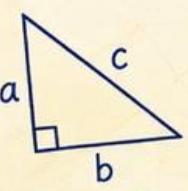


तत् त्वं पूषन् अपावृणु

$$\sqrt{a^2} = |a|$$



$$a^2 + b^2 = c^2$$



THINK
SOLVE
ACHIEVE



$$ax^2 + bx + c = 0$$



KENDRIYA VIDYALAYA SANGATHAN

★ REGIONAL OFFICE AGRA ★

STUDENT STUDY MATERIAL

(MATHEMATICS)

CLASS - 10

SESSION:- 2026-27



REAL NUMBERS
POLYNOMIALS



LINEAR EQUATIONS
IN TWO VARIABLES



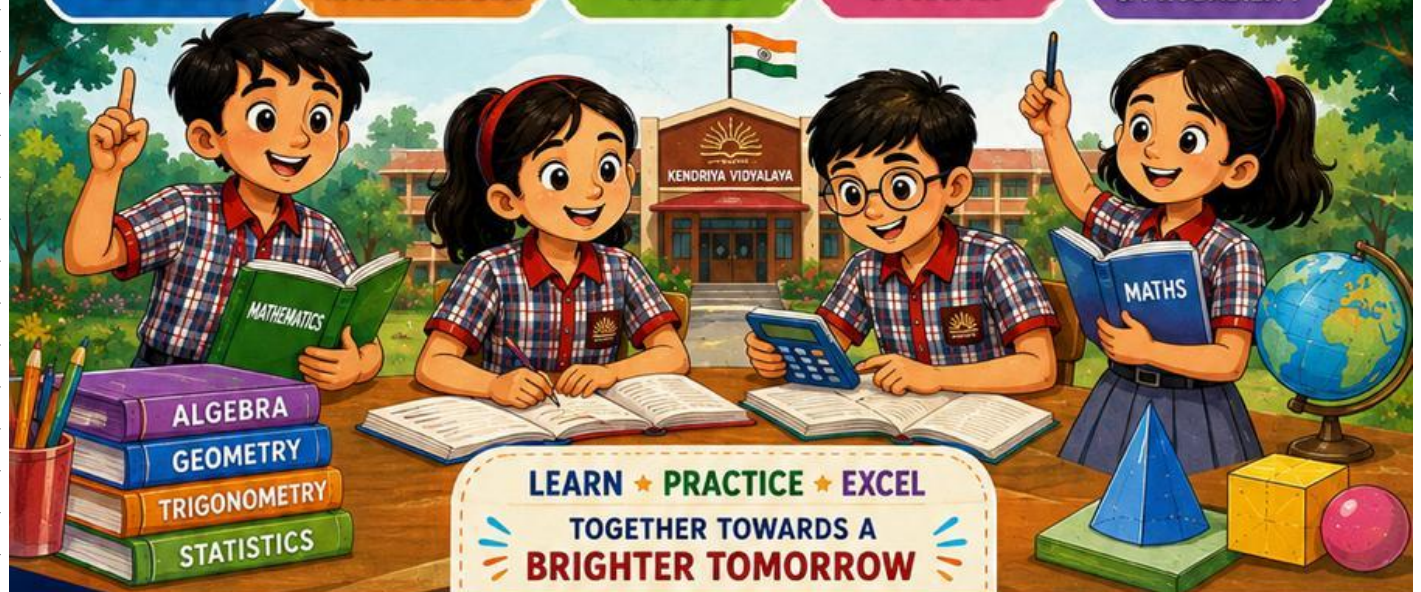
TRIANGLES
& CIRCLES



SURFACE AREAS
& VOLUMES



STATISTICS
& PROBABILITY



LEARN ★ PRACTICE ★ EXCEL

TOGETHER TOWARDS A
BRIGHTER TOMORROW



CONCEPTUAL
UNDERSTANDING



CRITICAL
THINKING



PROBLEM
SOLVING



CONFIDENCE
& SUCCESS

MATHEMATICS STUDY MATERIAL

(SESSION 2026-27)

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for their valuable guidance, constant encouragement
and visionary leadership.*

TEAM – MATHEMATICS

OUR TEAM

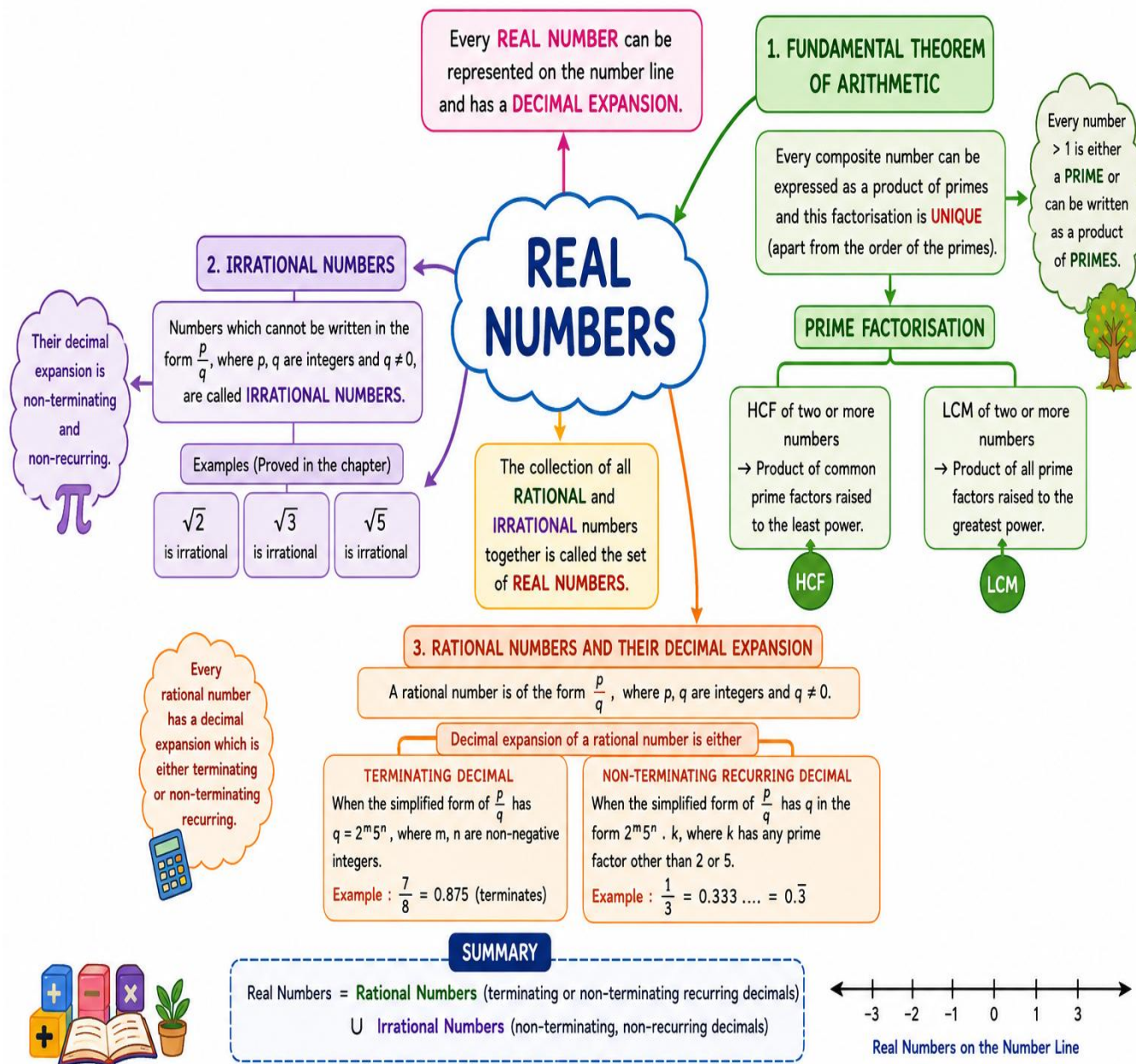
SL.NO.	Chapter (X Mathematics)	Teacher Allotted (TGT Maths)	School Name
1	REAL NUMBERS	MRS. MAMTA KUSHWAHA	PM SHRI KV NO.3 AGRA CANTT
2	POLYNOMIALS		
3	PAIR OF LINEAR EQUATIONS IN TWO VARIABLES	MS SAPNA RATWAYA	PM SHRI KV MORADABAD
4	QUADRATIC EQUATION		
5	ARITHMETIC PROGRESSION	MR. VIKASH KUMAR SINGHAL	PM SHRI KV MUZAFFARNAGAR (SH-I)
6	COORDINATE GEOMETRY		
7	INTRODUCTION TO TRIGONOMETRY	MR JEETENDRA BHATIA	PM SHRI KV SEC-24 NOIDA (SH-II)
8	APPLICATION OF TRIGONOMETRY		
9	TRIANGLES	MR HEMANT KUMAR MEENA	KV BAAD MATHURA
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CHAPTER 1

REAL NUMBERS



Concise notes-

The Fundamental Theorem of Arithmetic- Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

- The prime factorisation of a natural number is unique, except for the order of its factors.
- for any two positive integers a and b ,

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

PRIME FACTORISATION METHOD TO FIND HCF AND LCM-

Example 1: Find the LCM of 315 and 360 by the prime factorisation method. Hence, find their HCF.

Solution: $315 = 3 \times 3 \times 5 \times 7 = 3^2 \times 5 \times 7$

$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$$

$$\text{LCM} = 3^2 \times 5 \times 7 \times 2^3 = 2520$$

$$\text{HCF (315 and 360)} = \frac{315 \times 360}{\text{LCM (315} \times 360)}$$

Example 2: Find the HCF of 300, 360 and 240 by the prime factorisation method.

Solution: $300 = 2^2 \times 3 \times 5^2$

$$360 = 2^3 \times 3^2 \times 5$$

$$240 = 2^4 \times 3 \times 5$$

$$\text{HCF (300, 360, 240)} = 2^2 \times 3 \times 5 = 60$$

THEOREM 1.3: Let p be a prime number. If p divides a^2 , then p divides a , where a is a positive integer.

Example: Prove that $\sqrt{3}$ is irrational.

Solution : Let us assume, to the contrary, that $\sqrt{3}$ is rational. That is, we can find integers a and b ($\neq 0$) such that $\sqrt{3} = \frac{a}{b}$.

Suppose a and b have a common factor other than 1, then we can divide by the common factor, and assume that a and b are coprime.

$$\text{So, } b = \sqrt{3} a$$

Squaring on both sides, and rearranging, we get $3b^2 = a^2$.

Therefore, a^2 is divisible by 3, and by Theorem 1.3, it follows that a is also divisible by 3. So, we can write $a = 3c$ for some integer c . Substituting for a , we get $3b^2 = 9c^2$

$$b^2 = 3c^2$$

This means that b^2 is divisible by 3, and so b is also divisible by 3 (using Theorem 1.3 with $p = 3$). Therefore, a and b have at least 3 as a common factor. But this contradicts the fact that a and b are coprime. This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

Practice questions:-

Q. 1 Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Q. 2 Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.

Q. 3 Find the greatest number that will divide 445, 572 and 699 leaving remainder 4, 5 and 6 respectively.

Q. 4 A merchant has 120 litres of oil of one kind, 180 litres of another kind and 240 litres of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

Q. 5. 144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would

have?

Q. 6. In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

Q. 7. Show that the following numbers are irrational.

(i) $\frac{1}{\sqrt{7}}$ (ii) $2\sqrt{3}$ (iii) $6 + \sqrt{5}$ (iv) $3 - \sqrt{7}$

Answers

Solution 1: Let the HCF of 615 and 963 be x.

Since it is given remainder is 6 in each case, Therefore for the numbers to be completely divisible, 6

should be subtracted from both the numbers.

Therefore new numbers are:

$$615-6 = 609$$

$$963-6 = 957$$

$$\text{Prime factors of } 609 = 3 \times 3 \times 29$$

$$\text{Prime factors of } 957 = 3 \times 11 \times 29$$

$$\text{Therefore HCF of } 609 \text{ and } 957 \text{ is: } 3 \times 29 = 87$$

Hence the largest number which divides 615 and 963 leaving remainder 6 in each case is 87.

Solution 2: The new numbers after subtracting remainders are:

$$285-9 = 276$$

$$1249-7 = 1242$$

$$\text{Prime factors of } 276 = 2 \times 3 \times 23$$

$$\text{Prime factors of } 1242 = 2 \times 3 \times 3 \times 3 \times 23$$

$$\text{Therefore HCF of } 276 \text{ and } 1242 \text{ is: } 2 \times 3 \times 23 = 138$$

Hence the greatest number which divides 285 and 1249 leaving remainder 9 and 7 respectively is 138.

Solution 3: The new numbers after subtracting remainders are:

$$445-4 = 441$$

$$572-5 = 567$$

$$699-6 = 693$$

$$\text{Prime factors of } 441 = 3 \times 3 \times 7 \times 7$$

$$\text{Prime factors of } 567 = 3 \times 3 \times 3 \times 3 \times 7$$

$$\text{Prime factors of } 693 = 3 \times 3 \times 7 \times 11$$

$$\text{Therefore HCF of } 441, 567 \text{ and } 693 \text{ is: } 3 \times 3 \times 7 = 63$$

Hence the greatest number that will divide 445, 572 and 699 leaving remainder 4, 5 and 6 respectively is 63.

Solution 4: To find greatest capacity of tin we should find HCF of 120 and 180 and 240

$$\text{Prime factors of } 120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$\text{Prime factors of } 180 = 2 \times 2 \times 3 \times 3 \times 5$$

$$\text{Prime factors of } 240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

$$\text{Therefore HCF of } 120, 180 \text{ and } 240 \text{ is: } 2 \times 2 \times 3 \times 5 = 60$$

Therefore the greatest capacity of a tin is 60 Liters.

Solution 5: To find greatest number of cartons each stack would have, we should find HCF of 144 and 90

Prime factors of 144 = $2 \times 2 \times 2 \times 2 \times 3 \times 3$

Prime factors of 90 = $2 \times 3 \times 3 \times 5$

Therefore HCF of 144 and 90 is: $2 \times 3 \times 3 = 18$

Therefore the greatest number of cartons each stack would have is: 18

Solution 6: To calculate minimum distance Minimum distance we should calculate LCM

Prime factors of 80 = $2 \times 2 \times 2 \times 2 \times 5$

Prime factors of 85 = 5×17

Prime factors of 90 = $2 \times 3 \times 3 \times 5$

LCM of 80, 85 and 90 $\Rightarrow 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 17 = 12240$ Minimum distance is 12240 cm, As 1 m = 100 cm So 12240 = 120 m 40 cm

Therefore the minimum distance each should walk so that he can cover the distance in complete steps is 122 m 40 cm.

PREVIOUS YEAR QUESTIONS

Q. 1. If HCF (850, 325) is 25, then LCM (850, 325) is :

- (A) 442 (B) 11050 (1 Marks) (CBSE 2026-430-4-1 Basic)
(C) 8450 (D) 2210

Q. 2. $7 \times 29 \times 23 + 1$ is :

- (1 Marks) (CBSE 2026-430-4-1 Basic)
(A) a prime number. (B) divisible by 23.
(C) an odd number. (D) a composite number.

Q. 3. The HCF of 960 and 432 is:

- (1 Marks) (CBSE 2026-30-1-1 Standard)
(a) 48 (b) 54
(c) 72 (d) 36

Q. 4. The natural number 2 is:

- (1 Marks) (CBSE 2026-30-1-1 Standard)
(a) a prime number (b) a composite number
(c) prime as well as composite (d) neither prime nor composite

Q. 5. For any natural number n , 6^n ends with the digit:

- (1 Marks) (CBSE 2026-30-1-1 Standard)
(a) 0 (b) 6
(c) 3 (d) 2

Q. 6. If the HCF of two positive integers a and b is 1, then their LCM is : (1 Mark) (2025)

- (A) $a + b$ (B) a (C) b (D) ab

Q. 7. The number $3 + \sqrt{2}$ is :

- (1 Mark) (2025)
(A) a rational number (B) an irrational number
(C) an integer (D) a natural number

Q. 8. If $\text{HCF}(98, 28) = m$ and $\text{LCM}(98, 28) = n$, then the value of $n \times m$ is :

- (1 Mark) (2025)
(A) 7442 (B) 2744
(C) 4498 (D) 1927

Q. 9. If $(-1)^n + (-1)^8 = 0$, then n is: (1 Mark) (2025)

(A) any positive integer
(C) any odd number

(B) any negative integer
(D) any even number

Q. 10. The total number of factors of a prime number is

(a) 1 (b) 0 (c) 2 (d) 3 (2020)

Q. 11. The ratio of HCF to LCM of the least composite number and the least prime number is

(a) 1:2 (b) 2:1 (c) 1:1 (d) 1:3 (2023)

Q. 12. If $\text{HCF}(39, 91) = 13$, then $\text{LCM}(39, 91)$ is

(a) 91 (b) 273 (c) 39 (d) 3549 (Term I, 2021-22)

Q. 13. Two positive numbers have their HCF as 12 and their product as 6336. The number of pairs possible for the numbers, is

(a) 2 (b) 3 (c) 4 (d) 1 (Term I, 2021-22)

Q. 14. If 'n' is any natural number, then $(12)^n$ cannot end with the digit

(a) 2 (b) 4 (c) 8 (d) 0 (Term I, 2021-22)

Q. 15. The number 385 can be expressed as the product of prime factors as

(a) $5 \times 11 \times 13$ (b) $5 \times 7 \times 11$
(c) $5 \times 7 \times 13$ (d) $5 \times 11 \times 17$ (Term I, 2021-22)

Q. 16. The HCF and the LCM of 12, 21 and 15 respectively, are

(a) 3, 140 (b) 12, 420
(c) 3, 420 (d) 420, 3 (2020)

Q. 17. $\text{LCM}(850, 500)$ is: (2024)

(a) 850×50 (b) 17×500
(c) $17 \times 52 \times 22$ (d) $17 \times 53 \times 2$

Q. 18. If two positive integers p and q can be expressed as $p = 18 a^2 b^1$ and $q = 20 a^3 b^2$, where a and b are prime numbers, then $\text{LCM}(p, q)$ is:

(2024)

(a) $2 a^2 b^2$ (b) $180 a^2 b^2$
(c) $12 a^2 b^2$ (d) $180 a^3 b^4$

Q. 19. The natural number 1 is:

(A) a prime number. (B) a composite number.
(C) prime as well as composite. (D) neither prime nor composite.

Q. 20. For any natural number n, 5^n ends with the digit:

(A) 0 (B) 5 (C) 3 (D) 2

Q. 21. The LCM of 960 and 240 is:

(A) 960 (B) 240 (C) 60 (D) 15

Q. 22. If $a = 2^5 \times 3^3$, $b = 2^3 \times 3^2 \times 5$ and $c = 3^3 \times 5^n$ and $\text{L.C.M.}(a, b, \text{ then } n \text{ is equal to } c) = 2^0 \times 3^4 \times 5^1$,

(A) 7 (B) 5 (C) 3 (D) 1

Assertion and Reason

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

- c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q. 1. Assertion (A) : $4n$ can not end with the digit zero. (1 Marks) (CBSE 2026-430-4-1 Basic)
Reason (R) : Prime factorisation of $4n$ is unique.

Q. 2. Assertion (A) : For any two natural numbers a and b , the HCF of a and b is a factor of the LCM of a and b .
Reason (R) : HCF of any two natural numbers divides both the numbers.
(1 Mark) (2025)

Q. 3. Assertion (A): If the graph of a polynomial touches x -axis at only one point, then the polynomial cannot be a quadratic polynomial.

Reason (R): A polynomial of degree n ($n > 1$) can have at most n zeroes.

Q. 4. Assertion (A): If HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

Reason (R): For any two positive integers a and b , $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$

Q. 5. Assertion (A): $11 \times 4 \times 3 \times 2 + 4$ is a composite number.

Reason (R): Every composite number can be expressed as product of primes.

Previous Year Questions-

Q. 1. Prove that $\sqrt{5}$ is an irrational number. (3 Marks) (CBSE 2026-430-4-1 Basic)

Q. 2. Prove that $2\sqrt{3}$ is an irrational number. (3 Marks) (CBSE 2026-30-1-1 Standard)

Q. 3. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.
(2020)

Q. 4. The LCM of two numbers is 9 times their HCF. The sum of LCM and HCF is 500. Find the HCF of the two numbers.
(2019C)

Q. 5. If $\text{HCF}(336, 54) = 6$, find $\text{LCM}(336, 54)$.
(2019)

Q. 6. The HCF of two numbers a and b is 5 and their LCM is 200 Find the product ab .
(AI 2019)

Q. 7. What is the HCF of smallest prime number and the smallest composite number?
(2018)

Q. 8. The HCF of two numbers is 27 and their LCM is 162, if one of the number is 54, find the other number.
(Board Term 1, 2017)

Q. 9. The LCM of two numbers is 2079 and their HCF is 27. If one of the number is 297. Find the other number. (Board Term 1, 2015)SAI (2 marks)

Q. 10. Find the least number which when divided by 12, 16 and 24 leaves remainder 7 in each case.
(2023)

Q. 11. Two numbers are in the ratio 2 : 3 and their LCM is 180 What is the HCF of these numbers?
(2023)

Q. 12. Find HCF and LCM of 404 and 96 and verify that $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$.
(2018)

Q. 13. Find the largest possible positive integer that divides 125, 162 and 259 leaving remainder 5, 6 and 7 respectively.
(Board Term 1, 2017)

Q. 14. Prove that $6-4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number. (2024)

Q. 15. Show that $11 \times 19 \times 23 + 3 \times 11$ is not a prime number. (2024)

Q. 16. Prove that $5-2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number. (2024)

Q. 17. Show that the number $5 \times 11 \times 17 + 3 \times 11$ is a composite number. (2024)

Q. 18. An army contingent of 678 soldiers is to march behind an army band of 36 members in a Republic Day parade. The two groups are to march in the same number of columns. What is the maximum number of columns they can march? (Board Term 1, 2017)

Q. 19. On a morning walk, three persons steps off together and their steps measure 40 cm, 42 cm, and 45 cm respectively. What is the minimum distance each should walk so that each can cover same distance in complete steps?

(Board Term 1, 2015) LA (4/5/6 marks)

Q. 20. A sweet shopkeeper prepares 396 gulab jamuns and 342 ras-gullas. He packs them into containers. Each container consists of either gulab jamun or ras-gullas but have equal number of pieces. Find the number of pieces he should put in each box so that number of boxes are least.

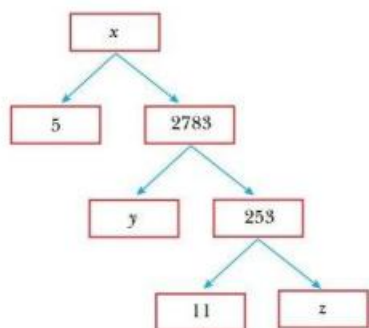
(Board Term 1, 2017)

Q. 21. In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject. (2024)

Case Study-based Questions

Case Study 1.

A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.



Observe the following factor tree and answer the following:

(i) What will be the value of x?

- (ii) What will be the value of y ?
- (iii) What will be the value of z ?

Or

Which type of the number 13915 is according to Fundamental Theorem of Arithmetic.

Case Study 2.

To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections-section A and section B of grade X. There are 32 students in section A and 36 students in section B.



- (i) What is the minimum number of books you will require for the class library, so that they can be distributed equally among students of Section A or Section B?
- (ii) If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32, 36) is
- (iii) Explain why $7 \times 11 \times 13 + 13$ is a composite number.

or

If p and q are positive integers such that $p = a \times b^2$ and $q = a^2 \times b$ where a, b are prime numbers, then the LCM (p, q) is.

Case Study 3.

A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.



- (i) In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number of participants that can accommodated in each room are.
- (ii) What is the minimum number of rooms required during the event?
- (iii) The LCM of 60, 84 and 108 is.

- (iv) The product of HCF and LCM of 60,84 and 108 is.
(v) 108 can be expressed as a product of its primes as.

Case Study 4.

Old age homes mean for senior citizens who are unable to stay with their families or destitute. These old age homes have special medical facilities for senior citizens such as mobile health care systems, ambulances, nurses and provision of well-balanced meals.

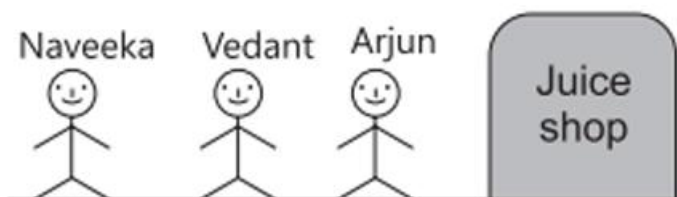


Himanshu, Gaurav and Gagan start preparing greeting cards for each person of an old age home on new year. In order to complete one card, they take 10, 16 and 20 min respectively. Based on the above information, solve the following questions:

- Q1. Co-prime numbers are those numbers which do not have any common factor other than 1. Is this statement true?
Q2. Find the sum of the powers of all different prime factors of the numbers 10, 16 and 20.
Q3. If all of them started together, then what time will they start preparing a new card together?
OR
What is the common time to make one card?

Case Study 5.

In a morning walk, Naveeka, Arjun and Vedant step off together, their steps measuring 240 cm, 90 cm, 120 cm respectively. They want to go for a juice shop for a health issue, which is situated near by them. Based on the above information, solve the following questions:



- Q1. Factor tree is a chain of factors, which is represented in the form of a tree. Is this statement true?

Q2. Find the sum of the powers of all common prime factors of the numbers 240, 90 and 120.

Q3. Find the minimum distance of shop from where they start to walk together, so that one can cover the distance in complete steps.

Or

Find the number of common steps covered by all of them to reach the juice shop.

ANSWERS

MCQS-

1. (B). 11050, 2. (d) Composite Number, 3. (a) 48, 4. (a) prime number, 5. (b) 6, 6. (d) ab, 7. (b) irrational number, 8. (b) 2744, 9. (c) any odd number, 10. (c) 2, 11. (a) 1:2, 12. (b) 273, 13. (a) 2, 14. (d) 0, 15. (b) $5 \times 7 \times 11$, 16. (d) 420, 3, 17. (b) 17×500 , 18. (d) $180 a^3 b^4$, 19. (d) neither prime nor composite, 20. (b) 5, 21. (a) 960, 22. (a) 7

Assertion and Reason

1. A, 2. A, 3. D, 4. A, 5. A

Previous Year Questions-

- 3.91, 4. 50, 5. 3024, 6. 1000, 7. 2, 8. 81, 9. 189, 10. 55, 11. 30, 12. HCF = 4, LCM = 9696, 13. 12, 14. 18. 6 COLUMNS, 19. 2520, 20. 18 Pieces, 21. 17 rooms,

Case Study

1. (i) $x = 13915$, (ii) $y = 11$, (iii) $z = 23$, or composite number

2. (i) 288, (ii) 4, (iii) composite number, or $a^2 b^2$

3. (i) 12, (ii) 21, (iii) 3780 or $2^2 3^3$

4. (i) True, (ii) $4 + 1 = 5$ (iii) 80 minutes, or 80 minutes

5. (i) True (ii) $1 + 1 + 1 = 3$ (iii) 720 cm, or 24 steps.

QUADRATIC POLYNOMIAL

Polynomial : An algebraic expression in the form of $a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$, (where n is a whole number and $a_0, a_1, a_2, \dots, a_n$ are real numbers) is called a polynomial in one variable x of degree n .

Value of a Polynomial at a given point : If $p(x)$ is a polynomial in x and ' a ' is any real number, then the value obtained by putting $x = a$ in $p(x)$, is called the value of $p(x)$ at $x = a$.

Zero of a Polynomial : A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.

Geometrically, the zeroes of a polynomial $p(x)$ are precisely the x -co-ordinates of the points, where the graph of $y = p(x)$ intersects the x -axis.

- (i) A linear polynomial has one and only one zero.
- (ii) A quadratic polynomial has at most two zeroes.
- (iii) A cubic polynomial has at most three zeroes.
- (iv) In general, a polynomial of degree n has at most n zeroes.

Graphs of Different types of Polynomials :

Linear Polynomial : The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects x -axis at one point only.

Quadratic Polynomial :

- (i) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if $a > 0$ and intersects x -axis at a maximum of two distinct points.
- (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects x -axis at a maximum of two distinct points.

Graph of a cubic polynomial : Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects x -axis at a maximum of three distinct points.

Relationship between Zeroes and Coefficients of a Polynomial

If α^* and β^* are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, Then you know that $x - \alpha$ and $x - \beta$ are the factors of $p(x)$. Therefore, $ax^2 + bx + c = k(x - \alpha)(x - \beta)$, where k is a constant

$$(a + \beta)x + a\beta = kx^2 - k(a + \beta)x + k\alpha\beta$$

Comparing the coefficients of x^2 , x and constant terms on both the sides, we get $a = k$, $b = -k(a + \beta)$ and $c = k\alpha\beta$.

$$\text{This gives } a + \beta = -b/a, \quad a\beta = c/a$$

$$\text{sum of zeroes} = \alpha + \beta = -b/a =$$

$$\text{product of zeroes} = \alpha\beta = c/a =$$

MCQS-

1. If α and β are the zeroes of polynomial $3x^2 + 6x + k$ such that (1 Mark) (2025)

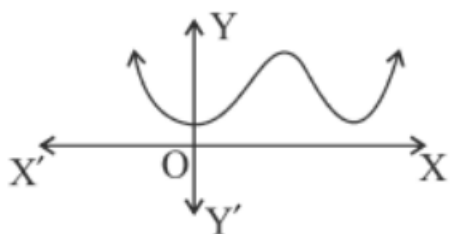
$$\alpha + \beta + \alpha\beta = \text{then the value of } k \text{ is}$$

(A) 8

(B) 8

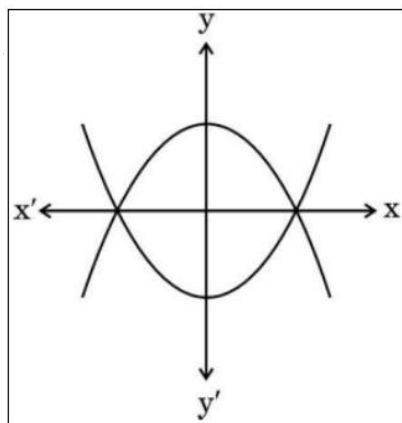
(C) 4

(D) 4



2. The graph of $y=f(x)$ is given. The number of zeroes of $f(x)$ is:

- (a) 0 (b) 1
(c) 2 (d) 4



3. Two polynomials are shown in the graph below. The number of common zeroes of both the polynomials is :

4. If one of the zeroes of the quadratic polynomial $(a-1)x^2+ax+1$ is -3 , then the value of a is: (2024)

- (A) (B) (C) (D) -

5. For what value of k , the product of zeroes of the polynomial kx^2-4x-7 is 2 ? (2024)

- (A) (B) (C) (D)

6. The zeroes of a polynomial $x^2 + px + q$ are twice the zeroes of the polynomial $4x^2- 5x - 6$. The value of p is: (2024)

- (A) (B) (C) (D)

The degree of polynomial having zeroes -3 and 4 only is

- (a) 2 (b) 1 (c) More than 2 (d) 3

8. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2 , then the value of k is (2020)

- (a) 10 (b) -10 (c) -7 (d) -2

9. If the sum of zeroes of the polynomial $p(x) = 2x^2 - k\sqrt{2}x+1$ is $\sqrt{2}$, then value of k is: (2024)

- (a) (b) 2 (c) 2 (d)

10. Which of the following is a quadratic polynomial with zero 3 and 0 ?

- (a) $3x(3x-5)$ (b) $3x(x-5)$ (c) $x^2 - 9$ (d) $x^2 - 15x + 15$

11. If α, β are the zeroes of a polynomial $p(x) = x^2 + x - 1$ then $\alpha + \beta$ is

- (a) 1 (b) 2 (c) -1 (d) -2

12. If α, β are zeroes of the polynomial $x^2 - 1$, then value of $(\alpha + \beta)$ is (2023)

- (a) 2 (b) 1 (c) -1 (d) 0

13. If α, β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$ then $\alpha + \beta$ is

- (A) (B) (C) (D)

14. If α and β are the zeros of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is (2022-23)

- (a) $-2/3$ (b) $2/3$ (c) $1/3$ (d) $-1/3$

15. If 2 and $1/2$ are the zeroes of $px^2 + 5x + r$, then

- (a) $p=r=2$ (b) $p=r=-2$

- (c) $p=2, r=-2$ (d) $p=-2, r=2$
16. quadratic polynomial having sum and product of its zeroes as 2 and -7 respectively, is:
- (A) $x^2 - 2x + 7$ (B) $4(x^2 + 2x - 7)$
- (C) $(x - 2)(x + 7)$ (D) $3(x^2 - 2x - 7)$

SAI (2 marks)

If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\alpha + \beta$

If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k . (2021C)

Form a quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. (2020)

Find the quadratic polynomial whose zeroes are 3 and -4 respectively. (Board Term 1, 2015)

Find the zeroes of the polynomial $p(x) = 3x^2 - x - 4$. Hence, verify the relationship between the zeroes and its coefficients.

Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Find the zeroes of the polynomial $x^2 + x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.

. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) -1 (ii) 1

Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$ and verify the relationship between the zeroes and the coefficients.

Find a quadratic polynomial whose sum and product of zeroes are 0 and -9, respectively. Also, find the zeroes of the polynomial so obtained.

Find the zeroes of quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients of the polynomial. (2025)

LA (3 marks)

If α, β are the zeroes of polynomial $p(x) = 6x^2 - 5x - 3$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$

One zero of the polynomial $4x^2 + 12x + (2k + 1)$ is five times the other. Find the value of k .

Find the zeroes of polynomial $x^2 + x - 6$.

If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k . (2023)

If α and β are zeroes of the polynomial $x^2 - p(x + 1) + c$ such that $(\alpha + 1)(\beta + 1) = 0$, then find the value of c . (Board Term 1, 2016)

If α and β are zeroes of $4x^2 + 3x + 7$, then find the value of $\alpha + \beta$

Find the value of k such that the polynomial $x^2 - (k+6)x + 2(2k - 1)$ has sum of its zeroes equal to half of their product. (Delhi 2019)

Find the quadratic polynomial, sum and product of whose zeroes are -1

and - 20 respectively. Also, find the zeroes of the polynomial so obtained.
(2019)

Assertion And Reason

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

- (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
- (c) Assertion (A) is true, but Reason (R) is false.
- (d) Assertion (A) is false, but Reason (R) is true.

Assertion (A): Zeroes of a polynomial $p(x) = x^2 - 2x - 3$ are -1 and 3.
Reason (R): The graph of polynomial $p(x) = x^2 - 2x - 3$ intersects x-axis at (-1, 0) and (3, 0).
(2024)

Answer. (a)

Assertion (A): The polynomial $p(y) = y^2 + 4y + 3$ has two zeroes.
Reason (R): A quadratic polynomial can have at most two zeroes.

Ans a

Assertion (A): The polynomial $p(x) = x^2 + 3x + 3$ has two real zeroes.
Reason (R): A quadratic polynomial can have at most two real zeroes.

Ans d

Assertion (A) : If the sum of the zeroes of the quadratic polynomial $x^2 - 2kx + 8$ is 2 then value of k is 1.

Reason (R) : Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $-b/a$.

Ans a

Assertion (A) : If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other, then $k = 2$.

Reason (R) : If $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$ i.e., a is a zero of $p(x)$.

Ans b

Assertion (A) : $x^2 + 7x + 12$ has no real zeroes.

Reason (R) : A quadratic polynomial can have at the most two zeroes.

Ans. d

Case study:

Case study: 1.



The figure given alongside shows the path of a diver, when she takes a jump from the diving board. Clearly, it is a parabola.

Annie was standing on a diving board, 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time 't' (in seconds) is given by the polynomial $h(t)$ such that $h(t) = -16t^2 + 8t + k$.

Based on above information, attempt any 4 out of 5 subparts.

At what time will she touch the water in the pool?

Rita's height (in feet) above the water level is given by another polynomial

$p(t)$ with zeroes -1 and 2. Then $p(t)$ is given by

A polynomial $q(t)$ with sum of zeroes as 1 and the product as -6 is modelling Anu's height in feet above the water at any time t (in seconds). Then $q(t)$ is given by

OR

The zeroes of the polynomial $r(t) = -12t^2 + (k - 3)t + 48$ are additive inverse of each other. Then k is

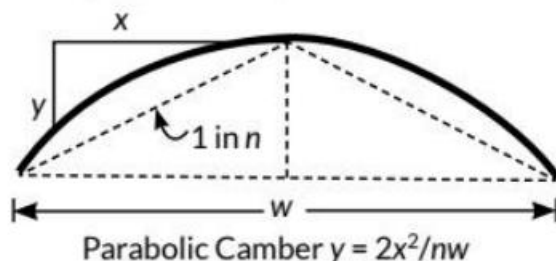
Case Study:2.

Applications of Parabolas-Highway Overpasses/Underpasses

A highway underpasses is parabolic in shape.



Shape of Cross Slope :



A parabola is the graph that results from $p(x) = ax^2 + bx + c$. Parabolas are symmetric about a vertical line known as the Axis of Symmetry.

The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the Vertex.

Based on above information Answer the following-

1. If the highway overpass is represented by $x^2 - 2x - 8$. Then its zeroes are
2. The highway overpass is represented graphically. Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial.
3. The representation of Highway Underpass whose one zero is 6 and sum of the zeroes is 0, is

OR

The number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is

Case Study: 3.

A group of school friends went on an expedition to see caves. One person remarked that the entrance of the caves resembles a parabola and can be represented by a quadratic polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, where a , b and c are real numbers.



Based on the given information, solve the following questions:

Q1. Draw a neat labelled figure to show above situation diagrammatically.

Q2. If one of the zeroes of the quadratic polynomial $(p-1)x^2 + px + 1$ is 4. Find the value of p .

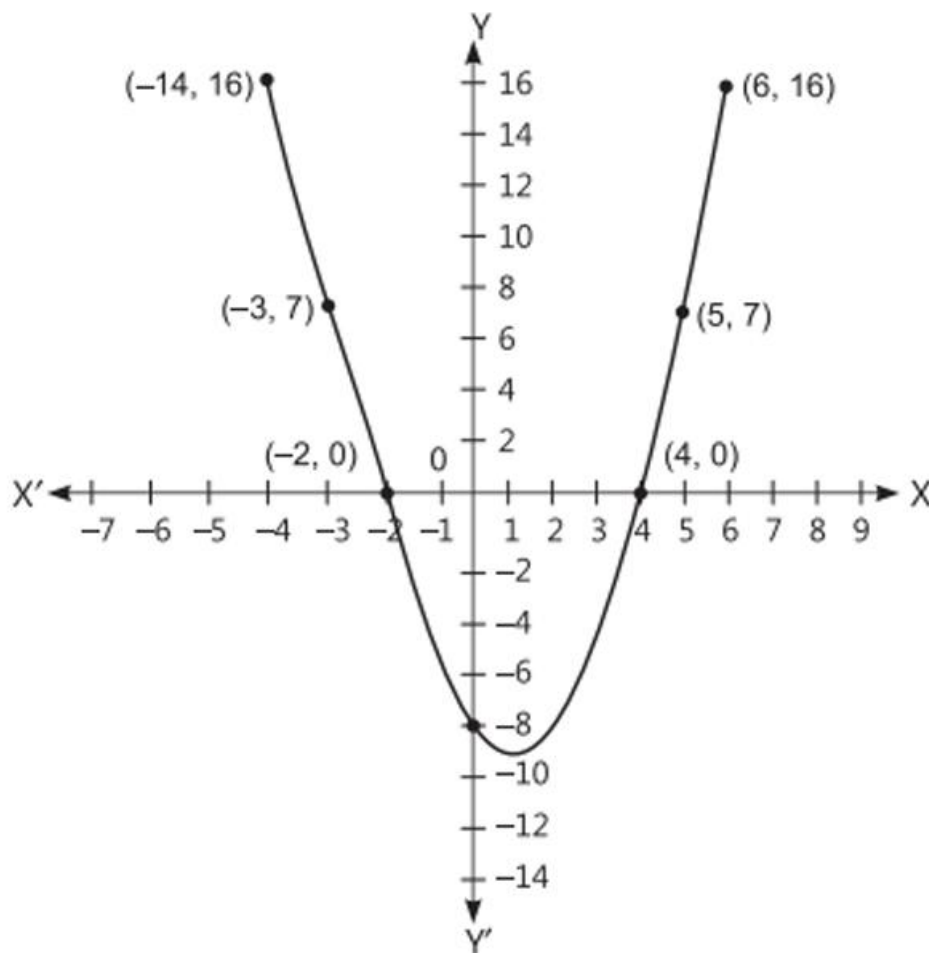
Q3. Find the quadratic polynomial whose zeroes are 5 and -12.

OR

If one zero of the polynomial $f(x) = 5x^2 + 13x + m$ is reciprocal of the other, then find the value of m .

Case Study: 4.

A student was given a task to prepare a graph of quadratic polynomial $p(x) = -8 - 2x + x^2$. To draw this graph, he take seven values of y corresponding to different values of x . After plotting the points on the graph paper with suitable values, he obtains the graph as shown below.



Based on the above graph, solve the following questions:

Q1. What is the shape of graph of a quadratic polynomial?

Q2. Find the zeroes of given quadratic polynomial.

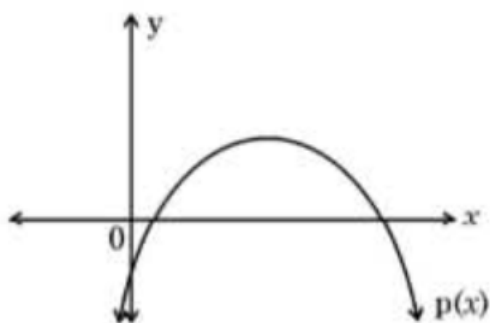
Q3. The graph of the given quadratic polynomial cut at which points on the X-axis?

OR

The graph of the given quadratic polynomial cut at which point on Y-axis?

Case Study: 5.

A parabolic fountain in a park is designed such that its shape follows the polynomial $p(x) = -x^2 + 6x - 5$ where $p(x)$ is the height (in metres) above ground level at a horizontal distance x metre from the origin.



Based on the above, answer the following questions:

What are the zeroes of $p(x)$?

What is the height of the fountain at a horizontal distance of 2 metres from origin? (1)

(a) If $q(x)$ is a reflection of $p(x)$ that is $q(x) = -p(x)$ write its value. as polynomial and draw its figure. (2)

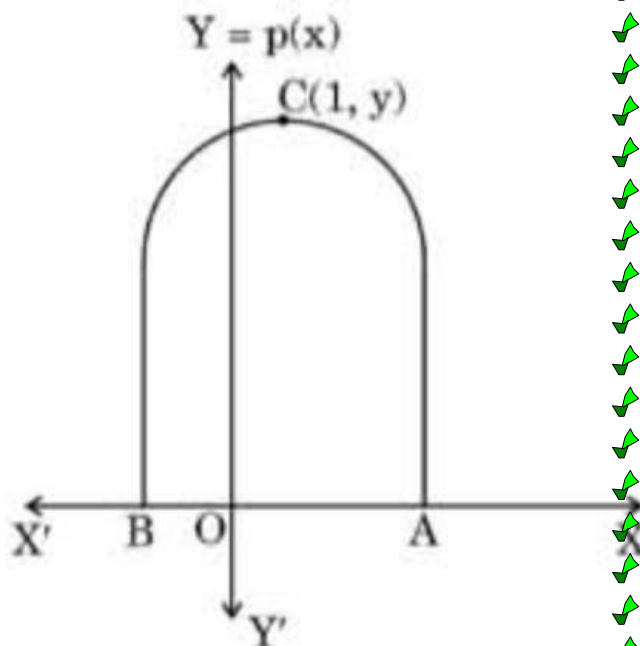
OR

(b) At what horizontal distance from the origin does water hit the ground?

(2)

Case Study: 6.

During a theatre drama, a backdrop of building arches was used. The shape of the curve shown below can be represented by the polynomial $p(x) = x^2 + 2x + 8$, where x is the length (in feet) on stage level.



Based on the figure given above, answer the following questions:

(i) Determine the height of the arch.

(iii) Write the coordinates of the point of intersection of the above curve with the y-axis.

(iii) (a) Find zeroes of the polynomial $p(x)$. Which points on the graph represent the zeroes? 2

OR

(iii) (b) Find the span of the arch on the stage floor.

ANSWERS-

MCQS-

- | | | | | | |
|-----------|---------------|------------------|---------------------------|-----------|-----------|
| 1. (c) 4, | 2. (d) 4, | 3. (d) 4, | 4. (c) , | 5. (b) | 6. (a) |
| 7. (a) 2 | 8. (b) -10 | 9. (b) 2 | 10. (a) $3x(3x-5)$ | 11. (a) 1 | 12. (d) 0 |
| 13. (D) | 14. (b) $2/3$ | 15. (b) $p=r=-2$ | 16. (D) $3(x^2 - 2x - 7)$ | | |

SAI (2 marks)

- | | | |
|--|---------------|--------------------------|
| 1. $+ - a \beta = 15/4$ | 2. $K = -10,$ | 3. $X^2 + 3x + 2$ |
| 4. $X^2 + x - 12$ | 5. $4/3, -1$ | 6. $\sqrt{3}, -\sqrt{3}$ |
| 8. (i) $x^2 - x - 4,$ (ii) $3x^2 - 3\sqrt{2}x + 1$ | 9. $1/2, 1/2$ | 7. $4/3, -3/2$ |

10. $X^2 - 9$, $x = +3, -3$

11. $X = -1/3, 3/2$

LA (3 marks)

1. $1 + + = ,$ 2. $K = 2,$ 3 $x = , -2,$ 4. $K = -4,$ 5. $C = -1$
6. $+ = -3/7,$ 7. $K = 7,$ 8. $X^2 + x - 20, x = 4, -5$

Assertion And Reason

a, 2. a, 3. d, 4. a, 5. b, 6. d

Case study:

- (i) $t = 2$, (ii) $-16t^2 + 16t + 32$, (iii) $t^2 - t - 6$ or $k = 3$
(i) $X = -2, 4$ (ii) x - axis (iii) $x^2 - 36$ or no real zeros
(ii) $p = 5/4$ (iii) $x^2 + 7x - 60$ or $m = 5$
(i) Parabola, (ii) $-2, 4$, (iii) $(-2, 0), (4, 0)$ or $(0, -8)$
(i) $x = 1, 5$ (ii) 3 metres, (iii) $x^2 - 6x + 5$ or 5 metres from origin
(i) height = 9 feet, (ii) $(0, 8)$, (iii) 4, -2 points $A(4, 0), B(-2, 0)$ or 6 feet

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. Chapter Summary & Key Concepts

Concept	Description & Formulae
Standard Form	A linear equation in two variables x and y is of the form $ax + by + c = 0$, where a, b, c are real numbers and $a^2 + b^2 \neq 0$.
System of Equations	A pair of linear equations in two variables: $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$
Consistent System	A system of equations that has at least one solution (Unique or Infinitely many).
Inconsistent System	A system of equations that has no solution.
Algebraic Methods	1. Substitution Method: Express one variable in terms of the other and substitute it into the second equation. 2. Elimination Method: Multiply equations by suitable constants to make the coefficients of one variable equal, then add/subtract to eliminate it.

2. Conditions for Solvability (Graphical & Algebraic Interpretation)

Ratio	Algebraic	Graphical	System
-------	-----------	-----------	--------

Comparison	Interpretation	Representation	Consistency
$a_1/a_2 \neq b_1/b_2$	Exactly one unique solution	Intersecting lines	Consistent
$a_1/a_2 = b_1/b_2 = c_1/c_2$	Infinitely many solutions	Coincident (overlapping) lines	Consistent (Dependent)
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	No solution	Parallel lines	Inconsistent

3. Solved Examples

Type	Example & Solution
Basic Level	Q: Solve: $x + y = 14$ and $x - y = 4$ Sol: Adding both equations: $2x = 18 \Rightarrow x = 9$. Substituting $x=9$ in eq 1: $9 + y = 14 \Rightarrow y = 5$. (Solution: $x=9, y=5$)
Intermediate	Q: Find k for which $kx + 3y = 7$ and $2x + 6y = 14$ have infinitely many solutions. Sol: For infinite solutions, $a_1/a_2 = b_1/b_2 = c_1/c_2 \Rightarrow k/2 = 3/6 = -7/-14 \Rightarrow k/2 = 1/2 \Rightarrow k = 1$.
HOTS	Q: The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits differ by 2,

find the number.

Sol: Let tens digit be x and
units digit be y . Number =
 $10x+y$. Reversed = $10y+x$.
Sum = $11x + 11y = 66 \Rightarrow x+y$
= 6.

Difference: $x-y = 2$ or $y-x = 2$.

Case 1: $x+y=6$, $x-y=2 \Rightarrow 2x=8$
 $\Rightarrow x=4$, $y=2$. (Number: 42).

Case 2: $x+y=6$, $y-x=2 \Rightarrow 2y=8$
 $\Rightarrow y=4$, $x=2$. (Number: 24).

GRADED WORKSHEET (MAX. MARKS: 80)

SECTION A: Multiple Choice Questions (1 Mark Each = 18 Marks)

1. The pair of equations $x+2y+5=0$ and $-3x-6y+1=0$ has:
 - (a) Unique solution
 - (b) Exactly two solutions
 - (c) Infinitely many solutions
 - (d) No solution

2. If a pair of linear equations is consistent, then the lines will be:
 - (a) Parallel
 - (b) Always coincident
 - (c) Intersecting or coincident
 - (d) Always intersecting

3. The value of c for which $cx-y=2$ and $6x-2y=3$ will have infinitely many solutions is:
 - (a) 3
 - (b) -3
 - (c) -12
 - (d) no value

4. Graphically, the pair of equations $6x-3y+10=0$, $2x-y+9=0$ represents two lines which are:
 - (a) Intersecting exactly at one point
 - (b) Intersecting at two points
 - (c) Coincident
 - (d) Parallel

5. The pair of equations $y=0$ and $y=-7$ has:
 - (a) One solution
 - (b) Two solutions

- (c) Infinitely many solutions
- (d) No solution

6. The pair of equations $x=a$ and $y=b$ graphically represents lines which are:

- (a) Parallel
- (b) Intersecting at (b,a)
- (c) Coincident
- (d) Intersecting at (a,b)

7. For what value of k , do the equations $3x-y+8=0$ and $6x-ky=-16$ represent coincident lines?

- (a) $1/2$
- (b) $-1/2$
- (c) 2
- (d) -2

8. If $x=a$, $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are:

- (a) 3 and 5
- (b) 5 and 3
- (c) 3 and 1
- (d) -1 and -3

9. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in years) are:

- (a) 4 and 24
- (b) 5 and 30
- (c) 6 and 36
- (d) 3 and 24

10. One equation of a pair of dependent linear equations is $-5x+7y=2$. The second equation can be:

(a) $10x+14y+4=0$

(b) $-10x-14y+4=0$

(c) $10x-14y=-4$

(d) $10x-14y=4$

11. Two numbers are in the ratio 5:6. If 8 is subtracted from each, the ratio becomes 4:5. The numbers are:

(a) 40, 42

(b) 40, 48

(c) 48, 40

(d) 50, 60

12. If $3x+4y : x+2y = 9:4$, then $3x+5y : 3x-y$ is:

(a) 4:1

(b) 1:4

(c) 7:1

(d) 1:7

13. The solution of the equations $0.4x+0.3y = 1.7$ and $0.7x-0.2y = 0.8$ is:

(a) $x=1, y=2$

(b) $x=2, y=3$

(c) $x=3, y=4$

(d) $x=5, y=3$

14. Aruna has only Rs 1 and Rs 2 coins. If the total number of coins is 50 and the amount is Rs 75, the number of Rs 1 and Rs 2 coins are:

(a) 35 and 15

(b) 35 and 20

(c) 15 and 35

(d) 25 and 25

15. The area of the triangle formed by the lines $x=3$, $y=4$, and $x=y$ is:

(a) $1/2$ sq unit

- (b) 1 sq unit
- (c) 2 sq units
- (d) 4 sq units

16. The value of k for which the system $kx - y = 2$, $6x - 2y = 3$ has a unique solution is:

- (a) $k = 3$
- (b) $k \neq 3$
- (c) $k \neq 0$
- (d) $k = 0$

17. The coordinates of the point where the line $2x - 3y = 6$ intersects the x -axis are:

- (a) $(0, -2)$
- (b) $(3, 0)$
- (c) $(2, 0)$
- (d) $(0, 3)$

18. If $2x - 3y = 7$ and $(a+b)x - (a+b-3)y = 4a+b$ represent coincident lines, then a and b satisfy:

- (a) $a + 5b = 0$
- (b) $5a + b = 0$
- (c) $a - 5b = 0$
- (d) $5a - b = 0$

SECTION B: Assertion and Reasoning (1 Mark Each = 2 Marks)

Directions: Read the Assertion (A) and Reason (R). Choose the correct option:

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is NOT the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

19. Assertion (A): The linear equations $x - 2y - 3 = 0$ and $3x + 4y - 20 = 0$ have exactly one

solution.

Reason (R): The system of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ has a unique solution if $a_1/a_2 \neq b_1/b_2$.

20. Assertion (A): The system of equations $2x+3y=5$ and $4x+6y=9$ is consistent.

Reason (R): If a system has no solution, it is called an inconsistent system.

SECTION C: Short Answer Type I Questions (2 Marks Each = 10 Marks)

21. Solve for x and y: $x+y=8$ and $2x-3y=1$.

22. For what value of p does the pair of linear equations $4x+py+8=0$ and $2x+2y+2=0$ have a unique solution?

23. Find whether the following pair of linear equations is consistent or inconsistent: $3x+2y=8$ and $6x-4y=9$.

24. In a cyclic quadrilateral ABCD, $\angle A = (4x+20)^\circ$, $\angle B = (3x-5)^\circ$, $\angle C = (4y)^\circ$, and $\angle D = (7y+5)^\circ$. Find x and y. (Hint: Sum of opposite angles is 180°)

25. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

SECTION D: Short Answer Type II Questions (3 Marks Each = 18 Marks)

26. Solve the following system of linear equations by the elimination method: $3x + 4y = 10$ and $2x - 2y = 2$.

27. Find the values of a and b for which the following system of linear equations has infinite solutions: $2x+3y=7$ and $(a-b)x + (a+b)y = 3a+b-2$.

28. The sum of a two-digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the original number, the new number is 4 more than 5 times the sum of the digits of the original number. Find the original number.

29. A fraction becomes $1/3$ when 1 is subtracted from the numerator and it becomes $1/4$ when 8 is added to its denominator. Find the fraction.

30. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

31. Solve for x and y: $x/a + y/b = 2$ and $ax - by = a^2 - b^2$.

SECTION E: Long Answer Questions (5 Marks Each = 20 Marks)

(Each question consists of three 1-mark sub-parts and one 2-mark sub-part)

32. The Highway Car Problem

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. Let the speed of car from A be x km/hr and from B be y km/hr ($x > y$).

(i) Write the algebraic equation when they travel in the same direction. (1 Mark)

(ii) Write the algebraic equation if they travel towards each other and meet in 1 hour. (1 Mark)

(iii) Find the speed of the faster car. (1 Mark)

(iv) Find the speed of the slower car and calculate how far the slower car travels if they travel towards each other. (2 Marks)

33. The Fixed and Variable Cost Problem

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days, she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180.

(i) Let fixed charge be x and cost of food per day be y . Form the equation for Student A. (1 Mark)

(ii) Form the equation for Student B. (1 Mark)

(iii) Find the cost of food per day. (1 Mark)

(iv) Find the fixed charges and calculate the total hostel bill for a student who stays for 30 days. (2 Marks)

34. The Geometrical Angle Problem

In $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$.

(i) Using the angle sum property of a triangle, write an equation relating $\angle A$, $\angle B$, $\angle C$. (1 Mark)

(ii) Using the given condition, express $\angle C$ in terms of $\angle B$. (1 Mark)

(iii) Find the measure of $\angle B$. (1 Mark)

(iv) Find the measures of $\angle A$ and $\angle C$. Check if it forms a right-angled triangle. (2 Marks)

35. Work and Wages Problem

2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Let 1 woman alone finish it in x days and 1 man alone finish it in y days. (Hint: Use substitution $1/x = u$ and $1/y = v$)

(i) Form the algebraic equation for the first condition. (1 Mark)

(ii) Form the algebraic equation for the second condition. (1 Mark)

(iii) Find the time taken by 1 woman alone to finish the work. (1 Mark)

(iv) Find the time taken by 1 man alone, and calculate how much time 1 woman and 1 man together would take. (2 Marks)

SECTION F: Case-Based Questions (4 Marks Each = 12 Marks)

36. Case Study 1: The Taxi Fare

The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155.

(i) Assuming the fixed charge as Rs x and per km charge as Rs y , form the equation for the 10 km journey. (1 Mark)

(ii) Form the equation for the 15 km journey. (1 Mark)

(iii) Find the fixed charge and the per km charge. How much does a person have to pay for traveling a distance of 25 km? (2 Marks)

37. Case Study 2: Library Books

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days.

(i) Let the fixed charge for the first 3 days be Rs x and the charge for each extra day be Rs y . Form the equation for Saritha. (1 Mark)

(ii) Form the algebraic equation for Susy. (1 Mark)

(iii) Find the fixed charge and the additional daily charge. (2 Marks)

38. Case Study 3: Competitive Examination

Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks.

- (i) Let the number of correct answers be x and wrong answers be y . Form the equation for the first condition. (1 Mark)
- (ii) Form the equation for the second condition. (1 Mark)
- (iii) Find the values of x and y . How many total questions were there in the test? (2 Marks)

ANSWER KEY & HINTS

Section A: MCQs

- 1. (d) [$1/-3 = 2/-6 \neq 5/1$]
- 2. (c) [Consistent means ≥ 1 solution]
- 3. (d) [For infinite: $c/6 = -1/-2 = 2/3$. Thus $c=3$ and $c=4$, which is impossible simultaneously]
- 4. (d) [$6/2 = -3/-1 \neq 10/9$]
- 5. (d) [Both are horizontal lines parallel to the x -axis, separated by a distance of 7]
- 6. (d) [$x=a$ is vertical, $y=b$ is horizontal; they meet at (a,b)]
- 7. (c) [$c_1=8, c_2=16 \Rightarrow 8/16 = 1/2$. Thus $k=2$]
- 8. (c) [Solve $x-y=2$ & $x+y=4 \Rightarrow 2x=6 \Rightarrow x=3, y=1$]
- 9. (c) [$x=6y$ and $x+4 = 4(y+4) \Rightarrow 2y=12 \Rightarrow y=6, x=36$]
- 10. (c) [Multiply by -2 : $10x - 14y = -4$]
- 11. (b) [$x/y = 5/6, (x-8)/(y-8) = 4/5 \Rightarrow$ numbers are 40 and 48]
- 12. (c) [$4(3x+4y) = 9(x+2y) \Rightarrow 3x = 2y \Rightarrow x/y = 2/3$. Substitute $x=2, y=3 \Rightarrow 21/3 = 7/1$]

13. (b) [Multiply by 10: $4x+3y=17$; $7x-2y=8 \Rightarrow x=2, y=3$]

14. (d) [$x+y=50$, $x+2y=75 \Rightarrow y=25, x=25$]

15. (a) [Vertices are (3,3), (3,4), (4,4). Base = 1, Height = 1. Area = $\frac{1}{2} \times 1 \times 1$]

16. (b) [$k/6 \neq -1/-2 \Rightarrow k \neq 3$]

17. (b) [Put $y=0 \Rightarrow 2x=6 \Rightarrow x=3$]

18. (c) [$2/(a+b) = -3/-(a+b-3) \Rightarrow a+b = -6$. Also $8a+2b = 7a+7b \Rightarrow a=5b \Rightarrow a-5b=0$]

Section B: Assertion and Reasoning

19. (a) [True, $1/3 \neq -2/4$. R is the correct explanation]

20. (d) [Assertion is false because $2/4 = 3/6 \neq 5/9$ means no solution / inconsistent.

Reason is true]

Section C: 2 Marks Questions (Hints)

21. Adding: $3x=9 \Rightarrow x=3$. $y=5$.

22. $4/2 \neq p/2 \Rightarrow p \neq 4$.

23. $3/6 = 2/-4 \Rightarrow 1/2 \neq -1/2$. Unique solution, therefore Consistent.

24. $\angle A + \angle C = 180 \Rightarrow 4x+20 + 4y = 180 \Rightarrow x+y=40$. $\angle B + \angle D = 180 \Rightarrow 3x+7y=180$.

Solve to get $x=25, y=15$.

25. $x = y+4$. $x+y=36$. Solving gives $x=20$ m (length), $y=16$ m (width).

Section D: 3 Marks Questions (Hints)

26. Multiply eq 2 by 2: $4x - 4y = 4$. Add to eq 1: $7x = 14 \Rightarrow x=2$. $y=1$.

27. $2/(a-b) = 3/(a+b) = 7/(3a+b-2)$. Solve first two: $2a+2b = 3a-3b \Rightarrow a=5b$.

Solve next two with substitution to get $a=5, b=1$.

28. $x+y=10$. Second condition: $(10x+y) - 10 = 5(x+y) + 4$. Original number is 73.

29. $3x - y = 3$ and $4x - y = 8$. Subtracting gives $x = 5, y = 12$. Fraction is $5/12$.

30. $x-5 = 3(y-5) \Rightarrow x-3y=-10$. $x+10 = 2(y+10) \Rightarrow x-2y=10$. Nuri (x) = 50, Sonu (y) = 20.

31. Eq 1: $bx + ay = 2ab$. Multiply Eq 2 by b : $abx - b^2y = a^2b - b^3$. By eliminating, $x = a$, $y = b$.

Section E: 5 Marks Questions

32. (i) $5x - 5y = 100 \Rightarrow x - y = 20$

(ii) $x + y = 100$

(iii) Add eq: $2x = 120 \Rightarrow x = 60$ km/hr.

(iv) $y = 40$ km/hr. Slower car distance in 1 hour = 40 km.

33. (i) $x + 20y = 1000$

(ii) $x + 26y = 1180$

(iii) Subtracting: $6y = 180 \Rightarrow y = 30$ (Rs 30 per day).

(iv) $x + 20(30) = 1000 \Rightarrow x = 400$ (Rs 400 fixed). 30 days bill = $400 + 30(30) = 1300$ Rs.

34. (i) $\angle A + \angle B + \angle C = 180^\circ$

(ii) $\angle C = 3\angle B$

(iii) From given: $3\angle B = 2(\angle A + \angle B) \Rightarrow \angle B = 2\angle A \Rightarrow \angle A = \angle B / 2$. Sum: $\angle B/2 + \angle B + 3\angle B = 180 \Rightarrow 4.5\angle B = 180 \Rightarrow \angle B = 40^\circ$.

(iv) $\angle A = 20^\circ$, $\angle C = 120^\circ$. Not a right-angled triangle.

35. (i) $2/x + 5/y = 1/4$

(ii) $3/x + 6/y = 1/3$

(iii) Solve to get $1/x = 1/18 \Rightarrow x=18$ days.

(iv) $1/y = 1/36 \Rightarrow y=36$ days. Together: $1/18 + 1/36 = 3/36 = 1/12 \Rightarrow 12$ days.

Section F: Case-Based Questions

36. (i) $x + 10y = 105$

(ii) $x + 15y = 155$

(iii) Solving gives $5y = 50 \Rightarrow y=10$ and $x=5$. Total for 25km = $5 + 25(10) = \text{Rs } 255$.

37. (i) $x + 4y = 27$

(ii) $x + 2y = 21$

(iii) Solving gives $2y = 6 \Rightarrow y=3$. $x = 15$. Fixed charge Rs 15, Extra day Rs 3.

38. (i) $3x - y = 40$

(ii) $4x - 2y = 50 \Rightarrow 2x - y = 25$

(iii) Subtracting: $x = 15$. Put in eq: $45 - y = 40 \Rightarrow y = 5$. Total questions = $x + y = 15 + 5 = 20$.

QUADRATIC EQUATIONS

- Standard Form: A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.
- Roots / Solutions: A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$. A quadratic equation has at most two roots.
- Methods to Solve:
 1. Factorization: Splitting the middle term.
 2. Quadratic Formula: Directly computing roots using the formula.
- Quadratic Formula: The roots of $ax^2 + bx + c = 0$ are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant (D): The expression $D = b^2 - 4ac$ is called the discriminant. It determines the nature of the roots.
- Nature of Roots:
 1. $D > 0$: Two distinct real roots.
 2. $D = 0$: Two equal real roots ($x = -b/2a$).
 3. $D < 0$: No real roots.

SOLVED EXAMPLES (Basic to High Order Thinking Skills)

Example 1 (Basic)

Question: Find the roots of the equation $2x^2 - 5x + 3 = 0$ by factorization.

Solution: $2x^2 - 5x + 3 = 0 \Rightarrow 2x^2 - 2x - 3x + 3 = 0$

$2x(x - 1) - 3(x - 1) = 0 \Rightarrow (2x - 3)(x - 1) = 0$

Roots are $x = 1$ and $x = 3/2$.

Example 2 (Intermediate)

Question: Find the value of k for which the quadratic equation $kx(x - 2) + 6 = 0$ has two equal roots. (NCERT)

Solution: Equation: $kx^2 - 2kx + 6 = 0$. Here $a=k, b=-2k, c=6$.

For equal roots, $D = 0 \Rightarrow b^2 - 4ac = 0$

$(-2k)^2 - 4(k)(6) = 0 \Rightarrow 4k^2 - 24k = 0$

$4k(k - 6) = 0$. Since $k \neq 0$ (as it's quadratic), $k = 6$.

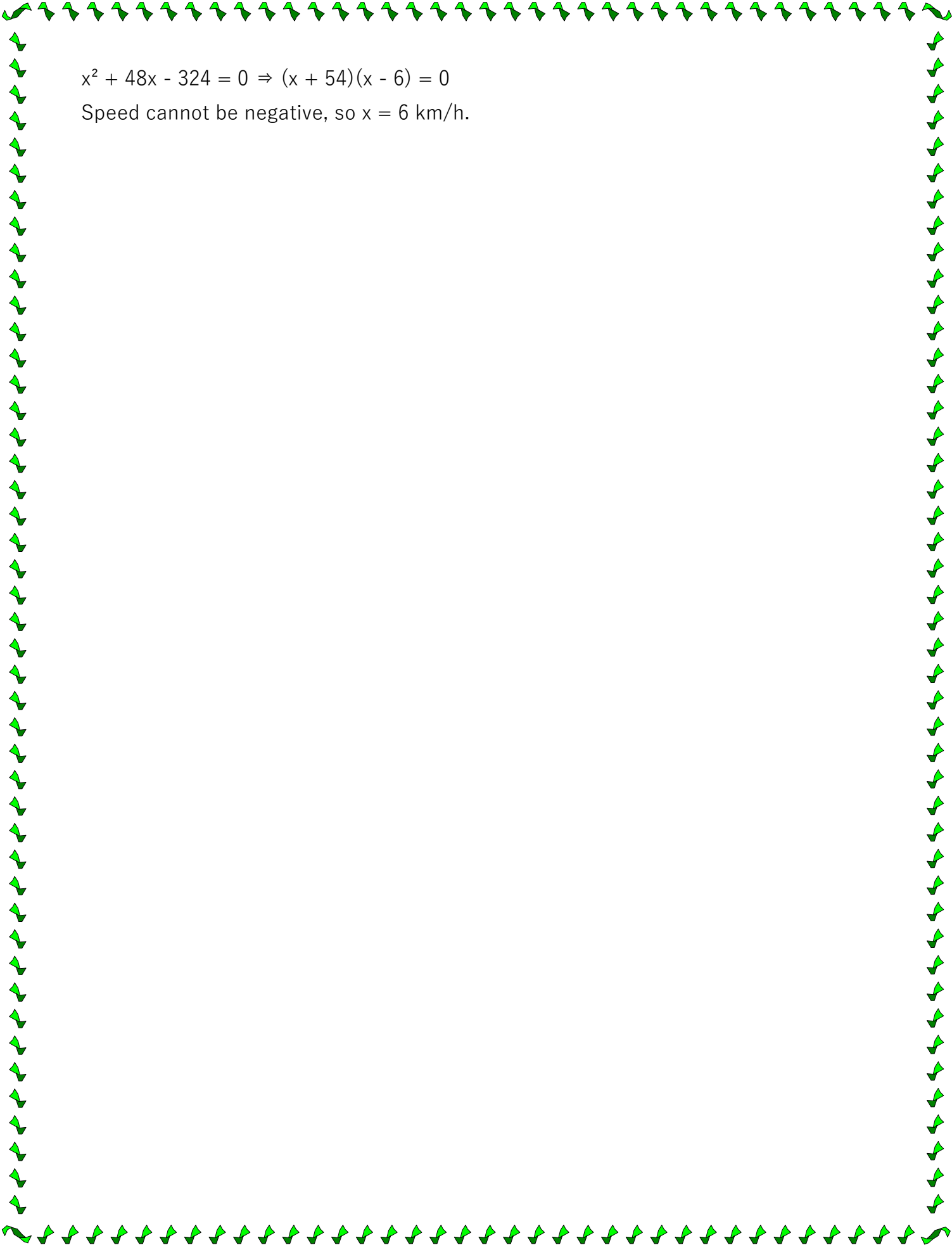
Example 3 (HOTS)

Question: A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. (PYQ)

Solution: Let stream speed = x km/h. Upstream speed = $(18 - x)$, Downstream = $(18 + x)$.

Time diff: $24/(18 - x) - 24/(18 + x) = 1$

$24(18 + x - 18 + x) = 1(18^2 - x^2) \Rightarrow 48x = 324 - x^2$


$$x^2 + 48x - 324 = 0 \Rightarrow (x + 54)(x - 6) = 0$$

Speed cannot be negative, so $x = 6$ km/h.

CLASS X MATHEMATICS WORKSHEET - QUADRATIC EQUATIONS
Maximum Marks: 80 | Time: 3 Hours

SECTION A: MULTIPLE CHOICE QUESTIONS (1 Mark Each)

1. Which of the following is a quadratic equation?
(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (b) $-2x^2 = (5 - x)(2x - 2/5)$
(c) $(k+1)x^2 + 3/2x = 7, k=-1$ (d) $x^3 - x^2 = (x - 1)^3$
2. The roots of the quadratic equation $x^2 - 3x - 10 = 0$ are:
(a) 2, -5 (b) -2, 5 (c) 2, 5 (d) -2, -5
3. The discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$ is:
(a) -8 (b) 8 (c) 0 (d) 10
4. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then:
(a) $k < 4$ (b) $k > 4$ (c) $k \geq 4$ (d) $k \leq 4$
5. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has:
(a) Two distinct real roots (b) Two equal real roots (c) No real roots (d) More than two real roots
6. Which of the following equations has 2 as a root?
(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$ (c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$
7. If $1/2$ is a root of the equation $x^2 + kx - 5/4 = 0$, then the value of k is:
(a) 2 (b) -2 (c) $1/4$ (d) $1/2$
8. The sum of the roots of the quadratic equation $3x^2 - 9x + 5 = 0$ is:
(a) 3 (b) -3 (c) $5/3$ (d) $-5/3$
9. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is:
(a) 0 only (b) 4 (c) 8 only (d) 0, 8
10. The product of two consecutive positive integers is 306. The quadratic equation to find the integers is:
(a) $x^2 + x - 306 = 0$ (b) $x^2 - x - 306 = 0$ (c) $x^2 + 2x - 306 = 0$ (d) $x^2 - 2x + 306 = 0$
11. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then:
(a) $b = c$ (b) $a = b$ (c) $ac = 1$ (d) $a = c$
12. A quadratic equation with integral coefficients has integral roots. Justify whether the discriminant is a perfect square.
(a) Always True (b) Always False (c) Sometimes True (d) None
13. The roots of the equation $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$ are:
(a) $\sqrt{3}, 7\sqrt{3}$ (b) $-\sqrt{3}, -7\sqrt{3}/3$ (c) $\sqrt{3}, 7/\sqrt{3}$ (d) None
14. If the roots of $px^2 + qx + 2 = 0$ are reciprocals of each other, then p is:
(a) 0 (b) -2 (c) 2 (d) 1
15. The area of a rectangular plot is 528 m². Length is one more than twice its breadth. If breadth is x, the equation is:
(a) $x(2x+1) = 528$ (b) $x(x+1) = 528$ (c) $x(2x-1) = 528$ (d) $x^2 + x = 528$
16. If $b^2 - 4ac < 0$ in the formula $x = (-b \pm \sqrt{b^2 - 4ac}) / 2a$, then the roots are:
(a) Real and equal (b) Real and distinct (c) Not real (d) Rational

17. Which of the following is not a quadratic equation?

- (a) $2(x-1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$ (c) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

18. If $x = 1$ is a common root of $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$, then ab is:
(a) 3 (b) 3.5 (c) 6 (d) -3

SECTION B: ASSERTION AND REASONING (1 Mark Each)

Directions: Choose (a) Both A and R are true and R is correct explanation (b) Both A and R are true but R is NOT correct explanation (c) A is true but R is false (d) A is false but R is true.

19. Assertion (A): The equation $x^2 + 3x + 5 = 0$ has no real roots.

Reason (R): A quadratic equation $ax^2 + bx + c = 0$ has no real roots if $b^2 - 4ac < 0$.

20. Assertion (A): If the roots of the quadratic equation $ax^2 + bx + c = 0$ are real and equal, then $b^2 = 4ac$.

Reason (R): The roots of $2x^2 - 4x + 2 = 0$ are real and distinct.

SECTION C: VERY SHORT ANSWER TYPE QUESTIONS (2 Marks Each)

21. Check whether the following is a quadratic equation: $(x + 2)^3 = 2x(x^2 - 1)$

22. Find the values of k for which the quadratic equation $x^2 + 5kx + 16 = 0$ has no real roots.

23. Solve the quadratic equation by factorization: $3x^2 - 2\sqrt{6}x + 2 = 0$

24. The product of two consecutive odd integers is 255. Formulate the quadratic equation to determine the integers.

25. Find the discriminant of the equation $3x^2 - 2x + 1/3 = 0$ and hence find the nature of its roots.

SECTION D: SHORT ANSWER TYPE QUESTIONS (3 Marks Each)

26. Find the roots of the following equation: $1/x - 1/(x-2) = 3, x \neq 0, 2$.

27. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

28. Find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2) = 0$.

29. A cottage industry produces a certain number of pottery articles in a day. The cost of production of each article was 3 more than twice the number of articles produced. If total cost was Rs 90, find the number of articles produced and cost of each.

30. Solve for x : $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$.

31. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k .

SECTION E: CASE-BASED QUESTIONS (4 Marks Each)

32. Case Study 1: Age Problems

The sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is $1/3$. Let present age be x .

- (i) Write Rehman's age 3 years ago and 5 years from now in terms of x . (1 Mark)
- (ii) Form the quadratic equation. (1 Mark)
- (iii) Solve the equation to find Rehman's present age. (2 Marks)

33. Case Study 2: Speed and Distance

An express train takes 1 hr less than a passenger train to travel 132 km. Passenger train speed = x km/h. Express train speed is 11 km/h more.

- (i) Write expression for time taken by passenger train. (1 Mark)
- (ii) Write expression for time taken by express train. (1 Mark)
- (iii) Formulate quadratic equation and find passenger train's speed. (2 Marks)

34. Case Study 3: Number Relations

Two numbers are such that their sum is 27 and product is 182. Let one number be x .

- (i) Write expression for the second number. (1 Mark)
- (ii) Form the quadratic equation for their product. (1 Mark)
- (iii) Find the two numbers by solving. (2 Marks)

SECTION F: LONG ANSWER TYPE QUESTIONS (5 Marks Each)

35. A flight left 30 minutes late. To reach its destination 1500 km away in time, it had to increase its speed by 250 km/h.

- (i) If usual speed is x km/h, write expression for scheduled time. (1 Mark)
- (ii) Write expression for actual time with increased speed. (1 Mark)
- (iii) Form the quadratic equation. (1 Mark)
- (iv) Solve to find the usual speed. (2 Marks)

36. The length of a rectangular park is 5 m more than its breadth. Its area is 150 sq m.

- (i) If breadth is x m, express length in terms of x . (1 Mark)
- (ii) Form the quadratic equation for area. (1 Mark)
- (iii) Find the discriminant. (1 Mark)
- (iv) Find dimensions of the park. (2 Marks)

37. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The larger tap takes 10 hours less than the smaller one to fill it separately.

- (i) If smaller tap takes x hours, write time taken by larger tap. (1 Mark)
- (ii) What is the portion filled by both in 1 hour? (1 Mark)
- (iii) Form the quadratic equation. (1 Mark)
- (iv) Find the time in which each tap can separately fill the tank. (2 Marks)

38. Sum of areas of two squares is 468 m^2 . The difference of their perimeters is 24 m.

- (i) If side of smaller square is x , what is its perimeter? (1 Mark)
- (ii) Express side of larger square in terms of x . (1 Mark)
- (iii) Form the quadratic equation representing sum of areas. (1 Mark)
- (iv) Solve to find sides of both squares. (2 Marks)

ANSWER KEY & HINTS

1. (d) $x^3 - x^2 = x^3 - 3x^2 + 3x - 1 \Rightarrow 2x^2 - 3x + 1 = 0$ (Degree is 2).
2. (b) $(x - 5)(x + 2) = 0 \Rightarrow x = 5, -2$.
3. (a) $D = (-4)^2 - 4(2)(3) = 16 - 24 = -8$.
4. (a) $D > 0 \Rightarrow 16 - 4k > 0 \Rightarrow 4k < 16 \Rightarrow k < 4$.
5. (c) $D = (\sqrt{5})^2 - 4(2)(1) = 5 - 8 = -3 < 0$.
6. (c) Substitute $x=2$ in $2x^2 - 7x + 6$: $2(4) - 7(2) + 6 = 8 - 14 + 6 = 0$.
7. (a) $(1/2)^2 + k(1/2) - 5/4 = 0 \Rightarrow 1/4 + k/2 - 5/4 = 0 \Rightarrow k/2 = 1 \Rightarrow k = 2$.
8. (a) Sum of roots $= -b/a = -(-9)/3 = 3$.
9. (d) $D = 0 \Rightarrow k^2 - 4(2)(k) = 0 \Rightarrow k^2 - 8k = 0 \Rightarrow k(k - 8) = 0 \Rightarrow k = 0, 8$.
10. (a) Let integers be $x, x+1$. $x(x+1) = 306 \Rightarrow x^2 + x - 306 = 0$.
11. (d) Product of roots $= 1 \Rightarrow c/a = 1 \Rightarrow a = c$.
12. (a) True. If roots are integers, they are rational, meaning D must be a perfect square.
13. (b) $D = 100 - 4(\sqrt{3})(7\sqrt{3}) = 16$. Roots are $-\sqrt{3}$ and $-7\sqrt{3}/3$.
14. (c) Reciprocal roots mean Product $= 1 \Rightarrow 2/p = 1 \Rightarrow p = 2$.
15. (a) Length $= 2x + 1$. Area $= x(2x + 1) = 528$.
16. (c) Not real.
17. (c) The x^2 terms cancel out, reducing it to a linear equation.
18. (a) $x=1$ in 1st eq: $a + a + 3 = 0 \Rightarrow a = -1.5$. In 2nd eq: $1 + 1 + b = 0 \Rightarrow b = -2$. $ab = 3$.
19. (a) $D = 9 - 20 = -11 < 0$. Reason correctly explains Assertion.
20. (c) A is true ($D=0 \Rightarrow b^2 = 4ac$). R is false (for $2x^2 - 4x + 2 = 0$, $D=0$, roots are real and equal, not distinct).
21. $x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x \Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$. It is a cubic, NOT a quadratic equation.
22. $D < 0 \Rightarrow 25k^2 - 4(1)(16) < 0 \Rightarrow 25k^2 < 64 \Rightarrow -8/5 < k < 8/5$.
23. $3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0 \Rightarrow x = \sqrt{6}/3$ (equal roots).
24. Let odd integers be x and $x+2$. Equation: $x(x+2) = 255 \Rightarrow x^2 + 2x - 255 = 0$.
25. $D = (-2)^2 - 4(3)(1/3) = 4 - 4 = 0$. Roots are real and equal.
26. $(x-2-x)/(x^2-2x) = 3 \Rightarrow 3x^2 - 6x + 2 = 0$. Roots: $x = (3 \pm \sqrt{3})/3$.
27. Base $= x$, Alt $= x-7$. $x^2 + (x-7)^2 = 13^2 \Rightarrow x^2 - 7x - 60 = 0$. Base $= 12$ cm, Alt $= 5$ cm.
28. Roots: $x = (4p \pm \sqrt{(16p^2 - 16(p^2 - q^2))})/8 = (p \pm q)/2$.
29. Articles $= x$, Cost $= 2x+3$. $x(2x+3) = 90 \Rightarrow 2x^2 + 3x - 90 = 0$. Articles $= 6$, Cost $=$ Rs 15.
30. Discriminant $D = 9(a-b)^2$. Roots: $x = (2a+b)/3$ and $x = (a+2b)/3$.
31. -5 is a root: $2(25) - 5p - 15 = 0 \Rightarrow p = 7$. Eq 2: $7x^2 + 7x + k = 0$. Equal roots: $49 - 28k = 0 \Rightarrow k = 7/4$.
32. (i) $(x-3)$ and $(x+5)$. (ii) $1/(x-3) + 1/(x+5) = 1/3 \Rightarrow x^2 - 4x - 21 = 0$. (iii) $x = 7$. Present age $= 7$ years.

33. (i) $T_p = 132/x$. (ii) $T_e = 132/(x+11)$. (iii) $132/x - 132/(x+11) = 1 \Rightarrow x^2 + 11x - 1452 = 0$.

Speed = 33 km/h.

34. (i) $27 - x$. (ii) $x(27 - x) = 182 \Rightarrow x^2 - 27x + 182 = 0$. (iii) $x = 13$ or 14 . The numbers are 13 and 14.

35. (i) $1500/x$. (ii) $1500/(x+250)$. (iii) $1500/x - 1500/(x+250) = 1/2 \Rightarrow x^2 + 250x - 750000 = 0$. (iv) Speed = 750 km/h.

36. (i) Length = $x + 5$. (ii) $x(x + 5) = 150 \Rightarrow x^2 + 5x - 150 = 0$. (iii) $D = 625$. (iv) Breadth = 10 m, Length = 15 m.

37. (i) $x - 10$. (ii) Portion = $8/75$. (iii) $1/x + 1/(x-10) = 8/75 \Rightarrow 4x^2 - 115x + 375 = 0$. (iv) Smaller = 25 h, Larger = 15 h.

38. (i) $4x$. (ii) $4x + 24$. Side of larger = $x + 6$. (iii) $x^2 + (x+6)^2 = 468 \Rightarrow x^2 + 6x - 216 = 0$. (iv) Sides are 12 m and 18 m.

ARITHMETIC PROGRESSION

Summary of the chapter:

Consider the following lists of numbers:

- (i) 1, 2, 3, 4, ...
- (ii) 100, 70, 40, 10, ...
- (iii) -3, -2, -1, 0, ...
- (iv) 3, 3, 3, 3, ...

Each of the numbers in the list is called a term.

In (i), each term is 1 more than the term preceding it.

In (ii), each term is 30 less than the term preceding it.

In (iii), each term is obtained by the adding 1 to the term preceding it.

In (iv), each term is obtained by adding 0 to the term preceding it.

In all the lists above, we see that successive terms are obtained by adding a fixed number to the preceding terms. Such list of numbers is said to form an Arithmetic Progression (AP). This fixed number is called the common difference of the AP. It can be positive, negative or zero.

You can see that $a, a + d, a + 2d, a + 3d, \dots$ represents an arithmetic progression where a is the first term and d the common difference. This is called the general form of an AP.

Let us denote the first term of an AP by a_1 , second term by a_2 , ..., nth term by a_n and the common difference by d .

Then $d = a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1}$

The nth term of an AP is given by the formula:

$$a_n = a + (n - 1)d$$

The sum of first n terms of an AP is given by the formula:

$$S_n = \frac{n}{2}\{2a + (n - 1)d\} \quad \text{or} \quad S_n = \frac{n}{2}\{a + a_n\}$$

The nth term from the end of the AP is given by the formula:

$$l_n = l - (n - 1)d, \text{ where } l \text{ is the last term of the AP.}$$

Note: $a_{10} = a + 9d$, $a_{20} = a + 19d$ and $a_{100} = a + 99d$, and so on.....

Solved Examples:

Example-1. Find the 15th term of the AP: 3, 8, 13, ...

Solution: Here $a = 3$, $d = 8 - 3 = 5$ and $n = 15$

We have $a_n = a + (n - 1)d$

$$a_{15} = 3 + (15 - 1)5 = 3 + 14 \times 5 = 3 + 70 = 73 \quad \text{Ans}$$

Example-2. Find the 10th term from the end of the AP: 10, 7, 4, ..., -62.

Solution: Here $l = -62$, $d = 7 - 10 = -3$ and $n = 10$

We have $l_n = l - (n - 1)d$

$$l_{10} = -62 - (10 - 1)(-3) = -62 + 9 \times 3 = -62 + 27 = -35 \quad \text{Ans}$$

Example-3. Which term of the AP: 3, 8, 13, 18, ... is 78?

Solution: Here $a = 3$, $d = 8 - 3 = 5$ and $a_n = 78$, $n = ?$

We have $a_n = a + (n - 1)d$

$$78 = 3 + (n - 1) \times 5$$

$$78 - 3 = (n - 1) \times 5$$

$$75 = (n - 1) \times 5$$

$$75 \div 5 = (n - 1)$$

$$15 = n - 1$$

$$16 = n \quad \text{hence 78 is its 16th term.} \quad \text{Ans}$$

Example-4. Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?

Solution: Let required term be its n th term then

$$a_n = a_{54} + 132$$

$$a + (n - 1)d = (a + 53d) + 132$$

$$(n - 1) \times 12 = 53 \times 12 + 11 \times 12$$

$$(n - 1) = 53 + 11$$

$$(n - 1) = 64$$

$$n = 65, \quad \text{hence required term is 65th term} \quad \text{Ans}$$

Example-5. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution: Let first term be a and common difference be d .

$$a_3 = 16$$

$$a + 2d = 16 \quad \text{-----(1)}$$

$$a_7 = a_5 + 12$$

$$a + 6d = a + 4d + 12$$

$$2d = 12$$

$$d = 6, \text{ put in equation (1) we will get } a + 2(6) = 16$$

$$a = 16 - 12$$

$$a = 4, \quad \text{hence AP is } a, a + d, a + 2d, \dots \dots \dots \text{i.e. } 4, 10, 16, \dots \dots \dots \quad \text{Ans}$$

Example-6. Find the sum: $(-5) + (-8) + (-11) + \dots + (-230)$

Solution: Here $a = (-5)$, $d = (-8) - (-5) = -8 + 5 = -3$, $a_n = (-230)$, $S_n = ?$

$$\text{We have } a_n = a + (n - 1)d$$

$$(-230) = (-5) + (n - 1)(-3)$$

$$-230 + 5 = (n - 1)(-3)$$

$$-225 \div (-3) = (n - 1)$$

$$75 = n - 1$$

$$76 = n$$

$$S_n = \frac{n}{2} \{a + a_n\}$$

$$S_{76} = \frac{76}{2} \{(-5) + (-230)\} = 38 \times (-235) = -8930 \quad \text{Ans}$$

Example-7. Find the sum of first 15 multiples of 8.

Solution: First 15 multiples of 8 are: 8, 16, 24,.....

As there is a fixed increment hence it is an AP.

Where $a = 8$, $d = 16 - 8 = 8$, $n = 15$, $S_{15} = ?$

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

$$S_{15} = \frac{15}{2} \{2 \times 8 + (15 - 1) \times 8\} = \frac{15}{2} \{16 + 112\} = \frac{15}{2} \times 128 = 15 \times 64 = 960 \quad \text{Ans}$$

Example-8. If the sum of first n terms of an AP is $4n - n^2$. Find AP.

Solution: Here $S_n = 4n - n^2$

$$S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$$

$$S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$$

$$S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3, \quad \text{and so on.....}$$

$$a_1 = S_1 = 3$$

$$a_2 = S_2 - S_1 = 4 - 3 = 1$$

$$a_3 = S_3 - S_2 = 3 - 4 = -1, \text{ and so on ...}$$

Hence AP is 3, 1, -1, ...

Ans

Example-9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

$$\text{Solution: } S_7 = 49$$

$$S_{17} = 289$$

$$\frac{7}{2} \{2a + (7 - 1)d\} = 49$$

$$\frac{17}{2} \{2a + (17 - 1)d\} = 289$$

$$\frac{7}{2} \{2a + 6d\} = 49$$

$$\frac{17}{2} \{2a + 16d\} = 289$$

$$2a + 6d = 14 \text{ -----(1)}$$

$$2a + 16d = 34 \text{ -----(2)}$$

Subtracting equation (1) from equation (2)

$$10d = 20 \Rightarrow d = 2 \text{ put in equation (1) we will get } 2a + 6(2) = 14 \Rightarrow 2a = 2$$

$$a = 1$$

$$\text{hence } S_n = \frac{n}{2} \{2a + (n - 1)d\} = \frac{n}{2} \{2 \times 1 + (n - 1) \times 2\} = \frac{n}{2} \{2 + 2n - 2\} = \frac{n}{2} \{2n\}$$

$$S_n = n^2$$

Ans

Example-10. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Solution: Let cost of first prize be Rs x

Then cost of second prize = Rs $x - 20$

And cost of third prize = Rs $x - 40$, and so on.....

As a constant value is decreasing in the succeeding terms so it is an AP.

$$\text{Where } a = x, \quad d = -20, \quad n = 7, \quad s_7 = 700$$

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

$$S_7 = \frac{7}{2} \{2x + (7 - 1)(-20)\}$$

$$700 = \frac{7}{2} \{2x - 120\} \Rightarrow 200 = 2x - 120 \Rightarrow 320 = 2x \Rightarrow x = 160$$

Hence cost of prizes first to seventh are Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, Rs 40

Ans

MCQs: Choose one correct option (1 mark each)

1. The sum of first 20 odd natural numbers is

(a) 100

(b) 210

(c) 400

(d) 420

2. If 18, a , b , -3 are in A.P., then $a+b =$

- (a) 19 (b) 7 (c) 11 (d) 15

3. The first term of an AP is p and the common difference is q , then its 10th term is

- (a) $q+9p$ (b) $p-9q$ (c) $p+9q$ (d) $2p+9q$

4. The value of x for which $2x, x+10$ and $3x+2$ are consecutive terms of an AP. Is

- (a) -6 (b) 18 (c) 6 (d) -18

5. The sum of first 16 terms of an AP : 10, 6, 2, ..., is

- (a) -320 (b) 320 (c) -352 (d) -400

6. If the first term of an AP is -5 and the common difference is 2, then the sum of first 6 terms is

- (a) 0 (b) 5 (c) 6 (d) 15

7. The 4th term from the end of the AP: -11, -8, -5, ..., 49 is

- (a) 37 (b) 40 (c) 43 (d) 58

8. Which term of AP 21, 42, 63, 84, ...is 210?

- (a) 9th (b) 10th (c) 11th (d) 12th

9. If the 2nd term of an AP. Is 13 and 5th term is 25, then its 7th term is

- (a) 30 (b) 33 (c) 37 (d) 38

10. If the first term of an AP is 2 and the common difference is 4, the the sum of its 40 terms is

- (a) 3200 (b) 1600 (c) 200 (d) 2800

11. The number of terms of the AP.: 3, 7, 11, 15, ... to be taken so that the sum is 406 is

- (a) 5 (b) 10 (c) 12 (d) 15

12. The next term of the series: $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$ is

- (a) $\sqrt{40}$ (b) $\sqrt{46}$ (c) $\sqrt{50}$ (d) $\sqrt{72}$

13. The next term of the series: $\sqrt{18}, \sqrt{50}, \sqrt{98}, \dots$ is

- (a) $\sqrt{146}$ (b) $\sqrt{128}$ (c) $\sqrt{162}$ (d) $\sqrt{200}$

14. If the common difference of an AP is 7, then $a_{25} - a_{21}$ is equal to

- (a) 14 (b) 28 (c) 20 (d) 35

15. The common difference of the AP having $a_n = 2n + 3$ as n^{th} term is

- (a) 2 (b) 3 (c) 5 (d) 1
16. If the n th term of an AP is $7n + 12$, then its common difference is
 (a) 5 (b) 7 (c) 12 (d) 19
17. The sum of first n odd natural numbers is
 (a) $2n$ (b) $2n + 1$ (c) n^2 (d) $n^2 + 1$
18. If n th term of an AP is $(2n+1)$, then the sum of first n terms of the AP is
 (a) $n(n-2)$ (b) $n(n+2)$ (c) $n(n+1)$ (d) $n(n-1)$

Assertion and Reasoning Question:

Each of the following questions containing Assertion (A) and Reason (R) has the following choices:

- (a) If Assertion (A) is true, Reason (R) is true; Reason (R) is correct explanation for Assertion (A).
 (b) If Assertion (A) is true, Reason (R) is true; Reason (R) is not correct explanation for Assertion (A).
 (c) If Assertion (A) is true, Reason (R) is false.
 (d) If Assertion (A) is false, Reason (R) is true.

19. Assertion (A): For an A.P., 3, 6, 9, ..., 198, 10th term from the end is 168.

Reason (R): If ' a ' and ' l ' are the first term and last term of an A.P. with common difference ' d ', then n th term from the end of the given A.P. is $l - (n - 1)d$

20. Assertion (A): Common difference of the AP : 5, 1, -3, -7, ... is -4. Reason (R): Common difference of the AP : $a_1, a_2, a_3, \dots, a_n$ is obtained by $d = a_n - a_{n-1}$

Short Type-1 (2 marks each)

- Find the value of ' p ' if the numbers $x, 2x + p, 3x + p$ are three successive terms of the AP.
- Find p and q such that: $2p, 2p + q, p + 4q, 35$ are in AP.
- Find a, b and c such that the following numbers are in A.P. : $a, 7, b, 23, c$.
- Find the 6th term of the AP 54, 51, 48... 11.
- Find the 8th term from the end of the AP : 7, 10, 13, ..., 184.

Short Type-2 (3 marks each)

1. The 6th term of an AP is -10 and its 10th term is -26 . Determine the 15th term of the A.P.
2. Which term of the AP : $5, 15, 25, \dots$ will be 130 more than its 31st term?
3. Find the sum of the first 30 positive integers divisible by 6.
4. If in an AP, the sum of its first ten terms is -80 and the sum of its next ten terms is -280 . Find the AP.
5. Solve the equation : $1 + 4 + 7 + 10 + \dots + x = 287$
6. In an A.P., the first term is 25, nth term is -17 and sum of first n terms is 60. Find 'n' and 'd', the common difference.

Long Questions:

1. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?
2. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see figure). A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket.
 - (a) What is the distance the competitor has to run for 1st potato?
 - (b) What is the distance the competitor has to run for 10th potato?
 - (c) What is the total distance the competitor has to run for all 10 potatoes?
3. A school is organizing a charity run to raise funds for a local hospital. The run is planned as a series of rounds around a track, with each round being 300 metres. To make the event more challenging and engaging, the organizers decide to increase the distance of each subsequent round by 50 metres. For example, the second round will be 350 metres, the third round will be 400 metres and so on. The total number of rounds planned is 10. Based on the information given above, answer the following questions:
 - (a) Write the fourth, fifth and sixth term of the Arithmetic Progression so formed.
 - (b) Determine the distance of the 8th round.

(c) Find the total distance run after completing all 10 rounds. 2

OR

(c) If a runner completes only the first 6 rounds, what is the total distance run by the runner? 2

4. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on.

Based on the information given above, answer the following questions:

(a) Check whether the obtained sequence is an AP? 1

(b) Find the number of logs in 6th row? 1

(c) In how many rows are the 200 logs placed? 2

OR

(c) How many logs are in the top row? 2

PYQs: (2025)

1. If $k + 7$, $2k - 2$ and $2k + 6$ are three consecutive terms of an A.P., then the value of k is:

(a) 15 (b) 17 (c) 5 (d) 1

2. Two A.P.s have the same first term. The common difference of the first A.P. is -3 and of the second A.P. is -5. The difference of the 6th term of the second A.P. from that of the first A.P. is :

(a) 2 (b) -8 (c) -10 (d) 10

3. How many terms are there in the A.P. given below?

14, 19, 24, 29, ..., 119

(a) 18 (b) 14 (c) 22 (d) 21

4. If the sum of the first 7 terms of an A.P. is -14 and that of 11 terms is -55, then find the sum of its first n terms.

5. In an A.P., the sum of first n terms is $3n^2 + n$. Find the first term and the common difference of the A.P. Hence, find its 15th term.

6. An A.P. consists of ' n ' terms whose n th term is 4 and the common difference is 2. If the sum of ' n ' terms of an A.P. is -14, then find ' n '. Also, find the sum of the first 20 terms.

7. The sum of first 6 terms of an AP is 42. The ratio of the 10th term to the 30th term is 1:3. Calculate the first and the 13th terms of an AP.

8. The sum of the third term and the seventh term of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP.

9. The minimum age of children eligible to participate in a painting competition is 8 years. It is observed that the age of the youngest boy was 8 years and the ages of the participants, when seated in order of age, have a common difference of 4 months. If the sum of the ages of all the participants is 168 years, find the age of the eldest participant in the painting competition.

10. A sum of 2,000 is invested at 7% per annum simple interest. Calculate the interests at the end of 1st, 2nd and 3rd year. Do these interests form an AP? If so, find the interest at the end of the 27th year.

Answers :

MCQs: 1-C, 2-D, 3-C, 4-C, 5-A, 6-A, 7-B, 8-B, 9-B, 10-A, 11-D, 12-C, 13-C, 14-B, 15-A, 16-B, 17-C, 18-B, 19-D, 20-A

Short type-1: 1. $p=0$, 2. $p=10, q=5$, 3. $a=-1, b=15, c=31$,
4. 39, 5. 163

Short type-2: 1. -46, 2. 44, 3. 2790, 4. 1, -1, -3, -5, ...
5. $n=14, x=40$, 6. $N=15, d=-3$

PYQs: 1-B, 2-C, 3-C, 4. $n(13-3n)/4$, 5. $a=4, d=6, a_{15}=88$
6. $a=-8, n=7, S_{20}=-40$, 7. $a=2, d=2, a_{13}=26$ 8. $d=\pm 1/2, a=1$ or 5 $S_{16}=76$ or 20
9. $n=16, a_{16}=8+15(1/3)=13$ years 10. 140, 280, 420, $a_{27}=3780$

Long Answers:

1. Number of classes = 12

Since each class has 3 sections.

$\therefore$ Number of plants planted by class I = $1 \times 3 = 3$

Number of plants planted by class II = $2 \times 3 = 6$

Number of plants planted by class III = $3 \times 3 = 9$

Number of plants planted by class IV = $4 \times 3 = 12$

.....
Number of plants planted by class XII = $12 \times 3 = 36$

The numbers 3, 6, 9, 12, ..., 36 are in A.P.

Here, $a = 3$ and $d = 6 - 3 = 3$

Since number of classes = 12 i.e., $n = 12$

$\therefore$ Sum of the n terms of the above A.P., is given by

$$S_{12} = 12/2 [2(3) + (12-1)3] \text{ using } S_n = \frac{n}{2} \{2a + (n-1)d\}$$
$$= 6 [6 + 11 \times 3] = 6 [6 + 33] = 6 \times 39 = 234$$

Thus, the total number of trees = 234.

2. Here, number of potatoes = 10

The up-down distance of the 1st potato = $[5] \times 2 = 10$ m

The up-down distance of the 2nd potato = $[(5+3)] \times 2 = 16$ m The up-

down distance of the 3rd potato = $[(5+3+3)] \times 2 = 22$ m The up-down

distance of the 4th potato = $[(5+3+3+3)] \times 2 = 28$ m

.....
10, 16, 22, 28, ... are in A.P. such that

$a = 10$ and $d = 16 - 10 = 6$

(a) 10 m

(b) $a_{10} = a + 9d = 10 + 9 \times 6 = 10 + 54 = 64$ m

(c) Using $S_n = n/2 [2a + (n-1)d]$,

we have: $S_{10} = 10/2 [2(10) + (10-1) \times 6]$

$$= 5 [20 + 9 \times 6]$$

$$= 5 [20 + 54] = 5 [74] = 5 \times 74 = 370$$

Thus, the sum of above distances = 370 m.

The competitor has to run a total distance of 370 m.

3. The length of tracks are 300, 350, 400, ...

(a) $4^{\text{th}} = 400 + 50 = 450$, $5^{\text{th}} = 450 + 50 = 500$, $6^{\text{th}} = 500 + 50 = 550$

(b) The distance of 8^{th} round $a_8 = a + 7d = 300 + 7 \times 50 = 300 + 350 = 650$

$$(c) S_{10} = \frac{10}{2} \{2 \times 300 + (10 - 1) \times 50\} = 5 \{600 + 450\} = 5 \times 1050 = 5250$$

OR

$$S_6 = \frac{6}{2} \{2 \times 300 + (6 - 1) \times 50\} = 3 \{600 + 250\} = 3 \times 850 = 2550 \text{ m}$$

4. Obtained seq is 20, 19, 18, ...

(a) $19 - 20 = 18 - 19 = -1$, yes it is an AP

(b) $a_6 = a + 5d = 20 + 5(-1) = 20 - 5 = 15$

(c) Now, using, $S_n = \frac{n}{2} [2a + (n - 1)d]$,
we get $S_n = \frac{n}{2} [2(20) + (n - 1) \times (-1)]$

$$\Rightarrow 200 = \frac{n}{2} [40 - (n - 1)]$$

$$\Rightarrow 2 \times 200 = n \times 40 - n(n - 1)$$

$$\Rightarrow 400 = 40n - n^2 + n$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow n(n - 16) - 25(n - 16) = 0$$

$$\Rightarrow (n - 16)(n - 25) = 0$$

Either $\Rightarrow n - 16 = 0 \Rightarrow n = 16$ or $n - 25 = 0 \Rightarrow n = 25$

$$T_n = 0 \Rightarrow a + (n - 1)d = 0 \Rightarrow 20 + (n - 1) \times (-1) = 0$$

$\Rightarrow n - 1 = 20 \Rightarrow n = 21$ i.e., 21st term becomes 0 $\therefore n = 25$ is not required.

Thus, $n = 16 \therefore$ Number of rows = 16

OR

$$\text{Now, } T_{16} = a + (16 - 1)d = 20 + 15 \times (-1) = 20 - 15 = 5$$

$\therefore$ Number of logs in the 16th (top) row is 15

END

COORDINATE GOEMETRY

Summary/Formulae of the chapter:

- The abscissa and ordinate of a given point are the distances of the point from y-axis and x-axis respectively.
- The coordinates of any point on x-axis are of the form $(x, 0)$.
- The coordinates of any point on y-axis are of the form $(0, y)$.
- The distance between points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The coordinates of the point which divides the join of points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

The coordinates of the mid-point of the line segment joining the points

$P(x_1, y_1)$ and $Q(x_2, y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- The coordinates of the centroid of a triangle formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are

$$\left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2} \right)$$

Solved Examples:

Example-1. Find the point on the x-axis which is equidistant from $(2, -5)$ and $(-2, 9)$.

Solution: We know that any point on x-axis has its ordinate = 0.

Let the required point be $P(x, 0)$.

Let the given points be $A(2, -5)$ and $B(-2, 9)$.

∴ Since, A and B are equidistant from P, ∴ $PA = PB \Rightarrow PA^2 = PB^2$

$$\Rightarrow (x - 2)^2 + (0 + 5)^2 = (x + 2)^2 + (0 - 9)^2$$

$$\Rightarrow x^2 - 4x + 29 = x^2 + 4x + 85$$

$$\Rightarrow x^2 - 4x - x^2 - 4x = 85 - 29$$

$$\Rightarrow -8x = 56 \Rightarrow x = 56/8 = -7$$

∴ The required point is $(-7, 0)$.

Ans

Example-2. Find the values of y for which the distance between the points $P (2, -3)$ and $Q (10, y)$ is 10 units.

Solution: The given points are $P (2, -3)$ and $Q (10, y)$. and $PQ = 10$

$$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{(10 - 2)^2 + (y + 3)^2}$$

Squaring both sides, $y^2 + 6y + 73 = 100$

$$\Rightarrow y^2 + 6y - 27 = 0$$

$$\Rightarrow y^2 - 3y + 9y - 27 = 0$$

$$\Rightarrow (y - 3)(y + 9) = 0 \Rightarrow \text{Either } y - 3 = 0 \Rightarrow y = 3 \text{ or } y + 9 = 0 \Rightarrow y = -9$$

$\therefore$ The required value of y is 3 or -9 . Ans

Example-3. Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$.

Solution: Let the required point be $P (x, y)$.

Here, the end points are: $(-1, 7)$ and $(4, -3)$ and Ratio = $2:3$

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$$

$$P(x, y) = \left(\frac{2 \times 4 + 3 \times (-1)}{2 + 3}, \frac{2 \times (-3) + 3 \times 7}{2 + 3} \right)$$

$$= \left(\frac{8-3}{5}, \frac{-6+21}{5} \right) = \left(\frac{5}{5}, \frac{15}{5} \right) = (1, 3)$$

Thus, the required point is $(1, 3)$. Ans

Example-4. If $(1, 2)$, $(4, y)$, $(x, 6)$ and $(3, 5)$ are the vertices of a parallelogram taken in order, find x and y .

Solution: We have the parallelogram vertices $A (1, 2)$, $B (4, y)$, $C (x, 6)$ and $D (3, 5)$. Since, the diagonals of a parallelogram bisect each other.

$\therefore$ The coordinates of P are:

$$\left(\frac{1+x}{2}, \frac{2+6}{2} \right) = \left(\frac{4+y}{2}, \frac{y+5}{2} \right) \Rightarrow 1+x = 7 \text{ and } 8 = y+5 \Rightarrow x = 6 \text{ and } y = 3$$

$\therefore$ The required values of x and y are: $x = 6$, $y = 3$ Ans

Example-5. Find the coordinates of a point A , where AB is the diameter of a circle whose centre is $(2, -3)$ and B is $(1, 4)$.

Solution: Here, centre of the circle is $O (2, -3)$. Let the end points of the diameter be $A (x, y)$ and $B (1, 4)$ The centre of a circle is the mid-point of the ends of

a diameter.

$$\text{So } \left(\frac{x+1}{2}, \frac{y+4}{2}\right) = (2, -3) \Rightarrow x + 1 = 4 \text{ and } y + 4 = -6 \Rightarrow x = 3 \text{ and } y = -10$$

Thus the coordinates of a point A are (3, -10)

Ans

MCQs: Choose one correct option (1 mark each)

21. The distance of the point P (2, 3) from the x-axis is
(a) 2 (b) 3 (c) 1 (d) 5
22. The points (-4, 0), (4, 0) and (0, 3) are the vertices of a
(a) right triangle (b) isosceles triangle (c) equilateral triangle (d) scalene triangle
23. The fourth vertex D of a parallelogram ABCD whose three vertices are A (-2, 3), B(6, 7) and C(8, 3) is
(a) (0, 1) (b) (0, -1) (c) (-1, 0) (d) (1, 0)
24. If the distance between the points (4, p) and (1, 0) is 5, then the value of p is
(a) 4 only (b) ± 4 (c) -4 only (d) 0
25. The points A(9,0), B(9, 6), C(-9, 6) and D(-9,0) are the vertices of a
(a) square (b) rectangle (c) rhombus (d) trapezium
26. The distance between the points $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$
(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 2 (d) 1
27. If x is a positive integer such that the distance between the points $P(x, 2)$ and $Q(3, -6)$ is 10 units then $x =$
(a) 3 (b) -3 (c) 9 (d) -9
28. A line segment is of length 10 units. If the coordinates of its one end are (2, -3) and the abscissa of the other end is 10, then its ordinate is
(a) 9, 6
(b) 3, -9 (c) -3, 9 (d) 9, -6
29. The line segment joining the points (-3, -4) and (1, -2) is divided by y-axis in the ratio
(a) 1 : 3 (b) 2 : 3 (c) 3 : 1 (d) 3 : 2
30. The ratio in which (4, 5) divides the join of (2, 3) and (7, 8) is
(a) -2:3 (b) -3:2 (c) 3:2 (d) 2:3
31. The ratio in which the x-axis divides the segment joining (3, 6) and (12, -3)

- (a) 2:1 (b) 1:2 (c) -2:1 (d) 1:-2
32. The distance of the point (4, 7) from the x-axis is
(a) 4 (b) 7 (c) 11 (d) $\sqrt{65}$
33. The distance of the point (4, 7) from the y-axis is
(a) 4 (b) 7 (c) 11 (d) $\sqrt{65}$
34. The coordinates of the point P dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2:1 are
(a) (2,4) (b) (3, 5) (c) (4, 2) (d) (5, 3)
35. The point on the x-axis which is equidistant from points (-1, 0) and (5, 0) is
(a) (0, 2) (b) (2, 0) (c) (3, 0) (d) (0, 3)
36. The coordinates of the point on x-axis which are equidistant from the points (-3, 4) and (2, 5) are
(a) (20, 0) (b) (-23, 0) (c) (4/5, 0) (d) (-1, 9/2)
37. If the point P (x,y) is equidistant from A (5, 1) and B (-1, 5), then
(a) $5x = y$ (b) $x = 5y$ (c) $3x = 2y$ (d) $2x = 3y$
38. The coordinates of the fourth vertex of the rectangle formed by the points (0, 0), (2, 0), (0, 3) are
(a) (3, 0) (b) (0, 2) (c) (2, 3) (d) (3, 2)

Assertion and Reasoning Question:

Each of the following questions containing Assertion (A) and Reason (R) has the following choices:

- (a) If Assertion (A) is true, Reason (R) is true; Reason (R) is correct explanation for Assertion (A).
- (b) If Assertion (A) is true, Reason (R) is true; Reason (R) is not correct explanation for Assertion (A).
- (c) If Assertion (A) is true, Reason (R) is false.
- (d) If Assertion (A) is false, Reason (R) is true.

39. Assertion (A): if the coordinates of the mid-points of sides AB and AC of $\triangle ABC$ are D(3, 5) and E(-3, -3) respectively, then BC = 20 units

Reason (R): the line segment joining the mid-points of two sides of a triangle is parallel to the third side.

40. Assertion (A): A triangle with vertices at (4, 0), (-1, -1) and (3, 5) is isosceles right angled triangle.

Reason (R): if ABC is isosceles triangle, then it is right angled.

Short Type-1 (2 marks each)

6. Find the distance between the points P (-6, 7) and Q (-1, -5).
7. If the points (2, 1) and (1, -2) are equidistant from the point (x, y) . Show that $x + 3y = 0$.
8. Find a point on y-axis which is equidistant from the point A (6, 5) and B (-4, 3).
9. Which point on x-axis is equidistant from (5, 9) and (-4, 6).
10. Find the value of x if the distance between the points $(x, -1)$ and $(3, 2)$ is 5.

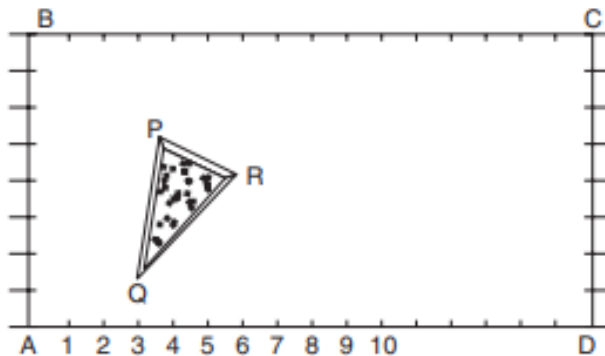
Short Type-2 (3 marks each)

7. Find the coordinates of the point which divides the line segment joining the points (6, 3) and (-4, 5) in the ratio 3:2.
8. Find the points of trisection of the segment joining the points (3, -2) and (-3, -4).
9. Find the ratio in which the point P $(x, 2)$ divides the line segment joining the points A(12, 5) and B(4, -3). Also, find the value of x .
10. Find the ratio in which the points $(-3, p)$ divides the line segment joining the points (-5, -4) and (-2, 3). Hence, find the value of p .
11. Find the ratio in which the line segments joining the points A (3, -3) and B (-2, 7) is divided by $x - axis$. Also find the coordinates of the point of division.
12. If the points A $(a, -11)$, B (5, b), C (2, 15) and D (1, 1) are the vertices of a parallelogram ABCD, find the values of a and b .

Long Questions:

1. The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a

triangular grassy lawn in the plot as shown in the Fig



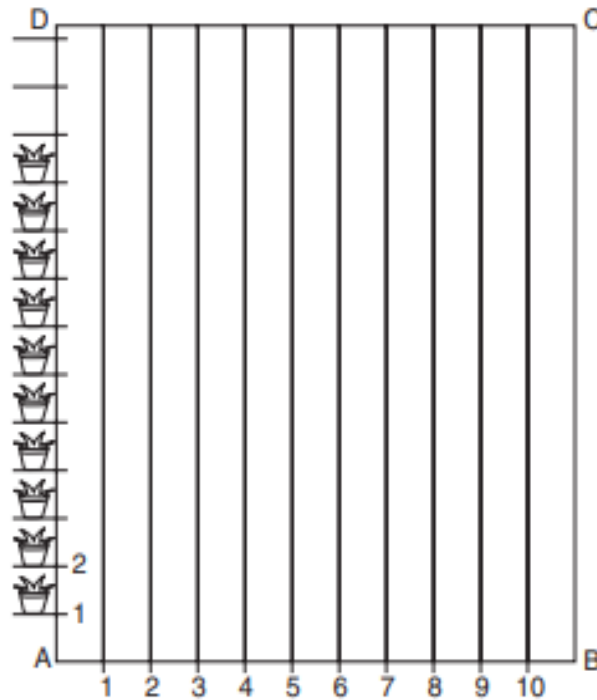
The students are to sow seeds of flowering plants on the remaining area of the plot.

- (i) Taking A as origin, find the coordinates of the vertices of ΔPQR .
- (ii) What will be the coordinates of the vertices of ΔPQR if C is the origin?
- (iii) Find the distance between P and Q

OR

- (iii) Find the distance between Q and R

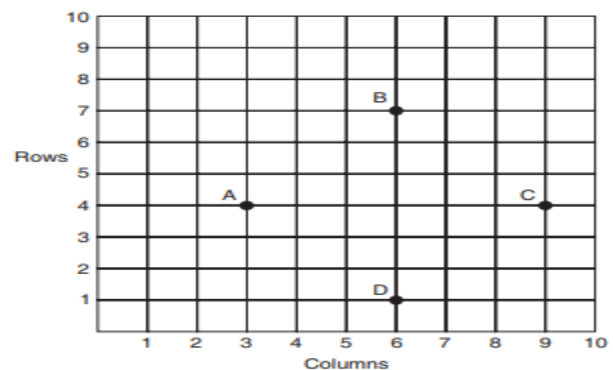
2. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the figure. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag.



(i) What is the distance between both the flags?

(ii) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

3. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, “Don’t you think ABCD is a square?” Chameli disagrees.



(i) Find the coordinates of A, B, C and D.

(ii) Find the distances AB, BC, CD and DA.

(iii) Find the distances AC and BD.

(iv) Check whether ABCD is a Square? Who is correct? Champa or Chameli ?

PYQs: (2025)

11. The coordinates of the end points of a diameter of a circle are (5,-2) and (5, 2). The length of the radius of the circle is :
(a) 2 (b) ± 2 (c) 4 (d) ± 4
12. The perimeter of the triangle formed by the vertices (0, 0), (2, 0) and (0, 2) is :
(a) 4 units (b) 6 units (c) $6\sqrt{2}$ units (d) $4 + 2\sqrt{2}$ units
13. AOBC is a rectangle whose three vertices are A(0, 2), O(0, 0) and B(4, 0). The square of the length of its diagonal is equal to :
(a) 36 (b) 20 (c) 16 (d) 4
14. If the mid-point of the line segment joining the points (a, 4) and (2, 2b) is (2, 6), then the value of (a + b) is given by :
(a) 6 (b) 7 (c) 8 (d) 16
15. The mid-point of the line segment joining the points P(4, 5) and Q(4, 6) lies on :
(a) x-axis (b) y-axis (c) origin (d) neither x-axis nor y-axis
16. If (a, b) is the mid-point of the line segment joining the points A(10, 6) and B(k, 4) and $a + 2b = 18$, then find the value of k.
17. If the mid-point of the line segment joining the points A (3,4) and B (k,6) is $P(x,y)$ and $x + y - 10 = 0$, find the value of k.
18. Find the ratio in which the y-axis divides the line segment joining the points (5, 6) and (-1, 4). Also find the point of intersection.
19. Find the coordinates of the points which divide the line segment joining A(2, 2) and B(2, 8) into four equal parts.
20. The coordinates of the centre of a circle are (2a, a-7). Find the value(s) of 'a' if the circle passes through the point (11, -9) and has diameter $10\sqrt{2}$ units.

Answers :

MCQs: 1-B, 2-B, 3-B, 4-B, 5-B, 6-B, 7-C, 8-B, 9-C, 10-B, 11-A, 12-B, 13-A, 14-B, 15-B, 16-B, 17-D, 18-C, 19-B, 20-C

Short type-1: 1. 13, 3. (0, 9), 4. (3, 0) 5. 7,-1

Short type-2: 1. (0, 21/5), 2. (1, -8/3) and (-1, -10/3), 3. 3:5, and x = 9, 4. 2:1, p = 2/3, 5. 3:7, (3/2, 0), 6. a=4, b=3

Long Answers:

1. (i) P(4, 5), Q(3, 1), R(6, 4) (ii) P(-12, -3), Q(-13, -7), R(-10, -4)
(iii) $PQ = \sqrt{17}$ or $QR = 3\sqrt{2}$

2. (i) $\sqrt{61}$, (ii) $(5, 45/2)$ in 5th row at a distance of 22.5 m above AB

3. (i) A(3, 4), B(6, 7), C(9, 4) D(6, 1) (ii) $AB=BC=CD=DA= \sqrt{18}$

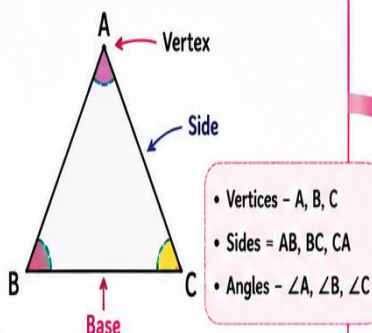
(iii) $AC=BD=6$ (iv) Yes it is a square so Champa is correct.

PYQs: 1-A, 2-D, 3-B, 4-A, 5-D, 6. $a=8, b=5, k=6$, 7. $k=7$, 8. $5:1$,
(0, $13/3$) 9. $(2, 7/2), (2, 5), (2, 13/2)$ 10. $a=3,5$

END

TRIANGLES

1. PARTS OF A TRIANGLE



KEY IDEA

Sum of interior angles
of a triangle = 180°

TRIANGLES

A polygon with three
sides, three vertices
and three angles.

2. TYPES OF TRIANGLES

(A) Based on Sides

- Equilateral Triangle
(All sides equal)
- Isosceles Triangle
(Two sides equal)
- Scalene Triangle
(All sides unequal)

(B) Based on Angles

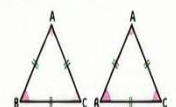
- Acute Angled Triangle
(All angles $< 90^\circ$)
- Right Angled Triangle
(One angle = 90°)
- Obtuse Angled Triangle
(One angle $> 90^\circ$)

3. CONGRUENCE OF TRIANGLES

Two triangles are congruent if their
corresponding parts are equal.

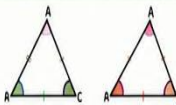
(i) SSS

(Side-Side-Side)



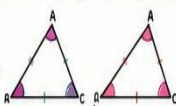
(ii) SAS

(Side-Angle-Side)



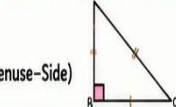
(iii) ASA

(Angle-Side-Angle)



(iv) RHS

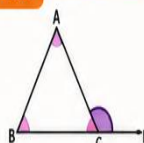
(Right angle-Hypotenuse-Side)



4. IMPORTANT THEOREMS

★ 1. Exterior Angle Theorem:

Exterior angle = Sum of two interior
opposite angles.

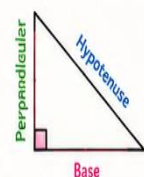


★ 2. Triangle Inequality Theorem:

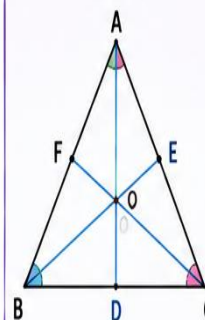
In any triangle, the sum of any two sides
is greater than the third side.

★ 3. Pythagoras Theorem:

In a right angled triangle,
(Hypotenuse)² = (Base)² + (Perpendicular)²



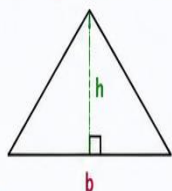
5. SPECIAL POINTS



- Centroid (O):
Intersection of medians.
- Incenter:
Intersection of angle bisectors.
- Circumcenter:
Center of circumcircle.
- Orthocenter:
Intersection of altitudes.

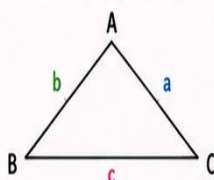
6. AREA OF A TRIANGLE

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$



7. PERIMETER OF A TRIANGLE

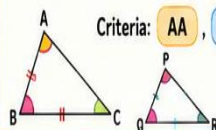
Perimeter = Sum of the three sides



8. SIMILARITY OF TRIANGLES

Two triangles are similar if their
corresponding angles are equal and
corresponding sides are proportional.

Criteria: AA, SAS, SSS



WHY IMPORTANT?

- ✓ Used in real life for measurement
of heights, distances and areas.
- ✓ Used in construction, navigation,
engineering, and design.
- ✓ Helps in solving many practical
problems.



★ TRIANGLES ARE THE BASIS OF SHAPES AND STRUCTURES AROUND US!

SIMILAR FIGURES

- Two figures having the same shape but not necessary the same size are called similar figures.
- All congruent figures are similar but all similar figures are not congruent.

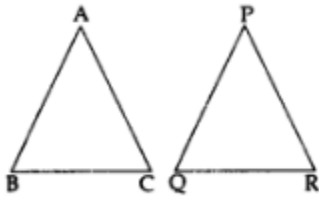
SIMILAR POLYGONS

Two polygons are said to be similar to each other, if:

- (i) their corresponding angles are equal, and
- (ii) the lengths of their corresponding sides are proportional

Example:

Any two triangles are similar, if their



- (i) Corresponding angles are equal

$$\angle A = \angle P$$

$$\angle B = \angle Q$$

$$\angle C = \angle R$$

- (ii) Corresponding sides are proportional

$$AB/PQ = AC/PR = BC/QR$$

THALES THEOREM OR BASIC PROPORTIONALITY THEOREM (BPT)

Theorem 1:

State and prove Thales' Theorem.

Statement:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB} \dots\dots(i) \text{ [Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{corresponding altitude}]$$

In $\triangle ADE$ and $\triangle CDE$,

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC}$$

$\because DE \parallel BC$...[Given]

$$\therefore \text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$$

...[$\because$ As on the same base and between the same parallel sides are equal in area]

From (i), (ii) and (iii),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given: In $\triangle ABC$, $DE \parallel BC$.

To prove: $AD \cdot DB = AE \cdot EC$

Const.: Draw $EM \perp AD$ and $DN \perp AE$. Join B to E and C to D .

Proof: In $\triangle ADE$ and $\triangle BDE$,

CRITERION FOR SIMILARITY OF TRIANGLES:

Two triangles are similar if either of the following three criterion's are satisfied:

- **AAA similarity Criterion.** If two triangles are equiangular, then they are similar.
- **Corollary (AA similarity).** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
- **SSS Similarity Criterion.** If the corresponding sides of two triangles are proportional, then they are similar.
- **SAS Similarity Criterion.** If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

Results in Similar Triangles based on Similarity Criterion:

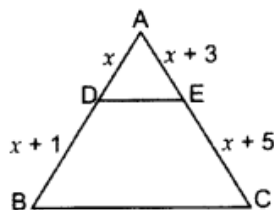
1. Ratio of corresponding sides = Ratio of corresponding perimeters
2. Ratio of corresponding sides = Ratio of corresponding medians
3. Ratio of corresponding sides = Ratio of corresponding altitudes
4. Ratio of corresponding sides = Ratio of corresponding angle bisector segments.

Multiple choice type questions:

1. If in two triangles ABC and DEF , $AB/DF = BC/FE = CA/ED$, then
 - (a) $\triangle ABC \sim \triangle DEF$
 - (b) $\triangle ABC \sim \triangle EDF$
 - (c) $\triangle ABC \sim \triangle EFD$
 - (d) $\triangle ABC \sim \triangle DFE$
2. If in triangles ABC and DEF , $AB/DE = BC/FD$, then they will be similar, when
 - (A) $\angle B = \angle E$
 - (B) $\angle B = \angle D$
 - (C) $\angle A = \angle D$
 - (D) $\angle A = \angle F$
3. In triangles ABC and DEF , $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3 DE$. Then, the two triangles are
 - (A) congruent but not similar
 - (B) similar but not congruent
 - (C) neither congruent nor similar
 - (D) congruent as well as similar

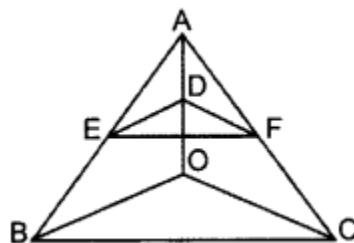
Very short answer type questions:

1. If $\triangle ABC \sim \triangle PQR$, perimeter of $\triangle ABC = 32$ cm, perimeter of $\triangle PQR = 48$ cm and $PR = 6$ cm, then find the length of AC .
2. $\triangle ABC \sim \triangle DEF$. If $AB = 4$ cm, $BC = 3.5$ cm, $CA = 2.5$ cm and $DF = 7.5$ cm, find the perimeter of $\triangle DEF$.
3. If $\triangle ABC \sim \triangle RPQ$, $AB = 3$ cm, $BC = 5$ cm, $AC = 6$ cm, $RP = 6$ cm and $PQ = 10$, then find QR .
4. In $\triangle DEW$, $AB \parallel EW$. If $AD = 4$ cm, $DE = 12$ cm and $DW = 24$ cm, then find the value of DB .
5. In $\triangle ABC$, $DE \parallel BC$, find the value of x .

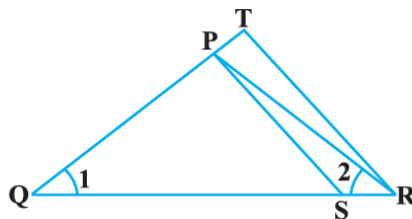


Short answer type questions:

1. In the figure, if $DE \parallel OB$ and $EF \parallel BC$, then prove that $DF \parallel OC$.



2. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.
3. In Fig. , $QR/QS = QT/PR$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$.



5. S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$.
Show that $\triangle RPQ \sim \triangle RTS$.

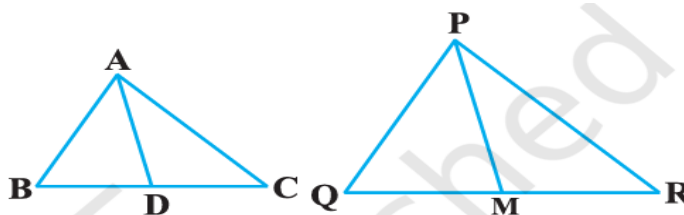
Long answer type questions:

Based on BPT:

1. ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show that $AO/BO = CO/DO$
2. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $AO/BO = CO/DO$. Show that ABCD is a trapezium.
3. State and prove BPT (Thales Theorem).

BASED ON SIMILARITY:

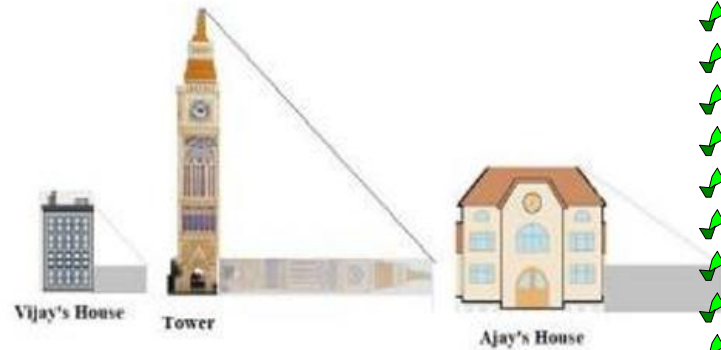
4. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see Fig. 6.41). Show that $\triangle ABC \sim \triangle PQR$.



5. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.
6. If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$, prove that $AB/PQ = AD/PM$.

CASE STUDY 1:

Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house is 20m when Vijay's house casts a shadow 10m long on the ground. At the same time, the tower casts a shadow 50m long on the ground and the house of Ajay casts 20m shadow on the ground.



1. What is the height of the tower?

- a) 20m b) 50m c) 100m d) 200m

2. What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12m?

- a) 75m b) 50m c) 45m d) 60m

3. What is the height of Ajay's house?

- a) 30m b) 40m c) 50m d) 20m

4. When the tower casts a shadow of 40m, same time what will be the length of the shadow of Ajay's house?

- a) 16m b) 32m c) 20m d) 8m

5. When the tower casts a shadow of 40m, same time what will be the length of the shadow of Vijay's house?

- a) 15m b) 32m c) 16m d) 8m

INTRODUCTION TO TRIGNOMETRY

KEY CONCEPTS

Trigonometric Ratios of an acute angle in a right angled triangle express the relationship between the angle and the length of its sides.

In any right angled triangle

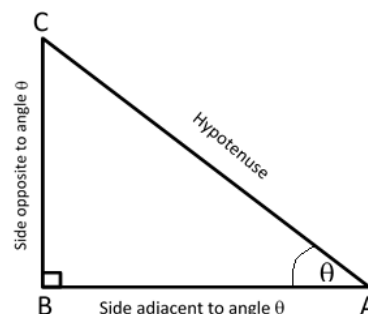
Side *opposite* to right angle = **Hypotenuse**

Side *opposite* to acute angle in consideration =

Perpendicular

Adjacent side in the triangle = **Base**

- Trigonometric ratios**



$$\sin = \frac{\text{Side opposite to } \theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\cos = \frac{\text{Side adjacent to } \theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\text{base}}{\text{hypotenuse}}$$

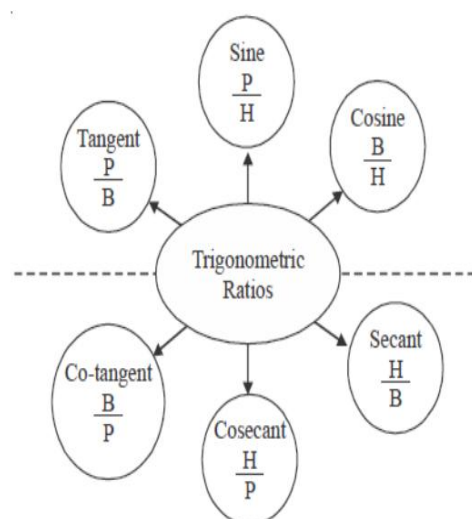
$$\tan = \frac{\text{Side opposite to } \theta}{\text{Side adjacent to } \theta} = \frac{BC}{AB} = \frac{\text{perpendicular}}{\text{base}} = \frac{\sin \theta}{\cos \theta} = \frac{1}{\cot \theta}$$

$$\operatorname{cosec} = \frac{\text{Hypotenuse}}{\text{side Opposite to } \theta} = \frac{AC}{BC} = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{1}{\sin \theta}$$

$$\sec = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \theta} = \frac{AC}{AB} = \frac{\text{hypotenuse}}{\text{base}} = \frac{1}{\cos \theta}$$

$$\cot = \frac{\text{Side Adjacent to } \theta}{\text{Side Opposite to } \theta} = \frac{AB}{BC} = \frac{\text{base}}{\text{perpendicular}} =$$

$$\frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$$



-
- **Trigonometric Identities**

➤ **Pythagorean Identities**

$$\circ \sin^2 \theta + \cos^2 \theta = 1$$

$$\circ \sec^2 \theta - \tan^2 \theta = 1 \quad 0^\circ \leq \theta < 90^\circ$$

$$\circ \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \quad 0^\circ < \theta \leq 90^\circ$$

➤ Reciprocal Identities

$$\circ \sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$\circ \cos \theta = \frac{1}{\sec \theta}$$

$$\circ \tan \theta = \frac{1}{\cot \theta}$$

$$\circ \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\circ \sec \theta = \frac{1}{\cos \theta}$$

$$\circ \cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities

$$\circ \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\circ \cot \theta = \frac{\cos \theta}{\sin \theta}$$

- The values of trigonometric ratios for angles 0° , 30° , 45° , 60° and 90° .

θ	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	2	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	N.D
Cosec	N.D	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	N.D
Cot	N.D	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

COMPETENCY BASED QUESTIONS (1 Marks)

- The value of $\cos 0^\circ + \cos 90^\circ$ is _____.
a) 0 b) $\frac{1}{2}$ c) 1 d) 2
- What is the minimum value of $\sin A, 0 \leq A \leq 90^\circ$
a) 0 b) $\frac{1}{\sqrt{2}}$ c) $\frac{\sqrt{3}}{2}$ d) 2
- If in $\triangle ABC$, $\angle C = 90^\circ$, then find $\sin (A + B)$
a) $\frac{\sqrt{3}}{2}$ b) $1/2$ c) 0 d) 1
- Find the value of $7 \tan^2 A - 7 \sec^2 A + 1$
a) 7 b) 6 c) -7 d) -6
- In $\triangle ABC$, right-angled at B , $AB = 5 \text{ cm}$ and $\angle ACB = 30^\circ$ then the length of the side AC is
a) $5\sqrt{3}$ b) $5\sqrt{2}$ c) 10 d) $2\sqrt{3}$
- If $\sin \theta = \cos \theta$, find the value of θ
a) 30° b) 45° c) 60° d) 90°
- Evaluate $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$
a) 0 b) 1 c) $3/2$ d) 2
- If $\tan \theta = 4/3$ then find the value of $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$
a) $7/3$ b) $7/3$ c) 7 d) $7/3$

9. Evaluate $\frac{1+\tan^2 \theta}{1+\cot^2 \theta}$

- a) $\cot^2 \theta$ b) $\tan^2 \theta$ c) 1 d) -1

10. If $\cos A = 7/25$, find the value of $\tan A + \cot A$

- a) 168/625 b) 625/168 c) 25/24 d) 24/7

11. In a right triangle ABC , right-angled at B , if $\tan A = 1$, then the value of $2\sin A \cos A =$ _____

- a) 0 b) 1/2 c) 1 d) none of these

12. The value of $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$ is

- a) $\sin 30^\circ$ b) $\sin 45^\circ$ c) $\sin 60^\circ$ d) $\sin 90^\circ$

13. If $5 \tan \theta - 4 = 0$, then value of $\text{If } \frac{5 \sin \theta - 4 \cos \theta}{5 \sin \theta + 4 \cos \theta}$

- a) $\frac{5}{3}$ b) 1 c) $\frac{5}{6}$ d) 0

14. If $\operatorname{cosec} \theta - \sin \theta = \sqrt{5}$, then find the value of $\operatorname{cosec} \theta + \sin \theta$.

- a) 3 b) 5 c) $\sqrt{3}$ d) $\sqrt{5}$

15. If $x \tan 60^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$, then $x =$

- a) $\cos 30^\circ$ b) $\tan 30^\circ$ c) $\sin 30^\circ$ d) $\cot 30^\circ$

16. The minimum value of $2\sin^2 \theta + 3\cos^2 \theta$

- a) 0 b) 1 c) 2 d) 3

17. If $\sin \theta + \cos \theta = \sqrt{2}$, then $\tan \theta + \cot \theta =$

- a) 1 b) 2 c) 3 d) 4

18. If $\tan \alpha = \cot \alpha$, then the value of $\sec \alpha$ is

- a) $\sqrt{2}$ b) $2/\sqrt{3}$ c) 2 d) 1

19. $\sin 2A = 2 \sin A \cos A$ is true when $A =$

- a) 30° b) 45° c) 60° d) any angle

20. If $\sin A = \frac{1}{2}$, then the value of $3\cos A - 4\cos^3 A$ is

- a) 0 b) 1 c) $\frac{1}{2}$ d) -1

DIRECTION: In the following questions, a statement of **assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

21. Assertion: If $x = 3 \sin^2 \theta + 1$ and $y = 3 \cos^2 \theta + 1$ then the value of $x + y = 4$.

Reason: For any value of θ , $\sin^2 \theta + \cos^2 \theta = 1$

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

22. Assertion: In a right angled triangle ABC right angled at B . If AB is opposite side, then BC is adjacent side.

Reason: Side opposite to right angle is hypotenuse

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

23. Assertion: $\sin \theta$ is the product of $\sin$ and θ

Reason: The value of $\sin \theta$ increases as θ increases.

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.

d) Assertion (A) is false but reason (R) is true.

24. Assertion: The value of $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$ is 1.

Reason: $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$.

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

25. Assertion: $\sin(A + B) = \sin A + \sin B$.

Reason: For any value of θ , $\sin^2 \theta + \cos^2 \theta = 1$

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

VERY SHORT ANSWER QUESTIONS (2 Marks)

26. If $\sin \theta = \cos \theta$, find the value of $(\sin \theta + \cos \theta)$

27. If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, find A and B

28. If $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$, find A and B

29. Find the value of $\operatorname{cosec}^2 30^\circ - \sin^2 45^\circ - \sec^2 60^\circ$

30. If $3 \cot A = 4$, find the value of $\operatorname{cosec} A + 1$

31. If $\tan(3x - 15^\circ) = 1$ then find the value of x .

32. Show that : $\frac{1-\sin 60^\circ}{\cos 60^\circ} = 2 - \sqrt{3}$

33. Find the value of θ , if $\frac{\cos \theta}{1-\sin \theta} + \frac{\cos \theta}{1+\sin \theta} = 4, \theta \leq 90^\circ$

34. If $\sin \theta - \cos \theta = 0$, find the value of $\sin^4 \theta + \cos^4 \theta$

35. If $\sec \theta + \tan \theta = 7$, then evaluate $\sec \theta - \tan \theta$.

SHORT ANSWER QUESTIONS (3 Marks)

36. Prove that: $(\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2 = 2$

37. Prove that $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} + \sin \theta \cdot \cos \theta = 1$

38. Prove that: $\frac{2 \cos^3 \theta - \cos \theta}{\sin \theta - 2 \sin^3 \theta} = \cot \theta$

39. Prove that: $\frac{\cos A}{\operatorname{cosec} A + 1} + \frac{\cos A}{\operatorname{cosec} A - 1} = 2 \tan A \sec A$

40. Prove that: $\frac{2 \tan \theta}{1 + \tan^2 \theta} = 2 \sin \theta \cos \theta$

41. Prove that: $\frac{2 \sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = 2 \tan \theta + \cot \theta$

42. Prove that: $\sqrt{\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta}} = \frac{1 - \sin \theta}{\cos \theta}$

43. Evaluate: $2(\sin^4 30^\circ + \cos^4 60^\circ) - (\tan^2 60^\circ + \cot^2 45^\circ) + 3 \operatorname{cosec}^2 60^\circ$

44. Evaluate: $\frac{\sin^2 45^\circ - \cos^2 60^\circ + \tan^2 30^\circ}{3 \operatorname{cosec}^2 30^\circ - 2 \sec^2 60^\circ}$

45. Evaluate: $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$

LONG ANSWER QUESTIONS (5 Marks)

46. Prove that: $(\sec A - \cos A) \cdot (\cot A + \tan A) = \tan A \cdot \sec A$

47. Prove that: $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

48. Prove that: $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\frac{1 - \tan A}{1 - \cot A}\right) = \tan^2 A$

49. Prove that: $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{\cos \theta}{1 - \sin \theta}$

50. Prove that: $\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$

51. Prove that: $\frac{\sec \theta + \tan \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\cos \theta}{1 - \sin \theta}$

52. If $A = 30^\circ$ and $B = 60^\circ$, Prove that $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$

53. If $\sec \theta = x + \frac{1}{4x}$, prove that $\sec \theta + \tan \theta = 2x$ or $\frac{1}{2x}$

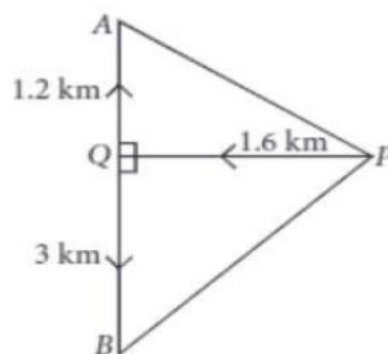
54. If $\tan \theta + \sin \theta = m$, $\tan \theta - \sin \theta = n$ then show that $m^2 - n^2 = 4mn$.

55. If $x = a \cos \theta - b \sin \theta$ and $y = a \sin \theta + b \cos \theta$, then prove that $a^2 + b^2 = x^2 + y^2$

CASE STUDIES/SOURCE BASED INTEGRATED QUESTIONS (4 Marks)

QUESTION -1

Two aero planes leave an airport, one after the other. After moving on runway, one flies due North and other flies due South. The speed of two aero planes is 400 km/hr and 500 km/hr respectively. Considering PQ as runway and A and B are any two points in the path followed by two planes, then answer the following questions.

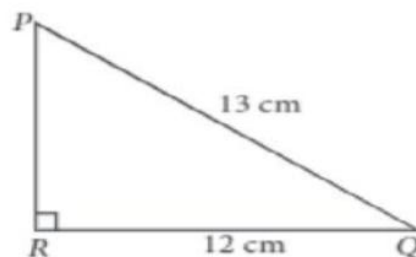


1. Find the distance between A and P
2. If $\angle APQ = \theta$, Find $\cot B - \tan \theta$
3. Find $PB - AP$

QUESTION -2

Yogita, a student of class 10th, has to make a project. She decides to make a bird house which is triangular in shape. She uses cardboard to make the bird house as

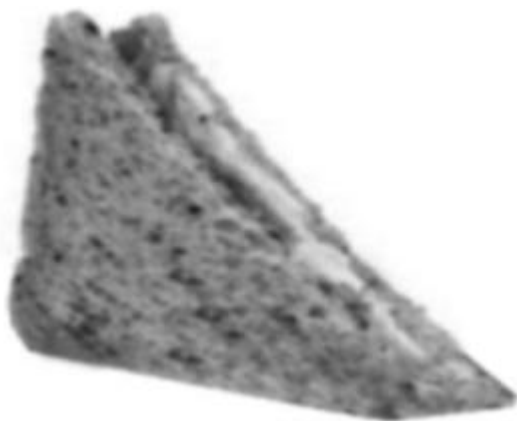
shown in the figure. Considering the front side of bird house as right angled triangle PQR , right angled at R , answer the following questions.



1. Find the length of PR
2. Find the value of $\frac{1+\tan^2 Q}{1+\cot^2 Q}$
3. Find the value of $\sin Q - \cos P$

QUESTION -3

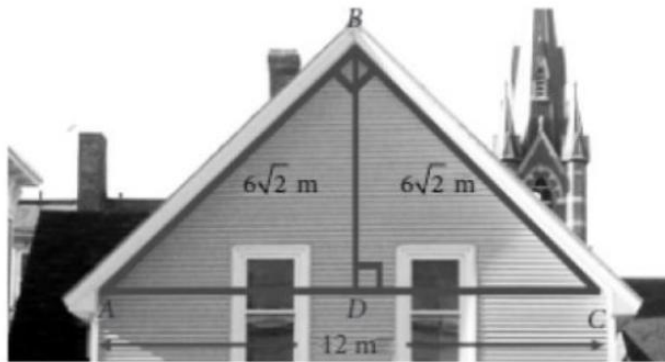
Ritu's daughter is feeling so hungry and so thought to eat something. She looked into the fridge and found some bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found that it forms a right angled triangle, with sides 4 cm , 8 cm . On the basis of above information, answer the following questions.



1. Find $\sin M \times \cos K$
2. Find the measure of $\angle KML$ and $\angle MKL$
3. Evaluate $\cos(M + K) - \tan K$

QUESTION -4

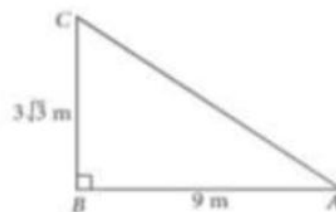
Aanya and her father go to meet her friend Juhi for a party. When they reached to Juhi's place, Aanya saw the roof of the house, which is triangular in shape. If she imagined the dimensions of the roof as given in the figure, If D is the midpoint of AC then answer the following questions :



1. The measure of $\angle A$ is ____
2. Find the value of $\tan^2 C - \tan^2 A$
3. Find the value of $\sin A + \cos C$

QUESTION -5

Three friends- Megha, Kristi and Joya are playing hide and seek in a park. Megha and Kristi hide in the shrubs and Joya have to find both of them. If Position of three friends are at A, B and C respectively as shown in the figure and forms a right angled triangle such that $AB = 9\text{ m}$, $BC = 3\sqrt{3}\text{ m}$ and $\angle B = 90^\circ$, then answer the following questions:



1. Find measure of $\angle C$
2. Evaluate $\tan^2 A + \cot^2 C$

3. Find the distance between the friends A and C.

ANSWERS

1. c) 1
2. a) 0
3. d) 90°
4. d) -6
5. c) 10
6. b) 45°
7. d) 2
8. c) 7
9. b) $\tan^2 \theta$
10. b) $625/168$
11. c) 1
12. a) $\sin 30^\circ$
13. d) 0
14. a) 3
15. b) $\tan 30^\circ$
16. c) 2
17. b) 2
18. a) $\sqrt{2}$
19. d) *any angle*
20. a) 0
21. a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

22. b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

23. d) Assertion (A) is false but reason (R) is true.

24. b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

25. d) Assertion (A) is false but reason (R) is true.

26. $\sqrt{2}$

27. $A = 45^\circ, B = 45^\circ$

28. $A = 45^\circ, B = 15^\circ$

29. $-\frac{1}{2}$

30. $\frac{8}{3}$

31. $x = 20^\circ$

33. $\theta = 60^\circ$

34. 0

35. $\frac{1}{7}$

43. $\frac{1}{4}$

44. $\frac{7}{48}$

45. 2

ANSWERS - CASE STUDIES/ SOURCE BASED INTEGRATED QUESTIONS

QUESTION 1

1. 2 km

2. $\frac{9}{8}$

3. 1.4 km

QUESTION 2

1. 5 cm

2. 0

3. $\frac{25}{144}$

QUESTION 3

1. $\frac{16}{64}$

2. $30^\circ, 60^\circ$

3. $-\sqrt{2}$

QUESTION 4

1. 60°

2. 2

3. $\sqrt{2}$

QUESTION 5

1. 30°

2. 0

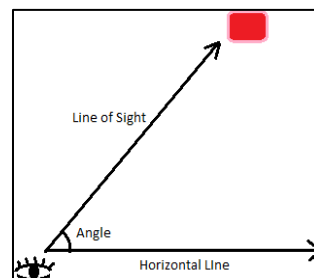
3. $\sqrt{108}$

APPLICATION OF TRIGNOMETRY

KEY CONCEPTS

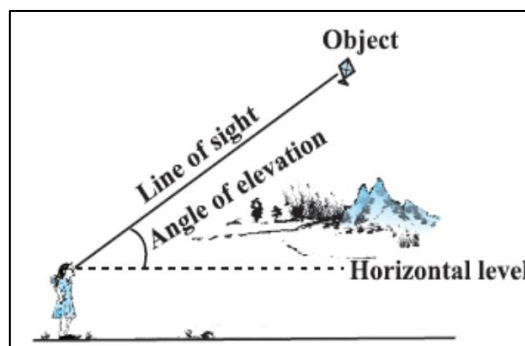
LINE OF SIGHT

Line joining the object to the eye of the observer is called the line of sight.



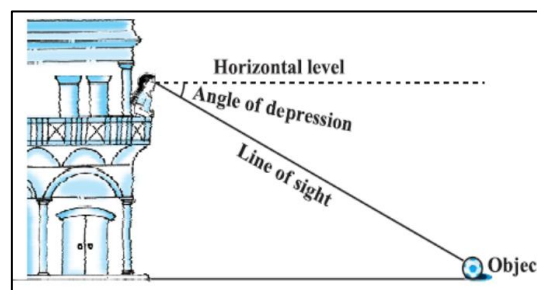
ANGLE OF ELEVATION

When an observer looks at an object which is at a greater height than the observer, the angle formed between the line of sight and the horizontal line from the observer's eye is called an angle of elevation.



ANGLE OF DEPRESSION

When an observer looks at an object which is at a lesser height than the observer, the angle formed between the line of sight and the horizontal line from the observer's eye is called an angle of depression.



Steps in Solving Questions

Step 1. In order to solve this problem, first write all the information given into the problem

Step 2. Convert the above statements into the diagram using following

- Represent *towers, poles, buildings and houses* standing on ground by vertical straight line segments.
- Represent *ground, rivers, rocks, boy, men(if height not given), car, and distances between two objects* by horizontal straight line segments.
- The angle of elevation or the angle of depression is always the angle between the line of sight and the horizontal line.

Step 3. Solve the above problem using trigonometric ratios using thus right triangles so formed.

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \quad \tan 45^\circ = 1 \quad \tan 60^\circ = \sqrt{3}$$

COMPETENCY BASED QUESTIONS (1 Marks)

1. A pole 6 m high casts a shadow $2\sqrt{3}\text{ m}$ long on the ground, then the Sun's elevation is
a) 30° b) 45° c) 60° d) 90°
2. The angle of elevation of the top of a tower is 30° . If the height of the tower is doubled, then the angle of elevation of its top will
a) also get doubled b) will get halved c) will be less than 60° d) none of these
3. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is :
a) 30° b) 45° c) 60° d) 90°
4. The tops of the poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an angle of 30° with the horizontal, the $l =$
a) 26 m b) 16 m c) 12 m d) 10 m
5. A tower is 50 m high. When the sun's altitude is 45° then what will be the length of its shadow?
a) 50 m b) $50\sqrt{3}\text{ m}$ c) $\frac{50}{3}\sqrt{3}\text{ m}$ d) 100 m
6. If a tower is 30 m height, casts a shadow $10\sqrt{3}\text{ m}$ long on the ground, then the angle of elevation of the sun is:
a) 30° b) 45° c) 60° d) 90°

7. If the length of the shadow of a tower is increasing, then the angle of elevation of the sun -

- a) is also increasing b) is decreasing c) remains unaffected d) Don't have any relation

8. A ladder 15 *metres* long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the distance between foot of ladder and wall is - ($\sqrt{2}1.414, 3 = 1.7$)

- a) 7.5 m b) 7.7 m c) 10.60 m d) 12.75 m

9. If the height of a tower and the distance of the point of observation from its foot, both, are increased by 10%, then the angle of elevation of its top -

- a) increases b) decreased c) remains same d) none of these

10. If the height and length of the shadow of a man are equal, then the angle of elevation of the sun is,

- a) 30° b) 45° c) 60° d) 90°

ASSERTION and REASONING QUESTIONS (1 Marks)

DIRECTION: In the following questions, a statement of **assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

11. Assertion: When we move towards the object, angle of elevation decreases.

Reason: As we move towards the object, it subtends large angle at our eye than before

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

c) Assertion (A) is true but reason (R) is false.

d) Assertion (A) is false but reason (R) is true.

12. Assertion: If the angle of elevation of Sun, above a perpendicular line (tower) decreases, then the shadow of tower increases.

Reason: It is due to decrease in slope of the line of sight.

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

13. Assertion: If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is decreasing.

Reason: The line the angle formed between the line of sight and the horizontal line from the observer's eye is called an angle of elevation

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

14. Assertion: The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Reason: Trigonometric ratios are used to find height or length of an object or distance between two distant

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

15. Assertion: If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°

Reason: The angle of depression. of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level.

- a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- c) Assertion (A) is true but reason (R) is false.
- d) Assertion (A) is false but reason (R) is true.

VERY SHORT ANSWER QUESTIONS (2 Marks)

- 16.** Two pillars of equal heights are on either side of a road, which is hundred metres wide. The angles of elevation of the tops of the pillars are 60° and 30° at a point on the road between the pillars. Find the position of the point between the pillars?
- 17.** A kite is flying at a height of $50\sqrt{3}m$ from the horizontal. It is attached with a string and makes an angle 60° with the horizontal. Find the length of the string.
- 18.** The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30° . Find the height of the tower.

19. Two poles of height 6m and 11m stands vertically on the ground. If the distance between their feet is 12m. Find the distance between their tops.
20. The length of the shadow of a pillar and its height are in the ratio 1:1. Find the angle of elevation of the source of light

SHORT ANSWER QUESTIONS (3 Marks)

21. If the horizontal distance of the boat from the bridge is 25 m and the height of the bridge is 25 m, then find the angle of depression of the boat from the bridge.
22. The angle of elevation of a tower at a point is 45° . After going 40 m towards the foot of the tower, the angle of elevation of the tower becomes 60° . Find the height of the tower
23. The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it was 60° . Find the height of the tower.
24. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.
25. The upper part of a tree broken over by the wind makes an angle of 30° with the ground and the distance of the root from the point where the top touches the ground is 25 m. What was the total height of the tree?

LONG ANSWER QUESTIONS (5 Marks)

26. Two ships are there in the sea on either side of a lighthouse in such a way that the ships and the base of the lighthouse are in the same straight line. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° . If the height of the lighthouse is 200m, find the distance between the two ships.

27. From the top of a 300 *metre* high light-house, the angles of depression of two ships, which are due south of the observer and in a straight line with its base, are 60° and 30° .Find their distance apart?

28. A Statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45° .Find the height of the pedestal?

29. An aero plane, at an altitude of 1200 *m*, finds that two ships are sailing towards it in the same direction. The angles of depression of the ships as observed from the aero plane are 60° and 30° respectively. Find the distance between the two ships?

30. The angles of depression of the top and bottom of an 8 m tall building from top of a multi-storied building are 30° and 45° , respectively. Find the height of multi-storied building and distance between two buildings

31. A 1.5 *m* tall boy is standing at some distance from a 30 *m* tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

32. An airplane, when 3 *km* high, passes vertically above another airplane at an instant when the angle of elevation of two airplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes.

33. The angle of elevation of the top of a tower 30 *m* high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30° . Find the distance between the two towers and also the height of the other tower.

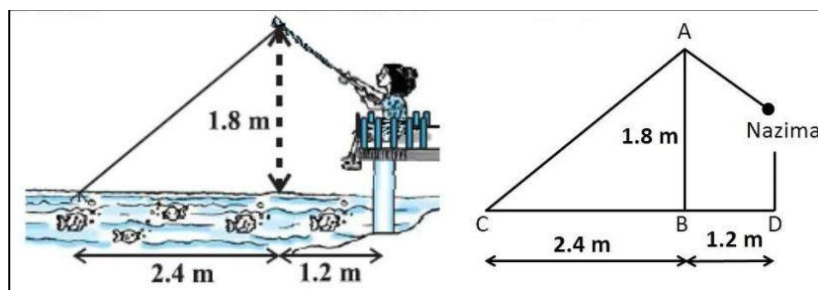
34. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.
35. The angle of elevation of a cloud from a point h metres above the surface of a lake is θ and the angle of depression of its reflection in the lake is ϕ . Find the height of the cloud above the lake.

CASE STUDIES/ SOURCE BASED INTEGRATED **QUESTIONS**

(4 Marks)

QUESTION -1

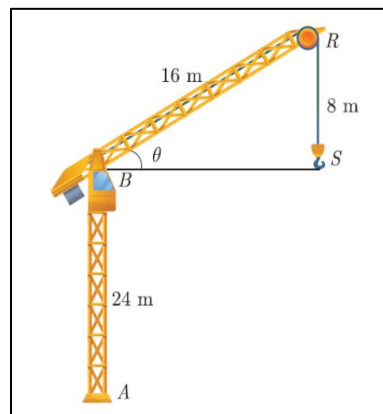
Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. She is pulling the string at the rate of 5 cm per second. Nazima's friend observe her position and draw a rough sketch by using A, B, C and D positions of tip, point directly under the tip of the rod, fish and Nazima's position (see the below figure). Assuming that her string (from the tip of her rod to the fly) is taut, answer the following questions:



1. What is the length AC?
2. What is the length of string after 12 seconds?
3. What will be the horizontal distance of the fly from her after 12 seconds?

QUESTION -2 Tower cranes are a common fixture at any major construction site. They're pretty hard to miss – they often rise hundreds of feet into the air, and can reach out far. The construction crew uses the tower crane to steel, concrete, large tools like acetylene torches generators, and a wide variety of other building materials

A crane stands on a level ground. It is represented by a tower AB, of height 24 m and a jib BR. The jib is of length 16 m and can rotate in a vertical plane about B. A vertical cable, RS, carries a load S. The diagram shows current position of the jib, cable and load.



just as
lift
and

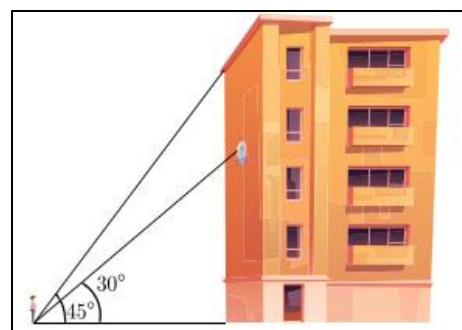
jib

On the basis of the above information, answer any four of the following questions:

1. What is the angle that the jib, BR, makes with the horizontal?
2. Now the jib BR, has been rotated and the length RS is increased. The load is now on the ground at a point 8 m from A. What is the angle through which the jib has been rotated ?
3. What is the length by which RS has been increased?

QUESTION -3

A clinometer is a tool that is used to measure the angle of elevation, or angle from the ground, in a right - angled triangle. We can use a clinometer to measure the height of tall things that you can't possibly reach to the top of, flag poles, buildings, trees. Shailesh got a clinometer from school lab



and started the measuring elevation angle in surrounding. He saw a building on which society logo is painted on wall of building. From a point P on the ground level, the angle of elevation of the roof of the building is 45° . The angle of elevation of the center of logo is from same point. The point P is at a distance of 24 m from the base of the building.

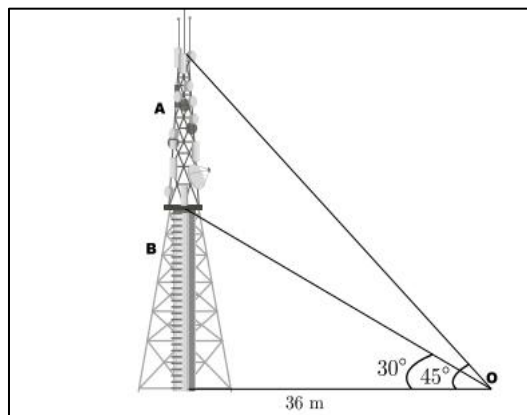
On the basis of the above information, answer any four of the following questions:

- What is the height of the building logo from ground?
- What is the aerial distance of the point P from the top of the building?

- If the point of observation P is moved 16 m towards the base of the building, find the angle of elevation θ of the logo on building.

QUESTION -4

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure, including microwave dishes. They are among the tallest human-made structures. There are 2 main types: guyed and self-supporting structures.



On a similar concept, a radio station tower was built in two sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 m. From point O, the angle of elevation of the top of section B is and the angle of elevation of the top of section A is.

On the basis of the above information, answer any four of the following questions:

2. What is the length of the wire structure from the point O to the top of section A?
3. What is the height of the section A?
4. What is the angle of depression from top of tower to point O?

ANSWERS

6. c) 60°

7. c) will be less than 60°

8. a) 30°

9. c) 12 m

10. a) 50 m

11. c) 60°

- 12.** b) is decreasing
- 13.** a) 7.5 m
- 14.** c) remains same
- 15.** b) 45°
- 16.** d) Assertion (A) is false but reason (R) is true.
- 17.** a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
- 18.** b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- 19.** b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- 20.** b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
- 21.** 25 m
- 22.** 100 m
- 23.** $10\sqrt{3}\text{m}$
- 24.** 13 m
- 25.** 45°
- 26.** 45°
- 27.** 94.8 m
- 28.** $25\sqrt{3}\text{m}$
- 29.** $20(\sqrt{31})\text{m}$
- 30.** 43.3 m

31. 315.33 m

32. $200\sqrt{3}m$

33. $0.8(\sqrt{31})m$

34. $800\sqrt{3}m$

35. $12 + 4\sqrt{3}m, 12 + 4\sqrt{3}m$

36. $19\sqrt{3}m$

37. 1268 m

38. $10\sqrt{3}m, 10 m$

39. 2.5 m

40.
$$\frac{h(\tan\phi + \tan\theta)}{\tan\phi - \tan\theta}$$

ANSWERS - CASE STUDIES/ SOURCE BASED INTEGRATED QUESTIONS

QUESTION 1

1. 3 m

2. 2.4 m

3. 2.78 m

QUESTION 2

1. 30°

2. 30°

3. $8(\sqrt{32})m$

QUESTION 3

1. $8\sqrt{3}m$

2. $24\sqrt{2}m$

3.

60°

QUESTION 4

1.

$36\sqrt{2}m$

2.

$12(3 - \sqrt{3})m$

3.

45°

CIRCLES

Circle: A circle is a collection of all points in a plane which are at a constant distance from a fixed point.

Centre: The fixed point is called the centre.

Radius: The constant distance from the centre is called the radius.

Chord: A line segment joining any two points on a circle is called a chord.

Diameter: A chord passing through the centre of the circle is called diameter. It is the longest chord.

Tangent: When a line meets the circle at one point or two coinciding The line is known as a tangent.

The tangent to a circle is perpendicular to the radius through the point of contact.

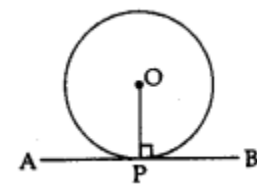
$$\Rightarrow OP \perp AB$$

lengths of the two tangents from an external point to a circle are equal.

$$= PB$$

Length of Tangent

Segment PB and PA are normally called the lengths of tangents from outside point P.



The
circle
 $\Rightarrow AP$

Properties of Tangent to Circle

Theorem 1: Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Given: XY is a tangent at point P to the circle with centre O.

To prove: $OP \perp XY$

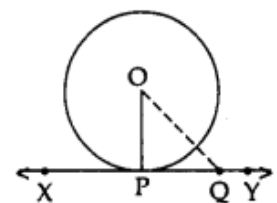
Construction: Take a point Q on XY other than P and join

Proof: If point Q lies inside the circle, then XY will become secant and not a tangent to the circle

$$> OP$$

This happens with every point on line XY except point P. OP is the shortest of all the distances of point O the points of XY

$OP \perp XY$...[Shortest side is the perpendicular]



OQ
a
OQ

Theorem 2: Prove that the lengths of tangents drawn from an external point to a circle are equal

Given: PT and PS are tangents from an external point P to the circle with centre O.

To prove: $PT = PS$

Construction: Join O to P, T and S.

Proof: In $\triangle OTP$ and $\triangle OSP$.

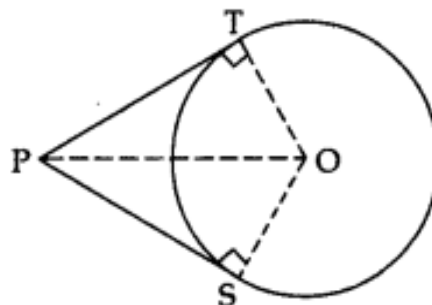
OS ...[radii of the same circle]

OP ...[common]

$\angle OSP$...[each 90°]

$\triangle OSP$...[R.H.S.]

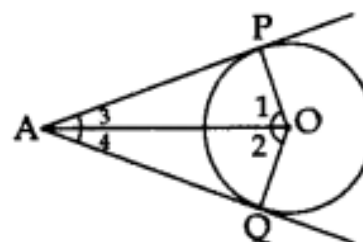
PS ...[c.p.c.t.]



OT =
OP =
 $\angle OTP$
 $\triangle OTP$
PT =

Note: If two tangents are drawn to a circle from an external point, then:

- They subtend equal angles at the centre i.e., $\angle 1 = \angle 2$.
- They are equally inclined to the segment joining the centre to that point i.e.,
 $\angle 3 = \angle 4$.
 $= \angle OAQ$



$\angle OAP$

Multiple choice type questions:

1. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is

(A) 3 cm (B) 9 cm (C) 6 cm (D) 1 cm

2. In Fig. 9.3, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to

(A) 62.5° (B) 45° (C) 35° (D) 55°

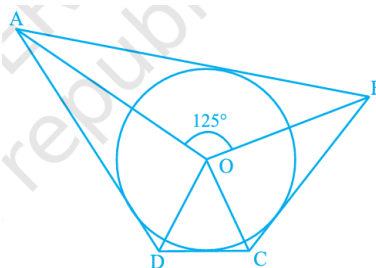


Fig. 9.3

3. In Fig. 9.4, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to
 (A) 65° (B) 50° (C) 60° (D) 40°

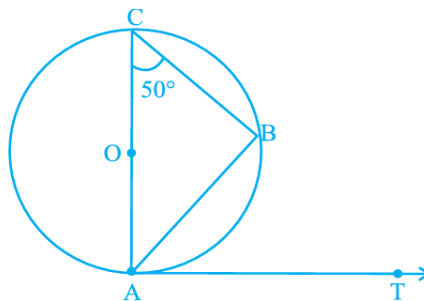


Fig. 9.4

4. In Fig. 9.7, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then $\angle OAB$ is equal to
 (A) 25° (B) 40° (C) 30° (D) 50°

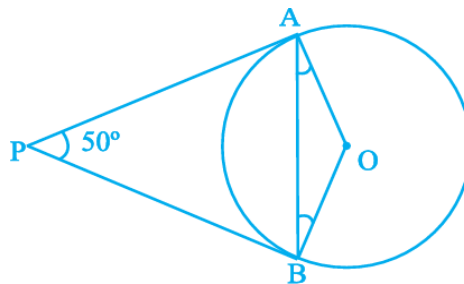


Fig. 9.7

5. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is
 (A) 4 cm (B) 5 cm (C) 6 cm (D) 8 cm

Very short answer type questions:

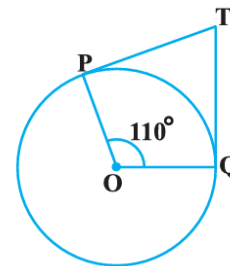
1. Fill in the blanks :

- (i) A tangent to a circle intersects it in _____ point (s).
- (ii) A line intersecting a circle in two points is called a _____.
- (iii) A circle can have _____ parallel tangents at the most.
- (iv) The common point of a tangent to a circle and the circle is called _____.

2. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12$ cm. find the Length of PQ.

3. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

4. In Fig., if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then find $\angle PTQ$.



Short answer type questions:

1. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.
2. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.
3. If a , b , c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by $r = (a + b - c)/2$

Long answer type questions:

(SIMILAR QUESTIONS: 1 & 2)

1. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that $AB + CD = AD + BC$

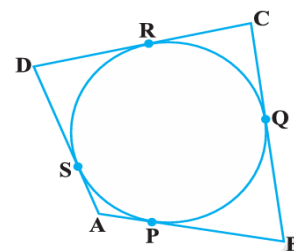


Fig. 10.12

2. Prove that the parallelogram circumscribing a circle is a rhombus.

3. In Fig. 10.13, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.

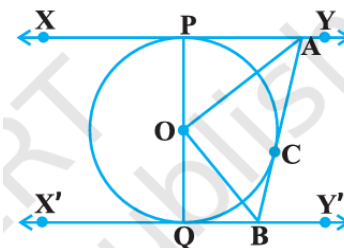


Fig. 10.13

4. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

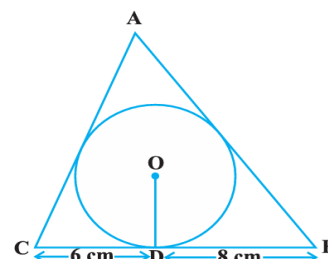


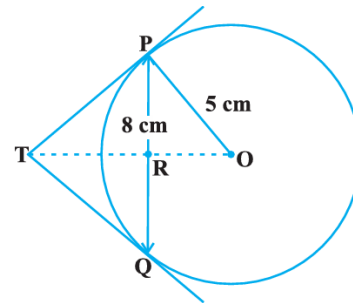
Fig. 10.14

5. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2} (BC + CA + AB)$

6. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2 \angle OPQ$.

HOTS:

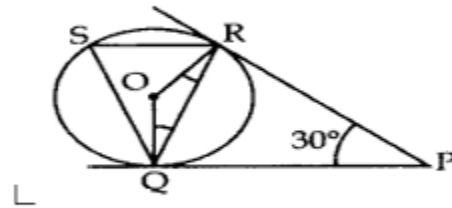
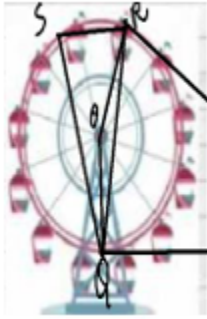
1. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see Fig.). Find the length TP.



CASE STUDY 1:

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.



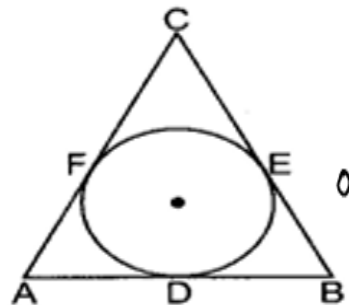
In the given figure

1. Find $\angle ROQ$.
2. Find $\angle RQP$.
3. Find $\angle RSQ$.

Or Find $\angle ORP$.

CASE STUDY 2:

Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a $\triangle ABC$, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.



1. Find the length of AD.
 2. Find the Length of BE.
 3. If radius of the circle is 4cm, Find the area of $\triangle OAB$
- Or Find area of $\triangle ABC$.

AREA RELATED TO CIRCLE

SUMMARY

Area of the sector of angle $\theta = \frac{\theta}{360} \pi r^2$, where r is the radius of the circle and θ is the angle in degrees.

Note: a) Area of semicircle = πr^2 ($\theta=180$)

b) Area of quadrant = $\frac{1}{4} \pi r^2$ ($\theta=90$)

Length of an arc of a sector of angle $\theta = \frac{\theta}{360} 2\pi r$,

where r is the radius of circle and

angle of the sector in degrees

segment APB = Area of the sector OAPB – Area of Δ OAB

$$= \frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin \theta$$

if triangle is right angled then area of triangle = $\frac{1}{2} \times b \times h$

the major sector OAQB = πr^2 - Area of the minor sector

segment AQB = πr^2 - Area of the minor segment APB

MULTIPLE CHOICE QUESTIONS

1. The length of an arc of a circle of radius 12 cm, subtended by an angle of 45° at the center is: a) 3π cm b) 6π cm c) 9π cm d) 12π cm **ANS -B**

2. The area of the sector of a circle of radius 5 cm, subtended by an angle of 60° at the center is:

a) $\frac{5\pi}{3}$ cm² b) $\frac{25\pi}{3}$ cm² c) $\frac{10\pi}{6}$ cm² d) $\frac{100\pi}{6}$ cm² **ANS -C**

3. The length of an arc of a circle of radius 21 cm, subtended by an angle of 120° at the center is: a) 22 cm b) 44 cm c) 88 cm d) 66 cm **ANS - B**

4. The area of a quadrant of a circle of radius 8 cm is: **ANS - A**

a) 16π cm² b) 32π cm² c) 64π cm² d) 128π cm²

5. The area of a circle circumscribing a square of side 14 cm is.

a) 77π cm² b) 308π cm² c) 154π cm² d) 196π cm² **ANS-B**

6. Find the area of a circle's sector if the radius is 20 units and the length of the arc is 8 units. a)

40 sq units b) 90 sq units c) 60 sq units d) 80 sq units **ANS -D**

7. The area of the square that can be inscribed in a circle of radius 8 cm is

(a) 256 cm² (b) 64 cm² (c) 642 cm² (d) 128 cm² **ANS-D**

8. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/h?

(a) 300 (b) 400 (c) 500 (d) 450 **ANS - C**

9. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20m $\times$ 16m, then the area of the field in which the cow can graze is:

(a) 154 m² (b) 156 m² (c) 158 m² (d) 160 m² **ANS -A**

10. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, then the cost of making the design at the rate of Rs. 0.35 per cm² is:

(a) Rs.146.50 (b) Rs.148.75 (c) Rs.152.25 (d) Rs.154.75 **ANS -B**

11. The perimeter of a sector of a circle of radius 15 cm is 80 cm. Find the area of the sector. **Ans 375 cm²**

12. The length of the minute hand of a clock is 6cm. Find the area swept by it when it moves from 7:05 p.m. to 7:40 p.m. Ans 66 cm^2

13. Find the circumference and area of circle of radius 4.2 cm.

Ans

Circumference = 26.4 cm and Area = 55.44 cm^2

14. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc Ans 22cm

(ii) area of the sector formed by the arc Ans 231 cm^2

(iii) area of the segment formed by the corresponding chord Ans $231 - \frac{441\sqrt{3}}{4}$

15. The length of the minute-hand of a clock is 14 cm. Find the area swept by this minute-hand in 5 minutes. Ans $\frac{154}{3} \text{ cm}^2$

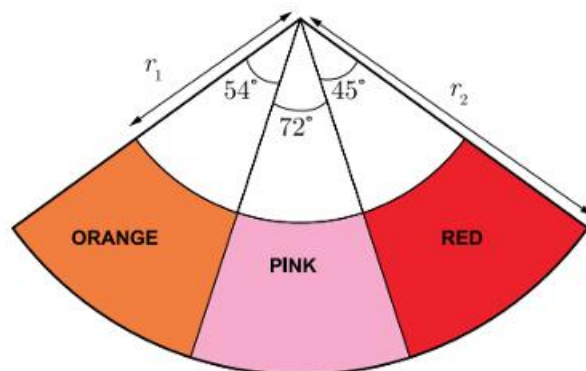
16. To warn ships for underwater rocks, a lighthouse throws a red coloured light over a sector of central angle 80° up to a distance of 16.5 km. Find the area of the sea over which the ships are warned. Ans 189.97 km^2

Case based question

17. Japanese fans are made of paper on a bamboo frame, usually with a design painted on them. A Japanese Fan symbolises friendship, respect and good wishes and are given on special occasions, as well as to help cool you down in hot weather. The fan is an immediately recognizable icon of Japanese culture. Today they remain an important artistic medium and stylish fashion accessory.



Lavanya hold a Japanese folding fan in her hand as shown in figure. It is shaped like a sector of a circle. The inner and outer radii are 14 cm and 21 cm. The fan has three colour material.



(i) If the region

containing the pink colour makes an angle of $\theta_2 = 72^\circ$ at the centre, then find the area of the region having pink colour. Ans = $616/5$

(ii) If the region containing the orange colour makes an angle of $\theta_1 = 54^\circ$ at the centre, then find the area of the region having orange colour. Ans $1848/20$

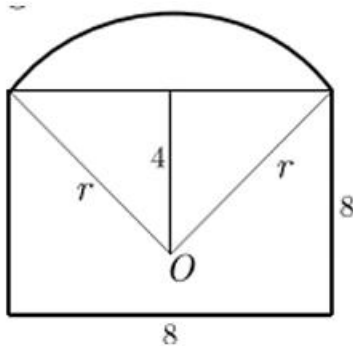
(iii) If the region containing the red colour makes an angle of $\theta_3 = 45^\circ$ at the centre, then find the perimeter of the region having red colour. Ans 39cm

OR

Find the area of the region having radius 14 cm.

Ans 616 cm^2

18. Barn : A barn is an agricultural building usually on farms and used for various purposes. A barn refers to structures that house livestock, including cattle and horses, as well as equipment and fodder, and often grain. Ramkaran want to build a barn at his farm. He has made a design for it which is shown above. Here roof is arc of a circle of radius r at centre O .



What is the value of radius of arc? Ans $4\sqrt{2}$

What is the curved width of roof? Ans $2\sqrt{2}\pi$

area of cross section of barn? Ans $48 + 8\pi$

OR

If the length of the barn is 12 meters, what is the curved surface area of roof?

$24\sqrt{2}\pi$

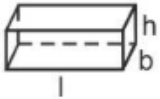
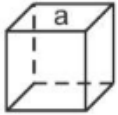
CHAPTER-13 SURFACE AREA AND VOLUME

Important Formula:

(i)

(ii)

(iii) What is

Name	Figure	Curved Surface area	Total surface area	Volume
Cuboid		$2h(l + b)$	$2(lb + bh + lh)$	lbh
Cube		$4a^2$	$6a^2$	a^3
Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$
Right circular cone		πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
Sphere		—	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$
Hemi-sphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$

Multiple Choice Questions (MCQ's)

1. If a marble of radius 2.1 cm is put into a cylindrical cup full of water of radius 5cm and height 6 cm, then how much water flows out of the cylindrical cup?

(A) 38.8 cm^3 (B) 55.4 cm^3 (C) 19.4 cm^3 (D) 471.4 cm^3 Ans -A

2. The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is

(A) 9.7 cm^3 (B) 77.6 cm^3 (C) 58.2 cm^3 (D) 19.4 cm^3 Ans -D

3. A cylindrical pencil sharpened at one edge is the combination of

(A) a cone and a cylinder (B) frustum of a cone and a cylinder

(B) a hemisphere and a cylinder (D) two cylinders. Ans A

4. A surahi is the combination of

(A) a sphere and a cylinder (B) a hemisphere and a cylinder

(C) two hemispheres (D) a cylinder and a cone Ans A

5. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is

(A) 0.36 cm^3 (B) 0.34 cm^3 (C) 0.33 cm^3 (D) 0.32 cm^3 Ans -A

6. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

(A) $4\pi r^2$ (B) $6\pi r^2$ (C) $3\pi r^2$ (D) $8\pi r^2$ Ans A

7. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is

(A) 3 : 4 (B) 4 : 3 (C) 9 : 16 (D) 16 : 9 Ans -D

8. The radius of the base of a cone is 5 cm and its height is 12 cm. Its curved surface area is

(a) $60\pi \text{ cm}^2$ (b) $65\pi \text{ cm}^2$ (c) $30\pi \text{ cm}^2$ (d) None of these Ans- B

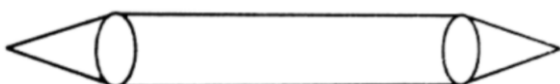
9. The diameter of the base of a cone is 42 cm and its volume is 12936 cm^3 . Its height is

(a) 35 cm (b) 21 cm (c) 28 cm (d) 14 cm Ans C

10. The radii of the base of a cylinder and a cone are in the ratio 3 : 4. If their heights are in the ratio 2 : 3, the ratio between their volumes is

(a) 9 : 8 (b) 3 : 4 (c) 8 : 9 (d) 4 : 3 Ans A

11. The shape of a gilli, in the gilli-danda game (fig. above), is a combination of



(a) two cylinders (b) a cone and a cylinder

(c) two cones and a cylinder (d) two cylinders and a cone Ans C

12. The total surface area of a hemisphere of radius 7 cm is

(a) $588\pi \text{ cm}^2$ (b) $392\pi \text{ cm}^2$ (c) $147\pi \text{ cm}^2$ (d) $98\pi \text{ cm}^2$ Ans C

13. If each edge of a cube is increased by 50%, the percentage increase in the surface area is

(a) 50% (b) 75% (c) 100% (d) 125% Ans D

14. The volume of largest sphere that can be cut out from a cube of edge 4.2 cm.

(a) 9.7 cm^3 (b) 77.6 cm^3 (c) 38.8 cm^3 (d) 19.4 cm^3 Ans C

15. The barrel of a fountain pen, cylindrical in shape, is 7cm long and 5mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a a bottle of ink containing one fifth of a litre?

(a) 580000 (b) 380000 (c) 480000 (d) 290000 Ans C

Direction for questions 16 to 20: In question numbers 16 to 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

16. Assertion : Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2 .

Reason : If r be the radius and h be the height of the cylinder, then total surface area of cylinder = $2\pi rh + 2\pi r^2$

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d) Assertion (A) is false but Reason (R) is true.

Ans A

17. Assertion : If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.

Reason : If r be the radius and h the slant height of the cone, then slant height $= \sqrt{h^2 + r^2}$

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

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Ans D

18. Assertion : The radii of two cones are in the ratio 2:3 and their volumes in the ratio 1:3. Then the ratio of their heights is 3:4

Reason : Volume of the cylinder $= \pi r^2 h$

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

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Ans B

19. Assertion : If the radius of a cone is halved and volume is not changed, then height remains same.

Reason : If the radius of a cone is halved and volume is not changed then height must become four times of the original height.

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Ans D

20. Assertion: The volume of two sphere are in the ratio 27: 8 then their surface area are in the ratio 9:4 .

Reason: Volume of hemisphere= $\frac{2}{3}\pi r^3$ and it's total surface area= $3\pi r^2$

- a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
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Ans B

VSA TYPE QUESTION EACH 2 MARKS

1. Two cubes of volume 64 cm^3 are joined end to end. Find the surface area of the Resulting Cuboid.

Ans 160 cm^2

2. A solid is in the shape of a cone surmounted on a hemisphere, the radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Ans $\frac{1001}{6}$ or 166.83

3. A cone of height 20 cm and radius of base 5 cm is made up modelling clay. A child. reshapes it in the form of a sphere. Find the diameter of the sphere.

Ans 10 cm

4. A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8:5, show that the ratio between the radius of their bases to their height is 3:4.

5. The volume of a hemisphere is 2425.5 cm^3 . Find its curved surface area?

Ans 693 cm^2

6. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid

Ans $d = 7 \text{ cm}$ and $\text{area} = 332.5 \text{ cm}^2$

SA TYPE QUESTION EACH 3 MARKS

1. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2 ? Ans- 18 cm^2

2. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π . Ans $\pi \text{ cm}^3$

3. A metallic sphere of radius 10.5 cm is melted and then recast into smaller cones, each of radius 3.5 cm and height 3 cm. How many cones are obtained?

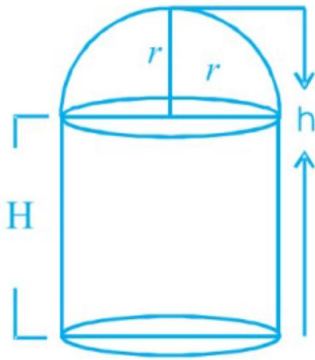
Ans 126

Long Answer Type Questions:

1. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. (311.04 cm^2)

2. A building is in the form of a cylinder surmounted by a hemispherical dome (see Fig). The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains 67121 m^3 of air.

Ans (60 m)

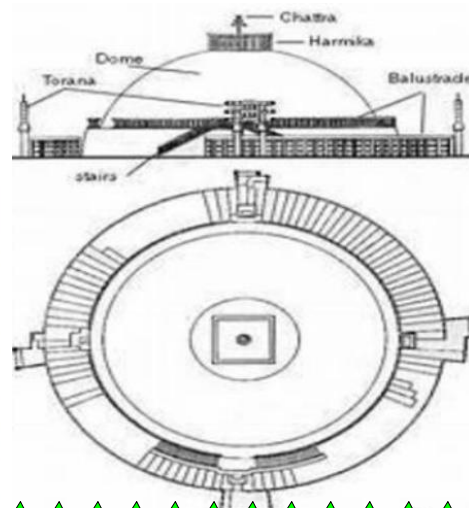


3. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at Rs 0.05 per dm^2 Ans cost = Rs 2250

4. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?
Ans vol= 148.5 and canvas cloth = 80.61 cm^2

CASE STUDY BASED

The Great Stupa at Sanchi is one of the oldest stone structures in India, and an important monument of Indian Architecture. It was originally commissioned by the emperor Ashoka in the 3rd century BCE. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. (Take $\pi = 22/7$)

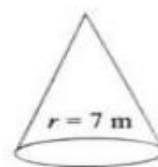
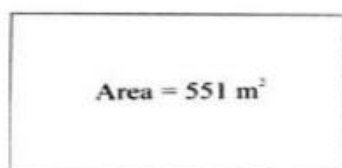
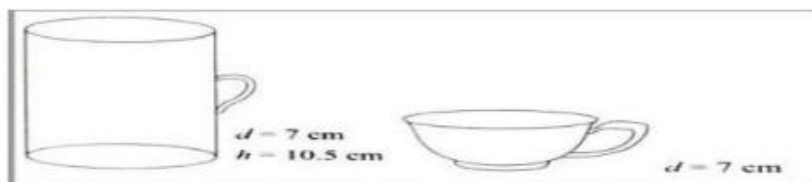


- (i) Calculate the volume of the hemispherical dome if the height of the dome is 21 m. Ans 19404 cm^3
- (ii) Find the cloth require to cover the hemispherical dome if the radius of its base is 14m. Ans 1232 m^2
- (iii) Find the total surface area of the combined figure i.e. hemispherical dome with radius 14m and cuboidal shaped top with dimensions 8m × 6m × 4m Ans 1344 cm^2

OR

Find the volume of combined figure Ans $\frac{17824}{3} = 5941.33$

2. Adventure camps are the perfect place for the children to practice decision making for themselves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below. After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area 551m^2 . Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to 1m^2 , the students put the tents. The radius of the tent is 7m.



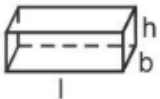
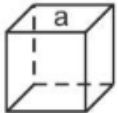
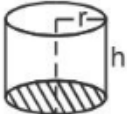
- (i) Find the volume of cylindrical cup. Ans 404.25 cm^3
- (ii) Find the volume of hemispherical cup. Ans 89.83 cm^3
- (iii) Which container had more juice and by how much?
Ans Cylindrical cup by 314.42 cm^3

OR

Find the height of the conical tent prepared to accommodate four students
Ans 24 cm

CHAPTER-13 SURFACE AREA AND VOLUME

Important Formula:

Name	Figure	Curved Surface area	Total surface area	Volume
Cuboid		$2h(l + b)$	$2(lb + bh + lh)$	lbh
Cube		$4a^2$	$6a^2$	a^3
Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$
Right circular cone		πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
Sphere		—	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$
Hemi-sphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$

Multiple Choice Questions (MCQ's)

- If a marble of radius 2.1 cm is put into a cylindrical cup full of water of radius 5cm and height 6 cm, then how much water flows out of the cylindrical cup?
(A) 38.8 cm^3 (B) 55.4 cm^3 (C) 19.4 cm^3 (D) 471.4 cm^3 Ans -A
- The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is
(A) 9.7 cm^3 (B) 77.6 cm^3 (C) 58.2 cm^3 (D) 19.4 cm^3 Ans -D
- A cylindrical pencil sharpened at one edge is the combination of
(A) a cone and a cylinder (B) frustum of a cone and a cylinder
(C) a hemisphere and a cylinder (D) two cylinders. Ans -A

4. A surahi is the combination of

- (A) a sphere and a cylinder (B) a hemisphere and a cylinder
(C) two hemispheres (D) a cylinder and a cone

Ans A

5. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of entire capsule is 2 cm. The capacity of the capsule is

- (A) 0.36 cm^3 (B) 0.34 cm^3 (C) 0.33 cm^3 (D) 0.32 cm^3

Ans -A

6. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

- (A) $4\pi r^2$ (B) $6\pi r^2$ (C) $3\pi r^2$ (D) $8\pi r^2$

Ans A

7. Volumes of two spheres are in the ratio 64:27. The ratio of their surface areas is

- (A) 3 : 4 (B) 4 : 3 (C) 9 : 16 (D) 16 : 9

Ans -D

8. The radius of the base of a cone is 5 cm and its height is 12 cm. Its curved surface area is

- (a) $60\pi \text{ cm}^2$ (b) $65\pi \text{ cm}^2$ (c) $30\pi \text{ cm}^2$ (d) None of these

Ans- B

9. The diameter of the base of a cone is 42 cm and its volume is 12936 cm^3 . Its height is

- (a) 35 cm (b) 21 cm (c) 28 cm (d) 14 cm

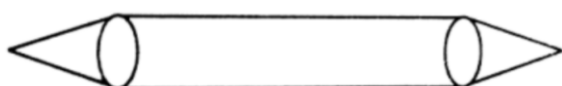
Ans C

10. The radii of the base of a cylinder and a cone are in the ratio 3 : 4. If their heights are in the ratio 2 : 3, the ratio between their volumes is

- (a) 9 : 8 (b) 3 : 4 (c) 8 : 9 (d) 4 : 3

Ans A

11. The shape of a gilli, in the gilli-danda game (fig. above), is a combination of



(a) two cylinders (b) a cone and a cylinder

(c) two cones and a cylinder (d) two cylinders and a cone Ans C

12. The total surface area of a hemisphere of radius 7 cm is

(a) $588\pi \text{ cm}^2$ (b) $392\pi \text{ cm}^2$ (c) $147\pi \text{ cm}^2$ (d) $98\pi \text{ cm}^2$ Ans C

13. If each edge of a cube is increased by 50%, the percentage increase in the surface area is

(a) 50% (b) 75% (c) 100% (d) 125% Ans D

14. The volume of largest sphere that can be cut out from a cube of edge 4.2 cm.

(a) 9.7 cm^3 (b) 77.6 cm^3 (c) 38.8 cm^3 (d) 19.4 cm^3 Ans C

15. The barrel of a fountain pen, cylindrical in shape, is 7cm long and 5mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a a bottle of ink containing one fifth of a litre?

(a) 580000 (b) 380000 (c) 480000 (d) 290000 Ans C

Direction for questions 16 to 20: In question numbers 16 to 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.

16. Assertion : Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2 .

Reason : If r be the radius and h be the height of the cylinder, then total surface area of cylinder = $2\pi rh + 2\pi r^2$

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Ans A

17. Assertion : If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.

Reason : If r be the radius and h the slant height of the cone, then slant height $l = \sqrt{h^2 + r^2}$

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Ans D

18. Assertion : The radii of two cones are in the ratio 2:3 and their volumes in the ratio 1:3. Then the ratio of their heights is 3:4

Reason : Volume of the cylinder = $\pi r^2 h$

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

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Ans B

19. Assertion : If the radius of a cone is halved and volume is not changed, then height remains same.

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20. Assertion: The volume of two sphere are in the ratio 27: 8 then their surface area are in the ratio 9:4 .

Reason: Volume of hemisphere= $\frac{2}{3}\pi r^3$ and it's total surface area= $3\pi r^2$

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Ans B

VSA TYPE QUESTION EACH 2 MARKS

1. Two cubes of volume 64 cm^3 are joined end to end. Find the surface area of the Resulting Cuboid.

Ans 160 cm^2

2. A solid is in the shape of a cone surmounted on a hemisphere, the radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Ans $\frac{1001}{6}$ or 166.83

3. A cone of height 20 cm and radius of base 5 cm is made up modelling clay. A child. reshapes it in the form of a sphere. Find the diameter of the sphere.

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6. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid

Ans $d = 7 \text{ cm}$ and $\text{area} = 332.5 \text{ cm}^2$

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1. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2 ? Ans- 18 cm^2

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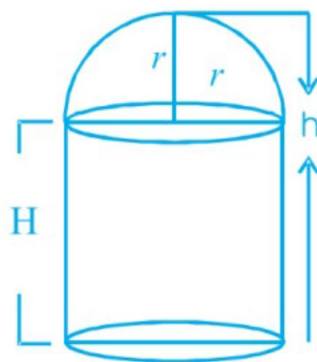
Ans 126

Long Answer Type Questions:

1. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. (311.04 cm^3)

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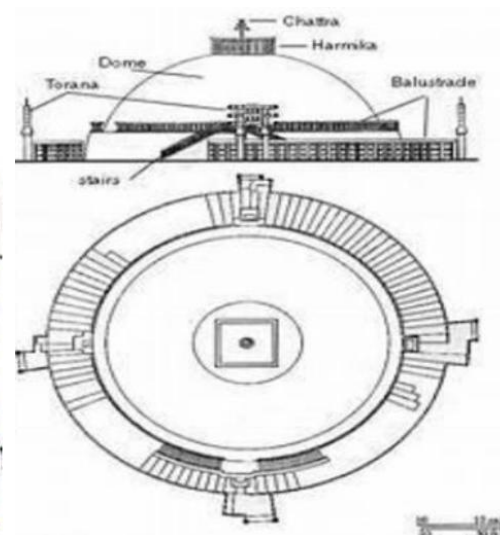
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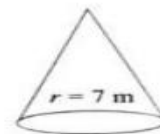
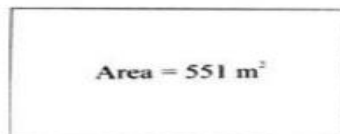


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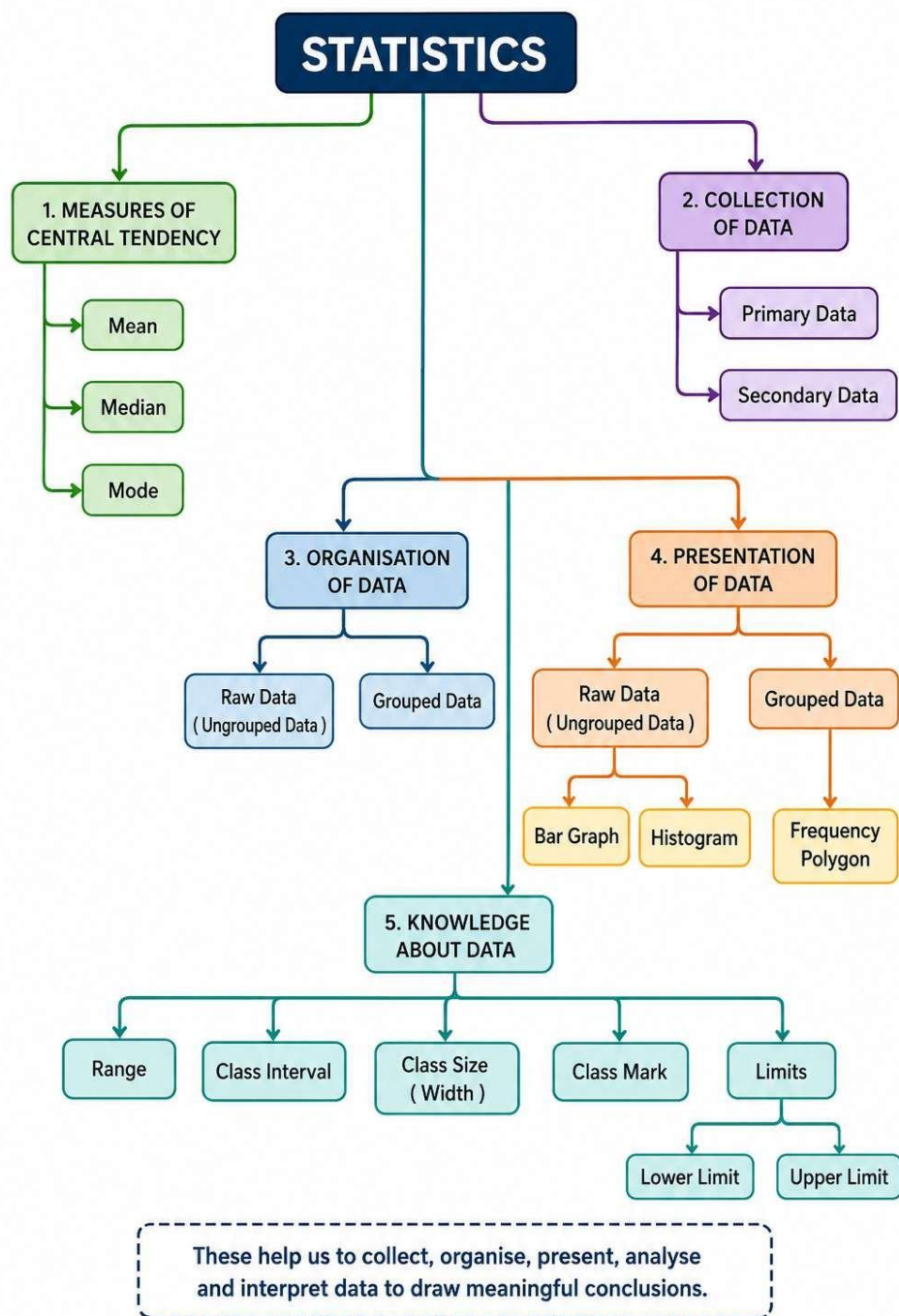
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Ans Cylindrical cup by 314.42 cm^3

OR

Find the height of the conical tent prepared to accommodate four students Ans 24

STATISTICS



What is Statistics?

Statistics is a branch of Mathematics concerned with how to collect, organize, and possibly interpret numerical data. In this chapter, we shall focus on:

- **Organising and representing data** using frequency distributions and suitable tables.

- **Calculating measures of central tendency** — Mean, Median, and Mode.
- **Finding the mean of grouped data** using appropriate methods
- **Determining the median and mode of grouped data** using standard formulas.
- **Representing and interpreting data visually** through graphs such as histograms and frequency polygons.
- **Analysing data** to draw meaningful conclusions and solve real-life problems.

Data Types: • Discrete Data: Data that can only take certain values (usually whole numbers) such as the number of students in a class.

1• Continuous Data: Data that can take any value within a range such as height or weight.

2. Collection of Data: • Methods of data collection, including surveys, experiments, and observational studies. • Primary Data: Data collected firsthand by the researcher. • Secondary Data: Data collected by someone else for another purpose.

3. Organization of Data: • Tabulation: Arranging data in rows and columns

. • Frequency Distribution: Organization of data according to the frequency of occurrence of each value. • Grouped Frequency Distribution: Data organized into intervals or classes.

4. Measures of Central Tendency

: • Mean: The average of a set of data

. • Median: The middle value when the data is arranged in ascending or descending order.

• Mode: The value that occurs most frequently in a data set. • $3\text{Median} = \text{Mode} + 2\text{Mean}$

Measures of Central Tendencies

In mathematics, a measure of central tendency is a statistical value that represents the centre or typical value of a data set. It consists of three mean measures: **Mean, median, and mode**. Each of these measures provides a summary representation of the data.

1. Mean:

For grouped data, the mean is a weighted average of the midpoints of the class intervals where frequency for each class is used as the weight. The mean is calculated with three methods:

- **Direct Method:** The direct method is quite straightforward, in which it sums all values in the data set and divides by the total number of observations.

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

- **Assumed Mean Method:** Assumed mean simplifies calculations by choosing a central value (assumed mean) and adjusting deviations of each observation from this assumed value.

$$d_i = x_i - a$$

$$\text{Mean} = \frac{\sum f_i d_i}{\sum f_i} + a$$

- **The Step Deviation Method:** This is obtained by adjusting the assumed mean based on the known standard deviation and is often applied in cases where data is known to have a known variance or spread

$$u_i = \frac{x_i - a}{h}$$

$$\text{Mean} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

The mode of grouped data is the value that occurs most often in the dataset. It can be found for grouped data by simply identifying which class contains the highest frequency and then using a formula to calculate the mode.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where:

- l is the lower boundary of the modal class,
- f_1 is the frequency of the modal class,
- f_0 is the frequency of the class before the modal class,
- f_2 is the frequency of the class after the modal class,
- h is the class width

2. Median:

The median for grouped data is the value that splits the data into two halves. It is found by obtaining the cumulative frequency corresponding to the middle position of the total data.

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \times h$$

where:

- L is the lower boundary of the median class,
- N is the total frequency,
- CF is the cumulative frequency of the class before the median class,
- f is the frequency of the median class,
- h is the class width.

Solved Problem

Question 1: Consider the following frequency distribution of marks obtained by class 10th in Maths.

Marks obtained	Frequency
20-30	15
30-40	13
40-50	10
50-60	8
60-70	9
70-80	5

Find the Mean of the following data with all the three methods mentioned above and also find the mode and median.

Class	Interv	f_i		f_i	d_i	f_id_i	u_i	f_i	C
al				x_i				u_i	F
20-30	15	25	375	-20	-300	-2	-30		15
30-40	13	35	455	-10	-130	-1	-13		28
40-50	10	45	450	0	0	0	0		38
50-60	8	55	440	10	80	1	8		46
60-70	9	65	585	20	180	2	18		55
Total	55		2305		-170		17		

$$\text{Mean from direct method} = \frac{\sum f_i \cdot x_i}{\sum f} = \frac{2305}{55} = 42$$

Let us assume $a = 45$

$$\text{Assumed mean method} = a + \frac{\sum f_i \cdot d_i}{\sum f} = 45 + \frac{-170}{55} = 42$$

$$\text{Step deviation method} = a + h \times \frac{\sum f_i \cdot u_i}{\sum f} = 45 + 10 \times \frac{-17}{55} = 42$$

Let, $f_0 = 0, f_1 = 15, f_2 = 13, L = 20$

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h = 20 + \left(\frac{15 - 0}{2 \times 15 - 0 - 13} \right) \times 10 = 28.8$$

Let $N = 55, N/2 = 27.5$ hence, $f = 13, L = 30, CF = 15$

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \times h = 30 + \left(\frac{27.5 - 15}{13} \right) \times 10 = 39.62$$

Q2 A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	Frequency
0-10	7
10-20	14
20-30	13
30-40	12
40-50	20
50-60	11
60-70	15

70-80

8

Solution:

From the given data:

Modal class = 40 – 50, $l = 40$,

class width (h) = 10, $f_m = 20$, $f_1 = 12$ and $f_2 = 11$

Substitute the values

Mode = $40 + (80/17) = 40 + 4.7 = 44.7$

Thus, the mode of the given data is 44.7 cars

Q3 Consider the following frequency distribution of the heights of 60 students of a class : Height (in cm) 150-155 155-160 160-165 165-170 170-175 175-180

Number of students 15 13 10 8 9
5

What is the sum of the lower limit of the modal class and upper limit of the median class ?

Q4 Consider the following distribution:

Marks	Above 0	Above 10	Above 20	Above 30	Above 40	Above 50
No. of Students	63	58	55	51	48	42

What is the frequency of the class 30 – 40 ?

Q 5 The mode and mean is given by 7 and 8, respectively. Then What is the median ?

Q 6: Find the median for the following frequency distribution.

Marks Obtained	No. of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

Solution:

Class interval	CF (no. of students)	f
Below 10	3	3
10-20	12	9
20-30	27	15
30-40	57	30
40-50	75	18
50-60	80	5

N = 80, N/2 = 40, modal class = 30-40, f = 30, L = 30, cf = 27

$$\text{Median} = L + \left(\frac{N - CF}{f} \right) \times h$$

$$= 30 + \left(\frac{40 - 27}{30} \right) \times 10$$

$$= 30 + 13/3$$

$$= 34.33$$

Q7 The following tables give the production yield per hectare of wheat of 100 farms of a village.

Production Yield	50-55	55-60	60-65	65-70	70-75	75-80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution . Solution:

Converting the given distribution to a more than type distribution, we get

Production Yield (kg/ha)	Number of farms
More than or equal to 50	100
More than or equal to 55	$100-2 = 98$
More than or equal to 60	$98-8= 90$
More than or equal to 65	$90-12=78$
More than or equal to 70	$78-24=54$
More than or equal to 75	$54-38 =16$

Q8 The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution :

Profit (Rs in lakhs)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Obtain the median profit.

median profit (in lakhs) is Rs. 17.5.

Q9 If the median of a distribution given below is 28.5 then, find the value of an x &y.

Class Interval	Frequency
0-10	5
10-20	x

20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Solution:

From the given data,

$$n = 60$$

Median of the given data = 28.5

$$\text{Where, } n/2 = 30$$

Median class is 20 – 30 with a cumulative frequency = 25 + x

Lower limit of median class = 20,

$$Cf = 5 + x ,$$

$$f = 20 \text{ \& } h = 10$$

Substitute the values

$$28.5 = 20 + 10(30 - 5 - x)/20$$

$$8.5 = (25 - x)/2$$

$$17 = 25 - x$$

Therefore, $x = 8$

Now, from cumulative frequency, we can identify the value of $x + y$ as follows:

Since,

$$60 = 5 + 20 + 15 + 5 + x + y$$

Now, substitute the value of x , to find y

$$60 = 5 + 20 + 15 + 5 + 8 + y$$

$$y = 60 - 53$$

$$y = 7$$

Therefore, the value of $x = 8$ and $y = 7$

Practice questions

ASSERTION-REASON BASED QUESTIONS

(a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

(b) If both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.

(c) If Assertion is correct but Reason is incorrect.

(d) If Assertion is incorrect but Reason is correct.

Q1. Assertion: If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.

Reason: Median = $(\text{mode} + 2 \text{ mean})/2$

Q2 Assertion: If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.

Reason: Median = $(n+1)/2$, if n is odd.

MCQ TYPE QUESTIONS

Q1 One of the methods for determining mode is

- (a) $\text{Mode} = 2 \text{ Median} - 3 \text{ Mean}$
- (b) $\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$
- (c) $\text{Mode} = 2 \text{ Mean} - 3 \text{ Median}$
- (d) $\text{Mode} = 3 \text{ Mean} - 2 \text{ Median}$

Q2 The mean of frequency distribution is 7.5 and $\sum f_i x_i = 120 + 3k$, $\sum f_i = 30$, then k is equal to:

- (a) 40
- (b) 35
- (c) 50
- (d) 45

Q3 The median of set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set

- (a) is increased by 2
- (b) is decreased by 2
- (c) is two times of the original number
- (d) Remains the same as that of the original set.

Q4. Mode is the

(a) middle most frequent value

(b) least frequent value

(c) maximum frequent value

(d) none of these

Q5. The mean of the following data 12, 22, 32,..... n^2 is -

(a) $(n+1)(2n+1)/6$

(b) $n(n-1)(2n+1)/6$

(c) $n(n+1)(2n-1)/6$

(d) $n(n-1)(2n-1)/6$

Q6. What should come in the blank? Mode= (.....) -2 (mean)

(a) 3 (median)

(b) 4 (median)

(c) 2 (median)

(d) 5 (median)

Q7. The mode and mean is given by 7 and 8, respectively. Then the median is:

(a) $1/13$

(b) $13/3$

(c) $23/3$

(d) 33

Q8 A batsman in his 12th innings makes a score of 63 runs and thereby increases his average score by 2. His average score after 12th innings is

- (a) 51
- (b) 45
- (c) 41
- (d) 60

Q9. If the mean of first n natural numbers is $5n/9$, then $n = ?$

- (a) 6
- (b) 7
- (c) 9
- (d) 10

Q10. Mean of n numbers $x_1, x_2, \dots, x_n$ is m . If x_n is replaced by x , then new mean is

- (a) $m - x_n + x$
- (b) $nm - x_n + x/n$
- (c) $(n-1)m + x/n$
- (d) $m - x_n + x/n$

Q11. While computing mean of grouped data, we assume that the frequencies are

- (a) centred at the upper limits of the classes

(b) centred at the lower limits of the classes

(c) centred at the class-marks of the classes

(d) evenly distributed over all the classes

Q12. If there are two class intervals 10-20 and 20-30, then in which interval will 20 fall?

(a) 10-20

(b) 20-30

(c) In both, 10-20 and 20-30

(d) Neither in 10-20 nor 20-30\

Q13. The median of the data 13, 15, 16, 17, 19, 20 is:

(a) $30/2$

(b) $31/2$

(c) $33/2$

(d) $35/2$

14. If the arithmetic mean of x , $x + 3$, $x + 6$, $x + 9$ and $x + 12$ is 10, then $x = ?$

(a) 1

(b) 2

(c) 6

(d) 4

15. Mode and mean of a data are 12k and 15k. Median of the data is

(a) 12k

(b) 14k

(c) 15k

(d) 16k

16. The algebraic sum of the deviations of a frequency distribution from its mean is always,

(a) greater than zero

(b) less than zero

(c) zero

(d) a non-zero number

17. The class mark of the class 15.5 – 20.5

(a) 15.5

(b) 20.5

(c) 18

(d) 5

18. The mean of the data: 4, 10, 5, 9, 12 is; (a)8

(b)10

(c)9

(d)15

SHORT TYPE-1 QUESTIONS

Q1. If the median of 12, 13, 16, $x + 2$, $x + 4$, 28, 30, 32 is 23, when $x + 2$, $x + 4$ lie between 16 and 30, then find the value of x .

2. If the mode of 12, 16, 19, 16, x , 12, 16, 19, 12 is 16, then find the value of x

Q3 In which measure of Central tendency Construction of cumulative frequency table is useful ?

Q4 The range of the data 14, 27, 29, 61, 45, 15, 9, 18 is: ---

Q5 The class mark of the class 120 – 150 is:-----

SHORT TYPE-2 QUESTIONS

Q1 From the following data of the marks obtained by students of class X

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	8	12	20	30	10	10

Find how many students, secured less than 40 marks?

Q2 Find the value of y from the following observations if these are already arranged in ascending order. The Median is 63.

20, 24, 42, y , $y + 2$, 73, 75, 80, 99

Q3 While checking the value of 20 observations, it was noted that 125 was wrongly noted as 25 while calculating the mean and then the mean was 60. Find the correct mean.

Q4 If the mean of first n natural numbers is 15, then find n .

Q5 A student scored the following marks in 6 subjects:

30, 19, 25, 30, 27, 30

Find his modal score.

Q6 If the mean of 4 numbers, 2, 6, 7 and a is 15 and also the mean of other 5 numbers, 6, 18, 1, a , b is 50. What is the value of b ?

LONG QUESTIONS:

Q1 A survey regarding the heights (in cm) of 50 girls of class Xth of a school was conducted and the following data was obtained. Find the mean, median and mode of the given data.

Heights (in cm)	120 – 130	130 – 140	140 – 150	150 – 160	160 – 170
No. of Girls	2	8	12	20	8

Q2 The median of the following data is 52.5. Find the values of x and y. if the total frequency is 100 .

Class	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency	2	5	x	12	17	20	y	9	7	4

Q3 The mean of the following frequency distribution is 25.2. Find the missing frequency x.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	x	10	11	9

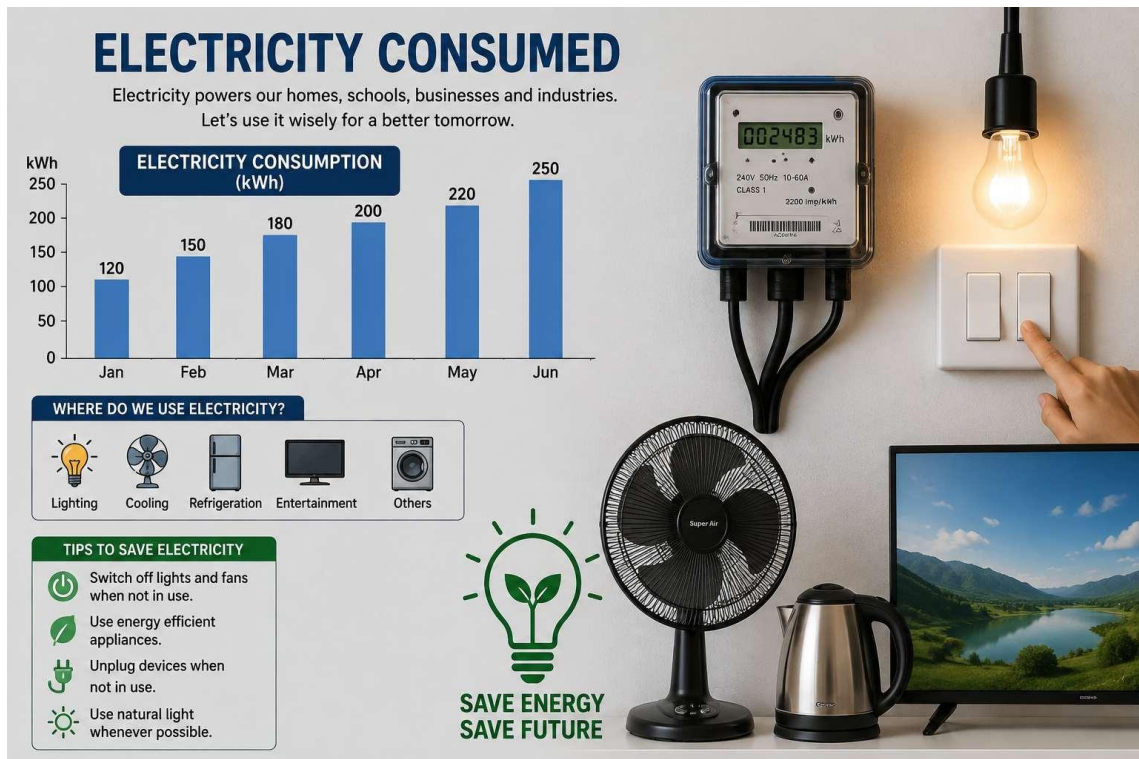
Q4 The mode of the following frequency distribution is 34.5. Find the value of x.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	4	8	10	x	8

CASE STUDY BASED QUESTIONS

Q1 Electricity energy consumption is the form of energy consumption that uses electric energy. Global electricity consumption continues to increase faster than world population, leading to an increase in the average amount of electricity consumed per person (per capita electricity consumption).

A survey is conducted for 56 families of a Colony A. The following tables gives the weekly consumption of electricity of these families.



Weekly consumption (in units) 0-10 10-20 20-30 30-40 40-50 50-60

No. of families 16 12 18 6 4 0

i). The median weekly consumption is

- a) 12 units
- b) 16 units
- c) 20 units
- d) None of these

ii). The mean weekly consumption is

- a) 19.64 units
- b) 22.5 units
- c) 26 units
- d) None of these

iii). The modal class of the above data is

- a) 0-10
- b) 10-20
- c) 20-30
- d) 30-40

Q2 100 Metres Race: The 100 metres is a sprint race in track and field competitions. The shortest common outdoor running distance, it is one of the most popular and prestigious events in the sport of athletics. It has been contested at the summer Olympics since 1896 for men and since 1928 for women. The World Championships 100 metres has been contested since 1983. The reigning 100 m Olympic or world champion is often named "the fastest man or woman in the world". A stopwatch was used to find the time that it took a group of students to run 100 m.



Time (in sec) 0-20 20-40 40-60 60-80 80-100

No. of students 8 10 13 6 3

Based on the above information, answer the following questions.

- (i) Estimate the mean time taken by a student to finish the race.
- (ii) What will be the upper limit of the modal class ?
- (iii) What is the sum of lower limits of median class and modal class ?

PREVIOUS YEAR QUESTIONS

Q1 Find the mean using the step deviation method. **CBSE 2024**

CLASS INTERVAL	FREQUENCY
0-10	6
10-20	10
20-30	15
30-40	9
40-50	10

Q2 Calculate mode of the following distribution:

CBSE 2024

Class:	5-10	10-15	15-20	20-25	25-30	30-35
Frequency:	5	6	15	10	5	4

Q3 Find the mean and median of the following data:

CBSE 2024

Class	85-90	90-95	95-100	100-105	105-110	110-115
Frequency	15	22	20	18	20	25

Q4 The monthly expenditure on milk in 200 families of a Housing Society is given below :

Monthly Expenditure (in Rs.)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of families	24	40	33	x	30	22	16	7

Find the value of x and also find the mean expenditure **CBSE 2024**

Q5 The following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
	60

2500-3000	86
3000-3500	74
3500-4000	62
4000-4500	48
4500-5000	

Find the average life time of a lamp.

CBSE2023

Q6 The length of 40 leaves of a plant are measured correct to nearest millimetre, and the data

obtained is represented in the following table.

Length [in mm] Number of leaves

118 – 126 3

127 – 135 5

136 – 144 9

145 – 153 12

154 – 162 5

163 – 171 4

172 – 180 2

Find the mean length of the leaves.

CBSE 2023

Q7 The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also find the mode of the data.

CBSE 2023

Marks obtained Number of students

20 – 30 p

30 – 40 15

40 – 50 25

50 – 60 20

60 – 70 q

70 – 80 8

Q 8 The following distribution shows the marks distribution of 80 students.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of students	2	12	28	56	76	80

What is the median class ?

CBSE

2023

Q9 Consider the frequency distribution of 45 observations.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	9	15	10	6

What is the upper limit of median class?**CBSE 2023**

Q10 In a grouped frequency distribution, the mid values of the classes are used to measure for finding ----- central tendency.**CBSE 2017**

ANSWER KEY

ASSERTION-REASON BASED ANSWERS

QUESTION	SOLUTION	
1	Ans c	
2	Ans a	

MCQ TYPE ANSWERS

QUESTION	SOLUTION	
1	b	
2	b	
3	d	
4	c	
5	a	
6	a	
7	c	
8	c	
9	c	
10	b	
11	c	
12	b	
13	c	
14	d	
15	b	
16	c	
17	c	
18	a	

SHORT TYPE -1 ANSWERS

1	X=20	
2	16	
3	Median	
4	52	

5	135	
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SHORT TYPE -2 ANSWERS		
1	70	
2	Y= 61	
3	65	
4	29	
5	30	
6	a=45 b=180	

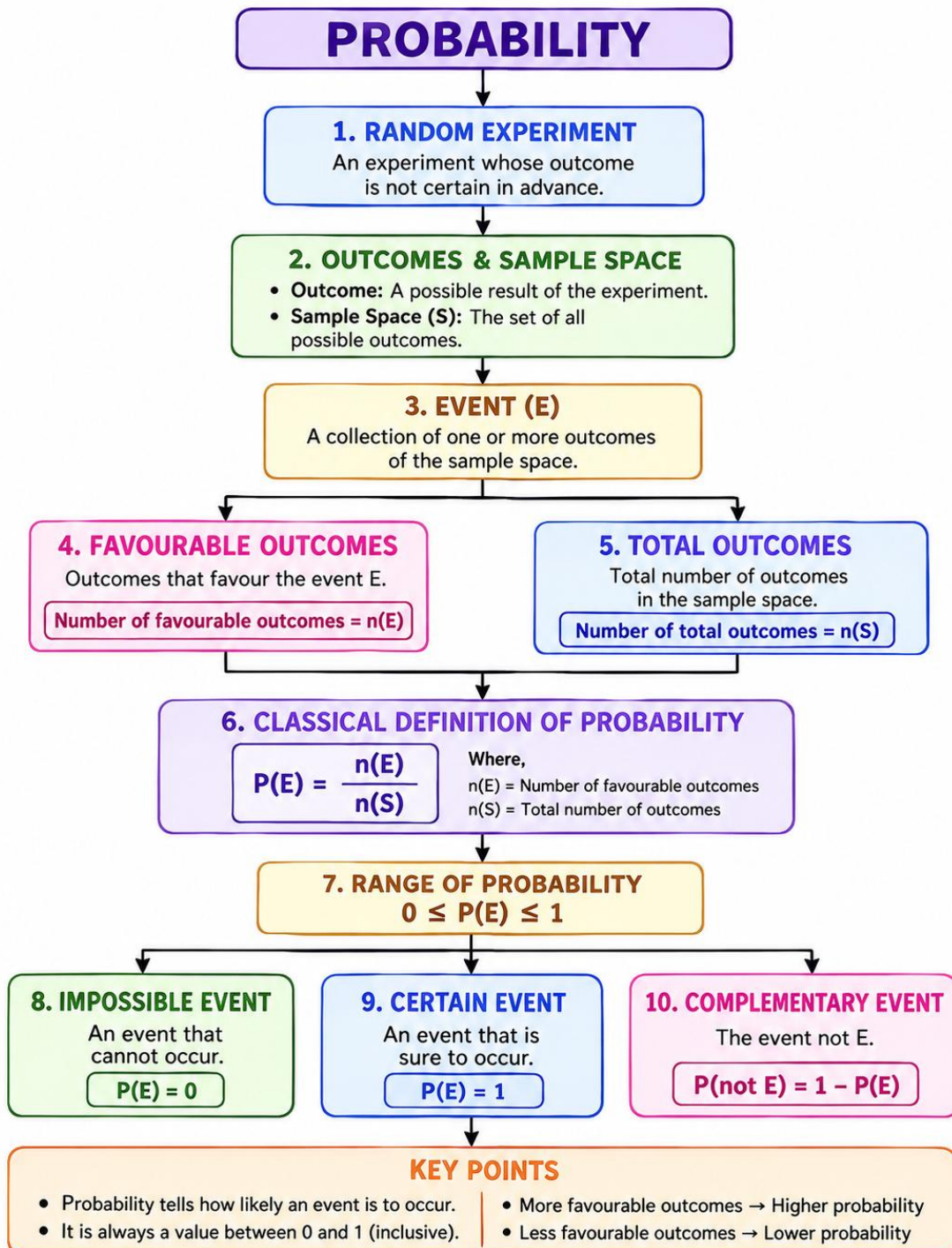
	LONG TYPE ANSWERS		
1		Mean =154.25 Mode = 154 Median =151.5	
2		X=9 Y=15	
3		X=12	
4		19	

	CASE STUDYBASED ANSWERS		
QUESTIONS	SOLUTION		
1	20,19.64,0-10		
2	43 SECONDS,60,80		
PREVIOUS YEAR SOLUTIONS			
1	26.4		
2	18.21		
3	Mean=100.875 Median=100.833		
4	X=28		

	Mean =rs 2662.5	
5	3410 hrs	
6	146.75	
7	P=5,Q=17,MODE=46.67	
8	MEDIAN CLASS =30-40	
9	30	
10	MEAN	

PROBABILITY

Class 10



FACTS ABOUT TOSSING COINS:

When we toss a coin, all the outcomes are H, T. So, number of all outcomes = 2
When we toss two coins, all the outcomes are (H,H), (H,T), (T,H), (T,T). So number of all outcomes are $2 \times 2 = 4$

When we toss three coins, all the outcomes are (H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T).

So number of all outcomes are $2 \times 2 \times 2 = 8$

FACTS ABOUT ROLLING DICES:

When we roll one dice, the number of all outcomes = 6.

Outcomes are 1,2,3,4,5,6.

When we roll two dices, the number of all outcomes = $6 \times 6 = 36$

Outcomes are

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6).

FACTS ABOUT PLAYING CARDS:



SOLVED EXAMPLES

(MINIMUM LEVEL OF LEARNING TYPE OF QUESTIONS)

Q1. Radha ,Rashmi and Rakhi are three friends .They are playing card games. A card is drawn at random from a well shuffled pack of 52 playing cards.

A. Find the probability of getting neither a red card nor a queen.

B. What is the probability of getting "a black-face card' from a well shuffled deck of 52 playing cards?

Solution:

A. Let E be the event of getting neither a red card nor a queen. No. of red cards = 26 (including 2 queens)

Remaining (black) queens = 2

Neither red nor queen = $52 - (26 + 2) = 52 - 28 = 24$

$P(E) = 24/52 = 6/13$

B. There are 6 black face cards. (2 red kings, 2 red queens and 2 jacks)

Total cards = 52 ∴

Required Probability = $6/52 = 3/26$

Q2 Probability that cannot exist among the following:

(a) $2/3$ (b) -2.6 (c) 25% (d) 0.7

Ans: (b) -2.6

Explanation: $0 \leq P(E) \leq 1$, for any event E.

Hence, it cannot be negative

Q3. Two coins are tossed simultaneously. The Probability of having exactly one head is

(a) $1/4$ (b) $1/2$ (c) $3/4$ (d) none of these

Ans : (b)

Total no. of outcomes = 4 (HT, TH, HH, TT)

No. of outcomes containing exactly one head = 2 (HT, TH)

Probability of outcomes of getting exactly one head = $2/4 = 1/2$

(HOTS QUESTIONS)

Q1 Sanvi and Vatan are friends. What is the probability that
both will have (i) different birthdays?
(ii) the same birthday? (ignoring a leap year).

SOLUTION:

Let Sanvi birthday is on x day.

(i) Then, for Vatan having different birthday, there can be other 364 days (other than x) as there are 365 days in a year.

So, probability of having them different birthdays = $364/365$

(ii) For having same birthday, Vatan birthday should also be on day x.

So, there is only single possibility of them having same birthday.

So, probability of having them same birthdays = $1/365$

2. Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is more than 9

Solution

: Total possible outcomes = $n(s) = 6 \times 6 = 36$.

Let event E be the event that sum of two numbers appearing on the top is more than 9.

Hence, Favourable outcomes of event E = $\{(4, 6), (5, 5), (5, 6), (6, 5), (6, 6), (6, 4)\}$

Hence, Total outcomes favourable to event E is $n(E) = 6$.

$\therefore P(E) = 6/36 = 1/6$

Q3 A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the drawn card is neither a king nor a queen.

Solution:

Total possible outcomes = 52

Let E be the event that the drawn card is neither a king nor a queen.

Total no. of kings and queens = $4 + 4 = 8$

Therefore $52 - 8 = 44$ cards that are neither king or queen.

No. of favourable outcomes = 44

Probability (E) = $44/52 = 11/13$

Q4. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

Solution:

When 2 dice are thrown at the same time, the overall possible outcomes are

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

Total number of possible outcomes = 36

Probability = number of favourable outcomes / number of possible outcomes

The favourable outcomes are

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4)

(3,1) (3,2)

(4,1) (4,2)

(5,1)

(6,1)

Number of favorable outcomes = 16

Number of possible outcomes = 36

Probability = number of favorable outcomes / number of possible outcomes

$$\begin{aligned}\text{Probability} &= 16/36 \\ &= 4/9\end{aligned}$$

Therefore, the probability of getting a product less than 9 is 4/9.

Q5. A letter is chosen from 26 alphabet find the probability that the letter chosen is from the word assassination?

SOLUTION:

The unique letters are: a, s, i, n, t, o.

Count the number of unique letters: There are 6 unique letters (a, s, i, n, t, o).

The total number of letters in the English alphabet: There are 26 letters.

$$\begin{aligned}\text{Probability} &= (\text{Number of favorable outcomes}) / (\text{Total outcomes}) \\ &= 6 / 26 = 3 / 13.\end{aligned}$$

MCQ TYPE QUESTIONS

(I). A bag contains 6 red and 5 blue balls. One ball is drawn at random. The probability that the ball is blue is:

$$\begin{array}{llll} \text{(A)} \frac{2}{11} & \text{(B)} \frac{5}{6} & \text{(C)} \frac{5}{11} & \text{(D)} \frac{6}{11} \end{array}$$

(II). One alphabet is chosen from the word MATHEMATICS. The probability of getting a vowel is:

$$\begin{array}{llll} \text{(A)} \frac{6}{11} & \text{(B)} \frac{5}{11} & \text{(C)} \frac{3}{11} & \text{(D)} \frac{4}{11} \end{array}$$

(III).

Two coins are tossed simultaneously. The probability of getting at most one head is

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

(IV) The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is

(A) $\frac{1}{5}$

(B) $\frac{2}{15}$

(C) $\frac{1}{3}$

(D) none of these

(V) If $P(E) = 0.07$, then what is the probability of 'not E'?

(A) 0.93

(B) 0.95

(C) 0.89

(D) 0.90

(VI) A fish tank has 5 male fish and 8 female fish. The probability of fish taken out is a male fish:

(A) $\frac{5}{8}$

(B) $\frac{5}{13}$

(C) $\frac{13}{5}$

(D) 5

(VII) The sum of the probabilities of all the elementary events of an experiment is

(A) 0.5

(B) 1

(C) 2

(D) 1.5

VIII). Someone is asked to take a number from 1 to 100. The probability that it is a prime is:

- A. $1/5$
- B. $6/25$
- C. $1/4$
- D. $13/50$

IX) An event is very unlikely to happen. Its probability is closest to:

- A. 0.0001
- B. 0.001
- C. 0.01
- D. 0.1

X). If the probability of an event is P , the probability of its complementary event will be:

- A. $P - 1$
- B. P
- C. $1 - P$
- D. $1 - 1/P$

XI. In a family of 3 children, the probability of having atleast one boy is:

- A. $7/8$
- B. $1/8$
- C. $5/8$
- D. $3/4$

XII. If $P(A)$ denotes the probability of an event then:

- A. $P(A) < 0$
- B. $P(A) > 0$
- C. $0 \leq P(A) \leq 1$
- D. $-1 \leq P(A) \leq 0$

XIII. The probability than a non-leap year selected at random will contain 53 Sundays is:

- A. $1/7$
- B. $2/7$
- C. $3/7$
- D. $5/7$

XIV. When a die is thrown, the probability of getting an odd number less them 3 is:

- A. $1/6$
- B. $1/3$
- C. $1/2$
- D. 0

XV. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favorable to E is:

- A. 4
- B. 13
- C. 48
- D. 51

XVI. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is:

- A. 7
- B. 14
- C. 21
- D. 28

XVII. A girl calculate that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought?

- A. 40
- B. 240
- C. 480
- D. 750

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SHORT TYPE- 1 QUESTIONS

Q1. From a well shuffled pack of playing cards, black jacks, black kings and black aces are removed. A card is then drawn from the pack. Find the probability of getting a red card?

Q2. A die is thrown once. Find the probability of getting a number greater than 5.

Q3. A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random. Find the probability of this ball being not a blue ball.

Q4. A bag contains three green marbles, four blue marbles and two orange marbles. If a marble is picked at random, then what is the probability that it is not an orange marble?

Q5. A box contains cards numbered 6 to 50. A card is drawn at random from the box. Calculate the probability that the drawn card has a number that is a perfect square.

SHORT TYPE -2 QUESTIONS

Q1. 100 tickets of a lottery were sold and there are 5 prizes on these tickets. If Saket has purchased one lottery ticket, what is the probability of winning a prize?

Q 2. Find the probability that a leap year selected at random will contain 53 Sundays.

Q3. One card is drawn from a well – shuffled deck of 52 cards. Find the probability of getting:

i) A face card ii) A spade iii) the jack of diamonds

Q4. A piggy bank contains hundred 50p coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a 50 p coin?

(ii) will not be a ₹5 coin?

Q5. A die is thrown once. Find the probability of getting

- (i) a prime number
- (ii) a number lying between 2 and 6
- (iii) an odd number

Q6. A die is thrown twice. What is the probability that

- (i) 5 will not come up either time?
- (ii) 5 will come up at least once?

LONG TYPE QUESTIONS

Q1. Apoorva throws two dice once and computes the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has the better chance of getting the number 36? Why?

Q2. 10 Cards numbered from 11-60 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn card is:

- i) A perfect square number
- ii) divisible by 5

Q3. A group consists of 12 persons, out of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is

- (i) extremely patient (ii) extremely kind or honest.

Q4. The probability of a defective egg in a lot of 400 eggs is 0.035. Calculate the number of defective eggs in the lot. Also calculate the probability of taking out a non defective egg from the lot.

ASSERTION-REASONING QUESTIONS

In the following questions 1 and 2, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

1. Assertion (A): The probability of getting exactly one head in tossing a pair of coins is $1/2$.

Reason (R): The sample space of two coin tossed is $= \{HH, TT, HT, TH\} = 4$

2. Assertion (A): The probability of winning a game is 0.4, then the probability of losing it, is 0.6.

Reason (R): $P(E) + P(\text{not } E) = 1$

CASE BASED QUESTIONS

1. A group of students conducted a survey to find out about the preferred mode of transportation to school among their classmates. They surveyed 200 students from their school. The results of the survey are as follows:
- 120 students preferred to walk to school.
 - 25% of the students preferred to use bicycles.
 - 10% of the students preferred to take the bus.
 - Remaining students preferred to be dropped off by car.



Based on the above information, answer the following questions:

- (a) What is the probability that a randomly selected student does not prefer to walk to school?
- (b) Find the probability of a randomly selected student who prefers to walk or use a bicycle.
- c) One day 50% of walking students decided to come by bicycle. What is the probability that a randomly selected student comes to school using a bicycle on that day?

1. A, B, C, D and E are five friends. They prepared some numbered cards with labeled from 11 to 60 and then they put all the number cards in the empty box. In this game, every friend was asked to pick the card randomly and after each draw, card was replaced back in the box.



- a) Find the probability that the number on the drawn card is an odd number
- (b) Find the probability that the number on the drawn card is a perfect square number
- (c) Find the probability that the number on the drawn card is divisible by 5

PREVIOUS YEAR QUESTIONS

1. One card is drawn at random from a pack of cards. Find the probability that it is a black king.

(CBSE 2020)

2. A box contains cards numbered from 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is:

[CBSE 2018]

(i) A square number

(ii) A multiple of 7.

3. In a family of three children. Find the probability of having at least two boys.

(CBSE 2020)

4. Two dice are rolled together bearing numbers 4, 6, 7, 9, 11, 12. Find the probability that the product of numbers obtained is an odd number

(CBSE 2024)

5. How many positive three digit integers have the hundredths digit 8 and unit's digit 5? Find the probability of selecting one such number out of all three digit numbers.

(CBSE 2024)

6. A bag contains 6 red, 4 black and some white balls. (i) Find the number of white balls in the bag if the probability of drawing a white ball is $\frac{1}{2}$. **(CBSE 2021)**

7. How many red balls should be removed from the bag for the probability of drawing a white ball to be $\frac{1}{2}$? **(CBSE 2023)**

8. A letter of English alphabet

is chosen at random. Determine the probability that chosen letter is a consonant. **(CBSE 2020)**

ANSWER KEY

ASSERTION-REASON BASED ANSWERS

QUESTION	SOLUTION	
1	(d)	
2	(a)	

MCQ TYPE ANSWERS

QUESTION	SOLUTION	
1	C	
2	D	
3	C	
4	A	
5	A	
6	B	
7	B	
8	C	
9	A	
10	C	
11	D	
12	C	

13	A	
14	A	
15	C	
16	B	
17	C	

SHORT TYPE -1 ANSWERS

1	1/26	
2	1/6	
3	17/22	
4	7/9	
5	1/9	

SHORT TYPE -2 ANSWERS

1	1/20	
2	2/7	
3	3/13, 1/4	
4	5/9, 17/18	
5	1/2, 1/2, 1/2	
6	25/36, 11/360	

	LONG TYPE ANSWERS		
1		PEEHU HAS BETTER CHANCE	
2		2/25,1/5	
3		1/4,3/4	
4		14,0.965	

	CASE STUDYBASED ANSWERS		
QUESTIONS	SOLUTION		
1	a) 7/20 b) 17/20 c) 1/25		
2	a) $\frac{1}{2}$ b) B) 2/25 c) 1/5,		
PREVIOUS YEAR SOLUTIONS			
1	1/52		
2	8/113,16/113		
3	$\frac{3}{4}$		
4	$\frac{1}{4}$		
5	1/90		

6	X = 10	
7	5	
8	21/25	

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