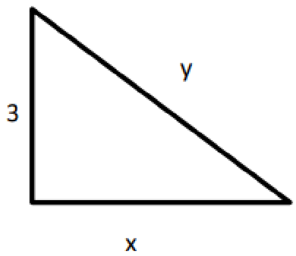
केन्द्रीय विद्यालय संगठन, बेंगलूरु संभाग
KENDRIYA VIDYALAYA SANGATHAN , BENGALURU REGION
प्रथम प्री-बोर्ड परीक्षा (2024-25)
FIRST PRE BOARD EXAMINATION (2024-25)

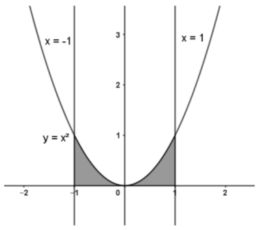
CLASS:XII MAX MARKS:80
SUBJECT : MATHEMATICS TIME : 3 HRS

MARKING SCHEME

SECTION-A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**			
1. c)± 12	6.c)5	11.d)I quadrant	16. d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)
2. a)4	7. a) $C(A+B')$	12.d)$-cotx - tanx + c$	17. c) 1.5
3. a)$(-\infty,-4)\cup(0,\infty)$	8.b)0.25	13.b)0	18. c) 2
4.d)8	9.c) $-\frac{1}{\sqrt{2}}\hat{\imath}-\frac{1}{\sqrt{2}}\hat{\jmath}$	14.b) not defined , 2	19. C
5.b)$\frac{1}{x}$	10.b)$\frac{7}{3}$	15. b) $\frac{\pi}{2}<y<\pi$	20 . A

Q No	Expected Answers/Value Points	Marks
21	$-1\le 3x-2\le 1$ $\Rightarrow \frac{1}{3}\le x\le 1$ or $\left[\frac{1}{3},1\right]$	1 1
22	$f'(x)=\dfrac{1-logx}{x^2}$ $,\therefore f'(x)=0\Rightarrow logx=1\ \Rightarrow x=e$ $f''(x)=\frac{2xlogx-3x}{x^4}$ $,f''(e)=-\frac{1}{e^3}<0$, $x=e$ is a point of local maximum	1 ½ ½
23	a) Let $x=\sin A$ and $y=\sin B$ $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1\Rightarrow \sin A cos\ B+\cos A sin\ B=1$ $\Rightarrow \sin(A+B)=1$ $\Rightarrow A+B=\dfrac{\pi}{2}$ $\Rightarrow \sin^{-1}x+\sin^{-1}y=\dfrac{\pi}{2}$ Differentiating w.r.t x , we obtain $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$ b) Let $y=\tan^{-1}x$ and $z=logx$ $\frac{dy}{dx}=\frac{1}{1+x^2}$ and $\frac{dz}{dx}=\frac{1}{x}$ $\dfrac{dy}{dz}=\dfrac{\frac{1}{1+x^2}}{\frac{1}{x}}=\dfrac{x}{1+x^2}$	½ ½ 1 ½ + ½ ½ + ½
24	a) $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12\Rightarrow |\vec{x}|^2-|\vec{a}|^2=12$ $\Rightarrow |\vec{x}|^2-1=12$ $\Rightarrow |\vec{x}|^2=13$ $\Rightarrow |x|=\sqrt{13}$	½ ½ ½

	OR b) $\vec{a}+\vec{b} = 6\hat{\imath} - 2\hat{\jmath} + (7+\lambda)\hat{k}$ and $\vec{a}-\vec{b} = -4\hat{\imath} + (7-\lambda)\hat{k}$ $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal if $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$ $\Rightarrow -24 + (7+\lambda)(7-\lambda) = 0$ $\lambda^2 = 25 \Rightarrow \lambda = \pm 5$	½ ½ 1 ½
25	$\overrightarrow{d_1} = \vec{a}+\vec{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, $\overrightarrow{d_2} = \vec{a}-\vec{b} = -6\hat{\jmath} - 8\hat{k}$ Area of the parallelogram $= \frac{1}{2}\left\|\overrightarrow{d_1} \times \overrightarrow{d_2}\right\| = \frac{1}{2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} =$ $\frac{1}{2}\left\|4\hat{\imath} + 32\hat{\jmath} - 24\hat{k}\right\| = \left\|2\hat{\imath} + 16\hat{\jmath} - 12\hat{k}\right\| = \sqrt{404} = 2\sqrt{101}$	½ ½ ½ + ½
26	$f(x) = sin3x \Rightarrow f'(x) = 3cos3x = 0 \Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}$ $f'(x) \geq 0$ for all $x \in \left[0, \frac{\pi}{6}\right] \Rightarrow f(x)$ is increasing on $\left[0, \frac{\pi}{6}\right]$ $f'(x) \leq 0$ for all $x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right] \Rightarrow f(x)$ is decreasing on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$	1 1 1
27	 $x^2 + 3^2 = y^2$ $When\ y = 5\ then\ x = 4, now\ 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ 4 (200) = 5 $\frac{dy}{dt}$ => $\frac{dy}{dt}$ = 160 cm/s	½ ½ 1 1
28	a) $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{c} \times \vec{d} - \vec{b} \times \vec{d}$ $\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} + \vec{b} \times \vec{d} - \vec{c} \times \vec{d} = 0$ $\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) + (\vec{b} - \vec{c}) \times \vec{d} = 0$ $\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) - \vec{d} \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow (\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow (\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$ OR b) Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-a}{4}$ is of the form $(2\lambda+1, 3\lambda+2, 4\lambda+a)$ and any point on the line $\frac{x-4}{5} = \frac{y-1}{2} = z$ is of the form $(5\mu+4, 2\mu+1, \mu)$. Solving $2\lambda+1 = 5\mu+4$ and $3\lambda+2 = 2\mu+1$ to get $\lambda = -1$ and $\mu = -1$. The lines will be skew if, $4\lambda + a \neq \mu$ $4(-1) + a \neq (-1)$ $a \neq 3$, $\therefore a \in R - \{3\}$	½ ½ ½ ½ ½ ½ 1 ½ + ½ ½ ½
29	a) Let $x^{\frac{3}{2}} = t \Rightarrow \frac{3}{2}x^{\frac{1}{2}}dx = dt$	1

	$\int \sqrt{\frac{x}{1-x^3}}\, dx \;= \frac{2}{3}\int \frac{dt}{\sqrt{1-t^2}}\, dx$ $= \frac{2}{3}\sin^{-1} t + c = \frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right) + c$ OR b) Using $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$ $\int_0^1 x(1-x)^n\, dx = \int_0^1 (1-x)x^n\, dx = \int_0^1 (x^n - x^{n+1})dx$ $= \frac{x^{n+1}}{n+1}\Big]_0^1 - \frac{x^{n+2}}{n+2}\Big]_0^1$ $= \frac{1}{n+1} + \frac{1}{n+2}$	1 ½ + ½ 1 + ½ ½ 1
30	Value of z at (15, 15) = 15p + q Value of z at (0, 20) = q 15p + q = q or p = 0	1 1 1
31	a) P(H) = $\frac{3}{4}$ P(T) = $\frac{1}{4}$ <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{9}{16}$</td><td>$\frac{6}{16}$</td><td>$\frac{1}{16}$</td></tr><tr><td>X P(X)</td><td>0</td><td>$\frac{6}{16}$</td><td>$\frac{2}{16}$</td></tr></table> Mean = $\frac{8}{16}$ or $\frac{1}{2}$ b) $P(A' \cup B') = \frac{1}{4} \Rightarrow 1 - P(A \cap B) = \frac{1}{4}$ $\Rightarrow P(A \cap B) = \frac{3}{4}$ Since $P(A \cap B) \neq 0$, A and B are not mutually exclusive. $P(A).P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \neq P(A \cap B)$ A and B are not independent	½ ½ 1 ½ ½ 1 1 1
32	 Required area = $2\int_0^1 x^2 dx$ $= 2\,\frac{x^3}{3}\Big]_0^1$ $= \frac{2}{3}$	1 1 2 1
33	Assumes the number of litres of orange juice, beetroot juice and kiwi juice as x, y and z, respectively to frame equations as follows: $500x + 20y + 800z = 1860$ $2x + 5y + 3z = 22$ $100x + 120y + 200z = 760$ Writes the above system of equations in the matrix form using AX = B as	1

	$$\begin{bmatrix}500 & 20 & 800\\ 2 & 5 & 3\\ 100 & 120 & 200\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1860\\ 22\\ 760\end{bmatrix}$$ Finds $\lvert A\rvert = 110000 \neq 0$ and hence writes that A is non-singular and has a unique solution.	½
	Finds adj A as: $\begin{bmatrix}640 & 92000 & -3940\\ -100 & 20000 & 100\\ -260 & -58000 & 2460\end{bmatrix}$	2
	Finds A^{-1}using $\lvert A\rvert$ and adj A as: $A^{-1}=\frac{1}{110000}\begin{bmatrix}640 & 92000 & -3940\\ -100 & 20000 & 100\\ -260 & -58000 & 2460\end{bmatrix}$	½
	Writes that X = A^{-1}B and finds X as $\begin{bmatrix}2\\ 3\\ 1\end{bmatrix}$	1
	Concludes that 2 litres of orange juice, 3 litres of beetroot juice and 1 litre of kiwi juice should go into the mixture.	
34	a) $(ax+b)e^{\frac{y}{x}}=x \Rightarrow \frac{y}{x}=\log\left(\frac{x}{a+bx}\right)=logx-\log(a+bx)$	½
	differenetaiting w.r.t. x , $\Rightarrow \frac{x\frac{dy}{dx}-y}{x^2}=\frac{1}{x}-\frac{b}{a+bx}=\frac{a}{x(a+bx)}$	1
	$\Rightarrow \frac{x\frac{dy}{dx}-y}{x}=\frac{a}{a+bx}$ ---(1)	½
	$\Rightarrow x\frac{dy}{dx}-y=\frac{ax}{a+bx}$	½
	Again differentiating w.r.t x , $\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{(a+bx)a-ax.b}{(a+bx)^2}=\frac{a^2}{(a+bx)^2}$	1
	$\Rightarrow x\frac{d^2y}{dx^2}=\frac{a^2}{(a+bx)^2}=\left[\frac{a}{a+bx}\right]^2=\left[\frac{x\frac{dy}{dx}-y}{x}\right]^2$ from (1)	1
	$\Rightarrow x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2.$	½
	b) $(x-a)^2+(y-b)^2=c^2$ Differentiating both sides w.r.t x , $2(x-a)+2(y-b)\frac{dy}{dx}=0$	½
	$\Rightarrow \frac{dy}{dx}=-\frac{x-a}{y-b}$	½
	Again differentiating w.r.t.x $\frac{d^2y}{dx^2}=-\left[\frac{(y-b)-(x-a)\frac{dy}{dx}}{(y-b)^2}\right]$	1
	$=-\left[\frac{(y-b)-(x-a)\left\{-\frac{x-a}{y-b}\right\}}{(y-b)^2}\right]=-\frac{c^2}{(y-b)^3}$	½+ ½
	$1+\left(\frac{dy}{dx}\right)^2=1+\left[-\frac{x-a}{y-b}\right]^2=\frac{c^2}{(y-b)^2}$	½

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$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{c^3}{(y-b)^3}$$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c$$, which is independent of a and b

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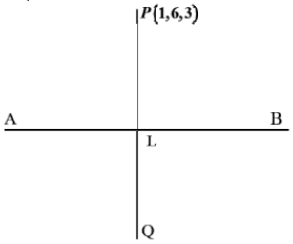
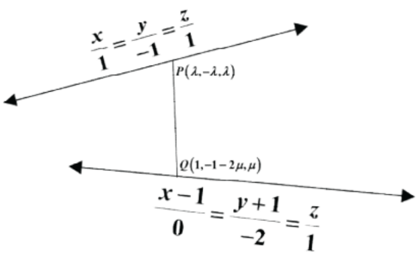
a) Let P(1,6,3) be the given point, and let ' ' L be the foot of the perpendicular from ' ' P to the given line AB (as shown in the figure below). The coordinates of a general point on the given line are given by



Coordinates of L is $(\lambda, 1+2\lambda, 2+3\lambda)$
So the direction ratios of PL are $\lambda-1, 2\lambda-5, 3\lambda-1$
Since PL ⊥ AB, $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$
$\Rightarrow \lambda = 1$
⇒ Coordinates of L are (1,3,5)
Let Q (α,β,γ) be the image , then L is the midpoint of PQ
$\left(\frac{\alpha+1}{2}, \frac{\beta+6}{2}, \frac{\gamma+3}{2}\right) = (1,3,5) \Rightarrow \alpha = 1, \beta = 0, \gamma = 7$
∴ Image of P(1,6,3) ,is (1,0,7)
The distance of (1,0,7) from y axis is $\sqrt{1^2+7^2} = \sqrt{50} = 5\sqrt{2}$ units
OR

b)



Let the coordinates of P be $(\lambda,-\lambda,\lambda)$) and Q be $(1,-2\mu-1,\mu)$
PQ is perpendicular to both the lines , direction ratio of PQ are
$\lambda-1, -\lambda+2\mu+1, \lambda-\mu$,
Since PQ is perpendicular to both the lines
$1(\lambda-1)+(-1)(-\lambda+2\mu+1)+1(\lambda-\mu)=0 \Rightarrow 3\lambda-3\mu=2$ --(1)

$0(\lambda-1)+(-2)(-\lambda+2\mu+1)+1(\lambda-\mu)=0 \Rightarrow 3\lambda-5\mu=2$ --(2)
Solving (1) and (2) we get $\lambda = \frac{2}{3}$ and $\mu = 0$
$\therefore P\left(\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}\right)$ and $Q(1,-1,0)$

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	So the required shortest distance is $\sqrt{\left(1-\frac{2}{3}\right)^2+\left(-1+\frac{2}{3}\right)^2+\left(-\frac{2}{3}\right)^2}$ $=\sqrt{\frac{2}{3}}$ units	1
36	(i) Let $(l_1, l_2)\epsilon\ R \Rightarrow l_1$ is parallel to $l_2 \Rightarrow l_2$ is parallel to l_1 $\Rightarrow (l_2, l_1)\epsilon\ R$, $\therefore R$ is symmetric. (ii) Let $(l_1, l_2), (l_2, l_3)\epsilon\ R \Rightarrow l_1$ is parallel to l_2 and l_2 is parallel to l_3 $\Rightarrow l_1$ is parallel to l_3 $\Rightarrow (l_1, l_3)\epsilon\ R$, $\therefore R$ is transitive. (iii) a) The set of rail lines in R related to the line $y = 3x + 2$ is the set of all lines parallel to it , $\{l: l$ is a line of type $y = 3x + c\ , c \in R\}$ OR (iii) b) Let $(l_1, l_2)\epsilon\ S \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1)\epsilon\ S$ $\therefore R$ is symmetric. Let $(l_1, l_2), (l_2, l_3)\epsilon\ R \Rightarrow l_1 \perp l_2$ and $l_2 \perp l_3$ $\Rightarrow l_1$ is parallel to l_3 $\therefore R$ is not transitive.	1 1 2 1 1
37	(i) When V = 40 km/h , F $=\frac{1600}{500}-\frac{40}{4}+14=\frac{36}{5}\ l/100km$ (ii) $\frac{dF}{dV}=\frac{2V}{500}-\frac{1}{4}=\frac{V}{250}-\frac{1}{4}$ (iii) a) For minimum $\frac{dF}{dV}=0 \Rightarrow \frac{V}{250}-\frac{1}{4}=0 \Rightarrow V=62.5$ km/h Also $\frac{d^2F}{dV^2}=\frac{1}{250}>0$ Hence F is minimum when V = 62.5 km/h OR (iii) b) $\frac{dF}{dV}=-0.01 \Rightarrow \frac{V}{250}-\frac{1}{4}=-0.01 \Rightarrow V=60$ km/h F $=\frac{3600}{500}-\frac{60}{4}+14=6.2\ l/100km$ Quantity of fuel required for 600 km = $6.2 * 6 = 37.2\ l$	1 1 1 ½ ½ ½ 1 ½
38	(i) Probability of a randomly chosen seed to germinate $=\frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}\times\frac{35}{100}=\frac{490}{1000}=0.49$ (ii) Probability that the randomly selected seed is of type of A_1 , given that it germinates $=\frac{\frac{4}{10}\times\frac{45}{100}}{\frac{490}{1000}}=\frac{18}{49}$	1 ½ + ½ 1 +1