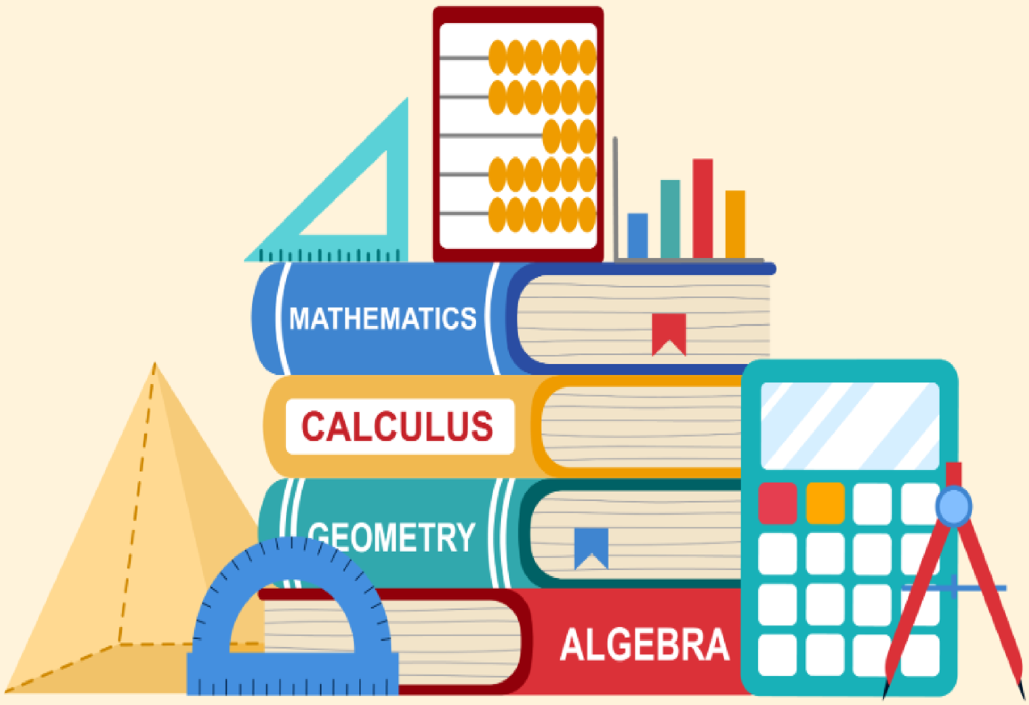
KENDRIYA VIDYALAYA SANGATHAN AGRA REGION

STUDENT SUPPORT MATERIAL

MATHEMATICS

CLASS 12

2026-2027



PREFACE

This study material is the culmination of an extensive collaborative effort involving a dedicated team of educators and subject matter experts. With meticulous care, we have meticulously designed this resource to provide students with a succinct yet comprehensive tool for consolidating their knowledge.

Under the diligent guidance of our esteemed subject experts and the unwavering enthusiasm of our team, we have incorporated the entire curriculum and an extensive collection of practice questions spanning all chapters. Our paramount objective has been to ensure perfect alignment with the latest curriculum and examination patterns as set forth by the CBSE.

We firmly believe that this material will prove to be an invaluable resource, serving as a clear and concise repository of essential information for effective subject revision. It encompasses all the critical components necessary to assist in students' preparation and enhance their understanding of the subject matter.

Our aspiration is that this study material will emerge as a dependable aid for swift and efficient revision, instilling confidence in students and ultimately contributing to their academic success. We strongly encourage you to actively engage with the content, pose questions, and fully utilize this resource in your educational journey.

We extend our heartfelt best wishes for your studies and sincerely hope that this material becomes your trusted companion on the path to academic excellence.

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Mathematics Subject Code – 041

Classes XII (2026 – 27)

The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. Senior Secondary stage is a launching stage from where the students go either for higher academic education in Mathematics or for professional courses like Engineering, Physical and Biological science, Commerce or Computer Applications. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in Focus Group on Teaching of Mathematics 2005 which is to meet the emerging needs of all categories of students. Motivating the topics from real life situations and other subject areas, greater emphasis has been laid on application of various concepts.

Objectives

The broad objectives of teaching Mathematics at senior school stage intend to help the students:

- to acquire knowledge and critical understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles, symbols and mastery of underlying processes and skills.
- to feel the flow of reasons while proving a result or solving a problem.
- to apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method.
- to develop positive attitude to think, analyze and articulate logically.
- to develop interest in the subject by participating in related competitions.
- to acquaint students with different aspects of Mathematics used in daily life.
- to develop an interest in students to study Mathematics as a discipline.
- to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases.
- to develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics.

COURSE STRUCTURE

CLASS – XII

(2026-27)

One Paper **Max. Marks: 80**

No.	Units	Marks
I.	Relations and Functions	08
II.	Algebra	10
III.	Calculus	35
IV.	Vectors and Three - Dimensional Geometry	14
V.	Linear Programming	05
VI.	Probability	08
	Total	**80**
	Internal Assessment	**20**

Unit-I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability

Continuity and differentiability, chain rule, derivative of composite functions, derivatives of inverse trigonometric functions like $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real- life situations).

3. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2+bx+c} \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \frac{px+q}{ax^2+bx+c}dx$$

$$\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx\,, \quad \int \sqrt{ax^2+bx+c}\, dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Application of the Integrals

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx} + py = q$, where p and q are functions of x or constants.
$\frac{dx}{dy} + px = q$, where p and q are functions of y or constants.

Unit-IV: Vectors and Three-dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three-dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming Problem

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem.

MATHEMATICS (Code No. – 041)

QUESTION PAPER DESIGN

CLASS – XII (2026-27)

Time: 3 hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	**80**	**100**

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper. However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note: For activities NCERT Lab Manual may be referred.

CHAPTER- 1 (RELATIONS AND FUNCTIONS)

SUMMARY

Relation

A relation R from set A to set B is a subset of the Cartesian product A × B obtained by describing a relationship between the first element x and the second element y of the ordered pairs in A × B.

Types of Relations

A relation R from A to A is also stated as a relation on A and it can be said that the relation in a set A is a subset of A × A. The empty set φ and A × A are two extreme relations. Below are the definitions of types of relations.

Empty Relation

If no element of A is related to any element of A, i.e. $R = \varphi \subset A \times A$**,** then the relation R in set A is called empty relation.

Universal Relation

If each element of A is related to every element of A, i.e. $R = A \times A$, then the relation R in set A is said to be universal relation.

Both the empty relation and the universal relation are called trivial relations.

A relation R in a set A is called-

Reflexive- if$(a, a) \in R$, for every $a \in A$,

Symmetric- if $(a, b) \in R$ implies that $(b, a) \in R$, for all $(a,b) \in R$,

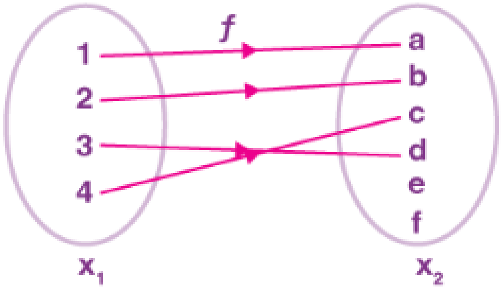
Transitive- if $(a,b) \in R$ and $(b,c) \in R$ implies that $(a,c) \in R$ for all . $(a,b), (b,c) \in R$

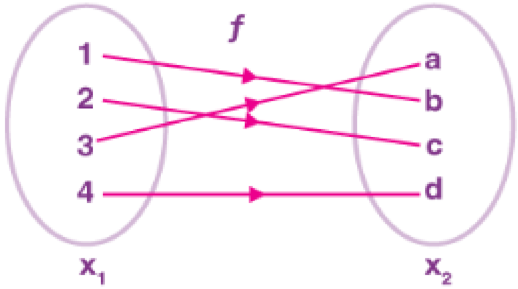
Equivalence Relation- A relation R in a set A is an equivalence relation if R is reflexive, symmetric and transitive.

Function
A relation f from set A to set B is said to be a function if every element of set A has exactly one image in set B.

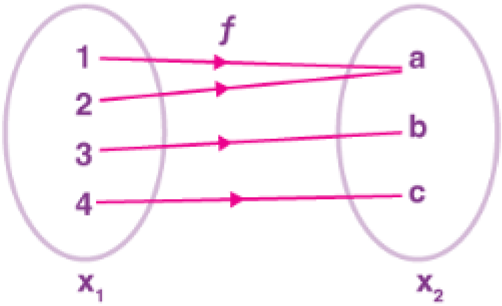
Types of Functions

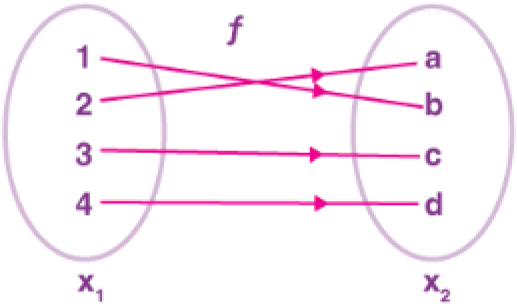
1. **One - one function:** A function $f : X \rightarrow Y$ is defined to be one-one (or injective), if the images of distinct elements of X under f are distinct, i.e., for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$. Otherwise, f is called many-one.



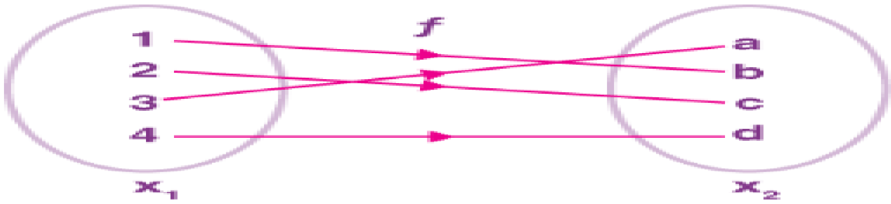


2. **Onto function:** A function f: X → Y is said to be onto (or surjective), if every element of Y is the image of some element of X under f, i.e. for every y ∈ Y, there exists an element x in X such that f(x) = y.





3. **One-one and Onto Function:** A function f: X → Y is said to be one-one and onto (or bijective), if f is both one-one and onto.



One-one and Onto function

Important points on Relations & Functions:

1. The total number of relations from a set A having 'm' elements to a set B having 'n' elements is 2^{mn}

2. The total number of relations in a set A having 'n' elements is 2^{n^2}

3. The number of reflexive relations in a set having n elements is $2^{(n^2-n)}$

4. The number of symmetric relations in a set having n elements is $2^{\frac{n(n+1)}{2}}$

5. The total number of functions from a set A having 'm' to another set B having 'n'elements is n^m.

6. The number of injective/one one function(s) from a set A having 'm' to another set B having 'n' elements with $n \geq m$ is ^nP_m . If (n <m) number of one one functions is zero.

7. The number of bijective functions in a set A having n elements is n!.

Solved Examples

Example 1: Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides a – b} is an equivalence relation.

Solution: R is reflexive, as 2 divides (a – a) for all a ∈ Z.
Further, if (a, b) ∈ R, then 2 divides a – b.
Therefore, 2 divides b – a.
Hence, (b, a) ∈ R so R is symmetric.

Similarly if (a, b) ∈ R and (b, c) ∈ R, then (a – b) and (b – c) are divisible by 2.
Now, a – c = (a – b) + (b – c) is even

From this, (a – c) is divisible by 2. (a,c) ∈ R
This shows that R is transitive.
Thus R is an equivalence relation in Z.

Example-2:-Check whether the relation S in the set of real numbers R defined by S = {(a, b): where a - b + √2 is an irrational number} is reflexive, symmetric or transitive.
Solution.
Reflexive:
Let a ∈ R.
Now, a-a + √2 = √2, which is an irrational number.
(a, a) ∈ S
Hence, S is reflexive.
Symmetric:
(√2,1) ∈ S as √2-1 + √2 = 2√2 - 1, which is an irrational number. But, (1,√2) ∉ S as 1 - √2 + √2 = 1, which is not an irrational number.
S is not symmetric.
Transitive:
(1,√3) ∈ S as 1-√3+√2 is an irrational number And, (√3,√2) ∈ S as √3-√2 + √2 = √3 is an irrational number But, (1,√2) ∉ S as 1-√2+√2=1 is not an irrational number S is not transitive.
Example 3:-Let A and B be sets. Show that f : A x B → B x A such that f(a, b) =(b,a) is one-one.
Solution:
Let (a, b), (c, d) ∈ A x B
f(a, b) = (b, a)
f(c, d) = (d, c)
Now, f(a, b) = f(c, d)
⇒(b, a) = (d, c)

b = d and a = c (Using equality of ordered pairs)

which can also be written as

a =c and b = d

(a, b) = (c, d) :

f(a, b) = f(c, d) ⇒ (a, b) = (c, d) f is one-one.

CASE STUDY BASED QUESTIONS

Q 4. Kriti and Kirat are two friends studying in class XII in a school at Chandigarh. While doing their mathematics project on Relations and Functions they have to collect the name of five metro cities and four cities other than metro cities of India; and present the name of cities in the form of sets. They have collected the name of cities and write in the form of sets given as follows: A={ five metro cities of India}= { Delhi, Mumbai, Bangalore, Calcutta, Pune} and B = {four non metro cities of India} = { Patiala, Agra, Jaipur, Ahmedabad}

Answer the following questions using the above information.

(i) Riya wants to know how many relations are possible from A to B

(ii) How many functions do exist from A to B.

(iii) Karan wants to know how many reflexive relation can be defined on set B.

Or How many symmetric relations can be defined on set A.

SOLUTION:-

n(A)=5

n(B)=4

Part(i)-The total number of relations from a set A having 'm' elements to a set B having 'n' elements is 2^{mn}

So number of relations from A to B is 2^{20}

Part(ii)-The total number of functions from a set A having 'm' to another set B having 'elements is n^m.

So number of functions from A to B is 4^5

Part (iii)- The number of reflexive relations in a set having n elements is $2^{(n^2 - n)}$

so number of reflexive relations in a set A having 5 elements is $2^{(5^2 - 5)}$=2^{20}

or

The number of symmetric relations in a set having n elements is $2^{\frac{n(n+1)}{2}}$

so number of symmetric relations in a set A having 5 elements is $2^{\frac{5(5+1)}{2}}$=2^{15}

MULTIPLE CHOICE QUESTIONS (1 MARK)

Q1.	Let f : R→R be a function defined by f(x) = [x] where [ ] denotes greatest integral Function then f is (a) Bijective function (b) one-one function (c) Many one function (d) onto function

Q2.	Given set A ={1, 2, 3} and a relation R = {(1, 2), (2, 1)}, the relation R will be (a) reflexive if (1, 1) is added (b) symmetric if (2, 3) is added (c) transitive if (1, 1) is added (d) symmetric
Q3.	Set A has 3 elements and the set B has 4 elements. Then the number of injective functions that can be defined from set A to set B is (a) 144 (b) 12 (c) 24 (d) 64
Q4.	Let A = {a, b }. Then number of one-one functions from A to A possible are (a) 2 (b) 4 (c) 1 (d) 3
Q5.	Consider set A = {1, 2, 3 } and the relation R= {(1, 2)}, then R is ? (a) Transitive relation (b) Equivalence relation (c) Reflexive relation (d) Symmetric relation
	ASSERTION AND REASON (Q6 AND Q7) Choose the correct answer out of the following choices. (a) Both A and R are true, and R is the correct explanation of A. (b) Both A and R are true, but R is not the correct explanation of A. (c) A is true, R is false. (d) A is false, R is true.
Q6	**Assertion (A):** A function f: R→R, given by $f(x) = x^2$ is one-one . **Reason (R):** The function f: X→Y is one – one if $x_1, x_2 \in X$, $f(x_1)=f(x_2) \Rightarrow x_1 = x_2$
Q7	**Assertion (A):** The relation R in the set {1,2,3} given by R = {(1,2),(2,1),(1,1)} is transitive. **Reason (R):**A relation R in set A is transitive if (a, b) ∈ R & (b , c) ∈ R ⇒ (a , c) ∈R, For every (a, b) , (b , c) ∈ R.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Q8.	Show that the function f: N→N given by f(x)=2x is one-one but not onto.
Q9.	Show that the greatest integer function f : R→R, given by f(x) = [x] is neither one-one nor onto.
Q10	Let A = {1,2,3}. Find the number of equivalence relations containing (1,2)
Q11	Let A ={1,2,3} and consider the relation R= {(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)} . Check whether R is reflexive ,symmetric and transitive or not .
Q12	Show that R defined as R = {(a, b): a ≤ b, a, b ∈R} is Reflexive and Transitive but not Symmetric

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

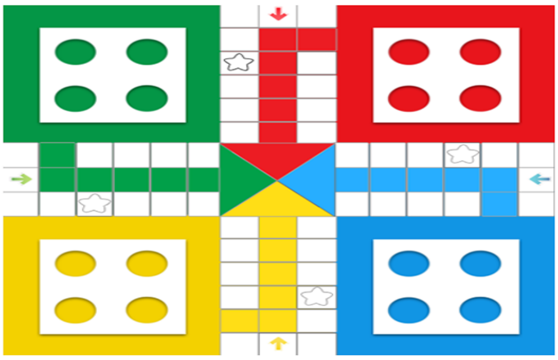
Q13	Show that the relation S in the set R of real numbers, defined as S = {(a,b) :a,b∈ R and $a \leq b^3$} is neither reflexive nor symmetric nor transitive.

Q14	Let A=R-{3} and B=R-{1}. Consider the function f: A→B defined by f(x) $=\frac{x-2}{x-3}$. Show that f isone-one and onto.
Q15	Show that the relation R on the set A = {x ϵZ : 0 ≤ x ≤ 12}, given by R = {(a,b): $\vert$a-b$\vert$ is a multiple of 4} is an equivalence relation.
Q16	Let L be the set of all lines in the XY plane defined as R= {(L1, L2): L1∥ L2}. Show that R is an equivalence relation.
Q17	Let R be the relation in $N \times N defined\ by$ $((a, b), (c, d))$ ϵ R If a+d=b+c *for (a,b), (c,d) in* $N \times N$. Prove that R is an equivalence relation.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

Q18	Show that the function f : A→A, A= {x: x∈R, -1 < x < 1} defined by f(x) $=\frac{X}{1+\vert X\vert}$, x ∈R is one- one and onto function.
Q19	Determine whether the relation R defined on the set of all real numbers as R ={(a, b) : a, b ∈R and a – b + √3 ∈S, where S is the set of all irrational numbers} is reflexive, symmetric and transitive or not.

CASE STUDY BASED QUESTIONS (4 Marks)

Q20	Aarti and Dev are playing Ludo by rolling the dice alternatively, it was observed that the possible outcomes of the die belongs to the set B={1,2,3,4,5,6}. Let A ={A,D} be the set of all players . Answer the following questions  (i) Let R:B→B defined as R ={(x,y): y is divisible by x} then R is Symmetric or not (ii) How many relations can be defined from A to B (iii) Let R1be a relation on B defined as R1={(1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)} then$R$1isReflexive, Symmetric and Transitive or not
Q21	In a Master chef competition final round, 3 chefS were selected and Judges assigned three dishes D={D1,D2,D3,} to the participants P={P1,P2, P3,} and asked them to prepare dishes as per the 

	following rules. Rule A: Everybody has to prepare exactly one dish Rule B:No two participant is allowed to prepare same dish Rule C: All the dish must be prepared in the competition Answer the following questions. (i) In how many ways all participants can choose a Dish following rule A only? Justify your answer (ii) In How many ways all participants can prepare exactly one dish as per rule C, Justify your answer

QUESTIONS FROM COMPETITIVE EXAM

Q22	Let a relation R on Nx N is defined as $(x_1,y_1)R\ (x_2,y_2)$ if and only if $x_1 \le x_2$ or $y_1 \le y_2$. Consider the two statements (i) R is reflexive but not symmetric (ii) R is transitive Then which of the following is true (a) Only (ii) is correct (b) Only (i) is correct (c)Both (i) and (ii) are correct (d) Neither (i) nor (ii) is correct **[JEE Mains 2024]**
Q23	If the function f : R→R defined by $f(x) = \|x\|(x-\sin x)$ then which of the following statement is true (a) f is one- one but not onto function (b) f is not one- one but onto function (c) f is both one- one and onto (d) f is neither one- one nor onto **[JEE Advanced 2020]**
Q24	The number of all onto functions from the set {1,2…,n} to itself is (a) 2^n (b) n^2 (c) n! (d) (2n)! [CUET 2023]
Q25	Let $f:(-\pi/2, \pi/2) \to R$ be given by $f(x)=[\log(\sec x + \tan x)]^3$ then (more then one correct) (a) f(x) is an odd function (b) f(x) is one-one function (c) f(x) is an onto function (d) f(x) is an even function **[JEE Advanced 2014]**
Q26	Which of the following is not an equivalence relation on Z (a) aRb⇔a+b is an even integer (b) aRb⇔a-b is an even integer (c) aRb⇔a=b

	(d) aRb⇔a≤b **[CUET 2023]**

ANSWERS :

Q1	**c**
Q2	d
Q3	**c**
Q4	**a**
Q5	**a**
Q6	**d**
Q7	**d**
Q9	Reflexive, Transitive but not symmetric
Q10	2
Q11	Reflexive yes, Symm no, Trans no
Q20	(i) Not symmetric (ii) 2^{12} (iii) Neither reflexive,nor symmetric not transitive
Q21	(i) 3^3=27 (ii)3!=6
Q22	B
Q23	C
Q24	C
Q25	a,b,c
Q26	D

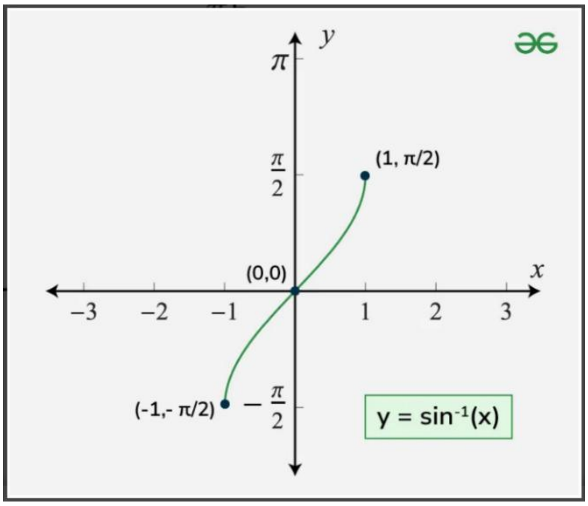
CHAPTER 2. INVERSE TRIGONOMETRIC FUNCTIONS

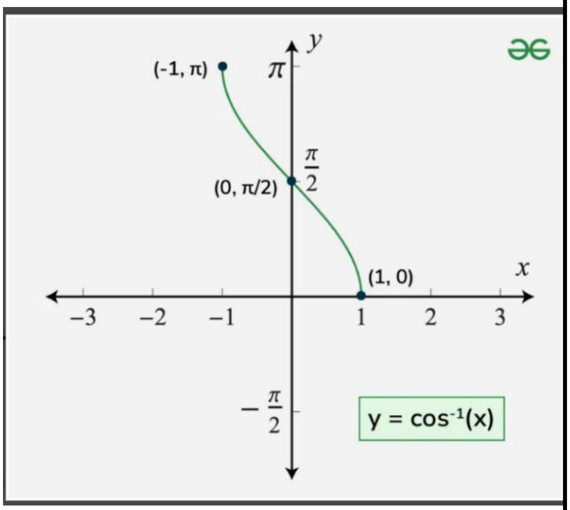
SUMMARY

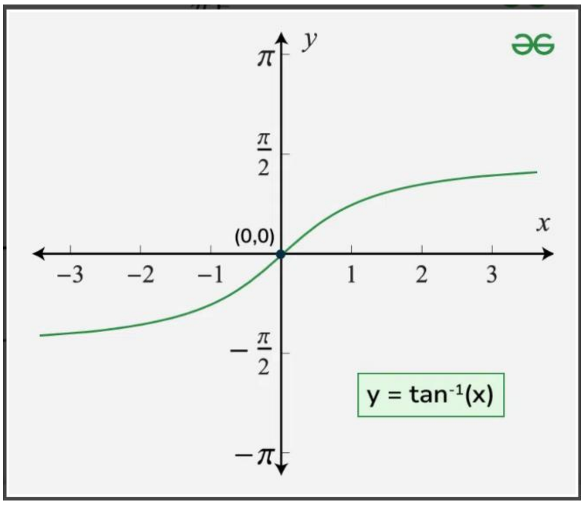
Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1} x$	$[-1,\ 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1,\ 1]$	$[0,\ \pi]$
$y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	R	$(0,\ \pi)$
$y = cosec^{-1} x$	$R-(-1,\ 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$
$y = \sec^{-1} x$	$R-(-1,\ 1)$	$[0,\ \pi]-\left\{\frac{\pi}{2}\right\}$

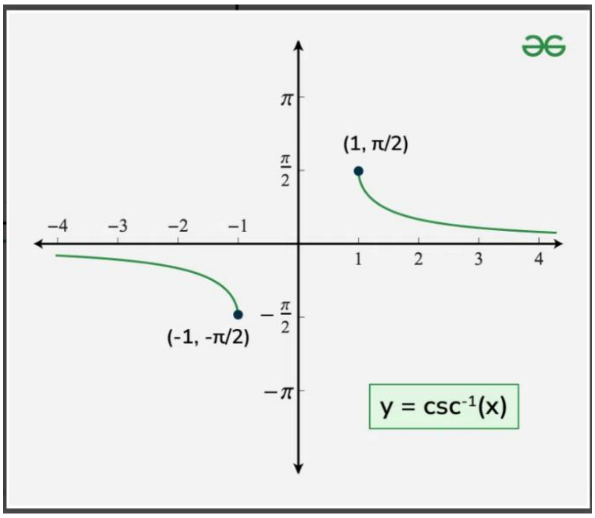
$\sin^{-1}\frac{1}{x} = cosec^{-1}x,\ x \in R-(-1,\ 1)$	$\sin^{-1}(-x) = -\sin^{-1}x,\ x \in [-1,\ 1]$
$\cos^{-1}\frac{1}{x} = \sec^{-1}x,\ x \in R-(-1,\ 1)$	$\tan^{-1}(-x) = -\tan^{-1}x,\ x \in R$
$\tan^{-1}\frac{1}{x} = \cot^{-1}x,\ x > 0$	$cosec^{-1}(-x) = -cosec^{-1}x,\ \lvert x\rvert \ge 1$
$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2},\ x \in [-1,\ 1]$	$\cos^{-1}(-x) = \pi - \cos^{-1}x,\ x \in [-1,\ 1]$
$cosec^{-1}x + \sec^{-1}x = \frac{\pi}{2},\ \lvert x\rvert \ge 1$	$\sec^{-1}(-x) = \pi - \sec^{-1}x,\ \lvert x\rvert \ge 1$
$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2},\ x \in R$	$\cot^{-1}(-x) = \pi - \cot^{-1}x,\ x \in R$

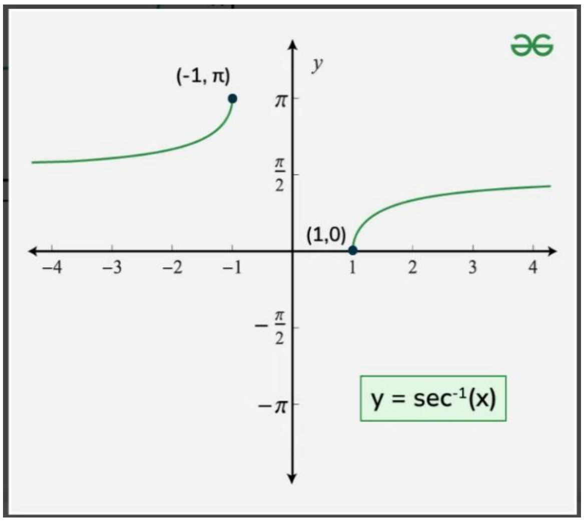
GRAPHS :

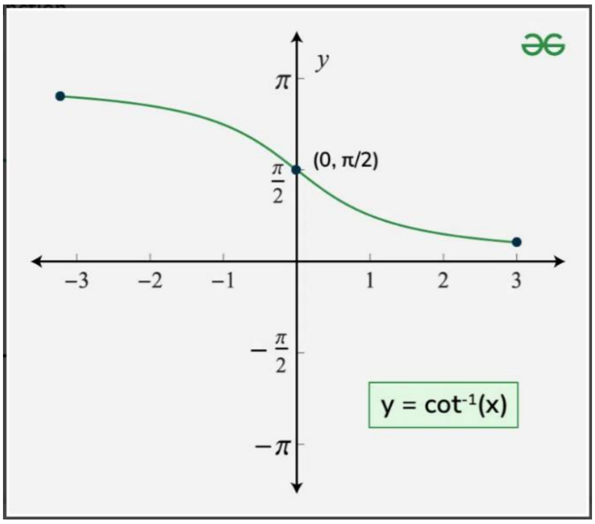












SOLVED EXAMPLES:

1. Find the principal value of $cos^{-1}\left\{cos\left(\frac{-7\pi}{6}\right)\right\}$

Ans. $cos^{-1}\left\{cos\left(\frac{-7\pi}{6}\right)\right\}$

$= cos^{-1}\left\{cos\left(\frac{7\pi}{6}\right)\right\}$ $\quad [\because\ cos(-x) = cos\,(x)]$

$= cos^{-1}\left\{cos\left(\pi + \frac{\pi}{6}\right)\right\}$ $\quad [\ \because\ \frac{7\pi}{6} \notin [0,\pi]\ ]$

$= cos^{-1}\left\{-\,cos\left(\frac{\pi}{6}\right)\right\}$

$= \pi - cos^{-1}\left\{cos\left(\frac{\pi}{6}\right)\right\}$ $\quad [\because\ cos^{-1}(-x) = \pi - cos^{-1}(x)]$

$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

2. Find the domain of the function $f(x) = cos^{-1}(x^2 - 4)$

Ans. Domain of $cos^{-1}x$ is $[-1,1]$.

$$\Rightarrow -1 \le x^2 - 4 \le 1$$

$$\Rightarrow 3 \le x^2 \le 5$$

$$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$$

3. Find the domain of $sin^{-1}(-x^2)$

Ans. Domain of $sin^{-1}x$ is $[-1,1]$.

$$\Rightarrow -1 \leq -x^2 \leq 1$$

$$\Rightarrow -1 \leq -x^2 \leq 0$$

$$\Rightarrow 0 \leq x^2 \leq 1$$

$$\Rightarrow x \in [-1, 1]$$

4. Find the principal value of $cos^{-1}\left(\frac{-1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

Ans. $cos^{-1}\left(\frac{-1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

$= \left[\pi - cos^{-1}\left(\frac{1}{2}\right)\right] + 2.\frac{\pi}{6}$ $\quad [\because cos^{-1}(-x) = \pi - cos^{-1}(x)]$

$= \left(\pi - \frac{\pi}{3}\right) + 2.\frac{\pi}{6}$

$= \pi$

5. Prove that $tan^{-1}\sqrt{x} = \frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0,1]$

Ans. Put $x = tan^2\theta \Rightarrow \theta = tan^{-1}\sqrt{x}$

R.H.S $= \frac{1}{2}cos^{-1}\left(\frac{1 - tan^2\theta}{1+tan^2\theta}\right)$

$= \frac{1}{2}cos^{-1}[cos\,(2\theta)]$

$= \frac{1}{2}(2\theta) = \theta = tan^{-1}\sqrt{x}$ = L.H.S

6. Evaluate : $sec^2\left\{tan^{-1}\left(\frac{1}{2}\right)\right\} + cosec^2\left\{cot^{-1}\left(\frac{1}{3}\right)\right\}$

Ans. $sec^2\left\{tan^{-1}\left(\frac{1}{2}\right)\right\} + cosec^2\left\{cot^{-1}\left(\frac{1}{3}\right)\right\}$

$= \left[1 + tan^2\left\{tan^{-1}\left(\frac{1}{2}\right)\right\}\right] + \left[1 + cot^2\left\{cot^{-1}\left(\frac{1}{3}\right)\right\}\right]$

$= \left[1 + \left(\frac{1}{2}\right)^2\right] + \left[1 + \left(\frac{1}{3}\right)^2\right] = \frac{85}{36}$

7. If $sin\left(sin^{-1}\left(\frac{1}{5}\right) + cos^{-1}x\right) = 1, then\ find\ the\ value\ of\ x.$

Ans. $sin\left(sin^{-1}\left(\frac{1}{5}\right) + cos^{-1}x\right) = 1$

$\Rightarrow sin^{-1}\left(\frac{1}{5}\right) + cos^{-1}x = sin^{-1}(1)$

$\Rightarrow sin^{-1}\left(\frac{1}{5}\right) + cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow sin^{-1}\left(\frac{1}{5}\right) = \frac{\pi}{2} - cos^{-1}x$ $\quad [\because sin^{-1}x + cos^{-1}x = \frac{\pi}{2}]$

$\Rightarrow sin^{-1}\left(\frac{1}{5}\right) = sin^{-1}(x)$

$\Rightarrow x = \frac{1}{5}$

8. Prove that $tan\left\{\frac{\pi}{4} + \frac{1}{2}cos^{-1}\left(\frac{a}{b}\right)\right\} + tan\left\{\frac{\pi}{4} - \frac{1}{2}cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$

Ans. L.H.S = $tan\left\{\frac{\pi}{4} + \frac{1}{2}cos^{-1}\left(\frac{a}{b}\right)\right\} + tan\left\{\frac{\pi}{4} - \frac{1}{2}cos^{-1}\left(\frac{a}{b}\right)\right\}$

Put $cos^{-1}\left(\frac{a}{b}\right) = \theta$

$\Rightarrow cos\,\theta = \frac{a}{b}$

L.H.S = $tan\left\{\frac{\pi}{4} + \frac{\theta}{2}\right\} + tan\left\{\frac{\pi}{4} - \frac{\theta}{2}\right\}$

$= \frac{tan\frac{\pi}{4} + tan\frac{\theta}{2}}{tan\frac{\pi}{4} - tan\frac{\theta}{2}} + \frac{tan\frac{\pi}{4} - tan\frac{\theta}{2}}{tan\frac{\pi}{4} + tan\frac{\theta}{2}}$

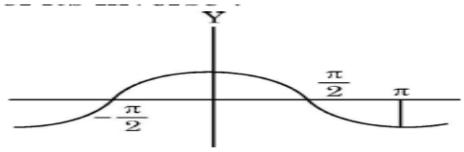
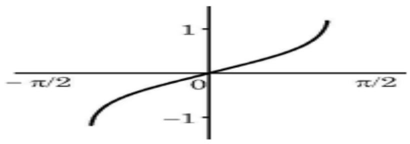
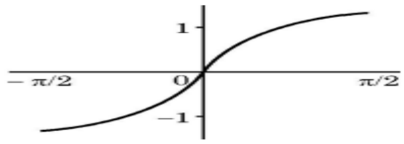
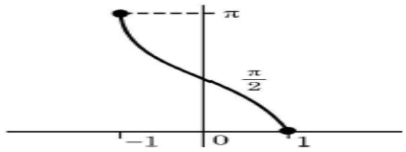
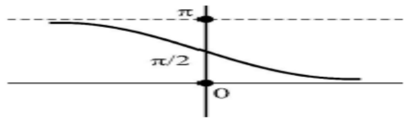
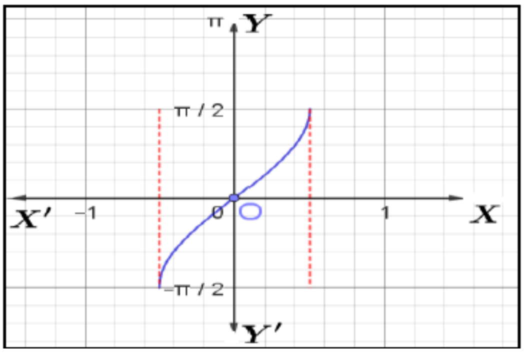
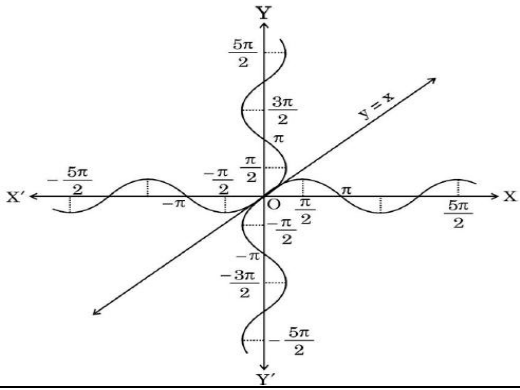
$= \frac{1 + tan\left(\frac{\theta}{2}\right)}{1 - tan\left(\frac{\theta}{2}\right)} + \frac{1 - tan\left(\frac{\theta}{2}\right)}{1 + tan\left(\frac{\theta}{2}\right)}$

$= \frac{\left\{1 + tan\left(\frac{\theta}{2}\right)\right\}^2 + \left\{1 - tan\left(\frac{\theta}{2}\right)\right\}^2}{\left(1 - tan\left(\frac{\theta}{2}\right)\right)\left(1 + tan\left(\frac{\theta}{2}\right)\right)}$

$= \frac{2\left(1 + tan^2\left(\frac{\theta}{2}\right)\right)}{1 - tan^2\left(\frac{\theta}{2}\right)}$ $\quad\left[\because\ \cos\theta = \frac{1 - tan^2\left(\frac{\theta}{2}\right)}{1 + tan^2\left(\frac{\theta}{2}\right)}\right]$

$= \frac{2}{cos\,\theta} = \frac{2}{a/b} = \frac{2b}{a}$ = R.H.S

S.NO	MCQ(1 mark each)
Q1.	The principal value branch of $sec^{-1}x$ is (a) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right] - \{0\}$ (b) $[0,\pi] - \left\{\frac{\pi}{2}\right\}$ (c) $(0,\pi)$ (d) $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$
Q2.	The principal value of $sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is (a) $\frac{-2\pi}{3}$ (b) $\frac{-\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{5\pi}{3}$
Q3.	The domain of the function $f(x) = cos^{-1}x + sin\,x$ is (a) $[-1,1]$ (b) $(-1,\ \pi + 1)$ (c)R (d) $\emptyset$
Q4.	The value of $tan^{-1}\left(tan\,\frac{7\pi}{6}\right)$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{7\pi}{6}$
Q5.	The domain of the function $sin^{-1}2x$ is (a) $[0,1]$ (b) $[-1,1]$ (c) $\left[\frac{-1}{2},\frac{1}{2}\right]$ (d) $[-2,2]$
Q6.	If $cos^{-1}x > sin^{-1}x$, then (a) $\frac{1}{\sqrt{2}} < x \leq 1$ (b) $0 \leq x < \frac{1}{\sqrt{2}}$ (c) $-1 \leq x < \frac{1}{\sqrt{2}}$ (d) $x > 0$
Q7.	The domain of the function defined by $f(x) = sin^{-1}\sqrt{(x-1)}$ is (a) $[1,2]$ (b) $[-1,1]$ (c) $[0,1]$ (d) None of these

Q8.	The graph of a trigonometric function is shown here. Which of the following will represent the graph of its inverse?  (A)  (B)  (C)  (D) 
Q9.	Identify the function shown in the graph? (a) $sin^{-1}x$ (b) $sin^{-1}2x$ (c) $sin^{-1}\frac{x}{2}$ (d) $2sin^{-1}x$ 
Q10	The value of the expression $(cos^{-1}x)^2$ is equal to (a) sec^2x (b) $\left(sec^{-1}\frac{1}{x}\right)^2$ (c) $cos^{-1}(x^2)$ (d) $\frac{\pi}{2} - (sin^{-1}x)^2$
Q11.	The following graph is a combination of (a) $y = sin^{-1}x$ and $y = cos^{-1}x$ (b) $y = cos^{-1}x$ and$y = cos\ x$ (c) $y = sin^{-1}x$ and $y = sin\ x$ (d) $y = cos^{-1}x$ and$y = sin\ x$ 
	Assertion Reason Questions: Directions for Q. No. 12 to 15. In the questions given below, there are two statements labelled as Assertion (A) and Reason (R). Read these statements and choose one correct answer from the given options (a) Both *A* and *R* are true and *R* is the correct explanation of *A* (b) Both A and R are true and R is not the correct explanation of A (c) *A* is true but *R* is false (d) *A* is false, but *R* is true

Q12.	Assertion (A): Set of values of $sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set. Reason(R): $sec^{-1}x$ is defined for $R - (-1, 1)$
Q13.	Assertion (A):The range of the function $f(x) = 2sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1,1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$. Reason(R):The range of the principal value branch of $sin^{-1}x$ is $[0, \pi]$.
Q14.	Assertion (A): Maximum value of $(cos^{-1}x)^2$ $is\ \pi^2$. Reason(R): Range of the principal value branch of $cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.
Q15.	Assertion (A):Range of $[sin^{-1}x + cos^{-1}x]$ is $[0, \pi]$ Reason(R):Principal value of $sin^{-1}x$ has range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

2 Marks Questions	
Q16.	Find the principal value of $cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$+$sin^{-1}\left\{sin\left(\frac{17\pi}{6}\right)\right\}$+$tan^{-1}(-1)$
Q17.	Find the domain of $cos^{-1}[x]$, where $[.]$ denotes the greatest integer function.
Q18.	Evaluate :$tan^{-1}\left[2\ sin\ \left\{2cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right]$
Q19.	Simplify :$sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$
Q20.	Find the value of $tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + tan^{-1}\left\{sin\left(\frac{-\pi}{2}\right)\right\}$
Q21.	Express $tan^{-1}\left(\frac{cos\ x}{1 - sin\ x}\right)$, $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.
Q22.	Find the value of k if $sin^{-1}\left[k\ tan\ \left\{2cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\right] = \frac{\pi}{3}$
Q23.	Evaluate :$sin^{-1}\left\{sin\left(\frac{3\pi}{4}\right)\right\} + cos^{-1}\left\{cos\left(\frac{3\pi}{4}\right)\right\} + tan^{-1}(1)$
Q24.	Prove that $tan^{-1}\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}cos^{-1}x$ where $\frac{-1}{\sqrt{2}} \le x \le 1$

Questions from competitive exams **MCQ with 1 correct answer**		**Reference**
Q25.	Using the principal values of the inverse trigonometric functions, the sum of the maximum and the minimum values of $16[(sec^{-1}x)^2 + (cosec^{-1}x)^2]$ is: (a) $18\ \pi^2$ (b) $31\ \pi^2$ (c) $24\ \pi^2$ (d) $22\ \pi^2$	JEE Main 22/01/25 Shift I
Q26.	If $y = cos\left\{\frac{\pi}{3} + cos^{-1}\left(\frac{x}{2}\right)\right\}$, then $(x-y)^2 + 3y^2 is\ equal\ to$ (a) 2 (b) 3 (c) 5 (d) 7	JEE Main 02/04/25

Q27.	What is $\sqrt{15 + cot^2\left(\frac{\pi}{4} - 2cot^{-1}3\right)}$ equal to? (a) 1 (b) 7 (c) 8 (d) 16	NDA I 2024
Q28.	What is $tan^{-1}\left(\frac{a}{b}\right) - tan^{-1}\left(\frac{a-b}{a+b}\right)$ equal to (a) $\frac{-\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $tan^{-1}\left(\frac{a^2 - b^2}{a^2 + b^2}\right)$ (d) $tan^{-1}\left(\frac{2ab}{a^2 + b^2}\right)$	NDA I 2024

ANSWERS:

1. b	2. b	3. a	4. a	5. c	6. c	7. a	8. c	9. b	10.b
11. c	12. a	13. c	14. c	15. d	16. $\frac{\pi}{12}$	17. $[-1,2)$	18. $\frac{\pi}{12}$	19. $tan^{-1}x$	20. $\frac{-\pi}{12}$
21. $\left(\frac{\pi}{4} + \frac{x}{2}\right)$	22. k=$\frac{1}{2}$	23. $\frac{5\pi}{4}$	24. Proof	25. d	26. b	27. c	28. b		

CHAPTER 3 (MATRICES)

Summary

A matrix is a rectangular array of $m \times n$ numbers arranged in m rows and n columns.

* **Row Matrix** : A matrix which has one row is called row matrix. $A = [a_{ij}]_{1\times n}$

* **Column Matrix** : A matrix which has one column is called column matrix. $A = [a_{ij}]_{m\times 1}$.

* **Square Matrix**: A matrix in which number of rows are equal to number of columns is called a square matrix $A = [a_{ij}]_{m\times m}$

* **Diagonal Matrix** : A square matrix is called a Diagonal Matrix if all the elements, except the diagonal elements are zero. $A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$,$i \neq j$. $a_{ij} \neq 0$,$i = j$.

* **Scalar Matrix**: A square matrix is called scalar matrix it all the elements, except diagonal elements are zero and diagonal elements are non-zero and equal. $A = [a_{ij}]_{n\times n}$,

where $a_{ij} = 0$ if $i \neq j$. and $a_{ij} = k$, $i = j$.

* **Identity or Unit Matrix** : A square matrix in which all the non-diagonal elements are zero and diagonalelements are unity is called identity or unit matrix.

* **Null Matrices** : A matrix in which all element are zero.

* **Equal Matrices** : Two matrices are said to be equal if they have same order and all their corresponding elements are equal.

* **Sum of two Matrices :**If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two matrices of the same order, say $m \times n$, then, the sum of the two matrices A and B is defined as a matrix
$C = [c_{ij}]_{m\times n}$, where $c_{ij} = a_{ij} + b_{ij}$, for all possible values of i and j.

* **Multiplication of a matrix by a number :**
$kA = k\,[a_{ij}]_{m\times n} = [k\,(a_{ij})]_{m\times n}$

* **Negative of a matrix** is denoted by –A. We define $-A = (-1)A$.

* **Difference of matrices :**If $A = [a_{ij}]$, $B = [b_{ij}]$ are two matrices of the same order, say $m \times n$, then difference A – B is defined as a matrix $D = [d_{ij}]$,
where $d_{ij} = a_{ij} - b_{ij}$,for all value of i and j.

* **Properties of scalar multiplication of a matrix :**If A and B be two matrices of the same order, and k and l are scalars, then
(i) $k(A+B) = kA + kB$,(ii) $(k+l)A = kA + lA$

***Product of matrices:** If A& B are two matrices, then product AB is defined, if number of column of A = number of rows of B.

i.e. $A = [a_{ij}]_{m\times n}$, $B = [b_{jk}]_{n\times p}$ then $AB = [C_{ik}]_{m\times p}$,where $C_{ik} = \sum_{j=1}^{n} a_{ij}.b_{jk}$

Properties of multiplication of matrices :

(i) Product of matrices is not commutative. i.e. $AB \neq BA$.

(ii) Product of matrices is associative. i.e. $A(BC) = (AB)C$

(iii) Product of matrices is distributive over addition i.e.(A+B) C = AC + BC

(iv) For every square matrix A, there exist an identity matrix of same order such that IA = AI = A.

* **Transpose of matrix**: If A is the given matrix, then the matrix obtained by interchanging the rows and columns is called the transpose of a matrix.

* **Properties of Transpose**:

If A & B are matrices such that their sum & product are defined, then

(i) $(A^T)^T = A$ (ii) $(A+B)^T = A^T + B^T$ (iii) $(KA^T) = K.A^T$ where K is a scalar.

(iv) $(AB)^T = B^T A^T$ (v) $(ABC)^T = C^T B^T A^T$.

* **Symmetric Matrix**: A square matrix is said to be symmetric if $A = A^T$

* **Skew symmetric Matrix** : A square matrix is said to be skew symmetric if $A^T = -A$.

SOLVED EXAMPLES	
1	The sum of a matrix and its transpose is $\begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$ .Find one such matrix for which this hold true. Solution: A= $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ be the required matrix. Then $A^T = \begin{bmatrix} x & z \\ y & w \end{bmatrix}$ $A + A^T = \begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$ $\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} x & z \\ y & w \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$ $\begin{bmatrix} 2x & y+z \\ y+z & 2w \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$ x=3, w =2, y =1, z= -2 A= $\begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$
2	Find matrix A such that . $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ A = $\begin{bmatrix} -1 & 8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ Solution: Given $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ A = $\begin{bmatrix} -1 & 8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ Since the product is 3×2 matrix and the prefactor is 3×2 matrix, the matrix A must be of other 2×2 Let A= $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ $\begin{bmatrix} 2x - z & 2y - w \\ x & y \\ -3x + 4z & -3y + 4w \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}, 2x - z = -1, 2y - w = -8, x = 1, y = -2, -3y + 4w = 22$

<table>
<tr><td></td><td>We get x=1,y=-2,z=3,w=4
Hence A= $\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$</td></tr>
<tr><td>3</td><td>There are three families A, B, C. the number of men, woman and children in these families are as under

	Men	Women	Children
Family A	2	3	1
Family B	2	1	3
Faimly C	4	2	6

Daily expenses of men , women and children are Rs 200, Rs 150 and Rs 200 respectively.
Using metrix multiplication, calculate the daily expenses of each family.</td></tr>
<tr><td></td><td>Solution: The number of men, women and children in families A , B and C can be represented by 3×3 matrix
$X= \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{bmatrix}$
Daily expenses $Y= \begin{bmatrix} 200 \\ 150 \\ 200 \end{bmatrix}$
Daily expenses of each family is given by matrix multiplication
$XY = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{bmatrix} \begin{bmatrix} 200 \\ 150 \\ 200 \end{bmatrix}$
$XY= \begin{bmatrix} 2 \times 200 + 3 \times 150 + 1 \times 200 \\ 2 \times 200 + 1 \times 150 + 3 \times 200 \\ 4 \times 200 + 2 \times 150 + 6 \times 200 \end{bmatrix}$
$XY= \begin{bmatrix} 1050 \\ 1150 \\ 2300 \end{bmatrix}$
Daily expenses of family A = Rs 1050
Daily expenses of family B=Rs 1150
Daily expenses of family C=Rs2300</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Q.NO.</td><td>MCQ/A-R QUESTIONS (1MARK)</td></tr>
<tr><td>1</td><td>The number of all scalar matrices of order 3, with entry -1 ,0 or 1 is
(a)1 (b)3 (c)2 (d) 3^9</td></tr>
<tr><td>2</td><td>Matrix A= $\begin{bmatrix} 0 & 0 & -5 \\ 0 & -5 & 0 \\ -5 & 0 & 0 \end{bmatrix}$ $is\ a$
(a)Square matrix (b) Diagonal matrix (c) Unit matrix (d) Scalar matrix</td></tr>
<tr><td>3</td><td>If A&B are two non zero square matrices of same order such that$(A+B)^2 = A^2 + B^2$, then
(a)AB=0 (b) AB= -BA (c) BA=0 (d) AB= BA</td></tr>
<tr><td>4</td><td>If the matrix A= $\begin{bmatrix} 0 & x+y & 1 \\ 3 & z & 2 \\ x-y & -2 & 0 \end{bmatrix}$ is a skew symmetric matrix, then
(a)x=2,y=1,z=0 (b) x=2,y=2,z=0 (c) x= -2,y = -1, z=0 (d) x=-2,y=-1,z=-1</td></tr>
<tr><td>5</td><td>A matrix A=[1 2 3],then the matrix AA’(Where A’ is a transpose of A) is
(a) 14. (b) [14] (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$</td></tr>
</table>

6	If $\begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\3\\2\end{bmatrix}$ Then the value of (2x+ y- z) is (a)1. (b)3. (c)2. (d) 5
7	If matrices A&B are of order 1×3 and 3×1 respectively, then the order of A'B' is a) 1×1. b) 3×1. c) 1×3. d) 3×3
8	If the sum of all the elements of 3×3 scalar matrix is 9, then the product of all its elements is (a)27. (b)0. (c) 9. (d) 729
	ASSERTION - REASON TYPE QUESTIONS: **Directions**: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below. (a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion. (b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion (c) Assertion is correct, Reason is incorrect (d) Assertion is incorrect, Reason is correct.
9	Assertion: The matrix A=$\begin{bmatrix}2 & 3\\6 & 4\end{bmatrix}$ Cannot be expressed as the sum of a symmetric and skew-symmetric matrix. Reason: The matrix A=$\begin{bmatrix}2 & 3\\6 & 4\end{bmatrix}$ is neither a symmetric matrix nor skew-symmetric.
10	Assertion: If order of A is 4×5 and order of B is 3×4 then AB is not define but BA is a 3×5 matrix. Reason: AB is defined if number of columns in A is equal to number of rows in B.
	2MARKS QUESTIONS
11	Find xy, if $\begin{bmatrix}x+y & 7\\9 & x-y\end{bmatrix}=\begin{bmatrix}2 & 7\\9 & 4\end{bmatrix}$
12	If A= $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$,show that (A-2I) (A-3I) = 0
13	If the matrix $\begin{bmatrix}-2 & x-y & 5\\1 & 0 & 4\\x+y & z & 7\end{bmatrix}$ is a symmetric matrix, find the value of x-2y+z.
14	If $A^T = \begin{bmatrix}-2 & 3\\1 & 2\end{bmatrix}$ and B = $\begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}$, find $(A+2B)^T$
	3MARKS QUESTIONS
15	If matrix A= $\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$,show that f(A)=0,where f(x)= x^2-2x-3.
16	Express the matrix as the sum of symmetric and skew-symmetric matrix. $\begin{bmatrix}2 & 4 & -6\\7 & 3 & 5\\1 & -2 & 4\end{bmatrix}$
	5 MARKS QUESTIONS
17	If A =$\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}$,Then show that $A^3-6A^2+7A+2I=0$
	CASE STUDY QUESTIONS
18	A trust fund has Rs. 35,000 that must invested in two different types of bonds, say X&Y. the first bond pays 10% interest.p.a which will be given to an old age home and second

	one pays 8% interest P.a. which will be given to WWA (woman welfare association). Let A be a 1×2 matrix and B be a 2×1matrix, representing the investment and interest rate on each bond respectively. Based on the above information, answer the following questions: i)Write the matrix which represent the total amount of interest received on both bonds. ii) What is the total amount of interest received on both the bonds if Rs. 15,000 is invested in bond X? iii) What amount should be interested the bond X&Y we do get an annual total interest of Rs. 32,00? or Calculate the amount of investment in bond Y if the amount of interest given to old age home is Rs. 500 .
	COMPETITIVE EXAMS QUESTIONS
1.	Which of the following statement is not correct? a)A row matrix has only one row. b) A diagonal matrix has all diagonal elements equal to 0. c) A symmetric matrix is a square matrix satisfying certain conditions. d) A skew symmetric matrix has all diagonal element equal to 0. **CUET2023**
2	If A= $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ Such that B=2A,then values of x and y are respectively: a)1,1b)1,-1 c) -1,-1 d)-1,1 **CUET 2023**
3	The matrix A has x rows and x + 5 columns. The matrix B has y rows and 11 y columns. Both AB and BA exist. What are the values of x and y respectively? i) 8 and 3 ii) 3 and 4 iii) 3 and 8 iv) 8 and 8. **NDA 2017**
4	If A = $\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, Then which one of the following is correct $i) A^2 = -2A$ ii)$A^2 = -4A$ $iii\ A^2 = -3\ iv) A^2 = 4A$
5	If A = $\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ Then what is $A^2 - 4A\ is$ equal to i) -5I. ii) -I. iii)I iv) 5I **NDA 2025**
	ANSWERS MCQ'S 1) c 2) a 3) b 4)c 5) b 6) d 7)d 8)a 9)d 10) a 11) -3 13 x-2y+z = 3 14)$\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$ 16)$\begin{bmatrix} 2 & \frac{11}{2} & -\frac{5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ -\frac{5}{2} & \frac{3}{2} & 4 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{3}{2} & -\frac{7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ 0 & 0 & 3 \end{bmatrix}$ 18.i) Will give the total amount of interest received on both bonds ii) Rs. 3100 iii) Rs 3200 or Rs 30000. **Competitive exams answer**: 1 c) 2) b 3) c 4) b5)d

CHAPTER-4 DETERMINANTS

Summary

* **Every square matrix** can be associated with a number (real or complex), is known as **determinant** .
* For a square matrix A, its determinant is denoted by det (A) or |A|
* |A| denotes determinant value not modules of A.
* Matrix being an arrangement of number has no value, but determinant has a fixed value. Determinants are not defined for non-square matrix.
* Determinant of square matrix A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ of order 2 is given by

  |A| = $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\, a_{22} - a_{21}\, a_{12}$
* Determinant of square matrix A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ of order 3 is given by

  |A| = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

This is know as the expansion of |A| along first row.

* |A| can be expanded along any of its row or column.
* **Singular Matrix:** |A| = 0
* **Non Singular Matrix:** |A| ≠ 0
* **Minors of a Matrix :** Minor of an element aij in a matrix is the determinant obtained by deleting i^{th} row and j^{th} column and denoted by Mij

  For example : If A = $\begin{bmatrix} 1 & 3 & 5 \\ -2 & 4 & 6 \\ -5 & -1 & -4 \end{bmatrix}$

  Then

  $M_{11} = \begin{vmatrix} 4 & 6 \\ -1 & -4 \end{vmatrix} = -10$, $M_{12} = \begin{vmatrix} -2 & 6 \\ -5 & -4 \end{vmatrix} = 38$ and so on.

* **Cofactors of a Matrix :** If Mij is the minor of element aij and Aij is cofactor of aij, then

$$A_{ij} = (-1)^{i+j} M_{ij}$$

* **Area of a Triangle:** Area of a Triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(since area is positive quantity, take absolute value of determinant for area)

If area of triangle is zero, then the points will be **collinear**.

* **Adjoint of Matrix :** If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Then $\text{adj}A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

Where A_{ij} is the cofactor of a_{ij}.

* Value of Determinants in Terms of cofactors :

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$= a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$ or $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$

* **Properties of Adjoint of a Matrix :** for two matrix A and B of order 'n'

- $\text{adj}(A') = (\text{adj } A)'$
- $\text{adj}(kA) = k^{n-1}(\text{adj } A)$, $k \in R$
- $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj}(\text{adj } A) = |A|^{n-2} . A$
- $|\text{adj}(\text{adj } A)| = |A|^{(n-1)2}$
- $A(\text{adj } A) = (\text{adj } A)A = |A| I_n$ (I_n is the identity matrix of order 'n')

* **Inverse of a Matrix:** for non-singular square matrix A and B, if

$AB = BA = I$ (I – identity matrix)

Then $A^{-1} = B$ and $B^{-1} = A$

If A is a non-singular matrix (i,e, $|A| \neq 0$)

Then $\mathbf{A^{-1} = \frac{1}{|A|}}$ **adjA**

* **Properties of Invesse of a Matrix :** for two square invertible matrices A and B of same order

- $(A^{-1})^{-1} = A$
- $(AB)^{-1} = B^{-1} A^{-1}$

- $(A')^{-1} = (A^{-1})'$
- $|A^{-1}| = |A|^{-1} = \frac{1}{|A|}$
- $AA^{-1} = A^{-1}A = I$
- $(KA)^{-1} = \frac{1}{K} A^{-1}$, $k \neq 0$

* **Some Important fact :**

- We cann't equate the corresponding elements of equal determinants like matrices .
- If A is a skew symmetric matrix of odd order, then $|A| = 0$
- The determinant of skew symmetric matrix of even order is perfect square.

* **Consistent and Inconsistent System :** A system of equation is said to be consistent , if its solution (one or more) exists.

If its solution doesn't exist then it is called inconsistent.

* **Solution of System of Linear Equations using inverse of a matrix:**

Let the system of linear equations be

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

It can be written in matrix form as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Or $AX = B$

Where, $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Now following cases arises :

Case-I If A is non-singular matrix ($|A| \neq 0$),

Then $\mathbf{X = A^{-1} B}$

Cases- II If $|A| = 0$

In this case, we calculate (adj A) B,

if (adj A) B $\neq$ O (here, O = zero matrix)

Then solution doesn't exist.

If (adj A)B = O

Then system of equation may be consistent or inconsistent.

Solved Examples

1. Find the values of k if area of triangle is 4 sq. units and vertices are (-2,0), (0,4), (0, k).

Sol. Given the vertices of triangle are (-2,0), (0,4), (0, k).

$\therefore$ Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\Rightarrow \frac{1}{2}\begin{bmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{bmatrix} = \pm 4 \Rightarrow \frac{1}{2}[-2(4-k)+0+0] =$

$\pm 4 \Rightarrow -8 + 2k =$

$\pm 8 \Rightarrow 2k = \pm 8 + 8 \Rightarrow$

So $2k = +8+8 \Rightarrow 2k = 16 \Rightarrow k = 8$

and $2k = +8+8 \Rightarrow 2k = 0 \Rightarrow k = 0$

$\therefore k = 8 \; and \; k = 0$

2. Without expanding the determinant, prove that

$$\begin{vmatrix} \mathbf{0} & \mathbf{2023} & \mathbf{-2021} \\ \mathbf{-2023} & \mathbf{0} & \mathbf{-2022} \\ \mathbf{2021} & \mathbf{2022} & \mathbf{0} \end{vmatrix} = \mathbf{0}$$

Sol. Let A = $\begin{bmatrix} 0 & 2023 & -2021 \\ -2023 & 0 & -2022 \\ 2021 & 2022 & 0 \end{bmatrix}$

Here A′ = -A

$\Rightarrow |A'| = |-A| = (-1)^3|A|$ [$|kA| = k^n|A|$ for matrix of order n]

$\Rightarrow |A| = -|A|$

$\Rightarrow |A| = 0$

$\Rightarrow \begin{vmatrix} 0 & 2023 & -2021 \\ -2023 & 0 & -2022 \\ 2021 & 2022 & 0 \end{vmatrix} = 0$

3. Solve the following system of equations by matrix method:

$$\boldsymbol{x + 2y + 3z = 6; \quad 2x - y + z = 2; \quad 3x + 2y - 2z = 3}$$

Sol. Given system of equations

$$x + 2y + 3z = 6$$

$$2x - y + z = 2$$

$$3x + 2y - 2z = 3$$

We can write this in matrix form as

	$\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 1\\ 3 & 2 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}6\\ 2\\ 3\end{bmatrix}$ $\downarrow \quad \downarrow \quad \downarrow$ A X B $\Rightarrow$ AX = B $\Rightarrow$ $X = A^{-1}B$ ....*(i)* Now, $A = \begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 1\\ 3 & 2 & -2\end{bmatrix}$ $\Rightarrow$ $\lvert A\rvert = (1 \times 0) - (2 \times -7) + (3 \times 7) = 14 + 21 = 35 \neq 0$ $\therefore$ A^{-1} exists Its co-factors are $A_{11} = 0$, $A_{21} = 10$, $A_{31} = 5$ $A_{12} = 7$, $A_{22} = -11$, $A_{32} = 5$ $A_{13} = 7$, $A_{23} = 4$, $A_{33} = -5$ $\therefore$ $C = \begin{bmatrix}0 & 7 & 7\\ 10 & -11 & 4\\ 5 & 5 & -5\end{bmatrix}$ $\Rightarrow$ adj $A = C^T = \begin{bmatrix}0 & 10 & 5\\ 7 & -11 & 5\\ 7 & 4 & -5\end{bmatrix}$ $\therefore$ $A^{-1} = \frac{\text{adj}A}{\lvert A\rvert} = \frac{1}{35}\begin{bmatrix}0 & 10 & 5\\ 7 & -11 & 5\\ 7 & 4 & -5\end{bmatrix}$ Putting in (i), we have $X = A^{-1}.B = \frac{1}{35}\begin{bmatrix}0 & 10 & 5\\ 7 & -11 & 5\\ 7 & 4 & -5\end{bmatrix}\begin{bmatrix}6\\ 2\\ 3\end{bmatrix} = \frac{1}{35}\begin{bmatrix}0+20+15\\ 42-22+15\\ 42+8-15\end{bmatrix} = \frac{1}{35}\begin{bmatrix}35\\ 35\\ 35\end{bmatrix}$ $\Rightarrow$ $x = 1,\ y = 1, z = 1$
4. Sol.	**If A =** $\begin{bmatrix}\mathbf{-3} & \mathbf{-2} & \mathbf{-4}\\ \mathbf{2} & \mathbf{1} & \mathbf{2}\\ \mathbf{2} & \mathbf{1} & \mathbf{3}\end{bmatrix}$ **, B =** $\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{0}\\ \mathbf{-2} & \mathbf{-1} & \mathbf{-2}\\ \mathbf{0} & \mathbf{-1} & \mathbf{1}\end{bmatrix}$ **, then find *AB* and use it to solve the following system of equations :** $\boldsymbol{x - 2y = 3}$ $\boldsymbol{2x - y - z = 2}$ $\boldsymbol{-2y + z = 3}$ Given, $A = \begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$ **,** $B = \begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$

	$\therefore \quad AB = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ $\begin{bmatrix} -3+4+0 & -6+2+4 & 0+4-4 \\ 2-2+0 & 4-1-2 & 0-2+2 \\ 2-2+0 & 4-1-3 & 0-2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\Rightarrow \quad AB = I$ By definition, $B^{-1} = A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ The given system of equations is $x - 2y = 3$ or $x - 2y + 0z = 3$ $2x - y - z = 2$ $-2y + z = 3$ or $0x - 2y + z = 3$ The above system of equations can be written in the matrix form as $\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ *i.e.,* $B'X = C$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ Pre-multiplying both sides by $(B')^{-1}$, we get $(B')^{-1} B' X = (B')^{-1} C$ $\Rightarrow \quad [(B')^{-1} B'] X = (B^{-1})' C$ $\Rightarrow \quad I X = (A)'C$ $\Rightarrow \quad X = A'C$ $\Rightarrow \quad X = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}' \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ $\Rightarrow \quad X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$ $\Rightarrow \quad X = \begin{bmatrix} -9+4+6 \\ -6+2+3 \\ -12+4+9 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\Rightarrow \quad x = 1,\ y = -1,\ z = 1$ Hence, the required solution is $x = 1,\ y = -1,\ z = 1$.
	MCQs
1.	$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ then the possible value(s) of 'x' is/are (a) 3 (b) √3 (c) -√3 (d) √3,-√3

2.	The area of a triangle with vertices (-3, 0),(3, 0) and (0, k) is 9 sq. units. The value of k will be (a) 9 (b) 3 (c) -9 (d) 6
3.	If f(x)= $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$,then (a) f(a) = 0 (b) f(b) = 0 (c) f(0) = 0 (d) f(1) = 0
4.	If |A| =|kA|, where A is a square matrix of order 2, then sum of all possible values of k is (a) 1 (b) -1 (c) 2 (d) 0
5.	If A is a square matrix of order 3 such that det (A) = 9, then det($9A^{-1}$) is equal to (a) 9 (b) 9^2 (c) 9^3 (d) 9^4
6.	Let A be a skew-symmetric matrix of order 3. If |A| = x, then $(2026)^x$ is equal to: (a) 2026 (b) 1 /2026 (c) $(2026)^2$ (d) 1
7.	If A is a square matrix of order 2 such that det (A) = 4, then det(4adj A) is equal to: (a) 16 (b) 64 (c) 256 (d) 512
8.	If A is a square matrix of order 3 such that the value of | adj A | = 8, then the value of |A′| is: (a) √2 (b) -√2 (c) 8 (d) 2√2
9.	If A is square matrix of order 3, |A′| = - 3, then |AA′ | is equal to: (a) 9 (b)-9 (c) 3 (d)-3
10.	S is a diagonal matrix with all the principal diagonal elements equal to p (p≠0). What is the determinant of S^{-1}? (a) 1/p (b) $1/p^3$ (c) p^3 (d)I/p^3
11.	If A = $\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{vmatrix}$, then the value of det (A^{-1}) is: (a) -1 (b) 1 (c) 0 (d) 2
12.	Given that A^{-1} = $\frac{1}{7}\begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix}$then matrix A is: (a) $7\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$ (c) $\frac{1}{7}\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$ (d) $\frac{1}{49}\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$
13.	If A and B are square matrices of the same order 3, such that |A| = 2 and AB = 2I, then the value of |B|: (a) 2 (b) 3 (c) 4 (d) 1

14.	If for any 2 x 2 square matrix A , A adj(A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$ then the value of |A| : (a) 64 (b) 8 (c) 4 (d) 32
15.	If A is a 3 x 3 invertible matrix, then what will be the value of k if det (A^{-1}) = (det A)^k? (a) k = + 1 (b) k = ± 1 (c) k = -1 (d) k = 2
2 Marks Questions	
16.	If area of triangle is 8 sq. units with vertices (1, 3), (k, 2) and (6, 0). Then, find the values of k.
17.	Find the values of x for which (x, 4), (2, 2) and (10, x) are collinear.
18.	If the matrix A =$\begin{bmatrix} 6 & x & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{bmatrix}$is a singular matrix then find the value of x.
5 Marks Questions	
19.	The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$; (where, 0≤ $x \leq 14$ and a,b,c ∈ R and a ≠ 0) with respect to a XY-coordinate system in the vertical plane. The ball passes through the points (2, 15), (4, 25) and (14, 15). Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball.
20.	Given A=$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$and B=$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$. Find AB, hence solve the following system of linear equations: x − y + z = 4 , x − 2y - 2z = 9 , 2x + y + 3z = 1
21.	Given A=$\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$and B=$\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$. Find AB, hence solve the following system of linear equations: x − 2y = 3 , 2x −y - z = 2 , -2y + z = 3
22.	If A=$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ find A^{-1}, hence solve the following system of linear equations: x + 2z = 7 , x +y+ z = 6 , 3x + y + z = 12

Case Study Questions	
23.	A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions: (a)The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club. (b)The number of students in Music club should be 20 more than half the number of students in Sports club. (c)The total number of students to be allocated in all three clubs are 180. Based on above answer the following questions:
(i)	Convert the above situation into a matrix equation of the form AX=B
(ii)	Find the number of students allocated to different clubs, using matrix method.
Competitive Exam Questions	
24.	Which one of the following options is incorrect? For a square matrix A in the matrix equation AX = B. (a) If $\|A\| \neq 0$, then there exists a unique solution. (b) If $\|A\| = 0$ and (adj A) B $\neq 0$, then there is no solution. (c) If $\|A\| \neq 0$ and (adj A) B $\neq 0$, then there is no solution. (d) If $\|A\| = 0$ and (adj A) B = 0 , then system has infinitely many solution. [CUET]
25.	Let A be the square matrix of order 3, then $\|kA\|$, where k is a scalar, is equal to: [CUET] (a) $3k\|A\|$ (b) $k^3\|A\|$ (c) $k^2\|A\|$ (d) $k\|A\|$
26.	Let A(-1, 1) and B(2, 3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is ax + by = 15, then 5a + 2b is: [JEE Mains] (a) - 12/5 (b)-6/5 (c) 4 (d) 6
27.	For any 3 x 3 matrix M, Let $\|M\|$ denotes the determinant of M. Let I be the 3 x 3 identity matrix. Let E and F be two 3 x 3 matrices such that (I-EF) is invertible. If $G = (I-EF)^{-1}$, then which of the following statement is (are) TRUE? [JEE Advanced] (a) $\|FE\| = \|I-FE\|\ \|FGE\|$ (b) $(I - FE)(I + FGE) = I$

	(c) EFG = GEF (d) (I - FE)(I - FGE) = I

Answers
1.(d), 2.(b), 3.(c), 4.(d), 5.(b), 6.(d), 7.(b), 8.(d), 9.(a), 10.(b), 11.(a), 12.(b), 13.(c), 14.(b), 15.(c), 16. k=4 , 17. x=-2,6 , 18. x=-3 , 19. $a = -1/2, b = 8, c = 1$,equation of path: $y = -1/2x^2 + 8x + 1$, 20. x = 1 , y = -1 , z = 1 , 21. x = 1/2 , y = 9/8 , z = 1/8 , 22. x = 3 , y = 1 , z = 2, 23. (i) $\begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}$ (ii) Sports=90, Music=65, Drama=25 , 24.(c), 25.(b), 26.(a), 27.(a) ,(b),(c)

CHAPTER 5 : CONTINUITY AND DIFFERENTIABILITY

SUMMARY OF THE CHAPTER

* A function f is said to be continuous at x = a if
Left hand limit = Right hand limit = value of the function at x = a
i.e. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$
i.e. $\lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(a-h) = f(a)$.

* A function is said to be differentiable at x = a
if $Lf'(a) = Rf'(a)$ i.e
$\lim_{h\to 0} \frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

IMPORTANT FORMULAE

(i) $\frac{d}{dx}(x^n) = n\, x^{n-1}$.

(ii) $\frac{d}{dx}(x) = 1$

(iii) $\frac{d}{dx}(c) = 0, \forall c \in R$

(iv) $\frac{d}{dx}(a^x) = a^x \log a,\ a > 0,\ a \neq 1$.

(v) $\frac{d}{dx}(e^x) = e^x$.

(vi) $\frac{d}{dx}(\log_a x) =, \frac{1}{x \log a}\ a > 0,\ a \neq 1,\ x$

(vii) $\frac{d}{dx}(\log x) = \frac{1}{x},\ x > 0$

(viii) $\frac{d}{dx}(\log_a |x|) = \frac{1}{x \log a},\ a > 0,$
$a \neq 1,\quad x \neq 0$

(ix) $\frac{d}{dx}(\log |x|) = \frac{1}{x},\ x \neq 0$

(x) $\frac{d}{dx}(\sin x) = \cos x, \forall x \in R$.

(xi) $\frac{d}{dx}(\cos x) = -\sin x, \forall x \in R$.

(xii) $\frac{d}{dx}(\tan x) = \sec^2 x, \forall x \in R$.

(xiii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x,$
$\forall x \in R$.

(xiv) $\frac{d}{dx}(\sec x) = \sec x \tan x,$
$\forall x \in R$.

(xv) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
$\forall x \in R$.

(xvi) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$.

(xvii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$.

(xviii) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2},$
$\forall x \in R$

(xix) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2},$
$\forall x \in R$.

(xx) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$.

(xxi) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$.

(xxii) $\frac{d}{dx}(|x|) = \frac{x}{|x|},\ x \neq 0$

(xxiii) $\frac{d}{dx}(ku) = k\frac{du}{dx}$

(xxiv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(xxv) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$

(xxvi) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

	SOLVED EXAMPLE
Q1	If $\log x^2 + y^2 = 2\tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$ (2 MARKS)
Ans.	$\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$ On differentiating both sides w.r.t . $x, we\ gey$ $$\frac{1}{x^2+y^2}\left(2x + 2y\frac{dy}{dx}\right) = 2 \times \frac{1}{1+\frac{y^2}{x^2}}\left(\frac{\frac{dy}{dx}x - y}{x^2}\right)$$ $$\frac{2\left(x + y\frac{dy}{dx}\right)}{x^2+y^2} = \frac{2x^2}{x^2+y^2}\left(\frac{\frac{dy}{dx}x - y}{x^2}\right)$$ $x + y \times \frac{dy}{dx} = \frac{dy}{dx}x - y$ $\frac{dy}{dx}(x - y) = x + y \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$ Hence proved
Q2	If$y = \sin^{-1}\left(6x.\sqrt{1-9x^2}\right)$, then find $\frac{dy}{dx}$ (2 MARKS)
Ans	Given , $y = \sin^{-1}\left(6x.\sqrt{1-9x^2}\right)$ On putting $3x = sin\theta$, then $y = \sin^{-1}\left(2\sin\theta.\sqrt{1-(\sin\theta)^2}\right)$ $y = \sin^{-1}(\sin 2\theta)$ $y = 2\theta$ $y = 2\sin^{-1}(3x)$ $\frac{dy}{dx} = \frac{2\times 3}{\sqrt{1-9x^2}}$ $\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$
Q3	If y = 2log $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, then show that $x(1+x)^2y_2 + (1+x^2)y_1 = 2$ (2 MARKS)
	Ans. $y = 2\log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ $y = 2\log\left(\frac{x+1}{\sqrt{x}}\right)$ $y = 2\log(x+1) - 2 \times \frac{1}{2}\log x$ $y = 2\log(x+1) - \log x$ $\frac{dy}{dx} = 2 \times \frac{1}{x+1} - \frac{1}{x}$ $y_1 = \frac{(x-1)}{x(x+1)}$ $y_2 = \frac{d\left(\frac{x-1}{x(x+1)}\right)}{dx} = \frac{(x-1)'x(x+1)-(x-1)\left(x(x+1)\right)'}{\left(x(x+1)\right)^2}$ $y_2 = \frac{x^2 + x - [2x^2 - x - 1]}{\left(x(x+1)\right)^2}$ $y_2 = \frac{-x^2+2x+1}{\left(x(x+1)\right)^2}$ For prove: - Taking LHS $x(x+1)^2y_2 + (x+1)^2y_1$ $\frac{-x^2+2x+1}{x} + (x+1) \times \frac{(x+1)}{x} = \frac{-x^2+2x+1+x^2-1}{x} = \frac{2x}{x}$ $= 2$ Hence proved.

Q4.	Find the derivative of $x^{\log x}\ w.r.t\ \log x$ (3 MARKS)
Ans.	Let $y = x^{\log x}$ On taking log both sides, we get $\log y = \log x\ \log x$ $=(\log x)^2$ On differentiating both sides w.r.t.x, we get $\frac{1}{y} \times \frac{dy}{dx} = 2\log x \,.\left(\frac{1}{x}\right)$ $\frac{dy}{dx} = y.\left(\frac{2\log x}{x}\right)$ $= x^{\log x}\left(\frac{2\log x}{x}\right)$ $= x^{\log x - 1}(2\log x)$ Again , let $z = \log x$ $\frac{dz}{dx} = \frac{1}{x}$ Now $\frac{dy}{dx} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$ $= \frac{x^{\log x - 1}(2\log x)}{\frac{1}{x}}$ $= x^{\log x}.(2\log x)$
Q5	If $y = x^{\sin x - \cos x} + \frac{x^2-1}{x^2+1}$, find $\frac{dy}{dx}$. (3 MARKS)
Ans,	$y = x^{\sin x - \cos x} + \frac{x^2-1}{x^2+1}$ Let $u = x^{\sin x - \cos x}$ and $v = \frac{x^2-1}{x^2+1}$ $Y = u + v$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ....(i) Now, $u = x^{\sin x - \cos}$ $logu = log(x^{\sin x - \cos x})$ $logu = (\sin x - \cos x)logx$ Differentiating w.r.t. x, we get $\frac{1}{u}.\frac{du}{dx} = (\sin x - \cos x).\frac{1}{x} + (\cos x + \sin x).logx$ $\frac{du}{dx} = u[(\sin x - \cos x).\frac{1}{x} + (\cos x + \sin x).logx]$ $\frac{du}{dx} = x^{\sin x - \cos}[(\sin x - \cos x).\frac{1}{x} + (\cos x + \sin x).logx]$ ....(ii)

	Now, $v = \frac{x^2-1}{x^2+1} = 1 - \frac{2}{x^2+1}$ $\frac{dv}{dx} = 0 - 2.(-1)(x^2+1)^{-2}.2x$ $\frac{dv}{dx} = \frac{4x}{(x^2+1)^2}$ ....(iii) From (i),(ii),(iii), we get $\boxed{\frac{dy}{dx} = x^{\sin x - \cos}[(\sin x - \cos x).\frac{1}{x} + (\cos x + \sin x).logx] + \frac{4x}{(x^2+1)^2}}$
Q 6	Find the relationship between a and b so the function f defined by $f(x)$ $=\begin{cases} ax+1, if\ x \le 3 \\ bx+3, if\ x > 3 \end{cases}$ is continuous at x=3 (3 MARKS)
Ans	Given ,$f(x) = \begin{cases} ax+1, if\ x \le 3 \\ bx+3, if\ x > 3 \end{cases}$ and $f(x)$ is continuous at $x = 3$ here $a\ and\ b$ are two unknowns. Now, at $x = 3$, $f(3) = a(3) + 1 = 3a + 1$ LHL = $f(x) = \lim_{x\to 3^-}(ax+1) = \lim_{h\to 0}\{a(3-h)+1\}$ [Put $x = 3 - h; when\ x \to 3^- then\ h \to 0$] = $\lim_{h\to 0}(3a - ah + 1)$ =$3a$+1 And RHL = $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}\{bx+3\}$ =$\lim_{h\to 0}\{b(3+h)+3\}$ [put $x = 3 + h; when\ x \to 3^+, then\ h \to 0$] =$\lim_{h\to 0}\{3b + bh + 3\}$ =$3b$ + 3 Since ,$f(x) is\ continuous\ at\ x = 3$. $\therefore f(x) = LHL = RHL$ i.e $3a + 1 = 3a + 1 = 3b + 3$, $a = b + \frac{2}{3}$
Q7	Differentiate the function with respect to x. $(xcosx)^x + (xsinx)^{\frac{1}{x}}$ (5 MARKS)
Ans.	Let $y = (xcosx)^x + (xsinx)^{\frac{1}{x}}$ Also, let $u = (xcosx)^x$ and $v = (xsinx)^{\frac{1}{x}}$ $y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (xcosx)^x$$

$$\log(u) = \ log(xcos\ x)^x\ or\ log(u)\ =\ x\ log(xcosx)$$

$$logu = x[logx + log(\cos x)]$$

$$logu = xlogx + xlog(cosx)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}\big(xlog(x)\big) + \frac{d}{dx}\ (x\log(\cos x))$$

$$\frac{du}{dx} = \ u[\Big\{log(x)\frac{d}{dx}(x) + \ x * \frac{d}{dx}\big(log(x)\big)\Big\} + \ \{\log(cosx) * \frac{d}{dx}(x)\ +\ x * \frac{d}{dx}(log(cos\ x))\}]$$

$$\frac{du}{dx} = (xcosx)^x\ [\Big(log(x) + \ x\ * \frac{1}{x}\ + \Big\{log(cos\ x) + \ x\ * \frac{1}{cos\ x}\frac{d}{dx}\ (cos\ x)\Big\}\Big]$$

$$\frac{du}{dx} = (x\ cos\ x)^x\ \Big[(log(x) + \ 1) + \Big\{log(cos\ x) + \frac{x}{cos\ x}\ (-\ sin\ x)\Big\}\Big]$$

$$\frac{du}{dx} = \ (x\ cos\ x)^x\ \big[\big(1\ +\ log(x)\big) + \ (log(cos\ x) - \ x\ tan\ x)\big]$$

$$\frac{du}{dx} = (x\ cos\ x)^x\ \big[1\ -\ x\ tan\ x\ +\ \big(log(x) + \ log(cos\ x)\big)\big]$$

$$\frac{du}{dx} = (xcosx)^x\ [1\ -\ x\ tan\ x\ +\ log(x\ cos\ x)]$$

$$v\ =\ (x\sin x)^{\frac{1}{x}}$$

$$log(v) = \ log(x\ sin\ x)^{\frac{1}{x}}$$

$$log(v) = \frac{1}{x}\ log(x\ sin\ x)$$

$$log(v) = \tfrac{1}{x}\big(log(x) + \ log(sin\ x)\big)$$

$$log(v) = \frac{1}{x}\ log(x) + \frac{1}{x}\ log(sin\ x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{d}{dx}(v) = \frac{d}{dx}\left(\frac{1}{x}log(x)\right) + \frac{d}{dx}\left[\frac{1}{x}log(sin\ x)\right]$$

$$\frac{1}{v}\ \frac{dv}{dx}\ = [log.x\frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{x} * \frac{d}{dx}(log\ x)] + \ \left[log(sin\ x)\frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{x} * \frac{d}{dx}\{log(sin\ x)\}\right]$$

$$\frac{1}{v}\frac{dv}{dx} = \ \left[log(x)\ \left(-\frac{1}{x^2}\right) + \frac{1}{x} * \frac{1}{x}\right] + \ \left[log(sin\ x)\left(-\frac{1}{x^2}\right) + \frac{1}{x} * \frac{1}{sin\ x} * \frac{d}{dx}(sin\ x)\right]$$

$$\frac{1}{v}\frac{dv}{dx}\ = \frac{1}{x^2}\ \big(1\ -\ log(x)\big)\ +\ \left[-\frac{log(sin\ x)}{x^2} + \frac{1}{x\ *\ sin\ x} *\ cos\ x\right]$$

$$\frac{dv}{dx} = (x\ sin\ x)^{\frac{1}{x}}\ \left[\frac{1\ -\ log(x)}{x^2} + \frac{-\ log(sin\ x) + \ x\ cot\ x}{x^2}\right]$$

$$\frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1\ -\ log(x) - \ log(sin\ x) + \ x\ cot\ x}{x^2}\right]$$

$$\frac{dv}{dx} = (x\ sin\ x)^{\frac{1}{x}}\ \left[\frac{1\ -\ log(x\ sin\ x) + \ x\ cot\ x}{x^2}\right]$$

	$\frac{dy}{dx} = (x\,cos\,x)^x\,[1 - x\ \ tan\,x + log(x\,cos\,x)]$ $+ (x\,sin\,x)^{\frac{1}{x}}\left[\frac{x * cot\,x + 1 - log(x * sin\,x)}{x^2}\right]$ MCQ
Q1.	Find $\lim_{x\to 0}(\frac{\sin x}{x})$ (a)-1 (b) 0 (c) 1 (d) 2
Q2.	Find $\lim_{x\to 0}\frac{e^x-1}{x}$ (a)-1 (b) 0 (c) 1 (d) 3
Q3.	Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$,where p is a constant .then the value of p for which derivative of $f\ at\ (x = 0) = 1$ is (a) R (b)1 (c)0 (d)-1
Q4.	Find $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$ (a)$\frac{1}{2}$ (b) 1 (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
Q5	$f(x) = x^x$ find critical point's abscissa: a) 1 b)e c)2e d) 1/e
Q6.	If y = $\log\sqrt{\sin\sqrt{x}},$ then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}\ is$ (a) $\frac{1}{\pi}$ (b) π (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
Q7	If $xe^y = 1, then\ the\ value\ of\ \frac{dy}{dx}\ at\ x = 1\ is$ (a) -1 (b) 1 (c) -e (d) $\frac{-1}{e}$
Q8	If $\sin(xy) = 1, then\ \frac{dy}{dx}\ is\ equal\ to$ (a) $\frac{x}{y}$ (b) $\frac{-x}{y}$ (c) $\frac{y}{x}$ (d) $\frac{-y}{x}$
Q9	If $y = xcosx, then$ second derivative of y is (a) $-xcosx - 2sinx$ (b)$xcosx + 2sinx$ (c) $xsinx + cosx$ (d) none of these
Q10	If x=3$\cos\theta\ and\ y = 5\sin\theta\,, then \frac{dy}{dx}\ is\ equal\ to$ (a) $\frac{-3}{5}\tan\theta$ (b) $\frac{-5}{3}\cot\theta$ (c) $\frac{-5}{3}\tan\theta$ (d) $\frac{-3}{5}\cot\theta$
	Answers 1-c 2-c 3-d 4-a 5-d 6-a 7-a 8 -d 9-a 10-d
	QUESTIONS FOR PRACTICE

Q1	If $f(x) = \begin{cases} x : x \in Q \\ -x : x\ not\ beloges\ to\ Q \end{cases}$, then $f(x)$ is continuous at (a) only at zero (b) only at 0,1 (c)all real numbers (d) all rational number
Q 2	If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and$g(x) = f^{-1}(x)$ then the value of $g'(1)$.
Q 3	If $f(x) = (x+1)(x+2)(x+3) \ldots \ldots \ldots \ldots (x+n), then\ find \frac{df(0)}{dx}$.
Q 4	If $f(x) = \frac{\sqrt{1-x^2}\ \ (2x+3)^{\frac{1}{2}}}{(x^2+2)^{\frac{2}{3}}}, then\ find \frac{df(0)}{dx}$
Q 5	Discuss the differentiability of $f(x) = \|x-1\| + \|x-2\|\ at\ x = 1,2$
	Answers Q1 –(a) Q2—(2) Q3- $n!\left[1 + \frac{1}{2} + \frac{1}{3} = \cdots \ldots . \frac{1}{n}\right]$ Q4-$\frac{1}{2^{\frac{2}{3}} . \sqrt{3}}$ Q5-not differentiable at x=1 &2

CHAPTER- 6 (APPLICATIONS OF DERIVATIVES)

Some important applications of Derivatives as per syllabus.

1. Rate of change
2. Increasing decreasing function
3. Maxima and minima

1. Rate of change

If a quantity y varies with another quantity x , satisfying some rule y=f(x), then $\frac{dy}{dx}$ or f'(x) represents the rate of change of y with respect to x

Its value at x=a is denoted as $[\frac{dy}{dx}]_{x=a}$

Marginal cost and Marginal revenue

Let C be the total cost of producing and marketing x units of a product, then marginal cost(MC) is

MC=$\frac{dC}{dx}$

The rate of change of total revenue R with respect to the quantity sold is the marginal revenue (MR)

MR=$\frac{dR}{dx}$

2. Increasing decreasing function

If I is an interval in the domain of the function f(x) , then f(x) is said to be

Increasing on I if $x_1<x_2$, $f(x_1) \leq f(x_2)$ $\forall$ $x_1,x_2 \in I$

Decreasing on I if $x_1<x_2$, $f(x_1) \geq f(x_2)$ $\forall$ $x_1,x_2 \in I$

In order to find the intervals in which a function is increasing or decreasing, following working rule may be used.

Step I : Obtain f(x).

Step II : Compute its derivative, f'(x).

Step III : Solve the equation f'(x) =0 to get the value(s) of x, which is/are critical points of f(x).(**note where function is not define also consider as critical points)**

Step IV : Plot the critical points on a number line in their correct order and use them to divide the domain of f(x) , or the interval under consideration, into open intervals.

Step V: Determine the sign of f'(x) within each of these intervals.

Intervals of function	Sign of f'(x)	Nature of function
I_1	Positive	Increasing
I_2	Negative	Decreasing

3. Maxima and minima

Maxima and minima of a function are the largest and smallest value of the function respectively either within a given range or on the entire domain. Collectively they are also known as extreme of the function.

Tests for identifying Local Maxima and Minima

First Order Derivative Test:

Consider f be the function defined in an open interval I. Also, f be continuous at critical point c in I such that f'(c) = 0.

If f'(x) changes sign from positive to negative as x increases through point c, then c is the point of local maxima, and f(c) is the maximum value.

If f'(x) changes sign from negative to positive as x increases through point c, then c is the point of local minima, and f(c) is the minimum value.

If f'(x) doesn't change sign as x increases through c, then c is neither a point of local minima nor a point of local maxima.

Second Derivative Test:

Consider f be the function defined on an interval I and it is twice differentiable at c.

If x = c will be the point of local maxima if f'(c) = 0 and f''(c)<0. Then f(c) will be having local maximum value.

If x = c will be the point of local minima if f'(c = 0 and f''(c) < 0. Then f(c) will be having local minimum value.

When both f'(c) = 0 and f''(c) = 0 the test fails. And that first derivative test will give us the value of local maxima and minima.

Solved examples

1	Given a curve $y=7x-x^3$ and x increases at the rate of 2 units per second. the rate at which the slope of the curve is changing when x=5 is (2024) (a)-60 units/sec (b) 60 units/sec (c) -70 units/sec (d)-140 units/sec Solution: - Given curve is $y=7x-x^3$ Differentiating both sides w.r.t. x, we get $\frac{dy}{dx} = 7-3x^2$ Let m be the slope, then $m=7-3x^2$ Differentiating both sides w.r.t. t, we get

	$\frac{dm}{dt}$= -6x$\frac{dx}{dt}$ It is given that $\frac{dx}{dt}$ = 2units/sec and x=5 $\Rightarrow \frac{dm}{dt}$ = -6(5)(2) = -60 units/sec
2	If f(x)=x^3-3x , , then the interval in which f(x) is **decreasing** is: (A)$(-\infty,-1)$ (B)$(-1,1)$ (C)$(1,\infty)$ (D) $(-\infty,\infty)$ **Solution:** (B) $(-1,1)$
3	The area of an expanding rectangle is increasing at the rate of 48cm^2/s. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth = 4.5 cm? (2025 3marks) Sol:- Given , $\frac{dA}{dt}$ =48cm^2/s Area of rectangle A= lb=$l^{3/2}$ [l=b^2] Differentiating both sides w.r.t. t, we get $\frac{dA}{dt} = \frac{3}{2} l^{\frac{1}{2}} \frac{dl}{dt}$ $48 = \frac{3}{2} l^{\frac{1}{2}} \frac{dl}{dt}$ $\Rightarrow \frac{32}{\sqrt{l}} = \frac{dl}{dt} \Rightarrow \frac{dl}{dt} = \frac{32}{4.5} = \frac{64}{9}$ cm/s
4	A spherical medicine ball when dropped in water dissolves in such way that the rate of decrease of volume at any instant is proportional to is surface area. Calculate the rate of decrease of its radius. (2025 3mark) Sol:- Let V be the volume of the spherical medicine ball , r be the radius of the spherical ball, t be the time and K be the constant of proportionality. $\therefore$ Volume of sphere =$\frac{4}{3}\pi r^3$ Surface area of sphere ,A =4πr^2 Given, $\frac{dV}{dt}$=-KA $\therefore$ $\frac{d}{dt}(\frac{4}{3}\pi r^3)$ = -K (4πr^2) $\Rightarrow \frac{4}{3}\pi$x3$r^2x\frac{dr}{dt}$ = -K (4πr^2) $\Rightarrow \frac{dr}{dt}$=-K This shows that the rate of decreasing of its radius is constant.
5.	Detemine the interval, where f(x) = sinx – cosx , $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing. Sol:- f(x) = sinx – cosx f(x) = √2(sinxcos$\pi/4$ – cosxsin $\pi/4$) f'(x)=√2[sin(x+π/4)] (0,3π/4) increasing (3π/4,7 π/4) Decreasing (7 π/4,2 π) Increasing
6.	**Case study** A company manufactures and sells pens. The profit (in ₹) obtained by selling x pens is given by: P(x)=$-2x^2+40x-150$ (i) Find the critical point of P(x). (ii) Determine the intervals where profit is increasing or decreasing. (iii) Find the maximum profit.

	Solution: $P'(x)=-4x+40$. Critical point: $-4x+40=0 \Rightarrow x=10$ Test intervals: For $x< 10$ e.g. $x=5$, $P'(5)=20>0 \rightarrow$ Increasing in $(-\infty,10)$ For $x>10$, e.g. $x=15$, $P'(15)=-20<0 \rightarrow$ Decreasing in $(10,\infty)$ Maximum profit at $x=10$: $P(10)=-2(100)+400-150=-200+400-150=50$.
	<u>Questions for practice</u>
	<u>MCQ/</u> Assertion- Reason
<u>1.</u>	Find the interval in which function $f(x) = x^2 - 4x + 5$ is increasing. a) $(2, \infty)$ b) $(-\infty, 2)$ c) $(3, \infty)$ d) $(-\infty, \infty)$
<u>2</u>	Find the interval in which function f(x) = sinx+cosx is increasing. a) $(5\pi/4, 2\pi)$ b) $[0, \pi/4)$ and $(5\pi/4, 2\pi]$ c) $(\pi/4, -5\pi/4)$ d) $(-\pi/4, \pi/4)$
<u>3</u>	Nature of the function $f(x) = e^{2x}$ is _______ a) increasing b) decreasing c) constant d) increasing and decreasing
<u>4</u>	If the rate of change of radius of a circle is 6 cm/s then find the rate of change of area of the circle when r=2 cm. a) 74.36 cm^2/s b) 75.36 cm^2/s c) 15.36 cm^2/s d) 65.36 cm^2/s
<u>5</u>	. The edge of a cube is increasing at a rate of 7 cm/s. Find the rate of change of area of the cube when x=6 cm. a) 578 cm^2/s b) 498 cm^2/s c) 504 cm^2/s d) 688 cm^2/s
<u>6</u>	The total cost P(x) in rupees associated with a product is given by P(x)=0.4x2+2x-10. Find the marginal cost if the no. of units produced is 5. a) Rs.3 b) Rs.4 c) Rs.5 d) Rs.6
<u>7</u>	If the circumference of the circle is changing at the rate of 5 cm/s then what will be rate of change of area of the circle if the radius is 6cm. a) 20 cm^2/s b) 40 cm^2/s c) 70 cm^2/s d) 30 cm^2/s
<u>8</u>	The total cost N(x) in rupees, associated with the production of x units of an item is given by N(x)=0.06x3-0.01x2+10x-43. Find the marginal cost when 5 units are produced. a) Rs. 1.44 b) Rs. 144.00 c) Rs. 14.4 d) Rs. 56.2
<u>9</u>	The length of the rectangle is changing at a rate of 4 cm/s and the area is changing at the rate of 8 cm/s. What will be the rate of change of width if the length is 4cm and the width is 1 cm. a) 5 cm/s b) 6 cm/s c) 2 cm/s d) 1 cm/s
<u>10</u>	At which point does $f(x) = \|x - 1\|$ has its local minimum? a) local minima at x = 1. b) no minima c) at x=0 d) Can't be predicted
<u>11</u>	In the following questions, a statement of Assertion (A) is followed by the statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not correct explanation of A. (c) A is true but R is false. (d) A is false and R is also false. Q1. **Assertion (A)**: The rate of change of area of a circular wave with respect to radius r where r = 6 cm is12 π cm^2/cm . **Reason (R)**: Rate of change of the circular wave with respect to radius r is dA/dr, where A is the area of cicular wave.

12	**Assertion (A)**: f(x)= tanx -x always increases **Reason (R)**: Any function y=f(x) is increasing if dy/dx >0
	(2 marks questions)
1	Show that function $f(x) = 2 - 3x + 3x^2 - x^3$ is decreasing function.
2	Show that function $f(x) = \frac{x-2}{x-1}$ is increasing function
3	Find the intervals in which function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or strictly decreasing.
4	Find the intervals in which function f(x) = sin x - cos x, is strictly increasing or strictly decreasing
5	Find the intervals in which function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is strictly increasing or strictly decreasing.
	(3 marks questions)
1	1. An open box with a square base in to be made out of a given quantity of sheet of area .Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ . .
2	Show that the surface area of closed cuboids with square base and given volume is minimum, when it is a cube.
3	If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of triangle is maximum when the angle between then is $\frac{\pi}{3}$.
4	The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When x =10cm and y = 6cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.
5	Find intervals in which the function given by $f(x) = \sin 3x$, $x \in [0, \frac{\pi}{2}]$is increasing or decreasing.
6	Find the absolute maximum and minimum values of a function f given by $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1, 5].
	(5 marks questions)
1	An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.
2	The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.
3	A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the maximum length of the hypotenuse is $(a^{\frac{3}{2}} + b^{\frac{3}{2}})^{\frac{3}{2}}$.
	Case study
1	. Read the following and answer the questions from (i) to (v). The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation y = $4x - \frac{1}{2}x^2$, where x is the number of the days exposed to sunlight.  Based on the above information answer the following: (i) The rate of growth of the plant with respect to sunlight is

(a) (b)4-x (c) x-4 (d) $x-\frac{1}{2}x^2$

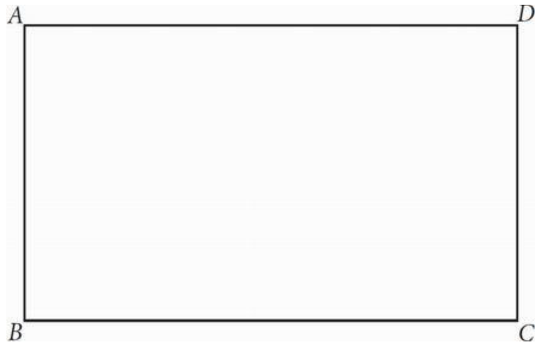
What are the numbers of the days it will take to grow to maximum height?

(a) (b) 6 (c) 7 (d) 10

(iii) What is the maximum height of the plant?

(a) 12cm (b) 10 cm (c) (d) 6 cm

2 . A rectangular hall is to be developed for a meeting of student parliament in Kendriya Vidyalaya Sangthan. It is given that the



floor has a fixed perimeter Pas shown below. Floor “A”

Based on the above information answer the following.

(i) If x and y represent the length and the breadth of the rectangular region, then the relation between the variables is

(a) x+y=P (b) $x^2+y^2= P$ (c) (d) x+2y= P

(ii) The area of the rectangular region “A” expressed as a function of x is

(a) $\frac{1}{2}$ $(P+x^2)=P$ (b) (c) $\frac{1}{2}$ $(px+2x^2)$ (d) $Px-2x^2$

(iii) The administration of Kendriya Vidyalaya Sangthan is interested in maximizing the area of floor “A”. For this to happen the value of x should be

(a) P (b) P/2 (c) 2P/3 (d) P/4

3. $P(x) = -5x^2 +125x + 37500$ is the total profit function of a company, where x is the production of the company.

1. What will be the production when the profit is maximum?
2. What will be the maximum profit?
3. Check in which interval the profit is strictly increasing

Questions from Competitive exams

1 Let the function $f(x) = \frac{x}{3}+\frac{3}{x}+3$, $x\neq 0$ be strictly increasing in $(-\infty,\alpha_1)\cup(\alpha_2,\infty)$ and strictly decreasing in $(\alpha_3,\alpha_4)\cup(\alpha_4,\alpha_5)$. Then $\sum_{i=1}^{5}\alpha_i^2$ is equal to

(JEE Mains 2025 ONLINE 8th April Evening Shift)

(a)48 (b) 40 (c) 36 (d) 28

2 Let a>0 . If the function $f(x)= 6x^3-45ax^2+108a^2x+1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1 x_2 =54$, then $a+ x_1+ x_2$ is equal to :

(JEE Main 2025 (Online) 4th April Evening Shift)

(a)15 (b) 13 (c) 24 (d) 18

3 Let **f :** $\mathbb{R}$ $\mathbb{R}$ be a polynomial function of degree four having extreme values at **x =4** and **x = 5**.

If $\lim_{x\to 0}\frac{f(x)}{x^2}=5$, then **f(2)** is equal to :

(JEE Main 2025 (Online) 7th April Evening Shift)

(a)8 (b) 10 (c) 12 (d) 14

4	Let x= -1 and x=-2 be the critical points of the function $f(x)=x^3+ax^2+b\log_e\|x\|+1, x\neq 0$. Let and M respectively be the absolute minimum and the absolute maximum values of in the interval $\left[-2,\frac{1}{2}\right]$. Then $\|M+m\|$ is equal to Take ($\log_e 2=0.7$) **JEE Main 2025 (Online) 7th April Morning Shift** (a)21.1 (b) 19.8 (c) 22.1 (d) 20.9
5	The number of critical points of the function $f(x)=(x-2)^{\frac{2}{3}}(2x+1)$ is **JEE Main 2024 (Online) 8th April Morning Shift** (a)2 (b) 1 (c) 0 (d) 3
	Answer
	MCQ **1.** a **2.** b **3.** A **4.** b **5.** c **6.** d **7.** d **8.** C **9.** D **10.** a **11.a** **12.a**
	2 marks **3.** strictly increasing in the intervals $(-\infty,2)U(3,\infty,)$ and strictly decreasing in the interval (2,3). **4. Increasing** $(0,\frac{3\pi}{4})\cup(\frac{7\pi}{4},2\pi)$ **Decreasing:** $(\frac{3\pi}{4},\frac{7\pi}{4})$ **5. Increasing** $(1,2)\cup(3,\infty)$ **Decreasing:** $(-\infty,1)\cup(2,3)$
	3 marks 4. - 2 cm/min, 2 cm^2/min **5.** Increasing$[0,\frac{\pi}{6}]$, decreasing $[\frac{\pi}{6},\frac{\pi}{2}]$) **6.** Absolute maximum value 56, absolute minimum value 24
	5 marks 1. $\sqrt{5}$
	Case study1. (i) $4x-\frac{1}{2}x^2$ (ii) 4 a (iii) 8 cm **2.** (i)c 2(x+y)= P (ii) $\frac{1}{2}$(px-2x^2) (iii) P **3.** 1. b) 12.5 2. b) Rs.38281.25 3. d) (0 , 12.5)
	Competition question 1. C (36) **2.** d (18) **3.b(10)** **4. a (21.1)** **5.** a (2)

CHAPTER - 7 INTEGRALS

SUMMARY

INTEGRATION: Integration is an inverse process of differentiation. It is also called antiderivative or primitive.

Types of Integrals: (i) Indefinite Integrals (ii) Definite Integrals

INDEFINITE INTEGRALS

Important Points of Indefinite Integrals:

- Integration as an Inverse Process of Differentiation.
- Integration by Substitution
- Integration by using trigonometric identities
- Integration by Partial fraction
- Integration by Parts
- Integration of some standard types of functions

$$\frac{d}{dx}[F(x)] = f(x) \Leftrightarrow \int f(x)dx = F(x) + c$$

Thus,F (x) + C is also an anti-derivative (Indefinite Integral) of f(x), Where C is an arbitrary constant and it is called constant of Integral.

$$\frac{d}{dx}[F(x) + c] = f(x)$$

$\int k\, f(x)dx = kF(x)$, where k is constant

$$\int \{f_1(x) \pm f_2(x)\}dx = \int F_1(x)dx \pm \int F_2(x)dx + c$$

Some Important Formulae:

S.N.	**Derivatives**	**Indefinite Integrals/Antiderivatives**
1	$\frac{d}{dx}(x^n)$=n x^{n-1}	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$;where n$\neq -1$
2	$\frac{d}{dx}(x)$ = 1	$\int 1$ dx= x + c
3	$\frac{d}{dx}$(sinx)=cosx	$\int cosx dx$= sin x + C
4	$\frac{d}{dx}$(cosx)= -sinx	$\int sinx dx$ = - cos x + C
5	$\frac{d}{dx}$(tanx) $=sec^2x$	$\int sec^2x\, dx$ = tan x + C
6	$\frac{d}{dx}$(cotx)= -$cosec^2x$	$\int cosec^2x\, dx$ = -cot x + C
7	$\frac{d}{dx}$(sec x)=secx tanx	$\int \sec x \tan x dx$ = sec x +C

8	$\frac{d}{dx}(cosec\ x) = -cosecx\ cotx$	$\int cosec\ x$ cotxdx = - $cosecx$ + C
9	$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C$
10	$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}dx = -\cos^{-1}x + C$
11	$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}dx = tan^{-1}x + c$
12	$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}dx = -tan^{-1}x + c$
13	$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{1-x^2}}$ $\frac{d}{dx}(cosec^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$	$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + c = -\cos ec^{-1}x + c$
14	$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
15	$\frac{d}{dx}(\log x) = \frac{1}{x}$	$\int \frac{1}{x}dx = \log x + C$
16	$\frac{d}{dx}(a^x) = a^x \log a$	$\int a^x dx = \frac{a^x}{\log a} + C$

Some Important Questions:

Evaluate the following:

(i) $\int 4^x 3^x dx$ (ii) $\int \frac{sin^2x - cos^2x}{sinx\ cosx}dx$

Solution- (i) $\int 4^x 3^x dx = \int 12^x dx = \frac{12^x}{log_e a} + c$

(ii) $\int \frac{sin^2x - cos^2x}{sinx\ cosx}dx = 2\int \frac{-cos2x}{2sinx\ cosx}$ dx $= 2\int -cot2x\ dx = -\log|sin2x| + c$

Integration by substitution: -

•Procedure to find integral by substitution method-

- Substitution Method is used to find integration of composite functions.

Like as: sin3x, e^{sinx} , sin (logx), ………. etc. Generally, substitution method is used if differentiation is given along with function

Let I = $\int g\{(x)\}.\ g'(x)dx$

Step-I Substitute g(x) = t,

Step- II Differentiate both sides; so that g'(x) dx = dt

Step-III Now I = $\int f\{t\}\ dt$ = F(t) + C = F{g(x)] + C

- **Integration of tan x, cot x, sec x and cosec x :**

(i) $\int tanx dx$ = - log |cos x| + C or log|secx| + C

(ii) $\int cotx dx$ = log|sinx| + C or - log|cosecx| + C

(ii) $\int secx dx$ = log|secx + tanx| + C or log|tan($\frac{\pi}{4}+\frac{x}{2}$)|+ C

(iii) $\int cosecx dx$ = log| cosecx – cotx| + C = log|tan$\frac{x}{2}$|+ C

Integration by using trigonometric identities: -

Some important trigonometric identities to find integrations	
2 sinA cosB = sin(A+B) + sin(A-B)	Cos2A= 1-2 $\sin^2$A ⇒ $\sin^2 A=\frac{1-\cos 2A}{2}$
2 cosA SinB = Sin(A+B) – Sin(A-B)	Cos2A= 2 $\cos^2$A-1 ⇒ $\cos^2 A=\frac{1+\cos 2A}{2}$
2 cosA cosB = cos(A-B) + cos(A+B)	Sin3A=3sinA-4$\sin^3$A⇒$\sin^3 A=\frac{3\sin A-\sin 3A}{4}$
2 sinA SinB = cos(A-B) – cos(A+B)	cos3A = $4\cos^3 A-3\cos A$⇒$\cos^3 A=\frac{\cos 3A+\cos A}{4}$

Some Standard Integrals: -

1. $\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\|+c$	2. $\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\|+c$	3. $\int\frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$
4. $\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$	5. $\int\frac{dx}{\sqrt{x^2-a^2}}=\log\left\|x+\sqrt{x^2-a^2}\right\|+c$	6. $\int\frac{dx}{\sqrt{a^2+x^2}}=\log\left\|x+\sqrt{a^2+x^2}\right\|+c$

7. $\int\frac{dx}{ax^2+bx+c}$ For finding the integral, make perfect square the quadratic polynomial ax^2 + bx + c ,
- To make perfect square the coefficient of x^2 should be + 1.
- After making perfect square, integrand will be changed in any one form given below
- $\int\frac{dx}{a^2+x^2}$ or $\int\frac{dx}{a^2-x^2}$ or $\int\frac{dx}{x^2-a^2}$

8. $\int\frac{dx}{\sqrt{ax^2+bx+c}}$ After making perfect square, integrand will be changed in any one form given below

$$\int\frac{dx}{\sqrt{A^2-x^2}} \quad \text{or} \quad \int\frac{dx}{\sqrt{A^2+x^2}} \quad \text{or} \quad \frac{dx}{\sqrt{X^2-A^2}}$$

9. $\int\frac{(px+q)dx}{ax^2+bx+c}$ first of all make derivative of ax^2+bx +c in numerator then integrate.

10. $\int\frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$ first of all make derivative of ax^2+bx +c in numerator then integrate.

11. $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} sin^{-1}\frac{x}{a}$ + C	
12. $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$	
13. $\int \sqrt{x^2 + a^2}\, dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$	

Integration by Partial Fraction:

- Partial fraction method is used to find integration of Rational functions.
- Rational Function: A function in the form of $\frac{f(x)}{g(x)}$, where f(x) and g(x) are polynomials in x and g(x) ≠ 0, is called rational function.

Types of Rational Functions:

(i)**Proper Rational Function**- If the degree of f(x) < Degree of g(x) then the function is called proper rational function.

(ii) **Improper Rational Function**- If the degree of f(x) ≥ Degree of g(x) then the function is called improper rational function. •

- If a function is improper rational function, then first convert it into Proper rational function using division method.
- Factories the denominator 'g(x)'. Factors of g(x) may be linear or quadratic
- A Proper rational function will decompose in the following forms –

Denominator g*(x)*	Form of given fraction	Format of Partial Fraction
(x+a) (x+b)	$\frac{P(x)}{(x+a)(x+b)}$	$\frac{A}{x+a} + \frac{B}{x+b}$
(x+a)^2 (x+b)	$\frac{P(x)}{(x+a)^2\,(x+b)}$	$\frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{x+b}$
(x+a) (x^2+b)	$\frac{P(x)}{(x+a)\,(x^2+b)}$	$\frac{A}{x+a} + \frac{Bx+C}{x^2+b}$
(x+a)(ax^2+bx+c) where ax^2+bx+c is non factorizable	$\frac{P(x)}{(x+a)(ax^2+bx+c)}$	$\frac{A}{x+a} + \frac{Bx+C}{ax^2+bx+c}$

Integration by Parts:

•Integration by parts is used to find integration of the following type of the functions

(i)Product of two functions: f(x). g(x), Example- x sinx,e^xsinx………etc.

I= $\int f(x)g(x)dx = \int u.vdx$ where u and v are function of x.

(ii) Logarithmic functions: logx, log(x+1)……etc

(iii) Inverse trigonometric functions : $\sin^{-1}x$, $\cos^{-1}x$……..etc

- First select the function as I & II with the help of the word **'ILATE'**

Select the first function which comes first as per alphabetical order in the word ILATE.	In the word **'ILATE'**
ILATE	**I**= Inverse trigonometric functions
	L = Logarithmic functions
	A = Algebraic Functions
	T = Trigonometric functions
	E= Exponential functions

- Now apply the formula to find integral of $\int f(x)g(x)dx = \int u.vdx$, where u and v are functions of x.

$$\int u.vdx = u\int vdx - \int \frac{du}{dx}\int vdx \quad \text{OR} \quad \int \{f(x).g(x)\}dx = f(x)\int g(x)dx - \int \{\frac{d}{dx}f(x).\int g(x)dx\}dx + c$$

- **Some Important Results:**

(i) $\int e^x \{(x) + f'(x)\}dx$ = e^x f(x) + C

(ii)$\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx) + c$

(iii) $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx) + c$

Some special integrals-

(i) to find integral of the form$\int \frac{a\ sinx+bcosx}{c\ sinx+d\ cosx}dx$:

Let asinx+bcosx= $\lambda \frac{d}{dx}(denominator) + \mu(denominator)$

Find $\lambda\ and\ \mu$ by comparing the coefficients of sinx and cosx. Then given integral is reduced to one of the known standard form.

(ii) Integral of the form $\int \frac{x^2 \pm 1}{x^4+kx^2+1}dx$, Divide the numerator and denominator by x^2 and put $(x+\frac{1}{x})$ =t or $(x-\frac{1}{x})$ =t. then integrate.

Solved examples :-

$\int tan^{-1}\sqrt{\frac{1-cos2x}{1+cos2x}}dx$

$= \int tan^{-1}\{\sqrt{\frac{2sin^2x}{2cos^2x}}\}dx = \int tan^{-1}(tanx)\,dx = \int xdx = \frac{x^2}{2} + c$

1. $\int 2^{2^{2^{2^x}}}\, 2^{2^x} 2^x dx$ (put $2^{2^{2^{2^x}}}$ = t)

$= \frac{2^{2^{2^{2^x}}}}{(log2)^3} + c$

2. $\int \frac{sin^6x+cos^6x}{sin^2xcos^2x}dx$ **(NCERT EXEMPLAR)**

Use $(a^3+b^3)= (a+b)^3-3ab(a+b)$ in numerator and after simplification ,we get

$= \int\{\frac{1}{sin^2x\, cos^2x} - 3\}dx$

$=\int\{\frac{sin^2x + cos^2x}{sin^2x\, cos^2x} - 3\}dx = \int sec^2xdx + \int cosec^2x\, dx - 3x$= tanx-cotx-3x+C

3. $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$ **(CBSE 2023)**

$put\ e^x = t$

$\int \frac{dt}{\sqrt{5-4t-t^2}} = \int \frac{dt}{\sqrt{9-(t+2)^2}}dx = sin^{-1}(\frac{t+2}{3})+C = sin^{-1}(\frac{e^x+2}{3})+C$

4. $\int \frac{dx}{(x-1)\sqrt{2x+3}}$

put $2x+3=t^2$

$=\int \frac{tdt}{(\frac{t^2-3}{2}-1)t} = 2\int \frac{dt}{t^2-5} = 2X\, \frac{1}{2\sqrt{5}}\, log\frac{t-\sqrt{5}}{t+\sqrt{5}} +C= \frac{1}{\sqrt{5}}\log\frac{\sqrt{2x+3}-\sqrt{5}}{\sqrt{2x+3}+\sqrt{5}} + C$

5. $\int \frac{dx}{1-3sinx}$ (use $sinx=\frac{2\, tanx/2}{1+tan^2x/2}$)

$= \int \frac{dx}{1-3(\frac{2\tan x/2}{1+tan^2\frac{x}{2}})} = \int \frac{sec^2x/2}{tan^2\frac{x}{2}-6tan\frac{x}{2}+1}dx$ now put tanx/2=t

$= \int \frac{2dt}{t^2-6t+1} = 2\int \frac{dt}{(t-3)^2-8} = \frac{1}{2\sqrt{2}}\log\frac{tan\frac{x}{2}-3-2\sqrt{2}}{tan\frac{x}{2}-3+2\sqrt{2}}+C$

6. $\int \frac{x+sinx}{1+cosx}dx$ **(NCERT EXEMPLAR)**

Use sinx= 2sinx/2.cosx/2 and cosx= 2 $cos^2x/2-1$, then divide separately

$=\int(\frac{1}{2}x\, sec^2\frac{x}{2} + tan\frac{x}{2})dx$ now use integration by parts in first integral

we have

$=xtan\frac{x}{2} + c$

7. $\int \frac{1+x^2}{1+x^4}\, dx$ divide numerator and denominator by x^2 , we get

$= \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+2-2}dx = \int \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2}dx$ put $x-\frac{1}{x}$= t then we get

$= \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\,tan^{-1}\frac{t}{\sqrt{2}} + C = \frac{1}{\sqrt{2}}\,tan^{-1}(\frac{x-\frac{1}{x}}{\sqrt{2}}) + C$

MCQ's

1	Evaluate $\int sec^2(7-x)dx$ is equal to (a) $\sec(7-x)$ +c (b) tan (7- x)+c (c) - tan (7 -x)+c (d)None of these
2	$\int e^x$secx (1+ tanx) dx = ... (a) e^xcosx + C (b) e^xsecx + C (c) e^xsinx + C (d) e^xtanx + C
3	Evalulate $\int \frac{e^x(x+1)}{cos^2(xe^x)}$dx (a) –cot(e$x^2$) +C (b) tan(x$e^x$) +C (c) tan($e^x$) +C (d) cot($e^x$) +C
4	$\int \frac{cos2x}{\sqrt{1+sin2x}}dx$ equals (a)sinx-cosx+c (b)cotx+cosx+c (c)tanx+secx+c (d) sinx+cosx+c
5	$\int \frac{1}{e^x+1}dx$ *is equal to* (a)log (e^x+1)+c (b) log (e^{-x}+1)+c (c) -log (e^{-x}+1)+c (d) none
6	$\int \frac{2}{1+cos2x}dx$ is equal to (a)tanx+c (b) –tanx+c (c) sinx+c (d) –sinx+c
7	$\int logx\,dx$ is equal to (a)log(ex)+c (b)x log X (C) x log ($\frac{e}{x}$)+c (d) x log (x/e)+c

2 MARKS QUESTIONS

Q1	$\int \frac{dx}{(e^x-1)(1-e^{-x})}$
Q2	$\int \frac{dx}{x^2-6x+13}$
Q3	$\int \frac{dx}{1+cosx}$
Q4	$\int \frac{sin2x}{\sqrt{9-cos^4x}}$dx
Q5	$\int \frac{(logx+1)^2}{x}dx$

3 MARKS QUESTIONS

Q1	$\int \frac{(1+x)^2}{\sqrt{x}}dx$
Q2	$\int \frac{x^3}{(x+1)}dx$
Q3	$\int sec^3x\,dx$
Q4	$\int \frac{(x-5)e^x}{(x-3)^3}dx$
Q5	$\int \frac{sin^3x+cos^3x}{sin^2x\,cos^2x}dx$
Q6	$\int \frac{e^{4x}-1}{e^{4x}+1}dx$ (CBSE 2024)

Q7	$\int \sqrt{1 - sin2x}dx, \frac{\pi}{4} < x < \frac{\pi}{2}$ (NCERT EXEMPLAR)
Q8	$\int \frac{2x^2+3}{x^2(x^2+9)} dx$, $x \neq 0$
Q9	$\int \frac{sin2x}{acos^2x+bsin^2x} dx$

5 MARKS QUESTIONS

Q1	$\int \frac{1}{9x^2+6x+5}$ dx
Q2	$\int \frac{sinx}{\sin(x+a)} dx$
Q3	$\int \frac{x^2}{1-x^4}$dx
Q4	$\int sinx\ sin2x\ sin3xdx$
Q5	$\int \sqrt{tanx}dx$

Answers

MCQ's

1c	2b	3b	4d	5c
6a	7d			

2MARKS QUESTIONS

1. -1/(e^x-1) +c 2. $\frac{1}{2}\tan^{-1}\frac{x-3}{2} + c$ 3. tan(x/2)+c 4. -$Sin^{-1}(\frac{1}{3}cos^2x)$+c
5. $(1+logx)^3$ /3+c

3MARKS QUESTIONS

1. $2\sqrt{x} + \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$ 2. x-$\frac{x^2}{2} + \frac{x^3}{2} - \log(x+1) + c$
3. $\frac{1}{2}secxtanx + \frac{1}{2}$log(secx+tanx)+c 4. $\frac{e^x}{(x-3)^2} + c$ 5. Secx-cosecx+c
6. $\frac{1}{2}$log($e^{4x} + 1$)+c 7. −cosx-sinx+c
8. - $\frac{1}{3x} + \frac{5}{9}\ tan^{-1}\left(\frac{x}{3}\right) + c$ 9. $\frac{1}{b-a}\log(acos^2x + bsin^2x) + c$

5 MARKS QUESTIONS

1. $\frac{1}{6}\ tan^{-1}\left(\frac{3x+1}{2}\right) + c$ 2. xcosa-sina logsin(x+a)+c
3. $\frac{1}{4}\log\left(\frac{1+x}{1-x}\right) - \frac{1}{2}tan^{-1}x + c$ 4. $-\frac{cos2x}{8} + \frac{cos6x}{24} - \frac{cos4x}{16} + c$
5. $2\sqrt{tanx}$-$\frac{1}{\sqrt{2}}tan^{-1}(\frac{tanx}{\sqrt{2}})$+c

DEFINITE INTEGRATION

First Fundamental Theorem of Calculus.

Theorem 1: Let f be a continuous function on the closed interval [a, b]and let $A(x)$be the area function. Then $A'(x) = f(x)$for all $x\epsilon[a\ ,\ b]$

Second Fundamental Theorem of Calculus.

Theorem 2: Let *f* be a Continuous function defined on the closed interval [a ,b] and F be an antiderivative of f. Then

$\int_a^b \boldsymbol{f(x)dx} = [\boldsymbol{F(x)}]_a^b$ **=F(b)-F(a)**

PROPERTIES OF DEFINITE INTEGRALS:

P_0: $\int_a^b f(x)dx = \int_a^b f(t)dt$

P_1: $\int_a^b f(x)dx = -\int_b^a f(x)dx$, in particular $\int_a^a f(x)dx$ =0

P_2: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where a<c<b

P_3: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

P_4: $\int_0^a f(x)dx = \int_0^a f(a-x)$dx

P_5: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

P_6: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx\ , if$ f(2a-x)=f(x)

= 0,if f(2a-x)=-f(x)

P_7: (i) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, , if f(x) is an even function i.e. f(-x)=f(x)

(ii) $\int_{-a}^a f(x)dx = 0$, f(x) is an odd function i.e. f(-x)=-f(x)

Solved Questions

1. $\int_2^4 \frac{x}{x^2+1}dx$

$= \frac{1}{2}[\log(x^2+1]_2^4 = \frac{1}{2}\log\left(\frac{17}{5}\right)$

2. $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{tanx}}dx$

I=$\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{tanx}}dx$..........................(1)

$= \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{cotx}}dx$

I =$\int_{\pi/6}^{\pi/3} \frac{\sqrt{tanx}}{1+\sqrt{tanx}}dx$...............................(2)

Adding (1) and (2), we get

I=$\pi/12$

3. $\int_0^{\frac{\pi}{4}} \sqrt{1-sin2x}dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{\sin^2 x + \cos^2 x - 2sinxcosx}dx$

$= \int_0^{\frac{\pi}{4}} \sqrt{(cosx-sinx)^2}dx$

$= [sinx + cosx]_0^{\pi/4} = \sqrt{2}-1$

4. $\int_0^{\pi} \frac{xtanx}{secx+tanx}dx$,use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

I=$\int_0^{\pi} \frac{xtanx}{secx+tanx}dx$ ....(i)

I=$\int_0^{\pi} \frac{(\pi-x)tanx}{secx+tanx}dx$ ....(ii)

Adding (i)and (ii), we get

2I= $\pi\int_0^{\pi} \frac{tanx}{secx+tanx}dx$

(multiplying and dividing by secx-tanx)

$2I= \pi \int_0^{\pi} (secx\, tanx - \tan^2 x)\, dx$

$2I= \pi [\, secx - tanx + 1]_0^{\pi}$

$2I=\pi(\pi - 2)$

$I= \frac{\pi}{2}(\pi - 2)$

5. $\int_2^8 |x - 5| dx$

$=\int_2^5 -(x - 5)dx + \int_5^8 (x - 5)dx$ =9

MCQ/A-R QUESTIONS

1	$\int_{-1}^{1} x^2\, tanx\, dx$ is equal to (a) 0 (b) 1 (c) 2 (d) -1
2	$\int_{-1}^{1} x\|x\|dx$ equal to (a)1 (b) 2 (c) -1 (d) 0
3	$\int_0^{\frac{\pi}{2}} \log cotx\, dx =$ (a) $\pi/4$ (b) $\pi/8$ log 2 (c) 0 (d) π log 8
4.	**Assertion Reason questions-** Each of the following questions contains two statements Assertion (A) and Reason(R).Select any one option – (a)A is true ,R is true ; R is correct explanation of A. (b) A is true ,R is true; R is not correct explanation of A. (c) A is true ,R is false. (d) A is false ,R is true. Assertion(A)- $\int_0^{2\pi} sin^3 x dx = 0$ Reason(R)- $sin^3 x$ is an odd function

2 MARKS QUESTIONS

Q1	$\int_0^{2\pi} \boldsymbol{cos^5 x dx}$
Q2	$\int_0^{a^3} \frac{x^2}{x^6 + a^6} dx$
Q3	$\int_1^{e} logx\, dx$
Q4	$\int_0^{3} \frac{1}{\sqrt{9 - x^2}} dx$

3 MARKS QUESTIONS

Evaluate-

Q1	$\int_{-2}^{2} \frac{x^2}{1+5^x} dx$
Q2	$\int_0^{3/2} \|x\, sin\pi x\| dx$

Q3	$\int_0^{\pi} \frac{e^{cosx}}{e^{cosx}+e^{-cosx}} dx$
Q4	$\int_0^{2\pi} \frac{dx}{1+e^{sinx}}$
Q5	$\int_0^{\frac{\pi}{4}} \frac{x}{1+cos2x+sin2x} dx$
Q6	$\int_0^{\frac{\pi}{2}} \sqrt{sinx} \cos^5 x \ dx$
Q7	$\int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{1/3}}{x^4}$dx

5 MARKS QUESTIONS

Q1	$\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}}dx$
Q2	Prove that $\int_0^{\pi/2} \frac{dx}{1+2cosx} = \frac{1}{\sqrt{3}} \log(2+\sqrt{3})$
Q3	. $\int_0^{\pi/4} \frac{(sinx+cosx)dx}{16+9\sin 2x}$
Q4	$\int_0^{\pi/4} (\sqrt{tanx}+\sqrt{cotx})\, dx$
Q5	$\int_{-1}^{2} \|x^3-3x^2+2x\|dx$
Q6	$\int_0^{\pi} \frac{xdx}{1+sinx}$
Q7	$\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}dx$
Q8	Prove that $\int_0^{\pi} \frac{x}{a^2\cos^2 x+b^2\sin^2 x} dx \quad = \frac{\pi^2}{2ab}$

ANSWER

MCQ-

1-a	2-d	3-c	4-b

2 MARKS QUESTIONS

1. 0 2. $\frac{1}{3a^3}\tan^{-1} a^6$ 3. 1 4. $\frac{\pi}{2}$

3 MARKS QUESTIONS

1. 8/3 2. $(2\pi+1)/\pi 2$ 3. $\pi/2$ 4. π 5. $\frac{\pi log2}{16}$ 6. 64/231 7. 6

5 MARKS QUESTIONS

1. $(\pi-2)/4$ 3. (log2)/15 4. $\pi/\sqrt{2}$ 5. 11/4 6. π 7. 2 π

Competitive Exams Questions

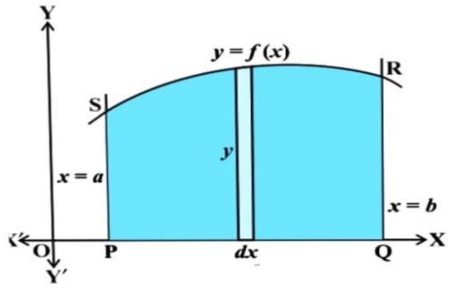
1	Evaluate $\int e^{(2logx+logx^2)}\, dx$ (NDA 2021) Answer {$x^5/5$ +c}
2	$\int e^x\{1+logx+xlogx\}dx$ (NDA 2022) Answer {xe^xlogx+c}

3	$\int \frac{3cosx+4sinx}{2cosx+5sinx}dx$ (NDA 2024) Answer($\frac{26x}{29}$+$\frac{7}{25}$log(2cosx+5sinx)+c
4	What is the value of I=$\int_0^{1.5}[x^2]dx$. where [] denotes the greatest integer function. (CUET 2024) [Hint $\int_0^1 0dx + \int_1^{\sqrt{2}} 1\ dx + \int_{\sqrt{2}}^{1.5} 2\ dx$] Answer 2-$\sqrt{2}$
5	Evaluate $\int \frac{\pi}{x^{n+1}-x}dx$ (CUET 2024) (Hint put x^n-1 =t) Answer (π/n)log{(x^n-1)/x^n}+c
6	Find the value of $\int_0^1 \frac{a-bx^2}{(a+bx^2)^2}dx$ (CUET 2024) Answer 1/(a+b)
7	$\int_2^3 \vert 2x-1\vert dx$ (CUET 2023) Answer 4
8	if I= $\int_0^{\pi/2} \frac{sin^{3/2}x}{sin^{3/2}x+cos^{3/2}x}$dx , then $\int_0^{2I} \frac{x\ sinx\ cosx}{sin^4x+cos^4x}dx$ equals…….(JEE MAINS 2025) Answer $\pi^2/16$
8.	If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{96\,x^2\cos^2 x}{(1+e^x)}dx = \pi(\alpha\pi^2+\beta), \alpha, \beta \in Z\ \ then\ (\alpha+\beta)^2$equals ……(JEE Main 2025) (Hint Apply King Property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ Answer-100
	$80\int_0^{\frac{\pi}{4}} \frac{sinx+cosx}{9+16sin2x}dx$ is equal to =…………..(JEE MAIN 2025) Answer 4 log3

CHAPTER – 8 APPLICATION OF INTEGRALS

SUMMARY OF CHAPTER

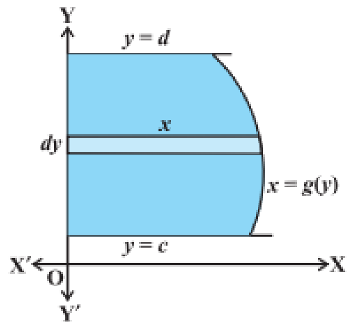
The total area A of the region between x-axis, ordinates x = a, x = b and the curve y = f (x) as the result of adding up the elementary areas of thin strips across the region PQRSP. Symbolically, we express



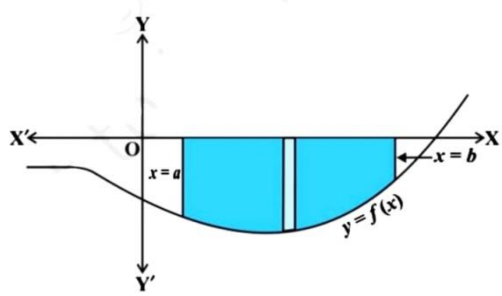
$$A = \int_a^b y\,dx = \int_a^b f(x)\,dx$$

The area A of the region bounded by the curve x = g (y), y-axis and the lines y = c, y = d is given by

$$A = \int_c^d x\,dy = \int_c^d g(y)\,dy$$

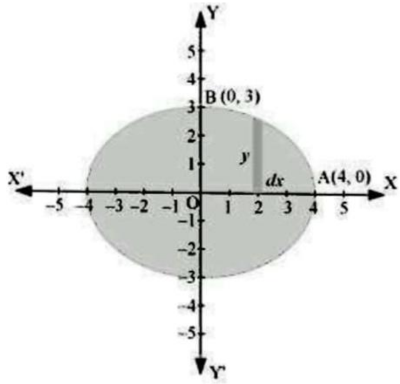
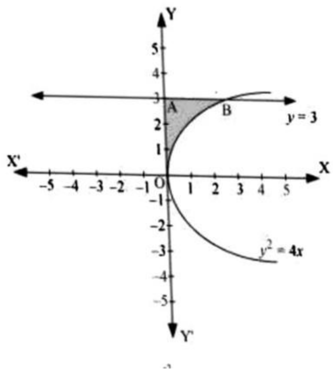
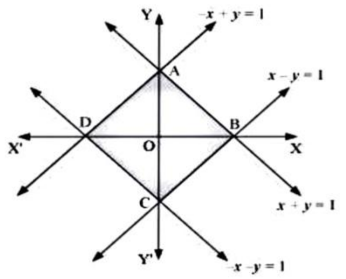
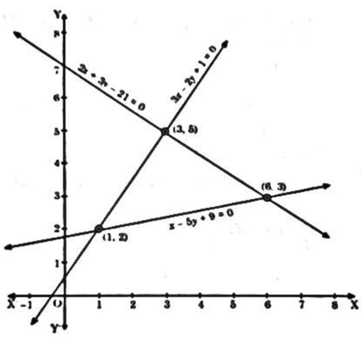


Remark If the position of the curve under consideration is below the x-axis, then since f (x) < 0 from x = a to x = b, as shown in Fig, the area bounded by the curve, x-axis and the ordinates x = a, x = b come out to be negative. But, it is only the numerical value of the area which is taken into consideration. Thus, if the area is negative, we take its absolute value, i.e.



$$A = \left|\int_a^b f(x)\,dx\right|$$

Q.NO.	**SOLVED EXAMPLES**
1	Using integration find the area enclosed by curve $9x^2 + 16y^2 = 144$

	Sol. $9\ x^{2} + 16\ y^2 = 144$ $\frac{X^2}{16} + \frac{y^2}{9} = 1$ Area of ellipse $= 4\int_0^4 y dx =$ $4\int_0^4 \frac{3\sqrt{16-x^2}}{4}\ dx$ = 12 л sq. units 
2	Using integration find the area of the region bounded by the curve $y^2 = 4x$, y-axis and the line y = 3. Sol. Required Area $= \int_0^3 x\ dy = \int_0^3 \frac{y^2}{4}\ dy$ $= [\frac{3^3-0}{12}] = \frac{9}{4}$ 
3	Using the method of integration find the area bounded by the curve $\|x\|+\|y\|=1$ Sol. Required area $= 4\int_0^1 y\ dx = 4$ $\int_0^1 (\ 1 - x\ )dx\ = 4(x - \frac{x^2}{2})_0^1$ = 2 sq.units 
4	Using the method of integration find the area of the lines 3x – 2y + 1 = 0, 2x + 3y – 21 = 0 and x – 5y + 9 = 0. Sol. On solving eqn i) & ii) and eqn ii) & iii) and eqn i) & iii) we get points of intersection as (3,5) , (6,3) & (1,2) Required Area $= \int_1^3 \frac{3x+1}{2}\ dx +$ $\int_3^6 \frac{21-2x}{3}\ dx - \int_1^6 \frac{x+9}{5}\ dx = \frac{13}{2}$ 
Q.NO.	**PRACTICE QUESTIONS**
	MCQ's(1 MARK)
1	The area bounded by the curves xy = 1, x = 1, x = 3 & y = 0 is a)log2 b) log3 c) log4 d) None of these

2	If A is the area lying between the curve y = sinx and x-axis, between x = 0 and x = $\frac{\pi}{2}$, then area of the region between the curve y = sin2x and x-axis, in the same interval is A b) 2A c) A/2 d) 3A/4
3	The area of the loop between the curve y = asinx & x-axis is a b) 2a c) 3a d) 4a
4	The area bounded by the curve $y = x\|x\|$, x-axis and the ordinates x = -1 and x = 1 is given by 0 b) 1/3 c) 2/3 d) 4/3
	ASSERTION - REASON TYPE QUESTIONS (1 MARK) **Directions**: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below. (a) Both Assertion and Reason are true and Reason is the correct explanation for assertion. (b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion. (c) Assertion is True but Reason is False. (d) Assertion is False but Reason is True.
5	**Assertion**: The area bounded by the curve y = cos x in I quadrant for $0 \leq x \leq \frac{\pi}{2}$ with x-axis is 1 sq. unit. **Reason**: $\int_0^{\frac{\pi}{2}} \cos x \, dx = 1$
6	**Assertion:** The area bounded by the lines y = x, x = -1 and x = 1 is 0 sq. units. **Reason:** Required Area = $2\int_0^1 y\,dx = 2\int_0^1 x\,dx = 1$ sq. units.
	2 MARKS QUESTIONS
7	Find the area bounded by the circle $x^2 + y^2 = a^2$.
8	Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
9	Using integration find the area bounded by the curve y = $\sqrt{16 - x^2}$ in the first quadrant.
10	The area enclosed between the graph of $y = x^3$ and the lines x = 0, y = 1, y = 8 is
	3 MARKS QUESTIONS
11	Using integration find the area bounded by the curves y = $\|x - 1\|$ and y = 1.

12	Sketch the graph of y = $\|x+3\|$ and evaluate $\int_{-6}^{0}\|x+3\|dx$.
13	Sketch the region bounded by the lines 2x + y = 8, y = 2, y = 4 and the y-axis. Hence, obtain its area using integration.
14	Using Integration find the area of the region $\{(x,y): y^2 \geq ax\ ,\ x \geq 0\ ,\ y \leq a\}$
	5 MARKS QUESTIONS
15	Using integration find the area of the region bounded by the triangle whose vertices are (-2,1),(0,4) and (2,3).
16	Using integration find the area of the triangular region whose sides have the equations y = 2x + 1 , y = 3x + 1 and x = 4.
17	Using integration find the area of the region bounded by the lines y = 2 + x , y = 2 - x and x = 2.
18	Sketch the curve y = 1 + $\|x+1\|$, x = -3 , x = 3 , y = 0 and find the area of the region bounded by them, using integration.
	COMPETITIVE EXAMS QUESTIONS
19	The area enclosed by the curves xy + 4y = 16 and x + y =6 (JEE MAINS 24) a)32 – 30 log2 b) 30 – 28 log2 c) 30 – 32 log2 d) none of these
20	If the area of the region bounded by the curve y^2 = 4ax,a > 0 and the line x = 4a is $\frac{256}{3}$ sq. units, then the value of a is (NDA 2022) 2 b) 4 c) 8 d) 16
21	The area of the region bounded by x - $\|y\| = 0$ & x – 2 = 0 is (NDA 2023) 1 b) 2 c) 4 d) 8
22	The area enclosed by the $y = \frac{\|x\|}{x}$ with x = a & x = b where $a < 0 < b$ (NDA 2019) ab b) a- b c) a+b d) none of these

Answer :

1) b

2) a

3) b

4) c

5) a

6) d

7) л a^2 sq. units

8) л ab sq. units

9) 4л sq. units

10) $\frac{45}{4}$ sq. units

11) 1 sq. units

12) 9 sq. units

13) 5 sq. units

14) $\frac{a^2}{3}$ sq. units

15) 4 sq. units

16) 8 sq. units

17) 4 sq. units

18) 16 sq. units

19) c

20) a

21) c

22) c

CHAPTER – 9 DIFFERENTIAL EQUATIONS

SUMMARY

An equation involving independent variable, dependent variable derivative or derivatives of dependent variable with respect to independent variable and constant is called differential equation.

e.g. $2x - 3y\frac{dy}{dx} = 5$ and $4\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = 0$

#ORDER OF DIFFERENTIAL EQUATION: The highest order of derivative involved in a differential equation is called order of differential equation.

e.g. $\frac{dy}{dx} + \sin x = 3$, is of first order and $\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} = 0$ is of third order differential equation.

#DEGREE OF DIFFERENTIAL EQUATION : The highest power (positive integral index) of the highest order derivative involved in differential equation , when it is written as polynomial in derivatives is called degree of differential equation.

e.g. $4\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = 0$,is of degree one.

IMPORTANT :In case of differential equation involving terms of the form $e^{dy/dx}$, $\log(\frac{dy}{dx})$,$\sin(\frac{dy}{dx})$,$\cos(\frac{dy}{dx})$ etc the degree is not defined, however if differential equation involving terms of the form e^x, e^y , log x, log y, sin x , sin y etc degree of differential equation is defined as usual.

DIFFERENTIAL EQUATION WITH VARIABLE SEPRABLE: A first order and first degree differential equation $\frac{dy}{dx} = f(x,y)$ is in the form of variable separable if it can be expressed as $\frac{dy}{dx} = h(y).g(x)$

If $h(y) \neq 0$ then write $\frac{1}{h(y)}\,dy = g(x)\,dx$

On integrating both sides $\int \frac{1}{h(y)}\,\mathrm{dy} = \int g(x)\,\mathrm{dx}$ or H(y)= G(x) + C is the required general solution where H(y) and G(x) are anti derivatives of $\frac{1}{h(y)}$ and $g(x)$, and C is arbitrary constant.

HOMOGENEOUS DIFFERENTIAL EQUATION: a differential equation $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ is said to be homogeneous , if f(x,y) ,g(x,y) are homogeneous functions of same degree i e it may be written as $\frac{dy}{dx} = \frac{x^n\ f(\frac{y}{x})}{x^n\ g(\frac{y}{x})} = \frac{f(\frac{y}{x})}{g(\frac{y}{x})} = F(\frac{y}{x})$

To check if the given equation is homogeneous or not we write equation as

$\frac{dy}{dx} = F(x,y)$ or $\frac{dx}{dy} = F(x,y)$ and replace x by λ x and y by λ y to get the function

$F(\lambda x, \lambda y) = \lambda^n F(x,y)$ here if n = 0 then differential equation is homogeneous otherwise not.

#SOLUTION OF DIFFERENTIAL EQUATION:If $\frac{dy}{dx} = F(x,y)$ is homogenous put y = vx

Hence $\frac{dy}{dx}$ = v + x $\frac{dv}{dx}$ then difeerential equation becomes variable separable.

After solving it put again v = $\frac{y}{x}$.

LINEAR DIFFERENTIAL EQUATION:A first order differential equation in which the degree of dependent variable and its derivative is **one** and they do not get multiply together is called linear differential equation of first order.

1. If general form of equation is $\frac{dy}{dx}$ +Py = Q ,where P,Q are functions of x or constant, then its solution is given by y X I.F. = $\int (Q\ X\ I.F.)\ \mathrm{dx}$ + C, where I.F. = $e^{\int P\,dx}$.
2. If general form of equation is $\frac{dx}{dy}$ +P'x = Q' ,where P',Q' are functions of y, then its solution is given by x X I.F. = $\int (Q'\ X\ I.F.)\ \mathrm{dy}$ + C, where I.F. = $e^{\int P'\,dy}$.

SOLVED EXAMPLES

Q 1	Verify that the function is a solution of the corresponding differential equation $x + y = tan^{-1} y\ ;\ y^2y' + y^2 + 1 = 0$.

Solution: $x + y = \tan^{-1} y$

$$1 + y^1 = \frac{1}{1+y^2} y^1$$

$$1 + y^2 + \cancel{y^1} + y^1y^2 = \cancel{y^1}$$

$$1 + y^2 + y^1y^2 = 0$$

Q 2	Solve the following differential equation: $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$

Solution: $x\,dy - y\,dx = \sqrt{x^2 + y^2}dx$

$$x\,dy = (y + \sqrt{x^2 + y^2}\,)dx$$

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} \quad . . . . (1)$$

Put $y = vx\ gives \frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute in equation (1), we get

$$v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$$

$$x\frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

Integrating on both sides

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\log\,|v + \sqrt{1+v^2}| = \ log|x| \ + \ log\ c$$

$$\log\,|v + \sqrt{1+v^2}| = \ log|cx|$$

$$v + \sqrt{1+v^2} = \ cx$$

$$\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = cx$$

$y \ + \ \sqrt{x^2+y^2} = cx^2$is the required solution.

Q 3	Show that the given differential equation is homogeneous and find the particular solution of the equation: $2xy + y^2 - 2x^2\frac{dy}{dx} = 0, y = 2\ when\ x = 1.$

Solution: Given differential equation is:

$$2xy + y^2 - 2x^2\frac{dy}{dx} = 0$$

$$\Rightarrow 2x^2\frac{dy}{dx} = 2xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy+y^2}{2x^2} \qquad ...(1)$$

$$\text{Let } F(x,y) = \frac{2xy+y^2}{2x^2}.$$

$$\therefore\ F(\lambda x, \lambda y) = \frac{2(\lambda x)(\lambda y)+(\lambda y)^2}{2(\lambda x)^2} = \frac{2xy+y^2}{2x^2} = \lambda^0 \cdot F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$y = vx$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting the value of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x\frac{dv}{dx} = \frac{2x(vx) + (vx)^2}{2x^2}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2v + v^2}{2}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + \frac{v^2}{2}$$

$$\Rightarrow \frac{2}{v^2}dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$2 \cdot \frac{v^{-2+1}}{-2+1} = \log|x| + C$$

$$\Rightarrow -\frac{2}{v} = \log|x| + C$$

$$\Rightarrow -\frac{2}{\frac{y}{x}} = \log|x| + C$$

$$\Rightarrow -\frac{2x}{y} = \log|x| + C \qquad ...(2)$$

Now, $y = 2$ at $x = 1$.

$$\Rightarrow -1 = \log(1) + C$$

$$\Rightarrow C = -1$$

Substituting $C = -1$ in equation (2), we get:

$$-\frac{2x}{y} = \log|x| - 1$$

$$\Rightarrow \frac{2x}{y} = 1 - \log|x|$$

$$\Rightarrow y = \frac{2x}{1 - \log|x|}, \ (x \neq 0, \ x \neq e)$$

Q 4	In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

Solution:Let y be the number of bacteria at any instant t.

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\therefore \frac{dy}{dt} \propto y$$

$$\Rightarrow \frac{dy}{dt} = ky \text{ (where } k \text{ is a constant)}$$

$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get:

$$\int \frac{dy}{y} = k \int dt$$

$$\Rightarrow \log y = kt + C \qquad ...(1)$$

Let y_0 be the number of bacteria at $t = 0$.

$\Rightarrow \log y_0 = C$

Substituting the value of C in equation (1), we get:

$$\log y = kt + \log y_0$$

$$\Rightarrow \log y - \log y_0 = kt$$

$$\Rightarrow \log\left(\frac{y}{y_0}\right) = kt$$

$$\Rightarrow kt = \log\left(\frac{y}{y_0}\right) \qquad ...(2)$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$$\Rightarrow y = \frac{110}{100} y_0$$

$$\Rightarrow \frac{y}{y_0} = \frac{11}{10} \qquad ...(3)$$

Substituting this value in equation (2), we get:

$$\frac{1}{2}\log\left(\frac{11}{10}\right)\cdot t = \log\left(\frac{y}{y_0}\right)$$

$$\Rightarrow t = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \qquad ...(4)$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1.

$\Rightarrow y = 2y_0$ at $t = t_1$

From equation (4), we get:

$$t_1 = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 100000 to 200000.

Q 5	Find the solution of the equation: $\frac{dy}{dx} - 3y cot x = sin\,2x$, when it is given that $y = 2 \;\; when \;\; x = \frac{\pi}{2}$.

SOLUTION:This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q$ (where $p = -3\cot x$ and $Q = \sin 2x$)

Now, I.F $= e^{\int p\,dx} = e^{-3\int \cot x\,dx} = e^{-3\log|\sin x|} = e^{\log\left|\frac{1}{\sin^3 x}\right|} = \frac{1}{\sin^3 x}$.

The general solution of the given differential equation is given by the relation,

$y.\frac{1}{sin^3 x} = \int[sin\,2x.\frac{1}{sin^3 x}]dx + c$

$y.cosec^3x = \int[2cot x cosec\, x.]dx$

$ycosec^3x = -2cosec\, x + c$

$y = \frac{-2}{cosec^2 x} + \frac{c}{cosec^3 x}$

$y = -2sin^2x + c\,sin^3x$

Now, $y = 2$ at $x = \frac{\pi}{2}$.

Therefore, we get:

$2 = -2 + C$

$\Rightarrow C = 4$

Substituting C = 4 in equation (1), we get:

$y = -2\sin^2 x + 4\sin^3 x$

$\Rightarrow y = 4\sin^3 x - 2\sin^2 x$

This is the required particular solution of the given differential equation.

	MULTIPLE CHOICE QUESTIONS
Q1	The integrating factor of the differential equation $(x\,log\,x)\frac{dy}{dx} + y = 2\log x$, is given by: (a) log(log x) (b) e^x (c) log x (d) x
Q2	The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is: (a) log y=k x (b) y= k x (c) x y=k (d) y=k log x
Q3	The number of arbitrary constants in the general solution of differential equation of fourth order is/ are:

	(a) 0 (c) 3	(b) 2 (d) 4
Q4	The number of arbitrary constants in the particular solution of a differential equation of third order is /are: (a) 3 (c) 1	 (b)2 (d)0
Q 5	If m and n are the order and degree of the differential equation $(y^2)^5 + \frac{4\,(y_2)^3}{y_3} + y^3 = x^2 - 1$, then: (a) m=3,n=3 (c) m=3,n=5	 (b)m=3,n=2 (d) m=3,n=1
Q 6	The solution of the differential equation $2x\,\frac{dy}{dx} - y = 3$ represents: (a) circles (c) ellipses	 (b)straight lines (d) parabolas
Q 7	Integrating factors of the differential equation $\cos x\,\frac{dy}{dx} + y\sin x = 1$, is: (a) sin x (c) tan x	 (b) sec x (d) cos x
Q 8	The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are: (a) (1,2/3) (c) (3,3)	 (b) (3,1) (d) (1,2)
Q9	The particular solution of the differential equation $log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that y=0 when x=0, is given by: (a) $4e^{3x}+3e^{-4y}-7=0$ (c) $4e^{3x}-3e^{-4y}-7=0$	 (b) $4e^{3x}+3e^{-4y+}7=0$ (d) $4e^{3x-}3e^{-4y+}7=0$
Q10	Which of the following is a homogeneous differential equation? (a) $(xy)\,dx - (x^3 + y^3)\,dy = 0$ (c) $y^2dx + (x^2 - xy - y^2)\,dy = 0$	 (b) $(x^3 + 2y^2)\,dx + 2xy\,dy = 0$ (d)$(4x + 6y + 5)\,dy - (3y + 2x + 4)\,dx = 0$
Q11	Which of the following equations has y=x as one of its particular solution? (a) $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$ (c) $\frac{d^2y}{dx^2} + x^2\frac{dy}{dx} + xy = 0$	 (b)$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$ (d)$\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} + xy = x$
Q 12	If p and q are the degree and order of the differential equation$\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} +$	

	$\frac{d^3y}{dx^3} = 4$, then the value of 2p – 3q is: (a) 7 (b) – 7 (c) 3 (d) – 3
	ASSERTION – REASON QUESTIONS
	In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices: (a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion. (b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion. (c) Assertion is correct statement but Reason is wrong statement. (d) Assertion is wrong statement but Reason is correct statement.
Q 1	**Assertion:** The differential equation of all circles in a plane must be of order 3. **Reason:** If three points are non-collinear, then only one circle passes through these points.
Q 2	**Assertion:** y = a sin x + b cos x is a general solution of y′′ + y = 0. **Reason:** y = a sin x + b cos x is a trigonometric function.
	2 MARKS QUESTIONS
Q 1	Find the general solution of the differential equation: x dy+(y-x^3) dx=0
Q 2	Solve the differential equation:$\frac{dy}{dx} + 2ytanx = sinx$
Q 3	Solve the differential equation: xy dy=(y+5) dx
Q 4	Find the solution of the differential equation: $\sqrt{a+x}\frac{dy}{dx} + x = 0$.
Q 5	Find the particular solution of the differential equation dy/dx =y tanx, given that y= 1 when x = 0
Q 6	Solve the differential equation $\frac{dy}{dx}$ = $e^{x-y} + x^3e^{-y}$
Q 7	Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$
	3 MARKS QUESTIONS
Q 1	Solve the following differential equation: $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$
Q 2	Solve the differential equation: $x^2\frac{dy}{dx} = 2xy + y^2$.

Q 3	Solve the differential equation: $x\frac{dy}{dx} = y - x\,tan\left(\frac{y}{x}\right)$.
Q 4	Find the general solution of the differential equation: $cos^2 x\ \frac{dy}{dx} + y = \ tan\,x\ ;\ \ 0 \le x \le \frac{\pi}{2}$.
Q 5	Find the particular solution of the differential equation $x\ (\ x^2 - 1)\frac{dy}{dx}\ \ = \ 1$ given that y=0 when x=2.
Q 6	Find the particular solution of the differential equation: $\frac{dy}{dx} - \frac{y}{x} + cosec\left(\frac{y}{x}\right) = 0; y = 0\ when\ x = 1.$
	5 MARKS QUESTIONS
Q 1	Show that the given differential equation is homogeneous and find the particular solution of the equation: $2xy + y^2 - 2x^2\frac{dy}{dx} = 0, y = 2\ when\ x = 1.$
Q 2	Find the general solution of the differential equation $\sqrt{1 + x^2 + y^2 + y^2x^2} + xy\frac{dy}{dx} = 0$.
Q 3	Find the particular solution of the differential equation $\frac{dy}{dx} = \ 1 + x + y + xy$, given that $y = 0\ \ when\ \ x = 1$.
	CASE BASED
Q 1	**CASE STUDY: 1** A women and child welfare society wants to spread a message about the welfare of children in a town. Message spreads in a population of 5000 people at a rate proportional to the product of number of people who have heard it and the number of people who have not. Also, it is given that 100 people initially heard the message and a total of 500 people know about the message after 2 hours. Based on above information, answer the following questions: Q.1 If p(t) denotes the number of people who know about the message at an instant t, then find the maximum value of p(t). Q 2 $\frac{dp}{dt}$is proportional to:------------- Q 3 Find the value of p(0). Q 4 Find the value of p(2).

Q 2	**CASE STUDY : 2**. A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation: $dT\ /dt \propto\ (T\ -\ 70)$, where 70°F is the room temperature and T is the temperature of the object at time t. Substituting the two different observations of T and t made, in the solution of the differential equation $dT\ /dt\ =$ k $(T\ -\ 70)$ where k is a constant of proportion, time of death is calculated. **Q 1**. State the degree of the above given differential equation. **Q 2.** Which method of solving a differential equation helped in calculation of the time of death? **Q 3**. If the temperature was measured 2 hours after 11.30pm, will the time of death change? (Yes/No) **Q 4**. Find the solution of the differential equation $dT\ /\ dt\ = k(T - 70)$.
	COMPETITIVE EXAM QUESTIONS
1.	Find the equation of curve which passes through the point(1,1) and whose slope is given by 2y/x. JEE MAINS 2022
2.	A function y = f(x) has the second order derivative $f''(x) = 6(x-1)$.If its graph passes through the point (2,1) and at the point the tangent to the graph is y=3x-5,then find the function. JEE MAINS 2016
3	Find the differential equation for all the straight lines which are at unit distance from origin. AIEEE 2011

ANSWERS:**MCQ**
1.(c),2.(b),3.(d),4.(d),5.(b),6.(d),7.(d),8.(c),9.(a),10.(c),11.(a),12.(b)

ASSERTION – REASON
Q1(b),Q2(b)

2MARKS ANSWERS

1.$xy=x^4/4+c \Rightarrow y=x^3/3+c$

2. $y=\cos x+C\cos^2 x$

3. $y-5\log|y+5|=\log|x|+c$

4. $y=-2/3(a+x)^{3/2}+2a\sqrt{a+x}+c$

5. $y = \sec x$

6.$e^y = e^x + \frac{x^4}{4} + C$

7. $\tan^{-1}y = \tan^{-1}x + C$

3 MARKS ANSWERS

1: $y + \sqrt{x^2+y^2} = cx^2$

2: $\frac{y}{y+x} = cx$

3: $Sin\frac{y}{x} = \frac{c}{x}$ $or\ x\ Sin\frac{y}{x} = c$

4:$y = (\tan x-1)+ C e^{-\tan x}$

5:$y =\frac{1}{2}\log[4(x^2-1)/3x^2]$

6: $cos\frac{y}{x} = log|x| + 1$

5MARKS ANSWERS

1:$y = \frac{2x}{(1-logx)}$, $(x\neq0, x\neq e)$

2: $\sqrt{1+y^2} = \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right| + \sqrt{1+x^2} + c$

3: $\log|1+x|= x+x^2/2-3/2$

CASE BASED

1. CASE STUDY:1

Q 1 : 5000, Q 2: p(5000-p), Q 3 :100 , Q 4: 500

3. CASE STUDY:2

Q1:Degree is 1,Q2:Variable separable method,Q3:No,Q4:$\log|T-70| = kt + C$

COMPETITIVE EXAM QUESTIONS

1.$y=x^2$

2.$f(x)=(x-1)^3$

3.$1+ (dy/dx)^2= [y-x(dy/dx)]^2$

CHAPTER-10 VECTORS

SUMMARY

* Position vector of point $A(x, y, z) = \vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$
* If $A(x_1, y_1, z_1)$ and point $B(x_2, y_2, z_2)$ then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
* If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$; $\left|\vec{a}\right| = \sqrt{x^2 + y^2 + z^2}$

*Unit vector parallel to $\vec{a} = \dfrac{\vec{a}}{\left|\vec{a}\right|}$

* Scalar Product (dot product) between two vectors: $\vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$ where θ is angle between the vectors

* $\cos\theta = \dfrac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$

* If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$

* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a}.\vec{b} = 0$

* $\vec{a}.\vec{a} = \left|\vec{a}\right|^2$

* Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}$

* Vector product between two vectors:

$\vec{a} \times \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta\,\hat{n}$; $\hat{n}$ is the normal unit vector

which is perpendicular to both $\vec{a}$ & $\vec{b}$

* $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$

* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a} \times \vec{b} = 0$

* Area of triangle (whose sides are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right|$

* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$) $= \left|\vec{a} \times \vec{b}\right|$

* Area of parallelogram (whose diagonals are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right|$

SOLVED EXAMPLES

1	The position vector of a point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$. in the ratio 1 : 2 internally is (a) $\frac{3\bar{a}+2\bar{b}}{3}$ (b) $\bar{a}$ (c) $\frac{5\bar{a}-\bar{b}}{3}$ (d) $\frac{4\bar{a}+\bar{b}}{3}$ **ANS: (*d*), as position vector=** $\frac{m\overrightarrow{OB}+n\overrightarrow{OA}}{m+n}$ $=\frac{2(\bar{a}+\bar{b})+1(2\bar{a}-\bar{b})}{1+2}$ $=\frac{4\bar{a}+\bar{b}}{3}$
2	If $\vec{a}+\vec{b}+\vec{c} = 0$ and $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = 5$ and $\lvert\vec{c}\rvert = 7$, show that angle between $\vec{a}$ and $\vec{b}$ is 60°. $Answer: GIVEN\ \vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{a}+\vec{b}=-\vec{c} \Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}).=(-\vec{c}).(-\vec{c})$ $\Rightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\vec{a}.\vec{b} = \lvert\vec{c}\rvert^2 \Rightarrow 9 + 25 + 2\vec{a}.\vec{b} = 49$ $\Rightarrow 2\ \vec{a}.\vec{b} = 15 \Rightarrow 2\lvert\vec{a}\rvert\ \lvert\vec{b}\rvert \cos\theta = 15 \Rightarrow 30\cos\theta = 15$ $\Rightarrow \cos\theta = 1/2 \Rightarrow \theta = 60^0$
3	If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{j}-\hat{k}$ find a vector $\vec{c}$ such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$ $Answer.\ Let\ \vec{c} = x\hat{i}+y\hat{j}+z\hat{k}$ $\vec{a}\times\vec{c}=\vec{b} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j}-\hat{k} \Rightarrow \hat{i}(z-y)+\hat{j}(x-z)+\hat{k}(y-x) = \hat{j}-\hat{k}$ $\Rightarrow z-y=0, x-z=1, y-x=-1$ $Also\ \vec{a}.\vec{c}=3 \Rightarrow x+y+z=3$ $\Rightarrow x+z+z=3 \Rightarrow x+2z=3$ $and\ x-z=1 \Rightarrow z=\frac{2}{3}, x=\frac{5}{3}, y=\frac{2}{3}$ $\therefore \vec{c} = \frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}\ or\ \frac{1}{3}(5\hat{i}+2\hat{j}+2\hat{k})$

4

Example 2 Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 2) and (–1, 0, 8), respetively.

Solution The vector with initial point P (1, 3, 2) and terminal point Q (–1, 0, 8) is given by

$$\overrightarrow{PQ} = (-1-1)\,\hat{i} + (0-3)\,\hat{j} + (8-2)\,\hat{k} = -2\,\hat{i} - 3\,\hat{j} + 6\,\hat{k}$$

Thus $\quad \overrightarrow{QP} = -\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$$\Rightarrow |\overrightarrow{QP}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$$

Therefore, unit vector in the direction of $\overrightarrow{QP}$ is given by

$$\widehat{QP} \quad \frac{\overrightarrow{QP}}{|\overrightarrow{QP}|} \quad \frac{2\hat{i} \quad 3\hat{j} \quad 6\hat{k}}{7}$$

Hence, the required vector of magnitude 11 in direction of $\overrightarrow{QP}$ is

$$11\,\widehat{QP} = 11\,\frac{2\hat{i} \quad 3\hat{j} \quad 6\hat{k}}{7} = \frac{22}{7}\hat{i} + \frac{33}{7}\hat{j} - \frac{66}{7}\hat{k}.$$

5

If the points (–1, –1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of m.

Solution Let the given points be A (–1, –1, 2), B (2, m, 5) and C (3, 11, 6). Then

$$\overrightarrow{AB} = (2+1)\,\hat{i} + (m+1)\,\hat{j} + (5-2)\,\hat{k} = 3\,\hat{i} + (m+1)\,\hat{j} + 3\hat{k}$$

and $\quad \overrightarrow{AC} = (3+1)\,\hat{i} + (11+1)\,\hat{j} + (6-2)\hat{k} = 4\,\hat{i} + 12\,\hat{j} + 4\hat{k}$.

Since A, B, C, are collinear, we have $\overrightarrow{AB} = \lambda\,\overrightarrow{AC}$, i.e.,

$$(3\,\hat{i} \quad (m \quad 1)\,\hat{j} \quad 3\hat{k}) \quad \lambda(4\hat{i} + 12\hat{j} + 4\hat{k})$$

$$\Rightarrow \quad 3 = 4\lambda \text{ and } m + 1 = 12\lambda$$

Therefore $\quad m = 8$.

SECTION A (MCQ- 1 MARKS EACH)

1	value of λ for which vectors $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-\lambda\hat{j}+4\hat{k}$ are orthogonal is (A)12 (B) 14 (C) 16 (D) none of these
2	If $\left\|\vec{a}\right\|$ = 5, $\|\bar{b}\|$ = 13 and $\|\bar{a}X\,\bar{b}\|$ = 25, then $\bar{a}\cdot\bar{b}$ is equal to (*a*) 12 (*b*) 5 (*c*) 13 (*d*) 60
3	If the projection of vector $\hat{i}-2\hat{j}+3\hat{k}$ on $2\hat{i}+\lambda\hat{k}$ is zero , find λ (*a*) 0 (*b*) 1 (*c*) -2/3 (*d*) 3/2
4	If $\vec{a}$ and $\vec{b}$ are such that $\|\vec{a}.\vec{b}\|$= $\|\vec{a}X\vec{b}\|$, then the angle between $\vec{a}$ and $\vec{b}$ is $(a)\,\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$
5	Value of the following is $\hat{\imath}.(\hat{\jmath}\times\hat{k})+\hat{\jmath}.(\hat{k}\times\hat{\imath})+\hat{k}.(\hat{\imath}\times\hat{\jmath})$. (a) 1 (b) 2 (c) 3 (d) 0
6	Let $\vec{a}\; and\; \vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if (a) $\theta=\frac{\pi}{4}$ (b) $\theta=\frac{\pi}{3}$ (c)$\theta=\frac{\pi}{2}$ (d) $\theta=\frac{2\pi}{3}$
7	The area of triangle formed by vertices formed by O,A,B where $\overrightarrow{OA}$= $\hat{i}+2j+3k$ and $\overrightarrow{OB}$= $-3\hat{i}-2\hat{j}+\hat{k}$ is (a) $3\sqrt{5}$ sq. unit (b) $5\sqrt{5}$ sq. unit (c)$6\sqrt{5}$ sq. unit (d) 4 sq. unit
8	The number of vectors of unit length perpendicular to vectors $\vec{a}$= $2\hat{i}+j+2k$ and $\vec{b}=j+k$ is (a) 1 (b) 2 (c) 3 (d) infinite
9	If $\vec{a}+\vec{b}=\hat{i}$ and $\vec{a}=2\hat{i}-2j+2k, then\ \|\vec{b}\|$ equals (a) $\sqrt{14}$ (b) $\sqrt{12}$ (c) 3 (d) $\sqrt{17}$
10	$\|\vec{a}\|=10\; and\; \|\vec{b}\|=2$ and $\bar{a}\cdot\bar{b}=12$ then value of $\|\vec{a}X\vec{b}\|$ is (a) 5 (b)10 (c) 14 (d) 16
11	If $\vec{a}$ = $3\hat{i}+2\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}+\hat{j}-3\hat{k}$ and $\vec{c}=6\hat{i}-\hat{j}+2\hat{k}$ are three given vectors, then $(2\vec{a}\,.\,\hat{i})\hat{i}-(\vec{b}\,.\,\hat{j})\hat{j}+(\vec{c}\,.\,\hat{k})\hat{k}$ is same as the vector (A) $\vec{a}$ (B) $\vec{b}+\vec{c}$ (C) $\vec{a}-\vec{b}$ (D) $\vec{c}$

Assertion- Reason question.

(A) Both (a) and (b) are correct and (b)is the correct explanation of (a)

(B) Both (a) and (b) are correct and (b)is not the correct explanation of (a)

(C) (a) is correct but (b) is false

(D) (a) is false but (b) is correct

1- Assertion: (a) Two vectors are said to be like vectors if they have the same direction but different magnitude.

Reason: (b) Vector quantities do not have a specific direction.

2- Assertion(A): Given two non-zero vectors $\vec{a}$ and $\vec{b}$. If $\vec{r}$ is another non-zero vector such that $\vec{r}\ .\ (\vec{a} + \vec{b}\ ) = \vec{0}$. Then $\vec{r}$ is perpendicular to $\vec{a} + \vec{b}$.

Reason (R): The vector $(\vec{a} + \vec{b}\ )$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$

3- Assertion(a): If the difference of two unit vectors is again a unit vector then angle between them is 60^0

Reason (b): If angle between $\vec{a}$ & $\vec{b}$ is acute than $|\ \vec{a}.\vec{b}\ | < |\ \vec{a}\ |\ |\vec{b}\ |$

SECTION B(2 MARKS)

1	Write the value of p for which, $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors.
2	If $\|\vec{a}\| = 13, \|\vec{b}\| = 5$ and $\vec{a}.\vec{b} = 60$, then find $\|\vec{a} \times \vec{b}\|$
3	If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a}\ .\ \vec{b} = \vec{a}\ .\vec{c}$ then find the angle between $\vec{a}$ and $\vec{b} - \vec{c}$.
4	For given vectors $\vec{a} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k})$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $find\ a$ unit vector in the direction of the vector $\vec{a} + \vec{b}$.
5	Find the position vector of a point R which divides the line joining two points with position vectors $(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $(-\hat{\imath} + \hat{\jmath} + \hat{k})$ in the ratio 2: 1
6	If $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$, such that $\|\vec{a}\| = 2, \|\vec{b}\| = 3, \|\vec{c}\| = 4$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then prove that $\vec{a} \cdot (\vec{b} \times \vec{c}\ ) = 12\sqrt{3}$.
7	If $4\hat{\imath} + \hat{\jmath} + 3\hat{k}, \hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ and $3\hat{\imath} + 4\hat{\jmath}$ are the position vectors of the vertices, A, B and C of the Δ ABC respectively. Find the angle between the median AD and the side BC of the ΔABC.
8	Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$, if $\|\vec{a}\| = 1, \|\vec{b}\| = 4$ and $\|\vec{c}\| = 2$.

9	Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.
10	The two vectors $\hat{i} + \hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 3\hat{k}$ represent the two sides OA and OB,respectively of a ΔOAB, where O is the origin.The point P lies on AB such that OP is a median. Find the area of the parallelogram formed by the two adjacent sides as OA and OP.

SECTION C(3 MARKS)

1	If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $\vec{b} = -1\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ then find the value of 'λ'.
2	Find the magnitude of vectors $\vec{a}$ and $\vec{b}$ having same magnitude such that angle between them is $60°$ and their scalar product is $\frac{1}{2}$.
3	Find a vector of magnitude 4 units, and parallel to the resultant of the vectors $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.
4	Find the value of the constant M such that the scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with the unit vector parallel to the sum of the vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $M\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one.
5	Three vectors $\vec{a}$,$\vec{b}$ and $\vec{c}$ satisfying the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ if $\lvert\vec{a}\rvert = 1$, $\lvert\vec{b}\rvert = 4$, $\lvert\vec{c}\rvert = 2$.
6	Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{d}.\vec{c} = 21$.

SECTION D(5 MARKS EACH)

1	Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = 4$ and $\lvert\vec{c}\rvert = 5$ and each one of them being perpendicular to the sum of other two, find $\lvert\vec{a} + \vec{b} + \vec{c}\rvert$.
2	Express the vector $5\hat{i} - 2\hat{j} + 5\hat{k}$ as the sum of two vectors such that one is parallel and other is perpendicular to the vector $3\hat{i} + \hat{k}$.

3	If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq 0$, then prove that $\vec{b}=\vec{c}$.

SECTION E(CASE BASED QUESTIONS)

1	The paths of two aeroplanes one taking off and the other just landing on the runway are respectively along the vectors $(\hat{\imath}+2\hat{\jmath}+\hat{k})$ and $(2\hat{\imath}-\hat{\jmath}-\hat{k})$  i) Find a vector perpendicular to both vectors ii) Find the projection off first vector on second

SECTION E(COMPTETIVE EXAM QUESTIONS)

1	If $\vec{a}$ is a unit vector and $(2\vec{x}-3\vec{a}).(2\vec{x}+3\vec{a}) = 91$, then write the value of $\lvert\vec{x}\rvert$. CUET 2024
2	If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then write the values of θ for which $\vec{a}\,.\,\vec{b} \geq 0$. NDA 2024
3	Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 4$ and $\lvert\vec{c}\rvert = 5$ and each one of them being perpendicular to the sum of other two, find $\lvert\vec{a}+\vec{b}+\vec{c}\ \rvert$. CUET 2025
4	A bird flies through a distance in a straight line given by the vector $\hat{i} + 2\hat{j} + \hat{k}$. A man standing beside a straight metro rail track given by $\vec{r} = (3 + \lambda)\hat{\imath} + (2\lambda - 1\ \hat{\jmath} + 3\lambda\hat{k}$ is observing the bird. The projected length of its flight on the metro track is NDA 2025 (A) 6/ √14 units (B) 14/√6 units (C) 8/√14 units (D) 5/√6 units
5	The dot product of a vector with the vectors $\hat{\imath} - 3\hat{k}$, $\hat{\imath} - 2\hat{k}$ and $\hat{\imath} + \hat{\jmath} + 4\hat{k}$ are 0 , 5 and 8 respectively. Find the vector. JEE MAINS 2023

ANSWERS (SECTION – A MCQ)

1- (B) 14	**2-**(d) 60	**3-(C) -2/3**	**4-**(c) $\frac{\pi}{4}$	**5-** (c) 3
6- (d) $\theta = \frac{2\pi}{3}$	**7-**(a) $3\sqrt{5}$ sq. unit	**8-**(b) 2	**9-**(c) 3	**10-**(d) 16
11- (D) $\vec{c}$				

ANSWERS: ASSERTION- REASONING

1- C **2- C** **3- B**

ANSWERS – SECTION B (2 MARKS QUESTIONS)

1- P=2/3	2 -$\left\|\vec{a} \times \vec{b}\right\| = 25$	$3 - \frac{\pi}{2}$	$4 - \frac{3\hat{\imath} + \hat{k}}{\sqrt{10}}$	$5\,\frac{-\hat{\imath} + 4\hat{\jmath} + \hat{k}}{3}$
6 – correct proof	$7 - \frac{\pi}{2}$	8 - -21/2	9 - $15\sqrt{2}$ Sq unit	10 - $2\sqrt{2}$ Sq. Unit

ANSWERS

SECTION C(3 MARKS)

1- $\boldsymbol{\lambda = 8}$	**2- X=1**	3- $\frac{\mathbf{8}}{\mathbf{3}}\hat{\imath} + \frac{\mathbf{8}}{\mathbf{3}}\hat{\jmath} + \frac{\mathbf{4}}{\mathbf{3}}\hat{k}$
4 - M=1	**5 - -21/2**	**6 -** $\mathrm{d} = 7\hat{\imath} - 7\hat{\jmath} - 7\hat{k}$

ANSWERS

SECTION D(5 MARKS)

1 - $5\sqrt{2}$

2 - PARALLEL VECTOR= $\widehat{6\imath} + 2\,\hat{k}$

PERPENDICULAR VECTOR= $-\hat{\imath} - 2\hat{\jmath} + 3\,\hat{k}$

Answer CASE STUDY

i) - $\hat{\imath} + 3\hat{\jmath} - 5\,\hat{k}$ (ii)$\frac{-1}{\sqrt{6}}$

ANSWERS

SECTION E(COMPTETIVE EXAM QUESTIONS)

1.- 5

2- 0$\leq \boldsymbol{\theta} \leq \boldsymbol{\pi}/\mathbf{2}$

3 - $5\sqrt{2}$

4 - $\frac{8}{\sqrt{14}}$ 5 - $15\hat{\imath} - 27\hat{\jmath} + 5\,\hat{k}$

CHAPTER- 11 : Three Dimensional Geometry
Summary

1.Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.

2. If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$

3. Direction cosines of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\frac{x_2 - x_1}{PQ}$, $\frac{y_2 - y_1}{PQ}$, $\frac{z_2 - z_1}{PQ}$ Where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

4.Direction ratios of a line are the numbers which are proportional to the direction cosines of a line.

5. Direction ratios of a line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

5.Vector equation of a line passing through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

6. Cartesian equation of a line that passes through a point A (x_1, y_1, z_1) and having direction ratios a,b,c is $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

7. A general point on the line is P $(a\lambda + x_1, b\lambda + y_1, c\lambda + z_1,)$.

8.Equation of a line passing through two-point whose position vectors are $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.

9. Cartesian equation of a line that passes through two points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) is, $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_2} = \frac{z - z_1}{z_2 - z_1}$.

10. For lines having angle θ between them

$\vec{r}_1 = \vec{a}_1 + \lambda\overrightarrow{b1}, \vec{r}_2 = a_2 + \mu\overrightarrow{b2}$ or $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ & $\frac{x - x_2}{a} = \frac{y - y_2}{b} = \frac{z - z_2}{c}$

$$cos\theta = \frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1||\vec{b}_2|} = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

* For perpendicular lines: $\vec{b}_1 . \vec{b}_2 = a_1a_2 + b_1b_2 + c_1c_2 = 0$

*For parallel lines: $\vec{b}_1 = \mu\vec{b}_2$ $i.e.$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

11. Shortest distance between two skew lines

$\vec{r}_1 = \vec{a}_1 + \lambda\overrightarrow{b1}, \vec{r}_2 = \vec{a}_2 + \mu\overrightarrow{b2}$

S.D. = $\left| \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2).}{|\vec{b}_1 \times \vec{b}_2|} \right|$ $(\vec{b}_1 \times \vec{b}_2 \neq 0)$

*Condition for two lines to intersect$[(\vec{a}_2 - \vec{a}_1).(\vec{m}_1 x \vec{m}_2)] = 0$

12. Shortest distance between two parallel line $\vec{r}_1 = \vec{a}_1 + \lambda\vec{b}, \vec{r}_2 = \vec{a}_2 + \mu\vec{b}$ is

SD= $\left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$, $(\vec{b}_1 \times \vec{b}_2 = 0)$

Solved examples:

.

1. **Write the vector equation of a line passing through point $(1, -1, 2)$ and parallel to the line whose equation is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.**

Sol: The Vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \mu\vec{b}$ $where\ \mu \in R$

Given point on the line is $(1,-1,2)$ then $\vec{a} = (\hat{\imath} - \hat{\jmath} + 2\hat{k})$
and a vector parallel to the line $\frac{(x-3)}{1} = \frac{(y-1)}{2} = \frac{(z+1)}{-2}$ is $b^{\rightarrow} = i^{\wedge} + 2j^{\wedge} - 2k^{\wedge}$
Hence, Direction Ratios of a required line $are\ 1,2\ and\ -2$
So, required vector equation of line is

$$\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} - 2\hat{k}), where\ \lambda \in R$$

and the corresponding vector equation is $\vec{r} = \hat{\imath} + 2\hat{\jmath} - \hat{k} + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$.

2.**Find the shortest distance between the Lines:** $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ **and** $\vec{r} = (5\hat{\imath} - \hat{\jmath}) + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$**. Also, find whether the lines are intersecting or not.**

Sol: The given two lines are
$\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = (5\hat{\imath} - \hat{\jmath}) + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$
By observing the given two lines, we have
$\overrightarrow{a_1} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$, $\overrightarrow{b_1} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$
$\overrightarrow{a_2} = 5\hat{\imath} - \hat{\jmath}$, $\overrightarrow{b_2} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$
Hence, Shortest distance between the above lines = $\left|\frac{(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = 8\hat{\imath} + 0\hat{\jmath} - 4\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{80}$

Now, S.D = $\left|\frac{(2\hat{\imath}-3\hat{\jmath}+4\hat{k}).(8\hat{\imath}+0\hat{\jmath}-4\hat{k})}{3\sqrt{80}}\right|$
$= \frac{0}{\sqrt{80}} = 0$

If shortest distance is zero then the given two lines are intersecting.

3.Find the equation of the line which bisects the line segment joining points $A(2,3,4)$ and $B(4,5,8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and

$$\boldsymbol{\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}}$$

Sol: The midpoint of line joining points $A(2,3,4)$ and $B(4,5,8)$ is $(3,4,6)$
So, the equation of line passing through $(3,4,6)$ with direction ratios a,b,c is $\frac{x-3}{a} = \frac{y-4}{b} = \frac{z-6}{c}$
Since this line is perpendicular to $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$$\Rightarrow 3a - 16b + 7c = 0$$
$$\Rightarrow 3a + 8b - 5c = 0$$

by using cross multiplication method, we have a=2,b=3,c=6
Hence, the required equation of a line passing through (3,4,6) with d.r's <2,3,6> is

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-6}{6}$$

4.Find the vector and Cartesian equations of the line which is perpendicular to the lines with equations $\boldsymbol{\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}\ and\ \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}}$

and passes through the point (1, 1, 1).

Sol: Let equation of line through (1, 1, 1) with direction ratios a,b, c be $\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c}$.................(i)

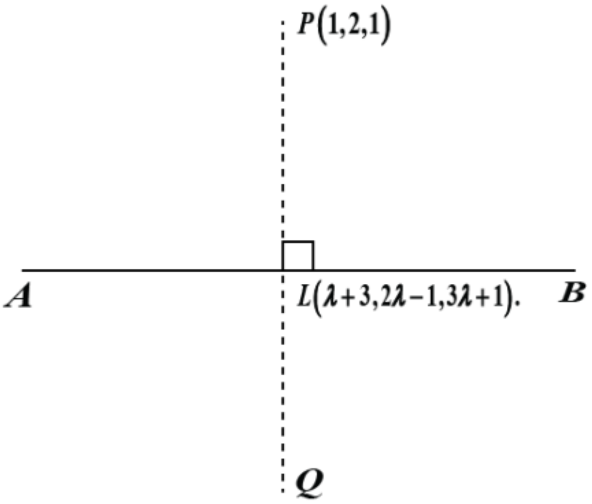
If Line(i) perpendicular to the lines $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ *and* $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

Therefore, a+2b+4c=0 and 2a+3b+4c=0

∴ dr's are -4, 4, -1 and 4, -4, 1

∴ Hence, the Cartesian equation of a line is $\frac{x-1}{4}=\frac{y-1}{-4}=\frac{z-1}{1}$ and vector equation of a line is $\vec{r}=(\hat{\imath}+\hat{\jmath}+\hat{k})+\lambda(4\hat{\imath}-4\hat{\jmath}+\hat{k})$

5. Find the image of the point (1,2, 1) with respect to the line $\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-1}{3}$. Also find the equation of the line joining the given point and its image.



Sol: Let $\mathrm{P}(1,2,1)$ be the given point and L be the foot of the perpendicular from P to the given line AB (as shown in the figure above).

Let's put $\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-1}{3}=\lambda$. Then, $x=\lambda+3, y=2\lambda-1, z=3\lambda+1$

Let the coordinates of the point L be $(\lambda+3,2\lambda-1,3\lambda+1)$.

So, direction ratios of PL are $(\lambda+3-1,2\lambda-1-2,3\lambda+1-1)$ *i.e.*, $(\lambda+2,2\lambda-3,3\lambda)$

Direction ratios of the given line are 1,2 and 3, which is perpendicular to PL.

Therefore, we have,

$$(\lambda+2)\cdot 1+(2\lambda-3)\cdot 2+3\lambda\cdot 3=0 \Rightarrow 14\lambda=4 \Rightarrow \lambda=2/7$$

Then, $\lambda+3=\frac{2}{7}+3=\frac{23}{7}$; $2\lambda-1=2\left(\frac{2}{7}\right)-1=-\frac{3}{7}$; $3\lambda+1=3\left(\frac{2}{7}\right)+1=\frac{13}{7}$

Therefore, coordinates of the point L are $\left(\frac{23}{7},-\frac{3}{7},\frac{13}{7}\right)$.

Let $\mathrm{Q}(x_1, y_1, z_1)$ be the image of $\mathrm{P}(1,2,1)$ with respect to the given line. Then, L is the mid-point of PQ.

Therefore, $\frac{1+x_1}{2}=\frac{23}{7}, \frac{2+y_1}{2}=-\frac{3}{7}, \frac{1+z_1}{2}=\frac{13}{7} \Rightarrow x_1=\frac{39}{7}, y_1=-\frac{20}{7}, z_1=\frac{19}{7}$

Hence, the image of the point $\mathrm{P}(1,2,1)$ with respect to the given line $\mathrm{Q}\left(\frac{39}{7},-\frac{20}{7},\frac{19}{7}\right)$.

The equation of the line joining $\mathrm{P}(1,2,1)$ and $\mathrm{Q}\left(\frac{39}{7},-\frac{20}{7},\frac{19}{7}\right)$ is

$$\frac{x-1}{32/7} = \frac{y-2}{-34/7} = \frac{z-1}{12/7} \Rightarrow \frac{x-1}{16} = \frac{y-2}{-17} = \frac{z-1}{6}$$

6. Find the shortest distance between the lines whose vector equations are

$$\vec{\mathbf{r}} = (\mathbf{1-t})\hat{\mathbf{i}} + (\mathbf{2-t})\hat{\mathbf{j}} + (\mathbf{3-2t})\hat{\mathbf{k}} \ \ \& \ \vec{\mathbf{r}} = (\mathbf{s+1})\hat{\mathbf{i}} + (\mathbf{2s-1})\hat{\mathbf{j}} - (\mathbf{2s+1})\hat{\mathbf{k}}$$

SOL: The equations of the given lines are

$$\vec{r} = (1-t)\hat{\imath} + (2-t)\hat{\jmath} + (3-2t)\hat{k} \ \ \& \ \vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$$

After writing standard equation of a line, then we have

$$\vec{r} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k} + t\left(-\hat{\imath} + \hat{\jmath} - 2\hat{k}\right) \quad \& \quad \vec{r} = \hat{\imath} - \hat{\jmath} - \hat{k} + s\left(\hat{\imath} + 2\hat{\jmath} - 2\hat{k}\right)$$

Shortest Distance between the lines = $\frac{\left|(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})\right|}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}$

$\overrightarrow{a_1} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k} \qquad \overrightarrow{a_2} = \hat{\imath} - \hat{\jmath} - \hat{k} \qquad \overrightarrow{a_2} - \overrightarrow{a_1} = 0\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$\overrightarrow{b_1} = -\hat{\imath} + \hat{\jmath} - 2\hat{k} \qquad \overrightarrow{b_2} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{\imath}(-2+4) - \hat{\jmath}(2+2) + \hat{k}\,(-2-1) = 2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{4+16+9} = \sqrt{29}$$

$$\text{SD} = \frac{\left|(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})\right|}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} = \left|\frac{(0\hat{\imath}+\hat{\jmath}-4\hat{k}) \cdot (2\hat{\imath}-4\hat{\jmath}-3\hat{k})}{\sqrt{29}}\right|$$

$$\text{SD} = \left|\frac{(0)(2)+(1)(-4)+(-4)(-3)}{\sqrt{29}}\right| = \left|\frac{-4+12}{\sqrt{29}}\right| = \frac{8}{\sqrt{29}} = \frac{8\sqrt{29}}{29}$$

MCQ/A-R Questions	
1	Find the direction cosines of the line that makes equal angles with the three axes in space. (a) ± 1/ √2 (b) ±1 (c) ± 1/√ 3 (d) √3
2	The equation of y-axis in space is (a) x = y = 0 (b) x = z = 0 (c) y = z = 0 (d) y = 0
3	The shortest distance between the two lines are zero if the lines are (a) Intersecting (b) parallel (c). Skew (d). none of these
4	The length of perpendicular from origin to the line $\vec{r} = \left(4\hat{\imath} + 2\hat{\jmath} + 4\hat{k}\right) + \lambda\left(\hat{3\imath} + 4\hat{\jmath} - 5\hat{k}\right)$ is (a) 2 (b) 2√3 (c) 6 (d) 7
5	If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to (a) −10 /7 (b) − 7/ 10 (c) −10 (d) −7
2 Marks Questions	
6	Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.
7	Write the vector equation of a line passing through the point (1, −1, 2) and parallel to the line whose equation is $\frac{(x-3)}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$
8	If a line has direction ratios proportional to 2, -1, -2, then what are its direction cosines?
9	Find the vector equation of the line passing through the points (-1, 0, 2)

	and (3, 4, 6).
10	If the angle between the lines $\frac{x-5}{\alpha}=\frac{y+2}{-5}=\frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1}=\frac{y}{0}=\frac{z}{1}$ is $\frac{\pi}{4}$. Find the relation between α and β.
3 Marks Questions	
11	Find the vector and Cartesian equations of the line through the point (5, 2, -4) and which is parallel to the vector $3\hat{\imath}+2\hat{\jmath}-8\hat{k}$
12	Find the points on the line $\frac{(x+2)}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3)
13	Find the shortest distance between the lines $\vec{r}=(4\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+2\hat{\jmath}-3\hat{k})$ and $\vec{r}=(\hat{\imath}-\hat{\jmath}+2\hat{k})+\mu(\widehat{2\imath}+4\hat{\jmath}-5\hat{k})$
14	Find the shortest distance between the lines whose vector equations are $\vec{r}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})+\lambda(\widehat{2\imath}+3\hat{\jmath}+4\hat{k})$and $\vec{r}=(2\hat{\imath}+4\hat{\jmath}+5\hat{k})+\mu(4\hat{\imath}+6\hat{\jmath}+8\hat{k})$
15	Find the angle between the pair of lines given by $\vec{r}=(3\hat{\imath}+2\hat{\jmath}-4\hat{k})+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=(\hat{\imath}-2\hat{\jmath})+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$
16	Find the distance between the lines l_1 and l_2 given by $\vec{r}=(\hat{\imath}+2\hat{\jmath}-4\hat{k})+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=(3\hat{\imath}+3\hat{\jmath}-5\hat{k})+\mu(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$
17	Find the angle between the following pair of lines: $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ *and* $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$
5 Marks Questions	
18	Prove that the lines: $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect each other and find the point of intersection.
19	Find the equation of a line passing through the point (2,1,3) and perpendicular to the lines: $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and : $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$
20	Find the image of the point (1, 6, 3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$
21	Let the point P(5, 9, 3) lies on the top of Qutub Minar, Delhi. Find the image of the point on the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$
22	Two bikers are running at the Speed more than allowed speed on the road along the Lines $\vec{r}=(\hat{\imath}+\hat{\jmath}-\hat{k})+\lambda(3\hat{\imath}-\hat{\jmath})$ and $\vec{r}=(4\hat{\imath}-\hat{k})+\mu(\hat{\imath}+3\hat{k})$ Using Shortest distance formula check whether they meet to an accident or not? While driving should driver maintain the speed limit as allowed. Justify?
23	Prove that the line through A(0, −1, −1) and B(4, 5,1) intersects the line

	through C(3, 9, 4) and D(−4, 4, 4).
24	Show that the lines $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$and $\vec{r} = (5\hat{\imath} - 2\hat{\jmath}) + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ are intersecting. Hence find their point of intersection.
25	Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{(x-2)}{1} = \frac{y-3}{3} = \frac{z+6}{5}$ intersect. Also find their point of intersection
26	Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. Also find the coordinates of the foot of the perpendicular and the equation of the perpendicular.
27	Find the equation of the line passing through the point (−1, 3, −2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$
Case Study Questions	
28	The equation of motion of a rocket are : x = 2t, y = -4t, z = 4t, where the time 't' is given in seconds, and the distance measured is in kilometres Based on the above information, answer the following questions  a. What is the path of the rocket? b. At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds? b. If the position of rocket at certain instant of time is (5, -8, 10), then what will be the height of the rocket from the ground, which is along the xy-plane?
Competitive exam questions	
29	If the direction cosines $< l, m, n >$ of a line are connected by the relation L+2m+n=0, 2l-2m+3n=0, then what is the value of $l^2 + m^2 - n^2$? a. 1/101 b.29/101 c. 41/101 d.92/101 **NDA-2024**
30	The shortest distance between the lines 2x+y+z=1, 3x+y+2z=2, x=y=z. a.$1/\sqrt{2}$ b. $\sqrt{2}$ c.$3/\sqrt{2}$ d. $\sqrt{3}/2$ **CUET-2024**
31	Find the angle between the pair of lines given by $\vec{r} = (3\hat{\imath} - 2\hat{\jmath} + \hat{k}) + \lambda(4\hat{\imath} + 6\hat{\jmath} + 12\hat{k})$and $\vec{r} = (7\hat{\imath} - 3\hat{\jmath} + 9\hat{k}) + \mu(5\hat{\imath} + 8\hat{\jmath} - 4\hat{k}))$ **a.** $\cos^{-1}\left(\frac{5}{21}\right)$ $b. \cos^{-1}\left(\frac{5}{21}\right)$ $c. \cos^{-1}\left(\frac{5}{21}\right)$ $d \cos^{-1}\left(\frac{5}{21}\right)$. **CUET-2024**
32	The point of intersection of the lines $\frac{(x-1)}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{(x-4)}{5} = \frac{y-1}{2} = z$ is

	a. (1,1,1) b. (1,-1,-1) c. (-1,1,-1) d. (-1,-1,-1) **CUET-2022**
33	The direction cosines of the line which is perpendicular to the lines with direction ratios 1, -2, -2 and 0,2,1 are: a. $\frac{2}{3}$, $\frac{-1}{3}$, $\frac{,2}{3}$ b. $-\frac{2}{3}$, $\frac{-1}{3}$, $\frac{2}{3}$ c. $\frac{2}{3}$, $\frac{-1}{3}$, $\frac{-,2}{3}$ d. $\frac{2}{3}$, $\frac{1}{3}$, $\frac{2}{3}$ **CUET-2024**

Answer

1. (c) ± 1/√ 3	2. x = z = 0	3. (a) intersecting	4. (d) 7
5. (a) -10/7	6. 70/11	**7.** $\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$	**8.** 2/3,-1/3,-2/3
9. $\vec{r} = (-\hat{\imath} + 2\hat{k}) + \lambda(\hat{\imath} + \hat{\jmath} + \hat{k})$	**10.** $\boldsymbol{\alpha\beta = \frac{25}{2}}$	**11.** $\vec{r} = (5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(3\hat{\imath} + 2\hat{\jmath} - 8\hat{k})$, $\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$	**12.(-2,-1,3) and (4,3,7)**
13. $\boldsymbol{6\sqrt{5}/5}$	**14.** $\boldsymbol{6/\sqrt{29}}$	**15.** $\boldsymbol{\cos^{-1}(\frac{19}{21})}$	**16.** $\boldsymbol{\sqrt{293}/7}$
17. $\boldsymbol{\cos^{-1}(\frac{26}{9\sqrt{38}})}$	**18. (5,-7,6)**	**19.** $\frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$	**20. (1,0,7)**
21.(1,1,11)	**22.will not meet**	**23. proof**	**24. (-1,-6,-12)**
25.(1/2,-1/2,-3/2)	**26.foot (3,-4,-2) , distance =2**$\boldsymbol{\sqrt{6}}$ **and equation** $\frac{x-1}{1} = \frac{y}{-2} = \frac{z}{-1}$	**27.** $\frac{x+1}{-8} = \frac{y-3}{-7} = \frac{z+2}{-5}$	**28.(a) straight line (b)** $\boldsymbol{\sqrt{650}}$ **(c) 10 km**
29.c. 41/101	**30.** a. $1/\sqrt{2}$	**31.** c. $\cos^{-1}(5/21)$	**32.** a.(1,1,1)
33. a.(2/3,-1/3,2/3)			

CHAPTER-12 (LINEAR PROGRAMING)
SUMMARY

**** An Optimisation Problem** A problem which seeks to maximise or minimise a function is called an optimisation problem. An optimisation problem may involve maximisation of profit, production etc or minimisation of cost, from available resources etc.

**** Linear Programming Problem (LPP)**
A linear programming problem deals with the optimisation (maximisation/minimisation) of a **linear function** of two variables (say x and y) known as **objectivefunction** subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (called **linear constraints)**. A linear programming problem is a special type of optimisation problem.

**** Objective Function** Linear function $Z = ax + by$, where a and b are constants, which has to be maximised or minimised is called a linear objective function.

**** Decision Variables** In the objective function $Z = ax + by$, x and y are called decision variables.

**** Constraints** The linear inequalities or restrictions on the variables of an LPP are called **constraints**. The conditions $x \geq 0$, $y \geq 0$ are called non-negative constraints.

**** Feasible Region** The common region determined by all the constraints including non-negative constraints $x \geq 0$, $y \geq 0$ of an LPP is called the feasible region for the problem.

**** Feasible Solutions** Points within and on the boundary of the feasible region for an LPP represent feasible solutions.

**** Infeasible Solutions** Any Point outside feasible region is called an infeasible solution.

**** Optimal (feasible) Solution** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

** Let R be the feasible region (convex polygon) for an LPP and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

** Let R be the feasible region for a LPP and let $Z = ax + by$ be the objective function. If R is **bounded**, then the objective function Z has both a maximum and a minimum value on R and each of these occur at a corner point of R. If the feasible region R is **unbounded**, then a maximum or a minimum value of the objective function may or may not exist. However, if it exits, it must occur at a corner point of R.

<u>SOLVED EXAMPLE</u>:

1- The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If Z = 18x + 9y be the objective function, then

(A) Z is maximum at (2, 72), minimum at (15, 20)
(B) Z is maximum at (15, 20), minimum at (40, 15)
(C) Z is maximum at (40, 15), minimum at (15, 20)
(D) Z is maximum at (40,15), minimum at (2, 72)

Answer: (C) Z is maximum at (40, 15), minimum at (15, 20)

Points	Value of Z = 18x + 9y
(2, 72)	684
(15, 20)	450
(40, 15)	855

2- If the feasible region of a L.P.P. with objective function Z = ax + by, is bounded, then which of the following is correct ?

(A) It will only have a maximum value.
(B) It will only have a minimum value.
(C) It will have both maximum and minimum value.
(D) It will have neither maximum nor minimum value.

Answer: (C) It will have both maximum and minimum value.

3-Solve the following linear programming problem graphically:

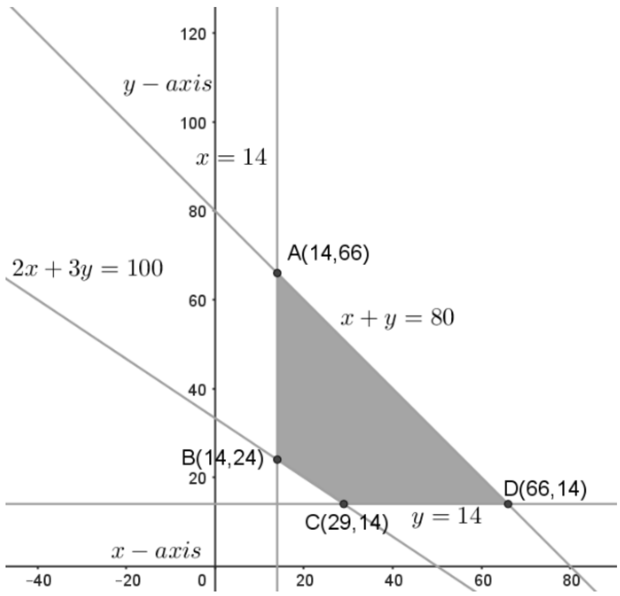
Maximise: Z = 20 x + 30 y

Subject to the constraints:

$x + y \leq 80$, $2x + 3y \geq 100$

$x \geq 14$, $y \geq 14$.

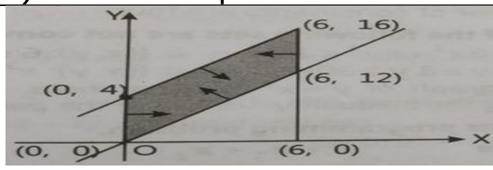
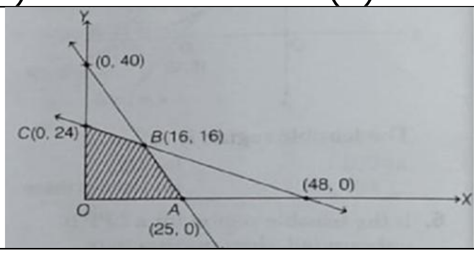
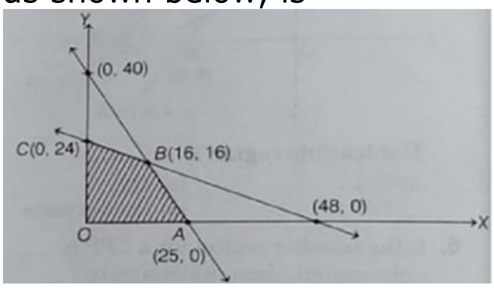
Solution:



CORNER POINT	Value of Z = $20x + 30y$
A(14,66)	2260
B(14,24)	1000
C(66,14)	1740
D(29,14)	1000

So maximum value of Z = 2260 at point (14,66).

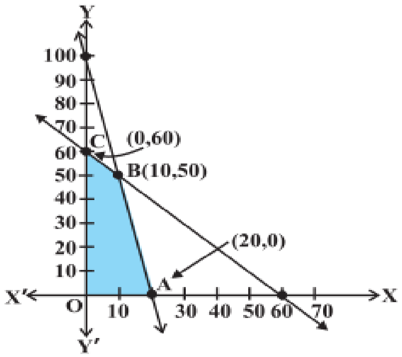
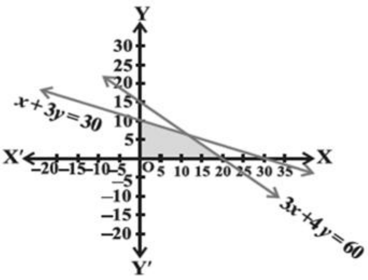
MCQs of (1 mark each)

1	The set of all feasible solutions of a LPP is a ____ set. (a) Concave (b) Convex (c) Feasible (d) None of these	1
2	In a LPP, if the objective function Z = ax + by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same……….value. (a) minimum (b) maximum (c) zero (d) none of these	1
3	In the feasible region for a LPP is ………, then the optimal value of the objective function Z = ax+by may or may not exist. (a) bounded (b) unbounded (c) in circled form (d) in squared form	1
4	Region represented by $x \geq 0$, $y \geq 0$ is: (a) First quadrant (b) Second quadrant (c) Third quadrant (d) Fourth quadrant	1
5	The feasible region for an LPP is shown shaded in the figure. Let Z = 3x- 4y be objective function. Maximum value of Z is  (a) 0 (b) 8 (c) 12 (d) -18	1
6	The maximum value of Z = 4x+3y, if the feasible region for an LPP is as shown below, is  (a) 112 (b) 100 (c) 72 (d) 110	1
7	Z = 250x + 75y is a linear objective function. Variables x and y are called ….. (a) Decision variables (b) Constraints (c) Constant (d) Objective function	1
8	Points within and on the boundary of the feasible region represent …… (a) Infeasible solution (b) Feasible solution (c) Objective solution (d) None	1
9	The maximum value of Z = 4x+3y, if the feasible region for an LPP is as shown below, is  (a) 112 (b) 72 (c) 110 (d) 100	1

10	Corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0). Let Z= px + qy, where p, q>0. Condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is (a) p=2q (b) p= q/2 (c) p=3q (d) p=q	1

ANSWERS: 1-(b), 2- (b), 3- (b), 4- (a), 5- (a), 6- (a), 7- (a), 8- (b), 9- (a), 10-(b).

QUESTIONS OF (3 MARKS EACH)

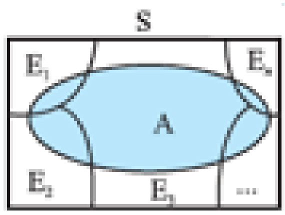
1	Solve the Linear Programming graphically: Maximize Z= 9x+3y subject to2x + 3y ≤ 13 , 3x + y ≤ 5 , x ≥0 , y ≥0	3
2	Find the minimum value of Z = 11x + 7y Subject to x + 3y ≤ 9 , x + y ≤ 5 , x ≥ 0 , y ≥ 0	3
3	Solve the following linear programming problem graphically: Maximise Z = 4x + y subject to the constraints:x + y ≤ 50 , 3x + y ≤ 90 , x ≥ 0, y ≥ 0	3
4	Solve the following Linear Programming Problem : Max. Z = x + 2y Subject to the constraints : x + y ≤ 50 , 3x + y ≤ 90 , x≥0 , y≥0	3
5	Solve the following Linear Programming Problem : Min Z = x + 2y Subject to the constraints : x + 2y ≥ 100 , 2x − y ≤ 0 , 2x + y ≤ 200 , x≥0 , y≥0	3
6	Solve the following Linear Programming Problems graphically: Maximise Z = 5x + 3y subject to 3x + 5y ≤ 15, 5x + 2y ≤ 10, x ≥ 0, y ≥ 0	3
7	Write the linear inequations for which the shaded area in the following figure is the solution set  	3
8	Write the linear inequations for which the shaded area in the following figure is the solution set.  	3

9	Solve the following Linear Programming Problem : Max. $Z = x + 2y$ Subject to the constraints : $x + y \leq 50$, $3x + y \leq 90$, $x \geq 0$, $y \geq 0$	3
10	Solve the following linear programming problem graphically: Maximise $Z = 3x + 4y$ subject to the constraints : $x + y \leq 4$, $x \geq 0$, $y \geq 0$.	3

<u>ANSWERS:</u> 1- **Max Z = 15 , 2- Min Z = 21 , 3- Max Z = 11 , 4- Max. Z =100, 5- Min Z= 100, 6- Maximum Z = $\frac{235}{19}$ at** $\left(\frac{20}{19},\frac{45}{19}\right)$, **7- $5x + y \leq 100$, $x + y \leq 60$, 8- $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq$, 9- Max. Z =100, 10- Max. Z =16 .**

CHAPTER- 13 PROBABILITY

SUMMARY		
SL	TOPIC	DEFINITION
1	Probability	Probability is defined as a quantitative measure of uncertainty – a numerical value that conveys the strength of our belief in the occurrence of an event.
	Probability of an event A	P(A) $=\frac{Number\ of\ times\ A\ occurs}{Total\ number\ of\ possible\ outcomes}$ ❖ **$0 \le P(A) \le 1$**
2	**Mutually exclusive and exhaustive events**	**If E1, E2, ..., E*n*** are *n* events of a sample space S such that they are mutually exclusive and exhaustive both ❖ **$E_i \cap E_j = \emptyset$ for every $i \neq j$** $\bigcup_{i=1}^{n} E_i = S$ ***Mutually exclusive and Exhaustive events form the partition of sample space S**
3	**Equally Likely Events**	The given events are said to be equally likely, if none of them is expected to occur in preference to the other.
4	**Addition rule**	❖ **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$** ❖ **$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$**
5	**Addition rule for mutually exclusive events**	❖ $P(A \cup B) = P(A) + P(B)$
6	**<u>Conditional Probability</u>**	Let A and B be two events associated with a random experiment, Then, the probability of occurrence of event A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional Probability.i.e.,

		P(A/B) = $\frac{P(A \cap B)}{P(B)}$
7	**Multiplication Rule of Probability**	❖ $P(E \cap F) = P(E)\ P(F\|E)$ $= P(F)\ P(E\|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$. ❖ $P(E \cap F \cap G) = P(E)\ P(F\|E)\ P(G\|(E \cap F))$ $= P(E)\ P(F\|E)\ P(G\|EF)$
8	**INDEPENDENT EVENTS**	Two events E and F are said to be independent, if ❖ $P(F\|E) = P(F)$ provided **P(E)** $\neq 0$ ❖ and $P(E\|F) = P(E)$ provided $P(F) \neq 0$ **$P(E \cap F) = P(E)\ P(F)$**
9		❖ **Three events A, B and C** are said to be mutually independent, if $P(A \cap B) = P(A)\ P(B)$ $P(A \cap C) = P(A)\ P(C)$ $P(B \cap C) = P(B)\ P(C)$ and $P(A \cap B \cap C) = P(A)\ P(B)\ P(C)$ ❖ **if E and F are independent events Then** (a) E and F′ are independent, (b) E′ and F are independent (c) E′ and F′ are independent
10	**<u>Important</u>**	**If A and B are two independent events, then the probability of occurrence of <u>at least one of A and B</u> is given by** **1– P(A′) P(B′)**
11	**<u>Partition of a Sample Space</u>**	The events $A_1, A_2, \ldots, A_n$ represent a partition of the sample space S, if they are pairwise disjoint, exhaustive and have non-zero probabilities. i.e., **(i)** $A_i \cap A_j = \varphi$; $I \neq j$; $I,j = 1,2, \ldots ,n$ (ii) $A_1 \cup A_2 \cup \ldots \cup A_n = S$ (iii) $P(A_i) > 0$, $\forall\ I = 1,2, \ldots ,n$
12	**<u>Total Probability</u>** 	Let S be the sample space and let $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or … or E_n then $P(A) = P(E_1)P(A / E_1) + P(E_2)P(A / E_2) + \ldots + P(E_n)\ P(A / E_n)$ $\sum_{r=1}^{n} P(Er)\ P(A/Er)$

SOLVED EXAMPLES

MCQ (1 MARK)

Q5	A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is : A) $\frac{2}{13}$ B) $\frac{3}{26}$ C) $\frac{19}{26}$ D) $\frac{3}{13}$ SOLUTION: These are independent events. P(Head on coin AND a face card from deck) = P(HEAD ON COIN) P(A FACE CARD) = (½)(12/52) = 3/26 CBSE(24-25)	B
Q8	Let X be a discrete random variable. The probability distribution of X is given below: X: 30, 10, -10; P(X): 1/5, 3/10, 1/2 Then E(X) is equal to: A) 6 B) 4 C) 3 D) -5 SOLUTION: E(X) = (30)(1/5) + (10)(3/10) + (-10)(1/2) = 6+3-5 = 4	B

X	30	10	-10
P(X)	1/5	3/10	1/2

ASSERTION AND REASON QUESTIONS

Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is **not** the correct explanation of the Assertion (A)

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Q1	**Assertion (A):** If A and B are two independent events with P(A)=1/5 and P(B)=1/5, then P(A′/B) is 1/5. Reason (R): P$(A'/B)=P(A'\cap B)/P(B)$ **Solution** P$(A'/B)=P(A'\cap B)/P(B)$ $=P(A').P(B)/P(B)$ $=P(A')$ $=1-P(A)$=4/5 $So\ A\ is\ false\ and\ R\ isTrue.$	b

(2 marks)

1	A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is

	selected is 0.6. Find the probability that B is selected…………………………….NCERT EXEMPLAR SOLUTION: Let p be the probability that B gets selected. P (Exactly one of A, B is selected) = 0.6 (given) P (A is selected, B is not selected OR B is selected, A is not selected) = 0.6 P (A∩B′) + P (A′∩ B) = 0.6 P (A) P (B′) + P (A′) P (B) = 0.6 (0.7) (1 – p) + (0.3) p = 0.6 p = 0.25 Thus the probability that B gets selected is 0.25.
(3 MARKS)	
2	A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee. SOLUTION: Let A = at least one girl will be chosen B = exactly 2 girls will be chosen. We require P (B | A). A′= no girl is chosen, i.e., 4 boys are chosen. $P(A') = \frac{8C_4}{12c_4} = \frac{70}{495} = \frac{14}{99}$ $P(A) = 1 - \frac{14}{99} = \frac{85}{99}$ $P(A \cap B) = P(2B\ and\ 2G) = \frac{8C_2 \cdot 4C_2}{12C_4} = \frac{6 \times 28}{495} = \frac{56}{165}$ $P(B\|A) = \frac{p(A \cap B)}{p(A)} = \frac{56 \times 99}{165 \times 85} = \frac{168}{42S}$
3	Probability of solving specific problem independently by A and B are ½ and 1/3 respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem. SOLUTION $P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$ $P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3}$ i) Probability that the problem is not solved by both $P(A' \cap B') = P(A')P(B')$ $= \frac{1}{2} x \frac{2}{3}$ $= \frac{1}{3}$ The probability that at least one solves the problem = 1- P(A′∩ B′) = 1-$\frac{1}{3}$ = 2/3

	ii) The probability that only one person will solve the problem $= P(A\cap B') + P(A' \cap B)$ $= P(A) \,.\, P(B') + P(A') P(B)$ $=\frac{1}{2}\times\frac{2}{3} + \frac{1}{2}\times\frac{1}{3}$ = 3/6 = 1/2

CASE STUDY QUESTIONS:

1	Based upon the results of regular medical check ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record. Let A1: People with good health A2: People with average health A3 : People with average health During a pandemic , the data expressed that the chances of people contracting the dieses From category A1,A2,A3 are 25%,35%and 50% respectively Based upon the above information, answer the following: i) A person was tested randomly. What is the probability that he/she has contracted the disease ii) Given that the person has not contracted the disease , what is the probability that the person is from category A2 SOLUTION: Let A = Person contracted disease $P(A) = P(A1)\, P(A\|A1) + P(A2)\, P(A\|A2) + P(A3)\, P(A\|A3)$ $= \frac{7}{10}\left(\frac{25}{100}\right) + \frac{2}{10}\left(\frac{35}{100}\right) + \frac{1}{10}\left(\frac{50}{100}\right) +$ $= \frac{295}{1000} = 0.295$ i) $P(A2\|A') = \frac{P(A2)P(A'\|A2)}{P(A1)P\left(A'\|A1\right) + (A2)P(A'\|A2)}$ $= \frac{\frac{2}{10}\times\frac{65}{100}}{\frac{7}{10}\times\frac{75}{100}+\frac{2}{10}\times\frac{65}{100}+\frac{1}{10}\times\frac{50}{100}}$ = 26/141 CBSE 24-25

MCQ(1 MARKER)

1	If A and B are two events such that $P(A)\neq 0$ and $P(B/A) = 1$, then A) $A\subset B$ B) $B \subset A$ C) $B = \emptyset$ D) $A = \emptyset$
2	If $P(A\|B) = P(A'\|B)$, then which of the following statements is true? A) $P(A) = P(A')$ B) $P(A) = 2P(B)$ C) $P(A\cap B) = (1/2)P(B)$ D) $P(A\cap B) = 2P(B)$
3	A person observed the first 4 digits of your 6-digit PIN. What is the

	probability that the person can guess your pin? A)1/81 B)1/100 C)1/90 D)1 SQP 25-26
4	CBSE 24-25 Let E be an event of a sample space S of an experiment, then P(S|E) = A)P(S∩E) B)P(E) C)1 D)0
5	Exemplar Let A and B be two events such that P (A) = 0.6, P (B) = 0.2, and P (A | B) = 0.5. Then P (A' | B') equals A)1/10 B)3/10 C)3/8 D)6/7

(2 MARKS)

1	A random variable can take all non – negative integral values and the probability that X takes the value r is proportional to 5^{-r}. Find P(X<3) SQP 24-25
2	The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\bar{A})+P(\bar{B})$ Examplar
3	Ashima can hit the target 2 out of 3 times. She tried to hit the target twice. Find The Probability that she missed the target exactly once.
4	A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes.
(3 MARKS)	
1	Two students Mehul and Rashi are seeking admission in a college. The probability that Mehul is selected is 0.4 and the probability of selection of exactly one of the them is 0.5. Chances of selection of them is independent of each other. Find the chances of selection of Rashi. Also find the probability of selection of at least one of them. SQP 25-26
2	(a) The probability that it rains today is **0.4**. If it rains today, the probability that it will rain tomorrow is **0.8**. If it does not rain today, the probability that it will rain tomorrow is **0.7**. If $P1$: denotes the probability that it does not rain today. $P2$: denotes the probability that it will not rain tomorrow, if it rains today. $P3$: denotes the probability that it will rain tomorrow, if it does not rain today. $P4$: denotes the probability that it will not rain tomorrow, if it does not rain today. **i)** Find the value of $P_1 \times P_4 - P_2 \times P_3$

	ii) Calculate the probability of rain tomorrow. SQP 24-25
3	The probability distribution of a random variable X is given below: <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X)</td><td>k</td><td>k/2</td><td>k/4</td><td>k/8</td></tr></table> i)Determine the value of K ii)Determine $P(X \leq 2)$ and $P(X>2)$ iii)Find $P(X \leq 2) + P(X > 2)$ EXAMPLAR
4	CBSE 24-25 A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.
5	An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question? NCERT
6	A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

CASE STUDY QUESTIONS

1	In a class of students of the age group 14 to 17, the students were categorised into three groups according to a feedback form filled by them. The first group constituted of the students who spent more than 4 hours per day on the mobile screen or the gaming screens, while the second group spent 2 to 4 hours /day on the same activities. The third group spent less than 2 hours /day on the same. The first group with the high screen time is 60% of all the students, whereas the second group with moderate screen time is 30% and the third group with low screen time is only 10% of the total number of students. It was observed that 80% students of first group faced severe anxiety and low retention issues, with 70% of second group, and 30% of third group having the same symptoms.

	I. What is the total percentage of students who suffer from anxiety and low retention issues in the class? [2] II. A student is selected at random, and he is found to suffer from anxiety and low retention issues. What is the probability that he/she spends screen time more than 4 hours per day? [2] SQP 25-26
2	Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage –II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously). Then two birds flew back from cage-II to cage-I(simultaneously). Assume that all the birds have equal chances of flying. On the basis of the above information, answer the following questions:- **(i)** When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I then find the probability that the owl is still in Cage-I. ***2Marks*** **(ii)** When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II? ***2Marks*** SQP 24-25
3	Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is 1/5, Jaspreet's selection is 1/3 and Alia's selection is ¼. The event of selection is independent of each other. CBSE 24-25 Based on the above information, answer the following questions: i) What is the probability that at least one of them is selected? ii) Find P(G|H') where G is event of Jaspreet's selection and H' denotes the event that Rohit is not selected iii) Find the probability that exactly one of them is selected OR iii) Find the probability that exactly two of them are selected.
4	CBSE 23-24 A departmental store send bills to charge its customers once a month. Past

	experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month. Based on the above information, answer the following questions: i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1)$ and $P(E_2)$ ii) Let A denotes the event the customer paying second month's bill in time,then find $P(A\| E_1)$ and $P(A\| E_2)$ iii) Find the probability of customer paying second month's bill in time OR iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time

Competitive level questions (CUET, NDA, JEE)

1	(NDA 2024) A die has two faces with number 4, three faces with number 5 and one face with number 6. If the die is rolled once, then what is the probability of getting 4 or 5? i) 1/3 ii)2/3 iii)5/6 iv)1/2
2	(NDA 2024) A can hit a target 5 times in 6 shots, B can hit 4 times in 5 shots and C can hit 3 times in 4 shots. What is the probability that A and C may hit but B may lose? i) 1/8 ii)1/6 iii)1/4 iv)1/3
3	(NDA 2024) A natural number x is chosen at random from the first 100 natural numbers. What is the probability that $x^2 + x > 50$? i)93/100 ii)47/50 iii)24/25 iv)23/25
4	(CUET) If A and B are two events such that P(A/B) = 2P(B/A) and P(A)+P(B) = 2/3, then P(B) is equal to i)2/9 ii) 7/9 iii)4/9 iv)5/9
5	(CUET) Three coins are tossed. If at least two coins show head, the probability of getting one tail is: a) ¾ b) 1/3 c) 1 d) 2/3
6	(CUET) A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. The probability of occurrence of A is.

	a) ½ b) 1/6 c) 5/6 d) 1/3
7	(JEE) Three ships A, B, and C sail from England to India. If the ratio of their arriving safely is 2:5, 3:7 and 6:11 respectively. Find the probability of all the ships arriving safely.

Answers(MCQ unsolved)

1	2	3	4	5
A	C	B	C	C

Answers(2Marks)

<table>
<tr><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td>124/
125</td><td>1.1</td><td>4/9</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>49/81</td><td>28/81</td><td>4/81</td></tr></table></td></tr>
</table>

Answers(2/3 Marks unsolved)

1	2	3	4	5	6
0.5, 0.7	0.04 0.74	i. 8/15 ii.14/15 ,1/15 iii.1	1/17	5/9	3/8

ANSWERS (case study):

1) I) 72% ii)2/3
2) i) ¾ ii) 1/9
3) i)1/3 ii)13/30 iii) 3/20
4) i) 0.3 ii) 0.4 iii) 0.68 or 14/17

ANSWERS(Competitive Exam Questions)

1. 5/6
2. 1/8
3. 47/50
4. 2/9
5. 3/4
6. 1/2
7. 18/595

MATHEMATICS – Code No. 041
SAMPLE QUESTION PAPER
CLASS - XII (2025-26)

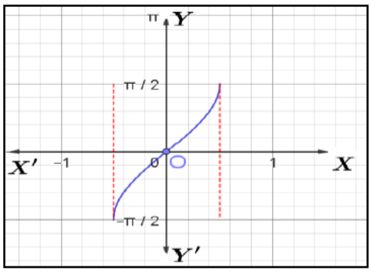
Maximum Marks: 80 **Time: 3 hours**

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed.

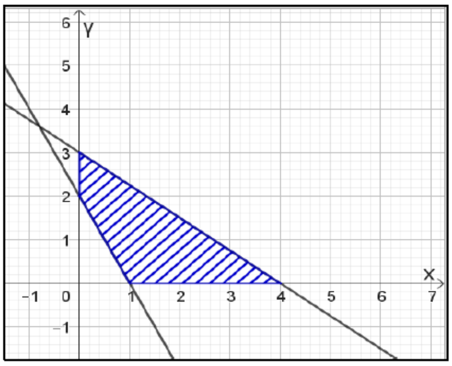
SECTION-A
This section comprises of multiple choice questions (MCQs) of 1 mark each.
Select the correct option (Question 1 - Question 18)

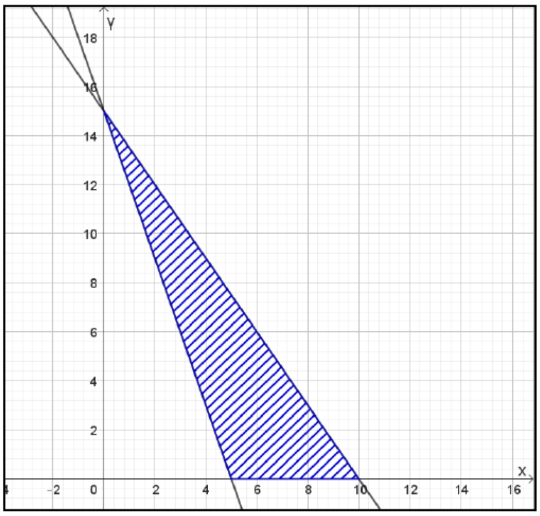
Q.No.	Questions	Marks
1.	Identify the function shown in the graph  (A) $\sin^{-1} x$ (B) $\sin^{-1}(2x)$ (C) $\sin^{-1}\left(\frac{x}{2}\right)$ (D) $2\sin^{-1} x$	**1**
	For Visually Impaired:	
1.	Inverse Trigonometric Function, whose domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$, is ... (A) $\cos^{-1} x$ (B) $\cos^{-1}\left(\frac{x}{3}\right)$ (C) $\cos^{-1}(3x)$ (D) $3\cos^{-1} x$	

*Please note that the assessment scheme of the Academic Session 2024-25 will continue in the current session i.e. 2025-26.

2.	If for three matrices $A = [a_{ij}]_{m\times 4}$, $B = [b_{ij}]_{n\times 3}$ and $C = [c_{ij}]_{p\times q}$ products AB and AC both are defined and are square matrices of same order, then value of m, n, p and q are: (A) $m = q = 3 \;\; and\, n = p = 4$ (B) $m = 2, q = 3 \;\; and\, n = p = 4$ (C) $m = q = 4 \;\; and\, n = p = 3$ (D) $m = 4, p = 2 \;\; and\, n = q = 3$	1
3.	If the matrix $A = \begin{bmatrix} 0 & r & -2 \\ 3 & p & t \\ q & -4 & 0 \end{bmatrix}$ is skew-symmetric, then value of $\frac{q+t}{p+r}$ is…. (A)-2 (B) 0 (C) 1 (D) 2	1
4.	If A is a square matrix of order 4 and $\lvert adj\, A \rvert = 27$, then $A\,(adj\, A)$ is equal to (A) 3 (B) 9 (C) $3\, I$ (D) $9\, I$	1
5.	The inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is… (A) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (C) $\begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{5} \end{bmatrix}$ (D) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$	1
6.	Value of the determinant $\begin{vmatrix} \cos 67^o & \sin 67^o \\ \sin 23^o & \cos 23^o \end{vmatrix}$ is (A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1	1
7.	If a function defined by $f(x) = \begin{cases} kx + 1, & x \leq \pi \\ \cos x \;, & x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of k is (A) π (B) $\frac{-1}{\pi}$ (C) 0 (D) $\frac{-2}{\pi}$	1
8.	If $f(x) = x \tan^{-1} x$, then $f'(1)$is equal to (A)$\frac{\pi}{4} - \frac{1}{2}$ (B) $\frac{\pi}{4} + \frac{1}{2}$ (C) $-\frac{\pi}{4} - \frac{1}{2}$ (D) $-\frac{\pi}{4} + \frac{1}{2}$	1
9.	A function $f(x) = 10 - x - 2x^2$ is increasing on the interval (A) $\left(-\infty, -\frac{1}{4}\right]$ (B) $\left(-\infty, \frac{1}{4}\right)$ (C) $\left[-\frac{1}{4}, \infty\right)$ (D)$\left[-\frac{1}{4}, \frac{1}{4}\right]$	1
10.	The solution of the differential equation $xdx + ydy = 0$ represents a family of (A) straight lines (B) parabolas (C) Circles (D) Ellipses	1

*Please note that the assessment scheme of the Academic Session 2024-25 will continue in the current session i.e. 2025-26.

11.	If $f(a+b-x)=f(x)$, then $\int_a^b x\,f(x)dx$ is equal to (A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}\int_a^b f(a-x)dx$ (C) $\frac{b-a}{2}\int_a^b f(x)\ dx$ (D) $\frac{a+b}{2}\int_a^b f(x)dx$	1
12.	If $\int x^3 \sin^4(x^4)\cos(x^4)\,dx = a\sin^5(x^4)$ + C, then a is equal to (A) $-\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$	1
13.	A bird flies through a distance in a straight line given by the vector $\hat{\imath}+2\hat{\jmath}+\hat{k}$. A man standing beside a straight metro rail track given by $\vec{r}=(3+\lambda)\hat{\imath}+(2\lambda-1)\hat{\jmath}+3\lambda\hat{k}$ is observing the bird. The projected length of its flight on the metro track is (A) $\frac{6}{\sqrt{14}}$ units (B) $\frac{14}{\sqrt{6}}$ units (C) $\frac{8}{\sqrt{14}}$ units (D) $\frac{5}{\sqrt{6}}$ units	1
14.	The distance of the point with position vector $3\hat{\imath}+4\hat{\jmath}+5\hat{k}$ from the y-axis is (A) 4 units (B) $\sqrt{34}$ units (C) 5 units (D) $5\sqrt{2}$ units	1
15.	If $\vec{a}=3\hat{\imath}+2\hat{\jmath}+4\hat{k}$, $\vec{b}=\hat{\imath}+\hat{\jmath}-3\hat{k}$ and $\vec{c}=6\hat{\imath}-\hat{\jmath}+2\hat{k}$ are three given vectors, then $(2\vec{a}.\hat{\imath})\hat{\imath}-(\vec{b}.\hat{\jmath})\hat{\jmath}+(\vec{c}.\hat{k})\hat{k}$ is same as the vector (A) $\vec{a}$ (B) $\vec{b}+\vec{c}$ (C) $\vec{a}-\vec{b}$ (D) $\vec{c}$	1
16.	A student of class XII studying Mathematics comes across an incomplete question in a book. Maximise $Z = 3x+2y+1$ Subject to the constraints $x\ge 0, y\ge 0, 3x+4y\le 12,$ He/ She notices the below shown graph for the said LPP problem, and finds that a constraint is missing in it: Help him/her choose the required constraint from the graph.  The missing constraint is (A) $x+2y\le 2$ (B) $2x+y\ge 2$ (C) $2x+y\le 2$ (D) $x+2y\ge 2$	1

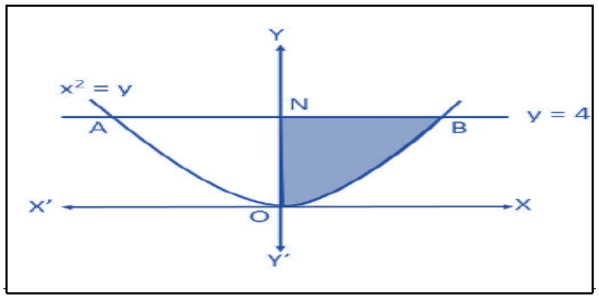
16.	**For Visually Impaired:** If $Z = ax + by + c$, where $a, b, c > 0$, attains its maximum value at two of its corner points (4,0) and (0,3) of the feasible region determined by the system of linear inequalities, then (A) $4a = 3b$ (B) $3a = 4b$ (C) $4a + c = 3b$ (D) $3a + c = 4b$	
17.	The feasible region of a linear programming problem is bounded but the objective function attains its minimum value at more than one point. One of the points is (5,0).  Then one of the other possible points at which the objective function attains its minimum value is (A) (2,9) (B) (6,6) (C) (4,7) (D) (0,0) **For Visually Impaired:** The graph of the inequality $3x + 5y < 10$ is the (A) Entire XY –plane (B) Open Half plane that doesn't contain origin (C) Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$ (D) Half plane that contains origin and the points of the line $3x + 5y = 10$	1
18.	A person observed the first 4 digits of your 6-digit PIN. What is the probability that the person can guess your PIN? (A) $\frac{1}{81}$ (B) $\frac{1}{100}$ (C) $\frac{1}{90}$ (D) 1	1

*Please note that the assessment scheme of the Academic Session 2024-25 will continue in the current session i.e. 2025-26.

	ASSERTION-REASON BASED QUESTIONS **(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)** (A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true but (R) is not the correct explanation of (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true.	
19.	**Assertion (A):** Value of the expression $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)+\tan^{-1}1-\sec^{-1}(\sqrt{2})$ is $\frac{\pi}{4}$. **Reason (R):** Principal value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and that of $sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.	1
20.	**Assertion(A):** Given two non-zero vectors $\vec{a}$ and $\vec{b}$. If $\vec{r}$ is another non-zero vector such that $\vec{r}\times(\vec{a}+\vec{b})=\vec{0}$. Then $\vec{r}$ is perpendicular to $\vec{a}\times\vec{b}$. **Reason (R):** The vector $(\vec{a}+\vec{b})$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$	1

SECTION B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21A	Evaluate $\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)$ **OR**	2
21B	Find the domain of $\cos^{-1}(3x-2)$	
22	If $y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$, then prove that $\boldsymbol{\frac{dy}{dx}-\sec x=0}$	2
23A	Find: $\int\frac{(x-3)}{(x-1)^3}e^x\,dx$ **OR**	2
23B	Find out the area of shaded region in the enclosed figure. 	

*Please note that the assessment scheme of the Academic Session 2024-25 will continue in the current session i.e. 2025-26.

23 B	**For Visually Impaired:** Find out the area of the region enclosed by the curve $y^2 = x$, $x = 3$ and x-axis in the first quadrant.	
24.	If $f(x+y) = f(x)f(y)$ for all $x, y \in$ R and $f(5)$= 2, $f'(0) = 3$, then using the definition of derivatives, find $f'(5)$.	2
25.	The two vectors $\hat{\imath} + \hat{\jmath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 3\hat{k}$ represent the two sides OA and OB, respectively of a ΔOAB, where O is the origin. The point P lies on AB such that OP is a median. Find the area of the parallelogram formed by the two adjacent sides as OA and OP.	2
	SECTION C **This section comprises of 6 short answer (SA) type questions of 3 marks each.**	
26A. 26B.	If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\log x}{(\log(xe))^2}$ and hence find its value at $x = e$. **OR** If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ find $\frac{d^2y}{dx^2}$.	3
27	A spherical ball of ice melts in such a way that the rate at which its volume decreases at any instant is directly proportional to its surface area. Prove that the radius of the ice ball decreases at a constant rate.	3
28A 28B 28A 28B	Sketch the graph $y = \|x+1\|$. Evaluate $\int_{-4}^{2}\|x+1\|dx$. What does the value of this integral represent on the graph? **OR** Using integration find the area of the region $\{(x, y) : x^2 - 4y \leq 0, y - x \leq 0\}$ **For Visually Impaired**: Define the function $y = \|x+1\|$. Evaluate $\int_{-4}^{2}\|x+1\|dx$. What does the value of this integral represent? **OR** Using integration find the area enclosed within the curve: $25x^2 + 16y^2 = 400$	3
29A 29B	Find the distance of the point $(2, -1, 3)$ from the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k})$ measured parallel to the z-axis. **OR** Find the point of intersection of the line $\vec{r} = (3\hat{\imath} + \hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + \hat{k})$ and the line through $(2, -1, 1)$ parallel to the z-axis. How far is this point from the z-axis?	3

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30.	Solve graphically: Maximise $Z = 2x + y$ subject to $x + y \leq 1200$ $x + y \geq 600$ $y \leq \frac{x}{2}$ $x \geq 0, y \geq 0$.	3
30	**For Visually Impaired:** The objective function $Z = 3x + 2y$ of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are $A(600,0), B(1200,0), C(800,400)$ and $D(400,200)$. Find the point at which Z is maximum and the point at which Z is minimum. Also, find the corresponding maximum and minimum values of Z.)	
31.	Two students Mehul and Rashi are seeking admission in a college. The probability that Mehul is selected is 0.4 and the probability of selection of exactly one of the them is 0.5. Chances of selection of them is independent of each other. Find the chances of selection of Rashi. Also find the probability of selection of at least one of them.	3
	SECTION D **This section comprises of 4 long answer (LA) type questions of 5 marks each**	
32.	For two matrices $A = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$, find the product AB and hence solve the system of equations: $3x - 6y - z = 3$ $2x - 5y - z + 2 = 0$ $-2x + 4y + z = 5$	5
33A	Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx$ **OR**	5
33B	Find $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - cos^2\theta - 4\sin\theta}\, d\theta$	
34A	Solve the differential equation: $y + \frac{d}{dx}(xy) = x(\sin x + x)$ **OR**	5
34B	Find the particular solution of the differential equation: $2\,y\,e^{x/y} dx + (y - 2\,x\,e^{x/y})\, dy = 0$ given that $y(0) = 1$	

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35. The two lines $\frac{x-1}{3} = -y, z+1 = 0$ and $\frac{-x}{2} = \frac{y+1}{2} = z+2$ intersect at a point whose y-coordinate is 1. Find the co-ordinates of their point of intersection. Find the vector equation of a line perpendicular to both the given lines and passing through this point of intersection. **5**

SECTION- E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each

36. **Case Study -1** **4**

A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals. The city has five major spots labelled $A, B, C, D, and\ E$.

The department has collected the following data regarding one-way traffic flow between spots:

1. Traffic flows from A to B, A to C, and A to D.
2. Traffic flows from B to C and B to E.
3. Traffic flows from C to E.
4. Traffic flows from D to E and D to C.

The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions:

I. Is the traffic flow reflexive? Justify. [1]

II. Is the traffic flow transitive? Justify. [1]

III A. Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation.

OR

III B. Does the traffic flow represent a function? Justify your answer. [2]

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37.	**Case Study -2**	4

LED bulbs are energy-efficient because they use significantly less electricity than traditional bulbs while producing the same amount of light. They convert more energy into light rather than heat, reducing waste. Additionally, their long lifespan means fewer replacements, saving resources and money over time.

A company manufactures a new type of energy-efficient LED bulb. The cost of production and the revenue generated by selling x bulbs (in an hour) are modelled as

$C(x) = 0.5x^2 - 10x + 150$ and $R(x) = -0.3x^2 + 20x$ respectively, where $C(x)$ and $R(x)$ are both in ₹.

To maximize the profit, the company needs to analyze these functions using calculus. Use the given models to answer the following questions:

I. Derive the profit function $P(x)$ [1]

II. Find the critical points of $P(x)$. [1]

III A. Determine whether the critical points correspond to a maximum or a minimum profit by using the second derivative test.

OR

III B. Identify the possible practical value of x (i.e., the number of bulbs that can realistically be produced and sold) that can maximize the profit, if the resources available and the expenditure on machines allows to produce minimum 10 but not more than 18 bulbs per hour. Also calculate the maximum profit. [2]

38. **Case Study -3** 4

Excessive use of screens can result in vision problems, obesity, sleep disorders, anxiety, low retention problems and can impede social and emotional comprehension and expression. It is essential to be mindful of the amount of time we spend on screens and to reduce our screen-time by taking regular breaks, setting time limits, and engaging in non-screen-based activities.

In a class of students of the age group 14 to 17, the students were categorised into three groups according to a feedback form filled by them. The first group constituted of the students who spent more than 4 hours per day on the mobile screen or the gaming screens, while the second group spent 2 to 4 hours /day on the same activities. The third group spent less than 2 hours /day on the same. The first group with the high screen time is 60% of all the students, whereas the second group with moderate screen time is 30% and the third group with low screen time is only 10% of the total number of students. It was observed that 80% students of first group faced severe anxiety and low retention issues, with 70% of second group, and 30% of third group having the same symptoms.

I. What is the total percentage of students who suffer from anxiety and low retention issues in the class? [2]

II. A student is selected at random, and he is found to suffer from anxiety and low retention issues. What is the probability that he/she spends screen time more than 4 hours per day? [2]

MATHEMATICS – Code No. 041
MARKING SCHEME
CLASS – XII (2025-26)

SECTION-A (MCQs of 1 mark each)		
Sol. N.	**Hint / Solution**	**Marks**
1	Clearly from the graph Domain is $\left[-\frac{1}{2},\frac{1}{2}\right]$ So graph is of the function $\sin^{-1}(2x)$ **Answer is (B)** $\sin^{-1}(2x)$	**1**
1 (V.I.)	Domain is $\left[-\frac{1}{3},\frac{1}{3}\right]$ So the function is $\cos^{-1}(3x)$ **Answer is (C)** $\cos^{-1}(3x)$	**1**
2	AB is defined so n=4 AC is defined so p=4 AB and AC are square matrices of same order so $m \times 3 = m \times q \Rightarrow q = 3 = m$ **Answer is (A)** $m = q = 3 \quad and\ n = p = 4$	**1**
3	As A is skew symmetric So $p = 0, q = 2, r = -3, t = 4$ So $\frac{q+t}{p+r} = \frac{6}{-3} = -2$ **Answer is (A) -2**	**1**
4	$\|adj\ A\| = 27 \Rightarrow \|A\|^3 = 27 = 3^3 \Rightarrow \|A\| = 3$ $A\ (adj\ A) = \ \|A\|\ I = 3\ I$ **Answer is (C) 3 *I***	**1**
5	Inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ **Answer is (B)**	**1**
6	$\begin{vmatrix} \cos 67^o & \sin 67^o \\ \sin 23^o & \cos 23^o \end{vmatrix} = \cos 67^o \cos 23^o - \sin 67^o \sin 23^o = \cos(67^o + 23^o) =$ $\cos 90^o = 0$ **Answer is (A) 0**	**1**
7	$f(x)$ is continuous at $x = \pi$ $\Rightarrow \lim\limits_{x\to\pi^-}(kx+1) = \lim\limits_{x\to\pi^+} cos\ x = f(\pi)$ $\Rightarrow \lim\limits_{h\to 0}[k(\pi - h) + 1] = \lim\limits_{h\to 0} cos(\pi + h) = k\pi + 1$ $\Rightarrow k\pi + 1 = -1 \qquad \Rightarrow k = \frac{-2}{\pi}$ **Answer is (D)** $\frac{-2}{\pi}$	**1**

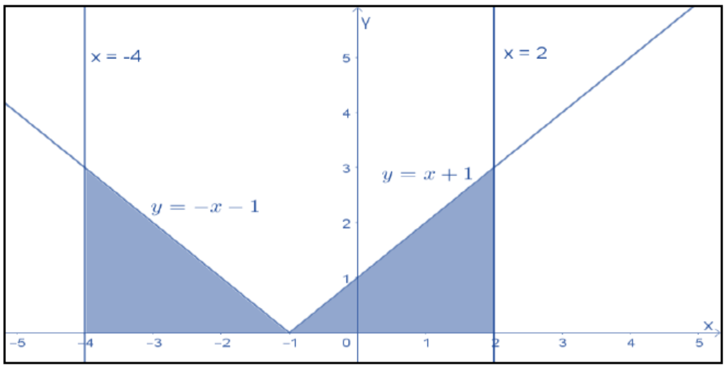
8	$f(x) = x\tan^{-1}x$ $f'(x) = 1.\tan^{-1}x + x.\frac{1}{1+x^2}$ $f'(1) = 1.\tan^{-1}1 + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}$ **Answer is (B)** $\frac{\pi}{4} + \frac{1}{2}$	1
9	$f(x) = 10 - x - 2x^2$ $\Rightarrow f'(x) = -1 - 4x$ For increasing function $f'(x) \geq 0$ $\Rightarrow -(1+4x) \geq 0$ $\Rightarrow (1+4x) \leq 0$ $\Rightarrow x \leq -1/4$ $\Rightarrow x \in \left(-\infty, -\frac{1}{4}\right]$ **Answer is (A)** $\left(-\infty, -\frac{1}{4}\right]$	1
10	$xdx + ydy = 0$ $\Rightarrow \int xdx = -\int ydy$ $\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$ $\Rightarrow x^2 + y^2 = 2k$ Solution is $x^2 + y^2 = 2k, k$ being an arbitrary constant. **Answer is (C) Circles**	1
11	$I = \int_a^b x\,f(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$ $\Rightarrow I = \int_a^b (a+b-x)f(x)dx$ (given $f(a+b-x) = f(x)$) $\Rightarrow I = \int_a^b (a+b)\,f(x)dx - \int_a^b x\,f(x)dx$ $\Rightarrow 2\,I = (a+b)\int_a^b f(x)dx$ $\Rightarrow I = \frac{1}{2}(a+b)\int_a^b f(x)dx$ **Answer is (D)** $\frac{a+b}{2}\int_a^b f(x)dx$	1
12	Let $I = \int x^3 sin^4(x^4)\cos(x^4)\,dx$ Let $\sin(x^4) = t \Rightarrow 4x^3\cos(x^4)\,dx = dt \Rightarrow x^3\cos(x^4) = \frac{1}{4}dt$ Thus $I = \int t^4\left(\frac{1}{4}dt\right) = \frac{1}{20}t^5 + C = \frac{1}{20}sin^5(x^4) + C$ $\Rightarrow I = \frac{1}{20}sin^5(x^4) + C = a\ sin^5(x^4) + C$ So, $a = \frac{1}{20}$ **Answer is (B)** $\frac{1}{20}$	1
13	The projection of the vector $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ on the line $\vec{r} = (3\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ is $\frac{1x1+2x2+1x3}{\sqrt{1^2+2^2+3^2}} = \frac{8}{\sqrt{14}}$ units **Answer is (C)** $\frac{8}{\sqrt{14}}$ **units**	1

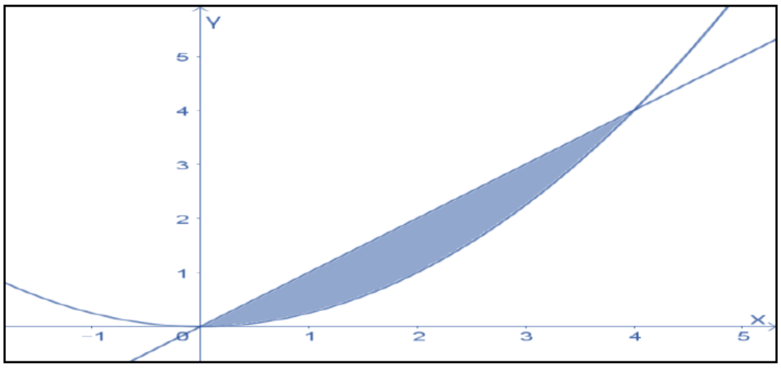
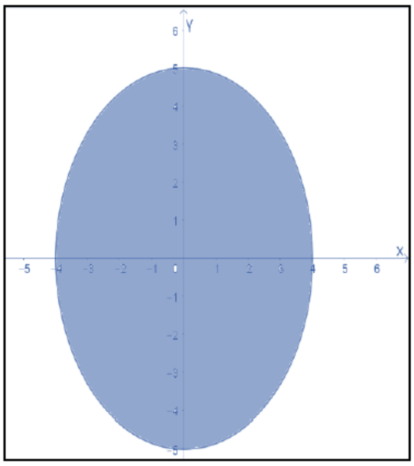
14	The distance of the point (a, b, c) from the y-axis is $\sqrt{a^2+c^2}$ So, the distance is $\sqrt{3^2+5^2} = \sqrt{34}$ units. **Answer is (B)** $\boldsymbol{\sqrt{34}}$ **units**	**1**
15	$(2\vec{a}.\hat{\imath})\hat{\imath} - (\vec{b}.\hat{\jmath})\hat{\jmath} + (\vec{c}.\hat{k})\hat{k} = (2\times 3)\hat{\imath} - (1)\hat{\jmath} + (2)\hat{k}$ $= 6\hat{\imath} - \hat{\jmath} + 2\hat{k} = \vec{c}$ **Answer is (D)** $\boldsymbol{\vec{c}}$	**1**
16	The points (1,0) and (0,2) satisfy the equation $2x + y = 2$ And shaded region shows that (0,0) doesn't lie in the feasible solution region So, the inequality is $2x + y \geq 2$ **Answer is (B)** $\boldsymbol{2x + y \geq 2}$	**1**
16 (V.I.)	(4,0) and (0,3) gives maximum value so $Z_{(4,0)} = Z_{(0,3)} \Rightarrow 4a + c = 3b + c \Rightarrow 4a = 3b$ **Answer is (A)** $\boldsymbol{4a = 3b}$	**1**
17	The student may read the point (2,9) from the line on the graph. The student may find the equation $3x + y = 15$ joining (5,0) and (0,15) and then verify the point (2,9) satisfies it. **Answer is (A) (2,9)**	**1**
17 (V.I.)	**Answer is (C)** Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$	**1**
18	**Answer is (B)** $\boldsymbol{\frac{1}{100}}$ The person knows the first 4 digits. So the person has to guess the remaining two digits. P (guessing the PIN)=1×1×1×1×$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$	**1**
19	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2}) = \frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{3} \neq \frac{\pi}{4}$ So, A is false. Principal Value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$. So, R is true **Answer is (D)Assertion is false, but Reason is true**	**1**
20	C. $\vec{r} \times (\vec{a} + \vec{b}) = \vec{0} \Rightarrow \vec{r}$ is parallel to $(\vec{a} + \vec{b})$ and $(\vec{a} + \vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$. So, $\vec{r}$ is parallel to the plane of $\vec{a}$ and $\vec{b} \Rightarrow \vec{r}$ is perpendicular to $(\vec{a} \times \vec{b})$. So, Assertion is true But $(\vec{a} + \vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$, so $(\vec{a} + \vec{b})$ is not perpendicular to the plane of $\vec{a}$ *and* $\vec{b}$ Therefore, Reason is false. **Answer is (C) Assertion is true, but Reason is false**	**1**

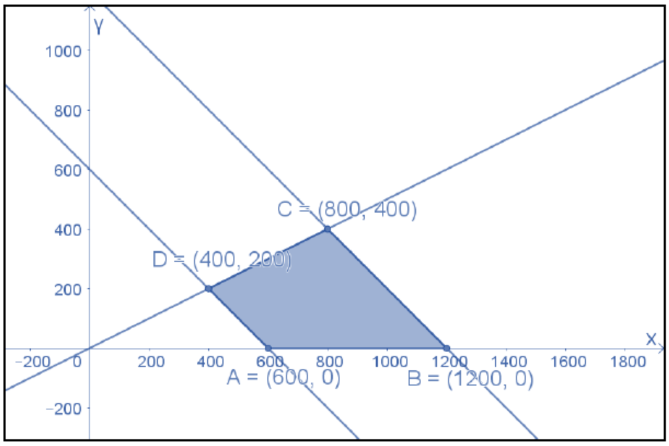
SECTION B
(VSA type questions of 2 marks each)

21A	$\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)=\tan\left(-\frac{\pi}{4}+\frac{\pi}{3}\right)$	½
	$=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$	1
	$=\frac{\sqrt{3}-1}{1+\sqrt{3}}\ or\ 2-\sqrt{3}$	½
	OR	OR
21B	For domain, $-1\leq 3x-2\leq 1$	½
	$\Rightarrow 1\leq 3x\leq 3$	½
	$\Rightarrow \frac{1}{3}\leq x\leq 1$	½
	So, domain of $\cos^{-1}(3x-2)$ is $\left[\frac{1}{3},1\right]$	½
22	$y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$	
	Differentiating with respect to x	
	$\frac{dy}{dx}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}.sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\frac{1}{2}$	½
	$=\frac{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}\cdot\frac{1}{cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}\cdot\frac{1}{2}$	
	$=\frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}=\frac{1}{\sin\left(\frac{\pi}{2}+x\right)}=\frac{1}{\cos x}$	1
	$\Longrightarrow \frac{dy}{dx}-\sec x=0$	½
23A	$\int\frac{(x-3)e^x}{(x-1)^3}dx=\int\frac{(x-1-2)e^x}{(x-1)^3}dx$	
	$=\int\left(\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right)e^x\,dx=\int\left(\frac{1}{(x-1)^2}+\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right)\right)e^x\,dx$	1
	$=\frac{e^x}{(x-1)^2}+c$ (as $\int(f(x)+f'(x))e^x\,dx=e^x f(x)+c$)	1
	OR	OR
23B	A = $\int_0^4 x\,dy = \int_0^4\sqrt{y}\,dy$	1
	$=\frac{2}{3}\times y^{3/2}\Big]_{y=0}^{y=4}=\frac{16}{3}$ sq. units	1
23B	**For Visually Impaired:**	
	A = $\int_0^3 y\,dx = \int_0^3\sqrt{x}\,dx$	1
	$=\frac{2}{3}\times x^{3/2}\Big]_{x=0}^{x=3}=2\sqrt{3}$ sq. units	1

24	Given $f(x+y)=f(x)f(y)$ $f(0+5)=f(0)f(5)$ $\Longrightarrow \quad f(0)=1$ $f'(5)= \lim_{h\to 0}\frac{f(5+h)-f(5)}{h} = \lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$ $[\because f(x+y)=f(x)f(y)]$ $= \lim_{h\to 0}\frac{2f(h)-2}{h}$ $[\because f(5)=2]$ $=2\lim_{h\to 0}\frac{f(h)-1}{h} = 2\lim_{h\to 0}\frac{f(h)-f(0)}{h} = 2f'(0)$ $=2(3)$ $[\because f'(0)=3]$ $=6$	½ 1 ½
25	The vector $\overrightarrow{OP}=\frac{1}{2}(4\hat{\imath}+4\hat{k})=2\hat{\imath}+2\hat{k}$ Area of the parallelogram formed by the two adjacent sides as OA and OP $=\left\|\left(\overrightarrow{OA}\text{ X }\overrightarrow{OP}\right)\right\| = \left\|\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & 1\\ 2 & 0 & 2\end{vmatrix}\right\|$ $=\left\|2\hat{\imath}-2\hat{k}\right\|$ $=2\sqrt{2}$ square units.	½ ½ ½ ½
	SECTION C **(SA type questions of 3 marks each)**	
26A	$x^y=e^{x-y}$ Taking log of both sides $y\log x=(x-y)\log e$ $y\log x + y = x$ (since $\log e=1$) $\Longrightarrow \quad y=\frac{x}{1+\log x}$ Differentiating with respect to x $\frac{dy}{dx}=\frac{(1+\log x).1-x.\frac{1}{x}}{(1+\log x)^2}$ $=\frac{\log x}{(\log e+\log x)^2}$ $=\frac{\log x}{(\log(x\,e))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e}=\frac{\log e}{(\log e^2)^2}=\frac{1}{(2\log e)^2}=\frac{1}{2^2}=\frac{1}{4}$ (as $\log e=1$) **Alternative Solution:** $x^y=e^{x-y}$ Taking log of both sides $y\log x=(x-y)\log e$ $y\log x + y = x$ (since $\log e=1$) Differentiating both sides w.r.t. x $\log x\frac{dy}{dx}+\frac{y}{x}+\frac{dy}{dx}=1$ $\Rightarrow \frac{dy}{dx}(1+\log x)=1-\frac{y}{x}$ $\Rightarrow \frac{dy}{dx}=\frac{x-y}{x(1+\log x)}=\frac{x-\frac{x}{1+\log x}}{x(1+\log x)}=\frac{x(1+\log x)-x}{x(1+\log x)^2}=\frac{x(1+\log x-1)}{x(\log e+\log x)^2}=\frac{\log x}{(\log(xe))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e}=\frac{\log e}{(\log e^2)^2}=\frac{1}{(2\log e)^2}=\frac{1}{2^2}=\frac{1}{4}$ (as $\log e=1$)	 1 1 1

	OR	OR
26B	$\frac{dx}{d\theta} = a(1-\cos\theta)$, $\frac{dy}{d\theta} = a(0+\sin\theta)$, $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2sin^2\left(\frac{\theta}{2}\right)} = \cot\frac{\theta}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}\, cosec^2\left(\frac{\theta}{2}\right)\frac{d\theta}{dx}$ $= -\frac{1}{2a}\, cosec^2\left(\frac{\theta}{2}\right)\frac{1}{2sin^2\left(\frac{\theta}{2}\right)}$ $= -\frac{1}{4a} cosec^4\left(\frac{\theta}{2}\right)$	1 1 1
27	Let r be the radius of ice ball at time t . $V = \frac{4}{3}\pi r^3$............ (1) S = $4\pi r^2$..............(2) Given $\frac{dV}{dt} \propto$ S $\Longrightarrow \frac{dV}{dt} = -$ k S (where k is some positive constant) ....(3) Differentiating (1) w.r.t. t, we get $\frac{dV}{dt} = \frac{4}{3}\pi.(3\,r^2)\frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ .... (4) $\Longrightarrow$ - k S = $4\pi r^2\frac{dr}{dt}$ (from (3) and (4)) $\Longrightarrow$ - k S = S $\frac{dr}{dt}$ (using (2)) $\Longrightarrow \frac{dr}{dt} = -$ k ⇒ radius of the ice-ball decreases at a constant rate	 ½ ½ 1 ½ ½
28A	 $\int_{-4}^{2}\lvert x+1\rvert dx = \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$ $= -\left.\frac{(x+1)^2}{2}\right]_{-4}^{-1} + \left.\frac{(x+1)^2}{2}\right]_{-1}^{2}$ $= -\left(0-\frac{9}{2}\right) + \left(\frac{9}{2}-0\right) = 9$ It represent the area of shaded region bounded by the curve $y = \lvert x+1\rvert$, $x -$ axis and the lines $x = -4$ and $x = 2$	1 ½ ½ ½ ½

28B	**OR**  Required Area = $\int_0^4 x\,dx - \int_0^4 \frac{x^2}{4}dx$ $= \left.\frac{x^2}{2}\right]_0^4 - \frac{1}{12}[x^3]_0^4$ $= \frac{1}{2}(16-0) - \frac{1}{12}(64-0) = 8 - \frac{16}{3} = \frac{8}{3}$ sq. units **For Visually Impaired**: $y = \lvert x+1\rvert = f(x) = \begin{cases} -x-1, & x < -1 \\ x+1, & x \geq -1 \end{cases}$ $\int_{-4}^{2}\lvert x+1\rvert dx = \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$ $= -\left.\frac{(x+1)^2}{2}\right]_{-4}^{-1} + \left.\frac{(x+1)^2}{2}\right]_{-1}^{2}$ $= -\left(0-\frac{9}{2}\right) + \left(\frac{9}{2}-0\right) = 9$ It represent the area of shaded region bounded by the curve $y = \lvert x+1\rvert$, $x -$ axis and the lines $x = -4$ *and* $x = 2$ **OR** 	1 1 ½ ½ 1 1 1

	$25x^2+16y^2=400 \Rightarrow \frac{x^2}{16}+\frac{y^2}{25}=1 \Rightarrow \frac{x^2}{4^2}+\frac{y^2}{5^2}=1 \Rightarrow y=\frac{5}{4}\sqrt{4^2-x^2}$	
	Required Area = $4\int_0^4 \frac{5}{4}\sqrt{4^2-x^2}\,dx$	1
	$=5\left[\frac{x\sqrt{4^2-x^2}}{2}+\frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$	1
	$=5[0+8\sin^{-1}(1)-0]$	
	$=40\times\frac{\pi}{2}=20\pi$ sq. units	1
29A	The line through $(2,-1,3)$ parallel to the z-axis is given by	
	$\vec{r}=\left(2\hat{\imath}-\hat{\jmath}+3\hat{k}\right)+\lambda\left(\hat{k}\right)$	1
	Any point on this line is P $(2,-1,3+\lambda)$	½
	Any point on the given line $\vec{r}=\left(2\hat{\imath}-\hat{\jmath}+2\hat{k}\right)+\mu(3\hat{\imath}+6\hat{\jmath}+2\hat{k})$ is	
	Q $(2+3\mu,-1+6\mu,2+2\mu)$	
	For the intersection point	½
	Q $(2+3\mu,-1+6\mu,2+2\mu)$ = P (2, -1, 3+ λ) $\Rightarrow 2=2+3\mu \Rightarrow \mu=0$	½
	The point of intersection is $(2,-1,2)$	½
	The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit.	
	Alternative Solution:	
	Any point on the line through $(2,-1,3)$ parallel to the z-axis is $(2,-1,\lambda)$	
	Any point on the given line is $(2+3\mu,-1+6\mu,2+2\mu)$	1
	Therefore, $2=2+3\mu \Rightarrow \mu=0$	1
	The point of intersection is $(2,-1,2)$	½
	The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit.	½
	OR	
29B	The line through $(2,-1,1)$parallel to the z-axis is $\vec{r}=\left(2\hat{\imath}-\hat{\jmath}+\hat{k}\right)+\lambda(\hat{k})$	1
	Any point on this line is P (2, -1, 1+ λ)	
	Any point on the given line is A (3+ μ, μ, 1+ μ)	
	A (3+ μ, μ, 1+ μ) = P (2, -1, 1+ λ) $\Rightarrow \mu=-1$	1
	The point of intersection is (2, -1,0)	½
	The distance of $(2,-1,0)$ from the z-axis is $\sqrt{2^2+(-1)^2}=\sqrt{5}$ units.	½
30	Sketching the graph	$1\frac{1}{2}$
		

30	Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z: $Z_A = 1200, Z_B = 2400, Z_C = 2000, Z_D = 1000$ Maximum $Z = 2400$ when $x = 1200$ and $y = 0$ **For Visually Impaired:** At Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z are $Z_A = 1800, Z_B = 3600, Z_C = 3200, Z_D = 1600$ Maximum Value of Z = 3600 at B(1200,0) And Minimum Value of Z= 1600 at D(400,200)	½ ½ ½ 1 1 1
31	Let the events be: A: Mehul is selected B: Rashi is selected Then according to the question, A and B are independent events and $P(A) = 0.4, \quad P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ Let $P(B) = x$ Then $P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ $\Rightarrow P(A)P(\bar{B}) + P(B)P(\bar{A}) = 0.5$ $\Rightarrow 0.4(1-x) + x(1-0.4) = 0.5$ $\Rightarrow 0.4 - 0.4x + 0.6x = 0.5$ $\Rightarrow 0.2x = 0.5 - 0.4 = 0.1$ $\Rightarrow x = \frac{0.1}{0.2} = \frac{1}{2} = 0.5$ So, probability of selection of Rashi = 0.5 Probability of selection of at least one of them $= 1 - P(\bar{A} \cap \bar{B})$ $= 1 - P(\bar{A})P(\bar{B})$ $= 1 - 0.6 \times 0.5$ $= 1 - 0.3 = 0.7$	1 1 1
	SECTION D **(LA type questions of 5 marks each)**	
32	$AB = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ So, $A^{-1} = B \text{ and } B^{-1} = A$ Given system of equations is $3x - 6y - z = 3, \quad 2x - 5y - z + 2 = 0, \quad -2x + 4y + z = 5$ In matrix form it can be written as: $AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $and\ C = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ Here $\|A\| = -3 - 0 + 2 = -1 \neq 0$ So, the system is consistent and has unique solution given by the expression $X = A^{-1}C = BC$ $\Rightarrow X = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$ Thus $x = 2, y = -3, z = 21$	1 ½ ½ ½ ½ ½ 1 ½

33A	Let $x = \tan\theta \Rightarrow dx = sec^2\theta\ d\theta$	½
	$I = \int_0^{\frac{\pi}{4}} \frac{\log(1+tan\theta)}{1+tan^2\theta} . sec^2\theta\ \ d\theta$	
	$I = \int_0^{\frac{\pi}{4}} log\ (1+tan\theta)d\theta \ = \ \int_0^{\frac{\pi}{4}} log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$	1
	$=\int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]\ d\theta$	1
	$=\int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right]$ dθ	
	$=\int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan\theta}\right]\ d\theta$	
	$=\int_0^{\frac{\pi}{4}} \log 2\ d\theta\ - \int_0^{\frac{\pi}{4}} \log[1+tan\theta]\ d\theta$	
	$= \log 2 \times x]_0^{\frac{\pi}{4}} - I$	1
	$\Rightarrow\ 2I = \frac{\pi}{4}\log 2$	1
	$\Rightarrow\ I = \frac{\pi}{8}\log 2$	½
33B	**OR**	OR
	I = $\int \frac{(3\ \sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta}\,d\theta = \int \frac{(3\ sin\,\theta-2)\,cos\,\theta}{5-(1-sin^2\theta)-4\,sin\,\theta}\,d\theta$	½
	$Let\ \ \ sin\,\theta = t\ \Longrightarrow cos\,\theta\ d\theta = dt$	
	$I = \int \frac{(3t-2)}{5-(1-t^2)-4t}\,dt$	
	$=\int \frac{(3t-2)}{t^2-4t+4}\,dt = \int \frac{3t-2}{(t-2)^2}\,dt$	1
	Let $\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}$	
	$3t-2 = A(t-2)+B$	
	Comparing the coefficients of t and constant terms on both sides	½
	$A\ =\ 3\,, -2A+B = \ -2\,, B\ =\ 4$	+½
	$\int \frac{(3\ \sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta}\,d\theta = \int \frac{3}{t-2}\,dt + \int \frac{4}{(t-2)^2}$ dt	
	$= 3\log\lvert t-2\rvert\ -\ \frac{4}{t-2} + C$	1+1
	$= 3\log\lvert sin\,\theta - 2\rvert\ - \frac{4}{\sin\theta-2}$ + C	½
34A	$y + \frac{d}{dx}(\,xy\,) = x\,(\sin x + x)$	
	$\Longrightarrow y + (x\frac{dy}{dx} + y\,) = x\,(\sin x + x)$	1
	$\Longrightarrow 2y + x\frac{dy}{dx} = x\,(\sin x + x)$	
	$\Longrightarrow \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$	
	This a linear differential equation of the form $\frac{dy}{dx} + Py = Q$	
	P=$\frac{2}{x}$,Q=$(\sin x + x)$	1
	I.F =$e^{\int \frac{2}{x}\,\mathrm{dx}}$= $e^{2\log x}$= $e^{\log x^2} = x^2$	
	Solution will be $\ y$. I.F =$\int Q.IF\ dx$	1
	$yx^2 = \int (sin\ x + x)\,x^2$ dx	
	$yx^2 = \int sinx.x^2\ dx + \int x^3\ dx$	

	$\Rightarrow\ yx^2 = -x^2\cos x + 2\int x\cos x\,dx\ + \frac{x^4}{4} + C$ $\Rightarrow yx^2 = -x^2 \cos x + 2\,(x\sin x + \cos x) + \frac{x^4}{4} + C$ Which is the required solution	1 1
	OR	
34B	$2y\,e^{x/y}dx + (\,y - 2x\,e^{x/y}\,)\,dy = 0$ $\Rightarrow \frac{dx}{dy} = \frac{2x\,e^{x/y} - y}{2y\,e^{x/y}} = \frac{2\frac{x}{y}e^{\frac{x}{y}} - 1}{2\,e^{\frac{x}{y}}}$ It is a homogeneous differential equation. Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$ $\Rightarrow y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v}$ $\Rightarrow\ y\frac{dv}{dy} = \frac{-1}{2e^v}$ $\Rightarrow 2e^v\,dv = -\frac{dy}{y}$ $\int 2e^v\,dv = -\int \frac{dy}{y}$ $\Rightarrow 2e^v = -\log\|y\| + C$ $\Rightarrow 2e^{\frac{x}{y}}$ +" $log\|y\| = C$ When $x = 0\,, y = 1$, C = 2 Required solution $2\,e^{\frac{x}{y}} + \log\|y\| = 2$	 1 1 1 1 1
35	Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \quad \Rightarrow$ Any point on it is $(3\lambda + 1, -\lambda, -1)$ For the point where $y = 1 \quad \Rightarrow\ \lambda = -1$ $\Rightarrow$ The point is $(-2, 1\,, -1)$ The directions of the two lines are $\vec{m} = 3\hat{\imath} - \hat{\jmath}$ and $\vec{n} = -2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $\vec{m}\times\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 0 \\ -2 & 2 & 1 \end{vmatrix} = -\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ The equation of the required line is $\vec{r} = \left(-2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \mu\left(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right)$ **Alternative Solution:** Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \ \Rightarrow$ Any point on it is $(3\lambda + 1, -\lambda, -1)$ For the point where $y = 1 \quad \Rightarrow\ \lambda = -1$ $\Rightarrow$ The point is $(-2, 1, -1)$ Let the direction ratios of the required line be a, b, c Then $3a - b = 0$ And $-2a + 2b + c = 0$ Solving we get $\frac{a}{-1} = \frac{-b}{3} = \frac{c}{4} \Rightarrow \frac{a}{-1} = \frac{b}{-3} = \frac{c}{4}$ The required line is $\frac{x+2}{-1} = \frac{y-1}{-3} = \frac{z+1}{4} = \mu$ In vector form $\vec{r} = \left(-2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \mu\left(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right)$	½ 1 ½ 1 ½ 1 ½ ½ 1 ½ 1 1 ½ ½

SECTION- E (3 case-study/passage-based questions of 4 marks each)		
36	I. Traffic flow is not reflexive as $(A, A) \notin R$ (or no major spot is connected with itself) II. Traffic flow is not transitive as $(A, B) \in R$ and $(B, E) \in R$, but $(A, E) \notin R$ III A. $R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, C)\}$ $\text{Domain} = \{A, B, C, D\}$ $\text{Range} = \{B, C, D, E\}$ **OR** III B. No, the traffic flow doesn't represent a function as A has three images.	1 1 1 ½ + ½ 1+1
37	I. $P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150)$ $= -0.8x^2 + 30x - 150$ II. For critical points $P'(x) = 0 \Rightarrow -1.6x + 30 = 0$ $\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75$ III A. Now $P''(x) = -1.6$ In particular $P''(18.75) = -1.6 < 0$ So, critical value $x = 18.75$ corresponds to a maximum profit. **OR** III B. As x is the number of bulbs, so practically 18 bulbs correspond to a maximum profit. Maximum profit is $P(18) = -0.8 \times 18^2 + 30 \times 18 - 150$ $= -259.2 + 540 - 150$ $= 540 - 409.2 = ₹130.80$	1 1 1 1 1 1
38	Let the events be E$_1$: the student is in the first group (time spent on screen is more than 4 hours) E$_2$: the student is in the second group (time spent on screen is 2 to 4 hours) E$_3$: the student is in third group (time spent on screen is less than 2 hours) A: the event of the student showing symptoms of anxiety and low retention $P(E_1) = \frac{60}{100}$ $P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{10}{100}$ $P(A\|E_1) = \frac{80}{100}$ $P(A\|E_2) = \frac{70}{100}$ and $P(A\|E_3) = \frac{30}{100}$ I. $P(A) = P(E_1) \times P(A\|E_1) + P(E_2) \times P(A\|E_2) + P(E_3) \times P(A\|E_3)$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100} = \frac{72}{100} = 72\%$ II. $P(E_1\|A) = \frac{P(E_1 \cap A)}{P(A)}$ $= \frac{\left(\frac{60}{100} \times \frac{80}{100}\right)}{\left(\frac{72}{100}\right)} = \frac{48}{72} = \frac{2}{3}$	2 2