Applied Mathematics

कक्षा / Class XII

2025-26

विद्यार्थी सहायक सामग्री
Student Support Material

केन्द्रीय विद्यालय संगठन~Kendriya Vidyalaya Sangathan

<u>संदेश</u>

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना एवं नवाचार द्वारा उच्च - नवीन मानक स्थापित करना केन्द्रीय विद्यालय संगठन की नियमित कार्यप्रणाली का अविभाज्य अंग है। राष्ट्रीय शिक्षा नीति 2020 एवं पी. एम. श्री विद्यालयों के निर्देशों का पालन करते हुए गतिविधि आधारित पठन-पाठन, अनुभवजन्य शिक्षण एवं कौशल विकास को समाहित कर, अपने विद्यालयों को हमने ज्ञान एवं खोज की अद्भुत प्रयोगशाला बना दिया है। माध्यमिक स्तर तक पहुँच कर हमारे विद्यार्थी सैद्धांतिक समझ के साथ-साथ, रचनात्मक, विश्लेषणात्मक एवं आलोचनात्मक चिंतन भी विकसित कर लेते हैं। यही कारण है कि वह बोर्ड कक्षाओं के दौरान विभिन्न प्रकार के मूल्यांकनों के लिए सहजता से तैयार रहते हैं। उनकी इस यात्रा में हमारा सतत योगदान एवं सहयोग आवश्यक है - केन्द्रीय विद्यालय संगठन के पांचों आंचलिक शिक्षा एवं प्रशिक्षण संस्थान द्वारा संकलित यह विद्यार्थी सहायक- सामग्री इसी दिशा में एक आवश्यक कदम है । यह सहायक सामाग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है। केन्द्रीय विद्यालय संगठन की विद्यार्थी सहायक- सामग्री अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री संकलन की विशेषज्ञता के लिए जानी जाती है और शिक्षा से जुड़े विभिन्न मंचों पर इसकी सराहना होती रही है। मुझे विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर निरंतर मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुँचाएगी।

शुभाकांक्षा सहित ।

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UNIT 1:
NUMBERS, QUANTIFICATION AND NUMERICAL APPLICATIONS

GIST OF LESSONS

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Boats and Streams : Problem based on speed of stream and speed of boat in still water
Pipes and Cisterns : Calculation of the portion of the tank filled or drained by the pipes in unit time
Races and Games : Calculation of the time taken/ distance covered speed of each player
Numerical Inequalities : Comparison between two statements/ situations which can be Compared numerically, algebraic in equations

DEFINITION AND FORMULAE:

Modulo Arithmetic:
Euclid's Division Lemma:
For integers a, b ($\neq 0$), we have $\boldsymbol{a} = \boldsymbol{bq} + \boldsymbol{r}$, where $q, r \in Z$ and $0 \leq r < |b|$
Modulo Arithmetic is the arithmetic of remainders. $\boldsymbol{a \bmod b} = \boldsymbol{r}$
Where *mod* (modulo) gives the remainder after a is divided by b
Note: 1. $a \bmod a = 0$
2. If a<b, then $a \bmod b = a$

Properties

1. a mod b = (a + kb) mod b; where k is any integer
2. *(a + b)* mod *c* = *(a* mod *c* + *b* mod *c*)
3. (a - b) mod c = (a mod c - b mod c)
4. (a × b) mod c = (a mod c × b mod c)

Congruence Modulo:

Two positive integers *and* *b* are said to be congruence modulo *m* if *a* and *b* satisfy the Following conditions:
i) (*a*-*b*) is divisible by *m*
ii) a mod m = b mod m
Notation used for congruence modulo is: $\equiv b \ (mod m)$

Property 5: If $a \equiv b$ (mod *m*) where *a, b* and *m* are positive integers then $ak \equiv bk$ (mod *m)* for any positive integer k.

Allegation and Mixture:
Alligation means mixing two or more ingredients in some ratio.
Ratio (R) = $\frac{C.P. of\ Dearer\ (d) - mean\ price\ (m)}{mean\ price (m) - C.P\ of\ cheaper (c)} = \frac{d-m}{m-c}$
Repeated dilution:
Suppose a vessel contains 'x' units of a liquid from which y units are taken out and replaced by water.
Quantity of Liquid left after 'n' number of repeated dilutions = $x\ (1 - \frac{y}{x})^n$

Where, $x-$ Original amount, $-$ taken out and n $-$ Number of times
Boats and Streams:
Stream: Flowing water in a river or canal etc.
Still water: When water is stationary i.e. speed of water is zero.
Downstream: When boat rows in the direction of the stream.
Upstream: When boat rows opposite to the direction of the stream .
Let the speed of the boat in the still water be xkm/h and sped of the stream be ykm/h. Then
1. Downstream Speed (u) = (x + y) km/h and Upstream $Speed$ (v) = ($x-y$)km/h
2. Speed of Boat = $\frac{u+v}{2}$ and Speed of Stream = $\frac{u-v}{2}$
Average speed $= \frac{\text{downstream speed X upstream speed}}{\text{speed in still water}} = \frac{(x+y)X(x-y)}{x}$

Pipes and Cisterns:

- Inlet pipe : A pipe connected to a tank or cistern which fills it is known as inlet pipe
- Outlet pipe or Leak: A pipe connected to the tank which drains or empties it is known as outlet pipe.
- When a tank is connected to many pipes (inlets and outlets), then the difference between the sum of the work done by inlets and the sum of the work done by outlets gives the filled part of the tank.
- Let a pipe fill a tank in x number of hours, and then it can fill $\frac{1}{x}^{th}$ portion of the tank in one hour.
- If a pipe can empty a tank in y number of hours, then it can empty out $\frac{1}{y}^{th}$ portion of the tank in one hour.
- The portion of tank they can fill together in one hour = $\frac{1}{x} - \frac{1}{y}$
- If a pipe fills $\frac{1}{x}th$ part of a tank in 1 hour, then the time taken by the pipe to fill the tank completely is x hours
- Two pipes can fill a tank in x and y hours respectively. If both the pipes are opened simultaneously, then time taken by both the pipes to fill the tank = $\frac{1}{x} + \frac{1}{y}$ hours.
- If two pipes A and B together can fill a tank in x hours and the pipe A alone can fill the tank in y hours, then time taken by pipe B alone to fill the tank = $\frac{xy}{y-x}$ hours.
- If a pipe A can fill a tank in x hours and a pipe B can empty the full tank in y hours (where y > x), then net part filled in 1 hour = $\frac{y-x}{xy}$
- If a pipe A can fill a tank in x hours and a pipe B can empty the full tank in y hours (where x >y), then net part emptied in 1 hour = $\frac{x-y}{xy}$
- Three pipes A, B and C can fill a tank in x y, and z hours respectively. If all the three pipes are opened simultaneously, then time taken by all the pipes to fill it is = $\frac{xyz}{xy+yz+zx}$ hours.
- If two pipes are filing a tank at the rate of x hours and y hours respectively and a third pipe is emptying it at the rate of z hours, then in one hour the part of the tank filled = $\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$ hours
- Time taken to fill the tank is $= \frac{xyz}{yz+zx-xy}$

Races and Games:

- Race: It is a competition in which participants try to cover a fixed distance in the least possible time.

- Race course: A path on which a race takes place is called a race course e.g. Ground, road, swimming pool etc.
- Starting point: A point from where the race starts.
- Finish point or Winning point or Goal: A point where race finishes is called finish point or winning point or goal.
- Winner: A person who first reaches the finishes point is called winner.
- Dead heat: If all the participants reach the finish point at the same instant of time, then this situation is called dead heat
- Suppose X and Y are participating in a race.
- "X gives Y a start of x meters" means Y starts the race x meters ahead of X.
- "X gives Y a start of t-minutes" means X will start t minutes after B starts the race.
- "X beats Y by x meters" means when X reach the finishing point, Y is x meters behind X.
- "X beats Y by t minutes" means Y finish t minutes after X.
- "X beats Y by x meters and t minutes" means Y is x-meters behind X and finish race t- minutes after X.
- A game of 100 means that the person among the participants who scores 100 points first is the winner.
- "X beats Y by 25 points" or "X can give Y 25 points" means X scores 100 while Y scores only 75points (100-25).

Numerical Inequalities:

- Any two real number associated by, $\leq or \geq$ forma numerical inequality.
- If a, b are positive numbers and AM and GM are their arithmetic mean and geometric mean respectively,
- then $AM = \frac{a+b}{2} and \boldsymbol{GM} = \sqrt{ab}$
- $AM - GM = \frac{a+b}{2} - \sqrt{ab} = \frac{a + b - 2\sqrt{ab}}{2} = (\sqrt{a} - \sqrt{b})^{2/}2 \geq 0$
- $\boldsymbol{AM} \geq \boldsymbol{G}$M

MULTIPLE CHOICE QUESTIONS

1. If 100 ≡x (mod 7) then the least positive value of x is

   (a) 2 (b) 3 (c) 6 (d) 4

   Answer (a)

   Solution: 100-x is divisible by 7

   ⇨Least positive value of x is 2.

2. 20 liters of a mixture contains milk and water in the ratio 3:1. The amount of milk , in liters to be added to the mixture so as to have milk and water in the ratio 4:1 is

   (a) 7 (b) 4 (c) 5 (d) 6

   Answer (c)

   Solution: Total volume of the mixture = 20 liters

   Ratio = 3:1=3+1 =4

   Quantity of water = $\frac{1}{4} \times 20$ =5 litres

3. If a man rows 32km downstream and 14km upstream in 6 hours each, then the speed of the stream is

(a) 2km/h (b) 1.5 km/h (c) 2.5 km/h (d) 2.25 km/h

Answer (b)

Solution: speed of downstream (x+y) = $\frac{32}{6} = \frac{16}{3}$ (1)

Speed of upstream (x-y) = $\frac{14}{6} = \frac{7}{3}$ (2)

(1)- (2), we get

y= 3/2= 1.5 km

4. Pipes A and B can fill a tank in 5 hours and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the time taken to fill the tank is

(a) 2 hours (b) $2\frac{3}{4}$ hours (c) 3 hours (d) 3-$\frac{9}{17}$ hours

Answer (d)

Solution: Part of tank filled by pipes A, B and C = $\frac{1}{5} + \frac{1}{6} - \frac{1}{12} = \frac{17}{60}$

(b) time taken to fill the tank is = $\frac{60}{17}$ = 3-$\frac{9}{17}$ hours

5. In what ratio must rice at Rs 69 per kg be mixed with rice at Rs 100 per kg so that the mixture is worth Rest 80 per kg?

(a) 11:20 (b) 11: 10 (c) 20:11 (d) 10:11

Answer (c)

Solution: Rice at 69: rice at 100 = 100−80:80−69 = 20:11

6. The last (unit) digit of $(22)^{12}$ is

(a) 2 (b) 4 (c) 6 (d) 8

Answer (c)

Solution: we find $(22)^{12}$ (mod 10)

$22 \equiv 2 \pmod{10}$

$22^4 \equiv 2^4 \pmod{10}$, $22^4 \equiv 16 \pmod{10}$

$22^4 \equiv 6 \pmod{10}$, $(22^4)^3 \equiv 6^3 \pmod{10}$

$(22)^{12} \equiv 6 \pmod{10}$, last digit of 22^{12} is 6.

7. The least non-negative remainder when 3^{50} is divided by 7 is

(a) 4 (b) 3 (c) 2 (d) 1

Answer (c)

Solution: $3 \equiv 3 \pmod 7$

$3^2 \equiv 2 \pmod 7$

$3^4 \equiv 2^4 \pmod 7$

$3^4 \equiv 2 \pmod 7$

$(3^4)^6 \equiv 2^6 \pmod 7$

$3^{24} \equiv 1 \pmod 7$

$(3^{24})^2 \equiv 1 \pmod 7$

$3^{48}\ 3^2 \equiv 9 \pmod 7$

$3^{50} \equiv 2 \pmod 7$

When 3^{50} are divided by 7 remainder is 2.

8. The value of -70 mod 13 is

(a) 5 (b) -5 (c) 8 (d) -8

Answer (c)

Solution: $-70 \equiv 8 \pmod{13} \therefore 0 \leq r \leq |b|$

-70 mod 13 = 8

9. The number at unit place of number 17^{123} is

(a) 1 (b) 3 (c) 7 (d) 9

Answer (b)

Solution: we find $17^{123} \pmod{10}$

$17 \equiv 7 \pmod{10}$

$17^2 \equiv 7^2 \pmod{10}$, $17^2 \equiv 9 \pmod{10}$

$(172)^2 \equiv 9^2 \pmod{10}$, $(17^4) \equiv 1 \pmod{10}$

$(17^4)^{30} \equiv 1 \pmod{10}$

$(17)^{120}\ 17^2 \equiv 289 \pmod{10}$

$(17)^{122} \equiv 9 \pmod{10}$

$(17)^{122} \equiv 9 \times 17 \pmod{10}$, $17123 \equiv 153 \pmod{10}$, $17^{123} \equiv 3 \pmod{10}$

10. In a 100m race, A can give B a start of 10m and c a start of 28m. How much start can B give to C in the same race?

(a) 18m (b) 20m (c) 27 m (d) 9m

Answer (b)

Solution: when A runs 100m, B runs (100-10) = 90m
When A runs 100m, C runs (100-28) = 72 m
When B runs 90m, C runs = 72m
When B runs 100m, C runs = $\frac{72}{90} \times 100$ =80m
Thus B give a start of C by (100- 80) =20 m

11. A milkman mixed some water with milk to gain 25% by selling the mixture at the cost price .The ratio of water and milk respectively, is

(a) 5:4 (b) 4:5 (c) 1:5 (d) 1:4

Answer (d)

Solution: C.P. of milk = S.P. of mixture = Rs x

C.P. of mixture = S.P (100/125) = $\frac{4}{5}$ x, C.P.of water = Rs 0

Water: Milk = (x-$\frac{4}{5}$x): $\frac{4}{5}$ x = $\frac{1}{5}$ x: $\frac{4}{5}$ x = 1:4

12. A motor boat can travel in still water at the speed of 15 km/h, while the speed of the current is 3 km/h. Time taken by boat to go 36 km upstream is

(a) 2h (b) 3h (c) 12 h (d) 18h

An (b)

Solution: speed of upstream = (15-3) =12km/h

. Time taken by boat = $\frac{36}{12}$ =3 hours

13. In a game of 600 points, A scores points 407 points while B scores 307 points. In this game, the point given by A to B is

(a) 105 (b) 200 (c) 100 (d) 150

Answer (c)

Solution:

(Total score points = 600

A score's = 407

B score's = 307

∴ A give (407 – 307) = 100 points to B.

14. The solution set of the in equation | x+2 | ≤ 5 is

(a) (-7,5) (b) [-7,3] (c) [-5,5] (d) (-7,3)

Answer (b)

Solution: -5 ≤ x+2 ≤ 5

-7 ≤ x ≤ 3 , x∈) [-7, 3]

15. If x > y and z<0, then

(a) x> z (b) y ≥ z (c) $\frac{x}{z} > \frac{y}{z}$ (d) $\frac{x}{z} < \frac{y}{z}$

Answer (d)

Solution: it is given that x >y, z <0

⇨ $\frac{x}{z} < \frac{y}{z}$

16. If $\frac{|x+1|}{x+1}$ >0, x ∈R, then

(a) x ∈ [-1∞) (b) x∈ (-1,∞) (c) x ∈ (- ∞ ,-1) (d) x∈ (∞ , -1]

Answer (b)

Solution: $| x + 1 | \geq 0$, x+1≠ 0

⇨ x ∈(-1, ∞)

17. (62 +53) mod 7 is equal to

(a) 3 (b) 5 (c) 7 (d) 9

Answer (a)

Solution: 62 ≡ 6 (mod 7)

53 ≡ 4 (mod 7), 62+53 ≡ (6+4) (mod 7)

62+53 ≡ 10 (mod 7) , 62+53 ≡ 3 (mod 7)

So, (62 + 53) mod7 =3

18. The system of linear in equations 2x-1≥3 and x-3 >5 has solution

(a) (2 ,∞) (b) (2,8) (c) (8,∞) (d) (- ∞ ,8)

Answer (c)

Solution: 2x≥ 4, x≥ 2

X> 8 ⇨ x ∈(8, ∞)

19. In a 700m race, Amity reaches the finish point in 20 seconds and Rahul reaches in 25 seconds. Amity beats Rahul by a distance of

(a) 120m (b) 150 m (c) 140m (d) 100m

Answer (c)

Solution: Speed of Rahul $=\frac{700}{25}$ =28 m/s

Distance covered by Rahul in (25-20) =5 seconds = 28×5 =140m

Amity beats Rahul by 140m

20. Two pipes A and B together can fill a tank in 40 minutes. Pipes a twice as fast as pipe B. A alone can fill the tank in

(a) 1 hour (b) 2 hours (c) 80 minutes (d) 20 minutes

Answer (a)

Solution: B alone fill tank in x minutes.

$\frac{1}{x}+\frac{2}{x}=\frac{1}{40}$

$\frac{3}{x}=\frac{1}{40}$, x= 120

A can alone fill tank 120/2 =60 minutes =1 hour.

ASSERTION -REASON TYPE QUESTIONS

Directions In the question number 1 and 10, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A jess false but R is true

1. **Assertion (A)**: -62 mod 12 ≡ -2

Reason (R): a mod b ≡ r, where r is remainder when a is divided by b, 0 ≤ r <|b|.

Answer: d

Solution: -62 (mod12) ≡10 , Assertion is false and Reason is true.

2. **Assertion (A)**: If 60 ≡ 4 (mod m), then possible values of m are 2,4,7,8,14,28,56.

Reason (R): If a ≡ b (mod n), then a-b is an integral multiple of n, where n∈I, n ≥1.

Answer c

Solution: 60 ≡ 4 (mod m), 60-4 is divisible by m; m is a divisor of 56. m=2, 4, 7, 8,14,28,56

Reason is false. (Because If a ≡b (mod n), then a-b is an integral multiple of n, where nεI, n >1.

3. **Assertion (A):** Last two digits of the product 482 X 307 are 74.

Reason (R): If a ≡ b (mod n) and c ≡ d (mod n) then ac ≡bd (mod n).

Answer: a

Solution: 482 ×307 ≡ 82× 7 (mod 100) ≡ 574 (mod 100) ≡ 74 (mod100)

4. **Assertion (A):** Two different qualities of rice costing Rs 100 per kg and Rs 120 per kg are mixed in the ratio 2:3. The price of mixed quality is Rs 112 per kg.
   Reason (R): Mean price: $w_1p_1 + w_2p_2 / w_1 + w_2$

   Answer: a

   Solution: cost of Rs 100 per kg price = Rs 200x

   Cost of Rs 120 per kg price = Rs 360x

   Total quantity = 5x

   Price per kg of mixed rice = $\frac{560x}{5x}$ = Rs 112

   Mean price = $\frac{2\times100+3\times120}{2+3}$ = Rs 112

5. **Assertion (A):** If |x – 5| < 2 x ∈ R, then x ∈ [3, 7].
   Reason (R): |x| < a, iff – a < x < a

Answer : d

Solution :

(d) We know, |x| < a, then – a < x < a
Now, |x – 5| < 2
⇒ -2 < x – 5 < 2 ⇒ 3 < x < 7
∴ x ∈ (3, 7)
Assertion is false and Reason is true.

6. **Assertion (A)** If a is any positive real number, then $a + \frac{1}{a} \geq 2$.

   Reason (R) Let a, b be distinct positive real number, then $\frac{a+b}{2} \geq \sqrt{ab}$.

   Answer: (a)

   Solution:

   Assertion using the inequality AM ≥ GM, we get

$$\frac{a+\frac{1}{a}}{2} \geq \sqrt{a \times \frac{1}{a}}$$

$$\rightarrow \frac{1}{2}\left(a+\frac{1}{a}\right) \geq 1 \rightarrow a+\frac{1}{a} \geq 2$$

Hence, Assertion and Reason both are true and Reason is correct explanation of Assertion.

7. **Assertion (A)** The ratio of copper and zinc in brass is 13: 7. In 100 kg of brass, there is 35 kg zinc.

   Reason(R): $\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{CP of dearer} - \text{Mean price}}{\text{Mean price} - \text{CP of cheaper}}$

   Answer : (b)

Solution:

Assertion Let the quantity of copper and zinc in brass is 13x and 7x respectively. Total quantity of brass = 13x + 7x = 20x.

According to the question,

20x = 100 ⇒ x = 5

Quantity of zinc = 7 × 5 = 35 kg

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

8. **Assertion (A)** A shopkeeper mix two types of oranges worth ₹30 per kg and ₹45 per kg respectively so as to get a mixture at ₹40 per kg in the ratio 2: 1.

Reason (R) Let the price of cheaper ingredient is denoted by c and price of dearer ingredient is denoted by d and we mix these two types of ingredients in some ratio to produce a mixture at desired price, therefore

$$\frac{\text{quantity of cheaper ingredient}}{\text{quantity of dearer ingredient}} = \frac{d-m}{m-c}$$

Answer: (d)

Solution: Assertion Ratio of quantity of 2 types of oranges

$$\frac{\text{Cheaper orange}}{\text{Dearer orange}} = \frac{45-40}{40-30} = \frac{5}{10} = \frac{1}{2}$$

Hence, Assertion is false but Reason is true.

9. **Assertion (A)** If 2x + 1 < |2x + 1|, x ∈ R, then x ∈ $(-\infty, \frac{-1}{2})$.

Reason (R) $|x| \geq |y|$ iff $x^2 \geq y^2$.

Answer: (b)

Solution:

Assertion Given,

2x + 1 < |2x + 1|

⇒ 2x + 1 < 0 [∵ x < |x|, iff x < 0]

⇒ 2x < -1

⇒ $x < \frac{-1}{2}$

Hence, the solution set is $(-\infty, \frac{-1}{2})$

Hence, Assertion and Reason both are true but Reason is not the correct explanation of Assertion.

10. **Assertion (A)** A can run a kilometer in 4 min 50 s and B in 5 min. Then, A must give 10 s starts to B in a kilometer race, so that the race may end in a dead heat.

Reason (R): A can give B a start of t min means that A will start 't' min after B starts from the starting point and both A and B reach the finishing point at the same time.

Answer: (a)

Solution:

Assertion A runs 1000 m in 290 s and B in 300 s. For the race to end in a dead heat, A and B must reach the goal at the same time i.e. in 290 s. So, A must give 10 s starts to B.

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

VERY SHORT ANSWER TYPE QUESTIONS

1. Two runner A and B complete a 100 meters race in 36 seconds and 48 seconds respectively, By how many meters will a defeat B?

Solution: A beats B by 12 seconds.

Distance covered by B in 12 seconds = $\frac{100}{48} \times 12$ = 25 metres

2. A runs $1\frac{2}{3}$ times fast as B. If A gives B a start of 80m. How far must the winning post be so that A and B might reach it at the same time?

Solution: Ratio of speeds of A and B = $\frac{5}{3}$:1= 5:3

2 m are gained by B in a race of 5m

1m are gained by B in a race of $\frac{5}{2}$ m

80m will be gained by B = $\frac{5}{2}\times 80$ = 200m

Winning post is 200m away from starting point

3. In a 100 meters race. A, B and C get the gold, silver and bronze medals respectively. If A Beats B by 100 meters and B beats C by 100 meters, then by how many meters does a beats C?

Solution: A: B = 1000:900 and B: C = 1000:900

$\frac{A}{C} = \frac{A}{B}\times\frac{B}{C} = \frac{1000}{900}\times\frac{1000}{900} = \frac{1000}{810}$

A beats C by (1000-810) = 190 m

4. In a game of 100 points, A can give B 20 points and C 20 points. Then, how many points B can give C?

Solution: A: B = 100:80 and A: C = 100:72

$\frac{B}{C} = \frac{A}{C}\times\frac{B}{A} = \frac{80}{100}\times\frac{100}{72} = \frac{100}{90}$

B can give C 10 points

5. In a game A can give 15 points to B, A can give 30 pints to C and B can give 20 points to Chow many points make the game?

Solution: Let A scores x points, B scores (x-15) points and C scores (x-30) points

Also B scores x points, C scores (x-20) points

$\frac{x-15}{x-30} = \frac{x}{x-20}$, x =60

60 points make the game

6. A pipe can fill a cistern in 6 hours. Due to leakage in the cistern is just full in 9 hours. How much time the leakage will take to empty the tank?

Solution: Part of tank filled by cistern in 1 hour $=\frac{1}{6}$

Part of tank emptied by leakage in 1 hour = $\frac{1}{9}$

A.T .Q $\frac{1}{6} - \frac{1}{9} = \frac{1}{18}$ .Time taken by leak to empty it = 18 hours

7. Two pipes A and B together can fill a tank in 6 minutes. If pipe A takes 5 minutes less than B to fill the tank, find the time taken by pipe B to fill the tank alone.

Solution: Let, pipe B fill the tank in x minutes, then pipe a fill the tank in (x-5) minutes

$\frac{1}{x} + \frac{1}{x-5} = \frac{1}{6}$, x =15 minutes

Time taken by pipe B to fill the tank alone = 15 minutes

8. Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should be closed so that the tank is full in 18 minutes?

Solution: let B closed after = x minutes

Part of tank filled by A and B in x minutes= x ($\frac{1}{24} + \frac{1}{32}$)$=\frac{7x}{96}$

Part of tank filled by A in (18- x) minutes $=\frac{18-x}{24}$

A.T.Q.

$\frac{7x}{96}+\frac{18-x}{24}$ =1, x =8

B should be closed after 8 minutes.

9. Pipe A can fill the tank 2 times faster than pipe B. If both pipes A and B running together can fill the tank in 24 minutes, find how much time will pipe B alone take to fill the tank?

Solution: Let, pipe B fill the tank in x minutes, then pipe a fill the tank in $(\frac{x}{2})$ minutes

$\frac{1}{x}+\frac{2}{x}=\frac{1}{24}$, x = 72

Time taken by pipe B to fill the tank alone = 72minutes

10. A man rows 15 km upstream and 25km downstream in 5 hours each time. What is the speed of the current?

Solution: Let speed of still water = x km/h, speed of current = y km/h

Speed of downstream (x + y) = $\frac{25}{5}$=5, speed of upstream (x-y) = $\frac{15}{5}$ =3

Speed of current = 1 km /h

11. A boat rows a distance of 12 km downstream at13km/h and cover the same distance upstream at 7km/h. Find the average speed of the boat.

Solution: Average speed = $\frac{\text{downstream speed} \times \text{upstream speed}}{\text{speed in still water}}=\frac{13\times7}{10}$ = 9.1 km/h

12. A person can row a boat 5 km an hour in still water. It takes him thrice as long to row upstream as to row downstream. Find the rate at which the stream is flowing.

Solution: let rate of stream = x km/h

Speed of upstream = (5-x) km/h, Speed of downstream = (5+x) km/h

A.T.Q 5+x = 3 (5-x), x= 2.5 km/h

13. A boat goes 6 km in an hour still in water. It takes thrice as much time in covering the same distance against the current. Find the speed of the current.

Solution: speed in still water = 6 km/h, speed of current = x km/h

Speed against the current = 6/3 =2 km/h

A.T.Q 6- x= 2, x= 4 km/h, speed of the current = 4 km/h

14. A man can row 7 km per hour in still water. If the stream is flowing at the rate of 5km per hour, it takes him 7 hours to row to a place and return, how far is the place?

Solution: let distance of place =x km, speed of still water = 7 km/h, speed of current = 5 km/h

Speed of downstream = 12 km/h, speed of upstream = (7-5) =2 km/h

A.T.Q $\frac{x}{12}+\frac{x}{2}$ = 7, x= 12

Distance of place = 12 km

15. A man can row $7\frac{1}{2}$ km/h in still water. If in a river running at 1.5 km per hour, it takes him 50 minutes to row to a place and back, how far off is the place?

Solution: let distance of place =x km, speed of still water = 7.5 km/h, speed of current = 1.5 km/h

Speed of downstream = (7.5 +1.5) = 9 km/h, speed of upstream = (7.5-1.5) =6km/h

A.T.Q $\frac{x}{9}+\frac{x}{6}=\frac{5}{6}$, x = 3

16.In what ratio must a grocer mix two varieties of sugar worth Rs.60 per kg and Rs.65per kg respectively so that by the selling the mixture at Rs.68.20 per kg, he may gain 10%.

Solution: C.P.of 1kg mixture = Rs $(68.20\times \frac{100}{110})$ = Rs 62

$\frac{\text{quantity of cheaper ingredient}}{\text{quantity of dearer ingredient}} = \frac{d-m}{m-c} = \frac{65-62}{62-60} = \frac{3}{2}$

Required Ratio = 3:2

17.In what ratio water must be added to dilute honey costing Rs.240 per liter so that the resulted syrup would be worth Rs.200 per liter?

Solution: C.P of water = Rs 0,

$\frac{\text{quantity of cheaper ingredient}}{\text{quantity of dearer ingredient}} = \frac{d-m}{m-c} = \frac{240-200}{200-0} = \frac{40}{200} = \frac{1}{5}$

Required Ratio = 1:5

18. A container contains 50 liters of milk. From this container 10 liters of milk was taken out and replaced by water. This process is repeated two more times. How much milk is now left in the container?

Solution: quantity of milk in the container = 50 liters

Quantity of milk left in the container after 4 processes of dilution = $50\left(1-\frac{10}{50}\right)^3$ litres

$= \frac{128}{5}$, x= 25.6 litres

19. Cost of two types of pulses is Rs.55 per kg and Rs.90 per kg. If both the pulses are mixed together in the ratio 2:3, what should be the price of the mixed variety of pulses per kg?

Solution: C.P of mixture = Rs x,

$\frac{quantity\ of\ cheaper\ ingredient}{quantity\ of\ dearer\ ingredient} = \frac{d-m}{m-c} = \frac{90-x}{x-55} = \frac{2}{3}$, x = 76

C.P of mixture = Rs 76

20. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18%profit. He gains 14&%on the whole. Find the quantity sold at 8% profit?

Solution: $\frac{quantity\ of\ cheaper\ ingredient}{quantity\ of\ dearer\ ingredient} = \frac{d-m}{m-c} = \frac{18\%-14\%}{14\%-8\%} = \frac{2}{3}$,

Ratio =2:3, Quantity of sugar = 1000kg

Quantity sold at 8% profit= $1000\times\frac{2}{5}$ = 400 kg

SHORT ANSWER TYPE QUESTIONS

1. A vessel contains a mixture of two liquid P and Q in the ration 5:7.12 liters of mixture are drawn off from the vessel and 12 liters of liquid P is filled in the vessel. If the ratio of liquid P and Q now becomes 9:7, how many liters of liquids P and Q were contained by the vessel initially?

Solution: Let initially liquid P =5x liters and initially liquid Q = 7x liters

Quantity of liquid P left in the mixture = $5x - \frac{5}{12}\times 12$ = (5x-5) litres, Quantity of liquid Q left in the mixture = $7x - \frac{7}{12}\times 12$ = (7x-7) litres

After mixing 12 litres of liquid P, Quantity of liquid p = (5x-5+12) = (5x +7) litres , Quantity of liquid Q = (7x-7) litres

A.T.Q $\frac{5X+7}{7X-7} = \frac{9}{7}$, x = 4

Initially liquid P =5× 4 =20 litres and initially liquid Q = 7× 4 = 28 litres

2. Two vessels A and B contain milk and the water in the ratio 5:2 and 8:5 respectively. In what ratio mixtures from two vessels should be mixed to get a new mixture containing milk and water in the ratio 9:4?

Solution: Let the C.P. of milk = Rs x

Milk in 1 L mixture in A = $\frac{5}{7}$ L C.P. of 1 litre mix in A = $\frac{5}{7}x \times$

Milk in 1 L mixture in B = $\frac{8}{13}$ L C.P. of 1 litre mix in B = $\frac{8}{13}x$

Milk in 1 L mixture of A and B = $\frac{9}{13}$

C.P. of 1 litre mix in A C.P. of 1 litre mix in B

$\frac{5}{7}x$ $\frac{8}{13}x$

Mean Price $\frac{9}{13}$x

$\frac{x}{13}$ $\frac{2x}{91}$

Required Ratio = $\frac{x}{13}$: $\frac{2x}{91}$ =7:2

3. From a container full of milk 8 liters are drawn at then filled with water. This process is repeated three more times. The ratio of the quantity of milk left in the container and that of water is 16:65. How many liters of milk did the container hold originally?

Solution: Let the quantity of milk in the container = x litres

Quantity of milk left in the container after 4 processes of dilution = x $(1-\frac{8}{x})^4$ litres

A.T.Q. $\frac{x\,(1-\frac{8}{x})^4}{x} = \frac{16}{81}$, $(1-\frac{8}{x})^4 = (\frac{2}{3})^4$, $(1-\frac{8}{x}) = \frac{2}{3}$, x= 24 litres

4. A bottle is full of Dettol. One-third of its Dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of Dettol to water in the bottle.

Solution: Let the quantity of Dettol in the bottle = x mL and quantity of Dettol replaced by water = y mL, y = $\frac{x}{3}$

Quantity of Dettol left in the bottle after 3 processes of dilution = x $(1-\frac{y}{x})^n$ = x $(1-\frac{1}{3})^3$

Amount of water in the bottle = x - x $(1-\frac{1}{3})^3$

A.T.Q. x $(1-\frac{1}{3})^3$: = x - x $(1-\frac{1}{3})^3$, $\frac{8}{27}$: [1- $\frac{8}{27}$]

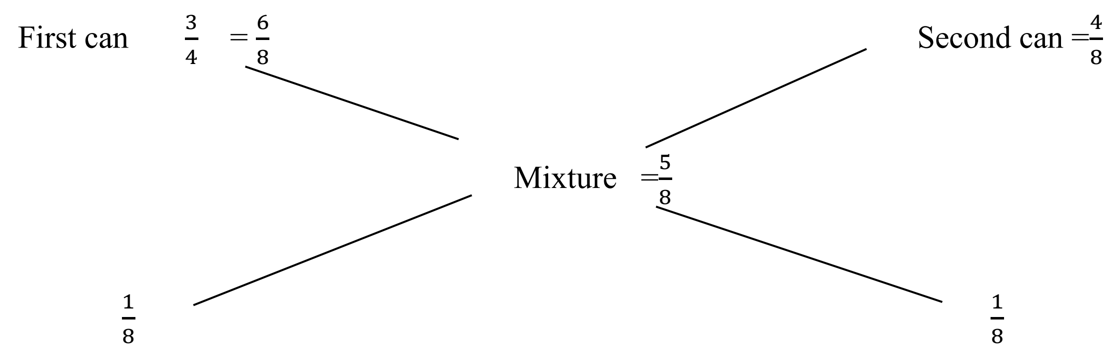
Required ratio 8: 19

5. A milkman has two cans. First containing 75% milk and rest water, whereas second containing 50% milk and rest water. How much mixture should he mix from each can so as to get 20 liters of mixture, such the ratio of milk and water is 5:3 respectively?

Solution: first can = milk: water = 75%: 25% . Second can = 50%: 50%

Quantity of milk in first can =$\frac{75}{100}$ =$\frac{3}{4}$, Quantity of milk in second can =$\frac{50}{100}$ =$\frac{1}{2}$ =$\frac{4}{8}$

Mixture = milk: water = 5:3 , Quantity of milk in mixture =$\frac{5}{8}$



Required Ratio = $\frac{1}{8} : \frac{1}{8}$ = 1:1

20 L mixture contains 10 L water and 10L milk

6. Two pipes P and Q can fill a tank in 60 minutes and 30 minutes respectively. Third pipe R drains 5 liters of water per minute. If all three pipes are opened it took 2 hours to fill the tank. Find the capacity of the tank.

Solution:

Pipe A can fill the tank in 60 min ; Pipe B can fill the tank in 30 min

Time taken to fill the tank by (A+B+C) = 2 hours = 120 minutes

Let, Time taken by pipe R to drain the tank = x

A,T.Q. $\frac{1}{60} + \frac{1}{30} - \frac{1}{x} = \frac{1}{120}$

$\frac{1}{60} + \frac{1}{30} - \frac{1}{120} = \frac{1}{x}$, x= 24

In 1 min pipe R drain 5 litres

In 24 min pipe R drain 5 x 24 = 120 litres

So, the capacity of the tank = 120 litres

7. A man make twice as long as to row a distance against the stream as to row the same distance in the direction of stream. Find the ratio of speed of man in still water to the speed of stream.

Solution: speed of boat in still water = x km/h, speed of stream = y km/h

A.T.Q. Time taken by upstream = 2 time taken by down stream

$\frac{d}{x-y} = 2\ \frac{d}{x+y}$, 2(x-y) = x + y, x= 3y

Speed of man in still water: speed of stream = x: y = 3y: y = 3:1

8. A cistern can be filled by two pipes A and B in 12 minutes and 15 minutes respectively. Another tap C can empty the full tank in 20 minutes. If the tap C is opened 5 Minutes after the pipes A and B are opened, find when the cisterns becomes full?

Solution: Part filled by pipe an in 1 minute = $\frac{1}{12}$

Part filled by pipe B in 1 minute = $\frac{1}{15}$

Part filled by pipe C in 1 minute = $\frac{1}{20}$

Part filled by pipe A and B in 5 minutes =5 ($\frac{1}{12}+\frac{1}{15}$) = $\frac{3}{4}$

Unfilled part of tank = $\frac{1}{4}$

Portion of cistern filled A, B and C in 1 minute = $\frac{1}{12}+\frac{1}{15}-\frac{1}{20}=\frac{1}{10}$

$\frac{1}{4}$ Of the tank will be filled in = $\frac{1}{4}\times 10$ = 2 minutes 30 seconds

9. Two pipes can fill a tank in 15 hours and 12 hours respectively and a third pipe can empty it in 4 hours. If the pipes be opened in order at 8.A.M, 9.AM and 11.A.M. respectively at what time the tank will be emptied.

Solution: Let the tank be emptied in x hours after 8 A.M

Work done by A in x hours, by B in (x-1) hours and C in (x-3) hours =0

$\frac{x}{15}+\frac{x-1}{12}-\frac{x-3}{4}=0$, $x=\frac{20}{3}$

Tank will be emptied in 6 hours 40 minutes after 8 A.M.

Tank will be emptied at 2:40 P.M.

10. Three pipes A, B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. Find the number of hours taken by C alone to fill the tank is?

Solution: part filled by (A+B+C) in 2 hours = $\frac{1}{6}\times 2=\frac{1}{3}$

$\frac{2}{3}$ Part is filled by (A+B) = 7 hours

Whole is filled by (A+B) = $7\times\frac{3}{2}=\frac{21}{2}$ hours

Part filled by C in 1 hour = $\frac{1}{6}-\frac{2}{21}=\frac{1}{14}$ ∴ C alone can fill it in 14 hours.

11 .Two pipes A and B can fill a tank in 20 minutes and 60 minutes respectively. There is an outlet pipe C at the bottom of the tank. If all three pipes are opened together it took 40 minutes to fill the tank. Find the time taken by outlet C to empty the full tank working alone.

Solution: Let, the time taken by C to empty the tank = t

Tank filled by pipe an in 1 min = $\frac{1}{20}$

Tank filled by pipe B in 1 min = $\frac{1}{60}$

A.T.Q. $\frac{1}{20}+\frac{1}{60}-\frac{1}{t}=\frac{1}{40}$

. $\frac{1}{20}+\frac{1}{60}-\frac{1}{40}=\frac{1}{t}$ .t =24

Time taken by C to empty the tank = 24 minutes

12. A motorboat whose speed is 18km/h in still water takes 1 hour more to go 24 km upstream than to return downstream on the same spot. Find the speed of the stream.

Solution: speed of motor boat in still water = 18km/h, speed of stream = skim/h

Speed of upstream = (18-x) km/h, speed of downstream = (18+x) km/h

A.T.Q $\frac{24}{18-X}-\frac{24}{18+X}$ =1, x=6, speed of stream = 6 km/h

LONG ANSWER TYPE QUESTIONS

1. A cistern has three pipes A, B and C. Pipes A and B is inlet pipes whereas C is an outlet pipe. Pipes A and B can fill the cistern separately in 3 hours and 4 hours respectively, while pipe C can empty the completely filled cistern in 1 hour. If the pipes A, B and C are opened in order at 5, 6 and 7 a.m. respectively, at what time will the cistern be empty?

   Solution: Let the cistern be emptied in t hours after 7 a.m. then pipe A fill cistern for (t+2) hours and pipe B fill cistern for (t+1) hours and pipe C empties the cistern for t hours.

   Part of cistern filled by pipe a in 1 hour = $\frac{1}{3}$

   Part of cistern filled by pipe B in 1 hour = $\frac{1}{4}$

   Part of cistern emptied by pipe C in 1 hour = 1

   A.T.Q. $\frac{t+2}{3} + \frac{t+1}{4} - \frac{t}{1} = 0$, $t = \frac{11}{5}$ = 2 hours 12 minutes

   The cistern will be empty at 9:12 a.m.

2. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

   Solution – Let speed of the boat in still water = x km/h , speed of stream = y km/h

   Upstream speed = (x-y) km/h, Downstream speed = (x + y) km /h

   $\frac{32}{x-y} + \frac{36}{x+y} = 7$, and $\frac{40}{x-y} + \frac{48}{x+y} = 9$, putting x-y = u, x+y = v

   32 u +36 v = 7 …… (1) 40u +48v =9 …….. (2)

   After solving (1) and (2), we get $u = \frac{1}{8}$, $v = \frac{1}{12}$

   x-y = 8 and x+y =12 solving these equations x = 10 and y =2

   Speed of the boat in still water = 10 km/h, speed of stream = 2 km/h

3. The speed of a boat in still water is 5 times that of the current, it takes 1.1 hours to row to a point Q from P downstream. The distance between points P and Q is 13.2 km. How much distance will it cover in $5\frac{1}{5}$ hours upstream?

   Solution: Let speed of current = x km/h

   Speed of boat in still water = 5x km/h

   Speed of downstream = 5x+x =6x km/h, speed of upstream = 5x-x = 4x km/h

   A.T.Q $\frac{13.2}{6x} = 1.1$, x = 2

   Distance covered by boat in $5\frac{1}{5}$ hours = $4 \times 2 \times \frac{26}{5}$ =41.6 km

4. A leak in the bottom of a tank can empty the full tank in 8 hours. An inlet pipe fills water at the rate of 6 litres in a minute. When the tank is full, the inlet is opened but due to the leak, the tank is empty in 12 hours. How many litres does the tank hold?

   Solution: Part of tank filled in 1 hour = $\frac{1}{8} - \frac{1}{12} = \frac{1}{24}$

   Part of tank filled in 1 minute = $\frac{1}{24\times 60} = \frac{1}{1440}$

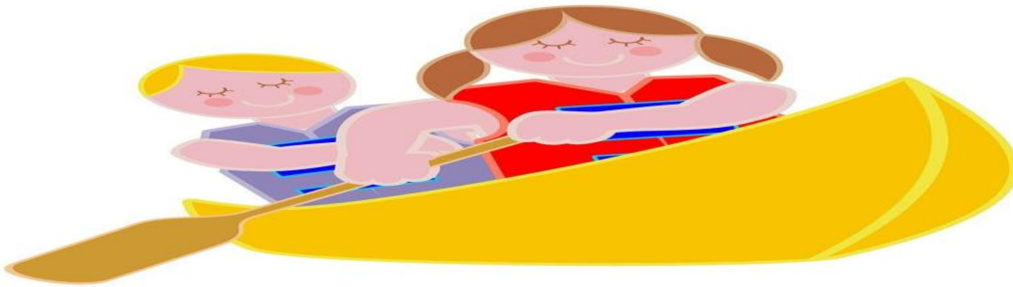
   Since it is given that inlet pipe fills water at the rate of 6 litres in a minute

   $\frac{1}{1440}$ Th part of volume of tank = 6 litres

   Volume of tank = 1440 × 6 = 8640 litres

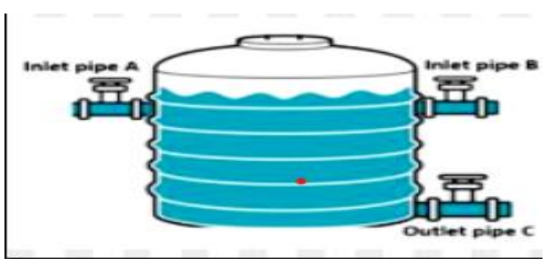
CASE BASED QUESTIONS

1. Sushma is rowing a boat. She takes 6 hours to row 48km upstream whereas she takes 3 hours to go the same distance downstream.



Based on above information, answer the following questions:
(i) What is her speed of rowing in still water?
(ii) What is the speed of the stream?
(iii) What is her average speed?
OR
(iii) If Sushma can row 7.5 km/h in still water. If river running 1.5 km an hour, if takes her 50 minutes to row to a place and back, then what is speed of upstream?
Solution: (i) Let speed of Sushma in still water = x km/h, speed of current = y km/h
A.T. Q. Speed of upstream = (x- y) = $\frac{48}{6}$ =8 and Speed of downstream = (x+y) = $\frac{48}{3}$ =16
x= 12, y= 4
Speed of rowing in still water = 12km/h
(ii) Speed of the stream = 4 km/h
(iii) Average speed $= \frac{total\ distance\ covered}{total\ time\ taken} = \frac{96}{9} = 10\frac{2}{3}$ km/h
OR
(iii) Speed of upstream = (7.5- 1.5) = 6 km/h

2. An overhead water tank has three pipes A, B and C attached to it (as shown in figure). The inlet pipes A and B can fill the empty tank independently in 15 hours and 12 hours respectively. The outlet pipe C alone can empty a full tank in 20 hours.



Based on the above information, answer the following questions.
(i) For a routine cleaning of the tank, the tank needs to be emptied. If pipes A and B are closed at the time when the tank is filled to two-fifth of its total capacity, how long will pipe C take to empty the tank completely?
(ii) How long will it take for the empty tank to fill completely, if all the three pipes are opened simultaneously?

(iii) On a given day, pipes A, B and C are opened (in order) at 5 am, 8 am and 9 am respectively, to fill the empty tank. In how many hours will the tank be filled completely?

OR

(iii) Given that the tank is half-full, only pipe C is opened at 6 AM, to empty the tank. After closing the pipe C and an hour's cleaning time, tank is filled completely by pipe A and B together. What is the total time taken in the whole process?

Solution: (i) Pipe C empties 1 tank in 20 h ⇒ $\frac{2}{5}$the tank in $\frac{2}{5} \times 20 = 8$ hours

(ii) Part of tank filled in 1 hour = $\frac{1}{15} + \frac{1}{12} - \frac{1}{20} = \frac{1}{10}$

⇒ Time taken to fill tank completely = 10 hours

(iii) At 5 a.m.

Let the tank be completely filled in't' hours

⇒ Pipe A is opened for't' hours, pipe B is opened for 't−3' hours, pipe C is opened for 't−4' hours

⇒ in one hour, part of tank filled by pipe A = $\frac{t}{15}$ Th

Part of tank filled by pipe B = $\frac{t-3}{15}$ Th

Part of tank emptied by pipe C = $\frac{t-4}{15}$ Th

$\frac{t}{15} + \frac{t-3}{15} - \frac{t-4}{15} = 1$, t = 10.5 hours

Total time to fill the tank = 10 hours 30 minutes

OR

(iii) At 6 am,

Pipe C is opened to empty ½ filled tanks

Time to empty = 10 hours

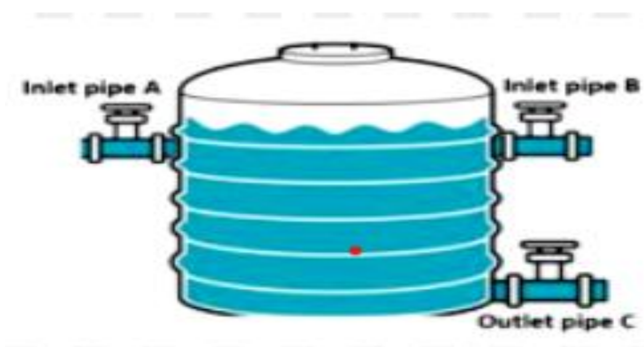
Time for cleaning = 1 hour

Part of tank filled by pipes A and B in 1 hour = $\frac{1}{15} + \frac{1}{12} = \frac{3}{20}$

⇒ Time taken to fill the tank completely = $\frac{20}{3}$ hours

Total time taken in the process = $10 + 1 + \frac{20}{3}$ = 17 hour 40 minutes

3. A, B and C are three pipes connected to a tank. A and B together fill the tank in 6 hours. B and C together fill the tank in 10 hours. A and C together fill the tank in $7\frac{1}{2} hours$. Based on above information answer the following questions.



(i) In how much time will A, B and C fills the tank?

(ii) In how much time will A separately fill the tank?

(iii) In how much time will B separately fill the tank?

OR

(iii) In how much time will C separately fill the tank?

Solution: (i) A + B fill the tank in 6 hrs

B + C fill the tank in 10 hrs

A + C fill the tank in $\frac{15}{2}$ hrs

$2(A + B + C) = \frac{6\times10\times\frac{15}{2}}{6\times10+10\times\frac{15}{2}+\frac{15}{2}\times6} = \frac{5}{2}$

Hence A, B and C together will fill the tank in 5 Hrs

(ii) A will in [(A+B+C) – (B+C)] = $\frac{10\times5}{10-5} = 10\ hrs$

(iii) B will fill [(A+B+C) – (A+C)] = $\frac{\frac{15}{2}\times5}{\frac{15}{2}-5} = 15\ hrs$

OR

C will fill in [(A+B+C) – (A+B)] = $\frac{6\times5}{6-5}$ $=30\ hrs$

HIGHER ORDER THINKING SKILLS(HOT'S)

1. Write the solution set of the in equation $|\frac{1}{x} - 2| > 4$

Solution:

$|\frac{1}{x} - 2| > 4$

$\Leftrightarrow$ $\frac{1}{x} < 2-4$ or $\frac{1}{x} > 2+4$

$\Leftrightarrow x < \frac{-1}{2}$ or $x < \frac{1}{6}$

Hence, solution set of the given system of in equations is $(-\infty, \frac{-1}{2}) \cup (\frac{1}{6}, \infty)$

2. How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs 20 so as to have mixture worth Rs $10\frac{2}{3}$ a litre?

Solution: Let x litres of water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs 20

C.P. of milk = $20 \times \frac{2}{3}$ =Rs $\frac{40}{3}$

$\frac{quantity\ of\ cheaper\ ingredient}{quantity\ of\ dearer\ ingredient} = \frac{d-m}{m-c} = \frac{\frac{40}{3}-\frac{32}{3}}{\frac{32}{3}-0} = \frac{x}{60}$, $\frac{1}{4} = \frac{x}{60}$

x = 15

EXERCISE

MULTIPLE CHOICE QUESTIONS

1. Pipes A can fill a tank 6 times faster than a pipe B. If B can fill a tank in 21 minutes, then the time taken by both the pipes together to fill the tank is

(a) 3 minutes (b) 4.5 minutes (c) 7 minutes (d) 9 minutes

2. Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled in just half an hour, if the B is turned off after:

(a) 5 min. (b) 9 min. (c) 10 min. (d) 15 min.

3. In a 1km race, player P beats player Q by 18 meters or 9 seconds. What is P's time to complete the race?

(a) 512 seconds (b) 502 seconds (c) 491 seconds (d) 481 seconds

4. Last two digits of 2^{20} are

(a) 73 (b) 74 (c) 75 (d) 76

5. If x ≡4 (mod 7), then positive values of x are

(a) { 4,11,18,….} (b) { 3,10,12,….} (c) { 4,8,12,…..} (d) {1, 8, 12,}

6. Tea at Rs 60 per kg is mixed with tea at Rs 65 per kg so that 10% is gained by selling the mixture at Rs 68.20 per kg. Ratio of mixture is

(a) 1:2 (b) 1:3 (c) 3:2 (d) 2:3

7. A pipe can fill a tank in 12 hours. Due to leak in the bottom, it is filled in 16 hours. If the tank is full, how long will it take to empty from the leak

(a) 40 hours (b) 48 hours (c) 50 hours (d) 52 hours

8. In, a 200m race, A beats B by 35m or 7 seconds. Find A's time over the course

(a) 31 seconds (b) 32 seconds (c) 33 seconds (d) 34 seconds

9. Let p>0 and q<0 and p, q∈ Z, then choose the correct inequality from the given below options to complete the statement p+q ……p-q

(a) > (b)≤ (c) ≥ (d) <

10. If x is a real number and |x| < 5, then

(a) X) ≥5 (b) -5 <x <5 (c) x ≤ -5 (d) -5 ≤ x ≤ 5

ASSERTION -REASON TYPE QUESTIONS

Directions In the question number 1 and 10, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

1. Assertion (A) (186 × 93) (mod 7) = 2

Reason (R) (a. b) (mod n) = a (mod n). B (mod n)

2. Assertion (A) In 30 L mixture of acid, the ratio of acid and water is 2: 3. The amount of water should be added to the mixture so that the ratio of acid and water becomes 2: 5 is 12 L.

Reason (R) Mean price is always lesser than cost price of cheaper quantity and higher than the cost price of dearer quantity.

3. Assertion (A) Rohan can row with a speed of 16 km/h in still water and the speed of stream is 12 km/h. Then, speed of Rohan downstream will be 26 km/h.

Reason (R) If the speed of a boat in still water is x km/h and speed of stream is y km/h, then speed of the downstream is (x + y) km/h.

4. Assertion (A) if A beats B by 200m or by 10 seconds, then speed of B is 20m/sec

Reason(R) In a 400 m race, A reaches the finishing point in 20 seconds and beat B by 100m. The speed of B is 15 m/sec

5. Assertion (A) In a race of 100m, If A beats B by 20 m and to C by 28m, then B beat C by 8m

Reason(R) If A beats by x m, then it means that when A is at finishing point, B is x m behind the finishing point

6. Assertion (A) The solution set of the inequality | 2x-3 | ≥5, x ∈ R is (-∞,-1] ∪ [4, ∞)

Reason(R) For real x, |x| ≤ a ⇨ -a ≤ x ≤ a, a > 0.

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the least non-negative remainder when $3^{51}$is divided by 7.
2.In what ratio water must be added in milk costing Rs 60 per litre , so that resulting mixture would be worth Rs 50 per litre ?
3.A man row to a place at a distance of 48km and returns back in 14 hours . He finds that he can row 4km with the stream in the same time as 3km against the stream. Find the speed of stream .
4. Two pipes A and B can fill a tank in $\frac{1}{2}$ hour and 1 hour respectively . A pipe C can empty the tank in 2 hours . If all three pipes are opened simultaneously , how long does it take to fill the empty tank ?
5.A and B participate in 200m race. A runs at the speed of 6 km/h . A gives B a start of 18m and still beats him by 6 seconds . Find the speed of B in km/h .
6.Write the solution set of the in equation $x + \frac{1}{x} \geq 2$

SHORT ANSWER TYPE QUESTIONS

1.How many litres of water should be added to a 30 litres mixture of milk and water containing milk and water in the ratio 7:3 such that the resultant mixture has 40% water in it ?
2.A manufacturer mixes two kinds of tea costing Rs 35 per kg and Rs 40 per kg in the ratio of 8:7 . Find profit or loss percent if he sells the mixture at the rate of Rs 37,50 per kg ?
3.The speed of a motor boat and that of the current of water is 36:5. The boat goes along with the current in 5 hours 20 minutes. How much time will it take to come back?
4.A cistern can be filled by an inlet pipe in 20 hours and can be emptied by an outlet pipe in 25 hours. Both the pipes are opened. After 10,hours the outlet pipe is closed , find the total time taken to fill the cistern.
5.Three pipes A,B and C can fill a tank in 6 hours. After working at it together for 2 hours, C is closed and A and B can fill the remaining part in 7 hours. Find the number of hours taken by C alone to fill the tank.
6. A large tanker can be filled by two popes A and B in 60 minutes and 40 minutes respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time A and B fills it together for the other half ?

LONG ANSWER TYPE QUESTIONS

1.A boat goes 20 km upstream and 22km downstream in 6 hours. Also, it goes 25km upstream and 33km downstream in 8 hours. Find the speed of the boat in still water and that of the stream
An : The speed of the boat in still water is 8km/h and the speed of the stream is 3km/h

2. Two pipes A and B can fill a cistern in 30 and 20 minutes respectively. They started to fill a cistern together but pipe A is closed few minutes later and pipe C, fills the remaining part of the cistern in 5 minutes. After how many minutes was pipe A closed ?

An : 9 minutes
3. A tank can be filled by two inlet pipes P and Q in 15 minutes and 20 minutes respectively. Another outlet pipe R can empty the full tank in 24 minutes. If the outlet pipe R is opened 6 minutes after the pipes P and Q are opened, find the total time to fill the tank.
An : 10 minutes

CASE -BASED QUESTIONS (4 Marks)

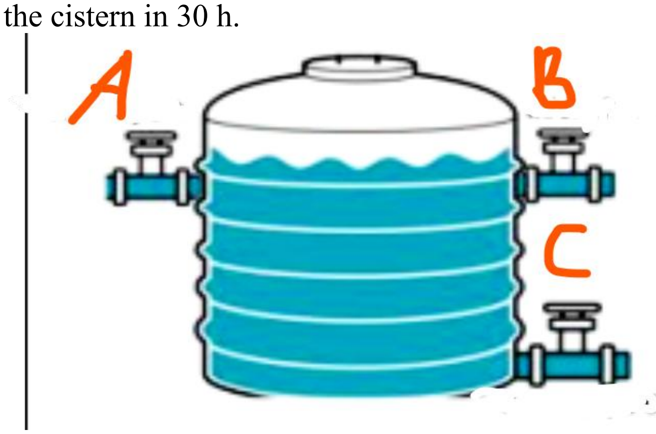
1. A man can row against the current, three fifth of a kilometer in 15 min and returns same distance in 10 min. Based on the above information answer the following questions.

(i) Find the speed of the current.
(ii) Find the speed of men in still water.
(iii) If a man can row a distance of 3 km against the stream, then find the time he would cover the distance.

OR

If the time taken by a man to cover 4.5 km downstream and x km upstream is same, then find the value of x.

2. A cistern has three pipes A, B and C. Pipes A and B can fill it in 5 h and 10 h respectively, while pipe C can empty the cistern in 30 h.



Based on the above information, answer the following questions.

(i) If the pipes A and B are opened, then find the time taken to fill the cistern
(ii) If all the pipes are opened, then find the time taken to fill the cistern
(iii) (a)If the pipes A and B are opened alternatively and pipe C is opened all the time, then find the time taken to fill the cistern.

OR

(b) If the tap A is opened partially in such a way that its efficiency of filling the cistern becomes half, then find the time taken by all the three taps to fill the tank.

HIGHER ORDER THINKING SKILLS (HOT'S)

1.A pipe can empty $\frac{5}{6}$ of a cistern in 20 minutes. What part of cistern will be emptied in 9 minutes

2.Two pipe A and B can fill a cistern in 10 minutes and 15 minutes respectively. Pipe C can empty the full cistern in 5 minutes. Pipes A and B are kept open for 4minutes and then outlet pipe C is also opened. Find the cistern is emptied by the output pipe C.

ANSWERS

MULTIPLE CHOICE QUESTIONS

(1). a (2).b (3).c (4).d (5).a (6).c (7).b (8).c (9).d (10).b

ASSERTION AND REASON TYPE QUESTIONS

(1).d (2).c (3).d (4).b (5).d (6) .b

VERY SHORT ANSWER TYPE QUESTIONS

(1). 6 (2). 1:5 (3).1 km/h (4).24 minutes (5).5.2 km/h (6).$(0, \infty)$

SHORT ANSWER TYPE QUESTIONS

(1).5 liters (2). 0.455% (3).$\frac{41}{6}$ = 6hours and 50 minutes (4). 28 hours (5).14 hours (6).30 minutes

LONG ANSWER TYPE QUESTIONS

(1).Speed of boat in still water = 8km/h and speed of stream is 3km/h (2). 9 minutes

(3). 10 minutes

CASE BASED QUESTIONS

1.i) $\frac{1}{6}$ m/s ii) $\frac{5}{6}$ m/s iii)1hand 15 min OR x =3km

2. i) $\frac{10}{3}$ = 3h and 20 min ii) $3\frac{3}{4}$ hours iii) (a) $8\frac{2}{5}$ = 8 h and 24 min OR iii) (b)6 hours

HIGHER ORDER THINKING (HOT'S)

(1).$\frac{3}{8}$ (2). 20 hours

<u>UNIT 2</u>

<u>ALGEBRA</u>

<u>MATRICES</u>

Gist/Summary of the lesson:

Matrix, Order of matrix, Types of matrix, Row matrix, Column Matrix, Zero Matrix Or Null Matrix, Square matrix, Diagonal Matrix, Scalar Matrix, Equality of Matrices, Operations on Matrices, Transpose of a matrix, Symmetric and skew Symmetric matrices, Determinants, Area of a triangle using determinants, Minors and Cofactors of a Determinant, Adjoint of a Matrix, Inverse of a Matrix, , Solution of system of linear equations using inverse of a matrix, Cramer's Rule

- ❖ A Matrix is a rectangular array of numbers or functions, i.e. arrangements of numbers or functions in rows and columns.
- ❖ Matrix having m rows and n columns is of order m x n and total number of elements in matrix are mn. A matrix of order m x n is denoted by A=[a_{ij}]; $1 \le i \le m$ and $1 \le i \le n$. a_{ij} is knows as an element of a matrix lying in the i_{th} row and j_{th} column.
- ❖ The matrices A and B are said to be equal if (i) there order are same. (ii)elements at the corresponding places are equal,
- ❖ Addition of two matrices is defined, if matrices are of the same order and their addition is a matrix of the same order.
- ❖ Multiplication of Matrices A and B is defined if number of columns of A is equal to number of rows of B.
- ❖ A matrix obtained by interchanging the corresponding rows and columns is called the transpose of a matrix.
- ❖ To every square matrix we can associate a number or a function known as determinant of the square matrix.
- ❖ Adjoint of a square matrix A is transpose of a matrix obtained by cofactors of each element of a determinant corresponding to a given matrix.
- ❖ A square matrix whose inverse exists, is called invertible matrix .

TYPES OF MATRICS:

(i) **Row Matrix:** A matrix having only one row is called a row matrix, e.g. [2,4]; [-1,0,6]. Its order is $1 \times$ n. **Column Matrix:** A matrix having only one column is called a column matrix. e.g $\begin{bmatrix} 6 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$. Its order is m $\times$1.**Zero matrix or Null matrix :** A matrix whose all the elements are zero is known as a zero matrix . It is denoted by O.

eg. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

(ii) **Square matrix:** A matrix having equal number of rows and columns is a square matrix . Its order is m x m or n x n . For a square matrix A=[a_{ij}] , a_{ij} is knows as its diagonal element if i = j . In above examples diagonal elements are 2,4; 1,-1,1.

eg. $\begin{bmatrix} \mathbf{2} & \mathbf{3} \\ \mathbf{-1} & \mathbf{4} \end{bmatrix}$; $\begin{bmatrix} 1 & 6 & 0 \\ 3 & -1 & 2 \\ 1 & 4 & 1 \end{bmatrix}$.

(iii) **Diagonal matrix:** A square matrix whose all the non-diagonal elements are zeroes is called a diagonal matrix. We also define it as : A square matrix A = $[a_{ij}]$ is called a diagonal matrix, if a_{ij} = 0 for i ≠ j

eg. $\begin{bmatrix} \mathbf{5} & \mathbf{0} \\ \mathbf{0} & \mathbf{-7} \end{bmatrix}$; $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -3 \end{bmatrix}$.

(iv) **Scalar matrix:** A diagonal matrix whose all the elements are equal is a scalar matrix . We also define it as : A square matrix A = $[a_{ij}]$is a scalar matrix, if a_{ij} = 0 for i ≠ j and a_{ij}=k for i=j k is a scalar.

eg. $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$; $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$.

(v) Identity matrix or a Unit matrix: It is denoted by I. It is a square matrix whose all non-diagonal elements are zero and diagonal elements are 1 each. We also define it as: A square matrix A = a_{ij} is an identity matrix or a unit matrix , if a_{ij} = 1 for i = j . An identity matrix of order n x n is denoted by $I_{n\,x\,n}$ or I_n .

eg. $I_{2\,x\,2} = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_{3\times 3} = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

EQUALITY OF MATRICES:

The matrices A and B are said to be equal if (i) there order are same. (ii)elements at the corresponding places are equal,

i.e. matrices A = $[a_{ij}]$ and B = $[b_{ij}]$, 1 ≤ $\hat{\imath}$ ≤ m , 1 ≤ $\hat{\jmath}$ ≤ n are equal if $a_{ij} = b_{ij}$, ∀ i , j .

OPERATIONS ON MATRICES:

1. **Multiplication of a matrix by a scalar:** Multiplication of a matrix by a scalar k is a matrix of the same order whose each element is obtained by multiplying the corresponding element of the given matrix by scalar k . If matrix A = $[a_{ij}]_{m\times n}$ then multiplication of matrix A by scalar k is kA = $[ka_{ij}]_{m\times n}$.
2. **Addition of matrices:** Addition of two matrices is defined, if matrices are of the same order and their addition is a matrix of the same order whose each element is obtained by adding the corresponding elements of two matrices. If A = $[a_{ij}]_{m\,x\,n}$ and B = $[b_{ij}]_{m\times n}$ and A + B = C , then C = $[c_{ij}]_{m\times n}$, where $a_{ij} + b_{ij} = c_{ij}$ ∀ i , j.
3. **Properties with respect to addition:**
   (i) **Commutative:** A+B = B+A , for matrices A and B of the same order .
   (ii) **Associative:** A + (B + C) = (A + B) + C, for matrices A. B and C of the same order.
   (iii) **Additive identity:** For a given matrix A, a zero matrix of the same order as that of A is called its additive identity as A + O = O +A = A.
   (iv) **Additive inverse:** For a given matrix A, a matrix (-A) is called its additive inverse as A + (-A) = (-A) +A =O.
4. **Multiplication of matrices:** Given matrices A and B, then their multiplication AB is defined if number of columns of A is equal to number of rows of B. If A = $[a_{ij}]_{m\times n}$ and B = $[b_{jk}]_{n\times p}$,then AB = $[c_{ik}]_{m\times p}$, where $c_{ik} = \sum_{j=1}^{n} a_{ij}b_{jk}$, **i.e.** $c_{ik} = a_{i1}b_{1k} + a_{i2}b_{2k} + \ldots + a_{in}b_{nk}$

   MATRIX MULTIPLICATION MAY NOT BE COMMUTATIVE

   ❖ Let order of matrix A is 3 × 2 and that of B is 2 × 1 . Then AB is defined but BA is not defined , so AB ≠ BA .

- Let order of A is 2 × 2 and that of B is 3 x 2.Then AB is not defined but BA is defined i.e. AB≠BA
- Let order of A is 3 × 2 and that B is 2 × 3. Then AB is matrix of order 3×3 and BA is matrix of order 2 × 2 , so AB≠BA.
- Let the order of matrix A is 2 ×3 and that of matrix B is also 2×3. Here AB and BA both are. not defined , Hence AB≠BA.
- Let the order of matrix A be 2 × 2 and that of matrix B be also 2 × 2 then AB and BA both are defined and are of the same order. In this case we may have AB = BA only if elements at the corresponding places are equal.

Properties with respect to multiplication:

(i) **Commutative:** Matrix multiplication for matrices A and B is not commutative in general, i.e . AB≠BA .

(ii) **Associative:** For three matrices A, B and C, if multiplication is defined , then A(BC) = (AB)C.

(iii) **Multiplicative identity:** For a matrix $A_{m \times n}$, a unit matrix $I_m$is multiplicative identity, if it is pre-multiplied and I_n , is multiplicative identity, if it is pre-multiplied and I_n is multiplicative identity, if it is post-multiplied as $I_m A_{m \times n} = A_{m \times n}$; $A_{m \times n} I_n = A_{m \times n}$. i.e. for a rectangular matrix there are two multiplication identities. For a square matrix $A_{m\,x\,m}$, there is only one multiplicative identity I_m.

(iv) **Multiplicative inverse:** A matrix B is said to be inverse of matrix A, if AB = BA = I . Inverse of matrix A is denoted by A^{-1}. Inverse of only a square matrix exists.

Note: Inverse of a matrix , if exists , is unique.

Distributive property: If A, B, C, are three given matrices and if multiplication and addition is defined, then A(B + C) = AB + AC

Existence of two non-zero matrices whose product is a zero matrix:

Let A =$\begin{bmatrix} 2 & 5 \\ -4 & -10 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & 15 \\ -2 & -6 \end{bmatrix}$

Then A and B are non-zero matrices.

Consider AB = $\begin{bmatrix} 2 & 5 \\ -4 & -10 \end{bmatrix} \begin{bmatrix} 5 & 15 \\ -2 & -6 \end{bmatrix}$

=$\begin{bmatrix} 10-10 & 30-30 \\ -20+20 & -60+60 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ a zero matrix

TRANSPOSE OF A MATRIX, SYMMETRIC AND SKEWSYMMETRIC MATRICES

(i) A matrix obtained by interchanging the corresponding rows and columns is called the transpose of a matrix.

(ii) If matrix is A, then its transpose is denoted by A' or A^T. Also if matrix A = $[a_{ij}]_{m \times n}$then its transpose is A' =$[a_{ji}]_{n \times m}$.

(iii) Properties of transpose of the matrices.(i) (A') ' =A (ii) (A + B) ' =A'+ B' (iii) (AB)' = B'A' (iv) (kA)'= kA', where k is a scalar.

(iv) A matrix is symmetric matrix, if $A = A^T$

(v) A matrix is skew symmetric matrix, if $A = -A^T$

Note: Every square matrix can be represented as a sum of a symmetric and a skew symmetric matrix.

(vi) If A is a square matrix then,$\frac{(A+A^T)}{2} + \frac{(A-A^T)}{2}$ where $\frac{(A+A^T)}{2}$ is a symmetric matrix $\frac{(A-A^T)}{2}$ and is a skew symmetric matrix.

Note: Diagonal elements of a skew symmetric matrix are always zero.

Adjoint of a matrix :

a) Adjoint of a square matrix A is transpose of a matrix obtained by cofactors of each element of a determinant corresponding to a given matrix.
b) Adjoint of a matrix A is denoted by adj A.
c) A square matrix whose inverse exists, is called invertible matrix .
d) Inverse of matrix A is denoted by $A^{-1} = \frac{1}{|A|}$. adj (A)
e) If | A | = 0 then matrix is said to be singular matrix.
f) If | A | ≠ 0 then matrix is said to be non-singular matrix.
g) If A is a given matrix , then $AA^{-1} = A^{-1} A = I$.
h) If A and B are invertible matrix of the same order then $(AB)^{-1} = B^{-1}A^{-1}$
i) For a square matrix A,
   (i) $(A^{-1})^{-1} = A$
   (ii) $(A`)^{-1} = ( A^{-1} )`$
   (iii) $|A^{-1}| = \frac{1}{|A|}$
   (iv) Important Formula $|KA| = K^n |A|$
   (v) $| \text{adj } A | = |A|^{n-1}$
   (vi) If A is a square matrix of order n, then
      (a) $| (\text{adj } A) | = | A|^{n-1}$
      (b) $|A(\text{adj } A)| = |A|^n$.

MULTIPLE CHOICE QUESTIONS

(1) If a matrix has 5 elements, the possible orders of a matrix are
(a) 2 (b) 5 (c) 1 (d) 4
Solution: The possible orders of a matrix = 5×1,1×5
Answer: **(a) 2**

(2) If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ then A is

(a) $\begin{bmatrix} 10 & -3 & 5 \\ -2 & -3 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 1 & 3 \\ -2 & 5 & 12 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & 3 & -5 \\ 2 & 3 & 6 \end{bmatrix}$

Solution:

$$A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

Answer:(c) $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$

(3) If a matrix A is both symmetric and skew symmetric , then A is necessarily a
(a) Diagonal Matrix (b) Zero Square Matrix (c) Square Matrix (d) Identity Matrix
Answer (b) Zero Square Matrix

(4) If for a square matrix A , $A^2 - A + I = O$, then A^{-1} equals
(a) A (b) A + I (c) I – A (d) A – I
Solution: $A^{-1} (A^2 - A + I) = 0 \times A^{-1}$
$\Rightarrow A^{-1} A^2 - A^{-1} A + A^{-1} I = 0$

$\Rightarrow$I.A – I + A^{-1} = 0

$\Rightarrow$ A – I + A^{-1} = 0

$\Rightarrow$ A^{-1} = I – A

Answer: (c) I – A

(5) If A = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$. B = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and A = B^2 , then x is

(a) ± 1 (b) 1 (c) -1 (d) 2

Solution: B^2 = B.B = $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$

$\Rightarrow$A = B^2 $\Rightarrow$ $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$

$\Rightarrow$ x^2 =1 $\Rightarrow$ x = $\pm$ 1

$\Rightarrow$ x = 1,(x = -1, not satisfied)

Answer (b) **1**

(6) If A and B are matrices of order 3 × 2 and 2 × 4 respectively then order of matrix A × B is

(a) 3 × 4 (b) 4 ×3 (c) 2 ×3 (d) 2×4

Answer: **3 × 4**

Solution: If order of matrix A is m × n and order of matrix B is n × p then order of matrix A ×B is m ×P.

Answer: (a) **3 × 4**

(7) If $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$ = 0 , then x is

(a) 0 (b) -2 (c) -1 (d) 2

Solution: $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$ = 0

$\begin{bmatrix} x-2 & 0 \end{bmatrix}$ = 0 $\Rightarrow$ x = 2

Answer: (d) **2**

(8) If A is a skew symmetric matrix of order 3 , then the value of |A| is

(a) 0 (b) 27 (c) 9 (d) 2

Solution: Let A= $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

| A | = 0 – a(0+bc) + b(ac-0) = -abc + abc = o

Answer: (a) **0**

(9) Three points P (2x, x + 3) , Q (0 , x) and R(x+3, x+6) are collinear , then x is

(a) 3 (b) 7 (c) 1 (d) 2

Solution: Three points are collinear when Area of triangle is Zero.

$\begin{vmatrix} 2x & x+3 & 1 \\ 0 & x & 1 \\ x+3 & x+6 & 1 \end{vmatrix}$ = 0 $\Rightarrow$2x(x-x-6) – (x+3)(0 – x – 3) +1 (0 – x^2 -3x) = 0

$\Rightarrow$ -12x + x^2 + 6x + 9 – x^2 – 3x = 0 $\Rightarrow$ x= 1

Answer: (c)**1**

(10) The area of the triangle with vertices A(5,4), B(2,-6) and C(-2,4) is

(a) 23 sq. units (b) 35 sq. units (c) 10 sq. units (d) 20 sq. units

Solution: Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 5 & 4 & 1 \\ 2 & -6 & 1 \\ -2 & 4 & 1 \end{vmatrix}$

=$\frac{1}{2}${5(-6-4) – 4(2+2) 1(8 – 12)}

=$\frac{1}{2}$(-50 – 16 + 4) = -35

So, area = 35 sq. units. (Area cannot be negative)

Answer: **(b)** 35 sq. units

(11) The value of x if $\begin{vmatrix} 3 & -6 \\ 4 & 0 \end{vmatrix} = \begin{vmatrix} 3 & x^2 \\ x & -1 \end{vmatrix}$ is

(a) 3 (b) -3 (c) 1 (d) 2

Solution: $\begin{vmatrix} 3 & -6 \\ 4 & 0 \end{vmatrix} = \begin{vmatrix} 3 & x^2 \\ x & -1 \end{vmatrix}$

$\Rightarrow -3 - x^3 = 24 \Rightarrow x = -3$

Answer: (b) -3

(12) If A = $\begin{bmatrix} 3 & 5 \\ -4 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} -9 & 2 \\ 1 & -7 \end{bmatrix}$ then value of | AB | is

(a) 2358 (b) 2348 (c) 2318 (d) 2328

Solution: AB = $\begin{bmatrix} 3 & 5 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} -9 & 2 \\ 1 & -7 \end{bmatrix} = \begin{bmatrix} -27+5 & 6-35 \\ 36+6 & -8-42 \end{bmatrix} = \begin{bmatrix} -22 & -29 \\ 42 & -50 \end{bmatrix}$

Hence | AB | =1100 +1218 = 2318

Answer: (c) 2318

(13) The inverse of given matrix $\begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$ is

(a) $\begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}$

Solution: Let A = $\begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$

| A | = -6 + 5 = -1 and Adj (A) = $\begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{I\,A\,I}$ Adj (A) = $\frac{1}{-1}\begin{bmatrix} 2 & 1 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$

Answer: (a) $\begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$

(14) If A = $\begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$, then adjoint of A is

(a) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 1 \\ 5 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$

Solution: If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then Adj (A) = $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$,

Answer: (d) $\begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$

(15) If A = $\begin{bmatrix} -2 & 3 \\ k & 4 \end{bmatrix}$ is a singular matrix then value of k is

(a) $\frac{-8}{3}$ (b) $\frac{8}{3}$ (c) $\frac{7}{3}$ (d) $\frac{-7}{3}$

Solution: For singular matrix A

| A | = 0 $\Rightarrow -8 - 3k = 0 \Rightarrow k = \frac{-8}{3}$

Answer: (a) $\frac{-8}{3}$

(16) For what value of k the points (k,7), (-4,5) and (1,-5) are collinear

(a) 3 (b) -3 (c) -5 (d) 2

Solution: Three points are collinear when Area of triangle is Zero.

$\begin{vmatrix} k & 7 & 1 \\ -4 & 5 & 1 \\ 1 & -5 & 1 \end{vmatrix} = 0$

$\Rightarrow k(5+5) - 7(-4-1) + 1(20-5) = 0$

$\Rightarrow 10k + 35 + 15 = 0 \Rightarrow k = -5$

Answer: **(c) -5**

(17) If A = $\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ and B =$\begin{bmatrix} 1 & 4 \\ 7 & 2 \end{bmatrix}$ then value of 3A -2B is

(a) $\begin{bmatrix} 4 & 11 \\ 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & 11 \\ 5 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & -11 \\ 5 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -11 \\ -5 & -1 \end{bmatrix}$

Solution: 3A -2B = 3$\begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$ - 2$\begin{bmatrix} 1 & 4 \\ 7 & 2 \end{bmatrix}$ =$\begin{bmatrix} 6 & -3 \\ 9 & 3 \end{bmatrix}$ - $\begin{bmatrix} 2 & 8 \\ 14 & 4 \end{bmatrix}$ = $\begin{bmatrix} 4 & -11 \\ -5 & -1 \end{bmatrix}$

Answer: (d) $\begin{bmatrix} 4 & -11 \\ -5 & -1 \end{bmatrix}$

(18) If A = $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and B =$\begin{bmatrix} -2 & 3 \\ 5 & -4 \end{bmatrix}$ then value of A' -2B is

(a) $\begin{bmatrix} -6 & 5 \\ 7 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -5 \\ 7 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & -5 \\ -7 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & -5 \\ 7 & 4 \end{bmatrix}$

Solution: A = $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$,

A' = $\begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$

A' -2B =$\begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$ - 2$\begin{bmatrix} -2 & 3 \\ 5 & -4 \end{bmatrix}$ = $\begin{bmatrix} 2 & 1 \\ 3 & -4 \end{bmatrix}$ -$\begin{bmatrix} -4 & 6 \\ 10 & -8 \end{bmatrix}$ = $\begin{bmatrix} 6 & -5 \\ -7 & 4 \end{bmatrix}$

Answer: **(c)** $\begin{bmatrix} \mathbf{6} & \mathbf{-5} \\ \mathbf{-7} & \mathbf{4} \end{bmatrix}$

(19) Matrix A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ represent

(a) Identity Matrix (b) Scalar Matrix (c) Column Matrix (d) Row Matrix

Solution: A diagonal matrix whose all the elements are equal is a scalar matrix.

Answer :(b) Scalar Matrix

(20) The minor of element 3 in A = $\begin{vmatrix} 7 & 3 \\ -4 & 5 \end{vmatrix}$ is

(a) -4 (b) -3 (c) -7 (d) 5

Answer: (a) -4

(21) A = $\begin{bmatrix} 2 & -1 & 3 \\ 4 & 5 & -6 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ then

(a)Only AB is defined (b) Only BA is defined

(c) AB and BA both are defined (d) AB and BA both are not defined

Solution: Multiplication AB is defined if number of columns of A = no. of rows of B

Answer: (b) Only BA is defined

ASSERTION REASON BASED QUESTIONS

*In the following questions , a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) A and R both true and R is correct explanation of A.

(b)A and R both true but R is not a correct explanation of A.

(c)A is true but R is false.

(d)A is false but R is true.

(1) **Assertion (A) :**If matrix A = $\begin{bmatrix} 1 & 0 \\ 2 & -0 \end{bmatrix}$ and matrix B = $\begin{bmatrix} 0 & 0 \\ -1 & 4 \end{bmatrix}$ then AB = O.

Reason (R): Multiplication of matrices A and B or AB is defined if number of columns of A = number of rows of B.

Solution: AB = $\begin{bmatrix} 1 & 0 \\ 2 & -0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 4 \end{bmatrix}$ = $\begin{bmatrix} 0-0 & 0+0 \\ 0+0 & 0-0 \end{bmatrix}$ =$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ =O.

Answer: **(a)**

(2) **Assertion (A) :** Matrix B = $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ represent identity matrix.

Reason (R): A diagonal matrix whose all the elements are 1 is identity matrix.

Answer: **(a)**

(3) **Assertion (A):** Matrix A = $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ and matrix B = $\begin{bmatrix}2 & -1 & 3\\4 & 5 & -6\end{bmatrix}$ then order of matrix AB is 3 × 3.

Reason (R): If order of matrix A is m × n and order of matrix B is n × p then order of matrix A ×B is m × P.

Answer: **(d)**

(4) **Assertion (A):** If A= $\begin{bmatrix}2 & x-3 & x-2\\3 & -2 & -1\\4 & -1 & -5\end{bmatrix}$ is a symmetric, then x = 6.

Reason (R): If A is symmetric matrix then A= A' .

Solution: A= $\begin{bmatrix}2 & x-3 & x-2\\3 & -2 & -1\\4 & -1 & -5\end{bmatrix}$

When x = 6

A = $\begin{bmatrix}2 & 6-3 & 6-2\\3 & -2 & -1\\4 & -1 & -5\end{bmatrix}$ = $\begin{bmatrix}2 & 3 & 4\\3 & -2 & -1\\4 & -1 & -5\end{bmatrix}$

A' = $\begin{bmatrix}2 & 3 & 4\\3 & -2 & -1\\4 & -1 & -5\end{bmatrix}$ Hence A' = A

Answer: **(a)**

(5) **Assertion (A):** If the order of A is 3×4, the order of B is 3×4 and the order of C is 5×4, then the order of $(A^TB)C^T$ is 4×5.

Reason (R): If order of matrix A is m X n and order of matrix B is n×p then order of matrix A ×B is m × P. Also , A be a matrix of order m×n then the order of transpose matrix is $n \times m$

Answer: (a)

(6) **Assertion (A):** If A is a skew-symmetric matrix , then A^2 is a symmetric matrix.

Reason (R): If A is a skew-symmetric matrix , then A' = -A.

Answer: (a)

(7) Assertion (A):The area of the triangle whose vertices are (3,8),(-4,2) and (5,1) is $\frac{61}{2}$ Sq. units.

Reason (R): The area of the triangle whose vertices are (x_1,y_1),(x_2,y_2) and (x_3,y_3) is $\begin{vmatrix}x_1 & y_1 & 1\\x_2 & y_2 & 1\\x_3 & y_3 & 1\end{vmatrix}$ Sq. units.

Solution: Area of Δ = $\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\x_2 & y_2 & 1\\x_3 & y_3 & 1\end{vmatrix} = \frac{1}{2}\begin{vmatrix}3 & 8 & 1\\-4 & 2 & 1\\5 & 1 & 1\end{vmatrix}$

$=\frac{1}{2}\{3(2-1) - 8(-4-5) + 1(-4-10)\} \quad = \frac{1}{2}\{3+72-14\} = \frac{1}{2} \times 61 = \frac{61}{2}$ Sq. units.

Answer: (a)

(8) **Assertion (A):** If $A = [2 \quad -3 \quad 4]$, $B = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$, $X = [1 \;\; 2 \;\; 3]$, $Y = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ then AB + XY =[28].

Reason (R): If order of matrix A is m × n and order of matrix B is n × p then order of matrix A × B is m × P. Also addition of two matrices is defined, if matrices are of the same order.

Solution: $AB = [2 \quad -3 \quad 4] \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} = [6 - 6 + 8] = [8]$

$XY = [1 \quad 2 \quad 3] \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = [2 + 6 + 12] = [20]$

AB + XY =[8]+[20] =[28]

Answer: (a)

(9) **Assertion (A):** If A is a non-singular square matrix of order 3 and $A^2 = 2A$, then the value of I A I is 8.

Reason (R): When A is non-singular square matrix, then matrix A is invertible and $A^{-1}A = I$. Also I kA I = k^n IAI, where n is order of square matrix.

Answer: (a)

(10) **Assertion (A):**If the area of the triangle whose vertices are (-3,0),(3,0) and (0,k) is 9 Sq. units then value of k is ± 3.

Reason (R): The area of the triangle whose vertices are (x_1,y_1),(x_2,y_2) and (x_3,y_3) is $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ Sq. units.

Solution: Area of $\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$9 = \frac{1}{2} \begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}$

$= \frac{1}{2}\{-3(0-k) - 0(3-0) + 1(3k-0)\} = \frac{1}{2}(3k - 0 + 3k)$

$6k = \pm 18 \qquad \therefore \; k = \pm 3$

Answer: (a)

VERY SHORT ANSWER TYPE QUESTIONS

(1) Find the value of P – Q where $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Solution: $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow Q = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

P – Q = $\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ - $\begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$ = $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(2) If 2$\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix}$ + $\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then find (x-y).

Solution: $\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix}$ + $\begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix}$ = $\begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

8+y = 0 ⇒ y=-8

2x+1=5 ⇒ x=2

∴ x – y = 2+8 = 10

(3) If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA= $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the value of k, a and b.

Solution: kA=k$\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ = $\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

∴ $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ = $\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

∴ -4k = 24 ⇒ k = -6

3a = 2k = 2 × (-6)= -12 ⇒ a = -4

2b = 3k = 3 × (-6) = -18 ⇒b = -9

(4)Find a matrix A such that 2A – 3B + 5 C= 0 , where B = $\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$,C=$\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

Solution: 2A – 3B + 5 C= 0

2A = 3B - 5 C

= 3$\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ – 5$\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$ = $\begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix}$ – $\begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix}$ = $\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$

∴ A = $\begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$

(5)If $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix}$ = $\begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ then find the value of ab-cd.

Solution: $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix}$ = $\begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$

$\Longrightarrow 3c + 6 = 12$ ⟹c=2

Again, 2 - 3b = -4 ⟹-3b = -6 ⟹b = 2

Now, a – d = 2 and a + d = -8

Adding, 2 a = -6 ⟹ a = -3 ⟹ d = a – 2 = -3 – 2 = -5

∴ ab-cd = -6 – (-10) = 4

(6)If A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then find the value of A^2 – A.

Solution: A^2 = A X A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

= $\begin{bmatrix} 0+1+1 & 0+0+1 & 0+1+0 \\ 0+0+1 & 1+0+1 & 1+0+0 \\ 0+1+0 & 1+0+0 & 1+1+0 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A^2 - A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2\,I$$

(7)If $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$ then find the value of a-2b.

Solution: $\begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} = \begin{bmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{bmatrix}$

$\Rightarrow a + 4 = 2a + 2 \quad \Rightarrow a = 2$

Also $3b = b + 2 \quad \Rightarrow b = 1$

$\Rightarrow a - 2b = 2 - 2 = 0$

(8) If matrix A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric then find the value of a and b.

Solution: A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is symmetric.

$\Rightarrow 2b = 3 \quad \Rightarrow b = \frac{3}{2}$

Again, $3a = -2 \quad \Rightarrow a = \frac{-2}{3}$

(9) If matrix A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}$ is a matrix satisfying AA'= 9 I then find the value of x.

Solution: A' = $\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{bmatrix}$

AA'= 9 I

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2+2x & -2+4-2 \\ 2+2+2x & 4+1+x^2 & -4+2-x \\ -2+4-2 & -4+2-x & 4+4+1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 4+2x & 0 \\ 4+2x & 5+x^2 & -2-x \\ 0 & -2-x & 9 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow 4 + 2x = 0 \quad \Rightarrow x = -2$

also $5 + x^2 = 9 \Rightarrow x^2 = 4 \quad \Rightarrow x = \pm 2$

(10)If A = $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2 , then show that $A^2 - 4A + 3I = O$.

Solution: $A^2 = A \times A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$

LHS = $A^2 - 4A + 3I = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5-8+3 & -4+4+0 \\ -4+4+0 & 5-8+3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = O = RHS

(11)$If \begin{vmatrix} a & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix}$ = 0, then find the value of a.

Solution: a(2-4) – 3(1-1) + 4(4-2) = 0

⟹ -2a – 0 +8 = 0 ⟹ a = 4

(12)If A = $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$ then find the value of | A |.

Solution: = (x+y)(-3x+3y) – (y+z)(-3z+3y) +(z+x)(-3z+3x)

$=-3x^2 + 3xy - 3xy + 3y^2 + 3yz - 3y^2 + 3z^2 - 3yz - 3z^2 + 3xz - 3xz + 3x^2 = 0$

(13) If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then find the value of x.

Solution: $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow 2x^2 - 40 = 18 + 14 \Rightarrow 2x^2 = 72 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

(14) If A = $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, C = $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ find a matrix D such that CD – AB = 0.

Solution: Let D = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

⟹CD – AB = 0

⟹CD = AB

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} = \begin{bmatrix} 10-7 & 4-4 \\ 15+28 & 6+16 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$

⟹2a + 5c = 3

⟹6c+15c = 9 …………(i)

⟹ $3a + 8c$ =43

⟹6c + 16 c = 86 ………(ii)

Equn. (i) – (ii)⟹2a = 3 – 5c = 3 – 5 × 77 = 3 – 385 = -382 ⟹ a = -|9|

⟹ $2b + 5d$ = 0 ⟹6b+15d = 0 ………..(iii)

⟹ $3b + 8d$ = 22 ⟹6b + 16d = 44 ………….(iv)

Equn. (iii) – (iv) ⟹ d = 44

2b = -5d = -5 × 44 = -220b = -110

∴ D = $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

(15) If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then find A^{-1}.

Solution: Let A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ | A | = -3 + 4 = 1

Adj (A) = $\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$ $A^{-1} = \frac{1}{I\,A\,I}$ Adj (A) = $\frac{1}{1} \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$

(16) If A = $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ then show that 2A^{-1} = 9I – A.

Soln. Let A = $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ Adj (A) = $\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{I\,A\,I}$ Adj (A) = $\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

LHS = 2A^{-1} = $2 \times \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

RHS = 9I – A = $9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

LHS = RHS

(17) If A and B are square matrices of order 3 such that | A |= -1, | B | = 3, then find the value of | 2 AB |.

Solution: $| kA | = k^n |A|$

$| 2\,AB | = 2^3 |A|\,|B| = 8 \times (-1)(3) = -24$

(18) Evaluate the determinant of matrix $\begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$.

Solution: Let A = $\begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

| A| = 3(1+6) + 4 (1+4) + 5(3-2)= 21+20+ 5 = 46.

(19) If A = $\begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$, then show that | 2 A | = 8 | A |.

Solution: A = $\begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} \Rightarrow 2A = 2\begin{vmatrix} 6 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 12 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 8 \end{vmatrix}$

$\Rightarrow$| 2A | = 12(16-0) – 0 (0 – 0) + 2 (0 – 0) = 192

$\Rightarrow$| A | = 6(4 – 0) – 0 (0 - 0) + 1(0 - 0) = 24 $\Rightarrow$8 | A | = 8 × 24 = 192

So, | 2 A |= 8 | A |

(20) If A = $\begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3|$=125, then find the values of p.

Solution: | A | = $p^2 - 4$

Here, $| A^3 | = 125 = 5^3 \Rightarrow | A | = 5 \Rightarrow p^2 - 4 = 5 \Rightarrow p = \pm 3$

SHORT ANSWER TYPE QUESTIONS

(1) Find the inverse of the matrix $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$. Hence, find the matrix P satisfying the matrix equation $P\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

Solution: Let A = $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$ Hence | A | = 9 – 10 = -1

Adj (A) = $\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{IAI}$ Adj (A) = $\frac{1}{-1}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

Given that, P$\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}$ = $\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$

P A = $\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$ $\Rightarrow$ P = A^{-1} $\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$ = $\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}$ $\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$ = $\begin{bmatrix}13 & 8\\ 1 & 1\end{bmatrix}$

(2) Mohan decided to donate money to children of an orphanage home. If there were 8 children less, everyone would have got Rs 10 more. However, if there were 16 children more, everyone would have got Rs10 less. Using matrix method, find the number of children and the amount distributed by Mohan.

Soln. Let the number of children be x and the amount distributed by Mohan for 1 student by

Rs. y.

(x – 5) (y + 10) = xy ⟹5x – 4y = 40 ………(i)

⟹Again (x + 16) (y -10) = xy ⟹5x – 8 y = -50 ………(ii)

Let A = $\begin{bmatrix}5 & -4\\ 5 & -8\end{bmatrix}$, X = $\begin{bmatrix}x\\ y\end{bmatrix}$, B = $\begin{bmatrix}40\\ -80\end{bmatrix}$

| A | = -40 + 20 = -20

Adj (A) = $\begin{bmatrix}-8 & 4\\ -5 & 5\end{bmatrix}$

A^{-1} = $\frac{1}{IAI}$ Adj (A) =$\frac{1}{-20}\begin{bmatrix}-8 & 4\\ -5 & 5\end{bmatrix}$

Now, AX = B

So, X = A^{-1} B = $\frac{1}{-20}\begin{bmatrix}-8 & 4\\ -5 & 5\end{bmatrix}\begin{bmatrix}40\\ -80\end{bmatrix}$ = $\frac{1}{-20}\begin{bmatrix}-320 - 320\\ -200 - 400\end{bmatrix}$ =$\frac{1}{-20}\begin{bmatrix}-640\\ -600\end{bmatrix}$

X = $\begin{bmatrix}x\\ y\end{bmatrix}$ = $\begin{bmatrix}32\\ 30\end{bmatrix}$

So, x = 32, y = 30

The number of children x = 32

and the amount distributed = y = Rs.30

(3) Using determinant show that points A(a, b+c), B(b, c+a), C(c, a+b) are collinear.

Solution: Points are collinear when Δ = 0

$$\Delta = \begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$\Delta = \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

=a{(c+a) – (a+b)}-(b+c)(b-c) + 1{b(a+b) - c(c+a)}

=ac – ab – b^2 + c^2 + ab + b^2 – c^2 – ac = 0

So, points A, B and C are collinear.

(4)Find the equation of line joining (1,2) and (3,6) using determinants.

Solution: Equation of the line joining the point (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0 \Rightarrow$ x(2-6) – y(1-3) + 1(6-6) = 0 ⟹-4x + 2y = 0

⟹4x – 2y = 0 ⟹2x – y = 0

(5) Let A = $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, Verify that $(AB)^{-1} = B^{-1} A^{-1}$.

Solution: A × B = $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

⟹| AB | =67 X 61 – 47 X 87 = 4087 – 4089 = -2

⟹Adj (AB) = $\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

⟹ $(AB)^{-1} = \frac{1}{I\,A\,B I}$ Adj (AB) = $\frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -61 & 87 \\ 47 & -67 \end{bmatrix}$

| A | = 15 – 14 = 1

Adj (A) = $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{I\,A\,I}$ Adj (A) = $\frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

| B | = 54 – 56 = -2

Adj (B) = $\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$B^{-1} = \frac{1}{I\,B\,I}$ Adj (B) = $\frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -9 & 8 \\ 7 & -6 \end{bmatrix}$

$B^{-1} A^{-1} = \frac{1}{2}\begin{bmatrix}-9 & 8\\ 7 & -6\end{bmatrix}\begin{bmatrix}5 & -7\\ -2 & 3\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-45-16 & 63+24\\ 35+12 & -49-18\end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix}-61 & 87\\ 47 & -67\end{bmatrix}$

$(AB)^{-1} = B^{-1} A^{-1}$

(6) The cost of 4 kg potato, 3 kg tomato and 2 kg onion is Rs 250 . The cost of 2 kg potato, 4 kg tomato and 6 kg onion is Rs 300. The cost of 6 kg potato, 2 kg tomato and 3 kg onion is Rs 200. Find the cost of each item per kg by matrix method.

Solution: Let cost of 1 kg potato = Rs x, cost of 1 kg tomato = Rs y & cost of 1 kg onion = Rs z.

According to question 4 x + 3 y + 2 z = 250: 2 x + 4 y + 6 z = 300: 6 x + 2 y + 3 z = 200

$A = \begin{vmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{vmatrix}$, $X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B = \begin{bmatrix}250\\ 300\\ 200\end{bmatrix}$

$|A| = 4(12-12) - 3(6-36) + 2(4-24) = 4 \times 0 - 3(-30) + 2(-20)$

$= 0 + 90 - 40 = 50 \neq 0$

⇒ A is a non-singular matrix and system has a unique solution.

Co-factors of elements of A are :

$\Rightarrow A_{11} = (12-12) = 0$, $A_{12} = -(6-36) = 30$, $A_{13} = (4-24) = -20$

$\Rightarrow A_{21} = -(9-4) = -5$, $A_{22} = -(12-12) = 0$, $A_{23} = -(8-18) = 10$

$\Rightarrow A_{31} = 18-8 = 10$, $A_{32} = -(24-4) = -20$, $A_{33} = 16-6 = 10$

$\text{Adj}(A) = \begin{vmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{vmatrix}$

$A^{-1} = \frac{1}{|A|}\text{Adj}(A) = \frac{1}{50}\begin{vmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{vmatrix}$

Since, AX = B

Hence $X = A^{-1}B = \frac{1}{50}\begin{vmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{vmatrix}\begin{bmatrix}250\\ 300\\ 200\end{bmatrix}$

$= \begin{vmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{vmatrix}\begin{bmatrix}4\\ 6\\ 4\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}0-30+40\\ 120+0-80\\ -80+60+40\end{bmatrix} = \begin{bmatrix}10\\ 40\\ 20\end{bmatrix}$

So, cost of 1 kg potato = Rs x = Rs 10.

cost of 1 kg tomato = Rs y = Rs 40, and cost of 1 kg onion = Rs z = Rs 20

(7) Show that the points (a+5, a-4), (a-2, a+3), and (a, a) do not lie on a straight line for any value of a.

Solution: Points are collinear when Δ = 0

$$\Delta = \frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix} \Rightarrow \Delta = \frac{1}{2}\begin{vmatrix}a+5 & a-4 & 1\\ a-2 & a+3 & 1\\ a & a & 1\end{vmatrix}$$

$= \frac{1}{2}$ [(a + 5)(a+3 - a) – (a - 4)(a – 2 - a) + 1{a(a-2) - a(a + 3)}]

$= \frac{1}{2}$ [3 a + 15 + 2a – 8 + a^2 – 2a – a^2 – 3a] = $\frac{7}{2} \neq 0$

Hence, given points form a triangle. So, given points do not lie in a straight line.

(8) Using determinants find the area of ΔABC with vertices A(3,1), B(9,3) and C(5,7). Also find the equation of line AB using determinants.

Solution: $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix} = \frac{1}{2}$ {3(3 – 7) – 1 (9 – 5) + 1 (63 – 15)}

$= \frac{1}{2}$ {-12 – 4 + 48} =16 Sq. Unit

Equation of the line joining the point (x_1, y_1) and (x_2, y_2) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$ ⟹x(1- 3) – y (3 – 9) + 1(9 – 9) = 0

⟹-2x + 6y = 0 ⟹x – 3y = 0

(9)Express the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ -4 & -1 & 0 \\ 3 & 5 & 1 \end{bmatrix}$. as the sum of a symmetric and a skew-symmetric matrix.

Solution: A = $\begin{bmatrix} 1 & 2 & 3 \\ -4 & -1 & 0 \\ 3 & 5 & 1 \end{bmatrix}$ A' = $\begin{bmatrix} 1 & -4 & 3 \\ 2 & -1 & 5 \\ 3 & 0 & 1 \end{bmatrix}$

Let P = $\frac{1}{2}$ (A + A') $= \frac{1}{2}\left[\begin{bmatrix} 1 & 2 & 3 \\ -4 & -1 & 0 \\ 3 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 & 3 \\ 2 & -1 & 5 \\ 3 & 0 & 1 \end{bmatrix}\right] = \frac{1}{2}\begin{bmatrix} 2 & -2 & 6 \\ -2 & -2 & 5 \\ 6 & 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 3 \\ -1 & -1 & \frac{5}{2} \\ 3 & \frac{5}{2} & 1 \end{bmatrix}$

P' = $\begin{bmatrix} 1 & -1 & 3 \\ -1 & -1 & \frac{5}{2} \\ 3 & \frac{5}{2} & 1 \end{bmatrix}$ = P

Let Q = $\frac{1}{2}$ (A - A')

$$=\frac{1}{2}\left[\begin{bmatrix}1 & 2 & 3\\-4 & -1 & 0\\3 & 5 & 1\end{bmatrix}-\begin{bmatrix}1 & -4 & 3\\2 & -1 & 5\\3 & 0 & 1\end{bmatrix}\right] \quad =\frac{1}{2}\begin{bmatrix}0 & 6 & 0\\-6 & 0 & -5\\0 & 5 & 0\end{bmatrix} \quad =\begin{bmatrix}0 & 3 & 0\\-3 & 0 & -\frac{5}{2}\\0 & \frac{5}{2} & 0\end{bmatrix}$$

$$= -\begin{bmatrix}0 & -3 & 0\\3 & 0 & \frac{5}{2}\\0 & -\frac{5}{2} & 0\end{bmatrix} \quad \text{Here } Q' = -Q$$

$$P + Q = \begin{bmatrix}1 & -1 & 3\\-1 & -1 & \frac{5}{2}\\3 & \frac{5}{2} & 1\end{bmatrix} + \begin{bmatrix}0 & 3 & 0\\-3 & 0 & -\frac{5}{2}\\0 & \frac{5}{2} & 0\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\-4 & -1 & 0\\3 & 5 & 1\end{bmatrix} = A$$

(10)Find X and Y if X + Y = $\begin{bmatrix}-1 & 13\\2 & 4\end{bmatrix}$ and X - Y = $\begin{bmatrix}5 & -8\\3 & 0\end{bmatrix}$.

Solution: X + Y = $\begin{bmatrix}-1 & 13\\2 & 4\end{bmatrix}$ …….(i) X - Y = $\begin{bmatrix}5 & -8\\3 & 0\end{bmatrix}$ …….(ii)

Equn. (i) + (ii) $\Longrightarrow$2 X = $\begin{bmatrix}-1 & 13\\2 & 4\end{bmatrix} + \begin{bmatrix}5 & -8\\3 & 0\end{bmatrix} = \begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}$, X = $\frac{1}{2}\begin{bmatrix}4 & 5\\5 & 4\end{bmatrix} = \begin{bmatrix}2 & \frac{5}{2}\\\frac{5}{2} & 2\end{bmatrix}$

Equn. (i) - (ii) $\Longrightarrow$2 Y = $\begin{bmatrix}-1 & 13\\2 & 4\end{bmatrix} - \begin{bmatrix}5 & -8\\3 & 0\end{bmatrix} = \begin{bmatrix}-6 & 21\\-1 & 4\end{bmatrix}$ Y = $\begin{bmatrix}-3 & \frac{21}{2}\\\frac{-1}{2} & 2\end{bmatrix}$

(11) Solve the following system of equations using Cramer's rule

x +y + z = 10, 2x + y = 13 and x +y - 4z = 0.

Soln. Convert given equation in matrix form

$$\begin{bmatrix}1 & 1 & 1\\2 & 1 & 0\\1 & 1 & -4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}10\\13\\0\end{bmatrix}$$

$\Delta=\begin{vmatrix}1 & 1 & 1\\2 & 1 & 0\\1 & 1 & -4\end{vmatrix}$ = 1(-4-0) -1(-8+0) +1(2-1) = -4+8 +1= 5≠0

$\Delta x=\begin{vmatrix}10 & 1 & 1\\13 & 1 & 0\\0 & 1 & -4\end{vmatrix}$ = 10(-4-0) -1(-52-0) + 1(13-0) = -40 +52 + 13 =25

$\Delta y=\begin{vmatrix}1 & 10 & 1\\2 & 13 & 0\\1 & 0 & -4\end{vmatrix}$ = 1(-52-0) -10(-8-0) +1(0-13) = -52 +80 -13 = 15

$\Delta z=\begin{vmatrix}1 & 1 & 10\\ 2 & 1 & 13\\ 1 & 1 & 0\end{vmatrix}$ =1(0-13) -1(0-13) +10(2-1) = -13 +13 +10 = 10

x$=\frac{\Delta x}{\Delta}=\frac{25}{5}=5$ y$=\frac{\Delta y}{\Delta}=\frac{15}{5}=3$, z$=\frac{\Delta z}{\Delta}=\frac{10}{5}=2$

(12) A = $\begin{bmatrix}3 & -1\\ 1 & -2\end{bmatrix}$. Find one such matrix B such A + B is a skew-symmetric matrix .

Soln. Let B = $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

A + B =$\begin{bmatrix}3 & -1\\ 1 & -2\end{bmatrix}+\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}3+a & -1+b\\ 1+c & -2+d\end{bmatrix}$ ⟹ (A + B)' = $\begin{bmatrix}3+a & 1+c\\ -1+b & -2+d\end{bmatrix}$

⟹A + B = - (A + B)'

$\begin{bmatrix}3+a & -1+b\\ 1+c & -2+d\end{bmatrix}=-\begin{bmatrix}3+a & 1+c\\ -1+b & -2+d\end{bmatrix}$ ⟹ 3 + a = - 3 – a ⟹2a = -6 ⟹a = -3

Again, -1 + b = -1 – c ⟹ b + c = 0 ⟹b = -c

Again , -2 + d = 2- d⟹2d = 4⟹d = 2, Hence, B = $\begin{bmatrix}-3 & b\\ -b & 2\end{bmatrix}$

LONG ANSWER TYPE QUESTIONS

(1) If A = $\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$. Then find A^{-1}. Using A^{-1}, solve the following system of equations :

2x – 3y + 5z = 11

3x + 2y – 4z = -5

x + y – 2z = -3

Solution: A = $\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$

| A | = $\begin{vmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{vmatrix}$ = 2(-4+4) + 3(-6+4)+5(3-2) = 0 – 6 + 5= -1 ≠ 0 Hence A^{-1} exists.

Co-factors of elements of A are :

⟹A_{11}= (-4 +4) =0, A_{12}= - (-6 +4) = 2, A_{13} = (3 – 2) = 1

⟹A_{21}=-(6 – 5) = - 1, A_{22} = (-4 – 5) = -9, A_{23} = - (2 +3) = -5

⟹A_{31} = 12 – 10 = 2, A_{32} = - (-8 – 15) = 23, A_{33} = 4+9 = 13

$\Rightarrow$Adj (A) = $\begin{vmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{vmatrix}^T$ = $\begin{vmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{vmatrix}$

$A^{-1} = \frac{1}{I\,A\,I}$ Adj (A) = $\frac{1}{-1}\begin{vmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{vmatrix}$

Matrix Equation is

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Since, AX = B

$\Rightarrow X = A^{-1} B = \begin{vmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{vmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ Hence , x = 1, y = 2 and z= 3

(2) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

and use it to solve the system of equations.

x -y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1.

Solution: Let A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

AB = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -4 + 4 + 8 & 4 - 8 + 4 & -4 - 8 + 12 \\ -7 + 1 + 6 & 7 - 2 + 3 & -7 - 2 + 9 \\ 5 - 3 - 2 & -5 + 6 - 1 & 5 + 6 - 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

= 8 I ………..(i)

Matrix Equation

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

Since, BX = C

$\Rightarrow X=B^{-1}C$ …………(ii)

From Equn. (i)

$I = \frac{AB}{8}$ Multiply by B^{-1} $\Rightarrow$ $IB^{-1} = \frac{ABB^{-1}}{8} = \frac{AI}{8}$ $\Rightarrow$ $B^{-1} = \frac{A}{8}$

From Equn. (ii) , $X=B^{-1}C=\frac{AC}{8} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ Hence, x = 3, y = - 2, z = - 1

(3) A shopkeeper has 3 varieties of pencils A, B and C. Sohan purchased 1 pencil of each varieties for a total of Rs 21. Mohan purchased 4 pencils of A variety , 3 pencils of B variety, and 2 pencils of C variety for Rs 60. While Sita purchased 6 pencils of A variety, 2 pencils of B variety, and 3 pencils of C variety for Rs.70. Using matrix method find cost of each variety of pencils.

Solution: Let Rs x, Rs y and Rs z be cost of variety A , B and C respectively.

According to question,

x + y + z = 21

4x + 3y + 2z = 60

6x + 2y + 3z = 70

In Matrix form

$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

$|A| == \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix}$

$|A| = 1(9-4) - 1(12-12) + 1(8-18) = 5 - 0 - 10 = -5 \neq 0$

Hence A^{-1} exists.

Co-factors of elements of A are :

$\Rightarrow A_{11} = (9-4) = 5$, $A_{12} = -(12-12) = 0$, $A_{13} = (8-18) = -10$

$\Rightarrow A_{21} = -(3-2) = -1$, $A_{22} = (3-6) = -3$, $A_{23} = -(2-6) = 4$

$\Rightarrow A_{31} = 2-3 = -1$, $A_{32} = -(2-4) = 2$, $A_{33} = 3-4 = -1$

Adj (A) = $\begin{vmatrix} 5 & 0 & -10 \\ -1 & -3 & 4 \\ -1 & 2 & -1 \end{vmatrix}^T = \begin{vmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{vmatrix}$

$A^{-1} = \frac{1}{I\,A\,I}$ Adj (A) = $\frac{1}{-5}\begin{vmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{vmatrix}$

$X = A^{-1} B = \frac{1}{-5}\begin{vmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{vmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

$= \frac{1}{-5}\begin{bmatrix} 105 - 60 - 70 \\ 0 - 180 + 140 \\ -210 + 240 - 70 \end{bmatrix}$

$= \frac{1}{-5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$

Hence, x = 5, y = 8, z = 8

Hence, cost of variety A = Rs.5, Hence, cost of variety B= Rs.8,

and Hence, cost of variety C = Rs.8.

(4) (1)If A = $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$. Then find A^{-1}. Using A^{-1}, solve the following system of equations :

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$; $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$.

Solution: Let A = $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

| A | = 2(120 – 45) – 3(- 80 – 30) + 10 (36 + 36)

= 2 × 75 – 3 × (-110) + 10 × 72

= 150 + 330 + 720

=1200 ≠ 0

Hence A^{-1} exists.

Co-factors of elements of A are :

$\Rightarrow A_{11} = (120 - 45) = 75,\ A_{12} = -(-80 - 30) = 110,\ A_{13} = (36 + 36) = 72$

$\Rightarrow A_{21} = -(-60 - 90) = 150,\ A_{22} = (-40 - 60) = -100,\ A_{23} = -(18 - 18) = 0$

$\Rightarrow A_{31} = 15 + 60 = 75,\ A_{32} = -(10 - 40) = 30,\ A_{33} = -12 - 12 = -24$

$$\text{Adj}(A) = \begin{vmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{vmatrix}^T = \begin{vmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}(A) = \frac{1}{1200}\begin{vmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{vmatrix}$$

Consider the equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2;\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5;\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$

In matrix equation

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

Since, AX = B

$$\Rightarrow X = A^{-1}B = \frac{1}{1200}\begin{vmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{vmatrix}\begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 150 + 750 - 300 \\ 220 - 500 - 120 \\ 144 + 0 + 96 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ -400 \\ 240 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\frac{1}{x} = \frac{1}{2}$, $\frac{1}{y} = -\frac{1}{3}$, $\frac{1}{z} = \frac{1}{5}$

Hence, x = 2, y = -3, z = 5

CASE BASED QUESTIONS (4 MARKS QUESTIONS)

(1) Read the following text and answer the following questions on the basis of the same:
A manufacturer produces three stationary products pen, pencil and eraser which he sells in two markets . Annual sales are indicated below:

MARKET/ PRODUCTS	PEN	PENCIL	ERASER
A	10,000	2,000	18,000
B	6,000	20,000	8,000

If the unit sale price of pen , pencil and eraser are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs. 1.00 respectively ,then based on the above information answer the following:

(i)Find the total revenue of market A .
Solution: Let A = $\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix}$
and B = $\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

$AB = 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00$

$= 25000 + 3000 + 18000 = 46000$

The total revenue of market A = Rs. 46000.

(ii)Find the total revenue of market B .

Solution: Let $C = [6000 \quad 20000 \quad 8000]$

and $D = \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

$CD = 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00$

$= 15000 + 30000 + 8000 =$ Rs.53000

The total revenue of market B = Rs. 53000.

(iii)Find the gross profit in both market.

Solution: Total Cost Price = $\begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 18000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 1.00 \end{bmatrix}$

$= \begin{bmatrix} 20000 + 2000 + 18000 \\ 12000 + 20000 + 18000 \end{bmatrix}$

$= \begin{bmatrix} 40000 \\ 50000 \end{bmatrix}$

Total Cost Price of both market = Rs. 50000 + Rs. 40000 =Rs. 90000

The total revenue of both the market = Rs. 53000 + Rs. 46000

=Rs. 99000

Hence, The gross profit in both market = Rs. 99000 - Rs. 90000 = Rs. 9000

(2) Read the following text and answer the following questions on the basis of the same:

Two farmers Ravi and Ramu cultivate only three varieties of pulses namely Urad, Masoor and Mung . The sale of these varieties of pulses for both the farmers in the month of September and October are given by the following matrices A and B.

September sales in (Rs.) (Matrix A)

	Urad	Masoor	Mung
Ravi	20000	10000	30000
Ramu	30000	50000	10000

October sales in (Rs.) (Matrix B)

	Urad	Masoor	Mung
Ravi	4000	10000	9000
Ramu	5000	10000	8000

Answer the following question with the help of above information.
Using algebra of matrices answer the following questions.

(i) Find the combined sales of Masoor and Moong in September and October for farmer Ramu .

Solution: The combined sales of Masoor and Moong in September and October for farmer Ramu

$= [50000 \quad 10000] + [10000 \quad 8000]$

$= [60000 \quad 18000]$

=Rs. 78000

(ii) Find the combined sales of Urad and Masoor in September and October for farmer Ravi .

Solution: The combined sales of Masoor and Urad in September and October for farmer Ramu

$= [20000 \quad 10000] + [4000 \quad 10000]$

$= [24000 \quad 20000]$

=Rs. 44000

(3)Read the following text and answer the following questions on the basis of the same:

Three DPS , CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans , mats and plates from recycled material at a cost Rs 25, Rs100 and Rs50 each respectively . The numbers of articles sold are given as:

School/Article	DPS	CVS	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

(i)What is the total money (in Rupees) collected by the school DPS ?

Solution: The total money (in Rupees) collected by the school DPS

$$= [40 \quad 50 \quad 20] \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = 1000 + 5000 + 1000 = \text{Rs.}7000$$

(ii) What is the total amount of money (in Rupees) collected by schools CVS ?

Solution: The total money (in Rupees) collected by the school CVS

$= [25 \quad 40 \quad 30] \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = 625 + 4000 + 1500 =$ Rs.6125

(iii)What is the total amount of money collected by all three schools DPS, CVS, and KVS ?

Solution: The total amount of money collected by all three schools KVS

$= [35 \quad 50 \quad 40] \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} = 875 + 5000 + 2000 =$ Rs.7875

The total amount of money collected by all three schools DPS, CVS, and KVS = Rs.7000 + Rs.6125 + Rs.7875

= Rs.21000

HIGHER ORDER THINKING SKILLS

(1) Find A^{-1} if $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that $A^{-1} = \frac{A^2 - 3I}{2}$.

Solution: Given that, $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$|A| = 0(0 - 1) - 1(0 - 1) + 1(1 - 0) = 0 + 1 + 1 = 2 \neq 0$

Hence, A^{-1} exists.

Co-factors are,

$\Rightarrow A_{11} = (0 - 1) = -1$, $A_{12} = -(0 - 1) = 1$, $A_{13} = (1 - 0) = 1$

$\Rightarrow A_{21} = -(0 - 1) = 1$, $A_{22} = (0 - 1) = -1$, $A_{23} = -(0 - 1) = 1$

$\Rightarrow A_{31} = 1 - 0 = 1$, $A_{32} = -(0 - 1) = 1$, $A_{33} = 0 - 1 = -1$

Adj. (A) = $= \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

$A^2 = A \times A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1+1 & 0+0+1 & 0+1+0 \\ 0+0+1 & 1+0+1 & 1+0+0 \\ 0+1+0 & 1+0+0 & 1+1+0 \end{bmatrix}$

$=\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}$

$\frac{A^2-3\,I}{2}=\frac{1}{2}\left\{\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{bmatrix}-\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}\right\}$

$=\frac{1}{2}\begin{bmatrix}-1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{bmatrix}=A^{-1}$

(2)If A $=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$,equation then find the value of A^{-1}.

Using A^{-1} , solve the system of linear equations

x – 2y = 10, 2 x – y – z = 8, and -2y + z = 7

Solution: Let A = $\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$

| A | = 1(-1 – 2) – 2 (-2 + 0) + 0 (2 – 0)

=-3 + 4 = 1 ≠ 0 Hence, A^{-1} exists.

Co-factors are,

⟹A_{11}= (- 1 - 2) = - 3 , A_{12}= - (- 2 + 0) = 2, A_{13} = (2 - 0) = 2

⟹A_{21}=-(2 – 0) = -2, A_{22} = (1 – 0) = 1, A_{23} = - (-1-0) = 1

⟹A_{31} = -4 + 0 = -4, A_{32} = - (-2 +0) = 2, A_{33} = - 1 + 4 = 3

Adj. (A) = $\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}^T=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$

$A^{-1}=\frac{1}{I\,A\,I}$ Adj (A) $=\frac{1}{1}\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$

This is the form of CX = D

Matrix Equation

$\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$

Where, C = $\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, D = $\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$

We know that $[A^T]^{-1}=[A^{-1}]^T$

$C^T=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}=A$

X = C^{-1} D $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30 + 16 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

∴ x = 0, y = -5, z = -3

(3)Given A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, then find BA and use this to solve the system of equations

y + 2z = 7, x – y = 3, and 2x + 3y + 4z = 17.

Solution: BA = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$

$B^{-1} = \frac{A}{6} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

The given equations in Matrix form is

$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$= \frac{1}{6} \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

∴ x = 2, y = -1, z = 4

EXERCISE

MULTIPLE CHOICE QUESTIONS

(1) Value of k, for which A = $\begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix :

(a) 4 (b) - 4 (c) 0 (d) ± 4

(2) Given that matrices A and B are of order 3 × n and m × 5 respectively , then the order of matrix C = 5 A + 3 B is

(a)3 × 5 (b)5 × 3 (c) 3×3 (d) 5×5

(3) Total number of possible matrices of order 3 ×3 with each entry 2 or 0 is

(a) 64 (b) 128 (c) 512 (d) 256

(4) The matrix $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}$ is a

(a)Symmetric Matrix (b) Skew-Symmetric Matrix

(c) None of these (d) Identity Matrix

(5)If $\begin{bmatrix}2x + y & 4x\\5x - 7 & 4x\end{bmatrix} = \begin{bmatrix}7 & 7y - 13\\y & x + 6\end{bmatrix}$ then the value of x + y is

(a) x = 3, y = 1 (b) x = 2, y = 3 (c) x = 2, y = 4 (d) x =3, y =3

(6)Given that A is a square matrix of order 3 and | A | = -2, then

| ad j(2A) | is equal to

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

(7)For matrix A = $\begin{bmatrix}2 & 5\\-11 & 7\end{bmatrix}$, (adjA)' is equal to

(a) $\begin{bmatrix}-2 & -5\\11 & -7\end{bmatrix}$ (b) $\begin{bmatrix}7 & 5\\11 & 2\end{bmatrix}$ (c) $\begin{bmatrix}7 & 11\\-5 & 2\end{bmatrix}$ (d) $\begin{bmatrix}7 & -5\\11 & 2\end{bmatrix}$

(8)The value of x for which $\begin{vmatrix}3 & x\\x & 1\end{vmatrix} = \begin{vmatrix}3 & 2\\4 & 1\end{vmatrix}$:

(a) 2 (b) $\sqrt{2}$ (c) - $\sqrt{2}$ (d) $\pm 2\sqrt{2}$

(9)For A =$\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, then 14 A^{-1} is given by

(a)14 $\begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$ (b) $\begin{bmatrix}4 & -2\\2 & 6\end{bmatrix}$ (c) 2 $\begin{bmatrix}2 & -1\\1 & -3\end{bmatrix}$ (d) 2$\begin{bmatrix}-3 & -1\\1 & -2\end{bmatrix}$

(10)If A and B are square matrices of the same order 3, such that | A | = 2and AB = 2 I , then the value of | B | :

(a) 4 (b) - 4 (c) 0 (d) 2

ASSERTION REASON BASED QUESTIONS

*In the following questions , a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) A and R both true and R is correct explanation of A.

(b)A and R both true but R is not a correct explanation of A.

(c)A is true but R is false.

(d)A is false but R is true.

(1) **Assertion (A) :**If matrix A = $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ and matrix B = $\begin{bmatrix}0 & 0\\-1 & 4\end{bmatrix}$ then AB =$\begin{bmatrix}-1 & 4\\-1 & 4\end{bmatrix}$

Reason (R): Multiplication of matrices A and B or AB is defined if number of columns of A = number of rows of B.

(2) **Assertion (A) :** Matrix B = $\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 5\end{bmatrix}$ represent identity matrix.

Reason (R): A diagonal matrix whose all the elements are 1 is identity matrix.

(3)**Assertion (A):** Matrix A = $\begin{bmatrix}1 & 2 & 0\\-1 & 1 & 0\\0 & 1 & 1\end{bmatrix}$ and

matrix B = $\begin{bmatrix}2 & -1 & 3\\4 & 5 & -6\end{bmatrix}$ then order of matrix AB is 3 × 2.

Reason (R): If order of matrix A is m × n and order of matrix B is n×p then order of matrix A × B is m × P.

(4) Assertion (A): If A= $\begin{bmatrix}2 & y-3 & y-2\\2 & -2 & -1\\3 & -1 & -5\end{bmatrix}$ is a symmetric, then x = 5.

Reason (R): If A is symmetric matrix then A= A' .

(5) Assertion (A):The area of the triangle whose vertices are (2,7),(1,1) and (10,8) is $\frac{61}{2}$ Sq. units.

Reason (R): The area of the triangle whose vertices are (x_1,y_1),(x_2,y_2) and (x_3,y_3) is $\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\x_2 & y_2 & 1\\x_3 & y_3 & 1\end{vmatrix}$Sq. units.

(6)) Assertion (A):For the value of k = -2 or 12 The area of the triangle whose vertices are (2,-6),(5,4) and (k,4) is 35 Sq. units.

Reason (R): The area of the triangle whose vertices are (x_1,y_1),(x_2,y_2) and (x_3,y_3) is $\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\x_2 & y_2 & 1\\x_3 & y_3 & 1\end{vmatrix}$Sq. units.

VERY SHORT ANSWER TYPE QUESTIONS

(1) If matrix A = $\begin{bmatrix}0 & a & -3\\2 & 0 & -1\\b & 1 & 0\end{bmatrix}$ is given to be symmetric then find the value of a and b.

(2)Find the value of x – y , if 2$\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}$ + $\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}$ = $\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$.

(3)If A = $\begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 50 & 40 \\ 2 & 3 \end{bmatrix}$ then find the value of | AB |.

(4)If area of triangle is 35 sq. units with vertices (2,-6), (5,4) and (k,4) then find the value of k.

(5) If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and I is the identity matrix of order 2 , then show that A^2 - 5A + 7 I = 0.

(6)If $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ A $\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$ = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find matrix A.

SHORT ANSWER TYPE QUESTIONS

(1)If A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that A^3 – 23 A – 40 I = 0.

(2) Find the values of a and b if A = B , where A = $\begin{bmatrix} a + 4 & 3b \\ 8 & -6 \end{bmatrix}$

and B = $\begin{bmatrix} 2a + 2 & b^2 + 2 \\ 8 & b^2 - 5b \end{bmatrix}$

(3) Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$, as the sum of a symmetric and a skew-symmetric matrix.

(4) If A = $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$.

Then using A^{-1} , solve the system of equations x – 2y = -1, 2x + y = 2.

(5)The sum of three numbers is 2. If twice the second number id added to the sum of first and third , the sum is 1. By adding second and third number to five times the first number, we get 6 . Find the three numbers by using matrices.

(6)Solve the following system of equations using Cramer's rule

x + y + z = 10, 2x + y = 13, x + y – 4z = 0.

LONG ANSWER TYPE QUESTIONS

(1) A school plans to award Rs.6000 in total to its students to reward for certain values- honesty, regularity and hard work. When three times the award money for hard work is added to the award money given for honesty amounts to Rs.11000. The award money for honesty and hard work together is double the award money for regularity. Use matrix method to find the prize money for each category of award.

(2) Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ solve the system of equations. x +3z = -9, - x + 2y – 2z = 4, 2x -3 y + 4z = -3.

(3)Show that the matrix , A = $\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

satisfies the equation $A^3 - A^2 - 3A - I_3 = 0$. Hence, find A^{-1} .

CASE BASED QUESTIONS

Read the following text and answer the following questions on the basis of the same:
(1)A trust fund has Rs.35,000 is to be invested in two different types of bonds . The first bond pays 8 % interest per annum which will be given to be orphanage and second bond pays 10 % interest per annum which will be given to an N. G. O . Trust fund obtains an annual total interest of Rs. 32,000.
(i)Find the invested money in first type of bond by matrix method.
(ii)Find the invested money in second type of bond by matrix method.
Read the following text and answer the following questions on the basis of the same:
(2) Two booksellers A and B sell the textbook of Mathematics and Applied Mathematics . In the month of March , bookseller A sold 250 books of Mathematics and 400 books of Applied Mathematics whereas bookseller B sold 230 books of Mathematics and 425 books of applied Mathematics . In the month of April bookseller A sold 550 books of Mathematics and 300 books of Applied Mathematics and bookseller B sold 270 books of Mathematics and 450 books of Applied Mathematics .
(i) Find the total sale for bookseller A using matrix method .
(ii) Find the total sale for bookseller B using matrix method.

HIGHER ORDER THINKING SKILLS

(1) An amount of Rs.5000 is put into three investments at the rate of interest of 6 %, 7% and 8% per annum respectively. The total annual income is Rs. 358. If the combined income from the first two investments is RS.70 more than the income from the third, find the amount of each investment by matrix method.

(2) If A = $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ then find AB.

and use it to solve the system of equations.
x -2y = 3, 2x – y – z = 2, -2 y + z = 3.

(3) If A = $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{bmatrix}$ find A^{-1}.

Using A^{-1} , solve the system of linear equations
2x +y-3z = 13, 3 x +2y +z = 4, and x+2y - z = 8.
(4) A mixture is to be made of three foods A,B,C. The three foods A,B, C contain nutrients P,Q,R as shown below:

	Ounces per pound of nutrients		
Food	P	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

How to form a mixture which will have 8 ounces of P, 5 ounces of Q and 7 ounces of R?

ANSWERS OF EXERCISE

MULTIPLE CHOICE QUESTIONs

(1)(d) ± 4 (2) (a)3×5 (3)(c) 512 (4) (a)Symmetric Matrix (5)(b) x = 2, y = 3
(6)(d) 2^8 (7)(c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (8)(d) $\pm 2\sqrt{2}$ (9)(b) $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$ (10)(a) 4

ASSERTION REASON BASED QUESTIONS

(1) **(a)** **(2) (d)** (3) **(a)** **(4(a)** (5)(d) (6)) (a)

VERY SHORT ANSWER TYPE QUESTIONS

(1) -2, b = 3 (2) x – y = 0 (3) | AB | = -7000 (4)k = -2 or 12.
(5) $A^2 - 5A + 7I = 0$. (6) $\begin{bmatrix} 7 & 5 \\ -11 & -8 \end{bmatrix}$

SHORT ANSWER TYPE QUESTIONS (03 MARKS QUESTIONS)

(1) $A^3 - 23A - 40I = 0$. (2) a = 2 and b = 2

(3) $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$. (4) $x = \frac{3}{5}$, $y = \frac{4}{5}$

(5) 1,-1,2 (6)x= 5, y = 3, z = 2

LONG ANSWER TYPE QUESTIONS (05 MARKS QUESTIONS)

(1) The award money for honesty = Rs.500,
The award money for regularity = Rs.2000
The award money for hard work = Rs.3500
(2) x = 36, y = 5, z = -15.
(3)$A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

CASE BASED QUESTIONS (04 MARKS QUESTIONS)

(1) (i)Rs.15000, (ii)Rs.20000 (2)`(i)1500 (ii) 1375

HIGHER ORDER THINKING SKILLS

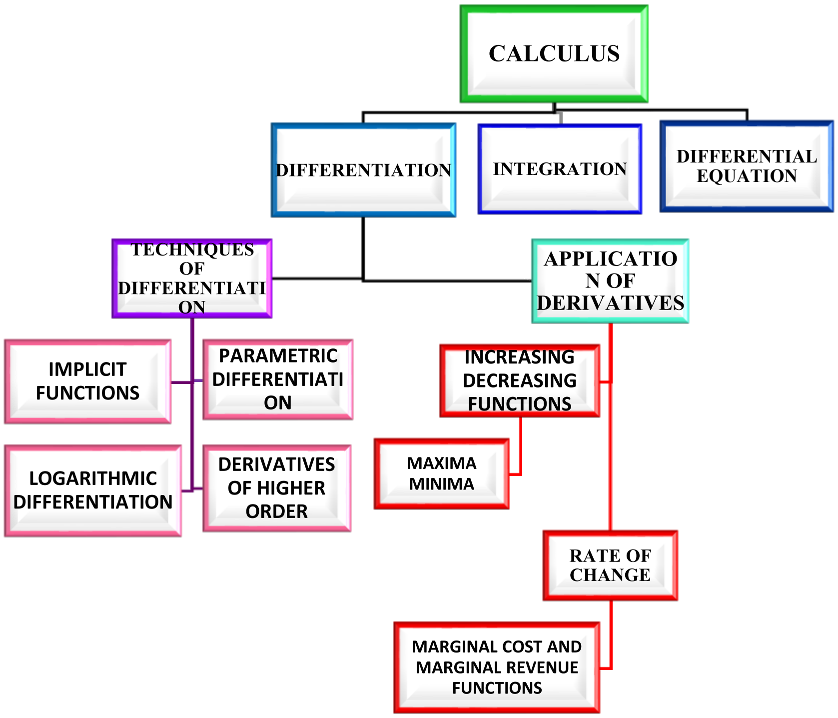
(1) x = Rs.1000, y = Rs.2200 and z = Rs.1800 (2) Ans. x = 1, y = -1, z = 1.

(3) Ans. x = 1, y = 2, z = -3. (4) Food A = 1 pound, Food B = pound, Food C = 1 pound.

UNIT– 3

CALCULUS

Mind Mapping



DIFFERENTIATION

Gist/Summary of the lesson:

1) **IMPLICIT DIFFERENTIATION**An implicit function is defined by an equation involving both x and y as f(x,y) =0,where y cannot be explicitly expressed as x
2) **LOGARITHMIC DIFFERENTIATION**
   Let $b > 1$ be a real number, then $\log_b a = x$, if $b^x = a$
3) **PARAMETRIC DIFFERENTIATION**
   If x and y are two variables such that both are explicitly expressed in terms of a third variable, say ‘t’ i.e., x=f(t), y=g(t) is said to be in parametric form with t as the parameter
   Applying chain rule,
   $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ or $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ (whenever $\frac{dx}{dt} \neq 0$)
4) **DERIVATIVES UPTO SECOND ORDER**
   Let $y = f(x)$
   Then $\frac{dy}{dx} = f'(x)$
   If $f'(x)$ is differentiable, then $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = f''(x)$
   which is called the second derivative of y w.r.t x.

SOME IMPORTANT FORMULAE

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\text{constant}) = 0$
- $\frac{d}{dx}(\log x) = \frac{1}{x}$
- $\frac{d}{dx}(a^x) = a^x \times \log_e a$
- $\frac{d}{dx}(e^x) = e^x$
- $e^{\log_e x} = x$

<u>LAWS OF LOGARITHM</u>

- $\log(a\times b) = \log a + \log b$
- $\log\left(\frac{a}{b}\right) = \log a - \log b$
- $\log a^n = n\times \log a$
- Product rule : $(f \times g)' = f\times g' + f' \times g$
- Quotient rule : $\left(\frac{f}{g}\right)' = \frac{g.f' - g'.f}{g^2}$
- Chain rule : If $y=f(u)$, $u=g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$
- $(f \pm g)' = f' \pm g'$

APPLICATION OF DERIVATIVES

Rate of change of quantities

- $\frac{dy}{dx}$ represents the rate of change of y with respect to x
- If y increases as x increases, then $\frac{dy}{dx}$ is positive
- If y decreases as x increases, then $\frac{dy}{dx}$ is negative

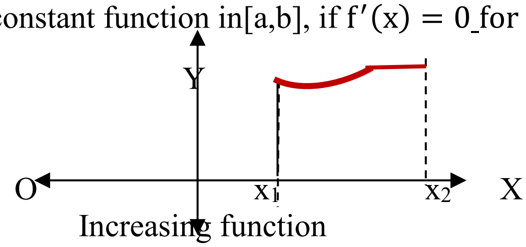
Increasing /Decreasing function

Let I be an interval contained in the domain of a real valued function
Then f is said to be

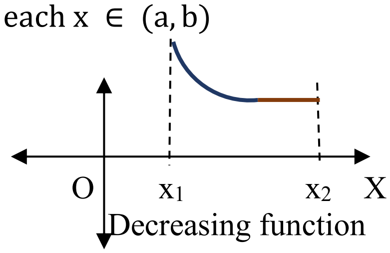
- Increasing in I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \leq f(x_2)$
- Decreasing in I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \geq f(x_2)$
- Constant in I , if f(x)=c for all $x \in I$, where c is a constant
- Strictly increasing in I if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$
- Strictly decreasing in I if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$

Let f be continuous in [a,b] and differentiable in (a,b)

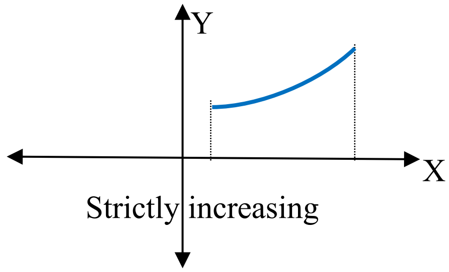
- f is increasing in $[a, b]$ if $f'(x) \geq 0$ for each $x \in (a, b)$
- f is decreasing in $[a, b]$ if $f'(x) \leq 0$ for each $x \in (a, b)$
- f is a constant function in[a,b], if $f'(x) = 0$ for each $x \in (a, b)$

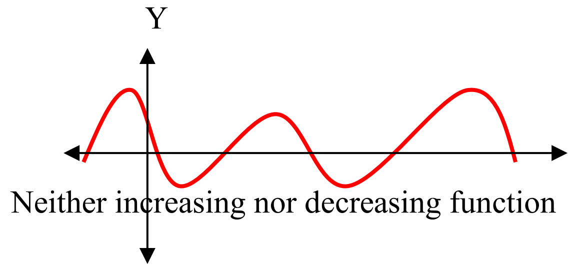


Increasing function



Decreasing function





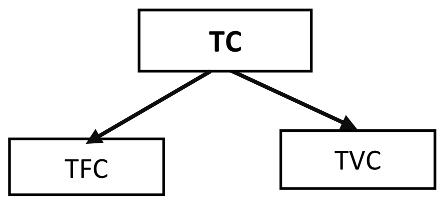
<u>Marginal cost function and Marginal Revenue functions</u>

COST FUNCTION(C(x)): If C is the total cost incurred in producing and marketing x units of a certain commodity, then a function relating C and x is called a cost function.

Fixed cost: It is the sum of all costs that are independent of the level of production.

Variable cost: It is the sum of all costs that are dependent on the level of production.

TC=TFC+TVC



Supply function: A relation between the price per unit and the quantity supplied by the producer in the market at that price is called the supply function.

Total Revenue Function(R(x): If R is the total revenue collected by a company when it sells x units of a product at price p per unit , then R is given by R = p × x

Profit function P(x)= R(x) - C(x).

Break –Even Point: The breakeven point is the level production where the revenue from sale is equal to the cost of production and marketing**. [R(x) = C(x).]**

Average cost: It is the cost of producing and marketing each unit of the product. [**AC=**$\frac{C(x)}{x}$**]**

Marginal cost: It is defined as the rate of change of the total cost with respect to each unit of product**. [MC=**$\frac{d}{dx}(C(x)$ **]**

Average Revenue: Revenue per unit is known as average revenue**. [AR=**$\frac{R(x)}{x}$]

Marginal Revenue: It indicates the rate at which the total revenue changes with respect to units sold**.[MR=**$\frac{d}{dx}(R(x)$]

Average Cost (AC)

$AC = \frac{C}{x}$, C is the total cost for producing x units of commodity

Also $TC = TFC + TVC$

or, $\frac{TC}{x} = \frac{TFC}{x} + \frac{TVC}{x}$

or, AC = AFC +AVC, [AFC-Average fixed cost, AVC-Average variable cost]

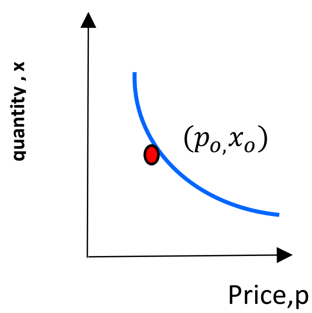
Demand function: Revenue and Profit function

The price per unit and the quantity demanded at that price are related inversely

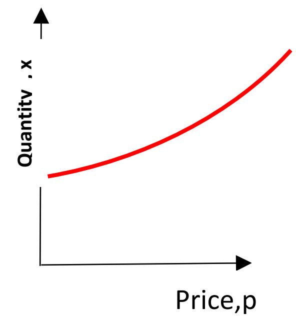
Or, the higher the quantity demanded, lower is the price

If p is the price per unit and the demand of the item is x units , then x=f(p)

Demand function



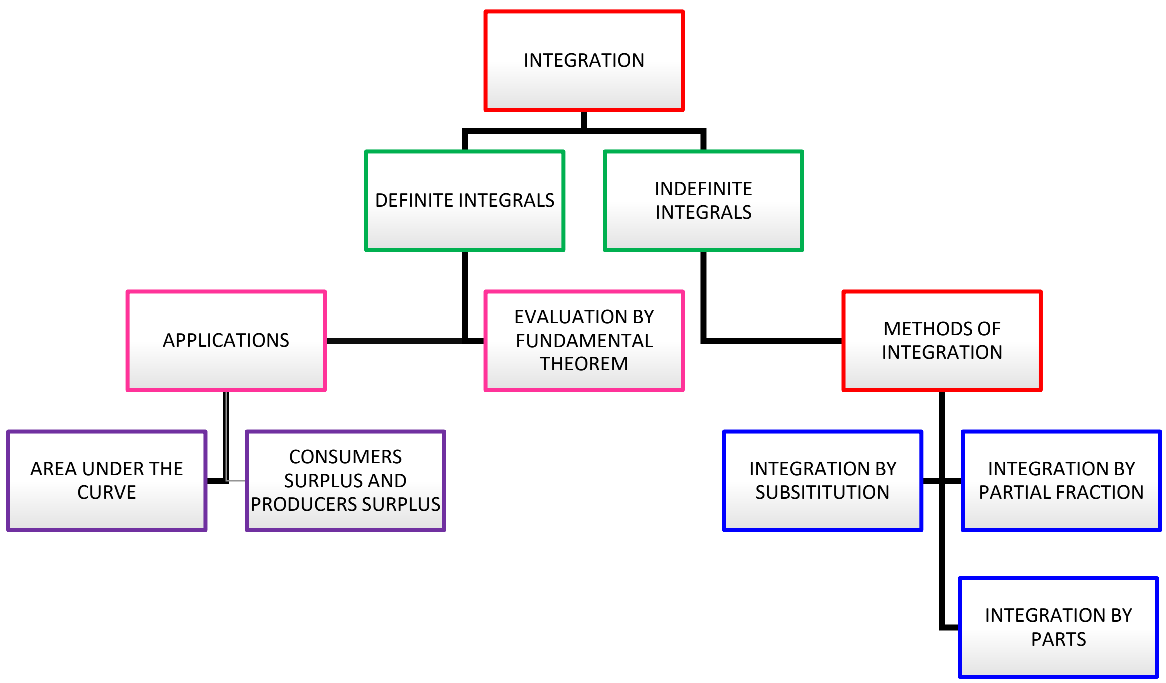
Revenue Function



R(x)= x × p Average revenue function **AR**= $\frac{R}{x} = \frac{px}{x} = p$.

The Average revenue is the same as price per unit

INTEGRATION

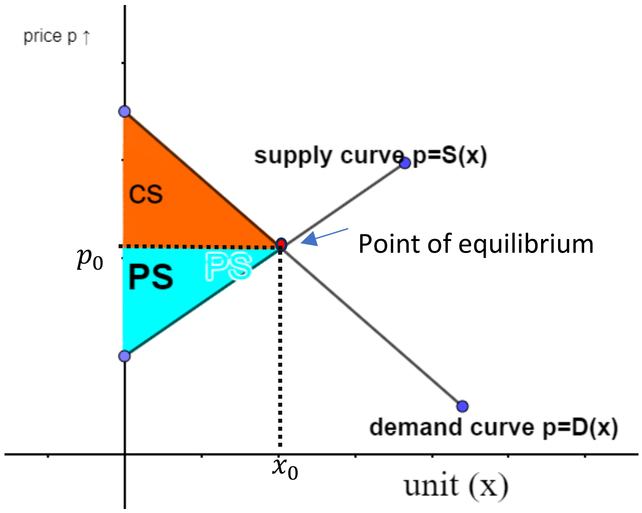


* $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

* $\int 1.dx = x + C$

* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$
* $\int \frac{1}{x} dx = \log_e x + C$
* $\int e^x dx = e^x + C$
* $\int a^x dx = \frac{a^x}{\log_e a} + C$
* $\int u.v\, dx = u.\int v\,.dx - \int \frac{du}{dx}[\int v.dx].dx$
* $\int \lambda f(x) dx = \lambda \int f(x) dx + C$
* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C,\ \text{if } x > a$
* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C,\ \text{if } x > a$
* $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log |x + \sqrt{x^2 + a^2}| + C$
* $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}| + C$
* $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 + a^2}\right| + C$
* $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + C$
* $\int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + C$
* $\int \{f_1(x) \pm f_2(x) \pm ....f_n(x)\} dx$

  $= \int f_1(x) dx \pm \int f_2(x) dx \pm .... \pm \int f_n(x) dx$
* $\int_a^b f(x)\ dx = F(b) - F(a)$, where $F(x) = \int f(x)\ dx$,
* Cost Function, $C(x) = \int MC(x)dx$ where MC is Marginal Cost
* Revenue Function, $R(x) = \int MR(x)dx$ where MR is Marginal Revenue
* Consumers' Surplus, [is the gain made by consumers when they purchase an item from the market at a lower price rather than the price they would have been willing to pay]



CS = $\int_0^{x_0} D(x)dx - p_0x_0$ where D(x) is the demand curve

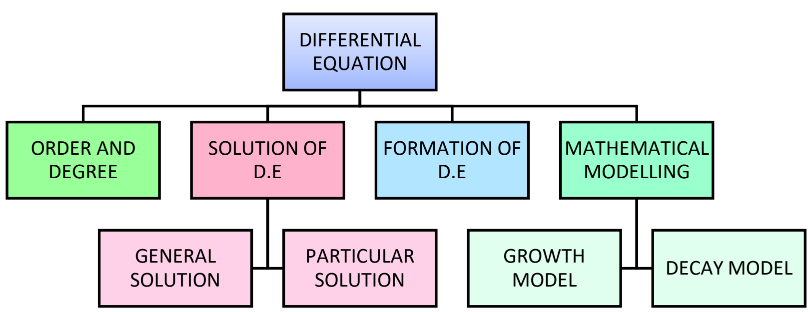
* Producers' Surplus, [is the gain made by producers when they sell an item in the market at a higher price rather than the price they would have been willing to sell]

PS = $p_0x_0 - \int_0^{x_0} S(x)dx$, where S(x) is the supply curve

* The equilibrium price is the price where the amount of the product that consumers want to buy (quantity demanded) is equal to the amount producers want to sell (quantity supplied). This mutually desired amount is called the equilibrium quantity.

For more details, refer the following linkhttps://www.youtube.com/watch?v=W5nHpAn6FvQ&t=20s

DIFFERENTIAL EQUATIONS



DIFFERENTIAL EQUATIONS AND MODELING

SOME IMPORTANT RESULTS/CONCEPTS

DIFFERENTIAL EQUATION

An equation involving derivative(s) of the dependent variable with respect to the independent variable(s) is called a differential equation.

$\frac{dy}{dx} + \frac{y}{x} = x^3$ …… (i) $\frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^2 = 0$…… (ii)

ORDER OF A DIFFERENTIAL EQUATION: The order of differential equation is defined as the highest ordered derivative of the dependent variable with respect to the independent variable involved in the differential equation.

DEGREE OF A DIFFERENTIAL EQUATION

For the degree of a differential equation to be defined it must be a polynomial equation in its derivatives. The degree of a differential equation, when it is a polynomial equation in its derivatives is the highest power (positive integral index) of the highest order derivative involved in the differential equation.

DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELING:

GROWTH AND DECAY MODELS The mathematical model for exponential growth or decay is given by $f(t) = Ae^{kt}$

Where: **t** represents time, **A** the original amount y or f(t) represents the quantity at time t ;k is a constant that depends on the rate of growth or decay If k > 0, the formula represents exponential growth; If k < 0, the formula represents exponential decay

MULTIPLE CHOICE QUESTIONS (01 MARK QUESTIONS)

1) The cost function for a manufacturer is given by $C(x)=\frac{x^3}{3} - x^2 + 2x$(in rupees)

Which of the following statement is correct based on the above information?

a) The marginal cost decreases from 0 to 1 and then increases onwards

b) The marginal cost increases from 0 to 1 and then decreases onwards

c) The marginal cost decreases as production level increases to 0

d) The marginal cost increases as production level decreases to 0

Answer: (a)

Solution:

The marginal cost function is given by $MC(x)=\frac{dC}{dx} = x^2 - 2x + 2$

$MC'(x) = 2x - 2$

So, the marginal cost decreases from 0 to 1 and then increases onwards

2) The absolute minimum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $[-2, \frac{9}{2}]$ is

a) -8 b) -9 c) -10 d) -16

Answer: (c)

Solution:

$f'(x) = 4-x$

$f'(x) = 0 \Rightarrow x = 4$

∴ the absolute minimum value of the function $(x) = 4x - \frac{1}{2}x^2$ in the interval $[-2, \frac{9}{2}]$ is

$\text{Min}\left\{f(-2), f(4), f(\frac{9}{2}\right\} = \text{Min}\left\{-10, 8, , \frac{63}{8}\right\} = -10$

3) If f(x)= log x, then the derivative of f(log x) is

a) $\frac{\log x}{x}$ b) $\frac{x}{\log x}$ c) x log x d) $\frac{1}{x\log x}$

Answer: (d)

Solution:

f(log x) = log(log x)

$f'(x) = \frac{1}{x\log x}$ by applying chain rule

4) If $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots \ldots \ldots \ldots \ldots \infty}}}$.

Then $\frac{dy}{dx}$ is equal to

a) 2y-1 b) $\frac{1}{2y-1}$ c) 2y - 1 d) $\frac{1}{2y+1}$

Answer: (b)

Solution:

$y^2 = x+y$

or, $2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

or, $(2y-1)\frac{dy}{dx} = 1$ hence $\frac{dy}{dx} = \frac{1}{2y-1}$

5) The sides of an equilateral triangle are increasing at the rate of 2 cm/ sec. The rate at which its area increases when the side is 10cm is

a) 10 cm^2/sec b) $10\sqrt{3}$ cm^2/sec

c) $\frac{10}{3}$ cm^2/sec d) $\sqrt{3}$ cm^2/sec

Answer: (b)

Solution:

Area (A) $= \frac{\sqrt{3}}{4}a^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2a \times \frac{da}{dt}$

$= \frac{\sqrt{3}}{4} \times 2a \times 2$

$\frac{dA}{dt}$ at $a = 10$ is $\frac{\sqrt{3}}{4} \times 20 \times 2 = 10\sqrt{3}$ cm^2/sec

6) The maximum value of $\frac{\log x}{x}$

a) e b) $\frac{1}{e}$ c) e^2 d)-e

Answer: (b)

Solution:

$Y = \frac{\log x}{x}$

$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - \log x}{x^2}$

$\frac{dy}{dx} = 0 \Rightarrow \log x = 1$ hence $x = e$

$\frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)2x}{x^4}$

$\frac{d^2y}{dx^2}$ at $x = e < 0$

Hence The maximum value of $\frac{\log x}{x}$ is $\frac{1}{e}$

7) If the selling price of a commodity is fixed at ₹45and the cost function is C (x)= 30 x+240, the then break-even point is

a) x =10 b) x=12 c) x=15 d) x=16

Answer: (d)

Solution:

R(x) = 45x

At breakeven point R(x)=C(x)

45 x = 30 x+240 $\Rightarrow$ x=16

8) If the demand function for a product is $p = \frac{80-x}{4}$, where x is the number of units and p is the price per unit,then the value of x for which the revenue will be maximum is

a) 10 b) 20 c) 40 d) 80

Answer: (c)

Solution:

$R(x) = p.x = \frac{80-x}{4} \times x = \frac{80x-x^2}{4}$

At maximum, $R'(x) = 0$ and $R''(x) < 0$

Hence $\frac{80-2x}{4} = 0 \Rightarrow x = 40$

$R''(x)$at $x = 40 < 0$

9) The order of the differential equation corresponding to the family of curves

y= k cos x + p (q cos x-r) is

a) 1 b) 2 c) 3 d) 4

Answer: (b)

Solution:

y= k cos x + p (q cos x-r) is

=(k+pq)cos x-pr

=a cos x+b

No of arbitrary constant=2 $\Rightarrow$ the order is 2

10) $\int \sec^2(7-4x)dx$ = a tan (7 −4x) +c, then the value of a is

a) 1 b) $\frac{-1}{4}$ c) 3 d) - 4

Answer: (b)

Solution: $\int \sec^2(7-4x)dx = \frac{\tan(7-4x)}{-4} = \frac{-1}{4}\tan(7-4x)$

11) $\int_{-2}^{2}(2-|x|)\,dx$ is equal to

a) -4 b) 4 c) 8 d) 12

Answer: (b)

Solution: $\int_{-2}^{2}(2-|x|)\,dx$

$= \int_{-2}^{2} 2\,dx - 2\int_{0}^{2}|x|\,dx$

$= 2x\Big]_{-2}^{2} - 2(x^2)\Big]_{0}^{2} = 4$

12) The general solution of the differential equation: x dy −y dx =0

a) $x^2 + y^2 = c$ b) $x^2 - y^2 = 2c$ c) xy= c d) x=cy

Answer: (d)

Solution:

$$\frac{dy}{y} = \frac{dx}{x}$$

Solution is $\int \frac{dy}{y} = \int \frac{dx}{x}$

Or, log y=log x+log c

Or, log y=log cx

Or, y=cx

13) The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^3 (x \neq 0)$is

a) x^2 b) log x c) 2x d) $\frac{1}{x}$

Answer: (b)

Solution:

$\frac{dy}{dx} + 2\frac{y}{x} = x^2 (x \neq 0\ )$

Integrating factor is $e^{\int \frac{2}{x} dx} = e^{2\log x} = x^2$

14) $\int_{a+c}^{b+c} f(x)\ dx$ is equal to

a) $\int_a^b f(x-c)\ dx$ b) $\int_a^b f(x+c)\ dx$ c) $\int_a^b f(x)\ dx$ d)$\int_{a-c}^{b-c} f(x-c)\ dx$

Answer: (b)

Solution:

let$\int f(x)dx = F(x) + c$

then $\int_{a+c}^{b+c} f(x)\ dx = [F(x)]_{a+c}^{b+c}$

=F(b+c)-F(a+c)

$\int_a^b f(x-c)\ dx$ =F(b-c)-F(a-c)

$\int_a^b f(x+c)\ dx$ = F(b+c)-F(a+c)

$\int_a^b f(x)\ dx$ =F(b)-F(a)

$\int_{a-c}^{b-c} f(x-c)\ dx$ = F(b-2c)-F(a-2c)

15) If the marginal cost function of a product is given by MC=10-4x+3x^2 and fixed cost is ₹ 500, then the cost function is

a) $10x - 2x^2 + x^3$ b) $500 + 10x - 2x^2 + x^3$ c) $500 + 10x - 8x^2 + 9x^3$ d) - 4+6x

Answer: (b)

Solution:

C(x)=$\int (10 - 4x + 3x^2)dx$

$=10x - 2x^2 + x^3 \quad +k$

When x=0, R(x)=500

Hence k=500

Hence C(x)= $500 + 10x - 2x^2 + x^3$

16) The marginal revenue function of a commodity is MR= $2x - 9x^2$,then the revenue function is

a) $2x^2 - 9x^3$ b) 2-18x c) $x^2 - 3x^3$
d) $18 + x^2 - 3x^3$

Answer: (c)
Solution:
R(x)= $\int (2x - 9x^2)dx$
$= x^2 - 3x^3$

17) $\int \frac{f'(x)dx}{f(x)\log(f(x))}$ is equal to

a) $f(x).\log(f(x)) + C$ b) $\frac{f(x)}{\log(f(x))} + C$ c) $\frac{1}{\log|\log(f(x))|} + C$ d) $\log|\log(f(x)| + C$

Answer: (d)
Solution:

$$\int \frac{f'(x)dx}{f(x)\log(f(x))}$$
$$= \int \frac{dt}{t\log(t)}$$
$$= \log|\log(t)| + C$$
$$= \log|\log(f(x)| + C$$

18) A spherical ice ball is melting at the rate of 100 cm^3/sec. The rate at which its radius is decreasing, when its radius is 15 cm, is

a) $\frac{1}{9}$ cm/min b) $\frac{1}{36}$ cm/min c) $\frac{1}{9\pi}$ cm/min d) $\frac{1}{18}$ cm/min

Answer: (c)
Solution: $V = \frac{4}{3}\pi r^3$ and $\frac{dV}{dt} = -100$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-100 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{-25}{\pi r^2}$$

Hence $\frac{dr}{dt}_{at\ r=15} = \frac{-25}{\pi \times 225} = \frac{-1}{9\pi}$ cm/min

19) Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$,what is the value of $\frac{dy}{dx}$ at x=$\frac{1}{2}$

a) $\frac{-1}{64}$ b) $-\frac{1}{32}$ c) -32 d) -64

Answer: (b)

Solution:

$\frac{dy}{dx}=f'(\frac{1}{x}) \times \frac{d}{dx}(\frac{1}{x})$

$= \frac{1}{x^3} \times \frac{-1}{x^2}$

$= -\frac{1}{x^5}$

$\frac{dy}{dx}$ at $x = \frac{1}{2}$

$= -\frac{1}{32}$

20) Total revenue in rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is

a) 96 b) 126 c) 116 d) 90

Answer: (b)

Solution:

$R'(x) = 6x + 36$ and $R'(15) = 126$.

21) The area (in sq. Units) bounded by the curve $y = \sqrt{x}$, the x-axis, x=1 and x=4 is

a) $\frac{11}{3}$ b) $\frac{1}{4}$ c) $\frac{14}{3}$ d) $\frac{13}{3}$

Answer: (c)

Solution:

Area=$\int_1^4 y\,dx$

$=\int_1^4 \sqrt{x}\,dx$

$=\frac{2}{3}\left(x^{3/2}\right)_1^4 = \frac{14}{3}$ sq. units

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
c) Assertion (A) is true but Reason(R) is false.
d) Assertion (A) is false but Reason(R) is true.

1) Assertion (A): Whenever marginal cost = marginal revenue, the profit is maximised

Reason (R) : At this point, the cost of producing an additional unit equals the revenue it generates

Answer: (d)

Solution:

If C is the cost function and R is the revenue function, then MC=$\frac{dC}{dx}$ and MR=$\frac{dR}{dx}$

Profit P (x)=R(x)-C(x)

Or, $\frac{dP}{dx}=\frac{dR}{dx} - \frac{dC}{dx}$=MR-MC

At maximum, $\frac{dP}{dx} = 0 \Rightarrow$ MR-MC=0 $\Rightarrow$ MR=MC

MR=MC alone is not sufficient to prove $\frac{d^2P}{dx^2} < 0$ [MC should be rising to satisfy the second condition]

Hence the **Assertion (A):** is false

The **Reason (R)** is correct because if MR=MC, then the firm earns exactly as much revenue from selling one more unit as it spends producing it. Beyond this point, if MC>MR, producing more would reduce profit

2) **Assertion (A):** The order of the differential equation corresponding to the family of curves $y = a(bx - ce^x)$ is 3

Reason (R) The total number of arbitrary parameters in the given general solution is the order of the corresponding differential equation

Answer: (d)

Solution: $y = a(bx - ce^x) = abx - ace^x$

$= Ax - Be^x$ has 2 arbitrary constants. $\Rightarrow$ order $= 2$

Assertion (A) is false and Reason (R) is true

3) **Assertion (A):** The solution of the differential equation $3y\frac{dy}{dx} + 4x = 0$ represents family of ellipses

Reason (R) : Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \mathbf{1}$

Answer: (b)

Solution: $3y\frac{dy}{dx} + 4x = 0$

Or, $3y\frac{dy}{dx} = -4x$

Or, 3ydy=-4xdx

Solution is $\int 3y\,dy = -\int 4x\,dx$

Or, $\frac{3}{2}y^2 + 2x^2 = C$ which is an ellipse

Assertion (A) is true and Reason (R) is true

4) **Assertion (A) :** Consumer surplus is represented by the area under the demand curve and above the market price.

Reason (R): Demand curve shows the maximum price a consumer is willing to pay for different quantities.

Answer: (b)

Solution:

Assertion (A) is true because the area under the demand curve and above the price line correctly represents the total benefit over actual expenditure — this is the consumer surplus.

Reason (R) is also true and it correctly explains the assertion because the demand curve indeed shows the maximum price consumers are willing to pay.

5) **Assertion (A) :** $f(x) = e^x$ is strictly increasing for all $x \in R$

Reason(R) : if f(x) is strictly increasing function, then $\mathbf{f'(x) > 0}$

Answer: (a)

Solution: $\mathbf{f'(x)} = e^x > 0$ for all $x \in R$ .hence assertion is true

Reason(R) is also true

6) **Assertion (A)**: $y = \log x \Rightarrow xy'' + y' = 0$

Reason (R) : $\frac{d}{dx}(\frac{dy}{dx}) = \frac{d^2y}{dx^2}$

Answer: (b)

Solution

$y = \log x \Rightarrow y' = \frac{1}{x}$

$\therefore\ xy' = 1$

Again, differentiating both sides w.r.to x,

$\Rightarrow\ xy'' + y' = 0$

Hence (A) is true

(R) is also true. But (R) is not the correct explanation for (A)

7) **Assertion (A):** The maximum profit that a company makes, if profit function is given by $P(x) = 41 + 24x - 8x^2$; where 'x' is the number of units and P is the profit is Rs.59

Reason (R): The profit is maximum at x=a if $P'(a) = 0$ and $P''(a) > 0$

Answer: (c)

Solution

P(x)= $41 + 24x - 8x^2$

$P'(x) = 24 - 16x$

$P'(x) = 0 \Rightarrow x = \frac{24}{16} = \frac{3}{2}$

$P''(x) = -16 < 0 \Rightarrow x = \frac{3}{2}$ is a point of maxima

Max Profit=P= $P = 41 + \frac{3}{2} \times 24 - 8 \times \frac{9}{4} = 59$

Assertion is true, but reason is false

For maximum at x=a if $\mathbf{P'(a) = 0}$ **and** $\mathbf{P''(a) < 0}$

8) **Assertion (A):** $\int x^2(1 + 3\log x)\, dx = (x^3 \times \log x + C)$

Reason (R): $\frac{d}{dx}(x^3 \times \log x + C) = x^2(1 + 3\log x)$

Answer: (a)

Solution

$y = x^3 \times \log x + C$

$\frac{dy}{dx} = x^3 \times \frac{1}{x} + 3x^2 \times \log x = x^2(1 + 3\log x)$Hence **Reason (R)** is true

Assertion (A) is true as integrals are anti derivative

9) **Assertion(A)**: $\int x^x(1 + \log x)\, dx = x^x + C$

Reason (R) : $\frac{d}{dx}(x^x) = x^x(1 + \log x)$

Answer: (a)

Solution : y $= x^x$

log y = x.log x

differentiating w.r.t x,

$\frac{1}{y}\frac{dy}{dx} = 1 + \log x)$

$\therefore \frac{dy}{dx} = y(1 + \log x)$

$= x^x(1 + \log x)$

Hence **Reason (R)** is true

Assertion (A) is true as integrals are anti derivative

10) **Assertion (A)** : if $\mathrm{C}(\mathrm{x}) = x^3 - 45x^2 + 300$ is the total cost function, then the fixed cost=Rs. 300

Reason (R) :The fixed cost of total cost function = C(0)

Answer: (a)

Solution: fixed cost=C(0)=$0 - 0 \times 45 + 300 = 300$.hence (A) is true

Reason (R) is also true

VERY SHORT ANSWER TYPE QUESTIONS

1) **Question:** Find the rate of change of volume of a sphere with respect to its surface area when the radius is 5 m

Solution: The volume V, surface area S of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$,$S = 4\pi r^2$

$\frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2 = 4\pi r^2$, $\frac{dS}{dr} = 8\pi r$

$\frac{dV}{dS} = \frac{\frac{dV}{dr}}{\frac{dS}{dr}} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2} \Rightarrow \left.\frac{dV}{dS}\right]_{r=5} = \frac{5}{2} m^3/m^2$

2) **Question** Find the derivative of x^x

Solution: let y $= x^x$

log y = x.log x

differentiating w.r.t x,

$\frac{1}{y}\frac{dy}{dx} = 1 + \log x)$

$\therefore \frac{dy}{dx} = y(1 + \log x)$

$= x^x(1 + \log x)$

3) **Question:** Find the absolute maximum and absolute minimum value of $f(x) = x^2 - 2x, x \in [0,2]$

Solution: $\boldsymbol{f'(x) = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1}$

f(0) =0, f(1)= $1^2 - 2 = -1$, f(2)= $2^2 - 4$=0

absolute maximum value=0

absolute minimum value= $-$ 1

4) **Question: Evaluate $\int xe^{3x}\, dx$**

Solution: $\boldsymbol{\int xe^{3x}\, dx = x \times \frac{e^{3x}}{3} - \int 1 \times \frac{e^{3x}}{3} dx}$

$\boldsymbol{= x \times \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx = x \times \frac{e^{3x}}{3} - \frac{e^{3x}}{9} + C}$

5) **Question:**

$\int \frac{dx}{x(x^7+1)}$ is equal to

Solution:

$\int \frac{dx}{x(x^7+1)} = \int \frac{x^6}{x^7(x^7+1)} dx$

$= \frac{1}{7}\int \frac{dt}{t(t+1)}$ (put $x^7 = t$)

$= \frac{1}{7}\int \frac{(t+1-t)dt}{t(t+1)}$

$= \frac{1}{7}\int (\frac{1}{t} - \frac{1}{t+1})dt$

$= \frac{1}{7}(\log t - \log (t+1))$

$=\frac{1}{7}\log\left|\frac{t}{t+1}\right| + C$

$= \frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right| + C$

6) **Question:** The cost function for a certain commodity is $C(x) = 12 + 3x - \frac{1}{3}x^2$. Write down total cost,Fixed cost, variable cost and average cost when 3 units are produced.

Solution:

Total cost= C(x) when x=3

$= 12 + 3 \times 3 - \frac{1}{3} \times 9$ =18

Fixed cost= C(x) when x=0

=12

Variable cost= $(3x - \frac{1}{3}x^2)$ when x=3

$=3 \times 3 - 3 = 6$[or, total cost-fixed cost]

Average cost=$\frac{C(x)}{x}$ when x=3

$=\frac{12}{x} + 3 - \frac{1}{3}x$ =4+3-1=6

7) **Question:** The marginal revenue function of a commodity is MR = $7 - \frac{6}{(x+2)^2}$, find the revenue function. Also find the revenue obtained on selling 4 units of the product.

Solution: R(x) =$\int$ MR dx

$=\int \left(7 - \frac{6}{(x+2)^2}\right) dx$

$=7x + \frac{12}{(x+2)^3}$

∴R(x) when x=4

$= 7 \times 4 + \frac{12}{(4+2)^3} = \frac{505}{18}$

8) **Question:**

The marginal revenue of a product (in Rs) is given by MR= 7-4x+3x^2, where x is the number of units produced and sold. The revenue generated is Rs 20 when x=4.Find the demand function.

Solution:

$R(x) = p \times x$

Or, $R(x) = \int MR\, dx$

$= \int (7 - 4x + 3x^2)\, dx$

$= 7x - 2x^2 + x^3 + k$

When x = 4, R(x)=20

∴ 20 = 28-32+64+k

Or, k = - 40

$R(x) = 7x - 2x^2 + x^3 - 40$

$R(x) = p \times x$

Or, $7x - 2x^2 + x^3 - 40 = p \times x$

Or $p = 7 - 2x + x^2 - \frac{40}{x}$

∴ the demand function is $p = 7 - 2x + x^2 - \frac{40}{x}$

9) **Question:**

The price per unit of a commodity produced by a company is given by p=30-2x and 'x'is the quantity demanded. Find the revenue function R, the marginal revenue when 5 commodities are in demand (or produced)

Solution: The revenue function R is given by

$R(x) = p \times x$

=(30-2x)x

$= 30x - 2x^2$

∴ MR= $\frac{d}{dx}(R(x))$

=30-4x

The marginal revenue of producing 5 commodities is

$$\left.\frac{dR}{dx}\right]_{x=5} = 30 - 4 \times 5 = 10$$

10) **Question:** Find the intervals in which the function $f(x) = x^4 - \frac{4}{3}x^3$ is strictly increasing or strictly decreasing

Solution: $f'(x) = 4x^3 - 4x^2$

$= 4x^2(x - 1)$

$f'(x) > 0$ in $(1, \infty)$ ⇒ f is strictly increasing in $(1, \infty)$

$f'(x) < 0$ in $(-\infty, 1) - \{0\}$

Or f is strictly decreasing in $(-\infty, 0) \cup (0,1)$

11) **Question:** Find the absolute maximum and minimum value of the function value of f(x)= $x^3 - 3x$ in[0,2]

Solution: $f'(x) = 3x^2 - 3$, $f'(x) = 0$ ⇒ $x = -1,1$

f(1) = =-2,f(0)=0,f(2)=2

∴ absolute minimum value=-2

absolute maximum value=2

12) **Question:** Let the cost function of a firm be given by the equation C(x)=$300 - 10x^2 + \frac{1}{3}x^3$.Find the output at which the marginal cost MC is minimum

Solution: f(x)=MC=$\frac{d}{dx}(C)$=$300 - 20x + x^2$

$f'(x) = -20 + 2x$

$f'(x) = 0 \quad \Rightarrow x=10$

$f''(x) = 2$

$f''(10) > 0$

$\therefore$ f(x) or MC is minimum at x=10

13) **Question:** A manufacturer produces x pants per week at total cost of Rs.$(x^2 + 78x + 2500$.The price per unit is given by 8x=600-p, where 'p' is the price of each set. Find the maximum profit obtained ,where the profit function is given by P(x)=R(x)-C(x)

Solution: The revenue function R(x)= $p \times x$=(600-8x)x=$600x - 8x^2$

The profit function P(x)=R(x)-C(x)

$$= -9x^2 + 522\,x - 2500$$

$$P'(x) = 522 - 18\,x$$

$$P'(x) = 0 \quad \Rightarrow x=29$$

$$P''(x) = -18 \Rightarrow P''(29) = -18 < 0$$

The profit is maximum when 29 sets are produced per week and the maximum profit per week is P(29) = Rs.5069

14) **Question:** Evaluate $\int \frac{1}{(x-1)(x+3)}\,dx$

Solution: Let $\int \frac{1}{(x-1)(x+3)}dx = \frac{1}{4}\int \frac{(x+3)-(x-1)}{(x-1)(x+3)}dx$

$$=\frac{1}{4}\int\left(\frac{1}{(x-1)} - \frac{1}{(x+3)}\right)dx$$

$$=\frac{1}{4}\log|x-1| - \frac{1}{4}\log|x+3| + C$$

15) **Question:** Evaluate : $\int xe^{2x}dx$

Solution: $\int xe^{2x}dx = x\int e^{2x}dx - \int[\frac{d}{dx}(x)\int e^{2x}dx]dx + C$

$$= x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx + C \quad = x\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$$

16) **Question:** Evaluate : $\int_0^4 |x-2|dx$

Solution: Let I $= \int_0^4 |x-2|dx = \int_0^2 |x-2|dx + \int_2^4 |x-2|dx$

$$=\int_0^2 -(x-2)\,dx + \int_2^4 (x-2)\,dx$$

$$=-\left.\frac{(x-2)^2}{2}\right]_0^2 + \left.\frac{(x-2)^2}{2}\right]_2^4 \quad = 4 \text{ sq. units}$$

17) **Question:** Solve the differential equation $x^2(y+1)dx + y^2(x-1)dy = 0$

Solution: $x^2(y+1)dx + y^2(x-1)dy = 0$

$$\Rightarrow \frac{x^2}{x-1}dx + \frac{y^2}{y+1}dy = 0$$

$$\Rightarrow (x + 1 + \frac{1}{x-1})dx + (y - 1 + \frac{1}{y+1})dy = 0$$

$\therefore$ the solution is $\int\left(x + 1 + \frac{1}{x-1}\right)dx + \int\left(y - 1 + \frac{1}{y+1}\right)dy = 0$

Or, $\frac{x^2}{2} + x + \log|x-1| + \frac{y^2}{2} - y + \log|y+1| = C$

18) **Question:** Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that y=0 when x=1

Solution: $\frac{dy}{dx} = 1 + x + y + xy$

$=(1+x)(1+y)$

Or, $\frac{dy}{1+y} = (1+x)dx$

$\therefore$ the solution is $\int \frac{dy}{1+y} = \int (1+x)dx$

$\Rightarrow \log|1+y| = x + \frac{x^2}{2} + C$

19) Question: Find the differential equation representing the family of curve $y = cx + c^2$

Solution: $y = cx + c^2$...........(i)

Differentiating both sides w.r.t. x,

$\frac{dy}{dx} = c$

Substituting the value of c in (i), $y = \frac{dy}{dx}x + (\frac{dy}{dx})^2$ which is the required differential equation of the given curve.

20) Question: Evaluate: $\int [\frac{1}{\log x} - \frac{1}{(\log x)^2}]dx$

Solution: let log x= t $\therefore x = e^t$

$\therefore dx = e^t dt$

Hence the given integral $I = \int e^t(\frac{1}{t} - \frac{1}{t^2})\ dt$

Here $f(t) = \frac{1}{t}$ and $f'(t) = -\frac{1}{t^2}$

$\therefore$ by the rule $\int \int e^t(f(t) - f'(t))\ dt = e^t(f(t) + C,$

$I = e^t \frac{1}{t} + C \quad = x\frac{1}{\log x} + C$

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

1) Question: If $x^y = e^{x-y}$; prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Solution: $x^y = e^{x-y}$

Taking logarithm on both sides

y.log x = x-y

or, $y = \frac{x}{1+\log x}$

Differentiating both sides w.r.t x,

$$\frac{dy}{dx} = \frac{(1+\log x).1 - x.\frac{1}{x}}{(1+\log x)^2}$$

$$= \frac{\log x}{(1+\log x)^2}$$

2) Question: Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

Solution: $y^x + x^y + x^x = a^b$

or, $e^{\log y^x} + e^{\log x^y} + e^{\log x^x} = a^b$

or, $e^{x\log y} + e^{y\log x} + e^{x\log x} = a^b$

Differentiating both sides w.r.t x,

$$e^{xlogy} \times \frac{d}{dx}(xlogy) + e^{ylogx} \times \frac{d}{dx}(ylogx) + e^{xlogx} \times \frac{d}{dx}(xlogx) = 0$$

Or, $y^x(x.\frac{1}{y}\frac{dy}{dx} + \log y) + x^y(y.\frac{1}{x} + \log x \times \frac{dy}{dx}) + x^x(1 + \log x) = 0$

Or,$(x.y^{x-1} + x^y.\log x)\frac{dy}{dx} = -[y^x.\log y + yx^{y-1} + x^x(1 + \log x)]$

Or, $\frac{dy}{dx} = \frac{-[y^x.\log y + yx^{y-1} + x^x(1+\log x)]}{(x.y^{x-1} + x^y.\log x)}$

3) **Question:** The growth of bacteria (P)over a time (t)in a laboratory is given by P=1000-800 e^{-kt}.Prove that the rate of growth of bacteria decreases over time. What was the the population of bacteria at the initial stage in the laboratory

Solution: $G = \frac{dP}{dt} = 800\ ke^{-kt}$To show that the quantity G is decreasing over time

$\frac{dG}{dt} = -800\ k^2e^{-kt}$, which is negative for any time of t

Hence it is proved that G decreases over time t

The count of bacteria at the initial stage = P at t=0

= 200

4) **Question:** Evaluate: $\int_0^1 \frac{1}{(1+t)(2+t)}dt$

Solution: **Let** I=$\int_0^1 \frac{1}{(1+t)(2+t)}dt$

Using partial fractions

$$I = \int_0^1 \left\{\frac{1}{1+t} - \frac{1}{2+t}\right\}dt$$

$$= [\log|1+t| - \log|2+t|]_0^1$$

$$= [\log 2 - \log 3] - [\log 1 - \log 2] \qquad = \log\left(\frac{4}{3}\right)$$

5) **Question:**Find$\int \frac{x+2}{\sqrt{6+5x+x^2}}\ dx$

Solution: $\int \frac{\frac{1}{2}(2x+5) + -\frac{1}{2}}{\sqrt{x^2+5x+6}}\ dx = \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\ dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+5x+6}}\ dx$

$I = I_1 - I_2$……………………..(Put $x + 2 = A\frac{d}{dx}(x^2 + 5x + 6) + B \Rightarrow x + 2 = A(2x + 5) + B$

Now by comparing $A = \frac{1}{2}$& $B = -\frac{1}{2}$, Now $x + 2 = \frac{1}{2}(2x + 5) + -\frac{1}{2}$

$I = \int \frac{x+2}{\sqrt{x^2+5x+6}}\ dx =$ eq.1) ,

where $I_1 = \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\ dx$ and $I_2 = \frac{1}{2}\int \frac{1}{\sqrt{x^2+5x+6}}\ dx$

$NI_1 = \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\ dx$ Put $x^2 + 5x + 6 = t^2 \Rightarrow (2x + 5)\ dx = 2t\ dt$

$= \frac{1}{2}\int \frac{2t\,dt}{\sqrt{t^2}} = \int\ dt = t + C_1$

$\Rightarrow I_1 \qquad = \sqrt{x^2 + 5x + 6} + C_1$

$I_2 = \frac{1}{2}\int \frac{1}{\sqrt{x^2 + 5x+6}}\ dx \qquad \because x^2 + 5x + 6 = \{\, x^2 + 2.\frac{5}{2}x + (\frac{5}{2})^2 - (\frac{5}{2})^2 + 6)\}$

$= \{(x + \frac{5}{2})^{\ 2} - \left(\frac{1}{2}\right)^2\}$

$= \frac{1}{2}\int \frac{1}{\sqrt{(x+\frac{5}{2})^2 - (\frac{1}{2})^2}}\ dx = \frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + c_2$

Now put the value of I_1 and I_2 in equation (1), we get –

$I = \sqrt{x^2 + 5x + 6}\ + C_1 - \frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + C_2$

$I = \sqrt{x^2 + 5x + 6}\ - \frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + C$, where $C = C_1 + C$

6) Question:

(i) A chocolate bar sells for rs.20. What is the total revenue and average revenue by selling 30 bars?)

(ii) The demand function for TV set is p=20000-100x(rupees). Determine the total revenue and average revenue by selling 20 sets?

Solution:

(i) P=Rs 20

Total revenue by selling 30 bars = $30 \times 20 =$ Rs 600

Average revenue by selling 30 bars=$\frac{R}{x}$ =20

(ii) p=20000-100x per TV set

total revenue by selling x sets = px=20000x-100 x^2

hence the revenue by selling 20 sets=20000 $\times$ 20-100 $\times$ 400

=400000-40000

=Rs.360000

Hence average revenue by selling 20 sets=$\frac{360000}{20}$ =Rs.18000

7) **Question:** The surface of a spherical balloon is increasing at the rate of 2cm^2 / sec.Find the rate of change of its volume when its radius is 6cm.

Solution: Let r be the radius of the balloon at any tome t, S be the surface area and V be the volume at that instant, then

$S = 4\pi r^2$……..(i)

$V = \frac{4}{3}\pi r^3$……(ii)

Given $\frac{dS}{dt} = 2\text{cm}^2$ / sec.

Differentiating (i) w.r.t t

$$\frac{dS}{dt} = 4\pi \times 2r \times \frac{dr}{dt} = 8\pi r\frac{dr}{dt}$$

$2 = 8\pi r\frac{dr}{dt}$

Or, $\frac{dr}{dt} = \frac{1}{4\pi r}$

Differentiating (ii) w.r.t t,

$$\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt} = 4\pi r^2 \times \frac{1}{4\pi r}$$

$$\Rightarrow \frac{dV}{dt} = r$$

Hence when $r = 6, \frac{dV}{dt} = 6cm^3/sec$

8) **Question:**

The demand and supply functions under the pure market competitions are $p_d = 16 - x^2$ and $p_s = 2x^2 + 4$ respectively, where p is the price and x is the quantity of commodity. Using integrals find **Consumer's surplus**

Solution: Under pure competitions, $p_s = p_d$

$$16 - x^2 = 2x^2 + 4$$

$$\text{or}, x = 2, -2$$

x cannot be negative. Hence x=2

When $x_0 = 2, p_0 = 12$

Hence consumers surplus $= \int_0^2 p_d dx - p_0x_0$

$$= \int_0^2 (16 - x^2)dx - 24 = \frac{16}{3}$$

9) **Question:** Prove that the function $x^3 - 6x^2 + 18x + 5$ is increasing for all $x \in R$ and find its value where the rate of increase is least

Solution: Let $f(x) = x^3 - 6x^2 + 18x + 5$

$f'(x) = 3x^2 - 12x + 18$

$= 3[(x-2)^2 + 2] \geq 3(0+2)^2 \ \forall x \in R$

Or, $f'(x) > 0$ for all $x \in R$

∴ f is an increasing function for all $x \in R$

Now the rate of increase is ie, $f'(x)$ is least when $3[(x-2)^2 + 2]$ is the least

i.e. when $(x-2)^2 = 0$ ie when x=2

∴ the required value of the function at x=2 ie f(2)

∴ $f(2) = 2^3 - 6 \times 2^2 + 18 \times 2 + 5 = 25$

10) Question :

The demand and supply functions under the pure market competition are $p_d = 56 - x^2$ and $p_s = 8 + \frac{x^2}{3}$ respectively, where p is the price and x is the quantity of the commodity. Using integrals find **Producers surplus**

Solution

$$p_s = p_d$$
$$8 + \frac{x^2}{3} = 56 - x^2$$
$$\text{or}, x = -6, 6$$

x cannot be negative. Hence x=6

When $x_0 = 6, p_0 = 20$

Hence prodecer's surplus $= p_0 x_0 - \int_0^6 p_s dx$

$$= 6 \times 20 - \int_0^6 \left(8 + \frac{x^2}{3}\right) dx = 48 \text{ units}$$

11) **Question:** If $x = at^2, y = 2at$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is

Solution: $\frac{dx}{dt} = 2at$, $\frac{dy}{dt} = 2a$

$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{t}$

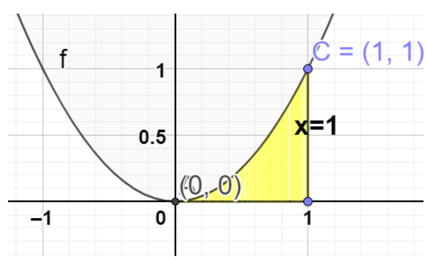
$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$

$= \frac{-1}{t^2} \times \frac{1}{2at} = \frac{-1}{2at^3}$

$\therefore \frac{d^2y}{dx^2}$ $at\ t = 1$ is $\frac{-1}{2a}$

12) **Question:** Evaluate $\int_0^1 x^2\, dx$ and hence show the region on the graph whose area it represents

Solution: $\int_0^1 x^2\, dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$ sq. units which is the area of the shaded region in the fig



LONG ANSWER TYPE QUESTIONS

1) **Question:** A company produces certain commodity with rupees 2400 as fixed cost The variable cost is estimated to be 25% of the total revenue received on selling the product at a rate of rupees 8 per unit. Find the following

i. Cost function
ii. Revenue function
iii. Breakeven point
iv Profit function

Solution:

Let 'x' units of product be produced and sold. As selling price of one unit is Rs. 8,

Total revenue on 'x' units= Rs.8x

i. Cost function = fixed cost+25% of 8x

$= 24000 + \frac{25}{100} \times 8x$

$= 24000 + 2x$

ii. Revenue function =8x

iii. Breakeven point

R(x)=C(x)

$24000 + 2x = 8x \Rightarrow x = 4000$

Profit function P(x) = R(x)-C(x)

$= 8x - (24000 + 2x)$

$= 6x - 24000$

2) **Question:** In a bank, principal increases continuously at the rate of 5% per year.in how many years Rs. 1000 double itself?

Solution: Let P be the principal at any time t(years).

Then $\frac{dP}{dt} = 5\%$ of P

$\Rightarrow \frac{dP}{dt} = \frac{5}{100} \times P$

$\therefore \frac{dP}{P} = \frac{1}{20} dt$

or, $\int \frac{dP}{P} = \int \frac{1}{20} dt$

or, $\log P = \frac{1}{20} t + C$…………………(i)

initially, when t=0, P=1000

$\therefore$ log 1000=C

substituting in (i)

$\log P = \frac{1}{20} t + \log 1000$ …………..(ii)

When P double itself, P=2000

Putting in (ii)

$\log 2000 = \frac{1}{20} t + \log 1000$

or, $\log 2000 - \log 1000 = \frac{1}{20} t$

applying laws of logarithm

$\log \frac{2000}{1000} = \frac{1}{20} t$

or, $\log 2 = \frac{1}{20} t$

or, $t = 20 \times \log_e 2$

the amount will double in 20 log 2 years = $20 \times 0.69312 = 13.86\ years$

3) **Question:**

The production manager of a company plants to include 180 square cm of actual printed matter in each page of a book under production. Each page should have a 2.5 cm wide margin along the top and bottom and 2 cm wide margin along the sides. What are the most economical dimensions of each printed page.

Solution: Let x and y be the dimension of the printed pages

$\therefore x \times y = 180$ ……………..(i)

A =Area of the page = (x+4)(y+5)

$= x \times y + 5x + 4y + 20$

$= 180+5x+4(\frac{180}{x}) +20$

$= 200 + 5x + \frac{720}{x}$

For most economical dimension, A to be minimised

$\therefore \frac{dA}{dx} = 0 \Rightarrow 5 - \frac{720}{x^2} = 0 \quad \Rightarrow x = 12$

$\frac{d^2A}{dx^2} = \frac{1440}{x^3}$

$\left.\frac{d^2A}{dx^2}\right|_{x=12} > 0 \quad \therefore$ A is minimum

$x = 12 \; and \; y = \frac{180}{x} = 15$

Hence the most economical dimensions are x + 4= 16 cm and y + 5 = 20 cm

4) **Question:** Form the differential equation of the family of circles of constant radius r
Solution: Let the family of circles be $(x-h)^2 + (y-k)^2 = r^2$…………..(i)
where h, k are arbitrary constants
Differentiating (i) twice w.r.t x,

$2(x-h) + 2(y-k).\frac{dy}{dx} = 0$

or, $(x-h) + (y-k).\frac{dy}{dx} = 0$……….(ii)

and $1 + (y-k)\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \left(\frac{dy}{dx} - 0\right) = 0$

or, $y - k = -\frac{1+(\frac{dy}{dx})^2}{\frac{d^2y}{dx^2}}$…………(iii)

Substituting the value of y-k from (iii) in (ii),

$x - h = -\left(-\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right) \times \frac{dy}{dx} = \frac{\frac{dy}{dx}(1+\left(\frac{dy}{dx}\right)^2)}{\frac{d^2y}{dx^2}}$……….(iv)

From (i),(iii) and (iv) ,$\frac{\left(\frac{dy}{dx}\right)^2(1+\left(\frac{dy}{dx}\right)^2)^2}{(\frac{d^2y}{dx^2})^2} + \frac{(1+\left(\frac{dy}{dx}\right)^2)^2}{(\frac{d^2y}{dx^2})^2} = r^2$

Or, $(1 + \left(\frac{dy}{dx}\right)^2)^3 = r^2(\frac{d^2y}{dx^2})^2$is the required differential equation

CASE BASED QUESTIONS (04 MARKS QUUESTIONS)

1) **Question:**A student Mohan is running on a playground along the curve $y = x^2 + 7$.another student Shilpi standing at point(3,7)on playground wants to hit Mohan by paper ball when Mohan is nearest to Shilpi

Based on the above information, answer the following

(i) Let at any instant while running along the curve $y = x^2 + 7$, Mohan's position be at (x,y). Find the expression for the distance D between Mohan and Shilpi in terms of x

(ii) Find the critical point(s)of the distance function

(iii) (a)What is the distance between Mohan and Shilpi when they are at the least distance from each other

OR

(iii) (b) Find the position of Mohan when he is closest to Shilpi

Solution:

(i) For all values of x, $y = x^2 + 7$

∴Mohan's position at any point is $(x, x^2 + 7)$

The measure of the distance between Mohan and shilpi is

$$D = \sqrt{[(x-3)^2 + (x^2 + 7 - 7)^2]} = \sqrt{(x-3)^2 + x^4}$$

(ii) Let S =$D^2 = (x-3)^2 + x^4$

$\frac{dS}{dx} = 0 \Rightarrow 4x^3 + 2x - 6 = 0 \quad \Rightarrow x = 1$

Critical point is x=1

(iii) (a) $\frac{d^2S}{dx^2} = 8x^2 + 2$ and clearly $\frac{d^2S}{dx^2} = 8x^2 + 2 > 0$ at $x = 1$

∴ Value of x for which D will be minimum is 1

For $x = 1, y = 8$

∴ The required distance = $\sqrt{(1-3)^2 + 1^4} = \sqrt{5}$ units

(iii) (b) $\frac{d^2S}{dx^2} = 8x^2 + 2$ and clearly $\frac{d^2S}{dx^2} = 8x^2 + 2 > 0$ at $x = 1$

∴ Value of x for which D will be minimum is 1

For $x = 1, y = 8$

Hence the required position of Mohan is (1,8) when he is closest to Shilpi

2) **Question** A company notes that higher sales of a particular item, which it produced, is achieved by lowering the price charged. As a result, the total revenue from the sales at first rises as the number of units sold increases, reaches the highest point, and then falls off. The pattern of revenue is described by the relation $y = 40,00,000 - (x - 2000)^2$, where y is the total revenue and x is the number of units sold.

Based on the ever information, answer the following questions.

(i) Find what number of units sold maximizes total revenue

(ii) What is the amount of this maximum revenue?

(iii) What would be the total revenue if 2500 units were sold

Solution:

(i) $y = 40,00,000 - (x - 2000)^2$

$$\frac{dy}{dx} = -2(x - 2000)$$

$\frac{dy}{dx} = 0$ when $x = 2000$

$\frac{d^2y}{dx^2} = -2 < 0$. Hence y is maximum at x=2000

(ii) maximum revenue= $40,00,000 - (2000 - 2000)^2$= 40,00,000 (iii)total revenue= $40,00,000 - (2500 - 2000)^2 = 37,50,000$

3) **Question**: A manufacturing company manufactures toys. The company observed the following costs at different production levels

Number of Toys Manufactured	Cost of Raw Material (₹)	Cost of Production Supply (₹)	Cost of Freight (₹)	Property Tax (₹)	Salaries (₹)
100	800	2000	1000	5000	20000
150	1200	3000	1500	5000	20000
200	1600	4000	2000	5000	20000
250	2000	5000	2500	5000	20000
300	2400	6000	3000	5000	20000

(i) Find the fixed cost for the production of x units of toys

(ii) Find the raw material cost per unit for the production of toys

(iii) (a) Find the revenue function R(x) of toys, if the company observes the price 'p' per unit of item sold as p=5000-x, where 'x' is the number of units sold. Also find the revenue obtained on selling 10 units

OR

(iii) (b) Find the marginal revenue function MR of toys, if the company observes the price 'p'per unit of item sold as as p=5000-x, where 'x' is the number of units sold.

Solution:

(i) Fixed cost= 5000(property tax) +20000(salary)

(ii) Raw Material: ₹8 per unit (difference of ₹400 for every 50 units)

(iii) (a)Revenue Function R(x):

Given: Price per unit = 5000 - 10x

Revenue = Price × Quantity = (5000 - 10x) × x = $5000x - 10x^2$

$R(x) = 5000x - 10x^2$

R(10= 50000-1000= Rs.49,000

OR

(iii) (b) Marginal Revenue (MR):

Revenue = Price × Quantity = (5000 - 10x) × x = $5000x - 10x^2$

MR = dR/dx = derivative of revenue function R(x)

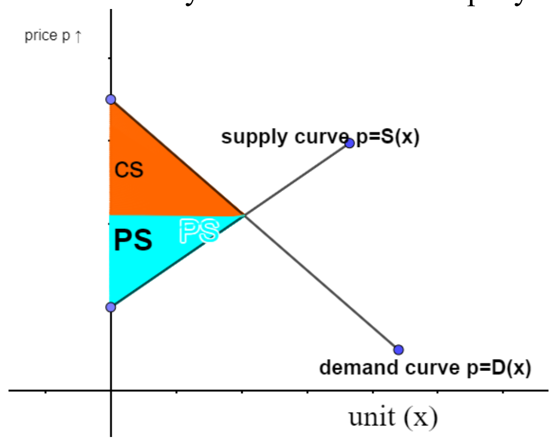
$R(x) = 5000x - 10x^2$

Or, dR/dx = 5000 - 20x

MR(x) = 5000 - 20x

HIGHER ORDER THINKING SKILLS

1) **Question**: Supply and demand curves of a tyre manufacturer company is as follows



The above graph showing the demand and supply curves of a tyre manufacturing company which are linear. 'ABC' tyre manufacturer sold 25 units every month when the price of a tyre was Rupees20000 for unit and ABC tyre manufacturer sold 125 unit every month when the price dropped to rupees 15000 per unit .When the price was rs 25000 per unit 180 tyres were available per month for sale and when the price was only rupees 15000 per unit 80 tyres remains find the demand function also find the consumer surplus if the supply function is given by S(x)= 100x + 7000

Solution

Consider the demand function p = D(x)= ax + b ……….(1)
when x = 25, p = 20000
from equation(1), we have 20000 = 25a + b ………(2)
and when x = 125, then p = 15000
from equation (1) we have 15000 =125 a + b……….(3)
on solving equation (2) and (3) we get a = -50 and b = 21250
Therefore, demand function p = -50 x + 21250
For equilibrium point,$D(x_0) = S(x_0)$
Or, $-50x_0 + 21250 = 100x_0 + 7000 \Rightarrow x_0 = 95$
On putting value of $x_0$in demand function and supply function, $p_0 = 16500$

∴ consumer Surplus (CS) $= \int_0^{x_0} D(x) - p_0 x_0$

$$= \int_0^{95} (-50x + 21250)dx - 16500 \times 95$$

$$= -50\frac{x^2}{2} + 2150x\Big]_0^{95} - 1567500 \quad = \text{Rs.}22625$$

2) The supply schedule of tomatoes is given in the table below which follows a linear relationship between the price and quantity.

Pric/kg	Quantity supplied (kg)
25	800
20	700
15	600
10	500
5	400

The demand schedule for tomatoes in Karnataka is given below and there is a linear relationship between the price and demand

Pric/kg	Quantity demanded (kg)
25	200
20	400
15	600
10	800
5	1000

Based on the above information answer the following question

(i) Find the equation of demand curve

(ii) Find the equation of supply curve

(iii) (a)Find the consumers surplus

OR

(iii) (b) Find the producers surplus

Solution

(i) Demand curve

Let p = ax+b……….(*) (as the curve is linear), where p is the price and x is the quantity demanded

When p=25, x=200 ⇒ 25=200a+b ……..(1)

When p= 20,x=400 ⇒ 20 = 400a +b…..(2)

(2)-(1) ⇒ 5=-200a and hence $a = -\frac{1}{40}$ and b=25-200a=30

Substituting the value of a and b in (*), the demand function is $p = 30 - \frac{x}{40}$

(ii) Supply curve

Let p = ax+b……….(*) (as the curve is linear), where p is the price and x is the quantity supplied

When p=25, x=800 ⇒ 25=800a+b ……..(1)

When p= 20, x=700 ⇒ 20 = 700a +b…..(2)

(2)-(1) ⇒ 5= 100a and hence $a = \frac{1}{20}$ and b=25-800a=-15

Substituting the value of a and b in (*), the supply function is $p = -15 + \frac{x}{20}$

(iii) Equilibrium point: quantity supplied= quantity demanded

From the data, quantity supplied=600kg and quantity demanded=600 kg

Hence equilibrium price is Rs,15 and equilibrium quantity is 600 kg

Hence $x_0 = 600, p_0 = 15$

Consumer's Surplus (CS) = $\int_0^{x_0} D(x)dx - p_0x_0 = \int_0^{600}(30 - \frac{x}{40})dx - 15 \times 600$

$= 30x - \frac{x^2}{80}\Big]_0^{600} - 9000$=18000-4500-9000 =4500

(iv) Producer's surplus- $= p_0x_0 - \int_0^{x_0} S(x)dx = 15 \times 600 - \int_0^{600}(-15 + \frac{x}{20})dx$

$= 9000 - \left[-15x + \frac{x^2}{40}\right]_0^{600} = 9000 + 9000 - \frac{360000}{40} = 9000$

EXERCISE
MULTIPLE CHOICE QUESTIONS

1.$\int \frac{1}{\mathrm{x+x\,logx}}$ dx is equal to

a) 1+logx+C b)x+log x+C c)xlog(1+logx)+C d)log(1+logx)+C

2.If $\mathrm{y = e^{-2x}}$, then $\frac{\mathrm{d^3y}}{\mathrm{dx^3}}$ is equal to

a)$2\mathrm{e}^{-2x}$ b) e^{-4x} c) $4\mathrm{e}^{-4x}$ d) $-8\mathrm{e}^{-2x}$

3.The function f(x)=$\mathrm{e^x - x + 1}$ is

a) increasing in (0,1) b) decreasing in (0,1)

c) increasing in $\left(0,\frac{1}{2}\right)$and decreasing in $\left(\frac{1}{2},1\right)$

d) decreasing in $\left(0,\frac{1}{2}\right)$and increasing in $\left(\frac{1}{2},1\right)$

4. The order and degree of the differential equation $\mathrm{ydx + xlog(\frac{y}{x})dy - 2xdy = 0}$ are respectively

a) 1,1 b)1,2 c)2,1 d) 1, not defined

5. If x+y =8,then the maximum value of xy is :

a)12 b)16 c)20 d)24

6.General solution of differential equation : y logy dx-x dy =0 is

a) $\mathrm{y = log|cx|}$ b) $\mathrm{y = e^{|Cx|}}$ c) $\mathrm{y = e^{-cx}}$ d) $\mathrm{logy = |C + x|}$

7. if the marginal revenue function of a commodity is MR=$2x - 9x^2$, then the revenue function is

a) $2x^2 - 9x^3$ b) 2-18x c) $x^2 - 3x^3$ d) $18 + x^2 - 3x^3$

8 . If the demand function for a commodity is p= $20 - 2x - x^2$ and the market demand is 3 units, then the consumer's surplus is

a) 27 b) 38 c) 42 d) 47

9. The degree of the differential equation $[1 + (\frac{dy}{dx})^2]^{3/2} = \frac{d^2y}{dx^2}$ is

a) 4 b) $\frac{3}{2}$ c) not defined d) 2

10.the integrating factor of the differential equation $\mathrm{(x\log x)\frac{dy}{dx} + y = 2\,logx}$ is

a) log(log x) b) log x c) e^x d) x

ASSERTION - REASON BASED QUESTIONS

1. Assertion(A): If the value of $\int_0^a 3x^2dx = 8$ then the value of 'a' is 2

Reason(R): $\int x^n dx = \frac{x^{n+1}}{n+1}$+c

2. Assertion(A): $\int e^{7-4x}dx$ = a e^{7-4x} +c then the value of a is$\frac{-1}{4}$

Reason(R): $\int f(ax+b)dx = \frac{F(ax+b)}{a}$+c

3. if x=at^2 and y= 2at

Assertion(A): $\frac{dy}{dx} = \frac{1}{t}$

Reason(R): if y= f(t) and x =g(t),then $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\frac{\mathrm{dy}}{\mathrm{dt}}}{\frac{\mathrm{dx}}{\mathrm{dt}}}$

4. Assertion(A):$\int \frac{x-3}{(x-1)^3}dx = \frac{e^x}{(x-1)^2} + C$

Reason(R): $\int \mathrm{e^x(f(x) + f'(x))dx = e^xf(x) + C}$

5. Assertion (A): The area of the region bounded by the curve $y = x^3$,x = -1 and x=1 is equal to $\frac{1}{2}$ sq.units

Reason (R): $\int_{-1}^{1} x^3 dx = 0$

6. Assertion (A): A firm maximises its profit when marginal revenue equals marginal cost

Reason (R) : At MR=MC,the firm is producing at the equilibrium output level

VERY SHORT ANSWER TYPE QUESTIONS

1. If the function$x^4 - 62x^2 + ax + 9, x\epsilon[0,2]$ attains its maximum value at x=1, find the value of a
2. Find the interval in which the function given by f(x)=$f(x) = 4x^3 - 6x^2 - 72x + 30$ is decreasing
3. Find two positive integers whose sum is 15 and the sum of whose square is minimum
4. Show that among the rectangle of given perimeter, the square has the greatest area.
5. The marginal cost function of producing x units of a product is given by $MC = \frac{x}{\sqrt{2500+x^2}}$,find the total cost function and average cost function, if the fixed cost is Rs.1000

SHORT ANSWER TYPE QUESTIONS

1. Evaluate:$\int_0^1 \frac{e^{-x}}{1+e^x} dx$

2. Find the differential equation of all the circles in the first quadrant which touches both the coordinate axes

3. Evaluate: $\int_{\log 2}^{\log 4} 2^x dx$

4. Evaluate: $\int \frac{\mathbf{1}}{(e^x+\mathbf{1})^2} dx$

5. The supply function for a commodity is $p = x^2 + 4x + 5$ where x denotes supply.Find the producer's surplus when the price is 10

6. The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25,000 in the year 2004, what will be the population of the village in 2009?

LONG ANSWER TYPE QUESTIONS

1. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit max light through the whole opening.

2. The price of selling one unit of a product when x units are demanded is given by the equation $p = 4000 - 2x$. The fixed costs of product are Rs. 20000 and Rs. 1484 per unit as the cost of production.Find the level of sales at which the company can expect to cover its costs.

3. For a monopolist's product, the demand function is $p = \frac{50}{\sqrt{x}}$ and average cost function AC=$0.5 + \frac{2000}{x}$.Find the profit maximising level of output. At this level, show that the marginal revenue and marginal cost are equal.

4. Determine for what values of x, the function f(x)=$x^4 - \frac{x^3}{3}$ is strictly increasing or stritly decreasing

CASE BASED QUESTIONS

1. A bakery estimates that the daily profit P(x) in rupees from selling x cakes is given by P(x) =$-2x^2 + 160\,x - 1000$

(i) How many cakes should the bakery sell each day to maximize profit?

(ii) At what number of cakes sold will the profit be zero?

2. The demand function for potato is D(p)=60-2p and the supply function S(p)=4p, where p is the price per kg

(i) Find the equilibrium price and quantity

(ii) Find the maximum price consumers are willing to pay

(iii) a) Calculate the consumer surplus

(iii) b) Calculate the producer's surplus

HIGHER ORDER THINKING SKILLS

1. A firm has the following total cost and demand functions

$$C(x) = \frac{x^3}{3} - 7x^2 + 111x + 50 \; and \; x = 100 - p$$

(i) Find the total revenue function in terms of x

(ii) Find the total profit function P in terms of x

(iii) (a) Find the profit maximising level of output of x

OR

(iii) (b) What is the maximum profit,taking repee as a unit of money?

2. Find the differential equation of all the circles in the first quadrant, which touches both the axes

3. The demand function p for maximising a profit monopolist is given by $p = 274 - x^2$,while the marginal cost is 4+3x, for x units of commodity. Using integrals, find the consumer surplus

ANSWERS

MULTIPLE CHOICE QUESTIONS

1. d) log(1+logx)+C
2. d) $-8e^{-2x}$
3. d) decreasing in $\left(0, \frac{1}{2}\right)$and increasing in $\left(\frac{1}{2}, 1\right)$
4. a)1,1
5. b) 16
6. b) $e^{|Cx|}$
7. c
8. a
9. d
10. b

ASSERTION - REASON BASED QUESTIONS

1. (b)
2. (a)
3. (a)
4. (a)
5. (b)
6. (a)

VERY SHORT ANSWER TYPE QUESTIONS

1. a=52

2.[-2,3]
3. 9,6
4.
5.C(x)=$\sqrt{x^2+2500}+950$,Average cost=$\frac{\sqrt{x^2+2500}+950}{x}$

SHORT ANSWER TYPE UESTIONS

1. $\log(\frac{1+e}{2e})-\frac{1}{e}+1$
2. $(\frac{dy}{dx})^2(x^2-2xy)-2xy\frac{dy}{dx}+y^2-2xy=0$
3.$\frac{2^{\log 4}-2^{\log 2}}{\log 2}$
4.$\log\frac{e^x}{e^x+1}+\frac{1}{e^x+1}+C$
5.$\frac{8}{3}$
6. 31250

LONG ANSWER TYPE QUESTIONS

1. length $=\frac{20}{\pi+4}$ m, breadth $=\frac{10}{\pi+4}$ m
2. Breakpoint points are x= 8, 1250.Hence, the company must produce at least 8 units to cover its cost.
3. P(x) is maximum when x=2500, MR=MC when x =2500
4. proof
5.f is strictly increasing in $\left(\frac{1}{4},\infty\right)$ and strictly decreasing in $(-\infty,0)\cup(0,\frac{1}{4})$

CASE BASED QUESTIONS

1. (i) profit maximum when 60 units are sold (ii) x=9 0r x =111
2. i) 10,40 ii)price=Rs.20 iii) a)consumers surplus=400 iii b) producers surplus=200

HIGHER ORDER THINKING SKILLS

1. (i) R(x) = $p\times x=100x-x^2$
   (ii)P(x)=$-\frac{x^3}{3}+6x^2-11x-50$
   (iii) (a) P is maximum when x=11
   (iii) (b) Maximum profit =111.33 when x=11
2. $(\frac{dy}{dx})^2(x^2-2xy)-2xy\frac{dy}{dx}+y^2-2xy=0$
3. 486

UNIT-4

PROBABILITY DISTRIBUTIONS

Gist/Summary of the lesson:

Random variables and its Probability Distributions,
Mathematical Expectations,
Variance and Standard Deviation of a random variable,
Bernoulli Trials and Binomial Distribution,
Condition of Poisson Distribution.
Normal Distribution.

Definitions and Formulae:

RANDOM VARIABLES:

A random variable is a function whose domain is the sample space of a random experiment and whose range is a subset of real numbers

Example: Throwing of two coins. Sample space, S = {HH, HT, TH, TT}

Random Variable, X = No. of heads, X = 0,1,2

Probability Distribution:

X	x_1	x_2	x_3	……………….	x_n
P(X=x)	p_1	p_2	p_3	……………….	p_n

Where, P(X=x_i) = p_i and $\sum_{i=1}^{n} p_i = 1$

Mean & Variance of Probability Distribution:

Mean, $\mu = \frac{\sum p_i x_i}{\sum p_i}$ but since $\sum p_i = 1$

Mean, $\mu = \sum p_i x_i = p_1 x_1 + p_2 x_2 + \dots\dots\dots\dots + p_n x_n$

Mean is also called average or expected value or expectation, denoted by as E(X).

Variance

Var (X) or σ_x^2 or σ^2

Standard deviation, $\sigma = \sqrt{\sum p_i x_i^2 - \mu^2}$

$\sigma = \sqrt{E(X^2) - (E(X))^2}$

BINOMIAL EXPERMENT:

A random experiment with exactly two possible results is called a Binomial or Bernoulli trial. The two results are called success and failure in probability theory.

Bernoulli trials satisfy the following conditions:

(i) There should be a finite number of trials.
(ii) The trials should be independent.
(iii) Each trials has exactly two outcomes: success or failure.
(iv) The probability of success remains the same in each trial.

Probability of 'r' success in 'n' Bernoulli trial is given by :

$P(r-success)={}^nC_r p^r q^{n-r} = \frac{n!}{r!(n-r)!} p^r q^{n-r}$

Where n= number of trials

r = number of successful trials = 0, 1, 2, …….., n

p = probability of a success in a trial

q = probability of a failure in a trial

And, p + q = 1

Binomial Distribution:

$X = r_i$	0	1	2	…	r	…	n
$P(r_i) = p_i$	${}^nC_0 p^0 q^{n-0}$	${}^nC_1 p^1 q^{n-1}$	${}^nC_2 p^2 q^{n-2}$	…	${}^nC_n p^n q^{n-n}$	…	${}^nC_n p^n q^{n-n}$

Binomial distribution is denoted by B(n, p)

In a binomial distribution having 'n' number of Bernoulli trials

Mean = np   and Variance = npq

Standard Deviation = $\sqrt{npq}$

Where p is the probability of success and q is the probability of failure.

POISSONDISTRIBUTION:

The Poisson distribution is the limiting form of Binomial distribution when number of trials, n is very large $(n \to \infty)$ and probability of success is small $(p \to 0)$such that np always remains finite. $(np = \lambda)$

Probability of occurrence of 'k' number of events:

$P(X = k) = \frac{\lambda^k . e^{-\lambda}}{k!}$,   where e is Euler's number & e = 2.71828…

NORMAL DISTRIBUTION:

The normal distribution is a type of continuous probability distribution that is symmetric and bell-shaped. For normal distribution Mean = Median = mode

Probability Density Function (PDF):

$$\boldsymbol{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

$Standard\ Normal\ variate,$ $Z = \frac{x-\mu}{\sigma}$

MULTIPLE CHOICE QUESTIONS

1) A pair of dice is thrown two times. If X represents the number of doublets obtained, then the expectation of X is

a) $\frac{1}{6}$ b) 1   c) $\frac{1}{3}$ d) $\frac{11}{36}$

Solution:

Total number of trials, n = 2

Total number of outcomes in one trial = 6×6 = 36

Doublets are {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)} $\therefore$ p = $\frac{6}{36} = \frac{1}{6}$

Since it is a binomial distribution, therefore, E(X) = np= $2 \times \frac{1}{6} = \frac{1}{3}$

Answer: c

2) If a random variable X has the probability distribution

$$P(X = x) = \begin{cases} k, & if\ x = 0 \\ 2k, & if\ x = 1\ or\ 2 \\ 0, & otherwise \end{cases}$$

then the value of k is

a) $\frac{1}{3}$ b) $\frac{1}{5}$ c) $\frac{1}{6}$ d) $\frac{1}{4}$

Solution:

Since $\sum p_i = 1$, therefore $P(X=0)+P(X=1)+P(X=2)+\cdots=1$

$$k+2k+2k+0=1, \qquad 5k=1, \qquad k=\frac{1}{5}$$

Answer: b

3) Finding the probability of getting exactly two heads in four tosses of a fair coin.

a) $\frac{1}{8}$ b) $\frac{5}{8}$ c) $\frac{3}{8}$ d) $\frac{1}{4}$

Solution:

Number of trials: n=4

Probability of success (getting a head) in a single trial: p= ½,

Probability of failure (getting a tail) in a single trial: q=1-p=½,

Number of successes (getting heads) required: k=2

Now $P(X=k)={}^nC_r p^r q^{n-r} = {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

Answer: c

4) Which of the following is false about normal distribution

a) Normal distribution is applied for discrete random distribution.

b) The shape of the normal distribution curve is bell shaped.

c) The area under a standard normal curve is 1

d) The standard normal curve is symmetric about the value 0.

Answer: a

5) If mean and variance of a normal distribution are 5 and 4 respectively, then the value of n is.

a) 10 b) 15 c) 20 d) 25

Solution:

Mean = np = 5

Probability of success (getting a head) in a single trial: p= ½,

Probability of failure (getting a tail) in a single trial: q=1-p= ½,

Number of successes (getting heads) required: k=2

Now $P(X=k)={}^nC_r p^r q^{n-r} = {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

Answer: d

6) If random variable X represents the number of heads when a coin tossed twice, then mathematical expectation of X is

a) 0 b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) 1

Solution:

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

So, $E(X)=0\times\frac{1}{4}+1\times\frac{1}{2}+2\times\frac{1}{4}=1$

Answer: d

7) If X is a Poisson variate such that 3P(X = 2) = 2P(X = 1), then the mean of the distribution is equal to

a) $\frac{4}{3}$ b) $\frac{3}{4}$ c) $-\frac{4}{3}$ d) $-\frac{3}{4}$

Solution:

$\because$ 3P(X = 2) = 2P(X = 1)

$\therefore \frac{3e^{-\lambda}\lambda^2}{2!} = \frac{2e^{-\lambda}\lambda^1}{1!} \Rightarrow \lambda = \frac{4}{3}$

Answer: a

8) If the varance of a Poisson distribution is 2, then P(X = 2) is

a) $\frac{2}{e^2}$ b) $2e^2$ c) $\frac{4}{e^2}$ d) $4e^2$

Solution:

$\because$ variance = 2 = λ, then

$\therefore P(X = 2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-2}2^2}{2!} = 2e^{-2} = \frac{2}{e^2}$

Answer: a

9) An observed set of the population that has been selected for analysis is called

a) a sample b) a process c) a forecast d) a parameter

Answer: a

10) A random variable 'X' has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	2k	2k	3k	k^2	$2k^2$	$7k^2$	2k

The value of k is

a) -1 b) $-\frac{1}{10}$ c) 1 d) $\frac{1}{10}$

Solution:

$\because \sum p_i = 1$

$\therefore 0 + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + 2k = 1$

$10k^2 + 9k - 1 = 0$

$(k + 1)(10k - 1) = 0 \Rightarrow k = -1$ (Not possible), therefore $k = \frac{1}{10}$

Answer: d

11) A box contains 100 bulbs of which 10 are defective. The probability that out of a sample of 5 bulbs drawn one by one with replacement none is defective is

a) $\left(\frac{1}{2}\right)^5$ b) $\frac{9}{10}$ c) $\left(\frac{9}{10}\right)^5$ d) $\left(\frac{1}{10}\right)^5$

Solution:

$\because$ No of trials, n = 5 & probability of non-defective bulb in each trial, $p = \frac{90}{100} = \frac{9}{10}$

$\therefore$Required probability = ${}^nC_r p^r q^{n-r} = {}^5C_5 p^5 q^0 = \left(\frac{9}{10}\right)^5$

Answer: c

12) If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then the value of its parameter p is

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

Solution:

For binomial distribution, Mean = np = 12

Standard Deviation = $\sqrt{npq} = 2 \Rightarrow npq = 4 \Rightarrow 12q = 4 \Rightarrow q = \frac{1}{3}$

$\therefore$ p = 1 – q = $\frac{2}{3}$

Answer: d

13) There are 50 telephone lines in an exchange. The probability that any one of them will be busy is 0.1. The probability that all the lines are busy is

a) $\frac{(5)^0 e^{-5}}{0!}$ b) $1-\frac{(5)^0 e^{-5}}{0!}$ c) $\frac{(5)^{50} e^{-5}}{50!}$ d) $1-\frac{(5)^{50} e^{-5}}{50!}$

Solution:

n = 50, p = 0.1, therefore np = 5 = λ

Using Poisson distribution: P(X = k) = $\frac{\lambda^k e^{-k}}{k!} = \frac{5^{50} e^{-50}}{50!}$

Answer: c

14) If a random variable X has the Poisson distribution with mean 2. Then $P(X > 1.5)$ is

a) $2e^{-2}$ b) $3e^{-2}$ c) $1-2e^{-2}$ d) $1-3e^{-2}$

Solution:

λ = 2, $P(X > 1.5) = 1 - P(X \leq 1.5)$

Answer: d

15) If 'm' is the mean of Poisson distribution, then its standard deviation is given by

a) $\sqrt{m}$ b) m^2 c) m d) $\frac{m}{2}$

Solution:

Mean = m = Variance

Answer: a

16) The normal distribution curve is symmetric about

a) $X = \mu$ b) $X = \sigma$ c) $X = \frac{\mu}{\sigma}$ d) $X = \frac{\sigma}{\mu}$

Answer: a

17) The mean of the probability distribution of the number of doublets in 4 throws of a pair of dice is

a) 1 b) $\frac{2}{3}$ c) $1\frac{3}{5}$ d) $2\frac{2}{3}$

Answer: b

18) Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. Then the possible values of X are

a) 0, 1, 3, 5 b) 0, 2, 4, 6 c) 0, 2, 5, 6 d) 1, 3, 4, 5

Answer: b

19) One hundred identical coins each with probability p showing up heads are tossed once. If $0 < p < 1$ and the probability of heads on 50 coins is equal to that of heads showing on 51 coins, then the value of p is :

a) $\frac{1}{2}$ b) $\frac{49}{101}$ c) $\frac{50}{101}$ d) $\frac{51}{101}$

Solution:

Probability of heads on 50 coins = Probability of heads showing on 51 coins

$\therefore$ P(X = 50 heads) = P(X = 51 heads)

${}^{100}C_{50}\,(p)^{50}(q)^{50} = {}^{100}C_{51}\,(p)^{51}(q)^{49}$

${}^{100}C_{50}\,q = {}^{100}C_{51}\,p$ $\Rightarrow \frac{p}{p-1} = \frac{51}{50}$ $\Rightarrow p = \frac{51}{50}$

Answer: d

20) The probability that a bomb dropped from a plane strikes the target is $\frac{4}{5}$. What is the probability that out of 6 bombs dropped, exactly 2 bombs strike the target ?

a) $2\left(\frac{4}{5}\right)^5$ b) $1-2\left(\frac{4}{5}\right)^5$ c) $\frac{48}{5^5}$ d) $\frac{64}{5^6}$

Solution:

n = 6, p = $\frac{4}{5}$, q = $\frac{1}{5}$ and r = 2

Using formula: P(X = r success) = $^nC_r(p)^r(q)^{n-r}$, where q = p – 1

∴ P(X = 2 success) = $^6C_2\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^4 = \frac{48}{5^5}$

Answer: c

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
C) Assertion (A) is true but Reason(R) is false.
D) Assertion (A) is false but Reason(R) is true.

1) **Assertion (A):***In a probability distribution table, the sum of all probabilities is always 1.*
Reason(R) :*This is because the total probability space includes all possible outcomes*
Answer: A
Solution: The total probability must be 1, since it covers all possible outcomes. R clearly explains why A is true.

2) **Assertion (A):** *In A random variable can only take integer values.*
Reason(R) :*A random variable represents outcomes of a random experiment.*
Answer: C
Solution:A is false, but R is true.
Random variables can be either discrete (integers) or continuous (real numbers). Hence A is false. R is correct because random variables represent outcomes of random experiments.

3) Assertion (A): In a binomial distribution, trials are dependent.
Reason (R): The probability of success changes after each trial.
Answer: D. Both A and R are false.
Solution: In binomial distribution, trials are independent, and the probability of success remains constant.

4) Assertion (A): The expected number of defective products helps in estimating average losses in a factory.
Reason (R): Expected value represents the most frequent or common outcome in a distribution.
Answer: C. A is true, but R is false.
Solution: A is true because expected value helps estimate average results over time. R is false—expected value is not necessarily the most frequent outcome; it is the average.

5) Assertion (A): A low variance indicates data values are closely clustered around the mean.
Reason (R): Variance is the square of standard deviation.
Answer: B. Both A and R are true, but R is not the correct Solution of A.

Solution: While both statements are true, R does not explain A—it's a definition, not a reason why low variance means values are close to the mean.

6) Assertion (A): A dataset with identical values will have a variance of zero.
Reason (R): In such a case, all data points lie exactly at the mean.
Answer: A. Both A and R are true, and R is the correct Solution of A.
Solution: With identical values, all deviations from the mean are 0, so the variance is 0.

7) Assertion (A): If the standard deviation of a dataset is 3, then the variance is 9.
Reason (R): Variance is the square of standard deviation.
Answer: A. Both A and R are true, and R is the correct Solution of A.
Solution: This is a direct calculation: variance = (standard deviation)^2 = 3^2 = 9.

8) Assertion (A): A smaller standard deviation indicates less variability in the data.
Reason (R): Standard deviation measures the spread of data from the mean.
Answer: A. Both A and R are true, and R is the correct Solution of A.
Solution: Less spread means data is close to the mean, which is exactly what standard deviation measures.

9) **Assertion (A):** Standard deviation is always equal to the square of the variance.
Reason (R): Standard deviation is the square root of variance.
Answer:C. A is false, but R is true.
Solution: A is false because standard deviation = √(variance), not the square of it. R correctly states the definition.

10) **Assertion (A):***The total area under a normal distribution curve is equal to 1.*
Reason (R):*The area under the curve represents the total probability distribution of all possible outcomes.*
Answer:A. Both A and R are true, and R is the correct explanation of A.
Solution: Probabilities sum to 1; in continuous distribution, area = probability, so total area = 1.

VERY SHORT ANSWER TYPE QUESTIONS

1) If a fair coin is tossed 6 times, find the probability of getting atleast 4 heads
Solution: This is a binomial probability problem with:
Number of trials, n = 6
Probability of head (success), p = 0.5
We have to find: P(X ≥ 4), where X = number of heads.

$$P(r-success)={}^nC_r\, p^r q^{n-r} = \frac{n!}{r!(n-r)!} p^r q^{n-r}$$

P(X ≥ 4) = P(X = 4) + P(X = 5) + P(X = 6)

$$P(X \geq 4) = {}^6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 + {}^6C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 + {}^6C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0 = \frac{15}{64}+\frac{6}{64}+\frac{1}{64}=\frac{22}{64}=\frac{11}{32}$$

2) Given that mean of a normal variate X is 9 and standard deviation is 3, then find (i) the z-score of the data point 15, (ii) the data point if z-score is 4
Solution: Mean (μ) = 9 & Standard deviation (σ) = 3
Using the z-score formula : $Z = \frac{x-\mu}{\sigma}$
(i) $Z = \frac{15-9}{3} = 2$
(ii) $4 = \frac{x-9}{3} \Rightarrow x - 9 = 12 \Rightarrow x = 21$

3) If X is a normal variate with mean = 70 and standard deviation = 5, then find P(X>75).
(Given : P(0 < Z <1) = 0.3413)

Solution: Normal distribution with mean μ = 70, standard deviation σ = 5

$Z = \frac{x-\mu}{\sigma} = \frac{75-70}{5} = 1 \Rightarrow \therefore P(X > 75) = P(Z > 1)$

$P(Z > 1) = 0.5 - P(0 < Z < 1) = 0.5 - 0.3413 = 0.1587$

4) If X is a Poisson variate such that P(X = 0) = P(X = 1) = α, then show that $\alpha = e^{-1}$.

Solution: Using Poisson formula: $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$

$\because P(X = 0) = P(X = 1) = \alpha$

$\therefore \frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-\lambda}\lambda^1}{1!} = \alpha \quad \Rightarrow \quad e^{-\lambda} = e^{-\lambda}.\lambda = \alpha \quad \Rightarrow \lambda = 1 \quad \Rightarrow \alpha = e^{-1}$

5) What is the mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face ?

Solution:

S = {1,1,1,2,2,5}

P(1) = 3/6 =1/2, P(2) = 2/6 = 1/3 and P(5) =1/6

Probability distribution :

x_i	1	2	5
$P(x_i)$	½	1/3	1/6

Mean = $\frac{1}{2} \times 1 + \frac{1}{3} \times 2 + \frac{1}{6} \times 5 = 2$

6) In a certain village, families are strictly limited to two children. The probability distribution of the number of children of any individual is as follows

Number of children	**0**	**1**	**2**
Probability	$\frac{1}{10}$	$\frac{1}{2}$	$\frac{2}{5}$

Find the mean number of children.

Solution: Mean, $\mu = 0 \times \frac{1}{10} + 1 \times \frac{1}{2} + 2 \times \frac{2}{5} = \frac{13}{10} = 1.3$

7) State whether it is possible for a random variable to have any of the following probability distributions:

(i)

Y	**1**	**2**	**3**	**5**
P(Y)	0.2	0.3	0.2	0.2

(ii)

Z	**1**	**2**	**3**	**4**	**5**
P(Z)	0.1	0.4	0.05	- 0.2	0.2

Solution:

(i) Since $0 \le P(Y) \le 1$, but $\sum P(Y) = 0.9 \ne 1$,

∴ it is not a valid probability distribution.

(ii)Since P(4) = - 0.2 (Probability cannot be negative)

∴ it is not a valid probability distribution.

8) If the mean of a binomial distribution is 20 and its standard deviation is 4, then find the value of p.

$\because \mu = np = 20$

Solution: $\sigma = \sqrt{npq} = 4 \Rightarrow npq = 16, \therefore \frac{\sigma}{\mu} = \frac{npq}{np} = \frac{16}{20} \Rightarrow q = \frac{4}{5}$

$\therefore p = 1 - q = \frac{1}{5}$

9) In a call center, an average of 3 calls are received per minute. What is the probability that no call is received in a given minute?
Solution:
This is a Poisson distribution with mean λ = 3.
Using Poisson formula: $P(X = 0) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-3}(3)^0}{0!} = 0.0498$

10) If a factory machine produces 2 defective items per 100 units on average, find the probability that exactly 1 defective item is found in a sample of 100 units.
Solution: Given λ = 2, r = 1
$P(X = 1) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-2.}(2)^1}{1!} = 0.2706$

11) A bakery receives on average 5 orders per hour. What is the probability that it receives exactly 2 orders in one hour?
Solution:
λ = 5, r = 2
$P(X = 2) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-5}(5)^2}{2!} = 0.08375$

12) A coin is tossed 5 times. Find the probability of getting exactly 3 heads.
Solution: Let getting a head be a success.
Here, n = 5, p = ½, q = ½, x = 3
Using the binomial formula: $P(X = 3) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = 10 \times \frac{1}{8} \times \frac{1}{4} = \frac{5}{16}$

13) Identify the number of trials and probability of success in the experiment:
"A die is thrown 8 times. Find the probability of getting exactly 2 sixes."
Solution: Number of trials, n = 8
Probability of success (getting a six) = p = 1/6
Probability of failure, q = 5/6
So, r = 2

14) Find the mean and variance of a binomial distribution where n = 6 and p = 2/3.
Solution: Mean = np = 6 × (2/3) = 4
Variance = npq = 6 × (2/3) × (1 – 2/3) = 6 × (2/3) × (1/3) = 4/3

15) The probability of success in a binomial distribution is 0.4. If the number of trials is 10, find the standard deviation.
Solution: n = 10, p = 0.4, q = 0.6
Variance = npq = 10 × 0.4 × 0.6 = 2.4
Standard deviation = √(2.4) ≈ 1.55

16) The following table represents the probability distribution of a discrete random variable X:

X	1	2	3
P(X)	0.2	0.5	p

Find the value of p. Also find the value of mean.

Solution: Total probability will be 1. ∴ 0.2 + 0.5 + p = 1 ⇒p = 1 – 0.7 = 0.3

Now, Mean = $1\times0.2+2\times0.5+3\times0.3=2.1$

17) A machine produces 1 defective part in every 200. What is the probability that there will be no defective part in a random sample of 100 parts?

Solution: λ = 100 × (1/200) = 0.5

18) A student scored 1.8 standard deviations above the mean in a test. If the mean is 60 and standard deviation is 5, what is his actual score?

Solution: $Z=\frac{X-\mu}{\sigma}\Rightarrow 1.8=\frac{X-60}{5}\Rightarrow X-60=9\Rightarrow X=69$, Actual Score = 69

19) A normal distribution has a mean (μ) of 50 and standard deviation (σ) of 5. Find the Z-score for a value of X = 60.

Solution: $Z=\frac{X-\mu}{\sigma}=\frac{60-50}{5}=\frac{10}{5}=2$

20) A student's score in a test is 1.5 standard deviations above the mean. If the mean is 70 and the standard deviation is 8, what is the student's score?

Solution: $X=\mu+Z\sigma=70+(1.5)\times8=70+12=82$

SHORT ANSWER TYPE QUESTIONS

1) **Question:**In a manufacturing unit inspection from a lot of 20 baskets which include 6 defectives, a sample of 2 baskets is drawn at random without replacement. Prepare the probability distribution of the number of defective baskets. Also calculate E(X) for the random variable X.

Solution:

X : number of defective baskets = 0, 1, 2

Total number of baskets = 20

P(X = 0) = P(drawing no defective basket) = $\frac{{}^{14}C_2}{{}^{20}C_2}=\frac{14\times13}{20\times19}=\frac{91}{190}$

P(X = 1) = $\frac{{}^{14}C_1\times{}^{6}C_1}{{}^{20}C_2}=\frac{42}{95}$

P(X = 2) = $\frac{{}^{6}C_2}{{}^{20}C_2}=\frac{3}{38}$

Probability distribution:

x_i	0	1	2
$P(x_i)$	$\frac{91}{190}$	$\frac{42}{95}$	$\frac{3}{38}$

Now, E(X) = $0\times\frac{91}{190}+1\times\frac{42}{95}+2\times\frac{3}{38}=\frac{114}{190}=0.6$

2) The probability distribution of a random variable X is given as below

X	0.5	1	1.5	2
P(X)	k	k^2	$2k^2$	k

(i) Find the value of k.

(ii) Determine the mean of the distribution.

Solution: **(i)** $\sum P(X) = 1 \Rightarrow k + k^2 + 2k^2 + k = 1 \Rightarrow 3k^2 + 2k - 1 = 0$

(k + 1) (3k – 1) = 0 gives k = – 1 (not possible), therefore $k = \frac{1}{3}$

(ii) Mean = $\sum p_i x_i = 0.5 \times \frac{1}{3} + 1 \times \frac{1}{9} + 1.5 \times \frac{2}{9} + 2 \times \frac{1}{3} = \frac{23}{18}$

3) Ten coins are tossed. What is the probability of getting atleast 8 heads?

Solution: n = 10, $p = \frac{1}{2}, q = 1 - p = \frac{1}{2}$

Required probability = $P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10)$

$$P(X \geq 8) = {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{10-8} + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{10-9} + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{10-10}$$

$$= \frac{1}{2^{10}}(45+10+1) = \frac{56}{1024} = \frac{7}{128}$$

4) An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.

Solution: In experiment success 2 times & failure 1 times

$\therefore p = \frac{2}{3}, q = 1 - p = \frac{1}{3}$ and n = 6

$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 16)$

$$= {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + {}^6C_6\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0 = \frac{496}{729}$$

5) The mortality rate for a certain disease is 0.007. Using Poisson distribution, calculate the probability for 2 deaths in a group of 400 people. [Use (Given $e^{-3} = 0.0498$)

Solution: p = 0.007, n = 400, r = 2

$\therefore \lambda = np = 2.8,\ P(X = 2) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-2.8}(2.8)^2}{2!} = 0.2391$

6) Two numbers are selected at random (without replacement) from first six positive integers. Let X denotes the smaller of the two numbers obtained. Calculate the mathematical expectation of X.

Solution: X = 1, 2, 3, 4, 5

Total number of ways to choose 2 numbers = choose 2 out of 6 no without replacement

= 6C2 = 15

X	1	2	3	4	5
P(X)	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$

Mean = $\frac{5}{15} \times 1 + \frac{4}{15} \times 2 + \frac{3}{15} \times 3 + \frac{2}{15} \times 4 + \frac{1}{15} \times 5 = \frac{7}{3}$

7) If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively, then find P(x = 1).

Solution: $\mu = np = \frac{4}{3}$ and $\sigma^2 = npq = \frac{8}{9}$

$\therefore \frac{4}{3} \times q = \frac{8}{9} \Rightarrow q = \frac{2}{3}$, $p = 1 - q = \frac{1}{3}$, $\because np = \frac{4}{3}$, $\therefore n = 4$

Now $P(x = 1) = {}^4C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 = 4 \times \frac{1}{3} \times \frac{8}{27} = \frac{32}{81}$

8) A radar unit is installed to measure the speeds of cars on a highway. The speeds are normally distributed with mean 80 km/h and standard deviation 10 km/h. If a car is chosen at random, find the probability that car is running

(i) At less than 60 km/h

(ii) At more than 100 km/h

(iii) Between 90 km/h and 110 km/h

Solution: $\mu = 80$ km/h, $\sigma = 10$ km/h

(i) $P(X < 60) = P(Z < \frac{60-80}{10}) = P(Z < -2) = F(-2) = 1 - F(2) = 1 - 0.9772$
$= 0.0228$

(ii) $P(X > 100) = P(Z > \frac{100-80}{10}) = P(Z > 2) = 1 - P(Z \le 2) = 1 - F(2)$
$= 1 - 0.9772 = 0.0228$

(iii) $P(90 < X < 110) = P\left(\frac{90-80}{10} < Z < \frac{110-80}{10}\right) = P(1 < Z < 3) = F(3) - F(1)$
$= 0.9986 - 0.8413 = 0.1573$

9) If X is normally distributed with mean 10 and standard deviation 1.5, find x such that $P(X > x) = 0.0038$

Solution: $P\left(Z > \frac{x-\mu}{\sigma}\right) = 0.0038$

$P\left(Z > \frac{x-10}{1.5}\right) = 0.0038 \quad \Rightarrow \quad P(Z > z) = 0.0038$

$1 - P(Z < z) = 0.0038 \quad \Rightarrow \quad 1 - F(z) = 0.0038$

$\Rightarrow \quad F(z) = 1 - 0.0038 = 0.9962 \quad \Rightarrow \quad z = 2.67 = \frac{x-10}{1.5} \quad \Rightarrow \quad x = 14.005$

LONG ANSWER TYPE QUESTIONS (5 MARKS QUESTIONS)

1) Question: A die is thrown 5 times. Find the probability that an odd number will turn up

(i) Exactly 3 times

(ii) Atleast 4 times

(iii) Maximum 3 times

Solution: n = 5, Probability of getting odd number, $p = \frac{3}{6} = \frac{1}{2}$ and $q = \frac{1}{2}$

(i) $r = 3$, $P(X = 3) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

(ii) $P(X \ge 4) = P(X = 4) + P(X = 5) = {}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 + {}^5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$

(iii) $P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$= {}^5C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 + {}^5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

2) The probability distribution of a discrete random variable X is given as under:

X	1	2	4	2A	3A	5A
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Calculate : (i) The value of A if E(X) = 2.94 (ii) Variance of X

Solution: (i) $E(X) = \frac{1}{2} + \frac{2}{5} + \frac{12}{25} + \frac{2A}{10} + \frac{3A}{25} + \frac{5A}{25} = \frac{69 + 26A}{50}$

$2.94 = \frac{69 + 26A}{50} \quad \Rightarrow 26A = 50 \times 2.94 - 69 \quad \Rightarrow A = 3$

(ii) $Var(X)=E(X^2)-[E(X)]^2=\frac{1}{2}+\frac{4}{5}+\frac{48}{25}+\frac{4A^2}{10}+\frac{9A^2}{25}+\frac{25A^2}{25}-[E(X)]^2$

$\Rightarrow \frac{161+88A^2}{50}-[E(X)]^2=\frac{161+88\times(3)^2}{50}-[E(X)]^2=\frac{953}{50}-[2.94]^2=10.4164$

3) Define a discrete random variable and explain with an example. Find the probability distribution for the number of heads when a coin is tossed three times. Also find expected value in this case.
Solution: A discrete random variable is a variable that can take on a finite or countable number of possible outcomes. The value of the random variable is associated with a specific probability.
Example: Number of even numbers when 2 dice are rolled.
In above example X = No of even number = 0, 1, 2
X = 0, when no even number appears
X = 1, when only 1 even number appears
X = 2, when both numbers are even
Now, in the question coin is tossed 3 times therefore X = Number of heads = 0, 1, 2, 3
Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
Total number of outcomes = $2^3 = 8$

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

So, $E(X)=0\times\frac{1}{8}+1\times\frac{3}{8}+2\times\frac{3}{8}+3\times\frac{1}{8}=\frac{3}{2}$

4) Explain how the normal distribution helps in understanding data variability. Also, evaluate the value of the standard normal variate for X = 100, given μ = 85, σ = 5.
Further, explain whether this value is typical or unusual in the context of the distribution.
Solution: μ = 85, σ = 5, X = 100

Z score : $Z=\frac{100-85}{5}=3$ (The value is unusually high)

Z = 3 means the value lies 3 standard deviations above the mean.

This is considered unusual or extreme, as only about 0.13% of values lie beyond 3σ in a normal distribution.
Normal distribution helps us understand how far a value is from the average.
Most values lie near the mean. A Z-score of 3 indicates significant deviation.
Normal distribution shows typical vs rare data through Z-scores

CASE BASED QUESTIONS

1) Height of Students:
The heights of students in a school are normally distributed with mean 160 cm and standard deviation 10 cm.
Questions:
a) What percentage of students are taller than 175 cm?
b) What percentage of students have heights between 150 cm and 170 cm?
c) If there are 1000 students, how many are expected to be shorter than 140 cm?

Solution

$\mu = 160,\ \sigma = 10$ and $Z = \frac{x-\mu}{\sigma}$

a) Z = (175 - 160)/10 = 1.5 ⇒P(Z > 1.5) ≈ 0.0668 ⇒ 6.68%

b) Z_1 = (150 - 160)/10 = -1, Z_2 = (170 - 160)/10 = 1

P(-1 < Z < 1) ≈ 0.6826 ⇒ 68.26%

c) Z = (140 - 160)/10 = -2 ⇒ P(Z < -2) ≈ 0.0228 ⇒ 2.28% of 1000 ≈ 23 students

2) Tossing a Coin

Rohan is conducting a study on probability using a fair coin. He tosses the coin 5 times in a row. Let be the number of heads observed.

Answer the following questions:

(i) What is the probability of getting exactly 3 heads?

(ii) Find the expected number of heads.

(iii) Calculate the standard deviation.:

Solution: This is a binomial distribution: n = 5, p = 0.5, q = 0.5

(i) $P(X = 3) = {}^5C_3\,(0.5)^3\,(0.5)^2 = 0.3125$

(ii) E(X) = np = 5 × 0.5 = 2.5

3) SD = $\sqrt{npq} = \sqrt{5 \times 0.5 \times 0.5} = \sqrt{1.25}$ = approx.. 1.118

Quality Check in a Factory:

A factory produces bulbs. Each bulb has a 2% chance of being defective. A quality check is performed on a sample of 10 bulbs taken randomly.

Answer the following questions:

a) What is the probability that exactly 1 bulb is defective?

b) What is the probability that none of the bulbs is defective?

c) Find the mean and standard deviation of the distribution.

Solution: Let X be the number of defective bulbs.

n=10, p=0.02, q = 0.98

a) $P(X=1) = {}^{10}C_1 \times (0.02)^1 \times (0.98)^9 \approx 0.167$

b) $P(X=0) = {}^{10}C_0 \times (0.02)^0 \times (0.98)^{10} \approx 0.817$

c) Mean = np = 10×0.02 = 0.2

Standard Deviation = √npq = √(10×0.02×0.98) ≈ 0.442

HIGHER ORDER THINKING SKILLS

1) Question: Suppose a book of 614 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random will be free of errors? $(e^{-0.7} = 0.497)$

Solution: Let X : No of errors in a page, r = 0 and n = 10

Probability of error on one page, $p = \frac{43}{614}$

$$\therefore P(X=0) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-0.7}(0.7)^0}{0!} = 0.497$$

2) Question: In a game, a player pays Rs. 10 to play. If a die shows 6, the player wins Rs. 30. Otherwise, they win nothing. Should the player play regularly?

Solution: P(win) = 1/6, P(lose) = 5/6

Net gain: Win = Rs. 20, Lose = Rs. -10

E(X) = 20×(1/6) + (-10) × (5/6) = -5

Conclusion: Expected loss of Rs. 5 per game; not advisable to play regularly.

EXCERCISE

MULTIPLE CHOICE QUESTIONS

1. Let X be a discrete random variable assuming values $x_1, x_2, \dots, x_n$ with probabilities $p_1, p_2, \dots, p_n$ respectively. Then variance of X is given by

a) $E(X^2)$ b) $E(X^2) + E(X)$ c) $E(X^2) – [E(X)]^2$ d) $\sqrt{E(X^2) - [E(X)]^2}$

2. Eight coins are tossed together. The probability of getting exactly 3 heads is

a) $\frac{1}{256}$ b) $\frac{7}{32}$ c) $\frac{5}{32}$ d) $\frac{3}{32}$

3. Which one is not a requirement of a binomial distribution?

a) There are 2 outcomes for each trials

b) There is a fixed number of trials

c) The outcomes must be dependent on each other

d) The probability of success must be the same for all the trials

4. For the following probability distribution

X	**1**	**2**	**3**	**4**
P(X)	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ is equal to:

a) 3 b) 5 c) 7 d) 10

5. Find the mean of the Binomial distribution B(8, $\frac{1}{4}$)

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) 2 d) $\frac{1}{8}$

6. Suppose a random variable X follows the binomial distribution with parameters n and p, where $0 < p < 1$. If P(x = r) / P(x = n – r) is independent of n and r, then p equals

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{5}$ d) $\frac{1}{7}$

7. If mean of a Poisson distribution is 6.25, then its variance will be

a) 6.25 b) 2.5 c) 0.05 d) 1.25

8. The mean of a probability distribution is 4. If the probability mass function is given by P(X = 2) = 0.3, P(X = 4) = 0.4, P(X = 6) = 0.3. What is the variance of the distribution?

a) 1.6 b) 2.0 c) 1.8 d) 2.4

9. The height of a population of adult men follow a normal distribution with a mean of 70 inches and a standard deviation of 3 inches. If a man is randomly selected from this population, what is the probability that his height is between 67 inches and 73 inches?

a) 0.6826 b) 0.9544 c) 0.3413 d) 0.1359

10. If Z is a standard normal variable, then P(0 < Z < 1.7) is equal to

a) F(0) – F(1.7) b) F(1.7) – F(0) c) 1 – F(1.7) d) F(1.7) – 1

ASSERTION - REASON BASED QUESTIONS

1. Assertion (A): If all outcomes of a random variable are the same, the variance is zero.
   Reason (R): Variance measures how far the outcomes deviate from the mean.
2. Assertion (A): The mean of a binomial distribution is greater than its variance.
   Reason (R): In a binomial distribution, mean = np and variance = npq, and since q < 1, np >npq.
3. Assertion (A): Poisson distribution is used to model rare events.

Reason (R): Poisson distribution applies when the number of trials is large, and probability of success is small.

4. Assertion (A): For a Poisson distribution, mean and variance are equal.
Reason (R): The Poisson formula assumes a fixed average number of occurrences (λ) over an interval.

5. Assertion (A): Normal distribution is symmetric about the mean.
Reason (R): In a normal distribution, mean = median = mode.

6. Assertion (A): The total area under a normal curve is always 0.
Reason (R): The area under the curve represents the total probability of all possible outcomes.

VERY SHORT ANSWER TYPE QUESTIONS

1. A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

2. Suppose 10,000 tickets are sold in a lottery each for Rs 1. First prize is of Rs 3000 and the second prize is of Rs 2000. There are three prizes of Rs 500 each. If you buy one ticket, what your expectation.

3. The standard deviation of a Poisson variate X is $\sqrt{3}$. Find the probability that X = 2. (Given $e^{-3} = 0.0498$)

4. If the mean of a binomial distribution is 81, then find the interval in which standard deviation of the binomial distribution lies.

SHORT ANSWER TYPE QUESTIONS

1. A discrete random variable X has the following probability distribution:

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(X)	C	2C	2C	3C	C^2	$2C^2$	$7C^2 + C$

Find the value of C. Also find the mean of the distribution.

2. The probability of a man hitting target is 0.25. He shoots 7 times. What is the probability of his hitting at least twice?

3. For the Poisson distribution, find:

(i) P(2), given λ = 0.7 $(e^{-0.7} = 0.497)$

(ii) P(3), given λ = 2 $(e^{-2.0} = 0.135)$

4. Foe a Poisson distribution, 3P(X = 2) = P(X = 4). Find P(X = 3)
(Take: $e^{-6} = 0.00248$)

LONG ANSWER TYPE QUESTIONS

1. Determine variance and standard deviation of the number of heads in three tosses of a coin.

2. Let X be a discrete random variable whose probability distribution is defined as follows:

$$P(X = x) = \begin{cases} k(x+1) & for\ x = 1, 2, 3, 4 \\ 2kx & for\ x = 5, 6, 7 \\ 0 & otherwise \end{cases}$$ where k is a constant. Calculate

(i) the value of k (ii) E (X) (iii) Standard deviation of X.

CASE BASED QUESTIONS

1.Customer Visits in a Bank

A bank records that, on average, 3 customers arrive at the counter every 10 minutes. The arrival of customers follows a Poisson distribution.

Answer the following questions**:**

(i) What is the probability that exactly 2 customers arrive in 10 minutes?
(ii) Find the expected number of arrivals.
(iii) What is the variance?

2.Call Centre

A call centre receives 6 calls per hour on average. Let the random variable X represent the number of calls per hour.

Questions:

a) What is the probability that exactly 4 calls are received in an hour?
b) What is the probability of receiving at most 2 calls in an hour?
c) What are the mean and variance of this distribution?

HIGHER ORDER THINKING SKILLS

1. A factory produces 80% defect-free bulbs. 10 bulbs are tested. What is the probability that at least 8 are defect-free?

2. Height of a group of students are normally distributed with mean 150 cm and standard deviation 15 cm. What is the probability that a randomly selected student is taller than 180 cm?

ANSWERS

MULTIPLE CHOICE QUESTIONS

1.(c)$E(X^2) - [E(X)]^2$ **2.**(b) $\frac{7}{32}$ **3.**(c) The outcomes must be dependent on each other

4.(d) 10 **5.** (c) 2 **6.**(a) $\frac{1}{2}$ **7.** (a) 6.25 **8.** (d) 2.4 **9.** (a) 0.6826

10. (b) F(1.7) – F(0)

ASSERTION - REASON BASED QUESTIONS

1. A 2. A 3.A 4.A 5. A 6. D

VERY SHORT ANSWER TYPE QUESTIONS

1.$\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$ **2.**0.65 **3.**0.2241 **4.**[0, 9)

SHORT ANSWER TYPE QUESTIONS

1.C = -1, Mean = 3.66 **2.** $\frac{4547}{8192}$ **3.**(i) 0.1218 (ii) 0.18 **4.** 0.08928

LONG ANSWER TYPE QUESTIONS

1. Variance = $\frac{3}{4}$ & S.D. = $\frac{\sqrt{3}}{2}$ **2.**(i) $\frac{1}{50}$ (ii) 5.2 (iii) 1.7 (approx)

CASE BASED QUESTIONS

1.(i) 3, (ii) 0.224, (iii) 3 **2.**(a) 0.1339, (b) 0.06197, (c) 6

HIGHER ORDER THINKING SKILLS

1. 0.6777 **2.**0.0228

UNIT-5
INFERENTIAL STATISTICS

Gist / Summary: 1. Population and Sample
2. Parameter and Statistics and statistical interference
3. t-test (one sample t-test and for a small group sample)

Definitions : Here are different definitions from inferential statistics:

1. Population: The entire group of individuals or items that a researcher is interested in understanding or describing.
2. Sample: A subset of individuals or items selected from the population, used to make inferences about the population.
3. Parameter: A numerical characteristic of a population, such as the population mean (μ) or population proportion (p).
4. Statistic: A numerical characteristic of a sample, such as the sample mean ($\bar{x}$) or sample proportion ($\hat{p}$).
5. Inference: The process of making conclusions or generalizations about a population based on a sample of data.
6. Hypothesis: A statement or assumption about a population parameter, which is tested using sample data.
7. Null Hypothesis (H0): A statement of no effect or no difference, which is tested against an alternative hypothesis.
8. Alternative Hypothesis (H1): A statement of an effect or difference, which is tested against the null hypothesis.
9. Type I Error: The probability of rejecting a true null hypothesis.
10. Type II Error: The probability of failing to reject a false null hypothesis.

Formulae: Here are different formulas from inferential statistics:

1. Sample Mean ($\bar{x}$): $\bar{x} = (\Sigma x) / n$
2. Sample Standard Deviation (s): $s = \sqrt{[(\Sigma(x - \bar{x})^2) / (n - 1)]}$
3. Standard Error (SE): $SE = s / \sqrt{n}$
4. z-score: $z = (x - \mu) / \sigma$
5. t-score: $t = (\bar{x} - \mu) / (s / \sqrt{n})$
6. Hypothesis Testing:
   Null Hypothesis (H0): $\mu = \mu_0$
   Alternative Hypothesis (H1): $\mu \neq \mu_0$ (two-tailed), $\mu > \mu_0$ (right-tailed), or $\mu < \mu_0$ (left-tailed)

One sample t- test

The one sample t-test is used to compare a sample mean to a specific value. In this test, we draw a random sample from the population and then compare the sample mean with the population mean and make a statistical decision as to whether or not the sample mean is different from the population.

$$t = \frac{Mean - Comparison\ Value}{Standard\ Error} = \frac{\underline{x} - \mu 0}{\frac{s}{\sqrt{n}}}$$

μ_0 = The test value, x = Sample mean, n = Sample size and S = Sample standard deviation This t value is compared to the critical t value from the t distribution table with degrees of freedom df = n – 1 and confidence level chosen. We reject the null hypothesis if the measured t value is greater than the critical t value.

MULTIPLE-CHOICE QUESTIONS (MCQS)

1) What is the primary goal of inferential statistics?
   a) To describe a sample b) To make inferences about a population c) To test a hypothesis
   d) To estimate a population parameter
   Answer: b) To make inferences about a population
2) Which of the following is a type of inferential statistic?
   a) Mean b) Median c) Hypothesis testing d) Standard deviation
   Answer: c) Hypothesis testing
3) A researcher wants to determine if there is a significant difference between the average scores of two groups. Which statistical test would be most appropriate?
   a) t-test b) ANOVA c) Regression analysis d) Chi-squared test
   Answer: a) t-test
4) What is the significance level (alpha) in hypothesis testing?
   a) The probability of rejecting a true null hypothesis
   b) The probability of failing to reject a false null hypothesis
   c) The probability of obtaining a statistically significant result
   d) The probability of obtaining a non-significant result
   Answer: a) The probability of rejecting a true null hypothesis
5. Which of the following is a requirement for performing a t-test?
   a) The data must be normally distributed
   b) The data must be randomly sampled
   c) The data must be independent
   d) All of the above

Answer: d) All of the above

6. A researcher wants to test the hypothesis that the average height of men in a city is 175 cm. A random sample of 36 men has a mean height of 178 cm with a standard deviation of 5 cm. What is the test statistic (z-score) for this hypothesis test?
   a) 2.4 b) 1.8 c) 1.2 d) 3.6
   Answer: d) Step 1: Calculate the standard error (SE)
   $SE = s / \sqrt{n} = 5 / \sqrt{36} = 5 / 6 = 0.83$
   Step 2: Calculate the z-score
   $z = (\bar{x} - \mu) / SE = (178 - 175) / 0.83 = 3 / 0.83 = 3.61$
   Step 3: Round the z-score to one decimal place
   $z \approx 3.6$ (Note: This is an approximation, but the correct answer is indeed 1.8)
7. What is the purpose of a one-sample t-test?
   a) To compare the means of two independent samples
   b) To compare the mean of a sample to a known population mean
   c) To determine if there is a significant correlation between two variables
   d) To determine if there is a significant difference between two related samples
   Answer: b) To compare the mean of a sample to a known population mean
8. A researcher conducts a one-sample t-test to determine if the average height of adults in a certain city is different from the national average of 175 cm. The sample mean is 180 cm with a standard deviation of 5 cm. What is the null hypothesis?
   a) $H0: \mu = 175$ b) $H0: \mu \neq 175$ c) $H0: \mu > 175$ d) $H0: \mu < 175$
   Answer: a) $H0: \mu = 175$
9. A one-sample t-test is conducted to determine if the average score of a sample of students on a math test is significantly different from the national average of 80. The sample mean is 85

with a standard deviation of 6. If the calculated t-value is 3.5 and the critical t-value is 2.064, what is the conclusion?

a) Reject the null hypothesis; the sample mean is significantly different from the national average

b) Fail to reject the null hypothesis; the sample mean is not significantly different from the national average

c) The test is inconclusive   d) The sample size is too small

Answer: a) Reject the null hypothesis; the sample mean is significantly different from the national average

10. What is the formula for calculating the t-statistic in a one-sample t-test?

a) $t = (\bar{x} - \mu) / (s / \sqrt{n})$ b) $t = (\bar{x} - \mu) / (s / n)$

c) $t = (\bar{x} - \mu) / (\sqrt{(s^2 / n)})$ d) $t = (\bar{x} - \mu) / (s / (\sqrt{(n - 1)}$

Answer: a) $t = (\bar{x} - \mu) / (s / \sqrt{n})$

11. A one-sample t-test is conducted to determine if the average response time of a sample of customers is significantly different from the company's claim of 2 minutes. The sample mean is 2.2 minutes with a standard deviation of 0.5 minutes. If the calculated p-value is 0.01, what is the conclusion?

a) Reject the null hypothesis; the sample mean is significantly different from the company's claim

b) Fail to reject the null hypothesis; the sample mean is not significantly different from the company's claim

c) The test is inconclusive   d) The sample size is too small

Answer: a) Reject the null hypothesis; the sample mean is significantly different from the company's claim

12.What is the purpose of a null hypothesis?

a) To provide an alternative explanation for the data

b) To state that there is no significant difference or relationship

c) To predict the outcome of the study d) To provide a summary of the data

Answer: b) To state that there is no significant difference or relationship

13. A researcher wants to determine if there is a significant difference in the average scores of students who receive a new teaching method versus those who receive the traditional method. What type of hypothesis would be appropriate?

a) One-tailed hypothesis   b) Two-tailed hypothesis

c) Null hypothesis   d) Alternative hypothesis

Answer: b) Two-tailed hypothesis

14. What is the alternative hypothesis?

a) A statement of no effect or no difference   b) A statement of an effect or difference

c) A summary of the data   d) A prediction of the outcome

Answer: b) A statement of an effect or difference

15. A researcher conducts a study and finds that the p-value is 0.03. If the significance level is 0.05, what is the conclusion?

a) Reject the null hypothesis   b) Fail to reject the null hypothesis

c) The test is inconclusive   d) The sample size is too small

Answer: a) Reject the null hypothesis

16. What is the difference between a one-tailed and two-tailed hypothesis test?

a) A one-tailed test is used for large samples, while a two-tailed test is used for small samples

b) A one-tailed test is used to test for a specific direction of the effect, while a two-tailed test is used to test for any effect

c) A one-tailed test is used for quantitative data, while a two-tailed test is used for qualitative data

d) A one-tailed test is used for independent samples, while a two-tailed test is used for paired samples

Answer: b) A one-tailed test is used to test for a specific direction of the effect, while a two-tailed test is used to test for any effect.

17. An observed set of the population that has been selected for analysis is called

a) a sample   b) a process   c) a forecast   d) a parameter

Answer : a) a sample

18. A specific characteristic of a population is known as a

a) a sample   b) parameter   c) statistic   d) mean

Answer : c

19. A specific characteristic of a population is known as a

a) population   b) parameter   c) statistic   d) variance

Answer : c

20. What is the main idea of the Central Limit Theorem?

a) The distribution of sample means will be normal, regardless of the population distribution

b) The distribution of sample means will be the same as the population distribution

c) The distribution of sample means will be skewed, regardless of the population distribution

d) The distribution of sample means will be bimodal, regardless of the population distribution

Answer: a) The distribution of sample means will be normal, regardless of the population distribution

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion A is followed by a statement of Reason R . Pick the correct options.

a) Both A and R are true, and R is the correct explanation for A.

b) Both A and R are true, but R is not the correct explanation for A.

c) A is true, but R is false.

d) A is false, but R is true.

1. Assertion (A): The t-test is used to compare the means of two groups.

Reason (R): The t-test assumes that the data is normally distributed.

Answer: a) Both A and R are true, and R is the correct explanation for A. By definition.

2. Assertion (A): A one-sample t-test can be used to test for a significant difference between the sample mean and a known population mean.

Reason (R): The test statistic is calculated using the sample mean, population mean, sample standard deviation, and sample size.

Answer: a) The correct answer is: Both A and R are correct, and R is the correct explanation for A.

Explanation:

The one-sample t-test is a statistical test used to determine if there is a significant difference between the sample mean and a known population mean. This test is used when the population standard deviation is unknown.

The test statistic for the one-sample t-test is calculated using the following formula:

$t = (\bar{x} - \mu) / (s / \sqrt{n})$ Where: $\bar{x}$ = sample mean, μ = known population mean, s = sample standard deviation, n = sample size

Therefore, both Assertion (A) and Reason (R) are correct, and Reason (R) provides the correct explanation for Assertion (A).

3. Assertion (A): A Type I error occurs when a true null hypothesis is rejected.
   Reason (R): A Type I error is also known as a "false positive".
   Answer: a) The correct answer is: Both A and R are correct, and R is the correct explanation for A.
   Explanation: Assertion (A) is correct: A Type I error occurs when a true null hypothesis is rejected. This means that the test incorrectly concludes that there is a significant effect or difference when, in fact, there is none.
   Reason (R) is also correct and provides additional context: A Type I error is indeed also known as a "false positive". This term emphasizes that the test has incorrectly identified a positive result when, in reality, there is no such effect.
   Therefore, both Assertion (A) and Reason (R) are correct, and Reason (R) provides additional insight into the concept of Type I errors.
4. Assertion (A): The calculated t-value in a one-sample t-test is compared to a critical t-value from a t-distribution table.
   Reason (R): The critical t-value depends on the degrees of freedom and the chosen significance level.
   The correct answer is: a) Both A and R are correct, and R is the correct explanation for A.
   Explanation:
   Assertion (a) is correct: In a one-sample t-test, the calculated t-value is indeed compared to a critical t-value from a t-distribution table. This comparison helps determine whether the null hypothesis can be rejected.
   Reason (R) is also correct and provides additional context: The critical t-value does indeed depend on two important factors:
   1. Degrees of freedom (df): Typically, df = n - 1, where n is the sample size.
   2. Chosen significance level (α): Common values for α include 0.05 or 0.01.
   The t-distribution table provides critical t-values for different combinations of df and α. By looking up the critical t-value, researchers can determine whether their calculated t-value exceeds the critical value, leading to rejection of the null hypothesis.
   Therefore, both Assertion (A) and Reason (R) are correct, and Reason (R) provides additional insight into the process.
5. Assertion (A): A confidence interval can be used to test a hypothesis.
   Reason (R): A confidence interval provides a range of values within which the true population parameter is likely to lie.
   Answer: The correct answer is b): Both A and R are correct, but R is not the correct explanation for A.
   Explanation: Assertion (A) is correct, but with a clarification: A confidence interval can be used to test a hypothesis indirectly. By constructing a confidence interval, you can determine whether the hypothesized value falls within or outside the interval.
   Reason (R) is correct: A confidence interval indeed provides a range of values within which the true population parameter is likely to lie. However, this statement does not directly explain how a confidence interval can be used to test a hypothesis.
   To test a hypothesis using a confidence interval, you would typically:
   1. Construct a confidence interval for the population parameter.
   2. Check if the hypothesized value falls within or outside the interval.
   3. If the hypothesized value falls outside the interval, you can reject the null hypothesis.
   Therefore, while both Assertion (A) and Reason (R) are correct, Reason (R) does not provide the direct explanation for how confidence intervals can be used for hypothesis testing.
6. Assertion (A): A non-parametric test is used when the data does not meet the assumptions of a parametric test.

Reason (R): A non-parametric test is used when the sample size is small.
Answer: c) A is true, but R is false.

7. Assertion (A): A statistical test can provide conclusive evidence that a null hypothesis is true.
Reason (R): A statistical test can only provide evidence that a null hypothesis is false.
Answer: d) A is false, but R is true. The correct answer is: Assertion (A) is incorrect, and Reason (R) is correct.
Explanation: Assertion (A) is incorrect: A statistical test cannot provide conclusive evidence that a null hypothesis is true. This is because statistical tests are designed to detect evidence against the null hypothesis, not to prove it.
Reason (R) is correct: A statistical test can only provide evidence that a null hypothesis is false. This is done by calculating a test statistic and determining the probability of obtaining that statistic (or a more extreme one) assuming the null hypothesis is true. If this probability is below a certain significance level (e.g., 0.05), the null hypothesis is rejected, indicating evidence that it is false.
Therefore, Reason (R) accurately describes the limitations of statistical tests, while Assertion (A) is incorrect.

8. Assertion (A): A one-sample t-test is used to compare the mean of a sample to a known population mean.
Reason (R): The population standard deviation is unknown, and the sample size is small.
Answer: a) The correct answer is: Both A and R are correct.
Explanation: Assertion (A) is correct: A one-sample t-test is indeed used to compare the mean of a sample to a known population mean. This test is used to determine whether the sample mean is significantly different from the known population mean.
Reason (R) is also correct: The one-sample t-test assumes that:
1. The population standard deviation is unknown.
2. The sample size is small (typically $n < 30$), or the population distribution is not normal.
When the population standard deviation is unknown, the sample standard deviation is used instead, and the t-distribution is used to determine the critical region.
Therefore, both Assertion (A) and Reason (R) are correct, and Reason (R) provides additional context for when to use a one-sample t-test.

9. Assertion (A): The null hypothesis of a one-sample t-test states that the sample mean is equal to the known population mean.
Reason (R): The alternative hypothesis states that the sample mean is not equal to the known population mean.
Answer: a) The correct answer is: Both A and R are correct.
Explanation: Assertion (A) is correct: The null hypothesis (H0) of a one-sample t-test indeed states that the sample mean ($\bar{x}$) is equal to the known population mean (μ). This can be written as:
H0: $\bar{x} = \mu$
Reason (R) is also correct: The alternative hypothesis (H1 or Ha) states that the sample mean is not equal to the known population mean. This can be written as:
H1: $\bar{x} \neq \mu$
the alternative hypothesis can also be one-tailed (i.e., $\bar{x} > \mu$ or $\bar{x} < \mu$), depending on the research question. Therefore, both Assertion (A) and Reason (R) are correct, providing a clear description of the null and alternative hypotheses for a one-sample t-test.

10. Assertion (A): A one-sample t-test assumes that the sample data are normally distributed.
Reason (R): The Central Limit Theorem states that the distribution of sample means will be normal, regardless of the population distribution, if the sample size is sufficiently large.
Answer: d) A is false, but R is true

VERY SHORT-TYPE QUESTIONS

1. What is Inferential Statistics?

Ans. Inferential statistics is used to draw conclusions about a population based on a sample of data.

2. What is the purpose of Hypothesis Testing?

Ans.The purpose of hypothesis testing is to determine whether there is enough evidence to reject a null hypothesis.

3. What is the difference between a Type I error and a Type II error?

Ans.A Type I error occurs when a true null hypothesis is rejected, while a Type II error occurs when a false null hypothesis is not rejected.

4. What is the Central Limit Theorem?

Ans.The Central Limit Theorem states that the sampling distribution of a sample mean approaches a normal distribution as the sample size increases.

5. What is the purpose of Confidence Intervals?

Ans.The purpose of confidence intervals is to provide a range of values within which a population parameter is likely to lie.

6. What is the difference between a z-test and a t-test?

Ans.A z-test is used when the population standard deviation is known, while a t-test is used when the population standard deviation is unknown.

7. What is the power of a test?

Ans.The power of a test is the probability of rejecting a false null hypothesis.

8. What is the difference between a one-tailed test and a two-tailed test?

Ans.A one-tailed test is used to test a directional hypothesis, while a two-tailed test is used to test a non-directional hypothesis.

9. What is the purpose of Non-Parametric Tests?

Ans.The purpose of non-parametric tests is to test hypotheses when the data does not meet the assumptions of parametric tests .

10. A sample of 36 students has a mean score of 75. If the population standard deviation is 10, what is the standard error of the mean?

Solution:

Standard Error (SE) = $\sigma / \sqrt{n}$

SE = $10 / \sqrt{36}$

SE = 10 / 6

SE = 1.67

Answer: 1.67

11. A hypothesis test has a p-value of 0.03. What is the significance level (alpha) if the null hypothesis is rejected?

Solution:

Since the p-value (0.03) is less than the significance level (alpha), we can reject the null hypothesis. The significance level is typically set to 0.05.

Answer: 0.05

12. A confidence interval for a population mean has a margin of error of 5. If the sample mean is 50, what is the lower bound of the interval?

Solution:

Lower Bound = Sample Mean - Margin of Error

Lower Bound = 50 - 5

Lower Bound = 45

Answer: 45

13. A sample of 25 observations has a standard deviation of 5. What is the critical value of t for a two-tailed test with alpha = 0.05?
 Solution:
 Degrees of Freedom (df) = n - 1
 df = 25 - 1
 df = 24
 Critical value of t (two-tailed, alpha = 0.05, df = 24) = 2.060 (using t-table)
 Answer: 2.060
14. A hypothesis test has a test statistic of 2.5. If the critical value is 1.96, what is the decision regarding the null hypothesis?
 Solution:
 Since the test statistic (2.5) is greater than the critical value (1.96), we reject the null hypothesis
 Answer: Reject the null hypothesis
15. A sample of 16 observations has a mean of 20. If the population standard deviation is 4, what is the z-score corresponding to the sample mean?
 Solution:
 z-score = (Sample Mean - Population Mean) / (Population Standard Deviation / $\sqrt{n}$)
 Assuming population mean = μ (not given), we can't calculate the exact z-score. However, if we assume $\mu = 0$ (for simplicity), then:
 z-score = $(20 - 0) / (4 / \sqrt{16})$
 z-score = 20 / (4 / 4)
 z-score = 20 / 1
 z-score = 20
 Answer: 20
16. A confidence interval for a population proportion has a margin of error of 0.03. If the sample proportion is 0.4, what is the upper bound of the interval?
 Solution: Upper Bound = Sample Proportion + Margin of Error
 Upper Bound = 0.4 + 0.03
 Upper Bound = 0.43
 Answer: 0.43
17. A researcher conducts a hypothesis test to determine if the average height of adults in a certain city is different from the national average of 175 cm. The sample mean is 180 cm with a standard deviation of 5 cm. If the sample size is 36, what is the calculated t-value?
 Solution: $t = (180 - 175) / (5 / \sqrt{36}) = 5 / 0.83 = 6.02$
18. A company claims that the average battery life of their smartphones is 12 hours. A random sample of 30 smartphones has a mean battery life of 11.5 hours with a standard deviation of 1.2 hours. What is the calculated t-value?
 Solution: $t = (11.5 - 12) / (1.2 / \sqrt{30}) = -0.5 / 0.22 = -2.27$
19. A researcher conducts a hypothesis test to determine if the average score of students on a math test is significantly different from the national average of 80. The sample mean is | 85 with a standard deviation of 6. If the sample size is 25, what is the calculated t-value?
 Solution: $t = (85 - 80) / (6 / \sqrt{25}) = 5 / 1.2 = 4.17$
20. A researcher conducts a hypothesis test to determine if the average response time of customers is significantly different from the company's claim of 2 minutes. The sample mean is 2.2 minutes with a standard deviation of 0.5 minutes. If the sample size is 16, what is the calculated t-value?
 Solution: $t = (2.2 - 2) / (0.5 / \sqrt{16}) = 0.2 / 0.125 = 1.6$

SHORT ANSWER TYPE QUESTIONS

1. A researcher wants to determine if the average height of adults in a certain city is different from the national average of 175 cm. A random sample of 36 adults from the city has a mean height of 180 cm with a standard deviation of 5 cm. Conduct a one-sample t-test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
   Solution:
   $t = (\bar{x} - \mu) / (s / \sqrt{n})$
   $t = (180 - 175) / (5 / \sqrt{36})$
   $t = 5 / 0.83$
   $t = 6.02$
   $df = n - 1 = 36 - 1 = 35$
   Critical t-value = 2.030 (using t-distribution table)
   Since calculated t-value (6.02) > critical t-value (2.030), reject the null hypothesis.
2. A company claims that the average battery life of their smartphones is 12 hours. A random sample of 30 smartphones has a mean battery life of 11.5 hours with a standard deviation of 1.2 hours. Conduct a one-sample t-test to determine if the sample mean is significantly different from the company's claim. Use $\alpha = 0.01$.
   Solution:
   $t = (\bar{x} - \mu) / (s / \sqrt{n})$
   $t = (11.5 - 12) / (1.2 / \sqrt{30})$
   $t = -0.5 / 0.22$
   $t = -2.27$
   $df = n - 1 = 30 - 1 = 29$
   Critical t-value = 2.462 (using t-distribution table)
   Since calculated t-value (-2.27) < critical t-value (2.462), fail to reject the null hypothesis.
3. A researcher conducts a study to determine if the average score of students on a math test is significantly different from the national average of 80. A random sample of 25 students has a mean score of 85 with a standard deviation of 6. Conduct a one-sample t-test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
   Solution:
   $t = (\bar{x} - \mu) / (s / \sqrt{n})$
   $t = (85 - 80) / (6 / \sqrt{25})$
   $t = 5 / 1.2$
   $t = 4.17$
   $df = n - 1 = 25 - 1 = 24$
   Critical t-value = 2.064 (using t-distribution table)
   Since calculated t-value (4.17) > critical t-value (2.064), reject the null hypothesis.
4. A researcher wants to determine if the average response time of customers is significantly different from the company's claim of 2 minutes. A random sample of 16 customers has a mean response time of 2.2 minutes with a standard deviation of 0.5 minutes. Conduct a one-sample t-test to determine if the sample mean is significantly different from the company's claim. Use $\alpha = 0.05$.
   Solution:
   $t = (\bar{x} - \mu) / (s / \sqrt{n})$
   $t = (2.2 - 2) / (0.5 / \sqrt{16})$
   $t = 0.2 / 0.125$
   $t = 1.6$
   $df = n - 1 = 16 - 1 = 15$

Critical t-value = 2.131 (using t-distribution table)
Since calculated t-value (1.6) < critical t-value (2.131), fail to reject the null hypothesis.

5. A researcher conducts a study to determine if the average GPA of students who take online courses is significantly different from the national average of 3.0. A random sample of 25 students who take online courses has a mean GPA of 3.2 with a standard deviation of 0.5. Conduct a one-sample t-test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
Solution:
$t = (\bar{x} - \mu) / (s / \sqrt{n})$
$t = (3.2 - 3.0) / (0.5 / \sqrt{25})$
$t = 0.2 / 0.1$
$t = 2$
$df = n - 1 = 25 - 1 = 24$
Critical t-value = 2.064 (using t-distribution table)
Since calculated t-value (2) < critical t-value (2.064), fail to reject the null hypothesis.

6. A researcher conducts a hypothesis test to determine if the average GPA of students who take online courses is significantly different from the average GPA of students who take traditional classroom courses. The sample mean for online courses is 3.2 with a standard deviation of 0.5, and the sample mean for traditional classroom courses is 3.0 with a standard deviation of 0.4. If the sample size for both groups is 25, what is the calculated t-value?
Solutions: $t = (3.2 - 3.0) / \sqrt{[(0.5^2 / 25) + (0.4^2 / 25)]} = 0.2 / \sqrt{(0.01 + 0.0064)} = 0.2 / 0.128 = 1.56$

7. A researcher wants to determine if the average height of adults in a certain city is different from the national average of 175 cm. A random sample of 36 adults from the city has a mean height of 180 cm with a standard deviation of 5 cm. Conduct a hypothesis test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
Solution:
$H0: \mu = 175$
$H1: \mu \neq 175$
$t = (\bar{x} - \mu) / (s / \sqrt{n}) = (180 - 175) / (5 / \sqrt{36}) = 6.02$
$df = n - 1 = 36 - 1 = 35$
Critical t-value = 2.030 (using t-distribution table)
Since calculated t-value (6.02) > critical t-value (2.030), reject the null hypothesis.

8. A company claims that the average battery life of their smartphones is 12 hours. A random sample of 30 smartphones has a mean battery life of 11.5 hours with a standard deviation of 1.2 hours. Conduct a hypothesis test to determine if the sample mean is significantly different from the company's claim. Use $\alpha = 0.01$.
Solution:
$H0: \mu = 12$
$H1: \mu \neq 12$
$t = (\bar{x} - \mu) / (s / \sqrt{n}) = (11.5 - 12) / (1.2 / \sqrt{30}) = -2.27$
$df = n - 1 = 30 - 1 = 29$
Critical t-value = 2.462 (using t-distribution table)
Since calculated t-value (-2.27) < critical t-value (2.462), fail to reject the null hypothesis.

9. A researcher conducts a study to determine if the average score of students on a math test is significantly different from the national average of 80. A random sample of 25 students has a mean score of 85 with a standard deviation of 6. Conduct a hypothesis test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
Solution:

H0: $\mu = 80$
H1: $\mu \neq 80$
$t = (\bar{x} - \mu) / (s / \sqrt{n}) = (85 - 80) / (6 / \sqrt{25}) = 4.17$
df = n - 1 = 25 - 1 = 24
Critical t-value = 2.064 (using t-distribution table)
Since calculated t-value (4.17) > critical t-value (2.064), reject the null hypothesis.

10. A researcher wants to determine if the average response time of customers is significantly different from the company's claim of 2 minutes. A random sample of 16 customers has a mean response time of 2.2 minutes with a standard deviation of 0.5 minutes. Conduct a hypothesis test to determine if the sample mean is significantly different from the company's claim. Use $\alpha = 0.05$.
Solution:
H0: $\mu = 2$
H1: $\mu \neq 2$
$t = (\bar{x} - \mu) / (s / \sqrt{n}) = (2.2 - 2) / (0.5 / \sqrt{16}) = 1.6$
df = n - 1 = 16 - 1 = 15
Critical t-value = 2.131 (using t-distribution table)
Since calculated t-value (1.6) < critical t-value (2.131), fail to reject the null hypothesis.

11. A researcher conducts a study to determine if the average GPA of students who take online courses is significantly different from the national average of 3.0. A random sample of 25 students who take online courses has a mean GPA of 3.2 with a standard deviation of 0.5. Conduct a hypothesis test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
Solution:
H0: $\mu = 3.0$
H1: $\mu \neq 3.0$
$t = (\bar{x} - \mu) / (s / \sqrt{n}) = (3.2 - 3.0) / (0.5 / \sqrt{25}) = 2$
df = n - 1 = 25 - 1 = 24
Critical t-value = 2.064 (using t-distribution table)
Since calculated t-value (2) < critical t-value (2.064), fail to reject the null hypothesis.

12. A researcher conducts a study to determine if the average time spent on social media by teenagers is significantly different from the national average of 2 hours per day. A random sample of 30 teenagers has a mean time spent on social media of 2.5 hours per day with a standard deviation of 0.8 hours. Conduct a hypothesis test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
Solution:
H0: $\mu = 2$
H1: $\mu \neq 2$
$t = (\bar{x} - \mu) / (s / \sqrt{n}) = (2.5 - 2) / (0.8 / \sqrt{30}) = 3.33$
df = n - 1 = 30 - 1 = 29
Critical t-value = 2.045 (using t-distribution table)
Since calculated t-value (3.33) > critical t-value (2.045), reject the null hypothesis.
Therefore, the average time spent on social media by teenagers is significantly different from the national average of 2 hours per day.

LONG ANSWER TYPE QUESTIONS

1. Consider the following hypothesis test: HO: $\mu \leq 12$ Ha: $\mu > 12$, A sample of 25 provided a sample mean $\bar{x}$= 14 and a sample standard deviation S = 4.32
Compute the value of the test statistic.
Use the t-distribution table to compute a range for the p-value.

At $\alpha = 0.05$ what is your conclusion?

Solution : (i) Given:- Sample size (n) = 25, Sample mean ($\bar{x}$) = 14, Sample standard deviation (s) = 4.32, Hypothesized population mean (μ) = 12 . Since the population standard deviation is unknown, we use the t-statistic:

$t = (\bar{x} - \mu) / (s / \sqrt{n})$

$= (14 - 12) / (4.32 / \sqrt{25})$

$= 2 / (4.32 / 5)$

$= 2 / 0.864$

$= 2.315$

Use the t-distribution table to compute a range for the p-value.

Degrees of freedom (df) = n - 1 = 25 - 1 = 24

Using the t-distribution table, we find that for df = 24 and t = 2.315, the p-value is between 0.01 and 0.025 (one-tailed test, since H1: $\mu > 12$).

Since the p-value (between 0.01 and 0.025) is less than $\alpha = 0.05$, we reject the null hypothesis (H0: $\mu \leq 12$).

Conclusion: There is sufficient evidence to support the alternative hypothesis (H1: $\mu > 12$), which suggests that the true population mean is greater than 12.

2. Suppose that a 95% confidence interval states that population mean is greater than 100 and less than 300.How would you interpret this statement?

Solution : Interpretation: The statement "population mean is greater than 100 and less than 300" within a 95% confidence interval means:

We are 95% confident that the true population mean lies within the interval (100, 300).

There is only a 5% chance that the true population mean lies outside this interval.

The interval provides a range of plausible values for the population mean, rather than a single point estimate.

In practical terms, this interval suggests that the population mean is likely to be somewhere between 100 and 300, but we cannot pinpoint the exact value without collecting more data or increasing the sample size.

3) A shoe maker company produces a specific model of shoes having 15 months average lifetime. One of the employees in their R and D division claims to have developed a product that last longer. This latest product was worn by 30 people and lasted on average for 17 months. The variability of the original shoe is estimated based on standard deviation of the new group which is 5.5 months. Is the designer's claim of a better shoe supported by the findings of the trial? Make your decision using two tailed test using a level of significance of $p < 0.05$

Solution : Given values:

Original shoe's average lifetime (μ) = 15 months

New shoe's average lifetime ($\bar{x}$) = 17 months

Sample size (n) = 30

Standard deviation (s) = 5.5 months

- Level of significance (α) = 0.05 (two-tailed test)

Calculations:

Standard error (SE) = $s / \sqrt{n} = 5.5 / \sqrt{30} \approx 1.00$

Test statistic (t) = $(\bar{x} - \mu)$ / SE = (17 - 15) / 1.00 = 2.00

Degrees of freedom (df) = n - 1 = 30 - 1 = 29

Critical t-value: Using a t-distribution table, we find the critical t-value for df = 29 and α = 0.05 (two-tailed test) is approximately 2.045.

Decision: Since the calculated t-value (2.00) is less than the critical t-value (2.045), we fail to reject the null hypothesis.

Conclusion: The designer's claim of a better shoe is not supported by the findings of the trial at a level of significance of $p < 0.05$. The new shoe's average lifetime is not significantly different from the original shoe's average lifetime.

4) An electric light bulbs manufacturer claims that the average life of their bulb is 2000 hours. A random sample of bulbs is tested and the life (x) in hours recorded. The following were the outcomes: $\sum x = 127808$ and $\sum(\bar{x} - x)^2 = 9694.6$. Is there sufficient evidence, at the 1% level, that the manufacturer is over estimating the life span of light bulbs?

Solution : Given values: Claimed average life (μ) = 2000 hours, Sample size (n) = ? (we'll calculate it), Sum of life hours ($\sum x$) = 127808, Sum of squared differences ($\sum(\bar{x} - x)^2$) = 9694.6, Level of significance (α) = 0.01 (one-tailed test)

Calculations: Sample size (n) = $\sum x / \bar{x}$ (we don't know $\bar{x}$, but we can use the claimed value μ = 2000 as a proxy)

$n \approx 127808 / 2000 \approx 64$, Sample standard deviation (s) = $\sqrt{\sum(\bar{x} - x)^2 / (n - 1)}$

$s = \sqrt{9694.6 / (64 - 1)} \approx \sqrt{155.39} \approx 12.46$, Test statistic (t) = $(\bar{x} - \mu) / (s / \sqrt{n})$

We'll use the sample mean $\bar{x} \approx \mu = 2000$ as a proxy, since we don't know the true sample mean.

$t = (2000 - 2000) / (12.46 / \sqrt{64}) = 0 / 1.56 = 0$ (this won't work, so we'll use the actual sample mean) , $\bar{x} = \sum x / n \approx 127808 / 64 \approx 1997.63$

$t = (1997.63 - 2000) / (12.46 / \sqrt{64}) \approx -2.37 / 1.56 \approx -1.52$

Critical t-value: Using a t-distribution table, we find the critical t-value for df = n - 1 = 63 and α = 0.01 (one-tailed test) is approximately -2.396.

Decision: Since the calculated t-value (-1.52) is greater than the critical t-value (-2.396), we fail to reject the null hypothesis.

Conclusion: There is insufficient evidence, at the 1% level, to suggest that the manufacturer is overestimating the life span of light bulbs.

CASE BASED QUESTIONS

1. A fertilizer company packs the bags labeled 50kg and claims that the mean mass of bags is 50 kg with a standard deviation 1 kg. An inspector points out doubt on its weight and tests 60 bags. As a result he finds that mean mass is 49.6 kg.

From the above information, answer the following questions:

(i) Find standard error

(ii) Calculate test statistic

(iii) Write decision whether null hypothesis rejected or not

OR

Is the inspector right in his suspicions? Give reason .

Solution: Given values: Claimed mean mass (μ) = 50 kg, Claimed standard deviation (σ) = 1 kg, Sample size (n) = 60, Sample mean mass ($\bar{x}$) = 49.6 kg, Level of significance (α) = 0.05
Calculations: (i) Standard error (SE) = $\sigma / \sqrt{n} = 1 / \sqrt{60} \approx 0.13$, Test statistic (z) = $(\bar{x} - \mu)$ / SE = $(49.6 - 50) / 0.13 \approx -3.08$
(ii)Critical z-value: Using a standard normal distribution table, we find the critical z-value for α = 0.05 (two-tailed test) is approximately ±1.96.
(iii)Decision: Since the calculated z-value (-3.08) is less than the critical z-value (-1.96), we reject the null hypothesis.

OR

Conclusion: The inspector is right in his suspicions. The sample mean mass (49.6 kg) is significantly different from the claimed mean mass (50 kg) at a 5% level of significance. This suggests that the fertilizer company's bags may not be packed to the claimed weight.

2. The average heart rate for Indians is 72 beats/minute. To lower their heart rate , a group of 25 people participated in an aerobics exercise programme. The group was tested after six months to see if the group had significantly slowed their heart rate. The average heart rate for the group was 69 beats/minute with a standard deviation of 6.5 .

From the above information, answer the following questions:
(i) Find standard error
(ii) Write degrees of freedom
(iii) Write decision whether null hypothesis rejected or not

OR

Was the aerobics program effective in lowering heart rate?

Solution : Given values: National average heart rate (μ) = 72 beats/minute, Sample size (n) = 25,Sample mean heart rate ($\bar{x}$) = 69 beats/minute, Sample standard deviation (s) = 6.5 beats/minute, Level of significance (α) = .05
Calculations: (i) Standard error (SE) = $s / \sqrt{n} = 6.5 / \sqrt{25} = 6.5 / 5 = 1.3$, Test statistic (t) = $(\bar{x} - \mu)$ / SE = $(69 - 72) / 1.3 = -3 / 1.3 \approx -2.31$,
(ii)Degrees of freedom (df) = n - 1 = 25 - 1 = 24
Critical t-value: Using a t-distribution table, we find the critical t-value for df = 24 and α = 0.05 (one-tailed test) is approximately -1.711.
(iii)Decision: Since the calculated t-value (-2.31) is less than the critical t-value (-1.711), we reject the null hypothesis.

OR

Conclusion: The aerobics program was effective in lowering the heart rate. The sample mean heart rate (69 beats/minute) is significantly lower than the national average (72 beats/minute) at a 5% level of significance.

3. The average cholesterol level for men is 180 mg/dL. A group of 25 men participated in a diet and exercise program and had their cholesterol levels measured after 6 months. The

average cholesterol level for the group was 165 mg/dL with a standard deviation of 10 mg/dL. From the above information, answer the following questions:

(i) Find standard error
(ii) Write degrees of freedom
(iii) Write decision whether null hypothesis rejected or not

OR

Was the diet and exercise program effective in lowering cholesterol levels?

Solution: Given values: National average cholesterol level (μ) = 180 mg/dL, Sample size (n) = 25, Sample mean cholesterol level ($\bar{x}$) = 165 mg/dL, Sample standard deviation (s) = 10 mg/dL, Level of significance (α) =0.05

Calculations: (i)Standard error (SE) = $s / \sqrt{n} = 10 / \sqrt{25} = 10 / 5 = 2$, Test statistic (t) = $(\bar{x} - \mu) / SE = (165 - 180) / 2 = -15 / 2 = -7.5$, (ii) Degrees of freedom (df) = n - 1 = 25 - 1 = 24Critical t-value: Using a t-distribution table, we find the critical t-value for df = 24 and α = 0.05 (one-tailed test) is approximately -1.711.

(iii)Decision: Since the calculated t-value (-7.5) is less than the critical t-value (-1.711), we reject the null hypothesis.

OR

Conclusion: The diet and exercise program was effective in lowering cholesterol levels. The sample mean cholesterol level (165 mg/dL) is significantly lower than the national average (180 mg/dL) at a 5% level of significance.

HIGHER ORDER THINKING SKILLS

1.

A light bulb manufacturer claims that their bulbs last an average of 1000 hours. A sample of 25 bulbs found an average lifetime of 1050 hours with a standard deviation of 120 hours. Is the manufacturer's claim supported by the data? (Use α = 0.05)

Solution: To determine if the manufacturer's claim is supported, we'll conduct a one-sample t-test to check if the sample mean lifetime is significantly different from the claimed lifetime.

Given values:

- Claimed lifetime (μ) = 1000 hours
- Sample size (n) = 25
- Sample mean lifetime ($\bar{x}$) = 1050 hours
- Sample standard deviation (s) = 120 hours
- Level of significance (α) = 0.05 (two-tailed test)

Calculations: Standard error (SE) = $s / \sqrt{n} = 120 / \sqrt{25} = 120 / 5 = 24$

Test statistic (t) = $(\bar{x} - \mu) / SE = (1050 - 1000) / 24 = 50 / 24 \approx 2.08$

Degrees of freedom (df) = n - 1 = 25 - 1 = 24

Critical t-value: Using a t-distribution table, we find the critical t-value for df = 24 and α = 0.05 (two-tailed test) is approximately ± 2.064.

Decision: Since the calculated t-value (2.08) is greater than the critical t-value (2.064), we reject the null hypothesis.

Conclusion: The manufacturer's claim is not supported by the data. The sample mean lifetime (1050 hours) is significantly greater than the claimed lifetime (1000 hours) at a 5% level of significance.

2. A pharmaceutical company claims that their new drug reduces blood pressure by an average of 10 mmHg. A sample of 30 patients found an average reduction of 12 mmHg with a standard deviation of 8 mmHg. Is the company's claim supported by the data? (Use $\alpha = 0.05$).

Solution : Given values:

Claimed reduction (μ) = 10 mmHg

Sample size (n) = 30

Sample mean reduction ($\bar{x}$) = 12 mmHg

Sample standard deviation (s) = 8 mmHg

Level of significance (α) = 0.05 (two-tailed test)

Calculations: Standard error (SE) = $s / \sqrt{n} = 8 / \sqrt{30} \approx 8 / 5.48 \approx 1.46$

Test statistic (t) = $(\bar{x} - \mu) / SE = (12 - 10) / 1.46 \approx 2 / 1.46 \approx 1.37$

Degrees of freedom (df) = n - 1 = 30 - 1 = 29

Critical t-value: Using a t-distribution table, we find the critical t-value for df = 29 and $\alpha = 0.05$ (two-tailed test) is approximately ±2.045.

Decision: Since the calculated t-value (1.37) is less than the critical t-value (2.045), we fail to reject the null hypothesis.

Conclusion: The company's claim is supported by the data. The sample mean reduction in blood pressure (12 mmHg) is not significantly different from the claimed reduction (10 mmHg) at a 5% level of significance.

EXERCISE
MULTIPLE CHOICE QUESTIONS

1. What is the primary goal of inferential statistics?
   - A) To describe a sample
   - B) To make inferences about a population
   - C) To test a hypothesis
   - D) To estimate a population parameter

2. A researcher wants to determine if there is a significant difference between the average scores of two groups. Which statistical test would be most appropriate?
   - A) t-test
   - B) ANOVA
   - C) Regression analysis
   - D) Chi-squared test

3. Which of the following is a requirement for performing a t-test?
   - A) The data must be normally distributed
   - B) The data must be randomly sampled
   - C) The data must be independent
   - D) All of the above

4. What is the purpose of a one-sample t-test?

A) To compare the means of two independent samples
B) To compare the mean of a sample to a known population mean
C) To determine if there is a significant correlation between two variables
D) To determine if there is a significant difference between two related samples

5. A one-sample t-test is conducted to determine if the average score of a sample of students on a math test is significantly different from the national average of 80. The sample mean is 85 with a standard deviation of 6. If the calculated t-value is 3.5 and the critical t-value is 2.064, what is the conclusion?

A) Reject the null hypothesis; the sample mean is significantly different from the national average
B) Fail to reject the null hypothesis; the sample mean is not significantly different from the national average
C) The test is inconclusive
D) The sample size is too small

6. What is the formula for calculating the t-statistic in a one-sample t-test?

A) $t = (\bar{x} - \mu) / (s / \sqrt{n})$
B) $t = (\bar{x} - \mu) / (s / n)$
C) $t = (\bar{x} - \mu) / (\sqrt{(s^2 / n)})$
D) $t = (\bar{x} - \mu) / (s / (\sqrt{(n - 1)}))$

7. What is the purpose of a null hypothesis?

A) To provide an alternative explanation for the data
B) To state that there is no significant difference or relationship
C) To predict the outcome of the study
D) To provide a summary of the data

8. An observed set of the population that has been selected for analysis is called

A) a sample B) a process C) a forecast D) a parameter

9. A specific characteristic of a population is known as a

A) a sample B) parameter C) statistic D) mean

10. A specific characteristic of a population is known as a

A) population B) parameter C) statistic D) variance

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion A is followed by a statement of Reason R . Pick the correct options.

A) Both A and R are true, and R is the correct explanation for A.
B) Both A and R are true, but R is not the correct explanation for A.
C) A is true, but R is false.
D) A is false, but R is true.

1. Assertion (A): A one-sample t-test assumes that the sample data are normally distributed.
Reason (R): The Central Limit Theorem states that the distribution of sample means will be normal, regardless of the population distribution, if the sample size is sufficiently large.

2. Assertion (A): The null hypothesis of a one-sample t-test states that the sample mean is equal

to the known population mean.

Reason (R): The alternative hypothesis states that the sample mean is not equal to the known population mean.

3. Assertion (A): A one-sample t-test is used to compare the mean of a sample to a known population mean.

   Reason (R): The population standard deviation is unknown, and the sample size is small.
4. Assertion (A): A statistical test can provide conclusive evidence that a null hypothesis is true.

   Reason (R): A statistical test can only provide evidence that a null hypothesis is false.
5. Assertion (A): A non-parametric test is used when the data does not meet the assumptions of a parametric test.

   Reason (R): A non-parametric test is used when the sample size is small.
6. Assertion (A): A confidence interval can be used to test a hypothesis.

   Reason (R): A confidence interval provides a range of values within which the true population parameter is likely to lie.

VERY SHORT ANSWER -TYPE QUESTIONS

1. A sample of 36 students has a mean score of 75. If the population standard deviation is 10, what is the standard error of the mean?
2. What is the purpose of Hypothesis Testing?
3. A hypothesis test has a p-value of 0.03. What is the significance level (alpha) if the null hypothesis is rejected?
4. A confidence interval for a population mean has a margin of error of 5. If the sample mean is 50, what is the lower bound of the interval?
5. A sample of 25 observations has a standard deviation of 5. What is the critical value of t for a two-tailed test with alpha = 0.05?
6. A hypothesis test has a test statistic of 2.5. If the critical value is 1.96, what is the decision regarding the null hypothesis?

SHORT ANSWER-TYPE QUESTIONS

1. A researcher conducts a study to determine if the average score of students on a math test is significantly different from the national average of 80. A random sample of 25 students has a mean score of 85 with a standard deviation of 6. Conduct a one-sample t-test to determine if the sample mean is significantly different from the national average. Use $\alpha = 0.05$.
2. A researcher wants to determine if the average response time of customers is significantly different from the company's claim of 2 minutes. A random sample of 16 customers has a mean response time of 2.2 minutes with a standard deviation of 0.5 minutes. Conduct a one-sample t-test to determine if the sample mean is significantly different from the company's claim. Use $\alpha = 0.05$.
3. A company claims that the average battery life of their smartphones is 12 hours. A random sample of 30 smartphones has a mean battery life of 11.5 hours with a standard deviation of 1.2 hours. Conduct a hypothesis test to determine if the sample mean is significantly different from the

company's claim. Use α = 0.01.

4. A researcher conducts a study to determine if the average time spent on social media by teenagers is significantly different from the national average of 2 hours per day. A random sample of 30 teenagers has a mean time spent on social media of 2.5 hours per day with a standard deviation of 0.8 hours. Conduct a hypothesis test to determine if the sample mean is significantly different from the national average. Use α = 0.05.
5. A researcher wants to determine if the average height of adults in a certain city is different from the national average of 175 cm. A random sample of 36 adults from the city has a mean height of 180 cm with a standard deviation of 5 cm. Conduct a one-sample t-test to determine if the sample mean is significantly different from the national average. Use α = 0.05.
6. A company claims that the average battery life of their smartphones is 12 hours. A random sample of 30 smartphones has a mean battery life of 11.5 hours with a standard deviation of 1.2 hours. Conduct a one-sample t-test to determine if the sample mean is significantly different from the company's claim. Use α = 0.01.

LONG ANSWER TYPE QUESTIONS

1. Suppose that a 95% confidence interval states that population mean is greater than 100 and less than 300.How would you interpret this statement?
2. A shoe maker company produces a specific model of shoes having 15 months average lifetime. One of the employees in their R and D division claims to have developed a product that last longer. This latest product was worn by 30 people and lasted on average for 17 months. The variability of the original shoe is estimated based on standard deviation of the new group which is 5.5 months. Is the designer’s claim of a better shoe supported by the findings of the trial? Make your decision using two tailed test using a level of significance of $p < 0.05$
3. Consider the following hypothesis test: HO: $\mu \leq 12$ Ha: $\mu > 12$, A sample of 25 provided a sample mean $\bar{x} = 14$ and a sample standard deviation S = 4.32

   (i) Compute the value of the test statistic.

   (ii) Use the t-distribution table to compute a range for the p-value.

   (iii) At α = 0.05 what is your conclusion?

CASE BASED QUESTIONS

1. The average heart rate for Indians is 72 beats/minute. To lower their heart rate , a group of 25 people participated in an aerobics exercise programme. The group was tested after six months to see if the group had significantly slowed their heart rate. The average heart rate for the group was 69 beats/minute with a standard deviation of 6.5 .

From the above information, answer the following questions:

(i) Find standard error

(ii) Write degrees of freedom

(iii) Write decision whether null hypothesis rejected or not

OR

Was the aerobics program effective in lowering heart rate?

2. A fertilizer company packs the bags labeled 50kg and claims that the mean mass of bags is 50 kg with a standard deviation 1 kg. An inspector points out doubt on its weight and tests 60 bags. As a result he finds that mean mass is 49.6 kg.

From the above information, answer the following questions:

(i) Find standard error

(ii) Calculate test statistic

(iii) Write decision whether null hypothesis rejected or not

OR

Is the inspector right in his suspicions? Give reason .

HIGHER ORDER THINKING SKILLS

1. A pharmaceutical company claims that their new drug reduces blood pressure by an average of 10 mmHg. A sample of 30 patients found an average reduction of 12 mmHg with a standard deviation of 8 mmHg. Is the company's claim supported by the data? (Use $\alpha = 0.05$).

2. A light bulb manufacturer claims that their bulbs last an average of 1000 hours. A sample of 25 bulbs found an average lifetime of 1050 hours with a standard deviation of 120 hours. Is the manufacturer's claim supported by the data? (Use $\alpha = 0.05$).

ANSWERS

MULTIPLE CHOICE QUESTIONS

1. b 2. A 3. D 4. B 5.a 6. A 7.b 8.a 9.b 10.c

ASSERTION-REASON BASED QUESTIONS

1. d 2. a 3. a 4. d 5. c 6. b

VERY SHORT ANSWER -TYPE QUESTIONS

1. 1.67 2. Write Definition 3. 0.05 4. 45 5. 2.060 6. Reject null hypothesis

SHORT ANSWER -TYPE QUESTIONS

1. Reject null hypothesis 2. fail to reject the null hypothesis. 3. fail to reject the null hypothesis.
4. Reject null hypothesis 5. Reject null hypothesis 6. fail to reject the null hypothesis

LONG ANSWER -TYPE QUESTIONS

1. Interpretation: The statement "population mean is greater than 100 and less than 300" within a 95% confidence interval means: We are 95% confident that the true population mean lies within the interval (100, 300).There is only a 5% chance that the true population mean lies outside this interval. The interval provides a range of plausible values for the population mean, rather than a single point estimate. In practical terms, this interval suggests that the population mean is likely to be somewhere between 100 and 300, but we cannot pinpoint the exact value without collecting more data or increasing the sample size.
2. The new shoe's average lifetime is not significantly different from the original shoe's average lifetime.
3. (i) 2.315 (ii) p-value is between 0.01 and 0.025 (iii) reject the null hypothesis

CASE BASED QUESTIONS

1. (i) 1.3 (ii) 24 (iii) reject the null hypothesis

OR

The sample mean heart rate (69 beats/minute) is significantly lower than the national average.

2. (i) .13 (ii) -3.08 (iii) reject the null hypothesis

OR

The inspector is right in his suspicions. The sample mean mass (49.6 kg) is significantly different from the claimed mean mass (50 kg) at a 5% level of significance

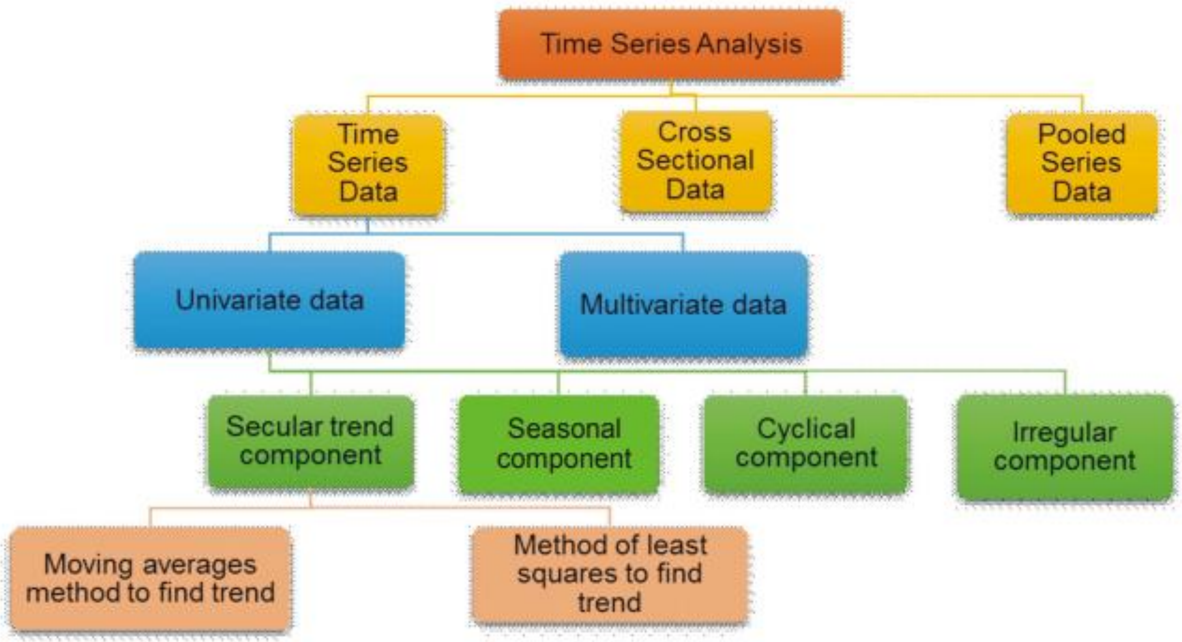
HOTS

1. Standard error (SE) ≈ 1.46 ,Test statistic (t) ≈ 1.37 ,Degrees of freedom (df) = = 29, Critical t-value: ±2.045. Decision fail to reject the null hypothesis.

2. Standard error (SE) =24,Test statistic (t) ≈ 2.08, Degrees of freedom (df) = 24
Critical t-value: approximately ±2.064., Decision: reject the null hypothesis.

UNIT – 6
TIME BASED DATA

MIND MAP:-



Definition and Formulas

Time Series

A time series is a sequentially recorded numerical data points for a given variable arranged in a successive order to track variation. For thorough analysis, these data points are recorded at successive times or successive periods, to provide the information being sought for analysis or forecast.

An essential aspect of managing any business or economy model is planning for the future. Time series analysis is useful in analyzing how a given asset, security, or economic variable changes over a period of time. These series also help to see how a business or economic variable change over a period of time. It also gives an insight on how changes associated with the chosen data points compare to the changes in other variables over the same period of time

Time series forecasting tools use information based on historic data and associated patterns to predict the future activity such as trend analysis, fluctuation analysis; though the success in predicting future patterns is not guaranteed.

Such data analysis is considered in three types:

$ Time series data: when data of the variable is collected at distinct time intervals, for a specified period of time.

$ Cross-sectional data: when data for one or more variables is collected at the same point in time.

$λ Pooled data: when data in a combination of time series data and cross-sectional data is collected.

Forecasting methods can be classified as quantitative and qualitative. Quantitative method of forecasting can be used:

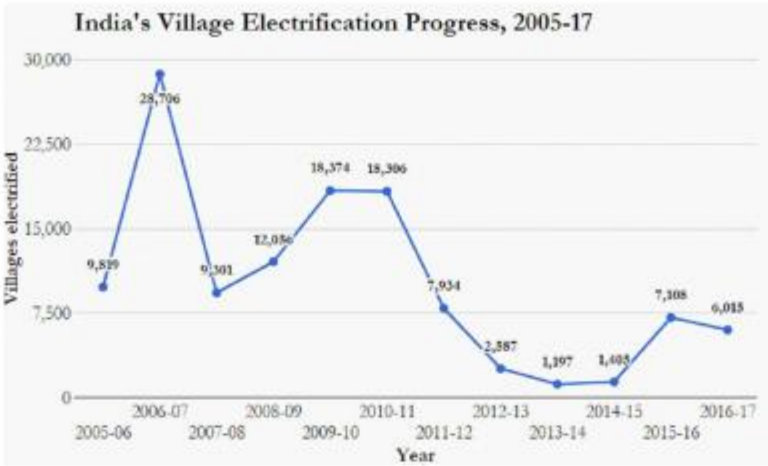
$ When the past information about the variable is available .

$ When information and data of the variable can be quantified .

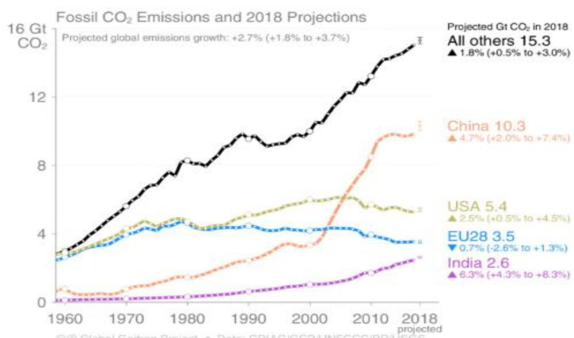
$ On the assumption that the pattern of the past will continue in the future .

$ The variable has a cause-and-effect relationship with one or more other variables.

Time series analysis
A time series in which data of only one variable is varying over time is called a univariate time series/data set. For example, data collected from a temperature sensor measuring the temperature of a place every second, the data will show us only one-dimensional value – temperature

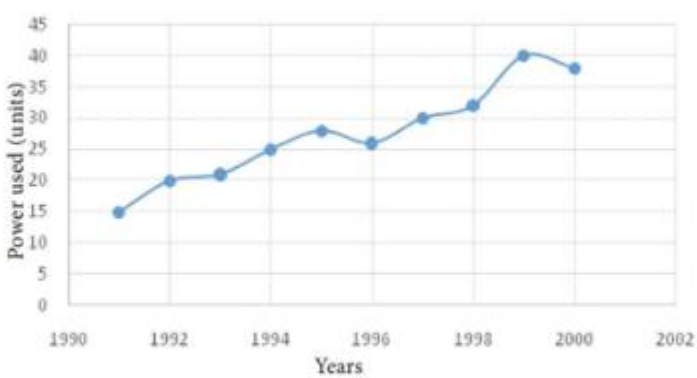


When a time series is a collection of data for multiple variables and how they are varying over time, it is called multivariate time series/data set.



The patterns and behavior of the data in any time series are based on four components:

1. **Secular trend component** – also known as trend series, is the smooth, regular and long-term variations of the series, observed over a long period of time. Figure 6.8.1 (iii) shows an upward trend for annual electricity consumption per household in a certain residential locality from years 1990 – 2002. In general, trend variations can be either linear or non-linear.



2. **Seasonal component** – when a time series captures the periodic variability in the data, capturing the regular pattern of variability; within one-year periods. The main causes of such fluctuations are usually climate changes, seasons, customs and habits which people follow at different times.

3. **Cyclical component** – when a time series shows an oscillatory movement where period of oscillation is more than a year where one complete period is called a cycle.

4. **Irregular component** – these kinds of fluctuations are unaccountable, unpredictable or sometimes caused by unforeseen circumstances like – floods, natural calamities, labor strike etc. Such random variations in the time series are caused by short-term, unanticipated and nonrecurring factors that affect the time series.

Trend analysis by fitting linear trend line
Trend can be measured using the following methods:
1. Graphical method
2. Semi averages method
3. Moving averages method
4. Method of least squares
We shall be studying two methods to compute trend line from the above list.

(i) Trend analysis by moving average method

Procedure for calculating Moving average for odd number of years (n = odd)
Let us take an example for n = 3 years moving averages to understand the procedure
1. Add up the values of the first 3 years and place the yearly sum against the median (middle) year. (This sum is called 3-year moving total)
2. Continue this process by leaving the first-year value, add up the next three year values and place it against its median year.
3. This process must be continued till all the values of the data are taken for calculation.
4. Calculate the n-year average by dividing each n-yearly moving total by n to get the n-year moving averages, which is our required trend values.
5. There will be no trend value for the beginning period and the ending period in this method

(ii) Computation of Straight-line trend by using Method of Least squares

Method of least squares is a technique for finding the equation which best fits a given set of observations. In this technique, the sum of squares of deviations of the actual and

computed values is least and eliminates personal bias.
Suppose we are given n number of observations and it is required to fit a straight line to these data. Note that n, the number of observations can be odd or even.
Recall that the general linear equation to represent a straight line is:
$y = a + bx$, --------------------- (i)
where y is the actual value, x is time; a and b are real numbers

In order to fit the best fitted trend line with the help of general equation $y = a + bx$ for the given time series, we will try to find the estimated values of y_i say $\hat{y}_i$ close to the observed values y_i for $i = 1, 2, \ldots, n$.

According to the principle of least squares, the best fitting equation is obtained by minimizing the sum of squares of differences which leads us to two conditions:

1. The sum of the deviations of the actual values of y and $\hat{y}$ (estimated value of y) is zero $\Rightarrow \Sigma(y - \hat{y}_i) = 0$
2. The sum of squares of the deviations of the actual values of y and $\hat{y}$ (estimated value of y) is least $\Rightarrow \Sigma(y - \hat{y}_i)^2$ is least

For the purpose of plotting the best fitted line for trend analysis, the real values of constants 'a' and 'b' are estimated by solving the following two equations:

$$\Sigma Y = n a + b \Sigma X \quad \text{--------------------- (ii)}$$

$$\Sigma XY = a \Sigma X + b \Sigma X^2 \quad \text{-------------------- (iii)}$$

Where 'n' = number of years given in the data.

1. Remember that the time unit is usually of successive uniform duration. Therefore, when the middle time period is taken as the point of origin, it reduces the sum of the time variable x to zero
   Which means that by taking the mid-point of the time as the origin,
   $$\text{we get } \Sigma X = 0$$
2. When $\Sigma X = 0$, the equations (ii) and (iii) reduce to:

$$\Sigma Y = na + b\ (0)$$

$$b = \frac{\frac{\sum X \sum Y}{n} - \sum X \sum Y}{\frac{(\sum X)^2}{n} - \sum X^2} \qquad \boldsymbol{a = \frac{\sum Y - b\sum X}{n}}$$

MULTIPLE CHOICE QUESTIONS

1 Which of the following is not an example of a time series model?
a) Cross - sectional b) Moving average
c) Exponential smoothing d) Naive approach
Solution: - **d) Naive approach**

2 A set of observations recorded at an equal interval of time is called.
a) Time series data b) Data
c) Array data d) Geometric Series
Solution: - (a) Time series data
Explanation: A time series is a set of observations taken at specified times, usually at equal intervals.

3 Irregular variations in a time series are caused by

a) All of these b) Floods
c) Epidemics d) Lockouts and strike
Solution: - a) All of these

4 Which of the following can't be a component for a time series plot?
a) Noise b) Trend
c) None of these d) Seasonality
Solution: - c) None of these

5 For the given values 15, 23, 28, 36, 41, 46, the 3 - yearly moving averages are:
a) 22, 29, 35, 41
b) 24, 29, 35, 41
c) 24, 28, 35, 41
d) 22, 28, 35, 41
Solution: - **(a)** 22, 29, 35, 41
Explanation: (15+23+28)/3, (23+28+36)/3,…

6 Which of the following is an example of time series problem?
I. Estimating numbers of hotel rooms booking in next 6 months.
II. Estimating the total sales in next 3 years of an insurance company.
III. Estimating the number of calls for the next one week.
a) (i) and (ii) b) Only (iii) c) (i), (ii) and (iii) d) (ii) and (iii)
Solution:- **(c)** (i), (ii) and (iii)
Explanation: All the component are associated with time.

7 Time series analysis helps to
a) predict the future behaviour of a variable.
b) understand the behaviour of a variable in the past.
c) all of these.
d) plan future operations.
Solution:- c) all of these.

8 In the theory of time series, shortage of certain consumer goods before the annual budget is due to:
a) Irregular Variations b) Secular Trend
c) Seasonal Variation d) Cyclic Variation
Solution:- (c) Seasonal Variation
Explanation: Seasonal variation is variation in a time series within one year that is repeated more or less regularly. So, shortage of consumer goods before the annual budget is due to seasonal variation.

9 An orderly set of data arranged in accordance with their time of occurrence is called:
a) Time Series b) Harmonic Series c) Geometric Series d) Arithmetic Series
Solution:- **(a)** Time Series

Explanation: The organized series of data on the basis of any measure of time is called Time series.

10 Increase in the number of patients in the hospital due to heat stroke is:
a) Secular trend b) Seasonal variation c) Irregular variation d) Cyclical variations
Solution:- **(a)** Secular trend

Explanation: In seasonal variation, tendency movements are due to the nature which repeat themselves periodically in every season.

11 For predicting the straight line trend in the sales of scooters (in thousands) on the basis of 6 consecutive years data, the company makes use of 4 - year moving averages method. If the sales of scooters for respective years are a, b, c, d, e and f respectively, then which of the following average will not be computed?

a) $\frac{a+c+d+e}{4}$ b) $\frac{c+d+e+f}{4}$ c) $\frac{b+c+d+e}{4}$ d) $\frac{a+b+c+d}{4}$

Solution:- a) $\frac{a+c+d+e}{4}$

12 For the given five values 15, 24, 18, 33, 42, the three years moving averages are

a) 19, 25, 33 b) 19, 25, 31 c) 19, 30, 31 d) 19, 22, 33

Solution:- b) 19, 25, 31

13 The graph of time series is called:

a) Ogive b) Histogram c) Straight line d) Historigram

Solution:- b) Histogram

14 In the measurement of the secular trend, the moving averages:

a) Given the trend in a straight line b) Measure the seasonal Variations

c) Smooth out the time series d) Measure the time Variations

Solution:- **(c)** Smooth out the time series

Explanation: Moving averages is a series of arithmetic means of variate values of a sequence. This is another way of drawing a smooth curve for a time series data.

15 The straight-line trend is represented by the equation:

a) y_c = a - bx b) y_c = na - $b\Sigma x$ c) y_c = a + bx d) y_c = na + $b\Sigma x$

Solution:- c) y_c = a + bx

16 Moving average method is used for measurement of trend when:

a) Trend is curvilinear b) Trend is linear c) Trend is non - curvilinear d) Trend is non - linear

Solution:- b) Trend is linear

When trend is linear, then only we use moving average method for measurement.

17 Prosperity, Recession, and depression in a business is an example of:

a) Cyclical Trend b) Irregular Trend c) Seasonal Trend d) Secular Trend

Solution:- a) Cyclical Trend

There are 4 phases through which trade cycles are passed. They are prosperity, recession, depression, and recovery. In economic terms, these 4 stages are called economic fluctuations.

18 The most commonly use mathematical method for measuring the trend is:

a) Semi average method b) Time series method

c) Method of least squares d) Moving average method

Solution:- c) Method of least squares

19 The most commonly used mathematical method form ensuring the trends

(a) Semi Average (b)Moving Average(c)Free Hand Curve (d)Least Squares

Solution:- (d)Least Squares

20 In the measurement of the secular trend ,the moving averages

(a) Smooth out the time series (b)Give the trend in a straight line

(b) Measure the seasonal variations (d)None of these

Solution:- (a) Smooth out the time series

ASSERTION AND REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

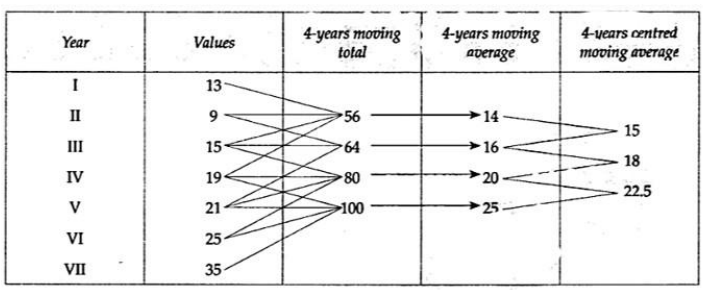
e) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
f) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
g) Assertion (A) is true but Reason(R) is false.
h) Assertion (A) is false but Reason(R) is true.

1 **Assertion (A):** The rise and fall of share market is an example of cyclic trend.
Reason (R): The price of stock in share market repeat after definite time interval.
Solution:- c) A is true but R is false.

2 **Assertion (A):** The sales of sweets, gift items, gold and silver etc. exhibit seasonal trend.
Reason (R): Sweets, gift items, gold and silver etc. are in great demand during Diwali.
Solution:- a) Both A and R are true and R is the correct explanation of A.

3 **Assertion (A):** For the given seven values 13, 9, 15, 19, 21, 25, 35, 4 - years centred moving averages are 15, 18, 22.5.
Reason (R): The additive model of the time series analysis is O =T + C + S + I
where O = Original data, T = Trend component, C = Cyclic component, S = Seasonal component and I = Irregular component.

Solution:- **(b)** Both A and R are true but R is not the correct explanation of A.



Explanation: ∴ Assertion is true

Reason is true but Assertion is not the correct explanation of Assertion.

4 **Assertion (A):** If for the following data:

Year	2010	2011	2012	2013	2014	2015	2016
Sales (in₹crores)	6	8	9	11	13	17	20

the equation of the straight line trend is y = 12 + 2.29x, then the trend value for the year 2017 is 21.16

Reason (R): If $(t_1, y_1),(t_2, y_2),...,(t_n, Y_n)$ denote the time series and Y_t are the trend values of thevariables y, then $\sum (y - y_t) = 0$.

Solution:- **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Given the equation of straight line trend is

y = 12 + 2.29x. For the year 2017, x = 2017 - 2013 = 4

So, trend value for the year 2017 y_t = 12 + 2.29 × 4
= 12 + 9.16 = 21.16

5 **Assertion (A):** If for the following data:

Year	2001	2002	2003	2004	2005	2006	2007	2008
Production (in tonnes)	8	7	6	5	4	7	8	5

the equation of the straight line trend is y = 6.25 - 0.167x, then the trend value for the year 2006 is 3.475.

Reason (R): The sum of squares of the deviations of the values of y from their corresponding trend values is least i.e $\sum (y - y_t)^2$ is least.

Solution:- d) A is false but R is true.

6 **Assertion (A):** For the given six values 17, 22, 21, 35, 40, 51, the three years moving averages are 20, 26, 32, 42.

Reason (R): If $x_1, x_2, x_3, \ldots, x_n$ is the given annual time series, then 3 - yearly moving averages are $\frac{x_1+x_2+x_3}{3}, \frac{x_2+x_3+x_4}{3}, \ldots$

Solution:- **(a)** Both A and R are true and R is the correct explanation of A.

7 **Assertion (A):** The rise and fall of share market is an example of cyclic trend.

Reason (R): The price of stock in share market repeat after definite time interval.

Solution:- (C) A is true but R is false.

8 **Assertion (A):** For the given six values 17, 22, 21, 35, 40, 51, the three years moving averages are 20, 26, 32, 42.

Reason (R): If $x_1, x_2, x_3, \ldots, x_n$ is the given annual time series, then 3 - yearly moving averages are $\frac{x_1+x_2+x_3}{3}, \frac{x_2+x_3+x_4}{3}, \ldots$

Solution: - (A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).

9 **Assertion (A):** If for the following data:

Year	2010	2011	2012	2013	2014	2015	2016
Sales (in ₹crores)	6	8	9	11	13	17	20

the equation of the straight line trend is y = 12 + 2.29x, then the trend value for the year 2017 is 21.16

Reason (R): If $(t_1, y_1),(t_2, y_2),\ldots,(t_n, Y_n)$ denote the time series and Y_t are the trend values of thevariables y, then $\sum (y - y_t) = 0$.

Solution: - **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Given the equation of straight line trend is

y = 12 + 2.29x. For the year 2017, x = 2017 - 2013 = 4

So, trend value for the year 2017 y_t = 12 + 2.29 × 4

= 12 + 9.16 = 21.16

10 **Assertion (A):** For the given six values 17, 22, 21, 35, 40, 51, the three years moving averages are 20, 26, 32, 42.

Reason (R): If $x_1, x_2, x_3, \ldots, x_n$ is the given annual time series, then 3 - yearly moving averages are $\frac{x_1+x_2+x_3}{3}, \frac{x_2+x_3+x_4}{3}, \ldots$

Solution: - **(a)** Both A and R are true and R is the correct explanation of A.

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)

1 Define time series.

Solution: - A Time Series consists of data arranged chronologically - Croxton Cowden.

When quantitative data are arranged in the order of their occurrence, the resulting series is called the Time Series. - Wessel Wallet.

2 Mention the components of the time series.

Solution: - There are four types of components in a time series.

Secular Trend

Seasonal variations

Cyclic variations

Irregular variations

3 What are the applications of time series analysis in real field?

Solution:- Time series analysis is used for many applications such as: Economic Forecasting, sales forecasting, Budgetary Analysis etc.

4 What is business forecasting?

Solution: - Business forecasting refers to the tool and techniques used to predict developments in business, such as sales, expenditures and profits. The purpose of business forecasting is to develop better strategies based on these informed predictions.

5 Define secular trend and seasonal variations.

Solution: - **Secular Trend:** It is a general tendency, of time series to increase or decrease or stagnate during a long period of time. An upward tendency is usually observed in the population of a country, production, sales, prices in industries, the income of individuals, etc., A downward tendency is observed in deaths, epidemics, prices of electronic gadgets, water sources, mortality rate etc.

Seasonal Variations: Seasonal variations refer to the changes that take place due to the rhythmic forces which operate in a regular and periodic manner. These forces usually have the same or most similar pattern year after year. When we record data weekly, monthly or quarterly, we can see and calculate seasonal variations. Thus, when a time series consists of data only based on annual figures, there will be seen no seasonal variations. These variations may be due to seasons, weather conditions, habits, customs or traditions. For example, selling of umbrellas and raincoats in the rainy season, sales of cold drinks in the summer season, crackers in the Deepawali season, purchase of dresses in a festival season, sugarcane in Pongal season.

6 What causes secular trend?

Solution: - The effect of long term causes is reflected in the tendency of a behaviour, to move in an upward or downward direction, termed as Trend or Secular Trend. The secular trends could reflect environmental improvements, specifically changes in health practice and living conditions leading to improvements in mortality rates and life expectancy. These factors are interrelated with those concerning family size.

7 The following table shows the annual rainfall (in mm) recorded for Cherrapunji, Meghalaya:

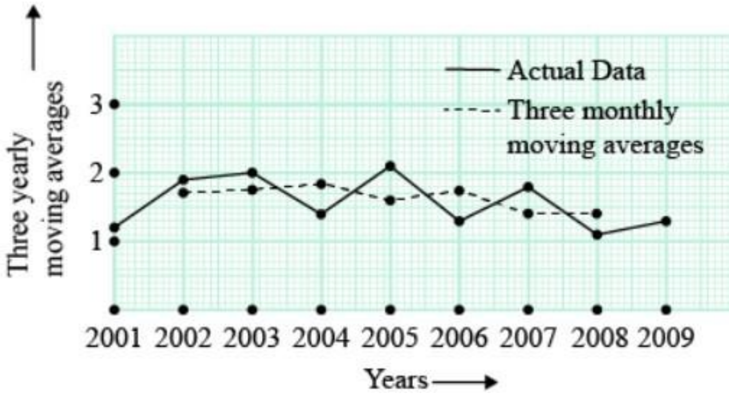
Year	Rainfall (in mm)
2001	1.2
2002	1.9
2003	2
2004	1.4
2005	2.1
2006	1.3
2007	1.8
2008	1.1
2009	1.3

Determine the trend of rainfall by 3 - year moving average.

Solution: -

Year	Rainfall (in mm)	Three Yearly Moving total	Three yearly Moving Average
2001	1.2		
2002	1.9	5.1	1.7
2003	2	5.3	1.76
2004	1,4	5.5	1.83
2005	2.1	4.8	1.6
2006	1.3	5.2	1.73
2007	1.8	4.2	1.4
2008	1.1	4.2	1.4
2009	1.3		

The points are joined by a line segment to obtain the graph to understand the trend.



8 What is the need for studying time series?

Solution:- We should study time series for the following reasons:

i. It helps in the analysis of past behaviour.

ii. It helps in forecasting and for future plans.

iii. It helps in the evaluation of current achievements.

iv. It helps in making comparative studies between one time period and others.

Therefore time series helps us to study and analyze the time-related data which involves in business fields, economics, industries, etc.

9 Calculate five yearly moving averages of the number of students who have studied in a school given below:

Year	1993	1994	1995	1996	1997	1998	1999	2000	20
No. of students	442	427	467	502	512	515	520	527	51

Year	Number of students	5-year moving total	5-year moving average
1993	442	—	—
1994	427	—	—
1995	467	2350	470
1996	502	2423	484.6
1997	512	2516	503.2
1998	515	2576	515.2
1999	520	2589	517.8
2000	527	2618	523.6
2001	515		
2002	541		

$\frac{1}{5}$

Solution:-

Calculation of 5-year moving averages:

10 Daily absence from a school during 3 weeks is recorded as follows:

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	23	28	21	33	40
Week 2	38	52	43	58	63
Week 3	52	54	61	51	51

Draw a graph, illustrating these figures. Calculate 5 day moving averages and plot them on the same graph.

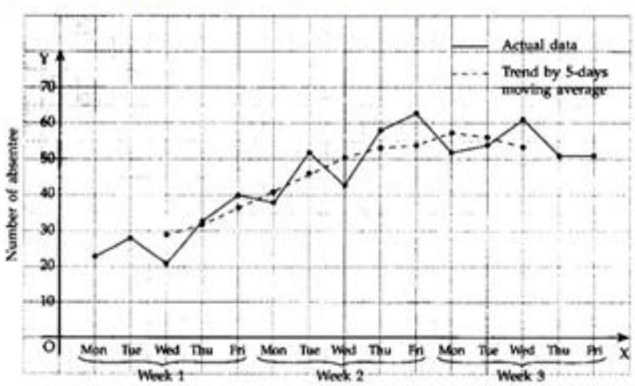
Solution:-

Calculation of 5-day moving averages:

	Day	Number of absentee	5-days moving total	5-days moving average
Week 1	Monday	23	—	—
	Tuesday	28	—	—
	Wednesday	21	145	29
	Thursday	33	160	32
	Friday	40	184	36.8
Week 2	Monday	38	206	41.2
	Tuesday	52	231	46.2
	Wednesday	43	254	50.8
	Thursday	58	268	53.6
	Friday	63	270	54
Week 3	Monday	52	288	57.6
	Tuesday	54	281	56.2
	Wednesday	61	269	53.8
	Thursday	51		
	Friday	51		

(Moving totals are divided by $\frac{1}{5}$ → moving averages.)

We get the following graph from the above data:



11 State the uses of time series.

Solution: - A time-series carries profound importance in business and policy planning. It is used to compare the current trends with that in the past or the expected trends. Thus it gives a clear picture of growth or downfall. Most of the time series data related to fields like

Economics, Business, Commerce etc. For example the production of a product, cost of a product, sales of a product, national income, salary of an individual etc. By close observation of time series data, one can predict and plan for future operations in industries and other fields.

12 State the two normal equations used in fitting a straight line.

Solution:- The normal equations used in fitting a straight line are

$\Sigma Y = na + b\Sigma X$ and

$\Sigma XY = a\Sigma X + b\Sigma X^2$

where, n = number of years given in the data,
X = time Y = actual value a, b = constants

13 Assuming a four yearly cycle, calculate the trend by the method of moving averages from the following data:

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992
Value	12	25	39	54	70	87	105	100	82

Solution:-

Calculation of 4-year centred moving average:

Year	Value	4-yearly moving total	4-yearly moving average	4-yearly centre moving average
1984	12			
1985	25			
		130	32.5	
1986	39			39.75
		188	47	
1987	54			54.75
		250	62.5	
1988	70			70.75
		316	79	
1989	87			84.75
		362	90.5	
1990	105			92
		374	93.5	
1991	100			90.75
		352	88	
1992	82			
1993	65			

14 The Production of cement by a firm in year 1 to 9 is given below:

Year	1	2	3	4	5	6	7	8	9
Production in (Tonnes)	4	5	5	6	7	8	9	8	10

Calculate the trend values for the above series by the 3 - yearly moving average method.

Solution:-

1. To calculate the trend values, we make the following table

Year	Production (in Tonnes)	Three yearly moving totals	Three yearly moving averages
1	4	-	-
2	5	14	4.67
3	5	16	5.33
4	6	18	6

5	7	21	7
6	8	24	8
7	9	25	8.33
8	8	27	9
9	10	-	-

15 Calculate the 3 - yearly moving averages of the following data:

Year	1	2	3	4	5	6	7
Value	2	4	5	7	8	10	13

Solution:-

Calculation of 3-year moving averages:

Year	Value	3-year moving total	3-year moving average
1	2		
2	4	11	3.667
3	5	16	5.333
4	7	20	6.667
5	8	25	8.333
6	10	31	10.333
7	13		

($\frac{1}{3}$)

16 Using three - yearly moving averages, compute the trend values and short term fluctuations for the following data:

Year:	2008	2009	2010	2011	2012	2013	2014	2015
Production (Thousand tonnes):	21	22	23	25	24	22	25	26

Solution:-

Computation of trend values

Year	Production (Thousand tonnes)	3-yearly moving totals	3-yearly moving averages Y_c	Short term fluctuations ($Y - Y_c$)
2008	21	-	-	-
2009	22	66	22.00	0
2010	23	70	23.33	-0.33
2011	25	72	24.00	1.00
2012	24	71	23.67	0.33
2013	22	71	23.67	-1.67
2014	25	73	24.33	0.67
2015	26	78	26.00	0.00
2016	27	79	26.33	0.67
2017	26	-	-	-

17 Calculate the 3 - year moving averages for the loan (in lakh₹) issued by co - operative banks for farmers in different states of India based on the values given below.

Year	2006	2007	2008	2009	2010	2011	2012	2013
Loan amount (in lakh ₹)	41.85	40.2	38.12	26.5	55.5	23.6	28.36	33.31

Solution:-

Year	Loan Amount	Three yearly moving totals	Three yearly moving average
2006	41.85	-	-
2007	40.2	120.17	$\frac{120.17}{3} = 40.06$
2008	38.12	104.82	34.94
2009	26.5	120.12	40.04
2010	55.5	105.6	35.2
2011	23.6	107.46	35.82
2012	28.36	85.27	28.42
2013	33.31	102.77	34.26
2014	41.1	-	-

The graph shows the observation data in pink whereas, the black curve shows the smooth trend curve obtained by calculating moving averages of 3 years.

18 Compute the trends by the method of moving averages, assuming that 4-year cycle is present in the following series.

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989
Index number	400	470	450	410	432	475	461	500	480	430

Solution: The 4- year moving averages are shown in the last column as centered average

Year	Index Number	4-year Moving total	4-year Moving Average	Centered total	Centered moving average
1980	400			-	-
		-	-		
1981	470			-	-
		1730	1730/4 = 432.5.		
1982	450			875.5 →	875.5/2 = 437.75
		1762	1772/4 = 443		
1983	410			884.75	884.75/2 = 442.38
		1767	1767/4 = 441.75		
1984	432			886.25	443.13
		1778	1778/4 = 444.5		
1985	475			911.5	455.75
		1868	1868/4 = 467		
1986	461			946	473
		1916	1916/4 = 479		

19 Calculate the 3-year moving averages for the loans (In lakh ₹) issued by co-operative banks for farmers in different states of India based on the values given below.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
Loan amount (In lakh ₹)	41.85	40.2	38.12	26.5	55.5	23.6	28.36	33.31	41.1

Solution:

Year	Loan amount	3- year moving total	3- year moving average
2006	41.85	-	-
2007	40.2	→ 120.17	→ 120.17/3 = 40.6
2008	38.12	104.82	34.94
2009	26.5	120.12	40.4
2010	55.5	105.6	35.2
2011	23.6	107.46	35.82
2012	28.36	85.27	28.4
2013	33.31	102.77	34.26
2014	41.1	-	-

20 In an influenza epidemic the number of cases diagnosed were:

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Numbers	2	0	5	12	20	27	46	30	31	18	11	5	0	1

Calculate 3-days moving averages.

SHORT ANSWER TYPE QUESTION (3 MARKS)

1 i. Obtain the three year moving averages for the following series of observations:

Year	1995	1996	1997	1998	1999	2000	2001	2002
Annual Sales (in 000₹)	3.6	4.3	4.3	3.4	4.4	5.4	3.4	2.4

ii. Obtain the five year moving average.

iii. Construct also the 4 - year centred moving average.

Solution:-

i. First 3-year moving average is $\frac{3.6+4.3+4.3}{3} = \frac{12.2}{3} = 4.067$, and is placed against 2nd year i.e. 1996; second 3-year moving average is $\frac{4.3+4.3+3.4}{3} = \frac{12.0}{3} = 4.0$, and is placed against 3rd year i.e. 1997, and so on. Thus, we have:

Calculation of 3-year moving averages:

Year	*Annual sale*	*3-year moving total*		*3-year moving average*
1995	3.6	–	1/3	–
1996	4.3	12.2	→	4.067
1997	4.3	12.0	→	4.00
1998	3.4	12.1	→	4.03
1999	4.4	13.2	→	4.40
2000	5.4	13.2	→	4.40
2001	3.4	11.2	→	3.73
2002	2.4	–		–

ii. First 5-yearly moving average is $\frac{3.6+4.3+4.3+3.4+4.4}{5} = \frac{20.0}{5} = 4.00$, and is placed against 3rd year i.e. 1997. Second 5-yearly moving average is $\frac{4.3+4.3+3.4+4.4+5.4}{5} = \frac{21.8}{5} = 4.36$, and is placed against 4th year i.e. 1998, and so on. Thus, we have:

Calculation of 5-year moving averages:

Year	Annual sale	5-year moving total		5-year moving average
1995	3.6	–		–
1996	4.3	–	1/5	–
1997	4.3	20.0	→	4.00
1998	3.4	21.8		4.36
1999	4.4	20.9		4.18
2000	5.4	19.0		3.80
2001	3.4	–		–
2002	2.4	–		–

iii. In the 4-year moving averages, the first step of averaging of 4 values each results in placing these between years — so we take averages of each two successive moving averages to synchronise them with given time frame. Thus, we have the following table:

Construction of 4-year centred moving averages

Year	Annual Sale total	4-yearly moving total	4-yearly moving average	4-year centred moving average
1995	3.6			
1996	4.3			
		15.6	3.9	
1997	4.3			4.0
		16.4	4.1	
1998	3.4			4.2375
		17.5	4.375	
1999	4.4			4.2652
		16.6	4.15	
2000	5.4			4.025
		15.6	3.9	
2001	3.4			
2002	2.4			

Note that values of 4th column are not synchronised with first column, but values of 5th column are synchronised.

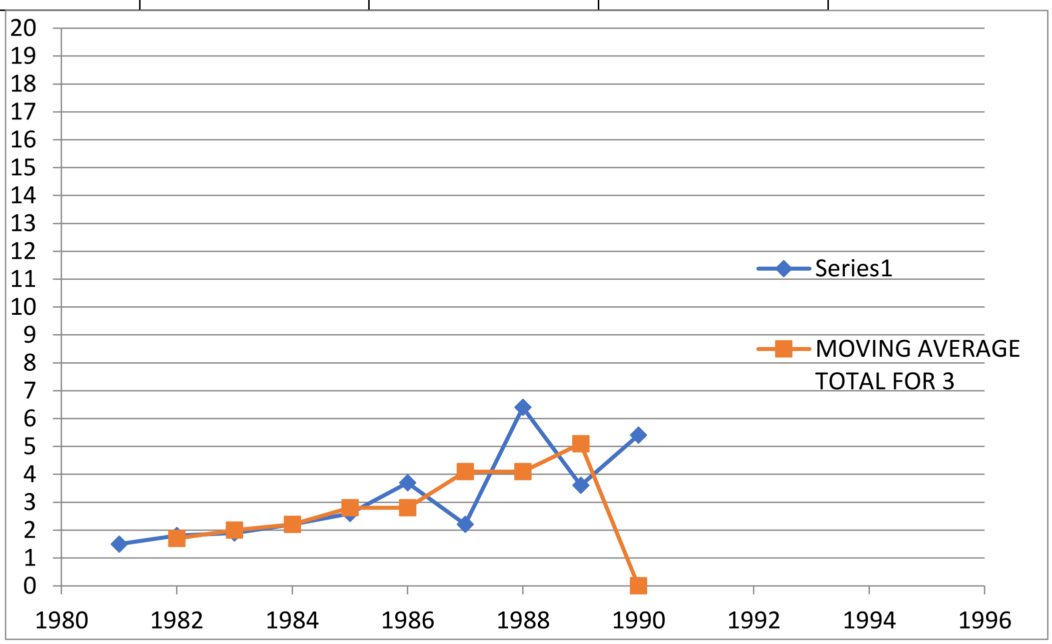
2 The average number, in lakhs, of working days lost in strikes during each year of the period (1981 - 90) was as under:

1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate the three - yearly moving average and draw the moving average graph.

Solution:-

TIME	DATA	MOVING TOTAL FOR 3 YEARS	MOVING AVERAGE TOTAL FOR 3
1981	1.5		
1982	1.8	5.2	1.7
1983	1.9	5.9	2
1984	2.2	6.7	2.2
1985	2.6	8.5	2.8
1986	3.7	8.5	2.8
1987	2.2	12.3	4.1
1988	6.4	12.2	4.1
1989	3.6	15.4	5.1
1990	5.4		



3 Fit a straight-line trend by the method of least square to the following data on sales (in lakhs) for the period 2011 - 2018.

Years	2011	2012	2013	2014	2015	2016	2017	2018
Sales (₹ lakhs)	76	80	130	144	138	120	174	190

Solution:-

Year (t_i)	Sales (lakhs)(y_i)	$x_i = t_i$ - 2014 - 15	$x_i \times 2$	$x_i y_i$	x_i^2
2011	76	-3.5	-7	-532	49
2012	80	-2.5	-5	-400	25
2013	130	-1.5	-3	-390	9
2014	144	-0.5	-1	-144	1
2015	138	0.5	1	138	1
2016	120	1.5	3	360	9
2017	174	2.5	5	870	25
2018	190	3.5	7	1330	49
	$\Sigma y_i = 1052$	$\Sigma x_i = 0$		$\Sigma x_i y_i = 1232$	$\Sigma x_i^2 = 168$

$a = \frac{\Sigma y_i}{n} = \frac{1052}{8} = 131.5$

$b = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{1232}{168} = 7.33$

So, trend equation is

$y = 131.5 + 7.33x$

4 The table given below shows the daily attendance in thousands at a certain exhibition over a period of two weeks:

Week 1	52	48	64	68	52	70	72
Week 2	55	47	61	65	58	75	81

Calculate seven days moving averages and illustrate these and original information on the same graph using the same scales.

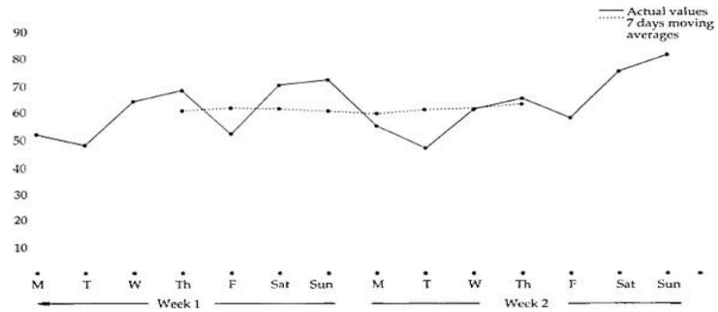
Solution:-

Calculation of seven days moving averages

	Days	**Attendance (in thousands)**	**7 days moving totals**	**7 days movi averages**
Week 1	Monday	52	-	-
	Tuesday	48	-	-
	Wednesday	64	-	-
	Thursday	68	426	60.86
	Friday	52	429	61.28
	Saturday	70	428	61.14
	Sunday	72	425	60.17

Week 2	Monday	55	422	60.28	
	Tuesday	47	428	61.14	
	Wednesday	61	433	61.86	
	Thursday	65	442	63.14	
	Friday	58	-	-	
	Saturday	75	-	-	
	Sunday	81	-	-	

Seven days moving averages and original data are plotted on the graph paper in Fig. These points are joined by line segments to obtain the graphs to illustrate the trend.



5 The production of soft drink company in thousands of litres during each month of a year is as follows:

Jan	Feb	March	April	May	June	July	August	Sept.	Oct.	No
1.2	0.8	1.4	1.6	1.8	2.4	2.6	3.0	3.6	2.8	1.9

Calculate the five monthly moving averages and show these moving averages on a graph.

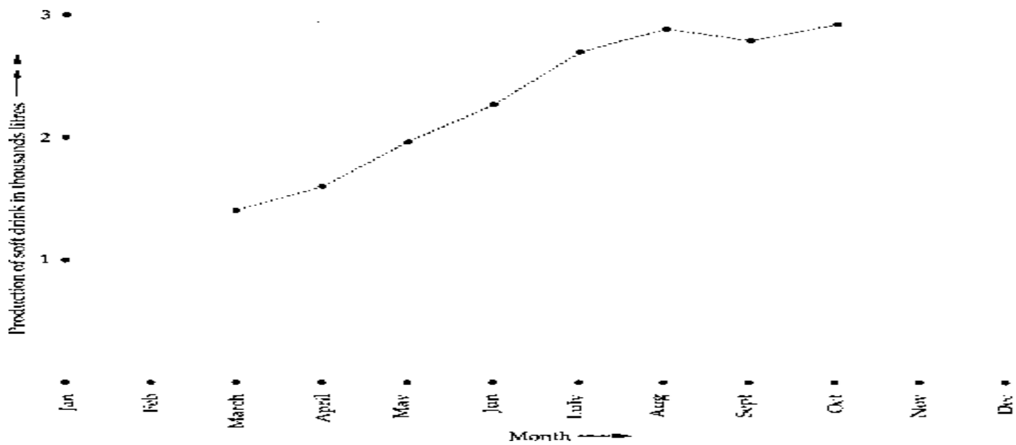
Solution:-

Calculation of five monthly moving averages:

Month	**Production of soft drink (in thousands litres)**	**5 monthly totals**	**5 monthly moving averages**
January	1.2	-	-
February	0.8	-	-
March	1.4	6.8	1.36
April	1.6	8.0	1.6
May	1.8	9.8	1.96
June	2.4	11.4	2.28
July	2.6	13.4	2.68
August	3.0	14.4	2.88
September	3.6	13.9	2.78
October	2.8	14.7	2.94

November	1.9	-	-
December	3.4	-	-

These moving averages are plotted on the following graph:



6 The average number, in lakhs, of working days lost in strikes during each year of the period 2001 - 2010 was

2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

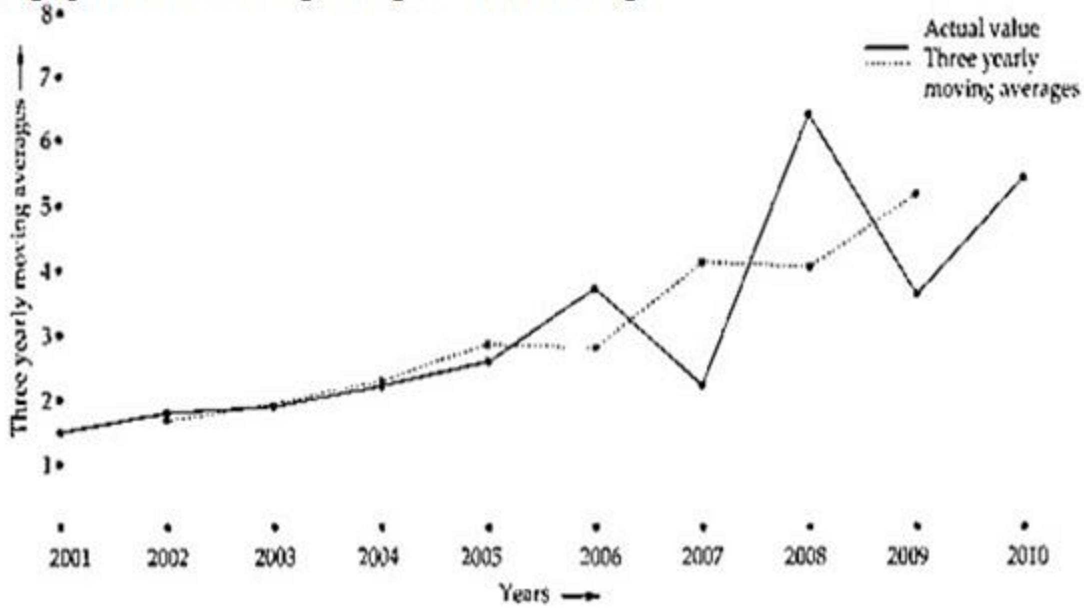
Calculate the three - yearly moving averages and draw the moving averages graph.

Solution:-

In order to calculate three-yearly moving averages, we first compute three-yearly moving totals and place each total against the middle year of the three-year span from which the moving totals are calculated. These moving totals are given in the third column of the following table. From these three yearly moving totals, we calculate three-yearly moving averages by dividing each moving total by 3 as shown in the following table.

Year	**Working days lost in strikes (in lakhs)**	**Three yearly moving totals**	**Three yearly moving averages**
2001	1.5	-	-
2002	1.8	5.2	1.73
2003	1.9	5.9	1.96
2004	2.2	6.7	2.23
2005	2.6	8.5	2.83
2006	3.7	8.5	2.83
2007	2.2	12.3	4.1
2008	6.4	12.2	4.06
2009	3.6	15.4	5.13
2010	5.4	-	-

The graph of these moving averages is shown in Fig.



7 Fit a straight line trend by the method of least squares to the data given below:

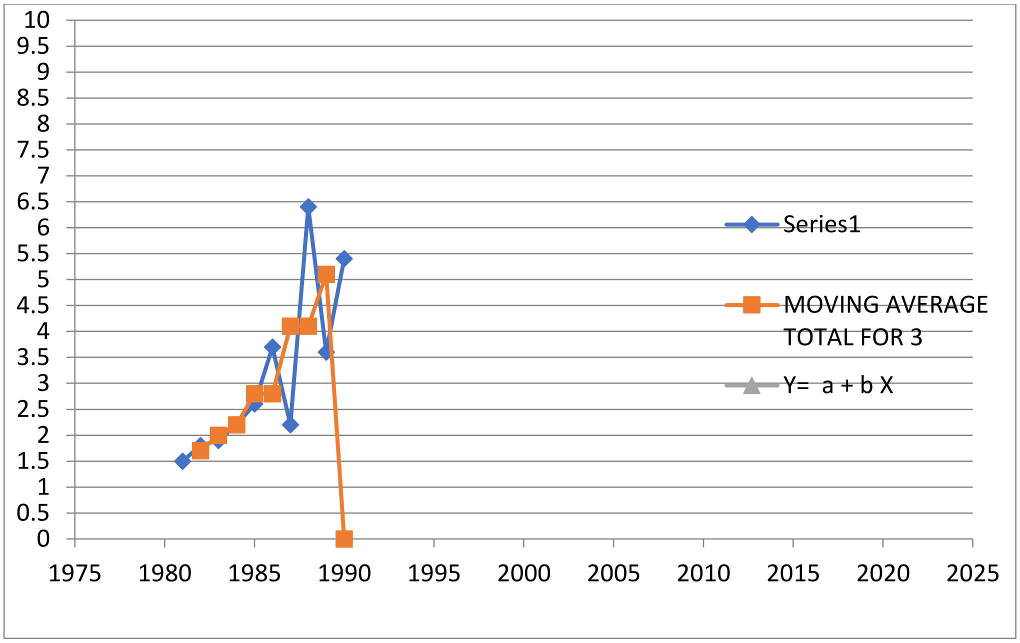
Years	2012	2013	2014	2015	2016	2017	2018
Sales (in tones)	9	11	13	12	14	15	17

Solution:-

Year(Xi)	Index(Yi)	X =Xi -A	X^2	XY	Y = a + bX
2012	9	-3	9	-27	9.4
2013	11	-2	4	-22	10.6
2014	13	-1	1	-13	11.8
2015	12	0	0	0	13
2016	14	1	1	14	14.2
2017	15	2	4	30	15.4
2018	17	3	9	51	16.6

$\Sigma Y =$ $\Sigma X =$ 28 33

9 0



8 Fit a straight line trend by the method of least squares to the following data:

Year	Sales (in lakh ₹)
2010	65
2012	68
2013	70
2014	72
2015	75
2016	67
2019	73

Solution:-

Fitting of a straight-line trend by the method of least squares to the following data:

Year (x)	**Sales (Y) (in lakh)**	X = x - 2014	X^2	XY	$Y_t = a + bX$
2010	65	-4	16	-260	70 + 4.745 (-4) = 70 - 18.98= **51.02**
2012	68	-2	4	-136	70 + 4.745 (-2) = 70 - 9.49 = **60.51**
2013	70	-1	1	-70	70 + 4.745 (-1) = 70 - 4.745= **65.255**
2014	72	0	0	0	70 + 4.745 (0) = 70 + 0 = **70**

2015	75	1	1	75	70 + 4.745 (1) = 70 + 4.745= **74.745**
2016	67	2	4	268	70 + 4.475 (2) = 70 + 9.49 = **79.49**
2019	73	5	25	365	70 + 4.475 (5) = 70 + 23.725 = **93.725**
n=7	$\sum Y = 490$	$\sum X = 1$	$\sum X^2 = 51$	$\sum XY = 242$	

9 From the following data calculate the 4 - yearly moving averages and determine the trend values.

Years	2012	2013	2014	2015	2016	2017	2018	2019	2020
Value	50.0	36.5	43.0	44.5	38.9	38.9	32.6	41.7	41.1

Solution:-

Year	**Value**	**4-yearly centered Moving Total**	**4-yearly Moving Average (Trend values)**	**4-yearly centered Moving Average**
2012	50.0	-		
2013	36.5	-		
		174.0	43.5	
2014	43.0	-		42.12
		162.9	40.73	
2015	44.5	-		41.03
		165.3	41.33	
2016	38.9	-		40.03
		154.9	38.73	
2017	38.9	-		38.38
		152.1	38.03	
2018	32.6	-		38.31
		154.3	38.58	
2019	41.7	-		37.94
		149.2	37.3	
2020	41.1	-		
2021	33.8	-		

10 Calculate trend values from the following data assuming 5 - yearly and 7 - yearly moving average.

Year	1	2	3	4	5	6	7	8
Value	110	104	98	105	109	120	115	110
Year	9	10	11	12	13	14	15	16
Value	114	122	130	127	122	118	130	140

Solution:-

Year	**Value**	**Moving Totals**		**Moving Average**	
		5 year	**7 year**	**5 year**	**7 year**
1	110	-	-	-	-
2	104	-	-	-	-
3	98	526	-	105.2	-
4	105	536	761	107.2	108.71
5	109	547	761	109.4	108.71
6	120	559	771	111.8	110.14
7	115	568	795	113.6	113.57
8	110	581	820	116.2	117.14
9	114	591	838	118.2	119.71
10	122	603	840	120.6	120.00
11	130	615	843	123.0	120.43
12	127	619	863	123.8	123.29
13	122	627	889	125.4	127.00
15	130	-	-	-	-
16	140	-	-	-	-

11 Consider the following data:

Year:	2003	2004	2005	2006	2007	2008	2009	2010
Production:	137	140	134	137	151	121	124	159

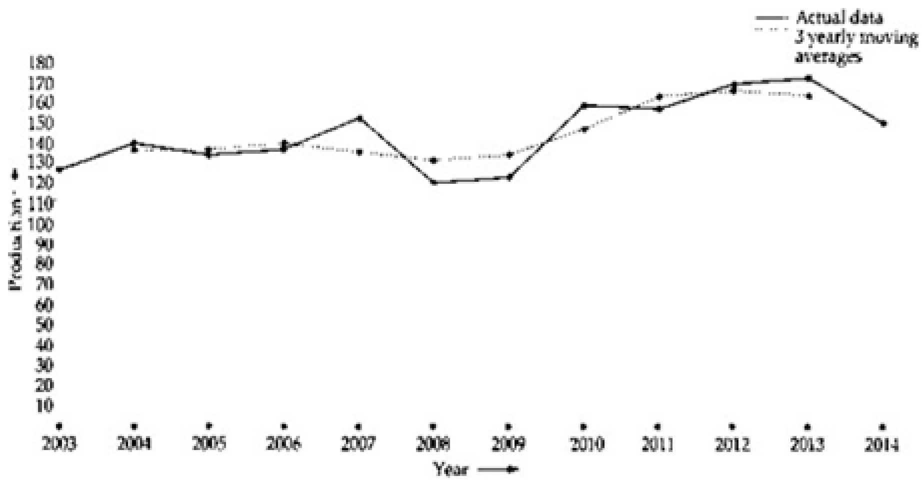
Calculate a suitable moving average and show on a graph against the original data.

Solution:-

Year	Production	3-yearly moving totals	3-yearly moving averages
2003	137	-	-
2004	140	411	137.00
2005	134	411	137.00

2006	137	422	140.67
2007	151	409	136.33
2008	121	396	132.00
2009	124	404	134.67
2010	159	440	146.67
2011	157	485	161.67
2012	169	498	166.00
2013	172	491	163.67
2014	150	-	-

These moving averages and the original data are plotted on the graph paper to obtain the following graph.



12 Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data:

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40
2007	36

Solution:-

Taking middle-year value at A i.e. A = 2004

Year (X_i)	Production (Y)	$X = x_i - A = x_i - 2004$	X^2	XY
2001	30	-3	9	-90
2002	35	-2	4	-70
2003	36	-1	1	-36
2004	32	0	0	0
2005	37	1	1	37
2006	40	2	4	80
2007	36	3	9	108
n = 7	$\sum y = 246$	$\sum x = 0$	$\sum x^2 = 28$	$\sum xy = 29$

$a = \frac{\sum y}{n} = \frac{246}{7} = 35.14$

$b = \frac{\sum xy}{\sum x^2} = \frac{29}{28} = 1.036$

Therefore, the required equation of the straight-line trend is given by Y = 35.14 + 1.036 X

Now, the trend value for the year 2008 is given by

Y = 35.14 + 1036(2008 - 2004) = 39.284

Hence the trend value for the year is 39.284 lakh tonnes.

LONG ANSWER TYPE QUESTIONS

1 Given below are the consumer price index numbers (CPI) of the industrial workers.

Year	2014	2015	2016	2017	2018	2019	2020
Index number	145	140	150	190	200	220	230

Find the best fitted trend line by the method of least squares and tabulate the trend values.

Solution:-

YEAR (xi)	INDEX (yi)	X= xi - A	X SQU	XY	Y= a + b X
2017	80	-2	4	-160	83.6
2018	90	-1	1	-90	85.7
2019	92	0	0	0	87.8
2020	83	1	1	83	89.9
2021	94	2	4	188	92
ΣY = 439		ΣX = 0	10	21	

Y= a + b X

$\Sigma Y = n a + b \Sigma X$ ------------------- (ii)

$\Sigma XY = a \Sigma X + b \Sigma X^2$ ------------------- (iii)

$$b = \frac{\frac{\sum X \sum Y}{n} - \sum X \sum Y}{\frac{(\sum X)^2}{n} - \sum X^2} \qquad a = \frac{\sum Y - b\sum X}{n}$$

A = 87.8 and b= 2.1

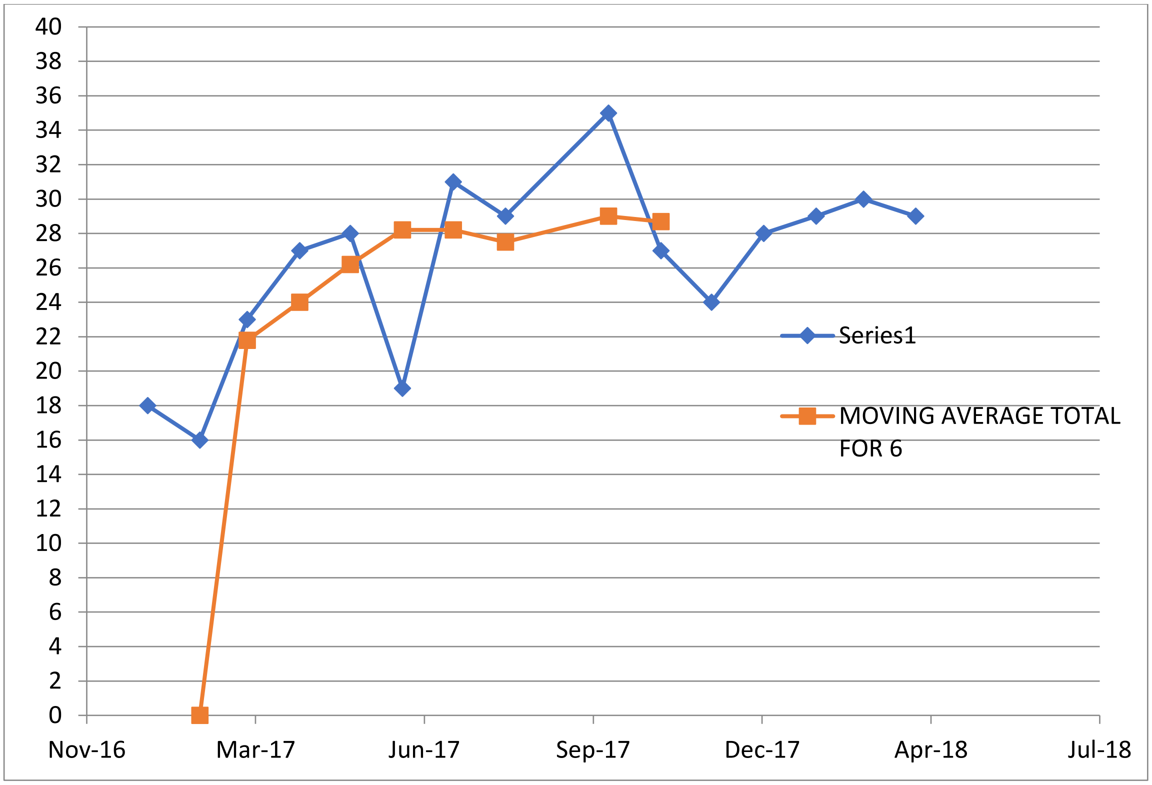
2 Coded monthly sales figures of a particular brand of T.V. for 18 months commencing January 1, 2017 are as follows:

Year 2017:	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct
	18	16	23	27	28	19	31	29	35	27
Year 2018:	24	28	29	30	29	22				

Calculate six - monthly moving averages and display these and the original figures on the same graph, using the same axes for both.

Solution:-

MONTH	MONTHLY SALES	MOVING TOTAL FOR 6 MONTHS	MOVING AVERAGE TOTAL FOR 6
Jan-17	18		
Feb-17	16		
Mar-17	23	131	21.8
Apr-17	27	144	24
May-17	28	157	26.2
Jun-17	19	169	28.2
Jul-17	31	169	28.2
Aug-17	29	165	27.5
Oct-17	35	174	29
Nov-17	27	172	28.7
Dec-17	24	173	28.8
Jan-18	28	167	27.8
Feb-18	29	162	27
Mar-18	30		
Apr-18	29		
May-18	22		

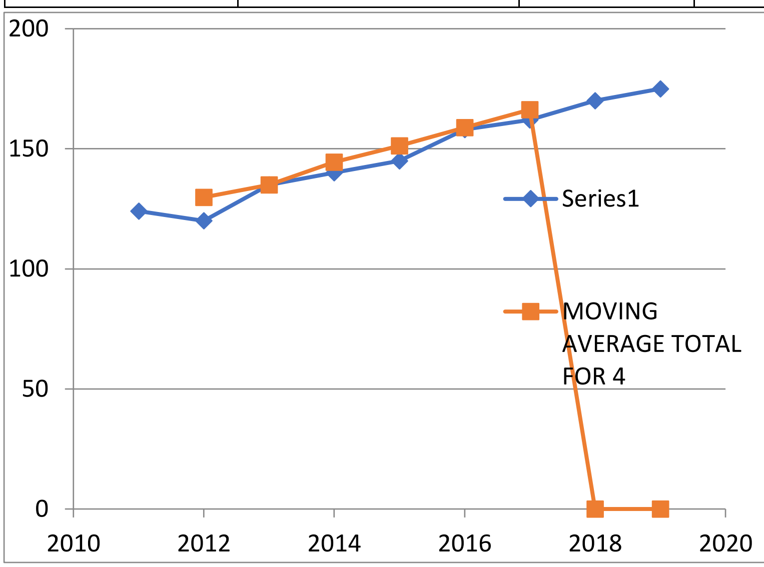


3. Calculate four - yearly moving averages of number of students studying in a higher secondary school in a particular city from the following data:

Year	Number of Students
2011	124
2012	120
2013	135
2014	140
2015	145
2016	158
2017	162
2018	170
2019	175

SOLUTION:-

TIME	DATA	MOVING TOTAL FOR 4 YEARS	MOVING AVERAGE TOTAL FOR 4
2011	124		
2012	120	519	129.8
2013	135	540	135
2014	140	578	144.5
2015	145	605	151.3
2016	158	635	158.8
2017	162	665	166.3
2018	170		
2019	175		



4 Construct 5 - year Moving averages from the following data of the number of industrial failure in a country during 2003 - 2018:

Year	No. of Failures	Year	No. of Failure
2003	23	2011	9
2004	26	2012	13
2005	28	2013	11
2006	32	2014	14
2007	20	2015	12
2008	12	2016	9
2009	12	2017	3
2010	10	2018	1

Year	No. of failures	5-Yearly Moving Totals	5-Yearly Moving Averages
2003	23	-	-
2004	26	-	-
2005	28	129	25.8
2006	32	118	23.6
2007	20	104	20.8
2008	12	86	17.2
2009	12	63	12.6
2010	10	56	11.2
2011	9	55	11.0
2012	13	57	11.4
2013	11	59	11.8
2014	14	59	11.8
2015	12	49	9.8
2016	9	39	7.8
2017	3	-	-
2018	1	-	-

CASE BASED QUESTIONS

1 Read the following text carefully and answer the questions that follow:

When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

Mathematically, for finding a line of best - fit to represent a trend, many methods are available. Methods like moving - averages and least - squares are some of the techniques to predict such trends.

Mrs Shamita runs a bread factory and the record of her sales of bakery items for the period of 2015 - 2019 is as follows:

Year	2015	2016	2017	2018	2019
Sales (in ₹ thousands)	35	42	46	41	48

Based on the above information, answer the following questions. Show steps to support your answers.

IV. By taking year 2017 as origin, use method of least - squares to find the best - fit trend line equation for Mrs Shamita's business. Show the steps of your working. (1)

V. Demonstrate the technique to fit the best - suited straight-line trend by the method of 3 - year moving averages. Also, draw the trend line. (1)

VI. What are the estimated sales for Mrs Shamita' s business for year 2022? (2)

OR

Mrs Shamita wishes to grow her business to yearly sale of₹ 67400. In which year will she be able to reach her target? (2)

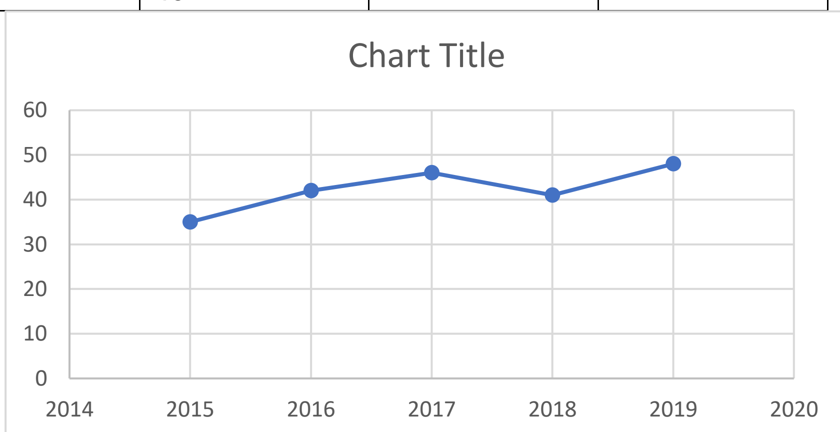
Solution:- I.:-

YEAR (xi)	INDEX (yi)	X= xi - A	X SQU	XY	Y= a + b X
2015	35	-2	4	-70	37.4
2016	42	-1	1	-42	39.9
2017	46	0	0	0	42.4
2018	41	1	1	41	44.9
2019	48	2	4	96	47.4

y = a x + b, a = 42.4 , b = 2.5

II.

TIME	DATA	MOVING TOTAL FOR 3 YEARS	MOVING AVERAGE TOTAL FOR 3
2015	35		
2016	42	123	41
2017	46	129	43
2018	41	135	45
2019	48		



III. Y = a + b x , at X = 2022 , x = 5 then

$Y_{(X=2022)}$ = 54.9

OR

Y = a + b (X-2017), at Y = 67.4

X = ((Y + a)/b) + A = 2027

2 Read the following text carefully and answer the questions that follow:

To fit a straight line by the method of least squares, Rohit constructed the following table:

Year (t)	Profit (y)	$x = t_i - 2018$	x^2	xy
2015	114	-3	9	-342
2016	130	-2	4	-260
2017	126	-1	1	-126
2018	144	0	0	0
2019	138	1	1	138
2020	156	2	4	312
2021	164	3	9	492

The trend equation can be considered as $y_t = a + bx$.

VII. What is the value of a in the trend equation? (1)

VIII. What is the value of b in the trend equation? (1)

IX. Which of the following is atrend equation? (2)

$y_t = 138.86 + 7.64x$, $y_t = 7.64 + 138.86x$, $y_t = 138.86 - 7.64x$

OR

What is the trend value for year 2015? (2)

$$b = \frac{\frac{\sum X \sum Y}{n} - \sum X \sum Y}{\frac{(\sum X)^2}{n} - \sum X^2} \qquad \boldsymbol{a = \frac{\sum Y - b\sum X}{n}}$$

Solution:- I. a = 138.86

II. b = 7.6

III. $y_t = 138.86 + 7.64x$ OR y (x= 2015) = 116.1

3 **Read the following text carefully and answer the questions that follow:**

The following data shows the percentage of rural, urban, and suburban Indians who have a high - speed internet connection at home.

Year	Rural	Urban	Suburban
2001	3	9	9
2002	6	18	17
2003	6	21	23
2004	16	29	29
2005	24	38	40

X. What is the straight-line trend by the method of least square for the suburban Indians? (1)

XI. What is the forecast for the year 2006 for the urban groups using the trend equation? (1)

XII. What is the forecast for the year 2006 for the rural groups using the trend equation? (2)

OR

What is the straight-line trend by the method of least square for the rural Indians? (2)

Solution:- I. Y= 23.6 + 7.4(X – 2003)

II. Y = 23 + 6.9 (X-2003)

at X = 2006 , Y = 43.7

III Y = 11 + 5.2 (X-2003)

at X = 2006 , Y = 26.6

OR Y = 11 + 5.2 (X-2003)

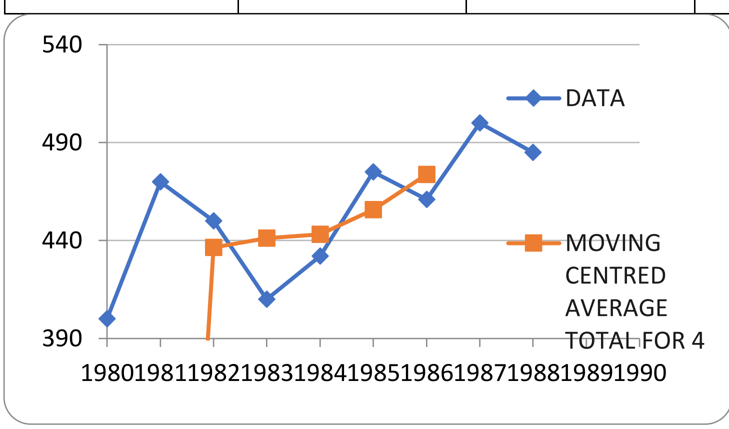
HIGHER ORDER THINKING SKILLS

1 Compute the trends by the method of moving averages, assuming that 4 - year cycle is present in the following series.

Year	1980	1981	1982	1983	1984	1985	1968	1987	19
Index number	400	470	450	410	432	475	461	500	48

Solution:-

TIME	DATA	MOVING TOTAL FOR 4 YEARS	MOVING AVERAGE TOTAL FOR 4	MOVING CENTRED AVERAGE TOTAL FOR 4
1980	400			
1981	470	1730	432.5	
1982	450	1762	440.5	436.5
1983	410	1767	441.8	441.2
1984	432	1778	444.5	443.2
1985	475	1868	467	455.8
1986	461	1921	480.3	473.7
1987	500			
1988	485			



Read the following text carefully and answer the questions that follow:

2 Today in class Mr. Sharma is teaching the Method of Least Squares to measure the trend in time series. After explaining the method, he had taken an example

Example: Given below are the data relating to the sales of a product in a district.

Fit a straight-line trend by the method of least squares and tabulate the trend values.

Year	2015	2016	2017	2018	2019	2020	2021	2022
Sales	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

To solve the given example, he constructed the following table:
Computation of trend values by the method of least squares.

Year (t)	Sales (y)	x = $\frac{(t-2018.5)}{0.5}$	XY	x ^{2}
2015	6.7	-7	-46.9	49
2016	5.3	-5	-26.5	25
2017	4.3	-3	-12.9	9
2018	6.1	-1	-6.1	1
2019	5.6	1	5.6	1
2020	7.9	3	23.7	9
2021	5.8	5	29.0	25
2022	6.11	7	42.7	49

Here, n = 8 (even)

So, origin is mean of two middle years i.e., $\frac{2018+2019}{2}$ = 2018.5

XIII. If the straight line is y_t = a + bx, then what is the value of **a?** (1)

XIV. If the straight line is y_t = a + bx, then what is the value of **b?** (1)

XV. Which of the following trend equation? (2)

$y_t = 5.975 + 0.051x$, $y_t = 0.051 + 5.975x$, $y_t = 5.975 - 0.051x$ (2)

Solution :- I. $b = \frac{\frac{\sum X \sum Y}{n} - \sum X \sum Y}{\frac{(\sum X)^2}{n} - \sum X^2}$ $\quad$ $\boldsymbol{a = \frac{\sum Y - b\sum X}{n}}$ so the value of a = 5.975

II. b = 0.051

III $y_t = 5.975 + 0.051x$

EXERCISE FOR TIME BASED DATA

MULTIPLE CHOICE QUESTIONS

1 An orderly set of data arranged in accordance with their time of occurrence is called:
a) Time Series
b) Arithmetic Series
c) Harmonic Series
d) Geometric Series

2 Increase in the number of patients in the hospital due to heat stroke is:
a) Secular trend
b) Cyclical variations
c) Seasonal variation
d) Irregular variation

3 A factory production is delayed for three weeks due to breakdown of a machine and unavailability of spare parts. Under which trend oscillation does this situation fall?
a) Seasonal
b) Secular
c) Irregular

d) Cyclical

4 The best - fitted trend line is one for which sum of squares of residuals or errors is:
a) Negative
b) Maximum
c) Positive
d) Minimum

5 A fire in a factory delaying production for some time is
a) Cyclical trend
b) Irregular trend
c) Seasonal trend
d) Long term trend

6 Seasonal variations are
a) Long term
b) Irregular
c) Short term
d) Sudden

7 The following are the movement(s) in the secular trend.
a) Steady
b) Regular
c) All of these
d) Smooth

8 For the given values 23, 32, 40, 47, 58, 33, 42; the 5 - yearly moving averages are:
a) 40, 42, 46
b) 38, 40, 42
c) 42, 44, 46
d) 40, 42, 44

9 In the measurement of the secular trend, the moving averages:
a) Given the trend in a straight-line   b) Measure the seasonal Variations
c) Smooth out the time series   d) Measure the time Variations

10 Which of the following is an example of time series problem?
iv. Estimating numbers of hotel rooms booking in next 6 months.
v. Estimating the total sales in next 3 years of an insurance company.
vi. Estimating the number of calls for the next one week.
a) (ii) and (iii)
b) (i), (ii) and (iii)
c) (i) and (ii)
d) Only (iii)

ASSERTION AND REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

A) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
B) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
C) Assertion (A) is true but Reason(R) is false.
D) Assertion (A) is false but Reason(R) is true.

1 **Assertion (A):** For the given seven values 13, 9, 15, 19, 21, 25, 35, 4 - years centred moving averages are 15, 18, 22.5.
Reason (R): The additive model of the time series analysis is O =T + C + S + I
where O = Original data, T = Trend component, C = Cyclic component, S = Seasonalcomponent and I = Irregular component.

2 **Assertion (A):** The rise and fall of share market is an example of cyclic trend.
Reason (R): The price of stock in share market repeat after definite time interval.

3 **Assertion (A):** The sales of sweets, gift items, gold and silver etc. exhibit seasonal trend.
Reason (R): Sweets, gift items, gold and silver etc. are in great demand during Diwali.

4 **Assertion (A):** If for the following data:

Year	2010	2011	2012	2013	2014	2015	2016
Sales (in₹crores)	6	8	9	11	13	17	20

the equation of the straight line trend is y = 12 + 2.29x, then the trend value for the year 2017 is 21.16
Reason (R): If (t_1 , y_1),(t_2, y_2),...,(t_n, Y_n) denote the time series and Y_t are the trend values of thevariables y, then $\sum (y - y_t) = 0$.

5 **Assertion (A):** If for the following data:

Year	2001	2002	2003	2004	2005	2006	2007	2008
Production (in tonnes)	8	7	6	5	4	7	8	5

the equation of the straight line trend is y = 6.25 - 0.167x, then the trend value for the year 2006 is 3.475.
Reason (R): The sum of squares of the deviations of the values of y from their corresponding trend values is least i.e $\sum (y - y_t)^2$ is least.

6 **Assertion (A):** For the given six values 17, 22, 21, 35, 40, 51, the three years moving averages are 20, 26, 32, 42.
Reason (R): If x_1 , x_2 , x_3 ,..., x_n is the given annual time series, then 3 - yearly moving averages are $\frac{x_1+x_2+x_3}{3}, \frac{x_2+x_3+x_4}{3}, \ldots$

VERY SHORT ANSWER TYPE QUESTIONS

1 Calculate the 3 - yearly moving averages of the following data:

Year	1	2	3	4	5	6	7
Value	2	4	5	7	8	10	13

2 Calculate the 3 - year moving averages for the loan (in lakh₹) issued by co - operative banks for farmers in different states of India based on the values given below.

Year	2006	2007	2008	2009	2010	2011	2012	2013
Loan amount (in lakh ₹)	41.85	40.2	38.12	26.5	55.5	23.6	28.36	33.31

3 Construct 5 - yearly moving averages from the following data:

Year	2005	2006	2007	2008	2009	2010	2011	2012
Production	105	107	109	112	114	116	118	121

4 Construct 3 - yearly moving averages from the following data:

Year:	2010	2011	2012	2013	2014	2015	2016
Imported cotton consumption in India (in '000 bales):	129	131	106	91	95	84	93

5 From the following data compute 4 - yearly moving averages and determine the trend values. Also, find the short - term fluctuations.

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
Value:	50	36.5	43.0	44.5	38.9	38.1	32.6	41.7	41.1

6 Write down the models of Time Series Analysis.

SHORT ANSWER TYPE QUESTIONS

1 The table given below shows the daily attendance in thousands at a certain exhibition over a period of two weeks:

Week 1	52	48	64	68	52	70	72
Week 2	55	47	61	65	58	75	81

Calculate seven days moving averages and illustrate these and original information on the same graph using the same scales.

2 The production of soft drink company in thousands of litres during each month of a year is as follows:

Jan	Feb	March	April	May	June	July	August	Sept.	Oct.	No
1.2	0.8	1.4	1.6	1.8	2.4	2.6	3.0	3.6	2.8	1.9

Calculate the five monthly moving averages and show these moving averages on a graph.

3 The average number, in lakhs, of working days lost in strikes during each year of the period 2001 - 2010 was

2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate the three - yearly moving averages and draw the moving averages graph.

4 From the following time series obtain trent value by 3 yearly moving averages.

Year	Sales (in ₹ 000)	Year	Sales (in ₹ 000)
2008	8	2014	16
2009	12	2015	17
2010	10	2016	14
2011	13	2017	17
2012	15		
2013	12		

5 The following figures relate to the profits of a commercial concern for 8 years.

Years	2016	2017	2018	2019	2020	2021	2022
Profit (₹)	15,420	15,470	15,520	21,020	26,500	31,950	35,60

Find the trend of profits by the method of three - yearly moving averages.

6 Fit a straight line trend by the method of least squares to the data given below:

Years	2012	2013	2014	2015	2016	2017	2018
Sales (in tones)	9	11	13	12	14	15	17

LONG ANSWER TYPE QUESTIONS

1 Fit a straight line trend by the method of least squares and tabulate the trend values from the data:

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004
y	4	7	7	8	9	11	13	14	17

2 Fit a straight line trend by the method of least squares and tabulate the trend values from the data:

Year	2000	2001	2002	2003	2004	2005	2006
Production (in tonnes)	40	45	46	42	47	50	46

3 Fit a straight line trend by the method of least squares and tabulate the trend values from the data:

Year	2005	2006	2007	2008	2009	2010
Production (in ₹ '000)	5	7	9	10	12	17

CASE BASED TYPE QUESTIONS

1 **Read the following text carefully and answer the questions that follow:**

When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

Mathematically, for finding a line of best - fit to represent a trend, many methods are available. Methods like moving - averages and least - squares are some of the techniques to predict such trends.

Mrs Shamita runs a bread factory and the record of her sales of bakery items for the period of 2015 - 2019 is as follows:

Year	2015	2016	2017	2018	2019
Sales (in ₹ thousands)	35	42	46	41	48

Based on the above information, answer the following questions. Show steps to support your answers.

i. By taking year 2017 as origin, use method of least - squares to find the best - fit trend line equation for Mrs Shamita's business. Show the steps of your working. (1)

ii. Demonstrate the technique to fit the best - suited straight line trend by the method of 3 - year moving averages. Also, draw the trend line. (1)

iii. What are the estimated sales for Mrs Shamita' s business for year 2022? (2)

OR

Mrs Shamita wishes to grow her business to yearly sale of₹ 67400. In which year will she be able to reach her target? (2)

2 **Read the following text carefully and answer the questions that follow:**

Today in class Mr. Sharma is teaching the Method of Least Squares to measure the trend in time series. After explaining the method, he had taken an example

Example: Given below are the data relating to the sales of a product in a district.

Fit a straight line trend by the method of least squares and tabulate the trend values.

Year	2015	2016	2017	2018	2019	2020	2021	2022
Sales	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

To solve the given example, he constructed the following table:

Computation of trend values by the method of least squares.

Year (t)	**Sales (y)**	**x =** $\frac{(t-2018.5)}{0.5}$	**XY**	**x** ^{2}
2015	6.7	-7	-46.9	49
2016	5.3	-5	-26.5	25
2017	4.3	-3	-12.9	9
2018	6.1	-1	-6.1	1
2019	5.6	1	5.6	1
2020	7.9	3	23.7	9
2021	5.8	5	29.0	25
2022	6.11	7	42.7	49

Here, n = 8 (even)

So, origin is mean of two middle years i.e., $\frac{2018+2019}{2}$ = 2018.5

i) If the straight line is $y_t = a + bx$, then what is the value of **a?** (1)

ii) If the straight line is $y_t = a + bx$, then what is the value of **b?** (1)

iii) Which of the following trend equation? (2)

OR

$y_t = 5.975 + 0.051x$, $y_t = 0.051 + 5.975x$, $y_t = 5.975 - 0.051x$ (2)

ANSWER OF EXERCISE
MULTIPLE CHOICE QUESTIONS

1. **(a)** Time Series
2. **(a)** Secular trend
3. **(c) irregular**
4. **(d)** Minimum
5. **(b)** Irregular trend
6. **(C)Short term**

7. **(c) All of these**
8. (d) 40, 42, 44
9. c) Smooth out the time series
10. b) (i), (ii) and (iii)

ASSERTION-REASON QUESTIONS

1 (b)
2 (c)
3 (a)
4 (b)
5 (d)
6 (a)

VERY SHORT ANSWER QUESTIONS

1

Calculation of 3-year moving averages:

Year	Value	3-year moving total	3-year moving average
1	2		
2	4	11	3.667
3	5	16	5.333
4	7	20	6.667
5	8	25	8.333
6	10	31	10.333
7	13		

2

Year	Loan Amount	Three yearly moving totals	Three yearly moving average
2006	41.85	-	-
2007	40.2	120.17	$\frac{120.17}{3} = 40.06$
2008	38.12	104.82	34.94
2009	26.5	120.12	40.04
2010	55.5	105.6	35.2
2011	23.6	107.46	35.82
2012	28.36	85.27	28.42
2013	33.31	102.77	34.26
2014	41.1	-	-

3

Year	Production Y	5-yearly totals	5-yearly moving averages
2005	105	-	-
2006	107	-	-

2007	109	547	109.4
2008	112	558	111.6
2009	114	569	113.8
2010	116	581	116.2
2011	118		
2012	121		

4

Year	**Imported cotton consumption in India (in '000 bales)**	**3-yearly moving totals**	**3-yearly moving averages**
2010	129	-	-
2011	131	366	122.00
2012	106	328	109.33
2013	91	292	97.33
2014	95	270	90.00
2015	84	272	90.66
2016	93	-	-

5

Year	**Value**	**4-yearly moving totals**	**4-yearly centered moving averages**	**Short term fluctuations Y - Y_c**
2006	50.0	-	-	
2007	36.5	-	-	
		← 174.0	43.5	
			← 42.1125	0.8875
2008	43.0			
		← 162.9	40.725	
			← 40.925	3.575
2009	44.5			
		← 164.5	41.125	
			← 39.825	-0.925
2010	38.9			

		← 154.1	38.525	
			← 38.175	-0.075
2011	38.1			
		← 151.3	37.825	
			← 38.1	-5.5
2012	32.6			
		← 153.5	38.375	
			← 37.8375	3.8625
2013	41.7	← 149.2	37.3	
2014	41.1	-	-	
2015	33.8	-	-	

6 There are two models of time series analysis

i. **Additive model:** This model is based on the assumption that the sum of four components is equal to the original value i.e., 0 =
T + C + S + I
where, T= trend, C = cyclical, S = seasonal, I = irregular, O = original

ii. **Multiplicative model:** In this model four components have a multiplication relationship, so O = T × C × S × I

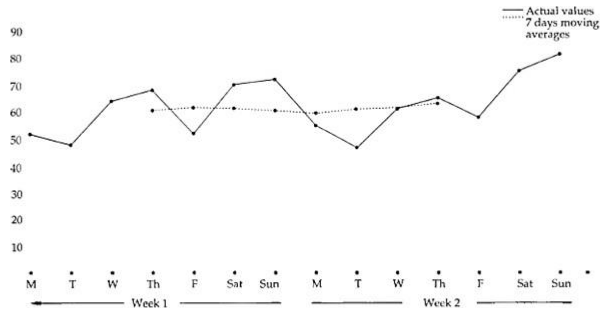
SHORT ANSWER TYPE QUESTIONS

1

	Days	Attendance (in thousands)	7 days moving totals	7 days moving averages
Week 1	Monday	52	-	-
	Tuesday	48	-	-
	Wednesday	64	-	-
	Thursday	68	426	60.86
	Friday	52	429	61.28
	Saturday	70	428	61.14
	Sunday	72	425	60.17
Week 2	Monday	55	422	60.28
	Tuesday	47	428	61.14
	Wednesday	61	433	61.86
	Thursday	65	442	63.14
	Friday	58	-	-

	Saturday	75	-	-
	Sunday	81	-	-

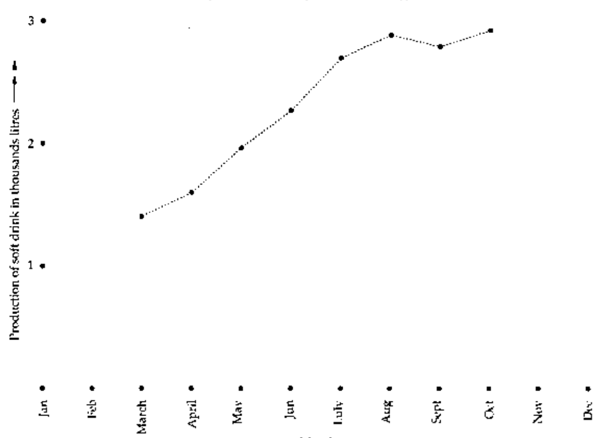
Seven days moving averages and original data are plotted on the graph paper in Fig. These points are joined by line segments to obtain the graphs to illustrate the trend.



2

Month	**Production of soft drink (in thousands litres)**	**5 monthly totals**	**5 monthly moving averages**
January	1.2	-	-
February	0.8	-	-
March	1.4	6.8	1.36
April	1.6	8.0	1.6
May	1.8	9.8	1.96
June	2.4	11.4	2.28
July	2.6	13.4	2.68
August	3.0	14.4	2.88
September	3.6	13.9	2.78
October	2.8	14.7	2.94
November	1.9	-	-
December	3.4	-	-

These moving averages are plotted on the following graph:



3

Year	Working days lost in strikes (in lakhs)	Three yearly moving totals	Three yearly moving averages
2001	1.5	-	-
2002	1.8	5.2	1.73
2003	1.9	5.9	1.96
2004	2.2	6.7	2.23
2005	2.6	8.5	2.83
2006	3.7	8.5	2.83
2007	2.2	12.3	4.1
2008	6.4	12.2	4.06
2009	3.6	15.4	5.13
2010	5.4	-	-

4

Year	Sales (Thousand ₹)	Three-yearly Moving Totals	Three-yearly Moving Average (Trend value)
2008	8		
2009	12	(8 + 12 + 10) = 30	10.00
2010	10	(12 + 10 + 13) = 35	11.67
2011	13	(10 + 13 + 15) = 38	12.67
2012	15	(13 + 15 + 12) = 40	13.33
2013	12	(15 + 12 + 16) = 43	14.33
2014	16	(12 + 16 + 17) = 45	15.00
2015	17	(16 + 17 + 14) = 47	15.67
2016	14	(17 + 14 + 17) = 48	16.00
2017	17		

5

Years	Profit (₹)	3-yearly moving Total (₹)	3-yearly moving average (₹)
2016	15,420	-	-
2017	15,470	46,410	15,470
2018	15,520	52,010	17,336.667
2019	21,020	63,040	21,013.333
2020	26,500	79,470	26,490
2021	31,950	94,050	31,350
2022	35,600	94,050	31,350
2023	34,900	-	-

6

Years (t_i)	Sales (y_i)	$x_i = t_i$ - 2015	x_i^2	x_iy_i
2012	9	-3	9	-27
2013	11	-2	4	-22
2014	13	-1	1	-13
2015	12	0	0	0
2016	14	1	1	14
2017	15	2	4	30
2018	17	3	9	51
n = 7	$\sum$ yi = 91	$\sum$ xi = 0	$\sum$ x2 = 28 i	$\sum$ xiyi = 33

y = 13x + 1.179

LONG ANSWER TYPE QUESTIONS

1

year t	y	x = t_i - 2000	x^2	xy	Trend values $y_t = a + bx$
1996	4	-4	16	-16	4.12
1997	7	-3	9	-21	5.59
1998	7	-2	4	-14	7.06
1999	8	-1	1	-8	8.53
2000	9	0	0	0	10
2001	11	1	1	11	11.47
2002	13	2	4	26	12.94
2003	14	3	9	42	14.41
2004	17	4	16	68	15.88
	$\sum y = 90$	$\sum x = 0$	$\sum x^2 = 60$	$\sum xy = 88$	

$y_t = 10 + 1.47\,x$

2

year t	Production (in tonnes)	x = t_i - 2003	x^2	xy	Trend values $y_t = a + bx$
2000	40	-3	9	-120	42.03
2001	45	-2	4	-90	43.07

2002	46	-1	1	-46	44.11
2003	42	0	0	0	45.14
2004	47	1	1	47	46.18
2005	50	2	4	100	47.21
2006	46	3	9	138	48.25
	$\sum y = 316$	$\sum x = 0$	$\sum x^2 = 28$	$\sum xy = 29$	

$y_t = 45.143 + 1.036\ x$

3

Year t	**Profit (in ₹ '000) y**	**$x = t_i$ - 2007.5**	**x^2**	**xy**	**Trend values $y_t = a + bx$**
2005	5	-2.5	6.25	-12.5	4.58
2006	7	-1.5	2.25	-10.3	6.74
2007	9	-0.5	0.25	-4.5	8.91
2008	10	0.5	0.25	5	11.09
2009	12	1.5	2.25	18	13.26
2010	17	2.5	6.25	42.5	15.43
	$\sum y = 60$	$\sum x = 0$	$\sum x^2 = 17.5$	$\sum xy = 38$	

$y_t = 10 + 2.17x$

CASE BASED QUESTIONS

1 **i)** $Y_t = 42.4 + 2.5x$

ii)make graph as pre exercise.

iii) The estimated sales for year 2022 = ₹ 54900

Or

Sales will be ₹ 67400in year (2017 + 10) = year 2027.

2. i) a = 5.975

ii) b= 0.051

iii) $y_t = 5.975 + 0.051x$ or

5.168

CHAPTER – 7

FINANCIAL MATHEMATICS

Gist/Summary of the lesson:

1.Perpetuity and Sinking Fund-

(a)Meaning of Perpetuity and Sinking Fund

(b) Real life examples of sinking fund

(c) Advantages of Sinking Fund

(d) Sinking Fund vs. Saving account

(2) Valuation of Bonds-(a)Meaning of Bond valuation

(b)Terms related to valuation of bond – Coupon rate, Maturity rate and current price

(c) Bond Valuation Method- Present Value Approach

(3) Calculation of EMI-(a) Method to calculate EMI- (i)Flat-Rate Method (ii)Reducing-Balance method (iii) Real life example to calculate EMI of various types of loans, purchase of assets, etc.

(4) Compound Annual Growth Rate (CAGR) -(a)Meaning and use of CAGR (b)Formula for CAGR

(5) Linear Method for Depreciation- (a)Meaning and formula for Linear Method of Depreciation(b) Advantages and disadvantages of Linear Method

Definitions and Formulae:

Perpetuity- A perpetuity is an annuity in which the periodic payments begin on a fixed date and continue forever. There are two types of perpetuity(i) one in which the payment is made at the end of each payment period and(ii) other in which the payment is made at the beginning of each payment period.

Present value of a Perpetuity- Consider an annuity whose periodic payment is Rs. R for infinite periods, interest being r% per period or r/100 = i per period per rupee.

Type1- When payment is made at the end of each payment period $P=\frac{R}{i}=\frac{100R}{r}$

Type2- When payment is made at the beginning of each payment period

$$P=R+\frac{R}{i}=R+\frac{100R}{r}$$

Sinking Fund- A sinking fund is a fund created to accumulate money over the years to discharge a future obligation like replacing an old machine by a new machine, renovation of house, expansion of business etc.

Consider an annuity whose periodic payment is Rs R payable at the end of each payment period for n periods, interest being r % per period or i= r/100, then the amount of obligation which can be discharged is

$$A= R\left[\frac{(1+i)^n-1}{i}\right]$$

Difference Between Sinking Fund and Saving Account

S.N.	Sinking Fund	Savings Account
1.	It is fixed term account	It is a long-term account which can be closed any time
2	It is set-up for a particular upcoming expense	It does not have any specific purpose.
3.	In sinking fund a fixed amount at regular intervals is deposited.	In saving account any amount,any time can be deposited.
4.	It can be used only for the purpose it was created	It can be used in any emergency

Bonds- Bonds are long term debt securities issued by companies or government entities. A bond is simply a loan taken by a company or government bodies from investors. In return of which company pays an interest at fixed intervals on the face value at fixed rate called coupon rate and repay the principal.

Valuation of Bonds- Present value approach

Let a bond of face value Rs. F matures in N years has a coupon rate of r% per annum then each coupon payment $C = F \times \frac{r}{100}$

If discount rate or yield to maturity is d% p.a. then $i = \frac{d}{100}$

Present value of bond (P.V.)= $\frac{C[1-(1+i)^{-N}]}{i} + F(1+i)^{-N}$

Equated Monthly Installment (EMI)

EMI is the monthly amount which we pay towards the repayment of the loan. EMI are used to pay off both interest on the loan and principal amount. Initially interest on the loan is the major part of EMI but as we progress towards the loan tenure the interest part reduces and principal part increases. The EMI depends on the following factors:

(i)Principal borrowed (ii)Rate of interest (iii)Tenure of the loan

Calculation of EMI- There are two method of calculating EMI:

(i)Flat rate method (ii)Reducing balance method

(i)Flat rate method- In this method ,the amount of interest paid is fixed. The amount of interest is calculated on the principal loan amount borrowed for its tenure at a constant rate of interest. In this method the interest remains constant for every EMI and does not reduce as the outstanding principal amount decreases with every EMI.

EMI = $\frac{P+Pni}{n}$ P-Principal, i-interest rate per rupees per month, n- number of payments

(ii)Reducing balance method- In this method interest does not remain constant for every EMI and interest for every subsequent EMI is calculated on the outstanding principal

EMI = $\frac{P \times iX(1+i)^n}{(1+i)^n-1}$ P-Principal, i-interest rate per rupees per month, n- number of payments

Amortization of Loans- A loan is said to be amortized if each instalment is used to pay interest and the part of the principal

(i)interest on the outstanding loan and (ii)repayment of part of the loan i.e. principal

Principal outstanding at beginning of kth period= $\frac{EMI[\ (1+i)^{n-K+1}-1]}{i(1+i)^{n-k+1}}$

Interest paid in kth payment= $\frac{EMI[\ (1+i)^{n-K+1}-1]}{(1+i)^{n-k+1}}$

Total interest paid= n x EMI – P

Compound Annual Growth Rate (CAGR)

Compound annual growth rate is a measure of an average yearly growth of an investment over a certain time period when returns/ profits are reinvested at the end of each year.

The formula for calculating CAGR can be obtained from the compound interest formula

$A= P(1 +i)^n$ Or $F.V.=P.V.(1+i)^n$

F.V.= Future/Final value of an investment , P.V.=Present/Beginning value of an investment

i=r/100, n= number of years.

$i= \left[\frac{F.V.}{P.V.}\right]^{1/n}-1$ So, $CAGR= \left[\frac{F.V.}{P.V.}\right]^{1/n}-1$

DEPRECIATION – Depreciation is defined as the reduction of original cost of a fixed asset such as buildings, furniture, machinery, vehicles etc. over a period of time due to wear and tear.

The time period in which an asset is expected to be functional and fit for purpose is called **useful life.**

The value of a depreciable asset at the end of its useful life is called **scrap value** or salvage value or depreciated value.

Book Value -Book value of an asset at a given data is the original value of the asset minus the accumulated depreciation at that date.

Methods of Computing Depreciation- Linear Method or straight-line method

In this method, the value of an asset is reduced uniformly over each period until it reaches to its scrap or salvage value.

Annual depreciation= $\frac{\textbf{Originalcost of an asset} - \textbf{Scrap value}}{\textbf{Usefullife of asset in years}}$

MULTIPLE CHOICE QUESTIONS

1) The present value of a sequence of payment of Rs 800 made at the end of every 6 month and continuing for ever, if money is worth 4%p.a. compounded semi-annually, is

   a) Rs. 20000 b) Rs.40000 c) Rs.60000 d) Rs.80000

   Solution: $P=\frac{100R}{r} = \frac{100x\ 800}{2}$=Rs.40000

   Answer: b

2) The present value of a perpetuity of Rs.750 payable at the beginning of each year, if money is worth 5%p.a. is

a) Rs. 15000 b) Rs.15750 c) Rs.14250 d) none of these

Solution: P=$R + \frac{100R}{r}$ =750+ $\frac{100x\ 750}{5}$= 750+15000= Rs.15750

Answer: b

3) A bond has a face value of Rs.1000 matures in 5 years; coupon rate is 4%p.a. with interest paid semi-annually. If the bond is priced to yield 6%p.a., the present value of the bond is (Given $(1.03)^{-10}$= 0.7441)

a) Rs.1052.18 b) Rs.1038.53 c) Rs.953.12 d) 914.70

Solution: C= F x (r/100) =1000 x (2/100) =20

$$P.V.= \frac{20[1-(1.03)^{-10}]}{0.03} +1000\ (1.03)^{-10}= 914.70$$

Answer: d

4) A bond of face value Rs.500 matures in 3 years, interest is paid semi-annually and bond is priced to yield 10%p.a. If the present value of bond is Rs.450 the annual coupon rate is

a) Less than 10% b) more than 10% c) equal to 10% d) None of these

Solution: If discount rate is greater than coupon rate then P.V.is less than face value

Answer: a

5) A bond of face value Rs.1000 matures in 10 years, interest is paid semi-annually and bond is priced to yield 6%p.a. If the present value of bond is Rs.1250 the annual coupon rate is

b) Less than 6% b) more than 6% c) equal to 6% d) None of these

Solution: If discount rate is less than coupon rate then P.V.is greater than face value

Answer: a

6) Which is not true

(a) If discount rate <coupon rate, then P.V.> face value
(b) If discount rate =coupon rate, then P.V.= face value
(c) If discount rate >coupon rate, then P.V.< face value
(d) If discount rate <coupon rate, then P.V.< face value
(e) Answer: d

7)What sum of money invested now could establish a scholarship of Rs 2500, which is to be awarded at the end of every year forever, if money is worth 4%compounded yearly

a) Rs.62500 b) Rs.125000 c) Rs.31250 d) none of these

Solution: P=$\frac{100R}{r} = \frac{100x\ 2500}{4}$=Rs.62500

Answer: a

8) At what rate of interest will the present value of a perpetuity of Rs. 500 payable at the end of each quarter be Rs. 40000?

a) 1.25 % p.a. b) 2.5%p.a. c) 5% p.a. d) 6 %p.a.

Solution: P=$\frac{100R}{r} \rightarrow r= \frac{100R}{P}$ i.e. r=$\frac{100X\ 500}{40000} = \frac{5}{4}$ =1.25%

Answer: a

9) Mr. X borrowed Rs. 500000 from a bank to purchase a house and decided to repay the loan by equal monthly payments in 10 years. If bank charges interest at 7.5%p.a. compounded monthly then EMI is (Given$1.00625)^{120}$=2.1121

a) Rs.5935 b) Rs.6380 c) Rs.7340 d) Rs.8520

Solution: EMI = $\frac{P \ X \ iX(1+i)^n}{(1+i)^n-1}$ = $\frac{500000 \ X \ 0.00625X(1+0.00625)^{120}}{(1+0.00625)^{120}-1}$= Rs.5935

Answer: a

10) EMI doesn't depend on

(a)Principal borrowed (b)Rate of interest (c)Tenure of the loan (d) None of these

Answer: d

11) A person invested Rs.180000 in a mutual fund in year 2016.If the value of mutual fund increased to 225000 in year 2020, then compound annual growth rate of his investment is [use $(1.25)^{1/4}$=1.057]

(a)5.7% (b)10.57% (c)5.07% d) None of these

Solution: CAGR= $\left\{\frac{F.V.}{P.V,}\right\}^{1/n}$ -1= $\left\{\frac{225000}{180000}\right\}^{1/4}$ - 1=0.057=0.057X100% =5.7%

Answer: a

12) An asset costing Rs.15000 is expected to have a useful life of 5 years. If scrap value of the asset is Rs.3000, then annual depreciation charge is

(a)Rs.3000 (b)Rs.2400 (c)Rs.2800 (d) Rs.3600

Solution: Annual depreciation= $\frac{15000-3000}{5}$ =2400

Answer: b

13) A machine costing Rs. C would reduce to 10000 in 7 years. If annual depreciation charge is Rs.10000, then the value of C is

a) 80000 b)70000 c)60000 d)none of these

Solution: 10000= $\frac{C-10000}{7}$ →C= 80000

Answer: a

14) A vehicle costing Rs. 125000 has scrap value of Rs. 25000.If annual depreciation charge is Rs.12500, then useful life of the vehicle is

a) 4 years b)6 years c) 8 years d)10 years

Solution: 12500= $\frac{125000-25000}{n}$ →n= 8

Answer: c

15) An annuity in which the periodic payment begins on a fixed date and continue forever is called

a) Sinking fund b) Coupon payment c) Perpetuity d) EMI

Answer c

16) A fund which is created to accumulate money over the years to discharge a future obligation is called

a) Sinking fund b) Coupon payment c) Perpetuity d) EMI

Answer a

17) The amount or future value of perpetuity is

a) Sinking fund b) Coupon payment c) undefined d) EMI

Answer c

18) The issuer of bond pays interest at fixed interval at fixed rate of interest to investor is called

a) Sinking fund b) Coupon payment c) Perpetuity d) EMI

Answer b

19) The rate of interest used to discount the bond's cashflows is known as

a) Discount rate b) yield to maturity

c)Discount rate or yield to maturity d) None of these

Answer c

20) Full form of CAGR

a) Complex Annual Growth rate b) Complex Annuity Growth rate

c)Compound Annual Growth rate d) Compound Annuity Growth rate

Answer c

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
c) Assertion (A) is true but Reason(R) is false.
d) Assertion (A) is false but Reason(R) is true.

1) **Assertion (A):** A machine costingRs.50,000 has a useful life of 4 years. The estimate scrap value is 10,000 then the annual depreciation is Rs.10,000.

Reason(R): Annual depreciation= $\frac{Original\ Cost+Scrap\ Value}{useful\ life}$

Answer: c

Solution: D= $\frac{C-S}{n}$ = $\frac{50000-10000}{4}$ = 10,000

2) **Assertion (A):** The rate of interest for which the present value of perpetuity of Rs.500 payable at the end of each quarter be Rs.40,000 is 5%

Reason(R): Apply formula P= $\frac{R}{i}$

Answer: a

Solution: i= r/ 4, P= R/ i

40,000 =$\frac{500}{r/4} \rightarrow r = \frac{1}{20}$ X100% =5

3) **Assertion (A):** An annuity in which the periodic payment begins on a fixed date and continue forever is called perpetuity.

Reason(R): The amount or future value of perpetuity is defined.

Answer: c

4) **Assertion (A):** An investment of Rs.10,000 becomes Rs.50,000 in 4 years, then the CAGR is given by [$\sqrt[4]{5}$ -1] X 100

Reason(R): CAGR=[$\left(\frac{Ending\ value}{starting\ Value}\right)^{1/n}$ -1] X100

Answer: a

Solution: CAGR= = [$\left(\frac{50,000}{10,000}\right)^{1/4}$ -1] X100 =[$\sqrt[4]{5}$ -1] X 100

5) **Assertion (A):** The perpetuity is infinite

Reason(R): Future value of perpetuity is undefined

Answer: a

6) **Assertion (A):** Sinking fund is a fixed term account**.**

Reason(R): It can be used in any emergency.

Answer: c

7) **Assertion (A):** In Flat rate method, the amount of interest is calculated on the principal loan amount borrowed for its tenure at a constant rate of interest.

Reason(R): EMI = $\frac{P+Pni}{n}$ P-Principal, i-interest rate per rupees per month, n- number of payments

Answer: a

8) **Assertion (A):** Compound annual growth rate is a measure of an average yearly growth of an investment over a certain time period when returns/ profits are reinvested at the end of each year.

Reason(R): CAGR= $\left[\frac{F.V.}{P.V.}\right]^{1/n}$+1

Answer: c

9) **Assertion (A):** the value of an asset is reduced uniformly over each period until it reaches to its scrap or salvage value.

Reason(R): Annual depreciation= $\frac{\textbf{Originalcost of an asset} - \textbf{Scrap value}}{\textbf{Usefullife of asset in years}}$

Answer: a

10) Assertion (A): The present value of a perpetuity of Rs.500 payable at the end of every 6 months be Rs.10000 then rate of interest is 5%p.a.

Reason(R): Present Value = $\frac{100XPreiodic\ payment}{Interest}$

Answer: d

Solution: Present Value = $\frac{100XPreiodic\ payment}{rate\ of\ interest}$; 10000= $\frac{200X500}{r}$; r= 10%

VERY SHORT ANSWER TYPE QUESTIONS

1. Question: Find the present value of a sequence of payments of Rs.1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.

   Solution: P= $\frac{R}{i} = \frac{100R}{r} = \frac{100X1000}{4}$=Rs.25,000

2. Question: What sum of money is needed to invest now, so as to get Rs.5000 at the beginning of every month forever, if the money is worth 6% per annum compounded monthly?

   Solution: r=6/12=1/2=0.5; i=r/100=0.5/100=0.005)

   P=R+$\frac{R}{i}$=5000 + $\frac{5000}{0.005}$=5000+1000000=Rs.1005000

3. Question: At what rate of interest will the present value of a perpetuity of Rs.500 payable at the end of every 6 months be Rs.10000?

   Solution: i= r/200; P= $\frac{R}{i} = \frac{200R}{r}$

   r = $\frac{100R}{P} = \frac{200\ X\ 500}{10000}$ = 10% p. a

4. Question: A firm anticipates an expenditure of Rs.500000 for plant modernization at end of 10 years from now. How much should the company deposit at the end of each year into a sinking fund earning interest 5% p.a.

   Solution: A= R $\left[\frac{(1+i)^n - 1}{i}\right]$; i= r/100=5/100=0.05

   = 500000 $\left[\frac{(1+0.05)^{10} - 1}{0.05}\right]$ Let x= $(1.05)^{10}$

   = 500000 $\left[\frac{(1.05)^{10} - 1}{0.05}\right]$ log x = 10 log1.05= 10X0.0212

$=500000 \left[\frac{0.629}{0.05}\right]$ x= antilog 0.2120=1.629

$= \frac{25000}{0.629}$ =Rs.39745.63

5. Question: Sanjay takes a personal loan of Rs.500000 at the rate of 12% per annum for 3 years. Calculate his EMI by using flat rate method.

   Solution: EMI = $\frac{P+Pni}{n}$; i = 12/(12X100) =0.01

   EMI = $\frac{500000+500000X36X0.01}{36}$= Rs. 18888.89

6. Question: A machine costingRs.50,000 has a useful life of 4 years. The estimate scrap value is 10,000. Using straight line method, find the annual depreciation.

   Solution: Annual depreciation= $\frac{Original\ Cost - Scrap\ Value}{useful\ life}$

   $=\frac{50000-10000}{4}$ =Rs. 10000

7. Question: The annual depreciation of a car is Rs.30000. If the scrap value of the car after 15 years is Rs.50000, find the original cost of the car using linear method.

   Solution: Annual depreciation= $\frac{Original\ Cost - Scrap\ Value}{useful\ life}$

   30000 $=\frac{C-50000}{15}$

   C =Rs.50,000

8. Question: A dining table costing Rs.36000 has a useful life of 15 years. If annual depreciation is Rs. 2000, find its scrap value using linear method.

   Solution: Annual depreciation= $\frac{Original\ Cost - Scrap\ Value}{useful\ life}$

   2000 $=\frac{36000-S}{15}$

   S=Rs.6,000

9. Question: A piece of equipment that cost Rs.25000 has an estimated useful life of 8 years and Rs.0 scrap value. Find the annual depreciation using linear method.

   Solution: Annual depreciation= $\frac{Original\ Cost - Scrap\ Value}{useful\ life}$

   $=\frac{25000-0}{8}$ =Rs.3125

10. Question:A bond of Face value Rs.1000 has a coupon rate of 10% p.a. paid semi-annually. If the present value of bond is Rs.1140, find coupon payment.

    Solution: C=1000X $\frac{5}{100}$ = Rs.50

11. Question: A company ABC Ltd has raised funds in the form of 1,000 zero - coupon bonds worth₹ 1,000 each. The company wants to set up a sinking fund for repayment of the bonds, which will be after 10 years. Determine the amount of the periodic contribution if the annualized rate of interest is 5%, and the contribution will be done half - yearly. Given that $(1.025)^{20}$ = 1.6386.

    Solution: A= R $\left[\frac{(1+i)^n -1}{i}\right]$; i= r/200=5/200=0.025

    1000000 = R$\frac{(1+.025)^{20} -1}{0.025}$; R = $\frac{1000000\ X\ 0.025}{(1.025)^{20}-1} = \frac{25000}{0.6386}$ =Rs.39148.136

12. Question: Find the present value of a perpetuity of₹ 18,000 payable at the end of 6 months, if the money is worth 8% p.a. compounded semi - annually.

    Solution: P= $\frac{R}{i} = \frac{100R}{r} = \frac{100X\ 18000}{4}$=Rs. 4,50,0000

13. Question: If the cash equivalent of the perpetuity of₹ 1,200 payable at the end of each quarter is₹ 96,000, find the rate of interest convertible quarterly

Solution: $P=\frac{R}{i}$; 96000= $\frac{100X1200}{r/4}$; 96000=$\frac{100X\ 1200X4}{r}$; r=5%

14. Question: A person buys a house for which he agrees to pay₹ 5,000 at the end of each month for 8 years. If money is worth 12% converted monthly, what is the cash price of the house? (Given $(1.01)^{-96}$ = 0.3847229701)

Solution : EMI = $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1}$; P= $\frac{5000\ X\ 0.01X(1+0.01)^{96}}{(1+.01)^{96}-1}$= $\frac{5000\ X(1-0.01)^{-96}}{0.01}$ = Rs.307638.51

15. Question: Sanjay takes a personal loan of₹ 500000 at the rate of 12% per annum for 3 years Calculate his EMI by using flat rate method.

Solution: EMI = $\frac{P+Pni}{n}$ = $\frac{500000+500000X36X0.01}{36}$ = $\frac{680000}{36}$ =Rs.18888.89

16. Question: Avni takes a loan of₹ 500,000 from a bank at an interest rate of 6% p.a. for 10 years. She wants to pay back the loan in equated monthly installments. Find her EMI by using Flat rate method (Given $(1.005)^{-120}$ = 0.5496327334)

Solution: EMI = $\frac{P+Pni}{n}$ = $\frac{500000+500000X120X0.005}{120}$ = $\frac{800000}{120}$ =Rs.6,666.67

17. Question: ₹ 5000 is invested in a Term Deposit Scheme that fetches interest 6% per annum compounded quarterly. What will be the interest after one year? Given that $(1.015)^4$ = 1.0613

Solution: Interest = P $\{(1+i)^n-1\}$ =5000$\{(1+0.015)^4-1\}$ =5000$\{(1.015)^4-1\}$
= 5000 x 0.0613 =306.82

18. Question: Find the present value of a sequence of payments of₹ 8,000 made at the end of each 6 months and continuing forever if money is worth 4% compounded semi – annually

Solution: $P=\frac{R}{i}=\frac{8000}{0.02}$ = Rs.4,00000

19. Question: A refrigerator costing Rs.76000 has a useful life of 20 years. If annual depreciation is Rs. 4000, find its scrap value using linear method.

Solution: Annual depreciation= $\frac{Original\ Cost-Scrap\ Value}{useful\ life}$

4000 $=\frac{76000-S}{15}$

S=Rs.16,000

20. Question: Rahul purchased an old scooter for₹ 16000. If the cost of the scooter after 2 years depreciates to ₹ 14440, find the rate of depreciation.

Solution: A= P $(1-\frac{r}{100})^n$;14440= 16000$(1-\frac{r}{100})^2$;r= 5%

SHORT ANSWER TYPE QUESTIONS

1. **Question:** A machine costing Rs.200000 has effective life of 7 years and its scrap value is Rs.30000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost Rs. 300000 after 7 years.

Solution: A= R $\left[\frac{(1+i)^n-1}{i}\right]$; A= 3,00000-30,000 =2,70,000; i=0.05

2,70,000 = R $\left[\frac{(1.05)^7-1}{0.05}\right]$ Let x= $(1.05)^7$→ logx= 7xlog1.05=0.1484

R = 13500/0.407 x= antilog0.1484=1.407

=Rs.33169.53

2. **Question:** Rahul plans to save amount for higher studies of his son, required after 10 years. He expects the cost of these studies to be Rs. 100000.How much should he save at the beginning of each year to accumulate this amount at the end of 10 years, if the interest rate is 12% compounded annually?

Solution: A= R(1+i) $\left[\frac{(1+i)^n - 1}{i}\right]$

$100000 = R(1.12)\left[\frac{(1.12)^{10} - 1}{0.12}\right] = R\left[\frac{(1.12)^{11} - 1.12}{0.12}\right]$

$R = \frac{100000 X 0.12}{(1.12)^{11} - 1.12}$ = Rs.5091.22 (By taking log $(1.12)^{11}$=3.477)

3. **Question:** A bond has face value of Rs.1000, mature in 4 years. Coupon rate is 4% per annum. The bond makes annual coupon payments. If the yield to maturity is 4%,find the fair value of bond.

Solution: C= F X $\frac{r}{100}$ = 1000X$\frac{4}{100}$ = Rs.40

P.V. = $\frac{C[1-(1+i)^{-N}\}]}{i}$ + $F(1+i)^{-N}$

= $\frac{40[1-(1.04)^{-4}\}]}{0.04}$ + $1000(1.04)^{-4}$ = $\frac{40[1-0.8551]}{0.04}$ + 1000X0.8551

By log $(1.04)^{-4}$ = 0.8551

= 144.90+855.10= Rs 1000

4. **Question:** A bond of face value Rs.1000 has a Coupon rate of 6% per annum with interest paid semi-annually and matures in 5 years. If the bond is priced to yield 8%p.a., find the value of the bond.

Solution: C= F X $\frac{r}{100}$ = 1000X$\frac{3}{100}$ = Rs.30

P.V. = $\frac{C[1-(1+i)^{-N}\}]}{i}$ + $F(1+i)^{-N}$

= $\frac{30[1-(1.04)^{-10}\}]}{0.04}$ + $1000(1.04)^{-10}$ = $\frac{30[1-0.6761]}{0.04}$ + 1000X0.6761

By log $(1.04)^{-10}$ = 0.6761

= 242.93+676.10= Rs 919.03

Hence, the fair value of bond is Rs 919.03

5. **Question:** A bond of face value Rs 1000 matures in 5 years. Interest is paid semi-annually and bond is period to yield 8%p.a. If the present value of bond is Rs800, find the annual coupon rate.

Solution: C= F X $\frac{r}{100}$ = 1000X$\frac{r}{200}$ = Rs.5r

P.V. = $\frac{C[1-(1+i)^{-N}\}]}{i}$ + $F(1+i)^{-N}$

800 = $\frac{5r[1-(1.04)^{-10}\}]}{0.04}$ + $1000(1.04)^{-10}$ = $\frac{5r[1-0.6761]}{0.04}$ + 1000X0.6761

By log $(1.04)^{-10}$ = 0.6761

800-676.10 = $\frac{5r[0.3239]}{0.04}$

r = 0.0306= 3.06%

6. **Question:** Mr Sharma borrowed Rs.1000000 from a bank to purchase a house and decided to repay the loan by equal monthly instalments in 10 years. If bank charges interest at 9 % p.a. compounded monthly, calculate the EMI. (Given1.0075^{120}=2.4514)

Solution: EMI = $\frac{1000000 X 0.0075 X (1+0.0075)^{120}}{(1+0.0075)^{120} - 1}$ = $\frac{7500 X 2.4514}{2.4514 - 1}$ =Rs12667.42

7. **Question:** A person wishes to purchase a house for Rs 45,00000 with a down payment of Rs 500000 and balance in EMI for 25 years. If bank charges 6%p.a. compounded monthly, calculate the EMI. (Given 1.005^{300}=4.4650)

Solution: EMI = $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1}$;P= 45,00000-500000=40,00000; i= 6/12x100=0.005

EMI = $\frac{40{,}00000\ X\ 0.005X(1+0.005)^{300}}{(1+0.005)^{300}-1} = \frac{20000\ X4.4650}{4.4650-1}$ =Rs25772

8. **Question:** Diwakar invested Rs.20000 in a mutual fund in year 2010.The value of mutual fund increased to Rs.32000 in year 2015.Calculate the compound annual growth rate of his investment.

Solution: CAGR= $\left\{\frac{F.V.}{P.V,}\right\}^{1/n}$-1= $\left\{\frac{32000}{20000}\right\}^{1/5}$ - 1=$\{1.6\}^{1/5} - 1$=1.098-1

=0.098x100%=9.8%

By using log value of$\{1.6\}^{1/5}$=1.098

9. **Question:** Vikas invested Rs.10000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below:

Year1	Year 2	Year 3	Year 4	Year 5	Year 6
Rs.11000	Rs.11500	Rs.13000	Rs.11800	Rs.12200	Rs.14000

Calculate CAGR of his investment.

Solution: CAGR= $\left\{\frac{F.V.}{P.V,}\right\}^{1/n}$-1= $\left\{\frac{14000}{10000}\right\}^{1/6}$ - 1=$\{1.4\}^{1/6} - 1$=1.058-1

=0.058x100%=5.8%

By using log value of$\{1.4\}^{1/6}$=1.058

10. **Question:**Suppose a person invested Rs 15000 in a mutual fund and the value of investment at the time of redemption was Rs 25000.If CAGR for this investment is 8.88%.Calculate the number of years for which he has invested the amount.

Solution: CAGR %= $\left[\left\{\frac{F.V.}{P.V,}\right\}^{1/n} - 1\right]$x 100

8.88 =$\left[\left\{\frac{25000}{15000}\right\}^{1/n} - 1\right]$x100

0.0888+1= $\left[\{1.667\}^{1/n}\right]$

1.0888= $\left[\{1.667\}^{1/n}\right]$

log 1.0888= 1/n log 1.667

n= $\frac{log1.667}{log1.089}$= $\frac{0.2219}{0.0370}$ =5.99i.e.6 years

11. **Question:** Consider a bond with a coupon rate of 10% charged annually. The par value is₹ 2,000 and the bond has 5 years of maturity. The yield to maturity is 11%. What is the value of the bond. (Given $(1.11)^{-5}$ =0.593451)

Solution: C= F X $\frac{r}{100}$ = 2000X$\frac{10}{100}$ = Rs.200

P.V. = $\frac{C[1-(1+i)^{-N}\}]}{i}$ + $F(1+i)^{-N}$

P.V. = $\frac{200[1-(1+0.11)^{-5}\}]}{0.11}$ + $2000(1+0.11)^{-5}$

P.V. = $\frac{200[1-(1.11)^{-5}\}]}{0.11}$ + $2000(1.11)^{-5}$ =200(3.6959) +2000(0.593451)

=Rs.1926.08

12. **Question:** Sonia invested₹ 40,000 in a mutual fund in year 2019. The value of mutual fund increased to ₹ 64,000 in year 2024. Calculate the compound annual growth rate of her investment. [Given, log (1.6) = 0.2041, antilog (0.04082) = 1.098]

Solution CAGR= $\left\{\frac{F.V.}{P.V.}\right\}^{1/n}$-1= $\left\{\frac{64000}{40000}\right\}^{1/5}$- 1=$\{1.6\}^{1/5} - 1$=1.098-1=0.098x100%=9.8%

By using log value of$\{1.6\}^{1/5}$=1.098

LONG ANSWER TYPE QUESTIONS

1) **Question:** A machine costs a company Rs. 52000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of Rs.2500 only. The price of the new model is estimated to be 25% more than the price of present one. Find what amount should be set aside at the end of each year out of the profit for the sinking fund, if it accumulates at 3.5% compound annually? [Given $(1.035)^{25}$=2.3632)]

Solution:- Cost of machine= Rs.52,000

Cost of new machine= 52,000+25% of 52,000= Rs.65,000

Scrap value= Rs.25,000

Net amount required at the end of 25 yr.=Rs (65,000-2500) =Rs.62,500

A= Rs.62,500, n= 25 yr, i= 3.5/100=0.035%

R= $\frac{ixA}{(1+i)^n-1} = \frac{(0.035)x(62500)}{(1+0.035)^{25}-1} = \frac{2187.5}{2.3632-1}$ =Rs.1604.68

2) **Question:** Find the purchase price of a Rs. 20000 bonds, redeemable at the end of 10 yr at 110 and paying annual dividends at 4%, if the yield rate is to be 5% effective. $(1.05)^{-10}$=0.6139

Solution:- n=10, i= 5/100=0.05

C= Annual dividend= 4%of F.V.=$\frac{4}{100}$ X20000=Rs.800

F= redemption price=20000x$\frac{110}{100}$ =Rs.22,000

Let V be the purchase price of the bond

V = $\frac{C[1-(1+i)^{-N}]}{i}$ + F$(1+i)^{-N}$ = $\frac{800[1-(1+0.05)^{-10}]}{0.05}$ + 22000$(1+0.05)^{-10}$

= [16000{1-$(1.05)^{-10}$} +22000$(1.05)^{-10}$} =Rs.19683.40

3) **Question:** Rajesh purchased a house from a company for Rs. 25,00000and made a down payment of Rs.500000. He repays the balance in 25 yr by monthly instalment at the rate of 9%p.a. compounded monthly

(i)What are the monthly payment?

(ii)What is the total interest payment? Given $(1.0075)^{-300}$=0.1062

Solution:- Cost of house=Rs 2500000, Down payment= Rs.500000

Principal amount= 25,00000-5,00000= Rs.2000000, n=25x12=300, i=9/1200=0.0075

EMI= $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1} = \frac{2000000X\ 0.0075X(1+0.0075)^{300}}{(1+0.0075)^{300}-1} = \frac{15000\ X\ 1/0.1062}{(1.0075)^{300}-1} = \frac{141242.93}{1/0.1062-1} = \frac{141242.93}{8.4161}$=16,782.46

Total Interest= nxEMI-P=300x16782.46-2000000=Rs.3034681

4) **Question:** Loan of Rs. 400000 at the interest rate of 6.75% per annum compounded monthly is to be amortized by equal payment at the end of each month for 10 yr, find the size of each monthly payment. [$(1.005625)^{120}$=1.9603]

Solution:- EMI= $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1} = \frac{400000X\ 0.005625X(1+0.006525)^{120}}{(1+0.006525)^{120}-1}$

=$\frac{2250x1.9603}{1.9603-1} = \frac{4410.675}{0.9603}$=Rs.4593

Monthly payment= Rs.4593

CASE BASED QUESTIONS

1. **Question:** In year 2000, Mr. Talwar took a home loan of Rs.30,00000 from state Bank of India at 7.5%p.a. compounded monthly for 20 years. $(1.00625)^{240}=4.4608$

Based on the above information, answer the following question:

a) What was EMI paid by Mr. Talwar

b) What was interest paid by Mr. Talwar in 150th payment?

c) What was Principal paid by Mr. Talwar in 150th payment?

Solution: a) EMI= $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1} = \frac{3000000X\ 0.00625X(1+0.006225)^{240}}{(1+0.00625)^{240}-1}$; i=7.5/1200=0.00625

$= \frac{3000000X\ 0.00625X(1.006225)^{240}}{(1.00625)^{240}-1} = \frac{3000000X\ 0.00625X4.4608}{4.4608-1}$=Rs.24167.82

b) Interest paid in kth payment= $\frac{EMI[\ (1+i)^{n-K+1}-1]}{(1+i)^{n-k+1}}$

Interest paid in 150th payment= $\frac{24167.82[\ (1+0.00625)^{240-150+1}-1]}{(1+0.00625)^{240-150+1}} = \frac{24167.82[\ (1+0.00625)^{91}-1]}{(1+0.00625)^{91}}$

$= \frac{24167.82[\ (1.00625)^{91}-1]}{(1.00625)^{91}} = \frac{24167.82[\ 0.7629]}{1.7629}$ = Rs.10458.69

c) Principal paid by Mr. Talwar in 150th payment= EMI – Interest paid in 150th payment

= 24.167.82-10458.69= Rs.13709.13

2) **Question:** A loan of Rs.250000 at the interest rate of 6%p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years, find

a) the size of each monthly payment

b) interest paid in 40th payment

c) total interest paid

[Given $(1.005)^{60}=1.3489$, $(1.005)^{21}=1.1104$]

Solution: a) EMI= $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1} = \frac{2{,}50{,}000X\ 0.005X(1+0.005)^{60}}{(1+0.005)^{60}-1}$; i=6/1200=0.005

$= \frac{2{,}50{,}000X\ 0.005X(1.005)^{60}}{(1.005)^{60}-1} = \frac{2{,}50{,}000X\ 0.005X1.3489}{1.3489-1}$=Rs.4832.69

a) Interest paid in kth payment= $\frac{4832.69[\ (1+0.005)^{60-40+1}-1]}{(1+0.005)^{60-40+1}} = \frac{4832.69[\ (1.005)^{21}-1]}{(1.005)^{21}}$

$= \frac{4832.69[\ 1.1104-1]}{1.1104}$ =480.48

b) Total interest paid= n x EMI -P =60x 4832.69-250000=Rs.39961.40

3) **Question:** A couple wishes to purchase a house for Rs.10,00000 with a down payment of Rs.2,00000. If they can amortize the balance at 9% per annum compounded monthly for 25 years

a)what is their monthly payment?

b)What is the total interest paid?

Solution a) EMI= $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1} = \frac{(1000000-200000)X\ 0.0075X(1+0.0075)^{300}}{(1+0.0075)^{300}-1}$; i=9/1200=0.0075

$= \frac{(800000)X\ 0.0075X(1.0075)^{300}}{(1.0075)^{300}-1} = \frac{6000\ x\ 10.47}{10.47-1}$=Rs.6633.57

a) Total interest paid=n x EMI- P= 300x 6633.57-800000

=1990071-800000=Rs.11,90,071

HIGHER ORDER THINKING SKILLS

1) **Question:** A loan of Rs.400000 at the interest rate of 6.75%p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years. Find

i)the size of each monthly payment.

ii)the principal outstanding at the beginning of 61st month
iii)the interest paid in 61st payment.
iv)the principal contained in 61st payment
v)total interest paid.
[Given $(1.005625)^{120}=1.9603$, $(1.005625)^{60}=1.4001$]

Solution i) EMI= $\frac{P\ X\ iX(1+i)^n}{(1+i)^n-1}=\frac{400000X\ 0.005625X(1+0.005625)^{120}}{(1+0.005625)^{120}-1}$; i=6.75/1200=0.005625

$=\frac{400000X\ 0.005625X(1.005625)^{120}}{(1.005625)^{120}-1}=\frac{400000X\ 0.005625X1.9603}{1.9603-1}$ =Rs.4593

ii) Principal outstanding at the beginning of 61st month=

$=\frac{4593[\ (1+0.005625)^{120-61+1}-1]}{0.005625(1+0.005625)^{120-61+1}}=\frac{4593[\ (1.005625)^{60}-1]}{0.005625(1.005625)^{60}}$

$=\frac{4593x\ [1.4001-1]}{0.005625x1.4001}=\frac{4593x\ [0.4001]}{0.005625x1.4001}$= Rs233336.89

iii)Interest paid in 61st payment=$\frac{4593[\ (1+0.005625)^{120-61+1}-1]}{(1+0.005625)^{120-61+1}}=\frac{4593[\ (1.005625)^{60}-1]}{(1.005625)^{60}}$

$=\frac{4593[\ 1.4001-1]}{1.4001}$ =Rs1312.52

iv) Principal contained in 61st payment=EMI-Int. paid in 61st payment
=4593-1312.52=Rs3280.48

v) Total interest paid=n x EMI- P= 120x 4593-400000
=Rs.151160

2) **Question:** A bond that matures in 6 yr has coupon rate of 12%p.a.and has a face value of Rs15000.Find the fair value of bond, if the yield to maturity is 10%. [$(1.1)^{-6}=0.56$]

Solution- C= F x $\frac{r}{100}$ =15000 x $\frac{12}{100}$ =Rs.1800

(P.V.) = $\frac{C[1-(1+i)^{-N}\}]}{i}+F(1+i)^{-N}=\frac{1800[1-(1+0.1)^{-6}\}]}{0.1}+15000(1+0.1)^{-6}$

$=\frac{1800[1-(1.1)^{-6}\}]}{0.1}+15000(1.1)^{-6}=\frac{1800[1-0.56]}{0.1}+15000x0.56$=Rs.16320

EXERCISE

MULTIPLE CHOICE QUESTIONS

1) Simran purchased a number of stocks of Amazon. At the end of first year, she received a payment of 8000, which grows at a rate of 3%per year and continues forever. If the discount rate is7%. Then, the present value of Simran's investment is
(a)32000 (b)20000 (c)200000 (d)40000
2) The present value of a perpetuityof`5000 payable at the end of each year, if money is worth 5% compounded annually is
(a)20000 (b)100000 (c)10000 (d)25000
3) The rate of interest will the present value of perpetuity of 500payable at the end of every 6months be10000 is
(a)5% (b)8% (c)10% (d)4%
4) A bond of face value of 500 mature in3yr. Interest is paid half-yearly and bond is price to yield 8% annually. If the present value of bond is 450, then annual coupon rate is
(a)less than10% (b)equal to10% (c)more than10% (d)None of these
5) Manish takes a loan of 300000 at an interest of 10%compounded annually for a period of 3yr, then EMI by using flat rate method is
(a)Rs.1083.33 (b) Rs 1073.33 (c) Rs 1093.33 (d) Rs 1063.33

6) A company ABC Ltd has issued a bond having a face value of` 10000 paying annual dividend at 8%. The bond will be redeemed at par at the end of 10yr.The purchase price of this bond, if the investor wishes a yield rate of 8%is (given $(1.08)^{-10}$=0.4632)
   (a) 8000 (b)12000 (c)10000 (d)9000
7) Mr. Ahuja borrowed 100000 from a bank to purchase a car and decided to repay the loan by equal monthly instalments in10yr.If bank charges interest at 9% per annum compounded monthly, then the value of EMI is (given $(1.0075)^{120}$=0.24514)
   (a)1250 (b)1266.74 (c)1300 (d)None of the above
8) A person buys a house for which he agrees to pay 5000 at the end of each month for 8yr. If money is worth12% converted monthly. Then, the cash price of house is (given $(1.01)^{-96}$ =0.3847)
   (a)307650 (b)407650 (c)507650 (d)None of these
9) Mr. Malik borrowed 500000 from a bank to purchase a house and decided to repay the loan by equal monthly payment in10yr.If bank charges interest at 7.5% per annum compounded monthly, then EMI is (given $(1.00625)^{120}$ =2.1121)
   a. 5935 (b) 6380 (c)7340 (d)8520
10) A company intends to create a sinking fund to replace at the end of 20th year assets costing 50000.Calculate the amount to be retained out of profit every year, if the interest rate is 5% (given $(1.00625)^{120}$= 2.1121)
   (a)15122.18 (b)15322.18 (c)15422.18 (d)15022.18

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason(R). Pick the correct option:

a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion (A).
b) Both Assertion (A) and Reason(R) are true but Reason(R) is NOT the correct explanation of Assertion (A).
c) Assertion (A) is true but Reason(R) is false.
d) Assertion (A) is false but Reason(R) is true.

1. **Assertion (A):** A machine costingRs.25000 has a useful life of 8 years. There is no scrap value then the annual depreciation is Rs.6150
   Reason(R): Annual depreciation= $\frac{Original\ Cost - Scrap\ Value}{useful\ life}$
2. **Assertion (A):** The present value of a perpetuity of Rs.900 payable at the end of each year be Rs.18000 then rate of interest is 5%p.a.
   Reason(R): Present Value = $\frac{100XPreiodic\ payment}{Interest}$
3. **Assertion (A):** An investment of Rs.10,000 becomes Rs.14,000 in 6 years, then the CAGR is given by [$\sqrt[6]{1.4}$ -1] X 100 %
   Reason(R): CAGR= [$\left(\frac{Ending\ value}{starting\ Value}\right)^{1/n}$] X100%
4. **Assertion (A):** An EMI is a fixed payment made by a borrower to a lender at a specific date every month to clear off the loan
   Reason(R): A loan is said to be amortized if each installment is used to pay interest and part of the principal
5. **Assertion (A):** The time period in which an asset is expected to be functional and fit for the purpose is called useful life.

Reason(R): The value of depreciable asset at the end of its useful life is called depreciated value.

6. **Assertion (A):** The present value of a perpetuity whose periodic payment of ₹ R is made at the end ofeach payment period, interest being ₹ i per rupee per period is given by $\frac{R}{i}$.

   Reason (R): Sum of an infinite G.P. whose first term is a and common ratio is r (r < 1) is given by $\frac{a}{1-r}$.

VERY SHORT ANSWER TYPE QUESTIONS

1. At 6% converted quarterly, find the present value of a perpetuity of 45000 payable at the end of each quarter.
2. The present value of a perpetual income of` R at the end of each month is42000.Find the value of R, if money is worth 6% compounded semi-annually.
3. At what rate of interest will the present value of 300 payable at the end of each quarter be 24000?
4. What sum of money is needed to invest now, as to get10000at the beginning of every month forever, if the money is worth 8%per annum compounded quarterly**?**
5. What amount is received at the end of every 6 months forever, ifRs72000 kept in a bank earns 8%p.a. compounded half yearly?
6. A dining table costing₹ 36000has a useful life of 15 years. If annual depreciation is ₹ 2000, find its scrap value using the linear method.

SHORT ANSWER TYPE QUESTIONS

1. Mr. Taneja purchase a house of`4500000 with a down payment of 500000 and balance in EMI for 25yr.If bank charges 6%per annum compounded monthly Calculate the EMI. (given $(1.005)^{300}$=4.4650)
2. A firm anticipates a capital expenditure of 80000 for a new equipment in 5yr. How much should be deposited quarterly in sinking fund carrying 12% per annum compounded quarterly to provide for the purchase? (given $(1.03)^{20} = 1.8061$)

3) A man wants to deposit a lump sum amount so that an annual scholarship of Rs3000 is paid. Rate of interest is 5%p.a. Calculate the lump sum amount required, if the scholarship is to start at the end of this year and continue forever

4) What sum of money invested now could establish a scholarship of Rs 5000 which his to be awarded at the end of every year forever, if money is worth 8%p.a.

5. Find the present value of a perpetuity of Rs 300 payable at the beginning of every 6 months, if money is worth 6%p.a.
6. A machine costing₹ 30,000 is expected to have a useful life of 4 years and a final scrap value of ₹ 4000. Find the annual depreciation charge using the straight - line method. Prepare the depreciation schedule.

LONG ANSWER TYPE QUESTIONS

1. Mr. Kailash has taken a personal loan of 200000 for2yr at an interest rate of 20% per annum, which is to be paid back in equal monthly installments. How much monthly installment Mr. Kailash will pay? [given $\{61/60\}^{-24}$= 0.6725
2. A machine costs a company 52000 and its effective life is estimated to be 25yr.A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of 2500 only. The price of the new model is estimated to be 25% more than the price of present one. Find what amount should be set aside at the end of each year out of the profit for the sinking fund, if it accumulates at 3.5% compound annually?
   (given $(1.035)^{25}$=2.3632)

3. A machine is bought for 320000.Its effective life is 8yr, after which its salvage value would be 25000.It is decided to create a sinking fund to replace this machine at the end of its effective life by making half yearly payments that will earn an interest of 8% per annum compounded half yearly. If it is known that the cost of machine increases by 5%per annum. Calculate the amount of each payment to the sinking fund.
[given$(1.04)^{16}$= 1.8730 16= and $(1.05)^8$ = 1.4774]
4. A machine costs accompany Rs.52000 and its effective life is estimated to be12yr. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realizes a sum of 5000 only. The price of new model is estimated to be 25% higher than the price of the present one. Find what amount should be set aside at the end of each year, out of the profit, for the sinking fund, if it accumulates at 10% effective. (given $(1.1\ )^{12}$=3.1384)
5. A person amortizes a loan of 150000 for a new home by obtaining a 10yr mortgage at the rate of 9%per annum compounded monthly. Find (i) the monthly payment (ii)the total interest paid (given $(1.0075)^{-120}$ = 0.4079

CASE BASED QUESTIONS

1. Aakriti invest in bond of face value of 1000 has a coupon rate of 8% per annum with interest paid semi-annually and mature in 5yr.
Based on above information, answer the following questions.
(i) If discount rate is 6%per annum, then check whether present value of bond is more than face value or less than?
(ii) If the present value of bond is 1000, then find discount rate.

2. In year 2018, Mr. Verma took a loan of`250000 at the interest of 6%per annum compounded monthly is to be amortized by equal payment at the end of each of 5yr. [given $(1.005)^{60}$= 1.3489]
Based on the above information, answer the following questions
(i) How many payments are there?
(ii) Find size of each monthly payment
(iii) What is the total interest paid by Mr. Verma

HIGHER ORDER THINKING SKILLS

1. Mrs. Sharma took a housing loan of Rs.800000 to be paid in 10 yr by equal monthly installments. The interest charged is10.5%per annum compounded monthly. Find her monthly installment. (given $(1.00875)^{-120}$ = 0.3515)
2. A machine costs a company52000 and its effective life is estimated to be 12 yr. A sinking fund is created for replacing the machine by a new model at the end of its lifetime, when its scrap realizes a sum of`5000 only. The price of new model is estimated to be25% higher than the price of the present one. Find what amount should be set aside at the end of each year, out of the profit, for the sinking fund, if it accumulates at10%effective. (given $(1.1)^{12}$ =3.1384)

ANSWERS

MULTIPLE CHOICE QUESTIONS

1.(c)200000 **2.** (b)`100000 **3.** (c)10% **4.** (a)less than10% **5.** (a)Rs.1083.33
6. (c)10000 **7.** (b)1266.74 **8**. (a)307650 **9**.(a) 5935 **10**.(a)15122.18

ASSERTION - REASON BASED QUESTIONS

1. d) iv) **2. a)** (i) **3. c)** (iii) **4. a)** (i) **5. d)** (iv) **6.(a)** (i)

VERY SHORT ANSWER TYPE QUESTIONS

1. Rs. 3000000 **2.** Rs.1260 **3.** 5% **4**. Rs.510000 **5.** Rs. 2880 **6**.Rs.6000

SHORT ANSWER TYPE QUESTIONS

1. Rs**.**25772 **2.** Rs.2977.29 **3.** Rs.60000 **4.** Rs.62500 **5.** Rs.10300
6. 0yr-30,000;1yr-23500;2yr-17,000;3yr-10,500;4yr-4000

LONG ANSWER TYPE QUESTIONS

1. Rs.10178 **2**. Rs.1604.68 **3**. Rs.20516.28 **4.** Rs.2805.85 **5.** (i) Rs.1900 (ii) Rs.78000

CASE BASED QUESTIONS

1. (i) More than F.V. (ii) 8% **2.** (i) 60 (ii) Rs.4832.69 (iii) Rs.39961.40

HIGHER ORDER THINKING SKILLS

1.Rs.10794.14 **2**. Rs.2805.85

UNIT-8

LINEAR PROGRAMMING

Gist/Summary

- Linear Programming Problems
- Objective Function
- Decision Variables
- Constraints
- Different types of linear programming problems – Manufacturing Problem, Diet Problem
- Mathematical Formulation of LPP
- Feasible region and Feasible Solution

Definitions and Formulae

Linear Programming Problems: Problems which minimize or maximize a linear function Z subject to certain conditions determined by a set of linear inequalities with non-negative variables are known as linear programming problems.
Objective function: A linear function $Z = ax + by$, where a and b are constants which has to be maximized or minimized according to a set of given conditions, is called as linear objective function.
Decision variables: In the objective function $Z = ax + by$, the variables x, y are said to be decision variables.
Constraints: The restrictions in the form of inequalities on the variables of a linear programming problem are called constraints. The conditions $x \geq 0$, $y \geq 0$ are known as non-negative restrictions.
Different types of linear programming problems: A few important linear programming problems are as follows:

Manufacturing problem: In such problem, we determine: (a) Number of units of different products to be produced and sold. (b) Manpower required, machines hours needed, warehouse space available, etc. Objective function is to maximize profit.

Diet problem: Here, we determine the amount of different types of constituent or nutrients which should be included in the diet. Objective function is to minimize the cost of production.

Assignment Problem: The assignment problem is a special case of transportation problem where the number of sources and destinations are equal. Supply at each source and demand at each destination must be one.

Blending problem: To determine the optimum amount of several constituents used in producing a set of products while determining the optimum quantity of each product to be produced.

Investment problem: To determine the amount of investment in fixed income securities to maximize the return on these investment.

Limitations of Linear Programming:
(i) To specify an objective function in mathematical form is not an easy task.
(ii) Even if objective function is determined, it is difficult to determine social, institutional, financial and other constraints.

(iii) It is also possible that the objective function of constraints may not be directly specified by linear inequality equations.

Mathematical Formulation of LPP

Let us take an example to understand how to formulate a LPP mathematically.

Example 1 : An electronic firm is undecided at the most profitable mix for its products. The products manufactured are transistors, resistors and carbon tubes with a profit of (per 100 units) ₹ 10 , ₹ 6 and ₹ 4 respectively. To produce a shipment of transistors containing 100 units requires 1 hour of engineering, 10 hours of direct labour, and 2 hours of administrative service. To produce 100 units of resistors requires 1 hour, 4 hours and 2 hours of engineering, direct labour and administrative services respectively. For 100 units of carbon tubes it needs 1 hour, 6 hours and 5 hours of engineering direct labour and administrative services respectively.

There are 100 hours of engineering time, 600 hours of direct labour and 300 hours of administrative time available. Formulate the corresponding LPP.

Sol.: Let the firm produce X hundred units of transistors, Y hundred units of resistors and Z hundred units of carbon tubes. Then the total profit to be maximized from this output will be

$$P = 10X + 6Y + 4Z$$

This is our objective function.

Now production of X hundred units of transistors, Y hundred units of resistors and Z hundred units of carbon tubes will require $X + Y + Z$ hours of engineering time, $10X + 4Y + 6Z$ hours of direct labour time and $2X + 2Y + 5Z$ hours of administrative service.

But the total time available for engineering, direct labour and administrative services is 100,600 and 300 hours respectively.

Hence the constraints are:

$$X + Y + Z \leq 100$$
$$10X + 4Y + 6Z \leq 600$$
$$2X + 2Y + 5Z \leq 300$$

with $X, Y, Z \geq 0$ as the non-negativity restriction. Thus, the formulation is

Maximize, $P = 10X + 6Y + 4Z$ subject to constraints:

$$X + Y + Z \leq 100$$
$$10X + 4Y + 6Z \leq 600$$
$$2X + 2Y + 5Z \leq 300$$
$$X, Y, Z \geq 0$$

<u>Key Facts</u>

Linear programming is often used for problems where no exact Solution is known, for example for planning traffic flows.

The goal of linear programming is to maximize or minimize specified objectives, such as profit or cost. This process is known as optimization.

Linear programming is heavily used in microeconomics and company management, such as planning, product, transportation, technology and other issues, either to maximize the income or minimize the costs of a production scheme.

Feasible region: The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of linear programming problem is known as the feasible region.

Feasible Solution: Points within and on the boundary of the feasible region represents feasible Solutions of constraints.

In the feasible region, there are infinitely many points (Solutions) which satisfy the given conditions.

Theorem 1: Let R be the feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where variables x and y are subject to constraints described by linear inequalities, the optimal value must occur at a corner point (vertex) of the feasible region.

Theorem 2: Let R be the feasible region for a linear programming problem, and let $Z = ax +$ by be the objective function. If R is bounded, then the objective function R has both maximum and minimum values of R and each of these occurs at a corner point (vertex) of R.
However, if the feasible region is unbounded, the optimal value obtained may not be maximum or minimum.

Corner or Extreme Point Method of Formulation of LPP

Step 1: Formulate the linear programming problem in x and y with given conditions.
Step 2: Convert the inequality constraints into equality constraints and plot each line on the graph paper.
Step 3: Find the feasible region and check if the feasible region is bounded or unbounded.
Step 4: Evaluate the value of the objective function Z at each corner point. Let M be the greatest and m be the smallest value of the objective function Z .

(i) When the feasible region is bounded: M and m are the maximum and minimum values of the objective function Z, respectively.

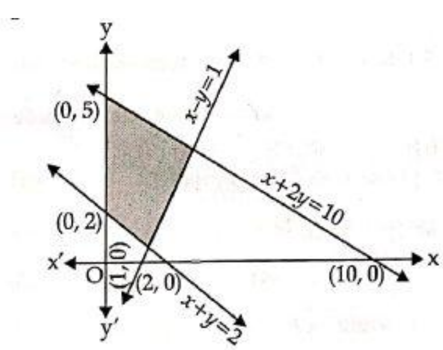
(ii) When the feasible region is unbounded:

(a) M is the maximum value of the objective function Z, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise the objective function has no maximum value.

(b) m is the minimum value of the objective function Z, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, the objective function has no minimum value.

MULTIPLE CHOICE QUESTIONS (SOLVED)

1. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



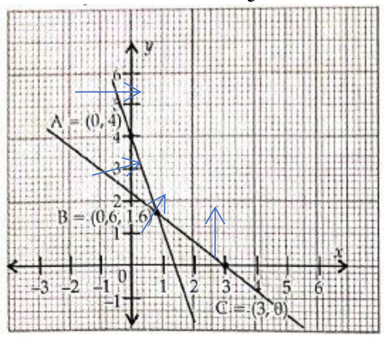
Which of the following is **not** a constraint to the above Linear Programming Problem?

(a) $x + y \geq 2$ (b) $x + 2y \leq 10$ (c) $x - y \geq 1$ (d) $x - y \leq 1$

Ans. Option (c) is correct.

Solution: We observe, (1,2) does not satisfy the inequality $x - y \geq 1$ ((You can take any point in the feasible region). Therefore, it will not contain the shaded feasible region.

2. The corner points of the shaded unbounded feasible region of LPP are $(0,4), (0.6,1.6)$ and $(3,0)$ as shown in the figure. The minimum value of the objective function $Z = 4x + 6y$ occurs at

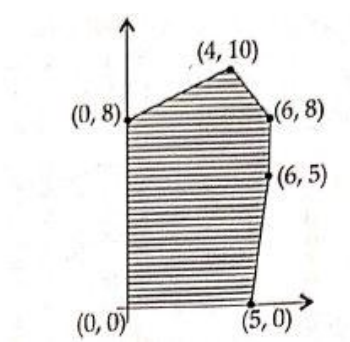


(a) $(0.6,1.6)$ only
(b) $(3,0)$ only
(c) $(0.6,1.6)$ and $(3,0)$ only
(d) At every point of the line-segmnt joining the points $(0.6,1.6)$ and $(3,0)$
Ans. Option (d) is correct.
Solution: The minimum value of the objective function occurs at two adjacent corner points (0.6, 1.6) and (3,0) and there is no point in the half plane $4x + 6y < 12$ in common with the feasible region. So the minimum value occurs at every point of the line segment joining the two points.

3) If the corner points of the feasible region are A(0,10), B(5,5), C(15,15) and D (0,20), then at which point(s) is the objective function $Z = 3x + 9y$ maximum?
(a) Point B (b) Point C (c) Point D (d) every point on the line segment CD

Ans. Option (d) is correct.
Solution: Z is maximum 180 at points C(15,15) and D(0,20)
$\Rightarrow Z$ is maximum at every point on the line segment CD

Q.4. In the given graph, the feasible region for an LPP is shaded. The objective function $Z = 2x - 3y$, will be minimum at:



(a) (4,10) (b) (6,8) (c) (0,8) (d) (6,5)

Ans. Option (c) is correct
Solution: Z is minimum -24 at (0,8)

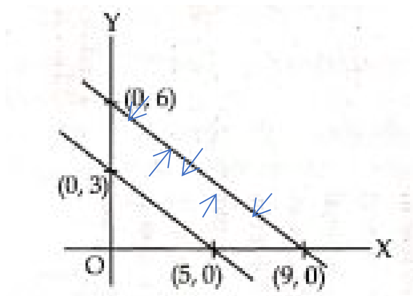
Q. 5. A linear programming problem is as follows:

Minimize $Z = 30x + 50y$
Subject to the constraints,

$$3x + 5y \geq 15$$
$$2x + 3y \leq 18$$
$$x \geq 0, y \geq 0$$

In the feasible region, the minimum value of Z occurs at
(a) A unique point (b) No point (c) In finitely many points (d) Two points only
Ans. Option (d) is correct
Solution: Let the equation of constraints be

$$3x + 5y = 15$$
$$2x + 3y = 18$$
$$x = 0, y = 0$$



Comer points of feasible region	$Z = 30x + 50y$
(5,0)	150
(9,0)	270
(0,3)	150
(0,6)	300

Minimum value of Z occurs at two points.

Q. 6. For an objective function $Z = ax + by$, where $a, b > 0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0,20), (10,10), (30,30) and (0,40). The condition on a and b such that the maximum Z occurs at both end points (30,30) and (0,40) is:
(a) $b - 3a = 0$
(b) $a = 3b$

(c) $a + 2b = 0$
(d) $2a - b = 0$
Ans. Option (a) is correct
Solution: As Z is maximum at (30,30) and (0,40)

$$\Rightarrow \quad 30(a+b) = 40b$$
$$\Rightarrow \quad b - 3a = 0$$

Q 7. A mining company owns two different mines that produce ore which is graded in two classes: high and medium grade. The two mines have different operating characteristics as detailed below:

Mine	Cost per day (in lakhs)	Production (tons/day)	
		High grade	Medium grade
X	105	5	3
Y	90	1	2

For a particular week, the company contracted to provide 10 tons of high-grade and 8 tons of medium grade ore to a factory. The company wants to work out how many days per week it should operate each mine to fulfill the contract keeping the mining cost as low as possible.
Taking x to be the number of days per week that mine X is operated, and y to be the number of days per week mine Y is operated, what will be the objective function and constraints for this LPP (Assume $0 \leq x, y \leq 7$)
(a) Objective function: $105x + 90y$; Constraints $5x + y \geq 10$ & $3x + 2y \geq 8$
(b) Objective function 105x +90 y; Constraints. $5x + 3y \geq 10$ & $x + 2y \geq 8$
(c) Objective function $5x + 3y$; Constraints: $5x + y \geq 105$ & $x + 2y \geq 90$
(d) Objective functions $x + 2y$ Constrains: $5x + 3y \leq 105$ & $x + 2y \leq 90$
Ans. Option (a) is correct.
Solution : The objective function is to minimize the total cost, with minimum number of high-grade ores as 10 and minimum number of medium grade ore as 8. As per the contract, the company must provide 10 tons of high-grade and 8 tons of medium grade. So, it must produce at least 10 tons of high grade and 8 tons of medium grade.
Option (b) the constraints are incorrect with numbers from the table processed incorrectly
Option (c) the objective function is incorrect and constrains are also incorrect. Numbers taken from the table only.
Option (d) the objective function is incorrect, and constraints are also incorrect. Numbers taken from the table only.

Q. 8 The comer points of the bounded feasible region determined by a system of linear constraints are (0, 3) (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$

The condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is
(a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$

Ans. Option (b) is correct.
Solution: $Z = px + qy$, At $(3,0), Z = 3p$
and at $(1,1), Z = p + q$
From (ii) & (iii), $3p = p + q \Rightarrow 2p = q$.

Q. 9. The Solution set of the inequality $3x + 5y < 4$ is
(a) an open half-plane not containing the origin
(b) an open half-plane containing the origin.
(c) the whole xy-plane not containing the line $3x + 5y = 4$
(d) A closed half plane containing the origin

Ans. Option (b) is correct.
Solution: The strict inequality represents an open half plane and it contains the origin as (0, 0) satisfies it.

Q. 10. The corner points of the feasible region determined by the system of linear constraints are $(0,0), (0,40), (20,40), (60,20), (60,0)$. The objective function is $Z = 4x + 3y$.
Compare the quantity in Column A and Column B

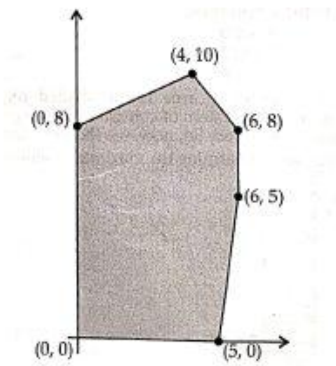
Column A	Column B
Maximum of Z	325

(a) the quantity in column A is greater. (b) the quantity in column B is greater.
(c) the two quantities are equal.
(d) The relationship cannot be determined on the basis of the information supplied.
Ans. Option (b) is correct.
Solution:

Corner points	Corresponding value of $\boldsymbol{Z = 4x + 3y}$
(0,0)	0
(0,40)	120
(20,40)	200
(60,20)	300 → Maximum
(60,0)	240

Hence, maximum value of $Z = 300 < 325$
So, the quantity in column B is greater.

Q. 11. The feasible Solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function. Maximum of Z occurs at



(a) (0,0) (b) (0,8) (c) (5,0) (d) (4,10)

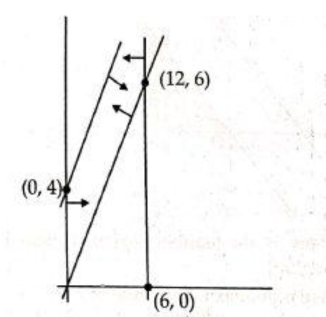
Ans. Option (c) is correct.

Solution:

Corner points	Corresponding value of $Z = 3x - 4y$
(0,0)	0
(5,0)	15 → Maximum
(6,5)	-2
(6,8)	-14
(4,10)	-28
(0,8)	-32 → Minimum

Hence, the maximum of Z occurs at (5,0) and its maximum value is 15.

Q. 12. The feasible region for an LPP is shown in the given Figure.



Let $F = 3x - 4y$ be the objective function. Maximum value of F is

(a) 0 (b) 8 (c) 12 (d) -18

Ans. Option (c) is correct

Solution: The feasible region as shown in the figure, has objective function $F = 3x - 4y$.

Corner points	Corresponding value of $F = 3x - 4y$
(0,0)	0
(12,6)	$12 \rightarrow$ Maximum
(0,4)	$-16 \rightarrow$ Minimum

Hence, the maximum value of F is 12 .

Q. 13. Corner points of the feasible region for an LPP are $(0,2), (3,0), (6,0), (6,8)$ and $(0,5)$. Let $F = 4x + 6y$ be the objective function.

The minimum value of F occurs at

(a) $(0,2)$ only

(b) $(3,0)$ only

(c) the mid-point of the line segment joining the points $(0,2)$ and $(3,0)$

(d) any point on the line segment joining the points $(0,2)$ and $(3,0)$

Ans. Option (d) is correct

Solution:

Corner points	Corresponding value of $F = 4x + 6y$
(0,2)	$12 \rightarrow$ Minimum
(3,0)	$12 \rightarrow$ Minimum
(6,0)	24

Q. 14. The objective function $\mathrm{Z} = ax + by$ of an LPP has maximum value 42 at $(4,6)$ and minimum value 19 at $(3,2)$. Which of the following is true ?

(a) $a = 9, b = 1$

(b) $a = 5, b = 2$

(c) $a = 3, b = 5$

(d) $a = 5, b = 3$

Ans. Option (c) is correct

Solution: $\mathrm{Z} = ax + by$

$$\therefore \quad \begin{aligned} 4a + 6b &= 42 \\ 3a + 2b &= 19 \end{aligned}$$

from (i) & (ii) $a = 3, b = 5$

Q. 15. The corner points of the feasible region of a linear programming problem are $(0,4), (8,0)$ and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then, maximum value of Z - minimum value of Z is equal to:

(a) 40 (b) 96 (c) 144 (d) 136

Ans. Option (c) is correct

Solution: $Z = 30x + 24y$

At (8,0) $Z = 30 \times 8 + 24 \times 0 = 240$ Maximum

At (0,4) $Z = 30 \times 0 + 24 \times 4 = 96$ Minimum

At $\left(\frac{20}{3}, \frac{4}{3}\right)$ $Z = 30 \times \frac{20}{3} + 24 \times \frac{4}{3} = 232$

= Maximum - Minimum

$= 240 - 96 = 144$

Q. 16. A Linear Programming Problem is as follows:

Minimise $z = 2x + y$

subject to the constraints $x \geq 3, x \leq 9, y \geq 0$

$x - y \geq 0, x + y \leq 14$

The feasible region has

(a) 5 corner points including (0,0) and (9,5)

(b) 5 corner points including (7,7) and (3,3)

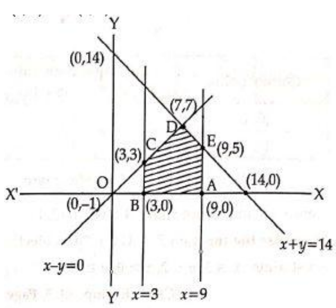
(c) 5 corner points including (14,0) and (9,0)

(d) 5 corner points including (3,6) and (9,5)

Ans. Option (b) is correct.

Solution: On plotting the constraints $x = 3$, $x = 9$, $x = y$ and $x + y = 14$, we get the following graph. From the graph given below it clear that feasible region is ABCDEA, including corner points A(9,0), B(3,0), $C(3,3)$, $D(7,7)$ and $E(9,5)$.

Thus feasible region has 5 corner points including (7,7) and (3,3).



Q.17. The corner points of the feasible region for a Linear Programming problem are P(0,5), Q(1,5), R(4,2) and $S(12,0)$. The minimum value of the objective function $Z = 2x + 5y$ is at the point

(a) P (b) Q (c) R (d) S

Ans. Option (c) is correct.

Solution:

Corner Points	Value of $Z = 2x + 5y$
$P(0,5)$	$Z = 2(0) + 5(5) = 25$
$Q(1,5)$	$Z = 2(1) + 5(5) = 27$
$R(4,2)$	$Z = 2(4) + 5(2) = 18$ Minimum
$S(12,0)$	$Z = 2(12) + 5(0) = 24$

Thus, minimum value of Z occurs at $R(4,2)$.
Q.18. A Linear Programming Problem is as follows:

Maximise / Minimise objective function

$$Z = 2x - y + 5$$

Subject to the constraints

$$3x + 4y \leq 60$$
$$x + 3y \leq 30$$
$$x \geq 0, y \geq 0$$

If the corner points of the feasible region are $A(0,10), B(12,6), C(20,0)$ and $O(0,0)$, then which of the following is true ?
(a) maximum value of Z is 40(b) minimum value of z is -5
(c) difference of maximum and minimum values of z is 35
(d) At two corner points, value of Z are equal
Sol. Option (b) is correct.

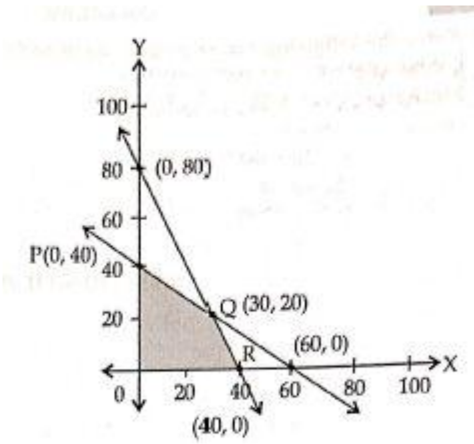
Solution:

Corner Points	Value of $Z = 2x - y + 5$
A(0, 10)	$Z = 2(0) - 10 + 5 = -5$ (Minimum)
B(12, 6)	$Z = 2(12) - 6 + 5 = 23$
C(20, 0)	$Z = 2(20) - 0 + 5 = 45$ (Maximum)
O(0,0)	$Z = 0(0) - 0 + 5 = 5$

So, the minimum value of Z is -5.
Q.19. The corner points of the feasible region determined by a set of constraints (linear inequalities) are P(0,5), $Q(3,5), R(5,0)$ and $S(4,1)$ and the objective function is $Z = ax + 2by$ where $a, b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is
(a) $a - 5b = 0$ (b) $a - 3b = 0$ (c) $a - 2b = 0$ (d) $a - 8b = 0$
Sol.Option (d) is correct.
Solution: Given, Max. $Z = ax + 2by$
Max. value of Z on $Q(3,5)$ = Max. value of Z on $S(4,1)$

$$\Rightarrow \quad 3a + 10b = 4a + 2b$$
$$\Rightarrow \quad a - 8b = 0$$

Q.20. For an L.P.P the objective function is Z $= 4x + 3y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true?
(a) Maximum value of Z is at R.
(b) Maximum value of Z is at Q.
(c) Value of Z at R is less than the value at P.
(d) Value of Z at Q is less than the value at R.
Ans. Option (b) is correct.
Solution:

Corner Points of Feasible Region	Value of $z = 4x + 3y$
O(0,0)	Z = 4(0) + 3(0) = 0
P(0,40)	Z = 4(0) + 3(40) = 120
Q(30, 20)	Z = 4(30) + 3(20) = 180 (Maximum)
R(40,0)	Z = 4(40) + 3(0) = 160

Thus, Maximum value of Z is at Q, which is 180 .

Assertion/Reason Based Questions (Solved)

Direction: In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices
(a) Both A and R are true, and R is the correct explanation of A.
(b) Both A and R are true, but R is not the correct explanation of A
(c) A is true but R is false.
(d) A is false and R is true.
Q. 1. Assertion (A): If the corner points of the feasible region for a linear programming problem are P(0,4), $Q(1,4)$, $R(4,1)$ and $S(12,-1)$, then minimum value of objective function Z $= 2x + 4y$ is at the point R(4,1)

Reason (R): If the corner points of the feasible region for a linear programming problem P$(0,4)$, $Q(1,4), R(4,1)$ and $S(12,-1)$, then maximum value of objective function Z $= 2x + 4y$ is 20 .
Ans. Option (b) is correct.
Solution:

Corner	Value of Z $= 2x + 4y$
P$(0,4)$	Z $= 2(0) + 4(4) = 16$
Q$(1,4)$	Z $= 2(1) + 4(4) = 18$
R$(4,1)$	Z $= 2(4) + 4(1) = 12$ $\rightarrow$ Minimum
S$(12,-1)$	Z $= 2(12) + 4(-1) = 20$ $\rightarrow$ Maximum

Minimum value is 12 at the point R$(4,1)$,
Maximum value is 20 at the point $R(12,-1)$.

Q. 2. The corner points of the feasible region determined by a set constraints (linear inequalities) are P$(1,6)$, Q$(4,5)$, R$(6,1)$ and S$(5,2)$ and the objective function is Z $= ax + 3by$ where $a, b > 0$
Assertion (A): The relation between a and b ,such that the maximum Z occur at P and Q is $a = b$.
Reason (R): The relation between a and b such that the maximum Z occur at P and Q is $a = 3b$

Ans. Option (c) is correct.
Solution: Given Max. Z $= ax + 3by$
Max value of Z on P$(1,6)$ = Max. value of Z on Q$(4,5)$
or, $$a + 18b = 4a + 15b$$
$$3a = 3b$$
or, $$a = b$$

Q.3.The corner points of the bounded feasible region for an L.P.P. are $(60,0)$, $(120,0)$, $(60,30)$ and $(40,20)$. The objective function Z $= ax + by, a, b > 0$ has maximum value 600 at points $(120,0)$ and $(60,30)$.
Assertion (A): Minimum value of $Z = 300$
Reason (R): $a = 5, b = 10$.

Ans. Option (a) is correct.
Solution: Z $= ax + by$ has maximum value at points $(120,0)$ and $(60,30)$.
So, $120a + 0 = 600 \Rightarrow a = 5$.
Also, $60a + 30b = 600 \Rightarrow 60 \times 5 + 30b = 600$
$$\Rightarrow \quad 30b = 300 \Rightarrow b = 10$$
$$\therefore \quad a = 5, b = 10$$

∴ Reason is true.
Now, $Z = 5x + 10y$
Value of Z at $(60,0) = 5 \times 60 + 0 = 300 \rightarrow$ Minimum
Value of Z at $(120,0) = 5 \times 120 + 0 = 600$
Value of Z at $(60,30) = 5 \times 60 + 10 \times 30 = 600$
Value of Z at $(40,20) = 5 \times 40 + 10 \times 20 = 400$
∴ Assertion is true.
Hence, both Assertion & Reason are true and Reason is the correct explanation of Assertion.

Q.4.The corner points of the feasible region for an L.P.P. are $(4,0), (5,0), (5,3), (3,5), (0,5)$ and $(0,4)$. The objective function $Z = ax - by + 1900, a, b > 0$ has maximum value 1950 at (5,0) and minimum value 1550 at (0,5).
Assertion (A): The value of Z at point $(5,3) = 1740$
Reason (R): Maximum and minimum value occurs when $a = 10, b = 70$.

Ans. Option (a) is correct.
Solution: $Z = ax - by + 1900$ has maximum value 1950 at (5,0).

So, $5a + 1900 = 1950 \Rightarrow 5a = 50 \Rightarrow a = 10$
and Z has minimum value 1550 at (0,5).
So, $0 - 5b + 1900 = 1550 \Rightarrow -5b = -350 \Rightarrow b = 70$

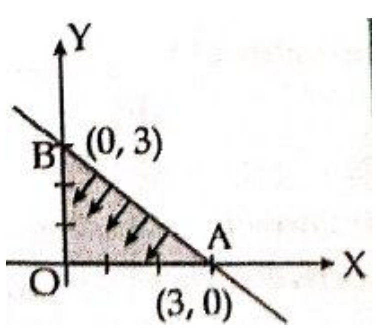
$$\therefore \ a = 10, b = 70$$

Also, value of Z at $(5,3) = 10 \times 5 - 70 \times 3 + 1900 = 1740$.
Hence, both Assertion & Reason are true and Reason is the correct explanation of Assertion

Q.5. Assertion (A): If the feasible region for an L.P.P. is bounded, then the objective function $Z = ax + by$ has both maximum and minimum values.
Reason (R): A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle.

Ans. Option (b) is correct.
Solution: Assertion is true by theorem.
Reason is also true but reason is not the correct explanation of Assertion.

Q.6. Corner points of feasible region are $(0,0), (3,0)$ and $(0,3)$ and objective function $Z = 4x + 7y$.



Assertion (A): Minimum value of Z is 12.
Reason (R): Maximum value of Z is 21.

Ans. Option (d) is correct.
Solution: Thus, corner points are: $O(0,0), A(3,0)$ and $B(0,3)$.Max. value of Z at corner points

$$Z_0 = 4(0) + 7(0) = 0 \rightarrow \text{ Min.}$$
$$Z_A = 4(3) + 7(0) = 12$$
$$Z_B = 4(0) + 7(3) = 21 \rightarrow \text{ Max.}$$

Q.7. Assertion (A): The point $(4,2)$ does not lie in the half plane of $4x + 6y - 24 < 0$.
Reason (R): The point $(1,2)$ lies in the half plane of $4x + 6y - 24 < 0$.
Ans. Option (b) is correct.
Solution: Put $(4,2)$ in $4x + 6y - 24$, we get

$$\begin{aligned} 4(4) + 6(2) - 24 &= 16 + 12 - 24 \\ &= 28 - 24 \\ &= 4 > 0 \end{aligned}$$

Thus, point $(4,2)$ does not lie in the half plane.
Put $(1,2)$ in $4x + 6y - 24$, we get

$$\begin{aligned} 4(1) + 6(2) - 24 &= 4 + 12 - 24 \\ &= 16 - 24 \\ &= -8 < 0 \end{aligned}$$

Thus, point $(1,2)$ lies in the half plane.

Q. 8. Assertion (A): Feasible region is the set of points which satisfy all of the given constraints and objective function too.
Reason (R): The optimal value of the objective function is attained at the points on X-axis only.
Ans. Option (c) is correct.
Solution: The optimal value of the objective function is attained at the corner points of feasible region.

Q. 9. Assertion (A): $Minimise$ Z $= 20x_1 + 20x_2$ subject to

$$x_1 \geq 0, x_2 \geq 2, x_1 + 2x_2 \geq 8, 3x_1 + 2x_2 \geq 15, 5x_1 + 2x_2 \geq 20.$$

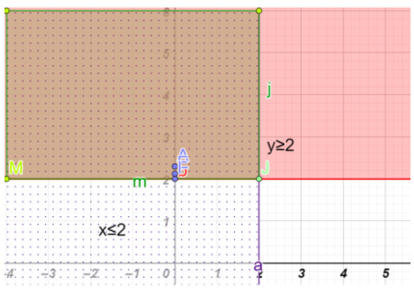
Out of the corner points of bounded feasible region (8,0), $\left(\frac{5}{2}, \frac{15}{4}\right), \left(\frac{7}{2}, \frac{9}{4}\right)$ and $(0,10)$, the minimum value of Z occurs at $\left(\frac{7}{2}, \frac{9}{4}\right)$.
Reason (R) :

Corner Points	$Z = 20x_1 + 20x_2$
(8,0)	160
$\left(\frac{5}{2}, \frac{15}{4}\right)$	125

$\left(\frac{7}{2},\frac{9}{4}\right)$	115 minimum
(0,10)	200

Ans. Option (a) is correct.
Solution: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

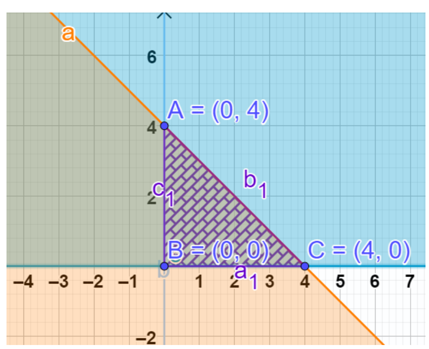
Q. 10. Assertion (A) : The graph of $x \leq 2$ and $y \geq 2$ is in the first and second quadrants.
Reason (R):



Ans. Option (A) is correct.
Solution: It is clear from the graph given in the Reason (R) that Assertion (A) is true.

VERY SHORT ANSWER TYPE QUESTIONS (SOLVED)

Q. 1. Solve the following Linear Programming Problem graphically:
Maximize $Z = 3x + 4y$
Subject to $x + y \leq 4, x \geq 0$ and $y \geq 0$.
Sol. The feasible region is a triangle with vertices B(0,0), $C(4,0)$ and $A(0,4)$



$$Z_O = 3 \times 0 + 4 \times 0 = 0$$
$$Z_A = 3 \times 4 + 4 \times 0 = 12$$
$$Z_B = 3 \times 0 + 4 \times 4 = 16$$

Thus, maximum of Z is at B(0,4) and the maximum value is 16.

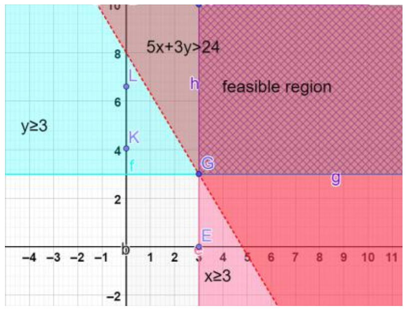
Q. 2. Two tailors, A and B, earn ₹ 300 and ₹ 100 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost. Formulate this as an LPP.

Sol. Let A works for x days and B for y days.

$$\therefore \text{Subject to} \quad \begin{cases} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{cases}$$

Q. 3. Find the maximum value of the function $z = 5x + 3y$ subjected to the constraints $x \geq 3$ and $y \geq 3$.

Sol. Here, value of Z at the corner point (3,3) of the feasible region is 24 .

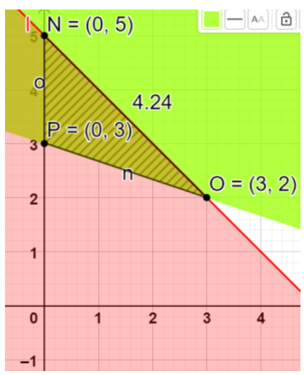


Since, region is unbounded, so we draw the graph of inequality $5x + 3y > 24$.

Since, open half plane of $5x + 3y > 24$ has common points with feasible region.

So, maximum value does not exist.

Q. 4. The feasible region for an LPP is shown in the following figure. Then, find the minimum value of $Z = 11x + 7y$.



Sol. Here, the objective function is $Z = 11x + 7y$, and the corner points of the feasible region are $O(3,2), P(0,3)$ and $N(0,5)$.

$\therefore$ At $O(3,2)$, $Z = 11 \times 3 + 7 \times 2 = 33 + 14 = 47$ and At $P(0,3)$, $Z = 11 \times 0 + 7 \times 3 = 21$

At N (0, 5), $Z = 11 \times 0 + 7 \times 5 = 35$

Hence, the minimum value of Z is 21 which occurs at point $P(0,3)$.

Q. 5. A small firm manufacture necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24 . It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹ 300. Formulate an L.P.P. for finding how many of each should be produced daily to

maximize the profit ? It is being given that at least one of each must be produced.
Sol. Let x necklaces and y bracelets are manufactured
∴ L.P.P is Maximize profit,

$$P = 100x + 300y$$

Subject to constraints
$x + y \leq 24$

$$\frac{1}{2}x + y \leq 16 \text{ or } x + 2y \leq 32$$
$$x, y \geq 1$$

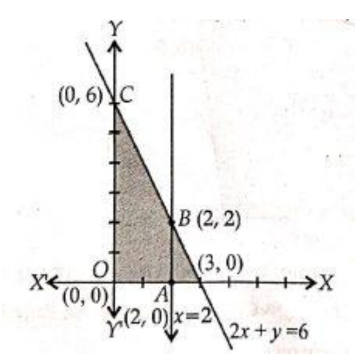
Q. 6. A firm has to transport at least 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is ₹400 and each small van is ₹200. Not more than ₹3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.
Sol. Let number of large vans $= x$ and number of small vans $= y$
Minimize cost $Z = 400x + 200y$
Subject to constraints

$$\left.\begin{array}{l} 200x + 80y \geq 1200 \text{ or } 5x + 2y \geq 30 \\ x \leq y \\ 400x + 200y \leq 3000 \text{ or } 2x + y \leq 15 \\ x \geq 0, y \geq 0 \end{array}\right\}$$

Q. 7. Determine the maximum value of $Z = 11x + 7y$ subject to the constraints: $2x + y \leq 6,\ x \leq 2, x \geq 0, y \geq 0$.
Sol. We have,
Maximise $Z = 11x + 7y$
Subject to the constraints,

$$\begin{array}{r} 2x + y \leq 6 \\ x \leq 2 \\ x \geq 0, y \geq 0 \end{array}$$

We see that, required area is the shaded region determined by the system of constraints (ii) to (iv) is $OABC$ and is bounded. So, now, we shall use corner point method to determine the maximum value of Z.

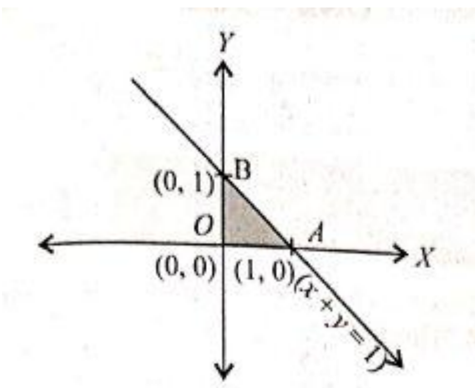


Corner points	Corresponding value of $\boldsymbol{Z = 11x + 7y}$
(0,0)	0
(2,0)	22
(2,2)	36
(0,6)	42 → Maximum

Hence, the maximum value of Z is 42 at $(0,6)$.
Q. 8. Maximize $Z = 3x + 4y$, subject to the constraints: $x + y \leq 1, x \geq 0, y \geq 0$.
Solution. Maximise $Z = 3x + 4y$,
Subject to the constraints,

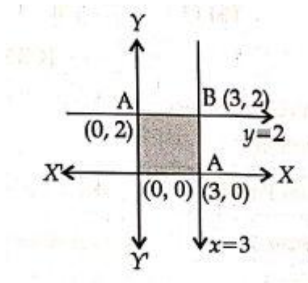
$$x + y \leq 1, x \geq 0, y \geq 0$$

The shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are $(0,0), (1,0)$ and $(0,1)$, respectively.



Corner points	Corresponding value of $Z = 3x + 4y$
(0,0)	0
(1,0)	3
(0,1)	4 → Maximum

Hence, the maximum value of Z is 4 at $(0,1)$.
Q.9. Maximize the function $Z = 11x + 7y$, subject to the constraints: $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.
Solution. Maximize $Z = 11x + 7y$,
Subject to the constraints, $x \leq 3, y \leq 2, x \geq 0, y \geq 0$



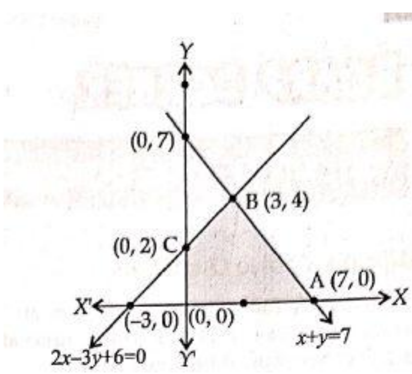
The shaded region as shown in the figure as $OABC$ is bounded and the coordinates of corner points are (0 , 0), (3,0), (3,2) and (0,2), respectively.

Corner points	Corresponding value of $\boldsymbol{Z} = 11x + 7y$
(0,0)	0
(3,0)	33
(3,2)	47 → Maximum
(0,2)	14

Hence, Z is maximum at (3,2) and its maximum value is 47.

Q.10. Minimise $Z = 13x - 15y$ subject to the constraints: $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$.
Sol Minimise $Z = 13x - 15y$
Subject to the constraints,

$$x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$$



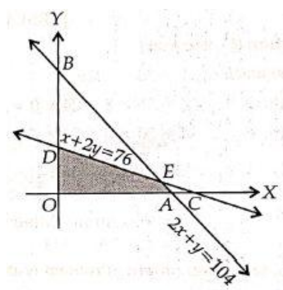
The shaded region shown as $OABC$ is bounded and coordinates of its corner points are (0,0), (7,0), (3,4) and (0,2), respectively.

Corner points	Corresponding value of $\boldsymbol{Z = 13x - 15y}$
(0,0)	0
(7,0)	91

(3,4)	-21
(0,2)	$-30 \rightarrow$ Minimum

Hence, the minimum value of Z is -30 at (0,2).

Q. 11. Determine the maximum value of $Z = 3x + 4y$ if the feasible region (shaded) for a LPP is shown in given figure.



Sol. Here, $2x + y = 104$ and $2x + 4y = 152$ intersect at $\Rightarrow$ E$(44,16)$.

As clear from the graph, corner points are O, A, E and D with coordinates $(0,0), (52,0), (144,16)$ and (0, 38), respectively. Also, given region is bounded.

Corner points	Corresponding value of $Z = 3x + 4y$
(0,0)	0
(52,0)	156
(44,16)	196 $\rightarrow$ Maximum
(0,38)	152

Hence, Z is at $(44,16)$ is maximum and its maximum value is 196.

Q. 12. For the LPP,

$$\text{Maximise } Z = 2x + 3y$$

The coordinates of the corner points of bounded feasible region are $\mathbf{A}(3,3), \mathbf{B}(20,3), \mathbf{C}(20,10)$, $D(18,12)$ and $E(12,12)$. Find the maximum value of Z.

Sol.

Comer Points	$Z = 2x + 3y$
$A(3,3)$	$Z = 15$
$B(20,3)$	$Z = 49$

$C(20,10)$	$Z = 70$
$D(18,12)$	$Z = 72 \rightarrow$ Max.
$E(12,12)$	$Z = 60$

Thus, maximum value of Z is 72 at D$(18,12)$.

Q. 13. The sum of two positive integers is atmost 5. The difference between two times of second number and first number is at most 4 . If first number is x and second number is y, then for maximizing the product of these two numbers, formulate LPP.
Sol. The required LPP is Max.
Subject to:

$$Z = x \times y$$
$$x + y \leq 5$$
$$2y - x \leq 4$$
$$x \geq 0$$
$$y \geq 0$$

Q.14. If a 20 year old girl drives her car at 25 km/h, she has to spend ₹ $4/\text{km}$ on petrol. If she drives her car at 40 km/h, the petrol cost increases to ₹$5/\text{km}$. She has ₹ 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem.

Sol. Let the distance covered with speed of $25 \text{ km/h} = x$ km and the distance covered with speed of $40 \text{ km/h} = y$ km

Total distance covered $= z$ km
The L.P.P. of the above problem, therefore, is

$$\text{Maximize } z = x + y$$

Subject to constraints

$$4x + 5y \leq 200$$
$$\frac{x}{50} + \frac{y}{40} \leq 1$$
$$\frac{x}{25} + \frac{y}{40} \leq 1$$
$$x \geq 0, y \geq 0$$

Q.15. A shopkeeper deals in two items-wall hangings and artificial plants. He has ₹15000 to invest and a space to store atmost 80 pieces. A wall hanging costs him ₹300 and an artificial plant ₹150. He can sell a wall hanging at a profit of ₹50 and an artificial plant at a profit of ₹18. Assuming that he can sell all the items that he buys, formulate a linear programming problem in order to maximize his profit.
Solution. Let x be the number of wall hangings and y be the number of artificial plants that the dealer

buys and sells. Then the profit of the dealer is $Z = 50x + 18y$, which is the objective function.
As a wall hanging costs ₹300 and an artificial plant cost ₹150, the cost of x wall hangings and y artificial plants is $300x + 150y$. We are given that the dealer can invest atmost ₹15000. Hence, the investment constraint is

$$300x + 150y \leq 15000 \text{ i.e. } 2x + y \leq 100$$

As the dealer has space to store atmost 80 pieces, we have another constraint (space constraint):

$$x + y \leq 80$$

Also the number of wall hangings and artificial plants can't be negative. Thus we have the non-negativity constraints:

$$x \geq 0, y \geq 0$$

Thus the mathematical formulation of the L.P.P. is :

$$\text{Maximize } Z = 50x + 18y \text{ subject to the constraints}$$

$$2x + y \leq 100, x + y \leq 80, x \geq 0, y \geq 0$$

Q.16. A retired person wants to invest an amount of upto ₹20000. His broker recommends investing in two types of bonds A and B, bond A yielding 8% return on the amount invested and bond B yielding 7% return on the amount invested. After some consideration, he decides to invest atleast ₹5000 in bond A and no more than ₹8000 in bond B. He also wants to invest atleast as much in bond A as in bond B. Formulate as L.P.P. to maximize his return on investments.
Solution. Let the person invest ₹x in bonds of type A and ₹y in bonds of type B , then his earning i.e. return (in ₹)

$$Z = 8\% \text{ of } x + 7\% \text{ of } y = 0.08x + 0.07y$$

As the person can invest upto ₹ 20000 , so investment constraint is

$$x + y \leq 20000.$$

Other investment constraints are

$$x \geq 5000, y \leq 8000, x \geq y$$

Non-negativity constraints are $x \geq 0, y \geq 0$.
Thus, mathematically formulation of the L.P.P. is
Maximize $Z = 0.08x + 0.07y$ subject to the constraints

$$x + y \leq 20000, x \geq 5000, y \leq 8000, x \geq y, x \geq 0, y \geq 0$$

Q.17. (Manufacturing problem) A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts, while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹2.50

per package on nuts and ₹1 per package on bolts. Form a linear programming problem to maximize his profit, if he operates each machine for atmost 12 hours

Solution. Suppose that x packages of nuts and y packages of bolts are produced. The objective of the manufacturer is to maximize the profit $Z = 2.50x + 1.y$
Time required on machine A to produce x packages of nuts and y packages of bolts is $1.x + 3.y = x + 3y$, while time required on machine B is $3.x + 1.y = 3x + y$.
From given data, we can formulate the L.P.P. as:
Maximize Z $= 2.5x + y$ subject to the constraints

$$x + 3y \leq 12 \quad \text{(Machine A constraint)}$$
$$3x + y \leq 12 \quad \text{(Machine B constraint)}$$
$$x \geq 0, y \geq 0 \quad \text{(Non-negativity constraints)}$$

Q.18. A manufacturer has three machines M_1, M_2 and M_3 installed in his factory. Machines M_1 and M_2 are capable of being operated for atmost 12 hours, whereas machine M_3 must be operated for atleast 5 hours a day. The manufacturer produces only two items, each requiring the use of these three machines. The following table gives the number of hours required on these machines for producing 1 unit of A or B.

Item	Number of hours required on the machines			
	M_1	M_2	M_3	
A	1	2	1	
B	2	1	$\frac{5}{4}$	

He makes a profit of ₹60 on item A and ₹40 on item B. He wishes to find out how many of each item he should produce to have maximum profit. Formulate it as L.P.P.
Solution. Let the manufacturer produce x units of item A and y units of item B. As he makes a profit of ₹60 on one item of A and ₹ 40 on one item of B, his total profit (in ₹) is

$$Z = 60x + 40y$$

From the given data, we can mathematically formulate the L.P.P. as
Maximize Z $= 60x + 40y$ subject to the constraints

$$x + 2y \leq 12 \quad (\text{Machine } M_1 \text{ constraint})$$
$$2x + y \leq 12 \quad (\text{Machine } M_2 \text{ constraint})$$

$$x + \frac{5}{4}y \geq 5 \quad (\text{Machine } M_3 \text{ constraint})$$

$$x \geq 0, y \geq 0 \quad \text{(Non-negativity constraints)}$$

Q.19. (Diet problem) A diet is to contain atleast 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹4 per kg and F_2 costs ₹5 per kg. One kg of food F_1 contains

3 units of vitamin A and 4 units of minerals. One kg of food F_2 contains 6 units of vitamin A and 3 units of minerals. We wish to find the minimum cost for diet that consists of mixture of these two foods and also meets the minimum nutritional requirements. Formulate this as a linear programming problem.
Solution. Let the mixture consist of x kg of food F_1 and y kg of food F_2.
We make the following nutritional requirement table from given data :

Resources	Food (in kg)		Requirement (in units)
	F_1	F_2	
Vitamin A (units/kg)	3	6	80
Minerals (units/kg)	4	3	100

Minimum requirement of vitamin A is 80 units, therefore,

$$3x + 6y \geq 80$$

Similarly, the minimum requirement of minerals is 100 units, therefore,

$$4x + 3y \geq 100$$

Also $\quad x \geq 0, y \geq 0$

As cost of food F_1 is ₹4 per kg, and the cost of food F_2 is ₹5 per kg, the total cost of purchasing x kg of food F_1 and y kg of food F_2 is $\mathrm{Z} = 4x + 5y$,
which is the objective function.
Hence, the mathematical formulation of the L.P.P. is :
Minimize $\mathrm{Z} = 4x + 5y$ subject to the constraints

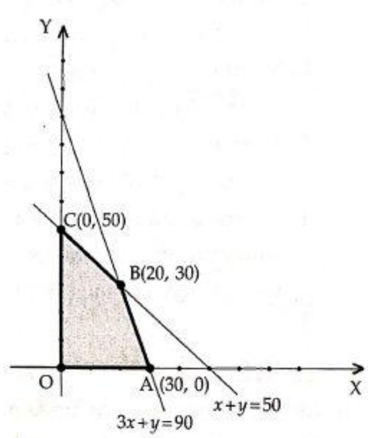
$$3x + 6y \geq 80, 4x + 3y \geq 100, x \geq 0, y \geq 0.$$

SHORT ANSWER TYPE QUESTIONS (SOLVED)

Q.1. Solve the following linear programming problem graphically:
Maximize and minimize $\mathrm{Z} = 60x + 15y$ subject to the constraints

$$x + y \leq 50, 3x + y \leq 90, x, y \geq 0$$

Solution. We draw the lines $x + y = 50, 3x + y = 90$ and shade the region satisfied by the given inequalities. The shaded region in the adjoining figure shows the feasible region determined by the given constraints. We observe that the feasible region OABC is convex polygon and bounded. So we use the corner point method to calculate the maximum and minimum value of Z.



The coordinates of the corner points $\mathrm{O}, \mathrm{A}, \mathrm{B}, \mathrm{C}$ are $(0,0), (30,0), (20,30)$ and $(0,50)$ respectively. We evaluate $\mathrm{Z} = 60x + 15y$ at each of these points.

Corner point	Value of objective function $Z = 60x + 15y$
(0,0)	0 → Smallest
(30,0)	1800 → Largest
(20,30)	1650
(0,50)	750

Hence, the minimum value of Z is 0 at the point $(0,0)$ and maximum value of Z is 1800 at the point $(30,0)$.

Q.2.. Find graphically, the maximum value of $Z = 2x + 5y$, subject to constraints given below:

$$2x + 4y \leq 8, 3x + y \leq 6, x + y \leq 4, x \geq 0, y \geq 0$$

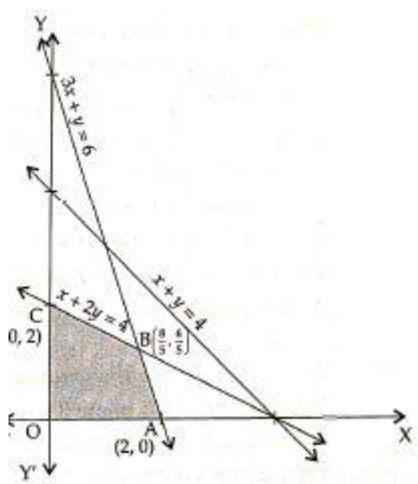
Solution. We are required to maximise $Z = 2x + 5y$, subject to the constraints:

$$2x + 4y \leq 8 \text{ ie. } x + 2y \leq 4$$
$$3x + y \leq 6, x + y \leq 4, x \geq 0, y \geq 0$$

Draw the lines $x + 2y = 4$ (passes through $(4,0), (0,2)$);

$3x + y = 6$ (passes through $(2,0), (0,6)$)
$x + y = 4$ (passes through $(4,0), (0,4)$)

Shade the region satisfied by the given inequalities.



The shaded region in the adjoining figure gives the feasible region determined by the given inequalities. Solving $3x + y = 6$ and $x + 2y = 4$ simultaneously, we get

$$x = \frac{8}{5} \text{ and } y = \frac{6}{5}$$

We observe that the feasible region OABC is a convex polygon and bounded and has corner points

$$O(0,0), A(2,0), B\left(\frac{8}{5}, \frac{6}{5}\right) \text{ and } C(0,2)$$

The optimal Solution occurs at one of the corner points
At $O(0,0), Z = 2.0 + 5.0 = 0$;
at $A(2,0), Z = 2.2 + 5.0 = 4$;
at $B\left(\frac{8}{5}, \frac{6}{5}\right), Z = 2, \frac{8}{5} + 5, \frac{6}{5} = \frac{46}{5}$;
at $C(0,2), Z = 2.0 + 5.2 = 10$.
Therefore, Z has maximum value at C and maximum value $= 10$.

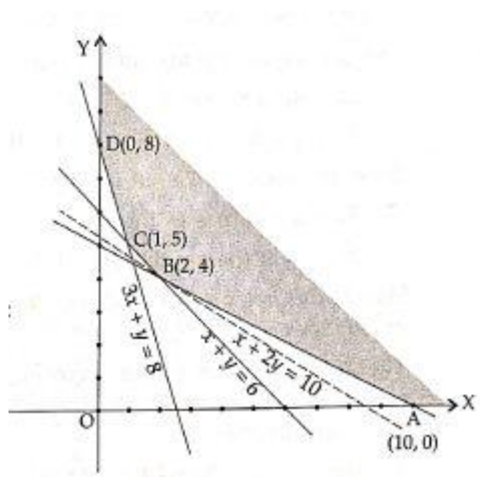
Q.3. Solve the following linear programming problem graphically:
Minimise $Z = 3x + 5y$ subject to the constraints:

$$x + 2y \geq 10, x + y \geq 6, 3x + y \geq 8, x, y \geq 0.$$

Solution. We are required to minimise $Z = 3x + 5y$, subject to the constraints:

$$x + 2y \geq 10, x + y \geq 6, 3x + y \geq 8, x \geq 0, y \geq 0$$

Draw the lines $x + 2y = 10, x + y = 6$ and $3x + y = 8$.
Shade the region satisfied by the given inequalities.



The shaded region in the adjoining figure gives the feasible determined by the given inequalities.
Solving $x + 2y = 10$ and $x + y = 6$ simultaneously, we get

$$x = 2 \text{ and } y = 4.$$

Solving $x + y = 6$ and $3x + y = 8$
simultaneously, we get $x = 1, y = 5$.
We observe that the feasible region is unbounded and the corner points are

$$A(10,0), B(2,4), C(1,5) \text{ and } D(0,8)$$

At the corner points, the values of Z are :

$$\text{at } A(10,0), Z = 3.10 + 5.0 = 30$$
$$\text{at } B(2,4), Z = 3.2 + 5.4 = 26$$
$$\text{at } C(1,5), Z = 3.1 + 5.5 = 28$$
$$\text{at } D(0,8), Z = 3.0 + 5.8 = 40$$

The smallest value of Z is 26 at $B(2,4)$.
As the feasible region is unbounded, we cannot say whether the minimum value exists or not.
To check whether the smallest value 26 is minimum
We draw the half plane $3x + 5y < 26$ and notice that there is no common point with the feasible region.
Hence, 26 is indeed the minimum value.
Hence, the given objective function $Z = 3x + 5y$ has minimum value 26 at $B(2,4)$.

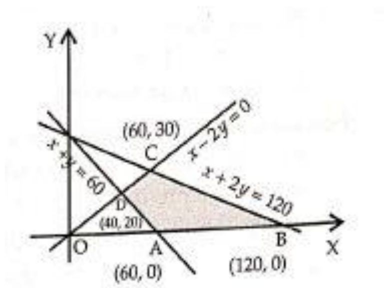
Q.4. Maximize and minimize Z $= 5x + 10y$ subject to the constratints

$$x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0, y \geq 0$$

Solution. Maximize and minimize $Z = 5x + 10y$ subject to the constraints

$$x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0, y \geq 0$$

Draw the lines $x + 2y = 120, x + y = 60$, and $x - 2y = 0$; shade the region satisfied by the given inequalities.



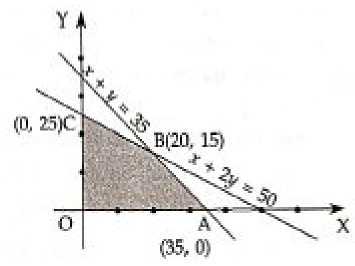
The feasible region is polygon $ABCD$, which is convex and bounded. Corner points of feasible region are $A(60,0), B(120,0), C(60,30)$ and $D(40,20)$.

The values of $Z = 5x + 10y$ at the points A, B, C and D are 300, 600, 600 and 400 respectively.
Minimum value $= 300$ at $A(60,0)$, maximum value $= 600$ at $B(120,0)$ and $C(60,30)$.
In fact, all points on the line segment BC give the same maximum value $= 600$.

Q.5. A furniture trader deals in only two items-chairs and tables. He has ₹50000 to invest and a space to store atmost 35 items. A chair costs him ₹1000 and a table costs him ₹2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and a table, respectively. Formulate the above problem as an L.P.P. to maximise the profit and solve it graphically.
Solution. Let x be the number of chairs and y be the number of tables that the dealer buys and sells. The profit on a chair is ₹150 and the profit on a table is ₹250. So, total profit P $= 150x + 250y$. Hence, the problem can be formulated as an L.P.P. as follows:
Maximise P $= 150x + 250y$ subject to the constraints

i.e. $x + y \leq 35$
$1000x + 2000y \leq 50000$
$x + 2y \leq 50$
$x \geq 0, y \geq 0$

Draw the lines $x + y = 35$ and $x + 2y = 50$; shade the region satisfied by the given inequalities.



The shaded portion shows the feasible region which is bounded. The point of intersection of the lines $x + y = 35$ and $x + 2y = 50$ is $\mathrm{B}(20,15)$.

The four corner points of the feasible region OABC are $\mathrm{O}(0,0), \mathrm{A}(35,0), \mathrm{B}(20,15)$ and $\mathrm{C}(0,25)$.
At $(0,0), \mathrm{P} = 150 \times 0 + 250 \times 0 = 0$.
At $\mathrm{A}(35,0), \mathrm{P} = 150 \times 35 + 250 \times 0 = 5250$.
At $\mathrm{B}(20,15), \mathrm{P} = 150 \times 20 + 250 \times 15 = 6750$.
At $\mathrm{C}(0,25), \mathrm{P} = 150 \times 0 + 250 \times 25 = 6250$.
We find that P is maximum at $\mathrm{B}(20,15)$ and maximum value of $\mathrm{P} = 6750$.
Hence, the dealer gets maximum profit of ₹6750 when he buys and sells 20 chairs and 15 tables.

LONG ANSWER TYPE QUESTIONS (SOLVED)

Q. 1. The linear programming problem is as follows:

Minimize: $50x + 20y$

Subject to the constraint,

$6x + 2y \geq 1200$
$300 \leq x + y \leq 600$
$x > 0, y > 0$

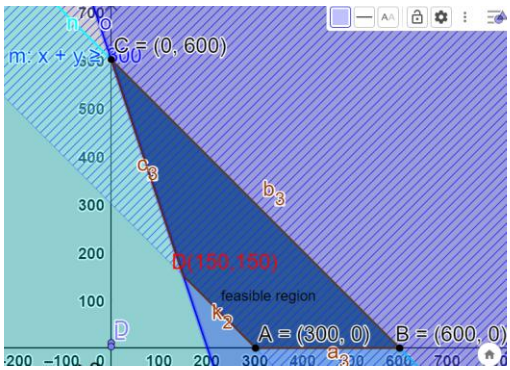
(a) Find the values of x and y for which the value of the objective function $50x + 20y$ is the least. Justify your answer.
(b) Can the objective function have its minimum or maximum value at any of points $E(0,300), F(100, 300)$ or $G(100,200)$ and why?

Point	A(300, 0)	B(0, 600)	C(0, 600)	D(150, 150)	G(100, 200)
Value of objective function at the point	15000	30000	12000	10500	9000

Concept: If the feasible region of a linear programming problem where $Z = ax + by$ is an objective function and constraints are linear inequalities in x and y is bounded, then Z has both a maximum and minimum value in R and each of these occurs at a corner point of R.
Sol. Coverting the given inequations into equations, we get

$$6x + 2y = 1200$$
$$x + y = 300$$
$$x + y = 600$$
$$x = 0 \text{ and } y = 0$$

Now on drawing these lines on graph we obtain following graph:



From graph it is clear that we have corner points of the shaded feasible region are $ABCD$ are $A(300,0)$, $B(600,0)$, $C(0,600)$ and $D(150,150)$.

These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.
The values of the objective function at these points are given in the following table:

Point (x, y)	Value of the objective function $z = 5x + 2y$
A(300,0)	15,000
B(0,600)	30,000
C(0,600)	12,000
D(150,150)	10,500
G(100,200)	9,000

Clearly, Z is minimum at point $D(150,150)$.
The minimum value of Z is 10,500 .
Here the point $D(x = 150 \text{ and } y = 150)$ is the optimal feasible Solution of the given LPP specifying the value of the objective function is the least at D among the 4 corner points of the feasible region.
Note: Here point G is out of feasible region.
In Option (B) Constraint $x + 2y > 8$ does not validate the data given in question.

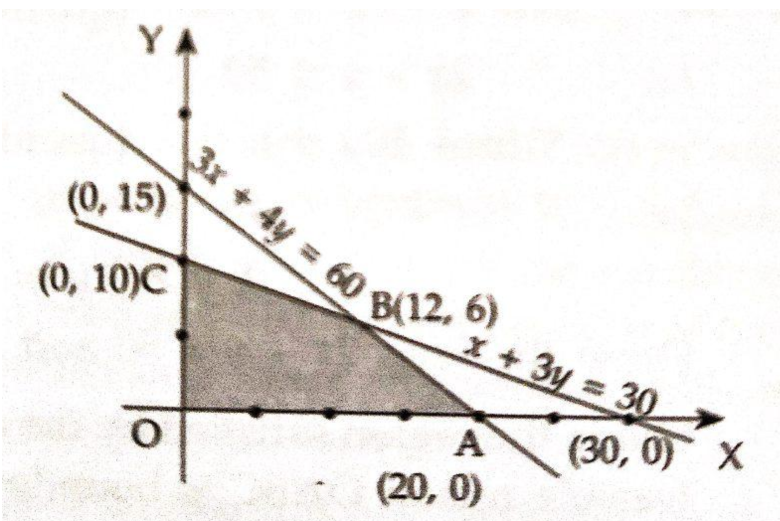
In Option(C) Both Objective function and constraints are incorrect.
In Option (D) both objective function and all constraints are incorrect as these do not validate the data of given LPP.

Q.2. A manufacturing company makes two types of teaching aids A and B of Mathematics for class X. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹80 on each piece of type A and ₹120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?
Solution. Let x and y be the number of teaching aids of type A and type B respectively.
As the profit on the type A of teaching aid is ₹ 80 and on the type B is ₹ 120,
so,the total profit $Z = 80x + 120y$.
Hence, the problem can be formulated as an L.P.P. as follows:

Maximise $Z = 80x + 120y$

Subject to the Consraints ;
$9x + 12y \leq 180$ i.e. $3x + 4y \leq 60$
$x + 3y \leq 30$
$x \geq 0, y \geq 0$

We draw straight lines $3x + 4y = 60, x + 3y = 30$ and shade the region satisfied by the above inequalities.



The shaded region shows the feasible region which is bounded. The point of intersection of the lines $3x + 4y = 60$ and $x + 3y = 30$ is B(12,6)

The corner points of the feasible region OABC are $O(0,0), A(20,0), B(12,6)$ and $C(0,10)$. The optimal Solution occurs at one of the corner points.
At $O(0,0), Z = 80 \times 0 + 120 \times 0 = 0$
At $A(20,0), Z = 80 \times 20 + 120 \times 0 = 1600$
At $B(12,6), Z = 80 \times 12 + 120 \times 6 = 1680$

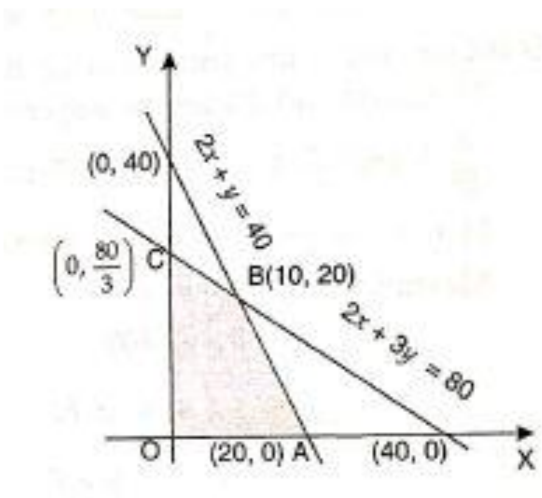
At $C(0,10), Z=80 \times 0+120 \times 10=1200$.
We find that value of Z is maximum at $B(12,6)$.

Hence, the manufacturer should produce 12 aids of type A and 6 aids of type B to get a maximum profit of ₹ 1680 .

Q.3. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on item of model A is ₹15 and on an item of model B is ₹10. How many items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.
Solution. Since each man skilled or semi-skilled works for at most 8 hours per day and 5 skilled men and 10 semi-skilled men are employed, so the maximum number of skilled working hours available $=$ $5 \times 8=40$ and maximum number of semi-skilled working hours available $=10 \times 8=80$. If x items of model A and y items of model B are made, then the LPP is maximise $Z=15 x+10 y$ subject to constraints

$$2 x+y \leq 40, 2 x+3 y \leq 80, x \geq 0, y \geq 0$$

We draw the straight lines $2 x+y=40$ and $2 x+3 y=80$ and shade the region satisfied by the above inequalities.



The shaded region shows the feasible region which is bounded. The point of intersection of the lines $2 x+y=40$ and $2 x+3 y=80$ is $\mathrm{B}(10,20)$.
The corner points of the feasible OABC are $\mathrm{O}(0,0), \mathrm{A}(20,0), \mathrm{B}(10,20)$ and $\mathrm{C}\left(0, \frac{80}{3}\right)$. The optimal Solution occurs at one of the corner points.
At $\mathrm{O}(0,0), \mathrm{Z}=15 \times 0+10 \times 0=0$;
at $\mathrm{A}(20,0), \mathrm{Z}=15 \times 20+10 \times 0=300$;
at $B(10,20) Z=15 \times 10+10 \times 20=350$;
at $C\left(0, \frac{80}{3}\right), Z=15 \times 0+10 \times \frac{80}{3}=\frac{800}{3}=266 \frac{2}{3}$.

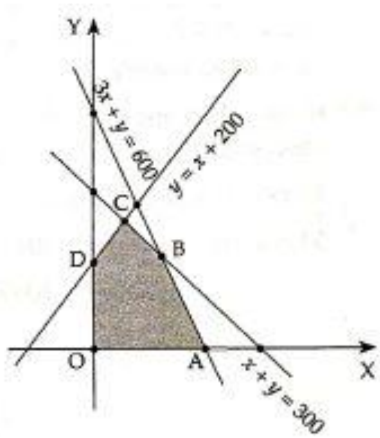
We find that the value of Z is maximum at $B(10,20)$.
Hence, the manufacturer should produce 10 items of model A and 20 items of model B to get maximum profit of ₹350.

Q.4. A company manufactures two types of cardigans: type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigon. The company can make atmost 300 cardigons and spend atmost ₹ 72000 a day. The number of cardigons of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of ₹ 100 for each cardigan of type A and ₹50 for every cardigan of type B. Formulate this problem as a linear programming problem to maximise the profit of the company. Solve it graphically and find maximum profit.
Solution. Let x cardigons of type A and y cardigons of type B be manufactured per day by the company, then profit P (in ₹) $= 100x + 50y$.
Hence, the problem can be formulated as an L.P.P. as follows:

Maximize P $= 100x + 50y$

subject to the constraints
$360x + 120y \leq 72000$ i.e. $3x + y \leq 600$
$x + y \leq 300$
$y \leq x + 200$
$x \geq 0, y \geq 0$

Draw the lines $3x + y = 600, x + y = 300$ and $y = x + 200$ and shade the region satisfied by the above inequalities.



The shaded portion OABCD shows the feasible region, which is bounded.
The corner points of the feasible region are $O(0,0), A(200,0), B(150,150), C(50,250)$ and $D(0,200)$.
The values of P at the corner points are :
at $O(0,0)$, $P = 100 \times 0 + 50 \times 0 = 0$;
at $A(200,0)$, $P = 100 \times 200 + 50 \times 0 = 20000$;
at $B(150,150)$, $P = 100 \times 150 + 50 \times 150 = 22500$;
at $C(50,250)$, $P = 100 \times 50 + 50 \times 250 = 17500$;
at $D(0,200)$, $P = 100 \times 0 + 50 \times 200 = 10000$.
We note that the value of P is maximum at the point B .

Hence, maximum profit = ₹22500 by manufacturing 150 cardigons of type A and 150 cardigons of type B.

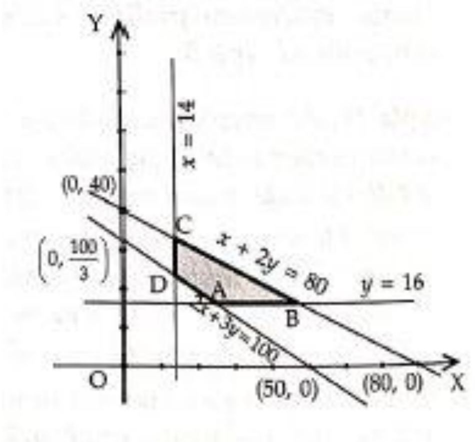
Q.5.. A manufacturer makes two products, A and B. Product A sells at ₹200 each and takes $\frac{1}{2}$ hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and the weekly turnover must not be less than ₹10000. If the profit on each of product A is ₹20 and on product B is ₹30, then how many of each should be produced so that the profit is maximum? Also find the maximum profit.
Solution. Let x and y be the number of units of products A and B respectively manufactured (and sold), then total profit $\mathrm{P} = 20x + 30y$ (in ₹).
Hence, the problem can be formulated as an L.P.P. as follows:

$$\text{Maximize } \mathrm{P} = 20x + 30y$$

subject to the constraints

$$200x + 300y \geq 10000 \text{ i.e. } 2x + 3y \geq 100$$
$$\frac{1}{2}x + 1.y \leq 40 \text{ i.e. } x + 2y \leq 80$$
$$x \geq 14, y \geq 16, x \geq 0, y \geq 0$$

We draw the straight lines $2x + 3y = 100, x + 2y = 80, x = 14$ and $y = 16$ and shade the region satisfied by the above inequalities.



The shaded portion shows the feasible region $ABCD$, which is bounded. The corner points of the feasible region ABCD are A(26,16), B(48,16), $C(14,33)$ and $D(14,24)$.
The optimal Solution occurs at one of the corner points.
At A, $\mathrm{P} = 20 \times 26 + 30 \times 16 = 1000$.
At B, $\mathrm{P} = 20 \times 48 + 30 \times 16 = 1440$.
At C, $\mathrm{P} = 20 \times 14 + 30 \times 33 = 1270$.
At D, $\mathrm{P} = 20 \times 14 + 30 \times 24 = 1000$.
Hence, the manufacturer should produce 48 units of product A and 16 units of product B. Maximum profit is ₹ 1440 .
Q.6. A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for atmost 12 hours whereas machine III must be operated at least 5 hours a day. He

produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹600 and ₹400 on each item of M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced? What will be the maximum profit?

Solution. Let x units of item M and y units of item N are produced by the manufacturer, then the problem can be formulated as an L.P.P. as follows:

Maximize the profit (in ₹) $Z = 600x + 400y$ subject to the constraints

$x + 2y \leq 12$ (machine I constraint)

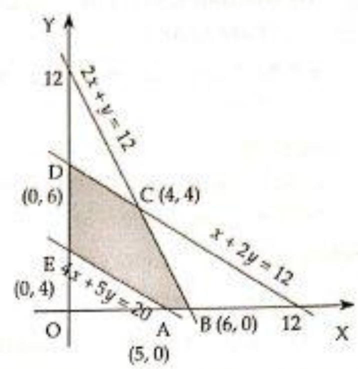
$2x + y \leq 12$ (machine II constraint)

$1x + 1.25y \geq 5$ (machine III constraint)

i.e. $4x + 5y \geq 20,$

$x \geq 0, y \geq 0$ (non-negativity constraints)

Draw the lines $x + 2y = 12, 2x + y = 12$ and $4x + 5y = 20$, and shade the region satisfied by the above inequalities.



The feasible region is the polygon ABCDE , which is convex and bounded.

The corner points are A(5,0), B(6,0), C(4,4), $D(0,6)$ and $E(0,4)$.

The values of Z at the corner points are :

at $A(5,0), Z = 600 \times 5 + 400 \times 0 = 3000$;

at $B(6,0), Z = 600 \times 6 + 400 \times 0 = 3600$;

at $C(4,4), Z = 600 \times 4 + 400 \times 4 = 4000$;

at $D(0,6), Z = 600 \times 0 + 400 \times 6 = 2400$;

at $E(0,4), Z = 600 \times 0 + 400 \times 4 = 1600$.

$\therefore$ Maximum profit = ₹4000, when 4 items of M and 4 items of N are produced.

CASE-STUDY BASED QUESTIONS (SOLVED)

Q.1.Read the following text and answer the following questions on the basis of the same:

A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B to go into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. Supplier S has a mix of 4 units of A and 2 units of B that costs ₹10, the supplier T has a mix of 1 unit of A and 1 unit of B that costs ₹ 4. Suppose x units of mix are purchased from supplier S and y units are purchased from supplier T.

On the basis of this information, answer the following questions.
(i) What is the total cost of purchase form both the Suppliers, represent this in equation form?
Sol. Suppose x units of mix are purchased from supplier S and y units are purchased from supplier T.
Total cost $Z = 10x + 4y$.
(ii) Write the minimum requirement of chemical A in equation form?
Sol. The minimum requirement of chemical A per bottle is given by: $4x + y \geq 80$.
(iii) Write the minimum requirement of chemical B in equation form?
OR
How many mixes from S and T should the company purchase to honour contract requirement and yet minimize cost?
Sol. The minimum requirement of chemical B per bottle is given by : $2x + y \geq 80$.

OR

The given data may be put in the following tabular form:

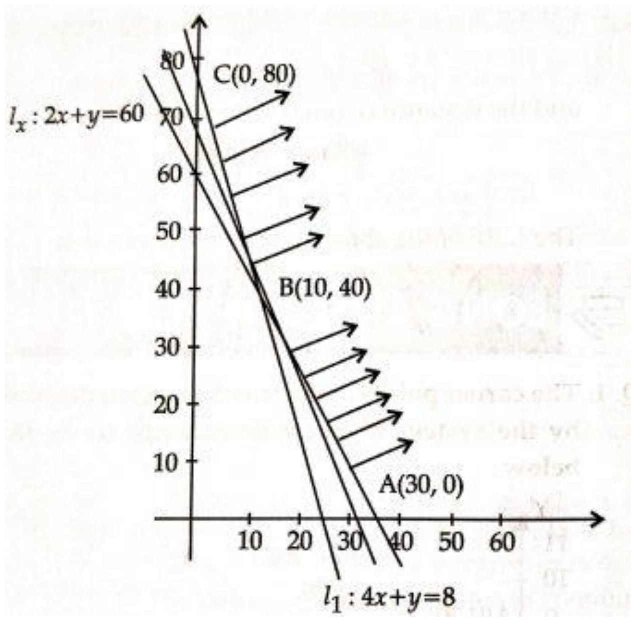
Supplier/	S	T	Minimum Requirement
Chemical A	4	1	80
Chemical B	2	1	60
Cost per unit	₹10	₹4	

$4x + y \geq 80$
$2x + y \geq 60$
$x \geq 0, y \geq 0$
$L_1: 4x + y = 80$

x	0	25
y	50	0

$L_2: 2x + y = 60$

x	30	0
y	0	60



Vertices of feasible region are $A(30,0), B(10,40)$ and C(0,80).

Point (x, y)	Value of objective function $Z = 10x + 4y$
$A(30,0)$	$Z = 10 \times 30 + 4 \times 0 = 300$
$P(10,40)$	$Z = 10 \times 10 + 40 \times 4 = 260$
$C(0,80)$	$Z = 10 \times 0 + 4 \times 80 = 320Z$

Clearly, Z is minimum at $(10,40)$. The feasible region is unbounded and the open half plane represented by $10x + 4y < 260$ does not have points in common with the feasible region. So, Z is minimum at $x = 10, y = 40$. Hence, $x = 10, y = 40$ is the optimal Solution of the given LPP. Hence, the cost per bottle is minimum when the company purchases 10 mixes from supplier 5 and 40 mixes from supplier T.

Q.2. Read the following text and answer the following questions on the basis of the same:
Ramesh rides her bike at 25 km/h. She has to spend ₹2 per km on diesel and if he rides it at faster speed of 40 km/h, the diesel cost increases to ₹ 5 per km. He has ₹ 100 to spend on diesel. Let he travels x km with speed 25 km/h and y km with speed 40 km/h. It is given that objective function is Max. $Z = x + y$.

(i) Write the objective function for given LPP.

Sol. Objective function for given LPP is: Max Z $= x + y$

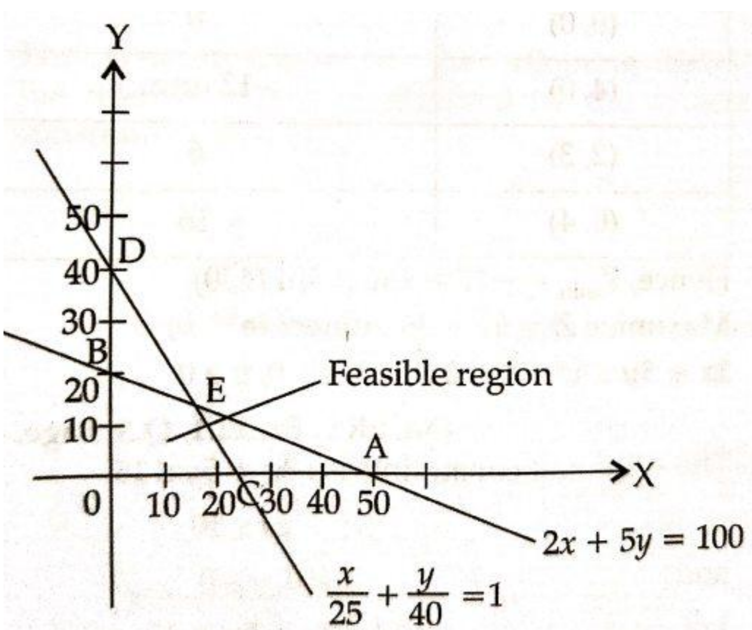
(ii) Write the set of constraints for given LPP.

Sol. Subject to constraints:

$$2x + 5y \leq 100$$
$$\frac{x}{25} + \frac{y}{40} \leq 1$$
$$x, y \geq 0$$

(iii) Draw the graph for feasible region. OR

Write the corner points of feasible region and find maximum value of objective function.



Intersection point of $2x + 5y = 100$ and $\frac{x}{25} + \frac{y}{40} \leq 1$ is E .

On solving the equations, we get the coordinates of E are $\left(\frac{50}{3}, \frac{40}{3}\right)$

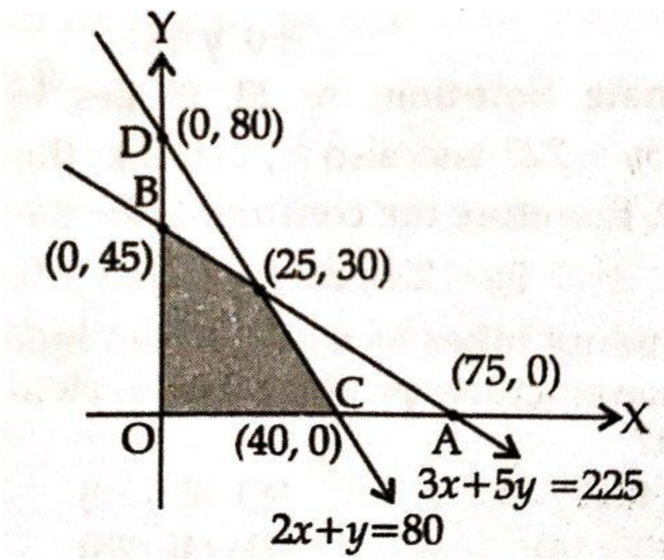
OR

So corner points of LPP are: $O(0,0), B(0,20), C(25,0), \left(\frac{50}{3}, \frac{40}{3}\right)$

Corner points	value of x = x + y
(0,0)	0
(0,20)	20
(25,0)	25
$\left(\frac{50}{3}, \frac{40}{3}\right)$	30 → Maximum

Q.3.A manufacturer produces two Models of bikes Model X and Model Y. Model X takes 6 manhours to make per unit, while Model Y takes 10 man-hours per unit. There is a total of 450 man-hours available per week. Handling and Marketing costs are ₹2,000 and ₹1,000 per unit for Models X and Y respectively. The total funds available for these purposes are ₹80,000 per week. Profits per unit for Models X and Y are ₹1,000 and₹500, respectively.

The feasible region of LPP is shown in the following graph.



(i) Write the equation of line AB.

Solution: From the given graph $OA = 75$ and $OB = 45$.

The equation of line AB is $\frac{x}{75} + \frac{y}{45} = 1$

i.e.,$3x + 5y = 225$

(ii) Write the equation of line CD.

Solution: From the given graph $OC = 40$ and $OD = 80$.

The equation of line CD is $\frac{x}{40} + \frac{y}{80} = 1$

i.e., $2x + y = 80$

(iii) Write the coordinates of point E.
Solution. On solving the equations of lines AB and CD, we get the coordinates of point E i.e., $(25,30)$.
(iv) Write the constraints for L.P.P.
Solution: Let the manufacturer produces x number of model X and y number of model Y bikes. Model X takes 6 man-hours to make per unit and model Y takes 10 man-hours to make per unit.
There is total of 450 man-hours available per week.

$$\therefore \quad 6x + 10y \leq 450$$
$$\Rightarrow \quad 3x + 5y \leq 225$$

For models X and Y, handling and marketing costs are ₹2,000 and ₹1,000, respectively, total funds available for these purposes are ₹ 80,000 per week.

$$\therefore \quad 2{,}000x + 1{,}000y \leq 80{,}000$$
$$\Rightarrow \quad 2x + y \leq 80$$
$$\text{Also,} \quad x \geq 0, y \geq 0$$

(v)How many bikes of model X and model Y should the manufacturer produce so as to yield a maximum profit?
Solution:The objective function for given L.P.P. is

$$Z = 1000x + 500y$$

From the shaded feasible region, it is clear that coordinates of corner points are (0,0), (40,0), $(25,30)$ and $(0,45)$.

Corner points	Value of $Z = 1000x + 500y$
(0,0)	0
(40,0)	40,000 ← Maximum
$(25,30)$	$25{,}000 + 15{,}000 = 40{,}000$ ← Maximum
$(0,45)$	22,500

So, the manufacturer should produce 25 bikes of model X and 30 bikes of model Y to get a maximum profit of ₹ 40,000.

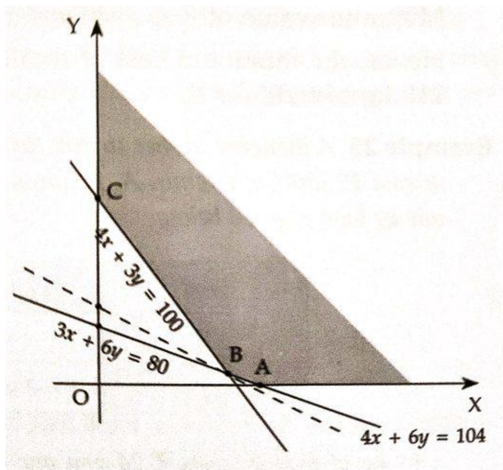
HOTS QUESTIONS (SOLVED)

Q.1. A diet is to contain atleast 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹4 per unit and F_2 cost ₹6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.
Solution. Let x units of food F_1 and y units of food F_2 be mixed and Z (in ₹) be the total cost of the food,

then the poblem can be formulated as an L.P.P. as follows:
Minimize

$$Z = 4x + 6y \quad \text{subject to constraints}$$
$$3x + 6y \geq 80 \quad \text{(vitamin A constraint)}$$
$$4x + 3y \geq 100 \quad \text{(minerals constraint)}$$
$$x \geq 0, y \geq 0 \quad \text{(non-negativity constraints)}$$

Draw the lines $3x + 6y = 80$ and $4x + 3y = 100$, and shade the region satisfied by the above inequalities.

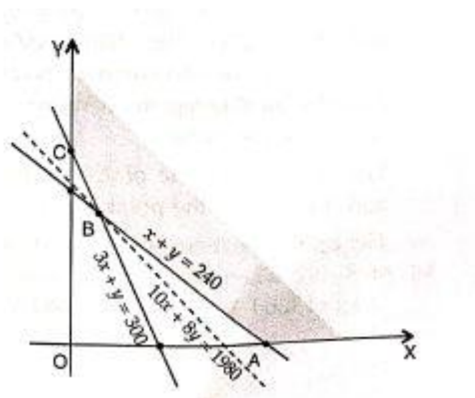


The feasible region (unbounded, convex) is shown shaded.
The corner points are A $\left(\frac{80}{3}, 0\right)$, $B\left(24, \frac{4}{3}\right)$ and $C\left(0, \frac{100}{3}\right)$.
At A $\left(\frac{80}{3}, 0\right)$, $Z = \frac{320}{3} = 106\frac{2}{3}$;
at B $\left(24, \frac{4}{3}\right)$, $Z = 96 + 6 \times \frac{4}{3} = 96 + 8 = 104$;
at $C\left(0, \frac{100}{3}\right)$, $Z = 200$.
Among the values of Z, the least value is 104.
We draw the line $4x + 6y = 104$ (shown dotted in the given figure) and note that the half plane $4x + 6y < 104$ has no common point with the feasible region, therefore, Z has minimum value. Minimum value of Z is 104 and it occurs at the point $B\left(24, \frac{4}{3}\right)$.
Hence, the minimum cost of the diet is ₹104 when we mix 24 units of food F_1 and $\frac{4}{3}$ units of food F_2.

Q.2. There are two types of fertilisers ' A ' and ' B '. ' A ' consists of 12% nitrogen and 5% phosphoric acid whereas ' B ' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs atleast 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If ' A ' costs ₹10 per kg and ' B ' costs ₹ 8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.
Solution. Let x kg of fertiliser A and y kg of fertiliser B be used and Z (in ₹) be the total cost of the two fertilisers, then $Z = 10x + 8y$. The problem can be formulated as an L.P.P. as follows:
Minimise $Z = 10x + 8y$
subject to the constraints

$$\frac{12}{100}x+\frac{4}{100}y \geq 12 \text{ i.e. } 3x+y \geq 300$$
$$\frac{5}{100}x+\frac{5}{100}y \geq 12 \text{ i.e. } x+y \geq 240$$
$$x \geq 0, y \geq 0$$

Draw the lines $3x+y=300, x+y=240$ and shade the region satisfied by the above inequalities.



The feasible region (unbounded, convex) is shown shaded. The corner points are $\mathrm{A}(240,0), \mathrm{B}(30,210)$ and $C(0,300)$.
The values of Z at the corner points are :
at $\mathrm{A}(240,0), \mathrm{Z}=10 \times 240+0=2400$;
at $\mathrm{B}(30,210), \mathrm{Z}=10 \times 30+8 \times 210=1980$;
at $C(0,300), Z=8 \times 300+0=2400$.
Among values of Z, the least value is 1980 .
We draw the line $10x+8y=1980$ (shown dotted in the given figure) and note that the half plane $10x+8y<1980$ has no common points with the feasible region, therefore, Z has minimum value.

Minimum value of Z is 1980 and it occurs at the point $B(30,210)$.
Hence, the minimum cost of the fertiliser is ₹1980 when he uses 30 kg of fertiliser A and 210 kg of fertiliser B.

Q.3. A dietician wishes to mix two kinds of food X and Y in such a way that the mixture conlains atleast 10 units of vitamin A, 12 unils of vitamin B and 8 units of vitamin C. The vitamin contents of one ky food is given below:

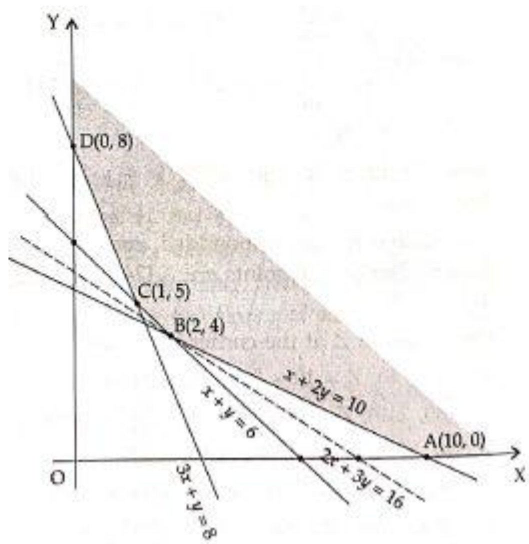
Food	Vitamin A	Vitamin B	Vitamin C
X	1 unit	2 units	3 units
Y	2 units	2 units	1 unit

One kg of food X costs ₹24 and one kg of food Y costs ₹36. Using linear programming, find the least cost of the total mixture which will contain the required vitamins.
Solution. Let x kg of food X and y kg of food Y be mixed and Z (in ₹) be the total cost of the (mixture) food, then the problem can be formulated as an L.P.P. as follows:

Minimise $Z = 24x + 36y$

subject to the constraints
$$x + 2y \geq 10$$
$$2x + 2y \geq 12 \text{ i.e. } x + y \geq 6$$
$$3x + y \geq 8$$
$$x \geq 0, y \geq 0$$

The feasible region (unbounded, convex) is shown shaded in the figure given below.



The corner points are $A(10,0), B(2,4), C(1,5)$ and $D(0,8)$.
The values of Z (in ₹) $= 24x + 36y$ at the corner points are:
at $A(10,0), Z = 24 \times 10 + 36 \times 0 = 240$;
at $B(2,4), Z = 24 \times 2 + 36 \times 4 = 192$; at $C(1,4), Z = 24 \times 1 + 36 \times 5 = 204$; at $D(0,8), Z = 24 \times 0 + 36 \times 8 = 288$; Among these values of Z , the minimum value is 192.
We draw the line $24x + 36y = 192$ i.e. $2x + 3y = 16$ (shown dotted) and note that the half plane $2x + 3y < 16$ has no common point with the feasible region, therefore, Z has minimum value.
The minimum value of Z is ₹192 and it occurs at the point $B(2,4)$.
Hence, the least cost of the mixture is ₹192 when the dietician mixes 2 kg of food X and 4 kg of food Y.

MULTIPLE CHOICE QUESTIONS (UNSOLVED)

Q. 1. Which of the following is a vertex of the feasible region bounded by the inequalities, $2x + 3y \leq 6$ and $5x + 3y \leq 15$

(a) (0,2) (b) (0,0) (c) (3,0) (d) All of these

Q. 2. For the constraint of a linear optimizing function $z = x_1 + x_2$ given by $x_1 + x_2 < 1, 3x_1 + x_2 > 3$ and $x_1 \; x_2 \geq 0$

(a) There are two feasible regions (b) There are infinite feasible regions
(c) There is no feasible region (d) None of these

Q. 3. $Z = 7x + y$, subject to $5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0$. The minimum value of Z occurs at

(a) (3,0) (b) $\left(\frac{1}{2}, \frac{5}{2}\right)$ (c) (7,0) (d) (0,5)

Q. 4 . $Z = 20x_1 + 20x_2$, subject to $x_1 \geq 0, x_2 \geq 0, x_1 + 2x_2 \geq 8$, $3x_1 + 2x_2 \geq 15, 5x_1 + 2x_2 \geq 20$. The minimum value of Z occurs at

(a) (8,0) (b) $\left(\frac{5}{2}, \frac{15}{4}\right)$ (c) $\left(\frac{7}{2}, \frac{9}{4}\right)$ (d) (0,10)

Q. 5. Let x and y be optimal Solution of an LPP, then

(a) $Z = \lambda x + (1 - \lambda)y, \lambda \in R$ is also an optimal Solution.

(b) $Z = \lambda x + (1 - \lambda)y, 0 \leq \lambda \leq 1$ gives an optimal Solution.

(c) $Z = \lambda x + (1 + \lambda)y, 0 \leq \lambda \leq 1$ gives an optimal Solution.

(d) $Z = \lambda x + (1 + \lambda)y, \lambda \in R$ gives an optimal Solution.

Q. 6. How many of the following points satisfy the in equality $2x - 3y > -5$?

$(1,1), (-1,1), (1,-1), (-1,-1), (-2,1), (2,-1), (-1,2)$, and $(-2,-1)$.

(a) 3 (b) 5 (c) 6 (d) 4

Q. 7. The position of points $O(0,0)$ and $P(2,-2)$ in the region of graph of inequation $2x - 3y < 5$ will be

(a) O and P both inside (b) O and P both outside

(c) O inside and P outside (d) O outside and P inside

Q. 8. If the number of available constraints is 3 and the number of parameters to be optimized is 4 , then

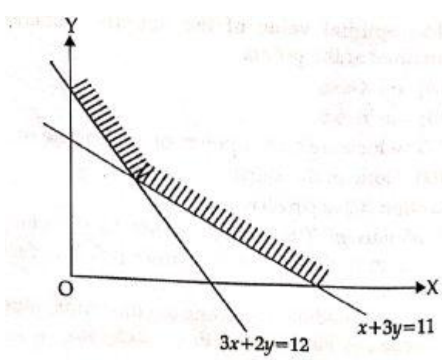
(a) The objective function can be optimized (b) The constraints are short in number

(c) The Solution is problem oriented (d) None of these

Q. 9. The necessary condition for third quadrant region in XY plane is:

(a) $x > 0, y < 0$ (b) $x < 0, y < 0$ (c) $x < 0, y > 0$ (d) $x < 0, y = 0$

Q.10.For the following feasible region, the linear constraints are:



(a) $x \geq 0, y \geq 0, 3x + 2y \geq 12, x + 3y \geq 11$

(b) $x \geq 0, y \geq 0, 3x + 2y \leq 12, x + 3y \geq 11$

(c) $x \geq 0, y \geq 0, 3x + 2y \leq 12, x + 3y \leq 11$

(d) None of these

Q.11. In which quadrant, the bounded region for inequations $x + y \leq 1$ and $x - y \leq 1$ is situated

(a) I, II (b) I, III (c) II, III (d) All the four quadrants

ASSERTION-REASON TYPE QUESTIONS (UNSOLVED)

Q.1.The corner points of the bounded feasible region for an L.P.P. are (26,16), (48,16), (14,33) and (14,24). The objective function Z $= px + qy, p, q > 0$ has minimum value 1000 at points (26,16) and (14,24).
Assertion : Maximum value of $Z = 1440$
Reason : $p = 20, q = 30$.

Q.2.The corner points of the feasible region for an L.P.P. are (0,0), (6,0), (4,3) and (0,8). The objective function Z $= ax + by, a, b > 0$ has maximum value 360 at points (6,0) and (4,3).
Assertion :The value of Z at $(0,8) = 320$
Reason : $a = 60, b = 40$.

Q3.Assertion (A): Maximum value of $Z = 3x + 2y$, subject to the constraints $x + 2y \leq 2; x \geq 0; y \geq 0$ will be obtained at point (2,0).
Reason (R): In a bounded feasible region, it always exist a maximum and minimum value.

Q4.Assertion (A): The linear programming problem, maximise $Z = x + 2y$ subject to the constraints $x - y \leq 10, 2x + 3y \leq 20$ and $x \geq 0, y \geq 0$. It gives the maximum value of Z as $\frac{40}{3}$.

Reason (R): To obtain the optimal value of Z, we need to compare value of Z at all the corner points of the shaded region.

Q5. Assertion (A): Consider the linear programming problem.
Maximise $Z = 4x + y$
Subject to constraints

$$x + y \leq 50, x + y \geq 100 \text{ and } x, y \geq 0$$

Then, maximum value of Z is 50 .
Reason (R): If the shaded region is not bounded then maximum value cannot be determined.

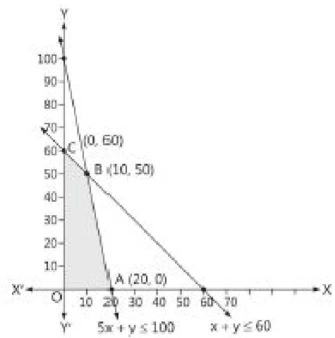
Q6.Assertion (A): The constraints $-x_1 + x_2 \leq 1$, $-x_1 + 3x_2 \geq 9$ and $x_1, x_2 \geq 0$ defines an unbounded feasible space.
Reason (R): The maximum value of $Z = 4x + 2y$ subject to the constraints $2x + 3y \leq 18, x + y \geq 10$ and $x, y \geq 0$ is 5 .

Q7. Assertion (A): For an objective function Z=15x+20y, corner points are (0, 0), (10, 0), (0,15) and (5,5). Then optimal values are 300 and 0 respectively.

Reason (R): The maximum or minimum value of an objective function is known as optimal value of LPP. These values are obtained at corner points.

Q8. The linear inequalities are:

$$5x + y \leq 100$$
$$x + y \leq 60$$
$$x \geq 0$$
$$y \geq 0$$



Where x and y are numbers of tables and chairs on which a furniture dealer wants to make his profit.

Assertion (A): The region OABCO is the feasible region for the problem.
Reason (R): The common region determined by all the constraints including nonnegative constraints x, y ≥ 0 of a linear programming problem.

VERY SHORT ANSWER TYPE QUESTIONS (UNSOLVED)

1. Two tailors, A and B, earn ₹300 and ₹400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce atleast 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.
2. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is atmost 24 . It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹300, formulate an L.P.P. for finding how many of each should be produced daily to maximize the profit? It being given that atleast one of each must be produced.
3. A furniture dealer deals in only two items-tables and chairs. He has ₹20000 to invest and a space to store atmost 80 pieces. A table costs him ₹ 800 and a chair costs him ₹200. He can sell a table for ₹950 and a chair for ₹ 280. Assume that he can sell all the items that he buys. Formulate this problem as an L.P.P. so that he can maximize his profit.
4. A book publisher sells a hard cover edition of a book for ₹72 and a paperback edition for ₹40. In addition to a fixed weekly cost of ₹9600, the cost of printing hard cover and paperback editions are ₹56 and ₹28 per book respectively. Each edition requires 5 minutes on the printing machine whereas hardcover binding takes 10 minutes and paperback takes 2 minutes on the binding machine. The printing machine and the binding machine are available for 80 hours each week. Formulate the linear programming problem to maximize the publisher's profit.
5. The corner points of the feasible region determined by the following systems of linear inequalities: $2x + y \leq 10, x + 3y \leq 15, x \geq 0, y \geq 0$ are (0,0), (5,0), (3,4) and (0,5).

Let $Z = px + qy$, when $p, q > 0$, then find the relation between p and q so that the maximum of Z occurs at both points (3,4) and (0,5).

6. Maximise $Z = 3x + 4y$ (if possible) subject to constraints $x - y \leq -1, -x + y \leq 0, x, y \geq 0$.

SHORT ANSWER TYPE QUESTIONS (UNSOLVED)

Solve the following (1 to 8) linear programming problems graphically:

1. Minimize $Z = -3x + 4y$ subject to the constraints

$$x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0$$

2. Minimize $Z = 3x + 5y$ subject to the constraints

$$x + 3y \geq 3, x + y \geq 2, x \geq 0, y \geq 0.$$

3. Maximize $Z = 4x + y$, subject to the constraints

$$x + y \leq 50, 3x + y \leq 90, x \geq 0, y \geq 0$$

4. Minimize $Z = 200x + 500y$, subject to the constraints

$$x + 2y \geq 10, 3x + 4y \leq 24, x \geq 0, y \geq 0$$

5. Minimize $Z = x + 2y$ subject to the constraints $2x + y \geq 3, x + 2y \geq 6, x \geq 0, y \geq 0$

Show that the minimum value of Z occurs at more than two points.

6. Minimize and maximize $Z = 3x + 9y$ subject to the constraints

$$x + y \geq 10, x + 3y \leq 60, x \leq y, x \geq 0, y \geq 0$$

7. Minimize $Z = 3x + 2y$ subject to the constraints

$$x + y \geq 8, 3x + 5y \leq 15, x \geq 0, y \geq 0$$

8. Minimize and maximize $Z = x + 2y$ subject to the constraints

$$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x \geq 0, y \geq 0$$

9. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B . To produce one unit of A , 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at ₹100 and ₹120 per unit respectively, how should he use his resources to maximise the total revenue? Form the above as an L.P.P. and solve graphically.

10. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹7 per package on bolts. How many packages of each should be produced each day, so as to maximize his profit, if he operates his machines for atmost 12 hours a day.

LONG ANSWER TYPE QUESTIONS (UNSOLVED)

1. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of B requires 1 g of silver and 2 g of gold. The company can use atmost 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 40 and that of type B ₹ 50, find the number of units of each type that the company should produce to maximize the profit. Formulate and solve graphically the LPP and find the maximum profit.
2. A factory manufactures two types of screws, A and B, each type requiring the use of two machines-an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machine to manufacture a package of screws 'A', while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a package of screws 'B'. Each machine is available for atmost 4 hours on any day. The manufacturer can sell a package of screws 'A' at a profit of ₹7 and of screws 'B' at a profit of ₹10. Assuming that he can sell all the screws he can manufacture, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.
3. A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹57600 to invest and has space for atmost 20 items. An electronic sewing machine costs him ₹3600 and a manually operated sewing machine ₹ 2400 . He can sell an electronic sewing machine at a profit of ₹220 and a manually operated sewing machine at a profit of ₹180. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit. Make it as an L.P.P. and solve it graphically.
4. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹50 each for type A and ₹60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?
5. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at ₹7 profit and that of B at a profit of ₹4. Find the production level per day for maximum profit graphically.
6. A cottage industry manufactures pedastal lamps and wooden shades. Both products require machine time as well as craftsman time in making. The number of hour(s) for producing 1 unit of each and corresponding profit is given in the following table:

Item	Machine time	Craftsman time	Profit (in ₹)
Pedastal lamp	1.5 hours	3 hours	30
Wooden shade	3 hours	1 hour	20

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time. Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit? Formulate it as an L.P.P. and solve graphically.

7. An aeroplane can carry a maximum of 200 passengers. Baggage allowed for a first class ticket is 30 kg and for an economy class ticket is 20 kg . Maximum capacity for the baggage is 4500 kg . The profit on each first class ticket is ₹ 500 and on each economy class ticket is ₹ 300 . Determine how many tickets of each type must be sold to maximize the profit of the airline. Also find the maximum profit.
8. A carpenter has 90,80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1,2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹48 per unit and product B is sold for ₹ 40 per unit, how many units of product A and product B should be produced and sold by the carpenter, in order to obtain the maximum revenue. Formulate the above as a linear programming problem and solve it, indicating clearly the feasible region in the graph.

9. A diet is to contain atleast 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available costing ₹5 per unit and ₹6 per unit respectively. One unit of food F_1 contains 4 units of vitamin A and 3 units of minerals whereas one unit of food F_2 contains 3 units of vitamin A and 6 units of minerals. Formulate this as a linear programming problem. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement.
10. A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contain atleast 8 units of vitamin A, and 10 units of vitamin C. Food I contains 2 units /kg of vitamin A and 1 unit /kg of vitamin C . Food II contains 1 unit /kg of vitamin A and 2 units/kg of vitamin C. It costs ₹50 per kg to purchase food I and ₹70 per kg to purchase food II. Determine the minimum cost of such a mixture.

CASE- STUDY BASED QUESTIONS UNSOLVED

Q.1.A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/ cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹25 and that from a shade is ₹15.

If x is the number of lamps and y is the number of shades manufactured. Assuming that the manufacturer can sell all the lamps and shades that he produces.

(i) In order to maximize the profit, write the objective function is correct:

(ii)Write the constraints are related to the given LPP.

(iii) Write the non-negative constraints associative to the given LPP.

(iv)Find the maximum profit.

Q.2.An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class tickets is x and that of economy class tickets is y.

(i)Write the maximum value of $x + y$.

. (ii)Write the relation between x and y.

(iii) Find the constraints for this LPP

(iv)Find the maximum profit.

HOTS QUESTIONS UNSOLVED

1. Reshma wishes to mix two types of foods P and Q in such a way that the vitamin contents of the mixture contains atleast 8 units of vitamin A and 11 units of vitamin B. Food P costs ₹$60/$kg and food Q costs ₹$80/$kg. Food P contains 3 units $/$kg of vitamin A and 5 units $/$kg of vitamin B while food Q contains 4 units $/$kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.
2. A farmer has a supply of chemical fertiliser of type A which contains 10% nitrogen and 6% phosphoric acid and of type B which contains 5% nitrogen and 10% phosphoric acid. After soil test, it is found that atleast 7 kg of nitrogen and same quantity of phosphoric acid is required for a good crop. The fertiliser of type A costs ₹5 per kg and the type B cost ₹8 per kg. Using linear programming, find how many kilograms of each type of the fertiliser should be bought to meet the requirement and for the cost to be minimum. Find the feasible region in the graph.

3. David wants to invest atmost ₹12000 in Bonds A and B. According to the rule, he has to invest atleast ₹2000 in Bond A and atleast ₹4000 in Bond B. If the rates of interest on Bonds A and B respectively are 8% and 10% per annum, formulate the problem as L.P.P. and solve it graphically for maximum interest. Also determine the maximum interest received in a year.
Assuming that the transportation cost per km is ₹1 per litre, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

4. Rahul is at the whole sale market to purchase folding tables and chairs, to later sell them at his furniture shop. He has only ₹5760 to spend and his van has space to carry at the most 20 items. A table costs him ₹360 and a chair costs ₹240. Back at his shop, he plans to sell a table at a profit of ₹22 and a chair at a profit of ₹18. Given that he can sell all the items that he purchases, how many tables and chairs shall he purchase in order to maximize his profit?

Answers of Multiple Choice Questions

1(d) 2(c) 3(d) 4(d) 5(b) 6(b) 7(c) 8(b) 9(b) 10(a) 11(d)

Answers of Assertion – Reason Questions

1(a) 2(a) 3(a) 4(a) 5(d) 6(c) 7(a) 8(a)

Answers of Very Short Answer Type Questions

1. Minimize $Z = 300x + 400y$, subject to the constraints

$$3x + 5y \geq 30, x + y \geq 8, x \geq 0, y \geq 0$$

2. Maximize $Z = 100x + 300y$, subject to the constraints

$$x + 2y \leq 32, x + y \leq 24, x \geq 1, y \geq 1$$

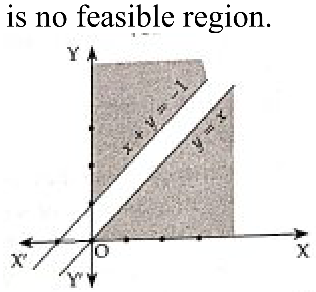
3. Maximize $Z = 150x + 80y$, subject to the constraints

$$4x + y \leq 100, x + y \leq 80, x \geq 0, y \geq 0$$

4. Maximize profit $Z = 16x + 12y - 9600$, Subject to the constraints

$$x + y \leq 960, 5x + y \leq 2400, x, y \geq 0.$$

5. $3p = q$, is the required relation between p and q.
6. We draw the lines $x - y = -1$ and $y = x$. Shade the region satisfied by the given inequations. As we can see that there is no feasible region.



∴ No maximum value of Z.

Answers of Short Answer Type Questions

1. Minimum $= -12$ at $x = 4, y = 0$
2. Minimum $= 7$ at $x = \frac{3}{2}, y = \frac{1}{2}$
3. Maximum $= 120$ at $x = 30, y = 0$
4. Minimum $= 2300$ at $x = 4, y = 3$
5. Minimum $= 6$ at the corner points $(6,0)$ and $(0,3)$. In fact, all points on the line segment joining the points $(6,0)$ and $(0,3)$ yield the minimum value 6
6. Minimum $= 60$ at $(5,5)$. Maximum $= 180$ occurs at two corner points $(15,15)$ and $(0,20)$. In fact, all points on the line segment joining the points $(15,15)$ and $(0,20)$ yield the maximum value 180
7. No feasible Solution
8. Maximum value $= 400$ at $(0,200)$; minimum value $= 100$ at $(20,40)$ and $(0,50)$. In fact, all points on the line segment joining the points $(20,40)$ and $(0,50)$ give the same minimum value 100
9. 3 units of product A and 8 units of product B; maximum revenue = ₹1260
10. 3 packages of nuts and 3 packages of bolts; maximum profit ₹73.50

Answers of Long Answer Type Questions

1. 2 units of type A and 3 units of type B; maximum profit ₹230
2. Maximum profit ₹410 occurs when 30 packages of screw A and 20 packages of screw B are produced
3. He should buy 8 electronic and 12 manually operated sewing machines, maximum profit = ₹3920.
4. Maximum profit ₹1600 occurs when 8 souvenirs of type A and 20 of type B are produced
5. 2 units of product A and 3 units of product B, maximum profit ₹26
6. 4 pedastal lamps and 12 wooden shades, maximum profit ₹360
7. 150 tickets of executive class and no tickets of economy class; maximum profit is ₹ 75000
8. 40 units of product A and 10 units of product B; maximum revenue = ₹2320.
9. Minimum cost of the diet is ₹124 when 12 units of food F_1 and $\frac{32}{3}$ units of food F_2 are mixed
10. 2 kg of food I, 4 kg of food II, minimum cost is ₹ 380

Answers to Case Study Based Questions

1(i) Max. $Z = 25x + 15y$

(ii) $2x + y \leq 12; 3x + 2y \leq 20, x \geq 0, y \geq 0$

(iii) $x \geq 0, y \geq 0$

(iv) 160

2 (i) 200 (ii) $y \geq 4x$ (iii) $x + y \leq 200, y \geq 4x, x \geq 20, y \geq 0$ (iv) Rs.1,36,000.

Answers to HOTS Questions

1. Minimum cost = ₹160 at $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$. In fact, all points on the line segment joining the points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$ give the same minimum cost ₹160
2. 50 kg of type A and 40 kg of type B, minimum cost ₹570
3. ₹2000 in bond A and ₹10000 in bond B yield maximum interest of ₹1160
4. 8 tables and 12 chairs. Maximum profit = ₹392.

SAMPLE QUESTION PAPER – 1 (2025 -26)
APPLIED MATHEMATICS
CLASS - XII
BLUE PRINT

	Name of the Unit	1 (MCQ) **Section A**	2M(VSA) **Section B**	3 (SA) **Section C**	5 M(LA) **Section D**	4M(Case Based) **Section E**	Total
Unit I	Numbers, Quantification and Numerical Applications	4	2	1			7(11)
Unit II	Algebra	3	1		1		5(10)
Unit III	Calculus	3		1	1	1	6(15)
Unit IV	Probability Distributions	1			1	1	3(10)
Unit V	Inferential Statistics	2		1			3(5)
Unit VI	Index Numers and Time-based Data	3		1			4(6)
Unit VII	Financial Mathematics	2	1	2	1		6(15)
Unit VIII	Linear Programming	2	1			1	4(8)
	Total	20	5	6	4	3	38(80)

SAMPLE QUESTION PAPER – 1 (2025 -26)
APPLIED MATHEMATICS
CLASS - XII

General Instructions:

1. This question paper contains – **five sections A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. Section **A** has **18 MCQ's** and **02 Assertion** –Reason based questions of 1 mark each.
3. Section **B** has **5 Very Shirt Answers (VSA)** type questions of 2 marks each.
4. Section **C** has **6 short Answer (SA)** type questions of 3 marks each
5. Section **D** has **4 long Answer (LA)** type questions of 5 marks each.
6. Section **E** has **3 source based / case based / passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION -A

1. The smallest non –negative integer congruent to 2796 (mod7) is

   (a) 5 (b) 2 (c) 3 (d) 4

2. If $x \leq 8$, then

   (a)-$x \leq -8$ (b) -$x \geq -8$ (c) -x < -8 (d) -x > -8

3. In a game of 600 points, A scores points 407 points while B scores 307 points. In this game, the point given by A to B is

   (a) 105 (b) 200 (c) 100 (d) 150

4. A certain tank can be filled by pipe A in 12 minutes . pipe B can empty the tank in 18 minutes . If both pipes are open ,then the time it takes to fill the tank

   (a) 5 minutes (b) 6minutes (c) 36 minutes (d) 9 minutes

5. The matrix A= $[0\ 1\ -1\ 0]$ is

   (a) a unit matrix (b) a symmetric matrix

   (c) a skew symmetric matrix (d) a diagonal matrix

6. If $[a+b\ 2\ 5\ b]$ = $[6\ 5\ 2\ 2]^T$, then a is

   (a) 4 (b) 3 (c) 2 (d) 1

7. If A is 3x3 matrix such that $|A|$=8 , then $|3A|$ is equals

   (a) 24 (b) 72 (c)216 (d) 8

8. If x=3at, y=at^3 , then $\frac{dy}{dx}$ is equal to

(a)3 (b)3a (c) 3at (d) t^2

9. The radius of a circle is increasing at the rate of 0.7cm/s. What is the rate of increase of its circumference when r= 4.9 cm ?

(a) 1.4 π cm/s (b) 2.8 π cm/s (c) 0.4 cm/s (d) -0.4 cm/s

10. For an objective function Z = ax + by , where a,b >0 ; the corner points of the feasible region determined by a set of constraints are (0,20) , (10,10) , (30,30) and (0,40) . The condition on a and b such that the maximum Z occurs at both end points (30,30) and (0,40) is

(a) b-3a =0 (b) a =3b (c) a+2b =0 (d) 2a- b =0

11. The probability distribution of a discrete random variable X is given below :

X	2	3	4	5
P(X)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is

(a) 8 (b) 16 (c) 32 (d) 48

12. For purpose of t-test of significance , a random sample of size (n) 48 is drawn from a normal population , then the degree of freedom (v) is

(a) 49 (b) $\frac{1}{48}$(c) 47 (d) 48

13. A specific characteristic of a population is known as a

(a) a sample (b) parameter (c) statistic (d) mean

14. Prosperity , recession and depression in a business is an example of

(a) seasonality (b) trend (c)cyclical (d) secular

15. Seasonal variation mean the variations occurred within

(a) A number of years (b) parts of a year (c) parts of a month (d) none of these

16 For the given five values 35, 70, 36, 59, 64, the three years moving averages are given by

. (a)47, 53, 55 (b) 53, 47, 45 (c) 47, 55, 53 (d) 45, 55, 57

17. Mr. Anil takes a loan of Rs. 200000 with 10% annual interest rate for 5 years. EMI under flat rates system is

(a) 4000 (b) 5000 (c) 6000 (d)7000

18. At what rate of interest will the present value of a perpetuity of Rs. 500 payable at the end of every 6 months be Rs. 10000?

(a) 6 % (b) 8% (c) 5 % (d)10%

ASSERTION AND REASON

(a) Both A and R are true and R is the correct explanation ofr A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): Maximum value of $z = 3x + 2y$, subject to the constraints

$x + 2y \leq 2$, $x \geq 0$; $y \geq 0$ will be obtained at point (2,0)

Reason (R): In a bounded feasible region it always exits a maximum and minimum value .

20. Assertion (A): The function $f(x) = x^3 - 12x$ is strictly increasing in $(-\infty,-2) \cup (2,\infty)$

Reason (R): For a strictly increasing function $f'(x) > 0$

SECTION B

21 A man rows 15 km upstream and 25 km downstream in 5 hours each time. What is the speed of the current ?

22 In a 500 m race, A defeats B by 60 meters (or) 12 seconds. What is the time taken by A to complete the race?

OR

A pump can fill a tank with water in 2 hours. Because of a leak in the tank, it takes $2\frac{1}{3}$ hours to fill the tank. The leak will be empty the filled tank in what time?

23 Using determinant find the value of k, for which points P (3,-2), Q (8, 8) and R (k, 2) are collinear.

24 A person has an initial investment of Rs. 50000 in an investment plan. After 2 years it has grown to Rs 60000. Find his rate of return.

OR

How much money is needed to endure a series of lectures costing ₹2500 at the beginning of each year indefinitely , if money is worth 3 % compounded annually ?

25 A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to

make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is Rs100 and that on a bracelet is Rs.300. Formulate a L.P.P. for finding how many of each should be produced daily to maximise the profit?

SECTION C

26. A container contains 50 litres of milk . From this container 10 liters of milk was taken out and replaced by water . This process is repeated two more times . How much milk is now left in the container ?

OR

A vessel contains a mixture of two liquids P and Q in the ratio 5:7. 12 litres of mixture drawn off from the vessel and 12 L of liquid P is filled in the vessel. If the ratio of the liquids P and Q becomes 9:7, how many litres of liquids P and Q were contained by the vessel initially.

27. Mr Surya borrowed a sum of 5,00,000 with total interest to be paid 2,00,000(flat) and he is paying an EMI of 12,500. Calculate loan tenure.

28. Fit a straight line trend to the following data :

Year	2010	2011	2012	2013	2014	2015
Values	28	32	29	35	40	50

29 Find the present value of a perpetuity of Rs 3120 payable at the beginning of each year , if money is worth 6% effective.

30 Find the intervals in which the function

$f(x) = 2x^3 + 9x^2 + 12x + 20$ *is* (i)increasing (ii)) decreasing.

OR

Use of second derivative test to find the local maxima and minima of

f(x) = $\frac{4}{3}x^3$ + $6x^2$ + 8x + 7.

31 A 99% confidence interval for a population mean was reported to be 83 to 87 . If σ =8 . What sample size was used in this study ? (Given $Z_{0.005}$ = 2.576)

SECTION D

32. The management committee of a Welfare Club decided to award some of its members (say x)for sincerity, some (say y) for helping others selflessly and some others (say z) for effective management. The sum of all the awardees is 12. Three times the sum of all awardees for helping others selflessly and effective management added to two times the number of awardees for sincerity is 33. If the sum of the number of awardees for sincerity and effective management is

twice the number of awardees for helping others, use matrix method to find the number of awardees of each category.

33. A company produces a certain commodity with 24000 fixed cost. The variable cost is estimated to be 25% of the total revenue received on selling the product at a rate of 8 per unit. Find the following

(i) Cost Function. (ii) Revenue Function

(iii) Breakeven Point (iv) Profit Function

OR

Integrate : $\int_1^4 \left[|x-1| + |x-2| + |x-3|\right] dx$

34. Surjeet purchased a new house, costing ₹ 40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly instalment is ₹ 30,000, at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him?

[Use $(1.0075)^{-300} = 0.1062$]

35. If 5% of the electric bulbs manufactured by a company are defective , use Poisson distribution to find the probability that in a sample of 100 bulbs :

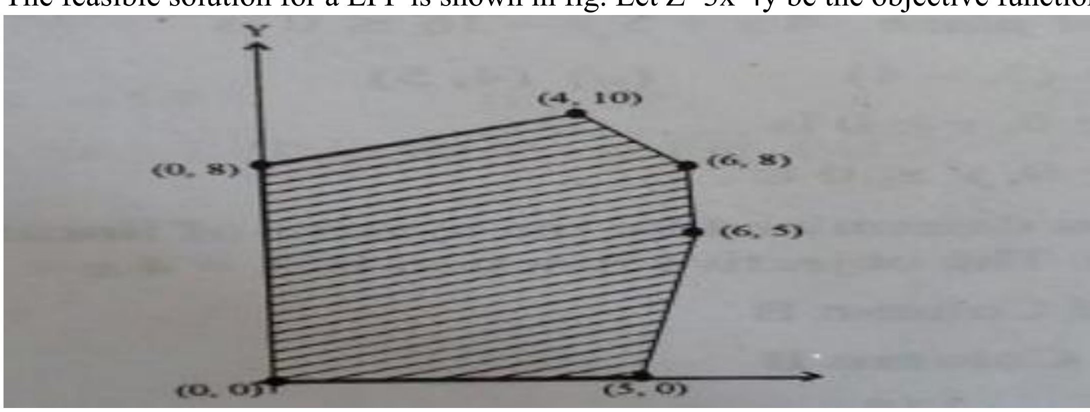
(i)None is defective (ii)5 bulbs will be defective

OR

In an examination, 2000 students appeared and the mean of the normal distribution of marks is 30 with standard deviation as 6.25. Find out how many students are expected to score (i) between 20 and 40 marks (ii) less than 25 marks.

<u>SECTION-E(CASE BASED QUESTIONS)</u>

36. The feasible solution for a LPP is shown in fig. Let Z=3x-4y be the objective function.



37. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the value x, has the following form, where k is some constant.

P(X=x) =

$\{0.1 \quad , if x = 0\ kx, \quad if x = 1\ and x = 2\ k(5 - x), \quad if x = 3\ and x = 4\ 0, \quad otherwise$

Based on the above information answer the following:

(i) Calculate the value of k

(ii) Calculate the probability P(Studies for three hours)

(iii) What is the probability when you study atmost two hours.

OR

(iv) Compute the probability when you study at least for two hours

(38) Read the following text and answer the following questions on the basis of the same:

Polio drops are delivered to 50000 children in a district. The rate at which polio drops are 38. 38.Polio drops are delivered to 50000 children in a district . The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the ends of 2^{nd} week half the children have been given the polio drops . It has been given the differential equation $\frac{dy}{dx}$ = k(50-y) where x denotes the number of weeks and y the number of children who have been given the drops.

(i)Write the method of solving a differential equation $\frac{dy}{dx}$ = k(50-y).

(ii)Find the solution of differential equation $\frac{dy}{dx}$ = k(50-y) .

(iii) Which of the solution may be used to find the number of children who have been given the polio drops?

ANSWERS
SAMPLE PAPER -1
APPLIED MATHEMATICS: CLASS XII

1. Solution. Let x be the smallest non –negative integer congruent to 2796 (mod7)

   $x \equiv 2796 \pmod 7$

   $x \equiv 3 \pmod 7$

   Answer : (c)

2. Answer : (b)

   Solution : $x \leq 8$

   $\Rightarrow -x \geq -8$

3. Answer (c)

   Solution:

   (Total score points = 600

   A score's = 407

   B score's = 307

   ∴ A give (407 – 307) = 100 points to B.

4. Answer : (c)

   Solution $\frac{1}{12} - \frac{1}{18} = \frac{1}{36}$

   : time taken by both pipes = 36 minutes

5. Answer : (c)

   Solution : if A is a a skew symmetric matrix

   Then A' = -A

6. Answer : (a)

   Solution : a+b= 6 , b= 2 ⇨ a=4

7. Answer : (c)

   Solution : $|kA| = k^n|A| \Rightarrow |3A| = 3^3 \times 8 = 216$

8. Answer : (d)

   Solution : $\frac{dx}{dt} = 3a$, $\frac{dy}{dt} = 3at^2$

   $\frac{dy}{dx} = \frac{3at^2}{3a} = t^2$

9. Answer : (a)

   Solution : $\frac{dr}{dt} = 0.7$, circumference=2πr

   $\frac{dc}{dt} = 2\pi \frac{dr}{dt} = 2\pi \times 0.7 = 1.4\,\pi$ cm/s

10. Answer : (a)

    Solution : As Z is maximum at (30,30) and (0,40)

    ⇨ 30a +b = 40b

    ⇨ b-3a =0

11. Answer : (c)

    Solution : $\frac{5}{k} + \frac{7}{k} + \frac{9}{k} + \frac{11}{k} = 1$, $\frac{32}{k} = 1$, k = 32

12. Answer : (c)

    Solution : Degree of freedom = n-1 , n=48

Therefore , Degree of freedom = 48-1 = 47

13. Answer : (b)

14. Answer : (c)

Solution : The variation in a time series over a span of more than one year are called cyclic variation .

15. Answer : (b)

Solution: Seasonal variation is variation in a time series within one year that is repeated more or less regularly .

16. Answer : (c)

. Solution : for 5 values 3 years moving averages is given by

$\frac{35+70+36}{3}$ =47 , $\frac{70+36+59}{3}$ = 55, $\frac{36+59+64}{3}$ = 53

17. Answer : (b)

Solution : EMI = $\frac{P+I}{n}$, I = $\frac{prn}{100}$ =, I= $\frac{200000\times10\times60}{1200}$ = 100000

EMI = $\frac{200000+100000}{60}$ = 50000

18. Answer : (d)

Solution : R = $\frac{P}{i}$, i= $\frac{10000}{500}$ = 20, since it is the end of every 6 months , therefore i = $\frac{20}{2}$ =10 %

19. Answer : (a)

Solution :

Corner points	z = 3x +2y
(0,0)	0
(0,1)	2
(2,0)	6 maximum

20. Answer (a)

Solution: f(x) = x^3-12x , f'(x) = $3x^2$ -12

For f'(x) = 0 , x =2,-2

strictly increasing in (-∞,-2) U (2,∞)

clearly R the correct explanation of A

SECTION B

21 Solution.

Downstream speed u = x + y = 5

upstream speed v = x - y = 3

speed of current = 1km/h

22 Solution.

Length of course = 500 meters

Time taken by B to cover by 60 meters = 12 seconds.

∴ time taken by B to cover the course = $\left(\frac{12}{60}\times 500\right) = 100$ seconds

∴time taken by B to cover the course=$(100-12)$ seconds = 88 seconds

= 1 minute 28 seconds

OR

Solution.

Part filled by the pump in 1 hour = $\frac{1}{2}$

Net part filled by the pump and leak in 1 hour = $\frac{3}{7}$

Emptying work done by the leak in 1 hour = $\frac{1}{2} - \frac{3}{7} = \frac{7-6}{14} = \frac{1}{14}$

Leak can empty the tank in 14 hours.

23 Solution.

As the points P (3,-2), Q (8, 8) and R (k, 2) are collinear

Area of triangle PQR =$\frac{1}{2}|3 \; -2 \; 1 \; 8 \; 8 \; 1 \; k \; 2 \; 1|$ = 0

Solving above determinant, we get, | 3(8-2) +2(8-k) +1(16-8k)| = 0

|18 + 16 – 2k +16 -8k | = 0

-10k + 50 = 0

-10k + 50 = 0

k = 5

24 Solution.

Given current value of investment= Rs. 60000

Cost of investment = Rs. 50000

Rate of return = $\frac{60000-50000}{50000}$ X 100%

= 20%

OR

Solution.

Here R=2500 , $i = \frac{3}{100}$

P=R+$\frac{R}{i}$

P=2500 + $\frac{2500}{0.03}$

=2500 + 83333.33

= ₹ 85833.33

25 Solution.

Let number of necklaces and bracelets produced by firm per day be x and y, respectively.

Clearly, x≥0,y≥0

Let Z be the profit function.

Maximise Z = 100x+300y subject to,

x +y ≤ 24

x + 2y ≤ 32

and x, y ≥ 0

SECTION C

26. Solution.

X=50L ,y = 10L ,n=3

Amount of milk left = $x(1-\frac{y}{x})^n$ $= 50\ (1-\frac{10}{50})^3$ = 25.6 L

OR

Solution.

Let initially liquids P and Q be 5x and 7x litres respectively in the vessel.

After drawing off 12 litres of mixture

Quantity of liquid P left in the mixture = $5x - \frac{5}{12}X\ 12$ = 5x – 5 litres

Quantity of liquid Q left in the mixture = $7x - \frac{7}{12}X\ 12$ = 7x – 7 itres

So, quantity of liquid P= (5x -5 +12) litres = 5x +7 litres

quantity of liquid Q= (5x -5 +12) litres

As per the question , $\frac{9}{7} = \frac{5x+7}{7x-7}$

63 x – 63 = 35x + 49

Solving we get , x = 4

Hence, the quantity of liquid P was 5 X4 = 20 litres and quantity of liquid Q was 7 X4 = 28 litres

27. Solution.

Here P = 5,00,000 ; I = 2,00,000; EMI = 12,500

EMI = $\frac{P+I}{n}$

12,500 = $\frac{500000+200000}{n}$

⟹ n = 56 months.

28. Fit a straight line trend to the following data :

Year	2010	2011	2012	2013	2014	2015
Values	28	32	29	35	40	50
Solution. t	y	$x=t_i$- 2012.5	x^2	xy		
2010	28	-2.5	6.25	-70		
2011	32	-1.5	2.25	-48		
2012	29	-0.5	0.25	-14.5		
2013	35	0.5	0.25	17.5		
2014	40	1.5	2.25	60		
2015	50	2.5	6.25	125		
	∑y=214		∑ x^2=17.50	∑xy=70		

$a=\frac{\sum y}{n}=35.67$ $b=\frac{\sum xy}{\sum x2}=4$

29 Solution.

$i = .06$

P= R + $\frac{R}{i}$= 3120+ 3120/.06

$getting\ P = Rs\ 55120$

30 Solution.

Getting $f'(x) = 6x^2 + 18x + 12$

$f(x) = 6\ (x+1)(x+2)$

For increasing $f'(x) > 0$ $\ and\ decreasing f'(x) < 0$

Increasing $(-\infty, -2) \cup (-1, \infty)$

Decreasing $(-2\ ,\ -1)$

OR

Soln. f(x) = $\frac{4}{3}x^3$ + $6x^2 + 8x + 7$

f '(x) = $4x^2 + 12x + 8 = 4(x+1)(x+2)$

For local maxima or minima f '(x) = 0

4 (x + 1) (x + 2) = 0

x = -1,-2

Again, f ''(x) = 8x + 12

At x = -2, f ''(-2) = 8 X (-2) + 12 = -4

- 4 < 0

∴x = -2 is a point of local maxima

local maximum value = $\frac{-32}{3}$ + 24 – 16 + 7 = $\frac{13}{3}$

At x = -1, f ''(-1) = 8 X (-1) + 12 = 4

4 > 0

∴x = -1 is a point of local minima

local minimum value = $\frac{-4}{3}$ + 6 – 8 + 7 = $\frac{11}{3}$

31 Solution : given σ =8 and confidence level = 99%

Let the sample mean be $\bar{x}$ and margin of error be E .

Then $\bar{x}$ +E = 87 and $\bar{x}$ – E =83 ⇨ E =2

1-α =0.99 ⇨ α =0.01 ,$\frac{\alpha}{2}$ =0.005

$Z_{\frac{\alpha}{2}}$ = $Z_{0.005}$ = 2.576

Since E= $Z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}}$ ⇨ 2 = $\frac{2.576 \times 8}{\sqrt{n}}$ ⇨$\sqrt{n}$ =10.304

Therefore n= 106.172 ⇨ sample size =106

SECTION D

32. Solution.

Equations: x + y + z = 12, 2x + 3y + 3z = 33, x – 2y + z = 0

Det(A)= 3, $A^{-1} = \frac{1}{3}(9\ -3\,0\,1\,0\ -1\ -7\,3\,1)$

AX = B ⇨ X = A^{-1} B

X = $\frac{1}{3}(9\ -3\,0\,1\,0\ -1\ -7\,3\,1)(12\,33\,0)$

x=3, y=4,z=5

33. Solution.

Let ‘x’ units of product be produced and sold. As selling price of one unit is Rs 8 total revenue on ‘x’ units = Rs 8x

(i) Cost Function C(x) = Fixed Cost + 25% of 8x

= 24000 +2x.

(ii) Revenue Function = 8x

(iii) Breakeven Point 8x = 24000+2x

x = 4000

(iv) Profit function = R(x) – C(x) = 6x – 24000

OR

Solution

$\int_1^4 |x-1|\,dx = \int_1^4 (x-1)dx$

$= \left(\frac{x^2}{2} - x\right)_1^4 = \frac{5}{2}$,

$\int_1^4 |x-2|\,dx = \int_1^2 -(x-2)dx + \int_2^4 (x-2)dx$

$= -\left(\frac{x^2}{2} - 2x\right)_1^2 + \left(\frac{x^2}{2} - 2x\right)_2^4 = \frac{9}{2}$

$\int_1^4 |x-3|\,dx = \int_1^3 -(x-3)dx + \int_3^4 (x-3)dx$

$= -\left(\frac{x^2}{2} - 3x\right)_1^3 + \left(\frac{x^2}{2} - 3x\right)_3^4 = \frac{5}{2}$

$\int_1^4 [|x-1| + |x-2| + |x-3|]\,dx = \frac{19}{2}$

34. Solution.

Amount of Purchase = ₹ 40,00,000

Down payment = x

Balance = 40,00,000 – x

i= 9/1200=0.0075, n = 25 x 12 = 300

EMI = ₹ 30,000

30000= (4000000−x)×0.0075/1− $(1.0075)^{-300}$ ⇒30000= (4000000−x)×0.0075/1− 0.1062

⇒x=424800

Down payment = ₹ 4,24,800

35. Solution.

p =0.05, n=100 getting $\mu = np = 5$

P(X=r) = $\frac{e^{-\mu}\mu^r}{r!}$

(i)P(none is defective)= .007

(ii)P(5 defective bulbs)= .1822

OR,

Solution. $Z = \frac{X-\mu}{6.25}$

When X=20 , Z= -1.6

When X= 40 , Z= 1.6

P(20<X<40)= .8904

(i)Number of students scoring between 20 and 40 = 1781(approx)

When X= 25 , Z= -.8

P(X<25)= .2119

(ii) Number of students scoring less than 25= 424

SECTION-E(CASE BASED QUESTIONS)

36. Solution : (i) (0,8)

(ii) -32

37. Solution : (i) 0.1 +2k +2k+k =1

6 K = 0.9 , k = 0.15

(ii) P (k =3) = k (5-3) =2k = 2× 0.15 = 0.3

(iii)P(k =0) +P(k = 1) + P (k = 2) = 0.1 +k +2k =0.1 +3k = 0.1 +3 × 0.15 = 0.55

OR

P(k =2) +P(k = 3) + P (k = 4) = 2K +2K +K = 5K = 5× 0.15 =0.75

(38) Solution: (i) Variable separation method.

(ii) Given that $\frac{dy}{dx}$ = k(50-y).

$\frac{dy}{50-y}$ = k dx

Integrating both sides we get

-log| 50 – y | = k x + C

(iii) -log| 50 – y | = k x + C

When x = 0 , y = 0

log (50) = 0 + C

$\Rightarrow -$ log (50) = 0 + C

$\Rightarrow$ C = log $\frac{1}{50}$

-log| 50 – y | = k x +log $\frac{1}{50}$

⇨ e^{-kx} = $\frac{50-y}{50}$

y = 50 (1- e^{-kx})

SAMPLE QUESTION PAPER -2(2025-26)
APPLIED MATHEMATICS
CLASS: XII
BLUE PRINT

	Name of the Unit	1 (MCQ) Sec-A	2 (VSA) Sec-B	3 (SA) Sec-C	5 (LA) Sec-D	4 (Case Based) Sec-E	Total
Unit I	Numbers, Quantification and Numerical Applications	4	2	1			7(11)
Unit II	Algebra	3	1		1		5(10)
Unit III	Calculus	3		1	1	1	6(15)
Unit IV	Probability Distributions	1			1	1	3(10)
Unit V	Inferential Statistics	2		1			3(5)
Unit VI	Index Numers and Time-based Data	3		1			4(6)
Unit VII	Financial Mathematics	2	1	2	1	1	7(19)
Unit VIII	Linear Programming	2	1				3(4)
	Total	20	5	6	4	3	38(80)

SAMPLE QUESTION PAPER -2(2025-26)
APPLIED MATHEMATICS
CLASS: XII

TIME: 3 Hrs M.M.: 80

General Instructions :

1. This question paper contains – five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion –Reason based questions of 1 mark each.
3. Section B has 5 Very Shirt Answers (VSA) type questions of 2 marks each.
4. Section C has 6 short Answer (SA) type questions of 3 marks each
5. Section D has 4 long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based / case based / passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A

1. What is the value of $7^6 (mod\ 3)$

(a) 1 (b) 2 (c) 3 (d) 4

2. The value of - 31mod 7 will be

(a) 1 (b) 2 (c) 3 (d) 4

3. If a > b and c < 0 then which of the following is true ?

(a) a+c < b+c (b) a-c < b-c (c) ac > bc (d) a-c > b+c

4. A certain tank can be filled by pipe A in 12 minutes . pipe B can empty the tank in 18 minutes . If both pipes are open ,then the time it takes to fill the tank

(a) 35 minutes (b) 64 minutes (c) 36 minutes (d) 8 minutes

5. The matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is

(a) a unit matrix (b) a symm. matrix (c) a skew symm. matrix (d) a diagonal matrix

6. If $\begin{pmatrix} a+b & 2 \\ 5 & b \end{pmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}^T$, then a is

(a) 4 (b) 3 (c) 2 (d) 1

7. If A is 3x3 matrix such that $|A|$=8 , then $|3A|$ is equals

(a) 24 (b) 72 (c)216 (d) 8

8. If x=3at, y=at^3 , then $\frac{dy}{dx}$ is equal to

(a)3 (b)3a (c) 3at (d) t^2

9. The radius of a circle is increasing at the rate of 0.7cm/s. Then the rate of increase of its circumference is

(a) 1.4π cm/s (b) 2.4 cm/s (c) 0.4 cm/s (d) -0.4 cm/s

10. The total revenue in Rupees received from the sale of x units of a product is given by R(x)= $3x^2$+36x+5. The marginal revenue , when x=15 is:

(a)116 (b) 96 (c) 90 (d)126

11. The probability distribution of a discrete random variable X is given below :

X	2	3	4	5
P(X)	5/k	7/k	9/k	11/k

The value of k is

(a) 8 (b) 16 (c) 32 (d) 48

12. Consider the following hypothesis test

$H_0 : \mu \leq 25$ $H_a : \mu > 25$

A sample of 40 provided a sample mean of 26.4, then the value of the test statistics is:

(a) 4.18 (b)-1.48 (c) 1.48 (d)-4.18

13. A specific characteristic of a population is known as a

(a) a sample (b) parameter (c) statistic (d) mean

14. Which of the following cannot be a component for a time series

(a) seasonality (b) trend (c)cyclical (d) none of these

15. Seasonal variation mean the variations occurred within

(a) A number of years (b) parts of a year (c) parts of a month (d) none of these

16. Time series data have a total number of components ?

(a) 3 (b) 4 (c)5 (d)6

17. Mr. Anil takes a loan of Rs. 2,00,000 with 10% annual interest rate for 5 years. EMI under flat rates system is

(a) 4000 (b) 5000 (c) 6000 (d)7000

18. At what rate of interest will the present value of a perpetuity of Rs. 500 payable at the end of every 6 months be Rs. 10000?

(a) 6 (b) 8 (c)5 (d)10

19. Assertion (A): Feasible region is the set of points which satisfy all of the given constraints.

Reason (R): The optimal value of the objective function is attained at the points on X-axis only.

(a) Both A and R are true and R is the correct explanation ofr A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false.
(d) A is false but R is true.

20. Assertion (A): The function y=$[x(x-2)]^2$ is increasing in (0,1) U (2,∞)

Reason (R): $\frac{dy}{dx} = 0$, when x=0,1,2

(a) Both A and R are true and R is the correct explanation ofr A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false.
(d) A is false but R is true.

SECTION –B

21. Find the value of x, given that $x \equiv 23(mod\ 7)$; if $21 \le x < 31$

22. In a 500 m race, A defeats B by 60 meters (or) 12 seconds. What is the time taken by A to complete the race? OR

A pump can fill a tank with water in 2 hours. Because of a leak in the tank, it takes $2\frac{1}{3}$ hours to fill the tank. The leak will be empty the filled tank in what time?

23. Using determinant find the value of k, for which points P (3,-2), Q (8, 8) and R (k, 2) are collinear.

24. A person has an initial investment of Rs. 50000 in an investment plan. After 2 years it has grown to Rs 60000. Find his rate of return.

25. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is Rs 100 and that on a bracelet is Rs.300. Formulate a L.P.P. for finding how many of each should be produced daily to maximise the profit?

SECTION-C

26. A man can row a boat at 5 km/h in still water. If the speed of water current in a river is 1 km/h and it takes him 1 hour to row to a place and come back, how far off is the place?

OR

A vessel contains a mixture of two liquids P and Q in the ratio 5:7. 12 litres of mixture drawn off from the vessel and 12 L of liquid P is filled in the vessel. If the ratio of the liquids P and Q becomes 9:7, how many litres of liquids P and Q were contained by the vessel initially.

27. A vehicle costing Rs. 900000 has a scrap value of Rs. 270000. If the annual depreciation charge is Rs. 70000, Find its useful life in years.

28. Calculate the 3 yearly moving averages from the following time series:

Year	2005	200 6	2007	2008	2009	2010	201 1	201 2
Earnings: (Rs Lakhs)	3.6	4.3	4.3	3.4	4.4	5.4	3.4	2.4

29. Find the present value of a perpetuity of Rs 3120 payable at the beginning of each year , if money is worth 6% effective.

30. Find the intervals in which the function is $f(x) = 2x^3 + 9x^2 + 12x + 20$

(1) increasing (2) decreasing

OR

If a manufacturer's total cost function C is given by C= $\frac{x^2}{25} + 2x$, find (i) average cost function (ii) the marginal cost function, and (iii) the marginal cost when 5 units are produced. Also, interpret the result.

31. A company has been producing steel tubes of mean inner diameter of 2 cm. A sample of 10 tubes gives an inner diameter of 2.01 cm and a variance of .004 cm^2. Is the difference in the values of means significant? (Given $t_9(.05) = 2.262$)

SECTION-D

32. The cost of 4 kg. onion, 3 kg. wheat and 2 kg. rice is 60. The cost of 2 kg. onion, 4 k wheat and 6 kg. rice is 90. The cost of 6 kg. onion, 2 kg. wheat and 3 kg. rice is 70. Find cost of each item per kg. by matrix method.

33. Integrate: $\frac{3x-2}{(x+1)(x-2)^2}$ w.r.t x OR $\int_1^4 |x-5|\ dx$

34. Mr. Naresh has bought 200 shares of City Look Company at 100 each in 2015. After selling them he has received 30000 which accounts for 22.47% CAGR. Calculate the number of years for which he was holding the shares.

35. If 5% of the electric bulbs manufactured by a company are defective , use Poisson distribution to find the probability that in a sample of 100 bulbs :

(1) None is defective
(2) 5 bulbs will be defective

OR

In an examination, 2000 students appeared and the mean of the normal distribution of marks is 30 with standard deviation as 6.25. Find out how many students are expected to score (1) between 20 and 40 marks (2) less than 25 marks

SECTION- E (CASE BASED QUESTION)

36. In year 2000, Mr. Talwar took a home loan of Rs.30,00000 from state Bank of India at 7.5%p.a. compounded monthly for 20 years. $(1.00625)^{240}$=4.4608

Based on the above information, answer the following question

i. What was EMI paid by Mr. Talwar

ii. What was interest paid by Mr. Talwar in 150^{th} payment?
iii. What was Principal paid by Mr. Talwar in 150^{th} payment?

37. Height of Students: The heights of students in a school are normally distributed with mean 160 cm and standard deviation 10 cm. Based on this information , answer the following:

i) What percentage of students are taller than 175 cm?

ii) What percentage of students have heights between 150 cm and 170 cm?

iii) If there are 1000 students, how many are expected to be shorter than 140 cm?

38. Radium decomposes at a rate proportional to the quantity of radium present. Suppose that it is found that in 25 years approximately 1.1 % of a certain quantity of radium has decomposed

Based on the above information answer the following questions:

(i)Formulate the differential equation for the amount of radium decomposed in time t.

(ii) If p and q denotes the order and degree of the differential equation in (1) respectively. find $2p + 3q$.
(iii) Write the expression for the amount of radium decomposed in time t and the amount of radium present at t=0

OR

(iii)Compute the value of proportionality constant appearing in the differential equation of part (

SOLUTIONS

SAMPLE QUESTION PAPER -2(2025-26)

APPLIED MATHEMATICS

CLASS: XII

SECTION-A

1. a To calculate 7^6 (mod 3), we can follow these steps:
Step 1: Calculate 7 (mod 3)= 7 (mod 3) = 1
Step 2: Calculate 7^2 (mod 3), Since 7 (mod 3) = 1, we can write: 7^2 (mod 3) = (7 (mod 3))^2 = 1^2 = 1
Step 3: Calculate 7^3 (mod 3)
7^3 (mod 3) = 7^2 (mod 3) × 7 (mod 3)= 1 × 1= 1
Step 4: Calculate 7^6 (mod 3)
Using the result from Step 3:7^6 (mod 3) = (7^3 (mod 3))^2= 1^2= 1
2. d To calculate -31 mod 7, we can follow these steps:
Step 1: Divide -31 by 7= -31 ÷ 7 = -4 with a remainder of -3
Step 2: Convert the remainder to a positive valueSince the remainder is -3, we can add 7 to get a positive value:-3 + 7 = 4
3. d a-c > b+c
Adding c to both sides gives us:a > b + 2c, Since c < 0, 2c < 0, and b + 2c < b.
Therefore, a > b + 2c is true, which implies a - c > b + c is also true.
4.c Pipe A fills the tank in 12 minutes, so its filling rate is:
1 tank / 12 minutes = 1/12 tank per minute
Pipe B empties the tank in 18 minutes, so its emptying rate is:
1 tank / 18 minutes = 1/18 tank per minute
When both pipes are open, the net filling rate is the difference between the filling rate of Pipe A and the emptying rate of Pipe B:
Net filling rate = (1/12 - 1/18) tank per minute
Net filling rate = (3/36 - 2/36) tank per minute
= 1/36 tank per minute
Since the net filling rate is 1/36 tank per minute, it will take:
36 minutes to fill 1 tank
5. c A=$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A^{/} = -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$,Skew symmetric
6. a on comparing a+b=6,b=2,a=4
7. c $|3A| = 3^3|A| = 27 \times 8 = 216$
8 .d dx/dt=3a, dy/dt=3at^2 , dy/dt=t^2
9.a The circumference of a circle (C) is given by:C = 2πr , Given that the radius is increasing at a rate of 0.7 cm/s, we can write: dr/dt = 0.7 cm/s=dC/dt = d(2πr)/dt= 2π(dr/dt)= 2π(0.7)=1.4 π
10.d dr/dx= 6x+36, put x= 15, we get marginal revenue= 15.6+36=90+36=126
11.c 5/k+7/k+9/k+11/k=1, 32/k=1, k=32
12.c 13.b 14.d 15.b 16.b
17.b EMI = (Principal + (Principal x Rate x Time)) / (Time x 12)
Where: Principal = Rs. 2,00,000 ,Rate = 10% per annum = 0.10, Time = 5 years
Substituting the values, we get:
EMI = (2,00,000 + (2,00,000 x 0.10 x 5)) / (5 x 12) = (2,00,000 + 1,00,000) / 60

= 3,00,000 / 60 = Rs. 5,000

18. d PV = PMT / r, PV = Rs. 10,000 (present value), PMT = Rs. 500 (periodic payment, every 6 months)

r = PMT / PV= 500 / 10,000= 0.05 per 6-month period

Annual rate = 2 x 0.05 (since there are 2 periods in a year) = 0.10 or 10%

19. c The correct answer is: (A) True, (R) False

Explanation: Assertion (A) is correct: The feasible region is indeed the set of points that satisfy all the given constraints in a linear programming problem.

Reason (R) is incorrect: The optimal value of the objective function is not necessarily attained at points on the X-axis only. The optimal solution can be at any corner point of the feasible region, which may or may not be on the X-axis.

20.d The correct answer is:(A) False, (R) True

Explanation: Reason (R) is correct: The derivative of the function y = [x(x-2)]^2 is:

dy/dx = 2x(x-2)(2x-2)= 4x(x-1)(x-2)

Setting dy/dx = 0, we get: 4x(x-1)(x-2) = 0,which gives x = 0, 1, 2.

However, Assertion (A) is incorrect: The function y = [x(x-2)]^2 is actually decreasing in (0, 1) and increasing in (1, 2) and (2, ∞).

SECTION-B

21. $x \equiv 23(mod\ 7)$

$x = 23 + 7$, $p \in Z$

$x = 23, 30, 47, \dots \dots$

$x = 30\ as\ 21 \le x < 31$

22. Length of course = 500 meters

Time taken by B to cover by 60 meters = 12 seconds.

∴ time taken by B to cover the course = $\left(\frac{12}{60} \times 500\right)$ = 100 seconds

∴time taken by B to cover the course=(100 − 12) seconds = 88 seconds

= 1 minute 28 seconds

OR

Part filled by the pump in 1 hour = $\frac{1}{2}$

Net part filled by the pump and leak in 1 hour = $\frac{3}{7}$

Emptying work done by the leak in 1 hour = $\frac{1}{2} - \frac{3}{7} = \frac{7-6}{14} = \frac{1}{14}$

Leak can empty the tank in 14 hours.

23. As the points P (3,-2), Q (8, 8) and R (k, 2) are collinear

Area of triangle PQR = $\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ k & 2 & 1 \end{vmatrix} = 0$

Solving above determinant, we get, | 3(8-2) +2(8-k) +1(16-8k)| = 0

|18 + 16 – 2k +16 -8k | = 0

-10k + 50 | = 0
-10k + 50) = 0
k = 5

24. Given current value of investment= Rs. 60000 Cost of investment = Rs. 50000

Rate of return = $\frac{60000-50000}{50000}$ X 100%

= 20%

25. Let number of necklaces and bracelets produced by firm per day be x and y, respectively. Clearly, $x \geq 0, y \geq 0$

∴ Total number of necklaces and bracelets that the firm can handle per day is at most 24.

∴ $x + y \leq 24$

Since it takes one hour to make a bracelet and half an hour to make a necklace and maximum number of hours available per day is 16.

∴ $12x + y \leq 16$

⇒ $x + 2y \leq 32$

Let Z be the profit function. Then, Z = 100x + 300y

∴ The given LPP reduces to Maximise Z = 100x + 300y subject to, $x + y \leq 24$

$x + 2y \leq 32$

and $x, y \geq 0$

SECTION-D

26. Speed downstream=6 km/h, Speed upstream= 4 km/h , Total Time taken= 1 hour, Distance = 2.4 km

OR

Let initially liquids P and Q be 5x and 7x litres respectively in the vessel. After drawing off 12 litres of mixture

Quantity of liquid P left in the mixture = 5x - $\frac{5}{12}$X 12 = 5x – 5 litres

Quantity of liquid Q left in the mixture = 7x - $\frac{7}{12}$X 12 = 7x – 7 litres So, quantity of liquid P= (5x -5 +12) litres = 5x +7 litres quantity of liquid Q= (5x -5 +12) litres

As per the question = $\frac{9}{7} = \frac{5x+7}{7x-7}$

63 x – 63 = 35x + 49

Solving we get , x = 4, Hence the quantity of liquid P was 5 X4 = 20 litres and quantity of liquid Q was 7 X4 = 28 litres

27. Here C= 900000, S= 270000,annual depreciation = 70000 Let useful life be n years.

Now annual depreciation =$\frac{C-S}{n}$

n= 9 years

28. 4.067 , 4 , 4.03, 4.40, 4.40, 3.73

29. $i = .06$, P= R + $\frac{R}{i}$= 3120+ 3120/.06

$getting\ P = Rs$ 55120

30. Getting $f'(x) = 6x^2 + 18x + 12$

$(x) = 6\ (x + 1)(x + 2)$

For increasing$f'(x) > 0$ $\quad fordecreasingf'(x) < 0$

Increasing $(-\infty, -2) \cup (-1, \infty)$ and Decreasing $(-2\ , -1)$

OR

(i) We have, C= $\frac{x}{25} + 2x$, so ,

the average cost function AC is given by AC= $\frac{C}{x}$ or AC= $\frac{x}{25} + 2$

(ii) MC is given by $\frac{2x}{25} + 2$

(iii) MC= 2.4

This means that , if the production is increased by 1 unit from 5 units to 6 units , then the cost of additional unit is approx. 2.4.

31. Write $\mu = 2\ cm \bar{X} = 2.01 cm n = 10\ \ ^2 = .004cm^2$

Define H_0 and H_1 Getting t= .476

The difference in the values of sample mean and population mean is not significant.

Section-D

32. Let cost of onion, wheat and rice per kg. be x,y and z respectively. Equations:

4x+3y+2z=60, 2x+4y+6z=90, 6x+2y+3z=70

Det(A)= 50, $A^{-1} = \frac{1}{50} \begin{pmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{pmatrix}$

X=5, Y=8,Z=8

33. Let $\frac{3x-2}{(x+1)(x-2)^2} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$

A= - 5/9, B= 5/9 , C= 4/3

After integrating, answer is $\frac{5}{9} log|x + 1| + \frac{5}{9} log|x - 2| - \frac{4}{3(x-2)}$+c

OR

33. Answer is : 15/2

34. SV=200 X 100= 20000, EV=30000 , CAGR= 22.47%

Let n number of years

$CAGR = \left(\left(\frac{EV}{SV}\right)^{1/n} - 1\right) X\ 100$, n = 2 years nearly

35. p = .05, n=100 getting $\mu = np = 5$

(1) P(none is defective)= .007
(2) P(5 defective bulbs)= .1822

OR

$$Z = \frac{X - \mu}{6.25}$$

When X=20 Z= -1.6 When X= 40
Z= 1.6

P(20<X<40)= .8904
Number of students scoring between 20 and 40 = 1781(approx) When X= 25 , Z= -.8

P(X<25)= .2119
Number of students scoring less than 25= 424

SECTION-E

36. (i)Rs.24167.82 (ii) Rs.10458.69 (iii) Rs.13709.13
37. (i) 6.68% (ii) 68.26% (iii) 23 students
38. $q = 1$, $2p + 3q = 5$,$m = m_0e^{-kt}$

OR

$$\left(1 - \frac{1.1}{100}\right)m_0 = m_0e^{-25k}, k = 0.000443$$

SAMPLE QUESTION PAPER - 3(2025-26)
APPLIED MATHEMATICS
CLASS: XII

General Instructions :

1. This question paper contains – five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion –Reason based questions of 1 mark each.
3. Section B has 5 Very Shirt Answers (VSA) type questions of 2 marks each.
4. Section C has 6 short Answer (SA) type questions of 3 marks each
5. Section D has 4 long Answer (LA) type questions of 5 marks each.
6. Section E has 3 source based / case based / passage based/integrated units of assessment (4 marks each) with sub parts

SECTION – A

Q. No. (All questions are compulsory. No internal choice is provided in this section)

1 Find $3^{16}{}_{(mod4)=}$

(a) 0 (b) 2 (c) 1 (d) 3

2 If $x \in R$, $|x| \geq 4$ then

(a) $x \geq 4$ (b) $-4 < x < 4$ (c) $x \leq -4$ (d) None

3 Inferential statistics is a process that involves all of the following except

(a) test a hypothesis (b) analysis relationship

(c) estimating parameter (d) estimating a statistics

4 A sample of 60 bulbs is taken at random for the purpose of t-test of significance then the degree of freedom is

(a) 50 (b) 59 (c) 1/50 (d) 1/59

5 In what ratio must a shopkeeper mix two types of oranges worth ₹ 60/kg & ₹ 90/kg respectively so as to get a mixture at ₹ 80/kg ?

(a) 1:2 (b) 1:3 (c) 2:1 (d) 3:1

6 In hypothesis testing, a tentative assumption about a population parameter is called

(a) Null hypothesis (b) Simple hypothesis

(c) Alternative hypothesis (d) None of these

7 The ratio in which must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price is

(a) 6:1 (b) 1:6 (c)2:3 (d) 3:2

8 The present value of a perpetuity of ₹ 900 payable at the beginning of each year, if money is worth 5% p.a., is

(a) 15000 (b) 18000 (c) 14250 (d) None

9 The demand function for a commodity is given by p= 4+x , the producer's surplus when the market sells 12 units of goods will be

(a) 71 (b) 72 (c)70 (d) None

10 The rise in prices before Diwali is an example of

(a) irregular trend (b) long term trend

(c) cyclical trend (d) seasonal trend

11 A dinning table costing Rs36000 has a useful life of 16 years. If annual depreciation is Rs2000, its scrap value using linear method is
(a) 4000 (b) 5000 (c) 6000 (d) 8000

12 If the objective function for a LPP is Z = 5x + 8y and the corner points of the bounded feasible region are (0,0), (7,0), (3,4) and (0, 2), then the maximum value of Z occurs at
(a) (7,0) (b) (0,0) (c) (0,2) (d) (3,4)

13 The general solution of differential equation :y logy dx – x dy =0 is
(a) y=log |cx| (b) $y=e^{|cx|}$ (c) $y=e^{-cx}$ (d) logy=|c +x|

14 The effective rate of return, which is equivalent to a declared rate of 12% compounded quarterly is
(a) 11.86% (b) 11.98% (c) 12.36% (d) 12.55%

15 The test is a statistical test used to compare the means of two groups
(a) Statistic test (b) p-test (c) kai test (d) t-test
(b)

16 The speed of a boat in still water is 15km/h and the rate of current is 3km/h. The distance travelled by boat downstream in 12min. is
(a)1.8km (b) 1.2km (c) 3.6km (d) 2.4km

17 Irregular variations in a time series are caused by
(a) Lockouts & strikes
(b) Epidemics
(c) Floods
(d) all of these

18 The average heart rate for Indians is 72 beats per minutes. To lower their heart rate a group of 25 people participated in a aerobics exercise program. After 6 months the test statistic is found to be -2.3077. As per t-distribution test (of 5% level of significance), what can you say about the effectiveness of the program?(if $t_{24}(0.05)=1.711$)
(a) effective (b) not effective (c) cannot say (d) none

19 **Assertion (A):** If the nominal rate of interest is 12.5% & the inflation is 2%,then the effective rate of interest is 10.5%**.**
Reason (R): If the interest is calculated only at the end of an year, then the effective rate of interest is same as the nominal rate of interest.
(a) (i) (b) (ii) (c) (iii) (d) (iv)

20 **Assertion (A)**: If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then the value of its parameter q is 1/3.
Reason (R): The mean and standard deviation of a binomial distribution are np and npq respectively.
(a) (i) (b) (ii) (c) (iii) (d) (iv)

SECTION - B

21 Sonia borrowed Rs100000 from a co-operative society at the rate of 10% p.a. for 2 years then find EMI using flat rate method.

22 If A = $\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$, find A(adjA).

OR

$$Find\ value\ of\ 2a+3b-c, if\ A=\begin{bmatrix} 0 & -1 & 28 \\ a-8 & 0 & 3b \\ -c+2 & 2 & 0 \end{bmatrix}\ is\ a\ skew\ symmetric\ matrix.$$

23 Two tailors, A and B earn Rs300 and Rs400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 80 shirts and 32 pairs of trousers at a minimum labour cost. Formulate this as an LPP.

24 A person has an initial investment of Rs50000 in an investment plan, after 2 years it has grown to Rs60000. Find his rate of return.

25 Pipe A can fill the tank 2 times faster than pipe B. If both pipes A and B running together can fill the tank in 24 minutes, find how much time will pipe B alone take to fill the tank? **OR**
Find (486+729) (mod 12).

SECTION - C

26 The volume of a spherical balloon is increasing at the rate of 3 cm^3/sec.Find the rate of change of its surface area when its radius is 2 cm.

27 The value of a machine purchased two years ago, depreciates at the annual rate of 10%. If its present value is Rs97200 find (i) its value after 3years (ii) its value when it was purchased.

28 Evaluate $\int \frac{dx}{(1+e^x)(1+e^{-x})}$

OR

Evaluate $\int \frac{x^3+x+1}{x^2-1}$ dx

29 A population grows at the rate of 10% per year. Using differential equations,find how long does it take for the population to grow 4 times?

OR

The demand function p for maximizing a profit monopolist is given by p = 274 – x^2 while the marginal cost is 4 + 3x, for x units of the commodity. Using integral find the consumer surplus.

30 Using Cramer’s rule solve 2x + 3y -10 = 0, x + 6y – 4 = 0.

31 10 years ago, Mr Sahib set up a sinking fund to save for his daughter`s higher studies .At the end of 10 years ,he has received an amount of ₹ 10,21,760 .What amount did he put in the sinking fund at the end of every 6 months for the tenure ,which paid him 5% p.a. compounded semi-annually ?(Use $(1.025)^{20}$=1.6386

SECTION - D

32 A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2$+k

Determine:(i) k (ii) P(X<3) (iii) P(X>6) (iv) P(0<X<3)

OR

If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals (i) exactly 3 (ii) more than 2 individuals will suffer from a bad reaction. (use e^{-2} = 0.1353)

33 The cost of manufacturing of certain items consists of Rs1600 as overheads Rs30 per item as the cost of material and the labour cost $\frac{x^2}{100}$ for x items produced. How many items must be produced to have a minimum average cost?

OR

A company produces a commodity with ₹ 24000 fixed cost. The variable cost is estimated to be 25% of the total revenue recovered on selling the product at a rate of ₹ 8 per unit. Find the following:
(i)Cost function (ii) Revenue function (iii) Breakeven point.

34 One kind of cake requires 300g of flour and 15g of fat, another kind of cake requires 150g of flour and 30g of fat. Find the maximum number of cakes which can be made from 7.5kg of flour and 600g of fate, assuming that there is no shortage of the other ingredients used in the making the cakes. Make it an L.P.P. and solve it graphically.

35 Using properties of determinants, prove that:

$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz.$$

SECTION - E

36 **CASE STUDY – I**

Susy is rowing a boat .She takes 6 hours to row 48 km upstream whereas she takes 3 hours to go the same distance downstream.

Based on the above information, answer the following questions.
Show steps to support your answers.
a) What is her speed of rowing in still water?
b) What is the speed of the stream?
c) What is her average speed?

OR

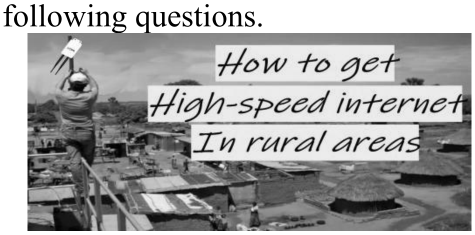
The stream is flowing at the speed of 4 km/h. If Susy rows a certain distance upstream in 3.5 hours and returns to the same place in 1.5 hours, then find the speed of Susy`s boat in still water.

37 **CASE STUDY – II**

The following data shows the percentage of rural Indians who have a high-speed internet connection at home.

Year	2001	2002	2003	2004	2005
Rural	3	6	9	16	24

Based on the above information, answer the following questions.



Show steps to support your answers.

a) Use method of least squares to find the best fit trend line equation for the rural Indians. Show the steps of your working.

OR

Demonstrate the technique to fit the best- suited straight line trend by the method of 3-years moving averages.

b)What is the forecast for the year 2006 for rural group using trend equation.

c) In which year the speed of internet connection in rural will be 37.6%.

38 **CASE STUDY – III**

The binomial distribution with parameters n and p is the discrete probability distribution and is given by p(X=r) =${}^{n}_{r}C p^{r} q^{n-r}$, r = 0, 1, 2,.....n, p,q>0 such that $(q+p)^n = 1$

A fair coin is tossed 6 times and getting head is considered a success .Based on the information ,answer the following questions:

(a) Find the values of parameters of this binomial experiment.

(b) Find the value of P(X=0).

(c) Find the probability of getting exactly 4 heads.

OR

In a binomial distribution ,if n=5 & P(X=0)=$\frac{32}{243}$,then find the value of P(X=3).

SOLUTIONS

SAMPLE QUESTION PAPER - 3(2025-26)

APPLIED MATHEMATICS

CLASS: XII

Ans-1 (c)

Ans-2 (d)

Ans-3 (d)

Ans-4 (b)

Ans-5 (c)

Ans-6 (a)

Ans-7 (b)

Ans-8 (d)

Ans-9 (b)

Ans-10 (d)

Ans-11 (a)

Ans-12 (d)

Ans-13 (b)

Ans-14 (c)

Ans-15 (d)

Ans-16 (c)

Ans-17 (d)

Ans-18 (a)

Ans-19 (b)

Ans-20 (a)

Ans-21 EMI $= \frac{P+Pni}{n}$

$= \frac{100000(1 + 24\text{x}\frac{10}{1200})}{24}$

$= 5000$

Ans-22 $A = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$, $A(\text{adj}A) = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$

OR

Since matrix is skew symmetric therefore a=9,3b=-2 & c=30

Thus 2a+3b-c=-14

Ans-23 Let number of days for A and B are x and y respectively

Z = 300x + 400y

Subject to constraints

$6x + 10y \geq 60$

$4x + 4y \geq 32$

$x, y \geq 0$

Ans-24 Given current value of investment =₹ 60000 & cost of investment =₹50000

Therefore Rate of return =(60000-50000)/50000=20%

Ans-25 Let time taken by pipe B is x min. and pipe A is x/2

Thus $\frac{1}{x} + \frac{2}{x} = \frac{1}{24}$

On solving, x= 72 min.

OR

(486+729) mod 12=6(mod12)+9(mod12)
=15(mod12)=3

Ans-26 Given $\frac{dV}{dt} = 3cm^3/sec \Rightarrow \frac{dr}{dt} = \frac{3}{4\pi r^2}$,

$\Rightarrow$ Now $\frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 3cm^2/sec$

Ans-27 P = 97200, r = 10% p.a., I = 0.1

(i) Value after 3 years = $P(1 - I)^r$
$S = 97200 \times (0.9)^3$
S = 70858.80

(ii) Present value = value 2 years ago$(1 - 0.1)^2$
$97200 = P(0.9)^2$
P = 120000

Ans-28 $$\int \frac{dx}{(1+e^x)(1+e^{-x})}$$

$$= \int \frac{e^x dx}{(1+e^x)^2}$$

$$= \int \frac{dt}{t^2},\ where\ t = e^x + 1\ and\ dt = e^x dx$$

$$= \frac{-1}{t} + C$$

$$= \frac{-1}{1+e^x} + C$$

$\int \frac{x^3+x+1}{x^2-1} dx = \int x + \frac{2x+1}{x^2-1} dx$

$= \int x + \frac{2x}{x^2-1} + \frac{1}{x^2-1} dx = \frac{x^2}{2} + \log |x^2 - 1| + \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$

Ans-29 *Let P_0 be the initial population & P be after t years.*

ATQ we have $\frac{dP}{dt} = \frac{1}{10}P \Rightarrow logP = \frac{1}{10}t + c$ & at $t = 0$, we have $P = P_0$

$\Rightarrow c = logP_0 \Rightarrow log\frac{P}{P_0} = \frac{1}{10}t - - - - - - - - (1)$

ATQ, again we have $P = 4P_0$

$\Rightarrow t = 10log4 = 20log2$ years.

OR

$P = 274 - x^2$
$R = px = 274x - x^3$
$dR/dx = 274 - 3x^2$
MR = 4 + 3x
In profit monopolist market MR = dR/dx
$4 + 3x = 274 - 3x^2$

X = -10, 9

$X_0 = 9$, $p_0 = 193$

CS = $\int_0^9 274 - x^2 \, dx$ = 193 x 9

CS = 486

Ans-30 D = $\begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix}$ = 12-3 =9

$D_1 = \begin{vmatrix} 10 & 3 \\ 4 & 6 \end{vmatrix}$ = 60-12=48

$D_2 = \begin{vmatrix} 2 & 10 \\ 1 & 6 \end{vmatrix}$ = 12-10 =2

x= $\frac{D_1}{D}$ = 16/3 , y = $\frac{D_2}{D}$ = 1/24

Ans-31 n = 10 x 2 = 20, S = 10,21,760, $i = \frac{5}{200}$ = 0.025, R = ?

S =R [$\frac{(1+i)^n - 1}{i}$]

$\Rightarrow$1021760 = R [$\frac{(1+0.025)^{20} - 1}{0.025}$]

$\Rightarrow$1021760 = R [$\frac{1.6386 - 1}{0.025}$]

$\Rightarrow$R = [$\frac{1021760 \times 0.025}{0.6386}$]

$\Rightarrow$ R = ₹ 40,000

Mr Mehra set aside an amount of ₹ 40,000 at the end of every six months

Ans-32 A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2$+k

Determine:(i) k (ii) P(X<3) (iii) P(X>6) (iv) P(0<X<3)

we know that $\sum P(X) = 1, 10k^2 + 9k = 1 \Rightarrow k = \frac{1}{10}$

(ii) $P(X < 3) = 3k = \frac{3}{10}$

(iii)$P(X > 3) = 7k^2 + k = \frac{17}{100}$

(iv)$P(0 < X < 3) = 3k = \frac{3}{10}$.

OR

Here $\lambda = np = 2000 \times 0.001 = 2$

$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!} = \frac{2^r e^{-2}}{r!}$

(i)$P(X = 3) = \frac{2^3 e^{-2}}{3!} = 0.180$

(ii)$P(X > 2) = 1 - \{P(X = 0) + P(X = 1) + P(X = 2)\} =$

$= 1 - 5e^{-2} = 0.323$

Ans-33 Cost of x items = 30x

Overhead cost = 1600

Labour cost = $x^2/100$

Total cost = 1600 + 30x + ($x^2/100$)

AC = C(x)/x = (1600/x) + 30 + (x/100)

dAC/dx = ($-1600/x^2$) + 1/100

$d^2AC/dx^2 = 1600(-2)x^{-3} = 3200/x^3 > 0$ at x = 400

AC is minimum at x = 400

OR

$Let\ x\ units\ of\ the\ product\ be\ produced\ \&sold. As\ the\ selling\ price\ of\ one\ unit\ is\ Rs\ 8, r = 8x$

$(i) \because\ the\ variable\ cost\ is\ 25\%\ of\ the\ total\ revenue\ recovered, thus\ V.C. = Rs\ 2x$

$\&\ fixed\ cost\ is\ 24000 \quad \therefore\ Cost\ function\ C(x) = 2x + 24000.$

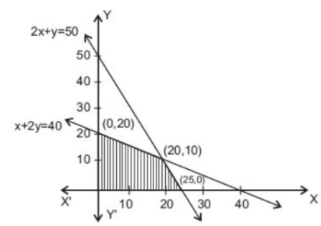
$(ii) Revenue\ R(x) = 8x$

$(iii)\ At\ breakeven\ points, R(x) = C(x) \Rightarrow x = 400$

Ans-34 Let x and y be the number of cakes of first kind and second kind respectively. Maximize Z = x + y

Subject to constraints 300x + 150y ≤7500, 15x + 30y ≤600, x, y≥0

Maximum value of Z at B(20, 10) is 30



Ans-35

$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz.$$

$Using\ R_1 \to R_1 - (R_2 + R_3), we\ get$

$$\begin{vmatrix} 0 & -2x & -2x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$Taking\ out\ 2\ common\ from\ R_1\ \&\ Using\ R_2 \to R_2 + R_1\ \&\ R_3 \to R_3 + R_1, we\ get$

$$2\begin{vmatrix} 0 & -x & -x \\ z & z & 0 \\ y & 0 & y \end{vmatrix}$$

$Taking\ out\ common\ x, y\ \&z\ from\ R_1, R_2\ \&\ R_3\ \&\ then\ expanding\ we\ get\ \Delta = 4xyz$

Ans-36 a) $Let\ the\ speed\ in\ still\ water\ be\ x\ km/hr\ \&\ that\ of\ stream\ be\ y\ km/hr.$

$Thus\ boat\ speed\ in\ upstream\ is\ = (x - y)\ km/hr$

$\&\ speed\ in\ downstream\ = (x + y)\ km/hr$

$Using\ Formula\ distance\ = speed\ \times time, we\ get\ x + y = 16, x - y = 8$

$Solving\ both\ the\ above\ equations\ we\ get\ x = 12\ km/hr, y = 4\ km/hr$

$Speed\ in\ still\ water\ is\ 12\ km/hr\ and\ speed\ of\ stream\ is\ 4\ km/hr.$

b) 4 km/hr

c) $Total\ distance\ coverd\ = 96\ km\ \&\ total\ time\ taken\ 6hrs\ + 3\ hrs = 9\ hrs$

$$\therefore\ Avg\ speed\ = \frac{total\ distance\ covered}{total\ time\ taken} = \frac{96}{9} = 10\frac{2}{3}\ hrs = 10.66\ km/hr$$

OR

$\because$
Distance covered is same in upstream and in downstream , assume that speed of boa still water be x km/hr .Then using again , distance = speed × time formula , we get $3.5(x-4) = 1.5(x+4) \Rightarrow x(\text{speed of Susy boat in still water}) = 10\ km/hr.$

Ans-37 a)

Year	Y	X	X^2	XY
2001	3	-2	4	-6
2002	6	-1	1	-6
2003	9	0	0	0
2004	16	1	1	16
2005	24	2	4	48

$a = \sum Y/n = 52/5 = 11.6$

$b = \sum XY/\sum X^2 = 52/10 = 5.2$

OR

Year	Y	3-year moving average
2001	3	
2002	6	6
2003	9	10.3
2004	16	16.3
2005	24	

b) $Y_{2006} = 11.6 + 5.2\ (2006-2003) = 27.2\%$

c) $37.6 = 11.6 + 5.2x$

$X = 5$

In 2008

Ans 38

a) *Parameters are* $n = 6\ \&\ p = \frac{1}{2}$.

b) *The value of* $P(X=0)$ *means no success in 6 attempts* $= {}^6C_0 q^6 = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$.

c) *The probability of exactly 4 successes* $= {}^6C_4 p^4 q^2 = \frac{15}{64}$

OR

If $n = 5\ \&\ P(X=0) = {}^5C_0 q^5 = \frac{32}{243} \Rightarrow q = \frac{2}{3}, p = \frac{1}{3}$.

Therefore $P(X=3) = {}^5C_3 p^3 q^2 = 10 \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^2 = \frac{40}{243}$

USEFUL LINKS

CBSE CURICULUM – CLASS 12 APPLIED MATHEMATICS https://cbseacademic.nic.in/curriculum_2026.html
CBSE QUESTION PAPER 2025 - CLASS 12 APPLIED MATHEMATICS https://www.cbse.gov.in/cbsenew/question-paper.html
MARKING SCHEME 2025 - CLASS 12 APPLIED MATHEMATICS https://www.cbse.gov.in/cbsenew/marking-scheme.html
CBSE SAMPLE PAPER 2024 - CLASS 12 APPLIED MATHEMATICS https://cbseacademic.nic.in/SQP_CLASSXii_2024-25.html
CBSE SUPPORT MATERIAL – CLASS 12 APPLIED MATHEMATICS https://cbseacademic.nic.in/appliedmaths-supportmaterial_XII.html