NEURAL NETWORK

(a Mathematical Model)

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PROJECT

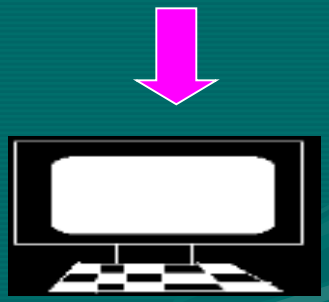
Mathematical Representation of the functional Behavior of the Biological Neuron

OUT LINE

- Man Vs Machine –a Comparison
- Use of Neural Network in Class room Learning Environment- a Motivation
- What is Neural Network and discussion of the terminologies used in the definition
- Mapping of Human Neuron to Artificial Neuron
- Application of FUNCTIONS in the Design of NN.
- Mathematical definition of Artificial NN
- Mathematical formulation of the sample problems and operations in detail
- Disadvantages of NN
- Conclusion
- Future work

Man Vs Machine

Number of Processing Elements



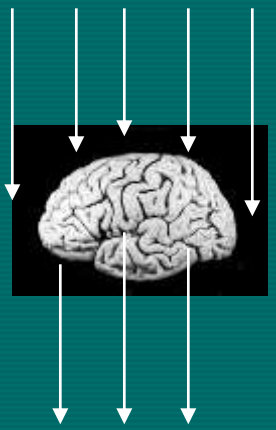
10^{14}
Synapses

10^{8}
Transistors

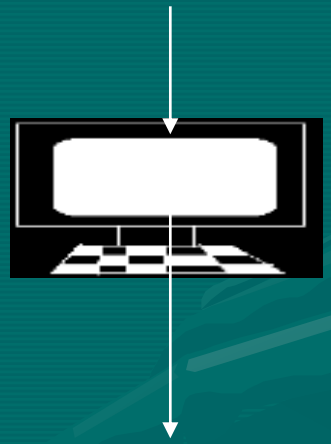
Human brain contains about 10 billion nerve cells (**Neurons**) and each neuron connected to other neurons through about 10000 **synapses**

Style of Computation

(Man Vs Machine)



Parallel & Distributed



Serial & Centralized

The brain's Network of Neurons forms a massively parallel **Information System**

Processing Speed

100 Hz ⟵ ⟶ 10^9 Hz

Man Vs Machine

- Processing speed of Brain is important because Speed & Efficiency of Information Processing are the basic components of Intelligence
- Brain is built with very slow h/w because Neurons operate at a max. speed of 100Hz
- Speed of a conventional PC is 10^9Hz
- A complex visual perception occurs <100ms this shows that Brain performs massive parallel and distributed computations

Learning Capability

Yes **a Little**

- How is Learning related to Intelligence ?
- Brain can learn (recognize itself) from **EXPERIENCE**
- Neural Networks learn by **EXAMPLE**
- Neural Networks and conventional algorithmic computer are not in competition but complement each other

Intelligence and Self-awareness

(Man Vs Machine)

- Man is usually intelligent and conscious

- Not (yet)

- Have the above definitions changed over time ?
- Do you believe a machine will ever be built that exhibits intelligence ?

Use of Neural Network in Class Room Learning Environment

- There is a correlation between the location of high and low-performing students in the room
- Low-performing students are seated in the front, their chance to do better increases.
- The results of high-performing students who are seated in the back are not affected.
- When high-performing students are seated in the outer four corners, the performance of the class as a whole increases

What is Neural Network ?

- It is a Computational or Mathematical model
- It is typically composed of a set of parallel and distributed processing elements called Nodes or Neurons.
- These nodes are interconnected by weighted signal channels, called connections or synaptic weights.

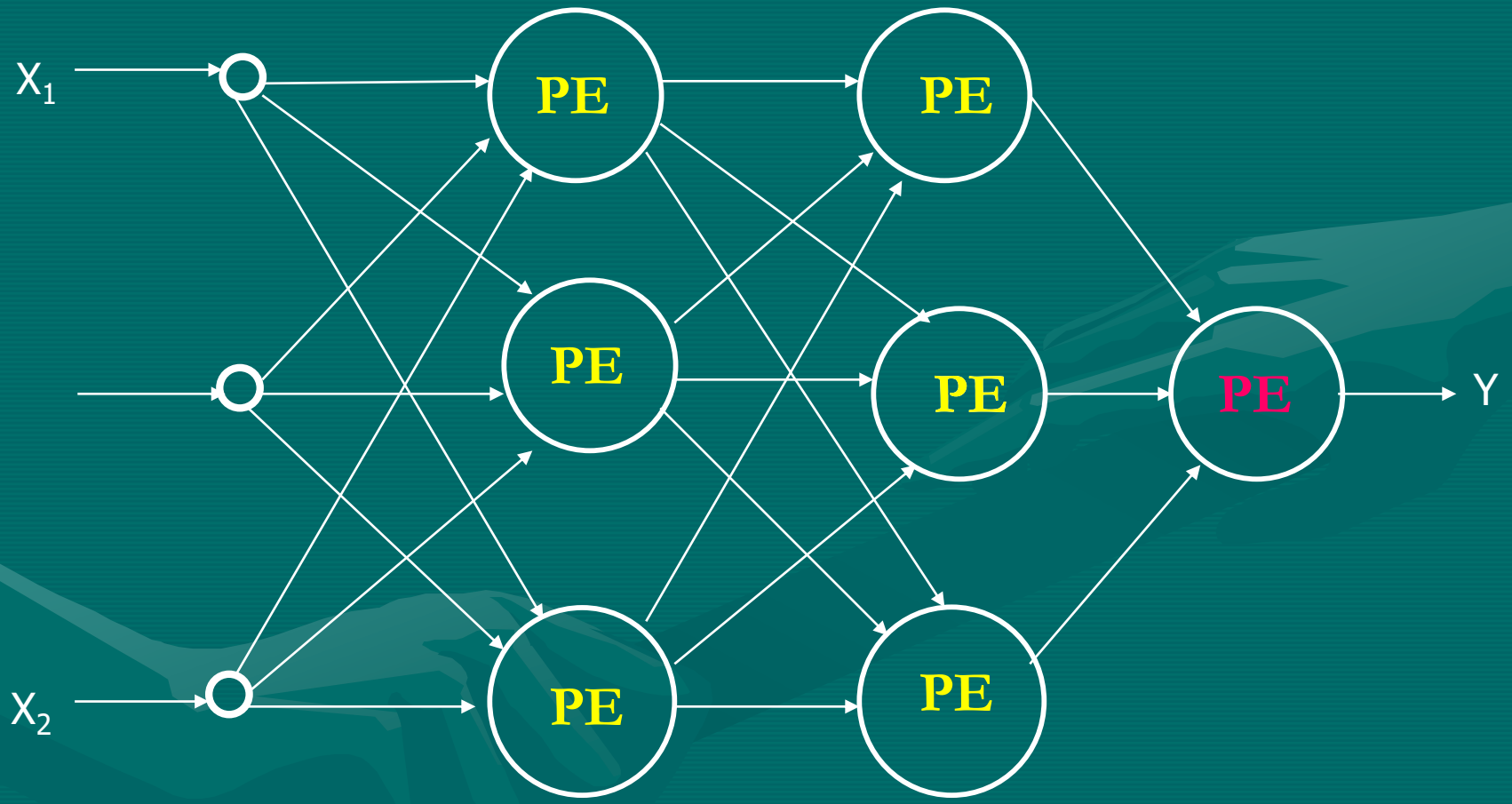
Concept of PARALLEL and DISTRIBUTED Processing

- A task is divided into number of processes .
- These processes are distributed in a number of CPUs,

  Then it is called distributed computing .
- If all these processes run in parallel

  Then it is said to be parallel computing in a distributed environment.

Diagram shows the connectionist approach of PARALLEL and DISTRIBUTED computing



Note:-Nodes are Interconnected appropriately

What is Neuron ?

- **Neurons are highly specialized nerve cells**
- **It is responsible for communicating information throughout the body in both chemical and electrical forms.**
- **It has 4 main regions :-**

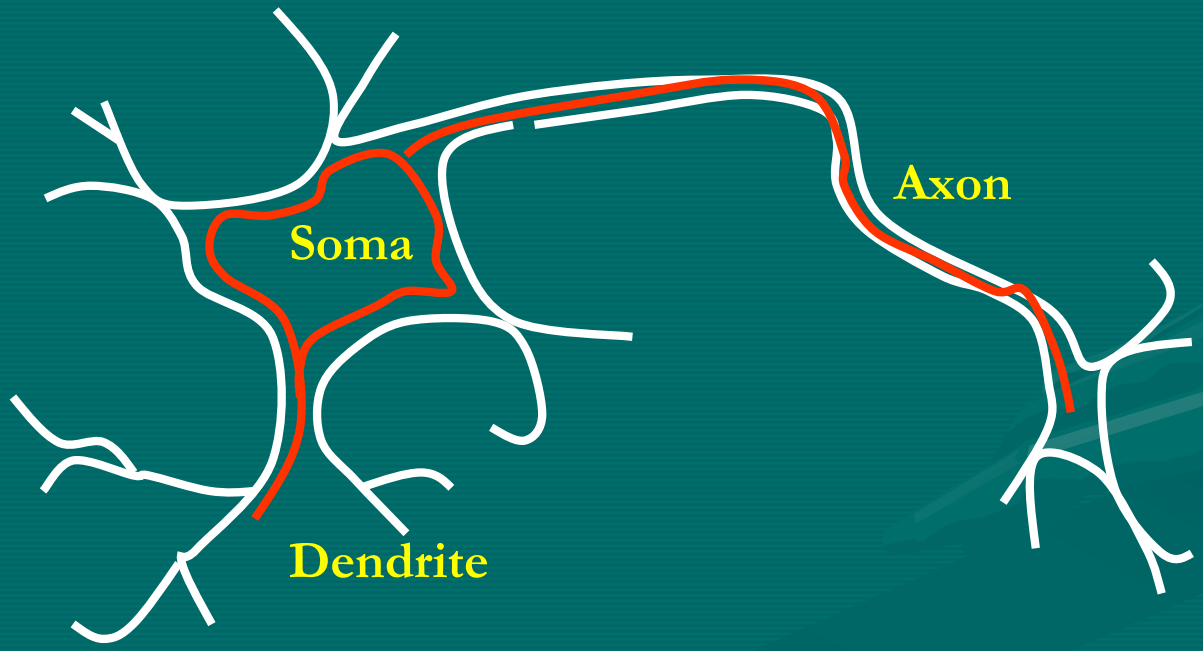
(1) Soma or The Cell Body

(2) Dendrites

(3) Axon

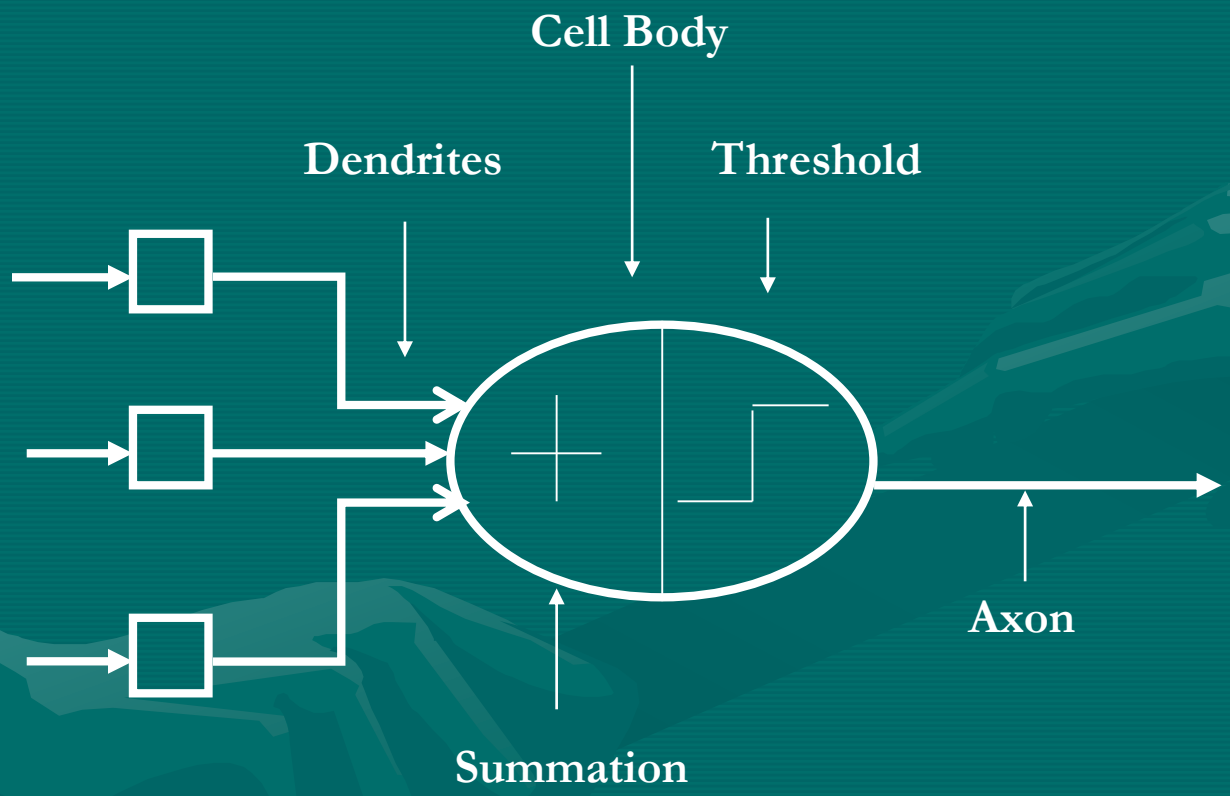
(4) Synapses

The Biological Model of Neuron



Note :-Information is received through Dendrite, get processed at the soma and transmitted through axon

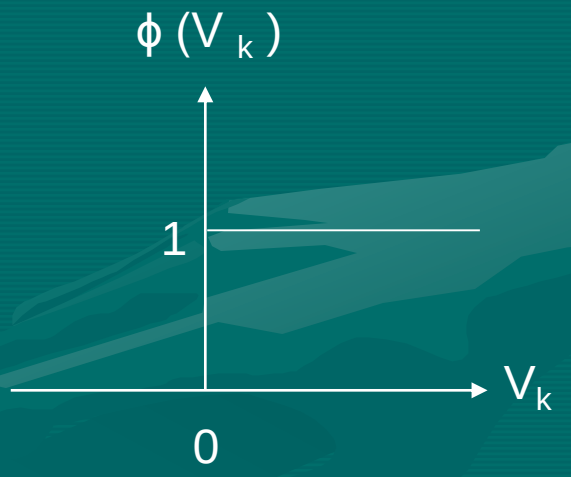
From HUMAN Neuron To Artificial Neuron



Application of FUNCTIONS in the design of Neural Network

(1) Threshold function

$$\phi(V_k) = \begin{cases} 0 & \text{If } V_k < \theta_k \\ 1 & \text{If } V_k \geq \theta_k \end{cases}$$

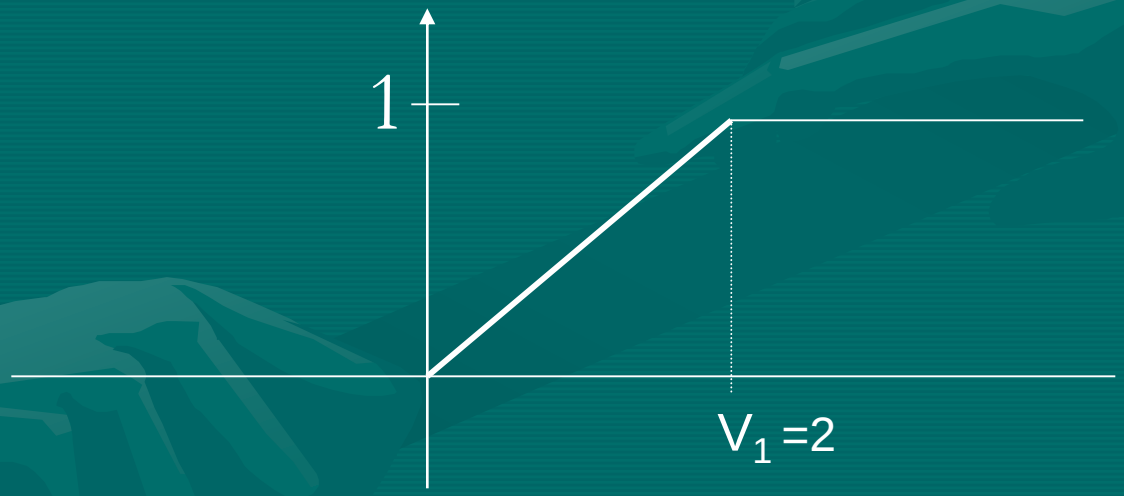


where V_k is the weighted sum of inputs

and θ_k is the Threshold value

(2) Piece-wise Linear function:

$$\Phi(V_k) = \begin{cases} 0 & \text{If } V_k \le 0 \\ \alpha V_k & \text{If } 0 < V_k < V_1 \\ 1 & \text{If } V_k \ge V_1 \end{cases}$$



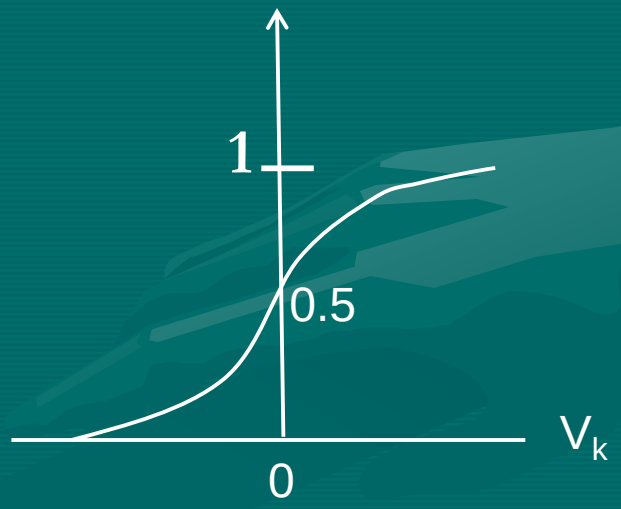
where V_k is the weighted sum of inputs

(3) Sigmoid function :

$$\phi(V_k) = \frac{1}{1 + e^{-V_k}}$$

$V \rightarrow -\infty$, $\phi(V) = 0$

$V \rightarrow +\infty$, $\phi(V) = 1$

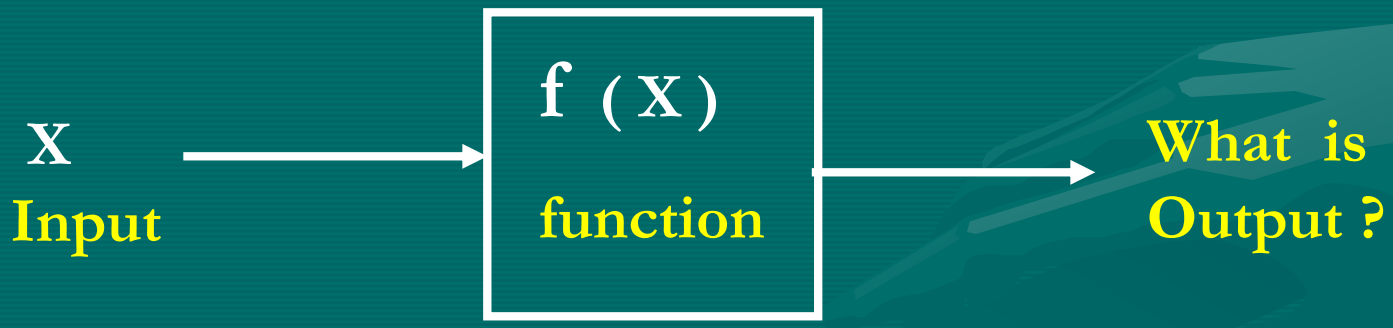


where V_k is the weighted sum of inputs

Note :-Sigmoid units bear a greater resemblance to real neurons

Input , Function and Output

Category I: Given Input and function , output is asked

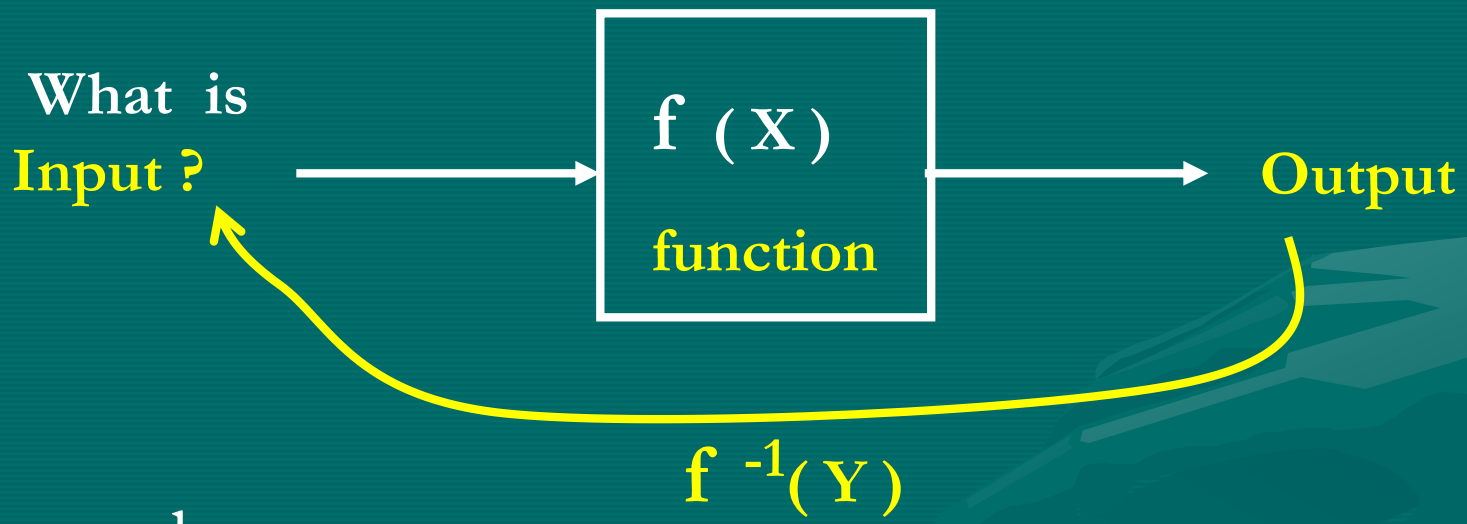


Example:-

Let x =1 , f(x) = 2 x + 1 , Determine Y=f (x)

ans: Y = 3

Category II: Given function and Output known, then determine its Inverse



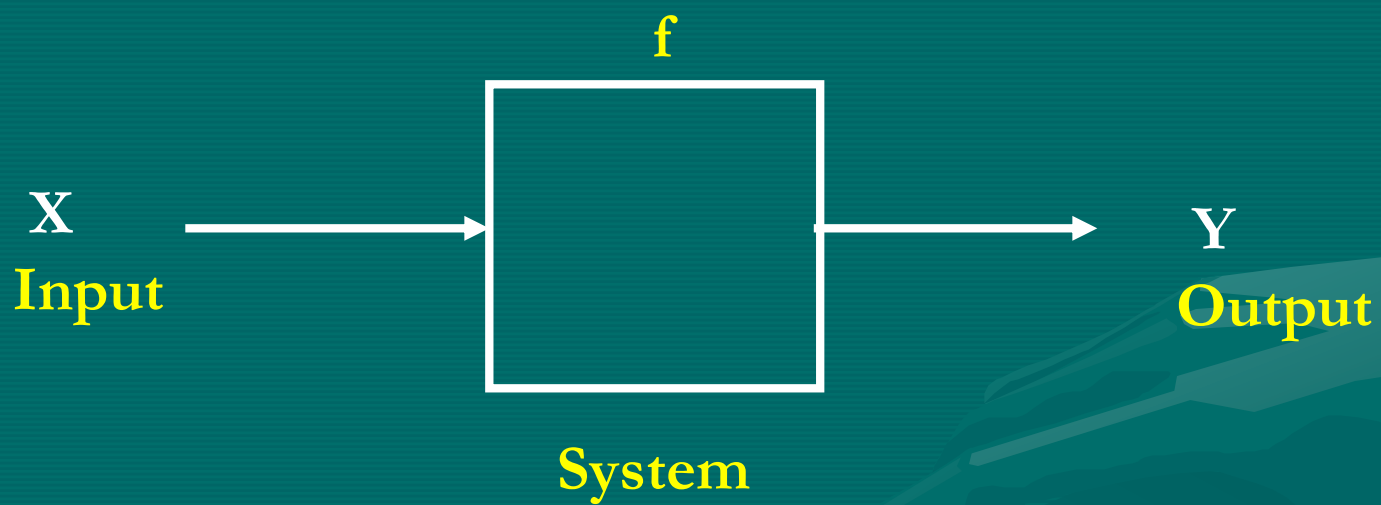
Example:-

Let $f(x) = 2x + 1$, $x = f^{-1}(y) = (y-1)/2$

For $Y = 3$, $f^{-1}(Y) = 1$ i.e Dom(f) = Range(f^{-1})

Note:-Its Application is in Cryptography

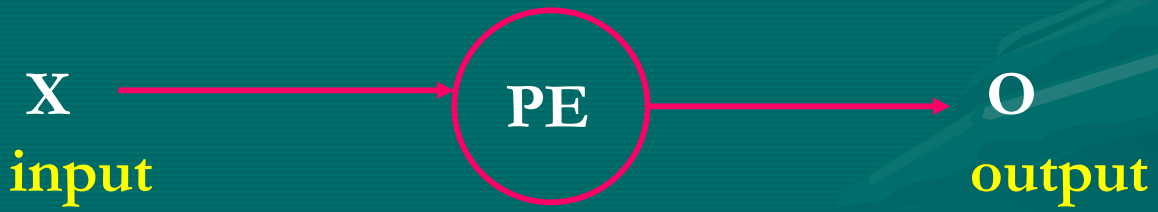
Category III: Given Input and Output then find f i.e determine the " System"



Design of Neuron Network (System)

Mathematical definition of ANN

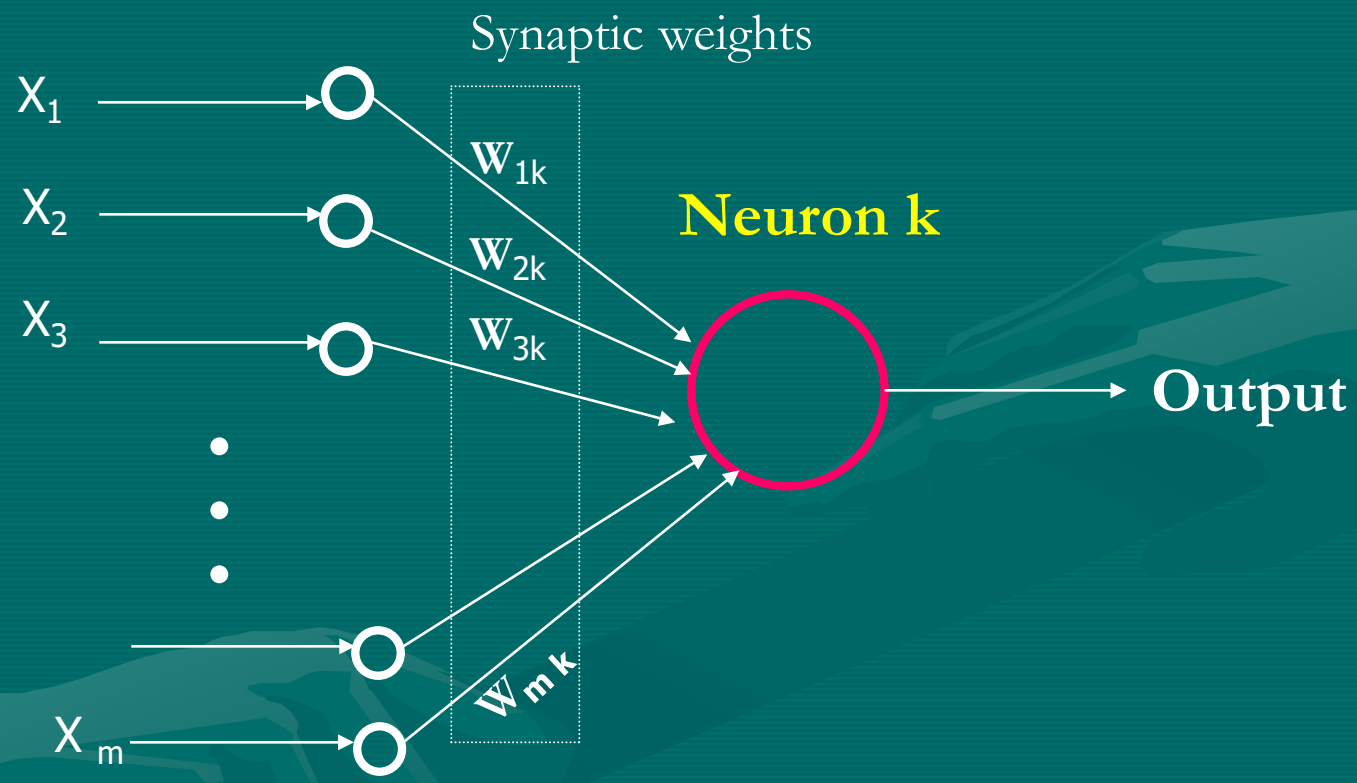
i. Set of computational elements called Neurons or Nodes or Processing Elements (P E s)



Neuron k

Neuron k receives input from neuron j , or perhaps from an external source ,processes it and produces output

ii. Each input has an associated weight which corresponds to brain synopses.

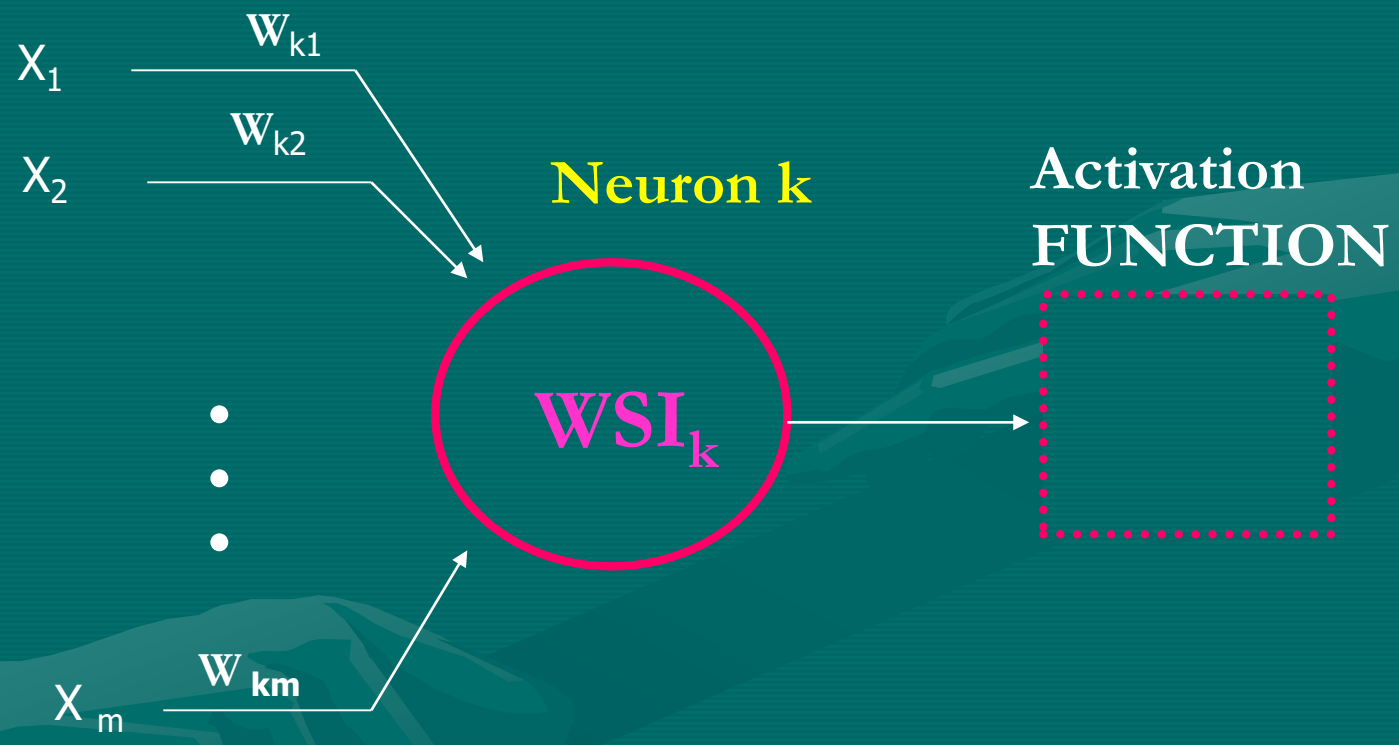


Learning occurs by changing the effectiveness of synopsis which are modeled as weights

iii. **Weighted sum of inputs (WSI) : is called the net input to a Neuron**

The net input to neuron' k is given by

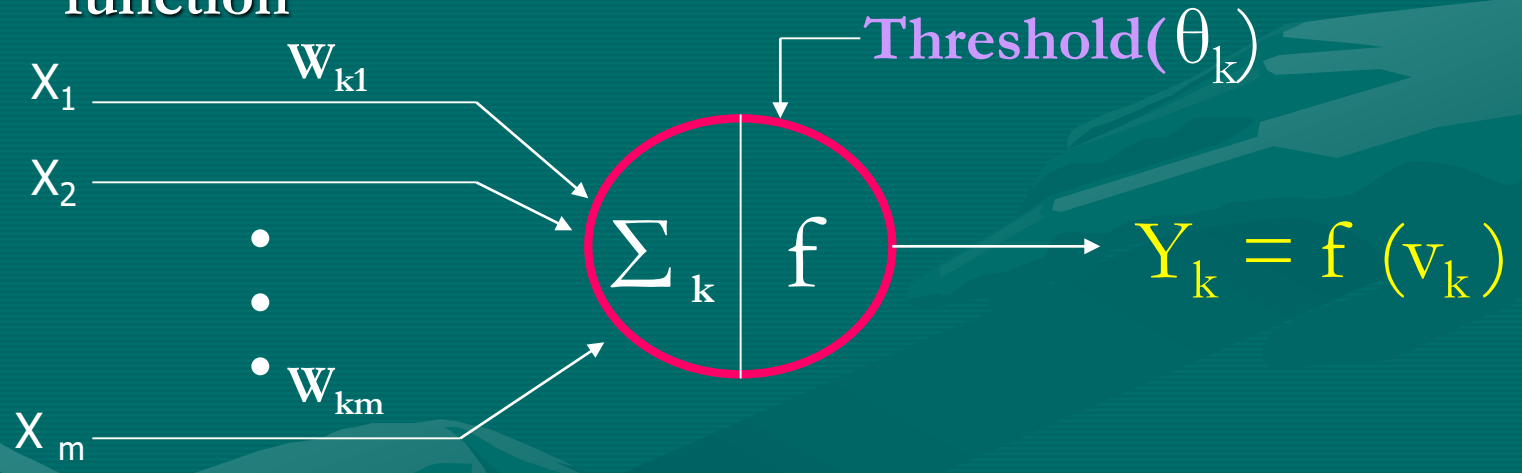
$$v_k = \sum_{j=1}^{m} w_{kj} x_j$$



WSI is specific for a Neuron

iv. Activation FUNCTION f : $f(v_k) = f(\Sigma w_{kj} x_j)$

- It activates the neuron by changing (transforming) its signal level from one state to other.
- It depends on weight and Input-Output transfer function



- This function falls typically into one of three categories: Linear, Threshold and Sigmoid

Output of this Neuron can serves as input to other Neurons

EXAMPLE :-To Construct an Artificial NN

Consider a single unit Neural Network .The network has 2 inputs , one output , all are binary. It finds the maximum between two binary digits

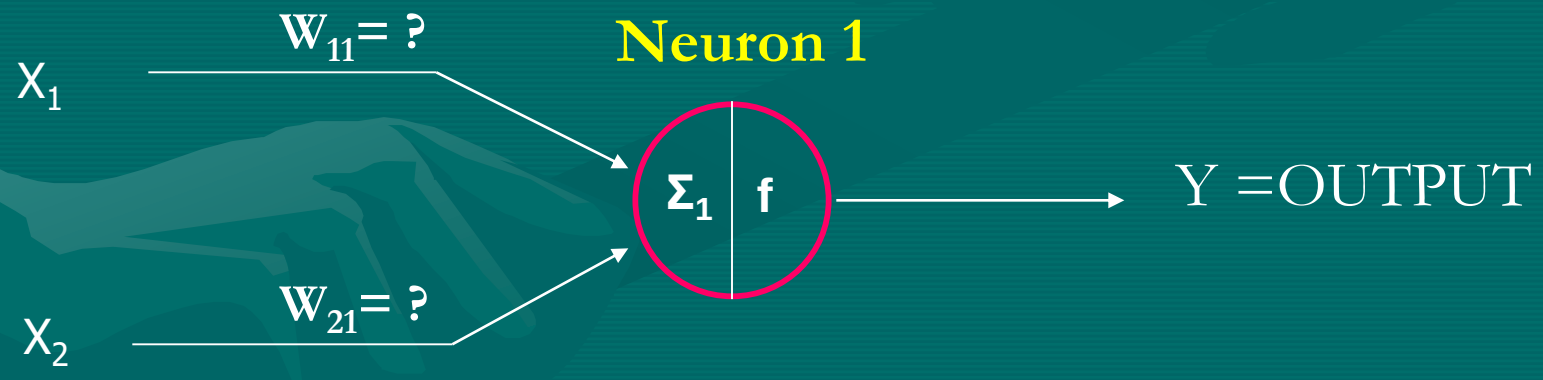
X1	X2	OUTPUT
0	0	0
0	1	1
1	0	1
1	1	1

Note: Given input and output ,find f i.e determine the "system"

SOLUTION :-

Step I: Define the parameters

- **Two inputs X1 and X2 (given)**
- **Make connection of Inputs to neuron with synaptic weights**



Note: The objective in the design of NN is adjusting the weights

Step II:- ADJUST WEIGHTS and Fix Threshold

- Assign values to Weights (Hit and Trial Method being followed)

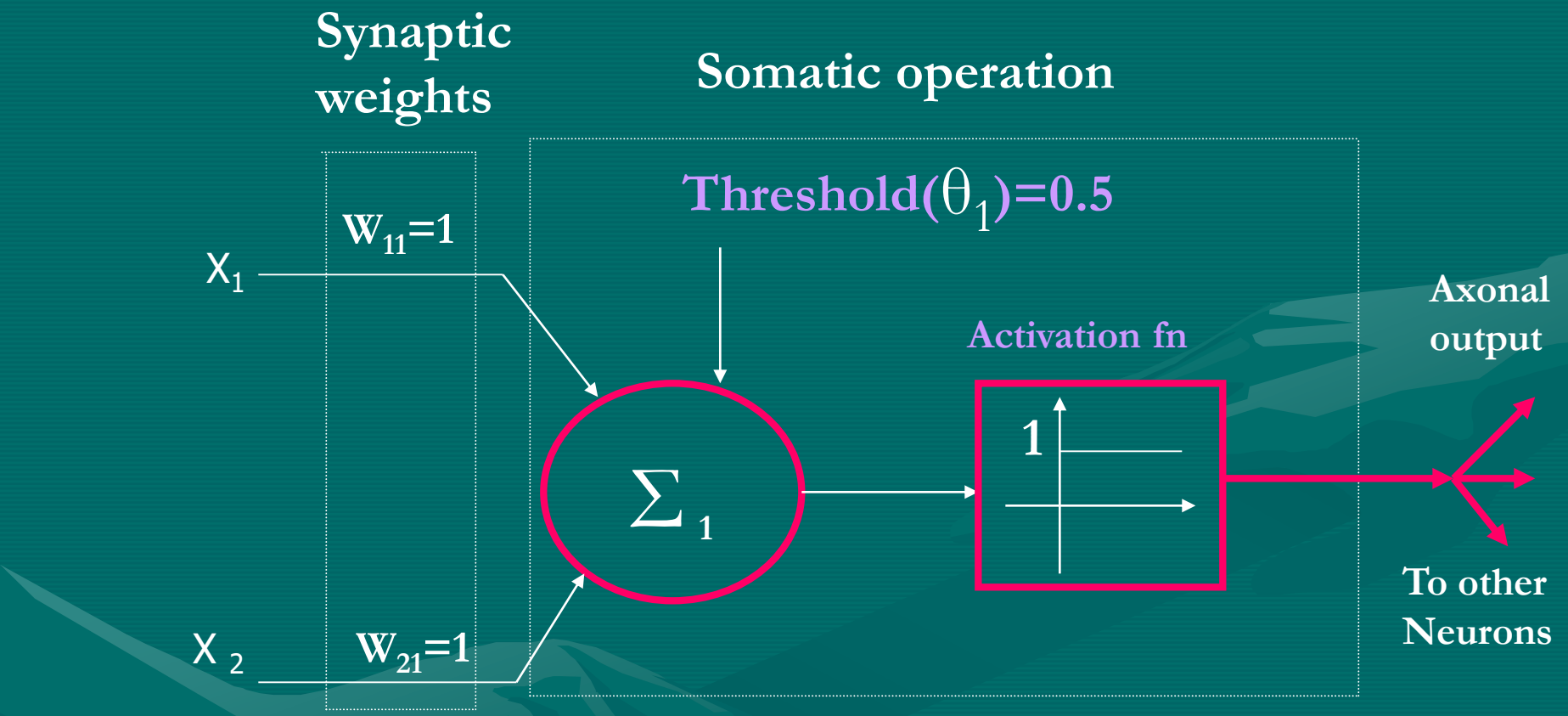
  Let W11=1 and W21=1

- Linear Combiner : Weighted Sum of Inputs (WSI) =X1.W11+X2.W21

- Apply Threshold Activation FUNCTION

  Let the threshold value for this Neuron is 0.5

The Operations at a Neuron of a NN



In this case there is No Hidden layer

Step III:- Operation to be Verified

i. **X1=0 , X2=0**

WSI=W11X1+W21X2=1×0+1×0 = 0 < 0.5, f(WSI)=0

Therefore Y = 0

ii. **X1=0 , X2=1**

WSI=W11X1+W21X2=1×0+1×1 = 1> 0.5, f(WSI)=1

Therefore Y = 1

iii. **X1=1 , X2=0**

WSI=W11X1+W21X2=1×1+1×0 = 1> 0.5, f(WSI)=1

Therefore Y = 1

iv. **X1=1 , X2=1**

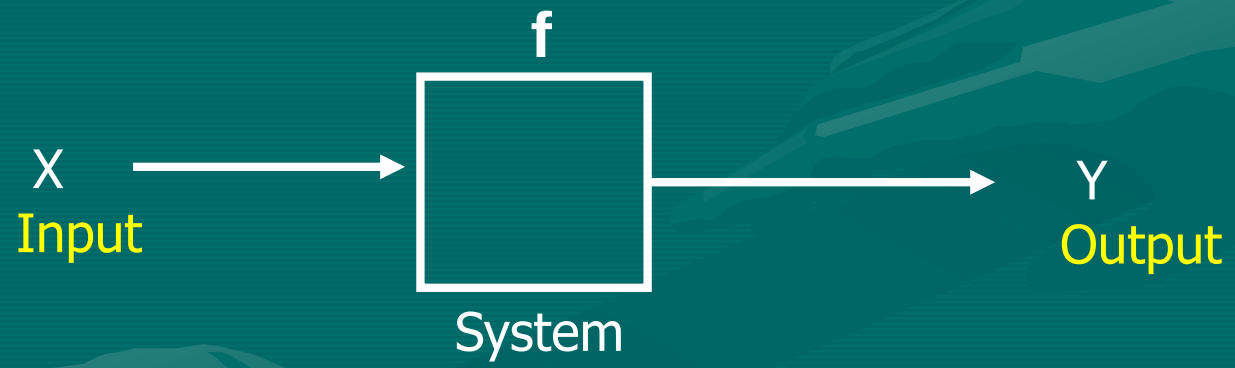
WSI=W11X1+W21X2=1×1+1×1 = 2> 0.5, f(WSI)=1

Therefore Y = 1

Problem Based on Artificial NN

Construct a Multilayered Feed-forward Neural Network which implements the Boolean Function XOR :

$Y = \overline{X_1} X_2 + X_1 \overline{X_2}$



Note: Given input and output ,find f i.e determine the "system"

SOLUTION

Step I: Construct the truth table for Boolean function XOR

X_1	X_2	$\overline{X_1}$	$\overline{X_2}$	$\overline{X_1}X_2$	$X_1\overline{X_2}$	$Y = \overline{X_1}X_2 + X_1\overline{X_2}$
0	0	1	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	0	0	0

Note: The above construction is to know the Input and Output

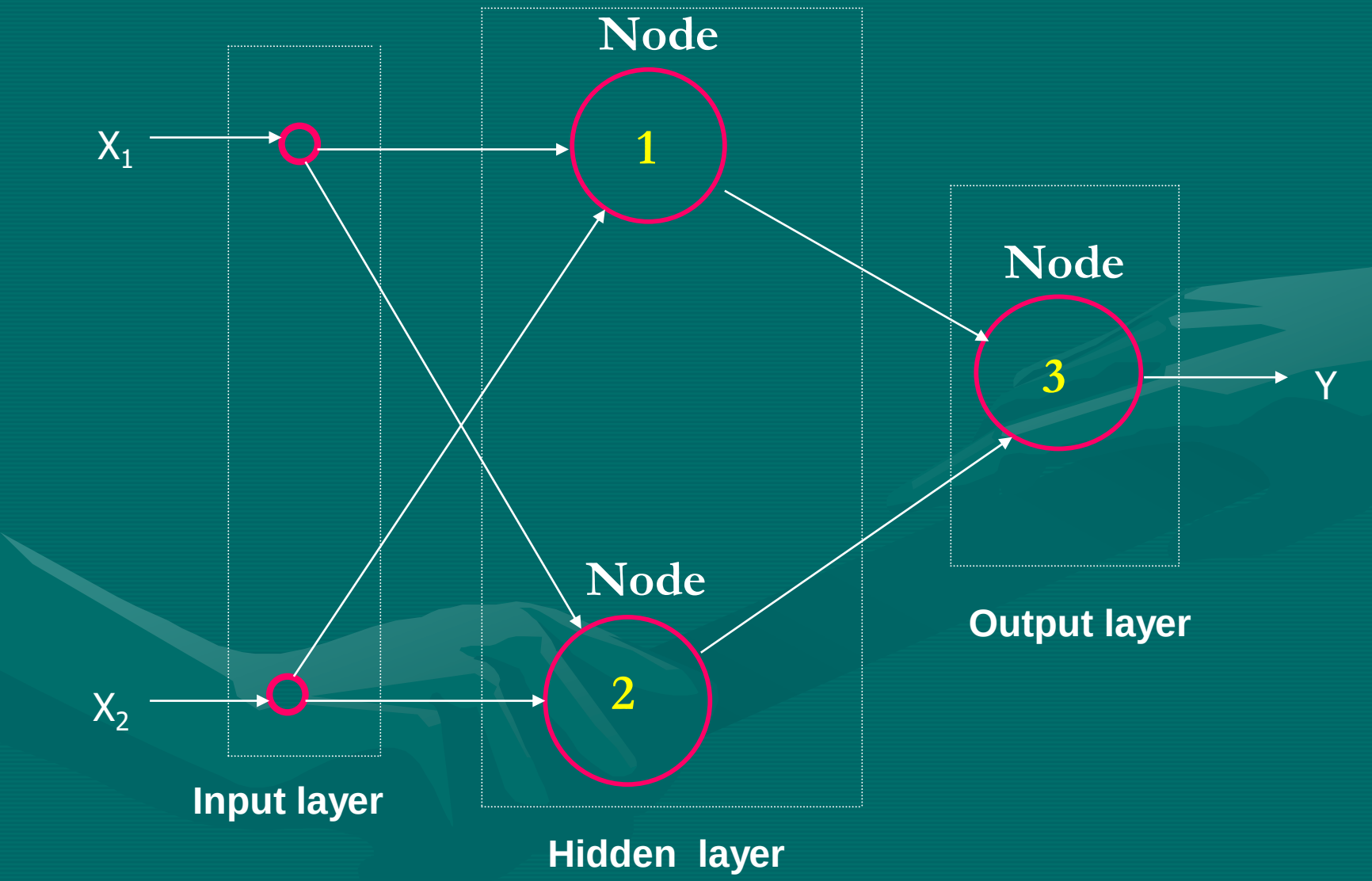
Step II. Define Parameters

i. Two inputs X1 and X2 (given)

ii. Let us take three Neurons :Node 1 , Node 2 an d Node3

iii. Join the Inputs X1 and X2 with Node1 and Node2 and Make output of these nodes as input to Node3

Representation of Artificial NN with 2 input & 1 output



Note:-Nodes are usually ordered into layers

Step III:- ADJUST WEIGHTS and Fix Threshold

- Assign values to Weights (Hit and Trial Method being followed)

  Let W11= -1, W12 =1, W21=-1,W22=-1

  W13= -1, W23= -1

- Linear Combiner : Weighted Sum of Inputs

  WSI for Node 1=X1.W11+X2.W21

  WSI for Node 2=X1.W12+X2.W22

  WSI for Node 3=O1.W12+O2.W23

- Apply Threshold Activation FUNCTION

  Threshold for Node 1=- 0.5, Node 2 = 1.5 and for Node3 = -0.5

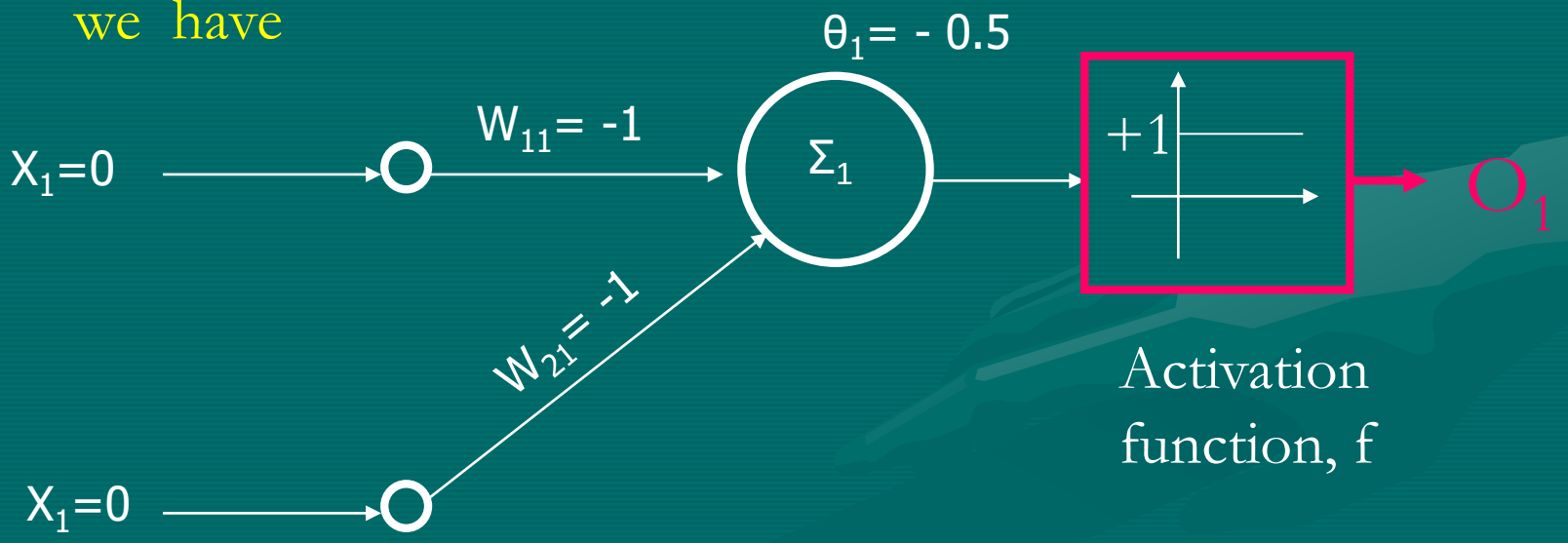
Note:-The Bias input is specific for a Neuron

Illustrating the Operation of Neuron 1:-

Net input to neuron1 designated as : Σ_1

For the Input X1 =0 , X2 = 0

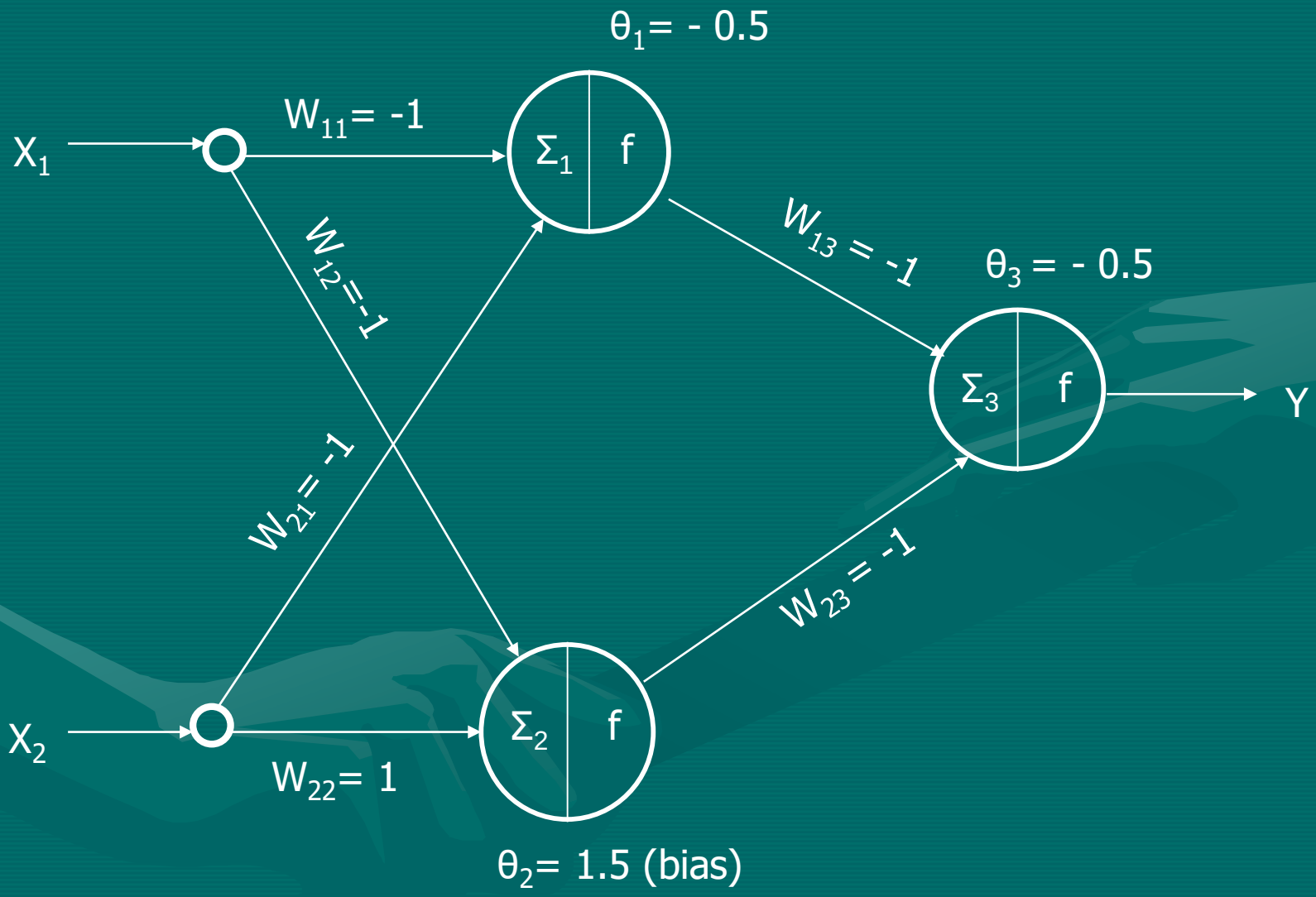
we have



$\Sigma_1 = W_{11}X_1 + W_{21}X_2 = -1 \times 0 - 1 \times 0 = 0 > -0.5$, $O_1 = f(\Sigma_1) = 1$

Note:-Output of this serves as input to Neuron3

Multilayer –Feed forward ANN with Weights and Threshold values



Verify input –output of XOR using ANN

i. X1=0 , X2=0

$\Sigma_1=W_{11}X_1+W_{21}X_2=-1\times0-1\times0 = 0 > -0.5$, $O_1=f(\Sigma_1) = 1$

$\Sigma_2=W_{12}X1+W22X2=1\times0+1\times0 = 0 < 1.5$, $O2= f(\Sigma2) = 0$

$\Sigma3=W13O1+W22O2=-1\times1+-1\times0 = -1< -0.5$, $O3= f(\Sigma3) = 0$

Therefore Y = 0

ii. X1=0 , X2=1

$\Sigma1=W11X1+W21X2=-1\times0-1\times1 = -1 < -0.5$, $O1=f(\Sigma1) = 0$

$\Sigma2=W12X1+W22X2=1\times0+1\times1 = 1 < 1.5$, $O2= f(\Sigma2) = 0$

$\Sigma3=W13O1+W22O2=-1\times0+-1\times0 = 0 > -0.5$, $O3= f(\Sigma3) = 1$

Therefore Y = 1

iii. X1=1 , X2=0

$\Sigma1=W11X1+W21X2=-1\times1-1\times0 = -1 < -0.5$, $O1=f(\Sigma1) = 0$

$\Sigma2=W12X1+W22X2=1\times1+1\times0 = 1 < 1.5$, $O2= f(\Sigma2) = 0$

$\Sigma3=W13O1+W22O2=-1\times0+-1\times0 = 0 > -0.5$, $O3= f(\Sigma3) = 1$

Therefore Y = 1

iv. X1=1 , X2=1

$\Sigma1=W11X1+W21X2=-1\times1-1\times1 = -2 < -0.5$, $O1=f(\Sigma1) = 0$

$\Sigma2=W12X1+W22X2=1\times1+1\times1 = 2> 1.5$, $O2= f(\Sigma2) = 1$

$\Sigma3=W13O1+W22O2=-1\times0+-1\times1 = -1< -0.5$, $O3= f(\Sigma3) = 0$

Therefore Y = 0

Comment 1:-

- In order to train a neural network to perform some task, we must adjust the weights of each unit in such a way that the error between the desired output and the actual output is reduced.
- That is NN computes the error changes when weigh is increased/decreased slightly.
- The **back propagation algorithm** is the most widely used method for updating the weights.

Comment 2:-

- A biological neuron may have as many as 10,000 different inputs, and may send its output (the presence or absence of a short-duration spike) to many other neurons.
- Neurons are wired up in a 3-dimensional pattern.
- Real brains, however, are orders of magnitude more complex than any artificial neural network so far considered

Disadvantage of NN:-

- Neural Network needs train to operate
- Its operation can not be predicted since the network finds out how to solve the problem by itself.

CONCLUSION

- Mathematical models of reality are the vastly more important type of representation. Essentially, any thing in the physical or biological world whether natural or involving technology & human intervention is subject to analysis by mathematical expressions. Neural Network is one among them.

- This project is an integration of ideas taken from **Biology, Mathematics** and **Computer science.**

FUTURE WORK

A market index is comprised of a weight average measure of the prices of the individual .The values of exchange –traded funds and many financial products (options & futures) are tied to the values of market indices. The ability to forecast the future value of index will help investors to take better decisions. Multilayer Perceptron (MLP) can be used to predict say 5 day% change in the value of an index

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THE END