\sqrt{2}$

5 What is the decimal expansion of the rational number 5/7?

A) 0.714285… B) 0.714286… C) 0.714287… D) 0.714288...

ASSERTION AND REASONING QUESTIONS

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

A). Both assertion(A) and reason(R) are true and reason (R) is the correct explanation of assertion (A).

B). Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

C) .Assertion (A) is true but reason (R) is false.

D). Assertion (A) is false but reason (R) is true.

1	Assertion: Rational number lying between two rational numbers x and y is (x+y)/2. Reason: There is one rational number lying between any two rational numbers.
2	Assertion: The rationalizing factor of 3+2√5 is 3–2√5. Reason: If the product of two irrational numbers is rational then each one is called the rationalising factor of the other.
3	Assertion: Sum of two irrational numbers 2+√3 and 4+√3 is an irrational number. Reason: Sum of two irrational numbers is always an irrational number.
4	Assertion: 0.329 is a terminating decimal. Reason: A decimal in which a digit or a set of digits is repeated periodically, is called a repeating, or a recurring, decimal.
5	Assertion: $13^5 \times 13^2 = 13^{10}$ Reason : If a > 0 be a real number and p and q be rational numbers. Then $a^p \times a^q = a^{p+q}$

	VERY SHORT ANSWER TYPE QUESTIONS		**SHORT ANSWER TYPE QUESTIONS**
1	Find 4 rationals between $\frac{3}{4}$ *and* $\frac{4}{5}$.	1	If $x = 3 + 2\sqrt{2}, find\ the\ value\ of\ x - \frac{1}{x}$.
2	Express $3.1\overline{7}\ in \frac{p}{q}\ form$	2	Express 12.2313131… in p/q form
3	Simplify $\left(\frac{243}{32}\right)^{\frac{-4}{5}}$	3	Find the value of x, if $5^{x-3} \times 3^{2x-8} = 225$
4	Rationalise $\frac{2\sqrt{5}}{3\sqrt{2}-\sqrt{5}}$	4	Express 0.003003003… as a rational number
5	Simplify $\frac{\sqrt{27}-\sqrt{12}}{\sqrt{3}}$.	5	If $a = 2 + \sqrt{3}\ and\ b = 2 - \sqrt{3}, find\ a^2 + b^2$.

	LONG ANSWER TYPE QUESTIONS
1	Locate $\sqrt{5.8}$ on the real number line.
2	Find the values of a and b in the following: $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$
3	If $x = 5 - 2\sqrt{3}$, then find $x - \frac{1}{x}$
4	Simplify: $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$
5	Show that 0.142857142857... = $\frac{1}{7}$

	CASE BASED TYPE QUESTIONS	
1	Mr. Roy, a Mathematics teacher explained some key points of unit 1 of class IX to his students. Some are given here. There are infinite rational numbers between any two rational numbers. Rationalisation of a denominator means to change the irrational denominator to rational form. A number is irrational if its decimal form is non-terminating non-recurring. On the basis of these key points, he asked the following questions to class. Can you help the class to solve these questions. i) What is the reciprocal of 4 - √15 with rational denominator? (1) ii)Is 2.201200120001…… a rational number? Give reasons. (1) iii) A vehicle started from point A with speed as a irrational number 50√2 km/hr to reach point B. Distance between points A and B is given by a rational number 350km . Tell whether the time will be given by a rational number or irrational number. Given reasons. (2)	

2 Two girls were playing with numbers to test each other's concept by asking each other riddles with numbers. Rohini says " A number x when raised to the power 5 of its 5th root, multiplied to its 5th root and the resultant is again multiplied to its power raised to $\frac{4}{5}$ then the result is 25." Mohini says " A number y is raised to the power $\frac{-4}{5}$ and then the new number's 5th root is raised to the power $\frac{-5}{4}$ and the resultant is raised to the power 5. The result comes out to be 5" Based on the above information answer the following questions:

(i) What can be the possible solutions of Rohini's riddle? (2)

(ii) What is the algebraic equation of Mohini's riddle ? (1)

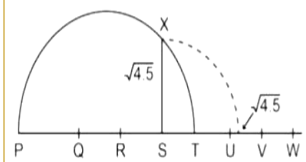
(iii) Is x = y if both x and y are natural numbers? (1)

3 Vasu represents √4.5 on the number line PW. The length of TS = 1 unit. His representation is shown below.

(i) Which letter represents 0 of the number line? (1)

(ii) What is the length of ST ? (1)

(iii) Between which two points does 5.2 lie on this number line? (2)



ANSWERS

1 B) 129/110

2 A) $2^6 \times 3^4$

3 A) 3

4 D) √2

5 A) 0.714285…

1 (C) Assertion (A) is true but reason (R) is false

2 Correct option: (A) Both assertion (A)and reason(R) are true and reason (R) is the correct explanation of assertion (A).

3 Correct option: (C) Assertion (A) is true but reason (R) is false.

4 Correct option: (B) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

5 Correct option: (D) Assertion (A) is false but reason (R) is true.

1	Any 4 rational numbers between ¾ and ⅘.	1	1. $4\sqrt{2}$	1	locate on number line as per step
2		2	122301/999	2	$a = \frac{9}{2}$ *and* $b = \frac{1}{2}$
3	314/99	3	x=5	3	(60 - 24√3)/13
4	16/81	4	1/333	4	-1
5	$\frac{6\sqrt{10}+10}{13}$	5	14	5	convert in p/q form
	1				

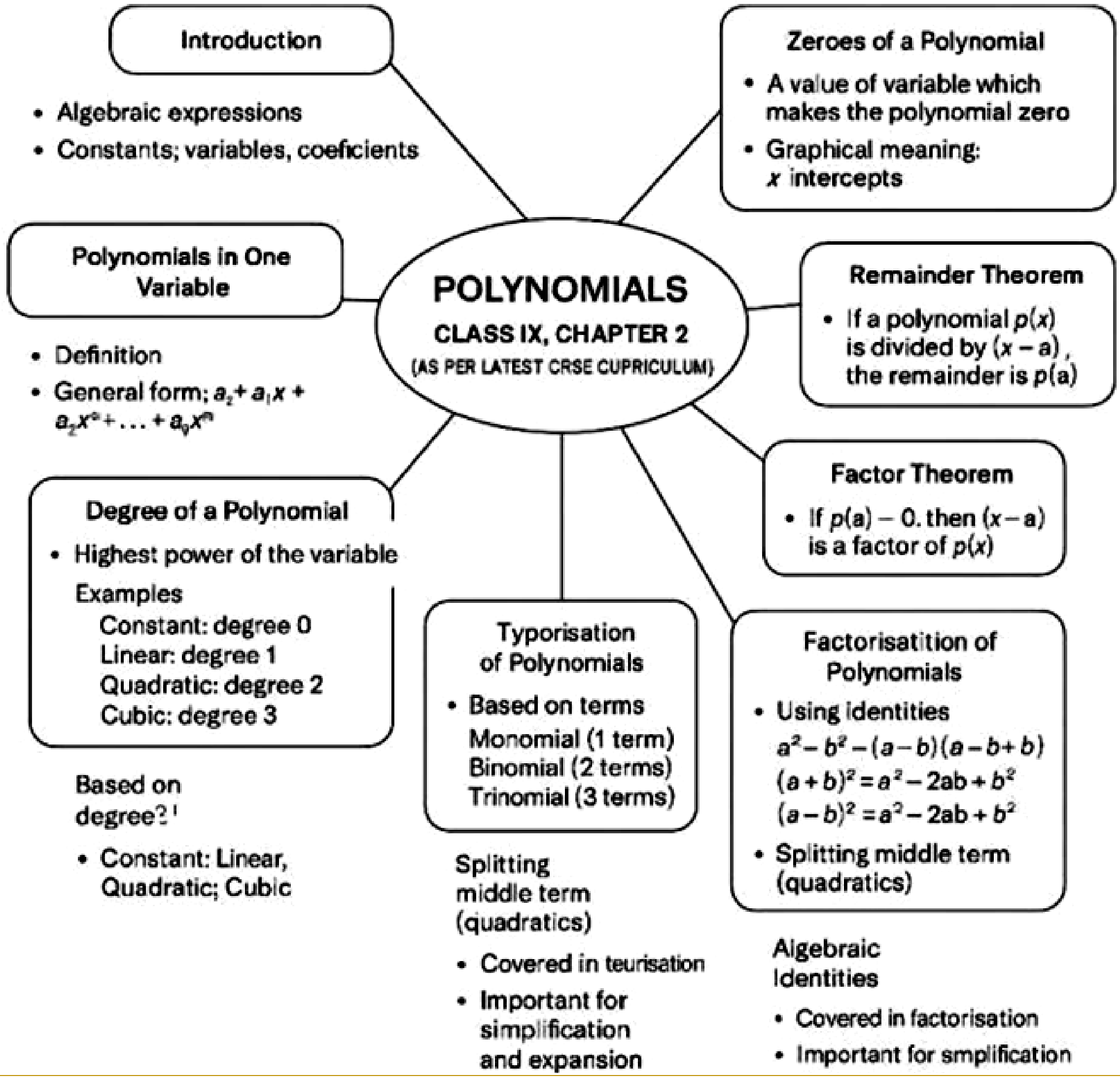
1 (i) Reciprocal of 4 - √15 = 4 + √15

(ii) No, 2.201200120001…… is not a rational number because decimal form of rational numbers is either terminating or non-termination recurring but it is non-terminating non-recurring. (iii) time = $\frac{7\sqrt{2}}{2}$ is an irrational number.

2 (i) x=-5 or x=5 (ii) $\left\{\left\{\left(y^{\frac{-4}{5}}\right)^{\frac{1}{5}}\right\}^{\frac{-5}{4}}\right\}^{5}$;y=5 (iii) if x and y are natural numbers, then x=y=5

3 (i) S (ii) 1 unit (iii) U and

CHAPTER – 2
POLYNIMIALS
MIND MAPPING



GIST/SUMMARY OF THE LESSON:
SUMMARY

- A polynomial is an algebraic expression made up of terms consisting of variables and coefficients, combined using addition, subtraction, and multiplication.
- Polynomials are classified based on:
- The number of terms (monomial, binomial, trinomial.
- The degree (linear, quadratic, cubic)
- Zero of a polynomial is a value of the variable for which the polynomial becomes zero.
- Remainder Theorem and Factor Theorem help in polynomial division and factorisation.
- Polynomials can be factorised using algebraic identities and by splitting the middle term.

DEFINITIONS AND FORMULAE:
DEFINITIONS

- **POLYNOMIAL**: An algebraic expression in which the exponents of the variable(s) are non-negative integers. Example: $4x^3 - 3x + 7$
- **DEGREE OF A POLYNOMIAL**: The highest power of the variable in the polynomial.
- **TYPES OF POLYNOMIALS**: - Monomial: 1 term (e.g., $5x$) ; - Binomial: 2 terms (e.g., $x + 2$) - Trinomial: 3 terms (e.g., $x^2 + 2x + 1$)
- **ZERO OF A POLYNOMIAL**: A number a such that $p(a) = 0$
- **REMAINDER THEOREM**: If $p(x)$ is divided by $(x - a)$, then the remainder is $p(a)$

	▪ **FACTOR THEOREM**: If p(a) = 0, then (x - a) is a factor of p(x)		
	FORMULAE AND IDENTITIES		
	ALGEBRAIC IDENTITIES: ▪ $(a + b)^2 = a^2 + 2ab + b^2$ ▪ $(a - b)^2 = a^2 - 2ab + b^2$ ▪ $a^2 - b^2 = (a - b)(a + b)$ ▪ $(x + a)(x + b) = x^2 + (a + b)x + ab$ ▪ $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ ▪ $(a – b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ ▪ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ ▪ $a^3 - b^3 = (a – b)(a^2 + ab + b^2)$ **GENERAL POLYNOMIAL FORM:** ▪ $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, where $a_n \neq 0$ ▪ Factorisation of Quadratic Polynomial: $ax^2 + bx + c = (px + q)(rx + s)$, using middle term splitting		
	MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)	**ANSWERS**	
1	A polynomial has a degree of 0. What is always true about it? A. It contains at least one variable B. It represents a constant C. It must be zero D. It has no terms	Answer: B	
2	The polynomial $p(x) = x^3 + ax + 1$ has no x^2 term. What is the coefficient of x^2? A. 1 B. a C. 0 D. Undefined	Answer: C	
3	Which of these is a binomial? A. $2x^2 + 3x + 1$ B. $x^2 + 4x$ C. $3x^3$ D. 7	Answer: B.	
4	A student adds two polynomials of degrees 3 and 2. The result will be a polynomial of degree: A. 6 B. 1 C. 5 D. 3	Answer: D.	
5	Which of these is a zero of the polynomial $p(x) = x^2 - 4$? A. 0 B. 2 C. 4 D. -1	Answer: B.	
6	The number of terms in the polynomial $x^2 + 2x - 3x + 7$ is: A. 2 B. 3 C. 4 D. 5	Answer: B.	
7	What is the degree of the polynomial $3x^4 + 0x^3 + 2x + 5$? A. 3 B. 4 C. 2 D. 1	Answer: B.	
8	If x = -1 is a zero of $p(x) = x^2 + kx + 1$, what is the value of k? A. 2 B. 1 C. -2 D. -1	Answer: A.	
9	Which polynomial has all zero coefficients except the constant term? A. x^3 B. $x^2 + x$ C. 5 D. 2x - 3	Answer: C.	
10	A polynomial is said to be monic if the coefficient of its highest degree term is: A. Zero B. One C. Negative D. Any rational number	Answer: B.	
	ASSERTION - REASON BASED QUESTIONS **DIRECTION** : In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R).Pick the correct option: A)Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A). B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A). C)Assertion (A) is true but Reason(R) is false. D)Assertion (A) is false but Reason(R) is true.		
1	Assertion: The degree of the polynomial $2x^3 + 3x^2 - 5x + 1$ is 3. Reason: The degree of a polynomial is determined by the highest power of the variable.	Solution: (A)	
2	Assertion: The polynomial $x^2 + 4x + 4$ can be factorized as $(x + 2)^2$. Reason: The polynomial $x^2 + 4x + 4$ is a perfect square trinomial.	Solution: (A)	
3	Assertion: The remainder when $x^3 + 2x^2 - 7x - 12$ is divided by x + 2 is -6.	Solution: (A)	

	Reason: According to the Remainder Theorem, the remainder is p(-2).	
4	Assertion: The polynomial $2x^2 + 5x - 3$ can be factored as $(2x - 1)(x + 3)$. Reason: The factors of -6 that add up to 5 are 6 and -1.	Solution: B).
5	Assertion: The degree of the polynomial 0 is undefined. Reason: The polynomial 0 has no terms with a variable.	Solution: A)
6	Assertion: The polynomial $x^3 - 8$ can be factored as $(x - 2)(x^2 + 2x + 4)$. Reason: $x^3 - 8$ is a difference of cubes.	Solution: (A)
7	Assertion: If $x + 1$ is a factor of p(x), then $p(-1) = 0$. Reason: According to the Factor Theorem, if $x - a$ is a factor of p(x), then $p(a) = 0$	Solution: (A)
8	Assertion: The polynomial $x^2 + 1$ has no real roots. Reason: The discriminant of $x^2 + 1$ is negative.	Solution: A)
9	Assertion: The polynomial $3x^2 + 2x - 5$ is a quadratic polynomial. Reason: The highest power of the variable in the polynomial is 2.	Solution: A)
10	Assertion: If $p(x) = x^3 + ax^2 + bx + c$, then $p(0) = c$. Reason: When $x = 0$, all terms with x become 0.	Solution: A)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

1	Find the value of p(2) if $p(x) = x^3 - 2x^2 + 5x - 1$. **SOLUTION**: $p(2) = (2)^3 - 2(2)^2 + 5(2) - 1= 8 - 8 + 10 - 1 = 9$
2	If $(x + 2)$ is a factor of $p(x) = x^3 + ax^2 + bx + 8$, find the values of a and b if $p(1) = 5$. **SOLUTION**: $p(-2) = 0 \Rightarrow (-2)^3 + a(-2)^2 + b(-2) + 8 = 0$; $-8 + 4a - 2b + 8 = 0 \Rightarrow 4a - 2b = 0$; $p(1) = 5 \Rightarrow 1 + a + b + 8 = 5 \Rightarrow a + b = -4$; Solving, $a = -4/3$, $b = -8/3$
3	If $(x - 1)$ is a factor of $p(x) = x^3 - ax^2 + bx - 1$, find the relation between a and b Solution: $p(1) = 0 \Rightarrow 1 - a + b - 1 = 0 \Rightarrow -a + b \Rightarrow a = b$
4	Simplify: $(x^2 + 2x + 1)/(x + 1) + (x^2 - 2x + 1)/(x - 1)$. **SOLUTION**: $(x + 1)^2/(x + 1) + (x - 1)^2/(x - 1)= x + 1 + x - 1 = 2x$
5	Find the value of k if $x + 1$ is a factor of $p(x) = x^3 + kx^2 + 2x + 1$. **SOLUTION**: $p(-1) = 0 \Rightarrow (-1)^3 + k(-1)^2 + 2(-1) + 1 = 0 \Rightarrow -1 + k - 2 + 1 = 0 \Rightarrow k = 2$
6	Factorize: $x^3 + 3x^2 + 3x + 1 - 8y^3$. **SOLUTION**: $(x + 1)^3 - (2y)^3 = (x + 1 - 2y)((x + 1)^2 + (x + 1)(2y) + (2y)^2)$
7	Find the remainder when $p(x) = x^4 + ax^3 + bx^2 + cx + d$ is divided by $x^2 + 1$. **SOLUTION**: Let $x^2 + 1 = 0 \Rightarrow x^2 = -1$ $p(x) = x^4 + ax^3 + bx^2 + cx + d = (x^2)^2 + ax(x^2) + bx^2 + cx + d = 1 - ax - b + cx + d = (d - b + 1) + x(c - a)$
8	If $(x - 2)$ is a factor of $p(x) = x^3 - 6x^2 + ax - 8$, find the value of a. **SOLUTION**: $p(2) = 0 \Rightarrow (2)^3 - 6(2)^2 + a(2) - 8 = 0$; $8 - 24 + 2a - 8 = 0 \Rightarrow a = 12$
9	Simplify: $(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$. **SOLUTION**: $x^3 + y^3 + z^3 - 3xyz$
10	Find the zeroes of the polynomial $p(x) = x^2 - 4$. **SOLUTION**: $x^2 - 4 = 0$; $(x - 2)(x + 2) = 0$; $x = 2$ or $x = -2$

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1	Find the value of m if $x + 1$ is a factor of $p(x) = x^3 + mx^2 + 2x + m$. **SOLUTION**: If $x + 1$ is a factor of p(x), then $p(-1) = 0$. $p(-1) = (-1)^3 + m(-1)^2 + 2(-1) + m= -1 + m - 2 + m = 2m – 3$; Since $p(-1) = 0$, we have: $2m - 3 = 0$; $2m = 3$; $m = 3/2$
2	Factorise: $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ **SOLUTION**: $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ $= (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) - 2(3y)(4z) - 2(2x)(4z) = (2x + 3y - 4z)^2$ $(2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$
3	If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$ **SOLUTION**: Given $x + y + z = 0$, we can rewrite this as: $z = -x - y$

	$x^3 + y^3 + z^3 = x^3 + y^3 + (-x - y)^3 = x^3 + y^3 - x^3 - 3x^2y - 3xy^2 - y^3$ $= -3x^2y - 3xy^2 = -3xy(x + y) = 3xy(-x - y) = 3xy(z) = 3xyz$ Therefore, $x^3 + y^3 + z^3 = 3xyz$.
4	Without finding the cubes, factorise $(x-2y)^3 + (2y-3z)^3 + (3z-x)^3$ **SOLUTION**: Notice that the sum of the terms $(x - 2y) + (2y - 3z) + (3z - x) = 0$. Using the identity $a^3 + b^3 + c^3 = 3abc$ when $a + b + c = 0$, we can factorize the expression: $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3 = 3(x - 2y)(2y - 3z)(3z - x)$
5	If (x-3) and (x-⅓) are the factors of the polynomial $px^2 + 3x + r$, show that p=r. **SOLUTION**: Let's rewrite the factors as $(x - 3) = 0$ and $(3x - 1) = 0$ (by multiplying the second factor by 3 to get rid of the fraction). The polynomial can be written in factored form as: $px^2 + 3x + r = p(x - 3)(x - 1/3)$ $= p(x - 3)(3x - 1)/3$ (to make the coefficient of x in the second factor match) $= (p/3)(x - 3)(3x - 1) = (p/3)(3x^2 - 10x + 3) = px^2 - (10/3)px + p$ Comparing coefficients of the original polynomial and the factored form: $px^2 + 3x + r = px^2 - (10/3)px + p$; The coefficient of x^2 is the same (p = p). The coefficient of x in the factored form is -(10/3)p, which should be equal to 3: $-(10/3)p = 3$; $-10p = 9$; $p = -9/10$ The constant term in the factored form is p, which should be equal to r: r = p Therefore, p = r = -9/10, which shows that p = r.
6	Factorise $(\frac{1}{x} + \frac{y}{3})^3$ **SOLUTION**: Using the identity $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$; $(\frac{1}{x} + \frac{y}{3})^3 = \left(\frac{1}{x}\right)^3 + 3\left(\frac{1}{x}\right)^2\left(\frac{y}{3}\right) + 3\left(\frac{1}{x}\right)\left(\frac{y}{3}\right)^2 + \left(\frac{y}{3}\right)^3 = \frac{1}{x^3} + \frac{y}{x^2} + \frac{y^2}{x} + \frac{y^3}{27}$
7	If $x^2 + \frac{1}{x^2} = 7$, find the value of $x^3 + \frac{1}{x^3}$ **SOLUTION**: $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})(x^2 - 1 + \frac{1}{x^2})$; $= (x + \frac{1}{x})(x^2 - 1 + \frac{1}{x^2}) = (x + \frac{1}{x})(x^2 - 1 + \frac{1}{x^2} + 3 - 3)$ $=(x + \frac{1}{x})(x^2 + 2 + \frac{1}{x^2} - 3) = (x + \frac{1}{x})((x + \frac{1}{x})^2 - 3)$; Find $(x + \frac{1}{x})$ from $x^2 + \frac{1}{x^2} = 7$ $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} = 7 + 2 = 9$; $x + \frac{1}{x} = \sqrt{9} \Rightarrow x + \frac{1}{x} = +3\ or\ -3$ $x^3 + \frac{1}{x^3} = (+3)(9 - 3) = +18$; $x^3 + \frac{1}{x^3} = (-3)(9 - 3) = -18$
8	If $p(x) = x^3 + 2x^2 + kx + 3$ and p(2) = 11, find the value of k. **SOLUTION**: $p(2) = 2^3 + 2(2)^2 + k(2) + 3 = 8 + 8 + 2k + 3 = 19 + 2k$ Since p(2) = 11, 19 + 2k = 11 ; 2k = -8 ; k = -4
9	Simplify: $(x + y + z)^2 - (x - y - z)^2$ **SOLUTION**: Given expression: $(x + y + z)^2 - (x - y - z)^2$ Using the identity $a^2 - b^2 = (a + b)(a - b)$, we can simplify: = [(x + y + z) + (x - y - z)][(x + y + z) - (x - y - z)]= (2x)(2y + 2z)= 4x(y + z)
10	Give expressions for the length and breadth of the following rectangle $25a^2 - 35a + 12$ **SOLUTION**: $25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$ = 5a(5a - 4) - 3(5a - 4) = (5a - 4)(5a - 3) ; The area of a rectangle is given by length × breadth. So, the possible expressions for length and breadth are: Length: 5a - 3 ; Breadth: 5a – 4 or Length: 5a - 4 ; Breadth: 5a - 3
	LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)
1	Verify (a) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$; (b) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ **SOLUTION**: (a) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ LHS = $x^3 + y^3$; RHS = $(x + y)(x^2 - xy + y^2) = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3$ Since LHS = RHS, the identity is verified. (b) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ LHS = $x^3 - y^3$; RHS = $(x - y)(x^2 + xy + y^2) = x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 = x^3 - y^3$ Since LHS = RHS, the identity is verified.
2	Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$.

SOLUTION: To prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$ without actual division, we can try to factorize $x^2 - 3x + 2$ and show that the roots of $x^2 - 3x + 2$ are also roots of $2x^4 - 5x^3 + 2x^2 - x + 2$

$x^2 - 3x + 2$= (x - 1)(x - 2)

Let's check if x = 1 and x = 2 are roots of $2x^4 - 5x^3 + 2x^2 - x + 2$:

For x = 1: $2(1)^4 - 5(1)^3 + 2(1)^2 - 1 + 2 = 2 - 5 + 2 - 1 + 2 = 0$

For x = 2: $2(2)^4 - 5(2)^3 + 2(2)^2 - 2 + 2 = 32 - 40 + 8 - 2 + 2 = 0$

Since x = 1 and x = 2 are roots of $2x^4 - 5x^3 + 2x^2 - x + 2$, and $x^2 - 3x + 2$ = (x - 1)(x - 2), we can conclude that $2x^4 - 5x^3 + 2x^2 - x + 2$is divisible by $x^2 - 3x + 2$.

3 If x+4 is a factor of the polynomial $x^3 - x^2 - 14x + 24$, find the other factors?

SOLUTION: Given that x + 4 is a factor of the polynomial $x^3 - x^2 - 14x + 24$.

Let's perform polynomial division or use synthetic division to find the other factors:

$x^3 - x^2 - 14x + 24$ = (x + 4)($x^2 - 5x + 6$)

Now, let's factorize the quadratic expression $x^2 - 5x + 6$:

$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$ = x(x - 2) - 3(x - 2)= (x - 2)(x - 3)

So, the polynomial $x^3 - x^2 - 14x + 24$. can be written as: $x^3 - x^2 - 14x + 24$. = (x + 4)(x - 2)(x - 3)

The other factors are (x - 2) and (x - 3).

4 If $(a + b + c) = 0$, find the value of $\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$

SOLUTION: Given that a + b + c = 0., We can rewrite this as: b + c = -a ; c + a = -b ; a + b = -c

Now, let's simplify the given expression: $\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$

$=\frac{(-a)^2}{bc} + \frac{(-b)^2}{ca} + \frac{(-c)^2}{ab} = \frac{(a)^2}{bc} + \frac{(b)^2}{ca} + \frac{(c)^2}{ab}$

Since a + b + c = 0, we can also write: a = -(b + c)

Now, let's simplify further: $=\frac{a^3+b^3+c^3}{abc}$

Using the identity $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ and a + b + c = 0, we get:

$a^3 + b^3 + c^3 = 3abc$

So, the expression becomes: $\frac{3abc}{abc} = 3$; The value of the given expression is 3.

5 If a + b + c = 5 and ab + bc + ca = 10, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

SOLUTION: Given that a + b + c = 5 and ab + bc + ca = 10.

We need to prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Using the identity: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Let's simplify $a^2 + b^2 + c^2$; $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$25 = a^2 + b^2 + c^2 + 2(10)$; $a^2 + b^2 + c^2 = 25 - 20$; $a^2 + b^2 + c^2 = 5$

Now, let's simplify $a^2 + b^2 + c^2 - ab - bc - ca$: = 5 - 10 = -5

Now, using the identity: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 5(-5) = -25$

Hence, proved that $a^3 + b^3 + c^3 - 3abc = -25$.

CASE BASED QUESTIONS (04 MARKS QUESTIONS)

1 Vetrivel and Vignesh start a new business together. The amount invested by both partners together is given by the polynomial $p(x) = 3x^2 + 10x + 7$. Which is the product of their individual shares.

i) Coefficient of x^2 in the given expression.

ii) Find the individual shares of both.

iii) Find the total amount invested by both if x = 200

iv) If another partner also came in the business with share $q(x) = 7x^2 + 12x + 9$. What will be the total amount of combined shares of all.

SOLUTION:

(i) $p(x) = 3x^2 + 10x + 7$ here coefficient of x^2 is 3.

(ii). ∴ The total amount invested is $p(x) = 3x^2 + 10x + 7$ and it is given that p(x) is the product of their individual shares. ∴ $we\ find\ the\ zeroes\ of\ p(x)$ by splitting the middle term method,

$p(x) = 3x^2 + (3+7)x + 7$; p(x) = 3x (x+1) + 7(x+1)= (x+1) (3x+7)
Hence, the individual shares are (x+1) and (3x+7)
(iii). if x = 200 then Total amount invested will be $p(200) = 3(200)^2 + 10(200) + 7$
= 3 x 40000 + 2000+7 = 120000 + 2000 + 7= Rs. 122007
(iv). another business partner came with share $q(x) = 7x^2 + 12x + 9$. ∴
The total amount = p(x) + q(x) = $3x^2 + 10x + 7 + 7x^2 + 12x + 9$. = $10x^2 + 22x + 16$

2 Ritesh lives in Delhi with his family. One day his father told him that we have a property in our village. Ritesh went to the village and found that he has a plot as ancestral property. The width of the plot was x m and length was 5 m less than 7 times of its breadth.

(a) Express the length as a polynomial.
(b) represent the perimeter as a polynomial.
(c) Form the polynomial to represent the area of the plot.
(OR)
(d) Express the perimeter if the length is increased by 2m.

SOLUTION:
(a) Given width = x m , Length = 7 times the breadth - 5 m = 7x - 5
(b)Perimeter = 2(length + breadth)= 2((7x - 5) + x) = 2(8x - 5) = 16x - 10
(c) Area = length × breadth= (7x - 5) × x = $7x^2 - 5x$
(d) New length = (7x - 5) + 2 = 7x - 3
New perimeter = 2(new length + breadth)= 2((7x - 3) + x) = 2(8x - 3) = 16x - 6

3 Simran is an engineer. She has a beautiful house. She made a beautiful rectangular garden and a swimming pool in her house. Area of the garden is $x^2 - 3x - 4$.

(a) What are the dimensions of the garden?
(b) Find the perimeter of the garden.
(c) If length is increased by x units then, what will be the total area of the garden?
(OR)
(d) What will be the total cost of preparing the garden, if the cost per sq unit is Rs.50

SOLUTION:
(a) Given area of the garden: $x^2 - 3x - 4$; Let's factorize the expression:
$x^2 - 3x - 4 = x^2 - 4x + x - 4$ = x(x - 4) + 1(x - 4)= (x - 4)(x + 1)
The dimensions of the garden are (x - 4) and (x + 1).
(b) Perimeter = 2(length + breadth)= 2((x - 4) + (x + 1)) = 2(2x - 3) = 4x - 6
(c) If length is increased by x units, new length = (x - 4) + x = 2x - 4
New area = (2x - 4)(x + 1) = $2x^2 - 2x - 4$
(d)Area of the garden: $x^2 - 3x - 4$
Total cost = Area × Cost per square unit = $(x^2 - 3x - 4)$ × 50 = $50x^2 - 150x - 200$

HOTS

1 If x + 1/x = 2√3, find the value of $x^5 + 1/x^5$

SOLUTION : $(x + 1/x)^2 = x^2 + 2 + 1/x^2 = (2\sqrt{3})^2 = 12$. ; $x^2 + 1/x^2 = 12 - 2 = 10$.
$x^3 + 1/x^3 = (x + 1/x)^3 - 3(x + 1/x) = (2\sqrt{3})^3 - 3(2\sqrt{3}) = 24\sqrt{3} - 6\sqrt{3} = 18\sqrt{3}$.
$(x^2 + 1/x^2)^2 = x^4 + 2 + 1/x^4 = 10^2 = 100$. L $x^4 + 1/x^4 = 100 - 2 = 98$.
S$x^5 + 1/x^5 = (x + 1/x)(x^4 + 1/x^4 - x^2 - 1/x^2 + 1) = (2\sqrt{3})(98 - 10 + 1) = 2\sqrt{3}(89)$; = 178√3.

2 Factor the following expression without using long division or direct expansion: $x^4 - 4x^2 + 3$, Then interpret the factorisation in terms of real zeroes of the polynomial.

SOLUTION: Step 1: Factor the expression as a quadratic in disguise
Let's treat x^2 as a variable. The expression $x^4 - 4x^2 + 3$ can be factored like a quadratic equation $ax^2 + bx + c$, where a = 1, b = -4, and c = 3, but with x^2 as the variable. So, $x^4 - 4x^2 + 3 = (x^2 - 3)(x^2 - 1)$.
Step 2: Factor each difference of squares $(x^2 - 1)$ can be factored into (x - 1)(x + 1).
$(x^2 - 3)$ can be factored into (x - √3)(x + √3).

Step 3: Write the final factorization $x^4 - 4x^2 + 3 = (x - \sqrt{3})(x + \sqrt{3})(x - 1)(x + 1)$.
Step 4: Interpret the factorization in terms of real zeroes
The real zeroes of the polynomial are the values of x that make each factor equal to zero. Setting each factor equal to zero gives us: $x - \sqrt{3} = 0 \Rightarrow x = \sqrt{3}$; $x + \sqrt{3} = 0 \Rightarrow x = -\sqrt{3}$; $x - 1 = 0 \Rightarrow x = 1$
$x + 1 = 0 \Rightarrow x = -1$; The final answer is: $\{(x - \sqrt{3})(x + \sqrt{3})(x - 1)(x + 1)\}$

3 You are given a cubic polynomial: $f(x) = x^3 - 6x^2 + 11x - 6$. Without performing long division, show that x=1, x=2, and x=3 are zeroes of the polynomial. Explain how patterns in coefficients (like sum or symmetry) might help you guess the factors quickly ?

SOLUTION: $f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$.
$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$.
$f(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$.
Since x = 1, x = 2, and x = 3 are zeroes, (x - 1), (x - 2), and (x - 3) are factors. Thus, $f(x) = (x - 1)(x - 2)(x - 3)$.
Expanding $(x - 1)(x - 2)(x - 3) = (x^2 - 3x + 2)(x - 3) = x^3 - 3x^2 - 3x^2 + 9x + 2x - 6 = x^3 - 6x^2 + 11x - 6$, which matches the given polynomial.
The coefficients of the polynomial are 1, -6, 11, -6. Notice that the sum of the coefficients is 1 - 6 + 11 - 6 = 0. This suggests that x = 1 might be a root, as f(1) = 0. Given the pattern and the fact that the polynomial is cubic, trying simple integer values like x = 1, x = 2, and x = 3 can quickly lead to finding the roots.
The final answer is: $\{(x - 1)(x - 2)(x - 3)\}$

4 A polynomial f(x) leaves a remainder of 5 when divided by x−1, and a remainder of 3 when divided by x+1. Find the remainder when f(x) is divided by $x^2 - 1$, and explain your reasoning using the Remainder Theorem.

SOLUTION: When f(x) is divided by $x^2 - 1 = (x - 1)(x + 1)$, the remainder will be of the form ax + b, because the divisor is a quadratic polynomial.
Given f(x) leaves a remainder of 5 when divided by x - 1, by the Remainder Theorem, f(1) = 5.
For the remainder ax + b, this implies a(1) + b = 5.
Given f(x) leaves a remainder of 3 when divided by x + 1, by the Remainder Theorem, f(-1) = 3.
For the remainder ax + b, this implies a(-1) + b = 3.
we have two equations:1). a + b = 5 2). -a + b = 3
Adding the two equations to eliminate a: 2b = 8, which implies b = 4.
Substitute b = 4 into one of the equations, a + 4 = 5, which implies a = 1.
The remainder when f(x) is divided by $x^2 - 1$ is ax + b = 1x + 4 = x + 4.
The final answer is: {x + 4}

5 A polynomial $f(x) = x^4 + px^2 + q$ is said to be symmetric if f(x)=f(-x)
What does symmetry mean in this context?
If f(1)=6 and f(2)=17 , find the values of p and q.

SOLUTION: Symmetry in the polynomial $f(x) = x^4 + px^2 + q$ means that the polynomial remains unchanged when x is replaced by -x. Since $f(-x) = (-x)^4 + p(-x)^2 + q = x^4 + px^2 + q = f(x)$, the polynomial is indeed symmetric with respect to the y-axis.
Given f(1) = 6 and f(2) = 17, we can set up two equations:
1. $(1)^4 + p(1)^2 + q = 6 \Rightarrow 1 + p + q = 6$
2. $(2)^4 + p(2)^2 + q = 17 \Rightarrow 16 + 4p + q = 17$

From equation 1: p + q = 5. ; From equation 2: 4p + q = 1.
Subtract the first equation from the second equation: (4p + q) - (p + q) = 1 - 5.
This simplifies to 3p = -4, so p = -4/3.
Substitute p = -4/3 into p + q = 5: -4/3 + q = 5.
q = 5 + 4/3 = (15 + 4)/3 = 19/3.
The final answer is: p = -4/3, q = 19/3

EXERCISE

MULTIPLE CHOICE QUESTIONS (1 MARK)

1 If xy = 6 and 3x+2y = 12, the value of $9x^2 + 4y^2$ will be
a)70 b) 72 c) -72 d) none of these

2 One of the factors of $(25x^2 - 1) + (154 + 5x)^2$ is:

	a) 5 + x b) 5 – x c) 5x – 1 d) 10x	
3	What is the degree of Polynomial √3? a) 0 b) 1 c) 2 d) 2	
4	If (x – a) is a factor of p(x) = a-x , then which of the following is true ? (a) p(a) = 2 (b) p(b) = 0 (c) p(a) = 0 (d) p(c) =2	
5	8 is a polynomial of degree (a) 0 (b) 1 (c) 2 (d) 3	
	ASSERTION - REASON BASED QUESTIONS A statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. a) Both A and R are true and R is the correct explanation of A b) Both true but R is false d) A is false but R is true	
1	Assertion (A): The factorisation of $z^3 + 125\ is\ (z+5)(z^2 - 5z + 25)$ Reason (R) : We know $x^3 + y^3 = (x+y)^3 + 3xy(x+y)$	
2	Assertion (A): The polynomial $p(x) = 4x^3 - 3x^2 + 5x - 6$ when divided by (x – 1) gives zero as the remainder. Reason (R) : (x – 1) is a factor of the polynomial $p(x) = 4x^3 - 3x^2 + 5x - 6$	
3	Assertion (A) : The degree of Polynomial of (x – 2) (x – 3) (x +4) is 4. Reason (R) : The number of zeros of a polynomial is the degree of that polynomial.	
4	Assertion (A) : (x + 1) is a linear polynomial. Reason (R) : Linear polynomials have one zero.	
5	Assertion : The constant polynomial 0 is called zero polynomial. Reason: √x+3 is a polynomial.	
	VERY SHORT ANSWER TYPE QUESTIONS	
1	Use suitable identity to find the following product (3-2x) (3+ 2x).	
2	Examine whether x+2 is a factor of $x^3 + 3x^2 + 5x + 6$.	
3	Find the zeroes of the polynomial :$p(x) = (x-2)^2 - (x+2)^2$	
4	If $x + \frac{1}{x} = 2$. What will be the value of $x^{100} + \frac{1}{x^{101}}$.	
5	Factorise the following: $4x^2 + 20x + 25$	
	SHORT ANSWER TYPE QUESTIONS	
1	Factorise :$2x^3 - 3x^2 - 17x + 30$	
2	If (x - 2) is a factor of $x^3 + kx^2 - 4x - 12.$, find the value of k.	
3	Factorize: $x^2 + 1/x^2 + 2 - 2x - 2/x$	
4	Find the following product:$(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$	
5	If (x – 2k) is a factor of $f(x) = x^4 - 4k^2x^2 + 2x + 3k + 3$. Find the value of k	
	LONG ANSWER TYPE QUESTIONS	
1	If $p(x) = x^4 + ax^3 + bx^2 + cx + d$ and p(1) = 10, p(-1) = 6, p(2) = 20, and p(-2) = 12, find the values of a, b, c, and d.	
2	If x + y + z = 8 and xy + yz + zx = 20, find the value of $x^3 + y^3 + z^3 - 3xyz$.	
3	If (x - 2) and (x + 3) are factors of the polynomial $p(x) = x^3 + ax^2 + bx - 12$, find the values of a and b.	
4	If $p(x) = x^3 - 2x^2 + kx + 5$ and p(2) = 11, find the value of k. Then, find p(-2).	
5	If (x + 1) is a factor of the polynomial $p(x) = 2x^3 + ax^2 + 2x + 1$, find the value of a. Then, find the other factors of p(x).	

CASE BASED QUESTIONS

1 Manoj is making a box using cardboard. He found a cardboard box with sides 9cm by 9cm. He cuts out four squares of equal size at corners and folds up the sides to make an open box. Manoj paints it beautifully and puts all his pens in it.
(a) Suppose the side of the square cut out is x cm, then find the polynomial to find the volume of the cuboid formed.
(b) Identify the degree of the polynomial.
(c) If the side of the square is 1 cm then what is the volume of the box? (OR)
(d) If the whole box is covered by a sheet of paper then what will be the area of paper?

2 Rahul is a landscape designer who is designing a rectangular garden bed. The length of the bed is 2 meters more than its width. If the area of the bed is given by the polynomial $x^2 + 2x$, where x is the width of the bed:
(i) What is the length of the garden bed in terms of x?
(ii) If the area of the bed is 15 square meters, find the width and length of the bed.
(iii)Find the value of the polynomial $x^2 + 2x$ when x = 4.
(iv)If the polynomial $x^2 + 2x$ represents the area of the bed, what type of polynomial is it?

3 A shipment service provider uses three types of containers for shipping materials. The height and width of the three containers are the same. The containers' height is 0.15 m more than their width, and the volume of the smallest container is 652 m^3
(i) Write a polynomial relating Container 1's length, breadth and height with its volume.
(ii) Which of the following statements is true?
The volume of the three containers is the same.
The length of the three containers is the same.
The volume of Container 3 is 2,608 m^3.
The length of Container 3 is 4 times the length of Container 2.
(iii) What is the height of each container?

HOTS

1 If both $(x + 1)$ and $(x - 1)$ are factors of $ax^3 + x^2 - 2x + b$,find a and b.

2 If $(x^2 - 1)$ is a factor of ax4$+bx^3 + cx^2 + dx + e$, show that $a + c + e = b + d = 0$

ANSWERS

MULTIPLE CHOICE QUESTIONS

1 b) 72.
2 d) 10x.
3 a) 0.
4 c) p(a) = 0
5 a) 0.

ASSERTION - REASON BASED QUESTIONS

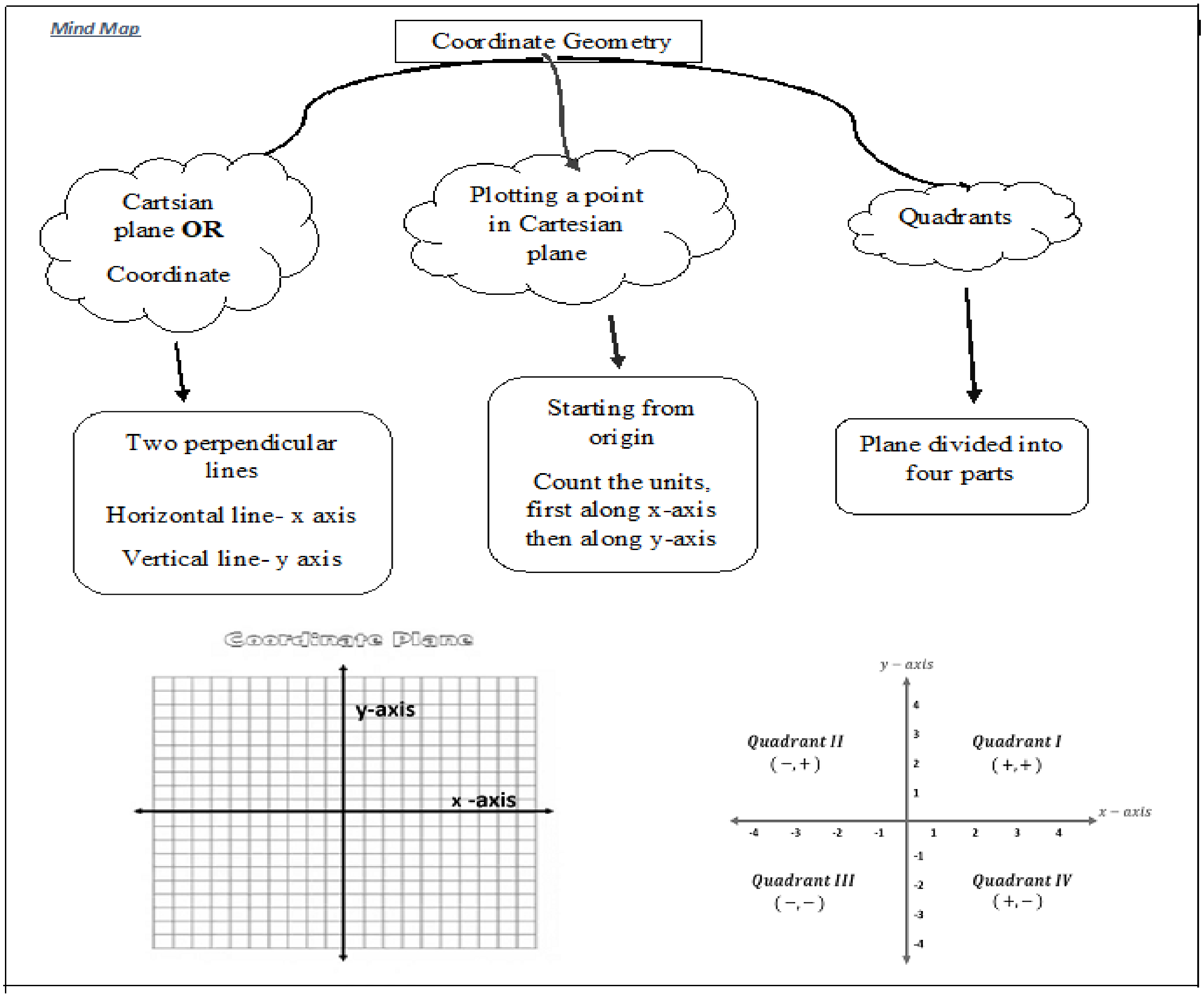
1 c) A is true but R is false.
2 a) Both A and R are true and R is the correct explanation of A.
3 d) A is false but R is true.
4 b) Both A and R are true but R is not the correct explanation of A.
5 c) A is true but R is false.

VERY SHORT ANSWER TYPE QUESTIONS

1 $9 - 4x^2$
2 Since p(-2) = 0, x + 2 is a factor of $x^3 + 3x^2 + 5x + 6$.
3 The zero of the polynomial is x = 0.
4 x = 1; $x^{100} + 1/x^{101} = 1^{100} + 1/1^{101} = 1 + 1 = 2$
5 The factorised form is $(2x + 5)^2$.

	SHORT ANSWER TYPE QUESTIONS
1 2 3 4 5	The factorised form is $(x - 2)(x + 3)(2x - 5)$. The value of k is 3. $(x - 1/x)^2$ $8x^3 - y^3 + 27z^3 + 18xyz$ The value of k is -3/7
	LONG ANSWER TYPE QUESTIONS
1 2 3 4 5	The values of a, b, c, and d are: $a = 0$; $b = -7/3$; $c = 2$; $d = 28/3$ the value of $x^3 + y^3 + z^3 - 3xyz$ is 32. The values of a and b are: $a = 3$; $b = -4$ The value of k is 3, and p(-2) is -17. The value of a is 3, and the other factor is $2x^2 + x + 1$.
	CASE BASED QUESTIONS
1	(a) The polynomial to find the volume is $4x^3 - 36x^2 + 81x$. (b) The degree of the polynomial $4x^3 - 36x^2 + 81x$ is 3. (c) The volume of the box is 49 cubic cm. (d) The area of the paper is $(9 - 2x)^2 + 4x(9 - 2x)$ or 77 square cm for $x = 1$.
2	(i) Length = x + 2 (ii) Width = 3 meters Length = 3 + 2 = 5 meters (iii) 24 (iv) The polynomial x^2 + 2x is a quadratic polynomial.
3	(i) $652 = l_1 \times x \times (x + 0.15)$ (ii) C. The volume of Container 3 is 2608 m^3. (iii) The height of each container is 7.15 m.
	HOT QUESTIONS
1	$a - b = 3$ and $a + b = 1$ solving we get a=2 and $b = -1$ Put p(1) we get $a + b + c + d + e = 0$, if p(-1) we get $a - b + c - d + e = 0$ comparing equations, we get $a + c + e = b + d = 0$

CHAPTER – 03
COORDINATE GEOMETRY
MIND MAPPING



GIST/SUMMARY OF THE LESSON:

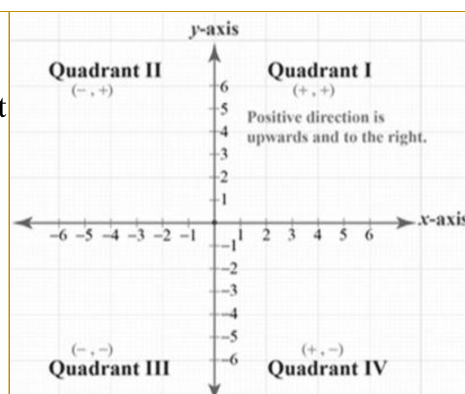
Gist of the lesson

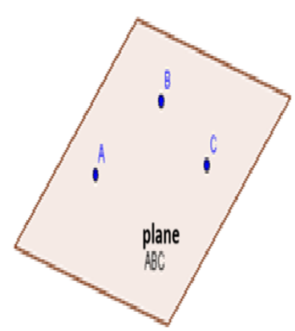
- CARTESIAN SYSTEM
- PLOTING OF POINTS

DEFINITIONS AND FORMULAE:

KEY POINTS

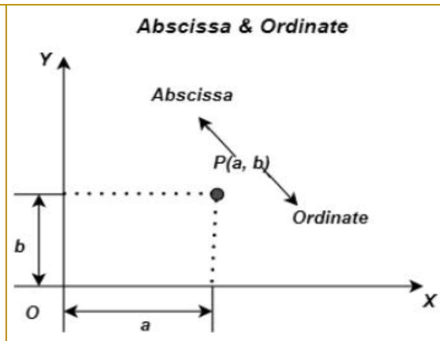
- Point – it defines the position of an object. It has no length and no thickness.
- Plane – A flat surface is known as plane.
- Cartesian plane – A plane on which one horizontal line and one vertical line intersecting each other at a point is known as Cartesian plane.
- Horizontal line - X- axis
- Vertical line - Y- axis
- Intersection point - Origin (O) (0,0)



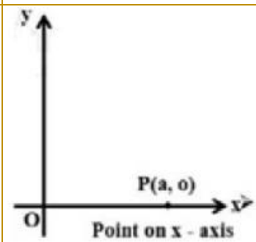


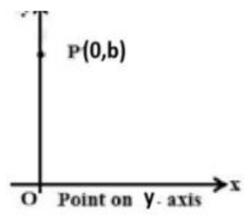
- **CONVENTIONAL SIGNS –**
- Right to the origin (+ve)
- Left to the origin (-ve)
- Above to the origin (+ve)
- Below to the origin (-ve)

- **QUADRANT**- x- axis and y-axis divides the plane into four parts,
- each part is called quadrant.
- Ist Quadrant - (+, +)
- IInd Quadrant - (- , +)
- IIIrd Quadrant - (- , -)
- IVth Quadrant - (+ , -)



- **POINT IN A CARTESIAN PLANE**- Any point in Cartesian plane is expressed in the form of ordered pair or fixed form (x , y).
- **ABSCISSA** – x- coordinate or perpendicular distance of a point from y-axis,
- **ORDINATE** – y – coordinate or perpendicular distance of a point from x-axis.
- Any point on the x-axis is of the form (a, 0)
- Any point on the y-axis is of the form (0, b)





MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)

1	The name of horizontal line in the cartesian plane which determines the position of a point is called: a) Origin b) X-axis c) Y-axis d) Quadrants	**SOLUTION**: b) x- axis
2	The name of vertical line in the cartesian plane which determines the position of a point is called: a) Origin b) X-axis c) Y-axis d) Quadrants	**SOLUTION**: c) y- axis
3	The section formed by horizontal and vertical lines determining the position of point in a cartesian plane is called: a) Origin b) X-axis c) Y-axis d) Quadrants	**SOLUTION**: d) Quadrants
4	The point of intersection of horizontal and vertical lines determining the position of point in a cartesian plane is called: a) Origin b) X-axis c) Y-axis d) Quadrants	**SOLUTION**: a) Origin
5	If x coordinate of a point is zero, then the point lies on: a) First quadrant b) Second quadrant c) X-axis d) Y-axis	**SOLUTION**: d) Y-axis
6	If the coordinates of a point are (3, 0), then it lies in: a) X-axis b) Y-axis c) At origin d) Between x-axis and y-axis	**SOLUTION**: a) X-axis
7	Abscissa of a point is positive in: a) I and II quadrants b) I and IV quadrants c) I quadrant only d) II quadrant only	**SOLUTION**: b) I and IV quadrants
8	If $x < 0$ and $y > 0$, then the point lies in a) I quadrant b) II quadrant c) III quadrant d) IV quadrant	**SOLUTION**: b) II quadrant
9	The point whose ordinate is 8 and lies on y-axis: a) (0, 8)b) (8, 0)c) (5, 8)d) (8, 5)	**SOLUTION**: a) (0, 8)
10	If the coordinates of a point are (0, -4), then it lies in: a) X-axis b) Y-axis c) At origin d) Between x-axis and y-axis	**SOLUTION**: b) y-axis

	ASSERTION - REASON BASED QUESTIONS		
	DIRECTIONS: In following questions, a statement of Assertion(A) is followed by a statement of Reason (R) Choose the correct option. a) Both assertion and reason are true and the reason is the correct explanation of assertion. b) Both assertion and reason are true but the reason is not the correct explanation of the assertion. c) Assertion is true and the reason is false. d) Assertion is false and the reason is true.		
1	Assertion(A): A point whose abscissa is -8 and ordinate is 5 lies in second Quadrant. Reason(R): Points of the type (- , +) lie in the second quadrant.	**SOLUTION**: a)	
2	Assertion(A): If the ordinate of a point is equal to its abscissa, then the point lies either in the first quadrant or in the second quadrant. Reason(R): A point both of whose coordinates are negative will lie in the third quadrants.	**SOLUTION**: d)	
3	Assertion(A): A point whose abscissa is 0 and ordinate is 2 lies on y-axis. Reason(R): Equation of y-axis is x = 0.	**SOLUTION**: a)	
4	Assertion(A): The perpendicular distance of the point A(3, 4) from the y-axis is 4. Reason(R): The perpendicular distance of a point from y-axis is called its x-coordinate.	**SOLUTION**: d)	
5	Assertion (A) : The abscissa of point (3,5) is 5. Reason (R) : The signs of points in quadrants I, II, III and IV are respectively (+ , +) ,(- , +) ,(-, -) and (+, -).	**SOLUTION**: d)	
6	Assertion(A): Point (2, -3) lies in IInd Quadrant. Reason (R) : A point is of the form (+, -) lies in IV Quadrant.	**SOLUTION**: d)	
7	Assertion(A): Intersection of horizontal line and vertical line is called origin. Reason(R): coordinate of the origin is (0, 0).	**SOLUTION**: b)	
8	Assertion(A): The perpendicular distance of a point(-2, -5) on the axis is -5. Reason(R): For any point, the perpendicular distance from y axis is known as abscissa.	**SOLUTION**: d)	
9	Assertion(A): For plotting any point on the Cartesian plane, the order of a point is (x, y). Reason(R): For a point (x, y), x coordinate is known as abscissa and y coordinate is known as ordinate.	**SOLUTION**: b)	
10	10. Assertion(A): A point (-2, -5) lies in IIIrd Quadrant. Reason(R): Any point is of the form (-, -) lies in the IIIrd Quadrant.	**SOLUTION**: a)	
	VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)		
1	If (x + 2, 4) = (5, y – 2), then find the value of (x, y) Solution: x + 2 = 5, 4 = y – 2 ; X = 5 – 2, y = 4 + 2 ; x = 3, y = 6 ; coordinates (x, y) are(3,6)		
2	P(3,2) and Q(7,7) are two points. Perpendiculars are drawn to the x-axis from P and Q meeting the x-axis at L and M respectively. i) Find the coordinates of L and M. ii) Find the length of LM.	**SOLUTION**: Coordinates of L is (3,0) Coordinates of M is (7,0) ii) **SOLUTION**: Length of LM = abscissa of M – abscissa of L = 7 – 3 units = 4 units	
3	Find the coordinates of the point i) whose ordinate is – 4 and which lies on y-axis. ii) whose abscissa is 5 and which lies on x-axis.	**SOLUTION**: (0,-4) **SOLUTION**: (5, 0)	
4	i) Find the coordinate of the point which lies on the x and y axes both. ii) Name the Quadrant if the ordinate is 1 and abscissa is – 1.	**SOLUTION**: Origin (0, 0) **SOLUTION**: IInd Quadrant	
5	Without plotting the points indicate the quadrant in which they will lie, if i) ordinate is 5 and abscissa is – 3 ii) abscissa is – 5 and ordinate is – 3	**SOLUTION**: II Quadrant **SOLUTION**: III Quadrant.	
6	i)What is the distance of the line y = -2 from x-axis. ii) . What is the intersection point of the line x=5 with x-axis.	**SOLUTION**: 2unit **SOLUTION**: (5, 0)	

7 i) Point (0, –2) lies on y-axis. (True/False)
ii) The perpendicular distance of the point (4, 3) from the x-axis is 4. (True/False)

SOLUTION: True
SOLUTION: False

8 A point lies on the x-axis at a distance of 7 units from the y-axis. What are its coordinates? What will be the coordinates if it lies on y-axis at a distance of -7 units from x-axis?
SOLUTION: A point lies on the x-axis at a distance of 7 units from the y-axis, then its coordinates are (7, 0) and if it lies on y-axis at a distance of –7 units from x-axis, its coordinates will be (0, -7).

9 Which of the following points lie on y-axis?
A (1, 1), B (1, 0), C (0, 1), D (0, 0), E (0, – 1), F (– 1, 0), G (0, 5), H (– 7, 0), I (3, 3).
SOLUTION: .We know that a point lies on the Y-axis if its x-coordinate is zero. Here, x-coordinate of points C(0,1), D(0, 0), E(0, -1) and G(0, 5) are zero. So, these points lie on Y-axis. Also, D(0, 0) is the intersection point of both the axes, so we consider that it lies on the y-axis as well as on x-axis.

10 i) The section formed by the horizontal line and vertical line on a Cartesian plane is known as…………..
SOLUTION: Quadrant.
ii) The horizontal line on the Cartesian plane is known as ……….
SOLUTION: x-axis

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

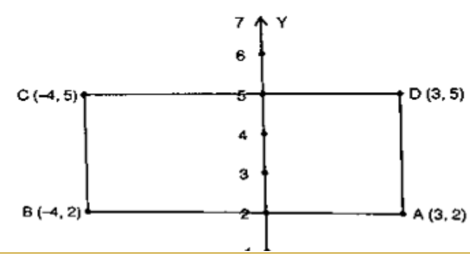
1 Name the quadrants and axis in which following points are lying

Point	Abscissa	Ordinate
A	5	-5
B	-1	-7
C	-8	2
D	4	6

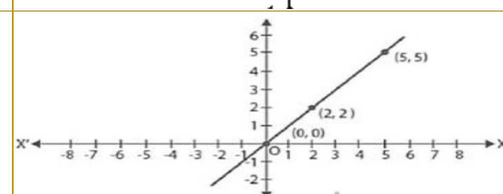
SOLUTION: A – IV Quadrant,
B – III Quadrant,
C – II Quadrant,
D – I Quadrant

2 Three vertices of a rectangle are (3,2), (-4,2), and(-4,5). Plot these points and find the coordinates of the fourth vertex.

SOLUTION:



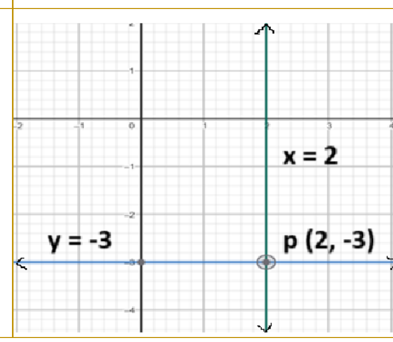
3 Plot the points (0, 0), (2, 2), (5, 5) and check whether they are collinear or not.
SOLUTION: yes, they are collinear.



4 Find the coordinate of point which are equidistant from these two points A(0, 2) and B(0, -2). How many points are possible to satisfy this condition?

SOLUTION: All the points on x- axis

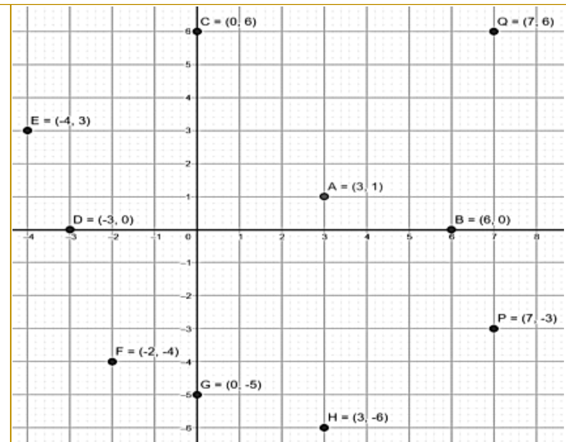
5 Write the point of intersection of the lines x = 2 and y = -3
SOLUTION: Intersection point p(2, -3)



6 Plot following points on the graph paper

A(3, 1),	B(6, 0),
C(0, 6),	D(-3, 0),
E(-4, 3),	F(-2, -4),
G(0, -5),	H(3, -6),
P(7, -3) ,	Q(7, 6).

SOLUTION:



7 A boy marks four points (4,0), (0.2), (-4,0) and (0,-2) on the graph paper and joins these points by Line segments and see that a quadrilateral figure is obtained.

i) plot these points on the graph paper

ii) Name the quadrilateral obtained after joining the points

iii) what is the area of the figure obtained?

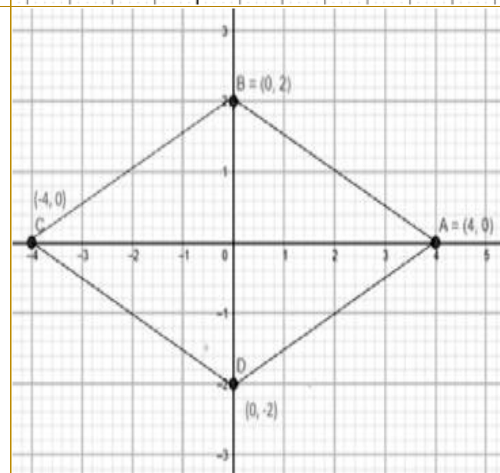
SOLUTION: i) Right Side

ii) Rhombus

iii) Area of Rhombus $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 8 \times 4$

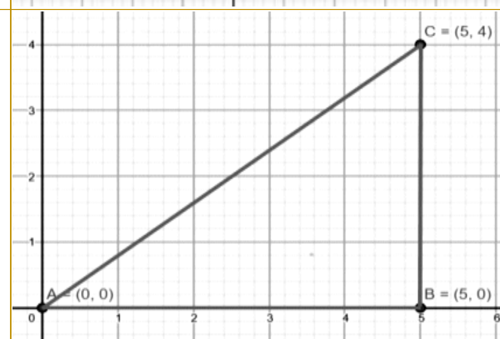
= 16 sq. unit



8 Draw a triangle with vertices (0, 0) , (5,0) and(5, 4). Find its area.

SOLUTION: It's a Right angled triangle

Area of triangle = $\frac{1}{2}$ x base x height = $\frac{1}{2}$ x 5 x 4 = 10 sq. unit



9 In the given figure , POQ is a triangle with coordinate of P and O as ($\sqrt{13}$, 0) and (0,0) respectively. If PQ = 7, find the coordinates of Q.

SOLUTION: Coordinate of P = ($\sqrt{13}$, 0) and Q = (0,0)

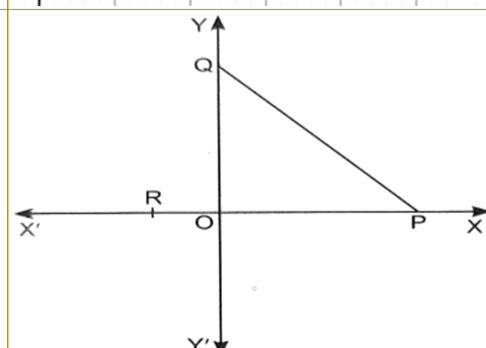
So, OP = $\sqrt{13}$ and PQ = 7 (given)

Using Pythagoras property

$(OP)^2 + (OQ)^2 = (PQ)^2$; $(\sqrt{13})^2 + (OQ)^2 = (7)^2$; $13 + (OQ)^2 = 49$

$(OQ)^2 = 49 – 13$; $OQ = \sqrt{36}$; OQ = 6

So, the coordinates of Q is(0, 6) because Q lies on y-axis.



10 The perpendicular distance of the point A(m,2n) from the x-axis and y-axis is 6 units. Given that m < 0 and n > 0, then what are the coordinates of point B(n+1, m) ?

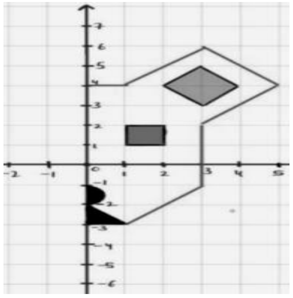
SOLUTION:: Since, the perpendicular distance of the point A(m,2n) from the x-axis and y-axis is 6 units Therefore, (m,2n) = (-6,6) [because m < 0 and n > 0 i.e II quadrant] ; m = -6 and 2n = 6

m = -6 and n $= \frac{6}{2}$ = 3 ; So, Coordinates of point

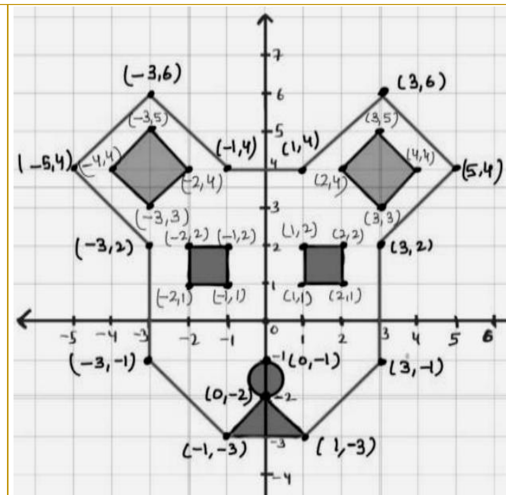
B(n+1, m) = (3+1, -6) = (4, -6)

LONGANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1 Ronny was reading a Maths magazine in which he saw an interesting figure which was half drawn as shown below. He wish to draw it completely. Draw again and help him to complete the figure and tell which line of symmetry you use. Along with that mark all the coordinates used in this drawing.



SOLUTION: We use y-axis as the line of symmetry to complete the figure



2 Lucky started from origin towards North-east direction to reach his school where east direction is represented by the positive x-axis. While coming back from school, he came with different path. He started from school 8km towards south direction and then turn right to walk 6km to reach home again. If speed of Lucky is 5 km/hr , how much time will he take to reach his school in the morning and what are the coordinates of school.

SOLUTION: Coordinates of School (B) = (6,8)

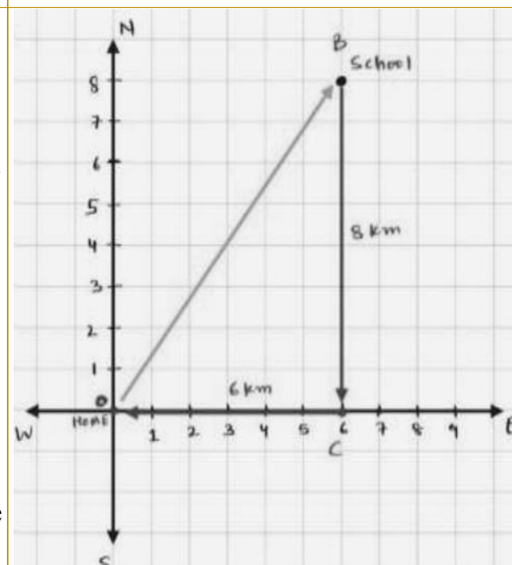
To find out the distance from Home to School (OB)

We use Pythagoras property,

$(OB)^2 = (BC)^2 + (CO)^2$; $(OB)^2 = 8^2 + 6^2$;

$(OB)^2 = 64 + 36$; $(OB)^2 = 100$; $OB = \sqrt{100}$; OB = 10 km

Since, the distance between home to school is 10km and speed of lucky is 5km/hr. So, Time taken to reach his school from home in the morning $= \frac{distance}{speed} = \frac{10}{5}$ = 2hrs



3 Once an Archaeologist was doing research in cave where he got a map to find treasure. So he went in search of treasure. But when he reached there his map fell into the water and everything vanished from the map. On the other side Bandit know the way to reach there but Police is behind him to catch. Tell archaeologist about the coordinates of Treasure and the length of shortest path to get the treasure before bandit (1 unit in map = 1Km).

SOLUTION: Coordinates of treasure (T) = (7, 14)

Coordinates of Archaeologist (A) = (2, 2)

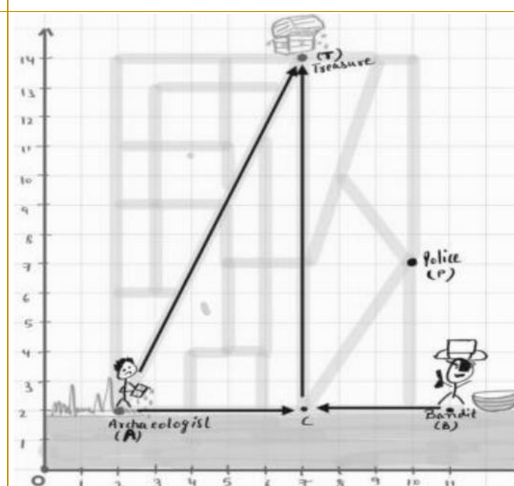
Coordinates of point C= (7, 2)

So, AC = 7 - 2 =5 ; And CT = 14 – 2 = 12

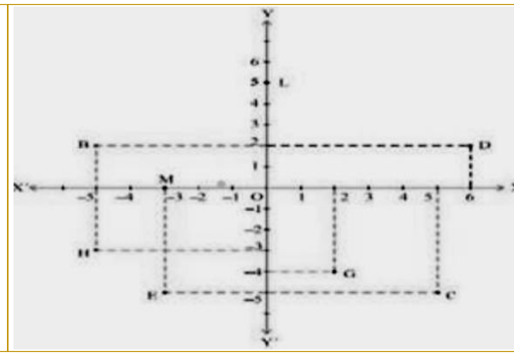
And the shortest path to reach there is AT , So, for finding length of shortest path, we use Pythagoras property

$(AT)^2 = (AC)^2 + (CT)^2$; $(AT)^2 = 5^2 + 12^2$

$(AT)^2 = 25 + 144$; $(AT)^2 = 169$; $AT = \sqrt{169}$; AT = 13km So, the length of shortest path AT is 13km.



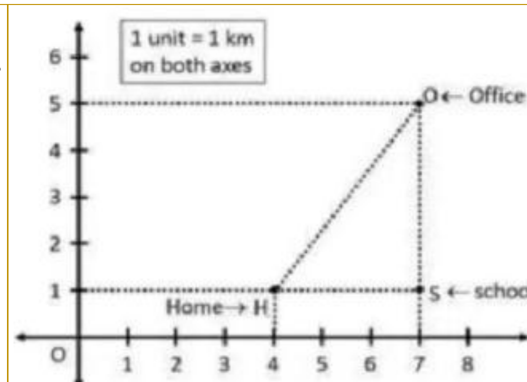
4 From the figure write the answer of the following:
(i)The coordinates of D.
(ii)The point identified by the coordinates (–5, 2).
(iii)The abscissa of the point B.
(iv)The ordinate of the point H.
The point identified by the coordinates (2,-4).
SOLUTION: i) D (6,2) ii)B iii) -5 iv) -3 v) G



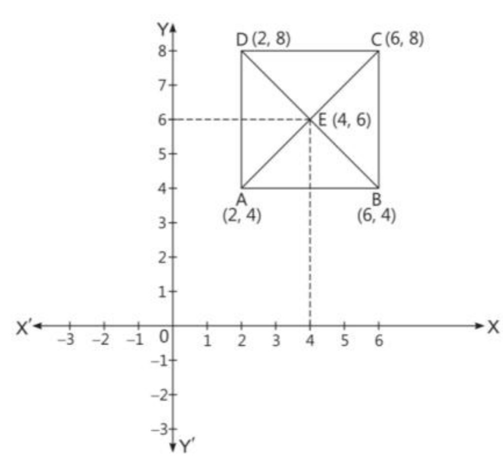
5 Without plotting the points indicate the quadrant in which they will lie, if
(i) the ordinate is 5 and abscissa is – 3
(ii) the abscissa is – 5 and ordinate is – 3
(iii) the abscissa is – 5 and ordinate is 3
(iv) the ordinate is 5 and abscissa is 3
(v) the abscissa is 2 and ordinate is -5
SOLUTION:
The point is (-3,5). Hence, the point lies in the II quadrant.
The point is (-5,-3). Hence, the point lies in the III quadrant.
The point is (-5,3). Hence, the point lies in the II quadrant.
The point is (3,5). Hence, the point lies in the I quadrant.
The point is (2,-5). Hence, the point lies in the IV quadrant.

CASE BASED QUESTIONS (04 MARKS QUUESTIONS)

1 Shristi has to reach her office every day at 8:00 am. On the way to her office, she drops her son at school. Now, the location of Shristi’ s home, her son’s school and her office are represented below by the map, Using the details given, answer the following questions
Find the coordinate of Shristi’s home?
Find the Area enclosed by triangle HOS?
Find the distance between Shristi’s home and his son’s school?
SOLUTION (i) (4, 1)
(ii) Area of triangle = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 3 × 4= 6 sq. km
(iii) 3km



2 Vivek wants to purchase a painting for drawing room. First of all, he puts a grid on the wall so that he could hang a painting on that area. The corner points of the grid are A (2, 4); B (6, 4); C (6, 8) and D (2, 8).
i) what is the area of the painting?
ii) Write the intersection point of the diagonal of the paintings.
iii) what is the perimeter of the painting?
OR
Write the image of the point B with respect to y-axis.
SOLUTION: i) Area = side x side = 4 x 4 = 16 sq unit
ii) intersection point of the diagonals is E Coordinate of the point E is (4, 6)
iii) Perimeter of the painting = perimeter of square
= 4 x side ; = 4 x 4 ; = 16 sq. unit
OR Image of point B with respect to y-axis is (-6, 4)



3 A teacher decides to teach some important concepts of geometry through playground activity so he takes all the students to the playground. There he divides playground into two small playing area, one is in the shape of rectangle and the other one is triangular in shape, which is shown in figure on the basis of the given information, students are asked some questions.

i) Find the altitude of the triangle ABC.

ii) Find the area of a triangle ABC.

iii) Find the coordinate of point G.

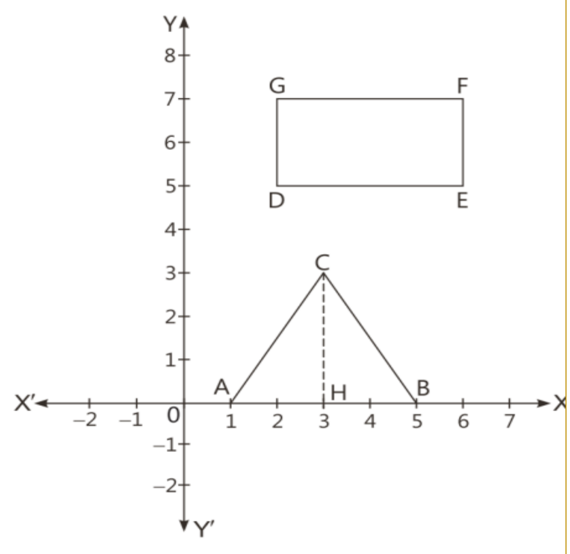
OR Find the area of a rectangle DEFG.

SOLUTION: i) Altitude CH = 3 unit

ii) Area of triangle = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 4 × 3 = 6 sq. unit

iii) Coordinate of G (2, 7)

iv) Area of rectangle = length x breadth = 4unit x 2 unit = 8 sq. unit



HOTS

1 There is a square lawn PQRS in the ground as shown in below figure

i) What is area of the lawn PQRS.

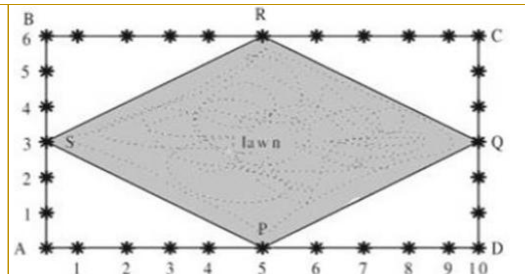
SOLUTION: Given AP=5m , AS=3m, PS =?

Using Pythagoras property , $(PS)^2 = (AP)^2+(AS)^2$

$(PS)^2 = 5^2 + 3^2$; $(PS)^2 = 25 + 9$; $(PS)^2 = 34$; PS = $\sqrt{34}$ m

So, side of square lawn = v34 m , Area of lawn PQRS = side x side

= $\sqrt{34}$ m x $\sqrt{34}$ m = 34 sq. m



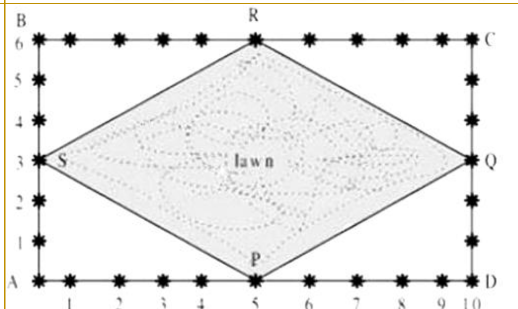
1b ii) Students want to fence lawn PQRS. If the rate of fencing is Rs $20\sqrt{34}$ per metre, what is the total cost of fencing ?

SOLUTION: For fencing lawn PQRS, we should perimeter first.

Perimeter of lawn PQRS = 4 x side = 4x $\sqrt{34}$m = $4\sqrt{34}$ m

Cost of fencing per metre = Rs $20\sqrt{34}$

Cost of fencing $4\sqrt{34}$ m = Rs $(20\sqrt{34})$ x $(4\sqrt{34})$ = Rs 2720



2a Today is Rohan's birthday. So he asked his friend to help him in shopping for the party. Rohan walked 3km from his house towards east direction and then turn left to meet his friend. From there, they together went to market and bought some items. After shopping, both of them came to Rohan's house to do arrangements.

Based on above information, answer the following questions:

How can Rohan reach to his friend early.

Suggest him the way and shortest distance to reach to his friend.

SOLUTION: Coordinates of A = (1,2) , B = (4,2) and C = (4,6)

Length of AB = abscissa of B – abscissa of A = 4 – 1 = 3 units

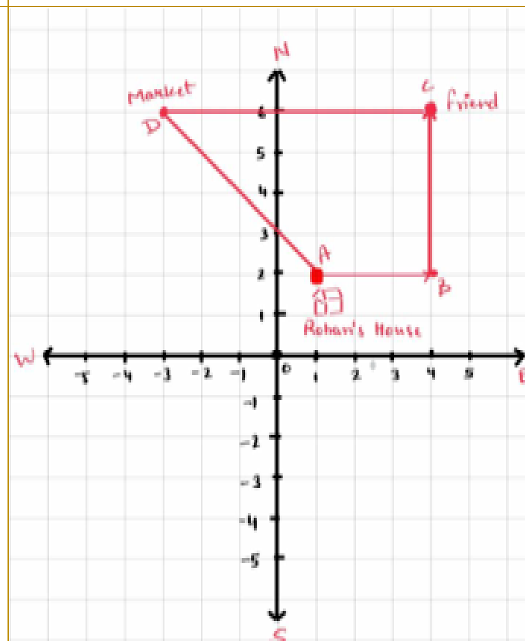
Length of BC = ordinate of C – ordinate of B = 6 – 2 = 4 units

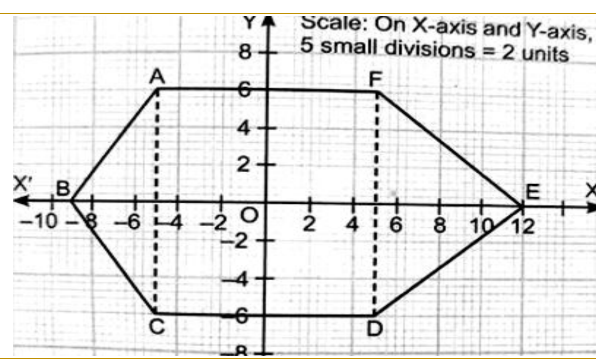
i) Rohan can reach to its early by following the path diagonally.

Length of diagonal AC can be find using Pythagoras property

$(AC)^2 = (AB)^2 + (BC)^2$; $(AC)^2 = 3^2 + 4^2$; $(AC)^2 = 9 + 16$

$(AC)^2 = 25$; AC = $\sqrt{25}$ = 5 km



2b	Is the figure formed in the picture is concave or convex polygon? Give reasons to support your answer. **SOLUTION**: Figure formed in the picture is convex polygon because polygons that are convex have no portions of their diagonals in their exteriors or any line segment joining any two different points, in the interior of the polygon, lies wholly in the interior of it
	EXERCISE
	MULTIPLE CHOICE QUESTIONS
1	Points (1,2), (-2,-3), (2,-3); a) First quadrant b) Do not lie in the same quadrant c) Third quadrant d) Fourth quadrant
2	The mirror of a point (3, 4) on y-axis is: a) (3, 4) b) (–3, 4) c) (3, –4) d) (–3, –4)
3	Signs of the abscissa and ordinate of a point in the second quadrant are respectively a) +, + b) +, – c) -, + d) -, -
4	If the coordinates of two points are A(-7,9) and B(3,4), then (abscissa of A) – (abscissa of B) is a) 4 b) -10 c) 10 d) -5
5	In which of the following points lies on the line y = - x? a) (2,2) b) (2,-2) c) (3,3) d) (-2, 3)
	ASSERTION REASON TYPE QUESTIONS
1	Assertion(A): Point (2,2) lies in second quadrant. Reason(R): Point of the type (+,+) lies in first quadrant
2	Assertion(A): The points (-7, 2) and (2,- 7) are at different positions in the coordinate plane. Reason(R): The perpendicular distance of the point A(5, 6) from the y-axis is 6
3	Assertion(A): The abscissa of a point (1, 8) is 1 Reason(R): The point (0,1) lies on y-axis.
4	Assertion(A): Abscissa of a point is positive in I and IV quadrant Reason(R): If (x + 2, 4) = (5, y – 2), then coordinates (x, y) are(3,6)
5	Assertion(A): ABCD is a quadrilateral in which P, Q, R and S are the mid points of AB, BC, CD and DA respectively. Then, PQRS is a parallelogram.. Reason(R): The line segment joining the mid points of any two sides of a triangle is parallel to the third side and equal to half of it.
	VERY SHORT ANSWER TYPE QUESTIONS
1	In which quadrant will a point lie if it's both the coordinates are positive?
2	Write the coordinates of the point at which two coordinate axes meet.
3	Write the coordinates of the point which lies at a distance of x-units from x-axis and y units from y-axis.
4	Find the coordinates of the point which lies on x-axis at a distance of 5 units from y-axis
5	Find the coordinates of the point which lies on y-axis at a distance of 9 units from x-axis in the negative direction
	SHORT ANSWER TYPE QUESTIONS
1	If we plot the points P(5, 0), Q(5,5), R(-5, 5) and S(-5, 0), which figure will we get? Name the axis of symmetry of this figure?
2	If the coordinates of two points are P(-2,3) and Q(-3,5), then find [(abscissa of P) – (abscissa of Q)] + [(ordinate of Q) – (ordinate of P)].
3	Write the coordinates of the point A, B, C, D, E and F of the figure formed on the graph. Also, write coordinates of the points of intersection of AC and DF with the x-axis. 

4 A rectangular field is of length 10 units & breadth 8 units. One of its vertex lie on the origin. The longer sideis along x-axis and one of its vertices lie in first quadrant. Find all the vertices.

5 Plot the point P(-5, 4) and from it draw PM and PN as perpendicular to x-axis and y-axis respectively. Write the coordinates of the points M and N.

LONG ANSWER TYPE QUESTIONS

1 The following table gives the relation between natural numbers and odd natural numbers . Plot the points and join them. Do you get a straight line by joining these points ? Analyse each coordinate and establish a relation between ordinate and abscissa.

x	1	2	3	4	5	6	7
y	3	5	7	9	11	13	15

2 Without plotting the points indicate the quadrant in which they will lie, if
(i) the ordinate is -9 and abscissa is 2 (ii) the abscissa is 7 and ordinate is 8
(iii) the abscissa is 3 and ordinate is -2 (iv) the ordinate is -5 and abscissa is 3
(v) the abscissa is -2 and ordinate is -5

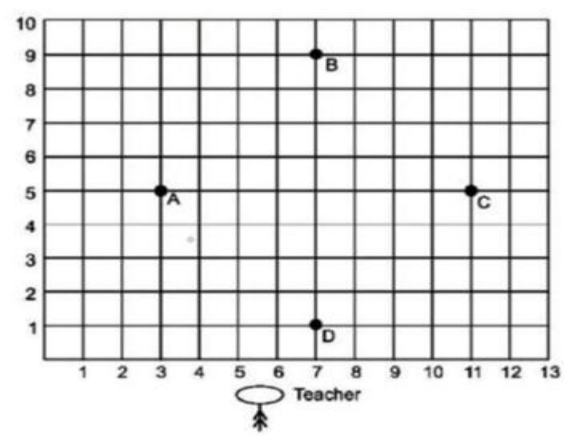
3 Plot the following points and write the name of the figure obtained by joining in order P(3,2), Q(-7, -3), R(6, -3) and S (2, 2). Find its area.

4 Rohit started from origin towards south-west direction to reach his school where west direction is represented by the negative x-axis. While coming back from school, he came with different path. He started from school 4km towards North direction and then turn right to walk 3km to reach home again. If speed of Lucky is 5 km/hr , how much time will he take to reach his school in the morning and what are the coordinates of school.

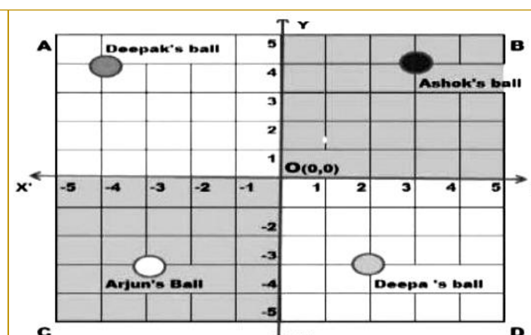
5 Plot the four points (3,0), (0,3), (-3,0) and (0,-3) on the graph paper and join them by line segments in an order and observe the figure formed.
i) Name the polygon formed. ii) Find the area of the polygon obtained?

CASE BASE STUDY QUESTIONS

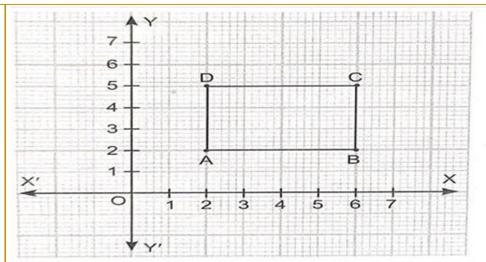
1 Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in the figure.
i) What are the coordinates of A and B respectively?
ii) What are the coordinates of C and D respectively?
iii) What is the distance between B and D? OR What is the distance between A and C?



2 There is a square park ABCD. Four children Ashok ,Deepa ,Arjun and Deepak went to play with their balls. The colour of the ball of Ashok ,Deepa ,Arjun and Deepak are Red, green, yellow and blue respectively. All four children roll their ball from centre point O in the direction of XOY, X'OY, X'OY' and XOY'. Their balls stopped as shown in the above image.
i)What are the coordinates of the ball of Ashok?
ii) What are the coordinates of the ball of Deepa ?
iii)What the line XOX' called? OR What the point O(0,0) called?

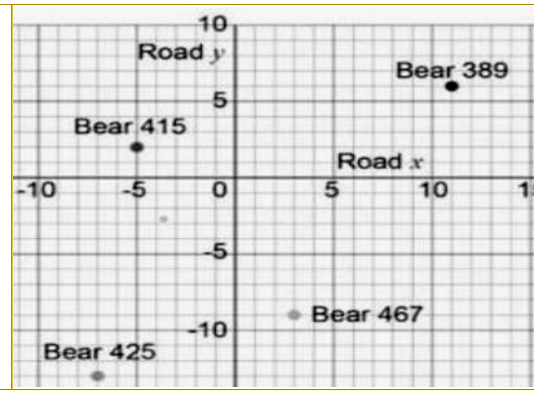


3 For a group activity of class IX, teacher divided the space as cartesian plane and chairs are placed at various points for the group of four students at points A, B, C and D as shown in the picture below:
Now answer the following questions:
i) Write the coordinates of A, B, C, D.
ii) perimeter of rectangle is ?
iii)Find the area of rectangle ABCD



HOTS

1 A forest ranger keeps track of bears in his area. He plotted their location on a graph. The origin represents the ranger's control room's location. To access and maintain equipment, Road x and Road y have been laid and paved inside the forest. They pass through the control room. One unit on the graph paper represents 1km.
Based on the information, answer the following questions:
i) A tiger is at (11, 4). How far from it is the nearest bear?
ii) In the forest, rain shelters are at an interval of 2km along paved roads. A forest ranger is travelling on Road x. He crosses a rain shelter located at (3,0). What is likely to be the location of the next shelter?



2 Plot the points P(0, -4) anQ(0, 4) on the graph paper. Now, plot the points R and S such that triangle PQR and triangle PQS are isosceles triangles.

Answers

MCQ Type	Assertion Reason Type	Very short Answer Type
1. b) Do not lie in the same quadrant 2. b) (–3, 4) 3. c) -, + 4. b) -10 5. b) (2,-2)	1. d 2. b 3. b 4. d 5. a	1. Ist Quadrant 2. origin 3. (y,x) 4. (±5, 0) 5. (0, -9)

Short Answer Type Questions

1. Rectangle, y-axis
2. 3
3. Coordinates of the points A, B, C, D, E and F are A(-5,6) ,B(-9,0), C(-5,-6), D(5,-6), E(12,0) andF(5,6) coordinates of the point of intersection of AC with the x-axis is (-5,0) and coordinates of the point of intersection of AC with the x-axis is (5,0)
4. (0, 0) (10, 0) (10, 8) (0, 8)
5. M(-5, 0) and N(0, 4)

Long Answer Type Questions

1. Yes, we get a straight line by joining these points. Relation y = 2x + 1
2. i) IV Quad ii) I Quad iii) IV Quad iv) IV v) III
3. Trapezium and Area = 45sq. unit
4. Time = 1hr coordinate of school is (-3, -4)
5. i) square ii) 32sq. unit

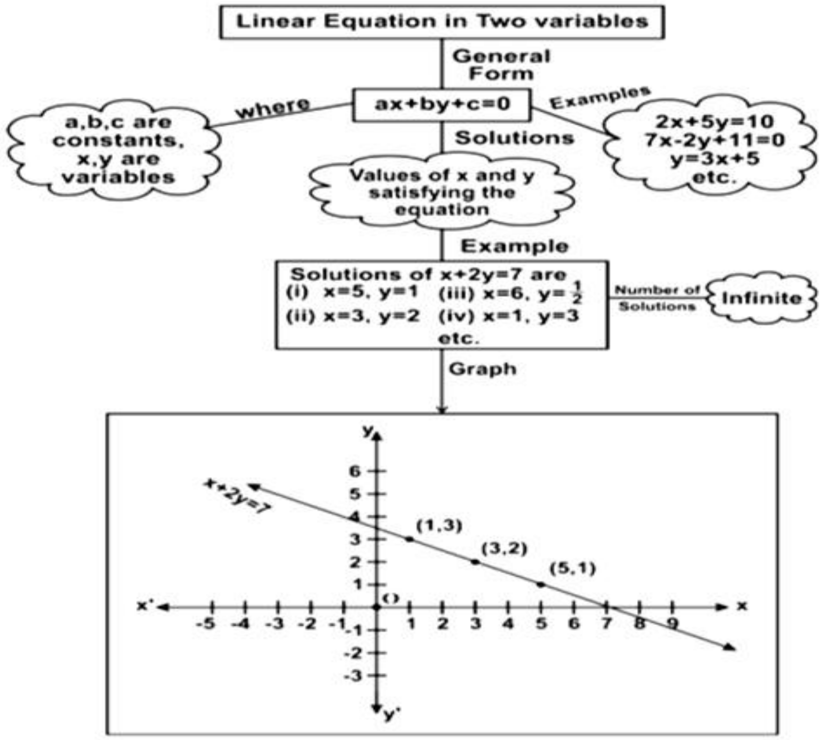
Case Base Study Questions

1	i) A(3, 5) ,B(7, 9)	ii) C(11, 5), D(7, 1)	iii) 8units	or	8 units
2	i) (3, 4)	ii) (2, -3)	iii) x-axis	or	origin
3	i) A(2, 2), B(6, 2), C(6, 5), D(2, 5)	ii) 14 unit	iii) 12 sq. unit		

Hots

1	i) 2km	ii) (5, 0)
2	Any point on the x-axis	

CHAPTER – 04
LINEAR EQUATION IN TWO VARIABLES
MIND MAPPING



GIST/SUMMARY OF THE LESSON:

- **LINEAR EQUATION**
- **SOLUTION OF LINEAR EQUATION**
- **GRAPHICAL REPRESENTATION**

DEFINITIONS AND FORMULAE:

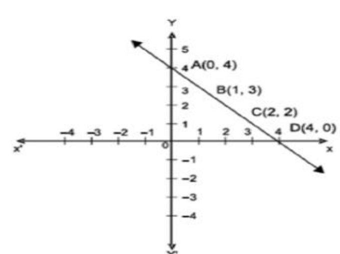
MEANING OF THE CHAPTER NAME :

- **GENERAL FORM** of linear equation in two variable $ax + b = 0$

  Linear Equation in two variables

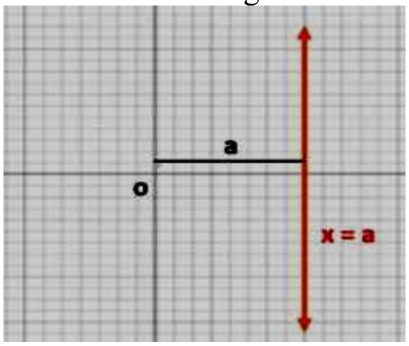
  Degree-1 Equality '=' two variables mainly x & y

  where a, b, c are real numbers and a & b are coefficients of x and y respectively such that a & b ≠ 0 and x, y are variables raising power 1
- **LINEAR EQUATION IN ONE VARIABLE**
- **GENERAL FORM : $\boldsymbol{ax + b} = 0$** ,where a and b are real numbers and a ≠ 0 only one variable
- **GRAPH**: A graph on a linear equation always show a straight line. (Hint: linear means straight)
- **SOLUTION**: Points lying on a line are known as Solution.
  In other words, the pair of values x and y which satisfies the equation are known as Solution.
  Example: x + y = 4 ; Solution of equation x + y = 4 are
  (0, 4) (1, 3) (2, 2) (4, 0) and many more.

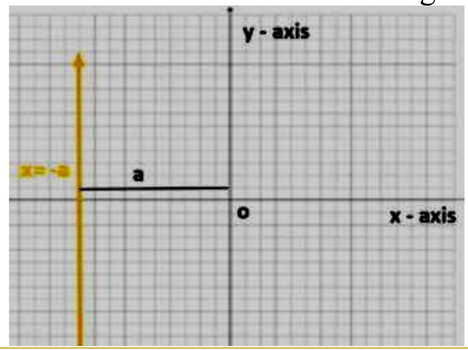


A line has infinitely many Solution as infinite points lie on a line.
Equation of x-axis: y = 0 (ordinate on x axis is zero)
Equation of y-axis: x= 0 (abscissa on y axis is zero)
Equations of lines parallel to x- axis and y- axis

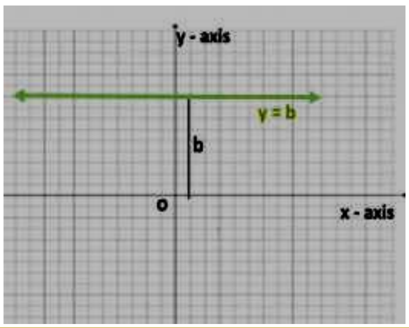
x = a ; Equation of a line parallel to y- axis at a distance of ‘a’ unit to the right side of the origin



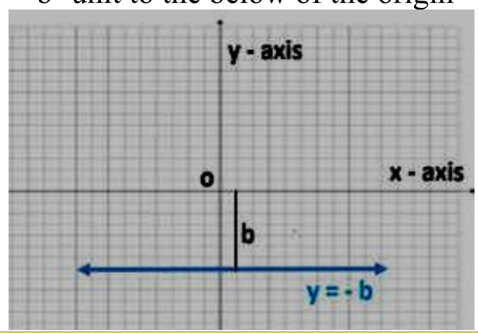
x = -a ; Equation of a line parallel to y- axis at a distance of ‘a’ unit to the left side of the origin



y = b ; Equation of a line parallel to x- axis at a distance of ‘b’ unit to the above of the origin



y = -b ; Equation of a line parallel to x- axis at a distance of ‘b’ unit to the below of the origin



MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)

1	A linear equation has a) unique Solution b) two Solution c) no Solution d) infinitely many Solution	**SOLUTION**: d
2	which of the following is not a linear equation a) $3x + 3 = 5x + 2$ b) $x - 5 = \frac{5}{3}x - 3$ c) $x + 5 = 3x^2 - 5$ d) $(x + 2)^2 = x^2 - 8$	**SOLUTION**: c
3	The equation of y-axis is a) x = a b) y = b c) x = 0 d) y = 0	**SOLUTION**: c
4	The graph of $7x + 11y + 13 = 0$ is a) a straight line parallel to x-axis b) a straight line parallel to y-axis c) a general straight line d) None of these	**SOLUTION**: c
5	If x = 2 and y = 3 is Solution of the linear equation 23ax + 37ay = 785 then the value of a is a) 0 b) -5 c) 1 d) 5	**SOLUTION**: d
6	Which of the linear equation has Solution as x = -3, y = 5? a) 2x + y = 8 b) x + 2y = 8 c) x + y = 8 d) –x + y = 8	**SOLUTION**: d
7	The equation y = 0 represents – a) x-axis b) y-axis c) a line parallel to x-axis d) a line parallel to y-axis	**SOLUTION**: a
8	The graph of x + y = 2 is a line which meets the x-axis at the point. a) (2, 0) b) (3, 0) c) (0, 2) d) (0, 3)	**SOLUTION**:a
9	The intersection point of the lines x = -5 and y = - 3 a) (5, 3) b) (5, - 3) c) (-5, -3) d) (-5, 3)	**SOLUTION**: c
10	Any point on the line y = x is of the form	**SOLUTION**:c

	a) (a, 0) b) (0, a) c) (a, a) d) (a, –a)		
	ASSERTION - REASON BASED QUESTIONS		
	DIRECTIONS: In following questions, a statement of Assertion(A) is followed by a statement of Reason (R) Choose the correct option. a) Both assertion and reason are true and the reason is the correct explanation of assertion. b) Both assertion and reason are true but the reason is not the correct explanation of the assertion. c) Assertion is true and the reason is false. d) Assertion is false and the reason is true.		
1	Assertion(A): The point (2, 2) is the Solution of x + y = 4. Reason(R): Every point which satisfies the linear equation is a Solution of the equation.	**SOLUTION**: a)	
2	Assertion(A): If x = 2k – 1 and y = k is a Solution of the equation 3x – 5y – 7 = 0, then value of k is 10. Reason(R): A linear equation in two variables has infinitely many Solution.	**SOLUTION**: b)	
3	Assertion(A): y = 3x represents a line passing through the origin. Reason(R): Any line parallel to the x-axis at a distance 'a' is y = a.	**SOLUTION**: c)	
4	Assertion(A): If x = -2, y = 1 is a Solution of the equation 2x + 3y = k, then the value of k is 7. Reason(R): The Solution of the line will satisfy the equation of the line.	**SOLUTION**: d)	
5	Assertion(A): An equation of the form ax+by+c = 0 where a, b and c are real numbers. Such that a and b are not both zero, is called a linear equation in two variables. Reason(R): A linear equation in two variables has infinitely many Solution.	**SOLUTION**: b)	
6	Assertion(A): x = 3 and y = 2 is a Solution of the linear equation 2x + 3y = 12 Reason(R): x = 4 and y = 2 is a Solution of the linear equation x + 3y = 10	**SOLUTION**: b)	
7	Assertion(A): If x = 4a and y = a + 5 is a Solution of the equation 3 x – 5y – 7 = 0, then the value of a is 10. Reason(R): A linear equation in two variables has infinitely many Solution.	**SOLUTION**: d)	
8	Assertion(A): Point (0, -3) is a Solution of the equation 2x + y + 3 = 0. Reason(R): A point which satisfies the equation is known as Solution.	**SOLUTION**: a)	
9	Assertion(A): A point lies on a linear equation is known as Solution. Reason(R): A linear equation has a unique Solution.	**SOLUTION**: c)	
10	Assertion(A): The intersection point of a line y = -3 with x-axis is (-3, 0). Reason(R): Y = -3 is a line parallel to x-axis at a distance of 3 unit to the negative direction of y- axis.	**SOLUTION**: d)	
	VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)		
1	If (1, -2) is a Solution of the equation 2x – y = p then find the value of P. **SOLUTION**: Because (1,2) is the Solution of equation 2x-y=p ∴ this value of x and y will satisfy the given equation. ; 2x1-(-2) = p ; 2+2=p ; P = 4		
2	Find the value of k, if x = 1, y = 1 is a Solution of the equation 2x + 3y = k. **SOLUTION**: Given equation is 2x+3y= k ; Now put values of x,y in the equation ⇒2+3= k ; ∴ k = 5		
3	i) Any point on the x- axis is of the form? ii) Any point on the y- axis is of the form?	**SOLUTION**:(a, 0) **SOLUTION**: (0, b)	
4	i)A point which satisfies a linear equation is known …………? ii)A linear equation has ……………. Solution. (infinite/unique)	**SOLUTION**: Solution **SOLUTION**: infinite	
5	i) what is the equation of x-axis? ii)) what is the equation of y-axis	**SOLUTION**: y = 0 **SOLUTION**: x = 0	
6	i) Equation of line parallel to x-axis at a distance 5units to the negative direction of y-axis? ii) Equation of line parallel to y-axis at a distance 3units to the positive direction of x-axis?	**SOLUTION**: y = -5 **SOLUTION**: x = 3	

7	Find two Solution for the equations 4x + 3y = 12. **SOLUTION**: For x = 0 ; 4(0)+3y = 12 ; 3y = 12 ; Y = 4 ; For y = 0 ; 4x + 3(0) = 12 ; 4x = 12 ; x = 3 ; The two Solution are (0,4) and (3,0).	
8	Find the coordinates of the points where the graph of the equation 7x – 3y = 4 cuts x- axis and y- axis. **SOLUTION**: 3x + 4y = 12 The point on y axis ; Let x = 0 , so 3(0) + 4y = 12 , 4y = 12, y = 3 Point is (0, 3) The point on x axis Let y =0, so 3x + 4(0) =12, 3x = 12, x= 4 Point is (4, 0)	
9	Write the equation $5x = \frac{7}{2}$ in the form of a linear equation in two variables. Compare with ax + by + c = 0 write the value of a, b and c. **SOLUTION**:$5x = \frac{7}{2}$; $5x$ X 2=7 ; 10 x = 7 ; 10x+0.y-7= 0 ; 10x+0.y-7= 0 Compare with ax + by + c = 0 ; a = 10, b = 0, c = -7	
10	Find the value of k, if x = 1, y = 1 is a Solution of the equation 2x + 3y = k. **SOLUTION**: Given equation is 2x+3y= k , Now put values of x,y in the equation ⇒2+3= k ; ∴ k = 5	
	SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)	
1	Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case: i) x – y/2 – 5 = 0 ii) 3x = -7y **SOLUTION**: i)x – y/2 – 5 = 0 ii) 3x = -7y 2x – y -10 = 0 Compare with general equation ax + by + c = 0 a = 2, b = -1, c = - 10 (ii) 3x + 7y = 0 Compare with general equation ax + by + c = 0 a = 3, b = 7, c = 0	
2	Write three Solution of the equation x = 6y **SOLUTION**: put y = 0 then x = 6×0 = 0 Solution is (0 , 0) ; put y = 1 then x = 6×1 = 6 Solution is (6 , 1) ; put y = -1 then x = 6×-1 = -6 Solution is (-6, -1) ; Two Solution are: (0, 0), (6, 1) and (-6 , -1)	
3	Check which of the followings are the Solution of the given linear equation 3x – 5y = 15 , Find i) (0, -3) ii) (-2, 2) iii) (5, 0) iv) (10, -3) **SOLUTION**: Equation 3x – 5y = 15	

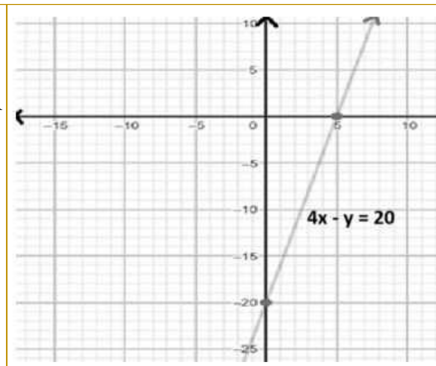
i) (0, -3)	ii) (-2, 2)	iii) (5, 0)	iv) (10, 3)
put x = 0 and y = -3 3x0 -5x-3 = 15 0+15 = 15 15 = 15 (True) Yes, it is a Solution.	put x = -2 and y = 2 3x-2 -5x2 = 15 -6 – 10 = 15 -16 = 15 (False) Not a Solution	put x = 5 and y = 0 3x5 -5x0 = 15 15 -0 = 15 15 = 15 (True) Yes, it is a Solution.	put x = 10 and y = 3 3x10 -5x 3 = 15 30 -15 = 15 15 = 15 (True) Yes, it is a Solution

4	If x = -7, y = 5 is a Solution of the equation 3x + 4y = k, find the value of k. **SOLUTION**: Equation 3x + 4y = k , Since, x = -7, y = 5 is a Solution of the above equation so it will satisfy it 3x-7 + 4x5 = k ; -21 + 20 = k ; -1 = k ; k = -1	
5	Let y varies directly as x. If y = 12 when x = 4, then write a linear equation. What is the value of y when x = 5 **SOLUTION**: Given that, y varies as x i.e. y = kx….(i) [where k is an arbitrary constant] , Given y = 12 and x = 4 (substituting these values in equation (i)) ; 12 = 4k ; k = 12/4 ; k = 3 ; putting the value of k in equation (i) y = 3x (Required linear equation) ; Now, when x = 5 ; y = 3 x 5 ; y = 15	
6	Write 3y – 4x = 0 in the form of ax + by + c = 0. Write x in terms of y. Find any two Solution of the equation. How many Solution you can find out? **SOLUTION**: $3y - 4x = 0$; $-4x + 3y = 0$ (form of ax + by + c = 0) ; $-4x = -3y$; x = $\frac{3}{4}$y (x in terms of y) . For Solutions, Put y = 0 then x = $\frac{3}{4} \times 0$ x = 0	

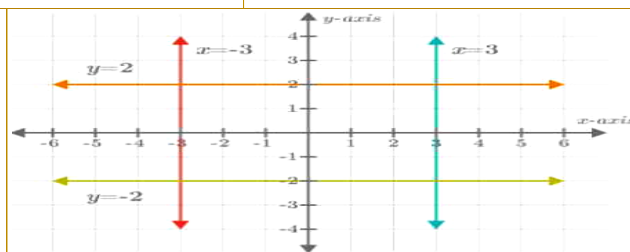
Solution (0, 0) , Put y = 4 then x = $\frac{3}{4}$ x 4 ; x = 3 Solution (3, 4) two Solutionutons are: (0, 0) and (3, 4) . we can find infinitely many Solution of any linear equation.

7 In an test, a boy gets 4 marks for each correct answer and loose 1 marks for each incorrect answer. If he gives x correct answer, y incorrect answers and he gets 20 as total marks. Write this situation in the form of linear equation. Also, Draw the graph for obtained linear equation.

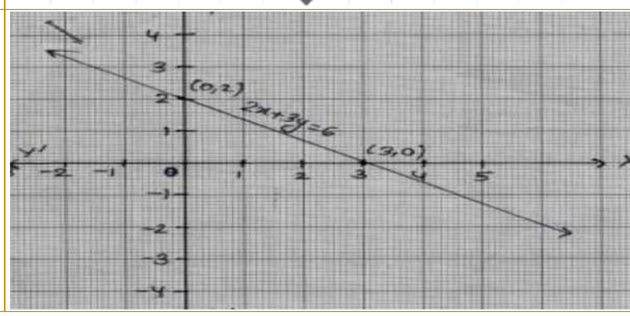
SOLUTION: correct answers = x ; Marks for each correct answer = 4
Incorrect answer = y ; Marks for each incorrect answer = -1
Total marks = 20 ; Linear equation is: 4x – y = 20



8 Draw the graph for the lines x = 3, x = -3 , y = 2 and y = -2.
SOLUTION:



9 Draw the graph for the linear equation 2x + 3 y = 6. Find the co-ordinate of the points where the line meet x- axis and y-axis.
SOLUTION:
2x + 3 y = 6
x- axis coordinate (3, 0)
y- axis coordinate (0, 2)



10 Sarita distributes chocolates on the occasion of children's Day. She gives 5 chocolates to each child and 20 chocolates to adults. If number of children is represented by 'x' and total distributed chocolates as 'y'.
i) write in the form linear equation in two variables.
ii) If she distributed 145 chocolates in total, find number of children?
SOLUTION: i) 5 x + 20 = y ; ii) put y = 145 in 5 x + 20 = y, we get x = 25

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1 A positive number is 5 times another number. If 21 is added to both the numbers, then the new larger numbers become twice the other smallest new number. What are the numbers?
SOLUTION: Let the first number = x
∴ the second number will be = 5 x , According to the given condition, 2 (x + 21) = (5 x + 21) ⇒ 2 x + 42 = 5 x +21 ⇒ 5 x – 2x = 42-21 ⇒ 3 x = 21 ⇒x = 7
∴ the First number is = 7 and the second number is 5 x 7 = 35.

2 The linear equation that converts Fahrenheit (F) to Celsius (C) is given by the relation c = $\frac{5f-160}{9}$
i) If the temperature is 86°F, what is the temperature in Celsius?
ii) If the temperature is 35°C, what is the temperature in Fahrenheit?
iii) If the temperature is 0°C what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?
iv) What is the numerical value of the temperature which is same in both the scales?

Solution: given equation is c = $\frac{5f-160}{9}$

i) Now, the temperature is 86°F

∴ the equation $c = \frac{5f-160}{9}$; $c = \frac{5\times86-160}{9}$; $c = \frac{430-160}{9}$; $c = \frac{270}{9}$; c = 30°

∴ the Temperature in Celsius is 30°

ii) now, the Celsius temperature = 35° ; equation $35 = \frac{5F-160}{9}$

⇒ 35x9 = 5f – 160 ⇒ 315 = 5f – 160 ⇒ 5f = 315+160 ⇒ 5f = 475 ⇒ f = 475/5 f = 95°

∴ the temperature in Fahrenheit is 95°F

iii) ∴ If temperature = 0ºC

by the equation $0 = \frac{5f-160}{9}$

⇒ 0 = 5F – 160

⇒ 5F = 160

⇒ F = 160/ 5

⇒ F = 32º

Again if temp. is 0ºF

by equation - ⇒ $C = \frac{5\times0-160}{9}$

⇒ $C = \frac{0-160}{9}$

⇒ C = 160/9

∴ If temp. is 0ºF then in Celsius = -17.7ºC

iv) Let the temp. which is same in both the scale is = xo by the equation $x = \frac{5x-160}{9}$

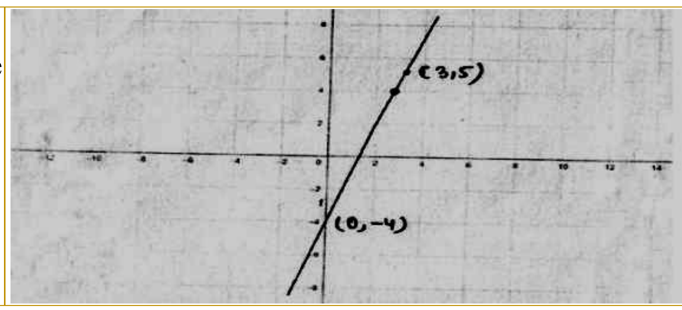
⇒ 9x = 5x – 160 ⇒ 9x – 5x = -160 ⇒ 4x = -160 ⇒ X = -160/ 4 ⇒ X = -40

Hence, -40o is the temperature which is same in both the scales.

3 Draw the graph of linear equation 3x - y = 4. From the graph find the value ofp and q if the graph passes through (p,-4) and (3, q)

SOLUTION: Clearly (0, -4) lies on the graph line

So, (0, -4) = (p, -4) P = 0 similarly, (3, 5) lies on the line so q = 5



4 The organizers of an essay competition decide that a winner in the competition, gets a prize, of Rs. 100 and a participant who does not win gets a prize of Rs.25. The total prize money distributed is Rs.3,000. Find the number of winners, if the total number of participants is 63.

SOLUTION: Let the participant who does not win = x

∴ the participant who win = 63 – x because total number of participants = 63

ATQ, Total prize win by winners = 100 (63 – x)

Total prize obtained by who does not win 25(x)

∴ The total prize money distributed = 3000

∴ 25x + 100 (63 – x) = 3000 ⇒ 25x + 6300 – 100x = 3000 ⇒ -75x = 3000 – 6300

⇒ -75x = -3300 $x = \frac{3300}{75}$ x = 44

∴ The total number of winners = 63 – 44 = 19

5 Write each of the following as an equation in two variables:

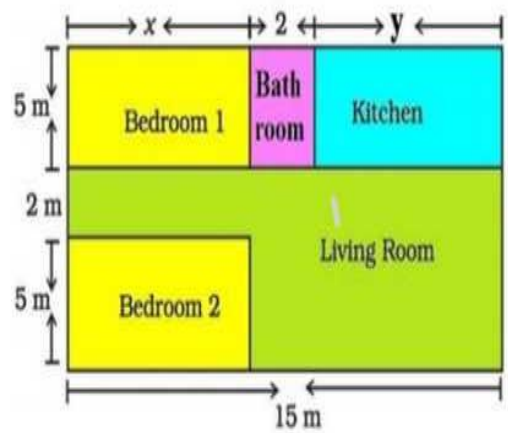
a) x = -5 b) y = 2 c) 2x = 3 d) y + 7 = 0 e) 5 - x = 0

SOLUTION:

	a)x= -5 x+0= -5 x+0.y+5 =0	b) y=2 0+y=2 0.x+y=2 0.x+y-2=0	c) 2x=3 2x+0=3 2x+0.y=3 2x+0.y-3=0	d) y + 7 = 0 0.x + 1.y + 7 = 0	e) 5 – x = 0 -x + 5 = 0 -x + 0.y + 5 = 0

CASE BASED QUESTIONS (04 MARKS QUUESTIONS)

An architecture makes a layout of a house . In the given layout the design and measurements has been made such that.
i) Find the area of the layout.
ii) If the area of two bedrooms and kitchen together is 75 sq. m. Represent this condition algebraically by using dimensions in the above layout.
iii) Form a linear equation if the area of the living room is 95 sq m.



SOLUTION: i) Area of the layout = 15 × 12 = 180 sq. m
ii) Area of two bedrooms = 2 × (× 5) = 10 x
Area of the kitchen = 5y ; It is given that area of two bedrooms and kitchen together is 75 square m. so, 10 x + 5y = 75 ; 2 x + y = 15
iii) Area of the living room = 95 sq. m ; 2 x + 7(2 + y) = 95
2 x + 14 + 7y = 95 ; 2 x + 7y = 95 – 14 ; 2 x + 7y = 81

2 Ashish and his friends decide to visit Delhi and he decides to travel via Delhi Metro. He starts from New Delhi metro station, if Ashish buys 3 tickets to Rajeev chowk from New Delhi and 2 tickets to Hawz khas from Rajeev chowk and the total cost is Rs. 76. If the fare of a ticket to Rajeev chowk from New Delhi is Rs. x and to Hawz Khas from Rajeev chowk is Rs. y. On behalf of this information, answer the following questions
i) Respresnt the Ist condition algebraically.
ii) How many Solution are there for the linear equation representing the above situation?
iii) If the fare of 1 ticket to Rajeev chowk from New Delhi is Rs. 18. What is the fare to Hawz Khas from New Delhi via Rajeev chowk

SOLUTION: i) 3 x + 2y = 76
ii) infinite many Solution
iii) fare of 1 ticket to Rajeev chowk from New Delhi is Rs. 18 i.e. x = 18
so, 3×18 + 2y = 76 ; 54 + 2y = 76 ; 2y = 76 – 54 ; 2y = 22 ; y = 11
Total fare = 18 + 11 = Rs. 29

3 A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Raj paid Rs.30 for a book kept for five days
i) Form a pair of linear equations in two variables from this situation if the fixed charge is x and additional charge for each day is y.
ii) Express the above linear equation in the form ax + by + c = 0 and indicate the values of a, b and c.
iii) If the fixed charge is Rs.6 and the book is kept for 15 days, then what is the total amount Raj has to pay?

	SOLUTION: i) $x + y(5-3) = 30$; $x + 2y = 30$ ii) $x + 2y - 30 = 0$; a = 1, b=2, c=-30 iii) $6 + 2y = 30$; y = 12 ; Total amount = 6 + 12 x 12 = Rs.150	

HOTS

1 Find the Solution of the forms x = a, y = 0 and x = 0, y = b for the following equations:
2x+ 5y = 10 and 2x+3y =6 Is there any common Solution?
SOLUTION: Substituting x = 0 in the equation 2x + 5y = 10, we get 2×0 + 5y = 10 ; 5y=10 ; y = 2
Thus, x = 0 and y = 2 is a Solution of 2x + 5y = 10.
Substituting y = 0 in 2x +5y = 10, we get 2x+5×0 =10 = 2x = 10 => x = 5
Thus, x = 5 and y = 0 is a Solution of 2x + 5y = 10. Thus, x=5, y=0 and x=0, y=2 are two Solution of 2x+5y =10.
Consider the equation 2x +3y = 6. Substituting x = 0, in this equation, we get 2x0+3y=6 ; 3y=6 ; => y = 2 ;
So, x = 0, y = 2 is a Solution of 2x + 3y = 6
Substituting y = 0 in 2x+3y = 6, we get 2x + 3×0=6 ; 2x = 6 ; X = 3 ; Thus, x=0, y=2 and x=3, y =0 are solution of 2x+3y=6 , Clearly, x = 0, y = 2 is common Solution of the given equations.

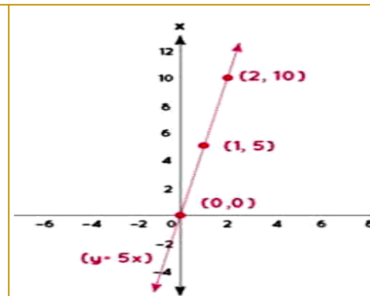
2 If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also, read from the graph the work done when the distance travelled by the body is (i) 2 units (ii) 0 units.

SOLUTION: Consider y as work done and x as distance travelled by the body
y = Cx, where C is a constant , It is given that the constant force is of 5 units, C= 5 ; Putting C = 5 in y = Cx, we get y = 5x This is the required equation in two variables. Putting x=0 in (i), we get y = 0 ; Putting x=1 in (i), we get y=5.

Thus, we have the following table exhibiting the values of y for different values of x. Plotting points O(0,0) and A (1,5)

x	0	1
y	0	5

on graph paper on a suitable scale and drawing a line passing through these two points, we obtain the graph of y = 5x as shown In order to find the work done in travelling 2 units distance, we first identify point A representing 2 units on X-axis and then draw a vertical line to cut y = 5x at P. From P draw perpendicular PM on Y-axis. Clearly, M represents (0,10). Hence, work done in travelling 2 units is 10 units.



ii)Clearly, y = 0 when x = 0 so, work done is zero when no distance is travelled.

EXERCISE

MULTIPLE CHOICE TYPE QUESTIONS

1 3x + 10 = 0 will have
a) Unique Solution b) Two Solution c) infinitely many Solution d)No Solution

2 The cost of ball pen (y) is Rs. 10 less than half of the cost of fountain pen(x) . The linear equation in two variable will be ……………………
a) x+y+10 = 0 b)2x+y+20 = 0 c)2x-y+20 = 0 d) x-2y-20 = 0

3 The Solution of equation x – 2y = 4 is
a) (0,2) b) (2,0) c) (4,0) d) (1,1)

4 Which option shows 5y – 8x = 7(x + y) – 9 expressed in the form of ax + by + c = 0?
a) -x + 6y – 9 = 0 b) -x + 12y – 9 = 0 c) 15x + 2y – 9 = 0 d)15x – 4y – 9 = 0

5 In an exhibition, the cost of tickets for an adult is Rs.5 more than thrice the cost of a ticket for child. Which equation relates the cost y, of adult tickets in terms of the cost x, of child tickets?
a) y = 5 + 3x b) y + 5 = 3x c) y = 3 + 5x d) y + 3 = 5x

ASSERTION REASON QUESTIONS

1 Assertion(A): A linear equation 5x - 3y = 2 has only (1,1) as a Solution .
Reason(R): A linear equation in two variables has infinitely many Solution.

2 Assertion(A) : The point (2, 2) is the Solution of x + y = 4.

	Reason(R): Every point which satisfy the linear equation is a Solution of the equation.	
3	Assertion(A): If $x = 2$, $y = 1$ is a Solution of the equation $2x + 3y = k$, then the value of k is 7. Reason(R): The Solution of the line will satisfy the equation of the line.	
4	Assertion(A) : The graph of the linear equation $2x – y = 1$ passes through the point (2, 3). Reason(R): Every point lying on graph is not a Solution of $2x – y = 1$.	
5	Assertion(A): The point (3, 0) lies on the graph of the linear equation $4x + 3y = 12$. Reason(R): (3, 0) satisfies the equation $4x + 3y = 12$.	
	VERY SHORT ANSWER TYPE QUESTIONS	
1 2 3 4 5	The equation of a line parallel to x-axis is…………………. =a, where a is any non-zero real number. The equation of a line parallel to y-axis is…………………. =a, where a is any non-zero real number. The graph of every linear equation in two variables is a …………………… The linear equation $7x + 9y = 8$ has a unique Solution. (True/False) The graph of the linear equation $x + 2y = 5$ passes through the point (0, 5). (True/False)	
	SHORT ANSWER TYPE QUESTIONS	
1	Find the value of P if $x = 2$, $y = 3$ is a Solution of equation $5x + 3Py = 55$.	
2	The cost of coloured paper is 7 more than 1/3 of the cost of white paper. Write this statements in linear equation in two variables.	
3	Write the equations of two lines passing through (3, 10).	
4	If the points A(3, 5) and B(1, 4) lies on the graph of line $ax + by = 7$, find the value of a.	
5	Determine the point on the graph of the equation $2x + 5y = 20$ whose x-coordinate is 5/2 times its ordinate.	
	LONG ANSWER TYPE QUESTIONS	
1	Draw the graph of equation $x + y = 5$. Also, find the area bounded by line and coordinate axes.	
2	Sarika distributes chocolates on the occasion of children's Day. She gives 5 chocolates to each child and 20 chocolates to adults. If number of children is represented by 'x' and total distributed chocolates as 'y'. Write it in the form of linear equation in two variables. If she distributed 145 chocolates in total, find number of children?	
3	Riya participates in Diwali Mela with her friends for the charity to centre of handicapped children. They donate ₹3600 to the centre from the amount earned in Mela. If each girl donates ₹150 and each boy donates ₹200, they Form the linear equation in two variables. If number of girls are 8, find number of boys.	
4	We know that C= 2 T, taking T=22/7, circumference as j units, radius as x units, form a linear equation. Draw the graph. Check whether the graph passes through (0, 0). From the graph read the circumference when radius is 2.8 units.	
5	Find five Solution of the line $y = x$.	
	CASE BASED QUESTIONS	
1	Rajan planned to celebrate his daughter's birthday in a small orphanage centre. He bought apples to give to children and adults working there. Rajan donated 3 apples to each children and 4 apples to each adult working there along with birthday cake. He distributed 100 total apples. i) How to represent the above situation in linear equations in two variables by taking the number of children as 'x' and the number of adults as 'y'? ii) If the number of children is 20, then find the number of adults iii) Find the value of b, if $x = 9$, $y = 10$ is a Solution of the equation $3x + 5y = 11b$	

2	Petrol is flowing into a tank at the rate of 25cm³/sec. The volume of petrol collected in x sec is y cm³ i) Represent the above situation in linear equations in two variables. ii) Express the above linear equation in the form ax + by + c = 0 and indicate the values of a, b and c. iii) Find the volume of the petrol after 4 seconds. OR After how many seconds the volume is 500cm³.	
3	Shikhar went for shopping in the evening by metro with his father who is expert in mathematics. He told Shikhar the path of the metro A is given by the equation 2x + 4y = 8 and Path of metro B is given by 3x + 6y = 18. His father put some equation to Shikhar. Help Shikhar to Solve the problem: i) What is the intersection points of the equation 2x + 4y = 8 with coordinate axes? ii) What is the intersection points of the equation 3x + 6y = 18 with coordinate axes? iii) What is the intersection point of these equation?	

HOTS

1	Draw the graphs of the linear equations 4x – 3y + 4 = 0 and 4x + 3y -20 = 0. Find the intersection point and area bounded by these lines and x –axis.
2	Aarushi was driving a car with uniform speed 60km/h. Draw distance- time graph. From the graph, find the distance travelled by Aarushi in i) $2\frac{1}{2}$ hours ii) $\frac{1}{2}$hours

ANSWERS

	MULTIPLE CHOICE	ASSERTION REASON	VERY SHORT	SHORT ANSWER
1	c) infinitely many	1.b)	1.y	1.p = 5
2	Solution	2. a)	2. x	2.cost of coloured paper = x , cost of white paper y;
3	d) x-2y-20 = 0	3.a)	3.Straight line	equation is 3x – y -21 = 0
4	c) (4,0)	4.c)	4.False	3.x + y -13 = 0 and 2x – y + 4 = 0
5	c) 15x + 2y – 9 = 0	5. a)	5.False	4.a = -1
	a) y = 5 + 3x			5.(5, 2)

LONG ANSWER TYPE QUESTIONS

1	12.5 Sq. unit
2	a) 5x + 20 = y b) x = 25
3	a) 150x + 200y = 3600 b) no. of boys = 12
4	7y = 44x , yes line passes through the origin. When r = 2.8cm circumference = 17.6 units
5	(1, 1), (0, 0), (-1, -1), (2, 2), (-2, -2)

CASE BASE STUDY QUESTIONS

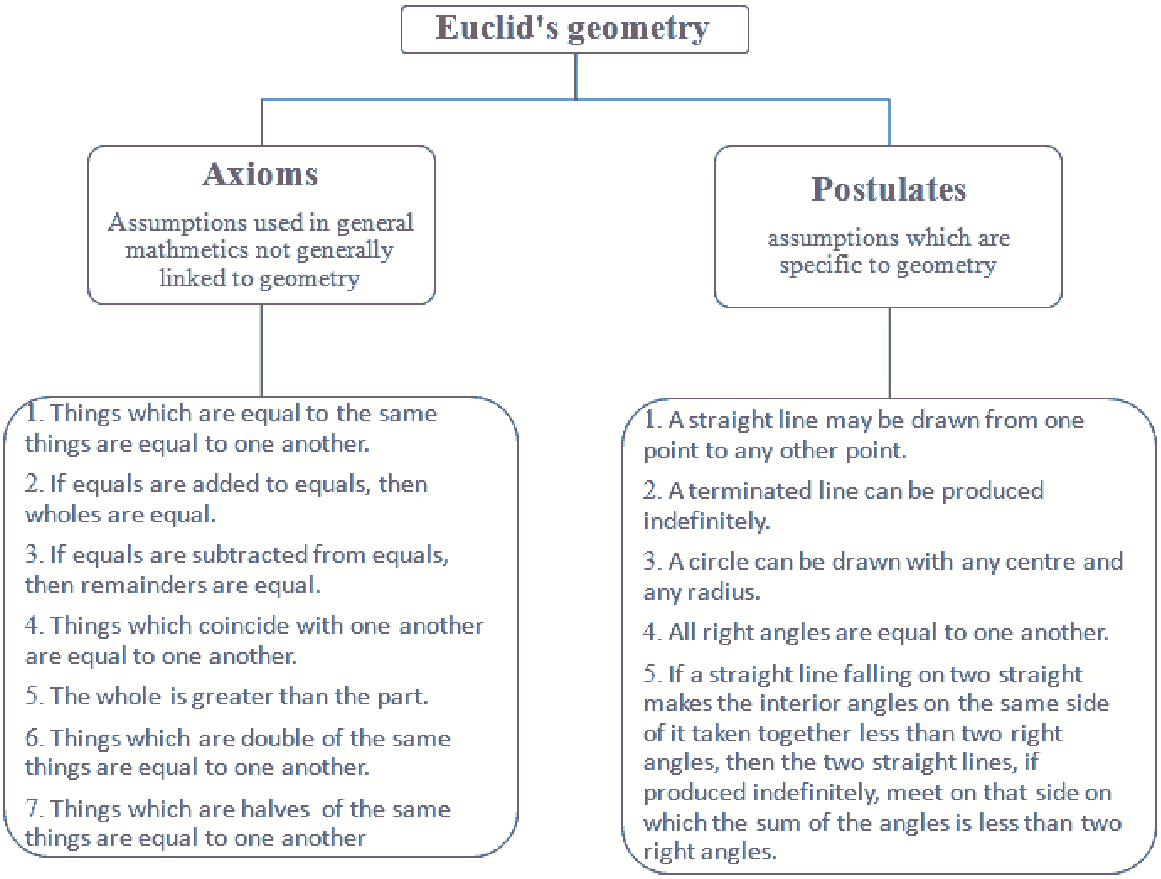
1	i) 3x+4y =100 ii) No. of adults = 10 iii) b = 7
2	i) y=25x ii) a = 25, b = -1, c = 0 iii) 100cm³ Or 20 seconds
3	i) (4, 0), (0, 2) ii) (6, 0), (0, 3) iii) No intersection point

HOTS

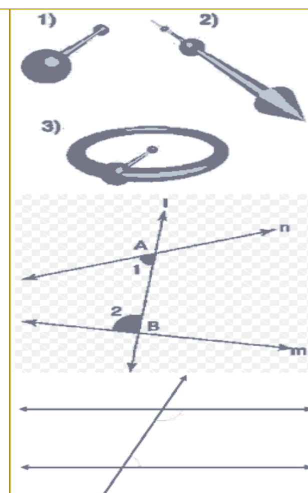
1	(3, 4) and Area is 12 sq. unit
2	i) 150km ii) 30km

CHAPTER 5
INTRODUCTION TO EUCLID'S GEOMETRY

MIND MAPPING



GIST/SUMMARY OF THE LESSON



- The word 'geometry' comes from the Greek words 'geo', meaning the 'earth', and 'metrein', meaning 'to measure' .
- Geometry originated from the need for measuring land.
- Euclid , the Greek mathematician is called the Father of Geometry.
- Euclid collected all the known works in his time and created his famous treatise – "Elements" . He divided the 'Elements' into thirteen chapters, each called a book.
- **EUCLID'S DEFINITIONS**

  - A point is that which has no part.
  - A line is a breadthless length.
  - The ends of a line are points.
  - A straight line is a line which lies evenly with the points on itself.
  - A surface is that which has length and breadth only.
  - The edges of a surface are lines.
  - A plane surface is a surface which lies evenly with the straight lines on itself.
- **UNDEFINED TERMS:**

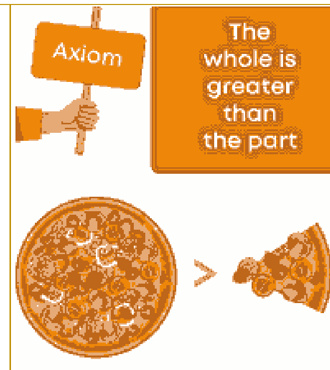
  - In geometry, a point, a line and a plane (in Euclid's words, a plane surface) are
  - considered as undefined terms.
  - Though Euclid defined a point, a line and a plane, the definitions are not accepted by mathematicians. Therefore, these terms are now taken as undefined.
  - Axioms and postulates are the assumptions which are obvious universal truth. They are not proved. Example Sun rises in the east and sets in the west is a

universal truth. Postulates are the assumptions which are specific to geometry whereas axioms are the assumptions used throughout mathematics and not specifically linked to geometry.

- Theorems are the statements which are proved using definitions, axioms, previously proved statements and deductive reasoning.

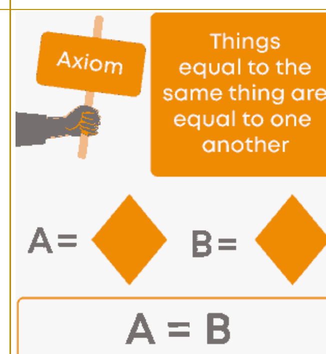
EUCLID'S AXIOMS

- Things which are equal to the same things are equal to one another.
  Example: if A=5 and B= 5, then A=B
- If equals are added to equals, then wholes are equal.
  Example: if A=B and C=D then A+C = B+D
- If equals are subtracted from equals, then remainders are equal.
  Example: if A=B and C=D then A-C = B-D
- Things which coincide with one another are equal to one another.
  Example: if you place one square exactly over another and they match perfectly, they are equal in shape and size.
- The whole is greater than the part
  Example: A full pizza is bigger than just one slice of it.
- Things which are double of the same things are equal to one another.
  Example: if two sticks are each twice as long as a 5cm stick, they are both 10cm and so are equal.
- Things which are halves of the same things are equal to one another.
  Example: if two pieces of chocolate are each half of a 100g bar, they both are 50g and equal.



EUCLID'S POSTULATES

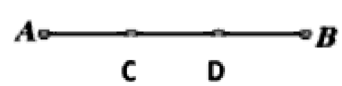
Postulate 1. A straight line may be drawn from one point to any other point.
Postulate 2. A terminated line can be produced indefinitely.
Postulate 3. A circle can be drawn with any centre and any radius.
Postulate 4. All right angles are equal to one another.
Postulate 5. If a straight line falling on two straight falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of the angles is less than two right angles.
Two equivalent version of fifth postulate are:
For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l. Two distinct intersecting lines cannot be parallel to the same line. Theorem

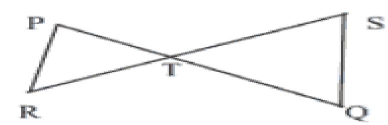
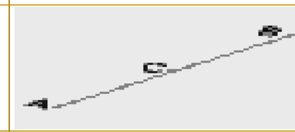
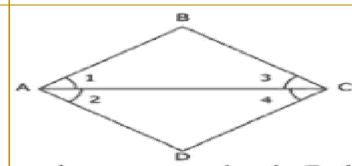
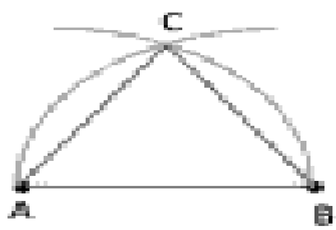
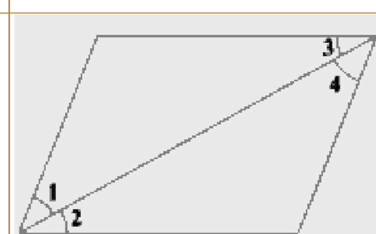
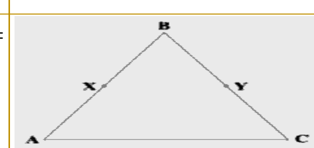


STATETMENT :Two distinct lines cannot have more than one point in common.
Given : Two lines l and m.
To prove : Lines l and m cannot have more than one point in common
Proof : Let us suppose that the two lines intersect in two distinct points, say P and Q.
Now there are two lines passing through two distinct points P and Q.
But this contradicts the axiom that only one line can pass through two distinct points.
So, the assumption that we started with, that two lines can pass through two distinct points is incorrect. From this, we conclude that two distinct lines cannot have more than one point in common.

MULTIPLE CHOICE QUESTIONS (1 MARKS QUESTIONS)

1	What does the Greek word 'Geo" mean ? a)Line B) Earth C) Point D) Ground	**ANSWER** :B)Earth
2	According to Euclid's definition, the ends of a line are a)Points B) edges C) dimensions D) surfaces	**ANSWER** :A) points

3	The assumptions which are specific to Geometry are called as : a)Definitions B) Postulates C) Elements D) Axioms	**ANSWER** :B) Postulates
4	Euclid stated that 'all right angles are equal to each other' in the form of A) an axiom B) a definition C) a postulate D) a proof	**ANSWER** :C) a postulate
5	How many lines can pass through a given point? A) Infinitely many B) Only 1 C) 2 D) 4	**ANSWER** :A) Infinitely many
6	Euclid collected all the known work in geometry and arranged in his famous treatise called A) Axioms B) Postulates C) Definitions D) Elements	**ANSWER** :D) Elements Explanation: By Euclid's axiom
7	If a point A lies between points B and C , then A) AC = 2 BC B) AC = BC C) AC = ½ BC d D) BC + AC = AB	**ANSWER** :d) BC + AC = AB
8	4 Things which coincide with one another are equal to one another. A proved statement is called : a) Definition b) Theorem c) Proposition d) Both (b) and (c)	**ANSWER** :b) Theorem
9	A pyramid is a solid shape , the base of which is a) a triangle b) a square c) a rectangle d) a polygon	**ANSWER** :b) square
10	How many lines can pass through 2 distinct points ? a) Infinite b) Only 1 c) 2 d) 4	**ANSWER** :b) Only 1
	ASSERTION REASON BASED QUESTION	
	In the following questions , a statement of ASSERTION is followed by a statement of REASON . Choose the correct option from the following : (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion(A) (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A) (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.	
1	Assertion: There can be infinite number of lines that can be drawn through a single point. Reason: Lines are made up of infinite number of points.	Answer :(B)
2	Assertion: Through two distinct points , only one line can be drawn. Reason: A line is formed by the joining only 2 points.	Answer : (B)
3	Assertion: If AB = PQ and PQ = XY, then AB = XY. Reason: Things which are equal to the same thing are equal to one another.	Answer :(A)
4	Assertion: If two circles are equal, then their radii are equal. Reason: Congruent circles have equal radii .	Answer :(A)
5	Assertion: Parallel lines are those lines which never intersect each other. Reason: Parallel lines can be two or more lines.	Answer :(B)
6	Assertion: A dimensionless dot which is drawn on a plane surface is known as point. Reason: A point is that which has no part.	Answer : (B)
7	Assertion: A line segment cannot be extended from both sides. Reason: A collection of points that has only length and no breadth is known as a line.	Answer :(B)
8	Assertion: A line can be extended on both sides . Reason: According to Euclid's second postulate , a terminated line can be produced indefinitely .	Answer :(A)
9	Assertion: An apple weighs 125 g but a part of it weighs more than 125 g. Reason: The whole is always greater than the part.	Answer :(D)
10	Assertion: Axioms are universal truths. Reason: Euclid stated only 6 axioms in his book "Elements".	Answer :(C)
	VERY SHORT ANWER TYPE QUESTIONS (2 MARKS)	
1	In the figure, AC = BD . Prove that AD =BC. **SOLUTION:** AC = BD ; AC + CD = BD + CD (equals added to equals) AD = BC hence proved	

2	In the given figure , PT = RT and TQ = TS , show that PQ = RS. **SOLUTION :** PT = RT (1) ; TQ = TS (2) ; Adding (1) and (2) PT + TQ = RT + TS (equals are added to equals whole are equal) So, PQ = RS hence proved	
3	If point C be the mid-point of a line segment AB , then write the relation between AC, BC and AB , with a suitable figure . **SOLUTION**: Relation : AC + CB = AB	
4	Define perpendicular lines. Are there any other words that need to be defined first? **SOLUTION**: Two lines which intersect at right angles Words that need to be defined : line, intersect, right angle	
5	In the figure, $\angle 1 = \angle 2$ and $\angle 2 = \angle 3$, show that $\angle 1 = \angle 3$ .Give reason . **SOLUTION** : Given $\angle 1 = \angle 2$ and $\angle 2 = \angle 3$ So, $\angle 1 = \angle 3$ (Things which are equal to the same thing are equal to one another)	
6	Write any two Euclid's postulate. **SOLUTION**: 1. A circle can be drawn with any centre and any radius. 2. All right angles are equal to one another.	
7	Prove that an equilateral triangle can be constructed on any given line segment. **SOLUTION**: Given : line segment AB of any length. Draw an arc with point A as the centre and AB as the radius. Similarly, draw an arc with point B as the centre and BA as the radius. The two arcs meet at a point, say C. Draw the line segments AC and BC to form Δ ABC . Now, AB = AC (equal radii) . ; Also , AB = BC (equal radii) By Euclid's axiom , things which are equal to the same thing are equal to one another Then AB = BC = AC. So, Δ ABC is an equilateral triangle	
8	What is a consistent system of axioms ? **SOLUTION**: A system of axioms is said to be consistent , if it is impossible to deduce from these axioms a statement that contradicts any axiom or previously proved statement .	
9	In the Fig, if $\angle 1 = \angle 3$,$\angle 2 = \angle 4$ and $\angle 3 = \angle 4$, write the relation between $\angle 1$ and $\angle 2$, using an Euclid's axiom. **SOLUTION**: Given$\angle 1 = \angle 3$ (1) ,$\angle 2 = \angle 4$ (2) $\angle 3 = \angle 4$ (3) From (1) ,(2) and (3) $\angle 1 = \angle 2$ (Things which are equal to the same things are equal to one another.)	
10	In the Fig, we have BX = ½ AB, BY = ½ BC and AB = BC. Show that BX = BY. **SOLUTION**: Given BX = ½ AB, (1) ; BY = ½ BC (2) ; BX = BY. (3) ; To Prove BX = BY ; Proof From (1), (2) and (3) BX = BY (Things which are equal to the same things are equal to one another.)	
	SHORT ANSWER TYPE QUESTIONS (3 MARKS)	
1	Read the following statement: "A square is a polygon made up of four line segments, out of which, length of three line segments are equal to the length of fourth one and all its angles are right angles".	

Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all angles and sides of a square are equal?

SOLUTION : The terms need to be defined are :

LINE : Undefined term Point

POLYGON : A simple closed figure made up of three or more line segments.

POINT: Undefined term ;

LINE SEGMENT : Part of a line with two end points.

RAY : Part of a line with one end point

RIGHT ANGLE : Angle whose measure is 90

ANGLE: A figure formed by two rays with a common initial point.

Undefined terms used are : line, point.

Euclid's fourth postulate says that "all right angles are equal to one another."

In a square, all angles are right angles, therefore, all angles are equal (From Euclid's fourth postulate).

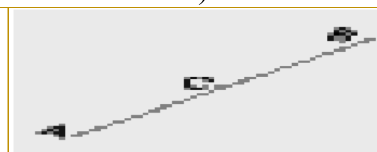
Three line segments are equal to fourth line segment (Given). Therefore, all the four sides of a square are equal. (by Euclid's first axiom "things which are equal to the same thing are equal to one another.")

2 If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$ AB. Explain by drawing the figure.

SOLUTION: Given AC= BC

AC+BC = AB (Things which coincide with one another are equal to one another.)

AC +AC =AB (AC =BC Given) ; 2AC = AB ; AC = $\frac{1}{2}$ AB.



3 In fig, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

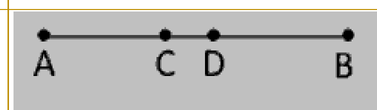
SOLUTION Let us consider AB has two mid point C and D

So AC = BC (1) AD = BD (2) ; Subtracting (1) from (2)

AD -AC = BD- BC ; CD= -CD ; 2CD = 0CD =0

So the distance between C and D is 0, i.e C and D coincides.

Thus every line has one and only one mid point.

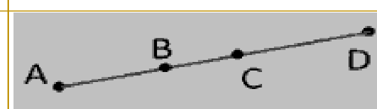


4 In Fig, if AC = BD, then prove that AB = CD

SOLUTION: Given AC = BD

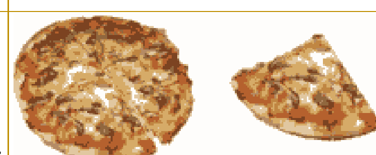
AC – BC = BD – BC (equals subtracted from equals remainder are equals)

AB = CD



5 Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'?

SOLUTION Axiom 5 says " whole is greater than the part" A part is always smaller than the whole thing. It is a universal truth which cannot be changed.



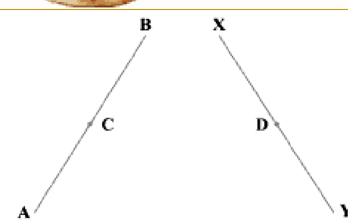
6 In Fig, we have : AC = XD, C is the mid-point of AB and D is the mid-point of XY. Using an Euclid's axiom, show that AB = XY.

SOLUTION: AB = 2AC (C is the mid-point of AB)

XY = 2XD (D is the mid-point of XY)

Also, AC = XD (Given)

Therefore, AB = XY, because things which are double of the same things are equal to one another.

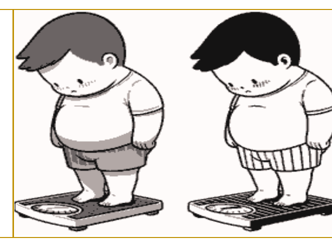


7 Solve the equation a – 15 = 25 and state which axiom do you use here.

SOLUTION: : a – 15 = 25. On adding 15 to both sides, we have a – 15 + 15 = 25 + 15 = 40 (If equals are added to equals, then wholes are equal).

8 Ram and Ravi have the same weight. If they each gain weight by 2 kg, how will their new weights be compared ?

SOLUTION: Let x kg be the weight each of Ram and Ravi. On gaining 2 kg, weight of Ram and Ravi will be (x + 2) each. According to Euclid's second axiom, when equals are added to equals, the wholes are equal. So, weight of Ram and Ravi are again equal.



9 Look at the Fig. Show that length AH > sum of lengths of AB + BC + CD.

SOLUTION: AB + BC + CD = AD

AD is a part of AH(Whole is always greater than part)

Therefore, AH > AD

AH > AB + BC + CD.

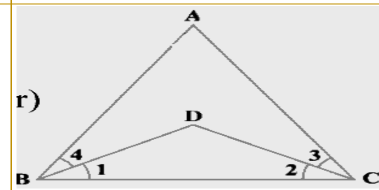
10 In the Fig, we have ∠ABC = ∠ACB, ∠3 = ∠4. Show that ∠1 = ∠2.

SOLUTION: Given ∠ABC = ∠ACB

∠1+ ∠3 = ∠2+∠4 (things coincide with one another are equal to one another)

∠1+ ∠3 = ∠2+∠3 (∠3 = ∠4 given)

∠1 = ∠2+∠3 - ∠3∠1 = ∠2.



LONG ANSWER TYPE (5 MARKS)

Discuss the contribution of Euclid to the development of geometry. How is his approach still relevant in modern mathematics?

EXPECTED RESPONSE: Brief biography of Euclid, Introduction to his book Elements , Description of axiomatic method, Influence on geometry in classrooms and logical structure in mathematics, Mention of other geometries that built upon or diverged from his model

How did the questioning of Euclid's Fifth Postulate lead to the birth of non-Euclidean geometry? Explain with examples.

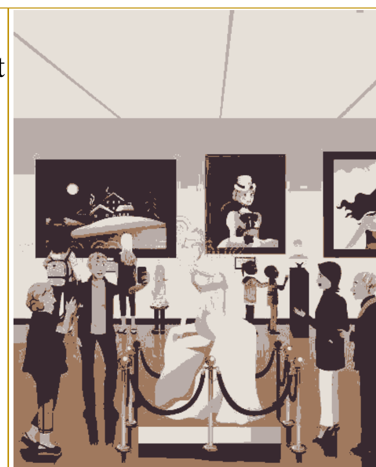
EXPECTED RESPONSE: State Euclid's 5th postulate , Explain why it was seen as complicated and hard to prove from the others , Development of new geometries like Spherical geometry (no parallel lines) , Hyperbolic geometry (many lines parallel) , Importance in modern fields like relativity theory and cosmology

State and explain any four Euclid's axioms. Give one example for each to show how they are used in geometry.

EXPECTED RESPONSE: State four axioms (e.g., "Things which are equal to the same thing are equal to one another.") , Explain each in your own words , Provide geometric or numerical examples , Include a small diagram if applicable

CASE BASED QUESTIONS

1 In a trip to a museum in Delhi, students observed that one of the halls is not a rectangle. They imagined that if the edges of the hall's floor are extended it will appear as lines exhibited in Euclid's 5th postulate . Later on the way back to school, they discussed Euclid's other axioms and postulates with their teacher. Next day they had a revision class in school where teacher asked them following questions:



- ? State the first postulate of Euclid .

Ans: A straight line may be drawn from one point to any other point.

- ? If PQ = QR and QR = RP , then PQ = RP. Identify the axiom used in this.

Ans: Things which are equal to the same thing are equal to one another.

- ? State Euclid's 5th postulate with a figure

Ans: If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of the angles is less than two right angles.

- ? OR Does Euclid's 5th postulate imply the existence of parallel lines ? Explain.

Ans: Yes , if the two straight lines make the interior angles on the same side of it taken together as equal to two right angles, then the two straight lines will never meet and will be parallel.

2 A National Public School organised an education trip to a museum. Almost all the students of class IX went to the trip with their teacher of Mathematics.

They saw many pictures of mathematicians and read about their contributions in the field of Mathematics. After visiting the museum, teacher asked the following questions from the students. On the basis of the above information, solve the following questions:

? Who is the teacher of Pythaoras?
ANS: Thales
? Name of the mathematician who is visible in the last picture.
ANS: Thales
? State any two Euclid's postulate
ANS: A circle can be drawn with any centre and any radius
All right angles are equal to one another.
? OR Define Axioms, postulates and Theorem
Axiom- Assumptions used in general mathmetics not generally linked to geometry
Postulate - assumptions which are specific to geometry
Theorem - Theorems are the statements which are proved using definitions, axioms, previously proved statements and deductive reasoning.

3 Rohan, a student of class 9th learnt that Labour day is celebrated on May 1st. He learnt the importance of house help and how everyone should respect each other's work. Rohan's maid has 2 children . Both of them have equal number of dresses .So Rohan, on his birthday, plans to give both of them same number of dresses

? If the children had x dresses each and Rohan gifts them 2 dresses , then how many dresses will each child have after Rohan's birthday?
ANS: Each child have after Rohan's birthday (x+2)
? Will the children have equal number of dresses after Rohan's birthday ? Which Euclid's axiom is used here ?
ANS: Yes, if equals are added to equals whole are equal.
? If $p + q = 7$, then $p + q - r = 7 - r$. Is any axiom used here? If yes, state the axiom.
ANS: Yes, if equals are subtracted from equals remainders are equal.
? OR If $\angle 1 = \angle 2$, then $2\angle 1 = 2\angle 2$. Is any axiom used here? If yes, state the axiom
ANS Yes, Things which are double of the same things are equal to one another.

HOTS

1 Euclid assumes that "a point is that which has no part." If a point has no size, how can it exists on a line or plane? Is this concept purely theoretical or does it have real world application?
SOLUTION: In mathematics, a point is a conceptual tool an abstract idea to denote location only, not physical size. Though it has no dimensions, it is defined to exist as a precise position in space. In a real world application (like graphs or blueprints) we represent points visually, but in theory they remain dimensionless. This abstraction is vital for constructing accurate models.

EXERCISE

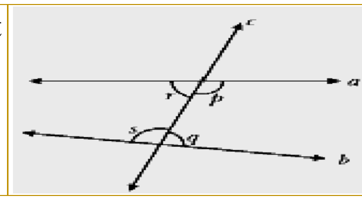
MULTIPLE CHOICE QUESTIONS

1 The side faces of a pyramid are :
(A) Triangles (B) Squares (C) Polygons (D) Trapeziums

2 Which of the following needs a proof ?
(A) Theorem (B) Axiom (C) Definition (D) Postulate

3 ‘Lines are parallel if they do not intersect’ is stated in the form of
(A) an axiom (B) a definition (C) a postulate (D) a proof

4 Observe the figure shown. A student claimed that the lines when extended meet at a point which lies on the left of the line c. Given that the student’s claim is true, which of these justifies the claim?
(a) $p + q < 180°$ (b) $p + r < 180°$
(c) $r + s < 180°$ (d) $s + q < 180°$



5 Two quantities P and Q are such that P = Q. Which of these equations illustrates the Euclid’s axiom “If equals are added to equals, the wholes are equals”?
(a) $P + x = Q - x$ (b) $P + x = Q + x$ (c) $P + x = Q$ (d) $P \times x = Q$

ASSERTION AND REASON QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the Answer :out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1 Assertion (A): If AB = MN and MN = PQ, then AB = PQ.
Reason (R): According to the Euclid’s first axiom, ‘Things which are equal to the same thing are also equal to one another’.

2 Assertion (A): If Rita and Reena are of same age that is 10 years then after 6 years also they will have the same age.
Reason (R): According to Euclid’s Axiom, when equals are subtracted from equals, remainders are equal.

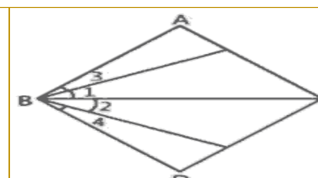
3 Assertion (A): A cake when it is whole, assume it measures 2kg but when a slice is taken out of it and measured, its weight will be smaller than the previous one
Reason (R): According to Euclid’s Axiom, whole is always greater than the part.

4 Assertion (A): Euclid’s fifth postulate implies the existence of parallel lines.
Reason (R): the sum of interior angles will be equal to the sum of two right angles then two lines will meet each other on either sides and therefore they will be parallel to each other.

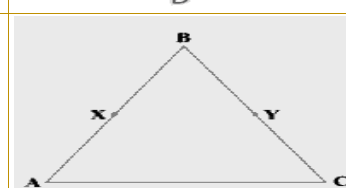
5 Assertion (A): an infinite number of lines can be drawn to pass through a given point.
Reason (R): A line segment has only two end points.

VERY SHORT ANSWER TYPE QUESTION (2MARKS)

1 In the given figure, we have $\angle 1 = \angle 2$, $\angle 3 = \angle 4$.
Show that $\angle ABC = \angle DBC$.
State the Euclid’s axiom used.



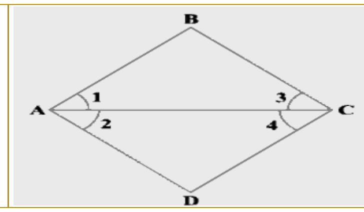
2 In the Fig., we have AB = BC, BX = BY.
Show that AX = CY.



3 Two salesmen make equal sales during the month of August. In September, each salesman doubles his sale of the month of August. Compare their sales in September.

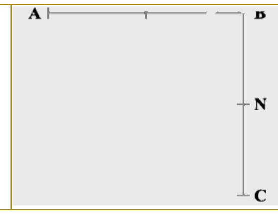
4 It is known that $x + y = 10$ and that $x = z$. Show that $z + y = 10$?

5 In the Fig, we have ∠1 = ∠2 = ∠3 Show that ∠1 = ∠3.



SHORT ANSWER TYPE QUESTIONS (3 MARKS)

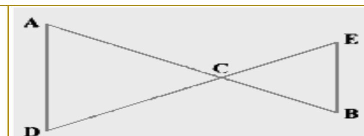
1 In the Fig.
(i)AB = BC, M is the mid-point of AB and N is the mid- point of BC. Show that AM = NC.
(ii) BM = BN, M is the mid-point of AB and N is the mid-point of BC. Show that AB = BC.



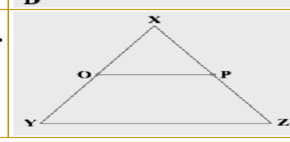
2 Read the following statement : An equilateral triangle is a polygon made up of three line segments out of which two line segments are equal to the third one and all its angles are 60° each. Define the terms used in this definition which you feel necessary. Are there any undefined terms in this? Can you justify that all sides and all angles are equal in a equilateral triangle.

3 Read the following two statements which are taken as axioms :
(1)If two line intersect each other, then the vertically opposite angles are not equal.
(2)If a ray stands on a line, then the sum of two adjacent angles so formed is equal to 180°. Is this system of axioms consistent? Justify your answer.

4 In the Fig, we have AC = DC, CB = CE. Show that AB = DE.



5 In the Fig., if OX = ½ XY and PX = ½ XZ and OX = PX, show that XY = XZ.



LONG ANSWER TYPE (5 MARKS)

1 Write Euclid 5th postulate and its equivalent version and explain with figure.
2 Write all Euclid postulate explain any one postulate with figure.
3 What is the difference between axiom, postulate and theorem give two examples of each.
4 What are undefined terms? Give examples
5 State and explain the significance of Euclid's fifth postulate. How did it lead to the development of other geometries?

HOTS

1 Euclid's fifth postulate states that if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, then the two lines will meet. Can you explain why this postulate is necessary and not derivable from the first four? What would geometry look like without it?

CASE BASED QUESTIONS(4 MARKS)

1 In last year, cyclone comes out in Andhra Pradesh. Due to this cyclone, many persons lost their lives and property. Deepak and Rohit decided to contribute equal amounts to National Disaster Relief Fund, so that the suffered person get some relief.
(a) In this process, which axiom is used. Also write their statement.
(b) If Deepak contributed ₹30,000, then how much contribution is done by Rohit?
(c) In the given figure,

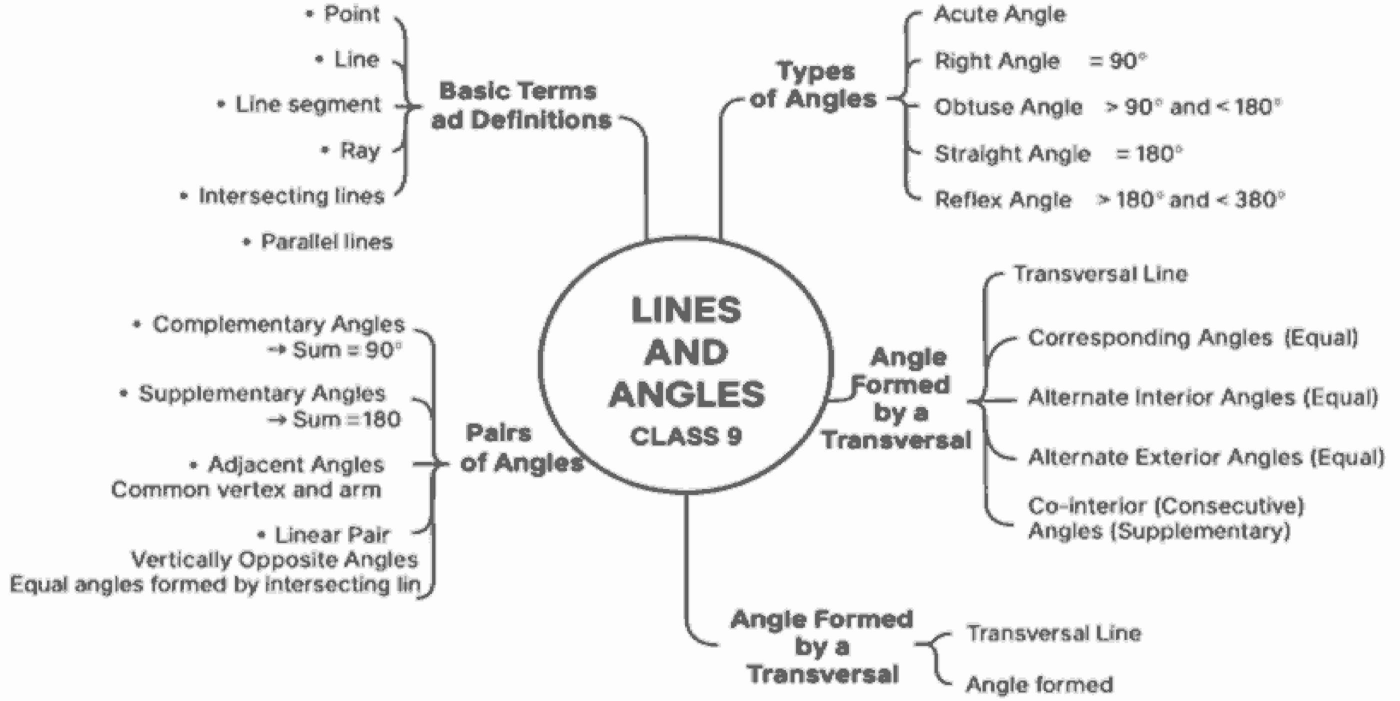
if PR = QS, then prove that PQ = RS.
OR Prove that every line has one and only one mid-point.

2	Rahul has a fantasy of collecting the old stamp. So, one day he went to collect old stamps from two different market stores of the Indira Nagar market. So, Rahul decides to take three from each store. (a) It is known that a + b = 20 and a = c. Show that c + b = 20. (b) How many stamps remain with each store after Rahul's purchase? (c) Solve the equation y + 12 = 15 and state the Euclid axiom used here. OR State any two Euclid's axiom.	
3	One day during their visit to mathematics laboratory , some students observed a solid which looked like an incomplete pyramid. They asked their teacher about the solid and she told them that it is a truncated pyramid. She also told them that how ancient Egyptians could find the area of such shapes using basic knowledge of geometry. The whole class was amazed to know the contributions of Greek Mathematician Thales and his pupil , Pythagoras and also about Euclid , the Father of Geometry, who collected all the known work and arranged them in his book called "Elements having 13 chapters. The teacher then explained about axioms and postulates. Later, the students were asked the following questions: a) Name the shape that forms the base of a pyramid . b) Name the famous book of Euclid and the number of chapters it has. c) What is an axiom ? State any one axiom. OR What is an Postulate ? State any one Postulate	

	ANSWERS OF EXERCISE	
	MCQs	1.A 2.A 3.C 4.C 5.B
	ASSERTION AND REASON	1.A 2.B 3.A 4.C 5.B
	VERY SHORT TYPE	3.Let x be sale in August. Sale in September 2x
	CASE STUDY QUESTIONS	A) Axiom 1 B) 30,000 B) x-3 c) y=3, axiom 3 A) triangle b) element 13 chapter

CHAPTER – 6
LINES AND ANGLES

MIND MAPPING



GIST/SUMMARY OF THE LESSON:

GIST OF THE CHAPTER: -

- Basic Definitions
- Types of Angles
- Types of Pairs of Angles
- Important Angle Properties
- Parallel Lines and a Transversal
- Axioms and Theorems

DEFINITIONS AND FORMULAE:

BASIC DEFINITIONS:

- Line: Extends indefinitely in both directions.
- Line segment: A part of a line with two endpoints.
- Ray: A part of a line that starts at one point and goes on forever
- Angle: Formed when two rays meet at a point.

TYPES OF ANGLES:

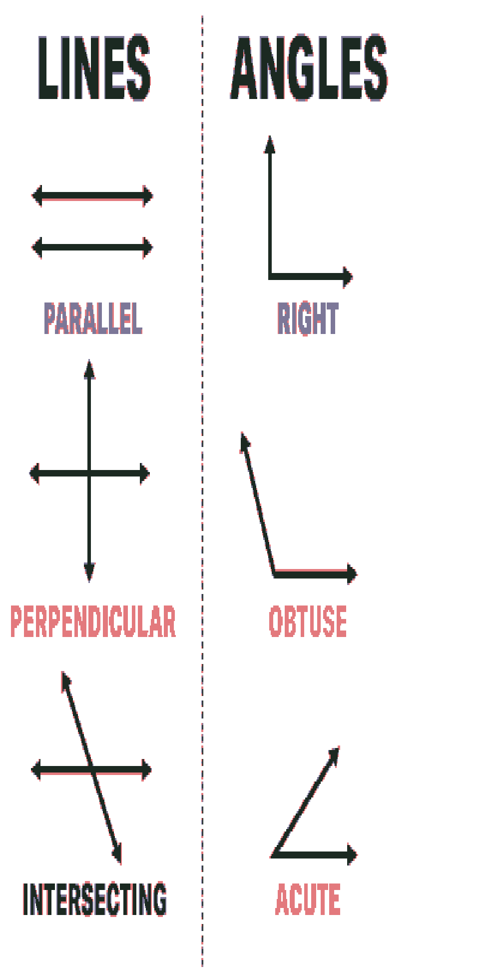
- Acute Angle: < 90° Right Angle: = 90°
- Obtuse Angle: > 90° and < 180°
- Straight Angle: = 180°
- Reflex Angle: > 180° and < 360°

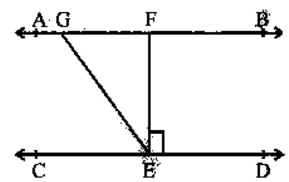
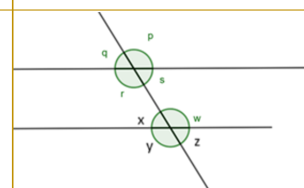
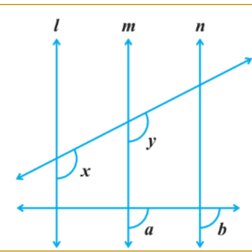
TYPES OF PAIRS OF ANGLES:

- Complementary Angles: Sum = 90°
- Supplementary Angles: Sum = 180°
- Adjacent Angles: Share a common arm and vertex.
- Linear Pair: Adjacent angles on a straight line (sum = 180°).
- Vertically Opposite Angles: Equal angles formed when two lines intersect.

IMPORTANT ANGLE PROPERTIES:

- Linear pair angles are supplementary.
- Vertically opposite angles are equal.
- If two lines intersect, opposite angles are equal.



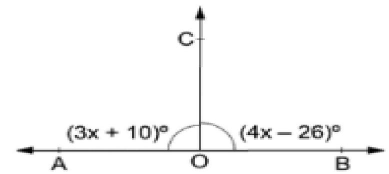
	MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)	
1	Intersecting lines cut each other at: a) One point b) Two points c) Three points d) Null **ANSWER**: (a)	
2	Explanation: Two lines always intersect each other at one point. Two parallel lines intersect at: a) One point b) Two points c) Three points d) Null **ANSWER**: (d),Explanation: If two lines are parallel to each other, they don't intersect each other. They meet each other at infinity.	
3	If AB || CD, EF ⊥ CD and ∠GED = 135° as per the figure given below. The value of ∠AGE is: a) 120° b) 140° c) 90° d) 135° **ANSWER**: (d) Explanation: Since AB || CD and GE is a transversal. Given, ∠GED = 135° , Hence, ∠GED = ∠AGE = 135° (Alternate interior angles)	
4	4. If two lines intersect each other, then the vertically opposite angles are: a) Equal b) Unequal c) Cannot be determined d) None of the above	**ANSWER**: (a)
5	5. A line joining two endpoints are called: a) Line segment b) A ray c) Parallel lines d) Intersecting lines	**ANSWER** (a)
6	An acute angle is: a) More than 90 ° b) Less than 90 ° c) Equal to 90 degrees d) Equal to 180 °	**ANSWER** (b)
7	A reflex angle is: a) More than 90 degrees b) Equal to 90 degrees c) More than 180 degrees d) Equal to 180 degrees	**ANSWER** (c)
8	A straight angle is equal to: a) 0° b) 90° c) 180° d) 360°	**ANSWER** (c)
9	Two angles whose sum is equal to 180° are called: a) Vertically opposite angles b) Complementary angles c) Adjacent angles d) Supplementary angles	**ANSWER** (d)
10	In the figure below, which of the following are corresponding angle pairs: a) ∠p and ∠q b) ∠p and ∠w c) ∠p and ∠x d) ∠p and ∠z	 **ANSWER** (b)
	ASSERTION - REASON BASED QUESTIONS	
	In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R).Pick the correct option: Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A). Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A). Assertion (A) is true but Reason(R) is false. Assertion (A) is false but Reason(R) is true.	
1	Assertion (A): In a figure, x = y and a = b. then line l || n. Reason(R) If a transversal intersects two lines in such a way that a pair of corresponding angles are equal, then the two lines are parallel. **ANSWER**. (a)x = y (Given) ,Therefore, l || m (Corresponding angles) (i) Also, a = b (Given) ,Therefore, n || m (Corresponding angles) (ii) From (i) and (ii), l || n (Lines parallel to the same line).	
2	Assertion (A) : Sum of the pair of angles 120° and 60° is supplementary.	

Reason(R) Two angles, the sum of whose measures is 180°, are called supplementary angles.
ANSWER: (a)

3 Assertion (A): The angles of a triangle are in the ratio 2:3: 4. The largest angle of the triangle is 80°.
Reason(R) The sum of all the interior angles of a triangle is 180°
ANSWER: (a)

4 Assertion (A) : If angles ‘a’ and ‘b’ form a linear pair of angles and a = 40°, then b = 150°.
Reason(R) Sum of linear pair of angles is always 180°.
ANSWER: (d)

5 Assertion (A): In the given figure, AOB is a straight line.
If ∠AOC = (3x + 10)°
and ∠BOC (4x – 26)°, then ∠BOC = 86^0
Reason(R) The sum of angles that are formed on a straight line is equal to 180°



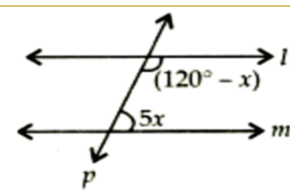
ANSWER: (a)

6 Assertion (A) : If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 5:4, then the greater of the two angles is 100^0
Reason(R) If a transversal intersects two parallel lines, then the sum of the interior angles on the same side of the transversal is 180^0
ANSWER: (a)

7 Assertion (A) : An angle is 14º more than its complementary angle, then angle is 52º.
Reason(R) Two angles are said to be supplementary if their sum of measure of angles is 180°
ANSWER: (b)

8 Assertion (A) : Supplement of angle is one fourth of itself. The measure of the angle is 144^0
Reason(R) Two angles are said to be supplementary if their sum of measure of angles is 180^0
ANSWER: (a)

9 Assertion (A): The value of x from the adjoining figure, if ℓ || m is 15º.
Reason(R) If two parallel lines are intersected by a transversal, then each pair of corresponding angles so formed is equal.

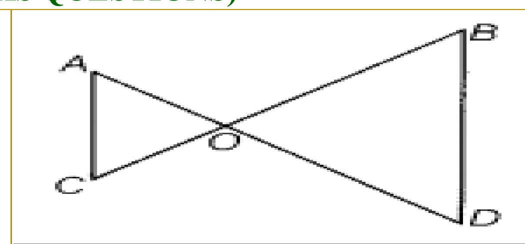


ANSWER: (b)

10 Assertion (A) : If two internal opposite angles of a triangle are equal and external angle is given to be 110°,
Reason(R) A triangle with one of its angle 90^0, is called a right triangle
ANSWER: (b)

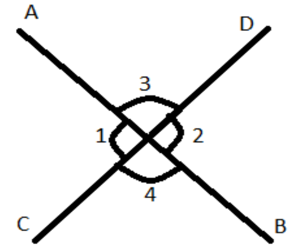
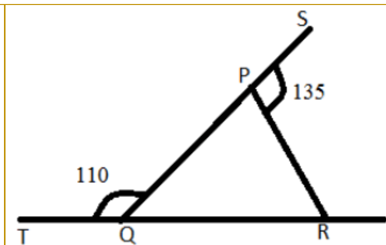
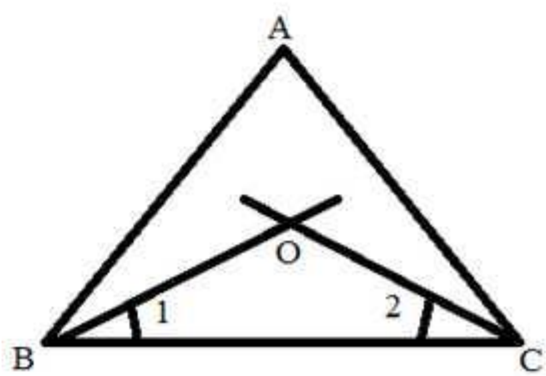
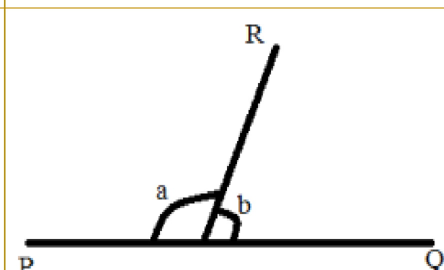
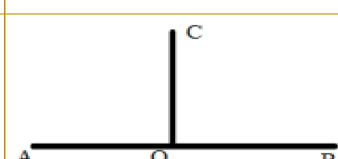
VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

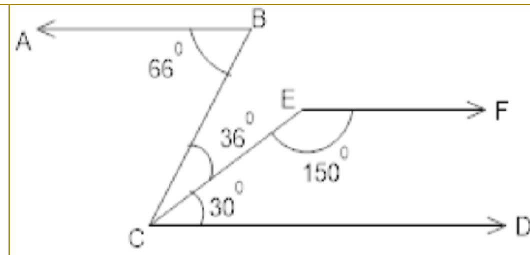
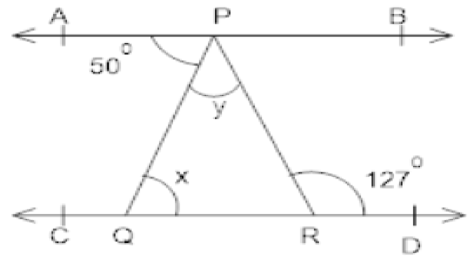
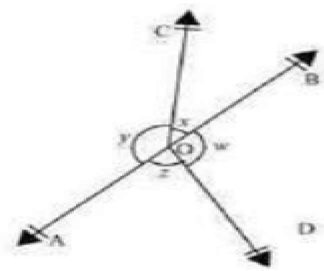
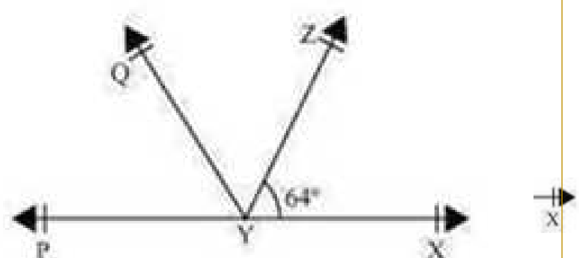
1 In the given fig ∠AOC =∠ACO and ∠BOD = ∠DBO prove that AC || DB

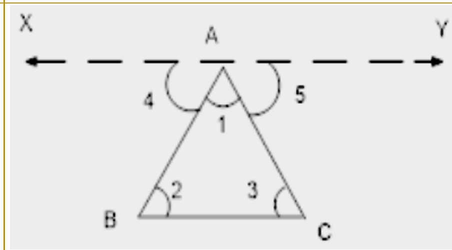
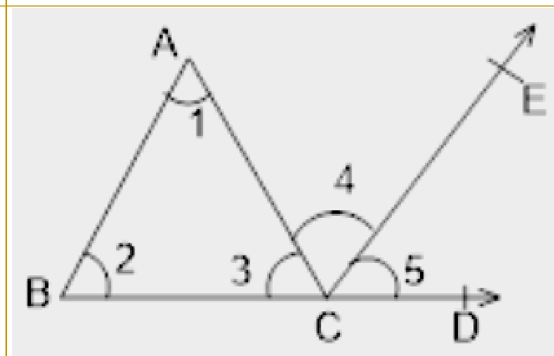
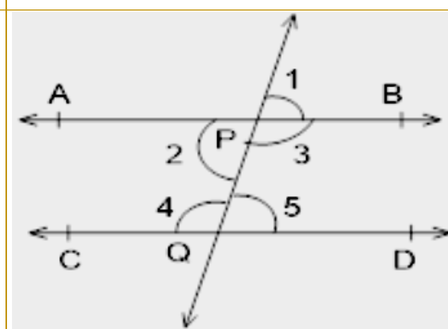
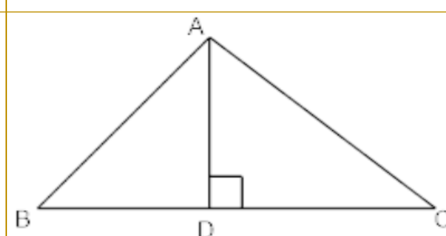
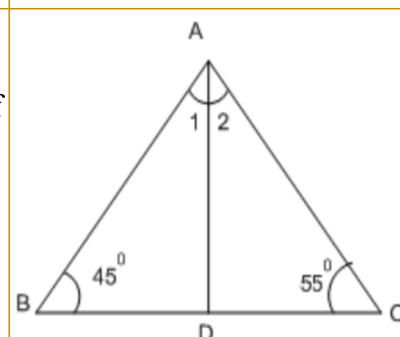


SOLUTION:.∠AOC =∠ACO and ∠BOD = ∠DBO [Given]
But, ∠AOC=∠BOD [vertically opposite angles]
∠ACO=∠DBO
∠BOD =∠BDO
AC || DB [By alternate interior angle property]

2 The measure of an angle is twice the measure of supplementary angle. Find measure of angles.
SOLUTION: Let the measure be x^0.
Then its supplement is 180^0- x^0
According to question
$x^0=180^0- x^0$,$x^0=360^0- 2x^0$
$3x^0=360^0$ implies $x^0=120^0$
The measure of the angles are 120^0 and 60^0

3	Prove that if two lines intersect each other than vertically opposite angler are equal. Solution: Given: AB and CD are two lines intersect each other at O. To prove: (i) $\angle 1=\angle 2$ and (ii) $\angle 3=\angle 4$ Proof: $\angle 1+\angle 4=180^0$……..(i) [By linear pair] $\angle 2+\angle 4=180^0$……..(ii) $\angle 1+\angle 4=\angle 2+\angle 4$ [By eq (i) and (ii)] $\angle 1=\angle 2$ Similarly, $\angle 3=\angle 4$	
4	Prove that if one angle of a triangle is equal to the sum of other two angles, the triangle is right angled. **SOLUTION**. Given in ΔABC, $\angle B=\angle A+\angle C$ To prove: ΔABC is right angled. Proof: $\angle A+\angle B+\angle C=180°$….. (1) [Sum of three angles of a ΔABC is 180°] $\angle B=\angle A+\angle C$ ….. (2) From (1) and (2) $\angle B+\angle B=180°$ $2\angle B=180° \angle B=90°$	
5	In fig. sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR=135°$ and $\angle PQT=110°$,find$\angle PRQ$. **SOLUTION**:$\angle PQT+\angle PQR=180°$ $110^0+\angle PQR=180°$ $\angle PQR=180°-110°$ $\angle PQR=70°$ Also $\angle SPR=\angle PQR+\angle PRQ$[Interior angle theorem] $135°=70°+\angle PRQ$ $135°-70°=\angle PRQ$ $\angle PRQ=65°$	 two
6	In fig the bisector of $\angle ABC$ and $\angle BCA$ intersect each other at point O prove that $\angle BOC=90^0+\frac{1}{2}\angle A$. **SOLUTION**: Given ΔABC such that the bisectors of $\angle ABC$ and $\angle BCA$ meet at a point O , To Prove :$\angle BOC=90°+\frac{1}{2}\angle A$ Proof: In ΔBOC $\angle 1+\angle 2+\angle BOC=180^0$.....(i) In ΔABC,$\angle A+\angle B+\angle C=180°$;$\angle A+2\angle 1+2\angle 2=180°$ [BO and CO bisects $\angle B$ and $\angle C$] $\frac{\angle A}{2}+\angle 1+\angle 2=90°$;$\angle 1+\angle 2=90°-\frac{\angle A}{2}$ [Divide forth side by 2] Substituting, 900-$\frac{\angle A}{2}$ + $\angle BOC$= 180° ;$\angle BOC=90°+\frac{1}{2}\angle A$	
7	In the given figure $\angle PQR$ and $\angle QOR$ form a linear pair if a – b =80°. Find the value of ‘a’ and ‘b’. **SOLUTION**: a + b =180°……(1) [by linear pair] $a – b =80^0$………(2) 2a=260° [Adding equations (1) and (2)] a =130° 130°+b=180°b=180°-130°=50°	
8	If ray OC stands on a line AB such that $\angle AOC=\angle BOC$, then show that $\angle AOC=90°$. **SOLUTION**. $\angle AOC=\angle BOC$ [Given];$\angle AOC+\angle BOC=180°$ [By linear pair] $\angle AOC+\angle AOC=180°$; $2\angle AOC=180°$;$\angle AOC=90°=\angle BOC$	

9	In the given figure show that AB||EF. **SOLUTION**: ∠BCD=∠BCE+∠ECD ; 36°+30°=66°=∠ABC AB||CD [Alternate interior angles are equal] Again ∠ECD=30° and ∠FEC=150° ∠ECD+∠FEC=30°+150°=180° EF||CD [sum of consecutive interior angle is 180^0] AB||CD and EF||CD; Hence, AB||EF.	
10	In figure if AB||CD, ∠APQ=50^0 and ∠PRD =127^0 Find x and y. **SOLUTION**: AB||CD and PQ is a transversal. ∠APQ=∠PQD [pair of alternate angles] 50^0=x Also AB||CD and PR is a transversal ∠APR=∠PRD 50^0+Y=127^0 Y=127^0-50^0=77^0	
	SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)	
1	In Fig. if x + y = w + z, then prove that AOB is a line. **SOLUTION** We need to prove that AOB is a line. We are given that $x + y = w + z$. We know that the sum of all the angles around a fixed point is 3600. Thus, we can conclude that ∠AOC+∠BOC+∠AOD+∠BOD=360^0 $x + y + z + w = 360^0$ But, x+y = w+z. (Given) 2(x+y)=360^0 (x+y)=180^0 From the given figure, we can conclude that y and x form a linear pair. We know that if a ray stands on a straight line, then the sum of the angles of linear pair formed by the ray with respect to the line is180^0. (x+y)=180^0 Therefore, we can conclude that AOB is a line.	
2	It is given that ∠XYZ=$64^0$and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP. **SOLUTION** We are given that ∠XYZ=64^0, *XY* is produced to *P* and *YQ* bisects ∠ZYP. We can conclude the given below figure for the given situation: We need to find ∠XYQ and reflex ∠QYP. From the given figure, we can conclude that ∠XYZ and ∠ZYP form a linear pair. We know that sum of the angles of a linear pair is180^0. ∠XYZ +∠ZYP=180^0 64^0 +∠ZYP=180^0 ∠ZYP=116^0 Ray *YQ* bisects ∠ZYP, or ∠QYZ=∠QYP = $\frac{116}{2}$ =58^0 ∠XYQ=∠QYZ+∠XYZ 58^0+64^0=122^0 reflex ∠QYP =360^0- ∠QYP 360^0-58^0=302^0	

	Therefore, we can conclude that $\angle XYQ=122^0$ and reflex $\angle QYP=302^0$		
3	Prove that sum of three angles of a triangle is 180^0. Draw a triangle ABC. Through point A, draw a line parallel to BC. Let angles at B and C be $\angle ABC$ and $\angle ACB$. Alternate interior angles formed with the transversal equal the angles of triangle. So: $\angle A + \angle B + \angle C = 180°$ (angle on a straight line) Hence, proved.		
4	The side BC of Δ ABC is produced from ray BD. CE is drawn parallel to AB, show that $\angle ACD=\angle A+\angle B$. Also prove that $\angle A+\angle B+\angle C= 180^0$ Solution $\because$ AB $\parallel$ CE and AC intersect them $\angle 1 = \angle 4$ (1) [Alternate interior angles] Also AB $\parallel$ CE and BD intersect them $\angle 2 = \angle 5$ (2) [Corresponding angles] Adding eq (1) and eq (2) $\angle 1 + \angle 2 = \angle 4 + \angle 5$;;Adding $\angle C$ on both sides, we get $\angle A + \angle B + \angle C = \angle C + \angle ACD$; $\angle A + \angle B + \angle C = 180^0$		
5	Prove that if a transversal intersect two parallel lines, then each pair of alternate interior angles is equal. **SOLUTION** Given: line AB‖CD intersected by transversal PQ To Prove: (i) $\angle 2=\angle 5$ (ii) $\angle 3=\angle 4$ Proof: $\angle 1=\angle 2$…………… (i) [Vertically Opposite angle] $\angle 1=\angle 5$………………….. (ii) [Corresponding angles] By (i) and (ii) $\angle 2 = \angle 5$ Similarly, $\angle 3=\angle 4$ Hence Proved.		
6	In the given figure Δ ABC is right angled at A. AD is drawn perpendicular to BC. Prove that $\angle BAD=\angle ACB$ **SOLUTION** $\because$ $AD \perp BC$;;;;;; $\angle BAD=90^0 - \angle ABD \rightarrow (1)$;;; $\angle C=90^0 - \angle B \rightarrow (2)$ From (1) and (2);; $\angle BAD=\angle ACB$ Hence proved.		
7	In ΔABC, $\angle B=45^0$,$\angle C=55^0$ and bisector of $\angle A$ meets BC at a point D. Find $\angle ADB$ and $\angle ADC$. **SOLUTION** ΔABC, $\angle A + \angle B + \angle C = 180°$ [Sum of three angle of a Δ = 180°] $\Rightarrow \angle A+45^0+55^0 =180^0$; $\angle A=180^0 - 100^0 = 80^0$ $\therefore$ AD bisects $\angle A$; $\angle 1=\angle 2=\frac{1}{2}\angle A = \frac{1}{2}\times 80^0 = 40^0$ Now in Δ ADB, We have $\angle 1+\angle B+\angle ADB=180^0$		

$\Rightarrow\ 40^0 + 45^0 + \angle ADB = 180^0$; $\Rightarrow\ \angle ADB = 180^0 - 85^0 = 95^0$

$\angle ADB + \angle ADC = 180^0$;Also $95^0 + \angle ADC = 180^0$

$\angle ADC = 180^0 - 95^0 = 85^0$; $\angle ADB = 95^0 \; and \; \angle ADC = 85^0$

8 In figure two straight lines AB and CD intersect at a point O. If $\angle BOD = x^0$ and $\angle AOD = (45-x)^0$. Find the value of x hence find

(a) ∠BOD (b) ∠AOD (c) ∠AOC (d) ∠BOC

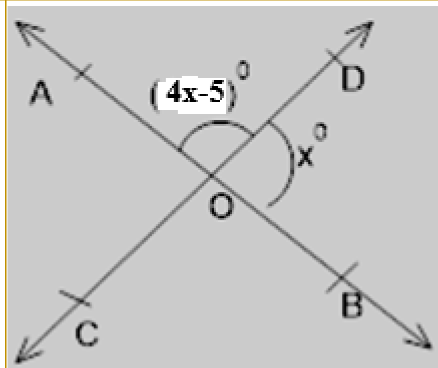
Solution: ∠ADB=∠AOD+∠DOB…….. By linear pair

$180^0 = 4x - 5 + x$;; 5x=185

$x = \frac{185}{5} = 37^0$;; $= 4 \times 37 - 5 = 148 - 5 = 143^0$

$\angle BOC = 143^0$;;

vertically opposite angles ; $\angle BOD = \angle AOC = 37°$



9 The side BC of a Δ ABC is produced to D. the bisector of ∠A meets BC at L as shown if fig. prove that ∠ABC+∠ACD=2∠ALC

Solution InΔABCwe have $\angle ACD = \angle B + \angle A \;\; \rightarrow (1)$ [Exterior angle property]

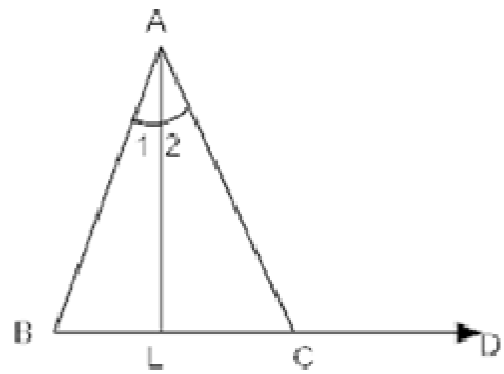
$\angle ACD = \angle B + 2\angle 1$

In ΔABL ; $\angle ALC = \angle B + \angle BAL$ [exterior angle property]

$\angle ALC = \angle B + \angle 1$; $\Rightarrow 2\angle ALC = 2\angle B + 2\angle 1$ …(2)

Subtracting (1) from (2)

$2\angle ALC - \angle ACD = \angle B$;; $\angle ACD + \angle ABC = 2\angle ALC$



LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1 In fig M and N are two plane mirrors perpendicular to each other; prove that the incident ray CA is parallel to reflected ray BD.

Solution Draw AP ⊥ M and BQ ⊥ N

$\therefore BQ \perp N \; and \; AP \perp M$ and $M \perp N$

$\therefore \angle BOA = 90^0$

$\Rightarrow$ BQ $\perp$ AP

In Δ BOA $\angle 2 + \angle 3 + \angle BOA = 180^0$ [By angle sum property]

$\Rightarrow \angle 2 + \angle 3 + 90^0 = 180^0$

$\therefore \angle 2 + \angle 3 = 90^0$

Also $\angle 1 = \angle 2$ and $\angle 4 = \angle 3$

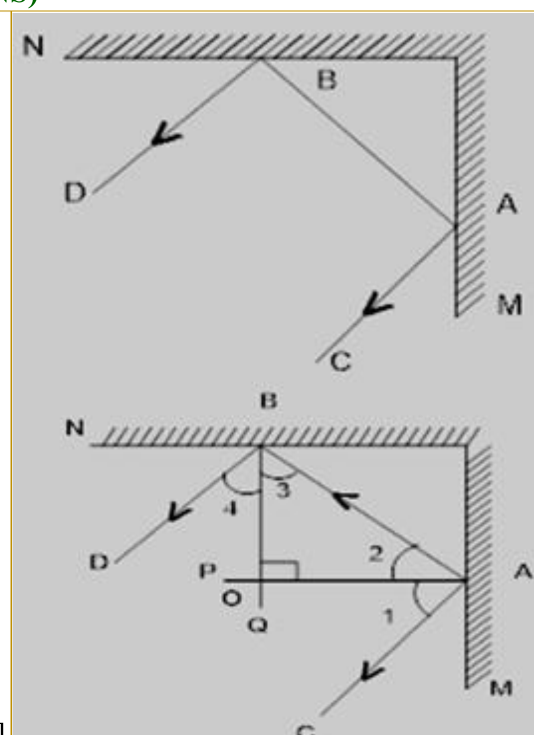
$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3 = 90^0$

$\therefore (\angle 1 + \angle 4) + (\angle 2 + \angle 3) = 90^0 + 90^0 = 180^0$

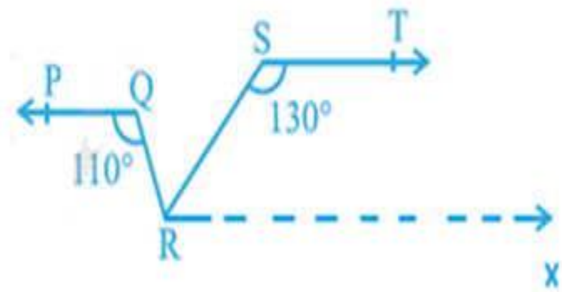
$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^0$

or $\angle CAB + \angle DBA = 180^0$

$\therefore$ CA || BD [By sum of interior angles of same side of transversal]



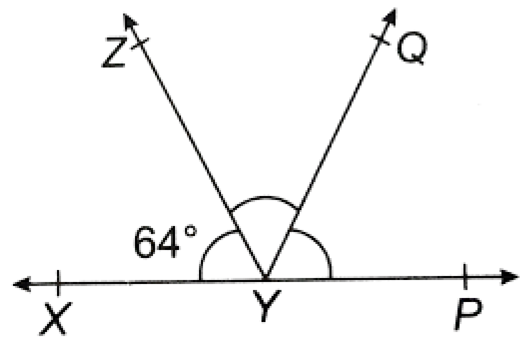
2 In the given figure, if PQ || ST, ∠PQR=110^0 ,∠RST=130^0, find ∠QRS.



SOLUTION We are given that $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$.

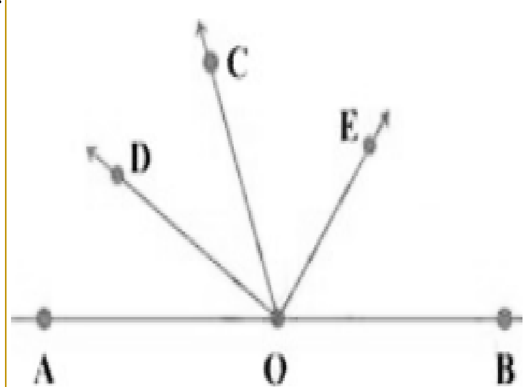
We need to find the value of $\angle QRS$ in the figure.
We need to draw a line RX that is parallel to the line ST, to get
Thus, we have $ST \parallel RX$.
We know that lines parallel to the same line are also parallel to each other.
We can conclude that $PQ \parallel ST \parallel RX$.
$\angle PQR = \angle QRX$ (Alternate interior angles), or $\angle QRX = 110^\circ$.
We know that angles on same side of a transversal are supplementary.
$\angle RST + \angle SRX = 180^\circ$; $\Rightarrow 130^\circ + \angle SRX = 180^\circ$
$\Rightarrow \angle SRX = 180^\circ - 130^\circ = 50^\circ$. ; From the figure, we can conclude that
$\angle QRX = \angle SRX + \angle QRS$; $\Rightarrow 110^\circ = 50^\circ + \angle QRS$
$\Rightarrow \angle QRS = 60^\circ$. Therefore, we can conclude that $\angle QRS = 60^\circ$.

3 It is given that ∠XYZ=64^0 and XY is produced to point P.
Draw a figure from the given information.
If ray YQ bisects ∠ZYP. Find ∠XYQ and reflex ∠QYP.



SOLUTION:
XYP is a straight line,
∠ XYZ + ∠ ZYP = 180° (Linear pair)
∠ ZYP = 180° – ∠ XYZ
∠ ZYP = 180° – 64°
∠ ZYP = 116°
Since YQ bisects ∠ ZYP
∠ ZYQ = ∠ QYP = $\frac{1}{2}$ ∠ ZYP

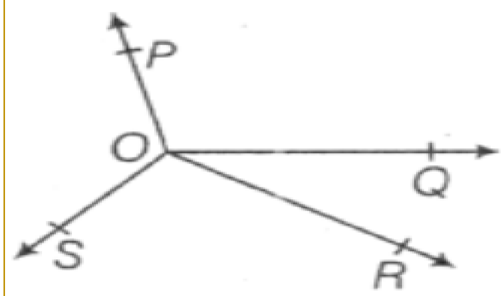
4 In the below fig. If OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that the points A, O and B are collinear.



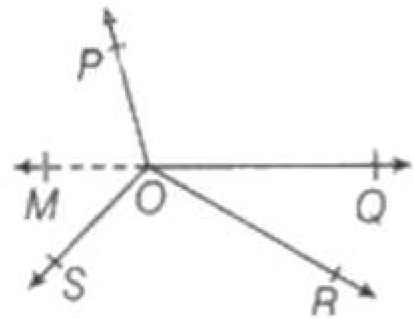
SOLUTION Given: In figure, OD ⊥ OE (i.e. ∠DOE=90^0), OD and OE are the bisectors of ∠AOC and ∠BOC.
To prove: points A, O and B are collinear i.e., AOB is a straight line.
Proof: From the fig. we have ∠AOB comprising ∠AOC and ∠BOC
such that OD and OE are the bisectors of these two angles
∠AOB = ∠AOC + ∠BOC .Since, OD and OE bisect angles ∠AOC and ∠BOC
respectively.∴∠AOC=2∠DOC …..............................(1)
And ∠COB=2∠COE …...(2)
On adding equations (1) and (2), we get

$\angle AOC+\angle COB=2\angle DOC+2\angle COE$
$;\Rightarrow \angle AOC+\angle COB=2(\angle DOC+\angle COE)$
$\Rightarrow \angle AOC+\angle COB=2\angle DOE$; $\Rightarrow \angle AOC+\angle COB=2\times 90^\circ [\because OD\perp OE]$
$\Rightarrow \angle AOC+\angle COB=180^\circ$; $\therefore \angle AOB=180^\circ$
So, $\angle AOC$ and $\angle COB$ are forming linear pair or AOB is a straight line.
Hence, points A, O and B are collinear.

5 In the given figure, OP, OQ, OR and OS are four rays. Prove that $\angle POQ + \angle ROQ + \angle SOR + \angle POS = 360^\circ$.

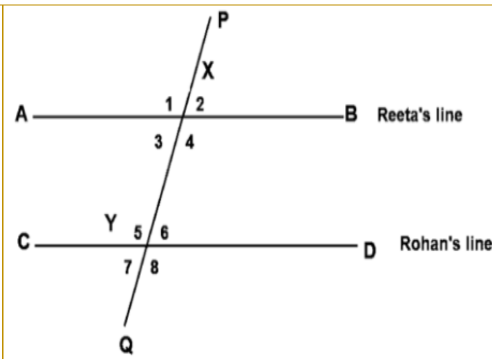


SOLUTION Let us produce a ray OQ backwards to a point M, then MOQ is a straight line. Now, OP is a ray on the line MOQ. Then, by linear pair axiom, we have $\angle MOP + \angle POQ = 180^\circ$ ......(i)
Similarly, OS is a ray on the line MOQ. Then, by linear pair axiom, we have
$\angle MOS + \angle SOQ = 180^\circ$ ....(ii)
Also, $\angle SOR$ and $\angle ROQ$ are adjacent angles.
$\therefore \angle SOQ = \angle SOR + \angle ROQ$ ...(iii)
On putting the value of $\angle SOQ$ from Eq.(iii) in Eq.(ii), we get
$\angle MOS + \angle SOR + \angle ROQ = 180^\circ$ ....(iv)
Now, on adding Eqs.(i) and (iv), we get
$\angle MOP + \angle POQ + \angle MOS + \angle SOR + \angle ROQ = 180^\circ + 180^\circ$
$\Rightarrow \angle MOP + \angle MOS + \angle POQ + \angle SOR + \angle ROQ = 360^\circ$ ....(iv)
But $\angle MOP + \angle MOS = \angle POS$
Then, from Eq.(v), we get $\angle POS + \angle POQ + \angle SOR + \angle ROQ = 360^\circ$
Hence proved.



CASE BASED QUESTIONS (04 MARKS QUUESTIONS)

1 Reeta and Rohan were playing a game on parallel lines and the angles formed with the transverse line (i.e., alternate angles, corresponding angle and interior angles).First Reeta drew a straight line AB, then Rohan drew another straight line CD||AB. Further, a transverse line PQ was drawn which intersects lines AB and CD at points X and Y respectively. Now they did toss with a coin and Rohan won the toss. Following were the rules of the game: Toss winner will ask a question and others will answer. If the answer is correct then person answering will ask question else questioner will ask next question.
Who wins the last question he/she will be the winner.
Based on the above paragraph please answer these questions:
(i)Which is the alternate angle to $\angle 6$?
(ii)Which is the corresponding angle to $\angle 1$?
(iii)If $\angle 4 = 120^\circ$ then what is measure of $\angle 6$?
(OR) What is the sum of $\angle 3$ and $\angle 5$?
ANSWERS :$\angle 3$,$\angle 5$,60°,180°

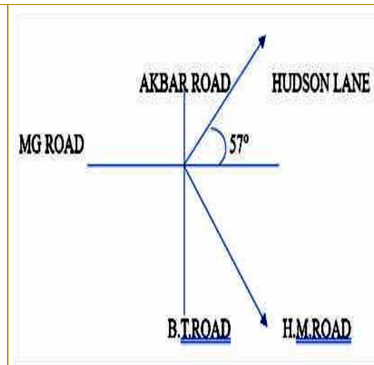


2 Ritesh and Sheetal are cousins and both went to visit Mughal Garden. Before going, they searched the location of their destination on a map. During searching, they found on map that Akbar Road and M.G. road form a right angle at their intersection point and Hudson lane form 57° angle with M.G. road.

(a) What is the measure of acute angle between Akbar Road and Hudson lane?

(b) If Ritesh is standing on M.G Road in the west direction and Sheetal is on H.M road, what is the shortest angle they can cover in order to meet?

(c) Find the measure of reflex angle formed between M.G Road [in east direction] with Hudson lane.



SOLUTION (a) From the given figure, Hudson Lane forms 57° with M.G road and Akbar Road and M.G Road form a 90° at their intersection point. Therefore, the required angle between Akbar Road and Hudson lane = 90° – 57° = 33°.

(b) Sheetal travels from H.M road to M.G road [East] to Hudson to Akbar road and then to M.G road west. So, the measure of angle she cover = 37° + 90° + 90° = 217°. But if she goes from H.M road to south of BT road and then to M.G road [west], Then, the measure of angle, she cover = 53° + 90° = 143° Hence, the shortest angle she has to cover will be 143°

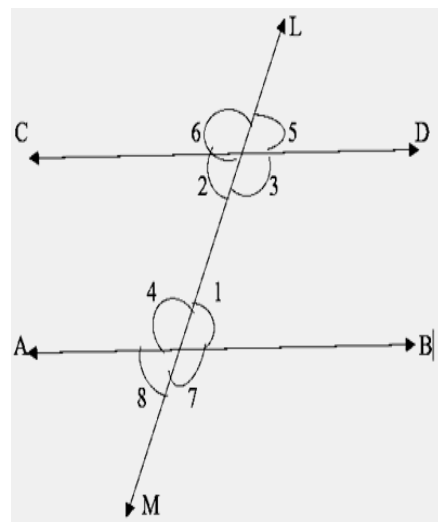
(c) The required measure of reflex angle formed between M.G Road [in east direction] with Hudson lane = 360° – 57° = 303°.

3 Two lines are parallel to each other, if the distance between these 2 lines always remains constant throughout and they never meet. There are various examples of parallel lines that we see in our daily life like railway line, 2 steps of ladder, opposite sides of a table etc. A line which cuts a pair of parallel lines is called a transversal as shown in the figure. Answer the following questions:

(a) If ∠5 = 65°. Then what is the ∠8? (1)

(b) If ∠6 = 2x and ∠1 = 70°. Then find the value of x. (1)

(c) If ∠6 :∠5 = 2 : 3 then find the value of ∠7. (2)



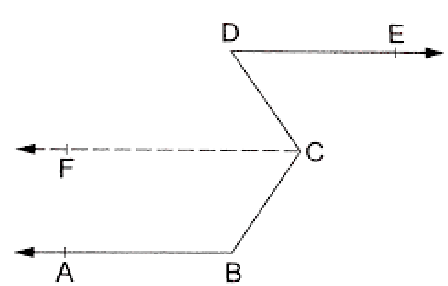
SOLUTION (a) Since CD | | AB and LM is transversal,
∠5 and ∠8 are the alternate exterior angles. ∠ 5 =∠ 8 or ∠8 = ∠5 = 65°

(b) Since CD | | AB and LM is transversal, ∠5 = 70° (Corresponding angles) and ∠6 + ∠5 = 180° (Linear pair axiom)
2x + 70° = 180°; 2x = 110° x = 55°.

(c) Let ∠6 = 2k and ∠5 = 3k;Now, ∠6 + ∠5 = 180° (Linear pair axiom)
2k + 3k = 180°;5k = 180° k = 36°;∠6 = 2k = 2 × 36° = 72°
Now, ∠6 and ∠7 are the alternate exterior angles. ;∠6 = ∠7 or ∠7 = ∠6 = 72°

HOTS

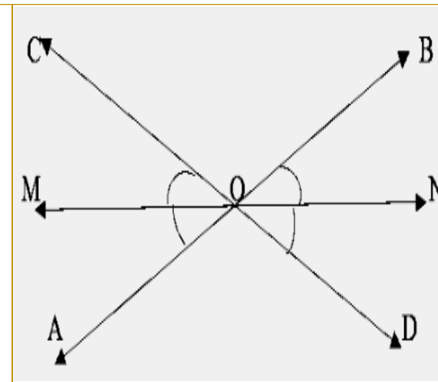
1 In Figure, AB || DE. Prove that ∠ABC+∠BCD=180°+∠CDE.



SOLUTION: Through C, draw CF parallel to both AB and DE. Since AB || CF and the transversal BC cuts them at B and C respectively.
∠ABC+1=180° [Consecutive interior angles are supplementary]………..(i)
Similarly, DE || CF and transversal CD intersects them at C and D respectively.
∠CDE = ∠2; Adding (i) and (ii), we get ∠ABC+∠1+∠2=180°+∠CDE
;∠ABC+∠BCD=180°+∠CDE

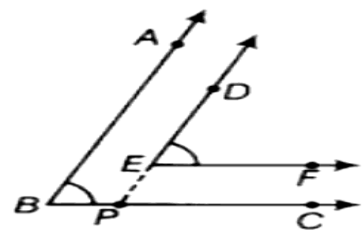
2 Prove that bisectors of pair of vertically opposite angles are in the same straight line.

SOLUTION: Given: Two lines AB and CD intersect at point O. Also, OM and ON are the bisectors of $\angle AOC$ and $\angle BOD$ respectively.
To prove: MON is a straight line. Prove: Since, the sum of all the angles around a point O is 360°, we have $\angle AOC + \angle BOC + \angle BOD + \angle AOD = 360°$
or, $2\angle MOC + 2\angle BOC + 2\angle BON = 360°$ [$\angle BOC = \angle AOD$ (vertically: opp. angles OM is bisector of $\angle AOC$, ON is bisector of $\angle BOD$]
or, $\angle MOC + \angle BOC + \angle BON = 180°$; or, $\angle MON = 180°$
$\angle MON$ is a straight angle. ; Hence, MON is a straight line.



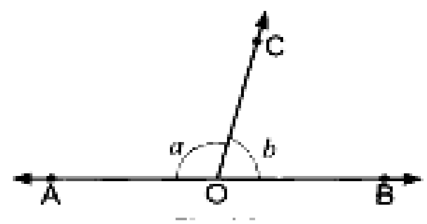
3 In a figure, BA || ED and BC || EF. Show that $\angle ABC = \angle DEF$

SOLUTION: Given BA || ED and BC || EF.
To show $\angle ABC = \angle DEF$. Construction Draw a ray EP opposite to ray ED.
Proof In the figure, BA || ED or BA || DP$\therefore \angle ABP = \angle EPC$
[corresponding angles]$\Rightarrow \angle ABC = \angle EPC$ ....(i)
Again, BC || EF or PC || EF$\therefore \angle DEF = \angle EPC$ [corresponding angles] ....(ii)
From eqs.(i) and (ii),$\angle ABC = \angle DEF$



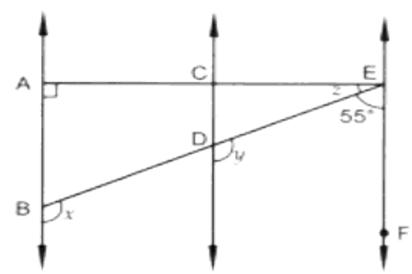
4 In Fig. $\angle AOC$ and $\angle BOC$ form a linear pair. If a - b = 20°, find the values of a and b.

SOLUTION: Since, total sum of angles on a line is equal to 180° .
$\therefore a + b = 180°$ (Linear pair) ...(i)&, $a - b = 20°$ (Given) ...(ii)
Adding (i) and (ii), we get$(a + b) + (a - b) = 180° + 20°$
Or, $2a = 200°$; $\therefore a = 100°$
Putting the value of a in equation (i), we get$100° + b = 180°$; $\therefore b = 80°$



5 Fig., AB || CD and CD || EF. Also, EA ⊥ AB. If $\angle BEF = 55°$, find the values of x, y, and z.

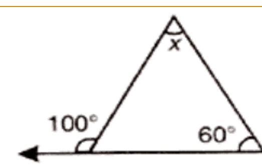
SOLUTION: Since corresponding angles are equal.$\therefore x = y$ ... (i)
We know that the interior angles on the same side of the transversal are supplementary.
$\therefore y + 55° = 180°$; $\Rightarrow y = 180° - 55o = 125°$
So, $x = y = 125°$; Since AB || CD and CD || EF.
$\therefore$ AB || EF ; $\Rightarrow \angle EAB + \angle FEA = 180°$ [$\because$ Interior angles on the same side of the transversal EA are supplementary]
$\Rightarrow 90° + z + 55° = 180°$; $\Rightarrow z = 35°$



EXERCISE

MULTIPLE CHOICE QUESTIONS

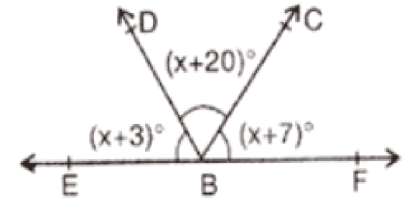
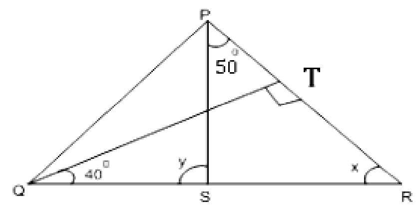
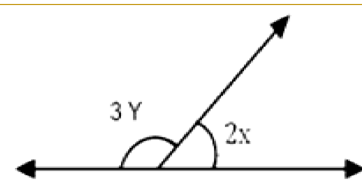
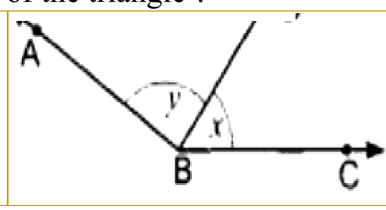
1 Value of x in the figure below is:
a) 20° b) 40o c) 80° d) 160°



2 If two complementary angles are in the ratio 13 :5, then the angles are:
a) 13x°, 5x° b) 25°, 65° c) 65°, 25° d) 65°, 35°

3 The diagonals of the rectangle ABCD intersect at O. If $\angle COD = 78°$, then $\angle OAB$ is:
a) 35° b) 51° c) 70° d) 110°

4 If AB = x + 3, BC = 2x and AC = 4x – 5, then for what value of 'x', B lies on AC?
a) 2 b) 3 c) 5 d) 8

5	In the given figure, find the value of x: a) 40° b) 50° c) 60° d) 80°	
	ASSERTION - REASON BASED QUESTIONS	
1	Assertion : A triangle can have two obtuse angles. Reason: The sum of all the interior angles of a triangle is 180°	
2	Assertion : The angles of a triangle are in the ration 3:5:7. The triangle is acute-angled Reason: The sum of angles that are formed on a straight line is equal to 180°.	
3	Assertion : line l is parallel to line m. Reason: If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.	
4	Assertion (A): If angles 'a' and 'b' form a linear pair of angles and a = 40° then b = 150°. Reason (R): Sum of linear pair of angles is always 180°.	
5	Assertion (A): An angle is 14° more than its complementary angle, then angle is 52°. Reason (R): Two angles are said to be supplementary if their sum is 180°	
	VERY SHORT ANSWER TYPE QUESTIONS	
1	In figure, $QT \perp PR, \angle TQR = 40^0 and$ $\angle SPR = 50^0$ *find x and y.*	
2	The three angles of a triangle, one is twice the smallest and another is three times the smallest. Find the angles.	
3	In fig x=300 the value of Y is	
4	The angles of a triangle are in the ratio 5 :3 : 7, what is the type of the triangle ?	
5	For what value of x + y in Fig., will ABC be a line? Justify your answer.	
	SHORT ANSWER TYPE QUESTIONS	
1	If two straight lines are perpendicular to the same line, prove that they are parallel to each other.	
2	Two unequal angles of a parallelogram are in the ratio 2:3. Find all its angles in degrees.	
3	The opposite sides of a quadrilateral are parallel. If one angle of the quadrilateral is 600, find the other angles.	
4	Two lines AB and CD intersect at O. If ∠AOC+ ∠COB+∠BOD=270°, find the measures of ∠AOC, ∠COB, ∠BOD and ∠DOA.	
	LONG ANSWER TYPE QUESTIONS	
1	In Figure, ∠AOF and ∠FOG form a linear pair. If ∠EOB = ∠FOC =90° and ∠DOC=∠FOG=∠AOB=30°. (i) Find the measures of ∠FOE, ∠COB and ∠DOE. (ii) Name all the right angles. (iii) Name three pairs of adjacent complementary angles.	

(iv) Name three pairs of adjacent supplementary angles.
(v) Name three pairs of adjacent angles.

2 (i)If two parallel lines are intersected by a transversal, then prove that the bisectors of any pair of alternate interior angles are parallel.
(ii)If two parallel lines are intersected by a transversal, then prove that the bisectors of any two corresponding angles are parallel.

3 Prove that "If two lines intersect each other, then the vertically opposite angles are equal." Using this theorem, find all the angles if ∠ POR: ∠ ROQ = 5 : 7 in the below figure where lines PQ and RS intersect each other at point O.

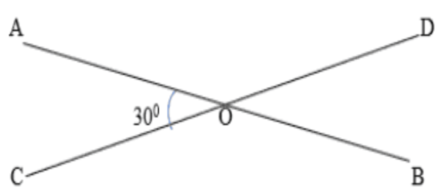
4 If two parallel lines are intersected by a transversal prove that the bisectors of the interior angles on the same side of transversal intersect each other at right angles.

5 Two parallel lines are intersected by a transversal. Prove that the bisectors of two pairs of interior angles enclose a rectangle.

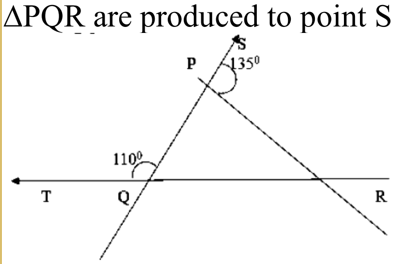
CASE BASED QUESTIONS

1 BSE stands for a disease called Bovine Spongiform Encephalopathy. "Bovine" means that the disease affects cows, "spongiform" refers to the way the brain from a sick cow cow looks spongy under a microscope, and "encephalopathy" indicates that it is a disease of the brain. This disease is commonly called "mad cow disease."
A farmer has a field ABCD formed by two pair of parallel roads as shown below in which 1 || m and p || q. His four cows suffering from BSE. Thus, he tied them at four corners of the field ABCD.
Q. 1. If ∠BAC = 30°, find ∠CAD.
Q. 2. ∠ABC + ∠BCD = 180° as:
Q.3. If cow at C and cow at Dis 2 km apart, then what is the distance between cow at A and cow at B?
Q. 4. If ∠B=45°, then ∠D =…………………
Q. 5. If we join BD such that BD meet AC at O and ∠BOC 30°, then what is the measure of ∠AOD?

2 Harry was going on a road trip with his father. They were travelling on a straight road. After riding for some distance, they reach a cross road where one straight road cuts the other at 30°.
Q. 1. Find the measure of ∠BOD.
Q. 2. Find the measure of ∠AOD
Q. 3. Which property is used in this case.



3 To Protect poor people from cold weather, Ramlal, has given his land to make a shelter home for them.In the given figure sides PQ and RQ of ΔPQR are produced to point S and T respectively.



If ∠PQT = 110° and ∠SPR = 135°

Q. 1. Find ∠QPR
Q. 2. Find ∠PRQ
Q. 3. Which property is used over here.

HOTS

1 If the complement of an angle is equal to the supplement of the thrice of it. Find the measure of the angle.
Ans.450.

2 If the supplement of an angle is three times its complement, find the angle.
Ans.450.

3 Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

4 If two straight lines intersect each other, prove that the ray opposite to the bisector of one of the angles thus formed bisects the vertically opposite angle.

5 If the bisectors of a pair of alternate angles formed by a transversal with two given lines are parallel, prove that the given lines are parallel.

ANSWERS

MCQ	ASSERTION - TYPE	VERY SHORT TYPE
1.Answer (b)	1.Answer: (d)	1.Answer: x=50o and y=80o.
2.Answer (c)	2.Answer: (b)	2.Answer: 30o ,60o ,and 90o.
3.Answer (b)	3.Answer: (d)	3.Answer: Y=40o.
4.Answer (d)	4.Answer: (d)	4.Answer: An acute angled triangle.
5.Answer (b)	5.Answer: (b)	5.Answer: x + y must be equal to 180°.

SHORT ANSWER TYPE QUESTIONS

1.Answer.Try to prove.
2.Answer:∠A=∠C=720, ∠B=∠D=1080.
3.Answer: 1200, 600, 1200.
4.AnswerEach equal to 900.
5.Answer. Try to prove.

LONG ANSWER TYPE QUESTIONS

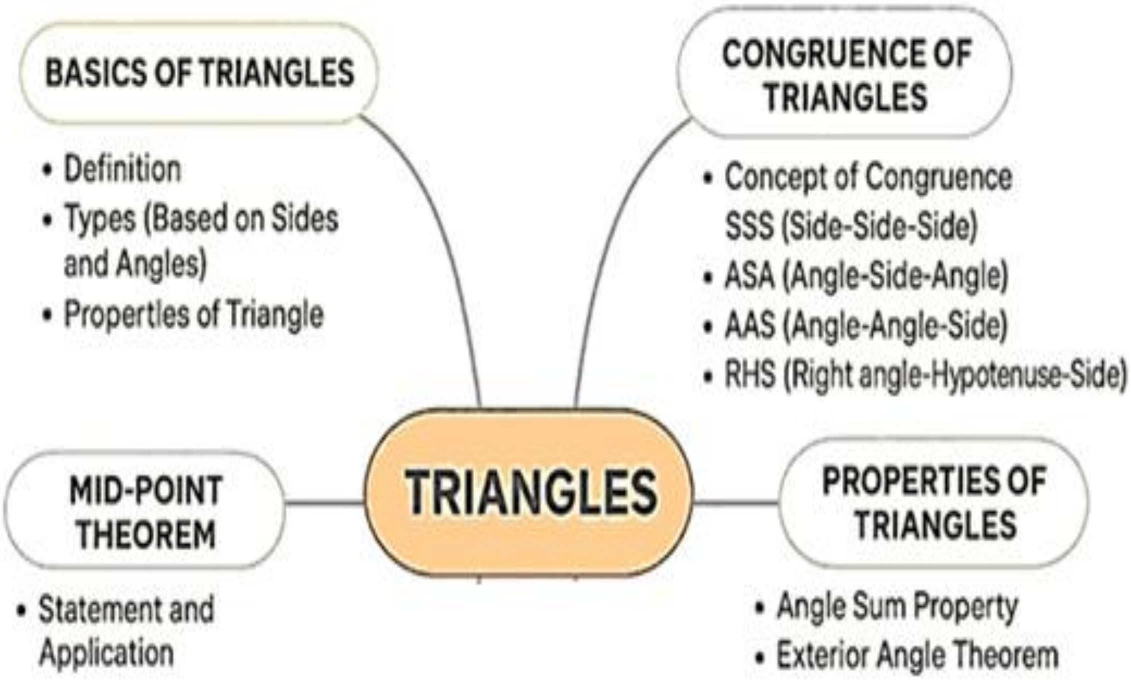
Answer: (i) ∠FOE = ∠COB = ∠DOE = 30°
(ii) ∠AOD, ∠BOE, ∠COF, ∠DOG,
(iii) ∠AOB, ∠BOD; ∠AOC, ∠COD; ∠BOC, ∠COE
(iv) ∠AOB, ∠BOG; ∠AOC, ∠COG; ∠AOD, ∠DOG
(v) ∠BOC, ∠COD; ∠COD, ∠DOE; ∠DOE, ∠EOF.
2..Answer. Try to prove.
3..Answer. Try to prove.
4..Answer. Try to prove.
5..Answer. Try to prove.

CASE BASED QUESTIONS

- ✅ 1.30°,Angles on the same side of a parallelogram are supplementary., 2 km, 45°, 30°
- ✅ 2.∠BOD=300, ∠AOD=1500, Ans. Intersecting lines property is used which states, that, "Two lines are said to be intersecting when the perpendicular distance between the two lines is not same everywhere. They meet at one point.",
- ✅ 3.∠QPR=450.∠PRQ=650.In this case exterior angle property is used which states that, "An exterior angle of a triangle is equal to the sum of interior opposite angles."

CHAPTER – 7
TRIANGLES

MIND MAPPING



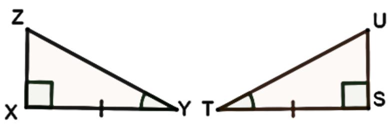
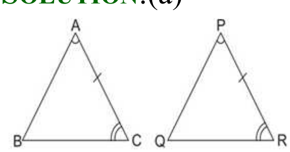
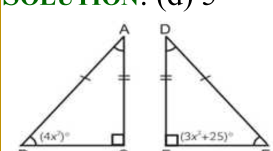
GIST/SUMMARY OF THE LESSON:

- Criteria for congruence of Triangles.
- Properties of Triangles.

DEFINITIONS AND FORMULAE:

Criteria for congruence of Triangles:

- **CONGRUENCE OF TRIANGLE**: The geometrical figures of same shape and size are congruent to each other i.e., two triangles ΔABC and ΔPQR are congruent if and only if their corresponding sides and the corresponding angles are equal. If two triangles ΔABC and ΔPQR are congruent under the correspondence A P, B Q and C R, then symbolically it is expressed as ΔABC ≅ ΔPQR
- **SAS CONGRUENCE RULE**: Two triangles are congruent if two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle.
- **ASA CONGRUENCE RULE**: Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.
- **AAS CONGRUENCE RULE**: Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.
- **SSS CONGRUENCE RULE**: If three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- **RHS CONGRUENCE RULE**: If in two right triangles, the hypotenuse and one side of a triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.
- Properties of Triangles:
  - ✓ (i)A triangle is isosceles if it's any two sides are equal. Here, we will discuss some properties related to isosceles triangle.
  - ✓ Angles opposite to equal sides of a triangle are equal. In figure, ∠B=∠C
  - ✓ (ii) The sides opposite to equal angles of a triangle are equal. In figure, AB=AC
  - ✓ In an isosceles triangle, bisector of the vertical angle of a triangle bisect the base.
  - ✓ The medians of an equilateral triangle are equal in length.
  - ✓ A point equidistant from two intersecting lines lies on the bisector of the angles formed by the two lines.

	MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)	
1	If ΔACB≅ΔEDF, then which of the following equation sis/are true? (I)AC=ED (II) ∠C=∠F (III)AB=EF (a) Only (I) (b) (I) and (III) (c) (II) and (III) (d)All of these	**SOLUTION**: (b)
2	Since, ΔACB≅ΔEDF. ∴AC=ED,CB=DF and AB=EF ,And ∠A=∠E,∠C=∠D and ∠B=∠F Therefore, equations(I)and(III)are true. In a triangle (as shown in fig).AB=CD, AD=BC and AC is the angle bisector of ∠A, then which among the following conditions is true for congruence of ΔABC and ΔCDA by SAS rule? (a) ∠A = ∠D (b) ∠B = ∠A (c) ∠B = ∠D (d)∠C=∠A	**SOLUTION**: (c)
3	As in ΔABC and ΔCDA, AB = CD and AD = BC For SAS Rule, if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, then triangles are congruent. Therefore, for ΔABC ≅ ΔCDA by SAS, ∠B must be equal to ∠D. If AB = QR, BC = PR and CA = PQ in ΔABC and ΔPQR, then: (a) ΔABC ≅ ΔPQR (b) ΔCBA ≅ ΔPRQ (c) ΔBAC ≅ ΔRPQ (d) ΔBCA ≅ ΔPQR	**SOLUTION**:(b)
4	Consider the triangles shown in the figure. Which of these is not true about the given triangles?  (a) ΔXYZ ≅ ΔSTU (by SSS congruence rule) ; (b) ΔXYZ ≅ ΔSTU (RHS congruence) (c) ΔXYZ ≅ ΔSTU (by ASA congruence rule) ; (d) ΔXYZ ≅ ΔSTU (SAS congruence)	**SOLUTION**:(c)
5	If ΔABC ≅ ΔPQR and ΔABC is not congruent to ΔRPQ, then which of the following is not true? (a) BC = PQ (b) AC = PR (c) QR = BC (d) AB = PQ	**SOLUTION**:(a) 
6	Given, ΔABC ≅ ΔPQR Thus, the corresponding sides are equal. Hence, AB = PQ, BC = QR and AC = PR. Therefore, BC = PQ is not true for the triangles. ΔLMN is an isosceles triangle such that LM = LN and ∠N = 65°. The value of ∠L is: (a) ∠L = 55° (b) ∠L = 45° (c) ∠L = 50° (d) ∠L = 65°	**SOLUTION**: (c)
7	Ritish wants to prove that ΔFGH ≅ ΔJKL using SAS rule. He knows that FG = JK and FH= JL. What additional piece of information does he need? (a) ∠F = ∠J (b) ∠H = ∠L (c) ∠G = ∠K (d) ∠F = ∠G **Solution: (a) ∠F = ∠J** We know for SAS, if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, then the triangles are congruent. So, ∠F = ∠J	**SOLUTION**: (a)
8	In the given figure, ΔABC ≅ ΔDEF by SAS congruence rule. The value of ∠x is: (a) 75° (b) 105° (c) 125° (d) 5°	**SOLUTION**: (d) 5° 
9	In ΔABC and ΔDEF,∠C=∠F, AB = DE∴ AC = DF, ΔABC≅ΔDEF[By RHS rule] ∠B = ∠E [By CPCT]⇒$(4x^2)°=(3x^2+25)°$⇒$x^2=25$⇒x= 5°	**SOLUTION**: (c) 4 cm

In ΔABC, ∠C = ∠A and BC = 4 cm and AC = 5 cm, then find length of AB.

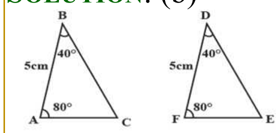
(a) 5 cm (b) 3 cm (c) 4 cm (d) 2.5 cm

10 It is given that ΔABC ≅ ΔFDE and AB = 5 cm, ∠B = 40° and ∠A = 80°. Then which of the following is true?(Two triangles are congruent if their corresponding sides are equal in length and their corresponding angles are equal in size.)

(a) DF = 5 cm, ∠F = 60° (b) DF = 5 cm, ∠E = 60°

(c) DE = 5 cm, ∠E = 60° (d) DE = 5 cm, ∠D = 40°

SOLUTION: (b)



ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R).Pick the correct option:

Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).

Assertion (A) is true but Reason(R) is false.

Assertion (A) is false but Reason(R) is true.

1 Assertion (A): If we draw two triangles with angles 40°, 60° and 80° and the length of the sides of one triangle be different than that of the corresponding sides of the other triangle, then two triangles are not congruent.

Reason (R):If two triangles are constructed which have all corresponding angles equal but have unequal corresponding sides, then two triangles cannot be congruent to each other.

ANSWER: (a) Assertion (A): There is no rule of congruency when both triangles have equal corresponding angle but sides are different. So, these two triangles are not congruent.

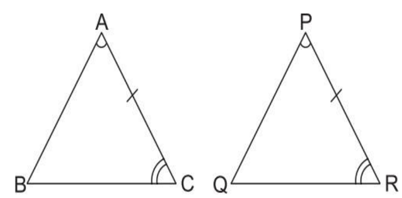
Therefore, Assertion (A) is true. Reason (R): It is also true.

2 Assertion (A): In triangles ABC and PQR, if ∠A= ∠P, ∠C = ∠R and AC = PR, then the two triangles are congruent by ASA congruence.

Reason (R):If two angles and included side of a triangle are equal to the corresponding angles and side of the other triangle, then the triangles are congruent by ASA congruence.

ANSWER: (a) Assertion(A) It is true to say that two angles and the included sides of two triangles are equal then they are congruent by ASA rule.

Reason(R): It is also true.



3 Assertion (A):In Δ PQR, PQ = QR and ∠R = 75°, then ∠P is 52°.

Reason (R):In a triangle, angles opposite to equal sides are equal.

ANSWER: (d) Assertion(A): in ΔPQR, PQ=QR, ∠R =∠P =75°, (Angles opposite to equal sides are equal).

Reason (R): It is true to say that angles opposite to equal sides of triangle are equal.

4 Assertion (A): In Δ ABC, P is the mid-point of BC. If PQ ⊥ AB and PR ⊥ AC, such that PQ = PR, then BQ = CR.

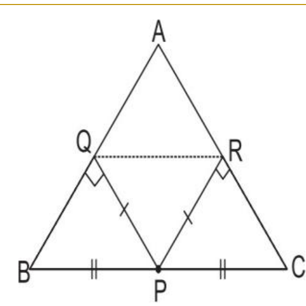
Reason (R): If two angles and the included side of one triangle are equal to two angles and the included side of other triangle, then two triangles are congruent by ASA rule.

ANSWER: (b)Assertion (A): in ΔPQB and ΔPRC,BP=PC (P is the mid -point of BC) ;

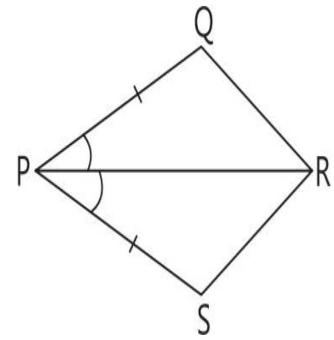
∠Q=∠R=90^0 (Angles opposite to equal sides are equal)And PQ=PR (Given) ;

ΔPQR ≅ ΔPRC (By RHS congruence)BQ=CR (By CPCT)

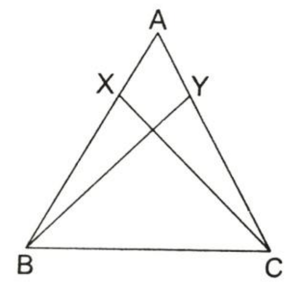
Reason (R):It is also true.



5 Assertion (A):In a quadrilateral PQRS, PQ = PS and PR bisects ∠P by SAS congruence rule.
Reason (R):Two triangles are congruent if one hypotenuse side and one of the perpendicular side of triangle is equal to the corresponding one hypotenuse side and one of the perpendicular side of the other triangle.
Answer :(b)Assertion (A):in ΔPQR and ΔPSR,
PQ=PS (Given) ;∠RPQ=∠RPS (PR bisects ∠P) ;And PR=PR (Common)
ΔPQR ≅ ΔPSR (By ASA congruence) ;So, Assertion (A) is true. ;
Reason (R): It is also true.



6 Assertion (A):In the adjoining figure ,X and Yare respectively two points one qual sides AB and AC of ΔABC such that AX = AY then CX = BY.
Reason (R) : If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent.
Answer. (a) We know that "If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent" - This is SAS Congruence Rule.



7 Assertion (A): Angles opposite to equal sides of a triangle are not equal.
Reason (R): Sides opposite to equal angles of a triangle are equal.
Answer. (d) We know that Angles opposite to equal sides of a triangle are equal.

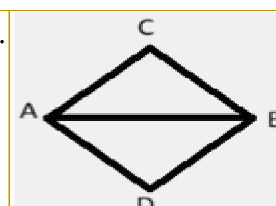
8 Assertion (A): ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at E, then ΔABD ≅ ΔACD
Reason (R): If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and one side of other triangle, then the two triangles are congruent.
Answer. (b) We know that "If in two right triangles, hypotenuse and one side of a triangle are equal to the hypotenuse and one side of other triangle, then the two triangles are congruent." – This is RHS Congruence Rule
So, Reason is correct.

9 Assertion (A): In ΔABC and ΔPQR, AB = PQ, AC = PR and ∠BAC= ∠QPR then ΔABC≅ ΔPQR
Reason (R): Both the triangles are congruent by SSS congruence.
Answer :(c) In ΔABC and ΔPQR,AB = PQ, AC = PR and ∠BAC = ∠QPR (Given)
then ΔABC ≅ ΔPQR by ASA Congruence Rule.

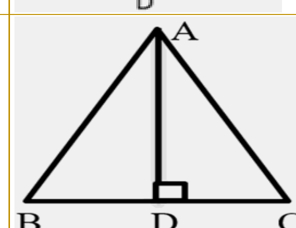
10 Assertion (A): In ABC, ∠A=∠C and BC=4 cm and AC = 3 cm then the length of side AB = 3 cm.
Reason (R): Sides opposite to equal angles of a triangle are equal.
Answer(d) We know that Sides opposite to equal angles of a triangle are equal. So, Reason is correct.

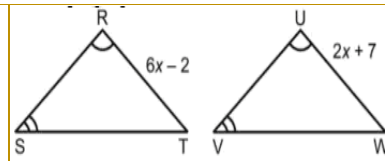
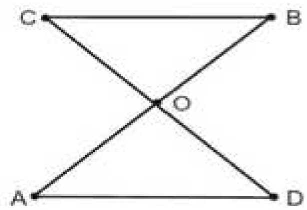
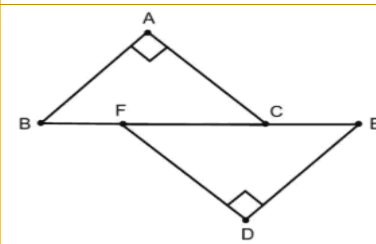
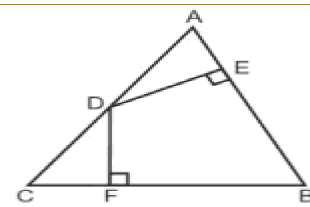
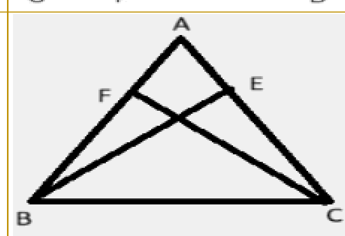
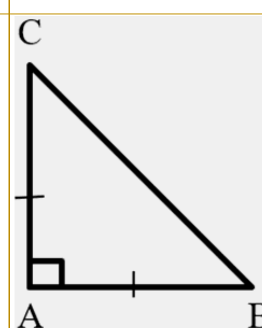
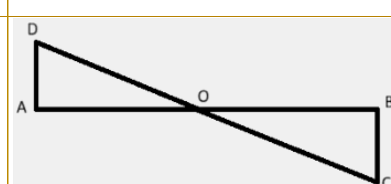
VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

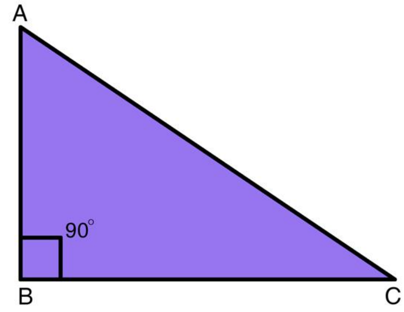
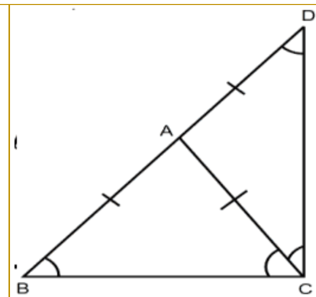
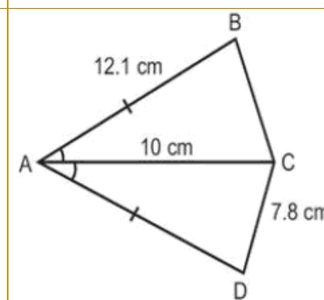
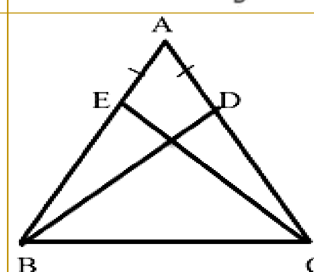
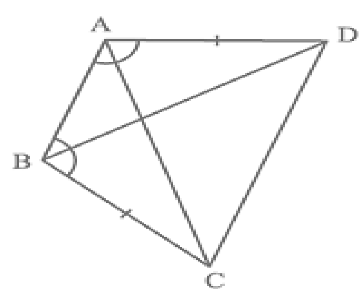
1 In quadrilateral ACBD, AC=AD and AB bisects ∠A. show that ΔABC≅ΔABD. What can you say about BC and BD?
Solution: In ΔABC and ΔABD, AC=AD [given];∠CAB=∠DAB [AB bisects ∠A] ; B=AB [common]
ΔABC≅ΔABD [SAS criterion] ; BC=BD [CPCT]



2 In ΔABC, the median AD is to BC. Prove that ΔABC is an isosceles triangle.
Solution: In ΔABD and ΔACD
BD =CD [D is mid-point of BC]
AD=AD [Common]
∠ ADB=∠ ADC [each 90^0]
ΔABD ≅ ΔACD [By SAS]
AB=AC [CPCT] Hence, triangle ABC is an isosceles triangle.



3	In $\Delta RST, RT=6x-2$. In $\Delta UVW, UW=2x+7, \angle R=\angle U$, and $\angle S=\angle V$. What must be the value of x in order to prove that $\Delta RST \cong \Delta UVW$? **SOLUTION**: Given that $\angle S=\angle V$ and $\angle R=\angle U$ $\angle T=\angle W$(by Angle sum property of triangle) For $\Delta RST \cong \Delta UVW, RT=UW$ using either ASA or AAS congruence rule $\Rightarrow 6x-2=2x+7 \Rightarrow 6x-2x=9 \Rightarrow 4x=9 \Rightarrow x=9/4 \Rightarrow x=2.25$		
4	In the given figure two lines AB and CD intersect each other at the point O such that BC||AD and BC = DA. Show that O is the midpoint of both the line-segment AB and CD. **SOLUTION**: BC || AD [Given]Therefore $\angle CBO = \angle DAO$ [Alternate interior angles];And $\angle BCO = \angle ADO$ [Alternate interior angles] Also, BC = DA[Given] So, $\Delta BOC \cong \Delta AOD$ [ASA congruence rule] ;Therefore, OB = OA and OC = OD, i.e. O is the mid-point of both AB and CD.		
5	In figure $BA \perp AC$, $DE \perp DF$. Such that BA=DE and BF=EC. Show that $\Delta ABC \cong \Delta DEF$. **SOLUTION**: According to the question, $BA \perp AC$, $DE \perp DF$ Such that BA = DE and BF =EC. In, ΔABC and ΔDEF ;BA = ED [Given];BF = EC [Given] ;$\angle A = \angle D$ [Both 90°];Now, BF = EC [Given] ;$\Rightarrow$ BF + FC = EC + FC $\Rightarrow$ BC = EF$\therefore \Delta ABC \cong \Delta DEF$ [By RHS Congruence Rule]		
6	In ΔABC, D is a point on side AC such that DE = DF and AD = CD and $DE \perp AB$ at E and $DF \perp CB$ at F, then prove that AB = BC. **SOLUTION**: In ΔAED and ΔCFD,AD = CD DE = DF ;$\Delta AED \cong \Delta CFD$ [By RHS congruence rule] ;$\angle A = \angle C$;$\therefore$ AB = BC [Sides opposite to equal angles are equal]		
7	ABC is an isosceles triangle in which altitudes BE and CF are drawn to sides AC and AB respectively (See figure). Show that these altitudes are equal. **SOLUTION**: In ΔABE and ΔACF, $\angle A = \angle A$ [Common];$\angle AEB = \angle AFC = 90^0$[Given] AB = AC [Given];$\Delta ABC \cong \Delta ACF$ [By ASA congruency] $\Rightarrow$ BE = CF [By C.P.C.T.] Altitudes are equal.		
8	ABC is a right angled triangle in which $\angle A = 90^0$ and AB = AC. Find $\angle B$ and $\angle C$. **SOLUTION**: ABC is a right triangle in which, $\angle A = 90^0$ And AB = AC In ΔABC,AB = AC $\angle C = \angle B$ ……….(i) We know that, in ΔABC, $\angle A + \angle B + \angle C = 180^0$ [Angle sum property] $\Rightarrow 90^0 + \angle B + \angle B = 180^0$ [$\angle A = 90^0$ (given) and $\angle B = \angle C$ (from eq. (i)] $\Rightarrow 2\angle B = 90^0$, therefore $\angle B = 45$ Also $\angle C = 45^0$ [$\angle B = \angle C$]		
9	If DA and CB are equal perpendiculars to a line segment AB. Show that CD bisects AB. **SOLUTION**: In ΔAOD and ΔBOC,AD=BC ……….. (Given) [$\angle A = \angle B$………each 90^0] ;And $\angle AOD = \angle BOC$ (vert opp. Angles $\Delta AOD \cong \Delta BOC$ (AAS rule) ;OA=OB(CPCT)Hence CD bisects AB.		

10	Show that in a right angles triangle, the hypotenuse is the longest side. **SOLUTION**: Given: Let ABC be a right angled triangle, right angled at B. To prove: Hypotenuse AC is the longest side. Proof: In right angled triangle ABC, $\angle A + \angle B + \angle C = 180^0$ $\angle A + 90^0 + \angle C = 180^0$ [$\angle B = 90^0$] ; $\angle A + \angle C = 180^0 - 90^0$ And $\angle B = 90^0$; $\angle B > \angle C$ and $\angle B > \angle A$;Since the greater angle has a longer side opposite to it.AC > AB and AC > AB Therefore $\angle B$ being the greatest angle has the longest opposite side AC, i.e. hypotenuse.	
	SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)	
1	ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that ΔBCD is a right angle triangle. **SOLUTION**: In ΔABC: AB = AC Also, given: AD = AB ; $\Longrightarrow$ AC = AB = AD ;In ΔABC: AB = AC ; $\Longrightarrow \angle ACB = \angle ABC = \angle 1$ … (i) [Angles opposite to equal sides are equal] In ΔACD: AC = AD ; $\Longrightarrow \angle ADC = \angle ACD = \angle 2$ … (ii) [Same reason] Now in ΔBCD: $\angle DBC + \angle BCD + \angle BDC = 180°$ … (iii) [Angle sum property] Substituting values: $\angle 1 + \angle 1 + \angle 2 + \angle 2 = 180°$; $\Longrightarrow 2(\angle 1 + \angle 2) = 180°$; $\Longrightarrow \angle 1 + \angle 2 = 90°$; $\Longrightarrow \angle BCD = 90°$	
2	Find the perimeter of the quadrilateral ABCD, if $\angle CAB = \angle CAD$ and also AB = AD. **SOLUTION**: Since AB = AD [Given]; $\therefore$ AD = 12.1 cm [AB = 12.1cm] Now, In ΔABC and ΔADC;AB = AD [Given]; $\angle BAC = \angle DAC$[Given] AC = AC [Given] ; $\therefore \Delta ABC \cong \Delta ADC$ [By SAS congruence rule] Hence BC = DC [By CPCT] ;BC = 7.8 cm Now, we have to calculate the perimeter of quadrilateral. Perimeter = AB + BC + CD + AD = 12.1 + 7.8 + 7.8 + 12.1 = 3	
3	ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE. **SOLUTION**: Given: AB = AC ;Also, BD and CE are two medians. $\therefore$ E is the mid-point of AB, D is the mid-point of AC ; $\Longrightarrow$ ½ AB = ½ AC $\Longrightarrow$ BE = CD In ΔBEC and ΔCDB:BE = DC [Given] $\angle EBC = \angle DCB$ [Angles opposite to equal sides are equal] BC = BC [Common side] $\Longrightarrow \Delta BEC \cong \Delta CDB$[By SAS congruence rule] $\Longrightarrow$ BD = CE [By CPCT]	
4	ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$. (See figure). Prove that:(i) $\Delta ABD \cong \Delta BAC$(ii) BD = AC (iii) $\angle ABD = \angle BAC$ **SOLUTION**: (i) In ΔABC and ΔABD, BC = AD [Given] ; $\angle DAB = \angle CBA$ [Given]AB = AB [Common] ; $\Delta ABC \cong \Delta ABD$ [By SAS congruency] Thus AC = BD [By C.P.C.T.] (ii) Since ΔABC ΔABDAC = BD [By C.P.C.T.] (iii) Since $\Delta ABC \cong \Delta ABD$ $\therefore \angle ABD = \angle BAC$ [By C.P.C.T.]	
5	Show that the angles of an equilateral triangle are 60° each. **SOLUTION**: Let ABC be an equilateral triangle. AB = BC = AC	

AB = BC

∠C = ∠A……….(i)

Similarly, AB = AC

⇒ ∠C = ∠B……….(ii)

From eq. (i) and (ii),

∠A = ∠B = ∠C……….(iii)

Now in ΔABC

∠A + ∠B + ∠C = 180^0……….(iv)

∠A + ∠A + ∠A = 180^0

3∠A = 180^0

⇒ ∠A = 60^0

Since ∠A = ∠B = ∠C [From eq. (iii)]

∴∠A = ∠B = ∠C = 60^0

Hence each angle of equilateral triangle is 60^0.

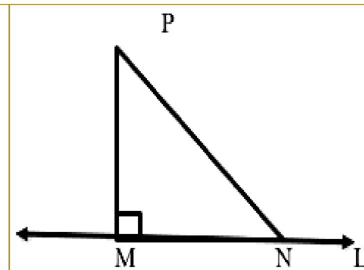
6 Show that all the line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

SOLUTION: Given: L is a line and P is point not lying on L. PM⊥ L

N is any point on L other than M. To prove: PM <PN Proof: In ΔPMN, ∠M is the right angle. N is an acute angle. (Angle sum property of Δ) ∴∠M >∠N

∴PN> PM [Side opposite greater angle] ⇒ PM <PN

Hence of all line segments drawn from a given point not on it, the perpendicular is the shortest.



7 In a huge park, people are concentrated at three points (See figure).

A: where there are different slides and swings for children.

B: near which a man-made lake is situated.

C: which is near to a large parking and exit.

D:Where should an ice cream parlour be set up so that maximum number of persons can approach it?

SOLUTION: The parlour should be equidistant from A, B and C.

For this let we draw perpendicular bisector say L of line joining points B and C also draw perpendicular bisector say M of line joining points A and C.

Let L and M intersect each other at point O. Now point O is equidistant from points A, B and C. Join OA, OB and OC.

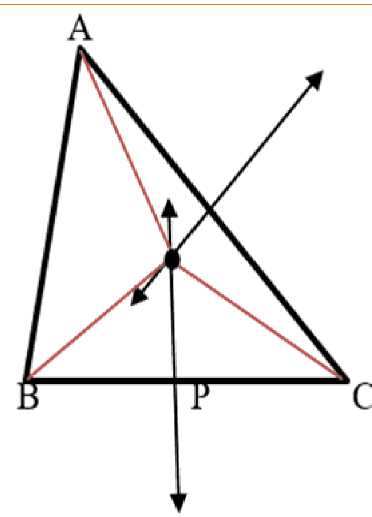
Proof: In ΔBOP and ΔCOP, OP = OP[Common]; ∠OPB = ∠OPC = 900

BP = PC [P is the mid-point of BC] ;∴ΔBOP ≅ ΔCOP [By SAS congruency]

⇒ OB = OC [By C.P.C.T.]…..(i) ; Similarly, ΔAOQ ≅ ΔCOQ

⇒ OA = OC [By C.P.C.T.]…..(ii) ; From eq. (i) and (ii),

OA = OB = OC . Therefore, ice cream parlour should be set up at point O, the point of intersection of perpendicular bisectors of any two sides out of three formed by joining these points.



8 Prove that if one angle of a triangle is equal to the sum of the other two angles, the triangle is right angled:

SOLUTION:∠A + ∠B + ∠C = 180^0 Sum of three angles of triangle is 180^0] …..(1)

Given that: ∠A + ∠C = ∠B…..(2);From (1) and (2) ;∠B + ∠B = 180^0 ;∠B=90^0

Hence ΔABC is right angled.

9 Prove that the Angle opposite to the greatest side of a triangle is greater than two- third of a right angle i.e. greater than 60^0.

SOLUTION: In ΔABCAB > BC [Given];∠C >∠A [angle opposite to large side is greater]….(i)

Similarly, AB > AC ; ∠C >∠B………(ii) ; Adding (i) and (ii)2∠C >∠A + ∠B

Adding ∠C to both sides,3∠C >∠A + ∠B + ∠C ; Δ3∠C > 180^0 [Sum of three angles of Δ is 180^0]

Or ∠C > 60^0.

LONGANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

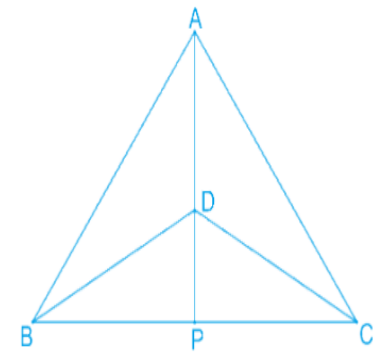
1 ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:

(i)ΔABD≅ ΔACD

(ii) ΔABP≅ΔACP

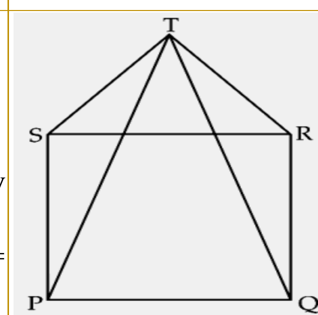
(iii) AP bisects ∠A as well as ∠D.

(iv) AP is the perpendicular bisector of BC.



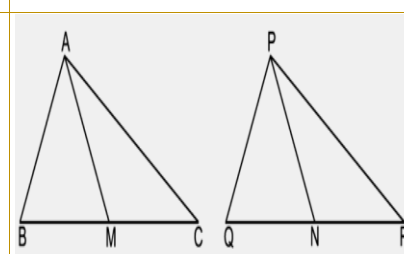
SOLUTION: i)ΔABC is an isosceles triangle. ∴ AB = AC ;
ΔDBC is an isosceles triangle. ∴ BD = CD ;Now in ΔABD and ΔACD,AB = AC[Given] ;BD = CD[Given] ;AD = AD[Common]
∴ΔABD ≅ΔACD[By SSS congruency] ;⇒ ∠BAD = ∠CAD[By C.P.C.T.]……….(i)
(ii)Now in ΔABP and ΔACP,AB = AC[Given] ;∠BAD = ∠CAD[From eq. (i)] AP = AP ; ∴ ΔABP≅ΔACP[By SAS congruency]
(iii)Since ΔABP≅ΔACP[From part (ii)] ;⇒ ∠BAP = ∠CAP[By C.P.C.T.]
⇒ AP bisects ∠A. ;Since ΔABD ≅ΔACD[From part (i)]
⇒ ∠ADB = ∠ADC[By C.P.C.T.]……….(ii)
Now ∠ADB + ∠BDP = 1800[Linear pair]……….(iii)
And ∠ADC + ∠CDP = 1800[Linear pair]……….(iv)
From eq. (iii) and (iv),∠ADB + ∠BDP = ∠ADC + ∠CDP
⇒ ∠ADB + ∠BDP = ∠ADB + ∠CDP[Using (ii)] ;⇒ ∠BDP = ∠CDP
⇒ DP bisects ∠D or AP bisects ∠D.
(iv) Since ΔABP≅ΔACP[From part (ii)] ; ∴ BP = PC[By C.P.C.T.]……….(v)
And ∠APB = ∠APC[By C.P.C.T.]……….(vi) ;
Now ∠APB + ∠APC = 1800[Linear pair] ;⇒ ∠APB + ∠APC = 1800[
⇒ 2∠APB = 1800 ;⇒ ∠APB = 900
⇒ AP ⊥ BC……….(vii)
From eq. (v), we have BP = PC and from (vii), we have proved AP ⊥ B. So, collectively AP is perpendicular bisector of BC.

2 In figure, PQRS is a square and SRT is an equilateral triangle; Prove that: (i) PT = QT; (ii) ∠TQR = 15°;



SOLUTION: PQRS is a square (Given); (i) SRT is an equilateral triangle (Given); ∴∠PSR = 90°, ∠TSR = 60°; ⇒∠PSR + ∠TSR = 150°; similarly, ∠QRT = 150°; In ΔPST and ΔQRT, we have PS = QR, ∠PST = ∠QRT = 150°, and ST = RT; By SAS, ΔPST ≅ ΔQRT; ⇒ PT = QT (CPCT); Hence Proved.
(ii) In ΔTQR, QR = RT (square and equilateral triangle on same base); ∴∠TQR = ∠QTR = x;∴x+x+∠QRT=180° ⇒2x+150°= 180°⇒2x=30° ∴x=15°. ⇒∠TQR=15°.

3 In the below figure, two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR; Show that ΔABC ≅ ΔPQR;



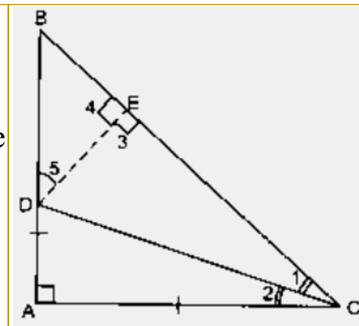
SOLUTION: In ΔABC and ΔPQR, BC = QR (Given); ⇒ ½ BC = ½ QR; ⇒ BM = QN; In triangles ABM and PQN, we have AB = PQ (Given); BM = QN (Proved above); AM = PN (Given); ∴ ΔABM ≅ ΔPQN (By SSS congruence criterion); ⇒∠B = ∠Q (By CPCT);Now, in triangles ABC and PQR, we have AB = PQ (Given); ∠B = ∠Q (Proved above); BC = QR (Given); ∴ ΔABC ≅ ΔPQR (By SAS congruence criterion);

4 Prove that if in two triangles, two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent.

SOLUTION: Given: Two triangles △ABC and △DEF such that ∠B = ∠E, ∠C = ∠F, and BC = EF;

To prove: △ABC ≅△DEF;
Proof: We prove the result in the following cases;
Case I: If AB = DE, then in △ABC and △DEF, we have AB = DE [by supposition]; BC = EF [given]; ∠B = ∠E [given]; Thus, △ABC ≅△DEF [SAS criterion];
Case II: If AB < DE, take a point G on ED such that EG = AB; join GF;
In △ABC and △GEF, we have AB = GE [by supposition]; BC = EF [given]; ∠B = ∠E [given];
Thus, △ABC ≅△GEF [SAS criterion]; ∠ACB = ∠GFE [CPCT]; But ∠ACB = ∠DFE [given];
Therefore, ∠GFE = ∠DFE;

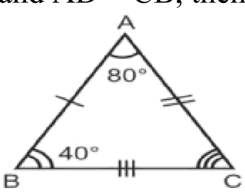
5 ABC is a right-angled triangle such that AB = AC and bisector of angle C intersects the side AB at D;
Prove: AC + AD = BC;

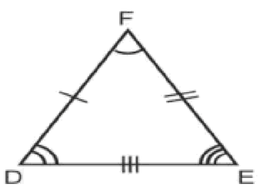


Solution: Given: ΔABC is right-angled such that AB = AC and CD is the angle bisector of ∠C; Construction: Draw DE ⊥ BC;
Proof: In right-angled triangle ABC, we have AB = AC [Given]; ∴ BC is hypotenuse (since hypotenuse is the largest side); ⇒∠A = 90° (angle opposite hypotenuse is 90°); In ΔDAC and ΔDEC, we have ∠A = ∠3 = 90°; ∠1 = ∠2 [Since CD is angle bisector of ∠C]; DC = DC [Common side];
So, by AAS criterion, ΔDAC ≅ ΔDEC; ∴ DA = DE ...(1) [CPCT]; and CA = CE ...(2) [CPCT]; In ΔBAC, AB = AC [Given] ⇒∠C = ∠B [Angles opposite equal sides are equal]; Now, ∠A + ∠B + ∠C = 180° [Angle sum property]; ⇒ 90° + ∠B + ∠B = 180° ⇒ 2∠B = 90° ⇒∠B = 45°; Now, in ΔBED, ∠4 + ∠5 + ∠B = 180°; ⇒ 90° + ∠5 + 45° = 180° ⇒∠5 = 45° ⇒∠B = ∠5;
⇒ DE = BE ...(3) [Sides opposite equal angles];
From (1) and (3), DA = DE = BE ...(4); Now, BC = CE + BE ⇒ BC = AC + AD [From (2), (3), and (4)]; ∴ AC + AD = BC; Hence proved;

CASE BASED QUESTIONS (04 MARKS QUUESTIONS)

1 Truss bridges are formed with a structure of connected elements that form triangular structures to make up the bridge; Trusses are the triangles that connect to the top and bottom cord and two end posts; You can see that some triangular shapes are shown in the picture and are represented as ΔABC, ΔCAD, and ΔBEA; If AB = CD and AD = CB, then prove ΔABC ≅ ΔCDA .

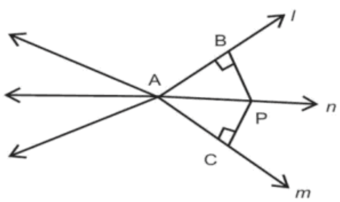


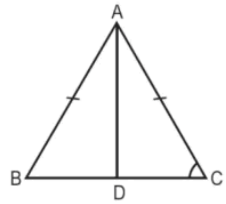


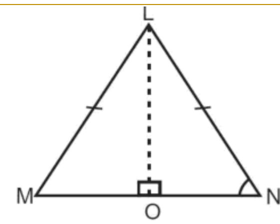
Solution(a): In ΔABC and ΔCDA, AB = CD [Given]; AD = CB [Given]; AC = CA [Common]; So, by SSS congruence rule, ΔABC ≅ ΔCDA;

(b) If AB = 7.5 m, AC = 4.5 m, and BC = 5 m, find the perimeter of ΔACD if ΔABC ≅ ΔCDA Perimeter of ΔABC = Perimeter of ΔCDA = (7.5 + 4.5 + 5) m = 17 m;
(c) If ΔABC ≅ ΔFDE, AB = 5 cm, ∠B = 40°, and ∠A = 80°, then find the length of DF and ∠E Given: ΔABC ≅ ΔFDE; AB = 5 cm; ∠B = 40°; ∠A = 80°;
Since ΔFDE ≅ ΔABC, DF = AB = 5 cm [By CPCT];
∠E = ∠C = 180° – (∠A + ∠B) = 180° – (80° + 40°) = 60°;
Hence, DF = 5 cm; ∠E = 60°;

2 To check the understanding of the students of Class IX on triangles, the Mathematics teacher wrote some questions on the blackboard and asked the students to read them carefully and answer the following:







SOLUTION:(a) In the figure, P is a point equidistant from the lines l and m intersecting at point A;
Let us consider ΔPAB and ΔPAC; Here, PB = PC [Perpendicular distances]; ∠PBA = ∠PCA = 90°; PA = PA [Common];
So, ΔPAB ≅ ΔPAC [By RHS congruence rule]; ⇒∠BAP = ∠CAP [By CPCT];
(b) In ΔABC, if AB = AC and BD = DC; We have AB = AC, BD = CD, and AD = AD [Common];
So, ΔABD ≅ ΔACD [By SSS congruence rule]; ⇒∠ADB = ∠ADC [By CPCT];
Since BDC is a straight line, ∠ADB + ∠ADC = 180° ⇒ 2∠ADC = 180° ⇒∠ADC = 90°;
(c) ΔLMN is an isosceles triangle where LM = LN and LO is the angle bisector of ∠MLN;
Given LM = LN ⇒∠M = ∠N ...(i); Also, ∠MLO = ∠NLO ...(ii);
In ΔMLO and ΔNLO: ∠OML = ∠ONL; LM = LN; ∠MLO = ∠NLO;
So, ΔMLO ≅ ΔNLO [By ASA congruence rule]; ⇒ OM = ON [By CPCT];
Hence, point O is the midpoint of side MN.

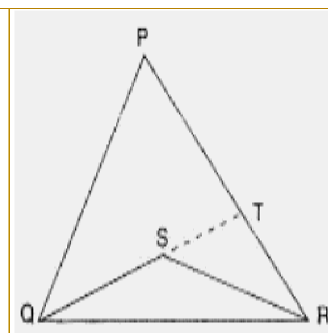
3 Ashok is studying in 9th class in Govt School, Chhatarpur. Once he was at his home and was doing his geometry homework. He was trying to measure three angles of a triangle using the Dee, but his dee was old and his Dee's numbers were erased and the lines on the dee were visible. Let us help Ashok to find the angles of the triangle. He found that the second angle of the triangle was three times as large as the first. The measure of the third angle is double of the first angle. Now answer the following questions:

- ? What was the value of the first angle?
- ? What was the value of the third angle?
- ? OR What was the value of the second angle?
- ? What was the sum of all three angles measured by Ashok using Dee?

ANSWER Key:
- ✅ 30°
- ✅ 60° OR 90°
- ✅ 180°

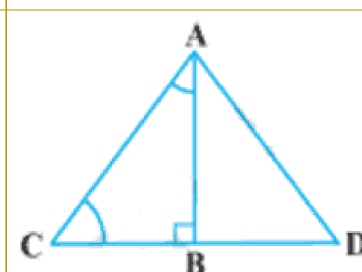
HOTS

1 S is any point in the interior of △ PQR. Show that SQ + SR < PQ + PR.



SOLUTION: Produce QS to meet PR in T In △PQT,
PQ + PT > QT ...[As the sum of any two sides of a triangle is greater than the third side]
PQ + PT > QS + ST ...[As QT = QS + ST] ...(1)
In △SRT,TR + ST > SR ...[As the sum of any two sides of a triangle is greater than the third side] ...(2)
Adding (1) and (2)
PQ + PT + TR + ST > QS = ST + SR ; ∴ PQ + PR + ST > QS + ST + SR
∴ PQ + PR > QS + SR ;∴ SQ + SR < PQ + PR.

2 In Fig., ABC is a right triangle and right-angled at B such that ∠BCA = 2∠BAC. Show that hypotenuse AC = 2BC

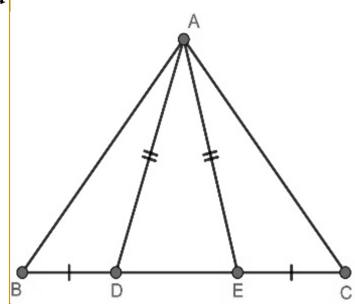


SOLUTION: Produce CB to a point D such that BC = BD and join AD.
In △ABC and △ABD, we have BC = BD (By construction)
AB = AB (Same side) ; ∠ABC = ∠ABD (Each of 90°)
Therefore, △ ABC ≅△ABD (SAS) ; So, ∠CAB = ∠DAB ....(1)
and AC = AD......(2)

Thus, ∠CAD = ∠CAB + ∠BAD = x + x = 2x [From (1)] ....(3)
and ∠ACD = ∠ADB = 2x [From (2), AC = AD] ...(4)
That is, △ ACD is an equilateral triangle. [From (3) and(4)]
or AC = CD, i.e., AC = 2 BC (Since BC = BD)

3 If △ABC ≅△FDE and AB =5 cm, ∠B=40° and ∠A = 80°. Then which of the following is true?
DF = 5 cm, ∠E = 60° 3. DF = 5 cm, ∠F = 60°
DE = 5 cm, ∠E = 60° 4.DE = 5 cm, ∠D = 40°
SOLUTION: (1) DF = 5 cm, ∠E = 60°Given that: In △ABC, AB = 5 cm, ∠B = 40° and ∠A = 80°
Using angles sum property of triangle, we have ∠A + ∠B + ∠C = 180°
⇒ 80° + 40° + ∠C = 180 ; ⇒ 120° + ∠C = 180° [∵∠B = 40° and ∠A = 80°] ; ⇒ ∠C = 180° – 120°⇒ ∠C = 60°
It is given that △ABC ≅△FDE, so we have AB = FD, BC = DE and AC = FE & ∠A = ∠F, ∠B = ∠D and ∠C = ∠E
⇒ AB = FD = 5cm and ∠C = ∠E = 60°.

4 In the given figure, D and E are points on side BC of a △ABC such that BD = CE and AD = AE. Show that △ABD≅△ACE.
SOLUTION: Given: △ABC in which BD = CE and AD = AE.
To Prove: △ABD ≅ △ACE Proof: In △ADE, we have AD = AE [Given] ⇒∠2=1
[∵ Angle opposite to equal sides of a triangle are equal]
Now, ∠1+∠3=180∘ ...(1) [Linear pair axiom]
∠2+∠4=180∘ …(2) [Linear pair axiom]
From equations (1) and (2), we get∠1+∠3=∠2+∠4⇒∠3=∠4 [∵∠1=∠2]
Now, in △ABD and △ACE, we have AD = AE [Given]∠3=∠4 [Proved above]
BD = CE [Given]So, by SAS criterion of congruence, we have
△ABD ≅ △ACE Hence, proved.

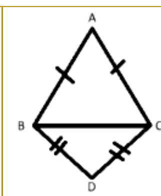
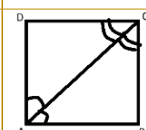
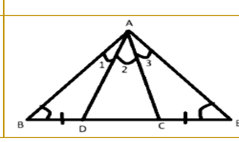


5 ABC is a right angled triangle such that AB = AC and bisector of angle C intersects the side AB at D.
Prove that AC + AD = BC.
SOLUTION: Given: Δ ABC is a right angled such that AB = AC and CD is the angular bisector of ∠C . To prove: AC + AD = BC; Construction: Draw DE⊥BC ;
Proof: In right angled triangle ABC, we have AB = AC. [Given]
∴ BC is hypotenuse (since, hypotenuse is the largest side)
⇒∠A=90∘(angle opposite to hypotenuse is 90°)
In ΔDAC and ΔDEC , we have :-∠A=∠3 [∵ Each Equal to 90º]
∠1=∠2. [Since, CD is angular bisector of ∠C] ; DC = DC. [Common side]
So, by AAS criterion of congruency of triangles, we have :-ΔDAC≅ΔDEC
∴ DA = DE ....(1) [CPCT] ; And, CA = CE…(2) [CPCT]
In ΔBAC , we have :-AB = AC. [Given]⇒∠C=∠B
[∵ Angles opposite to equal sides of a triangle are equal]
Now ,∠A+∠B+∠C=180∘ [Angle sum property of a triangle]
⇒90∘+∠B+∠B=180∘ [∵∠B=∠C] ;⇒2∠B=90∘⇒∠B=90∘2=45∘
Now, in ∠BED we have :-⇒∠4+∠5+∠B=180o [Angle sum property of a triangle]
;⇒90∘+∠5+∠45=180∘ [Since, ∠B. = 45°]⇒∠5=180∘−135∘⇒∠5=45∘∴∠B=∠5
⇒DE=BE …(3) [Since, sides opposite to equal angles of triangle are equal]
From (1) and (3), we get DA = DE = BE…(4)
Now, BC = CE + BE⇒BC=AC+AD. [From (2), (3) & (4)]
⇒AC+AD=BC Hence proved.

EXERCISE

MULTIPLE CHOICE QUESTIONS

1 In ΔABC, ∠B = ∠C and ray AX bisects the exterior angle ∠DAC. If ∠DAX = 70°, then ∠ACB=
(a) 35° (b) 90° (c) 70° (d) 55°

2	In a triangle, an exterior angle at a vertex is 95° and its one of the interior opposite angle is 550, then the measure of the other interior angle is (a) 55° (b) 85° (c) 40° (d) 9.0°	
3	If the sides of a triangle are produced in order, then the sum of the three exterior angles so formed is (a) 90° (b) 180° (c) 270° (d) 360°	
4	In ΔABC, if ∠A = 100°, AD bisects ∠A and AD ⊥ BC. Then, ∠B = (a) 50° (b) 90° (c) 40° (d) 1000	
5	An exterior angle of a triangle is 1080 and its interior opposite angles are in the ratio 4:5. The angles of the triangle are (a) 48°, 60°, 72° (b) 50°, 60°, 70° (c) 52°, 56°, 72° (d) 42°, 60°, 76°	
	ASSERTION - REASON BASED QUESTIONS Each of the following questions contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason) and has the following four choices (a), (b), (c) and (d) only one of which is the correct answer. Mark the correct choice. (a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. (b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. (c) Statement-1 is true, Statement-2 is false. (d) Statement-1 is false, Statement-2 is true	
1	Statement-1 (A): In an equilateral triangle ABC, if AD is the median, then AB + AC > 2 AD Statement-2 (R): In a right triangle, hypotenuse is the longest side.	
2	Statement-1 (A): The sum of any two sides of a triangle is greater than the third side. Statement-2 (R): It is possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm.	
3	Statement-1 (A): IFΔ ABC ≅Δ RPQ, then BC = QR Statement-2 (R): Corresponding parts of two congruent triangle are equal.	
4	Statement-1 (A): If any two angles and a non-included side of one triangle are equal to the corresponding angles and side of the another triangle, then the two triangles are congruent. Statement-2 (R): If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent.	
5	Statement-1 (A): If two sides and the included angle of one triangle are equal to the corresponding sides and the included angle of the other triangle, then the two triangles are congruent. Statement-2 (R):If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles are congruent.	
	VERY SHORT ANSWER TYPE QUESTIONS	
1	In the figure, ΔABC and ΔDBC are two isosceles triangles on the same base BC. Prove that ∠ABD =∠ ACD.	
2	In the figure below, the diagonal AC of quadrilateral ABCD bisects ∠BAD and ∠BCD. Prove that BC = CD.	
3	In figure ∠B = ∠E, BD = CE and ∠1 = ∠2. Show ΔABC ≅ΔAED.	
4	ABC is an isosceles triangle with AB = AC. Draw AP⊥BC. Show that ∠B = ∠	
5	PS is an altitude of an isosceles triangle PQR in which PQ = PR. Show that PS bisects ∠P.	
	SHORT ANSWER TYPE QUESTIONS	
1	PQR is a triangle in which PQ = PR and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.	
2	Ina ΔABC, it is given that AB = AC and the bisectors of ∠B and ∠C intersect at O. If M is a point on BO	

produced, prove that ∠MOC = ∠ABC.

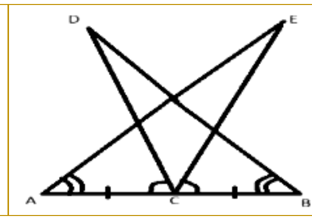
3 Pis a point on the bisector of an ∠ ABC. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.

4 A point O is taken inside an equilateral four sided figure ABCD such that its distances from the angular points D and B are equal. Show that AO and OC are in one and the same straight line.

5 ABCD is a quadrilateral such that AB = AD and CB = CD. Prove that AC is the perpendicular bisector of BD.

LONGANSWER TYPE QUESTIONS

1 In the given figure, if AC = BC, ∠DCA = ∠ECB and ∠DBC = ∠EAC, then prove that BD = AE.



2 ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) D is the mid-point of AC
(ii) MD⊥ AC
(iii) CM = MA = 1/2AB .

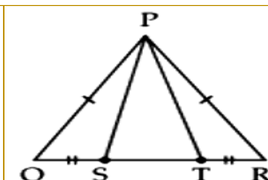
3 ABCD is a square and ABE is an equilateral triangle outside the square prove that∠ACE = $\frac{1}{2}$∠ABE.

4 ABC is a triangle in which ∠B=2∠C. D is a point on BC such that AD bisects ∠BAC and AB=CD. Prove that ∠BAC = 72°.

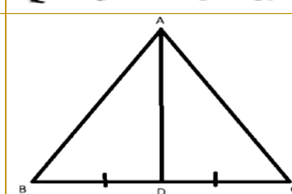
5 ABCD is a parallelogram, if the two diagonals are equal, find the measure of ABC.

CASE BASED QUESTIONS

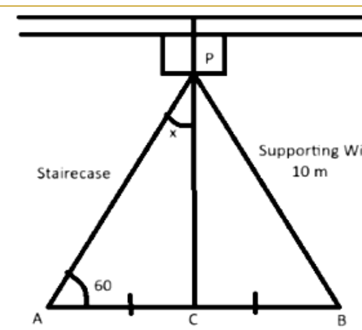
1 A children's park is in the shape of isosceles triangle say PQR with PQ = PR, S and T are points on QR such that QT = RS.
(i). Which rule is applied to prove that congruency of APQS and APRT.
(ii). If PQ = 6 cm and QR = 7 cm, then perimeter of ΔPQR is
(iii). If ∠QPR = 80°, find ∠PQR.



2 Neeraj has a plot in the shape of a triangle said ABC with AD as the perpendicular
bisector of BC such that BD = DC.
(i) Which rule is applied to prove the congruency of ΔABD and ΔACD?
(ii) If ∠ABD = 50°, then the value of ∠BAD.
(iii) In ΔADC find ∠ACD.



3 In Rajesh Village there was a big pole PC. This pole was tied with a strong wire of 10 m length. Once there was a big spark on this pole, thus wires got damaged very badly. Any small fault was usually repaired with the help of a rope which normal board electricians were carrying on bicycles. This time electricians need a staircase of 10 m, so that it can reach at point P on the pole and this should make 60° with line AC.
Q. 1. In ΔPAC and ΔPBC which side is common?
Q. 2. Find the value of ∠x?
Q. 3. In figure, ΔPAC and ΔPBC are congruent due to which criteria?



HOTS

1 ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC. If AD is extended to intersect BC at P, show that
(i) ΔABD≅ΔACD (ii) ΔABP ≅ΔACP
(iii) AP bisects ∠A as well as ∠D. (iv)AP is the perpendicular bisector of BC.

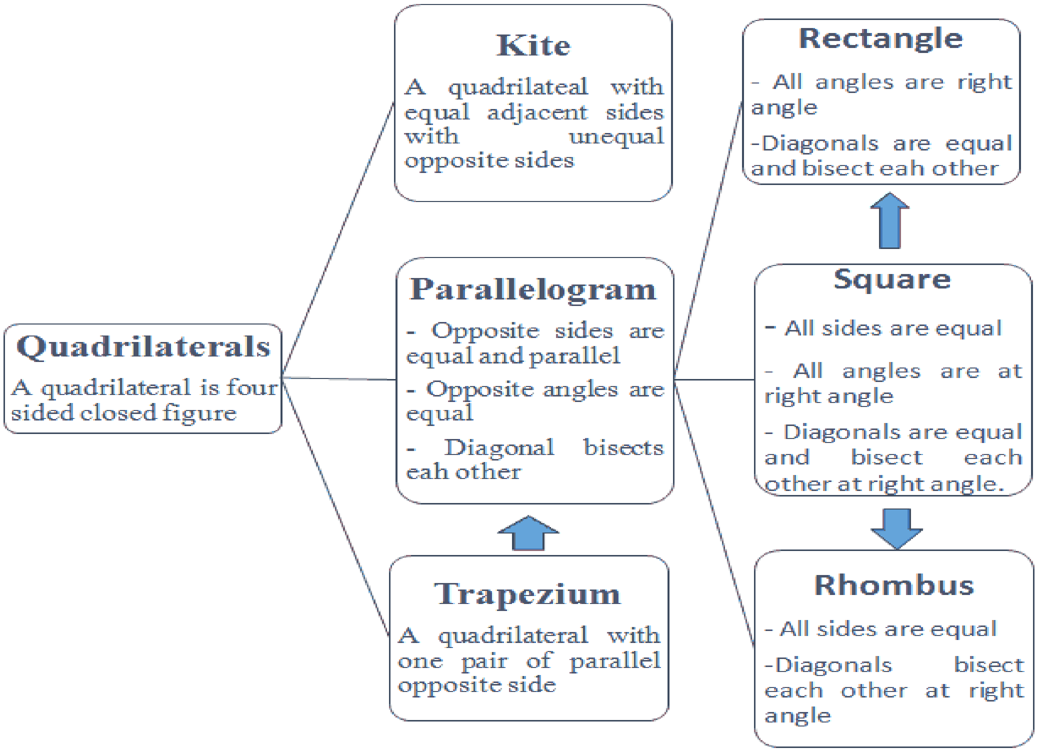
2 If two isosceles triangles have a common base, prove that the line joining their vertices bisects them at

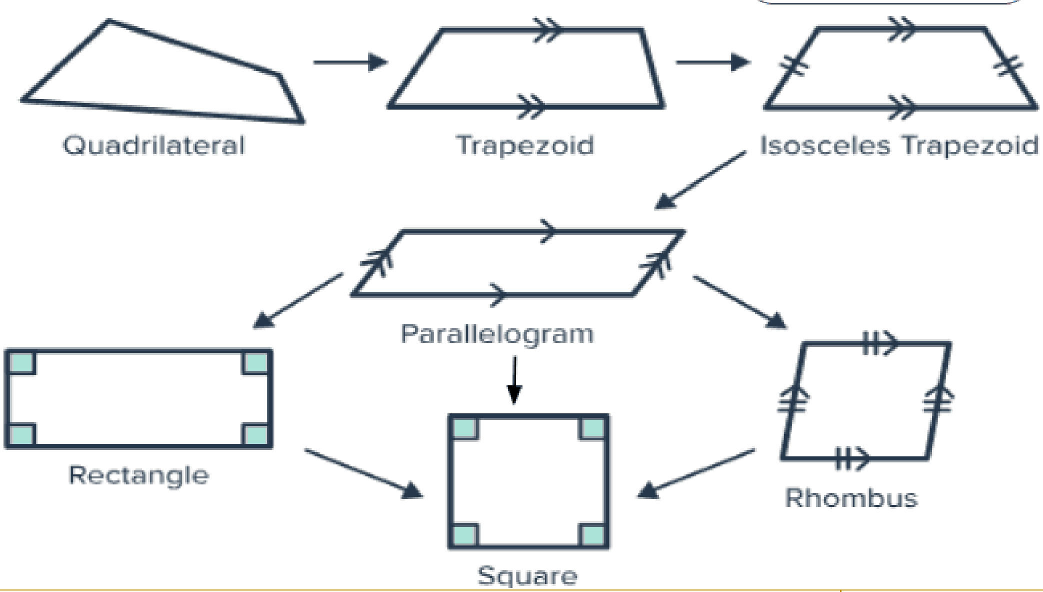
	right angles.
3	In a ΔPQR, if $PQ = QR$ and L, M and N are the mid-points of the sides PQ, QR and RP respectively. Prove that $LN = MN$.
4	Bisectors of angles B and C of an isosceles triangle ABC with $AB = AC$ intersect each other at O. Show that external angle adjacent to $\angle ABC$ is equal to $\angle BOC$.
5	Line segment joining the mid-points M and N of parallel sides AB and DC, respectively of trapezium ABCD is perpendicular to both the sides AB and DC. Prove that $AD = BC$.
	ANSWERS
	MCQs 1.Ans.(c) $70°$ 2.Ans.(c) $40°$ 3.Ans.(d)$360°$ 4.Ans.(c) $40°$ 5.Ans. (a) $48°, 60°, 72°$ — **ASSERTION - REASON BASED QUESTIONS** 1.Ans. (b) 2.Ans. (c) 3.Ans. (d) 4.Ans. (c) 5.Ans. (c)
	VERY SHORT ANSWER TYPE QUESTIONS
1	In ΔABC, $AB = AC$ (isosceles) $\Rightarrow \angle ABC = \angle ACB$ …(1) In ΔDBC, $DB = DC$ (isosceles) $\Rightarrow \angle CBD = \angle CDB$ …(2) Now, $\angle ABD = \angle ABC - \angle CBD$, $\angle ACD = \angle ACB - \angle CDB$ From (1) and (2): $\therefore \angle ABD = \angle ACD$
2	Given AC bisects $\angle BAD$ and $\angle BCD$. So in ΔABC and ΔADC: $\angle BAC = \angle DAC$ (AC bisects $\angle BAD$), $\angle BCA = \angle DCA$ (AC bisects $\angle BCD$) $AC = AC$ (common side) By ASA congruency: $\Delta ABC \cong \Delta ADC \Rightarrow BC = CD$ (CPCT)
3	In ΔABC and ΔAED: $\angle B = \angle E$ (given), $BD = CE$ (given), $\angle 1 = \angle 2$ (given) By ASA congruence: $\therefore \Delta ABC \cong \Delta AED$
4	In ΔABP and ΔACP: $AB = AC$ (given) $\angle APB = \angle APC = 90°$ ($AP \perp BC$), $AP = AP$ (common) By RHS congruence: $\Delta ABP \cong \Delta ACP \Rightarrow \angle B = \angle C$ (CPCT)
5	In ΔPQS and ΔPRS: $PQ = PR$ (given), $\angle PSQ = \angle PSR = 90°$ (altitude), $PS = PS$ (common) By RHS congruence: $\Delta PQS \cong \Delta PRS \Rightarrow \angle QPS = \angle RPS$ Hence, PS bisects $\angle P$
	SHORT ANSWER TYPE QUESTIONS
1	In ΔPQR, $PQ = PR \Rightarrow$ triangle is isosceles. $ST \parallel QR$ (given), so by Basic Proportionality Theorem (BPT) in ΔPQR: $PS/SQ = PT/TR$ Since $PQ = PR$ and $ST \parallel QR$, the segments PS and PT must also be equal. $\therefore PS = PT$
2	$AB = AC \Rightarrow \Delta ABC$ is isosceles $\Rightarrow \angle B = \angle C$ OB and OC are angle bisectors $\Rightarrow \angle OBC = \angle OCB$ In ΔBOC, OB and OC are angle bisectors meeting at O. Extend BO to M.$\angle MOC$ is an exterior angle to ΔBOC: $\angle MOC = \angle OCB + \angle OBC = 2\angle OBC = \angle ABC$
3	P lies on the bisector of $\angle ABC \Rightarrow \angle ABP = \angle CBP$ Line through P $\parallel$ AB $\Rightarrow \angle PQB = \angle ABP$ (alternate angles) $\angle PBQ = \angle CBP$ (since P is on bisector) $\therefore \angle PBQ = \angle PQB \Rightarrow \Delta BPQ$ is isosceles $\Rightarrow BP = PQ$
4	Let ABCD be equilateral $\Rightarrow$ All sides and angles equal. Given $OD = OB \Rightarrow$ O lies on perpendicular bisector of BD. Diagonals of equilateral quadrilateral bisect at $90°$.

	AO and OC lie along same diagonal AC. ∴ AO and OC lie on the same line.
5	In ΔABD and ΔACD: AB = AD, CB = CD, AC = AC (common) By SSS congruence ⇒ triangles are congruent. BD is bisected at intersection with AC Also, ∠BOC = 90° ∴ AC is the perpendicular bisector of BD
	LONGANSWER TYPE QUESTIONS
1	Given: AC = BC, ∠DCA = ∠ECB, ∠DBC = ∠EAC ;In ΔDBC and ΔAEC:∠DCA = ∠ECB (Given) ∠DBC = ∠EAC (Given), AC = BC (Given) ;⇒ By ASA congruence: ΔDBC ≅ ΔAEC ⇒ BD = AE
2	(i) D is the midpoint of AC: M is midpoint of AB ;MD ∥ BC ⇒ By midpoint theorem, D is midpoint of AC (ii) MD ⊥ AC:BC ⊥ AC (right triangle) and MD ∥ BC ⇒ MD ⊥ AC (iii) CM = MA = ½ AB: ;M is midpoint of AB in right triangle ⇒ MA = MB = MC ⇒ CM = MA = ½ AB
3	ABCD is a square ⇒∠ABC = 90° ;ABE is equilateral ⇒∠ABE = 60° ∠ACE lies between diagonal AC and AE ⇒ Symmetry implies ∠ACE = 30° ⇒∠ACE = ½ ∠ABE
4	Let ∠C = x, then ∠B = 2x ;∠A + 2x + x = 180° ⇒∠A = 180° - 3x AD bisects ∠A and AB = CD ⇒ Using triangle properties and angle bisector theorem ⇒∠A = 72°
5	In a parallelogram, diagonals are equal only if it's a rectangle ⇒ ABCD is a rectangle ⇒∠ABC = 90°
	CASE BASED QUESTIONS
1	1. (i) Ans. SAS (ii) Ans. 19 cm (iii) Ans. 50° 2. (i)Ans. SAS (ii) Ans. 40° (iii) Ans. 50° 3.(i) PC (ii) x= 60 quad 9 hots (iii) ΔPAC ≅ ΔPBC by the SAS (Side-Angle-Side) congruence criteria.
	HOTS
1	(i) Since AB = AC and DB = DC (isosceles triangles on same base), and AD is common, by SSS congruence, ΔABD ≅ ΔACD. (ii) From above, ΔABD ≅ ΔACD implies ∠ABD = ∠ACD. With BP = CP and AD common, ΔABP ≅ ΔACP by SAS. (iii) Since ΔABP ≅ ΔACP, ∠BAP = ∠CAP and ∠BDP = ∠CDP. Thus, AP bisects ∠A and ∠D. (iv) Since BP = CP and ∠BAP = ∠CAP, AP is perpendicular to BC and bisects it. So, AP is the perpendicular bisector of BC.
2	Let the triangles be ΔABC and ΔDBC with common base BC and vertices A and D. AB = AC and DB = DC (isosceles triangles).Let line AD intersect BC at M. By symmetry and congruent triangles (e.g., ABM ≅ ACM), ∠AMB = 90° and BM = MC. Hence, line joining vertices bisects BC at right angles.
3	Since PQ = QR and L, M are midpoints, then PL = QM.LN and MN are midlines in triangles ⇒ both are equal in length and parallel to side PR. Hence, LN = MN.
4	Since AB = AC, ∠B = ∠C. Their angle bisectors BO and CO divide ∠B and ∠C equally. The triangle formed by BO and CO at point O implies ∠BOC = external angle adjacent to ∠ABC, due to symmetry and equality of bisected angles.
5	Given: AB ∥ DC and MN ⊥ AB, DC. Since M and N are midpoints and MN is perpendicular to both bases, trapezium is symmetric. Hence, non-parallel sides AD and BC are equal ⇒ AD = BC

CHAPTER 8
QUADRILATERALS

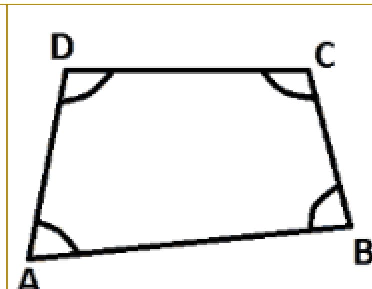
CONCEPT MAP





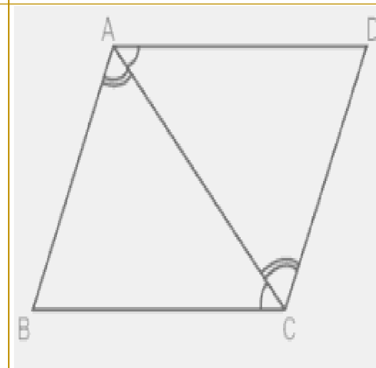
GIST/SUMMARY OF THE LESSON

- Sum of angle of quadrilateral is 360^0.
    - $\angle A + \angle B + \angle C + \angle D = 360^O$
- In a parallelogram
    - Opposite sides are equal
    - Opposite angles are equal
    - Diagonals bisect each other
- A quadrilateral is a parallelogram if
    - Opposite sides are equal or
    - Opposite angles are equal or
    - Diagonals bisect each other or
    - A pair of opposite sides is equal and parallel

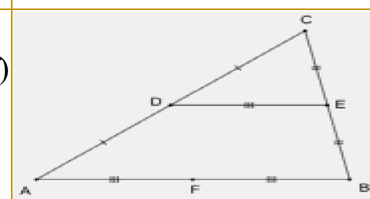


- A rectangle is a parallelogram in which all angles are 90^o and diagonals bisect each other and equal.
- A rhombus is a parallelogram in which diagonal bisect each other at right angles
- A square is a parallelogram in which all angles are equal and diagonal bisect each other at right angles
- Mid-point theorem: The line segment joining the midpoint of any two sides of the triangle is parallel to the third side and is half of it.
- Converse of midpoint theorem: A line through the mid-point of a side of a triangle parallel to another side bisects the third side.

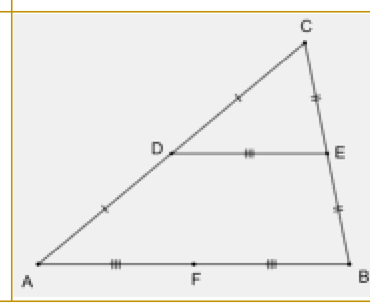
THEOREMS:



- A diagonal of a parallelogram divides it into two congruent triangles.
  Given llgm ABCD with diagonal AC
  To prove $\Delta ABC \cong \Delta CDA$
  Proof: In $\Delta ABC \cong \Delta CDA$
  ∠BAC = ∠ACD (Alternate Opp angles)
  ∠BCA =∠CAD (Alt. Int. angles)
  AC = AC (COMMON)
  Therefore $\Delta ABC \cong \Delta CDA$ (ASARULE)
- In a parallelogram, opposite sides are equal and conversely
- A quadrilateral is a parallelogram if a pair of its opposite sides are equal and parallel
- In a parallelogram the diagonals bisect each other and conversely.



- **MID-POINT THEOREM**: The line segment joining the midpoint of any two sides of the triangle is parallel to the third side and is half of it. (without proof)
  In Δ ABC , D and E is the midpoint of side AC and BC respectively
  i.e AD= DC and BE=EC then DE is parallel to AB i.e DE ∥ AB and DE = ½ AB

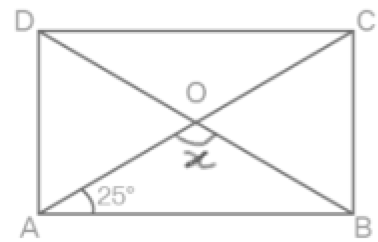
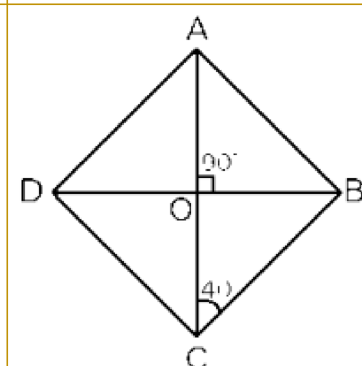
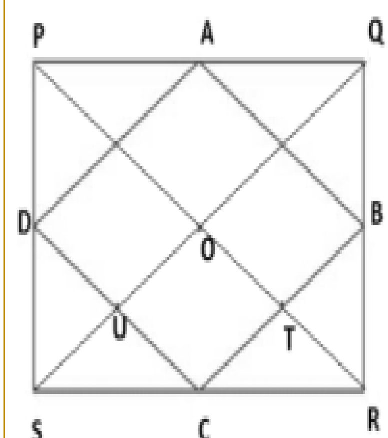


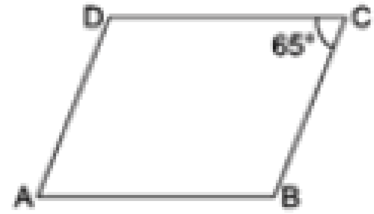
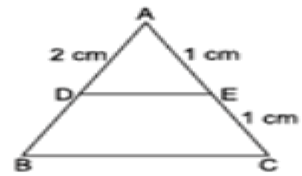
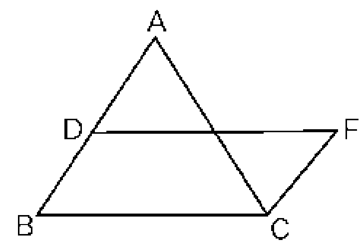
- **CONVERSE OF MID-POINT THEOREM** : A line through the mid-point of a side of a triangle parallel to another side bisects the third side. (without proof)
  In Δ ABC
  If DE is parallel to AB i.e DE ∥ AB
  Then D and E is the midpoint of side AC and BC respectively
  i.e AD= DC and BE=EC

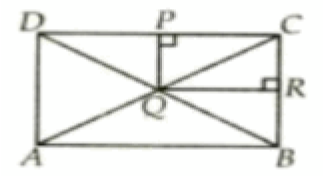
MULTIPLE CHOICE QUESTIONS (1 MARKS)

1 Three angles of a quadrilateral are 75^o, 90^o and 75^o. The fourth angle is
(A) 90^o (B) 95^o (C) 105^o (D) 120^o
SOLUTION: (D) Sum of angle of quadrilateral = 360^0
=>75^o+90^o +75^o+fourth angle=360^0
=> 240^0+ fourth angle=360^0 =>fourth angle=360^0- 240^0 =120^0

2 The angles of a quadrilateral are in the ratio 3 :4 :5 : 6. The respective angles of the quadrilaterals are
(A) 60^0, 80^0, 100^0, 120^0 (B) 120^0, 100^0, 80^0, 60^0
(C) 120^0, 60^0, 80^0, 100^0 (D) 80^0, 100^0, 120^0, 60^0
SOLUTION: (A) Sum of angle of quadrilateral = 360^0
=>3x+4x+5x+6x=360^0
=>18x=360^0
=>x=$\frac{360}{18}$ = 20

	angles are = 3 x 20 =60^0 4 x 20 = 80^0 5 x 20 = 100 6 x 20 = 120^0	
3	A diagonal of a rectangle is inclined to one side of the rectangle at 25º. The acute angle between the diagonals is (A) 55º (B) 50º (C) 40º (D) 25^0 **SOLUTION**: (B) AC = BD (diagonals of a rectangle are of equal) AC/2 = BD/2 OA = OB ∠ABO = 25^0 (angle opposite of equal sides) In Δ AOB, angle sum property of Δ ∠OAB+ ∠AOB + ∠ABO = 180^0 250+ ∠AOB + 250 = 180^0 500 + ∠AOB = 180^0 ∠AOB= 180 -50 = 130^0 Acute angle between the diagonal ∠AOB + ∠COB = 180 (straight angle) 130^0+ ∠COB = 180 ∠COB = 180 – 130 = 50^0	
4	ABCD is a rhombus such that ∠ACB = 40º. Then ∠ADB is (A) 40º (B) 45º (C) 50º (D) 60º **SOLUTION**: (C) As AD ‖ BC ∠DAC = ∠BCA = 40° (Alternate interior angles) ∠DAO = 40° ∠AOD = 90°(Diagonals of a rhombus are ⊥ to each other) In triangle DOA, angle sum property of triangle ∠AOD + ∠ADO + ∠DAO = 180° 90° + ∠ADO + 40° = 180° 130° + ∠ADO = 180° ∠ADO = 180° – 130° ∠ADO = 50° =>∠ADB = 50°	
5	The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if (A) PQRS is a rectangle(B) PQRS is a parallelogram (C) diagonals of PQRS are perpendicular (D) diagonals of PQRS are equal. **SOLUTION**: (C) In △QSR , B is the mid-point of QR and C is the mid-point of SR. BC= ½ SQ (mid-point theorem) BC∥SQ Hence, UO∥CT …….(1) In △PSR , C is the mid-point of SR and D is the mid-point of PS. CD= ½ PR (mid-point theorem) CD∥PR . Hence, UC∥OT …….(2) From (1)and(2), UOTC is a parallelogram Since ABCD is a rectangle, ∠C=90∘. ∠C=∠O=90∘ (opposite angle of llgm UOTC) Since∠O=90∘we can easily say that PR⊥QS(We all know that the diagonals of a	

	rhombus are perpendicular to each other) Hence, the diagonals of PQRS are perpendicular.	
6	The consecutive angles of a parallelogram are (A) Complementary (B) Supplementary (C) Equal (D) None **SOLUTION**:(B) Supplementary	
7	In the given figure, ABCD is a parallelogram. If ∠C = 65°, then (∠B + ∠D) is equal to (A) 180° (B) 115° (C) 155° (D) 230° **SOLUTION**: (D) Sum of angle of parallelogram = 360^0 =>65º+∠B+65º+ ∠D=360^0 => 130^0+ ∠B + ∠D=360^0 =>∠B + ∠D =360^0- 130^0 =230^0	
8	In the given figure, find BD, if DE | | BC. (A) 2 cm (B) 1 cm (C) 3 cm (D) none of these **SOLUTION**: (A) E is the midpoint of the triangle ABC, DE | | BC. By converse of midpoint theorem AD = BD = 2cm	
9	Two angles of a quadrilateral are 60° and 70° and other two angles are in the ratio 8 : 15, then the remaining two angles are (A) 140°, 90° (B) 100°, 130° (C) 80°, 150° (D) 70°, 160° **SOLUTION**: (C) Sum of angle of quadrilateral = 360^0 =>60º+70º +8x + 15x =360^0 ; => 130^0+ 23x=360^0 ; => 23x =360^0- 130^0 = 230^0 => x =$\frac{230}{23}$ = 10 ; Therefore other two angles are 8 x 10 = 80^0 and 15 x 10 = 150^0	
10	D and E are the mid-points of the sides AB and AC respectively of ΔABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is (A) ∠DAE = ∠EFC (B) AE = EF (C) DE = EF (D) ∠ADE = ∠ECF. **SOLUTION**:(C) DE = EF From the SAS congruence rule ΔADE ≅ ΔCFE	
	ASSERTION (A):AND REASON TYPE (1 MARKS)	
	In the following questions, a statement of Assertion (A):(A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true	
1	Assertion (A):: The angles of a quadrilateral are x^0, $(x – 10)^0$, $(x + 30)^0$ and $(2x)^0$, the smallest angle is equal to 58^0. Reason(R) : Sum of the angles of a quadrilateral is 360^0 **SOLUTION**:(A)	
2	Assertion (A):: If the diagonals of a parallelogram ABCD are equal, then ∠ABC = 90^0. Reason(R) : If the diagonals of a parallelogram are equal, it becomes a rectangle. **SOLUTION**: (A)	
3	Assertion (A):: A parallelogram consists of two congruent triangles. Reason(R) : Diagonal of a parallelogram divides it into two congruent triangles. **SOLUTION**:(A)	
4	Assertion (A):: ABCD is a square. AC and BD intersect at O. The measure of ∠ABC=90^0. Reason(R) : Diagonals of a square bisect each other at right angle **SOLUTION**:(B)	

5	Assertion (A):: ABCD and PQRC are rectangles and Q is a midpoint of AC . Then DP = PC. Reason(R) : The line segment joining the midpoint of any two sides of a triangle is parallel to the third side and equal to half of it. **SOLUTION**:(B)	
6	Assertion (A):: In ΔABC , median AD is produced to X such that AD = DX. Then ABXC is a parallelogram. Reason(R) : Diagonals AX and BC bisect each other at right angles. **SOLUTION**:(C)	
7	Assertion (A):: The consecutive sides of a quadrilateral have one common point. Reason(R) : The opposite sides of a quadrilateral have two common point. **SOLUTION**:(C)	
8	Assertion (A):: Two opposite angles of a parallelogram are $(3x-2)^0$ and $(50-x)^0$. The measure of one of the angle is 37^0. Reason(R) : Opposite angles of a parallelogram are equal. **SOLUTION**: (A)	
9	Assertion (A):: If the angles of a quadrilateral are in the ratio 2 :3 :7 : 6, then the measure of angles are 40^0, 60^0, 140^0 120^0, respectively. Reason(R) : The sum of the angles of a quadrilateral is 360^0. **SOLUTION**: (A)	
10	Assertion (A):: In ΔABC , E and F are the midpoints of AC and AB respectively. The altitude AP at BC intersects FE at Q. Then, AQ = QP. Reason(R) : Q is the midpoint of AP. **SOLUTION**: (A)	
	VERY SHORT ANSWER TYPE (2 MARKS)	
1	Angles of a quadrilateral are in the ratio 3 :4 :4 : 7. Find all the angles of the quadrilateral. **SOLUTION**: Sum of angle of quadrilateral = 360^0 ; =>3x +4x+4x+7x=360^0 ; => 18x=360^0 ; => x=$\frac{360}{18}$=20 First angle 3 x 20= 60^0 ; Second angle = 4 x 20 = 80^0 ; Third angle = 4 x 20 = 80^0 ; Fourth angle = 7 x 20 = 140^0	
2	Two consecutive angles of a parallelogram are (x + 60)° and (2x + 30)°. What special name can you give to this parallelogram? **SOLUTION**: Sum of consecutive angles of a parallelogram = 180^0 x + 60+ 2x + 30 = 180 ; 3x+ 90= 180 ; 3x= 180 -90 ; 3x = 90 ; x= $\frac{90}{3}$ = 30 first angle = 30 + 60 = 90^0 second angle = 2 x 30 + 30 = 90^0 this parallelogram is a rectangle.	
3	If one angle of a parallelogram is 30^O less than the smallest angle, then find the measure of each angle. **SOLUTION**: let the smallest angle be x Other angle = x – 30 Opposite angles of parallelogram are equal Sum of all angles of parallelogram = 360 x+ (x-30) + x + (x-30) =360 4x – 60 = 360 4x = 360+60 = 420 x= $\frac{420}{4}$ = 105 first angle = 105 second angle 105 – 30 = 75 third angle = 105 fourth angle = 105 – 30 = 75	

4 If the diagonals of a parallelogram are equal, then show that it is a rectangle.

SOLUTION:

Given: Let ABCD be a parallelogram in which AC= BD

To Prove : ABCD is a rectangle ; Proof: In Δ ABC and ΔDCB

AB= DC (Opposite side of parallelogram)

BC = BC (common) ; AC = BD (Given)

Δ ABC ≅ ΔDCB (SSS congruency)

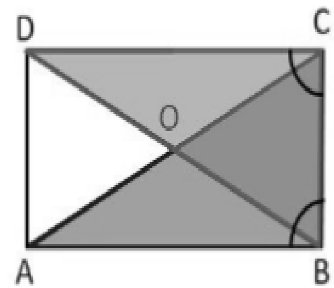
∠ABC = ∠DCB (CPCT) (1)

Now AB ll DC (Opposite sides of parallelogram)

∠B +∠C = 180 (adjacent angle of parallelogram are supplementary)

2∠B = 180 (from 1) ; ∠B= $\frac{180}{2}$ = 90

ABCD is a parallelogram with one angle 90^0 ; Therefore, ABCD is a rectangle.

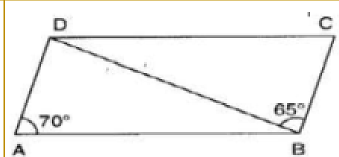


5 In the given Fig, ABCD is a parallelogram in which ∠DAB = 70 and $\angle DBC$ = 65 , then find the measure of $\angle CDB$.

SOLUTION:∠DAB = ∠DCB = 70^0 (Opposite angles of a parallelogram)

In Δ DCB, using angle sum property of triangle ∠DCB+$\angle DBC$+∠CDB= 180

70+65+∠CDB= 180 ; 135+∠CDB= 180 ; ∠CDB = 180-135 ; ∠CDB= 45



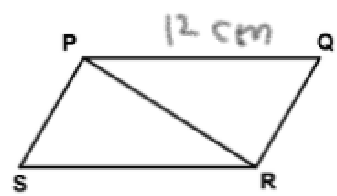
6 PQRS is a parallelogram, in which PQ = 12 cm and its perimeter is 40 cm. Find the length of each side of the parallelogram

SOLUTION:

PQ = SR (opp side of parallelogram) ; PQ+QR+SR+PS = 40cm

12 + QR+12+PS = 40 ; 24+QR+PS= 40 ; QR+PS = 40 -24 ; QR + PS = 16 ;

2QR= 16 (Opposite side of parallelogram) ; QR = $\frac{16}{2}$ = 8cm ; QR = PS =8cm



7 The diagonal AC and BD of a parallelogram ABCD intersect at point O. Find ∠DBC.

SOLUTION: Given ABCD is a parallelogram

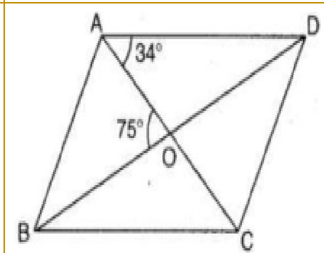
∠DAC = ∠ACB (Alternate interior angles) ; ∠AOB + ∠BOC= 180 (Straight angle)

75 + ∠BOC =180 ; ∠BOC = 180 – 75 ; ∠BOC =105

In Δ BOC, applying angle sum property

∠OCB + ∠BOC + ∠OBC = 180 ; =>34+ 105 + ∠OBC= 180 ; =>139 + ∠OBC= 180 ;

=>∠OBC = 180-139 ; =>∠OBC= 41^0 ; ∠DBC = 41^0



8 In the fig, ABCD is a rhombus, whose diagonals meet at O. Find x and y.

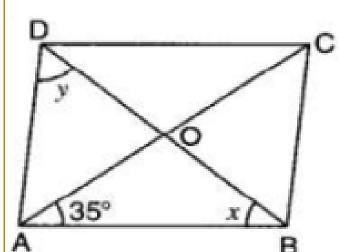
SOLUTION: Diagonals of rhombus bisect each other at right angle

∠AOB = 90^0 ; In Δ AOB, applying angle sum property of triangle

∠OAB + ∠AOB + x = 180 ; 35+ 90 + x = 180 ; 125 +x = 180

x= 180-125 = 55^0 ; ABCD is a rhombus, AB= AD (all sides of rhombus are equal)

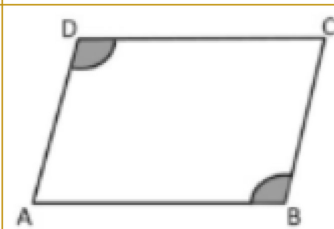
x= y = 55^0(angles opp to equal sides are equal)



9 If ABCD is a parallelogram, then what is the measure of ∠A – ∠C ?

SOLUTION: In parallelogram ABCD, ∠A and ∠C are opposite angles

We know, opposite angles are equal in parallelogram Therefore, ∠A -∠C = 0



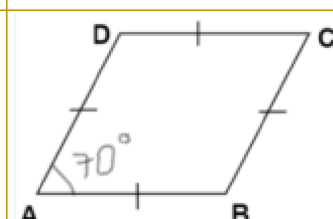
10 ABCD is a rhombus. If ∠A = 70°, find ∠B and ∠C.

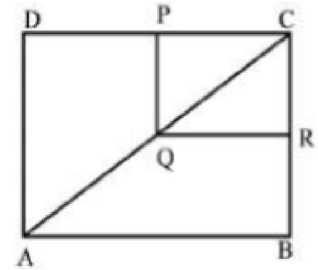
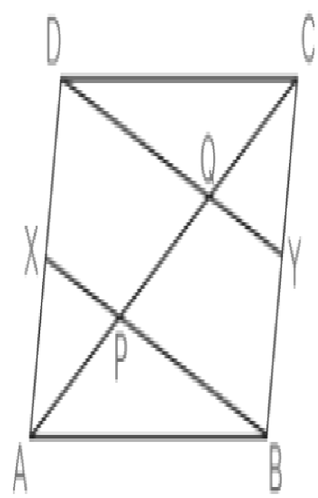
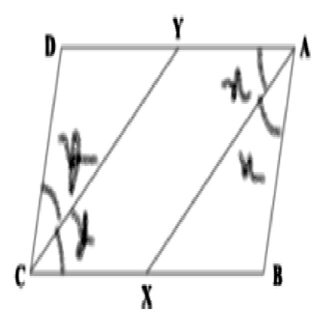
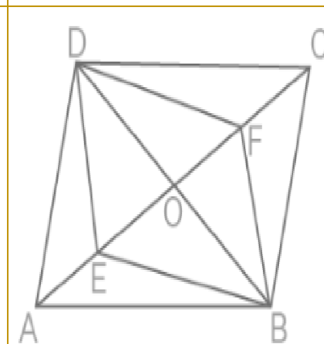
SOLUTION:

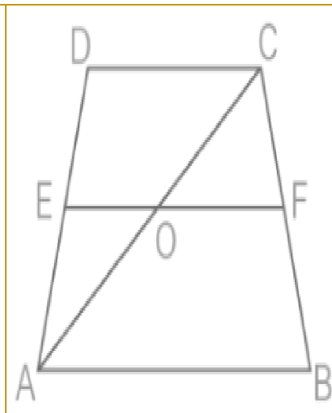
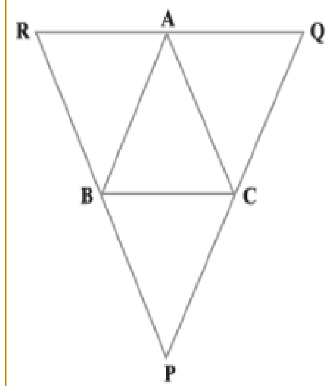
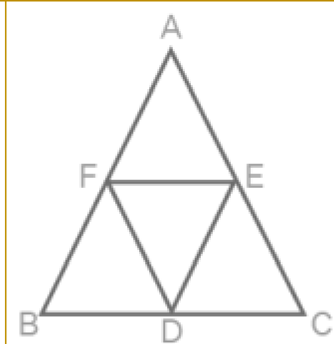
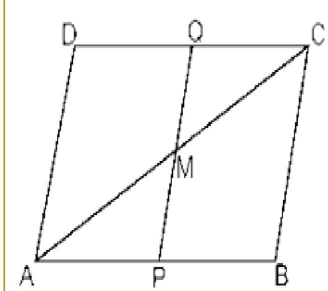
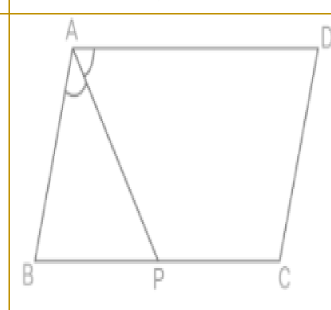
ABCD is a rhombus ; ∠A =∠C = 70^0 (opposite angles are equal)

∠A+∠B = 180 (adjacent angles) ; 70+ ∠B = 180

∠B = 180 -70 ; ∠B = 110^0

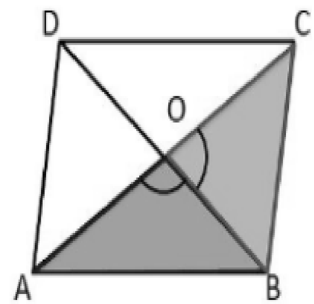


	SHORT ANSWER TYPE QUESTION (3 MARKS)	
1	In the given figure, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that: (i) DP = PC (ii) PR = ½ AC **SOLUTION**: (i)∠P = ∠D = 90 (angle of rectangle) Therefore, PQ ll AD In $\Delta\, DAC$, Q is the midpoint of AC and PQ ll AD Therefore, P is the midpoint of DC DP =DC (by converse of midpoint theorem) (ii) similarly, R is the midpoint of BC and Q is the midpoint of AC In Δ BDC RP ll BD and PR = ½ BD (bymid point theorem) PR = ½ BD (diagonal of rectangle are equal) PR= ½ AC	
2	In Fig, X and Y are respectively the mid-points of the opposite sides AD and BC of a parallelogram ABCD. Also, BX and DY intersect AC at P and Q, respectively. Show that AP = PQ = QC. **SOLUTION**: Given, ABCD is a parallelogram X and Y are the midpoints of the opposite sides AD and BC of the parallelogram BX and DY intersect AC at P and Q , To prove AP = PQ = QC Proof :AD = BC ----------- (1) (opposite sides of a parallelogram) AX = DX (X is the midpoint of AD) So, AD = AX + DX ; AD = DX + DX ; AD = 2DX --------------- (2) BY = CY (Y is the midpoint of BC) So, BC = BY + CY ; BC = BY + BY ; BC = 2BY ------------------ (3) Using (2) and (3) in (1), ; 2DX = 2BY ; DX = BY We know that a pair of opposite sides are equal and parallel in a parallelogram. So, DX || BY , This implies XBYD is a parallelogram So, PX || QD , From triangle AQD ; X is the midpoint of AD, and XP ll PQ By converse of midpoint theorem ; AP = PQ ------------ (4) Similarly, from triangle CPB ; CQ = PQ ------------ (5) From (4) and (5), AP = PQ = CQ ; Therefore, AP = PQ = QC	
3	In Fig, AX and CY are respectively the bisectors of the opposite angles A and C of a parallelogram ABCD. Show that AX || CY. **SOLUTION**: Given, ABCD is a parallelogram. ; AX and CY are the bisectors of the angles A and C. ; To prove AX || CY ; Proof: ∠DAB = 2x ∠DCB = 2y ; ∠A = ∠C (opposite angles of a parallelogram are equal) 2x = 2y ; x = y (1) ; As BC || AD, XC || AY ∠XCY = ∠CYD = y [Alternate angles] (2) ∠CYD = x (from 1 and 2) ; ∠XAY = x ; As ∠XAY and ∠CYD are corresponding angles ; AX || CY ; Therefore, AX is parallel to CY.	
4	E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram. **SOLUTION**: Given: ABCD is a parallelogram E and F are points on diagonal AC of parallelogram ABCD such that AE = CF To show: BFDE is a parallelogram. Construction: Join the other diagonal BD of the parallelogram. The diagonal BD meets AC at O Proof: OA = OC (diagonals of a parallelogram bisect each other) ; OD = OB AE = CF (given) ; OA - AE = OE ; OC - CF = OF ; OE = OF ; Therefore, BFDE is a parallelogram as the diagonals EF and BD bisect each other at O.	

5	E is the mid-point of the side AD of the trapezium ABCD with AB || DC. A line through E drawn parallel to AB intersect BC at F. Show that F is the mid-point of BC. **SOLUTION**: Given: ABCD is a trapezium ; E is the midpoint of the side AD with AB parallel to DC. ; A line through E drawn parallel to AB intersect BC at F. To show:F is the midpoint of BC ; Proof: AB ll CD and EF ll AB Therefore, EF ll DC ; In triangle ADC, ; E is the midpoint of AD EF ll DC (proved above) ; So, OE || DC O is the midpoint of AC (By converse of midpoint theorem) In triangle CBA, O is the midpoint of AC ; EF ll AB (given) So, OF || AB ; F is the midpoint of BC (By converse of midpoint theorem) Therefore, it is shown that F is the midpoint of BC.	
6	Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a Δ ABC as shown in Fig. Show that BC = ½ QR. **SOLUTION**: Given: ABC is a triangle , Lines RQ, PR and QP are drawn through A, B and C parallel to sides BC, CA and AB of the triangle ABC To show: BC = ½ QR. ; Proof: Given, RQ || BC ,PR || AC, QP || AB In quadrilateral BCAR, BR || CA, RA || BC We know the pair of opposite sides of a parallelogram are parallel. So, BCAR is a parallelogram. BC = AR ------------------ (1) In quadrilateral BCQA, BC || AQ, AB || QC ; So, BCQA is a parallelogram BC = AQ ------------------ (2) ; Adding (1) and (2), ; BC + BC = AR + AQ 2BC = AR + AQ ; From the figure, ; AR + AQ = RQ ; So, 2BC = RQ Therefore, BC = ½ QR	
7	D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that Δ DEF is also an equilateral triangle. **SOLUTION**: Given, ABC is an equilateral triangle D, E and F are the midpoints of the sides BC, CA and AB To show: DEF is also an equilateral triangle Proof: In triangle ABC, E and F are the midpoints AC and AB EF || BC (by midpoint theorem) ; EF = ½ BC ------- (1) Similarly DF || AC, DF = ½ AC ----------- (2) ; DE || AB, DE = ½ AB ----------- (3) Since ABC is an equilateral triangle ; AB = BC = AC Dividing by 2, ; ½ AB = ½ BC = ½ AC ; From (1), (2) and (3) DE = EF = DF ; Therefore, DEF is an equilateral triangle.	
8	Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ (see Fig.). Show that AC and PQ bisect each other. **SOLUTION**: Given, ABCD is a parallelogram, AP = CQ To show: PM = MQ ; Proof: In triangles AMP and CMQ, ∠PAM = ∠QCM ; AP = CQ (Given) ∠AMP = ∠CMQ (Vertically opp angles) ; Δ AMP ≅ Δ CMQ (By AAS rule) AP = CQ (CPCT)	
9	In Fig., P is the mid-point of side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2CD Solution: Given, ABCD is a parallelogram ; P is the midpoint of the side BC of the parallelogram ; ∠BAP = ∠DAP ; To prove that AD = 2CD Proof: AD || BC ---------------------- (1) (opp. Sides of parallelogram) ∠A + ∠B = 180° (adjacent angles are supplementary) ∠B = 180° - ∠A -------------- (2) ; In triangle ABP, ∠BAP + ∠B + ∠BPA = 180° (angle sum property) ∠BAP = ∠DAP (given) ; ∠BAP = ∠DAP = ½ ∠A ; ½ ∠A + ∠B + ∠BPA = 180°	

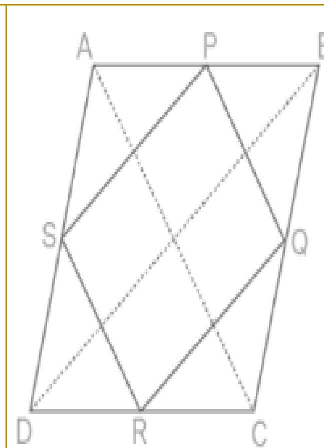
½ ∠A + 180° - ∠A + ∠BPA = 180° ; ∠BPA –½∠A = 0
∠BPA = ½ ∠A ; So, ∠BPA = ∠BAP
AB = BP (sides opposite to equal angles are equal)
Multiplying by 2 on both sides, 2AB = 2BP ; BP = CP(P is the midpoint of BC)
BC = BP + PC ; BC = BP + BP ; BC = 2BP ; So, 2AB = BC
ABCD is a parallelogram, AB =CD and AD = BC ; 2CD = AD HENCE PROVED

10 Show that the diagonals of a rhombus are perpendicular to each other.
Solution: Given: ABCD is a rhomus
To Prove: AC ⊥ BD ; Proof: Since ABCD is a rhombus AB = BC =CD =AD
In Δ AOB and Δ BOC ; OA = OC (diagonal of parallelogram bisect)
OB = OB (common) ; AB = CB (sides of a rhomus)
Δ AOB ≅ Δ BOC (By SSS rule) ; ∠AOB = ∠COB (CPCT) (1)
∠AOB+∠COB = 180 (straight angle)
2∠AOB = 180 ; ∠AOB = 180 / 2 = 90
∠AOB = ∠COD = 90 (Vertically opposite angle) ; ∠BOC = ∠AOD = 90
Therefore, diagonals of rhombus bisect each other at right angle

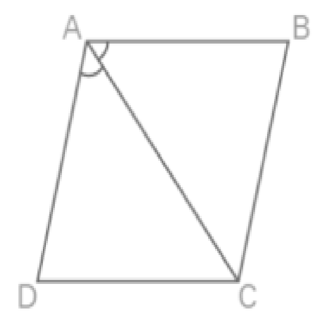


LONG ANSWER TYPE QUESTIONS (5 MARKS)

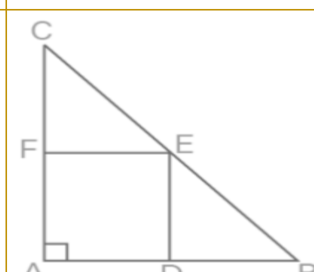
1 Show that the quadrilateral formed by joining the mid-points the sides of a rhombus, taken in order, form a rectangle.
Solution:
Given: a rhombus ABCD and P, Q, R and S are the midpoints of the sides AB, BC, CD and AD. To prove: PQRS is a rectangle
Construction: Join the diagonals AC and BD of the rhombus ABCD.
Proof: In triangle ABD, Since S and P are the midpoints of the sides AD and AB. By midpoint theorem
SP || BD ----------------- (1) ; SP = ½ BD --------------- (2) ; Similarly, RQ || BD
RQ = ½ BD -------------- (3) ; From (2) and (3),
SP = RQ Also, SP || RQ ; Therefore, PQRS is a parallelogram
We know that the diagonals of a rhombus are perpendicular.
So, AC⊥ BD (4) ; In triangle BAC, ; PQ || AC --------------- (5)
From (1), (4) and (5), ; SP ⊥PQ i.e., ∠SPQ = 90° ; We know that a rectangle is a quadrilateral with four right angles. The opposite sides are parallel and equal to each other. Therefore, PQRS is a rectangle.

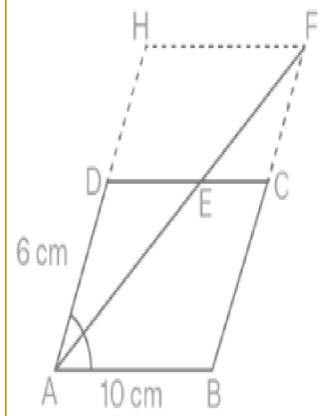
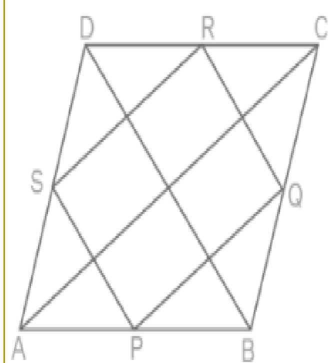
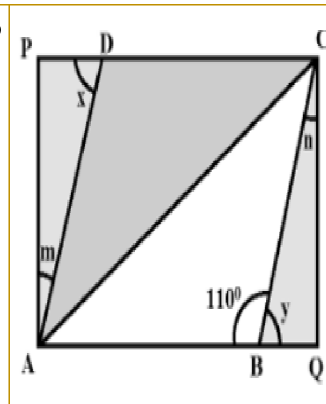


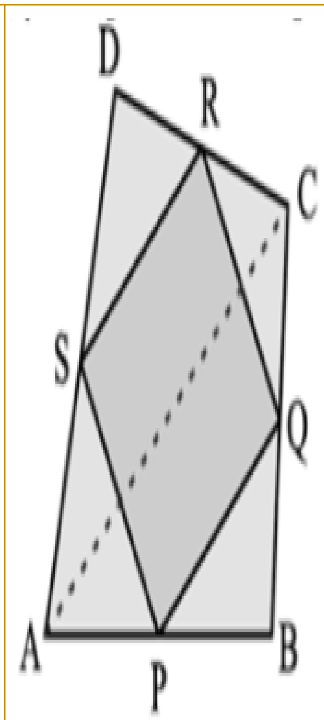
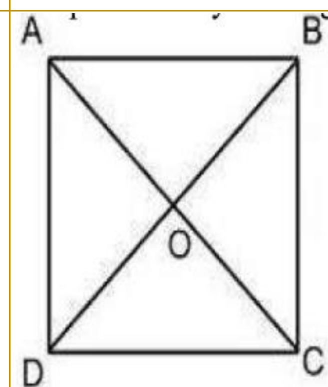
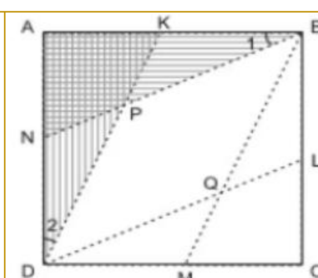
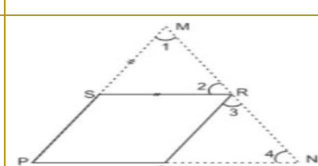
2 A diagonal of a parallelogram bisects one of its angle. Prove that it will bisect its opposite angle also.
Solution:
Given: ABCD is a parallelogram and .,∠DAC = ∠CAB
To prove: .,∠DCA = ∠ACB
Proof: ∠CAB = ∠CAD ------------------ (1) (AC bisects angle A)
The opposite sides of a parallelogram are parallel and equal.
AB||CD and AC is a transversal, AD||BC and AC is transversal
∠CAB = ∠ACD ----------------- (2) (Alternate interior angles)
∠DAC = ∠ACB ----------------- (3) (Alternate interior angles)
From (1), (2) and (3), ∠BCA = ∠DCA ; This implies AC bisects the opposite angle C.

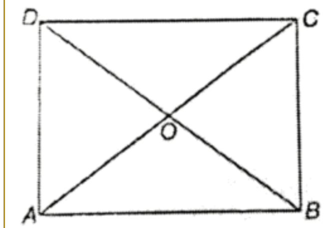
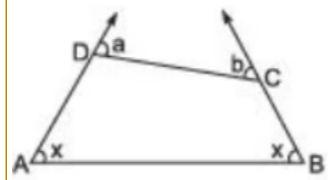
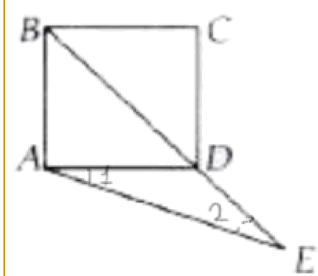


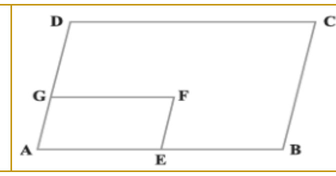
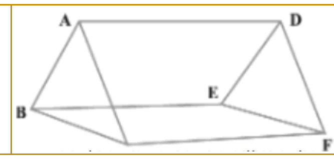
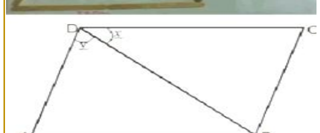
3 A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.
Solution: Given: CAB is an isosceles triangle and ADEF is a square and ∠A = 900
To prove: CE = BE ; Proof: ABC is an isosceles triangle
AB=AC -- (1) (sides of isosceles triangle) ; AD = AF (2) (sides of a square)
On subtracting (1) and (2), we get AB - AD = AC - AF ; BD = CF --------------- (3)



	In triangles CFE and BDE, ; DE = EF (side of a square) ; D = CF (proved above) ∠CFE = ∠EDB = 90° (angle of a square) ; ΔCFE ≅ Δ BDE (By SAS rule) CE = BE (CPCT) HENCE PROVED	
4	In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F. Find the length of CF. **Solution**: Construction: Extend AD to H and join HF Solution: ABFH is a parallelogram AB || HF and HF = AB ; ∠AFH = ∠FAB ------(1) (alternate interior angles) ∠HAF = ∠FAB ----- (2) (AF is the bisector of ∠A) From (1) and (2), ∠AFH = ∠HAF HF = AH (sides opposite to equal angles are equal) HF = AB (opp sides of parallelogram) HF = 10 cm ; Since HF = AH, AH = 10 cm ; From the figure, AH = AD + DH ; 10 = 6 + DH ; DH = 10 – 6 ; DH = 4 cm CFHD is a parallelogram , We know that the opposite sides of a parallelogram are parallel and equal. ; So, DH = CF ; Therefore, CF = 4cm	
5	P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus. **Solution**: Given: P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD ; To prove: PQRS is a rhombus Proof: In triangle ADC, S and R are the midpoints of AD and DC SR || AC, SR = ½ AC(1) (By midpoint theorem) ; In triangle ABC, P and Q are the midpoints of AB and BC ; PQ || AC, PQ = ½ AC --------(2) (By midpoint theorem) Comparing (1) and (2), SR = PQ = ½ AC ----------- (3) ; In triangle BCD, RQ || BD, RQ = ½ BD --------(4) (By midpoint theorem) , In triangle BAD, ; SP || BD SP = ½ BD -----------(5) (By midpoint theorem) Comparing (4) and (5), SP = RQ = ½ BD ----------- (6) AC = BD (given) Dividing by 2 on both sides, ½ AC = ½ BD ; From (3) and (6), SR = PQ = SP = RQ ; This implies all the sides of the quadrilateral are equal. Therefore, PQRS is a rhombus.	
	CASE STUDY QUESTIONS (4 MARKS)	
1	In the middle of the city, there was a park ABCD in the form of a parallelogram form so that AB = CD, AB||CD and AD =BC, AD || BC. Municipality converted this park into a rectangular form by adding land in the form of ΔAPD and ΔBCQ. Both the triangular shape of land were covered by planting flower plants. Which side is equal to PD? (1) **Solution**: PD = BQ Show that ΔABC and ΔCDA are congruent. **Solution**: In ΔABC and ΔCDA ; AB = CD (Opp side of llgm) AD = BC (Opp side of llgm) ; AC = AC (Common) ΔABC and ΔCDA (by SSS rule) ; Show that ΔAPD and ΔCQB are congruent. **Solution** In ΔAPD and ΔCQB ; AP = CQ (Opp side of rectangle) AD = BC (Opp side of llgm) ; ∠APD = ∠CQB (Each 900) ΔAPD and ΔCQB (by RHS rule) OR What is the value of ∠m? **Solution**:∠B = ∠D = 110^0 (Opp angles of parallelogram) In triangle APD ; m+ x + ∠APD = 180^0 ; m + (180 – 110) + 90 = 180^0 m + 160 = 180^0 ; m = 180 – 160 m = 20	

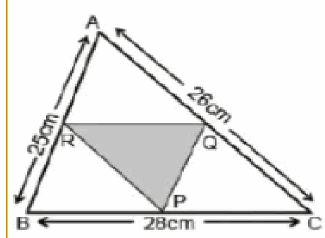
2	Activity-based learning- ensures active engagement of learner with concepts and instructional materials. Learning is hands-on and experiential, providing learners the opportunity of learning through manipulation of materials and objects. Teachers model the process, and students work independently to copy it. Mr. Shubham Maths teacher of class 9th wants to explain the properties of parallelograms in a creative way, so he gave students yellow colored paper in the shape of a quadrilateral and then ask the students to make a parallelogram from it by using paper folding and coloured it with green colour. How can a parallelogram be formed by using paper folding? **SOLUTION**: By joining mid points of sides of a quadrilateral one can make parallelogram. Now, S and R are mid points of sides AD and CD of ΔADC, P and Q are mid points of sides AB and BC of ΔABC, then by mid-point theorem $SR \parallel AC$ and $SR = \frac{1}{2}AC$ Similarly, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$. ; Therefore $SR \parallel PQ$ and $SR = PQ$ A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel. ; Hence PQRS is parallelogram. ; If $\angle RSP = 30°$, then find $\angle RQP$. **SOLUTION**:$\angle RQP = 30°$ (Opposite angles of a parallelogram are equal.) If SP = 3 cm, Find the RQ. ; Solution: RQ = 3cm (Opposite sides of a parallelogram are equal.) OR Find the value of $\angle R$ and $\angle S$; if $\angle P : \angle Q = 1 : 4$. **SOLUTION**: Since PQRS is a parallelogram, opposite angles are equal. $\Rightarrow \angle P = \angle R$ and $\angle Q = \angle S$ Also, $\angle P : \angle Q = 1 : 4 \Rightarrow \angle P = \angle R = k$ and $\angle Q = \angle S = 4k$ Now, $\angle P + \angle Q + \angle R + \angle S = 360°$ (Angle sum property of quadrilateral) $\Rightarrow k + 4k + k + 4k = 360° \Rightarrow 10k = 360° \Rightarrow k = 36°$ Hence, $\angle R = k = 36°$ and $\angle S = 4k = 144°$.	
3	Rajan is studying in IX standard. His father purchased a plot which is in a square shape. After visiting the land, few questions came in his mind. Give answers to his questions by looking at the figure. What is the measure of $\angle AOB$? **SOLUTION**:$\angle AOB = 90^0$ (diagonals of square bisect each other at right angle) Which is the correct congruence rule applicable to prove $\Delta ABC \cong \Delta BAD$ **SOLUTION**: In $\Delta ABC \cong \Delta BAD$ AB = AB (common) AC = BD (diagonals are equal) AD = BC (opp sides are equal)$\Delta ABC \cong \Delta BAD$ (By SSS rule) If OB = 5cm, find the value of BD **SOLUTION**: Diagonal of square bisect each other BD = 2OB= 2 x 5= 10cm Or c) If OA = 3 cm , find the value of OC. **SOLUTION**: Diagonal of square bisect each other OA = OC = 3cm	
	HOTS	
1	L,M,N and K are the midpoint of sides BC,CD, DA and AB respectively of a square ABCD. Prove that DL, DK, BM and BN enclose a rhombus. **SOLUTION**: BK = DM (halves of equal sides) Therefore, BM ll DK, Similarly BN ll DL Also $\Delta ABN \cong \Delta ADK$ (By SAS rule) =>$\angle 1 = \angle 2$ $\Delta PND \cong \Delta PKB$ (By AAS rule) =>PB = PD, Therefore ,DQBP is a rhombus	
2	PQRS is a parallelogram. PS is produced to M so that SM= SR and MR is produced to meet PQ produced at N. Prove that QN = QR. **SOLUTION**: In ΔSMR, SM = SR =>$\angle 1 = \angle 2$ (Angles opp to equal sides are equal =>$\angle 1 = \angle 3$ (QR ll PM, corresponding angles are equal)	

	Similarly =>∠2 = ∠4 (corresponding angles) =>∠3 = ∠4, Hence, In ΔQRN, QN = QR	
3	In a rectangle, one diagonal is inclined to one of its sides at 25°. Measure the acute angle between the two diagonals. **SOLUTION**: Let ABCD be a rectangle where AC and BD are the two diagonals which are intersecting at point O. ∠BDC = 25^0(Given) ; ∠BDA = 90-25 = 65^0 ; ∠DAC = ∠BDA (ΔDAC≅ ΔBDA) So, ∠DOA = acute angle between the diagonals= 180- 65- 65 = 50^0	
4	The sides AD and BC of a quadrilateral are produced as shown in the figure. Prove that x= $\frac{(a+b)}{2}$ **SOLUTION**:∠a + ∠ADC = 180^0 (Linear pair) ; Similarly ∠b + ∠BCD = 180^0 ; Adding a +b + ∠BCD +∠ADC = 360^0 (1) ; But x + x+ ∠ADC + ∠BCD = 360^0 (2) From (1) and (2) a +b + ∠BCD +∠ADC = x +x + ∠BCD +∠ADC ; x +x = a+b ; x= $\frac{(a+b)}{2}$	
5	In the fig. ABCD is a square, diagonal BD is extended through D to E. AD=DE and AD is drawn as shown in fig. What is the measure of ∠DAE? **SOLUTION**: ABCD is a square AD= DE (Given) ; ∠1 = ∠2 (angles opp to equal sides are equal) ∠ABD = ∠BDA =$\frac{90}{2}$(Diagonal of a square bisect an angle) ∠ADB + ∠ADE= 180^0 (Linear Pair) ; 45 + ∠ADE= 180^0 ∠ADE= 180^0- 45 = 135 In triangle ADE∠1 +∠2 + ∠ADE= 180^0 ; 2∠DAE + 135= 180^0 ∠DAE= (180^0-135)/2 = 22.5	
	EXERCISE	
	MULTIPLE CHOICE QUESTIONS (1 MARKS)	
1	Diagonals of a parallelogram ABCD intersect at O. If ∠BOC = 90º and ∠BDC = 50º, then ∠OAB is (A) 90º (B) 50º (C) 40º (D) 10º	
2	If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is a (A) rhombus (B) parallelogram (C) trapezium (D) kite	
3	If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a (A) rectangle (B) rhombus (C) parallelogram (D) quadrilateral whose opposite angles are supplementary	
4	If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form (A) a square (B) a rhombus (C) a rectangle (D) any other parallelogram	
5	D and E are the mid-points of the sides AB and AC of ΔABC and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is (A) a square (B)a rectangle (C) a rhombus (D) a parallelogram	
	ASSERTION (A):AND REASONING In the following questions, a statement of Assertion (A):(A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
1	Assertion: ABCD is a parallelogram in which P,Q,R and S are the midpoint of AB,BC,CD and DA respectively then PQRS is a parallelogram Reason: The line joining the midpoint of two sides of the triangle parallel to the third side and half of it.	

2	Assertion: In a rhombus ABCD the diagonal AC bisects $\angle A$ as well as $\angle C$ Reason: The diagonal of rhombus bisects each other at right angle.	
3	Assertion: Every parallelogram is a rectangle. Reason: The angle bisector of parallelogram forms a rectangle	
4	Assertion: The diagonal of parallelogram bisects each other Reason: If the diagonal of a parallelogram are equal and bisect each other at right angle then the parallelogram is a square.	
5	Assertion:the angles of a quadrilateral are110º, 80º, 70º and 95º Reason: The sum of angle of a quadrilateral is 360^0.	
	VERY SHORT ANSWER QUESTIONS (2 MARKS)	
1	ABCD and AEFG are two parallelograms. If ∠C = 55º, determine ∠F	
2	Diagonals of a quadrilateral ABCD bisect each other. If ∠A = 35º, determine ∠B.	
3	In ΔABC, AB = 5 cm, BC = 8 cm and CA = 7 cm. If D and E are respectively the mid-points of AB and BC, determine the length of DE.	
4	One angle of a quadrilateral is of 108º and the remaining three angles are equal. Find each of the three equal angles	
5	ABCD is a trapezium in which AB || DC and ∠A = ∠B = 45º. Find angles C and D of the trapezium.	
	SHORT ANSWER QUESTIONS (3 MARKS)	
1	Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.	
2	ABCD is a rectangle in which diagonal BD bisects ∠B. Show that ABCD is a square.	
3	E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD. Prove that EF || AB and EF = ½ (AB + CD).	
4	ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.	
5	The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram	
	LONG ANSWER QUESTIONS (5 MARKS)	
1	A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.	
2	P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram	
3	ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that ∠A = ∠B and ∠C = ∠D.	
4	In Fig, AB || DE, AB = DE, AC || DF and AC = DF. Prove that BC || EF and BC = EF.	
5	E is the mid-point of a median AD of ΔABC and BE is produced to meet AC at F. Show that AF = $\frac{1}{3}$ AC.	
	CASE STUDY QUESTIONS (4 MARKS)	
1	During Maths lab activity, each students were given with four broomsticks of length 8cm, 8cm, 5cm and 5cm to make different types of quadrilateral. Using above information, answer the following questions a) How many quadrilaterals can be formed using these sticks? (1) b) Name the quadrilaterals formed using these sticks. (1) c) One of the students made a parallelogram using the sticks as shown in the figure. What is the relation between x and y if BC< CD (2) OR	 

d) If P Q R and S are respectively midpoints of sides AB, BC, CD, and DA of quadrilateral ABCD in which AC=BD and AC⊥BD , PQRS is a _______. (2)

2 There is a Diwali Celebration in ABC School, New Delhi. Students are asked to prepare a rangoli in the form of a triangle. They made a rangoli in the form of triangle ABC with dimensions of triangle ABC are 26cm, 28cm and 25cm



a) In fig, R is the midpoint of AB and RQ ll BC then AQ is equal to which side.
b) In fig R and Q are midpoint of side AB and AC respectively. What is the length of RQ?
c) If garland is to be placed along the side of the triangle QPR which is formed by joining the midpoint, what is the length of the garland?
OR
d) In the above figure, R, P and Q are the midpoint of side AB, BC and AC respectively. What is the area of triangle PQR?

3 Due to frequent robberies in the colony during night. The secretary with the members together decides to attach more lights besides the street light set by municipality. There are poles on which lights are attached.
These 4 poles are connected to each other through wire and they form a quadrilateral. Light from pole B focus light on mid-point G of wire between pole C and B, from pole C focus light on mid-point F of wire between pole C and pole D. Similarly pole D and pole A focus light on the mid-point E and H respectively.
On the basis of the above information, solve the following questions:

a) If BD is the bisector of ∠B then prove that I is the mid-point of AC.
b) Is it true that every parallelogram is a rectangle?
c) Prove that quadrilateral EFGH is a parallelogram.
d) OR
e) If the distance between pole B and pole D is 50m then what is the length of GF?

HOTS

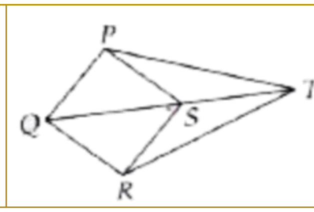
1 If the bisectors of angles of a quadrilateral enclose a rectangle, then show that it is a parallelogram.

2 In a ΔABC, DE is parallel to BC and D is the mid-point of side AB. Find the perimeter of ΔABC when AE = 4.5 cm, DE = 5 cm and DB = 3.5 cm

3 If an angle of a parallelogram is 4/5 of it adjacent angle, then find the measures of all the angles of the parallelogram.

4 The lengths of diagonals of a rhombus are 24 cm and 18 cm respectively. Find the length of each side of the rhombus.

5 In figure PQ= QR= RS= SP = SQ = 6cm and PT = RT = 14cm. Find the length of ST



ANSWERS

MCQs : 1. C 2.C 3.D 4.C 5.D

ASSERTION (A):AND REASONING (1 MARKS) : 1.A 2.B 3.D 4.D 5.D

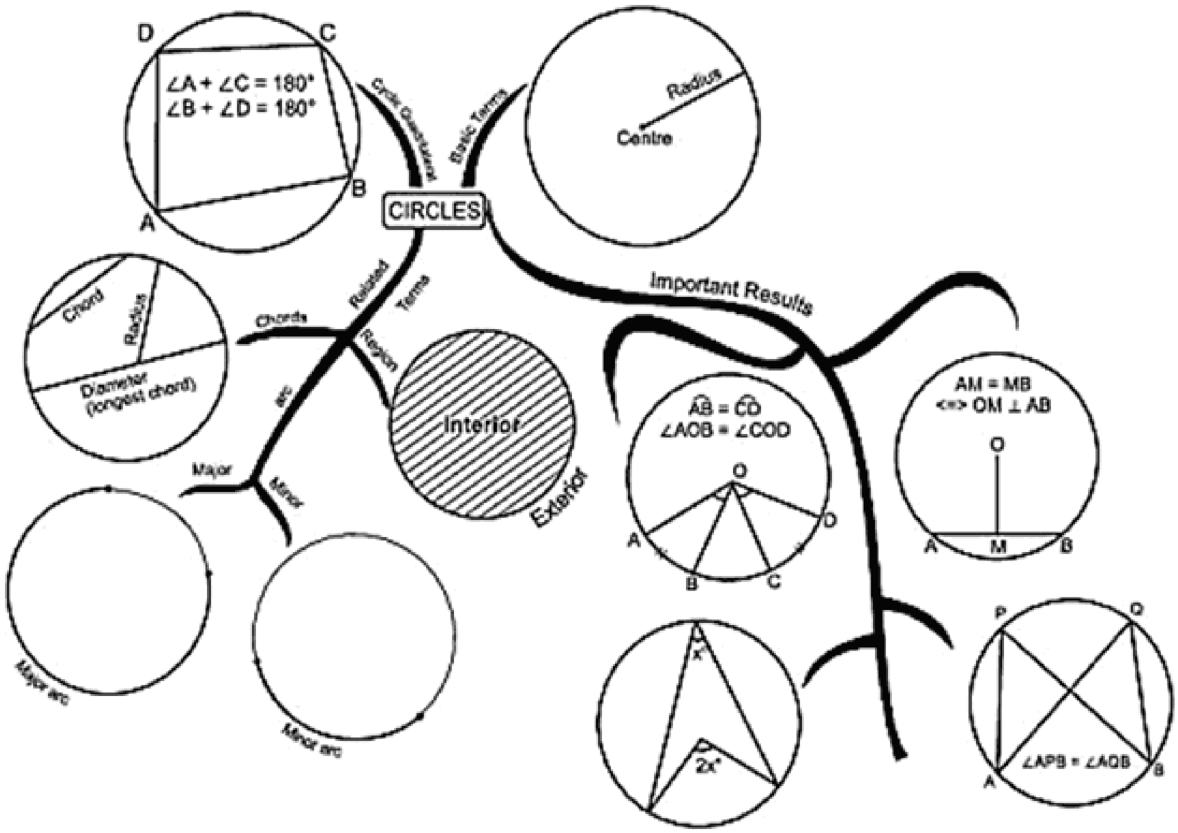
VERY SHORT ANSWER (2 MARKS) : 1. 55^0 2. 145^0 3. 3.5cm 4 . 24^0 5. Both angles are equal 135^0

CASE STUDY QUESTIONS :

A) Three types of quadrilateral can be formed
Parallelogram, Rectangle and Kite
x<y OR C) Square
A) QC B) 14cm C) 39.5cm OR C) ¼ ar(ABC) OR C) 25m

CHAPTER – 09
CIRCLES

MIND MAPPING

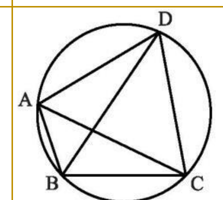
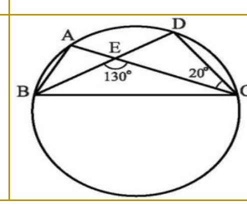


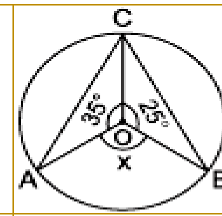
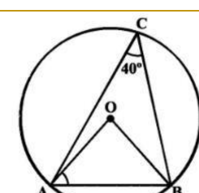
GIST/SUMMARY OF THE LESSON

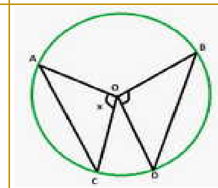
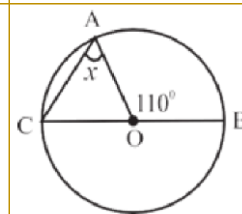
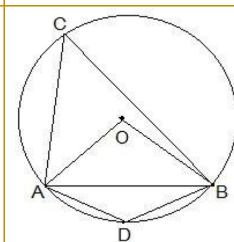
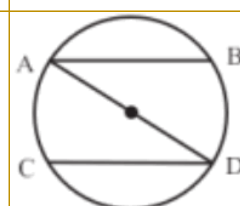
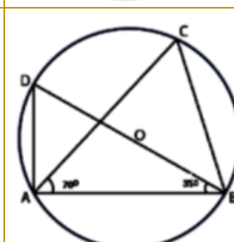
- The collection of all points lying inside and on the circle C(O,r) is called a circular disc with the centre O and radius r.
- Circles having the same centre and different radii are said to be concentric circles.
- A continuous piece of a circle is called an arc of the circle.
- The length of an arc is the length of the fine thread which just covers it completely.
- Let's C(O,r) be any circle. Then any angle whose vertex is O is called the centre angle.
- A minor arc of a circle is the collection of those points of the circle that lie on and also inside arc Central angle.
- A major arc of a circle is the set of points of the circle that lie on or outside a central angle.
- A line segment joining any two points on a circle is called a chord of the circle.
- A chord passing through the centre of a circle is known as its diameter.
- A diameter of a circle divides it into two equal parts which are arcs. Each of these two arcs is called a semicircle.
- Two circles are said to be congruent, if either of them can be superposed on the other so as to cover it exactly.
- If two arcs of a circle are congruent then corresponding chords are equal.
- If two chords of a circle are equal, then their corresponding arcs are congruent.
- The perpendicular from the centre of a circle to a chord bisect the chord.
- The line segment joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.
- There is one and only circle passing through three non-collinear points.
- If two circles intersect in two points, then the line through the centre is perpendicular to the common chord.
- Equal chords of a circle are equidistant from the centre.
- Chords of a circle which are equidistant from the centre are equal.
- Equal chords of a circle subtend equal angle at the centre.
- If the angles subtended by two chords of a circle at the centre are equal, the chords are equal.
- Of any two chords of a circle, the larger chord is nearer to the centre.
- Of any two chords of a circle the chord nearer to the centre is larger.

- The angle subtended by an arc of a circle at the centre is doubled angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- The angle in a semicircle is a right angle.
- If all vertices of a quadrilateral lie and a circle, it is called a cyclic quadrilateral.
- The opposite angles of a cyclic quadrilateral are supplementary.
- If the sum of any pair of opposite angles of a quadrilateral is 180º, then it is cyclic.
- The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.
- If two sides affect cyclic quadrilateral are parallel then the remaining two sides are equal and the diagonals are also equal.
- If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.
- An isosceles trapezium is cyclic.
- Altitudes of a triangle are concurrent.

MULTIPLE CHOICE QUESTIONS (1 MARK QUESTIONS)

1	If in a circle with centre O, AB and CD are two diameters perpendicular to each other, Then the relation between the lengths of chords AC and BD is…. (A)AC < BD (B) AC > BD (C) AC = ½ BD (D) AC = BD	**ANS: D**
2	Choose the one which is incorrect. (A) The length of the perpendicular from a point to a line is the distance of the line from the point. (B) If the point lies on the line, the distance of the line from the point is Zero. (C) The longer chord is nearer to the centre than the smaller chord. (D) The length of any diameter of a circle from the centre of the circle is equal to itself.	**ANS:D**
3	The sum of both pair of opposite angles of a quadrilateral is ……….. (A)360º (B) 180 º (C) 90º (D) 100 º **ANS:B**	
4	InbelowFig,ABCDisacyclicquadrilateralinwhichACandBDareitsdiagonals. If∠DBC= 60° and ∠BAC = 30°, find ∠BCD. (A) 80º(B)90º (C) 60º(D)none of these	**ANS: B**
5	Segment of a circle is the region between an arc andof the circle. (A)diameter (B)semicircle(C)chord (D)sector	**ANS: C**
6	The line drawn through the centre of a circle to a chord is perpendicular to the chord. (A)trisect (B)bisect (C)coincide (D) none of these.	**ANS: B**
7	Thelengthofachordofcircleofradius10cmis12cm.Determinethedistanceofthechord from the centre…….. (A)7 cm (B) 8 cm (C)6 cm (D)5cm	**ANS: B**
8	If AB is a chord of a circle, P and Q are the two points on the circle different from A and B, then ∠APB - ∠AQB = 90º (B) ∠APB + ∠AQB = 180º (C)∠APB - ∠AQB = 0º (D)∠APB + ∠AQB = 100º	**ANS: C**
9	In the above sided Fig.,A,B,C and their four points onacircle.AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC. (A)150º (B)110º (C) 90º(D)1000 **ANS:B**	

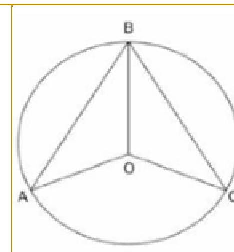
10	In the given figure, O is the centre of the circle. The value of x is (A) 300° (B) 70° (C) 10° (D) 210° **ANS:A**	
	ASSERTION – REASONING BASED QUESTIONS A) both Assertion and reason are correct and reason is correct explanation for Assertion B) both Assertion and reason are correct but reason is not correct explanation for Assertion C) Assertion is correct but reason is false D) both Assertion and reason are false	
1	Assertion(A): There can be infinite numbers of equal chords of a circle. Reason(R) : A circle has only finite number of equal chords.	**ANS:C**
2	Assertion(A): A circle has only one centre. Reason(R): A circle is defined as the path traced by a moving point that always remains at a fixed distance from a given fixed point	**ANS:A**
3	Assertion(A): With a given centre and a given radius, only one circle can be drawn. Reason(R): Because circles have only one parameter	**ANS:A**
4	Assertion(A): Two diameters of a circle will necessarily intersect. Reason(R): diameters will always intersect each other at the centre of the circle.	**ANS:A**
5	Assertion(A): A circle of radius 3cm can be drawn through two points A,B such that AB=6 cm. Reason(R): Diameter of circle =2x radius of circle	**ANS:A**
6	Assertion(A): If the sum of the circumferences of two circles with radii R1 and R2 is equal to the circumference of a circle of radius R, then R1+R2=R Reason(R): circumference of circle with radius =2ΠR	**ANS:A**
7	Assertion(A): There is only one tangent at a point of the circle. Reason(R): The tangents drawn at the extremities of the diameter of a circle are parallel	**ANS:B**
8	Assertion(A): The angle subtended by the diameter of a semi-circle is180° Reason: The semicircle is half of the circle; hence the diameter of the semicircle will be a straight-line subtending 180 degrees	**ANS:A**
9	Assertion(A): Angles in the same segment are equal. Reason(R): angle in a semi-circle is a Right angle.	**ANS:C**
10	Assertion(A): A semi-circle is one fourth part of the circle Reason(R): A semi-circle is obtained when a circle is divided into two unequal parts.	**ANS:D**
	VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)	
1	In the given figure,∠ACB=400.Find∠OAB. **ANS**. Ans: Given, ∠ACB = 40° We have to find ∠OAB We know that in a circle the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle. ∠AOB = 2∠ACB ∠AOB = 2(40°) ∠AOB = 80° ----------------------- (1)	

	Considering triangle AOB, OA = OB = radius of the circle We know that the angles opposite to the equal sides are equal. ∠OBA = ∠OAB ------------------- (2) By angle sum property of the triangle, ∠AOB + ∠OBA + ∠OAB = 180° From (1) and (2), 80° + ∠OAB + ∠OAB = 180° 2∠OAB = 180° - 80° 2∠OAB = 100° ∠OAB = 100°/2 Therefore, ∠OAB = 50°	
2	Prove that "Equal chords of a circle subtend equal angles at the centre". Given: A circle with centre O. AB and CD are equal chords of circle i.e. AB = CD To Prove : ∠AOB = ∠DOC Proof : In ΔAOB & ΔDOC AO = OD AB = CD OB = OC ∴ ΔAOB ≅ Δ DOC ∴ ∠ AOB = ∠ DOC Hence, Proved.	
3	Find the value of 'x' in the given figure. **ANS**:∠AOB=110^0, x = 110/2, x = 55^0	
4	A chord of a circle is equal to the radius of the circle. Find the subtended by the chord at any point on minor arc and also at any point on major arc. **ANS**: Here, the chord AB is equal to the radius of the circle. In the above diagram, OA and OB are the two radii of the circle. Now, consider the ΔOAB. Here, AB = OA = OB = radius of the circle So, it can be said that ΔOAB has all equal sides, and thus, it is an equilateral triangle. ∴ ∠AOC = 60° And, ∠ACB = ½ ∠AOB So, ∠ACB = ½ × 60° = 30° Now, since ACBD is a cyclic quadrilateral, ∠ADB +∠ACB = 180° (They are the opposite angles of a cyclic quadrilateral) So, ∠ADB = 180°-30° = 150° So, the angle subtended by the chord at a point on the minor arc and also at a point on the major arc is 150° and 30°, respectively.	
5	In the given figure, AB and CD are parallel chords and the length of AC arc is 14cm. What is the length of BD arc? **ANS**: Perpendicular distance between two parallel chord is same at any point, so AC=BD=14CM	
6	In figure,O is centre of circle. If, ∠ABD = 35° and ∠BAC = 70°, Find ∠ACB. **ANS**: We know that BD is the diameter of the circle. Angle in a semicircle is a right angle ∠BAD = 90° Consider △ BAD Using the angle sum property ∠ADB + ∠BAD + ∠ABD = 180° By substituting the values ∠ADB + 90o°+ 35° = 180° On further calculation ∠ADB = 180° – 90°– 35° By subtraction ∠ADB = 180° – 125° So we get ∠ADB = 55° We know that the angles in the same segment of a circle are equal ∠ACB = ∠ADB = 55° So we get ∠ACB = 55° Therefore, ∠ACB = 55°.	
7	AB and CB are two chords of circle. Prove that BO bisects ∠ABC. **SOLUTION**: Given: In the figure, AB = CB and O is the centre of the circle. To Prove: BO bisects ∠ABC. Construction: Join OA and OC.	

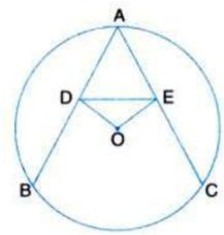
Proof : In ΔOAB and ΔOCB, OA = OC [Radii of the same circle] AB = CB [Given] OB = OB [Common]

∴ ΔOAB ≅ ΔOCB [By SSS]

∴ ∠ABO = ∠CBO [By CPCT] ⇒ BO bisects ∠ABC.



8 Prove that ADE is an isosceles triangle if AB=AC with OD ⊥ AB and OE ⊥ AC



SOLUTION: Given that AB and AC is two equal chords of the circle with centre O

OD ⊥ AB and OE ⊥ AC

OD =OE (Equal chords are equidistant)

∠ODE = ∠OED………..(i)

∠ODA =∠OEA……..(ii)

Subtracting (i) from (ii) ∠ODA - ∠ODE = ∠OEA -∠OED

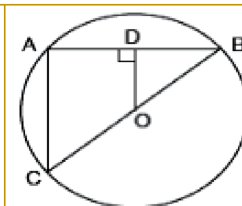
∠ADE = ∠AED

AD = AE

Δ ADE is an isosceles triangle.

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1 If BC is a diameter of a circle of centre O and OD is perpendicular to the chord AB of a circle, show that CA = 2OD.



SOLUTION: Since OD ⊥ AB

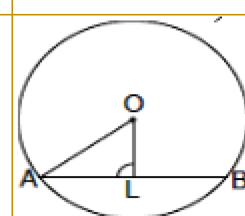
∴ D is the mid-point of AB (perpendicular drawn from the centre to a chord bisects the chord) O is centre ⇒ O is the mid-point of BC.

In ΔABC, O and D are the mid-points of BC and AB, respectively.

∴ OD || AC and OD = $\frac{1}{2}$ AC _ (mid-point theorem)

∴ *CA = 2OD*

2 In a circle of radius 5 cm having centre O, OL is drawn perpendicular to the chord AB. If OL = 3 cm, find the length of AB.



SOLUTION: Let AB be a chord of circle having centre O.

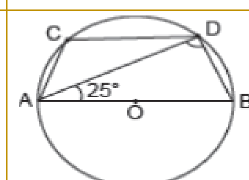
OL ⊥ AB and OA is the radius of the circle. So, OA = 5 cm

In ΔOAL, Applying Pythagoras theorem, we have $OA^2 = AL^2 + OL^2$

⇒ $(5)^2 = AL^2 + (3)^2$ ⇒ $25 = AL^2 + 9$ ⇒ $AL^2 = 25 – 9 = 16$ ⇒ $AL = \sqrt{16}$ cm = 4 cm

As we know that perpendicular drawn from the centre to the chord bisects the chord. ∴ AB = 2 (AL) = 2 (4) = 8 cm

3 3. In the given figure, AB is diameter of the circle with centre O and CD || AB. If ∠DAB = 25°, then find the measure of ∠CAD.



SOLUTION: ∠ADB = 90° [Angle in a semicircle]

∠BAD = ∠ADC = 25° [Alternate interior angles] ∴ ∠BDC = 90° + 25° = 115°

Now, ∠BDC + ∠BAC = 180° [opp. ∠s of cyclic quadrilateral] ⇒ 115° + ∠BAC = 180°

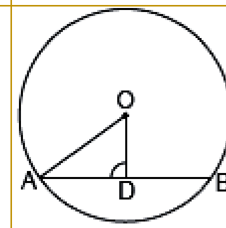
⇒∠BAC = 180° -115° = 65°

Now, ∠BAC = ∠BAD + ∠CAD

⇒ 65° = 25° + ∠CAD

$\therefore\ \angle CAD = 65° - 25° = 40°$

4 Distance of a chord AB of a circle from the centre is 12 cm and length of the chord AB is 10 cm. Find the diameter of the circle.



SOLUTION: Let in a circle having centre O, AB be the chord of length 10 cm.

Distance of chord AB from centre is OD $\therefore$ OD = 12 cm

OA = radius of circle In ΔAOD, OD $\perp$ AB

As we know perpendicular drawn from the centre to the chord bisects the chord. $\therefore$ AD = $\frac{1}{2}$ AB = $\frac{1}{2} \times 10 = 5$ cm

Now, in ΔAOD

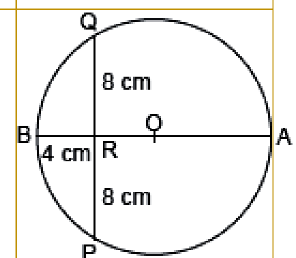
$OA^2 = AD^2 + OD^2$(Applying Pythagoras theorem) $\Rightarrow$ $OA^2 = (5)^2 + (12)^2 = 25 + 144$

$\Rightarrow OA^2 = 169$

OA = 13

$\Rightarrow diameter = 2\ (OA) = 2 \times 13\ cm = 26\ cm$

5 In the given figure, diameter AB of circle with centre O bisects the chord PQ. If PR = QR = 8 cm and RB = 4 cm, find the radius of the circle.



SOLUTION: Here AB is the diameter of the circle and AB bisects PQ.

Also, PR = RQ = 8 cm

OB, OP and OQ are radii of the circle. $\Rightarrow$ OB = OQ = OP = r (say)

Consider OR = x cm

OB = OR + BR = $(x + 4)$ cm

$\Rightarrow\ x = (OB - 4) = (r - 4)$ cm

As R is the mid point of PQ.

Also, RO is a line segment passing through centre O.

$\therefore$ OR $\perp$ PQ (line segment joining mid pt. of chord to the centre of circle is perpendicular to the chord)

In right-angled ΔOQR

$OQ^2 = OR^2 + QR^2$ (by Pythagoras theorem)

$= x^2 + (8)^2$

$\Rightarrow\ OQ^2 = x^2 + 64$

$r^2 = (r - 4)^2 + 64$ [Put $x = (r - 4)$]

$\Rightarrow r^2 = r^2 + 16$

$- 8\ r + 64 \Rightarrow$

$8r = 80$

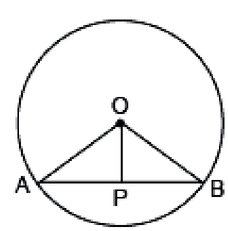
$\Rightarrow\ r = 10$ cm

6 Prove that the line drawn through the centre of a circle to the mid-point of a chord is perpendicular to the chord.

SOLUTION: Ans:

In circle, O is the centre and OP bisects the chord AB.

Join OA and OB (radius of circle)



In ΔAPO and ΔBPO

OA = OB (radius of circle)

OP = OP (common)

AP = BP (given)

$\therefore\ \Delta APO \cong \Delta BPO$ (by SSS)

$\therefore\ \angle OPA = \angle OPB$ ...(i) (by CPCT)

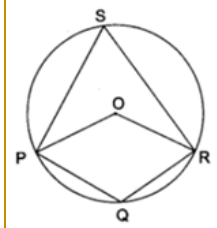
Also $\angle OPA + \angle OPB = 180°$ (Linear pair angles)

$\Rightarrow\ \angle OPA + \angle OPA = 180°$ [Using (i)]

$2\angle OPA = 180°$

$\Rightarrow \quad \angle OPA = 90° \Rightarrow OP \perp AB$.

7 Prove that the sum of both pair of the opposite angles of a cyclic quadrilateral is 180°.



To prove: $\angle P + \angle R = 180°$

$\angle Q + \angle S = 180°$

Construction: Join O, P and O, R.

Proof: By Central angle theorem

$\angle POR = 2\angle PSR$ [For the same segment angle formed at centre of circle is twice the angle formed at remaining part of the circle]

$\Rightarrow \quad \angle PSR = \frac{1}{2}\angle POR \quad ...(1)$

and Reflex $\angle POR = 2\angle PQR$

$\Rightarrow \quad \angle PQR = \frac{1}{2}$ Reflex $\angle POR \quad ...(2)$

Adding Equation (1) and (2)

$\angle PSR + \angle PQR = \frac{1}{2}\angle POR + \frac{1}{2}$ Reflex $\angle POR$

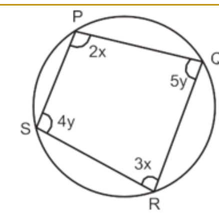
$= \frac{1}{2}[\angle POR + \text{Reflex } \angle POR]$

$= \frac{1}{2} \times 360° = 180°$

$\Rightarrow \quad \angle S + \angle Q = 180°$

Similarly, $\angle P + \angle R = 180°$

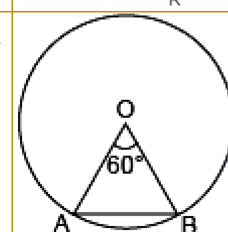
8 In the given figure, PQRS is a cyclic quadrilateral. Find the value of x and y.



Ans: $2x + 3x = 180°$, $5x = 180°$, $x = 36°$

$4y + 5y = 180°$, $9y = 180°$, $y = 20°$

9 In the given figure, chord AB subtends $\angle AOB$ equal to 60° at the centre O of the circle. If OA =5 cm. then find the length of AB.



Solution: In ΔAOB

$\angle AOB = 60°$ (Given)

$OA = OB$ (Equal radii)

$\therefore \angle OAB = \angle OBA$(Angles opposite to equal sides OA and OB) ...(i)

In ΔAOB $\angle OAB + \angle AOB + \angle OBA = 180°$(Angle sum property of triangle)

$60° + \angle OAB + \angle OAB = 180°$ ($\because \angle OAB = \angle OBA$, using (i))

$\Rightarrow 2\angle OAB = 180° - 60°$

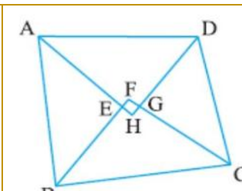
$\Rightarrow \quad \angle OAB = 60° \Rightarrow \angle OBA = 60°$

$\therefore \Delta AOB$ is an equilateral triangle Hence OA = OB = AB

$\Rightarrow$ AB = 5 cm (as OA = 5 cm)

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1 Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.



Solution:

Given: ABCD is a quadrilateral, AH, BF, CF and DH are the angle bisectors of internal angles A, B, C and D.

These bisectors form a quadrilateral EFGH. To Prove: EFGH is cyclic

Proof: In AEB.

∠EAB + ∠ABE + ∠AEB = 180° (Sum of angles of ΔABC) ⇒ AEB = 180° – (∠EAB + ∠ABE) ...(i)

Also ∠AEB = ∠FEH ...(ii) (Vertically opposite angles) By equating (i) and (ii) ∠FEH = 180° – (∠EAB + ∠ABE) ...(iii) Similarly, in ΔGDC ∠FGH = 180° – (∠GDC + ∠GCD)...(iv)

Adding (iii) and (iv)

∠FEH + ∠FGH = 360° – (∠EAB + ∠ABE + ∠GDC + ∠GCD)

$= 360° - \frac{1}{2}(\angle BAD + \angle ABC + \angle ADC + \angle BCD)$

(As AH, BF, CF and HD are bisectors of ∠ *A, ∠B, ∠C, ∠D)*

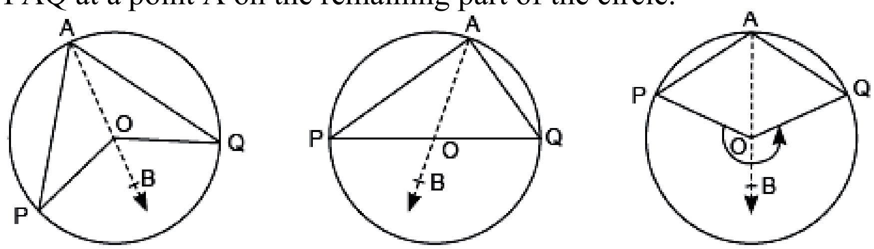
$= 360° - \frac{1}{2} \times 360°$ (Sum of angles of quadrilateral, ABCD)

∠FEH + ∠FGH = 360° – 180° = 180°

⇒ FEHG is a cyclic quadrilateral. (If the sum of opposite angles of quadrilateral is 180° then it is cyclic)

2 Prove that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

SOLUTION: Given : Given an arc PQ of a circle subtending angles POQ at the centre O and PAQ at a point A on the remaining part of the circle.



case (*i*) case (*ii*) case (*iii*)

To Prove: POQ = 2 PAQ

Construction: Join AO and extends it to B.

Proof: Consider three cases case (i): When arc PQ is a minor arc. case (ii): When arc PQ is a semicircle. case (iii): When arc PQ is a major arc. In all the three cases

Taking ΔAOQ

∠BOQ = ∠OAQ + ∠OQA (Exterior angle of Δ is equal to the sum of interior opposite angles) Also OA = OQ (radii of circle)

⇒ OAQ = ∠OQA (Angles opposite to equal sides) ⇒ ∠ BOQ = ∠OAQ + ∠OAQ

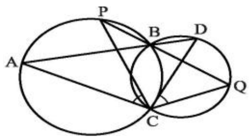
⇒ BOQ = 2∠OAQ ...(i)

Similarly ∠BOP = 2∠OAP ...(ii)

Adding (i) and (ii) we have

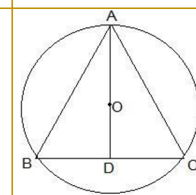
$\angle BOQ + \angle BOP = 2\angle OAQ + 2\angle OAP = 2(\angle OAQ + \angle OAP) \Rightarrow \quad POQ = 2\angle PAQ$
Specially for case (iii) we can write reflex $\angle POQ = 2\angle PAQ$

3 Two circles intersect a ttwo points B and C. Through B, two line-segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$.



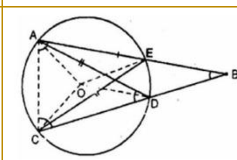
SOLUTION: **Ans:**
Join AP and DQ
$\angle ACP$ and $\angle ABP$ lie in the same segment.
Similarly, $\angle DCQ$ and $\angle DBQ$ lie in the same segment.
We know that angles in the same segment of a circle are equal.
So, we get $\angle ACP = \angle ABP$ and $\angle QCD = \angle QBD$
Also, $\angle QBD = \angle ABP$ (Vertically opposite angles)
Therefore, $\angle ACP = \angle QCD$.

4 A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.



SOLUTION: Here, the positions of Ankur, Syed and David are represented as A, B and C, respectively. Since they are sitting at equal distances, the triangle ABC will form an equilateral triangle.
$AD \perp BC$ is drawn. Now, AD is the median of ΔABC, and it passes through the centre O.
Also, O is the centroid of the ΔABC. OA is the radius of the triangle.
$OA = 2/3\ AD$
Let the side of a triangle a metres, then $BD = a/2$ m.
Applying Pythagoras' theorem in ΔABD,
$AB^2 = BD^2+AD^2$
$\Rightarrow AD^2 = AB^2 - BD^2$
$\Rightarrow AD^2 = a^2 - (a/2)^2$
$\Rightarrow AD^2 = 3a^2/4$
$\Rightarrow AD = \sqrt{3}a/2$
$OA = 2/3\ AD$
$20 \text{ m} = 2/3 \times \sqrt{3}a/2$
$a = 20\sqrt{3}$ m
So, the length of the string of the toy is $20\sqrt{3}$ m.

5 Let vertex of an $\angle ABC$ be located outside a circle and let the sides of The angle intersect chords AD and CE. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.



SOLUTION:
Vertex B of $\angle ABC$ is located outside the circle with centre O.
Side AB intersects chord CE at point E and side BC intersects chord AD at point D.
We have to prove that $\angle ABC = [\angle AOC - \angle DOE]$
Join OA, OC, OE and OD.
Now $\angle AOC = 2\angle AEC$ [Angle subtended by an arc at the centre of the circle is twice the angle subtended by the same arc at any point in the alternate segment of the circle]
$\angle AOC = \angle AEC$……….(i)
Similarly $\angle DOE = \angle DCE$……..ii)
Subtracting eq. (ii) from eq. (i), ½ $(\angle AOC - \angle DOE) = \angle AEC - \angle DCE$ ……(iii)

Now $\angle AEC = \angle ADC$ (Angles in same segment in circle) .....(iv)
Also $\angle DCE = \angle DAE$
Using eq. (iv) and (v) in eq.(iii), $1/2[\angle AOC - \angle DOE] = (\angle DAE + \angle ABD) - \angle DAE$ (exterior property)
$[\angle AOC - \angle DOE] = \angle ABD$

Or [∠AOC-∠DOE] = ∠ABC Hence proved.

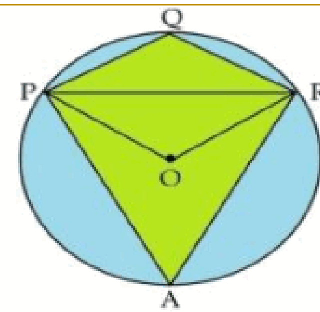
CASE BASED QUESTIONS (4 MARKS QUESTIONS)

1 Four Friends Rima, Mohan, Sohan and Sita are sitting on the circumference of a circular park full of water. Their locations are marked by points A, P, Q and R such that the APQR is a quadrilateral with greenery. Rohit joins them and sits at the centre of the circular park,so he is equidistant from all the other friends. His position is marked as O.They are sitting in such a way that ∠PQR = 110°.

a) What is measure of reflex ∠POR?(1)

b) What is the measure of ∠PAR?(2)

c) What is measure of ∠POR?(1)



SOLUTION: Ans: (i) Reflex ∠POR = 2∠PQR (∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.)

∴ Reflex ∠POR = 2 × 110° = 220°

(ii) ∠PAR + ∠PQR = 180° (∵ Sum of opposite angles of cyclic quadrilateral is 180°)

∴ ∠PAR = 180° – 110° = 70°

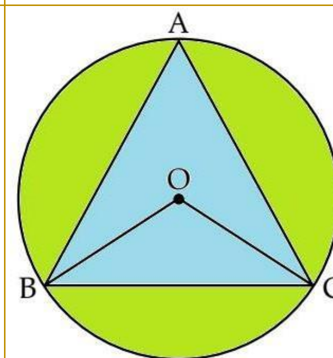
(iii) ∠POR = 2∠PAR (∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.) ∴ ∠POR = 2 × 70° = 140°

2 One triangular shaped pond is there in a park marked by ABC. Three friends are sitting positions at A, B and C. They are studying in Class IX in an International. A, B and C are equidistant from each other as shown in figure given below.

(i). What is the value of ∠BAC?

(ii). What will be the value of ∠BOC?

(iii).Which angle will be equal to ∠OBC?



SOLUTION:

Ans: (i) AB = BC = AC as per the given statement

∴ ΔABC is an equilateral triangle

∴ ∠BAC = 60° (Angles of a equilateral triangle)

(ii) ∠BOC = 2∠BAC (∵ Angle subtended by an arc at the centre is double the angle subtended by it in the remaining part of the circle.) ∴ ∠BOC = 2 × 60° = 120°

(iii) In ΔBOC, OB = OC (radii) ⇒ ∠OBC = ∠OCB (∠s opposite to equal sides are equal)

⇒ ∠OBC + ∠OCB + ∠BOC = 180° (Angle sum property of triangle)

⇒ ∠OBC + ∠OBC + 120° = 180° ⇒ 2∠OBC = 60° ⇒ ∠OBC = 30°

3 A farmer has a circular garden as shown in the picture above. He has a different type of tree , plants and flower plants in his garden. In the garden, there are two mango trees A and B at a distance of AB=10m. Similarly has two Ashok trees at the same distance of 10m as shown at C and D AB subtends ∠AOB=120° at the centre O, the perpendicular distance of AC from centre is 5m the radius of the circle is 13m.

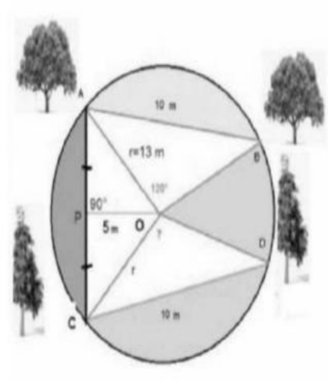
a) what is the value of ∠COD?

b) What is the distance between mango tree A and Ashok tree C?

c) What is the value of angle ∠OAB ?

d) What is the value of angle ∠OCD ?

ANSWERS : 1 (ii) 120° 2.(ii) 24m 3(iii) 30 4 (i) 30°



EXERCISE

MULTIPLE CHOICE QUESTIONS

1 AD is a diameter of a circle and AB is a chord of it. If AD = 34 cm and AB = 30 cm. Then distance

of AB from centre of the circle is
(A) 17cm. (B) 15 cm. (C) 4 cm. (D) 8 cm.

2 There is one and only one circle passing through given ………non-collinear points.
(A) two (B) three (C) four (D) five

3 A circle divides the plane, on which it lies, into ____ parts.
(A) two (B) three (C) four (D) five

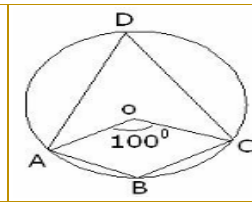
4 Angles in the same segment of a circle are _______.
(A) half (B) double (C) triple (D) equal

5 The angle in the semi circle is
(A)360º (B)270 º (C) 180º (D) 90º

ASSERTION – REASONING BASED QUESTIONS

A) both Assertion and reason are correct and reason is correct explanation for Assertion
B) both Assertion and reason are correct but reason is not correct explanation for Assertion
C) Assertion is correct but reason is false
D) both Assertion and reason are false

1 Assertion(A): circle is a plane figure
Reason(R): circle is a 2d figure and it can be drawn on a plane.



2 Assertion(A): A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
Reason(R) As we know that any chord whose length is twice as long as the radius of the circle always passes through the centre of the circle.

3 Assertion(A): The angles subtended by a chord at any two points of a circle are equal
Reason(R): Angles in the same segment of circle are equal.
4Assertion(A): A circle of radius 3 cm can be drawn through two points A, B such that AB=6cm.
Reason(R): Through three collinear points a circle can be drawn.

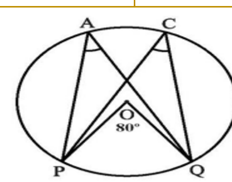
4 Assertion(A): If two chords AB and CD of a circle are each at a distance 4 cm from the centre, then AB = CD.
Reason(R): Chords equidistance from the centre of the circle are equal in length

VERY SHORT ANSWER TYPE QUESTIONS

1 Prove that a cyclic parallelogram is a rectangle.

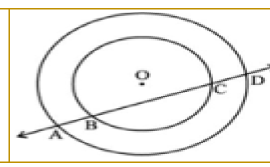
2 O is the centre of a circle and the measure of arc ABC is 100º. Find ∠ADC and ∠ABC.

3

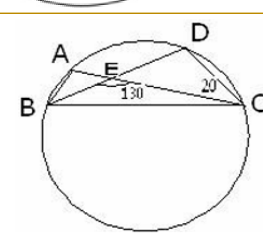


In the given figure, ∠POQ = 80º. Then find ∠PAQ.

4 Given are two concentric circles with centre O. A line cuts the circes at A, B, C ad D respectively. If AB = 10cm, then find CD .

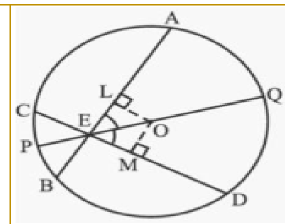


5 A, B, C and D are the four points on a circle. AC and BD intersect at point E such that ∠BEC = 130 º and ∠ECD = 20º. Find ∠BAC.



SHORT ANSWER TYPE QUESTIONS

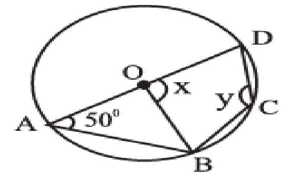
1 In the figure ABCD is a cyclic quadrilateral. AC and BD are its diagonals. If ∠DBC =55 º and ∠BAC = 55º, find ∠BCD.



2 Prove that "Equal chords of a circle subtend equal angles at the centre"

3 how that if two chords of a circle bisect each other they must be diameters.

4 In given figure O is the centre of the circle, ∠DAB = 50º, find the value of x and y.



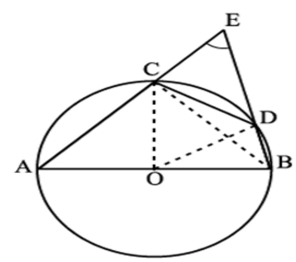
5 Prove that if opposite angles of a quadrilateral are supplementary then it is cyclic quadrilateral.

Answers: 1.∠BCD=70º 2. Proof 3. Proof 4. x=100 º y=130º 5. Proof

LONG ANSWER TYPE QUESTIONS

1 If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

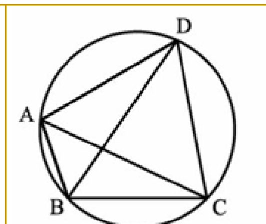
2 In the figure AB is the diameter of the circle.
CD is a Chord equal to the radius of the circle.
AC and BD when extended intersect at E.
Prove that ∠AEB = 60 º



3 Prove that "The angle subtended by an arc of a circle is double the angle subtended by it at any point in the remaining part of the circle".

4 In the non-parallel sides of a trapezium are equal, prove that it is cyclic.

5 A circular park of radius 20m is situated in a colony.
Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other.
Find the length of the string of each phone.

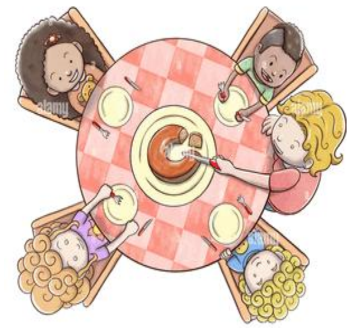
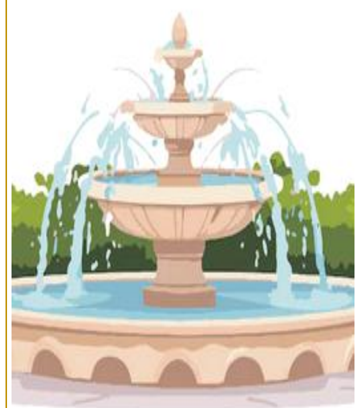


ANSWERS:

1. Proof
2. Proof
3. Proof
4. Proof
5. Use the properties of an equilateral triangle and also Pythagoras theorem.

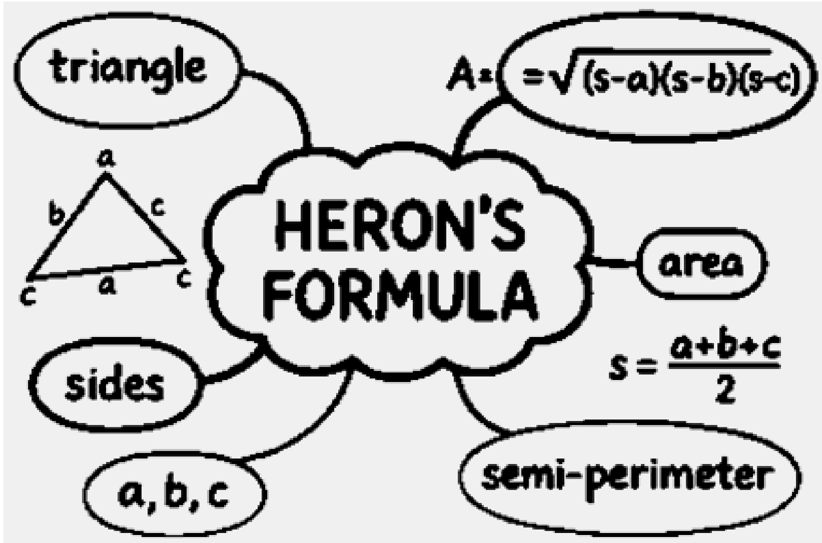
CASE BASED QUESTIONS

1 Ankit visited in a mall with his father. He sees that three shops are situated at P, Q, R as shown in the figure from where they have to purchase things according to their need. Distance between shop P and Q is 8 m, that of between shop Q and R is 10 m and between shop P and R is 6 m.
(i) Find the radius of the circle.
(ii) Measure of ∠QPR is (OR)
(iii) Area of Δ PQR is 18 sq.m (b) 20 sq.m 2 (c) 22 sq.m (d) 24 sq.m

	(iv) In figure, PSQP is known as..	
2	Four Friends are sitting in a circular table such that distance between first and third is equal to distance between second and fourth which is diameter of circular table 1. If distance between two consecutive friend is not equal and we join all points where friends are there which type of quadrilateral we get? 2.Angle subtended by first friend, second friend at the centre is – 3.If distance between two consecutive friend is equal If we join all points where friends are there which type of quadrilateral we get? (OR) 4. Which type of triangle ABC is	
3	3A circular park is there in a city in which there is a fountain in the centre, there are two trees in park from which gates are 20 m far each, there are two gates in park entrance(A) and exit gate(B) ,monika is walking on boundary of this circular park she is 15 m far from fountain. 1. distance between monika and fountain is called 2.Assume that distance between tree 1 and entrance gate is equal to distance between tree 2 and exit gate then the angle made by fountain with entrance gate and tree 1 and fountain with exit gate and tree 2 are_____. 3. If circular park is circle ,fountain is its (OR) 4. PQ in figure is (a) chord (b) diameter (c) centre (d) segment	
	EXERCISE ANSWERS	
	MCQS: 1.D 2.B 3.B 4.D 5.D **ARQS**:1.A 2.A 3.D 4.C 5.A **VSA**:1. Proof 2. ∠ADC=50º ∠ABC=130º 3. ∠PAQ=30º 4. CD=10 cm 5. ∠BAC=110º **SA** : 1. ∠BCD=70º 2. Proof 3. Proof 4. x=100 º y=130º 5. Proof **LA** : 1. Proof 2. Proof 3. Proof 4. Proof 5. Use the properties of an equilateral triangle and also Pythagoras theorem.	
	CBQ 1. 1. (a) 5 m 2. (b) 90° 3. (d) 24 sq.m (b) Minor segment 2.1.b) Rectangle 2. c) 90 3. c) square 4. a) right angled triangle 3. 1.a) Radius 2. b) equal 3. c) Centre 4. a) chord	
	HOTS 1) ∠BAC=60º 2)Proof 3) 30° 4) 40° 5) Proof	

CHAPTER – 10
HERON'S FORMULA

MIND MAPPING



GIST/SUMMARY OF THE LESSON:

- Heron's Formula
- Application of Heron's formula

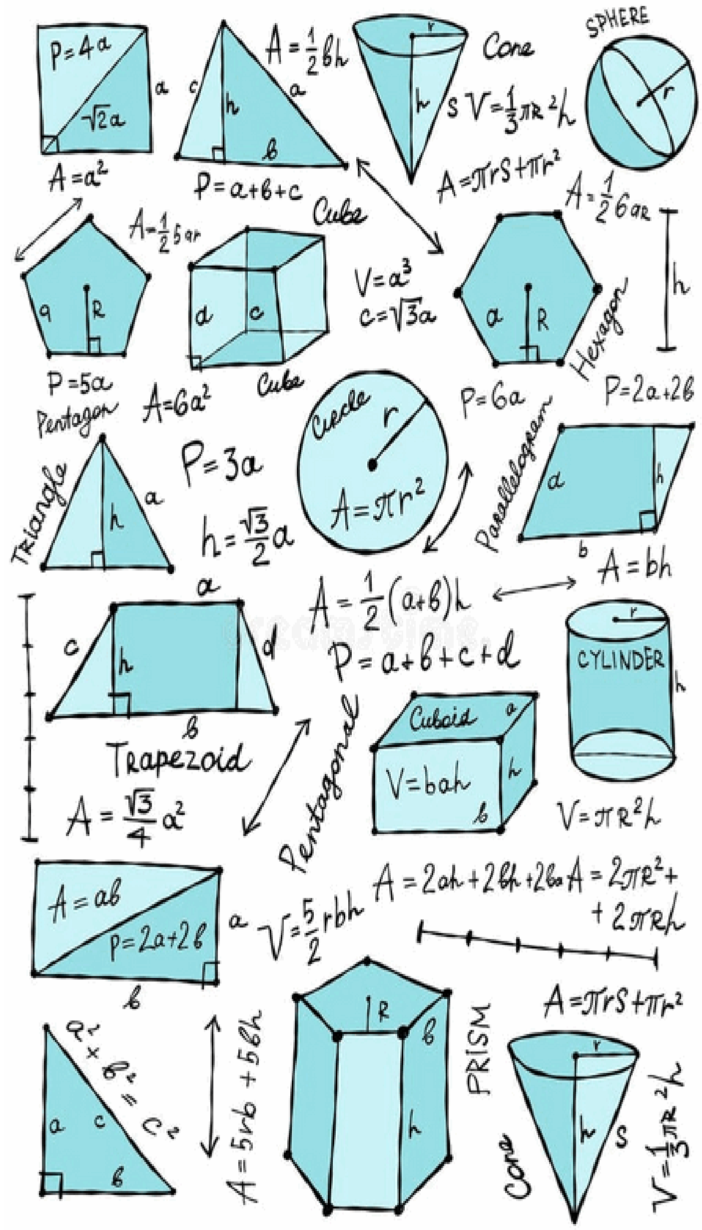
FORMULAE

- **RECTANGLE**
- Area = length × breadth
- Perimeter = 2 (length + breadth)

- **SQUARE**
- Area = (side)2
- Perimeter = 4 × side
- Diagonal = × side

- **TRIANGLE** with base (b) and altitude (h)
- Area = 1/2× b × h
- Triangle with sides as a, b, c
- Semi-perimeter = $\frac{a+b+c}{2}$ = s
- Area = $\sqrt{s(s-a)\,(s-b)\,(s-c)}$ (Heron's Formula)
- Isosceles triangle, with base a and equal sides b
- Area of isosceles triangle = $\frac{a}{4}\sqrt{4b^2-a^2}$
- Equilateral triangle with side a
- Area = $\frac{\sqrt{3}}{4}a^2$

- **PARALLELOGRAM** with base b and altitude h
- Area = bh
- **RHOMBUS** with diagonals, d1 and d2



- Area = $\frac{1}{2}$ d1 × d2
- Perimeter = $2\sqrt{d_1^2 + d_2^2}$
- **TRAPEZIUM** with parallel sides a and b, and the distance between two parallel sides as h
- Area = $\frac{1}{2}$ (a + b) × h
- **REGULAR HEXAGON** with side a
  - Area = 6 × Area of an equilateral triangle with side a = 6 ×$\frac{\sqrt{3}}{4}a^2 = \frac{3}{2}\sqrt{3}a^2$

MULTIPLE CHOICE QUESTIONS(01 MARK QUESTIONS)

1 The area of a equilateral triangle whose perimeter is 150m is

(a) 625m2 (b) 625 $\sqrt{3}$ m2 (c) 600 $\sqrt{3}$ m2 (d) 900m2

ANS: (b) 625 $\sqrt{3}$ m2

SOLUTION: Perimeter = 150m⇒a= 50m

Area = $\frac{\sqrt{3}}{4}$ × a2 = $\frac{\sqrt{3}}{4}$ × 50 ×50 = 625 $\sqrt{3}$ m2

2 The Perimeter of a triangle is 180cm. Its sides are in the ration 5:3:4 then its greatest side is

(a) 90cm (b) 75cm (c) 30cm (d) 10cm

ANS: (b) 75cm

SOLUTION: Let the sides be 5x, 3x, 4x ; $5x + 3x + 4x = 180 \Rightarrow x = 15$; Greatest side = 5x = 5 x15 = 75 cm

3 The area of triangle with given two sides 18cm and 10cm respectively and perimeter equal to 42cm is

(a) 20 $\sqrt{11}$ cm2 (b) 19 $\sqrt{11}$ cm2 (c) 22 $\sqrt{11}$ cm2 (d) 21 $\sqrt{11}$ cm2

ANS: (d) 21 $\sqrt{11}$ cm2

SOLUTION: 18 + 10 + x = 42 ⇒x = 42 – 28 = 14cm ; Area = = $\sqrt{21(21-18)(21-10)(21-14)}$ = 21 $\sqrt{11}$ cm2

4 The area of an equilateral triangle with side 6 $\sqrt{3}$ cm is

(a) 27cm2 (b) 27 $\sqrt{3}$ cm2 (c) 18 $\sqrt{3}$ cm2 (d) 54 $\sqrt{3}$ cm2

ANS: (b) 27 $\sqrt{3}$ cm2

SOLUTION: Area = $\frac{\sqrt{3}}{4}$ a2 = $\frac{\sqrt{3}}{4}\left(6\sqrt{3}\right)^2$ = 27 $\sqrt{3}$ cm2

5 The area of an equilateral triangle with altitude 2 $\sqrt{3}$ cm is

(a) 4cm2 (b) 4 $\sqrt{3}$ cm2 (c) 8cm2 (d) 8 $\sqrt{3}$ cm2

ANS: (b) 4 $\sqrt{3}$ cm2

SOLUTION: Area of an equilateral triangle with altitude 'P' is $\frac{P^2}{\sqrt{3}}$; Area = $\frac{\left(2\sqrt{3}\right)^2}{\sqrt{3}}$ =4 $\sqrt{3}$ cm2

6 The perimeter of an isosceles triangle is 64cm. The ratio of the equal side to base is 3:2. Its area is

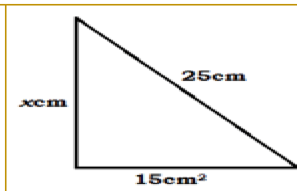
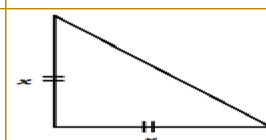
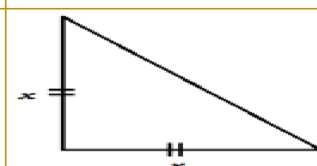
(a) 181cm2 (b) 32cm2 (c) 180cm2 (d) 96cm2

ANS: (a) 181cm2

SOLUTION: Let sides be 3x, 3x and 2x ; 3x + 3x + 2x = 64 ⇒ 8x = 64⇒ x=8; sides are 24cm, 24cm and 16cm

	S = 32 ; Area = = $\sqrt{32(32-24)(32-24)(32-16)}$= $128\sqrt{2}$cm2 =181cm2
7	The sides of a triangle are 13cm and 14cm and its semi perimeter is 18cm. Find the third side of the triangle. (a) 8cm (b) 6cm (c) 9cm (d) 10cm **ANS**: (c) 9cm **SOLUTION**: S = = 18 ⇒ x = 9cm
8	Find the area of a triangle having base 6cm and altitude 8cm. (a) 24cm2 (b) 48cm2 (c) 12cm2 (d) 16cm2 **ANS**: (a) 24cm2 **SOLUTION**: Area = b × h ; Area = × 6 ×8 = 24cm2
9	Each side of an equilateral triangle is 10cm. The height of the triangle is (a) $10\sqrt{3}$cm (b) $5\sqrt{3}$cm (c) 10cm (d) 5cm **ANS**: (b) $5\sqrt{3}$cm **SOLUTION**: height of an equilateral triangle = (side)x$\frac{\sqrt{3}}{2}$ = $\frac{\sqrt{3}}{2}$X 10 = $5\sqrt{3}$cm
10	If each side of an equilateral triangle of area A is doubled, then the area of new triangle is (a) 2A (b) 3A (c) 4A (d) 6A **ANS**: (c) 4A **SOLUTION**: Area = a2 But here side = 2a = (2a)2 =4A
11	The sides of a triangle are 7 cm, 24 cm, and 25 cm. What is the area of the triangle using Heron's formula? a) 70 cm² b) 84 cm² c) 96 cm² d) 100 cm² **ANS**: (c) A
	ASSERTION - REASON BASED QUESTIONS Direction: In the following questions a statement of assertion (A) is followed by a statement of reason (R). Choose the correct option. Both Assertion (A) and Reason (R) are true and reason (R) is correct explanation of Assertion (A) Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation for Assertion (A) Assertion (A) is true, but Reason (R) is false Assertion (A) is false, but Reason (R) is true
1	Assertion (A): If the sides of a triangle are 6cm, 8cm and 10cm then its Semi perimeter is 12 cm Reason (R): Semi perimeter = $\frac{a+b+c}{2}$ **ANS**: Both Assertion (A) and Reason (R) are true and reason (R) is correct explanation of Assertion (A)
2	Assertion (A): The area of an equilateral triangle whose altitude is 3cm is 3 $\sqrt{3}$ cm2 Reason (R): Area of an equilateral triangle with altitude 'P' cm is $\frac{P^2}{\sqrt{3}}$ **ANS**: Both Assertion (A) and Reason (R) are true and reason (R) is correct explanation of Assertion (A)
3	Assertion (A):The altitude 'p' of an equilateral triangle having each side 'a' is given by $\frac{a\sqrt{3}}{2}$ Reason (R): The area of an equilateral triangle having side 'a' is given by $\frac{\sqrt{3}}{4}$ a2 **ANS**: Both Assertion(A) and Reason(R) are true, but Reason(R) is not correct explanation for Assertion (A)
4	Assertion (A): The area of a given triangle and the area of a triangle obtained by doubling its sides are in the ratio 1:2 . Reason (R): If a, b, c are length of sides of a triangle with semi perimeter S, then its area is $\sqrt{s(s-a)(s-b)(s-c)}$ **ANS**: Assertion (A) is false, but Reason (R) is true **SOLUTION**: Semi Perimeter of triangle with side 2a, 2b, 2c is s I = = (a + b + c) = 2s

	Area of bigger triangle $=\sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$; $= 4\sqrt{s(s-a)(s-b)(s-c)}$= 4 × Area of smaller triangle ⇒Ratio is 1:4 ; Hence Assertion (A) is false but Reason (R) is true
5	Assertion (A): The area of an equilateral triangle with length of each side as positive integer, is an irrational number Reason (R): The area of an equilateral triangle with $\frac{\sqrt{3}}{4}$ a2 **ANS**: Both Assertion (A) and Reason (R) are true and reason (R) is correct explanation of Assertion (A) **SOLUTION**: If 'a' is a positive integer a2 is also a positive integer ; ⇒ $\frac{a^2}{4}$ is rational numbers ⇒$\sqrt{3} \times \frac{a^2}{4}$ is irrational number
6	Assertion (A): Area of a triangle with sides 3cm, 4cm and 5cm is 6cm2 Reason (R): Heron's formula for area of a triangle is × base × height **ANS**: Assertion (A) is true, but Reason (R) is false
7	Assertion (A): The area of an isosceles triangle each of whose equal side is 13cm and whose base is 24cm is 60cm2 Reason (R): The area of an isosceles triangle having base 'a' and each equal side 'b' is $\frac{a}{4}\sqrt{4b^2-a^2}$ **ANS**: Both Assertion (A) and Reason (R) are true and reason (R) is correct explanation of Assertion (A) **SOLUTION**: The area of an isosceles triangle with base 'a' and each equal side 'b' is given by Area = = $\frac{24}{4}\sqrt{4\ X\ (13)^2-(24)^2}$ ⇒ Area = 60cm2
8	Assertion (A): The sides of a triangle are 16 an, 30cm and 34cm then its area is 240cm2 Reason (R): Heron's formula to find then area of a triangle is$\sqrt{s(s-a)(s-b)(s-c)}$ **ANS**: Both Assertion (A) and Reason (R) are true and reason (R) is correct explanation of Assertion (A)
9	Assertion (A): Area of an equilateral triangle with altitude 12cm is 48 $\sqrt{3}$ Reason (R): Area of an equilateral triangle with altitude 'a' is $\frac{a^2}{\sqrt{3}}$ **ANS**: Both Assertion (A) and Reason (R) are true and reason (R) is correct explanation of Assertion (A)
10	Assertion (A): Heron's formula is for finding the area of scalene triangle Only Reason (R): The area of a triangle with sides a, b, c is given by Area: $\sqrt{s(s-a)(s-b)(s-c)}$ **ANS**: Assertion (A) is false, but Reason (R) is true
	VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)
1	Given three sticks of length 10cm, 5cm and 7cm. Find the area enclosed by these three as sides of a triangle **SOLUTION**: S = $\frac{10+5+7}{2} = \frac{22}{2}$ = 11cm ; Area = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{11(11-5)(11-7)(11-10)}$ = $2\sqrt{66}$ cm2
2	The perimeter of an isosceles triangle is 32cm. The ratio of the equal side to base is 3:2. Find the area. **SOLUTION**: Perimeter 3x + 3x + 2x= 32 ⇒x = 4cm Sides are 12cm, 12cm, 8cm Area = $\sqrt{S(S-a)(S-b)(S-c)} = \sqrt{16(16-12)(16-12)(16-8)} = 32\sqrt{2}$ cm2
3	The sides of a triangular are 13cm, 14cm and 15cm. Find the area of the field.

	SOLUTION: S = $\frac{13+14+15}{2} = \frac{42}{2}$ = 21cm ; Area = $\sqrt{S(S-a)(S-b)(S-c)} = \sqrt{21(21-13)(21-14)(21-15)}$; Area = 84cm2	
4	If the base of a right angled triangle is 15cm and its hypotenuse if 25cm. Find its area. **SOLUTION**: Using Pythagoras theory ; x = = 20 cm ; Area = Base × Height =$\frac{1}{2}$ 15 × 20 =150cm2	
5	AreaTraffic board is equilateral in shape, with words “SCHOOL AHEAD” on if the perimeter of the board is 180cm. Find area of the board. 3x = 180 ⇒x = 60cm **SOLUTION**: Area of equilateral triangle = $\frac{\sqrt{3}}{4}$ a2 = $\frac{\sqrt{3}}{4}$ × 60 × 60 = 900$\sqrt{3}$cm2	
6	The edges of a triangular board are 10cm, 24cm and 26cm. Find the cost of painting it at the rate of Rs.10 per cm2 **SOLUTION**: S = 30 ; Area = = = 120 cm2 ; Cost of painting 120 × 10 =Rs.1200 ;	
7	The sides of a triangle plot are in the ratio 3:5:7 and its perimeter is 300m. Find its area. ; **SOLUTION**: Let the sides be 3x, 5x and 7x 3x + 5x + 7x = 300 ⇒x = 20 ; Sides are 60m, 100m, 140m ; S = = 150 Area = = $\sqrt{150(150-60)(150-100)(150-140)}$; Area = 1500 $\sqrt{3}$ m2	
8	Find the area of a triangular board with base 8cm and height 5.5cm. **SOLUTION**: Area = × base × Height = × 8 × 5.5 = 22cm2	
9	If the area of an isosceles right angled triangle is 12.5cm2 then find the length of its hypotenuse. Area of isosceles right triangle **SOLUTION**: = × x2 = 12.5 ; x2 = 25 ; x = 5cm ; Hypotenuse = $\sqrt{5^2+5^2}$ = = $5\sqrt{2}$ cm	
10	Find the area of regular hexagon whose side is 4cm **SOLUTION**: Area of regular Hexagon = 6 × $\frac{\sqrt{3}}{4}$ a2 ; = 6 × $\frac{\sqrt{3}}{4}$ × 4 × 4 ; = 24 $\sqrt{3}$ cm2	
	SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)	
1	Find the area of a triangle whose perimeter is 180cm and its two sides are 80cm and 18cm. Calculate the altitude of triangle corresponding to its shortest side. **SOLUTION**: Given, a = 80, b = 18 ; Perimeter = a + b + c = 180 ; 80 + 18 + c = 180 ; c = 180 – 98 = 82m S = = 90 ; Area = $\sqrt{s(s-a)(s-b)(s-c)}$ = = $\sqrt{90(90-80)(90-18)(90-82)}$ Area = 720cm2 ; Shortest side of triangle = 18cm ; Area = × base × height 720 = × 18 × h ; h = = 80cm ; Altitude of triangle corresponding to its shortest side (18cm) is 80cm	

2 Find the area of shaded region in the given figure.

SOLUTION: Area of right angled ΔADB ; = × base × height ;= × 12 × 5 ; = 30cm2

Using Pythagoras theorem ; AB = $\sqrt{AD^2 + BD^2}$; AB = $\sqrt{12^2 + 5^2}$;

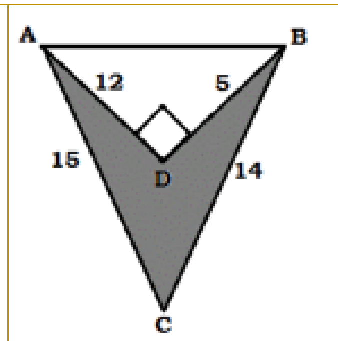
AB = $\sqrt{144 + 25}$; AB = = 13cm ; In ΔABC ; AB = 13cm, AC = 15cm, BC = 14cm

S = = = 21cm ; Area of ΔABC

= $\sqrt{S(S-a)(S-b)(S-c)}$; = $\sqrt{21(21-13)(21-15)(21-14)}$; = 84cm2

Area of shaded region = area of ΔABC – area of ΔABD

= 84 – 30 = 54cm2



3 Find the area of a trapezium whose parallel sides 25cm, 13cm and the other sides are 15cm and 15cm.

SOLUTION: Let ABCD be the trapezium

AB = 25cm ; CD = 13cm ; BC = AD = 15cm ; Draw CE||AD

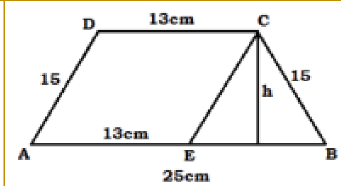
ADCE is a parallelogram ; AE = 13cm, EB = 12cm ; In ΔBCE

S = = 21 ; Area = $\sqrt{S(S-a)(S-b)(S-c)}$; Area = $\sqrt{21(21-15)(21-15)(21-12)}$

Area = $\sqrt{21 X 6 X 6 X 9}$; Area = 8 $\sqrt{21}$ cm2 ; Let h be the height of triangle ΔBCE

Area ΔBCE = × 12 × h = 6h ⇒6h = 18 $\sqrt{21}$ ⇒ h = = 3 $\sqrt{21}$ cm

Area of Trapezium ABCD =$\frac{1}{2}$ h (a+b) = $\frac{1}{2}$3 $\sqrt{21}$ x(25+13) =57 $\sqrt{21}$ cm2



4 How much area of triangle will increase in percentage if each side of the triangle is doubled?

SOLUTION: Let a, b, c be the sides of triangle , S = $\frac{a+b+c}{2}$; Area, $A_1 = \sqrt{S(S-a)(S-b)(S-c)}$

When the sides of triangle are doubled as 2a, 2b, 2c then its Area

$S_2 = \frac{2a+2b+2c}{2}$; S_2 = = 2S ; $A_2 = \sqrt{2S(2S-2a)(2S-2b)(2S-2c)}$

= 4x$\sqrt{S(S-a)(S-b)(S-c)}$; A2 = 4A1 ; % of increase in area = $\frac{Increase\ in\ area}{original\ area}$× 100 = $\frac{3A1}{A1}$× 100 = 300%

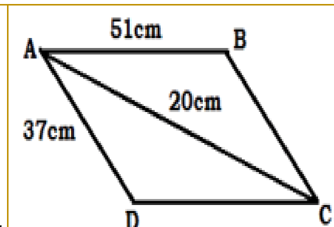
5 The length of two adjacent sides of a parallel are respectively 51cm and 37cm one of its diagonal is 20cm. Find the area of parallelogram.

SOLUTION:In ΔABC ; a = 37, b = 51, c = 20 ; S = = = 54

Area of ΔABC

= $\sqrt{S(S-a)(S-b)(S-c)}$ = $\sqrt{54(54-37)(54-51)(54-20)}$ = $\sqrt{54 X 17 X 3 X 34}$

= 306m2 ; Since diagonal divides the parallelogram into two congruent triangle of equal area ; Area of parallelogram ABCD = 2 × 306 ; = 612 cm2



6 Find the area of the given quadrilateral field if its perimeter is 120m.

SOLUTION: AD = 120 – (40 + 17 + 48) ; AD = 120 – 105 ; AD = 15m

In right triangle ΔADC ; AC2 = AD2 + DC2 ; AC = $\sqrt{15^2 + 40^2}$

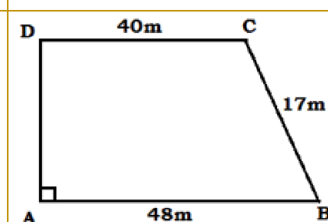
AC = $\sqrt{225 + 1600}$; AC = = 45m ; Area of ΔADC

Area = × base × height = ×15 × 40 = 300m2

Area of ΔABC ; a = 48m, b = 17m, c = 45m ; S = $\frac{48+17+45}{2} = \frac{110}{2}$ = 55

Area = $\sqrt{S(S-a)(S-b)(S-c)}$; = $\sqrt{55 X 7 X 38 X 10}$; = 2 X 5 $\sqrt{77 X 19}$

= 382.49 cm2



7 The lengths of the sides of a triangle are 5cm, 12cm and 13cm. Find the length of perpendicular from the opposite vertex to the side length is 13cm.

SOLUTION: S = $\frac{1}{2}$(a + b + c) = $\frac{1}{2}$ (5 + 12 + 13) = 15

Let Δ be the area of the given triangle. Then, Δ = $\sqrt{s(s-a)(s-b)(s-c)}$ Δ = $\sqrt{15(15-5)(15-12)(15-13)}$

Δ = = 30cm2 ; Let p be the length of the perpendicular from vertex A on the side Bc.
Then, Δ = × (13) × p

From (i) and (ii), we get $\frac{1}{2} \times 13 \times p = 30 \Rightarrow p = \frac{60}{13}$ cm

8 Calculate the area of the shaded region in Fig:

Solution: Lets and s be s[1] be the semi-perimeters of triangles ABC and DBC respectively. Then,
s = = 132 and s[!] = = 36 let $\Delta 1$ and $\Delta 2$ denote the areas of triangles ABC and DBC respectively. Then,

$\Delta 1 = \sqrt{s(s-120)(s-122)(s-22)}$

$\Delta 1 = \sqrt{132\ X\ 12\ X\ 10\ X\ 110}$

$\Delta 1 = \sqrt{12\ X\ 11\ X\ 12\ X\ 10\ X\ 10\ X\ 11}$

$\Rightarrow$ $\Delta 1$= 10 X 11 X 12cm2 = 1320m2

and $\Delta 2 = \sqrt{s'(s'-24)(s'-26)(s'-22)}$

$\Delta 2 = \sqrt{36(36-24)(36-26)(36-22)}$

$\Rightarrow$ $\Delta 2 = \sqrt{36\ X\ 12\ X\ 10\ X\ 14} = \sqrt{6^2\ X\ 2^2\ X\ 3\ X\ 2\ X\ 5\ X\ 2\ X\ 7}$

$\Rightarrow$ $\Delta 2 = 6\ X\ 22\sqrt{3\ X\ 5\ X\ 7} = 24\sqrt{105}$ m2

$\Delta 2$ = 24 × 10.25m2 = 246m2

Area of the shaded region = $\Delta 1$ -$\Delta 2$ = (1320 – 246)m2 = 1074m2

9 The sides of a triangular field are 41m, 40m and 9m. Find the number of rose beds that can be prepared in the field. If each rose bed, on an average needs 900cm2 space.

Solution: If a, b, c are the sides of a triangle and s is the semi perimeter, then its area Δ is given by

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Here, a = 41, b = 40 and c = 9 $\Rightarrow$ s = $\frac{1}{2}$ (a + b + c) = $\frac{1}{2}$ (41 + 40 + 9) = 45

$\Delta = = \sqrt{45(45-41)(45-40)(45-9)}$

$\Rightarrow$ $\Delta = \sqrt{45\ X\ 4\ X\ 5\ X\ 36} = \sqrt{3^2\ X\ 5\ X\ 2^2\ X\ 5\ X\ 2^2\ X\ 3^2}$

$\Rightarrow$ Δ= 32 × 22 × 5 = 180m2

Area of each rose beds = 900cm2 = $\frac{900}{10000}$ m2 = 0.09m2

Number of rose beds = = 2000

We observe that 412 = 402 + 92. So, the triangular field is in the form of a right triangle.

Area of the field × 40 × 9 = 180m2

10 An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see fig) each piece measuring 20cm, 50cm and 50cm. how much cloth of each colour is required for the umbrella?

Solution: sides of one triangular piece of cloth are of length a 20cm, b = 50cm and c = 50cm. Let s be the semi perimeter of the triangular piece. Then,

2s = a + b + c ; $\Rightarrow$ 2s = 20 + 50 + 50 ; $\Rightarrow$ s = 60

Let Δ be the area of the triangular piece. Then,

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$; $= \sqrt{60X(60-20)X(60-50)X(60-50)}$ cm2

$\Delta = \sqrt{60\ X\ 40\ X\ 10\ X\ 10}$ cm2 ;$\Delta = \sqrt{6\ X\ 4\ X\ 10\ X\ 10\ X\ 10}$ cm2 = 200 $\sqrt{6}$ cm2

Area of cloth of each colour = 5 × 200 $\sqrt{6}$ cm2 = 1000 $\sqrt{6}$ cm2

LONGANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1 International kite festival in Gujarat known as uttarayan is one of the biggest festival celebrated in Gujrat. It is celebrated on the auspicious day of makar sankrati every year. It is the sign for the farmers for the beginning of the summer season. On the day of uttarayan, Suresh a 16 year old boy wants to fly a kite. He ordered a triangle shaped kite of sides 10cm, 10 cm and 12 cm.

On the basis of the above information, solve the following questions:

i) Find the perimeter of a ΔABC

ii) Find the area of triangle ABC

iii) Find the length of AD

(OR)

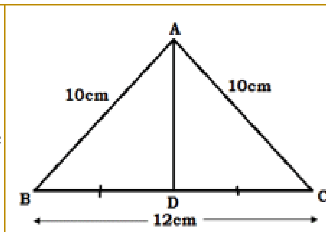
Find the area of ΔABD

Solution. i)Perimeter =10+10+12 =32

ii)areaofΔABC =$\sqrt{16X(16-10)(16-10)(16-12)}$ =48 cm2

iii)area =$\frac{1}{2}$ b ×h = $\frac{1}{2}$ x12x h =48 ⇒ h = 8 cm ⇒ AD =8 cm

OR area of ΔABD =$\frac{1}{2}$ b ×h = $\frac{1}{2}$×6 × 8 =24 cm2



2 Ajay bought some land for carrying out his wholesale business as shown in the figure below. He plans to divide this land into 3 parts for warehouse, inventory and canteen. Answer the following questions:

a) Find the semi perimeter of the area allotted for inventory

b) Find the area allotted for the warehouse

c) Find the cost of tiling the canteen floor at the rate of ₹500 per m²

d) (OR) Find the cost of tiling the inventory at the rate of ₹400 per m²

SOLUTION. a) sides of inventory are 5 m ,6m, 5m

Semi perimeter area allotted for inventory =8 m

b)Sides of warehouse are 5m ,5m,8 m

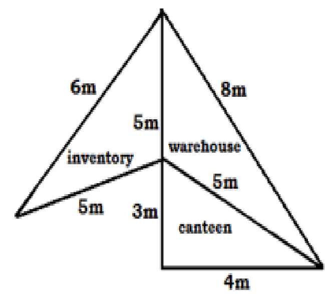
s=$\frac{a+b+c}{2}$ =$\frac{5+5+8}{2}$ =9 m

Δ = =$\sqrt{9\,X(9-5)(9-5)(9-8)}$ = 12 m2

c)Area of canteen = 6 m2 ; Cost =Rs.3000

OR For the inventory s =$\frac{5+5+6}{2}$ = $\frac{16}{2}$ =8 m ; Area =$\sqrt{8\,X(8-5)(8-5)(8-6)}$ = 12 m2

cost of tiling the inventory = 12 x 400 = ₹ 4800



3 To spread awareness about traffic signal. Central Board of Secondary Education (CBSE) decided to conduct an activity of making a traffic signal board and should take a photo if themselves telling about that particular traffic signal. One of the students decided to make a triangular sign board showing road work ahead. The dimensions of the sign board are 11 cm. 12 cm and 13 cm.

On the bases of the above information, solve the following questions:

If dimension of sign board is given, then which formula is used for finding the covered area.

a) Find the area of sign board

b) Find the smallest altitude of a triangle

c) What is the perimeter of the triangle?

SOLUTION.1) (a) Heron's formula

2) Area = 6 $\sqrt{105}$ cm2

3) (d) $\frac{12}{13}\sqrt{105}$ cm

4)Perimeter =36 cm

4 A quadrilateral is divided into two triangles by one of its diagonals. The lengths of the sides of the quadrilateral are 7cm, 8cm, 5cm, and 6 m, and the diagonal dividing it measures 9cm.

Calculate the area of each triangle formed by the diagonal.
find the area of the quadrilateral

SOLUTION: Given: Quadrilateral with sides: 7cm, 8cm, 5cm, and 6cm ; Diagonal dividing the quadrilateral = 9cm
The diagonal divides the quadrilateral into two triangles: Triangle 1: Sides = 7cm, 8cm, and 9cm (the diagonal)
Triangle 2: Sides = 5cm, 6 cm, and 9cm (the diagonal) ; Area of triangle 1:

Semi-perimeter: s = $\frac{a+b+c}{2} = \frac{7+8+9}{2} = \frac{42}{2} = 12$m ;

Area =$\sqrt{s(s-a)(s-b)(s-c)}$=$\sqrt{12(12-7)(12-8)(12-9)}$ = $\sqrt{720}$≈26.83 cm2

Area of triangle 2:

Semi-perimeter: s = $\frac{a+b+c}{2} = \frac{5+6+9}{2} = \frac{20}{2} = 20$m

Area=$\sqrt{s(s-a)(s-b)(s-c)}$=$\sqrt{10(10-5)(10-6)(10-9)}$ = $\sqrt{200}$≈14.14cm2

Area of quad =Area of triangle 1 + Area of triangle 2=26.83+14.14=40.97cm2

5 A triangular roadside garden has side lengths of 5 m, 12 m, and 13 m. A landscape architect wants to cover the entire garden with decorative stones. Each stone covers exactly 1m² of area. Find the area of the garden using Heron's formula. How many stones are needed to completely cover the garden? Suppose the architect mistakenly uses the formula $\frac{1}{2} \times 5 \times 12$ Is this result correct? Justify your answer.

SOLUTION.Sides: a =5 m, b=12 m, c=13 m

Semi-perimeter: s = $\frac{a+b+c}{2} = \frac{5+12+13}{2} = \frac{30}{2} = 15$

Area=$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{15(15-5)(15-12)(5-13)}$ = $\sqrt{900}$ = 30 cm2

Each stone covers 1 m², so:
Number of stones= 30

Architect uses:$\frac{1}{2} \times 5 \times 12$ = 30 cm2

This is correct only because the triangle is right-angled.
52+ 122 = 25+144 = 169 = 132

Yes, the triangle is a right triangle, so using $\frac{1}{2} \times base \times height$ is valid here.

CASE BASED QUSTIONS

1 Animals are an integral part of the nature. Animals also have a role to play in our daily lives. Every animal has a place in the ecosystem in the food chain to keep life in balance. 'Save Animals' must be a made into an awareness program for all to understand the value of animal life. Social workers started a campaign to protect animals. They prepared cardboard banners in the shape of equilateral triangles as shown in the figure.

(i) If the perimeter of a banner is 120 cm, then find the measure of one side.
(ii) Find the area of one cardboard banner.
(iii)If cardboard costs ₹ 1 per 10 $cm2$, find the total cost of 25. (Take √3 = 1.73)

SOLUTION.(i) side =$\frac{120}{3}$ = 40 cm

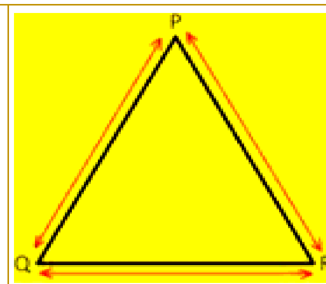
(ii)Area= $\frac{\sqrt{3}}{4}$ a2 =$\frac{\sqrt{3}}{4}$ x40 x40 =400$\sqrt{3}$ cm2

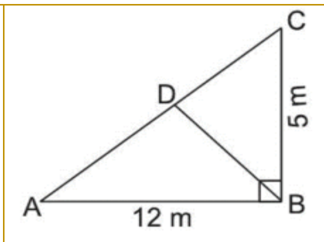
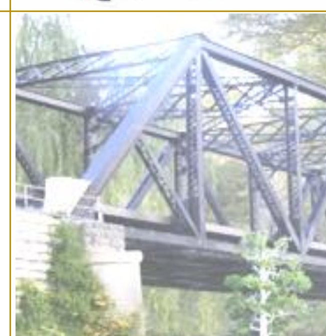
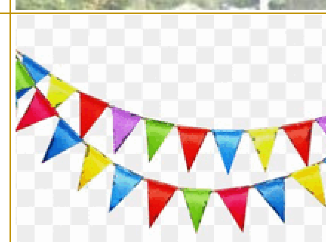
(iii)cost =400 x1.73x$\frac{1}{10}$x25 =Rs.1730

2 Rahul is fond of sceneries. He has decorated his home with many beautiful sceneries in various shapes. One of his friends visited his house and was impressed to see the triangular sceneries there. The dimensions of each triangular frame are 40 cm, 50 cm and 50 cm. Based on the above information answer the following questions:

(i) What is the total length of frame of scenery?
(ii) If the area of an equilateral triangle is 5√3 $m2$, find the length of each side of the triangle.
(iii) Find the area of the wall covered by two triangular scenery?

SOLUTION..i) length = 50+50+40 =140 cm

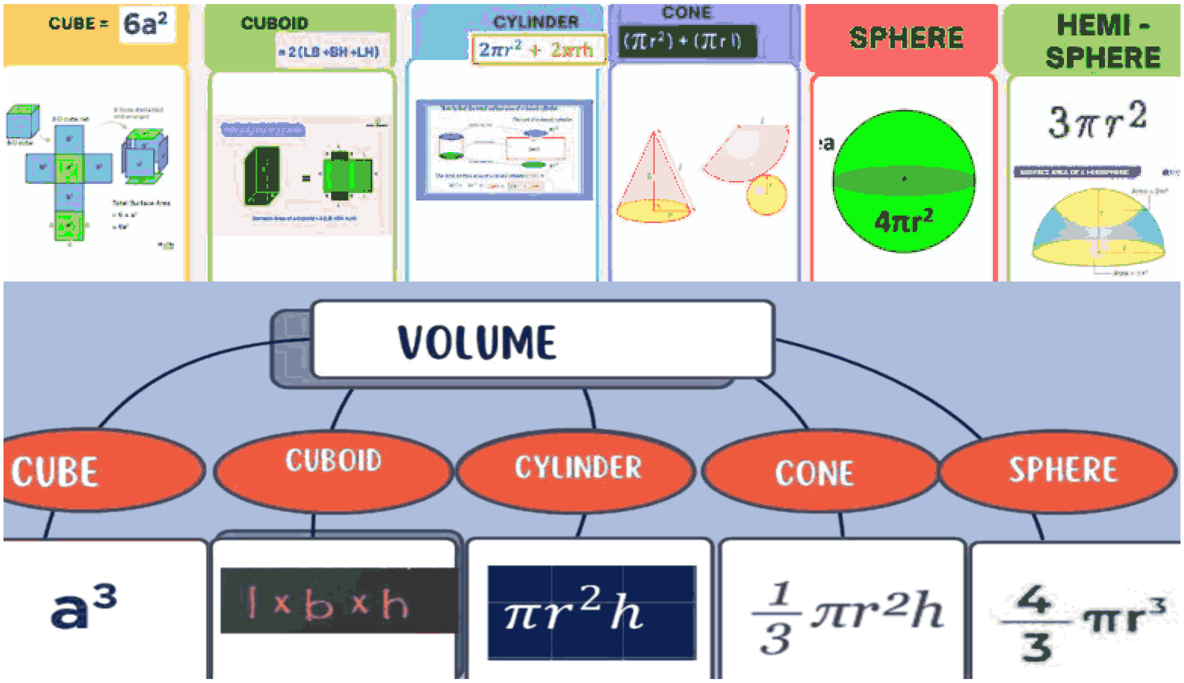
	(ii) $\frac{\sqrt{3}}{4}$ a2 = 5 $\sqrt{3}$ ⇒ a2 =20 ⇒ a = $\sqrt{20}$ =4.47 cm (iii) s = $\frac{140}{2}$ =70 ⇒ area of one triangle = $\sqrt{70(70-50)(70-50)(70-40)}$ = 200 $\sqrt{21}$ cm2 ⇒ area of 2 two scenery =400 $\sqrt{21}$ cm2	
3	In front of TRENDZ WHISPERING WOODS there is a triangular park whose sides are 50 m, 65 m and 65 m. A gardener Sidharaju has to put a fence all around it and also to plant grass inside. a)How much area does he get to plant? b)Find the cost of fencing it with barbed wire at the rate of Rs.7 per metre. Solution. a) s=90 ; Area= $\sqrt{\mathbf{s(s-a)(s-b)(s-c)}}$ = $\sqrt{90(90-65)(90-65)(90-50)}$; =Rs.1500 b) Perimeter =180 m Cost = 180x 7 = Rs 1260	
	EXERCISE	
1	Each side of an equilateral triangle is 6 cm long. The height of the triangle is (a) 10√3 cm (b) 3√3 cm (c) 10√3 cm (d) 5 cm	
2	The sides of a triangle are in the ratio 12 :17 : 25 and its perimeter is 540cm. The area is: a. 1000 cm2. b. 5000 cm2 c. 9000 cm2 d. 8000 cm2	
3	What is the perimeter of a triangle with sides 5 cm, 12 cm, and 13 cm? a) 12 cm B) 15 cm C) 16 cm D) 30 cm	
4	What is the area of a triangle with sides 9 cm, 12 cm, and 15 cm using Heron's formula? A) 54 sq. cm B) 72 sq. cm C) 108 sq. cm D) 216 sq. cm	
5	If the sides of a triangle are 8 cm, 10 cm, and 12 cm, then its area using Heron's formula will be: A) 30 sq. units B) 40 sq. units C) 50 sq. units D) 60 sq. units	
	ASSERTION - REASON BASED QUESTIONS	
1	Assertion(A): The area of an isosceles triangle each of whose equal side is 13 cm and whose base is 24cm is 60cm2 Reason(R): The area of an isosceles triangle having base a and each equal side b is $\frac{b}{4}\sqrt{4a^2-b^2}$	
2	Assertion: The height of a triangle is 18 cm and its area is 72cm2 then its base is 8cm Reason: The area of a triangle= $\frac{1}{2}$ x base x height	
3	Assertion:The side of an equilateral triangle is 6 cm then its area is 9 $\sqrt{3}$ cm2 Reason: All sides of an equilateral triangle are equal	
4	Assertion :The area of a triangle= $\sqrt{s(s-a)(s-b)(s-c)}$ Reason: s= $\frac{a+b+c}{2}$	
5	Assertion :In a right angled triangle if the hypotenuse is 5 $\sqrt{2}$ then the other two sides are equal to 5cm each Reason: In a right angled triangle $(\text{base})^2 + (\text{perpendicular})^2 = (\text{Hypotenuse})^2$	
	VERY SHORT ANSWER QUESTIONS(2 MARKS)	
1	The sides of triangle are 8 cm, 15cm, 17 cm. Find the area	
2	Find the Perimeter of an isosceles right angled triangle having an area 5000 m2 (use $\sqrt{2}$ = 1.41)	
3	Find the area of an isosceles triangle whose one side is 10cm greater than each of its equal sides and perimeter is 100 cm.	

4	The sides of triangle are 100cm, 120cm and 140cm. Find its area.(use $\sqrt{6} = 2.45$)	
5	If the base of an isosceles right angled triangle is 10cm , then find its area.	
	SHORT ANSWER QUESTIONS(3 MARKS)	
1	The perimeter of a triangular garden is 900 cm and its sides are in the ratio 3:5:4 find the area of triangular garden.	
2	Find the area of a triangle whose perimeter is 180cm and its two sides are 80cm 18cm. Calculate the altitude of triangle corresponding to its shortest side.	
3	Find the area of area of an equilateral triangle whose perimeter is 62 cm(use $\sqrt{3} = 1.73$)	
4	Find the area of triangle with sides 35 cm 54 cm 61cm	
5	**LONGANSWER TYPE QUESTIONS**	
1	Mayank bought a triangle shape field and wants to grow potato and wheat on his field. He divided his field by joining opposite sides. On the largest park he grew wheat and on the rest part he grew potato. The dimensions of a park are shown in the park. On the basis of the above information, solve the following questions: i) Find the length of AC in ΔABC. ii) Find the area of ΔABC. iii) If the cost of ploughing park is ₹5 per cm2, then find the total cost of ploughing the park.	
2	Find the formula of area of equilateral triangle using Herone's formula	
	CASE BASED QUESTIONS	
1	While selling clothes for making flags, a shopkeeper claims to sell each piece of cloth in the shape of an equilateral triangle of each side 10 cm while actually he was selling the same in the shape of an isosceles triangle with sides 10 cm, 10 cm and 8 cm i)Find the area of an equilateral triangular flag? ii) What is the semi-perimeter of an isosceles triangular flag. iii) Find the area of an isosceles triangular flag.	
2	Triangles are used in bridges because they evenly distribute weight without changing their proportions. When force is applied on a shape like rectangle it would flatten out. Before triangles were used in bridges, they were weak and could not be very big. To solve that problem engineers would put a post in the middle of a square and make it more sturdy. Isosceles triangles were used to construct a bridge in which the base and equal sides of an isosceles triangle are in the ratio 1:2:2 and its perimeter is 200 m. i)What are the measurements of the sides of an isosceles triangle? ii). Find the semi-perimeter of the above triangle. iii) What is the area of the above isosceles triangle?	
3	A craft mela is organized by Welfare Association to promote the art and culture for tribal people. Fairs and festivals are the custodians of our great cultural heritage. The pandal is to be decorated by using triangular flags around the field. Each flag has dimensions 25 cm, 25 cm and 22 cm. a) What is the semi-perimeter of the flag for the above mentioned dimensions? b) What is the area of the flag?(Use $\sqrt{14} \cong 3.74$) c) Find the area of cloth required for making 300 such flags in $m2$.	
	HOTS	
1	A triangular park has sides 40m, 30m, and 50m. Find the area of the park and the cost of fencing it at ₹50 per meter.	
2	A farmer has a triangular field with sides 100m, 120m, and 150m. Calculate the area of the field and the cost of irrigation at ₹20 per square meter.	

3	A triangular advertisement board has sides 10m, 12m, and 14m. Find the area of the board and the cost of painting it at ₹30 per square meter.
4	If the sides of a triangle are in the ratio 3:4:5, what can be inferred about the triangle? Use Heron's Formula to support your answer.
5	A triangle has sides a, b, and c. If a + b = c, what can be said about the area of the triangle? Use Heron's Formula to justify your answer.
	ANSWERS
	MCQ 1) b 2) c 3) d 4) a 5) c **ASSERTION- REASONING QUESTIONS** 1)A 2)A 3)B 4)B 5)A
	VERY SHORT ANSWER QUESTIONS 1) 60cm2 2) 341m 3) $200\sqrt{5}$cm2 4) 5880 cm2 5) 50 cm2
	SHORT ANSWER QUESTIONS 1)225 cm, 375 cm, 300cm are sides and area= 3.375 m2 2)80cm 3)8.948 cm2 4)939.68cm2
	CASE STUDY QUESTIONS i) 43.30cm2 ii)14cm iii)36.67 cm2 i) 40 m, 80 m, 80m ii)100m iii)$400\sqrt{5}$ m2 i)36 cm ii)246.84cm iii)7.4052m2
	5 MARK QUESTIONS 1)i) 13 m ii)30m2 iii) Rs.150 2) $\frac{\sqrt{3}}{4}$ a2 where a is length of the side
	HOTS 1) Area = 600 m2, cost ₹60002)area =6066.25m2, cost= ₹121325 3) area =58.8m2, cost= ₹1764 4) The triangle is a right-angled triangle ($3^2 + 4^2 = 5^2$). Using Heron's Formula, we can verify that the area is indeed $\frac{1}{2}\times$ base × height. 5)If a + b = c, the triangle is degenerate (has zero area). Using Heron's Formula, s = (a+b+c)/2 = c, and the area would be $\sqrt{(c(c-a)(c-b)(0))} = 0$.

CHAPTER – 11
SURFACE AREA AND VOLUME

MIND MAPPING



GIST/SUMMERY

Surface area of a solid object

Surface area and volume of cube, cuboid, cone, cylinder

DEFINITIONS AND FORMULAE:

- Lateral Surface Area of any prism= Perimeter of Base x height
- Volume of any prism = Base Area X Height
- THIS RULE WORKS FOR CUBE , CUBOID and CYLINDER

CUBE : LSA $=4a^2$ TSA $=6A^2$ VOLUME $=a^3$

CUBOID : LSA =2(lh + bh) TSA = 2 (lb + bh + hl) $VOLUME \ = l\ b\ h$

CYLINDER : LSA = $2\pi rh$ TSA = $2\pi r$ (r+h) VOLUME = $\pi r^2 h$

CONE : LSA = πrl TSA= πr (r +l) VOLUME $=\frac{1}{3}\pi r^2 h$

SPHERE : CSA = TSA= $4\pi r^2$ VOLUME $=\frac{4}{3}\pi r^3$

HEMISPHERE : CSA = $3\pi r^2$ TSA= $4\pi r^2$ VOLUME $=\frac{2}{3}\pi r^3$

- **SURFACE AREA**

This refers to total surface area covered by outer surface area of solid object. It is measured in square units (like cm^2 or m^2 or km^2).To find the area to be painted ,designing labels for c Solution: or boxes, determining the surface area of floors, roofs, or walls to estimate material costs (tiles, plaster, insulation) are based on the surface area.

- **VOLUME**

This represent the amount of space occupied m by solid. It is measured in cubic units (like cm^3 or m^3 or km^3)tTo find the capacity of a container /package and storage space/filling of a 0

Volume of a cone is one third of volume of a cylinder.

Volume of a hemisphere is half of volume of a sphere.

MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)

1 If the volume of a right circular cone of height 9cm is 48π cm^3 ,then the diameter of its base is:

a)4 b)8 c)12 d)6

	SOLUTION: b) V =48π cm^3 ,$\frac{1}{3}\pi r^2$ x9 =48 π ; r^2 = 16 then r =4 cm and diameter =8 cm
2	If the volume and surface area of a sphere are numerically equal , then its radius is: a) 1unit b) 2 unit c)3 unit d)4 unit **SOLUTION**: c)3 unit ; Volume = Surface Area $\Rightarrow\frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow$ r=3 unit
3	A cylindrical pencil sharpened at one edge is the combination of : a) a cone and a cylinder b) a cone and a hemisphere c) a frustom of a cone and cylinder c)a hemi sphere and a cylinder **SOLUTION**: a) a cone and a cylinder
4	In a cylinder, radius is doubled and height is halved , curved surface area will be: a)halved b)doubled c)no change d)four times **SOLUTION**:c) no change Here radius= 2r, height =$\frac{h}{2}$ CSA = 2πrh =2π (2r)$\frac{h}{2}$= 2 πrh
5	A cylindrical water tank has a radius of 7 meters and a height of 10 meters. How much water it can hold? a) 1540 m^3 b) 154 m^3 c) 15400 m^3 d) 154000 m^3 **SOLUTION**:a) 1540 m^3 Volume of a cylinder = $\pi r^2 h$ Here, r=7meters and h=10 meters. Volume = π×49×10 =$\frac{22}{7}$ x 49x10 =1540 m^3.
6	An ice cream is in the shape of a cone with a radius of 4 cm and a height of 9 cm. What is the volume of theicecream? a) 113.1 cm^3 b) 150.8 cm^3 c) 230.4 cm^3 d) 402.2 cm^3 **SOLUTION**:b) 150.8 cm^3 Volume of cone= $\frac{1}{3}\pi r^2 h = \frac{1}{3}$ x $\frac{22}{7}$ x 16 x 9 =144 x $\frac{1}{3}$ x $\frac{22}{7}$ =150.8 cm^3
7	Diameter of a sphere is 10 cm.Its surface area is:(consider π= 3.14) a)31400 cm^3 b)314 cm^3 c)3.14 cm^3 d)3140 cm^3 **SOLUTION**: b)314 Radius = $\frac{1}{2}$ x diameter =$\frac{1}{2}$ x10 =5cm Surface area = 4 π r^2 = 4 x 3.14 x 5 x 5 = 100 x 3.14 = 314
8	A hemisphere has a radius of 7 cm. What is the volume of the hemisphere? a)$\frac{700}{3}$ π cm^3 b) $\frac{700}{2}$ π cm^3 c) $\frac{686}{3}$ π cm^3 d)$\frac{343}{2}$ π cm^3 **SOLUTION**: c) $\frac{686}{3}$ π cm^3
9	A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is a) 1 :2 : 4 b) 2 :1 : 3 c) 2 :3 : 1 d) 1 :2 : 3 **SOLUTION**: d)1:2:3 ; Ratio is $\frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^2 h : \pi r^2 h$ but given that h =r So ratio is $\frac{1}{3}\pi r^2 r : \frac{2}{3}\pi r^2 r : \pi r^2 r$; That is $\frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 \Rightarrow$1:2:3
10	What is the radius of a hemisphere whose volume is 144π cm^3 a) 6 cmb) 8 cm c) 12 cm d) 10 cm **SOLUTION**: a)6 cm ; Volume of a hemisphere =$\frac{2}{3}\pi r^3$ =144 π ; Hence $r^3 = \frac{144 \times 3}{2}$ implies r =6 cm
	ASSERTION – REASON BASED QUESTIONS
1	Assertion : If the radius of right circular cone is halved and height is doubled the volume will be halved Reason :Volume of a cone is $\frac{1}{3}\pi r^2$ h **SOLUTION**:a)Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

	Here new r = $\frac{r}{2}$ new height h = 2h Volume = $\frac{1}{3}\pi(\frac{r}{2})^2(2h) = \frac{1}{2}(\frac{1}{3}\pi r^2h) = \frac{V}{2}$
2	Assertion (A): The total surface area of a sphere is given by the formula $4\pi r^2$. Reason(R): The sphere has only one curved surface, and its area is calculated using the formula for the lateral surface area of a cylinder **SOLUTION**:A
3	Assertion (A): The volume of a cylinder of radius r and height h is given by πr^2h. Reason (R): A cylinder can be thought of as a stack of circular discs, each with area πr^2, and the height h determines the number of such discs stacked together **SOLUTION**: A
4	Assertion (A): The total surface area of a sphere is given by the formula $4\pi r^2$ Reason (R): The formula for the curved surface area of a sphere is $2\pi r^2$ **SOLUTION**:C
5	Assertion (A): The surface area of a sphere increases when its radius increases. Reason (R): The formula for the surface area of a sphere is $4\pi r^2$, which depends on the square of the radius. **SOLUTION**: A
6	Assertion (A): The volume of a cylinder is greater than the volume of a cone having the same base radius and height. Reason (R): The volume of a cylinder is given by Πr^2h, whereas the volume of a cone is $\frac{1}{3}\pi r^2h$ **SOLUTION**: A
7	Assertion (A): The surface area of a cone is $\pi r(r+l)$, where l is the slant height. Reason (R):The curved surface area of a cone is πrl , and the base area is πr^2. **SOLUTION**: A
8	Assertion (A): The curved surface area of a cylinder is $2\pi rh$ Reason (R):The curved surface area is the area of the side of the cylinder, along with the top and bottom. **SOLUTION**: C
9	Assertion (A): The total surface area of a hemisphere is $3\pi r^2$ Reason (R):The total surface area includes the curved surface area $2\pi r^2$ and the base area πr^2 **SOLUTION**:A
10	Assertion (A): The curved surface area of a sphere is $4\pi r^2$ Reason (R):A sphere has no edges or vertices, and its surface area is uniform. **SOLUTION**:B
	VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)
1	A tent is in the shape of a cone. Its base radius is 6 m and slant height is 10m. Find the area of cloth required to make the tent. **SOLUTION**: r= 10m, l= 10m ; Area of cloth = $\pi rl = \frac{22}{7} \times 6 \times 10 = 188.57\ cm^2$
2	Neha wants to gift a spherical glass ball to her friend. The radius of the ball is 4.2 cm. Find the surface area of the glass ball. **SOLUTION**: r =4.2 cm ; Surface area = $4\pi r^2 = 4\times\frac{22}{7}\times 4.2\times4.2. = 221.76cm^2$
3	A juice can is in the shape of a cylinder. Its radius is 7 cm and height is 20 cm. Find the curved surface area of the can. **SOLUTION**: r=7cm, h= 20cm ; CSA of the cylinder is = $2\pi rh = 2\times\frac{22}{7}\times7\times20 = 880cm^2$
4	Ravi is painting the four walls of his room. The room is 5 m long, 4 m wide, and 3 m high. He does not paint the ceiling and floor. Find the total area he painted. **SOLUTION**: Here l=5m, b =4m, h =3m ; Area to be painted =CSA of cuboid= $2(l+b)\times h= 2(5+4)\times3= 54m^2$
5	A company packs soap bars in cuboidal boxes. Each box measures 12 cm × 8 cm × 5 cm.

Find the volume of the box.

SOLUTION:l = 12cm, b =8cm, h = 5cm ; volume of the Soap box = lbh =12×8×5 = 480cm^3

6 A water tank is in the shape of a cuboid of dimensions 2 m × 1.5 m × 1 m. It is full of water. How many litres of water does it hold? (1 m^3 = 1000 litres)

SOLUTION: l =2m, b= 1,5m, h=1m ; Volume = lbh = 2×1.5×1 =3 m^3 = 3× 1000liters = 3000 liters

7 Curved surface area of a cone is 308cm^2 and its slant height is 14cm.find theradius of the base.

SOLUTION: CSA of the cone = 308cm^{2}and l=14cm ; πrl = 308 ⇒$\frac{22}{7}$ ×r×14 = 308 ⇒ r= $\frac{308x\ 7}{22x14}$ = 7cm

8 The height of a cone is 15 cm. If its volume is 1570 cm^3. find the radius of the base (Take π=3.14)

Volume of the cone = 1570cm^3 , h= 15cm

SOLUTION : $\frac{1}{3}$πr^{2}h=1570cm^3⇒$\frac{1}{3}$ x3.14 xr^{2}x15 = 1570. ⇒ r= 10cm

9 A hollow spherical shell is made of a metal of density 4.5 g per cm3. If its internal and external radii are 8cm and 9cm respectively, find he weight of the shell .

SOLUTION: r = 8 cm R= 9cm. ; Difference in volume =$\frac{4}{3}$πR^3 -$\frac{4}{3}$πr^3 =$\frac{4}{3}$ x $\frac{22}{7}$(93-83) = 909.33cm^3

Weight = 909.33 X 4.5 = 4091.985g = 4092 g

10 Curved surface area of a cone is 308 cm^2 and its radius is 14cm. Find the slant height of its base.

SOLUTION: CSA of the cone is 308cm^2, , r= 14cm ; πrl =308 ⇒$\frac{22}{7}$x14xl = 308 ⇒ l= 7cm

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1 The radius and height of a cone are in the ratio 4:3. The area of the base is 154 cm^2. Find the area of the curved surface.

SOLUTION:-

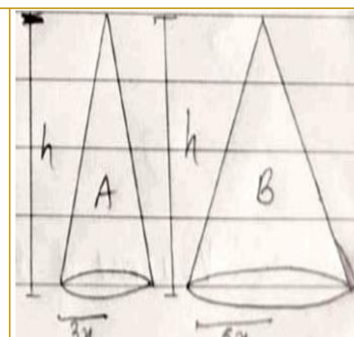
Let the radius & height be $4x$&$3x$ restively

Area of base = 154cm^2⇒ $\begin{aligned}\pi r^2 &= 154\\ \frac{22}{7}\times 4x\times 4x &= 154\end{aligned}$ ⇒ Radius = $4x$ = 7cm ; Height $-3x = \frac{21}{4}$ cm.

CSA of the cone = πrl ; $l = \sqrt{h^2+r^2} = \sqrt{\left(\frac{21}{4}\right)^2 + (7)^2} = \frac{35}{4}$ cm. ; $\pi rl = \frac{22}{7}\times 7\times\frac{35}{42} = 11\times 17.5$

∴ CSA of the cone = 192.5cm^2

2 The base radii of two right circular cones of the same height are in the ratio 3:5. Find the ratio of their volumes.



SOLUTION:

Let the two cones be A&B and their radius be $3x$ &$5x$ both of height ' h ' cm .

$$\frac{Vdl_A}{VdB} = \frac{\frac{1}{3}\pi(r_A)^2h}{\frac{1}{3}\pi(r_B)^2h}$$

$$= \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{3x}{5x}\right)^2$$

$$\therefore \frac{Vdl_A}{VdB} = \frac{9}{25}$$

3 How many spherical balls can be made of a lead cylinder 28cm high and base radius 6 cm. if each ball is 1.5 cm in diameter?

SOLUTION:

Let the number of balls made be ' n '. ; Then, Vol. of lead cylinder = n (Volume of sphere)

πr^{2}h = $nx\frac{4}{3}R^3$; $6\times 6\times 28 = n\left[\frac{4}{3}\times\frac{15}{20}\times\frac{15}{20}\times\frac{15}{20}\right]$; $6\times 6\times 28 = \frac{n\times 15\times 15}{20\times 20}$; $36\times 28 = \frac{n\times 9}{16}$

$n = \frac{36\times28\times16}{9}$; ∴ 1792 spherical balls can be made. ;

$n = 1792$

4 Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely immersed in water.

SOLUTION: Volume of displaced water = Volume of spherical ball ;

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times \frac{42}{20} \times \frac{42}{20} \times \frac{42}{20}$$
$$= \frac{441 \times 11}{125}$$
$$= \frac{4851}{125}$$
$$= 38.8\text{cm}^3$$

∴ Volume of displaced water $= 38.8$ml

5 A school provides milk to the students daily in a cylindrical glass of diameter 7 cm. If the glass is filled with milk up to a height of 12 cm, find how many litres of milk is needed to serve 1600 students.

SOLUTION: Volume of cylindrical glass $= \pi r^2 h$

Volume of 1 cylindrical glass $= 462\text{cm}^3$

$$= \pi r^2 h$$
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{8} \times 12$$
$$= 77 \times 6$$
$$= 462\text{cm}^3$$
$$\text{Required volume} = \overline{462 \times 1600}$$
$$= 739200\text{cm}^3$$
$$= 739200\text{ml}$$
$$\text{Required vel.} = 739.2\text{L}$$

6 A cylindrical road roller 2.5 m in length, 1.75 m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it make?

Solution: $5500\text{m}^2 = n$ (CSA of roller)

$$= n \times 2\pi rh$$
$$5500 = n \times \frac{22}{7} \times 2 \times 1.75 \times \frac{25}{10}$$
$$n = \frac{5500 \times 7 \times 14 \times 10}{22 \times 2 \times 175 \times 251}$$
$$= \frac{20}{1200 \times 7 \times 10}$$
$$n = 200 \text{ revolution es.}$$

7 A shopkeeper has one spherical laddoo of radius 5cm. With the same amount of material, how many laddoos of radius 2.5 cm can be made?

SOLUTION:Let no. of laddoos of 2.5 cm radius be ' n '.

Then,

volume of big laddoo= n (volume of smaller laddoo)

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$5 \times 5 \times 5 = n \times \frac{25}{10} \times \frac{25}{10} \times \frac{25}{10}$$

$$n = \frac{5 \times 5 \times 5 \times 10 \times 10^2 \times 10^2}{\frac{25}{5} \times \frac{25}{5} \times \frac{85}{5}}$$

$n =$ 8 laddoos.

8 A solid metal sphere with a radius of 6 meters is melted and recast into smaller spherical balls, each with a radius of 3 meters. How many such smaller spheres can be made?

SOLUTION: :No. of smalls spherical balls = 'n' (say)

Volume of bigger ball = n (Volume of smaller ball)

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{2}\pi r\gamma^3$$

$$6 \times 6 \times 6 = n \times 3 \times 3 \times 3$$

$n =$ 8 balls

9 A company is designing decorative steel balls to place in a garden. Each ball is a perfect sphere and is coated with a special paint that costs ₹88 per square meter. If the radius of each ball is 7 meters, how much will it cost to paint 3 such balls completely?

SOLUTION: ,TSA of 1 ball = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 7 \times 7$$

TSA of 1 ball = $88 \times 7 = 616\text{cm}^2$

TSA of 3 balls = $616 \times 3\text{cm}^2$

Cost of painting = $616 \times 3 \times 88 = 1{,}62{,}624$

10 A factory produces two types of spherical tanks: Type A and Type B. The radius of Type A tank is 6 meters. The radius of Type B tank is twice that of Type A. The outer surface of each tank needs to be coated with a special anti-corrosion material. The cost of coating is the same per square meter for both tanks. If coating 3 Type A tanks costs the same as coating 1 Type B tank, which of the following is most logically valid?

Options:

A) The surface area of Type B tank is 3 times that of Type A tank.

B) The surface area of Type B tank is 4 times that of Type A tank.

C) The surface area of Type B tank is 2 times that of Type A tank.

D) The surface area of Type B tank is equal to the total surface area of 3 Type A tanks.

SOLUTION: Type A TSA = $4\pi r^2 = 4\pi 6^2$

Type B TSA = $4\pi R^2 = 4\pi 12^2$

$$\frac{TSA}{TSA} = \frac{16\pi \cdot 6^2}{4\pi \cdot 6^2} = 4$$

⇒ (B) The surface area of Type B tank is 4 times that of Type A

LONGANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1 A company is designing eco-friendly water storage tanks in the shape of a closed cylinder (with top and bottom). Each tank has a radius of 1.5 meters and a height of 2 meters. The tanks are made of recycled aluminium, and the outer surface needs to be coated with a protective paint to prevent rust and heat absorption.

a) Calculate the total surface area of one cylindrical tank that needs to be painted.

b) How many litres of water can the tank hold?

SOLUTION: a) TSA of the tank = 2πr (h+r) ; = 2 X $\frac{22}{7}$ x1.5X(3.5) ; TSA of the tank is = 33m^2

b) 1m2= 1000L ; 33m^2= 33 X1000 = 33000 liters

2 A small-scale ice cream factory produces cone-shaped ice cream cones. Each cone has a radius of 3.5 cm and a height of 12 cm. The company wants to ensure accurate filling of the cones with ice cream.

a) Calculate the volume of ice cream that can fill one cone completely.

b) The company has to make labels for the ice cream cones with name of the company. Find the surface area of the name stickers required.

SOLUTION: :a) Volume of the cone = $\frac{1}{3}\pi r^2 h$; = $\frac{1}{3}$ X $\frac{22}{7}$ x $\frac{35}{10}$ x $\frac{35}{10}$ x 12

Volume of the cone = $154 cm^3$

b) l = $\sqrt{h^2 + r^2}$; = $\sqrt{12^2 + 3\cdot5^2}$; = 12.5cm

CSA of the cone = πrl ; = $\frac{22}{7}$ X3.5 X12.5 ; = $137.5 cm^2$

3 In a newly constructed park which is situated in the heart of a city Hyderbad, an architect has form a structure in the given shape. The shape has a cuboid, which is standing on the two cylindrical beams. The dimensions of the cuboid are 1.5 m, 3 m and 0.5 m. The dimensions of the cylinders are of height 2 m and diameter 0.6 m.

(i) As the structure is made from the concrete, how much volume of concrete is reqired to make the cuboidal shape?

(ii) What is formula for calculating the lateral surface area of the cylinder?

(iii) What is the volume of two cylinders?

SOLUTION: i) Volume of the cuboid = lbh

= 3 X1.5 X 0.5

Volume of the concrete required = $2.25 m^3$

ii) LSA of the cylinder is = $2\pi r^2 h$

iii) Volume = 2 X volume of the cylinder

= 2 X$\frac{22}{7}$x$\frac{3}{10}\times\frac{3}{10}\times 2$

= $1.131 m^3$

4 Mathematics teacher bring some green colour clay in the classroom to teach the topic menstruation first forms a cylinder of radius 6 cm and height 8 cm with the clay then he moulds that cylinder into sphere similarly he moulds sphere into other different shapes. Solution:wer the following questions.

a) What is the volume of the cylindrical shape?

b) What is the volume of sphere?

c) When clay changes into one shape to another which of the following remains same?

i) volume ii) area iii)CSA iv) Radius

d) What the radius of the sphere formed?

SOLUTION: a) volume of the cylinder = $\pi r^2 h$

=$\frac{22}{7}$ X6x6x8

Volume of the cylinder = $905.14 cm^3$

b) Volume of the Sphere = Volume of the cylinder 905.14cm3

c) (i) Volume

d) $\frac{22}{7}$ X6x6x8 = $\frac{4}{3}$ X$\frac{22}{7}$ XR^3

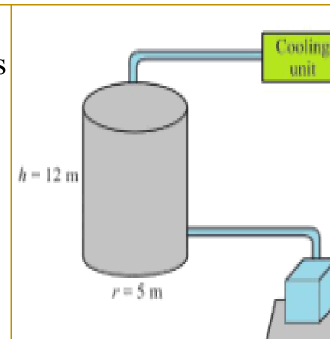
R = 6cm

5 One day teacher planned to take all the Class X students to the milk factory (Chilling plant) and asked the students to observe it carefully. Refer to this plant, its machinery is shown below. It is related to some solid shapes, which we study in our curriculum.

a) Refer to cylindrical container; calculate the quantity of milk it can store.

b) What is the formula for calculating the total surface area of the cylindrical container i.e., milk container?

c) If the cube shown in the picture is of dimension 6 cm each. Find the capacity of this cubic container.



SOLUTION: Volume of the cylinder is = $\pi r^2 h$

= $\frac{22}{7}$ X5X5x12

$= 942.85\ m^3$

$1m^3 = 1000$liters

$= 942850$liters

The cylindrical container can hold an amount of 942850 liters of milk

b) TSA of the cylinder $= 2\pi r\ (h+r)$

c) Volume of the cube $= a^3 = 6^3 = 216m^3$

$= 216\ X1000$liters

= The capacity of the cube is 216000 liters

CASE BASED QUESTIONS (4 MARK EACH)

1 A company is designing decorative paper cones to be used as packaging for party hats. Each cone has a radius of 7 cm and a slant height of 25 cm. The outer surface of each cone will be covered with colourful paper as a packaging designer, you are asked to: Calculate how much colourful paper (in cm²) is needed to cover the curved surface of one cone. If the company wants to include a circular base at the bottom of the cone, how much total surface area will be covered?
If the company to make 100 such cones, how much paper will be needed in total?

SOLUTION: a) CSA of the cone $= \pi rl = \frac{22}{7}$ x7 x 25

Required paper for colour paper

For one cone $= 550cm^2$

b) TSA of the cone $= \pi r\ (\ l+r)\ ; = \frac{22}{7}$ x7 (25+7); = 32x22; $= 704cm^2$.

c) Required area = CSA of the cone x 100 ; $= 5500cm^2$

2 A toy manufacturing company is designing rubber balls with a diameter of 14 cm. The entire surface of each ball is to be painted with a glossy finish. As a quality control supervisor, you are required to:

a) Calculate the surface area of one ball that needs to be painted.
b) If 100 such balls are to be painted, how much total surface area will be covered?
c) If 1 cm² of paint costs ₹0.10, calculate the total cost to paint 100 balls.

SOLUTION :

a) TSA of one ball $= 4\pi r^2$

$= 4X \frac{22}{7} X\ 7X7$

$= 616cm^2$

b) TSA of 100 balls $= 100X616 = 61600cm^2$

c) $1cm^2 = 0.10$

$61600cm^2 = 61600\ X0.10$

Cost = ₹6160

3 A company is producing cylindrical tin Solution: to store food items. Each can has a radius of 7 cm and a height of 20 cm. The outer surface of the can, including the top and bottom, will be labelled with printed material.
As part of the production team, you are assigned to:

a) Calculate the total surface area of one can that needs to be labelled.
b) If the company needs to produce 250 such Solution:, find the total surface area to be covered.
c) If 1 cm² of labelling costs ₹0.05, calculate the total labelling cost for all 250 cans

SOLUTION:

a) TSA of the Solution: $= 2\pi r(\ h+r)$

$= 2\ X\frac{22}{7}\ X\ 7X\ 27$

$= 44\ X\ 27$

$= 1188cm^2$

b) TSA of 250 cones $= 1188\ X\ 250$

$= 2, 97,0000\ cm^2$

$1cm^2 = 0.05$

$2,97,000cm^2 = 297000\ x0.05$

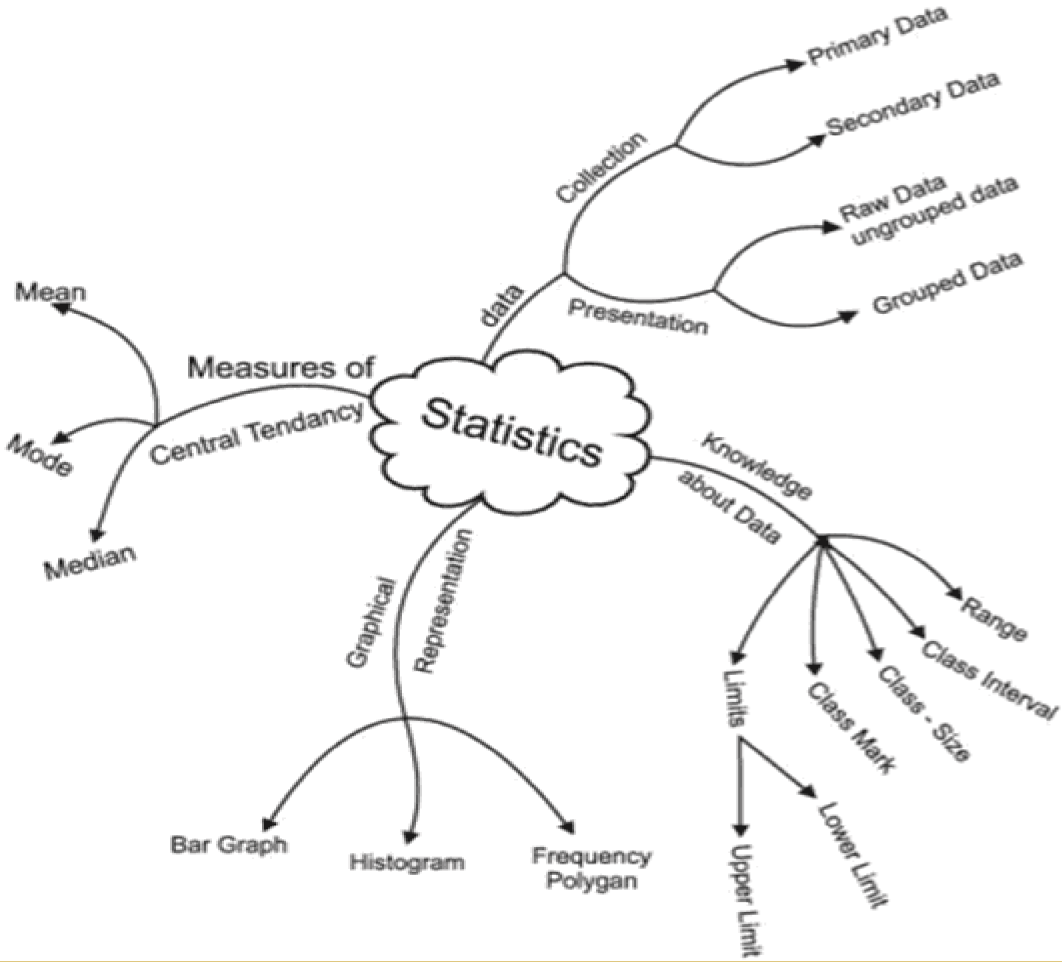
cost = ₹ 14850

	EXERCISE
	MULTIPLE CHOICE QUESTIONS
1	If surface area of a sphere is 676 π cm^2 find its radius. (a)12cm (b)13cm (c)5cm (d)17cm
2	Ibinu doesn't like the colour on the wooden ball he has. So he wants to scratch and remove the colour so that he can put the new one. How much area he has to scratch if diameter of ball is r cm. (a)πr^2 cm^2 (b) $2\pi r^2$ cm^2 (c) $4\pi r^2$ cm2 (d) $8\pi r^2$ cm^2
3	Anshul has a rectangular sheet of dimensions 14cm x 22cm. He wants to make a cylinder in such a way that volume is minimum. Find the height of the cylinder. (a)12cm (b)18cm (c)22cm (d)17cm
4	If the radius of right circular cone is halved and its height is doubled then the volume will be -------- (a)halved (b)doubled (c)trippled (d)unchanged
5	The largest sphere is carved out of a cube of side 7cm. Find the volume of the sphere. (a)127.67cm^3 (b)133.82cm^3 (c)179.67cm^3 (d)117cm^3
	ASSERTION – REASON BASED QUESTIONS
1	Assertion (A): The curved surface area of a right circular cylinder of radius r and height h is $2\pi rh$. Reason (R): The curved surface area of a cylinder is obtained by multiplying the circumference of the base with the height.
2	Assertion (A): The lateral surface area of a cone with radius r and slant height l is given by πrl Reason (R): The lateral surface of a cone is a sector of a circle whose radius is equal to the slant height
3	Assertion (A): The total surface area of a hemisphere of radius r is $3\pi r^2$ Reason (R): The total surface area of a hemisphere includes both the curved surface and the circular base.
4	Assertion (A): The volume of a sphere of radius r is given by $\frac{4}{3}\pi r^3$ Reason (R): The volume of a sphere is derived using integral calculus from the formula for the surface area of a sphere.
5	Assertion (A): The curved surface area of a hemisphere of radius r is $2\pi r^2$ Reason (R): The curved surface area of a hemisphere is half the total surface area of a sphere of the same radius.
	VERY SHORT SOLUTION:WERQUESTIONS(2 MARKS)
1 2 3 4 5	Find the surface area of a sphere with radius 7 cm. A cylinder has a height of 10 cm and a radius of 4 cm. Find its total surface area. Find the volume of a cone with radius 5 cm and height 12 cm. The volume of a cylindrical rode is 628 cm^3. If its height is 20cm, find the radius of its cross section The radii of two cylinders are in the ratio 2:3 and their heights are in the ratio 5:3. Find the ratio of their volumes.
	SHORT ANSWER QUESTIONS(3 MARKS)
1	Anandi has a piece of canvas, whose area is 551 m^2.She used to have a conical tent made with a base radius of 7m .Assuming that all the stitching margins and the wastage incurred while cutting amount to 1 sq meter.Find the slant height of conical tent.
2	A corn cob shaped somewhat like a cone, has the radius of its broadest end as 2.1cm and the length as 20cm. If each 1cm2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob
3	If the total surface area of sphere is 98.56 square centimeter, finds the radius of the sphere
4	The outer curved surface areas of hemisphere and sphere are in the ratio 2:9 find the ratio of their radii.
5	Into a conical tent of radius 8.4 m and vertical height 3.5 m how many full bags of wheat can be emptied if space for wheat in each bag is 1.96 cubic meter.
	CASE BASED QUESTIONS (4 MARK EACH)

1	At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up with a height of 32cm with orange juice. The juice is filled in small cylindrical glasses of radius 3cm up to a height of 8cm, and sold for Rs.3 each. How much money does the stall keeper receive by selling the juice completely? A semi-circular sheet of metal of diameter 28cm is bent to form an open conical cup. Find the capacity of the cup . The volume of two spheres are in the ratio 64:27 Find their radii, if the sum of their radii are 21cm. 
	LONG ANSWER TYPE QUESTIONS
1	A tent is of the shape of a right circular cylinder upto a height of 3m and then becomes a right circular cone with maximum height of 13.5 meters above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs.20 per square meter if the radius of the base is 14 meters
2	A bus stop is barricaded from the remaining part of the road by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1m. If the outer side of each of the cone is to be painted and the cost of painting is Rs.12 per m^2, what will be the cost of painting all these cones?
3	a) A Jocker's cap is in the form of a right circular cone of base radius 7 cm and height 24cm. Find the area of the sheet required to make 10 such caps.
4	A wall 6 m long 5 m high and 0.5 m thick is to be constructed with blocks, each having length 25cm, breath 12.5 cm and height 7.5 cm. Find the number of bricks required to construct the wall if it is given that cement and sand mixture occupy $\frac{1}{20}$ of the volume of the wall.
5	The barrel of a fountain pen cylindrical in shape, is 7cm long and 5mm in diameter . A full barrel of ink in the pen is used up on writing 330 words on an average. How many words would would use up a bottle of ink containing 1/5 of a litre?
	HOTS
1	A semi circular sheet of metal of diameter 28cm is bent into an open conical cup. Find the depth and capacity of cup.
2	The ratio of volumes of two cones are 4 :5 and the ratio of radii of their bases are 2:3. Find the ratio of their vertical heights.
3	A conical pit of top diameter 3.5 meter is 12 meter deep what is its capacity in Kiloliters?
4	A Sphere, a cylinder and a cone are of same radius and same height .Find the ratio of their curved surface areas.
5	A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 16cm and its height is 15cm and find the cost of painting the toy at Rs.7 per 100 square centimeter.
	SOLUTION OF EXERCISE QUESTIONS
	MCQ 1)b 2)a 3)c 4)d 5)c **ASSERTION AND REASONING** 1)A 2)A 3)A 4)B 5)A **VERY SHORT SOLUTION:** 1)196π =615.44 cm^2 2)112 π = 351.68 3)100 π=314cm^2 4)$\sqrt{10}$ cm 5)20:27
	SHORT SOLUTION: (3MARK) 1)25m 2)531 grains 3)2.8 cm 4)2:3 5)132 CASE STUDY QUESTIONS 1)Rs.300 2)622.16cm^2 3)5495cm^2
	LONG SOLUTION: QUESTIONS 1)Rs.20680 2)Rs.384.33 3)5500 4)6082 5)48000words **HOTS** 1)r =7cm, Depth =12.12cm, Capacity =622.26 cm^3 2)9:5 3)38.5kilo litres 4)4:4:$\sqrt{5}$ 5)Rs.58

CHAPTER – 12
STATISTICS

Mind Mapping



GIST/SUMMERY OF THE LESSON

Introduction to Statistics : A study dealing with the collection, presentation and interpretation and analysis of data is called statistics.

Data

- Facts/figures, numerical or otherwise, collected for a definite purpose are called data.
- Data collected first-hand data:- Primary
- Secondary data: Data collected from a source that already had data stored

Frequency The number of times a particular instance occurs is called frequency in statistics.

Ungrouped data Ungrouped data is data in its original or raw form. The observations are not classified into groups.

Grouped data In grouped data, observations are organized in groups.

Class Interval

- The size of the class into which a particular data is divided.
- Eg. divisions on a histogram or bar graph.
- Class width = upper class limit – lower class limit. It is also called Range or height of the
- class.

Regular and Irregular class interval

- Regular class interval: When the class intervals are equal or of the same sizes.
- Eg. 0-10, 10-20, 20-30….. 90-100
- Irregular class interval: When the class intervals are of varying sizes.
- Eg. 0-35, 35-45, 45-55, 55- 80, 80-90, 90-95, 95-100

Frequency table : A frequency table or distribution shows the occurrence of a particular variable in a tabular form.

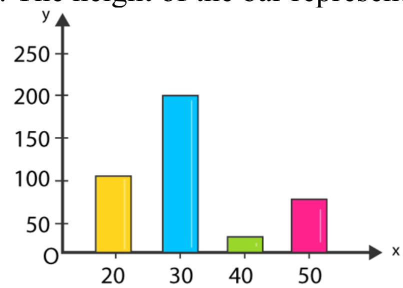
SORTING : Raw data needs to be sorted in order to carry out operations.- Sorting ⇒ ascending order or descending order

UNGROUPED FREQUENCY TABLE : When the frequency of each class interval is not arranged or organised in any manner.

GROUPED FREQUENCY TABLE : The frequencies of the corresponding class intervals are organised or arranged in a particular manner, either ascending or descending.

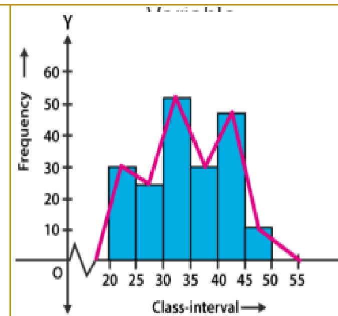
GRAPHICAL REPRESENTATION OF DATA / BAR GRAPHS : Graphical representation of data using bars of equal width and equal spacing between them (on one axis). The height of the bar represents the frequency of the data.

Savings (in %)	Number of Employees(Frequency)
20	105
30	199
40	29
50	73
TOTAL	400

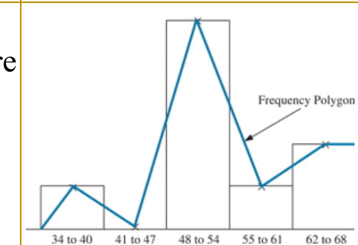


VARIABLE BEING A NUMBER : A variable can be a number such as 'no. of students' or 'no. of months'.
Can be represented by bar graphs or histograms depending on the type of data.
Discrete → bar graphs
Continuous → Histograms

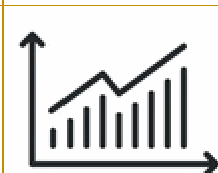
HISTOGRAMS
Like bar graphs, but for continuous class intervals.
Area of each rectangle is ∝ Frequency of a variable and the width is equal to the class interval.



FREQUENCY POLYGON
If the midpoints of each rectangle in a histogram are joined by line segments, the figure formed will be a frequency polygon.
Can be drawn without histogram. Need midpoints of class intervals.
Midpoint of class interval
The midpoint of the class interval is called a class mark
Class mark = (Upper limit + Lower limit)/2

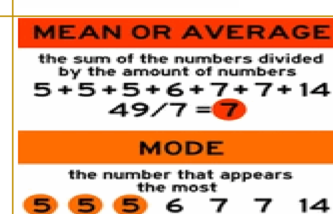


EQUALITY OF AREAS
The addition of two class intervals with zero frequency preceding the lowest class and succeeding the highest class intervals equates the area of the frequency polygon to that of the histogram(Using congruent triangles).
Measures of Central Tendency



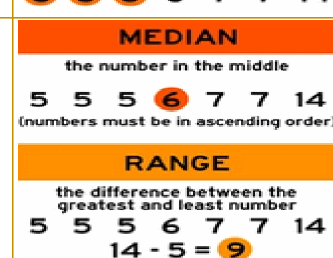
AVERAGE
The average of a number of observations is the sum of the values of all the observations divided by the total number of observations.

MEAN
Mean of ungrouped data mean $(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$



MODE
The most frequently occurring observation is called the mode.
The class interval with the highest frequency is the modal class

MEDIAN
Value of the middlemost observation.
If n(number of observations) is odd, Median =[(n+1)/2]th observation.
If n is even, the Median is the mean or average of (n/2)th and [(n+1)/2]th observation.



MULTIPLE CHOICE QUESTIONS – STATISTICS (1 MARK)

1	Classmarkofthe1stclassintervalis5andtherearefiveclasses.Iftheclasssizeis10thenthe last class interval is (A)20 – 30 (B) 30–40 (C)40 – 50 (D)50 – 60 **ANS**:C	
2	Intheclassintervals10–20,20–30,the number20isincludedin. (A)10 – 20 (B) 20–30 (C)both the intervals (D)none of these intervals **ANS**: B	
3	In Histogram the class intervals or the groups are taken along (A) X-Axis (B) Y-Axis (C) X-Axis and Y-Axis (D) In between X and Y – Axis **ANS**:A	
4	To draw a histogram to represent the following frequency distribution, the modified/adjusted frequency for the class 25-45 is Class Interval: 5-10, 10-15, 15-25, 25-45, 45-75 Frequency: 6, 12, 10, 8, 15 6 (B) 5 (C)2 (D)3 **ANS**:C	
5	In a histogram, which of the following is proportional to the frequency of the corresponding class? Width of the rectangle(B) Length of the rectangle (C)Perimeter of the rectangle (D)Area of the rectangle **ANS**:B	
6	Themeanof25observationsis36.Outoftheseobservationsifthemenoffirst13observations is 32 and that ofthe last 13 observations is 40, the 13th observation is (a)23 (b)36 (c)38 (d)40. **ANS**:B	
7	If 'm'be themed point and'l'the upper-class limit of a class in a continuous frequency distribution. Then lower-class limit of the class is (A)2m+l (B) 2m–l (C)m-l (D)m–2l **ANS**:B	
8	Theclassmarksofafrequencydistributionaregivenasfollows:15,20,25,……Theclass corresponding to the class mark 20 is (A)12.5 – 17.5 (B) 17.5–22.5 (C)22.5 – 27.5 (D)27.5 – 32.5 **ANS**:B	
9	Fordrawing a frequency polygon of a continuous frequency distribution, we plot the points whose ordinates are the frequency of the respective classes and abscissa are respectively (A) Upper limits of the classes (B)Lower limits of the classes (C)Class marks of the classes (D)Upper limits of preceding classes. **ANS**.C	
10	In a frequency distribution, the class width is 4 and the lower limit of first class is 10. If there are 6 Classes, the upper limit of last class is 22 (B) 26 (C)30 (D)34 **ANS**:D	
	ASSERTION AND REASONING QUESTIONS In questions a statement of Assertion(A) is followed by a statement of Reason (R). Choose the correct option. A). Both Assertion and Reason are correct and Reason is the correct explanation of Assertion. B). Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion. C). Assertion is correct but Reason is incorrect. D). Assertion is incorrect but Reason is correct	
1	Assertion(A):If the mean of five observations x, x+2, x+4, x+6 and x+8 is11, then mean of last three observations is 13. Reason(R): Mean of observations = Sum of all obsevations/Number of observstions	**ANS**:A
2	Assrtion(A):The range of first 5 multiples of 2 is 6	**ANS**:D

	Reason(R):Range= maximum value - minimum value.	
3	Assertion (A):The median of the observations 3, 5, 12,10, 7, 11, 4,1, 3, 8 is 6. Reason(R):When the number of observations is odd, The median is the value of the[n+1 2]th observation.	**ANS**:B
4	Assertion(A):The mode of 4, 6, 7,5, 9, 3, 2, 7, 7, 6, 5, 4, 9, 7,4 is 7. Reason(R): The mode is that value of the given number of observations which divides it into exactly two parts.	**ANS**:C
5	Assertion(A): Tally marks for 5 is ~~IIII~~ Reason(R): : Tally marks for 15 is ~~IIII~~ ~~IIII~~ ~~IIII~~ As15 = 5+5+5	**ANS**:B
6	Assertion(A): In bar graph frequency is taken only along y-axis. Reason(R): In bar graph frequency can be taken only along any axis.	**ANS**:C
7	Assertion(A):In a histogram the area of each rectangle is proportional to the class size of corresponding interval. Reason(R): To draw the histogram of a continuous frequency distribution with unequal class interval the frequencies of classes are adjusted by using the formula Adjusted frequency of a class (Minimum class size/class size) × frequency of the class	**ANS**:D
8	Assertion(A): Frequency polygon cannot be drawn for discontinuous data. Reason (R): Frequency polygon can be drawn for discontinuous data also by changing it into continuous data.	**ANS**:D
9	Assertion(A): Class mark for 20-30 is 25. Reason(R):Class mark of an interval is given by mean of upper limit and lower limit.	**ANS**:A
10	Assertion(A): The difference between the maximum and minimum values of a variable is called its range. Reason(R):: The number of times a variate (observation) occurs in a given data is called range.	**ANS**:C

VERY SHORT ANSWER TYPE QUESTIONS(2 MARKS QUESTIONS)

1 Find the true class limits of the first two classes of the distribution 1-9, 10-19,20-29….

ANS:

Given Class Intervals	1-9	10-19	20-29
True class intervals	0.5-9.5	9.5-19.5	19.5-29.5

2 The following are the marks obtained by 20 students in a class test.
40, 20, 36, 27, 30, 12, 15, 20, 25, 31, 34, 36, 39, 41, 43, 48, 46, 36, 37, 40
Arrange the above data in frequency distribution with equal classes, one of them being 0-10.

ANS:

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	0	2	4	8	6

3 In a bar graph, the height of a bar is 5 cm and it represents 40 units. What is the height of the bar representing 56 units.

ANS: Scale : 1 cm = 8 units , 7 cm=56 Units

4 A bar graph is drawn to the scale of 1 cm = k units, if the length of a bar representing a quantity of 702 units is 3.6 cm, then find the value of 'k'.

ANS: k= $\frac{702}{3.6}$ = 195

5 Write the class marks of the given following for the following frequency distribution table.

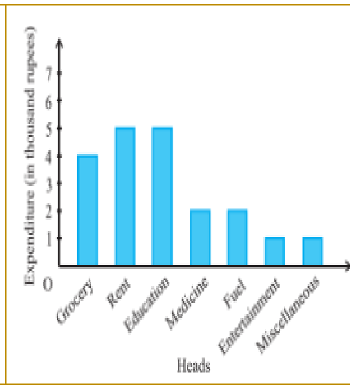
Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	10	15	20	17	12

Ans:

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	10	15	20	17	12
Class Mark	10	30	50	70	90

6 Write about Histograms-Graphical representation of data.

ANS: Histogram or frequency histogram is program is a graphical representation of a frequency distribution in the form of rectangles with class intervals as basis and heights proportional to corresponding frequences such that there is no gap between any two successive rectangles. What are differences between Bar graph and Histogram.



7 Define class-marks of a frequency distribution of data.

SOLUTION: The class mark (also called the midpoint) of a class interval in a frequency distribution is the average of the lower limit and upper limit of that class.

$$Classmark = \frac{Lower\ limt + upper\ limit}{2}$$

8 What is a kink? When it is required to represent data graphically.

ANS: Kink is a break on X-Axis or Y-Axis, where the class interval is not started from 0 in a frequency distribution.

9 Write any four real life situations where we use graphical methods to represent data.

SOLUTION: Any related real life situations.

SHORT ANSWER TYPE QUESTIONS(3 MARKS QUESTIONS)

1 Modify the given frequency distribution table into continuous class intervals data.

Class Interval	21-30	31-40	41-50	51-60	61-70	71-80
Frequency	6	9	13	14	8	7

SOLUTION :

Class Interval	21-30	31-40	41-50	51-60	61-70	71-80
Frequency	6	9	13	14	8	7
Continuous Class intervals	20.5-30.5	30.5-40.5	40.5-50.5	50.5-60.5	60.5-70.5	70.5-80.5

2 A family with a monthly income of Rs 15,000 had planned the following expenditures per month under various heads: Draw a bar graph for the data above.

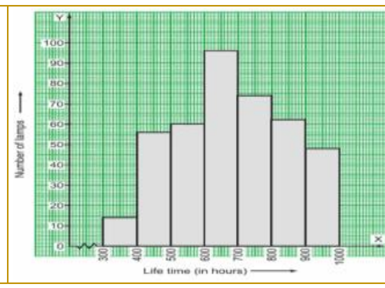
SOLUTION: Bar –graph

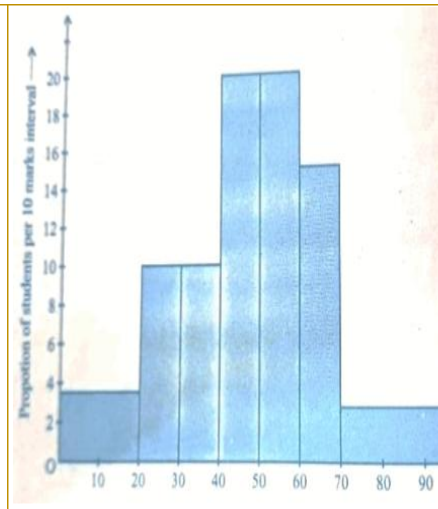
Marks	Grocery	Rent	Education of children	Entertainment	Medicine	Fuel	Misc
Expenditure in Thousand rupees	4	5	5	1	2	2	1

3 The following table give the life times of 400 neon lamps. Represent the graph information with the help of histogram.

Life time in Hours	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
No of lamps	14	56	60	86	74	62	48

SOLUTION:



4	A teacher wanted to analyse the performance of two sections of students in a mathematics test of 100 marks. Find the length of each rectangle of concerned class interval if the same data represents in a histogram.	

Marks	0-20	20-30	30-40	40-50	50-60	60-70	70-100
Frequency	7	10	10	20	20	15	8

Ans:

Class Interval	Frequency	Width	Length of the rectangle (Height)
0–20	7	20	7 / 20 = 0.35x10=3.5
20–30	10	10	10 / 10 = 1.0x10=10
30–40	10	10	10 / 10 = 1.0x10=10
40–50	20	10	20 / 10 = 2.0x10=20
50–60	20	10	20 / 10 = 2.0x10=20
60–70	15	10	15 / 10 = 1.5x10=15
70–100	8	30	8 / 30 ≈ 0.27x10=2.7

5 Give any three methods to represent a particular data graphically.

SOLUTION: Here are three common methods to represent data graphically:

Bar Graph

Used to compare quantities across different categories.

Each category is represented by a bar, and the length or height of the bar corresponds to the data value.

Pie Chart

Used to show proportions or percentages of a whole.

The circle is divided into sectors, each representing a category's contribution to the total.

Line Graph

Useful for showing trends over time.

Data points are plotted and connected with lines, making it easy to see increases or decreases.

6 The value of π up to 50 decimal places is given below:

3.14159265358979323846264338327950288419716939937510

(i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

(ii) What are the most and the least frequently occurring digits?

ANS:

Digit	0	1	2	3	4	5	6	7	8	9
No of times	2	5	5	8	4	5	4	4	5	8

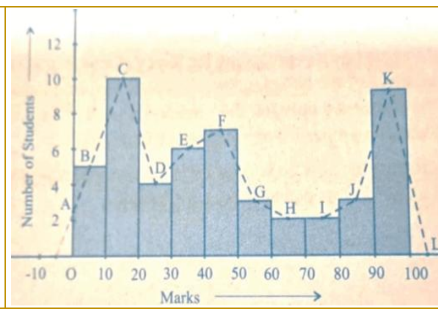
(i) [Frequency distribution to be created based on the digits.]

(ii) Digit 3 and 9 occurred 8 times.

7 Consider the marks (out of 100) obtained by 51 students of a class in a test, given in the below table. Draw a frequency polygon corresponding to this frequency distribution table.

Ans.

Cost of living index	140-150	150-160	160-170	170-180	180-190	190-200	Total
Number of week	5	10	20	9	6	2	52



8 In a city, the weekly observations made in a study on the cost of living index are given below in the following table: Draw a frequency polygon for the data above (without constructing a histogram).

Classes	Class Marks	Frequency
140-150	145	5
150-160	155	10
160-170	165	20
170-180	175	9
180-190	185	6
190-200	195	2

ANS:

9 The following observations have been arranged in ascending order as: 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95; If the median of the data is 63, find the value of x.

SOLUTION: The data is already in ascending order; since the number of terms is 10 (even), the median is the average of the 5th and 6th terms: ⇒ Median = (x + x + 2)/2 = 63 ⇒ (2x + 2)/2 = 63 ⇒ x + 1 = 63 ⇒ x = 62

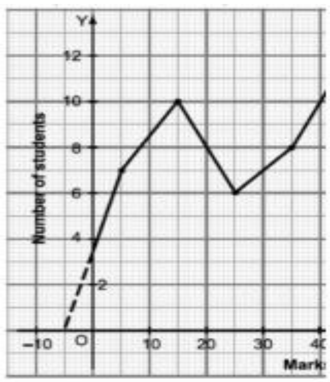
LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1 Draw a frequency polygon for the following distribution.

Marks	0-10	10-20	2030	30-40	40-50	50-60	60-70	70-80
No.of Students	7	10	6	8	12	3	2	2

SOLUTION

x	f	(x,f)
5	7	(5,7)
15	10	(15,10)
25	6	(25,6)
35	8	(35,8)
45	12	(45,12)
55	3	(55,3)
65	2	(65,2)
75	2	(75,2)

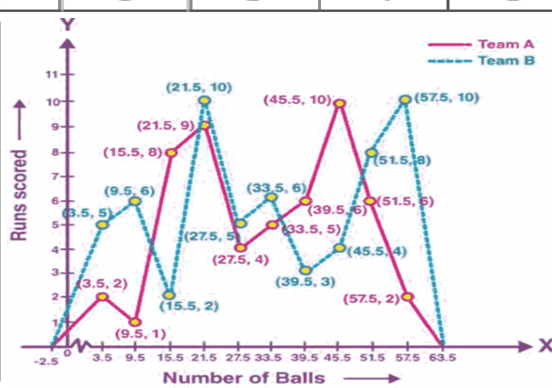


2 The runs scored by two teams A and B on the first 60 balls in a cricket match are given below: Represent the data of both the teams on the same graph by frequency polygons; Solution: Plot the class intervals (like 0–10, 10–20, etc.) on the x-axis and frequencies for both teams on the y-axis; mark the midpoints of each class and join them with lines for both teams to form two frequency polygons on the same graph.

Number of balls	01-06	07-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60
Team A	2	1	8	9	4	5	6	10	6	2
Team B	5	6	2	10	5	6	3	4	8	10

SOLUTION:

Number of Balls	Class mark	Team A	Team B
1-6	3.5	2	5
7-12	9.5	1	6
13-18	15.5	8	2
19-24	21.5	9	10
25-30	27.5	4	5
31-36	33.5	5	6
37-42	39.5	6	3
43-48	45.5	10	4
49-54	51.5	6	8
55-60	57.5	2	10

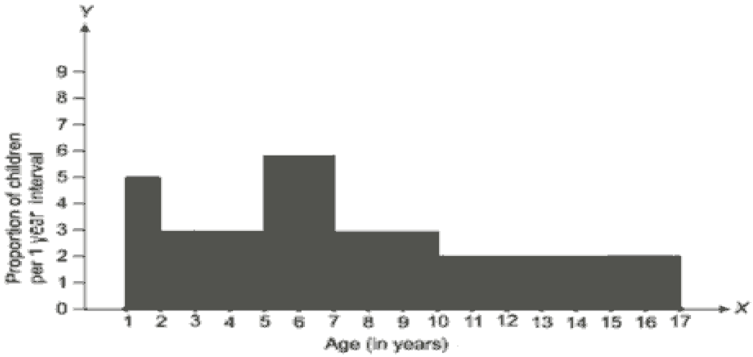


3 A random survey of the number of children of various age groups playing in a park was found as follows: Draw a histogram to represent the data above; Solution: Draw the age groups on the x-axis and number of children on the y-axis; use continuous bars without gaps for each age group with height proportional to frequency.

Age(in years)	1-2	2-3	3-5	5-7	7-10	10-15	15-17
Number of children	5	3	6	12	9	10	4

SOLUTION:

Age (in Years)	Number of children(frequency)	Width of the class	Length of the class
1-2	5	1	(5/1)x1=5
2-3	3	1	(3/1)x1=3
3-5	6	2	(6/2)x1=3
5-7	12	2	(12/2)x1=6
7-10	9	3	(9/3)x1=3
10-15	10	5	(10/5)x1=2
15-17	4	2	(4/2)x1=2



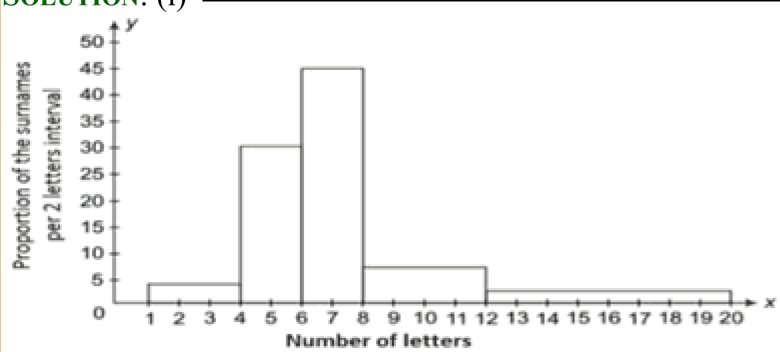
4 Hundred surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows.

No.of letters	Number of surnames
1-4	6
4-6	30
6-8	44
8-12	16
12-20	4

i)Draw a histogram to depict the given information.
ii)Write the class interval in which the maximum number of surnames lie.

SOLUTION: (i)

Number of letter	Number of surnames	Width of class	Length of rectangle
1-4	6	3	(6/3)x2=4
4-6	30	2	(30/2)x2=30
6-8	44	2	(44/2)x2=44
8-12	16	4	(16/4)x2=8
12-20	4	8	(4/8)x2=1



(ii) Maximum number of surnames lie in the class interval 6-8.

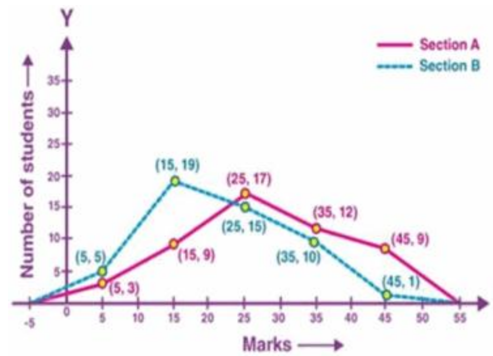
5 In a Mathematics test given to 15 students, the following marks (out of 100) are recorded.
41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60
Find the mean, median and mode of this data.
SOLUTION: Mean=Average = Sum of all the observations/Total number of observations

= (41+39+48+52+46+62+54+40+96+52+98+40+42+52+60)/15
= 822/15
= 54.8
Median
To find the median, we first arrange the data in ascending order.
39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98
Here,
Number of observations (n) = 15
Since the number of observations is odd, the median can be calculated as
Median = [(n+1)/2]th observation
= [(15+1)/2]th observation
= (16/2)th observation
= 8th observation
= 52
Mode
To find the mode, we first arrange the data in ascending order.
39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98
Here,
We find that 52 occurs most frequently (3 times)
∴ Mode = 52

CASE BASED QUESTIONS

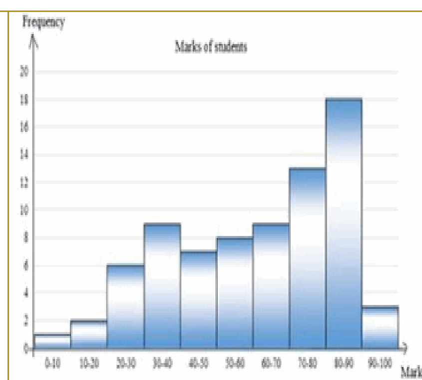
1 The following table gives the distribution of students in two sections according to the marks obtained by them. The marks of the students of both sections were represented on the same graph by two frequency polygons. Observe the two polygons and answer the questions.

Section-A		Section-B	
Marks	Frequency	Marks	Frequency
0-10	3	0-10	5
10-20	9	10-20	19
20-30	17	20-30	15
30-40	12	30-40	10
40-50	9	40-50	1



(i) For the class mark 25 which section performed well?
(ii) For which class marks section B performed better than section A?
(iii) For which class marks section A performed better than section B?
(iv) Which section performed better overall?
SOLUTIONS: i)Section-A ii)15 iii)25 iv) Section-A

2 Bhaskar is a Mathematics teacher in Hyderabad. After Periodic test 1, he asks students to collect the mathematics marks of all the students of Class IX- A, B and C. He prepares the frequency distribution table using the collected marks and draws Histogram using the table as shown in figure.



i)What is the width of the class?
(ii)What is the total number of students in the Histogram?
(iii)How many students scored less than 50% marks?
(iv)What is the range of the collected marks?
SOLUTIONS:i)10 ; ii)76 ; iii)25 ; iv)100

9	A group of students decided to make a project. They are collecting the heights (in cm) of their 50 girls of Class IX-A, B and C of their school. After collecting the data, they arranged the data in the following frequency distribution table form <table><tr><td>Height in cm</td><td>135-140</td><td>140-145</td><td>145-150</td><td>150-155</td><td>155-160</td><td>160-165</td></tr><tr><td>Number of students</td><td>5</td><td>6</td><td>17</td><td>11</td><td>7</td><td>4</td></tr></table>Based on the information, answer the following questions. (i) Write the lower limit of the class with highest frequency. (ii) How many students of the height 150 cm below are there? (iii) How many students of the height 145 cm and above? (iv) Write the class mark of the class with lowest frequency. **SOLUTIONS**:i)145 ii)28 iii)39 iv)162.5
	HOTS
1	Ten Observations 6, 14, 15, 17, x+1, 2x-13, 30, 32, 34, 43 are written in ascending order. The median of the data is 24. Find the value of the x. **SOLUTION**: Given, ten observations are written in an ascending order. The observations are 6, 14, 15, 17, x + 1, 2x - 13, 30, 32, 34, 43 The median of the data is 24 We have to find the value of x. Total number of observations, n = 10 Here, n is even The formula for median when n is even is given by Median = [(n/2)th observation + ((n/2) + 1)th observation] / 2 So, median = [(10/2)th observation + ((10/2) + 1)th observation] / 2 24 = [5th observation + (5 + 1)th observation] / 2 24 = [5th observation + 6th observation] / 2 24 = [(x + 1) + (2x - 13)]/2 48 = x + 2x + 1 - 13 48 = 3x - 12 3x = 48 + 12 3x = 60 x = 60/3 x = 20 Therefore, the value of x is 20.
2	The mean of 13 observations is 14.If the mean of the first 7 observations is 12 and that of last 7 Observations is 16 find the 7th observation? **SOLUTION**: Total sum of 13 observations =14×13=182 Sum of 14 observation =7×12+7×16=84×112 =196 So, the 7th observation =196−182=14
3	School has two sections the mean mark of one section of size 40 is 60 and mean mark of other section of size 60 is 80. Find the combined mean of all the students of the school. **SOLUTION**: Combined Mean= $\frac{(40X60)+(60X80)}{40+60} = \frac{7200}{100}$ =72
4	The mean weight of 180 students in a school is 50 kg the mean weight of boys is 60 kg while that of the girls is 45 kg. Find the number of the boys and girls in the school. **SOLUTION**: We are given: Total number of students = 180 Mean weight of all students = 50 kg Mean weight of boys = 60 kg

	Mean weight of girls = 45 kg Let: Number of boys = x Number of girls = 180−x We'll use the formula for the combined mean: Combined Mean Substitute the given combined mean (50 kg):50 => 50= 180X 50 = 15x+8100 =>900=15x => x=60 No. of boys are 60 and girls are 120
	EXERCISE
	MULTIPLE CHOICE QUESTIONS
1	A grouped frequency distribution table with class intervals of equal sizes using 250–270 as one of the class intervals is constructed for the following data: 268, 220, 368, 258, 242, 310, 272, 342, 310, 290, 300, 320, 319, 304, 402, 318, 406, 292, 354, 278, 210, 240, 330, 316, 406, 215, 258, 236; The frequency of the class 310–330 is: (a) 4 (b) 5 (c) 6 (d) 7
2	If each observation of the data is increased by 5 then their mean: (a) remains the same (b) becomes 5 times the original mean (c) is decreased by 5 (d) is increased by 5
3	The median of the data 78, 56, 22, 34, 45, 54, 39, 68, 54, 84 is: (a) 45 (b) 49.5 (c) 54 (d) 56
4	What is the class mark of the class interval 90–120? (A) 90 (B) 105 (C) 115 (D) 120
5	In a Histogram, if the first-class interval is not starting from zero, we show it on the graph by (A) marking a kink or break on the axis (B) drawing from Zero on the axis (C) neglecting from zero to starting value (D) considering frequency as zero till starting value
	ASSERTION AND REASONING QUESTIONS
1	Assertion(A): According to statistics more female children are born each year than male children in India. Reason(R):In India the death rate of male child is higher than female child.
2	Assertion(A): Range = Maximum value – Minimum value Reason(R):: The range of the first 6 multiples of 6 is 9.
3	Assertion(A): The class intervals 10-20, 20-30, 20 is included in interval 20-30 Reason(R): The lower limit of the class interval is included in the interval.
4	Assertion(A): The class size for the grouped frequency distribution of the class intervals 0-20 is 20 Reason(R): Class size is given by mean of upper limit and lower limit.
5	Assertion(A): In a bar graph, the height of the bar is proportionate to the value of the component. Reason(R): The class mark of 20-30 is 25.
	VERY SHORT ANSWER TYPE QUESTIONS
1	The following number of goals were scored by a team in a series of 10 matches: 2, 3, 4, 5, 0, 1, 3, 3, 4, 3. Find the mean, median and mode of these scores.
2	In a bar graph, the height of a bar is 5 cm and it represents 40 units. What is the height of the bar representing 56 units.
3	Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18. A grouped frequency distribution table with class intervals of equal sizes using 250–270 as one of the class intervals is constructed for the following data: 268, 220, 368, 258, 242, 310, 272, 342, 310, 290, 300, 320, 319, 304, 402, 318, 406, 292, 354, 278, 210, 240, 330, 316, 406, 215, 258, 236; Find the frequency of the class 310–330.
4	The median of 12, 13, 16, x+2, x+4, 28, 30, 32 is 23. When x+2 and x+4 lie between 16 and 30, find the value of x
	SHORT ANSWER TYPE QUESTIONS

1 1.Give five examples of data that you can collect from your day-to-day life.

2 A random survey of the number of children of various age groups playing in a park was found as follows: Draw a histogram to represent the data above.

Age(inyears)	Numberofchildren
1 –2	5
2 –3	3
3 –5	6
5 –7	12
7 –10	9
10 – 15	10
– 17	4

3 In a mathematics test given to 15 students, the following marks (out of 100) are recorded: 41, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60; Find the mean, median and mode of this data.

4 Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows: 0, 1, 2, 2, 1, 2, 3, 1, 3, 0, 1, 3, 1, 1, 2, 2, 0, 1, 2, 1, 3, 0, 0, 1, 1, 2, 3, 2, 2, 0; Prepare a frequency distribution table for the data given above.

5 Given below are the seats won by different political parties in the polling outcome of a state assembly election:

Political Party	A	B	C	D	E
Seats Won	75	55	37	29	37

(i) Draw a bar graph to represent the polling results.
(ii) Which political party won the maximum number of seats?

LONG ANSWER TYPE QUESTIONS

1 For the following data, draw a histogram and a frequency polygon.

X	0 –10	10 – 20	20 – 30	30 – 50	50 – 60	60 – 80	80 – 90	90–100
F	5	12	15	20	18	10	6	4

2 Following are the marks of a group of students in a test of reading ability test:
Construct a histogram and frequency polygon for the above data.

Marks:	50 – 52	47 – 49	44 – 46	41 – 43	38 – 40	35 – 37	32 – 34	Total
No. ofstudents	4	10	15	18	20	12	13	92

3 The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:
Represent the data of both the teams on the same graph by frequency polygons.

Number of balls	01-06	07-12	13-18	19-24	25-30	31-36	37-42	43-48	49-54	55-60
Team-A	2	1	8	9	4	5	6	10	6	2
Team-B	5	6	2	10	5	6	3	4	8	10

4 In a city, the weekly observations made in a study on the cost-of-living index are given below in the following table: Draw a frequency polygon for the data above (without constructing a histogram).

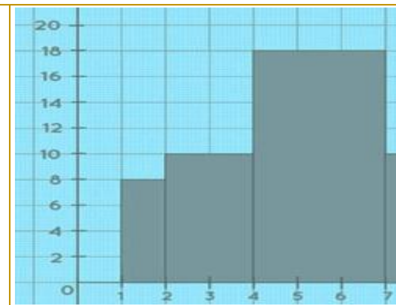
Cost of living index	Number of weeks
140-150	5
150-160	10
160-170	20
170-180	9
180-190	6
190-200	2
Total	52

5 100 surnames were randomly picked up from a local telephone directory and frequency distributions of the number of letters in the English alphabet in the surnames was found as follows: Draw a histogram to depict the given information. Write the class interval in which the maximum number of surnames lie.

Numberofletters	Numberofsurnames
1 –4	6
4 –6	30
6 –8	44
8 –12	16
12 – 20	4

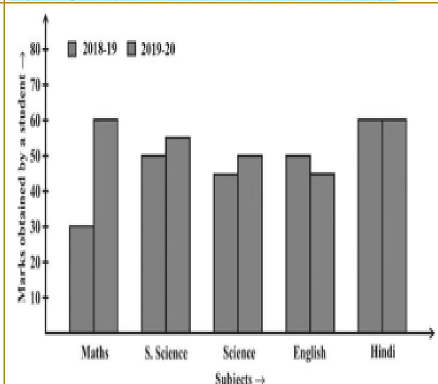
CASE BASED QUESTIONS

1 State government is planning to improve the facilities in government-maintained parks. A random survey is done on the number of children belonging to different age groups who play in government parks and the information is given in the form of a histogram given below.



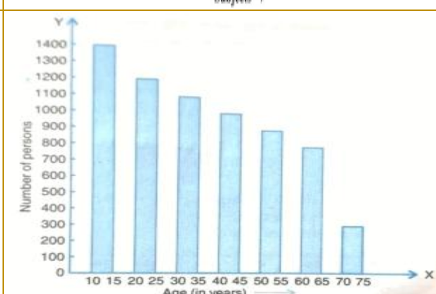
i)How many children of age group 9-11 are playing in the government parks?
ii)Which age group has the least frequency?
(iii) Which age group children has highest frequency?
(iv)How many children of age less than 4 years go to government parks?

2 The Class teacher of Class X preparing result analysis of a student. She compares the marks of a student obtained (out of 100) in Class IX (2018-19) and Class X (2019-20) using the double bar graph as shown below;



Read the above graph and answer the questions.
(i)In which subject was the performance at par?
(ii)What was the difference of marks in Maths Subject?
iii)In which subject the performance of 2018-19 was better than that of 2019 -20?
(iv)In which subject has the performance deteriorated?

3 Population census in India is conducted every 10 years the first complete census was taken in 1881 and the 15th decennial census was taken in 2011 the 16th Disney Lenses was to be conducted in 2021 but due to Covid it will be taken in 2022 the data obtained from the census of a town.



i)What is the total number of persons living in the town in the age groups 10 to 15 and 60 to 65?
ii)How many persons are more in the age group 10 to 15 than in the age group 30 to 35.
iii)What is the total population of the town?
iv)What is the number of persons in the age group 60 to 65?

HOTS

1 In a school 90 boys and 30 girls appeared in a public examination The mean marks of boys was found to be 45% where as the main marks of girls was 70% determine the average marks percentage of the school.

2 The median of the following observations arranged in ascending order 8, 9, 12, 18, x + 2,x + 4, 30 31, 34, 39 is 24.Find x.

3 The mean of 25 observations is 36.If the mean of the first 13 observations is 32 and that of the last 13 observations is 39, find the 13th observation.
Find the median of the data, 33, 31, 48, 45, 41, 92, 78, 51 and 61, if 92 is replaced by 29. What will be the new median?

4 The mean of 8 numbers is 15 if each number is multiplied by 2 what will be the new mean?

ANSWERS

MULTIPLE CHOICE QUESTIONS

1.C 2.D 3.C 4.B 5.A

ASSERTION AND REASONING QUESTIONS

1.C 2.C 3.A 4.C 5.B

VERY SHORT ANSWERQUESTIONS

1.Mean=2.8 Median =3 and Mode=3
2. 7 cm=56 Units
3.14
4.6
5.x=20

SHORT ANSWER QUESTIONS

1. Five examples from day-to-day life:
Number of students in our class.
Number of fans in our school.
Electricity bills of our house for last two years.
Election results obtained from television or newspapers.
Literacy rate figures obtained from Educational Surv
2.Draw Histogram
3.Mean=15,Median=52, Mode=52
4.Draw frequency distribution table
5.i)Draw Bar graph ii)Party-A

LONG ANSWER QUESTIONS

1.Draw Histogram and Frequency polygon
2.Draw Histogram and Frequency polygon
3.Draw Frequency polygon
4.Draw Frequency polygon
5.i) Draw Histogram ii) Class interval 6-8

CASE BASED QUESTIONS

1)i) 12 ii) 11-15 iii)4-7 iv)18
2)i) Hindi ii) 30 iii)English iv)English
3)i) 2200 ii) 300 iii)6700 iv)800

HOTS

1.51.25%
2.24
3.23
4.45
5.30

FORMATIVE ASSESMENT MATERIAL

PRACTICE QUESTION PAPERS(2025 – 26)

USEFUL LINKS

1	CBSE CURRICULUM https://cbseacademic.nic.in/curriculum_2025.html
2	NCERT TEXT BOOKS https://ncert.nic.in/textbook.php
3	NCERT YOUTUBE LESSIONS https://www.youtube.com/@NCERTOFFICIAL/search?query=CLASS%209%20MATHEMATICS
4	MATHEMTICIANS https://www.youtube.com/@NCERTOFFICIAL/search?query=MATHEMATICS%20SCIENCTISTS
	GEOGEBRA LESSIONS https://www.youtube.com/@NCERTOFFICIAL/search?query=GEOGEBRA%20LESSIONS