केन्द्रीय विद्यालय संगठन

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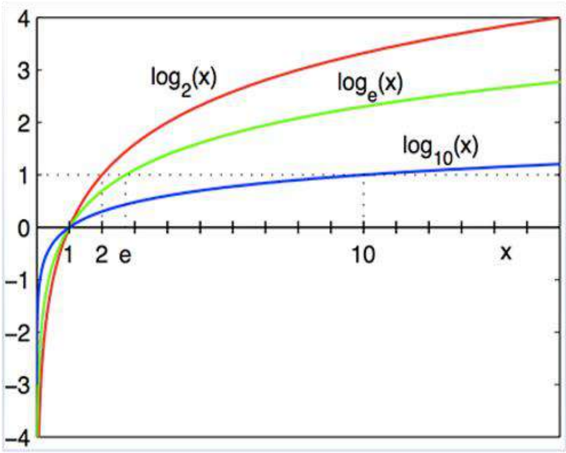
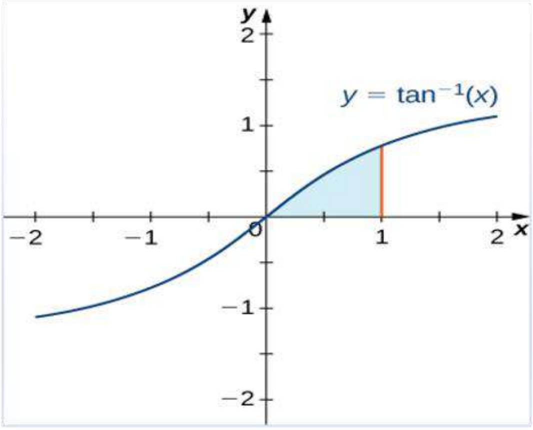
स्वच्छ भारत

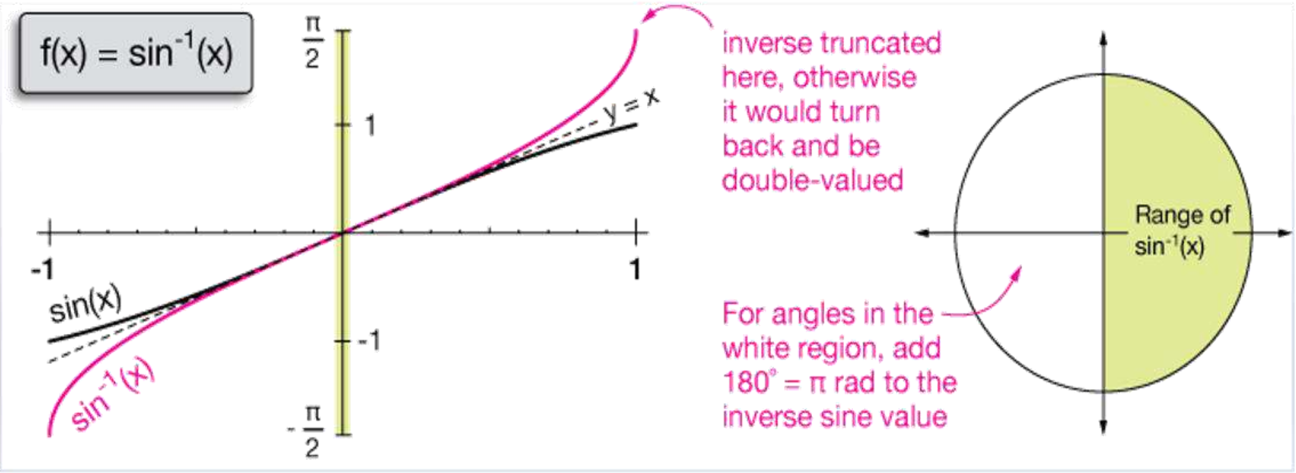
एक कदम स्वच्छता की ओर

बेंगलुरू संभाग
BENGALURU REGION

अध्ययन सामग्री / STUDY MATERIAL
शिक्षण सत्र / SESSION - 2024-25
कक्षा / CLASS – बारहवीं / TWELFTH
विषय/ SUBJECT- गणित /MATHEMATICS
विषय कोड / SUBJECT CODE - 041







PREFACE

This study material is the culmination of an extensive collaborative effort involving a dedicated team of educators and subject matter experts. With meticulous care, we have meticulously designed this resource to provide students with a succinct yet comprehensive tool for consolidating their knowledge.

Under the diligent guidance of our esteemed subject experts and the unwavering enthusiasm of our team, we have incorporated the entire curriculum and an extensive collection of practice questions spanning all chapters. Our paramount objective has been to ensure perfect alignment with the latest curriculum and examination patterns as set forth by the CBSE.

We firmly believe that this material will prove to be an invaluable resource, serving as a clear and concise repository of essential information for effective subject revision. It encompasses all the critical components necessary to assist in students' preparation and enhance their understanding of the subject matter.

Our aspiration is that this study material will emerge as a dependable aid for swift and efficient revision, instilling confidence in students and ultimately contributing to their academic success. We strongly encourage you to actively engage with the content, pose questions, and fully utilize this resource in your educational journey.

We extend our heartfelt best wishes for your studies and sincerely hope that this material becomes your trusted companion on the path to academic excellence.

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MESSAGE FROM THE DEPUTY COMMISSIONER

Dear students and teachers !

It is a matter of great pride and delight that KVS Bengaluru Region is putting forward the Students' Support Material (SSM) for class XII subject MATHAMATICS or the session 2024-25. I believe firmly that, the subject experts have left no stone unturned to enable our students to add on more to their quality of performance by deep rooting more towards accessing required understating in the subject. Certainly, use of this SSM will help students in empowering themselves as one of the tools and will lead in bringing success.

With devotion, dedication & persistent hard work the team of experts has crafted out this SSM meticulously to complement the classroom learning experience of the students as well as to cope up with the Competency Based Questions as per the new pattern of examinations aligned with NEP-2020 and NCFSE-2023. This SSM, being well-structured and presented in a manner which makes it to be comprehended easily, will definitely serve as a precious supplement for self-study of students.

I am pleased to place on record my appreciation and commendation for the commitment and dedication of the team comprising of the subject experts in carving out such a useful edition of Students' Support Material for the students.

Wishing all the best !

(DHARMENDRA PATLE)
DEPUTY COMMISSIONER
KVS BENGALURU REGION

केन्द्रीय विद्यालय संगठन
KENDRIYA VIDYALAYA SANGATHAN
बेंगलुरू संभाग / BENGALURU REGION
STUDY MATERIAL SESSION (2024 -25)
CLASS XII MATHEMATICS

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7	SH MATHIALAGAN	PMSHRI KV, DRDO BENGALURU

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CLASS-XII (2024-25)

One Paper **Max Marks: 80**

No.	Units	No. of Periods	Marks
I.	Relations and Functions	30	08
II.	Algebra	50	10
III.	Calculus	80	35
IV.	Vectors and Three - Dimensional Geometry	30	14
V.	Linear Programming	20	05
VI.	Probability	30	08
	TOTAL	240	80
	Internal Assessment		20

Unit-I: Relations and Functions

1. Relations and Functions 15 Periods

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions 15 Periods

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices 25 Periods

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. On commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants 25 Periods

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability 20 Periods

Continuity and differentiability, chain rule, derivative of inverse trigonometric functions, *like* $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives 10 Periods

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals 20 Periods

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them. $\int dx / x^2 \pm a^2$, $\int dx / \sqrt{(x^2 \pm a^2)}$, $\int dx / \sqrt{(a^2 - x^2)}$, $\int dx / ax^2 + bx + c$, $\int dx / \sqrt{(ax^2+bx+c)}$, $\int (px + q)/(ax^2 + bx + c)\, dx$, $\int (px + q)/\sqrt{(ax^2+bx + c)}\, dx$, $\int \sqrt{(a^2 \pm x^2)}\,dx$, $\int \sqrt{(x^2 - a^2)}\, dx$ $\int \sqrt{(ax^2 + bx + c)}\,dx$, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals 15 Periods

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations 15 Periods

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$dy/dx + py = q$, where p and q are functions of x or constants.

$dx/dy + px = q$, where p and q are functions of y or constants.

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors 15 Periods

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry 15 Periods

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming

1. Linear Programming 20 Periods

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability 30 Periods

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean of random variable.

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

❑ Meaning of Mathematical Symbols

Symbol	**Meaning**
$\Rightarrow$	Implies
$\in$	Belongs to
$A \subset B$	A is a subset of B
$\Leftrightarrow$	Implies and is implied by
$\notin$	Does not belong to
s.t.(: or |)	Such that
$\forall$	For every
$\exists$	There exists
iff	If and only if
&	And
$a \mid b$	a is a divisor of b
N	Set of natural numbers
I or Z	Set of integers
R	Set of real numbers
C	Set of complex numbers
Q	Set of rational numbers

Some important results on number of elements in sets

If A, B and C are finite sets and U be the finite universal set, then

(1) $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

(2) $n(A\cup B) = n(A) + n(B) \Leftrightarrow A, B$ are disjoint non-void sets.

(3) $n(A - B) = n(A) - n(A\cap B)$ *i.e.*, $n(A - B) + n(A\cap B) = n(A)$

(4) $n(A\Delta B)$ = Number of elements which belong to exactly one of A or B $= n((A - B) \cup (B - A)) = n(A - B) + n(B - A)$

[$\because$ $(A - B)$ and $(B - A)$ are disjoint]

$= n(A) - n(A\cap B) + n(B) - n(A\cap B) = n(A) + n(B) - 2n(A\cap B)$

(5) $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

(6) n (Number of elements in exactly two of the sets A, B, C) $= n(A\cap B) + n(B\cap C) + n(C\cap A) - 3n(A\cap B\cap C)$

(7) n(Number of elements in exactly one of the sets A, B, C) $= n(A) + n(B) + n(C) - 2n(A\cap B) - 2n(B\cap C) - 2n(A\cap C) + 3n(A\cap B\cap C)$

(8) $n(A'\cup B') = n(A\cap B)' = n(U) - n(A\cap B)$

(9) $n(A'\cap B') = n(A\cup B)' = n(U) - n(A\cup B)$

Laws of algebra of sets

(1) **Idempotent laws :** For any set A, we have

(i) $A\cup A = A$ (ii) $A\cap A = A$

(2) **Identity laws :** For any set A, we have

(i) $A\cup \phi = A$ (ii) $A\cap U = A$

i.e., ϕ and U are identity elements for union and intersection respectively.

(3) **Commutative laws :** For any two sets A and B, we have

(i) $A\cup B = B\cup A$ (ii) $A\cap B = B\cap A$

(iii) $A\Delta B = B\Delta A$

i.e., union, intersection and symmetric difference of two sets are commutative.

(iv) $A - B \neq B - A$ (v) $A\times B \neq B\times A$

i.e., difference and cartesian product of two sets are not commutative

(4) **Associative laws :** If A, B and C are any three sets, then

(i) $(A\cup B)\cup C = A\cup (B\cup C)$

(ii) $A\cap (B\cap C) = (A\cap B)\cap C$

(iii) $(A\Delta B)\Delta C = A\Delta (B\Delta C)$

i.e., union, intersection and symmetric difference of two sets are associative.

(iv) $(A - B) - C \neq A - (B - C)$ (v) $(A\times B)\times C \neq A\times (B\times C)$

i.e., difference and cartesian product of two sets are not associative.

(5) **Distributive law :** If A, B and C are any three sets, then

(i) $A\cup (B\cap C) = (A\cup B)\cap (A\cup C)$

(ii) $A\cap (B\cup C) = (A\cap B)\cup (A\cap C)$

i.e., union and intersection are distributive over intersection and union respectively.

(iii) $A\times (B\cap C) = (A\times B)\cap (A\times C)$

(iv) $A\times (B\cup C) = (A\times B)\cup (A\times C)$

(v) $A\times (B - C) = (A\times B) - (A\times C)$

(6) **De-Morgan's law :** If A, B and C are any three sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(iii) $A - (B \cap C) = (A - B) \cup (A - C)$

(iv) $A - (B \cup C) = (A - B) \cap (A - C)$

(7) If A and B are any two sets, then

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

(iii) $A - B = A \Leftrightarrow A \cap B = \phi$

(iv) $(A - B) \cup B = A \cup B$

(v) $(A - B) \cap B = \phi$

(vi) $A \subseteq B \Leftrightarrow B' \subseteq A'$

(vii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

(8) If A, B and C are any three sets, then

(i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Cartesian product of sets

Cartesian product of sets : Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = [(a, b) : a \in A \text{ and } b \in B]$

If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$.

***Example*:** Let $A = \{a, b, c\}$ and $B = \{p, q\}$.

Then $A \times B = \{(a, p), (a, q), (b, p), (b, q), (c, p), (c, q)\}$

Also $B \times A = \{(p, a), (p, b), (p, c), (q, a), (q, b), (q, c)\}$

Important theorems on cartesian product of sets :

Theorem 1 : For any three sets A, B, C

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Theorem 2 : For any three sets A, B, C

$A \times (B - C) = (A \times B) - (A \times C)$

Theorem 3 : If A and B are any two non-empty sets, then

$A \times B = B \times A \Leftrightarrow A = B$

Theorem 4 : If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$

Theorem 5 : If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.

Theorem 6 : If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

Theorem 7 : For any sets A, B, C, D

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Theorem 8 : For any three sets A, B, C

(i) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$

(ii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$

Relations

Definition: Let A and B be two non-empty sets, then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.

Let $R \subseteq A \times B$ and $(a, b) \in R$. Then we say that a is related to b by the relation R and write it as $a R b$. If $(a, b) \in R$, we write it as $a R b$.

(1) **Total number of relations :**Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subset of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines relation from A to B, so total number of relations from A to B is 2^{mn}. Among these 2^{mn} relations the void relation ϕ and the universal relation $A \times B$ are trivial relations from A to B.

(2) **Domain and range of a relation :**Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Dom $(R) = \{a : (a, b) \in R\}$ and Range $(R) = \{b : (a, b) \in R\}$.

Inverse relation

Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$

Clearly $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$. Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom (R^{-1})

Example **:** Let $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $R = \{(a, 1), (a, 3), (b, 3), (c, 3)\}$.

Then, (i) $R^{-1} = \{(1, a), (3, a), (3, b), (3, c)\}$

(ii) Dom $(R) = \{a, b, c\}$ = Range (R^{-1})

(iii) Range $(R) = \{1, 3\}$ = Dom (R^{-1})

<u>Types of relations</u>

(1) **Reflexive relation :**A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R is reflexive $\Leftrightarrow (a, a) \in R$ for all $a \in A$.

Example **:** Let $A = \{1, 2, 3\}$ and $R = \{(1, 1); (1, 3)\}$

Then R is not reflexive since $3 \in A$ but $(3, 3) \notin R$

A reflexive relation on A is not necessarily the identity relation on A.

The universal relation on a non-void set A is reflexive.

Note: The number of Reflexive relations on a finite set having n elements is $2^{n(n-1)}$.

(2) **Symmetric relation:** A relation R on a set A is said to be a symmetric relation *iff*

$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

i.e., $aRb \Rightarrow bRa$ for all $a, b \in A$.

it should be noted that R is symmetric iff $R^{-1} = R$

The identity and the universal relations on a non-void set are symmetric relations.

A reflexive relation on a set A is not necessarily symmetric.

The number of Symmetric relations on a finite set having n elements is $2^{n(n+1)/2}$.

(3) **Anti-symmetric relation:** Let A be any set. A relation R on set A is said to be an anti-symmetric relation *iff* $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$.

Thus, if $a \neq b$ then a may be related to b or b may be related to a, but never both.

(4) **Transitive relation :**Let A be any set. A relation R on set A is said to be a transitive relation *iff*

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$ *i.e.,* aRb and $bRc \Rightarrow aRc$ for all $a, b, c \in A$.

Transitivity fails only when there exists a, b, c such that $a R b$, $b R c$ but $a \not R c$.

Example**:** Consider the set $A = \{1, 2, 3\}$ and the relations

$R_1 = \{(1, 2), (1, 3)\}$; $R_2 = \{(1, 2)\}$; $R_3 = \{(1, 1)\}$;

$R_4 = \{(1, 2), (2, 1), (1, 1)\}$

Then R_1, R_2, R_3 are transitive while R_4 is not transitive since in $R_4, (2, 1) \in R_4; (1, 2) \in R_4$ but $(2, 2) \notin R_4$.

The identity and the universal relations on a non-void sets are transitive.

(5) **Identity relation :** Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only. Every identity relation will be reflexive, symmetric and transitive.

***Example* :** On the set = {1, 2, 3}, $R = \{(1, 1), (2, 2), (3, 3)\}$ is the identity relation on A .

It is interesting to note that every identity relation is reflexive but every reflexive relation need not be an identity relation.

(6) **Equivalence relation :** A relation R on a set A is said to be an equivalence relation on A *iff*

(i) It is reflexive *i.e.* $(a, a) \in R$ for all $a \in A$

(ii) It is symmetric *i.e.* $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$

(iii) It is transitive *i.e.* $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

***Congruence modulo* (*m*) :** Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo m if $a - b$ is divisible by m and we write $a \equiv b$ (mod m).

Thus $a \equiv b$ (mod m) $\Leftrightarrow a - b$ is divisible by m. For example, $18 \equiv 3$ (mod 5) because 18 – 3 = 15 which is divisible by 5. Similarly, $3 \equiv 13$ (mod 2) because 3 – 13 = –10 which is divisible by 2. But 25 ≠ 2 (mod 4) because 4 is not a divisor of 25 – 3 = 22.

The relation "Congruence modulo *m*" is an equivalence relation.

Equivalence classes of an equivalence relation

Let R be equivalence relation in $A (\neq \phi)$. Let $a \in A$. Then the equivalence class of a, denoted by $[a]$ or $\{\bar{a}\}$ is defined as the set of all those points of A which are related to a under the relation R. Thus $[a] = \{x \in A : x\ Ra\}$.

It is easy to see that

(1) $b \in [a] \Rightarrow a \in [b]$

(2) $b \in [a] \Rightarrow [a] = [b]$

(3) Two equivalence classes are either disjoint or identical.

Composition of relations

Let R and S be two relations from sets A to B and B to C respectively. Then we can define a relation SoR from A to C such that $(a, c) \in SoR \Leftrightarrow \exists b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

This relation is called the composition of R and S.

For example, if $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$, $C = \{p, q, r, s\}$ be three sets such that $R = \{(1, a), (2, b), (1, c), (2, d)\}$ is a relation from A to B and $S = \{(a, s), (b, r), (c, r)\}$ is a relation from B to C. Then SoR is a relation from A to C given by $SoR = \{(1, s)\ (2, r)\ (1, r)\}$

In this case RoS does not exist.

In general $RoS \neq SoR$. Also $(SoR)^{-1} = R^{-1}oS^{-1}$.

Tips & Tricks

- Equal sets are always equivalent but equivalent sets may need not be equal set.
- If A has n elements, then $P(A)$ has 2^n elements.

- The total number of subset of a finite set containing n elements is 2^n.
- If $A_1, A_2, \ldots\ldots, A_n$ is a finite family of sets, then their union is denoted by $\bigcup\limits_{i=1}^{n} A_i$ or $A_1 \cup A_2 \cup A_3 \ldots\ldots \cup A_n$.
- If $A_1, A_2, A_3 \ldots\ldots, A_n$ is a finite family of sets, then their intersection is denoted by $\bigcap\limits_{i=1}^{n} A_i$ or $A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n$.
- $R - Q$ is the set of all irrational numbers.
- Let A and B two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.
- The identity relation on a set A is an anti-symmetric relation.
- The universal relation on a set A containing at least two elements is not anti-symmetric, because if $a \neq b$ are in A, then a is related to b and b is related to a under the universal relation will imply that $a = b$ but $a \neq b$.
- The set $\{(a, a) : a \in A\} = D$ is called the diagonal line of $A \times A$. Then "the relation R in A is antisymmetric iff $R \cap R^{-1} \subseteq D$".
- The relation 'is congruent to' on the set T of all triangles in a plane is a transitive relation.
- If R and S are two equivalence relations on a set A , then $R \cap S$ is also an equivalence relation on A.
- The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.
- The inverse of an equivalence relation is an equivalence relation.

Function:

A relation from a set A to another set B is called a function if it relates each element of set A to a unique element of set B.

One-One Function / Injective Function:

A function f from set A to set B is called an injective function or into function if no two distinct element of set A be related to the same element of set B under this map f, i.e. $x \neq y \Rightarrow f(x) \neq f(y)$

i.e. $f(x) = f(y) \Rightarrow x = y$.

Surjective Function or Onto Map:

A function f from set A to set B is called a Surjective function or Onto function if every element of set B has a pre-image in set A under the function f, i.e. Range f = B.

Bijective Map or One-One Onto Map:

A function f from set A to set B is called a Bijective Map if it is both injective as well as surjective.

MCQ's

1. Let A be a set containing 10 distinct elements, then the total number of distinct functions from A to A is

a) 10^{10} b) 101

c) 2^{10} d) $2^{10} - 1$

2. If $f : [2, 3] \to R$ is defined by $f(x) = x^3 + 3x - 2$, then the range $f(x)$ is contained in the interval

a) [1, 12] b) [12, 34]

c) [35, 50]
d) [-12, 12]

3. Let $[x]$ denote the greatest integer $\leq x$. If $f(x) = [x]$ and $g(x) = |x|$, then the value of $f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right)$ is
a) 2
b) -2
c) 1
d) -1

4. If R be a relation defined as aRb iff $|a - b| > 0$, then the relation is
a) Reflexive
b) Symmetric
c) Transitive
d) Symmetric and transitive

5. The domain of the real function $f(x) = \frac{1}{\sqrt{4-x^2}}$ is
a) The set of all real numbers
b) The set of all positive real numbers
c) (-2, 2)
d) [-2, 2]

6. Let $A = \{x, y, z\}$ and $B = \{a, b, c, d\}$. Which one of the following is not a relation from A to B?
a) $\{(x, a), (x, c)\}$
b) $\{(y, c), (y, d)\}$
c) $\{(z, a), (z, d)\}$
d) $\{(z, b), (y, b), (a, d)\}$

7. If $f: R \to R$ and is defined by $f(x) = \frac{1}{2-\cos 3x}$ for each $x \in R$, then the range f f is
a) (1/3, 1)
b) [1/3, 1]
c) (1, 2)
d) [1, 2]

8. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}; x \in R$ is
a) $(1, \infty)$
b) (1, 11/7)
c) [1, 7/3]
d) (1, 7/5)

9. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is
a) A function
b) Transitive
c) Not symmetric
d) Reflexive

10. The range of the function $f(x) = x^2 - 6x + 7$ is
a) $(-\infty, 0)$
b) $[-2, \infty)$
c) $(-\infty, \infty)$
d) $(-\infty, -2)$

11. The number of onto mappings from the set $A = \{1, 2, \dots, 100\}$ to set $B = \{1, 2\}$ is
a) $2^{100} - 2$
b) 2^{100}
c) $2^{99} - 2$
d) 2^{99}

12. The number of reflexive relations of a set with four elements is equal to
a) 2^{16}
b) 2^{12}
c) 2^{8}
d) 2^{4}

13. Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then which of the following relations is a function from A to B?
a) {(1, 2), (2, 3), (3, 4), (2, 2)}
b) {(1, 2), (2, 3), (1, 3)}
c) {(1, 3), (2, 3), (3, 3)}
d) {(1, 1), (2, 3), (3, 4)}

14. Let $f: N \to N$ defined by $f(x) = x^2 + x + 1, x \in N$, then f is
a) One-one onto
b) Many-one onto
c) One -one but not onto
d) None of these

15. Let Wdenotes the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W$: the world x and y have at least one letter in common$\}$. Then, R is
a) Reflexive, symmetric and not transitive
b) Reflexive, symmetric and transitive
c) Reflexive, not symmetric and transitive
d) Not reflexive, symmetric and transitive

16. R is relation on N given by $R = \{(x, y): 4x + 3y = 20\}$. Which of the following belongs to R?
a) (-4, 12)
b) (5, 0)
c) (3, 4)
d) (2, 4)

17. Range of the function $f(x) = \frac{x^2}{x^2+1}$ is
a) (-1, 0)
b) (-1, 1)
c) [0, 1)
d) (1, 1)

18. Which of the following functions is one-to -one?
a) $f(x) = \sin x, x \in [-\pi, \pi]$
b) $f(x) = \sin x, x \in \left[-\frac{3\pi}{2}, -\frac{\pi}{4}\right]$

c) $f(x)=\cos x, x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ d) $f(x)=\cos x, x\in\left[\pi,\frac{3\pi}{2}\right]$

19. If R denotes the set of all real numbers, then the function $f: R\to R$ defined by $f(x)=|x|$ is
a) One-one only b) Onto only
c) Both one-one and onto d) Neither one-one nor onto

20. $A=\{1,2,3,4\}, B\{1,2,3,4,5,6\}$are two sets, and function $f: A\to B$ is defined by $f(x)=x+2\ \forall x\in A$, then the function f is
a) Bijective b) Onto
c) One-one d) Many-one

21. The function $f: R\to R$ given by $f(x)=x^3-1$ is
a) A one-one function b) An onto function
c) A bijection d) Neither one-one nor onto

22. The relation R defined on the set of natural numbers as $\{(a,b): a$ differs from b by 3$\}$ is given by
a) {(1, 4), (2, 5), (3, 6), ...} b) {(4, 1), (5, 2), (6, 3), ...}
c) {(1, 3), (2, 6), (3, 9), ...} d) None of the above

23. A relation from P to Q is
(a) A universal set of $P\times Q$ (b) $P\times Q$
(c) An equivalent set of $P\times Q$ (d) A subset of $P\times Q$

24. Let $A=\{a,b,c\}$ and $B=\{1,2\}$. Consider a relation R defined from set A to set B. Then R is equal to set
(a) A (b) B (c) $A\times B$ (d) $B\times A$

25. Let $n(A)=n$. Then the number of all relations on A is
(a) 2^n (b) $2^{(n)!}$ (c) 2^{n^2} (d)None of these

26. Let R be a reflexive relation on a finite set A having n-elements, and let there be m ordered pairs in R. Then
(a) $m\ge n$ (b) $m\le n$ (c) $m=n$ (d) None of these

27. The relation R defined on the set $A=\{1,2,3,4,5\}$ by $R=\{(x,y):\left|x^2-y^2\right|<16\}$ is given by
(a) $\{(1,1),(2,1),(3,1),(4,1),(2,3)\}$ (b) $\{(2,2),(3,2),(4,2),(2,4)\}$
(c) $\{(3,3),(3,4),(5,4),(4,3),(3,1)\}$ (d) None of these

28. A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by $xRy \Leftrightarrow x$ is relatively prime to y. Then domain of R is
(a) {2, 3, 5} (b){3, 5} (c) {2, 3, 4} (d) {2, 3, 4, 5}

29. Let R be a relation on N defined by $x+2y=8$. The domain of R is
(a) {2, 4, 8} (b) {2, 4, 6, 8} (c) {2, 4, 6} (d) {1, 2, 3, 4}

30. If $R=\{(x,y)\,|\,x,y\in Z, x^2+y^2\le 4\}$ is a relation in Z, then domain of R is
(a) $\{0,1,2\}$ (b) $\{0,-1,-2\}$ (c) $\{-2,-1,0,1,2\}$ (d) None of these

31. R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y=x-3$. Then R^{-1} is
(a)$\{(8,11)\ ,\ (10,13)\}$ (b) $\{(11,18)\ ,\ (13,10)\}$ (c)$\{(10,13),(8,11)\}$ (d) None of these

32. If $A=\{x: x^2-5x+6=0\}$, $B=\{2,4\}$, $C=\{4,5\}$, then $A\times(B\cap C)$is
(a)$\{(2,4),(3,4)\}$ (b)$\{(4,2),(4,3)\}$ (c)$\{(2,4),(3,4),(4,4)\}$ (d)$\{(2,2),(3,3),(4,4),(5,5)\}$

33. Let $A=\{1,2,3\}$. The total number of distinct relations that can be defined over A is
(a) 2^9 (b) 6 (c) 8 (d) None of these

34. Given two finite sets A and B such that $n(a)=2, \mathrm{n}(b)=3$. Then total number of relations from A to B is
(a) 4 (b) 8 (c) 64 (d) None of these

35. Let $=\{1,2,3\}$, $B=\{1,3,5\}$. A relation $R: A\to B$ is defined by $R=\{(1,3),\ (1,5),\ (2,1$Then R^{-1} is defined by

(a) $\{(1,2),(3,1),(1,3),(1,5)\}$ (b) $\{(1,2),(3,1),(2,1)\}$

(c) $\{(1,2),(5,1),(3,1)\}$ (d) None of these

36. Given the relation $R=\{(1,2),(2,3)\}$ on the set $A=\{1,2,3\}$, the minimum number of ordered pairs which when added to R make it an equivalence relation is

(a) 5 (b) 6 (c) 7 (d) 8

37. Let R be the relation on the set R of all real numbers defined by a R if $|a-b|\le 1$. Then R is

(a) Reflexive and Symmetric (b) Symmetric only

(c) Transitive only (d) Anti-symmetric only

38. The relation "less than" in the set of natural numbers is

(a) Only symmetric (b) Only transitive (c) Only reflexive (d) Equivalence

39. Let N denote the set of all natural numbers and R be the relation on $N\times N$ defined by (a,b) R(c, d) if $ad(b+c)=bc(a+d)$, then R is

(a) Symmetric only (b) Reflexive only (c) Transitive only (d) An equivalence

40. For real numbers x and y, we write $xRy \Leftrightarrow x-y+\sqrt{2}$ is an irrational number. Then the relation R is

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

41. If $R\subset A\times B$ and $S\subset B\times C$ be two relations, then $(SoR)^{-1}=$

(a) $S^{-1}oR^{-1}$ (b) $R^{-1}oS^{-1}$ (c) SoR (d) RoS

42. If R be a relation from $A=\{1,2,3,4\}$ to $B=\{1,3,5\}$ *i.e.*, $(a,b)\in R\Leftrightarrow a<b$, then RoR^{-1} is

(a) $\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$ (b) $\{(3,1)(5,1),(3,2),(5,2),(5,3),(5,4)\}$

(c) $\{(3,3),(3,5),(5,3),(5,5)\}$ (d) $\{(3,3)(3,4),(4,5)\}$

43. Let $P=\{(x,y)\,|\,x^2+y^2=1, x,y\in R\}$. Then P is

(a) Reflexive (b) Symmetric (c) Transitive (d) Anti-symmetric

44. *LetR* be a relation on the set N of natural numbers defined by $nRm\Leftrightarrow n$ is a factor of m $(i.e., n\,|\,m)$. Then R is

(a) Reflexive and symmetric (b) Transitive and symmetric

(c) Equivalence (d) Reflexive, transitive but not symmetric

2 MARK QUESTIONS

1. Check whether the relation R defined in the set {1, 2, 3, 4} as
R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)} is Reflexive, Symmetric and Transitive.

Ans: Not Reflexive and Symmetric but Transitive)

2. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor Transitive.

ANS: (1, 1) does not belongs to R)

3. Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor Transitive. ((1,1) not belongs to R)

ANS: Relation R = {(1, 2), (2, 1)} is symmetric as (a, b)∈R⇒(b, a)∈R,

It is not transitive as (1, 2)∈R and (2, 1)∈R but (1, 1)∈R.

It is not reflexive as 1∈{1, 2, 3} but (1, 1)$\notin \boldsymbol{R}$.

4. Relation R = {(a, b): a ≤ b, a, b ∈R} is reflexive as

$$\boldsymbol{x \le x i.e. \quad xRx \qquad \forall x \in R.}$$

It is transitive as $x \leq y, \;\; y \leq z \Rightarrow x \leq z i.e. \quad xRy, yRz \Rightarrow xRz$.

But, it is not symmetric as $2 \leq 5 but 5 > 2$.

5. Show that R defined as R = {(a, b): a ≤ b, a, b ∈R} is Reflexive and Transitive but not Symmetric.

6. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. whether f is one-one? Whether it is onto?

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)}.

The f is one-one as distinct elements of set A is related to distinct elements of B.

This function f is not onto as 7∈ $\boldsymbol{B}$ which do not have pre-image in $\boldsymbol{A}$.

7. Let **R** be the set of real numbers. Prove that the mapping f :**R** →**R** given by f (x) = |x|, ∀x∈**R**, is neither one-one nor onto.

ANS: Given, by f (x) = |x|, ∀x∈**R**

f (1) = |1| = 1

f (-1) = 1

f (1) = f(-1)

but $1 \neq -1$ So, f (x) is not one-one.

Also, f (x) is not onto as there is no pre-image for any negative element of **R** under the Mapping f (x).

8. If A = {1, 2, 3, 4} define relations on A which have properties of being:

(a) reflexive, transitive but not symmetric

(b) reflexive, symmetric and transitive.

ANS: Given that A = {1, 2, 3, 4}

(a) Let R_1 = { (1,1) , (1, 2) , (2,3) , (2,2) , (1,3) , (3,3)}.

R_1 is reflexive, since, (1,1) (2,2) (3,3) lie is R_1

Now, (1, 2) ∈ R_1 (2, 3) ∈ R_1⇒ (1, 3) ∈ R_1

Hence, R_1 is also transitive, but (1, 2) ∈ R_1, ⇒ (2, 1) ∉ R So, it is not symmetric.

(b) Let R_2 = {(1,2) , (2,1) ,(1,1), (2, 2) , (3,3) , (1,3) , (3,1) , (2,3) }

Hence, R_2 is reflexive, symmetric and transitive.

9. Let R be relation defined on the set of natural number N as follows: R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

ANS: Given that, R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}

Domain = {1, 2, 3……….20}

Range = {1, 3, 5, 7…………………39}

R = {(1, 39), (2, 37), (3, 35) ...... (19, 3), (20,1)}

R is not reflexive as (2, 2) ∉ R $\quad 2 \times 2 + 2 \neq 41$

So, R is not symmetric As (1, 39) ∈ R but (39, 1) ∉ R So, R is not transitive.

As (11, 19) ∈ R, (19, 3) ∈ R But (11,3) ∉R

Hence, R is neither reflexive, nor symmetric and nor transitive.

10. **1.** In each of the following cases, state whether the function is one-one , onto or bijective. Justify your answer:

(i) $f: R \to R$ defined by $f(x) = 3 - 4x$

(ii) $f: R \to R$ defined by $f(x) = 1 + x^2$

Sol. (i)$f: R \to R$ defined by $f(x) = 3 - 4x$

Let $x_1, x_2 \in R$be such that$f(x_1) = f(x_2)$

⇨ $3 - 4x_1 = 3 - 4x_2 \Rightarrow -4x_1 = -4x_2 \Rightarrow x_1 = x_2$

$\therefore f is one - one$

Let $y \in R$be any real number.

$\therefore f(x) = y \Rightarrow 3 - 4x = y$

$\Rightarrow x = \frac{3-y}{4}$

$\therefore f\left(\frac{3-y}{4}\right) = y$

∴ Corresponding to every element $y \in R$, there exists $\left(\frac{3-y}{4}\right)$ such that $f\left(\frac{3-y}{4}\right) = y$

⇒ f is onto.

ii) $f: R \to R$ defined by $f(x) = 1 + x^2$

Let $x_1, x_2 \in R$be such that$f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x = \pm x_2$

$\Rightarrow f(x_1) = f(-x_1)$

$\Rightarrow f is one - one$

Let $y \in R$be any real number

$\therefore f(x) = y \Rightarrow 1 + x^2 = y$

As $x^2 \geq 0 \Rightarrow 1 + x^2 \geq 1 \Rightarrow y \geq 1$

∴ Range of f is greater than or equal to 1

$\therefore R_f \neq R$

⇒f is onto.

Thus, fis neither one-one nor onto

3 MARKS QUESTIONS & 5 MARKS QUESTIONS

1. Prove that the relation R in the set A = {1,2,3,4,5} given by R = {(a, b): a, b∈A and $|a - b|$is even} is an equivalence relation

ANS: Consider any a, b, c$\in A$ where A = {1, 2, 3, 4, 5}

Since |a – a| = 0, which is even $\Rightarrow |a-a|$ is even

$\Rightarrow (a,a) \in R \Rightarrow R$ is reflexive. R is Symmetric

$(a,b) \in R \Rightarrow |a-b|$ is even $\Rightarrow |-(b-a)|$ is even $\Rightarrow |b-a|$ is even

$\Rightarrow (b,a) \in R \Rightarrow R$ is symmetric R is transitive

$(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even

$\Rightarrow a-b$ is even and $b-c$ is even

$\Rightarrow a-b+b-c$ is even

$\Rightarrow a-c$ is even $\Rightarrow |a-c|$ is even

$\Rightarrow (a,c) \in R \quad \Rightarrow R$ is transitive

Hence, R is an equivalence relation.

2. Check whether the function f: R→R defined by f(x) = 4+ 3x is one – one and onto.

ANS: f is One-one function if $f(x_1) = f(x_2) \rightarrow x_1 = x_2$ for $x_1, x_2 \in R$

$f(x_1) = f(x_2) \quad 4 + 3x_1 = 4 + 3x_2$

$x_1 = x_2$ f is one-one

Let $y \in R$ and $y = f(x)$

$y = 4 + 3x$,

$x = \frac{y-4}{3}$ is a real number.

For every element in co-domain there exists at least one pre image in domain such that f(x)=y.

Hence, f is onto

3. Define the relation R in the set $N \times N$ as follows: For (a, b), (c, d) $\in N \times N$, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in $N \times N$.

ANS: Let $(a, b) \in N \times N$. Then we have ab = ba (by commutative property of N)

$\Longrightarrow (a, b)\ R\ (a, b)$ Hence, R is reflexive.

Let $(a, b), (c, d) \in N \times N$ such that (a, b) R (c, d).

Then ad = bc$\Longrightarrow cb = da$ (by commutative property of multiplication of N)

$\Longrightarrow (c, d)\ R\ (a, b)$ Hence, R is symmetric.

Let $(a, b), (c, d), (e, f) \in N \times N$

(a, b) R (c, d) then ad = bc ……. (1)

(c, d) R (e, f) then cf = de ……….(2)

From (1), (2) $adcf = bcde \Longrightarrow af = be \Longrightarrow (a, b)(e, f)$ Hence, R is transitive.

Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$.

4. Let A = R – {3}, B = R {– 1}. If f: A → B be defined by f (x) = $\frac{x-2}{x-3} \forall$ x ∈ A. Then, show that f is bijective.

ANS: Given A = R – {3}, B = R {– 1} f: A → B be defined by f (x) = $\frac{x-2}{x-3} \forall$ x ∈ A

For injectivity if $f(x_1) = f(x_2) \rightarrow x_1 = x_2$ for $x_1, x_2 \in R$

Let f (x_1) = f $(x_2) \Longrightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Longrightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Longrightarrow x_1 = x_2$ So, f (x) is an injective

For surjection:

Let $y = \frac{x-2}{x-3} \Longrightarrow xy - 3y = x - 2 \Longrightarrow x = \frac{3y-2}{y-1}$ is a real number except x=3

$\Longrightarrow$ So, f (x) is surjective function. Since f is one & Onto f(x) is a bijective function.

5). Show that the function f : R –{-4/3) → R –{4/3) defined by f(x) = $\frac{4x+3}{3x+4}$ is one-one and onto.Hence, find inverse of f and find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

ANS: Let $x_1, x_2 \in R$ –{-4/3) and $f(x_1) = f(x_2)$

$\frac{\mathbf{4x1+3}}{\mathbf{3x1+4}} = \frac{\mathbf{4x2+3}}{\mathbf{3x2+4}}$

$(4x_1+3)(3x_2+4) = (3x_1+4)\,(4x_2+3)$ $x_1 - x_2 = 0$ $x_1 = x_2$ Hence f is one-one.

Let y∈ R – {-4/3) and $y = \frac{4x+3}{3x+4}$

$3xy + 4y = 4x + 3$ $x = \frac{4y-3}{4-3y}$

Hence f is onto and so bijective

$f^{-1}(y) = \frac{\mathbf{4y-3}}{\mathbf{4-3y}}$ $f^{-1}(0) = -3/4$

$f^{-1}(x) = 2 \Rightarrow \frac{\mathbf{4x-3}}{\mathbf{4-3x}} = 2 \Rightarrow 4x - 3 = 8 - 6x$ $x = 11/10$

6. Show that the relationR in the set A={ x∈ Z: 0≤ x ≤ 12} given by R={|a − b|is a multiple of 4} is an equivalence relation. And the equivalence class of 2 i.e. [2]

Sol :

Reflexive. $|a - a| = 0 \in A$,

Symmetry.

let (a,b)∈R

$|a - b|$is multiple of 4

$= |-(b - a)|$is multiple of 4

$|(b - a)|$is multiple of 4

hence (b,a)∈R

Transitive:

let $(a, b), (b, c) \in R$

then , $|a - b| = 4m$ and $|b - c| = 4n$.

$a - b = \pm 4m$ and $b - c = \pm 4n$

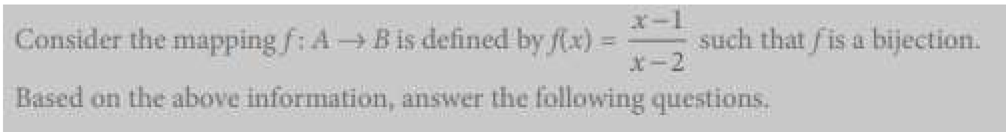
$a - b + b - c = \pm 4m \pm 4n = \pm 4M$

$|a - c|$is multiple of 4

hence (a,c)∈R

[2]= {0, 4, 8, 12}

Case Based Questions:



Consider the mapping $f : A \rightarrow B$ is defined by $f(x) = \frac{x-1}{x-2}$ such that f is a bijection.

Based on the above information, answer the following questions.

i) Find the domain of f

ii) 

If $g : R - \{2\} \rightarrow R - \{1\}$ is defined by $g(x) = 2f(x) - 1$, then $g(x)$ in terms of x is

Pratice Questions:

1. Consider a relation R on N X N, defined by (a,b)R(c,d)⇔a+d = b+c. show that the relation R is an equivalence relation.

2. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R={(a, b): } , is an equivalence relation. Find the equivalence class of [1].

3. Show that the relation on the set A={x∈Z:0≤x≤12} given by R={(a,b):$|a-b|$is divisible by 4} is an equivalence relation Find all elements related to 1, equivalence class [1]

4. Let R be the relation on N×N defined by (a, b) R (c, d) if and only if ad(b+ c) =bc (a+ d), Prove that R is an equivalence relation

5. Show that the relation R defined on the set N×N defined as (a, b)R(c, d) if and only if $a^2+d^2=b^2+c^2$ is an equivalence relation

6. Let A = {1, 2, 3, ... 9} and R be the relation in A ×A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A ×A. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].

7. Show that the relation S in the set R of real numbers defined as S = { (a, b) : a , b εR and a $\leq b^3$} is neither reflexive nor symmetric nor transitive.

8. Show that the relation R in the set Z of integers given by R = {(a, b): 2 divides a – b} is an equivalence relation.

9. Show that the function f: R →R given by $f(x) = x^3 + x$ is a bijection.

10. If f: N → N be a function defined by $f(x) = 9x^2 + 6x - 15$. Check whether f is one-one and onto or not.

11. Show that the function f : R → R defined by $f(x) = \frac{x^2}{1+x^2}$, x ∈ R is neither one-one nor onto function.

12. Show that the function f : R → {x ∈ R : – 1 < x < 1} defined by $f(x) = \frac{1}{1+|x|}$, x ∈ R is one-one and onto function.

CHAPTER 2 – INVERSE TRIGONOMETRIC FUNCTIONS

SOME IMPORTANT RESULTS AND CONCEPTS

FUNCTIONS	DOMAIN	RANGE (Principal Value Branch)
Sin^{-1}:	[-1 1]	[-π/2 π/2]
Cos-^{1}:	[-1 1]	[0 π]
Cosec-^{1}:	R-(-1 1)	[-π/2 π/2] -{0}
Sec^{-1}:	R-(-1 1)	[0 π]- { π/2 }
Tan^{-1}:	R	(-π/2 π/2)
Cot^{-1}:	R	(0 π)

PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

Sin^{-1}(sinx) = x if x Ɛ[-π/2, π/2] Cos-^{1} (cosx)= x if x Ɛ[0, π]

Tan^{-1} (tanx) = x if x Ɛ(-π/2 , π/2) Cosec-^{1}(cosecx)= x if x Ɛ [-π/2, π/2] -{0}

Sec^{-1} (secx) = x , if x Ɛ[0 π]- { π/2 } Cot^{-1}(cotx)= x, if x Ɛ (0 π)

Sin (Sin^{-1} x) = x , if x Ɛ[-1 1] cos(Cos-^{1} x)= x, f x Ɛ[-1 1]

Tan (Tan^{-1} x)= x if x ƐR cosec(Cosec-^{1} x)= x, if x Ɛ [-∞, -1] U [1, ∞)

Sec (Sec^{-1} x)= x if x Ɛ [-∞ -1] U [1 ∞) cot (Cot^{-1} x)= x, if x Ɛ R

sin^{-1}(x) = cosec^{-1}(1/x) & cosec^{-1}(x)= Sin^{-1}(1/x)

cos $^{-1}$(x) = sec^{-1} (1/x) & sec^{-1} (x)= Cos $^{-1}$(1/x)

tan^{-1}(x)= cot^{-1} (1/x) & cot^{-1} (x) = Tan^{-1}(1/x)

sin^{-1}(-x)= - Sin^{-1}x tan^{-1}(-x)= - Tan^{-1}x

cosec^{-1}(-x)= - cosec^{-1}x cos $^{-1}$(-x)= π - Cos $^{-1}$(x)

sec^{-1} (-x)= π - sec^{-1} (x) cot $^{-1}$(-x)= π - Cot $^{-1}$(x)

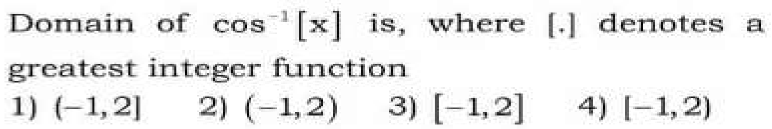
sin^{-1}(x)+ Cos $^{-1}$(x)= π/2 tan^{-1} x+ cot^{-1} x= π/2 sec^{-1} x+ cosec^{-1} x= π/2

2 tan^{-1} x = tan^{-1} (2x/1- x^2) = Cos $^{-1}$ (1- x^2 / 1+ x^2) = tan^{-1} (2x /1+ x^2)

MCQ’s

1. If $\sum_{i=1}^{20} \sin^{-1} x_i = 10\pi$,then $\sum_{i=1}^{20} x_i$ is equal to
   a) 20 b) 10
   c) 0 d) None of these

2.


Domain of $\cos^{-1}[x]$ is, where [.] denotes a greatest integer function

1) $(-1,2]$ 2) $(-1,2)$ 3) $[-1,2]$ 4) $[-1,2)$

3. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ is equal to

a) $-\frac{1}{\sqrt{10}}$ b) $\frac{1}{\sqrt{10}}$

c) $-\frac{1}{10}$ d) $\frac{1}{10}$

4. The value of $\sin\left(\sin^{-1}\frac{1}{3}+\sec^{-1}3\right)+\cos\left(\tan^{-1}\frac{1}{2}+\tan^{-1}2\right)$ is

a) 1 b) 2

c) 3 d) 4

5. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi,$ then$x^4+y^4+z^4+4x^2y^2z^2=k(x^2y^2+y^2z^2+z^2x^2)$ Where k is equal to

a) 1 b) 2

c) 4 d) none of these

6. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi,$ then

a) $x^2+y^2=z^2$ b) $x^2+y^2+z^2=0$

c) $x^2+y^2+z^2=1-2xyz$ d) None of the above

7. If $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\frac{3\pi}{2}$,then $\alpha\beta+\alpha\gamma+\beta\gamma$ is equal to

a) 1 b) 0

c) 3 d) -3

8. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, then the value of $x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$is

a) 0 b) 1

c) 2 d) 3

9. $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$is equal to

a) $\frac{2a}{1+a^2}$ b) $\frac{1-a^2}{1+a^2}$

c) $\frac{2a}{1-a^2}$ d) None of these

10. If $y=\cos(3\cos^{-1}x)+\cos(2\cos^{-1}x)$, $\frac{dy}{dx}=$

a) 4-24x b) -4-24x c) 6x+4 d) -4+24x

11. If $3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1+x}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$, then value of x is

a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$

c) 1 d) None of these

12. One branch of $\cos^{-1}(x)$ other than the principal value branch corresponds to

a) $[\frac{\pi}{2}, 3\frac{\pi}{2}]$ b) $[\pi, 2\pi]$-$\{3\frac{\pi}{2}\}$ c) $(0, \pi)$ d) $[2\pi, 3\pi]$

13. If $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$, then x is

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$

c) $\frac{-1}{2}$ d) None of these

14. If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$, then $\cos^{-1}x+\cos^{-1}y$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$

c) π d) $\frac{3\pi}{4}$

15. $\sin^{-1}x+\sin^{-1}\frac{1}{x}+\cos^{-1}x+\cos^{-1}\frac{1}{x}$ is equal to

a) π b) $\frac{\pi}{2}$

c) $\frac{3\pi}{2}$ d) None of these

16. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$

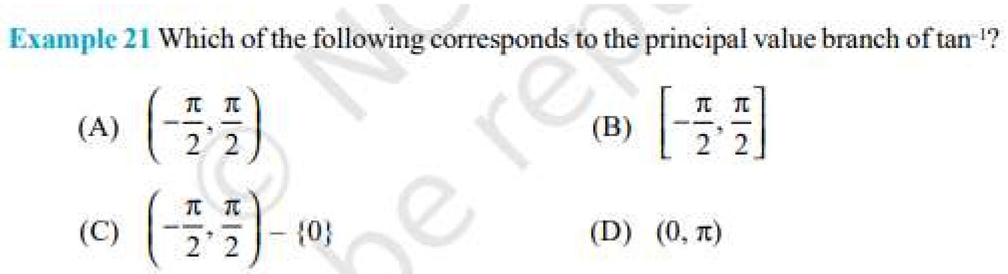
c) $\frac{2\pi}{3}$ d) $\frac{\pi}{4}$

17. $\cos^{-1}\left(\frac{-1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right) + 3\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) - 4\tan^{-1}(-1)$ equals

a) $\frac{19\pi}{12}$ b) $\frac{35\pi}{12}$

c) $\frac{47\pi}{12}$ d) $\frac{43\pi}{12}$

18.



Example 21 Which of the following corresponds to the principal value branch of tan⁻¹?

(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (D) $(0, \pi)$

19. The value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)$ is

a) 45° b) 90°

c) 15° d) 30°

20. $\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$ is equal to

a) 1 b) 5

c) 10 d) 15

21. Which one of the following is correct?

a) $\tan 1 > \tan^{-1} 1$ b) $\tan 1 < \tan^{-1} 1$

c) $\tan 1 = \tan^{-1} 1$ d) None of these

22. If x, y, z are in AP and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in AP, then

a) $x = y = z$ b) $x = y = -z$

c) $x = 1, y = 2, z = 3$ d) $x = 2, y = 4, z = 6$

23. The value of $\cos(2\cos^{-1} 0.8)$ is

a) 0.48 b) 0.96

c) 0.6 d) None of these

24. The value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is

a) $\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}$

c) $\frac{1}{2}$ d) $-\frac{1}{2}$

25. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then x is equal to

a) 0 b) 1/2

c) -1/2 d) 1

26. The value of $\cos^{-1}\left(-\frac{1}{2}\right)$ among the following, is

a) $\frac{9\pi}{3}$ b) $\frac{8\pi}{3}$

c) $\frac{5\pi}{3}$ d) $\frac{11\pi}{3}$

27. If $\cos^{-1} x > \sin^{-1} x$, then

a) $x < 0$ b) $-1 < x < 0$

c) $0 \le x < \frac{1}{\sqrt{2}}$ d) $-1 \le x < \frac{1}{\sqrt{2}}$

28. The domain of the function $f(x) = \frac{\cos^{-1} x}{[x]}$ is

a) $[-1, 0) \cup \{1\}$ b) [-1, 1]

c) [-1, 1) d) None of these

29. The domain of the real valued function $f(x) = \sqrt{1-2x} + 2\sin^{-1}\left(\frac{3x-1}{2}\right)$ is

a) $\left[-\frac{1}{3}, 1\right]$ b) $\left[\frac{1}{2}, 1\right]$
c) $\left[-\frac{1}{2}, \frac{1}{3}\right]$ d) $\left[-\frac{1}{3}, \frac{1}{2}\right]$

30. The domain of the function $\sin^{-}1\left(\log_2 \frac{x^2}{2}\right)$ is
a) [-1, 2]-{0} b) [-2, 2]-(-1, 1)
c) [-2, 2]-{0} d) [1, 2]

31. The domain of $\sin^{-1}(\log_3 x)$ is
a) [-1, 1] b) [0, 1]
c) $[0, \infty]$ d) $\left[\frac{1}{3}, 3\right]$

32. Let $f: (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval
a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
c) $[0, \frac{\pi}{2})$ d) $\left(0, \frac{\pi}{2}\right)$

33. The domain of the function
$f(x) = \frac{\sin^1(x-3)}{\sqrt{9-x^2}}$ is
a) [2, 3] b) [2, 3)
c) [1, 2] d) [1, 2)

34. The domain of the function $f(x) = \sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$ is
a) (-3, 3) b) [-3, 3]
c) $(-\infty, -3) \cup (3, \infty)$ d) $(-\infty, -3] \cup [3, \infty)$

35. The domain of the function $f(x) = \frac{\sqrt{4-x^2}}{\sin^{-1}(2-x)}$ is
a) [0, 2] b) [0, 2)
c) [1, 2) d) [1, 2]

36. The range of the function $\sin(\sin^{-1} x + \cos^{-1} x), |x| \leq 1$ is
a) [-1, 1] b) [1, -1]
c) {0} d) {1}

37. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$
(a) $-\frac{1}{\sqrt{10}}$ (b) $\frac{1}{\sqrt{10}}$ (c) $-\frac{1}{10}$ (d) $\frac{1}{10}$

38. The domain of the function defined by $f(x) = cos^{-1}\sqrt{x-1}$
(a) [1,2] (b) [0,2] (c) [-1,1] (d) [0,1]

39.
$$If\ a + \frac{\pi}{2} < 2tan^{-1}x + 3cot^{-1}x < b, then\ the\ values\ of\ a\ and\ b\ are\ respectively$$
(a) 0 and π (b) 0 and 2π (c) $\frac{\pi}{2}$ and 2 π (d) $\frac{-\pi}{2}$ and $\frac{\pi}{2}$

LEVEL 1: FIND THE PRINCIPAL VALUE OF THE FOLLOWING

1.Sin^{-1}(-1/2) ANS: Sin^{-1} {sin(-π/6)

.i.e - π/6 as this value belongs to the range of Sin^{-1} i.e[-π/2,π/2]

2. $\cos^{-1}(\sqrt{3}/2)$

ANS: $\text{Cos}^{-1}(\cos \pi/6)$ i.e $\pi/6$ whichis belongs to the range of Cos^{-1}i.e $[0, \pi]$

3. $\text{Tan}^{-1}(-\sqrt{3})$

ANS: Let $\text{Tan}^{-1}(-\sqrt{3}) = y$

Then $\tan y = -\sqrt{3}$ and $\tan y = -\tan \pi/3$

$\tan y = \tan(-\pi/3)$ i.e $(-\pi/3)$ which belongs to the range of $\tan^{-1}x$ i.e $(-\pi/2, \pi/2)$

4. $\text{Cos}^{-1}(-1/\sqrt{2})$

ANS: Let . $\text{Cos}^{-1}(-1/\sqrt{2}) = y$ therefore $\cos y = (-1/\sqrt{2})$

$\text{Cos } y = -\cos \pi/4$ but does not belongs to the rage $[0, \pi]$.

Hence, $\cos(\pi - \pi/4) = 3\pi/4$

LEVEL 2: SIMPLIFY THE FOLLOWING

1. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = x/2$, $x \in (0, \pi/4)$

ANS: $\sqrt{(1+\sin x)} = \sqrt{(\sin^2 x/2 + \cos^2 x/2 + 2 \sin x/2 \text{ X } \cos x/2)} =$

$=$

$\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2} = \left|\cos\frac{x}{2}+\sin\frac{x}{2}\right| = \cos\frac{x}{2}+\sin\frac{x}{2}$

Similarly,

$= \sqrt{(1-\sin x)} = \sqrt{(\sin^2 x/2 + \cos^2 x/2 - 2 \sin x/2 . \cos x/2)} = \sqrt{(\cos\frac{x}{2}-\sin\frac{x}{2})^2}$

: simplifying $\cot^{-1}\left(\frac{(\cos\frac{x}{2}+\sin x/2)+(\cos\frac{x}{2}-\sin\frac{x}{2})}{(\cos\frac{x}{2}+\sin x/2)-+(\cos\frac{x}{2}-\sin\frac{x}{2})}\right)$

simplifying $\cot^{-1}(2\cos x/2 \div 2 \sin x/2)$ i.e $\cot^{-1}(\cot x/2) = x/2$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right) = \pi/4 - x/2$

ANS: substitute $x = \cos 2\theta$ therefore $2\theta = \cos^{-1} x$ or $\theta = 1/2 \cos^{-1} x$

$\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$

$1+\cos 2\theta = 1+2\cos^2\theta - 1 = 2\cos^2\theta$ and $1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$

$\tan^{-1}\{(\sqrt{2}\cos^2\theta + \sqrt{2}\sin^2\theta)/(\sqrt{2}\cos^2\theta - \sqrt{2}\sin^2\theta)\}$

$\tan^{-1}\{(\cos\theta - \sin\theta)/(\cos\theta + \sin\theta)\}$

dividing the numerator and denominator by $\cos\theta$

we get $\tan^{-1}\{(1-\tan\theta)/(1+\tan\theta)\}$

$\tan^{-1}(\tan(\pi/4 - \theta)$ i.e$\pi/4 - \theta$

$\pi/4 - 1/2 \cos^{-1} x$

3. If $\sin(\sin^{-1} 1/5 + \cos^{-1} x) = 1$, find the x .

ANS: $\sin^{-1} 1/5 + \cos^{-1} x = \sin^{-1} 1$

$\sin^{-1} 1/5 + \cos^{-1} x = \pi/2$, comparing with $\sin^{-1} x + \cos^{-1} x = \pi/2$ $x = 1/5$

4) If $\sin^{-1} x + \sin^{-1} y = 2\pi/3$, then find the value of $\cos^{-1} x + \cos^{-1} y$

Solution: $(\pi/2 - \cos^{-1} x) + (\pi/2 - \cos^{-1} x) = 2\pi/3$, implies $\cos^{-1} x + \cos^{-1} y = \pi/3$

5) Find the domain of $\sin^{-1}(x^2 - 4)$

Solution: $-1 \le x^2 - 4 \le 1 \quad \Rightarrow \quad 3 \le x^2 \le 5 \quad \Rightarrow \quad x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

6) Find the value of $\sin^{-1} (\cos 33\pi/5)$

Solution: $\sin^{-1} (\cos 33\pi/5) = \sin^{-1} (\cos (6\pi + 3\pi/5)) = \sin^{-1} (\cos (3\pi/5)) = \pi/2 - \cos^{-1} (\cos (3\pi/5))$

$= \pi/2 - 3\pi/5 = -\pi/10$

Case Based Questions:

The domain and range of inverse circular function are defined as follows **:**

	Domain	**Range**
$\sin^{-1}x$	$[-1, 1]$	$\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(i) If $\sin^{-1}x < \frac{3\pi}{4}$ then find the solution set of x (A) $\left(\frac{1}{\sqrt{2}}, 1\right]$

(ii) If $\sin^{-1}x + \text{cosec}^{-1}x$ at $x = -1$ is then find the value of x. (A) 3π

(iii) If $x \in [-1, 1]$, then find the range of $\tan^{-1}(-x)$. (A) $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$

Practice Questions:

Each question carries 2 marks

1) Find the principal value of $\cos^{-1} (\cos(-680^0))$
2) Find the value of $\tan^2 (\sec^{-1}2) + \cot^2 (\text{cosec}^{-1}3)$
3) If $\cos^{-1}x > \sin^{-1}x$, then find x
4) Find the domain of $\cos x + \sin^{-1}x$
5) If $\sin^{-1}x + \sin^{-1}y = \pi$, then find $x^{2023} + y^{2023}$
6) Find the value of $\cot (\sin^{-1}x)$
7) Find the value of $\sin(\cos^{-1}(-3/5))$
8) Find the value of $\sin^{-1}(\cos(\sin^{-1} 1/2))$
9) Find the domain of $\sin^{-1}2x$
10) If $\tan^{-1}x = \pi/10$, then find the value of $\cot^{-1}x$

11) Write the value of $\mathbf{\tan^{-1} 2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)}$

12) Prove that $\text{sce}^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3) = 15$

13) Express $\mathbf{\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}}$ in the simplest form

14) Write the simplest form of $\mathbf{\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)}$

15) Write the simplest form of $\mathbf{\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)}$

16) Find the value of $\cot(\cos^{-1} x)$

17) Find the value of $\cos^{-1}(\cos 10)$

18) If $\cos^{-1}(-5/12) = \tan^{-1} x$, then find the value of x

19) Find the domain of $\tan^{-1}(3x + 4)$

20) Find the principal value of $2\cos^{-1}(1) + \sin^{-1}(1/\sqrt{2})$

21) The solution set of $\sin^{-1} x \leq \cos^{-1} x$

22) Find the value of $\cos(\pi/3 + \cos^{-1}(-1))$

23) Find the value of $\sin^{-1}(\sin 5\pi/3)$

24) Find the value of $\cos(\tan^{-1}\{\cot(\sin^{-1} 1/2)\})$

25) Write the value of $\sin(\pi/3 - \sin^{-1}(-1/2))$.

Answers

1) $2\pi/9$ 2) 11 3) $x \in (-1, 1/\sqrt{2})$ 4) $[-1, 1]$ 5) 2

6) $\frac{\sqrt{1-x^2}}{x}$ 7) $-24/25$ 8) $\pi/3$ 9) $[-1/2, 1/2]$ 10) $2\pi/5$

11) $\pi/3$ 12) $\frac{x}{\sqrt{1-x^2}}$ 13) $\frac{1}{2}\tan^{-1}x$ 14) $3\tan^{-1} x$

15) $\pi/4 - x/2$ 16) $4\pi - 10$ 17) No solution 18) R 19) $5\pi/4$

20) $(-1/\sqrt{2}, 1/\sqrt{2})$ 21) $4\pi/3$ 22) $-\pi/3$ 23) $1/2$ 24) 1

25)

CHAPTER: MATRICES

SYLLABUS: Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Definitions and Formulae:

- Matrix representation and order of matrix
- Types of Matrices
- Operations on Matrices
- Transpose of a Matrix
- Symmetric and Skew Symmetric Matrices
- Invertible Matrices

Order of Matrix:

If a matrix has $mrowsandncolumns$, then it is known as the matrix of order $m \times n$.

Representation of matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{pmatrix}$$

A general matrix of order $m \times n$ can be written as

$$= [a_{ij}]_{m\times} ,wherei = 1,2, \ldots mandj = 1,2, \ldots n$$

Types of Matrices:

Depending upon the order and elements, matrices are classified as:

- Column matrix
- Row matrix
- Square matrix
- Diagonal matrix
- Scalar matrix
- Identity matrix
- Zero matrix

Type of Matrix	Definition	Example
COLUMN MATRIX	A matrix is said to be a column matrix if it has only one column	$\begin{bmatrix} 2 \\ -9 \end{bmatrix}$ $Order 2 \times 1$ $\begin{bmatrix} -\sqrt{5} \\ 0 \\ -12 \end{bmatrix}$ $order\ 3 \times 1$
ROW MATRIX	A matrix is said to be a row matrix if it has only one row	$[14 \quad 26]$order1×2 $[0 \quad \sqrt{7} \quad 12]$ order1×3
SQUARE MATRIX	A matrix in which the number of rows is equal to the number of columns, is said to be a square matrix.	$\begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix}$ $\begin{bmatrix} 5 & 0 & -8 \\ 0 & 1 & 14 \\ 7 & -8 & 4 \end{bmatrix}$
DIAGONAL MATRIX	A square matrix A is said to be a diagonal matrix if all its non-diagonal elements are zero.	$\begin{bmatrix} 6 & 0 & 0 \\ 0 & \sqrt{6} & 0 \\ 0 & 0 & 9 \end{bmatrix}$

SCALAR MATRIX	A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal	$\begin{bmatrix}5 & 0 & 0\\0 & 5 & 0\\0 & 0 & 5\end{bmatrix}$
IDENTITY MATRIX	A square matrix in which all the elements in the diagonal are all equal to one and rest are all zero is called an identity matrix. And generally it is denoted by I.	$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
ZERO MATRIX	A matrix is said to be zero matrix or null matrix if all its elements are zero.	$A=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$ $B=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$

OPERATION OF MATRICES:

ADDITION OF MATRICES: Let $A = [a_{ij}]_{m\times n} and B = [b_{ij}]_{m\times n}$ be two matrices of the same order. Then $A + B$ is defined to be the matrix of order of $m \times n$ obtained by adding corresponding elements of $A and B$ i.e $A + B = [a_{ij} + b_{ij}]_{m\times}$.

DIFFERENCE OF MATRICES:

Let $A = [a_{ij}]_{m\times n} and B = [b_{ij}]_{m\times n}$ be two matrices of the same order. Then $A - B$ is defined to be the matrix of order of $m \times n$ obtained by subtracting corresponding elements of $A and B$ i.e $A - B = [a_{ij} - b_{ij}]_{m\times}$.

MULTIPLICATION OF MATRICES:

The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let $A= [a_{ij}]_{m\times n} and B = [b_{jk}]_{n\times}$. Then the product of the matrices A and B is the matrix C of order $m \times p$.

Example: Let $A = \begin{bmatrix}2 & 3 & 5\\1 & 6 & 8\end{bmatrix}$ $and\ B = \begin{bmatrix}4 & 3\\6 & 9\\5 & 8\end{bmatrix}$

$$AB = \begin{bmatrix}2\times 4+3\times 6+5\times 5 & 2\times 3+3\times 9+5\times 8\\1\times 4+6\times 6+8\times 5 & 1\times 3+6\times 9+8\times 8\end{bmatrix}$$

$$= \begin{bmatrix}8+18+25 & 6+27+40\\4+36+40 & 3+54+64\end{bmatrix} = \begin{bmatrix}51 & 73\\80 & 121\end{bmatrix}$$

MULTIPLICATION OF A MATRIX BY A SCALAR:

Let $A = [a_{ij}]_{m\times n}$ and k is a scalar, then $kA = k\,[a_{ij}]_{m\times n} = [k.\ a_{ij}]_{m\times n}$

Example: $\boldsymbol{A} = \begin{bmatrix}\mathbf{2} & \mathbf{4} & \mathbf{-5}\\ \boldsymbol{y} & \boldsymbol{z} & \boldsymbol{x}\end{bmatrix} \Rightarrow \mathbf{3}\boldsymbol{A} = \begin{bmatrix}\mathbf{3(2)} & \mathbf{3(4)} & \mathbf{3(-5)}\\ \mathbf{3}\boldsymbol{y} & \mathbf{3}\boldsymbol{z} & \mathbf{3}\boldsymbol{x}\end{bmatrix} =$

$\begin{bmatrix}\mathbf{6} & \mathbf{12} & \mathbf{-15}\\ \mathbf{3}\boldsymbol{y} & \mathbf{3}\boldsymbol{z} & \mathbf{3}\boldsymbol{x}\end{bmatrix}$

TRANSPOSE OF A MATRIX: If $A = [a_{ij}]_{m\times n}$ be an $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. Transpose of the matrix A is denoted by A'or AT.

If $A = [a_{ij}]_{m\times n}$, then $A' = [a_{\boldsymbol{j}}]_{n\times m}$

Example: $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 7 & 9 \\ 5 & 1 & 0 \end{pmatrix}$

$A^T = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 7 & 1 \\ 3 & 9 & 0 \end{pmatrix}$

SYMMETRICMATRIX: AsquarematrixIfA=[a_{ij}]issaidtobesymmetricif A^T=A

Example: $\begin{pmatrix} \mathbf{0} & \mathbf{5} & \mathbf{12} \\ \mathbf{5} & \mathbf{0} & \mathbf{3} \\ \mathbf{12} & \mathbf{3} & \mathbf{0} \end{pmatrix}$

SKEW-SYMMETRICMATRIX: Asquarematrix$A$=[a_{ij}]issaidtobeskew

symmetricmatrixifA^T=$-A$.

Example: $\begin{pmatrix} \mathbf{0} & \mathbf{5} & \mathbf{-12} \\ \mathbf{-5} & \mathbf{0} & \mathbf{-3} \\ \mathbf{12} & \mathbf{3} & \mathbf{0} \end{pmatrix}$

INVERTIBLE MATRICES: If A is a square matrix of order m, and if there existsanother square matrix B of the same order m, such that AB = BA = I, then B is calledtheinversematrixofAand it isdenoted byA^{-1}.In that caseAis said tobeinvertible.

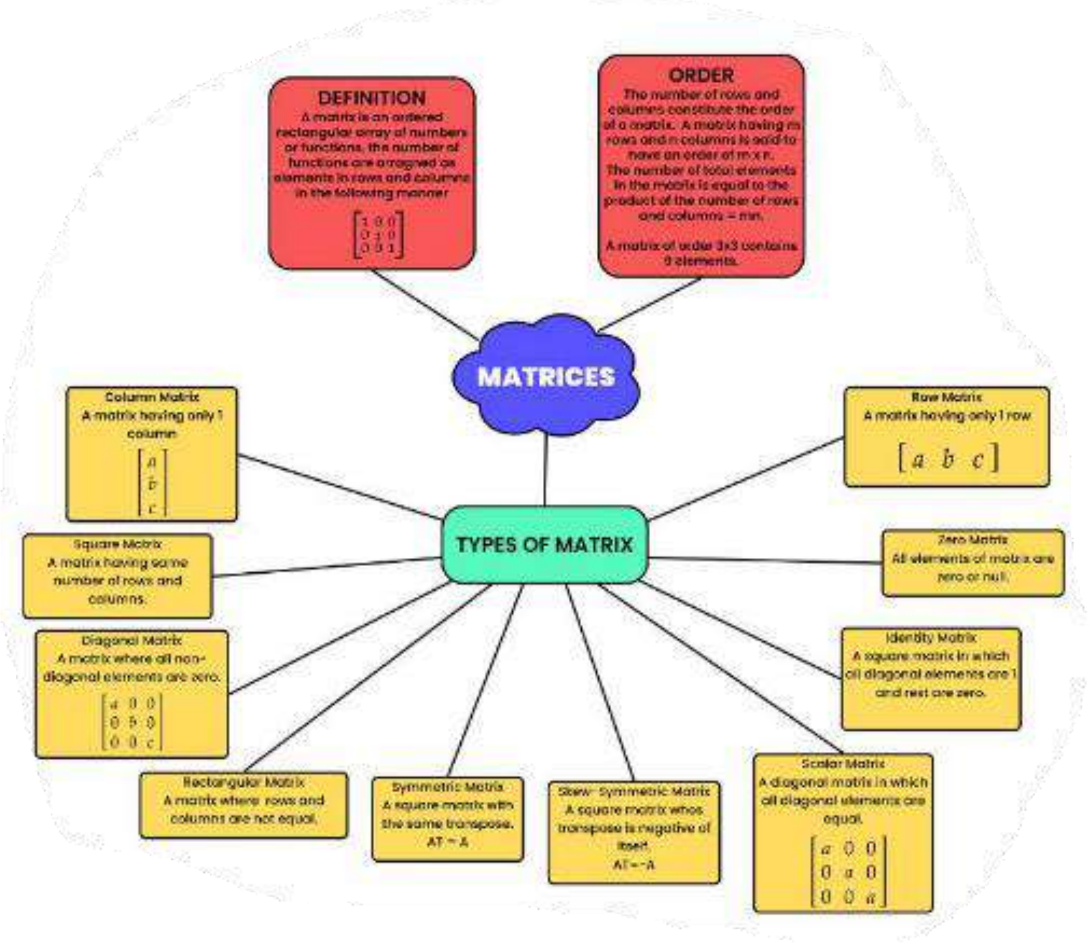
I PROPERTIES OF MATRICES:

- $A+B=B+A$
- $A-B\neq B-A$
- $AB\neq BA$
- $(AB)C=A(BC)$
- $(A')'=A$
- $AI=IA=A$
- $AB=BA=I, then A^{-1}=B and B^{-1}=A$

- $AB=0\Longrightarrow itisnotnecessarythatoneofthematrixiszero.$
- $A(B+C)=AB+AC$
- Every square matrix can possible to express as the sum of symmetric andskew-symmetricmatrices.

$A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$, where $\frac{1}{2}(A+A')$ is symmetric matrix and $\frac{1}{2}(A-A')$ is skew-symmetric matrices.

MIND MAPPING



DEFINITION
A matrix is an ordered rectangular array of numbers or functions, the number of functions are arragned as elements in rows and columns in the following manner

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

ORDER
The number of rows and columns constitute the order of a matrix. A matrix having m rows and n columns is said to have an order of m x n.
The number of total elements in the matrix is equal to the product of the number of rows and columns = mn.

A matrix of order 3x3 contains 9 elements.

Column Matrix
A matrix having only 1 column

$$\begin{bmatrix}a\\b\\c\end{bmatrix}$$

Row Matrix
A matrix having only 1 row

$$\begin{bmatrix}a&b&c\end{bmatrix}$$

Square Matrix
A matrix having same number of rows and columns.

Zero Matrix
All elements of matrix are zero or null.

Diagonal Matrix
A matrix where all non-diagonal elements are zero.

$$\begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}$$

Identity Matrix
A square matrix in which all diagonal elements are 1 and rest are zero.

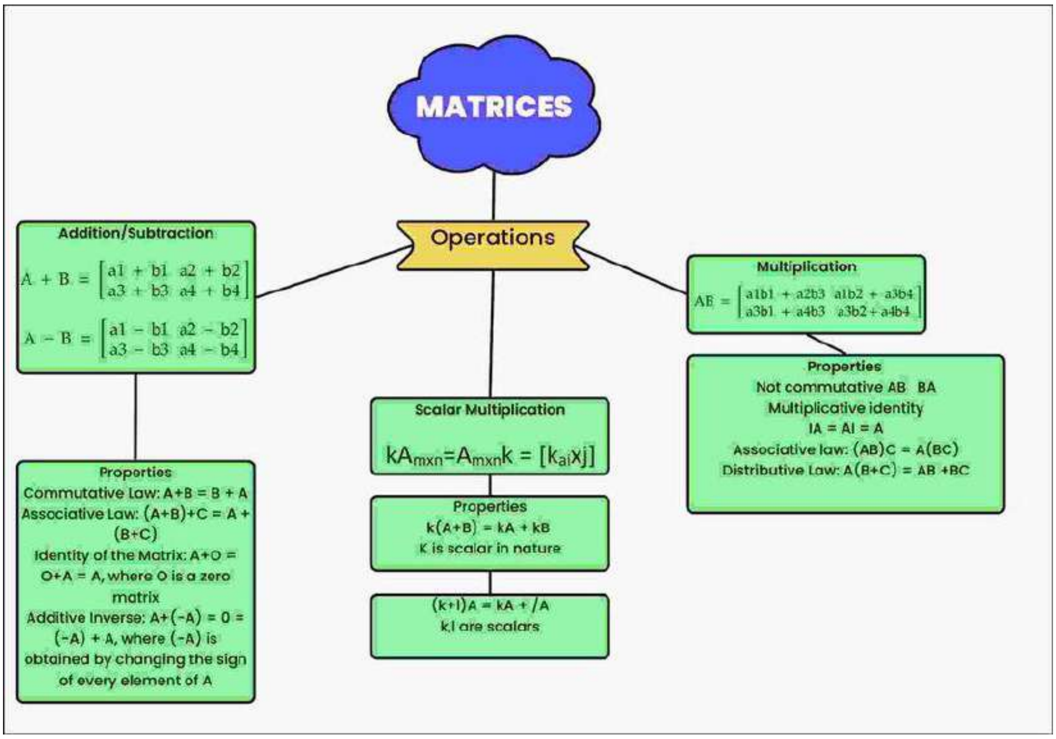
Rectangular Matrix
A matrix where rows and columns are not equal.

Symmetric Matrix
A square matrix with the same transpose.
AT = A

Skew- Symmetric Matrix
A square matrix whos transpose is negative of itself.
AT = -A

Scalar Matrix
A diagonal matrix in which all diagonal elements are equal.

$$\begin{bmatrix}a&0&0\\0&a&0\\0&0&a\end{bmatrix}$$



MATRICES

Operations

Addition/Subtraction

$A + B = \begin{bmatrix} a1 + b1 & a2 + b2 \\ a3 + b3 & a4 + b4 \end{bmatrix}$

$A - B = \begin{bmatrix} a1 - b1 & a2 - b2 \\ a3 - b3 & a4 - b4 \end{bmatrix}$

Properties
Commutative Law: A+B = B + A
Associative Law: (A+B)+C = A + (B+C)
Identity of the Matrix: A+O = O+A = A, where O is a zero matrix
Additive Inverse: A+(-A) = 0 = (-A) + A, where (-A) is obtained by changing the sign of every element of A

Scalar Multiplication

$kA_{mxn}=A_{mxn}k = [k_{ai}xj]$

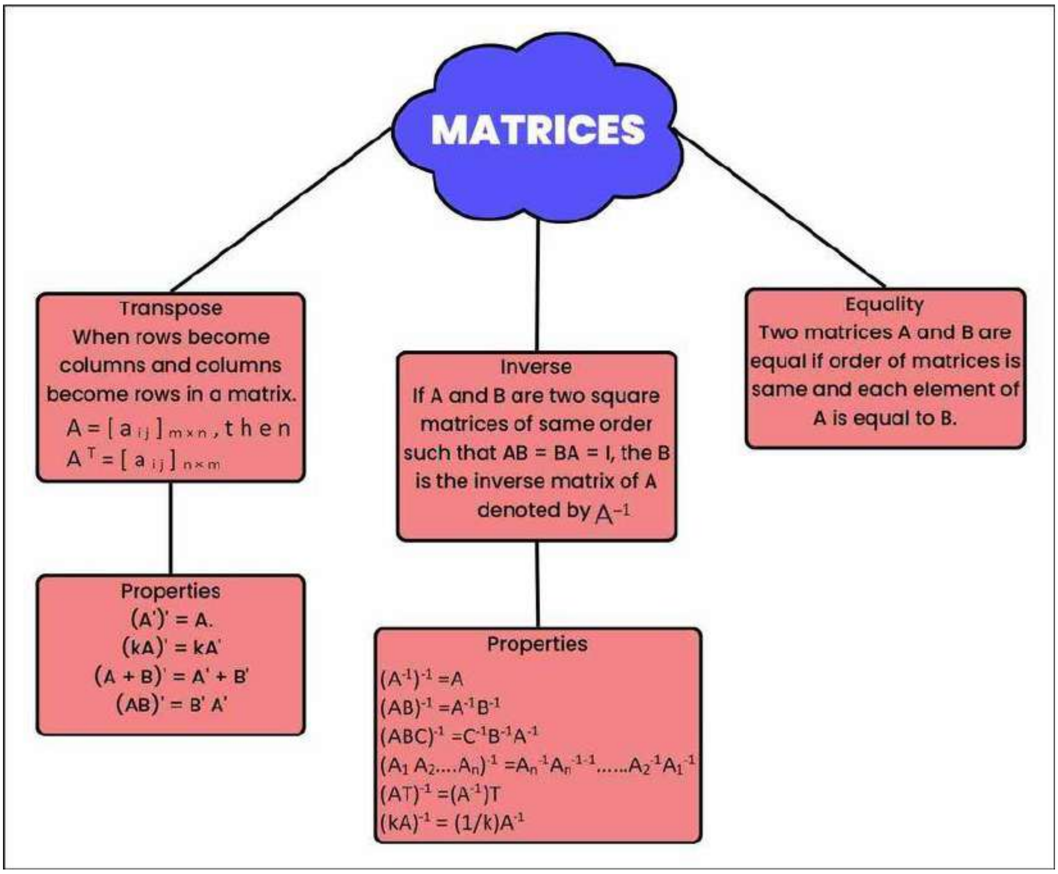
Properties
k(A+B) = kA + kB
K is scalar in nature

(k+l)A = kA + /A
k,l are scalars

Multiplication

$AB = \begin{bmatrix} a1b1 + a2b3 & a1b2 + a3b4 \\ a3b1 + a4b3 & a3b2 + a4b4 \end{bmatrix}$

Properties
Not commutative AB BA
Multiplicative identity
IA = AI = A
Associative law: (AB)C = A(BC)
Distributive Law: A(B+C) = AB +BC



MATRICES

Transpose
When rows become columns and columns become rows in a matrix.
$A = [a_{ij}]_{m \times n}$, then
$A^T = [a_{ij}]_{n \times m}$

Properties
(A')' = A.
(kA)' = kA'
(A + B)' = A' + B'
(AB)' = B' A'

Inverse
If A and B are two square matrices of same order such that AB = BA = I, the B is the inverse matrix of A denoted by A^{-1}

Properties
$(A^{-1})^{-1} =A$
$(AB)^{-1} =A^{-1}B^{-1}$
$(ABC)^{-1} =C^{-1}B^{-1}A^{-1}$
$(A_1 A_2....A_n)^{-1} =A_n^{-1}A_n^{-1-1}......A_2^{-1}A_1^{-1}$
$(AT)^{-1} =(A^{-1})T$
$(kA)^{-1} = (1/k)A^{-1}$

Equality
Two matrices A and B are equal if order of matrices is same and each element of A is equal to B.

MULTIPLE CHOICE QUESTIONS

1) If $\begin{pmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{pmatrix}$ is a scalar matrix , then the value of a +2b+3c+4d is

A)0 B)5 C)10 D)25

Solution:

In scalar matrix $a_{ij}=0$ if $i \neq j$ and $a_{ij}=k$ if $i=j$.

b=c=0 and a=d=5

a +2b+3c+4d = 5+2(0)+3(0) +4(5) =25

Ans: D

2)If A=$\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ then the value of I-A +A^2 –A^3+… is

A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ B)$\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$ C)$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ D)$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Solution:

$A^2 = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$A^2 = A^3 = \ldots = O$

$I-A+A^2-A^3+\ldots = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$

Ans: A

3) Given that [1 x] $\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix}$ = O, then the value of x is

A) -4 B)=-2 C)2 D)4

Solution:

[1x4+x(-2) 1x0 + x(0)] = O

4-2x=0

x=2

Ans:C

4) If A = $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then A^{10} is

A) 10A B)9A C)2^9A D) 2^{10} A

Solution :

$A^{2=}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ = 2I, A^3 =4I=2^2I … and so A^{10}=2^9 I

Ans:C

5) The number of all possible matrices of order 2 × 3 with each entry 0 or 1 is

A) 32 B) 64 C) 512 D) none of these

Solution:

There are six entries. Each can be filled in two ways.

Total number of possible matrices= 2^6=64 ways

Ans: B

6) If A= $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and A^2+7I = kA, then the value of k is

A)1 B) 2 C) 5 D) 7

Solution:

$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$A^2 + 7I = kA$

$\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = k\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} = \begin{bmatrix} 3k & k \\ -k & 2k \end{bmatrix}$

k=5

Ans: C

Assertion – Reason Based Questions

Questions No. 7 and 8 are Assertion(A) and Reason (R) basquestions. Choose the correct answer out of the following choices.

A) Both(A) and (R) are true and (R) is the correct explanation of (A).

B) Both(A) and (R) are true and (R) is not the correct explanation of (A)

C) (A) is true and (R) is false

D) (A) is false and (R) is true

7) Assertion (A): For any two matrices of the same order, $(A + B)^T = A^T + B^T$.

Reason (R): For any two matrices such that AB is defined, then $(AB)^T = A^T B^T$

Solution:

By definition assertion is true and reason is false.

Ans: C

8) Assertion (A):If A is a square matrix and $A^2 = A$, then$(A + I)^3 - 7A = I$

Reason (R): $AI = I = AI$ where I is unit matrix.

Solution:

$(I + A)^3 - 7A = I^3 + 3A^2 + 3A + A^3 - 7A = I + 3A + 3A + A - 7A = I$.

Since $A^2 = A$ and $A^3 = A$ So both the statements are correct. Second statement is the correct explanation for first. **Ans**: A

2 Mark Questions

1) If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$ Find x and y

Solution:

Equating the corresponding entries, we get

$x + 3 = 5,\ y - 4 = 3,\ x + y = 9$

Ans: $x = 2$ and $y = 7$

2) If $\begin{pmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{pmatrix}$ is a symmetric , then find a and b

Solution:

A=A^T (*Since* symmetric)

$$\begin{pmatrix}0 & 2b & -2\\ 3 & 1 & 3\\ 3a & 3 & -1\end{pmatrix}=\begin{pmatrix}0 & 3 & 3a\\ 2b & 1 & 3\\ -2 & 3 & -1\end{pmatrix}$$

Equating the corresponding entries, we get

Ans: a= -2/3 and b= 3/2

3) If A=$\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}$ and A+ A^T =I. then find x

Solution :

A+ A^T =I

$$\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}+\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}2\cos x & 0\\ 0 & 2\cos x\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

2cosx = 1

cosx= ½

Ans:x=$\pi/3$

4) Let A=$\begin{bmatrix}p & 0\\ 1 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$. Find the value of p, if A^2=B.

Solution: A^2=B.

$$\begin{bmatrix}p & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}p & 0\\ 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$$

$$\begin{bmatrix}p^2 & 0\\ p+1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}.$$

p^2=1 and p+1 = 5 .

p = ±1 and p = 4

Ans:p has no common value.

5) Construct a 2x3 matrix whose elements in ith row and jthcolumn are given by

$a_{ij}=\begin{cases} i-j, i\geq j\\ i+j,\ i<j\end{cases}$

Solution: a_{11}=0, a_{12}= 3, a_{13}=4

a_{21}=1, a_{22}= 0, a_{23}=5

Ans: $\begin{bmatrix}0 & 3 & 4\\ 1 & 0 & 5\end{bmatrix}$

6) If A and B is symmetric matrices of same order, show that AB is symmetric iff$AB = BA$

Solution: $(AB)^T=B^T A^T$ = BA

$(AB)^T$ = AB iff AB = BA

7) If A=$\begin{pmatrix}1 & 2 & x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ and B=$\begin{pmatrix}1 & -2 & y\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ and AB is identity matrix of order 3x3, find x+y.

Solution:

$$\begin{pmatrix}1 & 2 & x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & -2 & y\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 0 & x+y\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Ans: x+y=0

8) If $\begin{bmatrix}2a+b & a-2b\\ 5c-d & 4c+3d\end{bmatrix}=\begin{bmatrix}4 & -3\\ 11 & 24\end{bmatrix}$ Find a+b-c+2d

Solution:

Equating the corresponding entries, we get

$2a+b = 4$

$a-2b=-3$

Solving these two equations, we get , $a=1$, $b=2$

$5c-d=11$

$4c+3d=24$

Solving these two equations, we get, $c=3$, $d=4$

Ans: $a+b-c+2d= 8$

3 Mark Questions

1) Express A = $\begin{bmatrix} 7 & -3 \\ 5 & -4 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

Solution: P = $\frac{1}{2}(A + A^T) = \frac{1}{2}\left[\begin{bmatrix} 7 & -3 \\ 5 & -4 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ -3 & -4 \end{bmatrix}\right] = \frac{1}{2}\begin{bmatrix} 14 & 2 \\ 2 & -8 \end{bmatrix}$

$P^T = \frac{1}{2}\begin{bmatrix} 14 & 2 \\ 2 & -8 \end{bmatrix}$ = P, P is a symmetric matrix

Q = $\frac{1}{2}(A - A^T) = \frac{1}{2}\left[\begin{bmatrix} 7 & -3 \\ 5 & -4 \end{bmatrix} - \begin{bmatrix} 7 & 5 \\ -3 & -4 \end{bmatrix}\right] = \frac{1}{2}\begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$

$Q^T = \frac{1}{2}\begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix} = -Q$, Q is a skew symmetric matrix

$P + Q = \frac{1}{2}\begin{bmatrix} 14 & 2 \\ 2 & -8 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix} = \frac{1}{2}\left[\begin{bmatrix} 14 & 2 \\ 2 & -8 \end{bmatrix} + \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}\right] = \frac{1}{2}\begin{bmatrix} 14 & -6 \\ 10 & -8 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ 5 & -4 \end{bmatrix} = A$

Thus A represented as the sum of symmetric and skew symmetric matrices.

2) If $A=\begin{pmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{pmatrix}$ is a skew symmetric matrix, then find a and b

Solution:

Since A is a skew symmetric matrix $A^T= -A$

$$\begin{pmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{pmatrix} = -\begin{pmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{pmatrix}$$

Equating the corresponding entries, we get

Ans: $a= - 2$ and $b = 3$

3) Show that A= $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 I = 0$

Solution:

$A^2 = \left[\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}\right] = \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix}$

$6A == \begin{bmatrix} 12 & -18 \\ 18 & 24 \end{bmatrix}$ and $17I = = \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$

$A^2 - 6A + 17 I = \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix} - \begin{bmatrix} 12 & -18 \\ 18 & 24 \end{bmatrix} + \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix} = 0$

∴ A satisfies the equation $x^2 - 6x + 17 I = 0$

4) A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds,if the trust fund must obtain an annual total interest of Rs 1,800.

Solution:

Let Investment in 1st bond = Rs x

So, Investment in 2nd bond = Rs 30,000−x

$\begin{bmatrix} x & 30000-x \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [1800]$

$x=15000$

∴ The trust must be invested Rs 15000 in each bond to obtain an annual interest of Rs.1800.

5) If X+Y $=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and X − Y$=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, Find matrices X and Y.

Solution:

Adding both, we get

$2X=\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$X=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Substituting X in any one of the equation we get,

$Y=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Ans: $X=\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$ and $Y=\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

6) Let A and B be symmetric matrices of the same order, then show that

(i) $A + B$ is symmetric

(ii) $AB - BA$ is skew symmetric

(iii) $AB + BA$ is symmetric

Solution: $A^T = A$, $B^T = B$

(i) $(A + B)^T = A^T + B^T = A + B$

$\therefore A + B$ is symmetric

(ii) $(AB - BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA)$

$\therefore AB - BA$ is skew symmetric

(iii) $(AB + BA)^T = (AB)^T + (BA)^T = B^T A^T + A^T B^T = BA + AB = (AB + BA)$ $\therefore AB + BA$ is symmetric

7) Find the matrix X such that

$X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}=\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Solution: As per Matrix multiplication Rule , the order of the matrix X should be 2x2.

Let X be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}=\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix}=\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

a+4b=-7,2a+5b=-8,

Solving these two equations, we get, a=1, b=-2.

c+4d=2, 2c+5d=4

Solving these two equations, we get, c=2, d=0.

Ans: a=1, b=−2, c=2, d=0.

8) Let $A=\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$. Verify $(AB)^T=B^T A^T$

Solution:

$AB=\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 22 & 16 \\ 30 & 22 \end{bmatrix}$, $(AB)^T=\begin{bmatrix} 22 & 30 \\ 16 & 22 \end{bmatrix}$,

$B^T A^T=\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix},\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 22 & 30 \\ 16 & 22 \end{bmatrix}$

Hence Verified.

5 Mark Questions

1)Find the value of x, if $[x\ \ -5\ \ -1]\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=O$.

Hence find $[x\ \ -5\ \ -1]\begin{bmatrix}x\\4\\1\end{bmatrix}$.

Solution:

$[x\ \ -5\ \ -1]\begin{pmatrix}1&0&2\\0&2&1\\2&0&3\end{pmatrix}=[x-2\ \ \ -10\ \ \ 2x-8]$

$[x-2\ \ \ -10\ \ \ 2x-8]\begin{bmatrix}x\\4\\1\end{bmatrix}=[x^2-48]$

$[x^2-48]=0$

$x=\pm4\sqrt{3}$

$[x\ \ -5\ \ -1]\begin{bmatrix}x\\4\\1\end{bmatrix}=[x^2-20-1]=[48-20-1]=[27]$

2) Find a matrix A such that $\begin{bmatrix}2&-1\\1&0\\-3&4\end{bmatrix}A=\begin{pmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{pmatrix}$

Solution:

As per Multiplication Rule , the order of the matrix A should be 2x3.

Let $A=\begin{bmatrix}x&y&z\\a&b&c\end{bmatrix}$

$\begin{bmatrix}2&-1\\1&0\\-3&4\end{bmatrix}\begin{bmatrix}x&y&z\\a&b&c\end{bmatrix}=\begin{pmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{pmatrix}$

$\begin{pmatrix}2x-a&2y-b&2z-c\\x&y&z\\-3x+4a&-3y+4b&-3z+4c\end{pmatrix}=\begin{pmatrix}-1&-8&-10\\1&-2&-5\\9&22&15\end{pmatrix}$

Equating the corresponding entries, getting equations

2x-a=-1, x=1, -3x+4a=9, 2y-b=-8,y=-2,-3y+4b=22

2z-c=-10, z=-5, -3z+4c=15

x = 1, y = -2, z = -5

a = 3, b = 4, c = 0

$\therefore A=\begin{bmatrix}1&-2&-5\\3&4&0\end{bmatrix}$

3)Express $\begin{pmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{pmatrix}$ as the sum of symmetric and skew symmetric matrices.

Solution:

$P=\frac{1}{2}(A+A^T)=\frac{1}{2}\left[\begin{pmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{pmatrix}+\begin{pmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{pmatrix}\right]=\frac{1}{2}\begin{pmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{pmatrix}$

$P^T=\frac{1}{2}\begin{pmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{pmatrix}=P$, P is a symmetric matrix

$Q=\frac{1}{2}(A-A^T)=\frac{1}{2}\left[\begin{pmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{pmatrix}-\begin{pmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{pmatrix}\right]=\frac{1}{2}\begin{pmatrix}0&-5&-3\\5&0&-6\\3&6&0\end{pmatrix}$

$Q^T=\frac{1}{2}\begin{pmatrix}0&5&3\\-5&0&-6\\-3&6&0\end{pmatrix}=-\frac{1}{2}\begin{pmatrix}0&-5&-3\\5&0&-6\\3&6&0\end{pmatrix}=-Q$, Q is a skew symmetric matrix

$P+Q=\frac{1}{2}\begin{pmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{pmatrix}+\frac{1}{2}\begin{pmatrix}0&-5&-3\\5&0&-6\\3&6&0\end{pmatrix}=\frac{1}{2}\begin{pmatrix}6&-4&-8\\6&-4&-10\\-2&2&4\end{pmatrix}=\begin{pmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{pmatrix}=A$

Hence proved

Case-Study Questions

1)Ashish wants to purchase a rectangular plot from his neighbour to construct a house. He asked about the dimensions of the plot , his neighbour told that if the length is decreased by 20m and breadth is increased by 30 m ,the area will increase by 1400m^2 ,
but If the length is decreased by 50m and the breadth is increased by 50m, then the area will remain the same.

Based on the information given above, answer the following questions

(i) Let $xandy$ denote the length and breadth of the plot, find equations in terms of x .

Solution:

$(x - 20)(y + 30) = xy + 1400 \quad (x - 50)(y + 50) = xy$

on simplification we get

$3x - 2y = 200$ and $x - y = 50$

(ii) Represent the information in matrix form.

Solution: $3x - 2y = 200\ andx - y = 50$

So$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 200 \\ 50 \end{bmatrix}$

(iii) If A=$\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$, Find AA^T

(OR)

If P=$\begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$ and Q =$\begin{bmatrix} 200 \\ 50 \end{bmatrix}$Find PQ and QP.

Solution:

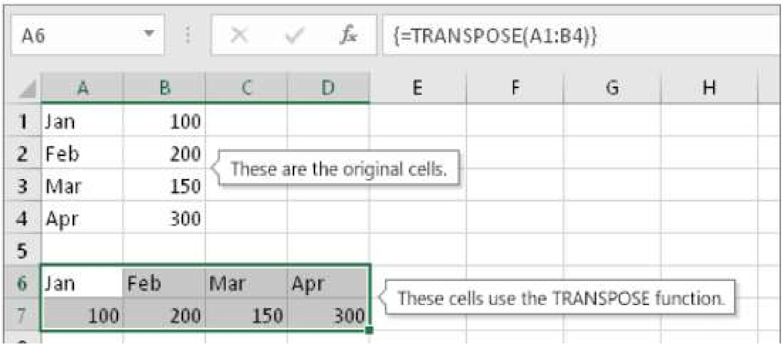
$$AA^T= \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & -1 \end{bmatrix}=\begin{bmatrix} 13 & 5 \\ 5 & 2 \end{bmatrix}$$

(OR)

$$PQ = \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 200 \\ 50 \end{bmatrix}=\begin{bmatrix} 150 \\ 500 \end{bmatrix}.$$

Number of columns in Q is not equal to number of rows in P. So QP is not possible.

2) While working with excel, we need to switch or rotate cells. You can do this by copying, pasting, and using the Transpose option. But doing that creates duplicated data. If you don't want that, you can type a formula instead using the TRANSPOSE function. For example, in the following picture the formula =TRANSPOSE(A1:B4) takes the cells A1 through B4 and arranges them horizontally.



A6 | {=TRANSPOSE(A1:B4)}

	A	B	C	D	E	F	G	H
1	Jan	100						
2	Feb	200						
3	Mar	150						
4	Apr	300						
5								
6	Jan	Feb	Mar	Apr				
7	100	200	150	300				

These are the original cells.

These cells use the TRANSPOSE function.

i. A square matrix A is expressed as sum of symmetric and skew symmetric matrices, and Find symmetric part of A.
Solution:
½(A+A^T)

ii. A square matrix A is expressed as sum of symmetric and skew symmetric matrices, and find the skew-symmetric part of A .
Solution:
½(A − A^T)

iii. Symmetric part of A= $\begin{pmatrix} 1 & 2 & 5 \\ 3 & 2 & 1 \\ 4 & 5 & 7 \end{pmatrix}$

(OR)

Skew Symmetric part of A = $\begin{pmatrix} 1 & 2 & 5 \\ 3 & 2 & 1 \\ 4 & 5 & 7 \end{pmatrix}$

Solution:

Symmetric part of A= $\begin{pmatrix} 1 & \frac{5}{2} & \frac{9}{2} \\ \frac{5}{2} & 2 & 3 \\ \frac{9}{2} & 2 & 7 \end{pmatrix}$

(OR)

Skew-Symmetric part of A= $\begin{pmatrix} 0 & \frac{-1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & -2 \\ \frac{-1}{2} & 2 & 0 \end{pmatrix}$

PRACTICE QUESTIONS

MCQs

1) A and B are non- singular square matrices of the same order, then $AB^T - BA^T$ is a :
   A) Skew-symmetric matrix B) Symmetric matrix C) Null matrix D) Non-singular matrix

2) If A is a matrix of order mxn and B is a matrix such that AB^T and BA^T are both defined, then order of matrix B is
   A) mxm B) nxn C) nxm D) mxn

3 A square matrix A = $[a_{ij}]_{nxn}$ is called a diagonal matrix if $a_{ij} = 0$ for

A) i = j B) I < j C) I >j D) i≠ j

4) If A = $\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - xA - 5I = 0$, then value of x is
   A) 5 B) 3 C) 7 D) 9

5) **Assertion (A):** If A= $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$, then B is the inverse ofA

Reason(R): If A is a square matrix of order m and if there exists another square matrix B of the same order m, such that AB=BA=I, then B is called the inverse of A.

2 Mark Questions

6) If A =$\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b so that $A^2 + aI = bA$
7) Find the value of x – y, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

8) If $[2\ \ 1\ \ 3]\begin{pmatrix}-1 & 0 & -1\\ -1 & 1 & 0\\ 0 & 1 & 1\end{pmatrix}\begin{bmatrix}1\\ 0\\ -1\end{bmatrix}$ = A, then write the order of matrix A

9) If B is skew symmetric matrix, check where the matrix (ABA^T) is symmetric or skew symmetric .

10) If $\begin{bmatrix}9 & -1 & 4\\ -2 & 1 & 3\end{bmatrix}$ = A + $\begin{bmatrix}1 & 2 & -1\\ 0 & 4 & 9\end{bmatrix}$, then find the matrix A

3 Mark Questions

11) Given 3$\begin{bmatrix}x & y\\ z & w\end{bmatrix}$ = $\begin{bmatrix}x & 6\\ -1 & 2w\end{bmatrix}$ + $\begin{bmatrix}4 & x+y\\ z+w & 3\end{bmatrix}$, determine the values of x, y, z and w.

12) Express $\begin{pmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{pmatrix}$ as the sum of symmetric and skew symmetric matrices.

13) Let $A = \begin{pmatrix}p & 0\\ 1 & 1\end{pmatrix}$, $B= \begin{pmatrix}16 & 0\\ -3 & 1\end{pmatrix}$ find the value of p, if $A^2 = B$

14) Write the value of x – y + z from the following equation:

$$\begin{bmatrix}x+y+z\\ x+z\\ y+z\end{bmatrix} = \begin{bmatrix}9\\ 5\\ 7\end{bmatrix}$$

15) Find the value of x, if $[1\ \ 2\ \ 1]\begin{pmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{pmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = O$

Case-study Questions

16) Two schools P and Q want to award their selected students on the values of Tolerance, Kindness, and Leadership. The school P wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with total award money of Rs. 2200.
School Q wants to spend Rs 3100 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each value is Rs1200, using matrices, find the following:

i. What is award money for Tolerance?

a) 350
b) 300
c) 500
d) 400

ii. If a matrix A is both symmetric and skew-symmetric, then

a) A is a diagonal matrix
b) A is a scalar matrix
c) A is a zero matrix
d) A is a square matrix

5 Mark Questions

17) If A = $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, then prove that $A^3 - 6A^2 + 9A - 4\,I = O$.

18) If F(x)=$\begin{pmatrix} cos\ x & -\sin x & 0 \\ \sin x & cos\ x & 0 \\ 0 & 0 & 1 \end{pmatrix}$, show that F(x) F(y) = F(x+y).

19) If A= $\begin{bmatrix} 0 & -tan\frac{\alpha}{2} \\ tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that I +A = (I-A)$\begin{bmatrix} cos\alpha & -\sin\alpha \\ \sin\alpha & cos\ \alpha \end{bmatrix}$

ANSWERS

- 1)B 2) D 3) D4) A5)A) Both A and R are true and R is the correct explanation of A

6) a=8, b=8 7) 0 8) 1x1 9) skew symmetric matrix

10) $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$11) x=2, y= 4 , z= 1, w = 3 13) −4 14)1 15)x=-1 16) i)b ii)c

Determinants

Toeverysquare matrixwecanassigna numbercalleddeterminant

I. **Determinant:**

- $LetA=\begin{bmatrix} a & b \\ c & d \end{bmatrix},thendet(A)=|A|=ad-bc$

1) For easier calculations, we shall expand the determinant along that row orcolumnwhich containsmaximum numberof zeros

- Theareaof atrianglewhoseverticesare(x1,y1), (x2,y2)and(x3,y3),is

$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$

$x_3 \qquad y_3 \qquad 1$

Since area is a positive quantity, we always take the absolute value of thedeterminant

- Theareaofthetriangleformed bythreecollinear points iszero.
- Equationoflinejoiningthepoints$(x_1,y_1)$and$(x_2,y_2)$is

$x \qquad y \qquad 1$
$|x_1 \qquad y_1 \qquad 1|=0$
$x_2 \qquad y_2 \qquad 1$

Minors: Minor of an element $a_{ij}$of a determinant is the determinant obtained bydeletingitsi^{th}rowandj^{th}columninwhichelementa_{ij}lies.Minorofanelement$a_{ij}$is denoted byM_{ij}.

Co-Factors: Cofactorofanelementa_i,denotedbyA_{ij}isdefinedby

$$A_{ij}=(-1)^{i+j}.M_{ij}\text{ ,where}M_{ij}\text{isminorof}a_{ij}$$

The value of a determinant Δ = sum of the product of elements of any row (or column)with theircorrespondingcofactors. If elementsofarow (orcolumn) aremultiplied withcofactors ofanyotherrow(orcolumn),then theirsum iszero.
For example $a_{11}A_{31}+a_{12}A_{32}+a_{13}A_{33}=0$

AdjointofaMatrix: The adjointofasquarematrix$A=[a_{ij}]$isdefinedasthetransposeofthematrix$[A_{ij}]$,whereA_{ij}isthecofactoroftheelementa_{ij}.

Adjointof thematrixAis denoted byadjA.

To find adjoint of a 2×2 matrix interchange the diagonal elements and change the signofnon – diagonal elements.

Inverseof a Matrix:LetA beasquarematrix.

$$A^{-1}=\frac{1}{|A|}adj$$

II. Solutionof systemof linearequationsbyusingmatrixmethod:

Letthesystemof linearequationsbe

$$a_1x+b_1y+c_1z=d_1$$

$$a_2x+b_2y+c_2z=d_2$$

$$a_3x+b_3y+c_3z=d_3$$

Theseequationscanbewrittenas

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX=B$$

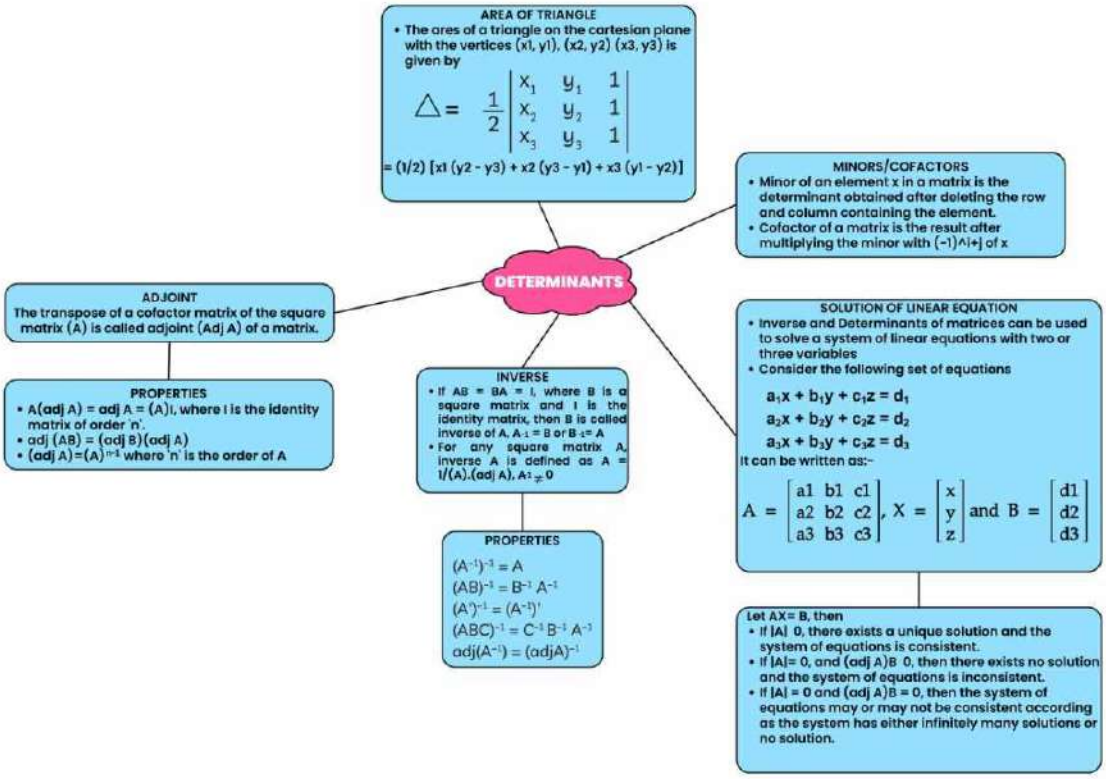
$$X=A^{-1}B$$

- $A^{-1} exists, if |A|\neq 0 i.e the solution exists and it is unique.$
- $The system of equations is said to be consistent if the solution exists.$
- $if |A|=0, then we calculate (adj A)B.$
- $If |A|=0 and (adj A)\neq 0$, (Obeingzero matrix), then solution does notexistand the system ofequations is called inconsistent.
- $If |A|=0$ $and (adj A)=O$, then systemmaybeeitherconsistent orinconsistentaccordingasthe systemhaveeither infinitelymanysolutionsorno solution.

III. Importantnotes:

- ThematrixA issingular if$|A|=0$
- AsquarematrixAis said to benon-singular if$|A|\neq 0$ –
- If AandBarenonsingularmatricesof thesameorder,thenABandBA arealso nonsingularmatrices of thesame order
- If A is an invertible matrix, then$|A|\neq 0$and $(A^{-1})^T=(A^T)^{-1}$
- $|\lambda A|=\lambda^n|A|, where n=order of matrix A$
- $A(adj A)=(adj A)A=|A|I$
- $|adj A|=|A|^{n-1}, where n=order of matrix A$

- $|A(adjA)|=|A|^{n}, wheren=orderofmatrixA$
- $|AB|=|A||B|$
- $(AB)^{-1}=B^{-1}A^{-1}$
- $|A^{-1}|=|A|^{-1}$
- $|A^{T}|=|A|$
- IfA andBaresquarematrices of thesameorder,then$a(AB)=(adjB).(adjA)$



DETERMINANTS

AREA OF TRIANGLE

- The ares of a triangle on the cartesian plane with the vertices (x1, y1), (x2, y2) (x3, y3) is given by

$$\triangle = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$= (1/2)\ [x1\ (y2 - y3) + x2\ (y3 - y1) + x3\ (y1 - y2)]$

MINORS/COFACTORS

- Minor of an element x in a matrix is the determinant obtained after deleting the row and column containing the element.
- Cofactor of a matrix is the result after multiplying the minor with (-1)^i+j of x

ADJOINT

The transpose of a cofactor matrix of the square matrix (A) is called adjoint (Adj A) of a matrix.

PROPERTIES

- A(adj A) = adj A = (A)I, where I is the identity matrix of order 'n'.
- adj (AB) = (adj B)(adj A)
- $(adj\ A)=(A)^{n-1}$ where 'n' is the order of A

INVERSE

- If AB = BA = I, where B is a square matrix and I is the identity matrix, then B is called inverse of A, $A^{-1} = B$ or $B^{-1} = A$
- For any square matrix A, inverse A is defined as A = 1/(A).(adj A), $A^{1} \neq 0$

PROPERTIES

$(A^{-1})^{-1} = A$

$(AB)^{-1} = B^{-1}A^{-1}$

$(A')^{-1} = (A^{-1})'$

$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

$adj(A^{-1}) = (adjA)^{-1}$

SOLUTION OF LINEAR EQUATION

- Inverse and Determinants of matrices can be used to solve a system of linear equations with two or three variables
- Consider the following set of equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

It can be written as:-

$$A = \begin{bmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d1 \\ d2 \\ d3 \end{bmatrix}$$

Let AX= B, then

- If |A| 0, there exists a unique solution and the system of equations is consistent.
- If |A|= 0, and (adj A)B 0, then there exists no solution and the system of equations is inconsistent.
- If |A| = 0 and (adj A)B = 0, then the system of equations may or may not be consistent according as the system has either infinitely many solutions or no solution.

MULTIPLE CHOICE QUESTIONS

1. Find the values of x if $\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$

A) 4 B) 8 C) 2 D) ± 4

Solution:

$2x^2 - 15 = 32 - 15$

$x^2 = 32$

$x = \pm 4$

Ans : D

2. If A is a square matrix of order 3 and $|\,adj\ A\,| = 64$, then $|\,A\,|$ is

A)64 B) 8 C) -8 D) ± 8

Solution: $|\,adj\ A\,| = 64$

$|\,A\,|^2 = 64$

$|\,A\,| = \pm 8$

Ans: D

3. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of x +y is

A)-2 B) 2 C) 3 D) -3

Solution: $A^2 - 3A + I = O$

Multiply by A^{-1}

$A - 3I + A^{-1} = O$

$A^{-1} = -A + 3I$

Hence x = -1, y = 3

x + y = 2

Ans: B

4. If *the determinant of* $\begin{pmatrix} 1-x & 2 & 3 \\ 0 & x & 0 \\ 0 & 0 & x \end{pmatrix}$ is 0, then its roots are

A)0 B) 1 C) 0,1 D) 0,1,-1

Solution: On expanding the determinant, we get

$(1 - x)x^2 = 0$

Hence, x = 0,1

Ans: C

5. If A (3,4), B (-7, 2), C (x , y) are collinear, then which of the following is correct?

A) x + 5y +17 = 0 B) x + 5y +13 = 0 C) x- 5y +17 = 0 D) x - 5y -17 = 0

Solution: If the points are collinear, then are of the triangle ABC = 0

$$\begin{pmatrix} 3 & 4 & 1 \\ -7 & 2 & 1 \\ x & y & 1 \end{pmatrix} = 0$$

$$3(2 - y) - 4(-7 - x) + 1(-7y - x) = 0$$

$$x - 5y + 17 = 0$$

Ans: C

6. IfA = $\begin{pmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{pmatrix}$,then the value of $|\,A\,(adj\ A)\,|$

A) 1001 B) 101 C) 10 D) 1000

Solution: $|A| = -2(-2-3) = 10$

$|A\,(adj\,A)| = |A|^3 = 1000$

Ans: D

Assertion – Reason Based Questions

Questions No. 7 and 8 are Assertion(A) and Reason (R) based questions. Choose the correct answer out of the following choices.

A) Both(A) and (R) are true and (R) is the correct explanation of (A).

B) Both(A) and (R) are true and (R) is not the correct explanation of (A)

C) (A) is true and (R) is false

D) (A) is false and (R) is true

7) Assertion (A): If the matrix A=$\begin{pmatrix} 1 & 3 & k+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{pmatrix}$ is singular, then k=4.

Reason (R) : If A is a singular matrix, then $|A| = 0$.

Solution: $|A| = 0$, k-4=0, k=4

Both(A) and (R) are true and (R) is the correct explanation of (A).

Ans: A

8) Assertion (A): If every element of a third order determinant of value D is multiplied by 5, then the value of the new determinant is 125D.

Reason (R) : If k is a scalar and A is an n x n matrix, then $|kA| = k^n |A|$

Solution:

A) Both(A) and (R) are true and (R) is the correct explanation of (A).

2 Mark Questions

1. If A=$\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix}$ and B=$\begin{bmatrix}1 & -4\\3 & -2\end{bmatrix}$ find $|AB|$

Solution:

AB=$\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}1 & -4\\3 & -2\end{bmatrix}$

=$\begin{bmatrix}7 & -8\\0 & -10\end{bmatrix}$

$|AB|$= -70

2. If A=$\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$write A^{-1} in terms of A

Solution:

$A^{-1} = \frac{Adj\,(A)}{|AB|}$

$=\frac{-1}{19}\begin{bmatrix}-2 & -3\\-5 & 2\end{bmatrix}$

$=\frac{-1}{19}\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$

$=\frac{-1}{19}A$

3. For what value of x, is the following matrix singular?

$\begin{bmatrix}3-2x & x+1\\2 & 4\end{bmatrix}$

Solution:

A matrix is singular if$|A|$ =0

(3 - 2x) 4 – (x+1)2=0

On solving we get x=1

4. If for any 2 X 2 square matrix A, A(adj A)=$\begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}$, then write the value of$|A|$.

Solution:

A(adj A) =$|A|$

$\begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}=|A|\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$\begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}=\begin{bmatrix}|A| & 0\\0 & |A|\end{bmatrix}$

$|A|$= 8

5. If A is a non singular matrix of order 3 and $|A| = -4$, find $|A.adj\,A|$

Solution:

$|A..adj\,A| = |A||A.adj\,A|$

$= |A|\,|A|^2$

$= |A|^3$

$=(-4)^3$

$= -64$

6. Find the equation of the line joining A(1, 3) and B(0,0) using determinants

Solution:

Let P(x, y) be any point on the line AB

Then area of ΔPAB = 0

$$\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

Equation of line AB is $y = 3x$

7. If $A = [a_{ij}]$ is a matrix of order 2x2 such that $|A| = -15$ and A_{ij} is the cofactor of the element a_{ij} then find $a_{21}A_{21} + a_{22}A_{22}$

Solution:

$a_{21}A_{21} + a_{22}A_{22} = |A|$

$= -15$

8. If A is a square matrix of order 2 and $|A| = 2$, then find $|4A^{-1}|$.

Solution:

$|4A^{-1}| = 4^2 \times \frac{1}{|A|} = 16\left(\frac{1}{2}\right) = 8$

3 Mark Questions

1. A coaching institute of Mathematics conduct classes in two batches, I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection of Rs.9000/-, where as in batch II 5 poor and 25 rich children and the monthly collection is Rs.26,000/-. Using matrix method finds the monthly fees paid by each child of the two types.

Solution:-

Let x and y be the fees paid by rich and poor children respectively.

According to the question,

5x+20y=9000

25x+5y=26000

Which can be written as $AX = B$, where A=$\begin{bmatrix} 5 & 20 \\ 25 & 5 \end{bmatrix}$,X=$\begin{bmatrix} x \\ y \end{bmatrix}$,B=$\begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$

$$|A| = -475 \neq 0$$

adj A=$\begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix}$

$\therefore\ A^{-1} = \frac{-1}{475}\begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix}$

X=$\begin{bmatrix} x \\ y \end{bmatrix}$, A^{-1}B=$\begin{bmatrix} 1000 \\ 200 \end{bmatrix}$

$x = 1000,\ y = 200$

2.

Using determinants, find the area of ΔPQR with vertices P(3,1),Q(9,3)and R(5,7). Also find the equation of line PQ using determinants.

Solution:-

Area $=\frac{1}{2}\begin{vmatrix}3 & 1 & 1\\9 & 3 & 1\\5 & 7 & 1\end{vmatrix} = 16$ sq.units

Equation of PQ is $\begin{vmatrix}x & y & 1\\3 & 1 & 1\\9 & 3 & 1\end{vmatrix}=0$

$-2x + 6y = 0$

OR

$x - 3y = 0$

3.

If A.(adjA)$=\begin{bmatrix}3 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}$,then find the value of $|A| + |adjA|$

Solution:-

$|A.adjA|=|A||adjA|=|A||A|^2=|A|^3$

$27=|A|^3$

$|A|=3$

$|adjA|=3^2=9$

$|A| + |adjA|=12$

4.

$If\ x = -9$ is a root of $\begin{vmatrix}x & 3 & 7\\2 & x & 2\\7 & 6 & x\end{vmatrix}=0$ then find the other 2 roots

Solution:-

$\begin{vmatrix}x & 3 & 7\\2 & x & 2\\7 & 6 & x\end{vmatrix}=0$

$x^3 - 67x + 126 = 0$

$(x + 9)(x - 7)(x - 2) = 0$

$x = -9,7,2$

Hence the other two roots are $7\ and\ 2$

5.

Show that the matrix A =$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$,

where I is 2 × 2 identity matrix and O is 2 × 2 zero matrix. Using this equation, find A^{-1}.

Solution:-

$A^2 - 4A + I$=$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$-$\begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix}$+$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$=$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$=0

$A^2 - 4A + I = O$

AA-4A+I=0

AA-4A= -I

Multiplying by A^{-1},we get

$AI - 4I = -A^{-1}$

$A^{-1} = 4I - A$

$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

6.

If A is a skew symmetric matrix of order 3, then prove that $det\ A = 0$

Solution:-

If A is a skew symmetric matrix of order 3,then $A= - A^T$

$|A| = |-A^T|$

$= - |A^T|$

$= - |A|$ (Since $|A^T| = |A|$)

$2|A| = 0$

Hence$|A|$=0

7.

If $A = \begin{vmatrix} 1 & sinx & 1 \\ -sinx & 1 & sinx \\ -1 & -sinx & 1 \end{vmatrix}$, where $0 \le x \le 2\pi$. Then prove that $|A| \epsilon [2,4]$

Solution:-

$|A| = 2 + 2\sin^2 x$

We know that $0 \le \sin^2 x \le 1$

i.e. $0 \le 2\sin^2 x \le 2$

i.e $2 \le 2 + 2\sin^2 x \le 4$

i.e. $2 \le |A| \le 4$

Hence $|A| \epsilon [2,4]$

8. If $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, find A^{-1} and hence so that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

Solution:

$|A| = 2$

$A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

$A^2 - 3I = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

$\frac{1}{2}(A^2 - 3I) = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

Hence proved

5 Mark Questions

1. Use the product of matrix $\begin{pmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{pmatrix}$ $\begin{pmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{pmatrix}$ to solve the system of equations. x+3z=9
x+2y-2z=4
2x-3y+4z=-3

Solution:

AB=

$$\begin{pmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{pmatrix}\begin{pmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{pmatrix}=\begin{pmatrix}-2-9+12 & -2-9+12 & 1+3-4\\ 0+18-18 & 0+4-3 & 0-6+6\\ -6+18+24 & 0-4+4 & 3+6-8\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

AB=I
A^{-1}=B

$$\begin{pmatrix}1 & 0 & 3\\ -1 & 2 & -2\\ 2 & -3 & 4\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}9\\ 4\\ -3\end{pmatrix}$$

A’ x=C
x=$(A')^{-1}C=(A^{-1})'C$
x=B’ C

$$x=\begin{pmatrix}-2 & 9 & 6\\ 0 & 2 & 1\\ 1 & -3 & -2\end{pmatrix}\begin{pmatrix}9\\ 4\\ -3\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}-18+36-18\\ 0+8-3\\ 9-12+6\end{pmatrix} = \begin{pmatrix}0\\ 5\\ 3\end{pmatrix}$$

$x = 0, y = 5, z = 3$

2. Using matrix, solve the following system of equation
$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4, \frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$

$$\begin{pmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{x}\\ \frac{1}{y}\\ \frac{1}{z}\end{pmatrix}=\begin{pmatrix}4\\ 0\\ 2\end{pmatrix}$$

AX=B
$|A|$=1(1+3)+1(2+3)+1(2-1)=10≠0

A^{-1}=$\frac{1}{10}\begin{pmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{pmatrix}$

X= A^{-1} B

$=\frac{1}{10}\begin{pmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{pmatrix}\begin{pmatrix}4\\ 0\\ 2\end{pmatrix}$

$\begin{pmatrix}\frac{1}{x}\\ \frac{1}{y}\\ \frac{1}{z}\end{pmatrix}=\frac{1}{10}\begin{pmatrix}16+0+4\\ -20+0+10\\ 4+0+6\end{pmatrix}=\frac{1}{10}\begin{pmatrix}20\\ -10\\ 10\end{pmatrix}=\begin{pmatrix}2\\ -1\\ 1\end{pmatrix}$

$\frac{1}{x}=2 \Rightarrow x=½$

$\frac{1}{y}=-1 \Rightarrow y=-1$

$\frac{1}{z}=1 \Rightarrow z=1$

3. If A=$\begin{pmatrix}2 & 1 & -3\\ 3 & 2 & 1\\ 1 & 2 & -1\end{pmatrix}$,find A^{-1} and hence solve the following system of equations:

2x+y-3z=13
3x+2y+z=4
X+2y-z=8

Solution:

For Matrix A=$\begin{pmatrix}2 & 1 & -3\\ 3 & 2 & 1\\ 1 & 2 & -1\end{pmatrix}$, $|A|$= -16≠0 so, A^{-1} exists.

Adj A=$\begin{pmatrix}-4 & -5 & 7\\ 4 & 1 & -11\\ 4 & -3 & 1\end{pmatrix}$

Thus, A^{-1}=$\frac{-1}{16}\begin{pmatrix}-4 & -5 & 7\\ 4 & 1 & -11\\ 4 & -3 & 1\end{pmatrix}$

So, given equation can be written into a matrix equation as

$$\begin{pmatrix}2 & 1 & -3\\ 3 & 2 & 1\\ 1 & 2 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}13\\ 4\\ 8\end{pmatrix}$$

A X=B

$\Rightarrow$X=A^{-1}.B

$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{-1}{16}\begin{pmatrix}-4 & -5 & 7\\ 4 & 1 & -11\\ 4 & -3 & 1\end{pmatrix}\begin{pmatrix}13\\ 4\\ 8\end{pmatrix}=\frac{-1}{16}\begin{pmatrix}-16\\ -32\\ 18\end{pmatrix}=\begin{pmatrix}1\\ 2\\ -3\end{pmatrix}\Rightarrow$ x=1, y=2, z=-3

4. Use the product of matrices $\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{pmatrix}$ to solve the following system of equations:

$x+2y-3z=6$
$3x+2y-2z=3$
$2x-y+z=2$

Solution:

$$AB=\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{pmatrix}=\begin{pmatrix}7 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 7\end{pmatrix}=7\,I$$

$A(\frac{1}{7}B)=I$

Thus, $A^{-1}=\frac{1}{7}B=\frac{1}{7}\begin{pmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{pmatrix}$

So, Given equations can be written into a matrix equation as

$$\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -2\\ 2 & -1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}6\\ 3\\ 2\end{pmatrix}$$

$$A \quad X = C$$

$$\Rightarrow X=A^{-1}.C$$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{7}\begin{pmatrix}0 & 1 & 2\\ -7 & 7 & -7\\ -7 & 5 & -4\end{pmatrix}\begin{pmatrix}6\\ 3\\ 2\end{pmatrix}=\frac{1}{7}\begin{pmatrix}7\\ -35\\ -35\end{pmatrix}=\begin{pmatrix}1\\ -5\\ -5\end{pmatrix}\Rightarrow x=1,\ y=-5,\ z=-5$$

5. Find A^{-1}, if $A=\begin{pmatrix}1 & 2 & 1\\ 2 & 3 & -1\\ 1 & 0 & 1\end{pmatrix}$, Hence , solve the following system of equations:

$x+2y+z=5$
$2x+3y=1$
$x-y+z=8$

Solution:

for matrix $A=\begin{pmatrix}1 & 2 & 1\\ 2 & 3 & -1\\ 1 & 0 & 1\end{pmatrix}$, Adjoint of Matrix A is

$|A|=-6\neq 0$ so, A^{-1} exists.

$$adjA=\begin{pmatrix}3 & -2 & -5\\ -3 & 0 & 3\\ -3 & 2 & -1\end{pmatrix}$$

Thus, $A^{-1}=\frac{-1}{6}\begin{pmatrix}3 & -2 & -5\\ -3 & 0 & 3\\ -3 & 2 & -1\end{pmatrix}$

So, given equation can be written into a matrix equation as

$$\begin{pmatrix}1 & 2 & 1\\ 2 & 3 & 0\\ 1 & -1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}5\\ 1\\ 8\end{pmatrix}$$

A^{T} X = B

→ $x = (A^{T})^{-1}.B = X = (A^{-1})^{T}.B$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{-1}{6}\begin{pmatrix}3 & -3 & -3\\ -2 & 0 & 2\\ -5 & 3 & -1\end{pmatrix}\begin{pmatrix}5\\ 1\\ 8\end{pmatrix}=\frac{-1}{6}\begin{pmatrix}-12\\ 6\\ -30\end{pmatrix}=\begin{pmatrix}2\\ -1\\ -5\end{pmatrix}$$

→ x = 2, y = -1, z = 5

PRACTICE QUESTIONS

MCQ

1) If A is a square matrix of order 3, $|A'| = -3, then\ |A\ A'|\ is$
   A) 9 B) -9 C)-3 D) 3
2) If $\begin{vmatrix}x & 2\\ 18 & x\end{vmatrix} = \begin{vmatrix}6 & 9\\ 4 & 6\end{vmatrix}$, then x equal to
   A) 6 B) -6 C) ± 6 D) None of these
3) The value of k, for which A = $\begin{pmatrix}k & 8\\ 4 & 2k\end{pmatrix}$ is a singular matrix.
   A) 4 B) -4 C)6 D) ± 4
4) The maximum value of determinantof $\begin{pmatrix}1 & 1 & 1\\ 1 & 1+\sin x & 1\\ 1 & 1 & 1+\cos x\end{pmatrix}$ is
   A) $\frac{\sqrt{3}}{2}$ B)$\frac{1}{2}$ C)$\frac{\sqrt{2}}{2}$ D)$2\sqrt{3}$
5) Let A be a 2 x 2 matrix

   Assertion(A):adj(adj(A) = A

   Reason(R):|adjA| = |A|

2 Mark Questions

6) If x,y,z are non-zero real numbers and A = $\begin{pmatrix}x & 0 & 0\\ 0 & y & 0\\ 0 & 0 & z\end{pmatrix}$, then prove that

   $A^{-1} = \begin{pmatrix}1/x & 0 & 0\\ 0 & 1/y & 0\\ 0 & 0 & 1/z\end{pmatrix}$
7) If A = $\begin{vmatrix}a & 2\\ 2 & a\end{vmatrix}$ and $|A|^{3}=125$, then find the value of a.
8) Verify A (adjA) = (adj A) A = $|A|\ I$, where A = $\begin{bmatrix}2 & 3\\ -4 & -6\end{bmatrix}$
9) If A = $\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$, show that $A^{2} - 5A - 14\ I = O$, Hence find A^{-1}
10) If $\begin{vmatrix}1 & -x & 2024\\ 0 & x & 0\\ 0 & 0 & x\end{vmatrix} = 0$, then find the value of x.

3 Mark Questions

11) If A = $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, then prove that $A^3 - 6A^2 + 9A - 4I = O$. Hence find A^{-1}.

12) Determine the value of k for which the following system of the equations fail to have a unique solution.

kx+3y-z=1 , x+2y+z=2 and –kx+y+2z=-1.

13) Find the values of k , if the area of the triangle is 4 square units whose vertices are (-2,0), (0,4) and (0,k)

14) Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ .

15) Solve the following system of equations by matrix method

3x-5y=-1

2x+3y=7

5 Mark Questions

16) Using matrix, solve the following system of equation

$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4, \frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2$

17) If A=$\begin{pmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$, find A^{-1}. Hence , solve the following system of equations:

x-y=3

2x+3y+4z=17

y+2z=7

18) Determine the product $\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$ and use it to solve the following system of equations:

x-y+z=4

x-2y-2z=9

2x+y+3z=1

19) If A=$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$,find A^{-1} and hence solve the following system of equations:

x+2y+z=4

-x+y+z=0

x-3y+z=4

20) Solve the following system of equations by matrix method.

x+ 2y + 3z = 6

2x – y +z = 2

3x + 2y -2z = 3

CASE STUDY BASED QUESTION

21) Each triangular face of the square pyramid of Peace in Kazakhstan is made up of 25smaller equilateral triangles as shown in the figure.

Using the above information and concept of determinants, answer the followingquestions.

i) Find the lateral surface area of the Pyramid.

ii) If (2, 4), (2, 6) are two vertices of a smaller equilateral triangle, then find the third vertex.

ANSWERS

1)A 2) C 3) C 4) B 5)**B Both A and R are true and R is the not correct explanation of A**

7) ± 3 9) $\frac{-1}{14}\begin{bmatrix}2 & 5\\ 4 & 3\end{bmatrix}$ 10) 0

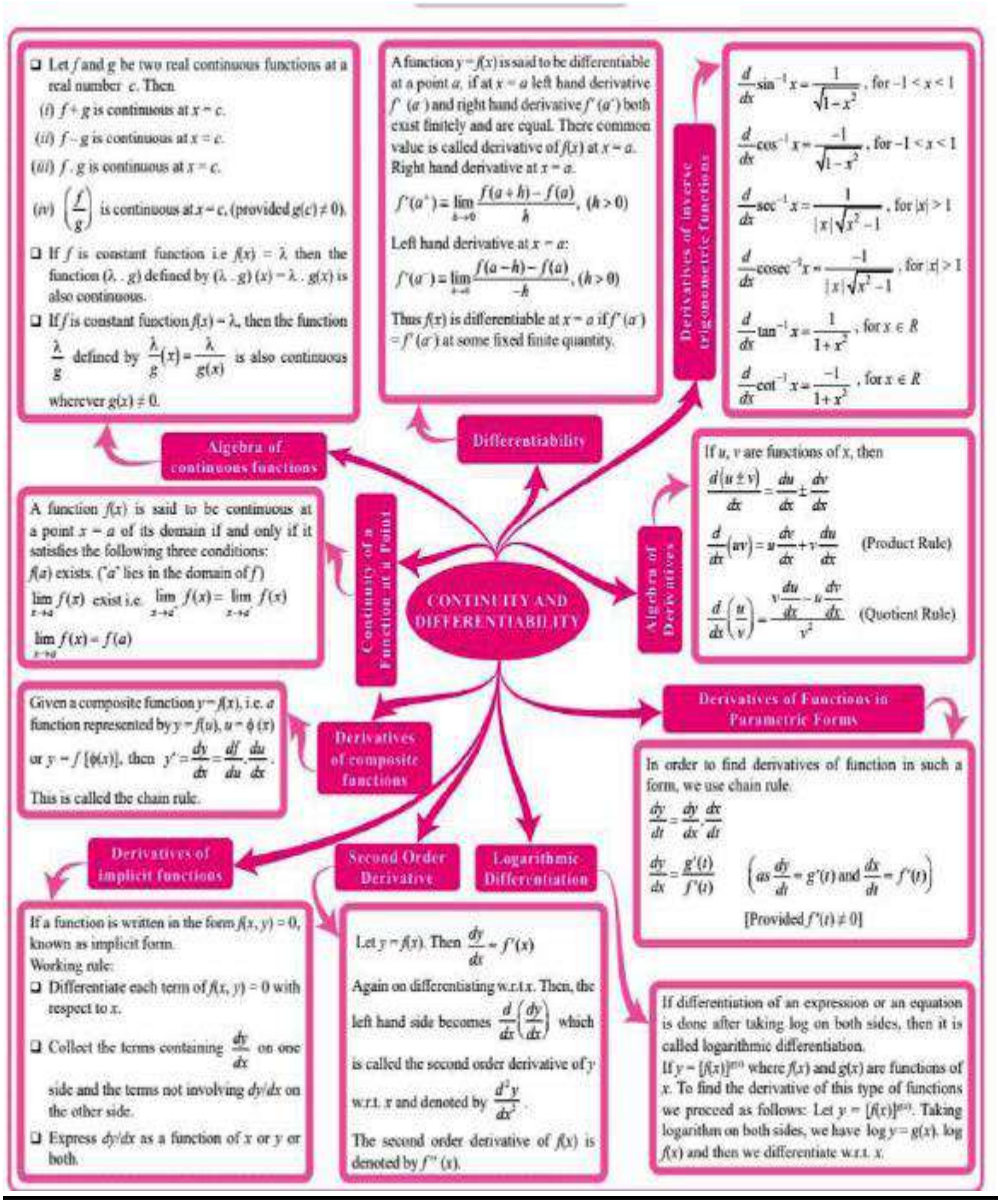
11)1/4 $\begin{pmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{pmatrix}$ 12) -7/2 13) 0,8 15) x=2,y=1 16) x = ½, y=-1,z=1

17) x = 2, y=-1, z=4 18) x = 3, y=-2, z=-1 19) x = 9/5, y= 2/5, z=7/5

20) x = 1, y= 1, z= 1 21)i)$300\sqrt{3}$ sq. unit (ii) $(2\pm\sqrt{3}\ , 5)$

5.CONTINUITY AND DIFFERENTIABILITY

MIND MAP



CONTINUITY AND DIFFERENTIABILITY

Algebra of continuous functions

- ❑ Let f and g be two real continuous functions at a real number c. Then
  - (i) $f + g$ is continuous at $x = c$.
  - (ii) $f - g$ is continuous at $x = c$.
  - (iii) $f \cdot g$ is continuous at $x = c$.
  - (iv) $\left(\frac{f}{g}\right)$ is continuous at $x = c$, (provided $g(c) \neq 0$).
- ❑ If f is constant function i.e $f(x) = \lambda$ then the function $(\lambda \cdot g)$ defined by $(\lambda \cdot g)(x) = \lambda \cdot g(x)$ is also continuous.
- ❑ If f is constant function $f(x) = \lambda$, then the function $\frac{\lambda}{g}$ defined by $\frac{\lambda}{g}(x) = \frac{\lambda}{g(x)}$ is also continuous wherever $g(x) \neq 0$.

Differentiability

A function $y = f(x)$ is said to be differentiable at a point a, if at $x = a$ left hand derivative $f'(a^-)$ and right hand derivative $f'(a^+)$ both exist finitely and are equal. There common value is called derivative of $f(x)$ at $x = a$.
Right hand derivative at $x = a$.

$$f'(a^+) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \ (h > 0)$$

Left hand derivative at $x = a$:

$$f'(a^-) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}, \ (h > 0)$$

Thus $f(x)$ is differentiable at $x = a$ if $f'(a^+) = f'(a^-)$ at some fixed finite quantity.

Derivatives of inverse trigonometric functions

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

$\frac{d}{dx}\text{cosec}^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$, for $x \in R$

$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$, for $x \in R$

Continuity of a Function at a Point

A function $f(x)$ is said to be continuous at a point $x = a$ of its domain if and only if it satisfies the following three conditions:
$f(a)$ exists. ('a' lies in the domain of f)
$\lim_{x \to a} f(x)$ exist i.e. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$
$\lim_{x \to a} f(x) = f(a)$

Algebra of Derivatives

If u, v are functions of x, then

$\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ (Product Rule)

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (Quotient Rule)

Derivatives of composite functions

Given a composite function $y = f(x)$, i.e. a function represented by $y = f(u)$, $u = \phi(x)$ or $y = f[\phi(x)]$, then $y' = \frac{dy}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$.
This is called the chain rule.

Derivatives of Functions in Parametric Forms

In order to find derivatives of function in such a form, we use chain rule.

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$\frac{dy}{dx} = \frac{g'(t)}{f'(t)}$ $\left(as \ \frac{dy}{dt} = g'(t) \text{ and } \frac{dx}{dt} = f'(t)\right)$

[Provided $f'(t) \neq 0$]

Derivatives of implicit functions

If a function is written in the form $f(x, y) = 0$, known as implicit form.
Working rule:

- ❑ Differentiate each term of $f(x, y) = 0$ with respect to x.
- ❑ Collect the terms containing $\frac{dy}{dx}$ on one side and the terms not involving dy/dx on the other side.
- ❑ Express dy/dx as a function of x or y or both.

Second Order Derivative

Let $y = f(x)$. Then $\frac{dy}{dx} = f'(x)$
Again on differentiating w.r.t x. Then, the left hand side becomes $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ which is called the second order derivative of y w.r.t. x and denoted by $\frac{d^2y}{dx^2}$.
The second order derivative of $f(x)$ is denoted by $f''(x)$.

Logarithmic Differentiation

If differentiation of an expression or an equation is done after taking log on both sides, then it is called logarithmic differentiation.
If $y = [f(x)]^{g(x)}$ where $f(x)$ and $g(x)$ are functions of x. To find the derivative of this type of functions we proceed as follows: Let $y = [f(x)]^{g(x)}$. Taking logarithm on both sides, we have $\log y = g(x) \cdot \log f(x)$ and then we differentiate w.r.t. x.

IMPORTANT POINTS

- A function f is said to be continuous at x=a if $\lim_{x \to a} f(x) = f(a)$

i.e, LHL at a = RHL at a = f(a) or $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

- A function f is said to be differentiable at x =a if LHD at a = RHD at a

i.e, $\lim_{h \to 0} \frac{f(a-h)-f(a)}{-h} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

- Standard results in differentiation:-

- Product Rule-

  If y= u v then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

- Quotient Rule-

  If y= $\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

- Chain Rule-

  If $y = f(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

- Derivative of implicit functions- Let $f(x, y) = 0$ be an implicit function of x, then to find $\frac{dy}{dx}$, first differentiate both sides of the equation w.r.t x and then take all the terms containing $\frac{dy}{dx}$ to LHS and remaining terms to the right , then find $\frac{dy}{dx}$.

- Logarithmic Differentiation-

  Used to differentiate functions of the form $u(x)^{v(x)}$

- Parametric Differentiation-

If x=f (t) and y= g (t) then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Standard Derivatives

Sl. No	Function	Derivative
1	x^n	nx^{n-1}
2	K (constant)	0
3	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
4	$sin\ x$	$cos\ x$
5	$cos\ x$	$-sin\ x$
6	$tan\ x$	sec^2x
7	$sec\ x$	$sec\ x\ tan\ x$

8	$cosec\ x$	$-cosec\ x\ cot\ x$
9	$cot\ x$	$-cosec^2 x$
10	e^x	e^x
11	$\log_e x$	$\frac{1}{x}$
12	$sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
13	$cos^{-1}x$	$\frac{-1}{\sqrt{1-x^2}}$
14	$tan^{-1}x$	$\frac{1}{1+x^2}$
15	$sec^{-1}x$	$\frac{1}{x\sqrt{x^2-1}}$
16	$cosec^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$
17	$cot^{-1}x$	$\frac{-1}{1+x^2}$
18	a^x	$a^x \log_e a$

MULTIPLE CHOICE QUESTIONS

1. The derivative of sin x with respect to cos x is :
   (a) tan x (b) $\sec^2$ x (c) – cot x (d) cosec^2 x
   Answer : c
2. If $f(x) = \frac{sin(e^{x-2}-1)}{\log((x-1)}$ $, x \neq 2\ andf(x) = k$ for x =2,then the value of k for which f is continuous is :
   (a) -2 (b) -1 (c) 0 (d) 1
   Answer : d
3. The number of points where $f(x) = |x-2| + |x+3|$ is not differentiable :
   (a) 0 (b) 1 (c) 2 (d) 3
   Answer : c

4. If $y = \sin^{-1}\left(\frac{3x}{2} - \frac{x^3}{2}\right)$, then $\frac{dy}{dx}$is :

(a) $\frac{3}{\sqrt{4-x^2}}$(b) $\frac{-3}{\sqrt{4-x^2}}$ (c) $\frac{1}{\sqrt{4-x^2}}$ (d) $\frac{-1}{\sqrt{4-x^2}}$

Answer : a

5. If $x = at^2 , y = 2at , then \frac{d^2y}{dx^2}$ is

(a) $\frac{1}{t}$(b) $\frac{-1}{t^2}$ (c)$\frac{-1}{2at^2}$ (d) $\frac{-1}{2at^3}$

Answer : d

6. If $y = 3^{\sqrt{x}} then \frac{dy}{dx} =$

(a) $\frac{3^{\sqrt{x}}\log 3}{2\sqrt{x}}$(b) $\frac{2^{\sqrt{x}}\log 3}{2\sqrt{x}}$ (c) $\frac{3^{\sqrt{x}}\log 3}{3\sqrt{x}}$ (d) $\sqrt{x}3^{\sqrt{x}-1}$

Answer : a

<u>ASSERTION & REASONING</u>

Below are given two statements – one labeled Assertion (A) and other labeled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

(i) Both A and R are true and R is the correct explanation of the assertion
(ii) Both A and R are true but R is not the correct explanation of the assertion
(iii) A is true, but R is false
(iv) A is false, but R is true

7. Assertion (A) :The function $f(x) = \begin{cases} 2x , if x < 0 \\ 0, if\ 0 \leq x \leq 1 \\ 4x, if x \geq 1 \end{cases}$ is continuous everywhere except at x = 1.

Reason (R) : Polynomial and constant functions are always continuous.

Answer : (ii)

8. Assertion (A) :The function $f(x) = [x]$is not differentiable at integral points.

Reason (R) : The function $f(x) = [x]$is not continuous at integral points.

Answer : (i)

<u>SHORT ANSWER TYPE QUESTIONS -2 MRKS</u>

9. If $y = x|x|$,find $\frac{dy}{dx}$ for x <0.

Solution:

$|x| = -x, for x < 0$. So $= -x^2$.

$$\frac{dy}{dx} = -2x$$

10. For what value of k ,the function $f(x) = \begin{cases} \frac{x^2+3x-10}{x-2}, x \neq 2 \\ k \quad , x = 2 \end{cases}$ is continuous at x =2 ?

Solution:

If f is continuous at x = 2, $\lim_{x\to 2} f(x) = f(2)$

$$\therefore \lim_{x\to 2} \frac{x^2 + 3x - 10}{x - 2} = k$$

i.e,$\lim_{x\to 2} \frac{(x+5)(x-2)}{x-2} = k$

$$\Rightarrow \lim_{x\to 2}(x + 5) = k$$

$\Rightarrow k = 7$

11. If $f(x) = \sin 2x - \cos 2x, find f'(\frac{\pi}{6})$.

Solution:

$$f'(x) = 2\cos 2\text{x} + 2\sin 2\text{x}$$

$f'\left(\frac{\pi}{6}\right) = 2\cos\frac{\pi}{3} + 2\sin\frac{\pi}{3} = 1 + \sqrt{3}$

12. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{sinx}\right)$ with respect to x.

Solution:

$$\tan^{-1}\left(\frac{1+\cos x}{sinx}\right) = \tan^{-1}\left(\frac{2cos^2\frac{x}{2}}{2sin\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(cot\frac{x}{2}\right) = \tan^{-1}\left[tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right]$$

$$\therefore y = \frac{\pi}{2} - \frac{x}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2}$$

13. Find the derivative of $sin^2 xw.r.te^{\cos x}$.

Solution:

Let $u = sin^2 xandv = e^{\cos x}$

$$\frac{du}{dx} = 2sinxcosxand\frac{dv}{dx} = -sinxe^{\cos x}$$

$$\frac{du}{dt} = \frac{du}{dx} \div \frac{dv}{dx} = -2cosx.e^{-\cos x}$$

14. If $y = log\left(\frac{x^2}{e^2}\right), find\frac{d^2y}{dx^2}$

Solution:

$$y = log\left(\frac{x^2}{e^2}\right) = logx^2 - loge^2 = 2logx - 2$$

$$\frac{dy}{dx} = \frac{2}{x}$$

$$\frac{d^2y}{dx^2} = -\frac{2}{x^2}$$

15. Find $\frac{dy}{dx}$ at $x = 1, y = \frac{\pi}{4} if sin^2 y + \cos(xy) = k$.

Solution:

Differentiate w.r.t x on both sides :$2sinycosy\frac{dy}{dx} - \sin(xy)\left(x\frac{dy}{dx} + y\right) = 0$

$\Rightarrow \frac{dy}{dx}(\sin 2y - xsin(xy) = y\sin(xy))$

$$\therefore \frac{dy}{dx} = \frac{y\sin(xy)}{\sin 2y - xsin(xy)}$$

Substituting $x = 1, y = \frac{\pi}{4}$ we get $\left.\frac{dy}{dx}\right]_{x=1, y=\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$.

16. Find whether the function $f(x) = \begin{cases} x, if x < 1 \\ 2 - x, if\ 1 \le x \le 2 \\ -2 + 3x - x^2, if x > 2 \end{cases}$ is differentiable at $x = 2$

Solution:

L$f'(2) = -1\ andRf'(2) = 3 - 2 \times 2 = -1$

Since L$f'(2) = Rf'(2)$, f is differentiable at $x = 2$

SHORT ANSWER TYPE QUESTIONS -3 MRKS

17. Find the values of p and q ,for which the function $f(x) = \begin{cases} \frac{1-sin^3x}{3\,cos^2x} ifx < \frac{\pi}{2} \\ pifx = \frac{\pi}{2} \\ \frac{q(1-\sin x)}{(\pi-2x)^2} ifx > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

Solution:

LHL at $\frac{\pi}{2} = \lim_{x\to\frac{\pi}{2}^-} \frac{1-sin^3x}{3\,cos^2x} = \lim_{x\to\frac{\pi}{2}^-} \frac{(1-sinx)(1+sinx+sin^2x)}{3\,(1-sinx)(1+sinx)} = \frac{1}{2}$ ....(I)

RHL at $\frac{\pi}{2} = \lim_{x\to\frac{\pi}{2}^+} \frac{q(1-\sin x)}{(\pi-2x)^2} = \lim_{h\to0} \frac{q(1-\sin(\frac{\pi}{2}+h))}{\left(\pi-2(\frac{\pi}{2}+h)\right)^2} = \lim_{h\to0} \frac{q(1-\cosh\ )}{4h^2}$

$= \lim_{h\to0} \frac{q(1-\cos^2 h\ )}{4h^2(1+\cosh\ )} = \frac{q}{4} \lim_{h\to0} \frac{\sin^2 h\ )}{h^2(1+\cosh\ )} = \frac{q}{4} \times 1 \times \frac{1}{1+1} = \frac{q}{8}$.............(II)

$f\left(\frac{\pi}{2}\right) = p$ ....(III)

Since LHL at $\frac{\pi}{2}$ = RHL at $\frac{\pi}{2}$ $=f\left(\frac{\pi}{2}\right)$,from II,II,and III,

$\frac{1}{2} = \frac{q}{8} = p \Rightarrow q = 4\ andp = \frac{1}{2}$

18. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), x^2 < 1, thenfind \frac{dy}{dx}$.

Solution:

Let $x^2 = \cos 2\theta\ .Theny = \tan^{-1}\left(\frac{\sqrt{2cos^2\theta}+\sqrt{2sin^2\theta}}{\sqrt{2cos^2\theta}-\sqrt{2sin^2\theta}}\right) = \tan^{-1}\left(\frac{1+\tan\theta}{1-tan\theta}\right)$

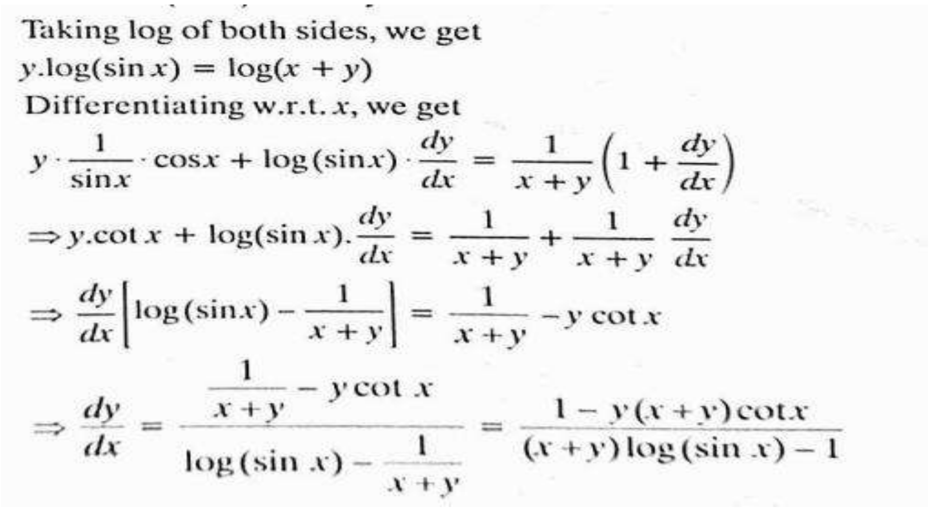
y $= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right) = \frac{\pi}{4}+\theta$

$\Rightarrow y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$$

19. If $(sinx)^y = x + y, find \frac{dy}{dx}$.

Solution:



Taking log of both sides, we get

$y.\log(\sin x) = \log(x + y)$

Differentiating w.r.t. x, we get

$$y \cdot \frac{1}{\sin x} \cdot \cos x + \log(\sin x) \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$$

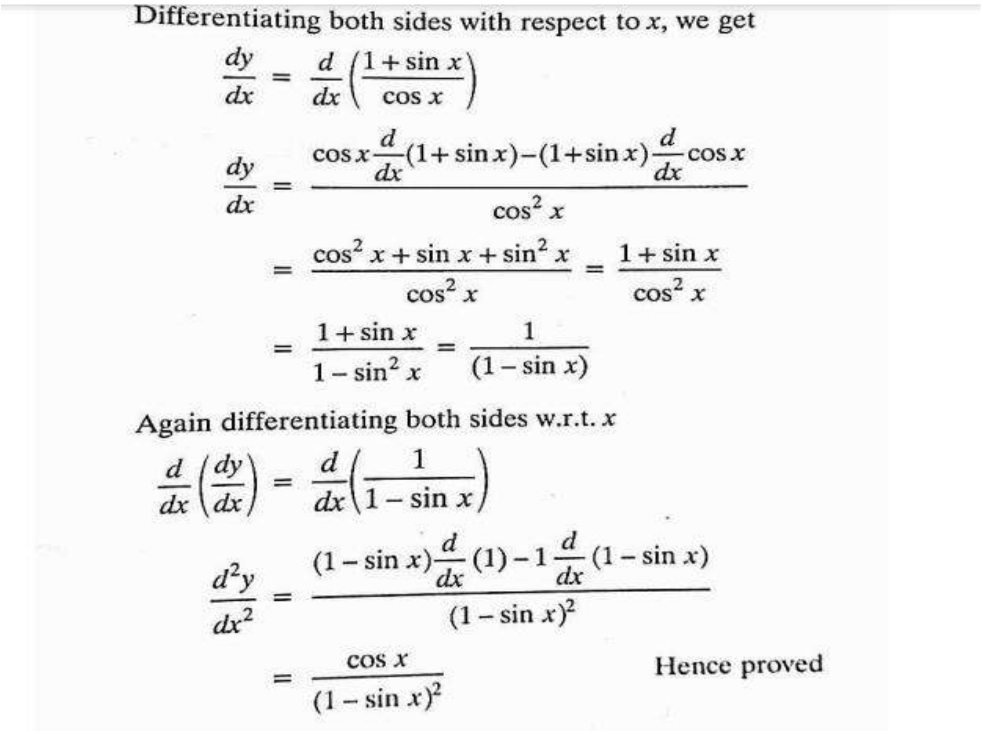
$$\Rightarrow y.\cot x + \log(\sin x).\frac{dy}{dx} = \frac{1}{x+y} + \frac{1}{x+y}\ \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left[\log(\sin x) - \frac{1}{x+y}\right] = \frac{1}{x+y} - y\cot x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log(\sin x) - \frac{1}{x+y}} = \frac{1 - y(x+y)\cot x}{(x+y)\log(\sin x) - 1}$$

20. If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{cosx}{(1-sinx)^2}$

Solution:



Differentiating both sides with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\sin x}{\cos x}\right)$$

$$\frac{dy}{dx} = \frac{\cos x\frac{d}{dx}(1+\sin x)-(1+\sin x)\frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{\cos^2 x+\sin x+\sin^2 x}{\cos^2 x} = \frac{1+\sin x}{\cos^2 x}$$

$$= \frac{1+\sin x}{1-\sin^2 x} = \frac{1}{(1-\sin x)}$$

Again differentiating both sides w.r.t. x

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{1-\sin x}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(1-\sin x)\frac{d}{dx}(1)-1\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$$

$$= \frac{\cos x}{(1-\sin x)^2}$$ Hence proved

21. If $x = sint\ , y = \sin pt, thenprovethat (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

Solution:

$x = sint \Rightarrow t = \sin^{-1}x \& y = \sin pt \Rightarrow \sin^{-1}y = pt$

$$\therefore \sin^{-1}y = p\sin^{-1}x$$

Differentiating w.r.t x, $\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = p\frac{1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\frac{dy}{dx} = p\sqrt{1-y^2} \Rightarrow (1-x^2)(\frac{dy}{dx})^2 = p^2(1-y^2)$

Differentiating again w.r.t x and dividing by $2(\frac{dy}{dx})$ we get

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$

22. If $y = x^x, provethat \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

Solution:

Taking log on both sides :$\log y = x\log x$

Differentiating w.r.t x, $y\frac{dy}{dx} = 1 + \log x \Rightarrow \frac{dy}{dx} = y(1+\log x)$......(I)

Differentiating again w.r.t x :$\frac{d^2y}{dx^2} = \frac{y}{x} + (1+\log x)\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y}\frac{dy}{dx}\cdot\frac{dy}{dx}$ from (I)

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$$

23. If $x = \cos t(3-2cos^2t), y = sint(3-2sin^2t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

Solution:

$$\frac{dx}{dt} = 3sint(2\ cos^2t - 1) = 3sint\cos 2t$$
$$\frac{dy}{dt} = 3cost(1 - 2\ sin^2t) = 3cost\cos 2t$$

$$\frac{dy}{dx} = \cot t$$
$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = 1$$

24. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) w.r.t \cos^{-1}\left(2x\sqrt{1-x^2}\right)$.

Solution:

Let $y = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) \ldots..(I)\ and t = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$......(II)

$$put x = cos\theta in (I) and simplify, we get y = \cos^{-1} x$$

Then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$

$$put x = cos\theta in (II) and simplify, we get t = \frac{\pi}{2} - 2\cos^{-1} x$$

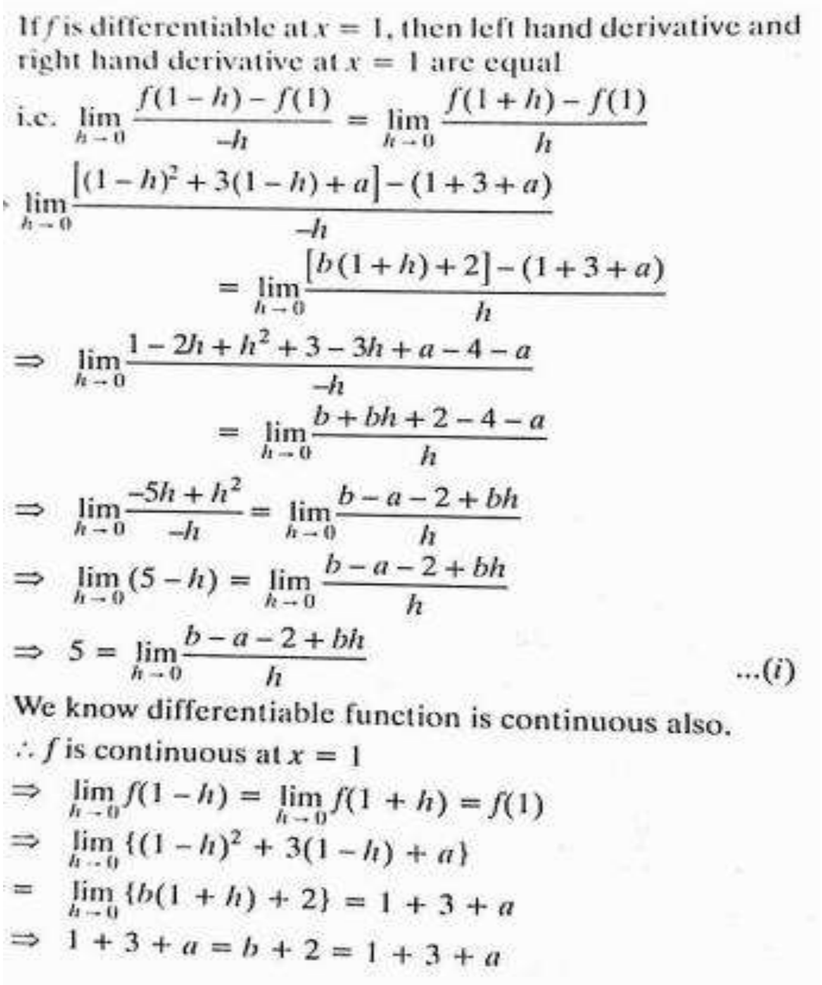
Then $\frac{dx}{dt} = \frac{2}{\sqrt{1-x^2}}$

Now $\frac{dy}{dt} = \frac{dy}{dx} \div \frac{dx}{dt} = -\frac{1}{2}$

LONG ANSWER TYPE QUESTIONS -5 MRKS

25 Find the values of $a\ and\ b$ if $f(x) = \begin{cases} x^2 + 3x + a\ , x \leq 1 \\ bx + 2\ , x > 1 \end{cases}$ is differentiable at $x = 1$.

Solution:



If f is differentiable at $x = 1$, then left hand derivative and right hand derivative at $x = 1$ are equal

i.e. $\lim_{h\to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$$\lim_{h\to 0} \frac{\left[(1-h)^2 + 3(1-h) + a\right] - (1+3+a)}{-h}$$

$$= \lim_{h\to 0} \frac{\left[b(1+h) + 2\right] - (1+3+a)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{1 - 2h + h^2 + 3 - 3h + a - 4 - a}{-h}$$

$$= \lim_{h\to 0} \frac{b + bh + 2 - 4 - a}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{-5h + h^2}{-h} = \lim_{h\to 0} \frac{b - a - 2 + bh}{h}$$

$$\Rightarrow \lim_{h\to 0} (5 - h) = \lim_{h\to 0} \frac{b - a - 2 + bh}{h}$$

$$\Rightarrow 5 = \lim_{h\to 0} \frac{b - a - 2 + bh}{h} \qquad \ldots(i)$$

We know differentiable function is continuous also.

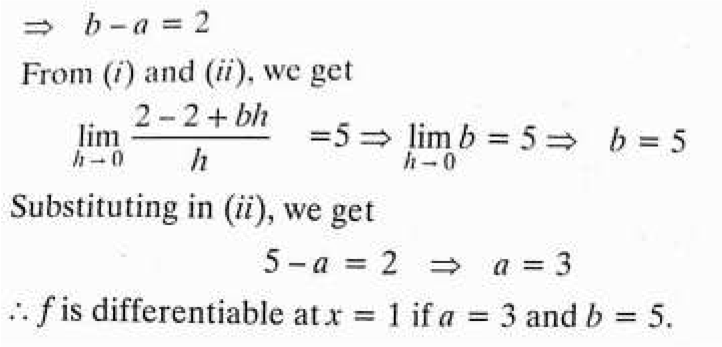
$\therefore f$ is continuous at $x = 1$

$$\Rightarrow \lim_{h\to 0} f(1-h) = \lim_{h\to 0} f(1+h) = f(1)$$

$$\Rightarrow \lim_{h\to 0} \{(1-h)^2 + 3(1-h) + a\}$$

$$= \lim_{h\to 0} \{b(1+h) + 2\} = 1 + 3 + a$$

$$\Rightarrow 1 + 3 + a = b + 2 = 1 + 3 + a$$



$\Rightarrow \quad b - a = 2$

From (i) and (ii), we get

$$\lim_{h \to 0} \frac{2 - 2 + bh}{h} = 5 \Rightarrow \lim_{h \to 0} b = 5 \Rightarrow \quad b = 5$$

Substituting in (ii), we get

$$5 - a = 2 \quad \Rightarrow \quad a = 3$$

$\therefore f$ is differentiable at $x = 1$ if $a = 3$ and $b = 5$.

26. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $thenshowthat$ $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$

Solution:

Differentiating $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ w.r.t x, we get $\frac{x}{a^2} + \frac{y}{b^2}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{b^2x}{a^2y}$ ....(1)

Differentiating (1) w.r.t x,

$$\frac{d^2y}{dx^2} = -\frac{b^2}{a^2}\left(\frac{y - x\frac{dy}{dx}}{y^2}\right)$$

Substituting $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$, $\frac{d^2y}{dx^2} = -\frac{b^2}{a^2}\left(\frac{y - x\left[-\frac{b^2x}{a^2y}\right]}{y^2}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b^2}{a^2}\left(\frac{a^2y^2 + b^2x^2}{a^2y^3}\right)$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b^2}{a^2}\left(\frac{a^2b^2}{a^2y^3}\right)\left(since\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2y^2 + b^2x^2 = a^2b^2\right)$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$$

CASE BASED QUESTIONS -4 MRKS

27. The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight. Answer the following questions:

(i) Find the rate of growth in height of the plant with respect to the number of days exposed to sunlight .

(ii) In how many days, the plant attains its maximum height?

(iii) What is the maximum height?

Solution:

(i)
$$\frac{dy}{dx} = 4 - x$$

(ii) Value of x when $\frac{dy}{dx} = 0 \Rightarrow 4 - x = 0 \Rightarrow x = 4. \therefore$ $in\ 4\ days$ the plant attains its maximum height.

(iii) Value of y when x=4 .i.e 8 cm

28. A ball thrown into the air was moving along the path given by the equation $y = -15x^2 + 3x + 10$ where y refers to the **vertical position** of the ball in meters and x refers to the horizontal position of the ball in meters. *y* is maximum if its instantaneous rate of change with respect to *x* is zero.

Based on the above information, answer the following questions:

(i) Find the horizontal distance covered by the ball ,when it reaches the maximum height.

(ii) What is the maximum height attained by the ball?

Solution:

(i) $\frac{dy}{dx} = -30x + 3$x

$$\frac{dy}{dx} = 0 \Rightarrow x = \frac{1}{10}m = 10cm$$

(ii)
$$y]_{x=1/10} = 10.15m$$

QUESTIONS FOR PRACTICE

1 If $siny = x\sin(a + y)$then $\frac{dy}{dx}$is equal to

$(a)\frac{\sin(a+y)}{sin^2 a}$ (b) $\frac{\sin^2(a+y)}{sina}$ (c) $\frac{\sin^2(a+y)}{sinaa}$ (d) $\frac{\sin(a+y)}{cos^2 a}$

Answer: b

2 The derivative of $f f(x) = |x - 1| - |x - 4| at\ x = 3$

(a) 3 (b) -3 (c) 0 (d) 1

Answer: c

3 If *x*=*a*sin *pt* ,*y* =*b*cos *pt* , then the value of $\frac{d^2y}{dx^2}$ at t =0 is:

(a) $\frac{a}{b^2}$ (b)$-\frac{a}{b^2}$ (c)$\frac{b}{a^2}$ (d)$-\frac{b}{a^2}$

Answer: d

4 The set of points where the function $f(x) = |2x - 1|\sin x$ is differentiable is :

(a)R (b)$R - \left\{\frac{1}{2}\right\}$ (c)$R - \{1\}$ (d) $R - \{0\}$

Answer: b

5 If $f(x) = \begin{cases} mx + 1\ if x \leq \frac{\pi}{2} \\ sinx + n if x > \frac{\pi}{2} \end{cases}$ is continuous at $\frac{\pi}{2}$,then

(a) $m = 1, n = 0$ (b)$m = n\frac{\pi}{2} + 1$ (c)$n = m\frac{\pi}{2}$ (d) $m = n = \frac{\pi}{2}$

Answer: c

6 If $f(x) = \begin{cases} x^2 if x \geq 1 \\ x if x < 1 \end{cases}$,then show that f is not differentiable at $x = 1$. (2 marks)

7 Find the points at which $f(x) = \frac{x^2+4}{4x-x^3}$ is discontinuous, (2 marks)

Answer: 0 ,± 2

8. If $y = \sin^{-1}(6x\sqrt{1-9x^2}), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$,then find $\frac{dy}{dx}$. (2 marks)

Answer:$\frac{6}{\sqrt{1-9x^2}}$

9. If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

(2 marks)

10.) If $y = \sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 = 0$.

(2 marks)

11. If $f(x) = \begin{cases} ax + b & ; \quad 0 < x \leq 1 \\ 2x^2 - x & ; \quad 1 < x < 2 \end{cases}$ is a differentiable function in (0, 2), then find the values of a and b.

(2 marks)

Answer: $a = 3, b = -2$

12. If $e^x + e^y = e^{x+y}, provethat \frac{dy}{dx} + e^{y-x} = 0$(3 marks)

13. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right), thenshowthat \frac{dy}{dx} = \frac{x+y}{x-y}$(3 marks)

14. If $y = (cosx)^x + \sin^{-1}\sqrt{3x}$,find $\frac{dy}{dx}$. (3 marks)

Answer:$(cosx)^x[-xtanx + \log(cosx)] + \frac{3}{2\sqrt{3x}\sqrt{1-3x}}$

15. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t), thenshowthat \frac{dy}{dx} = \frac{x+y}{x-y}$

(3 marks)

16. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$(3 marks)

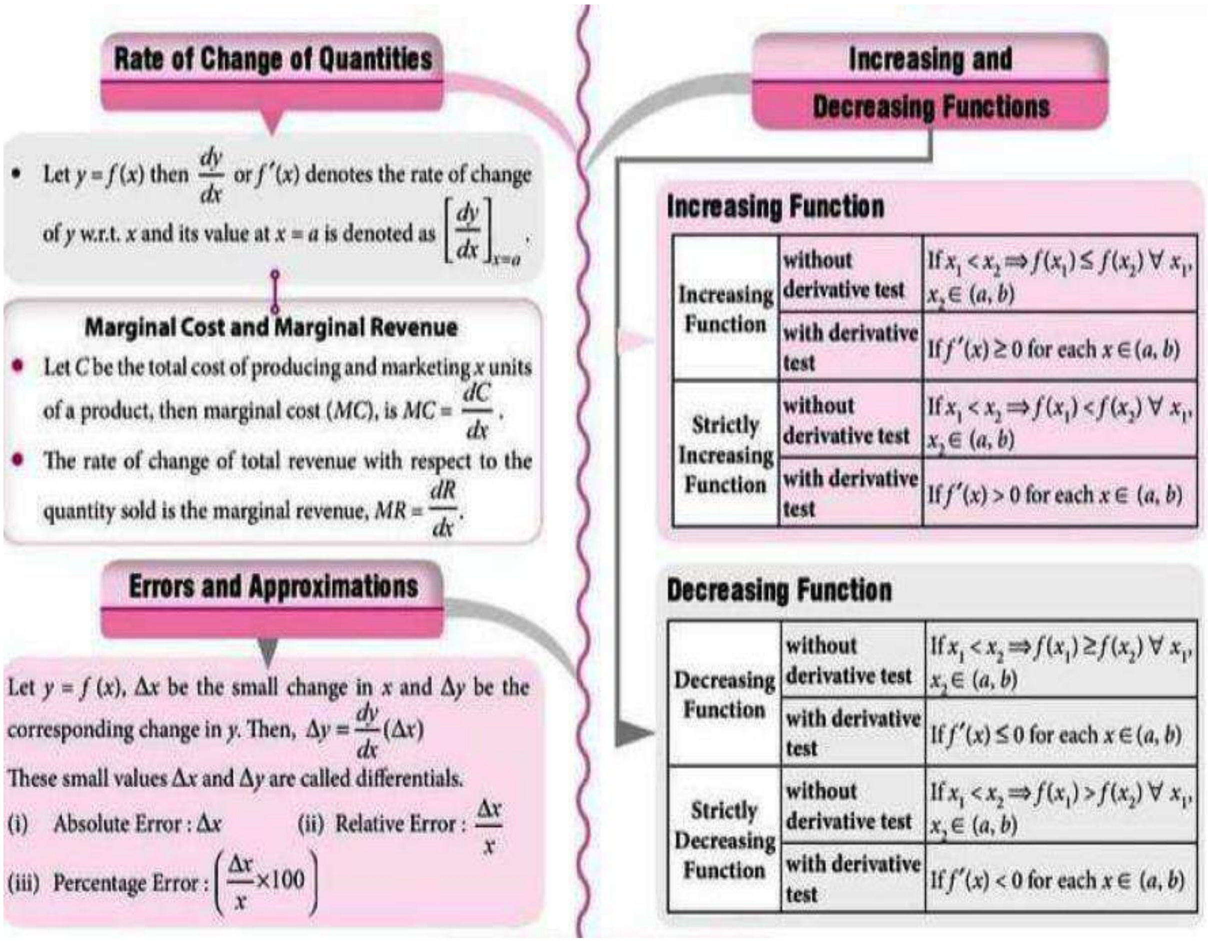
Answer:$\frac{1}{4}$

17. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}(x^2)$ (5 Marks)

Answer:$-\frac{1}{2}$

18. If $y = \left(x + \sqrt{1+x^2}\right)^n, thenshowthat(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$(5 Marks)

6.APPLICATIONS OF DERIVATIVES



Rate of Change of Quantities

- Let $y = f(x)$ then $\frac{dy}{dx}$ or $f'(x)$ denotes the rate of change of y w.r.t. x and its value at $x = a$ is denoted as $\left[\frac{dy}{dx}\right]_{x=a}$.

Marginal Cost and Marginal Revenue

- Let C be the total cost of producing and marketing x units of a product, then marginal cost (MC), is $MC = \frac{dC}{dx}$.
- The rate of change of total revenue with respect to the quantity sold is the marginal revenue, $MR = \frac{dR}{dx}$.

Errors and Approximations

Let $y = f(x)$, Δx be the small change in x and Δy be the corresponding change in y. Then, $\Delta y = \frac{dy}{dx}(\Delta x)$

These small values Δx and Δy are called differentials.

(i) Absolute Error : Δx (ii) Relative Error : $\frac{\Delta x}{x}$

(iii) Percentage Error : $\left(\frac{\Delta x}{x} \times 100\right)$

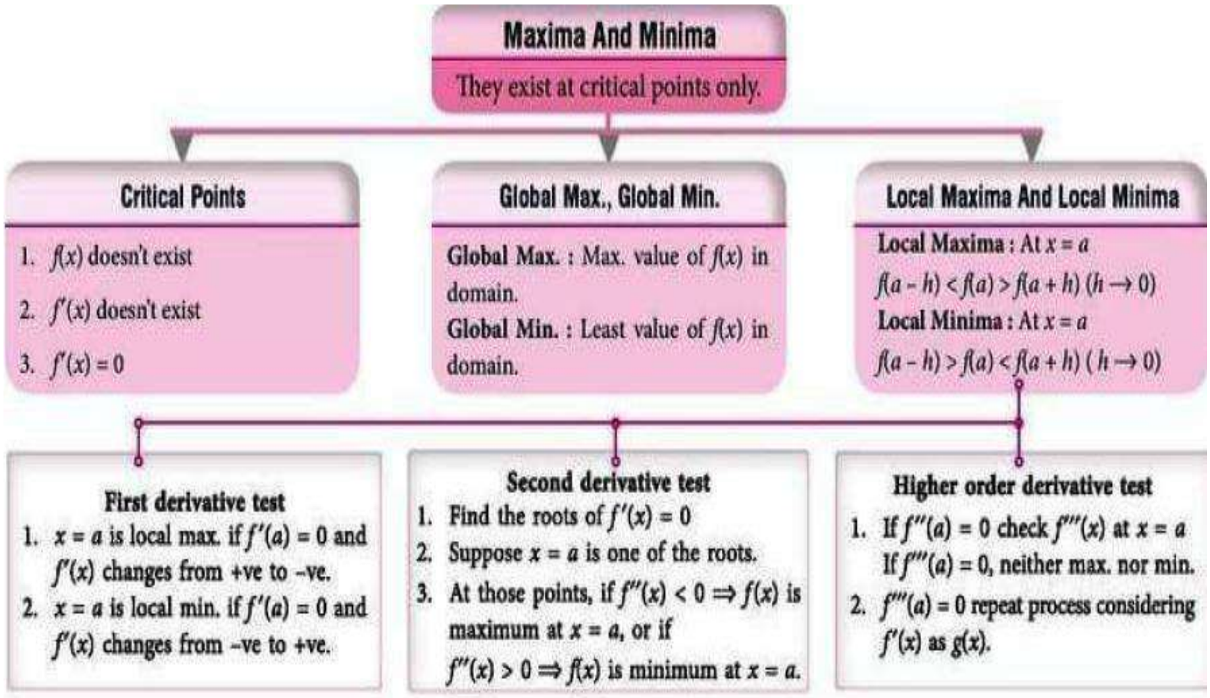
Increasing and Decreasing Functions

Increasing Function

Increasing Function	without derivative test	If $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)\ \forall\ x_1, x_2 \in (a, b)$
	with derivative test	If $f'(x) \ge 0$ for each $x \in (a, b)$
Strictly Increasing Function	without derivative test	If $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in (a, b)$
	with derivative test	If $f'(x) > 0$ for each $x \in (a, b)$

Decreasing Function

Decreasing Function	without derivative test	If $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)\ \forall\ x_1, x_2 \in (a, b)$
	with derivative test	If $f'(x) \le 0$ for each $x \in (a, b)$
Strictly Decreasing Function	without derivative test	If $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)\ \forall\ x_1, x_2 \in (a, b)$
	with derivative test	If $f'(x) < 0$ for each $x \in (a, b)$



Maxima And Minima

They exist at critical points only.

Critical Points

1. $f(x)$ doesn't exist
2. $f'(x)$ doesn't exist
3. $f'(x) = 0$

Global Max., Global Min.

Global Max. : Max. value of $f(x)$ in domain.

Global Min. : Least value of $f(x)$ in domain.

Local Maxima And Local Minima

Local Maxima : At $x = a$

$f(a - h) < f(a) > f(a + h)\ (h \to 0)$

Local Minima : At $x = a$

$f(a - h) > f(a) < f(a + h)\ (h \to 0)$

First derivative test

1. $x = a$ is local max. if $f'(a) = 0$ and $f'(x)$ changes from +ve to –ve.
2. $x = a$ is local min. if $f'(a) = 0$ and $f'(x)$ changes from –ve to +ve.

Second derivative test

1. Find the roots of $f'(x) = 0$
2. Suppose $x = a$ is one of the roots.
3. At those points, if $f''(x) < 0 \Rightarrow f(x)$ is maximum at $x = a$, or if $f''(x) > 0 \Rightarrow f(x)$ is minimum at $x = a$.

Higher order derivative test

1. If $f''(a) = 0$ check $f'''(x)$ at $x = a$
   If $f'''(a) = 0$, neither max. nor min.
2. $f'''(a) = 0$ repeat process considering $f'(x)$ as $g(x)$.

IMPORTANT POINTS

- Let $y = f(x)$ be a function. Then $\frac{dy}{dx}$ denotes the rate of change of y $w.r.t$ x.
- The value of $\frac{dy}{dx}$ at $x = x_0$ i.e $\left(\frac{dy}{dx}\right)_{x=x_0}$ i.e. represents the rate of change of y $w.r.t$ x at $x = x_0$
- If two variables x and y are varying with respect to another variable t, i.e., if $x = f(t)$ and y= g(t), then by Chain Rule $\frac{dy}{dx} = \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)$, provided $\frac{dx}{dt} \neq 0$
- $\frac{dy}{dx}$ is positive if y *increases as* x increases and is negative if y *decreases as* x *increases*.

Increasing and Decreasing Functions

- A function y = f (x) is said to be **increasing** on an interval (a, b) if x $_1$ < x $_2$ in (a, b) ⇒ f(x$_1$) ≤ f(x$_2$) for all $x_1, x_2 \in (a, b)$

Alternatively, a function y = f (x) is said to be increasing if f ′(x) ≥ 0 for each x in (a, b)

(a) **strictly increasing** on an interval (a, b) if x $_1$ < x $_2$ in (a, b) ⇒ f(x$_1$) < f(x$_2$) for all x $_1$, x $_2$ ∈ (a, b).

Alternatively, a function y = f (x) is said to be strictly increasing if f ′(x) > 0 for each x in (a, b)

(b) **decreasing** on (a, b) if x $_1$ < x$_2$ in (a, b) ⇒ f(x$_1$) ≥ f(x $_2$) for all x$_1$, x$_2$ ∈ (a, b). Alternatively, a function y = f (x)) is said to be decreasing if f ′(x) ≤ 0 for each x in (a, b)

(c) **strictly decreasing** on (a, b) if x $_1$ < x$_2$ in (a, b) ⇒ f(x $_1$) > f(x $_2$) for all x $_1$, x$_2$ ∈ (a, b).

Alternatively, a function y = f (x) is said to be strictly decreasing if f ′(x)<0 for each x in (a, b)

(d) **constant** function in (a, b), if f (x) = c for all x ∈ (a, b), where c is a constant.

Alternatively , f(x) is a constant function if f ′ (x)= 0.

A point c in the domain of a function f at which either f ′(c) = 0 or f is not differentiable is called **a critical point** of f.

Maxima and Minima

Definition: Let f be a function defined on an interval I. Then

1) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) > f(x)$, for all $x \in I$. The number f(c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.
2) f is said to have a minimum value in I, if there exists a point c in I such that $f(c) < f(x)$, for all $x \in I$. The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.
3) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I. The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

Local Maxima and Local Minima

Definition: Let f be a real valued function and let c be an interior point in the domain of f. Then

(a) c is called a point of local maxima if there is an $h > 0$ such that $f(c) \geq f(x)$, for all x in $(c - h, c + h), x \neq c$. The value f(c) is called the local maximum value of f.

(b) c is called a point of local minima if there is an $h > 0$ such that $f(c) \leq f(x)$, for all x in $(c - h, c + h)$. The value f(c) is called the local minimum value of f.

Theorem: (***First Derivative Test***) Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

1) If f ′(x) changes sign from positive to negative as x increases through c, i.e., if f ′(x) > 0 at every point sufficiently close to and to the left of c, and f ′(x) < 0 at every point sufficiently close to and to the right of c, then c is a point of local maxima.
2) If f ′(x) changes sign from negative to positive as x increases through c, i.e., if f ′(x) < 0 at every point sufficiently close to and to the left of c, and f ′(x) > 0 at every point sufficiently close to and to the right of c, then c is a point of local minima.
3) If f ′(x) does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. In fact, such a point is called **point of inflection**.

Theorem: ***(Second Derivative Test)*** Let f be a function defined on an interval I and c ∈ I. Let f be twice differentiable at c. Then

1) x = c is a point of local maxima if f ′(c) = 0 and f ″(c) < 0 The value f (c) is local maximum value of f.
2) x = c is a point of local minima if f ′(c) 0 = and f ″(c) > 0 In this case, f (c) is local minimum value of f.
3) The test fails if f ′(c) = 0 and f ″(c) = 0. In this case, we go back to the first derivative test and find whether c is a point of local maxima, local minima or a point of inflexion.

Absolute Maximum/Minimum

Working rule for finding absolute maximum value and/or absolute minimum value

Step 1: Find all critical points of f in the interval, i.e., find points x where either f ′(x) = 0 or f is not differentiable.

Step 2: Take the end points of the interval.

Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3 This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

MULTIPLE CHOICE QUESTIONS-1 MARK

1. The function $f(x) = \tan x - x$ is:
   (a) Always increasing (b) always decreasing (c) not always decreasing
   (d) neither increasing nor decreasing.
   Answer: a
2. If the function $f(x = kx^3 - 9x^2 + 9x + 3$ is monotonically increasing in every interval, then:
   (a) $k < 3$ (b) $k \leq 3$ (c) $k > 3$ (d) $k \geq 30$.
   Answer: c
3. Side of an equilateral triangle expands at rate of 2 cm / sec. The rate of increase of its area when each side is 10 cm is
   (a) $10\sqrt{2} cm^2/sec$ (b) $10\sqrt{3} cm^2/sec$ (c) 10 cm^2/sec (d) 5 cm^2/sec
   Answer: b
4. A cone whose height is always equal to its diameter is increasing in volume at the rate of $40cm^3$/ sec . At what rate is the radius increasing when its circular base area is $1m^2$?
   (a) 1 mm/sec (b) 0.001 cm/sec (c) 2 mm/sec (d) 0.002 cm/sec
   Answer: d
5. The maximum value of $f(x) = \frac{\log x}{x}$ is:
   (a) $\frac{1}{e}$ (b) $\frac{2}{e}$ (c) e (d) 1
   Answer: a
6. If the function $f(x) = x^3 + ax^2 + bx + 1$ is maximum at $x = 0$ and minimum at $x = 1$, then:
   (a) $a = \frac{2}{3}, b = 0$ (b) $a = -\frac{3}{2}, b = 0$ (c) $a = 0, b = \frac{3}{2}$ (d) $a = 0, b = -\frac{3}{2}$
   Answer: b

ASSERTION & REASONING

Below are given two statements – one labeled Assertion (A) and other labeled Reason

(R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

(i) Both A and R are true and R is the correct explanation of the assertion
(ii) Both A and R are true but R is not the correct explanation of the assertion
(iii) A is true, but R is false
(iv) A is false, but R is true

7. **Assertion (A) :** f(x) = x^2 – x + 1 is neither increasing nor decreasing on $(-1, 1)$
Reason (R) : The function f(x) is strictly increasing on I if $f'(x) > 0$ for all $x \in I$
Answer: ii

8. **Assertion (A):** f(x) = e^x has no minimum and maximum values .
Reason (R): f(x) has no critical points
Answer :i

SHORT ANSWER TYPE QUESTIONS -2 MRKS

9. The bottom of a rectangular swimming tank is 25 m by 40 m. Water is pumped into the tank at the rate of 500 cubic metres per minute. Find the rate at which the level of water in the tank is rising.

Solution:

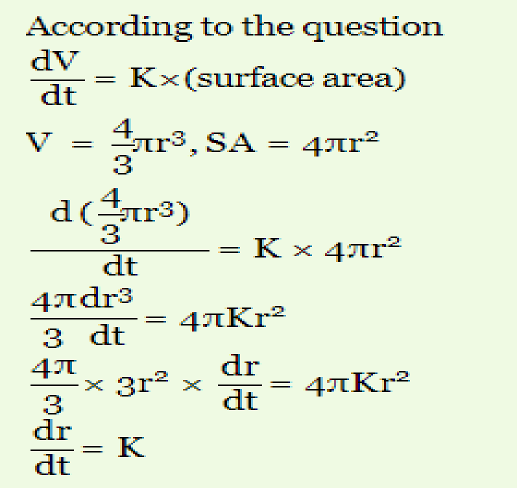
$$V = 25 \times 40 \times h$$

$$\frac{dV}{dt} = 25 \times 40 \times \frac{dh}{dt} \Rightarrow 500 = 1000 \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.5\ m/min$$

10. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

Solution:



According to the question

$$\frac{dV}{dt} = K \times (\text{surface area})$$

$$V = \frac{4}{3}\pi r^3, SA = 4\pi r^2$$

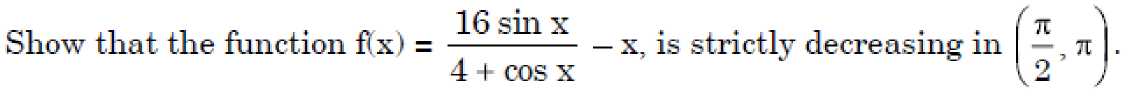
$$\frac{d\left(\frac{4}{3}\pi r^3\right)}{dt} = K \times 4\pi r^2$$

$$\frac{4\pi}{3}\frac{dr^3}{dt} = 4\pi K r^2$$

$$\frac{4\pi}{3} \times 3r^2 \times \frac{dr}{dt} = 4\pi K r^2$$

$$\frac{dr}{dt} = K$$

11.



Show that the function f(x) = $\frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Solution:

$$f'(x) = \frac{16[4 + \cos x]\cos x + 16\sin^2 x}{(4 + \cos x)^2} - 1$$

$$= \frac{\cos x\,(56 - \cos x)}{(4 + \cos x)^2}$$

in $(\frac{\pi}{2}, \pi)$, $\cos x < 0 \Rightarrow f'(x) < 0$

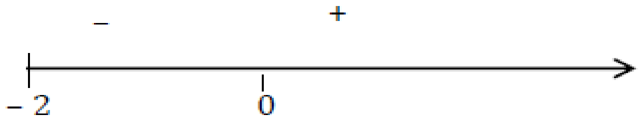
$\therefore$ f(x) in strictly decreasing in $(\frac{\pi}{2}, \pi)$

12. Find the sub-intervals in which $f(x) = \log(2 + x) - \frac{x}{2 + x}$, $x > -2$ is increasing or decreasing.

Solution:

$$f'(x) = \frac{1}{2 + x} - \frac{2}{(2 + x)^2} = \frac{x}{(2 + x)^2}$$

Sign of $f'(x)$



f(x) is decreasing in (– 2, 0)
and increasing in $(0, \infty)$

13. If the product of two positive numbers is 9, find the numbers so that the sum of their squares is minimum.

Solution:

Let numbers be x and $\frac{9}{x}$ $\therefore$ required sum $= x^2 + \frac{81}{x^2} = f(x)$ say

$$f'(x) = 2x - \frac{162}{x^3}$$

$f'(x) = 0 \Rightarrow x = 3$

showing f(x) is minimum at x = 3

sum is minimum when both numbers are 3.

14. A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x–coordinate 1, y–coordinate is changing twice as fast at x–coordinate. Find the value of a.

Differentiating equation $3y = ax^3 + 1$ with respect to 'x', $3\frac{dy}{dx} = 3ax^2$

Taking $x = 1$, $\frac{dy}{dx} = 2$, $3(2) = 3a(1)^2 \Rightarrow a = 2$

15. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.

Solution:

Let 'r' be the radius, C the circumference and A the area of the circle.

Then, $\frac{dC}{dt} = k$ **(Constant), also** $C = 2\pi r \Rightarrow \frac{dC}{dt} = 2\pi\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{k}{2\pi}$

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi\frac{dr}{dt} = 2\pi r \cdot \frac{k}{2\pi} = kr$, $\therefore$ **the rate of change of area is directly proportional to its radius**

16. If $f(x) = a(tanx - cotx)$, where $a > 0$, then find whether f(x) is increasing or decreasing function in its domain.

Solution:

$f'(x) = a(\sec^2 x + \mathrm{cosec}^2 x)$

As $a > 0$ and $\sec^2 x$, $\mathrm{cosec}^2 x$ are squares, $f'(x) > 0$

$\therefore$ f(x) is an increasing function in its domain.

SHORT ANSWER TYPE QUESTIONS -3 MRKS

17. Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (a) strictly increasing (b) strictly decreasing

Solution:

$$f'(x) = x^3 - 3x^2 - 10x + 24 = (x-2)(x-4)(x+3)$$

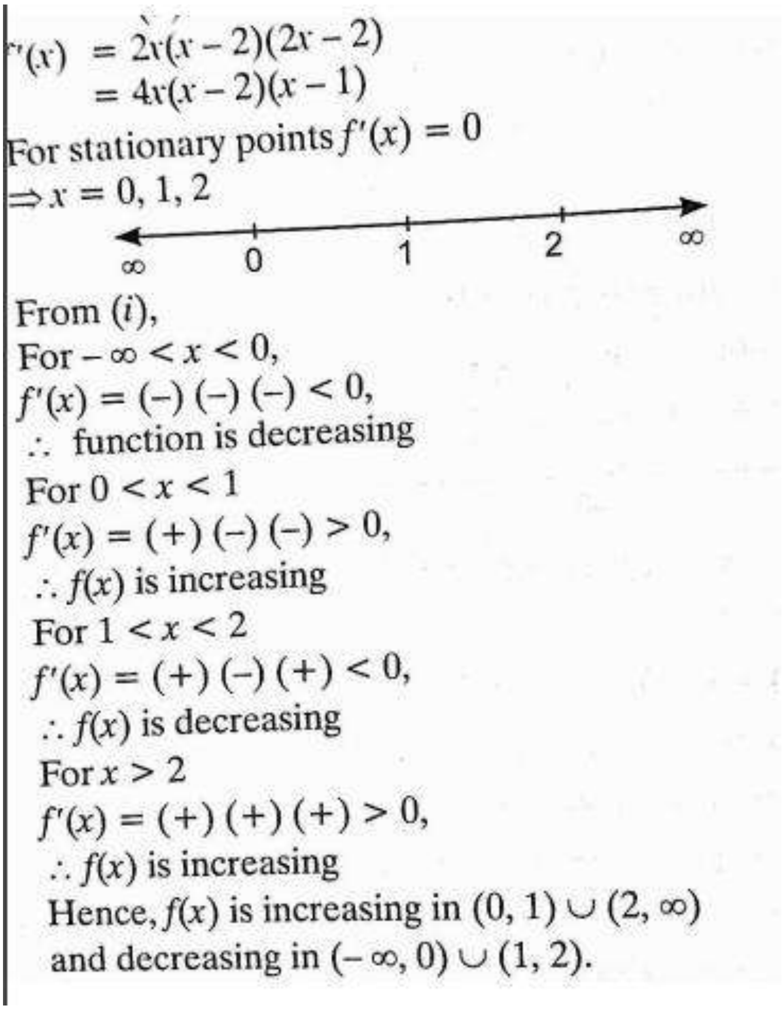
$f'(x) = 0 \Rightarrow x = 2,4,-3$

$$f'(x) < 0 \; on \; (-\infty,-3) \cup (2,4) \& f'(x) > 0 on \; (-3,2) \cup (4,\infty)$$

$$\therefore f(x) \; is \; strictly \; decreasing \; on \; (-\infty,-3) \cup (2,4)\&$$

strictly increasing on $(-3,2) \cup (4,\infty)$

18. Find the value of x for which the function $y = [x(x-2)]^2$ is an increasing function.



$'(x) = 2x(x-2)(2x-2)$
$= 4x(x-2)(x-1)$

For stationary points $f'(x) = 0$
$\Rightarrow x = 0, 1, 2$

From (*i*),
For $-\infty < x < 0$,
$f'(x) = (-)\,(-)\,(-) < 0$,
$\therefore$ function is decreasing
For $0 < x < 1$
$f'(x) = (+)\,(-)\,(-) > 0$,
$\therefore f(x)$ is increasing
For $1 < x < 2$
$f'(x) = (+)\,(-)\,(+) < 0$,
$\therefore f(x)$ is decreasing
For $x > 2$
$f'(x) = (+)\,(+)\,(+) > 0$,
$\therefore f(x)$ is increasing
Hence, $f(x)$ is increasing in $(0, 1) \cup (2, \infty)$
and decreasing in $(-\infty, 0) \cup (1, 2)$.

19 Find the rate of change of the volume of a sphere with respect to its surface area when the radius is 2 cm.?

Radius of sphere(r) = 2cm
$V = 4/3\ \pi r^3$
$\frac{dV}{dr} = 4\pi r2$
$A = 4\pi r2$
$\frac{dA}{dr} = 8\pi r$

$\frac{dV}{dA} = \frac{\frac{dV}{dr}}{\frac{dV}{dr}}$

$= \frac{4\pi r^2}{8\pi r} = \frac{r}{2}$

$\frac{dV}{dA_{r=2}} = \frac{2}{2} = 1$ cm

20. . Find the maximum value of $f(x) = (x - 1)^{\frac{1}{3}}(x - 2)$ in [1,9]

Solution:

$f(x) = (x - 1)^{\frac{1}{3}}(x - 2)$

$f'(x) = (x - 1)^{\frac{1}{3}}(1) + (x - 2) \times 1/3(x - 1)^{\frac{-2}{3}}$

$= (x - 1)^{-2/3}\left(x - 1 + \frac{x-2}{3}\right) = (x-1)^{-2/3}\frac{(3x - 3 + x - 2)}{3} = \frac{(4x-5)}{3}(x-1)^{-2/3}$

$f'(x) = 0 \Longrightarrow x = 5/4$

Find the value of fatx= 5/4, x= 1 , &at x= 9

$f(1) = 0,$

$f(9) = 14,$

$f(5/4) < 0$

Maximum value of $f(x)$ is 14

21. Find two numbers whose sum is 24 and whose product is as large as possible?

Let x & y be the required nos.

$x + y = 24$ (given)

$$P = xy = x(24 - x)$$

$=24x - x^2$

$$\frac{dp}{dx} = 24 - 2x$$

$$\frac{dp}{dx} = 0 \rightarrow 2x = 24\ or x = 12$$

$$Secondderivative of Pis - 2\ whichis - ve$$

$$SoPisamax. whenx = 12$$

$$Whenx = 12\ , y = 12$$

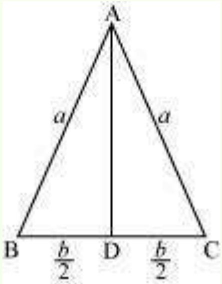
$$Sotherequirednumbersarex = 12\ \& y = 12$$

22. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

Let ΔABC be isosceles where BC is the base of fixed length *b*.

Let the length of the two equal sides of ΔABC be *a*.

Draw AD⊥BC.



Now, in ΔADC, by applying the Pythagoras theorem, we have:

$$AD = \sqrt{a^2 - \frac{b^2}{4}}$$

∴ Area of triangle $(A) = \frac{1}{2}b\sqrt{a^2 - \frac{b^2}{4}}$

The rate of change of the area with respect to time (*t*) is given by,

$$\frac{dA}{dt}=\frac{1}{2}b\cdot\frac{2a}{2\sqrt{a^2-\frac{b^2}{4}}}\frac{da}{dt}=\frac{ab}{\sqrt{4a^2-b^2}}\frac{da}{dt}$$

It is given that the two equal sides of the triangle are decreasing at the rate of 3 cm per second.

$$\therefore \frac{da}{dt}=-3\text{ cm/s}$$

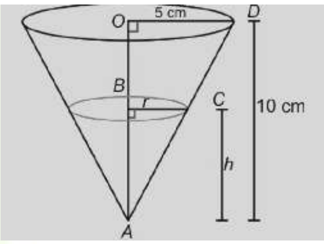
$$\therefore \frac{dA}{dt}=\frac{-3ab}{\sqrt{4a^2-b^2}}$$

Then, when $a = b$, we have:

$$\frac{dA}{dt}=\frac{-3b^2}{\sqrt{4b^2-b^2}}=\frac{-3b^2}{\sqrt{3b^2}}=-\sqrt{3}b$$

Hence, if the two equal sides are equal to the base, then the area of the triangle is decreasing at the rate of $\sqrt{3}\,b\text{ cm}^2/\text{s}$.

23. Water is leaking from a conical funnel at the rate of 5 cm /s. If the radius of the base of funnel is 5 cm and height 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top.



Let r be the radius and h be the height of surface of water at time t.

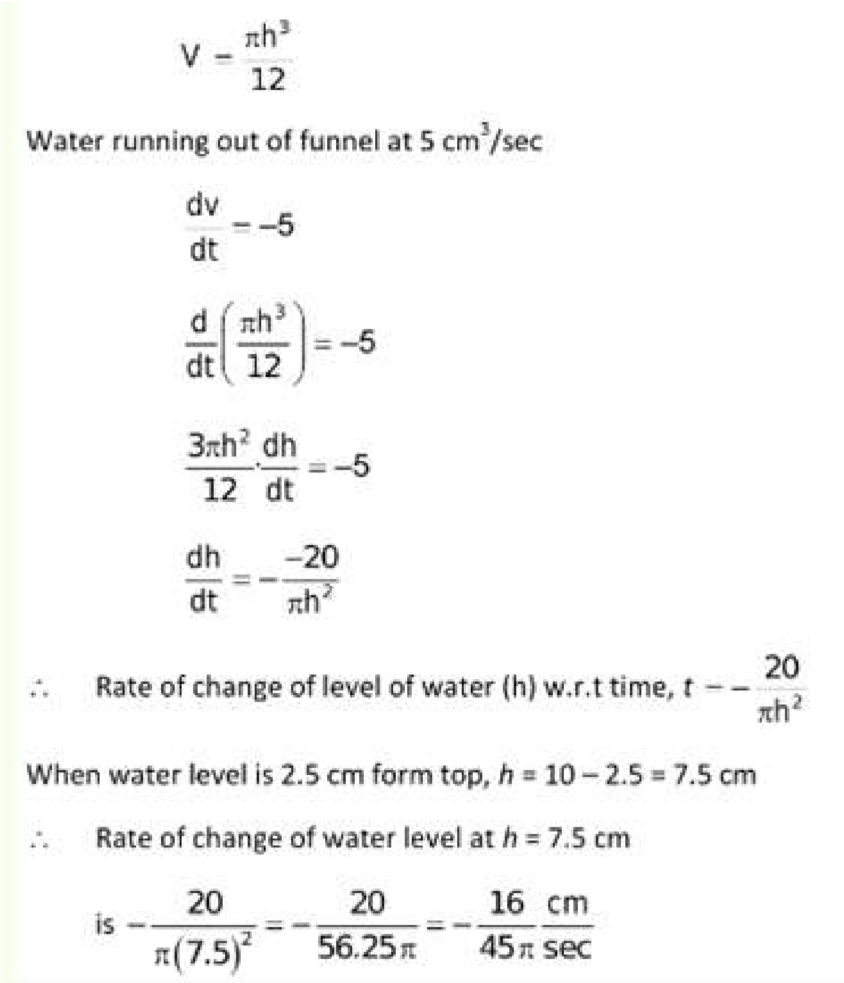
Let V be the volume of water in funnel.

$$V=\frac{1}{3}\pi r^2 h \qquad \text{...(i)}$$

$$\frac{r}{h}=\frac{5}{10}\ \text{(from the fig,we have)}$$

$$r=\frac{1}{2}h \qquad \text{...(ii)}$$

Substituting equation (ii) in equation (i) we have



$$V = \frac{\pi h^3}{12}$$

Water running out of funnel at 5 cm^3/sec

$$\frac{dv}{dt} = -5$$

$$\frac{d}{dt}\left(\frac{\pi h^3}{12}\right) = -5$$

$$\frac{3\pi h^2}{12}\cdot\frac{dh}{dt} = -5$$

$$\frac{dh}{dt} = -\frac{-20}{\pi h^2}$$

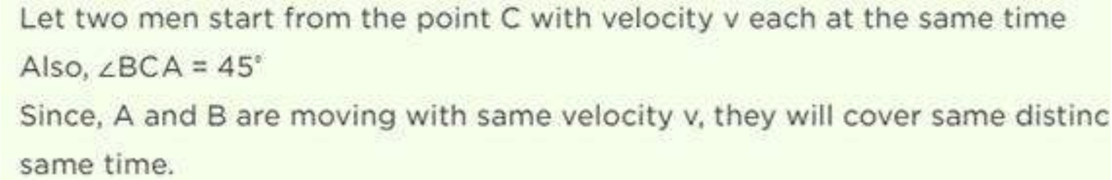
∴ Rate of change of level of water (h) w.r.t time, $t = -\frac{20}{\pi h^2}$

When water level is 2.5 cm form top, $h = 10 - 2.5 = 7.5$ cm

∴ Rate of change of water level at $h = 7.5$ cm

is $-\frac{20}{\pi(7.5)^2} = -\frac{20}{56.25\pi} = -\frac{16}{45\pi}\frac{cm}{sec}$

24. Two men A and B start with velocities v at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, find the rate at which they are being separated.

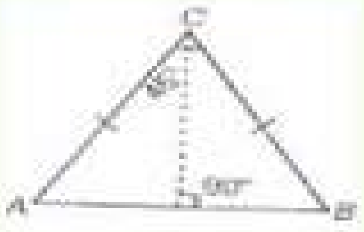


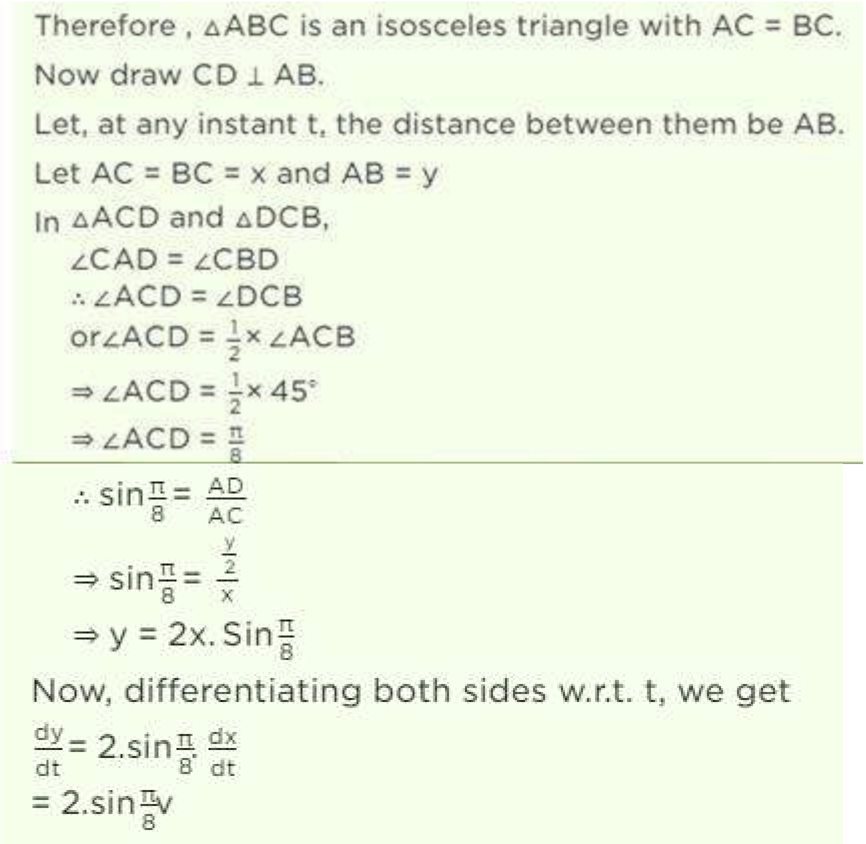
Let two men start from the point C with velocity v each at the same time

Also, ∠BCA = 45°

Since, A and B are moving with same velocity v, they will cover same distinc

same time.





Therefore , △ABC is an isosceles triangle with AC = BC.

Now draw CD ⊥ AB.

Let, at any instant t, the distance between them be AB.

Let AC = BC = x and AB = y

In △ACD and △DCB,

$\angle CAD = \angle CBD$

$\therefore \angle ACD = \angle DCB$

or$\angle ACD = \frac{1}{2} \times \angle ACB$

$\Rightarrow \angle ACD = \frac{1}{2} \times 45^\circ$

$\Rightarrow \angle ACD = \frac{\pi}{8}$

$\therefore \sin\frac{\pi}{8} = \frac{AD}{AC}$

$\Rightarrow \sin\frac{\pi}{8} = \frac{\frac{y}{2}}{x}$

$\Rightarrow y = 2x.\,\text{Sin}\frac{\pi}{8}$

Now, differentiating both sides w.r.t. t, we get

$\frac{dy}{dt} = 2.\sin\frac{\pi}{8}.\frac{dx}{dt}$

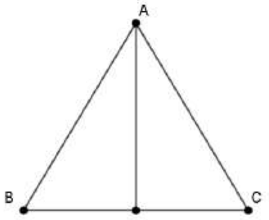
$= 2.\sin\frac{\pi}{8}v$

LONG ANSWER TYPE QUESTIONS -5 MRKS

25. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

Solution:

In an equilateral triangle, median is same as altitude. Let ‘h’ denote the length of the median (or altitude) and ‘x’ be the side of Δ ABC.



Then, $h = \frac{\sqrt{3}}{2}x$ or $x = \frac{2h}{\sqrt{3}}$ ________ (i)

It is given that $\frac{dh}{dt} = 2\sqrt{3}$ So, by (i) we have

$$\frac{dx}{dt} = \frac{2}{\sqrt{3}}\frac{dh}{dt} \Rightarrow \frac{dx}{dt} = 4$$

Thus, the side of Δ ABC is increasing at the rate of 4 cm/sec.

26. Sum of two numbers is 5. If the sum of the cubes of these numbers is least, then find the sum of the squares of these numbers.

Let the two numbers be x and y. Then, x + y = 5 or y = 5 – x

Let S denote the sum of the cubes of these numbers. Then

$$S = x^3 + y^3 = x^3 + (5 - x)^3$$

$$\frac{dS}{dx} = 3x^2 - 3(5 - x)^2 = 15(2x - 5)$$

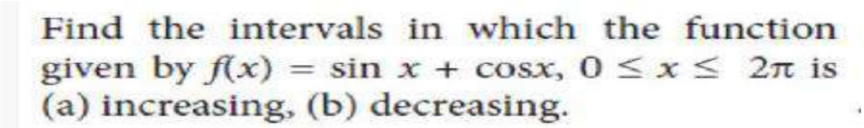
Now $\frac{dS}{dx} = 0$, gives $x = \frac{5}{2}$

Showing S is minimum at $x = \frac{5}{2}$

So, the two numbers are $\frac{5}{2}$ and $\frac{5}{2}$

$$\Rightarrow x^2 + y^2 = \frac{25}{4} + \frac{25}{4} = \frac{25}{2}$$

27.



Find the intervals in which the function given by $f(x)$ = sin x + cos$x$, $0 \le x \le 2\pi$ is (a) increasing, (b) decreasing.

The given function is

$f(x) = \sin x + \cos x;\ 0 \le x \le 2\pi$

$\Rightarrow\quad f'(x) = \cos x - \sin x$

Now $f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

$\Rightarrow\quad \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

Thus $f'(x) > 0$ in $\left[0, \frac{\pi}{4}\right)$,

$f'(x) < 0$ in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ and $f'(x) > 0$ in $\left(\frac{5\pi}{4}, 2\pi\right]$

$\Rightarrow$ The function f is decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ and it is increasing in $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$.

CASE BASED QUESTIONS -4 MRKS

28. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.

Based on the above information, answer the following questions :

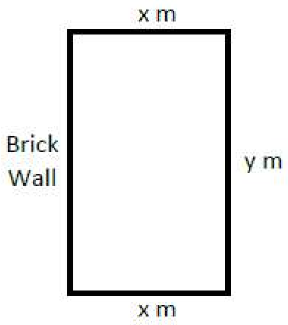
(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden.

(ii) Determine the maximum value of A(x).

Solution:

(i)$(a)\ 2x + y = 200$

$(b) A(x) = xy = x(200 - x)$



(ii) From (a) and (b) of (1) we have

$A(x) = x(200 - 2x)$

$= 200x - 2x^2$

From max./min of A(x)

$\frac{dA}{dx} = 0$ i.e. $200 - 4x = 0$

$\Rightarrow x = 50.$

$\frac{d^2A}{dx^2} = -4$

$\left(\frac{d^2A}{dx^2}\right)_{x=50} < 0$

Hence, A(x) is maximum at x = 50

Thus, Max $A(x) = 200(50) - 2(50)^2$

$= 10000 - 5000$

$= 5000$ sqm.

29. A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.

Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify.

(ii) Find the time at which the height of the ball is maximum.

(i) $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$

Clearly h(t) is a polynomial function, hence continuous.

Hence h(t) is a continuous function.

(ii)For maximum height ,

$\frac{dh}{dt} = 0 \Rightarrow -7t + \frac{13}{2} = 0$

$\Rightarrow t = \frac{13}{14}$

$\frac{d^2h}{dt^2} = -7 < 0 \quad \therefore$ height is maximum at $t = \frac{13}{14}$

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30. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x.

(ii) Find $\frac{dC}{dx}$.

(iii) (a) Find the value of x for which cost C is minimum.

OR

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$.

(i) $C = 40000h^2 + 5000x^2$

as $x^2h = 250$

$\Rightarrow C = \frac{40000\,(250)^2}{x^4} + 5000x^2$

(ii) $\frac{dC}{dx} = \frac{-160000\,(250)^2}{x^5} + 10000x$

(iii)(a) For minimum cost $\frac{dC}{dx} = 0$

$\Rightarrow 10000x^6 = 250 \times 250 \times 160000$

$\Rightarrow x = 10$

showing $\frac{d^2C}{dx^2} > 0$ at $x = 10$

$\therefore$ cost is minimum when $x = 10$

OR

(iii)(b) $\frac{dC}{dx} = \frac{-160000\,(250)^2}{x^4} + 10000x$

$\frac{dC}{dx} = 0$ gives $x = 10$

$\frac{dC}{dx} > 0$ in $(10, \infty)$ and $\frac{dC}{dx} < 0$ in $(0, 10)$.

Hence, cost function is neither increasing nor decreasing for $x > 0$

31.. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

(ii) Find $\frac{dV}{dr}$.

(iii) (a) Find the radius of cylinder when its volume is maximum.

OR

(b) For maximum volume, $h > r$. State true or false and justify.

(i) $\pi r^2 + 2\pi r h = 75\pi \Rightarrow h = \frac{75 - r^2}{2r}$, $\therefore V = \pi r^2 h = \frac{\pi}{2}\left(75r - r^3\right)$

(ii) $\frac{dV}{dr} = \frac{\pi}{2}\left(75 - 3r^2\right)$

(iii) $\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0 \therefore$ **volume is maximum when** $r = 5$

Or

False,

$\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0 \therefore$ **volume is maximum when** $r = 5$

As volume is maximum at $r = 5 \Rightarrow h = \frac{75 - 5^2}{2(5)} = 5 \Rightarrow h = r$

32. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.

A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45°.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm.

(iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm.

OR

(iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm.

(i) $v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ [as $\theta = 45°$ gives $r = h$]

(ii) $\frac{dv}{dt} = \pi r^2 \frac{dr}{dt}$

$\Rightarrow \left(\frac{dr}{dt}\right)_{r=2\sqrt{2}} = -\frac{1}{4\pi}$ cm/sec

(iii)(a) $C = \pi r l = \pi r \sqrt{2}\, r = \sqrt{2}\,\pi r^2$

$\frac{dC}{dt} = \sqrt{2}\,\pi\, 2r \frac{dr}{dt}$

$\left(\frac{dC}{dt}\right)_{r=2\sqrt{2}} = -2\ cm^2/sec$

OR

(iii)(b) $l^2 = h^2 + r^2$

$l = 4 \Rightarrow r = h = 2\sqrt{2}$

$h = r \Rightarrow \frac{dh}{dt} = \frac{dr}{dt} = -\frac{1}{4\pi}$ cm/sec

QUESTIONS FOR PRACTICE

1. The interval in which the function y = $x^2e^{-x}$is increasing is:

   $A)\ (-\infty, 0)\quad B)\ (0,2)\quad C)\ (2,\infty)\quad D)\ (-\infty,\infty)$

   Ans:B
2. The total revenue received from the sale of x units of a product is given by R(x) = $10x^2$ + 13x + 24. Find the marginal revenue when x = 5

   A) 113 Rupees B) 223 Rupees C) 93 Rupees D) 339 Rupees

   Ans:A
3. The bottom of a rectangular swimming tank is 25 m by 40 m water is pumped into the tank at the rate of 500 cubic meters per minute. Find the rate at which the level of water in the tank is rising.?

   A) ¼ m/min B) 2/3 m/min C) 1/3 m/min D) ½ m/min

   Ans:D
4. A particle moves along the curve $y = x^2 + 2x$. At what point(s) on the curve are the x and y coordinates of the particle changing at the same rate?

   A) (1/3, 2/3) B) (-1/3, 2/3) C) (-1/3, -2/3) D) (-1/2, -4/3)

   Ans:D
5. The volume of a cube is increasing at the rate of 8 cm3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

   A) 5/3 Sq.cm/sec B) 7/3 Sq.cm/sec C) 8/3 Sq.cm/sec D) 5/7 Sq.cm/sec

   Ans: C
6. Find an angle x which increases twice as fast as its sine. (2 marks)

   Ans:$\frac{\pi}{3}$
7. Show that the function $f(x) = \tan^{-1}(sinx + cosx)$ is decreasing at all $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

   (2 marks)
8. Find the absolute maximum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in $\left[-2, \frac{9}{2}\right]$

   Ans:8

   (2 marks)
9. Show that the function $f(x) = \cot^{-1} x + x$ increases in $(-\infty, \infty)$. (2 marks)
10. A particle moves along the curve y = x^2 + 2x. At what point(s) on the curve are the x and y coordinates of the particle changing at the same rate?

    Ans:$\left(-\frac{1}{2}, -\frac{3}{4}\right)$

    (2 marks)
11. The total cost associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 5 units are produced, where by marginal cost we mean the instantaneous rate of change of the total cost at any level of output.

    Ans: Rs. 15.495

    (3 marks)
12. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}(0.5)$ water is poured into it at a constant rate of 5cm^3/hr. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.

    Ans:$\frac{35}{88}\ m/hr$

    (3 marks)

13. A man is moving away from a tower 41.6 m high at the rate of 2 m/s. Find the rate at which the angle of elevation of the top of tower is changing when he is at a distance of 30 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.

Ans:$\frac{4}{25} radian/$ sec

(3 marks)

14. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is strictly increasing or decreasing.

Ans: Increasing on $(-1,0) \cup (2,\infty)$ decreasing on $(-\infty,-1) \cup (0,2)$

(3 marks)

15. A ladder 13 m long is leaning against a wall.The bottom of the ladder is being pulled away from the wall along the ground at the rate of 2cm/sec.How fast is the height on the wall decreasing when the foot of the ladder is 5m away from the wall?

Ans:$\frac{5}{6}\ cm/sec$

(3 marks)

16. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs 70 per sqmetres for the base and ₹ 45 per square metre for sides. What is the cost of least expensive tank?

Ans:Rs.1000/-

(5 marks)

17. Find the intervals in which: f(x) = sin3x – cos3x, 0 < x < π, is strictly increasing or strictly decreasing.

Ans: strictly increasing for $\left(0,\frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$ and strictly decreasing for $\left(\frac{\pi}{4},\frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12},\pi\right)$

(5 marks)

18. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle. (5 marks)

19. Mr.Suresh who is a popular businessman consulted a share market expert to know about the investment in a particular company. He predicted that the trend of the company would be governed by the function
$f(x) = x^3 - 4x^2 + 15x + 8$ where x is the years of investment in the company.

i. In the first ten years when will he get growth in his investment?

ii. There is going to be a lean patch in the investment. When is it going to happen? When will the market pick up again?

Ans: (i) x = 3 (ii) After 3 years to 5th year ,after 5th year

20. To reduce global warming environmentalists and scientists came up with an innovative idea of developing a spherical bulb that would absorb harmful gases and thereby reduce global warming.But during the process of absorption the bulb would get inflated and it's radius would be increasing at 1cm/sec.

i) Find the rate at which the volume increases when radius is 6 cm.

ii) At an instant when volume was increasing at the rate of 400πcm3/sec find the rate at which it's surface area is increasing?

Ans:(i)144 π cm^3/sec (ii) 80 π cm^2/sec

21. To honor the scientists associated with the success of chandrayaan-3, A school management decided to felicitate them. The students were asked to stand in a path governed by $y = x^2$. The scientists would be asked to move in a vehicle waving at the children along the path y = x – 2.
i) A student nearest to their path was given an opportunity to garland the dignitaries. Where on the curve does the student stand?
ii) What is the distance of the student from the line?

Ans:(i) (1/2 ,1/4) (ii) $\frac{7}{4\sqrt{2}}$units

22. A company was given contract to manufacture two varieties of bulbs A & B which will be sold at profits of Rs. 60 & Rs. 80 respectively. There was a condition that sum of squares of the number of bulbs of each type is a constant k.
i) What is the ratio of production of 2 bulbs for maximum profit?
ii) What is the maximum profit if k = 100?

Ans:(i) 3:4 (ii) Rs 1000 /-

23. A store has been selling calculators at Rs. 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold The relation between the price and quantity sold is given by demand function

$p = 450 - \frac{x}{2}$.

Based on the above information, answer the following questions:

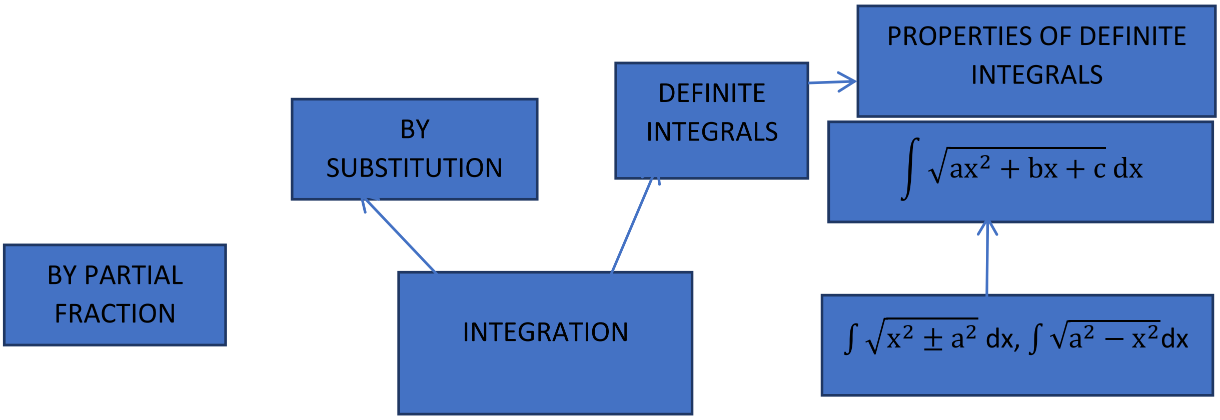
(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also verify the result.

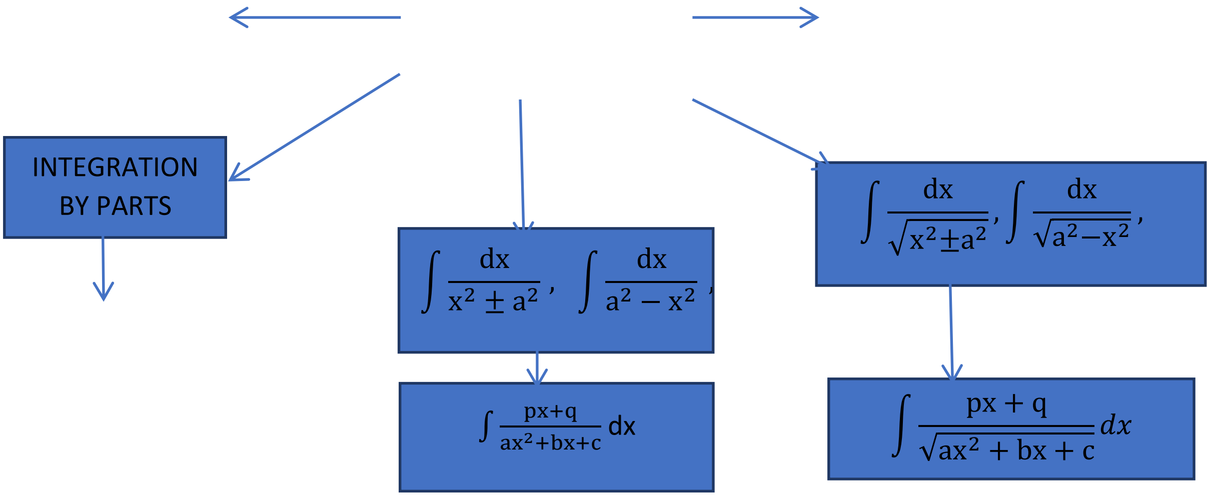
(ii) What rebate in price of calculator should the store give to maximise the revenue?

Ans: (i) x= 450 (ii) Rs.125 per calculator

CHAPTER: INTEGRALS

MIND MAP





SYLLABUS:

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{a^2 - x^2}, \quad \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \sqrt{x^2 \pm a^2}\, dx, \int \sqrt{a^2 - x^2}\, dx,$$

$$\int \sqrt{ax^2 + bx + c}\, dx \int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

IMPORTANT FORMULAE

Indefinite Integrals

1. $\int 1dx = x + c$
2. $\int x\, dx = x^2 + c$
3. $\int sinx\, dx = -cosx + c$
4. $\int cosxdx = sinx + c$
5. $\int tanx\, dx = \log secx + c$
6. $\int cosecx\, dx = \log|(cosecx - cotx)| + c$
7. $\int secx\, dx = \log|secx + tanx| + c$

8. $\int cotxdx = log|sinx| + c$

9. $\int sec^2xdx = tanx + c$

10. $\int cosec^2xdx = -cotx + c$

11. $\int secx.tanxdx = secx + c$

12. $\int cosecx.cotxdx = -cosecx + c$

13. $\int e^xdx = e^x + c$

14. $\int \frac{dx}{x} = \log x + c$

15. $\int a^xdx = \frac{a^x}{\log a} + c$

16. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

17. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

18. $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

19. $\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c$

20. $\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c$

21. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} + c$

22. $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

23. $\int \sqrt{x^2 + a^2}dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

24. $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

25. $\int e^x\left(f(x) + f^l(x)\right)dx = e^xf(x) + c$

26. $\int u.vdx = u\int v\,dx - \int u^l\,[\int v\,dx]dx$

Partial fractions

- The rational function $\frac{P(x)}{Q(x)}$ is said to be proper if the degree of P(x) is less than the degree of Q(x)
- Partial fractions can be used only if the integrand is proper rational function

S.No	Form of rational function	Form of Partial fraction
1	$\frac{1}{(x-a)(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$

2	$\frac{px+q}{(x-a)(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}$
3	$\frac{px^2+qx+c}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{C}{(x-c)}$
4	$\frac{1}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{C}{(x-c)}$
5	$\frac{1}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
6	$\frac{px+q}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
8	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$ where x^2+bx+c cannot be factorized further	$\frac{A}{(x-a)}+\frac{Bx+C}{x^2+bx+c}$

Definite Integrals

Properties of Definite Integrals

1. $\int_a^a f(x)dx$=0
2. $\int_a^b f(x)dx$=$\int_a^b f(t)dt$
3. $\int_a^b f(x)dx$=$-\int_b^a f(x)dx$
4. $\int_a^b f(x)dx$=$\int_a^c f(x)dx$+$\int_c^b f(x)dx$
5. $\int_a^b f(x)dx$=$\int_a^b f(a+b-x)dx$
6. $\int_0^a f(x)dx$=$\int_0^a f(a-x)dx$
7. $\int_0^{2a} f(x)dx$=$\int_0^a f(x)dx$+$\int_0^a f(2a-x)dx$
8. $\int_0^{2a} f(x)dx$x=$\begin{cases} 2\int_0^a f(x)dx & if\ f(2a-x)=f(x) \\ 0 & if\ f(2a-x)=-f(x) \end{cases}$
9. $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ f(-x)=f(x) \\ 0 & if\ f(-x)=-f(x) \end{cases}$

MULTIPLE CHOICE QUESTIONS

Qn. No	Questions and answers
1	$\int 2^{3x+4}\ dx = ?$ (a)$\frac{3\,.2^{3x+4}}{log2}+c$ (b) $\frac{2^{3x+4}}{3log2}+c$ (c) $\frac{2^{3x+4}}{2log3}+c$ (d) None of these Ans (b)
2	$\int \frac{cosx}{\sqrt{1+sinx}}$ dx = ? $a)$ $1 + sinx$ b) $1 - sinx$ c) $2\sqrt{1+sinx}$ d) $2\sqrt{1-sinx}$

	Ans(c)
3	$\int \frac{dx}{x\,cos^2(1+logx)}$ = ? (a) tan(1+logx) + c(b) cot (1+logx) + c (c) sec (1+logx) + c (d) None of these Ans (a)
4	$\int e^x\, secx(1+tanx)dx$ (a)$e^x secx$ + c(b) $e^x tanx$ + c (c) $e^x(1+tanx)$ + c (d) None of these Ans(a)
5	$\int_0^4 \lvert x-1\rvert$ dx =? (a)5 (b) 4 (c) 2 (d) None of these Ans(a)
6	If $\frac{d}{dx}$(f(x)) = logx , then f(x) equals : $a)$ $\frac{1}{x}$+C b) x (logx + x)+C c) x (log x -1) +C d) - $\frac{1}{x}$+C Ans (c)
7	$\int \frac{dx}{e^x-1}$= ? a) log |e^x – 1| +C b) log|1 – e^{-x}|+C c) log|1 – e^x|+C d) log| $e^{-x}-1$|+C Ans: (b)
8	$\int \sqrt{4-9x^2}$ dx=? a)$\frac{x}{2}$. $\sqrt{4-9x^2}$ - $\frac{2}{3}$sin^{-1}($\frac{3x}{2}$)+C b)$\frac{x}{2}$. $\sqrt{4-9x^2}$ +$\frac{2}{3}$sin^{-1}($\frac{3x}{2}$)+C c) $\frac{3x}{2}$. $\sqrt{4-9x^2}$ +$\frac{2}{3}$sin^{-1}($\frac{3x}{2}$)+C d)None of these Ans: (b)

ASSERTIONS AND REASONING QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

A) Both Assertion (A) andReason (R) true and Reason (R) is the correct explanation of Assertion (A).

B) Both Assertion (A) andReason (R) true and Reason (R) is not the correct explanation of Assertion (A)

C) Assertion (A) is true but Reason (R) is false

D) Assertion (A) is false but Reason (R) is true

Qn. No	Questions and answers
1	Assertion:$\int(sin^{-1}x+cos^{-1}x)dx=\frac{\pi}{2}x+c$ Reason: $sin^{-1}x+cos^{-1}x=\frac{\pi}{2}$

	Answer: A) both assertion and reasoning are correct and reason is the correct explanation
2	Assertion:$\int e^x(\cos x + \sin x)\ dx = e^x \cos x + c$ Reason:$\int e^x(f(x) + f'(x))\ dx = e^x f(x) + c$ **Answer:** D) Assertion is False and reasoning is correct
3	Assertion: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx = 0$ Reason: If f(x) is odd function $\int_{-a}^{a} f(x)dx = 0$ **Answer:** A) both assertion and reasoning are correct and reason is the correct explanation

2 MARK QUESTIONS

Qn No	Questions and Answers
1	Evaluate$\int \frac{dx}{5-8x-x^2}$ **Solution** $\int \frac{dx}{5-8x-x^2} = \int \frac{dx}{(\sqrt{21})^2-(x+4)^2} = \frac{1}{2\sqrt{21}} \log\left\|\frac{\sqrt{21}+(x+4)}{\sqrt{21}-(x+4)}\right\| +C$
2	Evaluate $\int \frac{1}{Cos\ (x-a\ )Cos\ (x-b\ )}\ dx$ **Solution:**$\int \frac{1}{Cos\ (x-a\ )Cos\ (x-b\ )}\ dx$ $= \frac{1}{Sin\ (a-b\ )} \int \frac{Sin(\ a-b\ )}{Cos\ (x-a\ )Cos\ (x-b\ )}\ dx$ $= \frac{1}{Sin\ (a-b\ )} \int \frac{Sin[\ (x-b)-(x-a\ )]}{Cos\ (x-a\ )Cos\ (x-b\ )}\ dx$ $= \frac{1}{Sin\ (a-b\ )}\ [\ \int tan\ (x-b)\ dx - \int tan(\ x-a\ )\ dx$

	$= \frac{1}{Sin\ (a-b)}$ log ($\frac{Cos\ (x-a)}{Cos\ (x-b)}$) + C
3	Find: $\int \frac{x+5}{3x^2+13x-10}$dx **Solution** $\int \frac{x+5}{3x^2+13x-10}$dx $=\int \frac{x+5}{(x+5)(3x-2)}$dx= $\int \frac{1}{(3x-2)}$dx= $\frac{1}{3}$log$\lvert 3x-2\rvert$+c
4	Find$\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}$dx **Solution:-** Let I=$\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}$dx Put x$^{3/2}$=t⟹ $\sqrt{x}$dx = $\frac{2}{3}$dt I=$\frac{2}{3}\int \frac{dt}{\sqrt{(a^{\frac{3}{2}})^2 - t^3}}$ =$\frac{2}{3}\sin^{-1}(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}})$+C
5	Find $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$dx **Solution** I=$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$dx Put $\sin^{-1} x$=t , x= sint, $\frac{dx}{\sqrt{1-x^2}}$ =dt I=$\int t\ sint\ dt$ = (- cost) t - $\int (-cost).1\ dt$ =-tcost + sint+C
6	$Evaluate \int sin2x\ sin3x\ dx$ **Solution** $\int sin2x\ sin3x\ dx =\int \frac{(cosx-cos5x)}{2}$ dx $=\frac{sinx}{2} - \frac{sin5x}{10}$+c
7	$Evaluate \int \frac{5x+3}{\sqrt{(x^2+4x+10)}}$ dx **Solution** 5x + 3 = A (2x +4) +B , 5= 2A , A =5/2 3= 4A + B , B =-7

	I = $5/2 \int \frac{2x+4}{\sqrt{(x^2+4x+10)}}$ dx - 7 $\int \frac{1}{\sqrt{(x^2+4x+10)}}$ dx $5/2 \int \frac{dt}{\sqrt{t}}$ (t=$\sqrt{(x^2+4x+10)}$) -7 $\int \frac{1}{\sqrt{((x+2)^2+6)}}$dx $5/2\frac{t^{1/2}}{\frac{1}{2}}$ - 7 log (x + 2 + $\sqrt{((x+2)^2+6)}$) +c $5\sqrt{(x^2+4x+10)}$ -7 log (x + 2 + $\sqrt{((x+2)^2+6)}$) +c.
8	$Evaluate \int \frac{cosx - sinx}{1+sin2x} dx$ **Solution** $\int \frac{cosx-sinx}{1+sin2x} dx$ =$\int \frac{cosx-sinx}{(\sin x+cosx)^2} dx$ =$\int \frac{dt}{t^2}$ (Substituting sinx+ cosx= t) = $\frac{-1}{sinx+cosx}$+ C

3 MARKS QUESTIONS

Qn No	Questions and Answers
1	Find $\int \frac{sin\theta}{(4+cos^2\theta)(2-sin^2\theta)}$ dθ **Solution:-** I= $\int \frac{sin\theta}{(4+cos^2\theta)(2-sin^2\theta)}$ dθ =$\int \frac{sin\theta}{(4+cos^2\theta)(1+cos^2\theta)}$ dθ Put cosθ=t $sin\theta$ dθ= -dt We get I=$-\int \frac{dt}{(4+t^2)(1+t^2)}$

	I= 1/6 $\tan^{-1} t/2 - 1/3\tan^{-1} t$+c = 1/6 $\tan^{-1}(cos\theta/2)$ −1/3$\tan^{-1}(cos\theta)$+c
2	Evaluate $\int \frac{e^x dx}{\sqrt{5-4e^x-e^{2x}}}$ Ans: $e^x = t$, $\frac{dt}{dx} = e^x$ dt = e^x dx$\int \frac{dt}{\sqrt{5-4t-t^2}}$= $\int \frac{dt}{\sqrt{5-(4t+t^2)}}$ =$\int \frac{dt}{\sqrt{5-[(t+2)^2-4]}}$ $\frac{dt}{\sqrt{9-(t+2)^2}}$ = $\sin^{-1}\left(\frac{t+2}{3}\right)$ + c = $\sin^{-1}(\frac{e^x+2}{3})$ + c
3	Evaluate $\int_0^\pi \frac{xtanx}{secx+tanx}dx$ **Solution** I=$\int_0^\pi \frac{xtanx}{secx+tanx}dx$ =$\int_0^\pi \frac{\pi-xtanx}{secx+tanx}dx$ 2I=$\pi\int_0^\pi \frac{tanx}{secx+tanx}dx$ = π$\int_0^\pi tanx(secx-tanx)dxdx$ =π/2[secx-tanx+x]$_0^\pi$ =π(π-2)/2
4	Find $\int \frac{x^2}{(x-1)(x^2+1)}$dx Solution: Let $\frac{x^2}{(x-1)(x^2+1)}$ =$\frac{A}{x-1}$ + $\frac{Bx+C}{x^2+1}$ A=1/2, B=1/2,C=1/2 $\int \frac{x^2}{(x-1)(x^2+1)}$dx = $\frac{1}{2}$log$\lvert x-1\rvert$+ $\frac{1}{4}$log$\lvert x^2+1\rvert$+$\frac{1}{2}\tan^{-1}x$+ C
5	Find$\int \frac{(3sin\theta-2)cos\theta\, d\theta}{5-cos^2\theta-4sin\theta}$ **Solution:-** I=$\int \frac{(3sin\theta-2)cos\theta\, d\theta}{5-(1-sin^2\theta)-4sin\theta}$ Put sinθ=t ⟹cosθdθ=dt

	I=$\int\frac{3t-2}{(t-2)^2}$dt On solving we get 3 log$\|t-2\|$-$\frac{4}{(t-2)}$+C 3 log$\|sin\theta-2\|$-$\frac{4}{(sin\theta-2)}$+C
6	Evaluate$\int_{-1}^{2}\|x^3-x\|\,dx$ **Solution:-** Let I=$\int_{-1}^{2}\|x^3-x\|\,dx$ =$\int_{-1}^{0}(x^3-x)dx$+$\int_0^1-(x^3-x)dx$+$\int_1^2(x^3-x)dx$ On solving we get =-(1/4-1/2)+(1/2-1/4)+(4-2)-(1/4-1/2) =1/4+1/4+2+1/4 =11/4
7	$Evaluate\int\frac{sinx-xcosx}{x(x+sinx)}$dx **Solution:-** I=$\int\frac{x+sinx-x(1+cosx)}{x(x+sinx)}$dx =$\int\frac{1}{x}dx$ - $\int\frac{1+cosx}{(x+sinx)}$dx put x+sinx=t⟹ (1+cosx)dx=dt =log$\|x\|$-log$\|x+sinx\|$+C
8	Evaluate:$\int_0^{\pi/4(}\frac{sinx+cosx\,)}{3+sin2x}$ dx **Solution:-** I=$-\int_0^{\pi/4}\frac{sinx+cosx}{(sinx-cosx)^2-2^2}$dx Put sinx-cosx=t, (cosx+sinx)dx=dt And t varies from -1 to 0 I=$-\int_{-1}^{0}\frac{dt}{t^2-2^2}$ =$\left(\frac{1}{4}\,log\frac{t-2}{t+2}\right)_{-1}^{0}$

	$= \frac{1}{4}$ log3

5 MARK QUESTIONS

Qn No	Questions and Answers
1	Evaluate: $\int_0^{\pi} \frac{xsinx}{1+3cos^2x}dx$ **Solution:-** Let I=$\int_0^{\pi} \frac{xsinx}{1+3cos^2x}dx$-----------(1) $=\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+3cos^2(\pi-x)}dx$ $=\int_0^{\pi} \frac{\pi sinx}{1+3cos^2x}dx$-$\int_0^{\pi} \frac{xsinx}{1+3cos^2x}dx$------(2) Adding (1) and (2) 2I=$\int_0^{\pi} \frac{\pi sinx}{1+3cos^2x}dx$ Put cosx =t, sinx dx= -dt 2I= - $\pi\int_1^{-1} \frac{dt}{1+3t^2}$ =π/3$\int_{-1}^{1} \frac{dt}{(\frac{1}{\sqrt{3}})^2+t^2}$ On solving we get I = $\frac{\sqrt{3}\,\pi}{3}.\frac{\pi}{3}= \frac{\sqrt{3}\pi^2}{9}$
2	$Evaluate \int_0^{\frac{3}{2}} \|x\cos\pi x\|\,dx$ Ans: In 0 < x<1/2 ,$x\cos\pi x > 0$ In 1/2 < x<3/2 ,$x\cos\pi x < 0$ $\int_0^{\frac{3}{2}}\|x\cos\pi x\|\,dx = \int_0^{\frac{1}{2}} x\cos\pi x\,dx$ - $\int_{\frac{1}{2}}^{\frac{3}{2}} x\cos\pi x\,dx$ $=\left(x\frac{\sin\pi x}{\pi}\right)_0^{\frac{1}{2}}$ - $\int_0^{\frac{1}{2}} 1\,\frac{\sin\pi x}{\pi}dx$ - $\left(x\frac{\sin\pi x}{\pi}\right)_{\frac{1}{2}}^{\frac{3}{2}} + \int_{\frac{1}{2}}^{\frac{3}{2}} 1\,\frac{\sin\pi x}{\pi}dx$ $= \left(\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right)_0^{\frac{1}{2}}$ - $\left(\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right)_{\frac{1}{2}}^{\frac{3}{2}}$

	$=\frac{1}{\pi}\left[\frac{5}{2}-\frac{1}{\pi}\right]$
3	Find: $\int\frac{2x+1}{(x^2+1)(x^2+4)}$dx **Solution:-** I=$\int\frac{2x+1}{(x^2+1)(x^2+4)}$ Let $\frac{2x+1}{(x^2+1)(x^2+4)}=\frac{Ax+B}{(x^2+1))}+\frac{Cx+D}{(x^2+4)}$ Getting A=2/3, B=1/3, C=-2/3,D=-1/3 I=$\frac{1}{3}\log\lvert x^2+1\rvert+\frac{1}{3}\tan^{-1}x-\frac{1}{3}log\lvert x^2+4\rvert-\frac{1}{6}\tan^{-1}\frac{x}{2}$ +C
4	$Evaluate\ \int_0^a\frac{1}{(x+\sqrt{a^2-x^2})}$ dx Solution: x = asinθ , dx = a cos$\theta d\theta$ $\int_0^a\frac{1}{(x+\sqrt{a^2-x^2})}$dx= $\int_0^{\frac{\pi}{2}}\frac{1}{(a\sin\theta+a\cos\theta)}$ a cos $\theta d\theta$ $I=\int_0^{\frac{\pi}{2}}\frac{\cos\theta}{(\sin\theta+\cos\theta)}d\theta$………………..(1) By property $\int_0^a f(x)\ dx\ =\int_0^a f(a-x)\ dx$ I= $\int_0^{\frac{\pi}{2}}\frac{\sin\theta}{(\sin\theta+\cos\theta)}d\theta$………………………………(2) (1) + (2) gives, 2I = $\int_0^{\frac{\pi}{2}}\frac{(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)}d\theta$ 2I= $\frac{\pi}{2}$ I=$\frac{\pi}{4}$
5	Evaluate $\int_0^\pi\frac{x}{1+\sin\alpha\sin x}dx$ Ans: Let I = $\int_0^\pi\frac{x}{1+\sin\alpha\sin x}dx$ Apply property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ to get

	$I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin\alpha \sin(\pi - x)} dx$ $2I = \pi \int_0^{\pi} \frac{dx}{1 + \sin\alpha \sin x}$ $= 2\pi \int_0^{\pi/2} \frac{dx}{1 + \sin\alpha \sin x}$ $I = \pi \int_0^{\pi/2} \frac{dx}{1 + \sin\alpha \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}}$ $I = \pi \int_0^{1} \frac{2dt}{1 + t^2 + 2t\sin\alpha}$ Put $\tan\frac{x}{2} = t$ $\Rightarrow I = 2\pi \int_0^{1} \frac{dt}{(t + \sin\alpha)^2 + \cos^2\alpha}$ $= \frac{2\pi}{\cos\alpha} \left[\tan^{-1}\left(\frac{t + \sin\alpha}{\cos\alpha}\right)\right]_0^1$ $\Rightarrow I = \frac{\pi}{\cos\alpha}\left(\frac{\pi}{2} - \alpha\right)$

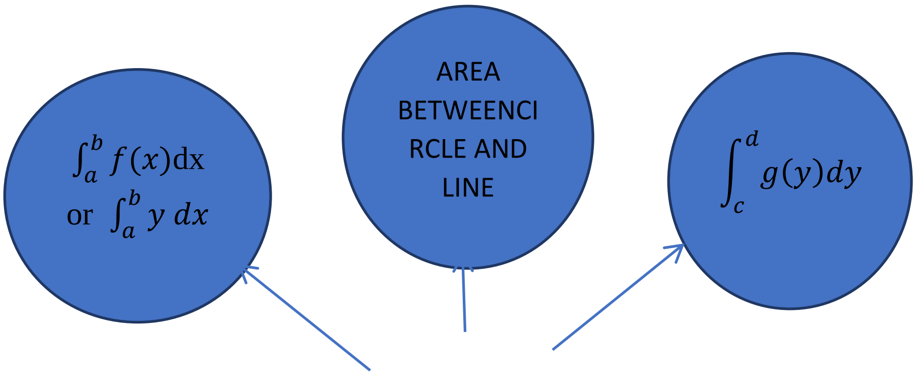
QUESTIONS FOR PRACTICE

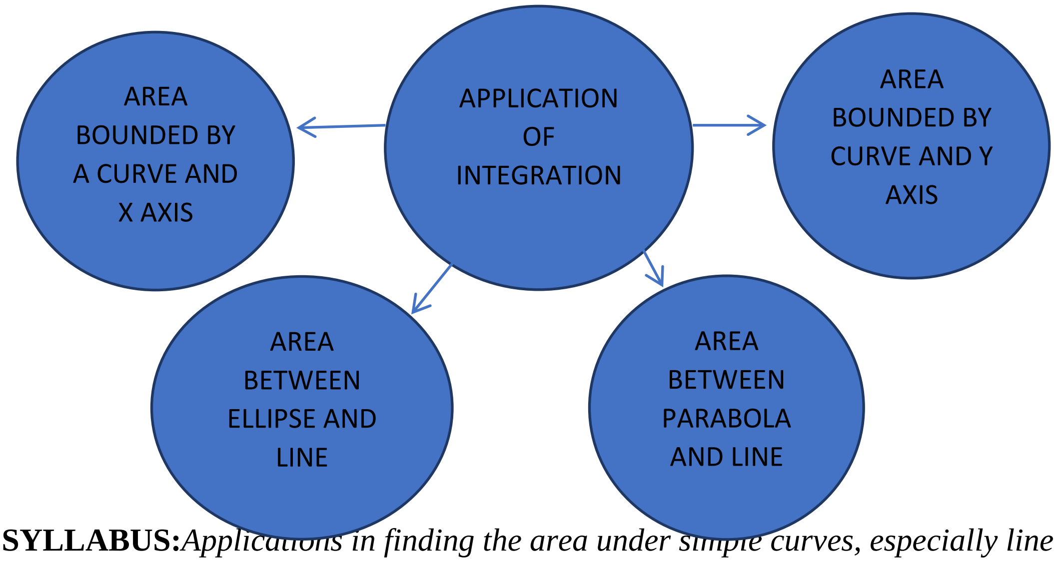
Q No	Questions	Answers	Marks
1	The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$ is : a) $\pi/6$ b) $\pi/4$ c) $\pi/2$ d) $\pi/18$	$\pi/2$	1
2			

	If $\int_0^a \frac{1}{1+4x^2}dx=\frac{\pi}{8}$, then a is a. $\frac{1}{2}$ b. $\frac{3}{5}$ c. $\frac{-1}{2}$ d. $\frac{3}{4}$	½	1
3	$\int \cos 4x \cos x\, dx=?$ (a) $\frac{1}{5}\sin 5x+\frac{1}{3}\sin 3x+c$ (b) $\frac{1}{5}\cos 5x-\frac{1}{3}\cos 3x+c$ (c) $\frac{1}{10}\sin 5x+\frac{1}{6}\sin 3x+c$ (d) none of these	(c) $\frac{1}{10}\sin 5x+\frac{1}{6}\sin 3x+c$	**1**
4	$\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx = a\sin 2x+c$, then a = ? a) -1/2 b) ½ c) -1 d) 1	-1/2	**1**
5	Q.Evaluate$\int$ [$\frac{1}{\log x}-\frac{1}{(log x)^2}$] dx ? $a)\frac{logx}{x}$ b) $\frac{x}{\log x}$ c) $\frac{x-1}{logx}$ d) $\frac{\log x}{x-1}$	$\frac{x}{\log x}$	**1**
6	Evaluate$\int\frac{xe^x}{(x+1)^2}$dx	$\frac{e^x}{x+1}$ +C	2
7	Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^3+x\,cosx+\tan^5 x+1)\,dx$	π	2
8	Find $\int\frac{1}{x(1+logx)}dx$	log(1+logx) +C	2
9	Evaluate $\int\frac{dx}{\sin^2 x\cos^2 x}$	tanx- cotx +C	2
10	Find $\int\sqrt{(x^2-2x)}dx$	$\frac{(x-1)}{2}\sqrt{(x^2-2x)}$ -$\frac{1}{2}$log$\left\|x-1+\sqrt{(x^2-2x)}\right\|+c$	2
11	Evaluate$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{1+\sqrt{tanx}}dx$	$\frac{\pi}{12}$	3
12	Evaluate$\int\frac{x^2+1}{(x^2+2)(x^2+3)}dx$	$-\frac{1}{\sqrt2}tan^{-1}\left(\frac{x}{\sqrt2}\right)+\frac{2}{\sqrt3}tan^{-1}\left(\frac{x}{\sqrt3}\right)+C$	3
13	Evaluate $\int\frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx$	6 $\sqrt{x^2-9x+20}$ + 34 log [x - $\frac{9}{2}$ + $\sqrt{x^2-9x+20}$] + C	3
14	Evaluate $\int\frac{1}{x\,(x^4-1)}$ dx?	$\frac{-1}{4}$ log ($\frac{x^4}{x^4-1}$) + C	3
15	Evaluate $\int\frac{2+Sin2x}{1+Cos2x}e^x\,.dx$	$e^x.tanx$ + C	3

16	Evaluate $\int_0^{\pi} \frac{xtanxdx}{secx+tanx}$	$\frac{\pi}{2}[\pi - 2]$	5
17	Evaluate $\int_0^{\pi/2} \frac{sin^2xdx}{sinx+cosx}$	$\frac{\log(1+\sqrt{2})}{\sqrt{2}}$	5
18	Evaluate $\int \frac{1}{1+x^4}\, dx$	$\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)-\frac{1}{4\sqrt{2}}\log\left\lvert\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right\rvert+C$	5
19	Evaluate$\int_1^4 \{\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-4\rvert\}dx$	$\frac{23}{2}$	5
20	Evaluate$\int_0^{\pi} e^{2x} \sin\left(\frac{\pi}{4}+x\right) dx$	I=$\frac{-1}{5\sqrt{2}}(e^{2\pi}+1)$	5

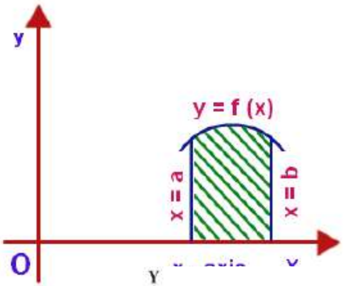
CHAPTER: APPLICATION OF INTEGRALS



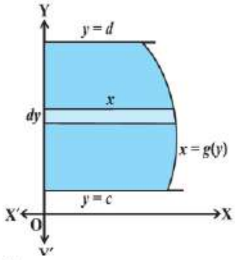


SYLLABUS:*Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)*

Definitions and Formulae:

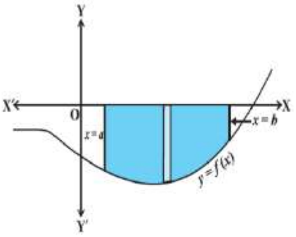


Let f(x) be a function defined in [a, b] , then the area bounded by the curve y = f(x) , x – axis and the ordinates x = a and x = b is given by $\int_a^b f(x)$dx or $\int_a^b y\,dx$



Let $g(y)$ be a function defined in [c, d] , then the area bounded by the curve $x = g(y)$, y – axis and the ordinates$y = c$ and $y = d$ is given by $\int_c^d g(y)dy$

If the curve y = f(x) lies below x- axis ,then the area bounded by the curve y= f(x) , x-axis and the ordinates x = a and x= b is –ve. So the area is $\left|\int_a^b f(x)\,\mathrm{dx}\right|$



MULTIPLE CHOICE QUESTIONS

Qn No	Questions
1	Find the area bounded by y = sinx between x = 0 and x = 2π. (a) 4 sq.unit (b) 4πsq. unit (c) 2 sq.unit (d) 1 sq.unit Ans: (a)
2	The area under the curve y =$\sqrt{a^2 - x^2}$ included between the lines x =0and x = a is (a)$\frac{\pi a^2}{4}$ sq. unit (b) $\frac{\pi}{4}$sq. unit

	(c)$\frac{a^2}{4}$ sq. unit (d) $\frac{\pi a^2}{12}$ sq.unit Ans: (a)
3	Area lying in the first quadrant and bounded by the circle $x^2 + y^2$ = 4 and the lines x =0 and x = 2 is (a) π sq. unit (b) $\frac{\pi}{2}$ sq. unit (c) $\frac{\pi}{3}$ sq. unit (d) $\frac{\pi}{4}$ sq. unit Ans: (a) π sq. unit
4	The area of the region bounded by the curve $y = x^2$ and the line y = 16 is (a) $\frac{32}{3}$ sq. unit (b) $\frac{256}{3}$ sq. unit (c) $\frac{64}{3}$ sq. unit (d) $\frac{128}{3}$ sq.unit Ans: (b) $\frac{256}{3}$ sq. unit
5	The area bounded by the parabola y^2 =8x , the x-axis and the latus rectum is: (a) 16/ 3 (b) 23 /3 (c) 32/ 3 (d) 62/ 3 Ans: (c) 32/ 3
6	The area bounded by curve y =sin 2x , x-axis and the lines x=π/ 4 and 3π/ 4 is: (a) 1 sq. units (b) 2 sq. units (d) 4 sq. units (d) 3/2 sq. units Ans: (a) 1 sq. units
7	The area of the region bounded by the straight line x = 2y + 3 , y axis and the lines y = 1 and y = -1 is (a)4 sq. unit (b) $\frac{3}{2}$ sq. unit (c) 6 sq. unit (d) 8 sq. unit Ans: (c) 6 sq. unit
8	The area of the region bounded by the curve x^2=4y and the line $x - 4y + 2 = 0$ is: (a) 3/8 (b) 5/8 (c) 7/8 (d) 9/8 Ans: (d) 9/8

ASSERTIONS AND REASONING QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

A) Both Assertion (A) andReason (R) true and Reason (R) is the correct explanation of Assertion (A).

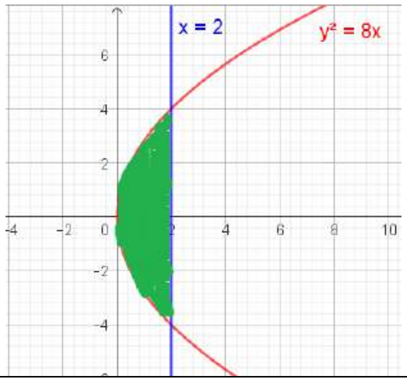
B) Both Assertion (A) andReason (R) true and Reason (R) is not the correct explanation of Assertion (A)

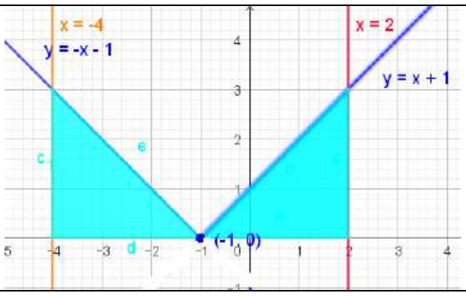
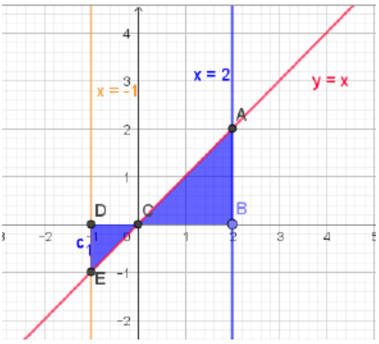
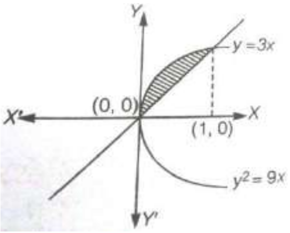
C) Assertion (A) is true but Reason (R) is false

D) Assertion (A) is false but Reason (R) is true

Qn No	**Questions**
1	**Assertion:** The area of the region bounded by y= cosx and the ordinates $x = 0\ and\ x = \pi\ is\ 2sq.unit$ **Reason:** cos x is an increasing function in the first quadrant **Answer:**(c) Assertion (A) is true but Reason (R) is false
2	**Assertion:** The area bounded by the curve x =y^2 , y-axis and the lines y =3 and y= 4 is$\frac{37}{2}$ **Reason:** Area = $\int_a^b f(\,y)$ dy **Answer:**(a)Both Assertion (A) andReason (R) true and Reason (R) is the correct explanation of Assertion (A).
3	**Assertion:** The area between x-axis and y = cosx when$0 \leq$ x$\leq 2\pi$ is 4sq.unit **Reason:**Area= $\int_0^{2\pi} cosx\ dx\ = 4\int_0^{\frac{\pi}{2}} cosx\ dx$ Ans:(a)

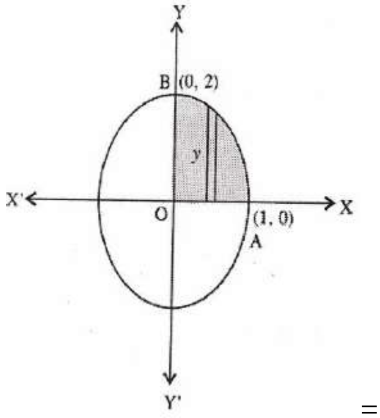
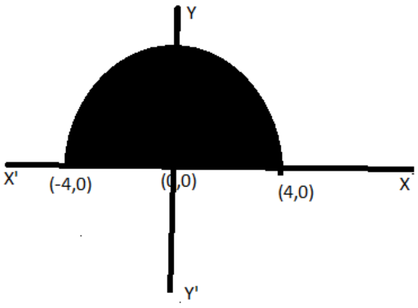
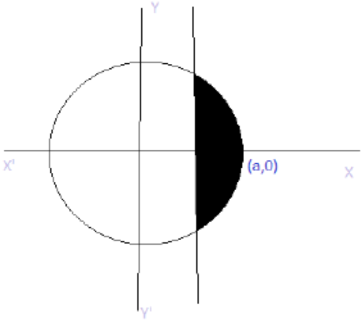
2 MARK QUESTIONS

Qn.No	Questions and Answers
1	Using integration find the area of the region bounded between the line $x = 2$, and the parabola y^2 = 8x. **Solution:** A = $2\int_0^2 \sqrt{8x}$ dx= $\frac{32}{3}$sq.units 
2	Draw the graph of $y = \|x + 1\|$ and find the area between $x -\ axis\ x = -4\ and\ x = 2.$ **Solution:** $A = \left\|\int_{-4}^{-1}(x + 1)\ dx\right\| + \int_{-1}^{2}(x + 1)$dx = 9 sq.units

	
3	Find the area bounded by $y = x$, the $x -$ axis and the ordinate $x = -1$, $x = 2$.  **Solution:** $A = \int_0^2 x\,dx + \left\|\int_{-1}^0 x\,dx\right\| = \frac{5}{2}.$
4	Find the area bounded by the curve $y^2 = 4x$, $y -$ axis and the line $y = 3$ **Solution:** Area = $\int_0^3 x\,dy = \int_0^3 \frac{y^2}{4}dy = 9/4$
5	Find the area of the region bounded by the curves $y^2 = 9x$ and $y = 3x$ **Solution:** Points of intersection are (0,0) and (1,3)  Required area= $\int_0^1 \sqrt{9x}\,dx - \int_0^1 3x\,dx = 2 - \frac{3}{2} = \frac{1}{2}$ sq. units

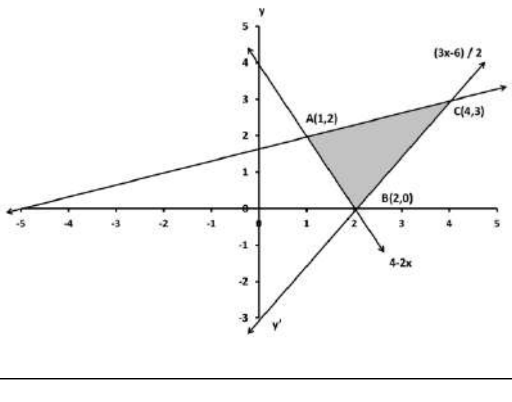
3 MARK QUESTIONS

Qn.No	Questions and Answers
1	Find the area of the region bounded by the curve $4x^2 + y^2 = 1$ **Solution:** Required area= $4\int_0^1 y\,dx = 4\int_0^1 2\sqrt{1-x^2} = 8\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\frac{x}{1}\right]_0^1$ $= 2\pi$ sq.units

	 =
2	Find the area bounded by $y =$ cos x, $x\ axis$and between x = 0 and x= π **Solution:** Required area = $\int_0^{\frac{\pi}{2}} cosx\ dx + \left\|\int_{\frac{\pi}{2}}^{\pi} cosx\ dx\right\|$ = 2 sq. units
3	Using integration, find the area bounded by the curve $y = \sqrt{16 - x^2}$ and the x axis  Required Area = $2\int_0^2 \sqrt{16 - x^2}$ dx =$2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$ = 8πsq.units
4	Using integration,find the area of smaller part of the circlex^2 + y^2 = a^2 cut off by the line x = $\frac{a}{\sqrt{2}}$. **Solution:**  Required area = $2\int_{a/\sqrt{2}}^{a} \sqrt{a^2 - x^2}$ dx = $2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{a/\sqrt{2}}^{a}$

	= $(\frac{\pi}{4} - \frac{1}{2})a^2$ sq.units
6	Find the area bounded by the curve y = sinx between$x = 0 \; and \; x = 2\pi$ **Solution:** $$\int_0^{\pi} sinx\; dx = 2\; sq.units.$$ Area formed below the axis= $\left\lvert\int_{\pi}^{2\pi} sinx\; dx\right\rvert$=2 sq.units. Required area = 4 sq.units
7	Find the area between the curves y= sinx and y= cosxformed between x = 0 and x $=\frac{\pi}{4}$ **Solution:** A = $\int_0^{\frac{\pi}{4}}(cosx - sinx)$ dx = √2 - 1sq.units
8	Find the area of the region bounded by the curve $y = \lvert x - 1\rvert \,, y = 1.$ **Solution:** $A = \int_0^2 1$ dx $-2\int_0^1 (x-1)\; dx$ = 1 sq.units

5 MARK QUESTIONS

Qn.No	Questions and Answers
1	Using the method of integration find the area of the region bounded by lines: 2x + y = 4, 3x – 2y = 6 and x – 3y + 5 = 0 

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Area of the triangle = $\int_1^4 \frac{x+5}{3}\,dx - \int_1^2 4-2x\,dx - \int_2^4 \frac{3\,x-6}{2}dx$

$= [\frac{15}{6} + 5] - [4-3] - [9-6]$

$= \frac{15}{2} - 1\text{-}3 = \frac{7}{2}$
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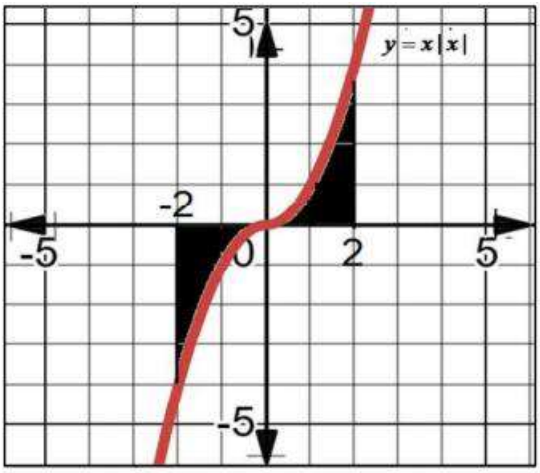
Sketch the graph of y = $x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

<u>Solution:</u>

Area of the shaded region = $\int_{-2}^{2} y\,dx = 2\int_{0}^{2} y\,dx = 2\int_{0}^{2} x^2\,dx$

$= 2\left(\frac{x^3}{3}\right)_0^2$

$= 2\left(\frac{8}{3}\right) = \frac{16}{3}$



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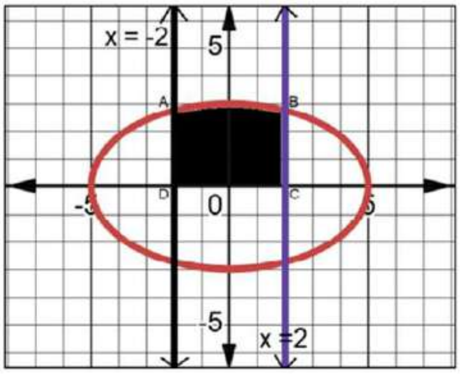
Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

<u>Solution:</u>



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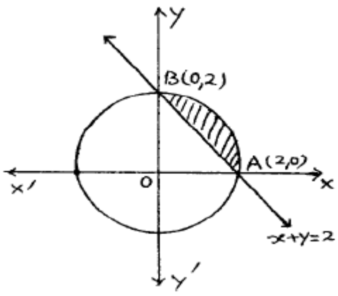
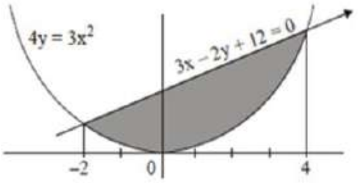
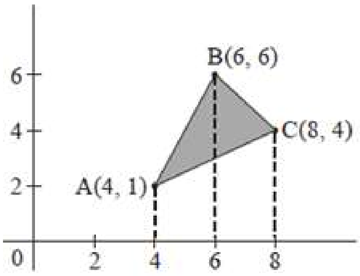
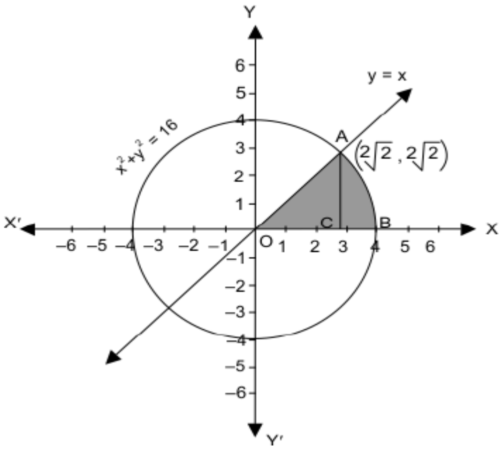
	As, $9x^2+25y^2=225 \Rightarrow \boxed{y=\pm\frac{3}{5}\sqrt{5^2-x^2}}$ Required Area $= \int_{-2}^{2}\frac{3}{5}\sqrt{5^2-x^2}\ dx = \frac{6}{5}\int_{0}^{2}\sqrt{5^2-x^2}\ dx$ $=\frac{6}{5}\left(\frac{x\sqrt{5^2-x^2}}{2}+\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right)_0^2$ $=\frac{6}{5}\left(\frac{2\sqrt{21}}{2}+\frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)\right)$ $=\left(\frac{6\sqrt{21}}{5}+15\sin^{-1}\left(\frac{2}{5}\right)\right)$
4	Find the area of the smaller part bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$ **Solution:-** Required area $=\boldsymbol{A}=\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\ dx - \int_0^a \frac{b}{a}(a-x)\ dx$ For simplifying and getting the result A $=\frac{ab}{2}\left(\frac{\pi}{2}-1\right)$ sq.units
5	Find the area of the region {(x,y): $x^2+y^2 \le 4, x+y \ge 2$} **Solution:-**  Finding the point of intersection as x=0,2 Required area= $\int_0^2 \sqrt{4-x^2}dx - \int_0^2 (2-x)dx$ $=[\frac{1}{2}\ x\ \sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}]_0^2 - [2x - \frac{x^2}{2}]_0^2$ = [0+2(π/2)] – (4-2) $= (\pi-2)\ sq.units.$

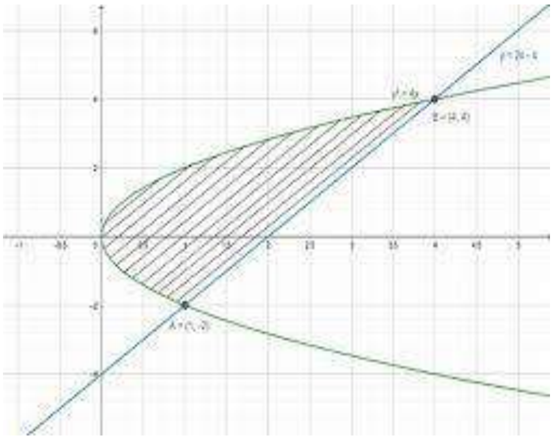
6	Find the area bounded between the curve $4y = 3x^2$ and the line 3x-2y+12=0 **<u>Solution:-</u>**  $4y = 3x^2$ and $3x - 2y + 12 = 0 \Rightarrow 4\left(\frac{3x+12}{2}\right) = 3x^2$ $\Rightarrow 3x^2 - 6x - 24 = 0$ or $x^2 - 2x - 8 = 0 \Rightarrow (x - 4)(x + 2) = 0$ $\Rightarrow$ x-coordinates of points of intersection are $x = -2$, $x = 4$ $\therefore$ Area (A) $= \int_{-2}^{4}\left[\frac{1}{2}(3x+12) - \frac{3}{4}x^2\right]dx$ $= \left[\frac{1}{2}\frac{(3x+12)^2}{6} - \frac{3}{4}\frac{x^3}{3}\right]_{-2}^{4}$ $= 45 - 18 = 27$ sq.units
7	Using integration, find the area of the triangle whose vertices are (4,1), (6,6) and (8,4) **<u>Solution:-</u>** 

	$\left.\begin{array}{l}\text{Equation of AB}: y=\frac{5}{2}x-9\\ \text{Equation of BC}: y=12-x\\ \text{Equation of AC}: y=\frac{3}{4}x-2\end{array}\right\}$ $\therefore$ Area (A) $= \int_4^6\left(\frac{5}{2}x-9\right)dx+\int_6^8(12-x)\,dx-\int_4^8\left(\frac{3}{4}x-2\right)dx$ $=\left[\frac{5}{4}x^2-9x\right]_4^6+\left[12x-\frac{x^2}{2}\right]_6^8-\left[\frac{3}{8}x^2-2x\right]_4^8$ $= 7 + 10 - 10 = 7$ sq.units
8	Find the area bounded by the curve $x^2 + y^2 = 16$and the line $y = x$ in the first quadrant. **Solution:-** Equation of the circle is $x^2 + y^2 = 16$ and the line $y = x$. Solving these two equations, we obtain $x = y = \pm 2\sqrt{2}$. Hence, the line intersects the circle at the point$(2\sqrt{2},2\sqrt{2})$ in the first quadrant. The area enclosed between the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant is as shown by the shaded area OABCO  $=\int_0^{2\sqrt{2}} x\,dx+\int_{2\sqrt{2}}^{4}\left(\sqrt{16-x^2}\right)dx$ $=\left[\frac{x^2}{2}\right]_0^{2\sqrt{2}}+\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{2\sqrt{2}}^{4}$ $=\left[\frac{\left(2\sqrt{2}\right)^2}{2}\right]+\left[\frac{4}{2}\sqrt{16-(4)^2}+\frac{16}{2}\sin^{-1}\frac{4}{4}-\frac{2\sqrt{2}}{2}\sqrt{16-\left(2\sqrt{2}\right)^2}-\frac{16}{2}\sin^{-1}\frac{2\sqrt{2}}{4}\right]$

	$= 4 + \left[8\sin^{-1}(1) - \sqrt{2} \times \sqrt{16-8} - 8\sin^{-1}\frac{1}{\sqrt{2}} \right]$ $= 4 + \left[8 \times \frac{\pi}{2} - 4 - 8 \times \frac{\pi}{4} \right]$ $= 4 + [4\pi - 4 - 2\pi]$ $= 2\pi$

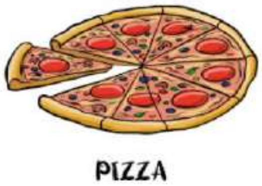
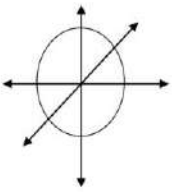
CASE BASED QUESTIONS

Qn No	Questions and Answers
1	A student designs an open air honeybee nest on the branch of a tree, whose Plane figure is parabolic, whose equation is $y^2 = 2x$ and the branch of tree is given by a straight line x- y = 4  Based on the above passage answer the following questions 1. Draw the rough diagram of parabola and straight line 2. Find point of intersection of the parabola and straight line 3. Find the area enclosed by the parabola and straight line **Solution:**

	1.  2 Solve the equations y^2 = 2x and x- y =4 to get the point of intersection as (2, -2) and (8,4) 3$\int_{-2}^{4} (y + 4 - \frac{y^2}{2})$ dy = 18 sq. units
2	A mirror is in the shape of an ellipse represented by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$was hanging on the wall. Arun and his sister were playing with ball inside the house , even their mother refused to do so . All of a sudden, ball hit the mirror and got a scratch in the shape of line represented by $\frac{x}{a} + \frac{y}{b} = 1$ Based on the above information answer the following questions.  1..Find the point of intersection of mirror and scratch 2. The value of $\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}$ dx 3. The area of the smaller region bounded by the mirror and the scratch **Solution:** 1) Solve the equations $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and$\frac{x}{a} + \frac{y}{b} = 1$. The points are (a,0) (0,b) 2). $\frac{b}{a}\int_0^a \sqrt{a^2 - x^2} = \frac{\pi ab}{4}$ 3).$\frac{b}{a}\int_0^a \sqrt{a^2 - x}$dx -$\frac{b}{a}\int_0^a a - x$ dx = $\frac{ab}{2}(\frac{\pi}{2}$ - 1).

QUESTIONS FOR PRACTICE

Qn.No	Question	Answer	Marks
1	The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y – 2$ is A) 3/8 sq.units B) 5/8sq. units C) 7/8 sq.units D)9/8sq.units	(D)	1
2	The area of the region bounded by the curve $y=\sqrt{16-x^2}$ and x-axis is (A) 8 sq units (B) 20π sq units (C) 16π sq units (D) 256π sq un	(A)	1
3	The area of the region bounded by the circle $x^2 + y^2 = 1$ is (A) 2π sq units (B) π sq units (C) 3π sq units (D) 4π sq units.	(B)	1
4	The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x=3$ is (A) 7/2 (B) 9/2 (C) 11/2 (D)13/2	(A)	1
5	The area of the region bounded by the curve $x = 2y + 3$ and the y lines. $y = 1$ and $y = –1$ is (A) 4 sq units (B) 3/ 2 sq units (C) 6 sq units (D) 8 sq unit	(C)	1
6	Find the area lying between the curves $y=x^2$ and $y=x$	1/6 sq units	2
7	Find the area bounded by the curve $x^2 = 4y$ and the line $x=4y-2$	9/8 sq.units	2
8	Find the area enclosed by the circle $x^2 + y^2 = 2$	2πsq.units	2
9	Find the area of the region bounded by the curve $y = x^2$ and the line $y = 1$	$\frac{256}{3}$ sq.units	2
10	Find the area of the region bounded by the curve $x = y^2$, y-axis and the lines $y = 3$ and $y = 4$	37/3 sq units	2
11	Using integration, find the area of the region bounded by the line $2y = 5x + 7$, x- axis and the lines $x=2$ and $x=8$	96 sq units	3
12	Draw a rough sketch of the curve $y = \sqrt{x-1}$ in the interval [1, 5]. Find the area under the curve and between the lines $x = 1$ and $x = 5$.	$\frac{16}{3}$sq.units	3
13	Find the area bounded by the curve $y = \sqrt{x}$, $x = 2y + 3$ in the first quadrant and the x axis.	9 sq. units	3
14	Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$	4 sq. units	3
15	Find the area of the region bounded by the curve $y = x^3$ and $y = x + 6$ and $x = 0$	10 sq. units	3
16	Find the area of a minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = a/2$	$\frac{a^2}{12}\left(4\pi-3\sqrt{3}\right)$ sq units.	5
17	Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x – y = 4$	18 sq.units	5
18	Find the area of region bounded by the triangle whose vertices are (–1, 1), (0, 5) and (3, 2), using integration	15/2 sq. units	5
19	Find the area of the region enclosed by the curve $x^2 = -y$ and the straight line $x + y + 2 = 0$.	9/2 sq.units	5
20	Using integration find the area of the region **$\{ (x,y) : 0\le y\le x^2+1 , 0\le y\le x+1, 0\le x\le 2\}$**	23/6 sq.units	5
21	**CASE BASED QUESTION**	1) $(1,\sqrt{3}\,)$	

	A child cut a pizza with a knife. Pizza is circular in shape which is represented by $x^2 + y^2 = 4$ and knife represents x $= \sqrt{3}y$, On the basis of the above information answer the following.   1) Find the point of intersection of circle and straight line. 2) Find the area enclosed by the circle, line and x-axis.	(-1, - $\sqrt{3}$) 2)$\frac{2\pi}{3}$.	

CHAPTER:DIFFERENTIALEQUATIONS

SYLLABUS: *Definition, order and degree, general and particular solutions of a differentialequation. Solution of differential equations by method of separation of variables, solutions ofhomogeneous differential equations of first order and first degree. Solutions of lineardifferentialequation of thetype:*

— dy/dx*+py=q,wherepandqarefunctionsofxorconstants.*

— dy/dx *+ px=q,wherepandqarefunctionsofyorconstants.*

IV. DefinitionsandFormulae:

Methodsof solvingFirstOrderandFirstDegreeDifferentialEquations

➤DifferentialEquationswithVariablesseparable

➤Homogeneousdifferentialequations

➤Lineardifferential equations

DifferentialEquationswithVariablesseparable

To solve the differential equation in variable separable form, write the differential equation as(xterms) X dx=(yterms) Xdythen integrateboth sides.

- **Letthedifferentialequation** $\frac{dy}{dx}=\frac{f(x)}{g(x)}$

  then g(y)dy=f(x)dx

  thenintegrateonbothsides$\int g(y)dy=\int f(x)dx$

- Letthedifferentialequation$\frac{dy}{dx}=\frac{g(x)}{f(x)}$

  Then $\frac{dy}{g(y)}=\frac{dx}{f(x)}$

thenintegrateonbothsides $\int\frac{dy}{g(y)}=\int\frac{dx}{f(x)}$

- Letthedifferentialequation $\frac{dy}{dx}$ = f(x).g(y)

  Then $\frac{dy}{g(y)}$f(x)dx

Then integrate on both sides $\int\frac{dy}{g(y)}=\int f(x)dx$

Homogeneous Differential equations

- A function$F(x,y)$is saidtobehomogeneousfunctionofdegreenif
  $F(\lambda x,\lambda y)=\lambda^n F\lambda$
  $^nF(x,y)$
  Adifferentialequationoftheform$\frac{dy}{dx}$=$F(x,y)$issaidtobehomogeneousifF(x,y) isa homogeneousfunction ofdegreezero
  i.e.if$F(\lambda x,\lambda y)=\lambda^0 F(x,y)$

- Substitute $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in $\frac{dy}{dx} = f(\frac{y}{x})$
- Then use variables and separable in terms of y and v only

Steps to solve the homogeneous differential equation of the type: $\frac{dx}{dy} = f(\frac{x}{y})$

- Let $x = vy$
- $\frac{dx}{dy} = v + y\frac{dv}{dy}$
- Substitute $x = vy$ and $\frac{dx}{dy} = v + y\frac{dv}{dy}$ in $\frac{dx}{dy} = f(\frac{x}{y})$
- Then use variables and separable in terms of x and v only

Linear differential equation

Steps to solve the Linear differential equation of the type:$\frac{dy}{dx} + P(x)y = Q(x)$

- $\frac{dy}{dx} + P(x)y = Q(x)$
- $Integratin\ Factor\ (IF) = e^{\int p(x)dx}$
- $Solution\ is\quad y.(IF) = \int(IF).Q(x)dx$

Steps to solve the Linear differential equation of the type:$\frac{dx}{dy} + P(y)x = Q(y)$

- $\frac{dx}{dy} + P(y)x = Q(y)$
- $Integratin\ Factor\ (IF) = e^{\int p(y)dy}$
- $Solution\ is\quad x.(IF) = \int(IF).Q(y)dy$

DIFFERENTIAL EQUATIONS

1

Differential Equation

An equation containing an independent variable, dependent variable & differential coefficients of dependent variable w.r.t. independent variable is called a differential equation.

For example,

(i) $\frac{dy}{dx}=\sin x$ (ii) $\frac{dy}{dx}+xy=\cot x$ (iii) $\frac{d^2y}{dx^2}-\frac{5dy}{dx}+6y=x^2$

A differential equation involving derivatives of the dependent variable w.r.t only one independent variable is called an ordinary differential equation. Above equations are all ordinary differential equations.

2

Order of Differential Equation

The order of a differential equation is the order of the highest derivative occuring in the differential equation.

For example

$\frac{d^2y}{dx^2}+y=0$ is a second order differential equation.

$\left(\frac{d^3y}{dx^3}\right)+x^2\left(\frac{d^2y}{dx^2}\right)^3=0$ is a third order differential equation.

3

Degree of Differential Equation

The degree of a differential equation is the highest degree of the highest derivative occuring in the differential equation when it is a polynomial of the differential coefficients i.e., differential coefficients free from radicals & fractions.

For example

Since, $\frac{d^3y}{dx^3}+x^2\left(\frac{d^2y}{dx^2}\right)^3=0$ as order = 3

$\therefore$ its degree = 1, as $\frac{d^3y}{dx^3}$ has power 1.

4

Solution of Differential Equations

Any relation between the dependent & independent variables (not involving the derivatives) which, when substituted in the differential equation reduces it to an identity is called a 'solution of the differential equation'.

General Solution : The solution which contains a number of independent arbitrary constants equal to the order of the equation is called general solution.

Particular Solution : Solutions obtained from the general solution by giving particular values to independent arbitrary constants are called particular solutions.

5

Formation of Differential Equation

Formation of a differential equation from a given equation representing a family of curves means finding a differential equation whose solution is the given equation.

The order of a differential equation representing a family of curves is same as the number of arbitrary constants present in the equation corresponding to the family of curves.

6

Differential Equations with Variables Separable

If a first order-first degree equation can be expressed in such a manner that coefficient of dx is f(x) & coefficient of dy is g(y), then we say that variables are separable. A first order-first degree differential equation is of the form $\frac{dy}{dx}=F(x,y)$

Above equation can also be written as:

$\frac{dy}{dx}=h(y)\cdot g(x)$ [if F(x, y) can be expressed as product of g(x) & h(y)]

Separating the variables, we have $\frac{dy}{h(y)}=g(x)\cdot dx$

$\therefore$ Integrate both sides $\int\frac{dy}{h(y)}=\int g(x)\cdot dx$

which is the required solution.

7

Homogeneous Differential Equations

An equation in x & y is said to be homogeneous if it can be put in the form $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$ where f(x, y) & g(x, y) are homogeneous functions of the same degree in x & y.

Here, $(x-y)\frac{dy}{dx}=x+2y$

or $\frac{dy}{dx}=\frac{x+2y}{x-y}$ is an example of homogeneous differential equation.

To solve the homogeneous differential equation $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$,

Substitute $y=vx$ & so $\frac{dy}{dx}=v+x\frac{dv}{dx}$

Thus $v+\frac{xdv}{dx}=F(v)\Rightarrow\frac{dx}{x}=\frac{dv}{F(v)-v}$

Therefore, solution is $\int\frac{dx}{x}=\int\frac{dv}{F(v)-v}+c$

8

Linear Differential Equations

A differential equation of the form $\frac{dy}{dx}+Py=Q$, where P&Q are constants or functions of x only, is known as a First Order Linear Differential Equation.

$\frac{dy}{dx}+y=\sin x, \frac{dy}{dx}+\left(\frac{1}{x}\right)y=e^x$

are some examples of Linear differential equations.

Steps to Solve First Order Linear Differential Equation :

(i) Write the given differential equation in the form $\frac{dy}{dx}+Py=Q$

(ii) Find the Integrating Factor (I.F) $=e^{\int Pdx}$

(iii) Write the solution of the given differential equation as

$$y(I.F)=\int(Q\times I.F)dx+c$$

Note that if the first order differential equation is in the form $\frac{dx}{dy}+P'x=Q'$ where P' & Q' are constants or functions of y only. Then I.F $=e^{\int Pdy}$ & the solution of the differential equation is given by

$x(I.F)=\int(Q'\times I.F)dy+c$

MULTIPLE CHOICE QUESTIONS

1) Integratingfactorforthedifferentialequation
   (xlogx) $dy/dx+y=2logx$is
   A) log(logx) B) logx C)e^x D)x

   Solution:

   Equationisdy/dx+ $\left(\frac{1}{xlogx}\right)$y $= 2/x$

   Here p(x) = 1/xlogx

   Integrating factor $e^{\int\left(\frac{1}{xlogx}\right)dx}$ = $e^{\log(logx)}$ = logx

   Ans:B

2) If m and n respectively, are the order and the degree of the differential equation
   $\frac{d}{dx}[(\frac{dy}{dx})^4]$= 0, then m+n =

   A)1 B)2 C)3 D) 4

   Solution:

   The given differential equation is $\frac{d}{dx}[(\frac{dy}{dx})^4]$= 0

   Differentiate w.r.t x , we get

   $4(\frac{dy}{dx})\frac{d^2y}{dx^2}=0$

   Here, m =2 and n=1

   Hence ,m+n = 3

   Ans:C

3) General solution of the differential equation

   $\log(\frac{dy}{dx})$= 2x+y

   A) $e^{-y}=\frac{1}{2}e^{2x}$+C B)$\frac{1}{e^y}= \frac{1}{2}e^{2x}$+C

   C)$-e^{-y}=\frac{1}{2}e^{2x}$+C D)$e^y=\frac{1}{2}e^{2x}$+C

   Solution:

   $\frac{dy}{dx} = e^{2x+y}=e^{2x}.e^y$

   $\int e^{-y}dy = \int e^{2x}dx$

   $-e^{-y}=\frac{1}{2}e^{2x}$+C

   Ans:C

4) The Particular solution of the differential equation

$\frac{dy}{dx}$=ytanx , given that y=1 when x=0 is

$A) y = cosx$B)y=secx

$C) y = tanx$ D) y=secxtanx

$Solution$: $\frac{dy}{dx}$=ytanx

$$\int \frac{dy}{y} = \int tanx \, \mathrm{dx}$$

$\log|y| = \log|secx| + logC$

y=Csecx , Given y=1, x=0

1=Csec0, C=1

Ans:B

5) Integrating factor of the differential equation $\frac{dy}{dx}$= $\frac{cosy}{1-xsiny}$ is

A)cosy B)-secy C)secy D)tany

Solution:

$\frac{dy}{dx}$= $\frac{cosy}{1-xsiny}$

$\frac{dx}{dy}$= $\frac{1-xsiny}{cosy}$= secy-xtany

$\frac{dx}{dy}$ + tany.x= secy

$Now, P(y) = tany: Q(y) = secy$

I.F $e^{\int (P\ dy)}$ = $e^{\int (tany\ \ dy)}$= secy

Ans:C

6) Differential equation x $\frac{dy}{dx}$= y(logy-logx+1) can be solved using method of

A) Separating the variable

B) Homogenous equation

C) Linear differential equation of first order

D) None of these

Solution: B

Assertion – Reason Based Questions

In the following questions, a statement of assertion (A) is followed by astatementofReason(R).Choosethecorrectansweroutofthefollowingchoices.

(A) BothAand Raretrueand Risthecorrectexplanationof A.
(B)BothAandRaretruebut R is not thecorrect explanation ofA.
(C)Ais truebut R is false.
(D)Ais false but R istrue

7) Assertion(A): Solutionofthedifferentialequation
$\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$ is $\frac{e^{2y}}{3}=\frac{e^{3x}}{3}+\frac{x^2}{2}$ +C

Reason(R):
$\frac{dy}{dx}=e^{3x-2y}+x^2e^{-2y}$

$\frac{dy}{dx}=e^{-2y}(e^{3x}+x^2)$

Solution:

Separating the variables

$e^{2y}dy=(e^{3x}+x^2)dx$

$$\int e^{2y}dy = \int (e^{3x} + x2)dx$$

$\frac{e^{2y}}{3}=\frac{e^{3x}}{3}+\frac{x^2}{2}$ +C

Solution:D) A is true but R is fasle

8)Assertion(A): Orderofdifferentialequationis$\frac{dy}{dx}+4y=\sin x$is1

Reason(R): The order ofthedifferentialequationis definedasorder ofthe highestderivativeoccurringinthedifferentialequation.
Solution :

Givendifferentialequationcontainsonly first orderderivative$\frac{dy}{dx}$withvariableand constants.Order of the differential equation is 1 .

BothAand Raretrueand Risthecorrectexplanationof A.
Ans:A

2 Mark Questions

1) Question:Findtheintegratingfactorofthedifferentialequation:

$\sqrt{x}\frac{dy}{dx}+y=e^{-2\sqrt{x}}$

Solution:Thegivendifferentialequationis:

$\frac{dy}{dx}+(1/\sqrt{x})y=\frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

I.F.=$e^{\int(1/\sqrt{x})dx}=e^{2\sqrt{x}}$

2) Findthegeneralsolutionofthedifferentialequation
ydx– (x+2y^2)dy=0

Solution:Givendifferentialequationcanbewrittenas
ydx= (x+2y^2)dy
y$\frac{dx}{dy}$ = x+2y^2

y$\frac{dy}{dx}-x=2y^2$

$\frac{dx}{dy}$–1/y.$x=2y$

Integratingfactorise^{-logy}= 1/y

Solution isx.1/y =$\int 2dy$= 2y+ C orx= 2y^2 +Cy

3) Solvethedifferentialequation xydy= (y+5)dx, giventhaty(5)=0

Solution:

$\frac{ydy}{y+5}=\frac{dx}{x}$

$\frac{(y+5-5)dy}{y+5}=\frac{dx}{x}$

$1-\frac{5}{y+5}=dx$

$$\int 1-\frac{5}{y+5}dy=\int\frac{dx}{x}$$

y-5log(y+5)=x+c ---(1)

substituting x=5,y=0 in (1)

-5log5=log5+c

C=-1log5

y– 5log|y+5|=log|x|-6log5 is required solution.

4) Forthedifferentialequation, $(\sqrt{a}+x)\frac{dy}{dx}+x=0$

Findthegeneralsolution

Solution:
dy=$\frac{-x}{\sqrt{a}+x}dx \Rightarrow \int dy=-\int\frac{x}{\sqrt{a}+x}dx$
$\Rightarrow\int dy=-\int(\sqrt{a}+ x-\frac{a}{\sqrt{a}+x} \quad)dx$
3
$\Rightarrow$y=-x$\sqrt{a}$ +$x^2/2$ $+2a\sqrt{(a+x)}$+Cistherequiredsolution.

5)
Forthedifferentialequation,$e^x\sqrt{(1-y^2)}dx+:\frac{y}{x}dy=0$
x
Solution:$\int xe^x dx+\int\frac{y}{\sqrt{1-y2}}dy=0$

$\Rightarrow xe^x-\int 1.e^x dx-\underline{½ \ x2\sqrt{(1}}-y^2)=C$
$\Rightarrow$x$e^x-e^x-\sqrt{1-y^2}$=Cistherequiredsolution.

6)
Findthegeneralsolutionofthedifferentialequation
$(x-y^3)dy+ydx=0$
Solution:$\frac{dx}{dy}=\frac{y3-x}{y}$
$\underline{1}$
$\Rightarrow\frac{dx}{dy}+\frac{1}{y}.x=y^2;I.F.=e^{\int y^{dy}}=y$
If x = $\int$Q If dy
Solutionis$y.x=\int y.y^2dy=\int y^3dy$
4
$\Rightarrow$yx=^{y} $/4+C$
X= $y^3/4$ +C/y

7)
Findthe generalsolutionofthedifferentialequation
$(x^2-1)\frac{dy}{dx}+2xy=\frac{2x}{x^2-1}$

Solution:$\frac{dy}{dx}+ \frac{2x}{x^2-1}.y= \frac{2x}{(x^2-1)^2}$
I.F.=$e^{\int}x2\frac{2x}{-1}dx=e^{log(x}{}^{2}-1)=(x^2-1)$
If x = $\int$Q If dy
Solutionis $(x^2-1)y=\int \frac{2x}{x^2-1}dx$

$\Rightarrow(x^2-1)y=log($ x^2 − 1) +C
y=$\frac{\log(x^2-1)}{x^2-1}+\frac{C}{x^2-1}$

8) Findthe generalsolutionofthedifferentialequation

$$\frac{dx}{dy}=+x=1+e^{-y}.$$

Solution:$\frac{dx}{dy}+x=1+e^{-y}$,$I.F.=e^{\int 1.dy}=e^{y}$,

If x = ∫Q If dy

Solution is$e^{y}.x=\int e^{y}(1+e^{-y})dy=\int(e^{y}+1)dy$
$\Rightarrow e^{y}.x=e^{y}+y+C$

x=1+y e^{-y}+C e^{-y} istherequiredsolution.

3 Marks Questions

1) Find the particular solution of the differential equation
$\frac{dy}{dx}$ = y cot2x. given that y(π/4)=2
Soluton:
$\int\frac{dy}{y}=\int$cot2x dx
log|y| = ½ log|sin2x| + logC
y=C√(sin2x)
when y(π/4)=2 , gives C=2
y=2$\sqrt{sin2x}$ is the required particular solution of the given differential equation.

2) Find the particular solution of the differential equation
(x $e^{y/x}$ +y)dx = xdy, given that y=1 when x=1

Solution:

$\frac{dy}{dx}=e^{y/x}$ + y/x = f(y/x) , so it's a homogenous differential equation
Let y=vx → $\frac{dy}{dx}$= v + x$\frac{dv}{dx}$
v+x$\frac{dv}{dx}$ = e^{v} +v
$\int e^{-v}$dv=∫(1/x) dx
-e^{-v} = log|x| +C
-$e^{-y/x}$ = log|x| +C
Now,x=1, y=1 , gives C= e^{-v}
log|x|+ $e^{-y/x}$ = e^{-1} is the required particular solution of the given differential equation.

3) Findtheparticularsolutionofthedifferentialoftheequation$\frac{dy}{dx}$=1+x+y+xy

giventhaty=0,whenx=1.
Solution:Considerequation $\frac{dy}{dx}$=1+x+y+xy

$\frac{dy}{dx}$=1(1+x)+y(1+x)=(1+x)(1+y)

$\frac{dy}{1+y}$= (1+x) dx

onintegratingbothsides
$\Rightarrow\int\frac{dy}{1+y}\;=\int(1+x)dx$

$\Rightarrow \log|1+y| = x + x^2/2 + C$

Given y=0, when x=1

$\log|1+0| = 1 + ½ + C \qquad \Rightarrow C = -3/2$

substitutingin (i),weget

$\log|1+y| = x + x^2/2 - x^3/3$isthe requiredsolution.

4) Solve the differential equation

$y + x.\sin\frac{y}{x} = x\frac{dy}{dx}$

Solution:$\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v + \sin v \Rightarrow \int \frac{1}{\sin v}dv = \int \frac{dx}{x}$

$\Rightarrow \int cosecv\ dv = \int \frac{dx}{x}$

$\Rightarrow \log|\mathrm{cosec}v - \cot v| = \log|x| + \log C$

$\Rightarrow \mathrm{cosec}v - \cot v = xC$

$\Rightarrow (\mathrm{cosec}\frac{y}{x} - \cot\frac{y}{x}) = xC$ is the required solution.

5) Solve the differential equation

$(1 + e^{\frac{y}{x}})dx + e^{\frac{y}{x}}(1 - \frac{x}{y})dy = 0$

Solution: Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

We get $(1 + e^v)(v + y\frac{dv}{dx}) + e^v(1 - v) = 0$

$\Rightarrow v + ve^v + y(1 + e^v)\frac{dv}{dy} + e^v - ve^v = 0$

$\Rightarrow \frac{dv}{dy} = \frac{-(e^v + v)}{y(1 + e^v)} \Rightarrow \int \frac{(1 + e^v)}{e^v + v}dv = -\int \frac{1}{y}dy$

$\Rightarrow \log|e^v + v| = -\log|y| + logC$

$\Rightarrow e^{\frac{y}{x}} + \frac{x}{y} = \frac{c}{y}$

6) Question: Solve the differential equation: $(x + y + 1)\frac{dy}{dx} = 1$

Solution : The given equation $(x + y + 1)\frac{dy}{dx} = 1$

$\Rightarrow \frac{dx}{dy} = x + y + 1$

$\Rightarrow \frac{dx}{dy} - x = y + 1$

It is a Linear Differential Equation

I.F. = $e^{\int -dy} = e^{-y}$

Now solution is

$xe^{-y} = \int (y+1)\, e^{-y} dy$

$xe^{-y} = \frac{\int (y+1)e^{-y}}{-1} - \frac{e^{-y}}{-1} dy$

$xe^{-y} = -(y+1)e^{-y} - e^{-y} + C$

$x = Ce^{y} - (y+2)$

7) Find the particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that y=0 when $x = \frac{\pi}{3}$

Solution:

$\frac{dy}{dx} + (2\tan x)y = \sin x$

$\text{IF} = e^{\int 2\tan x}dx = e^{2\log\sec x} = \sec^2 x$

$y\sec^2 x = \int \sec 2x. \sin x dx + C$

$= \int \frac{\sin x}{\cos^2 x} dx + c$

$= \int \tan x \sec x dx + C$

$y\sec^2 x = \sec x + C$

8) Find the general solution of the differential equation $\frac{dy}{dx} - y = \sin x$

Solution: Given differential equation is

$\text{IF} = e^{\int -1 dx} = e^{-x}$

Solution is

$y.e^{-x} = \int \sin x. e^{-x} dx = 1$

$I_1 = -\sin x. e^{-x} + \int \cos x. e^{-x} dx$

$= -\sin x. e^{-x} + [-\cos x e^{-x} - \int \sin x. e^{-x} dx]$

$I_1 = 1/2[-\sin x - \cos x]e^{-x}$

Solution is $y.e^{-x} = 1/2[-\sin x - \cos x]e^{-x} + c$

$y = -1/2[-\sin x - \cos x] + c.e^{-x}$

5 Marks Questions

1) Question: Find the particular solution of the differential equation: $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ given $y = 1, when\ x = 1$

Solution: The given differential equation is

$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

$\Rightarrow \frac{dy}{dx} = (1 + x^2) + y^2(1 + x^2)$

$\Rightarrow \frac{dy}{dx} = (1 + x^2)(1 + y^2)$

It is the variable separable differential equation

$\Rightarrow \int \frac{dy}{(1+y^2)} = \int (1 + x^2)dx$

$\Rightarrow tan^{-1}y = \frac{x^3}{3} + x + C$ $\Rightarrow C = \frac{\pi}{4} - \frac{4}{3}$

The particular solution of the given variable separable differential equation is

2) Question: Solve: $(x\log x)\left(\frac{dy}{dx}\right) + y = \frac{2logx}{x}$

Solution: Give differential equation: $(x \log x)(\frac{dy}{dx}) + y = \frac{2logx}{x}$

$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{xlogx}\right) y = \frac{2}{x^2}$

This is a linear equation of the form: $\frac{dy}{dx} + f(x)y = g(x)$

I.F. $= e^{\int \frac{1}{xlogx}dx} = e^{\log(\log x)} = \log x$

Now the solution of the differential equation will be:

$y(I.F.) = \int g(x)(I.F.)dx$

Now on integrating RHS by parts, we get

$y\log x = \log x\left(\frac{2x^{-1}}{-1}\right) - \int \frac{1}{x}\left(\frac{2x^{-1}}{-1}\right) dx$

$\Rightarrow y\log.x = \frac{-2logx}{x} + \int 2x^{-2}dx$

$\Rightarrow y\log x = \frac{-2logx}{x} + 2\frac{x^{-1}}{-1} + C$

$\Rightarrow y\log x = \frac{-2logx}{x} - \frac{2}{x} + C$

3) Question: Solve the differential equation

$(tanx^{-1}x - y)dx = (1 + x^2)dy$

Solution: Given differential equation can be written as

$(1+x^2)\frac{dy}{dx} + y = tan^{-1}x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{1+x^2}y = \frac{tan^{-1}x}{1+x^2}$

$\Rightarrow$I.F. $= e^{\int\left(\frac{1}{1+x^2}\right)dx} = e^{tan^{-1}x}$

∴Solution is

$y.\ e^{tan^{-1}x} = \int tan^{-1}x.\ e^{tan^{-1}x}.\frac{1}{1+x^2}dx$

$\Rightarrow y.\ e^{tan^{-1}x} = e^{tan^{-1}x}.(tan^{-1}x - 1) + C$

Or $y = (tan^{-1}x - 1) + C\ .\ e^{-tan^{-1}x}$

4) Question: Find the particular solution of the differential equation

$2y\, e^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$ given that $x = 0$ $when$ $y = 1$.

Solution: the given differential equation

$2y\, e^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$

On dividing Nr and Dr of RHS by y,we get

$\frac{dx}{} = \frac{-\left[1-2\left(\frac{x}{y}\right)e^{\left(\frac{x}{y}\right)}\right]}{}$

It is homogenous differential equation of second type

∴Put x =vy and $\frac{dx}{dy} = v + y\left(\frac{dv}{dy}\right)$

$v + y\left(\frac{dv}{dy}\right) = \frac{2ve^{v}-1}{2e^{v}}$

$\Rightarrow y\left(\frac{dv}{dy}\right) = \frac{2ve^{v}-1}{2e^{v}} - v$

$\Rightarrow y\left(\frac{dv}{dy}\right) = -\frac{1}{2e^{v}}$

$\Rightarrow 2\int e^{v} dv = \int \frac{dy}{y}$

$\Rightarrow 2\, e^{v} = -log|y| + C$

$\Rightarrow 2\, e^{\frac{x}{y}} = -log|y| + C$

Put x = 0 and y =1

$2e^{0} = -\log 1 + C$

$\Rightarrow 2 = 0 + C \qquad \Rightarrow C = 2$

Question: For the differential equation xy $\left(\frac{dy}{dx}\right)$ = (x+2)(y+2), find the solution curve passing through the point (1,-1)

5) Solution: The given differential equation xy $\left(\frac{dy}{dx}\right)$ = (x+2)(y+2)

It is a variable separable differential equation.

$\therefore \int \left(\frac{y}{y+2}\right) dy = \int \left(\frac{x+2}{x}\right) dx$

$\Rightarrow \int \left(\frac{(y+2-2)}{y+2}\right) dy = \int \left(\frac{x+2}{x}\right) dx$

$\Rightarrow \int \left(1 - \frac{2}{y+2}\right) dy = x + 2logx + C$

⇒ y -2log(y+2) = x+2log(x) + C

This curve passing through a point (1, -1)

Put x =1 and y = -1 in it.

-1 -2log (1) = 1 + 2log(1) + C

⇒ -1 – 0 =1 + 2 × 0 + C

⇒ -1 = 1+ C $\quad \Rightarrow C = -2$

On putting $C = -2$ in the curve equation:

$y - 2log|y + 2| = x + 2log|x| - 2$

CASE STUDYBASED QUESTIONS

1) Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx}$= k(50 − y)where x denotes the number of weeks and y the number of children who have been given the drops.

   (a) Find the order and degree of the above given differential equation and Which method of solving a differential equation can be used to solve

   Solution: Order is 1, Degree is 1 and Variable separable method can be used.

   (b) Find the particular solution of the differential equation $\frac{dy}{dx}$= k(50 − y) is given that y(0)=0.

   Solution:

   $\Rightarrow\int\frac{dy}{50-y}=\int \mathrm{x}dx$

   $\Rightarrow$-log|$50-y$ |=kx+C

   -log50=C

   -log|50-y|=kc-log50

   log$\frac{50}{50-y}$=kx

2) A function of the form y = Ø(x) + C which satisfies given differential equation, is called the solution of the differential equation. The solution which contains as many arbitrary constants as the order of the differential equation, is called the general solution of the differential equation. A solution obtained by giving particular values to arbitrary constants in the general solution of a differential equation, is called the particular solution.

   (a) Find thesolutionofthedifferentialequation$sec^2x.tanydx+sec^2ytanxdy=0$.

   Solution :$\frac{\sec 2y}{\tan y}dy$=-$\frac{\sec 2x}{\tan x}dx$.

   Integrating on both sides, we get log|tan y| = - log|tan x| + log C

   $tanx.tany=C$

(b) Find thesolutionofdifferentialequation $e^x\tan y\,dx+(1+e^x)sec^2y\,dy=0$

Solution:: $\frac{\sec 2y}{\tan y}dy = -\frac{e^x}{1+e^x}$

Integrating on both sides, we get $\log|\tan y| = -\log|1+e^x| + \log C$

$\tan y = C(1+e^x)^{-1}$

PRACTICE QUESTIONS

1 Mark Questions

1) The number of arbitrary constants in the particular solution of the differential equation

   $\log(\frac{dy}{dx}) = 3x + 4y$; $y(0) = 0$ is/are

   (A) 2 (B)1

   (C)0 (D)3

2) The integrating factor of the differential equation $(x+2y^2)\frac{dy}{dx} = y$ $(y>0)$ is :

   A) 1/x B)x C)y D) 1/y

3) The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0$, $(x\neq0)$ is :

   A) 2/x B) x^2 C) $e^{\frac{2}{x}}$ D) $e^{\log}$

4) The general solution of the differential equation xdy + ydx = 0 is :

   A) xy=c B)x+y=c C) $x^{2+}y^2= c^2$ D)logy=logx+c

5) **Assertion(A):**To solve the differentialequation$y^2dy+(xy+y^2)dx=0$,
   we have to put$y=vx$
   Reason(R): Alldifferentialequationoffirstorderfirstdegreebecomeshomogeneous if weputy=vx

2 Mark Questions

6) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ given that $y(0) = \sqrt{3}$.
7) Find the order and degree of the differential equation: $\log(\frac{d^2y}{dx^2})=(\frac{dy}{dx})^3 + x$
8) Solve the differential equation $\cos(\frac{dy}{dx})=a$,where a ϵ R
9) Find the general solution of the differential equation $\frac{dy}{dx} + 2y = e^{3x}$
10) Find the general solution of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = x$

3 Mark Questions

11) Find the particular solution of the following differential equation.

$\cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0$ given that y(0) = π/4

12) Show that the differential equation $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1- \frac{x}{y})dy = 0$ is homogeneous and its general solution.

13) Solve the differential equation $(x^2 - y^2)dx + 2xydy = 0$

14) Solve the following differential equation:$(\tan^{-1}y - x)dy = (1 + y^2)dx$.

15) Solve : $\cos^{-1}(\frac{dy}{dx}) = x + y$

5 Mark Questions

16) Show that the family of curves for which $\frac{dy}{dx} = \frac{x2 + y2}{2xy}$ is given by $x^2 - y^2 = cx$

17) Find the particular solution of the differential equation$[\, x \sin^2\left(\frac{y}{x}\right) - y \,]dx + x\, dy = 0$,

given that y = π/4 and x = 1.

18) Find the particular solution of the differential equationdy = cos x (2 – y cosec x)dx , given that y=0 when x= π/2

19) Find the particular solution of the differential equation $\frac{dx}{dy} + y \cot x = 2x + x^2 \cot x$, given that y = 0 and x= π/2

20) Find the equation of the curve passing through origin if the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates.

CASE STUDYBASED QUESTIONS

21) A thermometer reading 80°F is taken outside. Five minutes later the thermometer reads 60°F. After another 5 minutes the thermometer reads 50°F. At any time t the thermometer reading be T°F and the outside temperature be S °F.

i) **If λ is positive constant of proportionality, find dT/dt.**
ii) **Find the general solution of differential equation formed in given situation.**
iii) **Find the particular solution when T(10).**
Or
Find the particular solution when T(5).

Answers

1)C 2)D 3)B 4)A 5)C 6)$\tan^{-1} y = \tan^{-1} x + \sqrt{3}$ 7) order is 2 and degree is not defined

8) $y = x \cos^{-1} a + C$ 9) $y = \frac{e^{5x}}{5} + ce^{-2x}$ 10) $y = \frac{x^2}{3} + \frac{c}{x}$ 11) $\cos y = \frac{1}{\sqrt{2}} e^x$ 12) $ye^{\frac{x}{y}} + x = C$ 13) $\frac{Y^2}{X} + x^2 = C$ 14) $x = (\tan^{-1} y - 1) + Ce^{\tan^{-1} y}$ 15) $\tan(x + y) = 2x + C$ 17) $\cot(\frac{y}{x}) = \log|ex|$ 18) $y = -\frac{1}{2} \cos 2x \operatorname{cosec} x + \frac{1}{2\sqrt{2}} \operatorname{cosec} x$ 19) $y = x^2 - \frac{\pi^2}{4 \sin x}$ 20) $x + y + 1 = e^x$

21)(i) $-\lambda (T - S)$ (ii)$\log(T-S) = -\lambda t + C(iii)\log(T-S) = -\lambda t + 50$ (or) $\log(T-S) = -\lambda t + 60$

VECTOR ALGEBRA

Important Points :

1. Position vector of a point P(x, y, z) is given as $\overrightarrow{OP}\ (\vec{r}) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, and itsmagnitude by$\sqrt{x^2 + y^2 + z^2}$

2. The vector sum of the three sides of a triangle taken in order is$\vec{0}$

3. For a given vector$\vec{a}$, the vector$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the directionof $\vec{a}$.

4. The scalar product of two given vectors having angle θ betweenthem is defined as $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

5. Also, when $\vec{a}.\vec{b}$ is given, the angle ‘θ’ between the vectors may bedetermined by

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

6. If θ is the angle between two vectors , then their cross product isgiven as $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$, where$\hat{n}$ is a unit vector perpendicular to the plane containing . Suchthat$\vec{a}$,$\vec{b}$ and $\hat{n}$ form right handed system of coordinate axes.

7. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

8. Area of triangle with sides $\vec{a}$ and $\vec{b}$ is equal to $\frac{1}{2}|\vec{a} \times \vec{b}|$

9. Area of parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$ is equal to $|\vec{a} \times \vec{b}|$

10. Area of parallelogram with diagonals $\vec{d_1}$ and $\vec{d_2}$ is equal to $\frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

SOLVED QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $sin\ \theta = \frac{3}{5}$ Then $\hat{a}.\hat{b}$ is equal to
   a) $\pm\frac{3}{5}$ b)$\pm\frac{4}{3}$ c)$\pm\frac{4}{5}$ d)$\pm\frac{3}{4}$

Ans:c)

2. The vector with terminal point $A(2, -3, 5)$ and initial point $B(3, -4, 7)$ is
   a) $\hat{\imath} + \hat{\jmath} + \hat{k}$ b) $\hat{\imath} - \hat{\jmath} + \hat{k}$
   c) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$ d) $-\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Ans: c)

3. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true
   a) $\vec{a}.\vec{b} \geq |\vec{a}||\vec{b}|$ b) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|$
   c) $\vec{a}.\vec{b} \leq |\vec{a}||\vec{b}|$ d) $\vec{a}.\vec{b} < |\vec{a}||\vec{b}|$

Ans : c)

4. The unit vector perpendicular to both vectors)$\hat{\imath} + \hat{k}$ and) $\hat{\imath} - \hat{k}$ is
   a) $2\hat{\jmath}$ b)$\hat{\jmath}$ c)$\frac{\hat{\imath}-\hat{k}}{\sqrt{2}}$ d) $\frac{\hat{\imath}+\hat{k}}{\sqrt{2}}$

Ans :b)

5. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and$\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are
   a) collinear vectors which are not parallel b) parallel vectors
   c) unit vectors d) perpendicular vectors

Ans : d)

6. If $|\vec{a}| = 2$ and $-3 \leq k \leq 2$, then $|k\vec{a}| \in$

a)$[-6,4]$ b)$[0,4]$ c)$[4,6]$ d)$[0,6]$

Ans : b)

7. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + |\vec{a} \times \hat{k}|^2$ is

a) a^2 b)$2a^2$ c)$3a^2$ d) 0

Ans : b)

8. The vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $\vec{c} = -3\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ represents the sides of

a) an equilateral triangle b) an obtuse angled triangle

c) an isosceles triangle d) a right angled triangle

Ans :d)

9. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{5\pi}{6}$ d) $\frac{11\pi}{6}$

Ans : c)

10. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively . The point R divides line segment PQ in the ratio $3:1$ and S is the mid-point of line segment PR. The position vector of S is :

a) $\frac{\vec{p}+3\vec{q}}{4}$ b) $\frac{\vec{p}+3\vec{q}}{8}$ c) $\frac{5\vec{p}+3\vec{q}}{4}$ d) $\frac{5\vec{p}+3\vec{q}}{8}$

Ans : d)

ASSERTION REASONING QUESTIONS

.Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

11. Assertion (A) :$(\vec{b}.\vec{c})\vec{a}$ is a scalar quantity .

Reason (R) : Dot product of two vectors is a scalar quantity .

Ans : d)

12. Assertion (A) : Projection of $\vec{a}\, on\, \vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason (R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically
.

Ans : d)

13. Assertion (A) : The vectors $\vec{a} = 6\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$, $\vec{b} = 10\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ and $\vec{c} = 4\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$ represent the side of a right angled triangle .

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third .

Ans : b)

14. Assertion (A) : Quadrilateral formed by vertices $A(0,0,0)$, B(3,4,5) ,C(8,8,8) and D(5,4,3) is a rhombus .
Reason (R) : ABCD is a rhombus if $AB = BC = CD = DA\ , AC \neq BD$.

Ans : a)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)

15. Express the vector $\vec{a} = 5\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{\imath} + \hat{k}$ and the other is perpendicular to $\vec{b}$.

Soln: Let $\vec{a} = \vec{c} + \vec{d}$,

$$\vec{c} \parallel \vec{b}\vec{c} = \lambda\vec{b} \therefore \vec{c} = 3\lambda\hat{\imath} + \lambda\hat{k}$$

$\vec{d} = (\,5 - 3\lambda)\hat{\imath} - 2\hat{\jmath} + (5 - \lambda)\hat{k}$

$$\vec{b}.\vec{d} = 0 \Rightarrow 15 - 9\lambda + 5 - \lambda = 0 \Rightarrow \lambda = 2$$

$\vec{a} = (6\hat{\imath} + 2\hat{k}) + (-\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

16. If three non zero vectors are $\vec{a}, \vec{b}\ and\ \vec{c}$ such that $\vec{a}.\vec{b} = \vec{a}\,.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$

Soln: $\vec{a}.\vec{b} = \vec{a}\,.\vec{c} \Rightarrow \vec{a}.\left(\vec{b} - \vec{c}\right) = 0 \Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$

$$\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times \left(\vec{b} - \vec{c}\right) = 0 \Rightarrow \vec{a} \parallel (\vec{b} - \vec{c})$$

$\vec{a}$ cant be parallel and perpendicular to $\vec{b} - \vec{c}$ unless $\vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$

17. If $\vec{a}, \vec{b}\ and\ \vec{c}$ are three vectors such that $|\vec{a}| = 7, \left|\vec{b}\right| = 24\ , |\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

Soln:$\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \left(\vec{a} + \vec{b} + \vec{c}\right)^2 = \left(\vec{0}\right)^2$

$$|\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = 0$$

$$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -625$$

18. For the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, verify that the angle between $\vec{a}$ and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$

Soln :$\vec{a} \times \vec{b} = \hat{\imath} + 11\hat{\jmath} + 7\hat{k}$

$$\vec{a}.\left(\vec{a} \times \vec{b}\right) = 1 - 22 + 21 = 0 \Rightarrow \vec{a} \perp \left(\vec{a} \times \vec{b}\right)$$

$\Rightarrow$ angle between $\vec{a}$ and $\vec{a} \times \vec{b}$ is $\frac{\pi}{2}$

19. Find the projection of the vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k}$ on the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

Soln :projection of the vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k}$ on the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k} = \frac{7-2+16}{3} = \frac{21}{3} = 7$

20. In a parallelogram ABCD , the sides AB and AD are represented by the vectors $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ respectively . Find the unit vector parallel to its diagonal $\overrightarrow{AC}$.

Soln :$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{AD} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$

Required unit vector $= \frac{1}{7}(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k})$

21. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{\imath} + \hat{\jmath} + \hat{k}$

Soln : Unit vector along $\hat{\imath} + \hat{\jmath} + \hat{k} = \frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$

Required vectors are $3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$ or $-3\hat{\imath} - 3\hat{\jmath} - 3\hat{k}$

22. Position vectors of the points A , B and C as shown in the figure below are $\vec{a}, \vec{b}\ and\ \vec{c}$ respectively .

If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}\, and\, \vec{b}$

Soln :$\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB} \Rightarrow \vec{c} - \vec{a} = \frac{5}{4}\left(\vec{b} - \vec{a}\right)$

$$\Rightarrow \vec{c} = \vec{a} + \frac{5}{4}\left(\vec{b} - \vec{a}\right) = \frac{5}{4}\vec{b} - \frac{\vec{a}}{4}$$

23. If the vectors $\vec{a}$and $\vec{b}$ are such that $|\vec{a}| = 3, \left|\vec{b}\right| = \frac{2}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector , then find the angle between $\vec{a}$and $\vec{b}$.

Soln : Let θ be the angle between $\vec{a}$and $\vec{b}$

$\vec{a} \times \vec{b}$ is a unit vector $\Rightarrow \left|\vec{a} \times \vec{b}\right| = 1 \Rightarrow |\vec{a}|\left|\vec{b}\right| \sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2}$

$$\Rightarrow \theta = \frac{\pi}{6}$$

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

24. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$.

Soln : Vector perpendicular to $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 3\hat{\jmath} + 3\hat{k}$

Unit Vector perpendicular to $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k} = \frac{3\hat{\jmath}+3\hat{k}}{3\sqrt{2}} = \frac{\hat{\jmath}+\hat{k}}{\sqrt{2}}$

vector of magnitude 4 units perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k} = 4.\frac{\hat{\jmath}+\hat{k}}{\sqrt{2}}$

25. ABCD is a parallelogram such that $\overrightarrow{AC} = \hat{\imath} + \hat{\jmath}$ and $\overrightarrow{BD} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$. Find $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Also find the area of the parallelogram .

Soln : Let $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$

$$\overrightarrow{AC} = \hat{\imath} + \hat{\jmath} = \vec{a} + \vec{b}$$
$$\overrightarrow{BD} = 2\hat{\imath} + \hat{\jmath} + \hat{k} = \vec{b} - \vec{a}$$
$$\overrightarrow{AB} = -\frac{1}{2}\hat{\imath} - \frac{1}{2}\hat{k}$$
$$\overrightarrow{AD} = \frac{3}{2}\hat{\imath} + \hat{\jmath} + \frac{1}{2}\hat{k}$$

Area $= \left|\vec{a} \times \vec{b}\right| = \frac{\sqrt{3}}{2}$

26. If $\vec{a}$and $\vec{b}$ are two vectors such that $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}$, then find the vector $\vec{c}$, given that $\vec{a} \times \vec{c} = \vec{b}$ and$\vec{a}.\vec{c} = 4$.

Soln : Let $\vec{c}$ =$x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$$\vec{a}.\vec{c} = 4 \Rightarrow x - y + z = 4$$
$$\vec{a} \times \vec{c} = (-z - y)\hat{\imath} + (x - z)\hat{\jmath} + (x + y)\hat{k} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}$$

Solving we get $x = 0$, $y = -3, z = 1$

$\vec{c}$ =$-3\hat{\jmath} + \hat{k}$

27. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that the vector| $\vec{a} + \lambda\vec{b}$ is perpendicular to vector $\vec{c}$, then find the value of λ.

Soln :($\vec{a} + \lambda\vec{b}$**).**$\vec{c} = 0 \Rightarrow (2-\lambda)3 + (2+2\lambda)1 = 0$

$$\Rightarrow \lambda = 8$$

28. If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude , then prove that the vector $(2\vec{a} + \vec{b} + 2\vec{c})$ is equally inclined to both $\vec{a}$ and $\vec{c}$. Also find the angle between $\vec{a}$ and $(2\vec{a} + \vec{b} + 2\vec{c})$

Soln : $|\vec{a}| = |\vec{b}| = |\vec{c}| = m$

$$|2\vec{a} + \vec{b} + 2\vec{c}|^2 = 4|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2 + 4\vec{a}.\vec{b} + 4\vec{b}.\vec{c} + 8\,\vec{a}.\vec{c} = 9m^2$$

$$|2\vec{a} + \vec{b} + 2\vec{c}| = 3m$$

Let θ be the angle between $(2\vec{a} + \vec{b} + 2\vec{c})$ and $\vec{a}$

Then $\cos\theta = \frac{2m^2}{3m\times m} = \frac{2}{3}$

Let α be the angle between $(2\vec{a} + \vec{b} + 2\vec{c})$ and $\vec{c}$

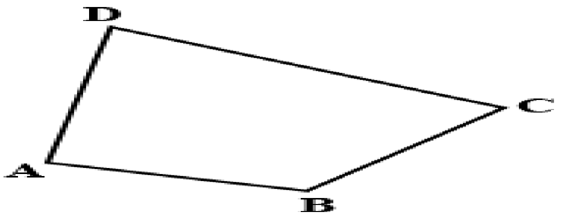
Then $\cos\alpha = \frac{2m^2}{3m\times m} = \frac{2}{3}$

$$\cos\theta = \cos\alpha$$

Required Angle $= \cos^{-1}\left(\frac{2}{3}\right)$

CASE BASED QUESTIONS (4 MARKS)

29. A garden is in shape of quadrilateral. A student wants to finds its area by using



Vector algebra. points are given as A(0,1,1), B(2,3,-2), C(22,19,-5),and D(1,-2,1).

From above information solve following questions

(i) Find $\overrightarrow{AC}$.

(ii) Find $\overrightarrow{BD}$.

(iii) Using this find area of quadrilateral.

SOLUTION:

(i) $\overrightarrow{AC} = 22\hat{\imath} + 18\hat{\jmath} - 6\hat{k}$

(ii) $\overrightarrow{BD} = -\hat{I} - 5\hat{J} + 3\hat{K}$

(iii) AREA=$\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right| = 6470$

30. . Tiranga point A(0,0,0) and shiv shakti point B(25,2,0)on moon are very important place for India unity and integrity. If Pragyan Rover is at a instant to a point C(0,1/2.0).

From these information solve the following questions

(i) Find $\overrightarrow{AB}$.

(ii) Find $\overrightarrow{AC}$.

(iii) Find area of triangle ABC .

Solution

(i) $\overrightarrow{AB} = 25\hat{\imath} + 2\hat{\jmath}$

(ii) $\overrightarrow{AC}$=1/2$\hat{J}$.

(iii) $Area = 25/2$.

31. There is a 9-meter tree in a field. Shape of field is in parallelogram whose two Adjacent side are given $\vec{a} = 4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = -2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

From above information solve following questions

(i) Find area of field.

(ii) Find height in vector form.

SOLUTION:

(i) Area= $\left|\vec{a} \times \vec{b}\right| = \left|-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right|$ =3

(ii) $9\,\frac{(-\hat{\imath}+2\hat{\jmath}+2\hat{k})}{3}$

32. Three birds are siting on tree at positions A(4,6,8) , B (6, 7 ,7) , C(5,6, 9) .

A student of class XII wants apply vector algebra concept to find different

component of triangle.

solve the problem that he finds in following questions.

(i) Find vector $\overrightarrow{AB}\,and\,\overrightarrow{AC}$.
(ii) Find centroid of triangle.
(iii) Find angle between vector $\overrightarrow{AB}\,and\,\overrightarrow{AC}$.

SOLUTION :

(i) $\overrightarrow{AB} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, $\overrightarrow{AC} = \hat{\imath} + \hat{k}$.
(ii) (5,19/3,8) .
(iii) $\cos^{-1}\frac{1}{\sqrt{12}}$.

33. A student is observing the launch of Chandrayan- 3. He assumed a frame Of reference that is x-axis, y-axis and z-axis . he finds the launching pad A(5,10,0) in km units . The Chandranan 3 starts with net acceleration 5km/s^2 And

He uses the formula $\vec{h} = 1/2\vec{a}t^2$.

Solve the following questions

(i) Find height of Chandrayan after 10 second .
(ii) Find position vector of Chandrayan- 3 in 10 second from .
(iii) What is angle of elevation by student if height of student neglected.

SOLUTION:

(i) $\vec{h} = \frac{1}{2} 5 \times 10^2\, \hat{k}$ or h= 250 km .
(ii) $\vec{r} = (5\hat{i} + 10\hat{j}\,) + 250\hat{k}$
(iii) $\theta = \tan^{-1} 10\sqrt{5}$

PRACTICE QUESTIONS

1. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.
2. Find a unit vector in the direction $\vec{a} = \hat{i} - 2\hat{j}$ whose magnitude is 7.
3. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$.
4. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$and the angle between them is 60° then find $\vec{a}\,.\vec{b}$
5. For what value of $\lambda$are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other ?
6. Write the value of p, for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors .
7. If $\vec{p}$ is a unit vector and($\vec{x} - \vec{p}$) . ($\vec{x} + \vec{p}$) =80 , then find $|\vec{x}|$.
8. Find the value of p if $(2\hat{i} + 6\hat{j} + 27\hat{k})x(\hat{i} + 3\hat{j} + p\hat{k})$=0.
9. If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a}\,.\vec{b} = \sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.
10. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$ and $\vec{a}$ x $\vec{b}$ is a unit vector , then find the angle between $\vec{a}$ and $\vec{b}$.
11. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}$ x $\vec{b}|$ = $|\vec{a}\,.\vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.
12. For what value of p , $(\hat{i} + \hat{j} + \hat{k})$p is a unit vector.
13. Write the projection of the vector $2\hat{i} + 3\hat{j} - \hat{k}$ along the vector $\hat{i} + \hat{j}$.
14. Write the value of $\hat{i}\,.(j\ X\ \hat{k}) + \ \hat{j}\,.(\ \hat{k}X\ \hat{i}) + \hat{k}\,.(\ \hat{i}X\ \hat{j})$
15. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a}$ x $\vec{c}$ = $\vec{b}$ and $\vec{a}\,.\vec{c} =$ 3.
16. If $\vec{a} + \vec{b} + \vec{c}$ =0 and $|\vec{a}| = 3, |\vec{b}| = 5\ and\ |\vec{c}| = 7$, show that the angle between $\vec{a}$ and $\vec{b}$ is 60°.
17. If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}, 3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$are the position vectors of the points A, B, C and D, find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.
18. If $\vec{a}$ x $\vec{b} = \vec{c}$ x $\vec{d}$ and $\vec{a}$ x $\vec{c} = \vec{b}$ x $\vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$.
19. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.
20. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}$. $\vec{b}$ = $\vec{a}$. $\vec{c}$ and $\vec{a}$ x $\vec{b}$ = $\vec{a}$ x $\vec{c}$, then show that $\vec{b}$ = $\vec{c}$.

21. Find the position vector of a point *R* which divides the line joining two points *P* and *Q* whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$respectively, externally in the ratio 1 : 2. Also, show that *P* is the mid-point of the line segment *RQ*.
22. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k},$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k},$ find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \overrightarrow{b} + 3\vec{c}$.
23. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}$. $\vec{d}$ = 18.
24. Find a unit vector perpendicular to each of the vectors $\vec{a}$ + $\vec{b}$ and $\vec{a} - \vec{b}$ where $\overline{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.
25. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$.
26. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 1$and $\vec{a}$. $\vec{b}$ =1 , then find the value of $(2\vec{a} + 7\vec{b}).(3\vec{a} - 5\vec{b})$
27. Using vectors, find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).
28. If vectors $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}, then\ find\ the\ value\ of\ \lambda$.
29. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 5$, $|\vec{b}| = 12\ and\ |\vec{c}|$ =13, $\vec{a} + \vec{b} + \vec{c}$ =0 , then find the value of $\vec{a}$. $\vec{b}$ +$\vec{b}$. $\vec{c}$ + $\vec{a}$. $\vec{c}$
30. If vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 7\hat{k}$ and $\vec{b} = 5\hat{\imath} - \hat{\jmath} + \hat{k}\lambda$, then find the value of λ so that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

THREE DIMENSIONAL GEOMETRY

Important Points

1. Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.
2. If l, m, n are the direction cosines of a line , then $l^2 + m^2 + n^2 = 1$.
3. Direction ratios of a line are the numbers which are proportional to the direction cosines of a line.
4. Vector equation of a line that passes through the given point whose position Vector$\vec{a}$ is and parallel to a given vector$\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.
5. If θ is the acute angle between $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, then
$$\cos\theta = \left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}\right|$$
6. Shortest distance between$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by
$$\left|\frac{\overrightarrow{(b_1 \times b_2)}.(\overrightarrow{a_2} - \overrightarrow{a_1})}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right|$$
7. Two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ are perpendicular to each other if $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$.

SOLVED QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. If the direction cosines of a line are $\sqrt{3}k$, $\sqrt{3}k$, $\sqrt{3}k$, then value of k is

   a) ±1 b)±3 c)$\pm\sqrt{3}$ d) $\pm\frac{1}{3}$

Ans: d)

2. The distance of point $P(a, b, c)$ from y axis is

   a) b b)b^2 c)$\sqrt{a^2 + c^2}$ d) $a^2 + c^2$

Ans : c)

3. The coordinates of the foot of the perpendicular drawn from the point $(0,1,2)$ on the x axis are given by

   a) $(1,0,0)$ b)$(2,0,0)$ c)$(\sqrt{5}\,,0,0)$ d) $(0,0,0)$

Ans : d)

4. If a line makes an angle of $30°$ with the positive direction of $x$axis ,$120°$ with the positive direction of y axis , then the angle which it makes with the positive direction of z axis is

   a) $90°$ b)$120°$ c)$60°$ d)$0°$

Ans : a)

5. Direction ratios of a vector parallel to $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are

   a) $2, -1, 6$ b)$2,1,3$ c)$2, -1, 3$ d)$2,1,6$

Ans : c)

6. If α, β and γ are the angles which a line makes with positive direction of , x , y and z axes respectively , then which of the following is **not** true ?

a) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ b) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

c) $cos2\alpha+\cos 2\beta+\cos 2\gamma=-1$ d) $cos\,\alpha+\cos\beta+\cos\gamma=1$

Ans : d)

7. The Cartesian equation of a line passing through the point with position vector $\vec{a}=\hat{\imath}-\hat{\jmath}$ and parallel to the line $\vec{r}=\hat{\imath}+\hat{k}+\mu(2\hat{\imath}-\hat{\jmath})$ is

a) $\frac{x-2}{1}=\frac{y+1}{0}=\frac{z}{1}$ b) $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{0}$

c) $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z}{0}$ d) $\frac{x-1}{2}=\frac{y}{-1}=\frac{z-1}{0}$

Ans : b)

8. The lines $\frac{1-x}{2}=\frac{y-1}{3}=\frac{z}{1}$ and $\frac{2x-3}{2p}=\frac{y}{-1}=\frac{z-4}{7}$ are perpendicular to each other for p equal to

a)$-\frac{1}{2}$ b)$\frac{1}{2}$ c) 2 d)3

Ans : c)

9. The angle which the line $\frac{x}{1}=\frac{y}{-1}=\frac{z}{0}$ makes with the positive direction of Y -axis is :

a) $\frac{3\pi}{4}$ b) $\frac{5\pi}{4}$ c) $\frac{5\pi}{6}$ d) $\frac{7\pi}{4}$

Ans : a)

10. The angle between the lines $2x=3y=-z$ and $6x=-y=-4z$ is

a) 0° b) 30° c)45° d)90°

Ans : d)

ASSERTION REASONING QUESTIONS

.Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

11. Assertion (A) : A line in space cannot be drawn perpendicular to x,y and z axes simultaneously.

Reason(R): If α,β and γ are the angles which a line makes with positive direction of , x , y and z axes respectively , then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$.

Ans :b)

12. Assertion (A) : The vector equation of a line passing through the points $A(-1,0,2)$ and $B(3,4,6)$ is $\vec{r}=-\hat{\imath}+2\hat{k}+\lambda(\hat{\imath}+\hat{\jmath}+\hat{k})$.

Reason(R): The equation of a line that passing through a point with positionVector $\vec{a}$ and parallel to a vector $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$.

Ans : a)

13. Assertion (A) : Equation of a line passing through the points $(1,2,3)$ and $B(3,-1,3)$ is $\frac{x-3}{2}=\frac{y+1}{3}=\frac{z-3}{0}$.

Reason (R) : Equation of a line passing through the points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$.

Ans : d)

14. Assertion (A) : If α, β and γ are the angles which a line makes with positive direction of , x , y and z axes respectively , then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

    Reason (R) : The sum of the squares of the direction cosines of a line is 1 .

Ans : a)

15. Assertion (A) : Aline through the points $(4,7,8)$ and $(2,3,4)$ is parallel to aline through the points $(-1,-2,1)$ and $(1,2,5)$.

    Reason (R) : Lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are parallel if $\vec{b_1}.\vec{b_2} = 0$.

Ans :c)

16. Assertion (A) : Two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ are perpendicular to each other if $\vec{b_1}.\vec{b_2} = 0$.

    Reason (R): If θ is the angle between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, then $\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$.

Ans : a)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)

17. Find the angle between the pair of lines given by $\frac{x-3}{1} = \frac{y+2}{-2} = \frac{z+4}{2}$; $\frac{x-5}{-3} = \frac{y+2}{2} = \frac{z}{6}$

Soln: $\cos\theta = \frac{-3-4+12}{3\times7} = \frac{5}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{21}\right)$

18. If the lines $\frac{x-1}{-3} = \frac{2y-2}{4k} = \frac{3-z}{-2}$ and $\frac{x-1}{3k} = \frac{3y-1}{6} = \frac{z-6}{-5}$ are perpendicular to each other , find the value of k .

Soln: The equation of the lines in standard form are

$\frac{x-1}{-3} = \frac{y-1}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-\frac{1}{3}}{2} = \frac{z-6}{-5}$

Since they are perpendicular $-9k + 4k - 10 = 0 \Rightarrow k = -2$

19. If α, β and γ are the angles which a line makes with positive direction of ,x , y and z axes respectively , then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

Soln :$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$
$$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

20. Find the angle between the pair of lines given by $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$. and $\vec{r} = 3\hat{\imath} - 5\hat{\jmath} + \hat{k} + \mu(3\hat{\imath} + 2\hat{\jmath} - 6\hat{k})$.

Soln :$\cos\theta = \frac{3-4+12}{3\times7} = \frac{11}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\right)$

21. Find the coordinates of points on line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin .

Soln : General point on given line is $P(\lambda, 2\lambda + 1, 2\lambda - 1)$

$$OP = \sqrt{11} \Rightarrow OP^2 = 11$$
$$\Rightarrow \lambda^2 + (2\lambda + 1)^2 + (2\lambda - 1)^2 = 11 \Rightarrow \lambda = \pm 1$$

Coordinates of points are $(1,3,1)$ and $(-1,-1,-3)$

22. If the equation of a line is $x = ay + b$, $z = cy + d$, then find the direction ratios of the line and a point on the line .

Soln : Equation of line can be written as $\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$

The direction ratios are a ,1, c.

A point on the line is $b, 0, d$

23. If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$. Find the relation between α and β .

Soln : $cos\frac{\pi}{4} = \frac{\alpha+\beta}{\sqrt{\alpha^2+\beta^2+25\sqrt{2}}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{\alpha+\beta}{\sqrt{\alpha^2+\beta^2+25\sqrt{2}}} \Rightarrow \alpha\beta = \frac{25}{2}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

24. Find the coordinates of the foot of the perpendicular drawn from point $P(5,7,3)$ to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Soln : A general point on the given line is $M(3\lambda + 15, 8\lambda + 29, -5\lambda + 5\ )$

DRs of MP are $(3\lambda + 10, 8\lambda + 22, -5\lambda + 2\ )$

M will be foot of perpendicular if $PM\ \perp$Line

$i.e\ (3\lambda + 10)(3) + (8\lambda + 22)(8) + (-5\lambda + 2)(-5) = 0 \Rightarrow \lambda = -2$

Hence $M(9,13,15)$ is the required foot of the perpendicular .

25. Find the distance between the lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$. and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(4\hat{\imath} + 6\hat{\jmath} + 12\hat{k})$.

Soln : Given lines are parallel

$$d = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right| = \frac{\sqrt{293}}{7}$$

26. Find the vector and Cartesian equations of a line that passes through the point $A(1,2,-1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

Soln: The given line is $\frac{x-5}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}}$

Cartesian equation of a line passing through $A(1,2,-1)$ and parallel to this is

$$\frac{x-1}{\frac{1}{5}} = \frac{y-2}{-\frac{1}{7}} = \frac{z+1}{\frac{1}{35}}\ or\ \frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$$

and the corresponding vector equation is $\vec{r} = \hat{\imath} + 2\hat{\jmath} - \hat{k} + \lambda(7\hat{\imath} - 5\hat{\jmath} + \hat{k})$.

27. Find the coordinates of the point where the line through the points (–1, 1, –8) and (5, –2, 10) crossesthe ZX-plane.

Soln : Equation of line through (–1, 1, –8) and (5, –2, 10) is $\frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} = \lambda$

Any point on this line is (6λ – 1, –3λ + 1, 18λ – 8)

Line crosses ZX-plane i.e. y = 0⇒–3λ + 1= 0 ⇒ λ= $\frac{1}{3}$

Required point is (1, 0, –2)

28. Check whether the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are skew or not .

Soln: Let $\overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\overrightarrow{a_2} = 4\hat{\imath} + \hat{\jmath}$

$\overrightarrow{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\overrightarrow{b_2} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$$\overrightarrow{a_2} - \overrightarrow{a_1} = 3\hat{\imath} - \hat{\jmath} - 3\hat{k}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = -5\hat{\imath} + 18\hat{\jmath} - 11\hat{k}$$

Hence $(\overrightarrow{a_2} - \overrightarrow{a_1}).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 0$

Hence given lines are not skew lines .

LONG ANSWER QUESTIONS (5 MARKS)

29.Equations of sides of a parallelogram ABCD are as follows

$$AB: \frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$$

$$BC: \frac{x-1}{3} = \frac{y+2}{-5} = \frac{z-5}{3}$$
$$CD: \frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$$

$DA: \frac{x-2}{3} = \frac{y+3}{-5} = \frac{z-4}{3}$. Find the equation of diagonal BD .

Soln : Any point on line AB is (λ – 1, –2λ + 2, 2λ +1)

Any point on line BC is (3μ+ 1, –5μ −2, 3μ+5)

B is the point of intersection of AB and BC , so coordinates of B are $(1, -2, 5)$

Any point on line CD is (λ +4, –2λ −7, 2λ+8)

Any point on line DA is (3μ+2, –5μ −3, 3μ+4)

D is the point of intersection of CD and DA , so coordinates of B are $(2, -3, 4)$

Equation of BD is $\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z-5}{-1}$

30. Find the equation of a line l_2 which is the mirror image of the line l_1 with respect to line $l: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that line l_1 passes through the point P(1,6,3) and parallel to line l.

Soln : Since l_1 passes through the point P(1,6,3) and parallel to line l equation of l_1 is

$$\frac{x-1}{1} = \frac{y-6}{2} = \frac{z-3}{3}$$

Since line l_2 is the mirror image of the line l_1 with respect to line l , l_2 is parallel to l.

Foot of perpendicular of P(1,6,3)to line l is (1,3,5)

So point on l_2 is (1,0,7) , image of P(1,6,3) with respect to line l

So equation of l_2 which passes through (1,0,7) and parallel to l is

$$\frac{x-1}{1} = \frac{y}{2} = \frac{z-7}{3}$$

31. Find the equation of the line which bisects the line segment joining points $A(2,3,4)$ and $B(4,5,8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

Soln: The midpoint of line joining points $A(2,3,4)$ and $B(4,5,8)$ is (3,4,6)

Equation of line passing through (3,4,6) is $\frac{x-3}{a} = \frac{y-4}{b} = \frac{z-6}{c}$

Since this line is perpendicular to $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$$\Rightarrow 3a - 16b + 7c = 0$$
$$\Rightarrow 3a + 8b - 5c = 0$$

Using cross multiplication method and solving the required line is

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-6}{6}$$

32. Find the value of p for which the lines $\vec{r} = \lambda\hat{\imath} + (2\lambda+1)\hat{\jmath} + (3\lambda+2)\hat{k}$ and $\vec{r} = \hat{\imath} - 3\mu\hat{\jmath} + (p\mu+7)\hat{k}$ are perpendicular to each other and also intersect . Also find the point of intersection of the given lines .

Soln : The lines are perpendicular to each other if $1 \times 0 + 2 \times (-3) + 3 \times p = 0$

$$\Rightarrow p = 2$$

Any point on the line $\vec{r} = \lambda\hat{\imath} + (2\lambda+1)\hat{\jmath} + (3\lambda+2)\hat{k}$ is $(\lambda, 2\lambda+1, 3\lambda+2)$

Any point on the line $\vec{r} = \hat{\imath} - 3\mu\hat{\jmath} + (p\mu+7)\hat{k}$ is $(1, -3\mu, 2\mu+7)$

Equating the coordinates and solving we get $\lambda = 1$ and $\mu = -1$

The point of intersection is $(1,3,5)$

PRACTICE QUESTIONS

1. Prove that the line through A(0, –1, –1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(–4, 4, 4).

2. Find the vector and Cartesian equations of the line through the point $((1,2,-4)$ and perpendicular to the two lines$\vec{r}=8\hat{\imath}-19\hat{\jmath}+10\hat{k}+\lambda(3\hat{\imath}-16\hat{\jmath}+7\hat{k})$ and $\vec{r}=15\hat{\imath}+29\hat{\jmath}+5\hat{k}+\mu(3\hat{\imath}+8\hat{\jmath}-5\hat{k})$.
3. Find the values of a so that the following lines are skew :$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-a}{4}$

$$\frac{x-4}{5}=\frac{y-1}{2}=z$$

4. Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$intersect . Find their point of intersection .
5. Find the coordinates of the foot of the perpendicular drawn from the point $(2,3,-8)$ to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$.
   Also find the perpendicular distance of the given point from the line .
6. Find the shortest distance between the lines $L_1\&L_2$ given below :
   L_1: The line passing through $(2,-1,1)$ and parallel to $\frac{x}{1}=\frac{y}{1}=\frac{z}{3}$
   L_2:$\vec{r}=\hat{\imath}+(2\mu+1)\hat{\jmath}-(\mu+2)\hat{k}$.
7. Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point $(4,0,-5)$.
8. If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{-1}=\frac{z-6}{-7}$ are perpendicular to each other , find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3,-4,7)$.
9. Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}$ and perpendicular to these given lines .
10. Two vertices of the parallelogram ABCD are given as $A(-1,2,1)$ and $B(1,-2,5)$. If the equation of the line passing through C and D is $\frac{x-4}{1}=\frac{y+7}{-2}=\frac{z-8}{2}$, then find the distance between sides AB and CD .Hence , find the area of parallelogram ABCD.
11. The vertices of ΔABC are $A(1,1,0)$, $B(1,2,1)$ and $C(-2,2,-1)$. Find the equations of the medians through A and B . Use the equations so obtained to find the coordinates of the centroid.
12. Find the shortest distance between the lines whose vector equations are :
    $\vec{r}=\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=3\hat{\imath}-3\hat{\jmath}-5\hat{k}+\mu(-2\hat{\imath}+3\hat{\jmath}+8\hat{k})$.
13. Find the vector and the Cartesian equations of a line passing through the point $(1,2,-4)$ and parallel to the line joining the points $A(3,3,-5)$ and $B(1,0,-11)$. Hence find the distance between the two lines.
14. Find the equations of the line passing through the points $A(1,2,3)$ and $B(3,5,9)$. Hence find the coordinates of the points on this line which are at a distance of 14 units from point B.
15. Show that the lines $\vec{r}=\hat{\imath}+2\hat{\jmath}+\hat{k}+\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})$ and $\vec{r}=\hat{\imath}+\hat{\jmath}+\hat{k}+\mu(\hat{\imath}-\hat{\jmath}+2\hat{k})$ do not intersect.
16. Find the coordinates of the foot of perpendicular drawn from the point $(3,-1,11)$ to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Hence write the equation of this perpendicular line .

17. Find the equations of the diagonals of the parallelogram PQRS whose vertices are $P(4,2,-6)$, $Q(5,-3,1)$,$R(12,4,5)$ and $S(11,9,-2)$. Use these equations to find the point of intersection of diagonals .
18. A line l passes through point $(-1,3,-2)$ and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. Find the vector equation of the line l . Hence , obtain its distance from origin.
19. Find the image of the point $(2,-1,5)$ in the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.
20. Vertices B and C of ΔABC lie on the line $\frac{x+2}{2}=\frac{y-1}{1}=\frac{z}{4}$. Find the area of ΔABC given that point A has coordinates $(1,-1,2)$ and the line segment BC has length of 5 units .

LINEAR PROGRAMMING PROBLEMS

Formulae/Important Points

- **Linear Programming Problem:** A linear programming problem is one in which we have to find optimal value (maximum or minimum) of a linear function of several variables (called objective function) subject to certain conditions that the variables are non-negative and satisfying by a set of linear inequalities with variables, are sometimes called division variables.
- **Terms related to Linear Programming**
  Objective Function: A linear function z = px + qy (p and q are constants) which has to be maximised or minimised, is called an objective function.
- **Feasible Region:** The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of a linear programming problem is called the feasible region for the problem. The region other than the feasible region is called an infeasible region. The feasible region is always a convex polygon.
- **Optimal Feasible Solution:** Any point in the feasible region that gives the optimal value of the objective function is called the optimal feasible solution.
- **Bounded and Unbounded Region:** A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle. Otherwise, it is called unbounded.
- **Fundamental Theorems for Solving Linear Programming**
  Theorem 1: Let R be the feasible region for a linear programming problem and let z = ax + by be the objective function. When z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities. This optimal value must occur at a corner point (vertex) of the feasible region.
  Note: A corner point of a feasible region is a point in the region which is the intersection of two boundary lines.
- **Theorem 2:** Let R be the feasible region for a linear programming problem and let z = ax + by be the objective function. If R is bounded, then z has both a maximum and a minimum value on R and each of these recurs at a corner point of JR.
  Note: Maximum or a minimum may not exist,- if the feasible region is unbounded.

- **Steps for Applying Corner Point Method**
  Find the feasible region of the linear programming problem and determine its corner points either by inspection or by solving the two equations of the lines intersecting at that point.

- Evaluate the objective function z = ax + by at each corner point. Let M and m be, respectively denote the largest and the smallest values of these points.

- If the feasible region is bounded, then M and m respectively are the maximum and minimum values of the objective function at corner points.

- If the feasible region is unbounded , then
  (a) M is the maximum value of objective function z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, z has no maximum value.
  (b) m is the minimum value of z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, z has no minimum value.

- If two corner points of the feasible region are both optimal solutions of the same type, i.e both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

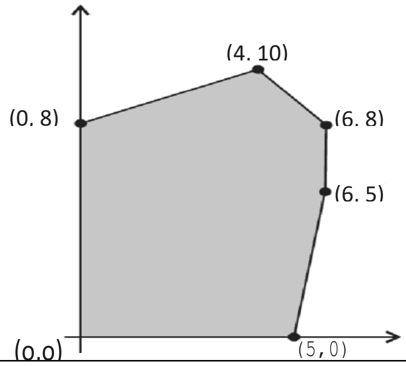
MULTIPLE CHOICE QUESTIONS

1 The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0,40), (20,40),(60,20),(60,0).The objective function is Compare the quantity in Column A and Column *B*

Column A	Column *B*
Maximum of Z	325

(a) The quantity in column A is greater	(b)The quantity in column *B* is greater
(c) The two quantities are equal.	(d) The relationship cannot be determined on the basis of the information supplied.

2 The feasible solution for a LPP is shown in given figure. Let Z=3x-4y be the objective function. Minimum of Z occurs at



a)(0,0)	(b) (0,8)
(c) (5,0)	(d) (4,10)

3 Corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0). Let Z= px+qy, where p, q>0. Condition on p and q so that the minimum of Z occurs at (3,0) and (1,1) is

(a) p=2q	(b) p=q/2
(c) p=3q	(d) p=q

4 The set of all feasible solutions of a LPP is a ____ set.

(a) Concave	(b) Convex
(c) Feasible	(d) None of these

5 Corner points of the feasible region for an LPP are (0,2), (3,0), (6,0), (6,8) and (0,5). Let F=4x+6y be the objective function. Maximum of F – Minimum of F =

(a) 60	(b) 48
(c) 42	(d) 18

6 In a LPP, if the objective function Z = ax+by has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same……….value.

(a) minimum	(b) maximum
(c) zero	(d) none of these

7 In the feasible region for a LPP is ………, then the optimal value of the objective function Z = ax+by may or may not exist.

(a) bounded	(b) unbounded
(c) in circled form	(d) in squared form

8 A linear programming problem is one that is concerned with finding the …A … of a linear function called …B… function of several values (say x and y), subject to the conditions that the variables are …C… and satisfy set of linear inequalities called linear constraints.

(a) Objective, optimal value, negative	(b) Optimal value, objective, negative
(c) Optimal value, objective, non-negative	(d) Objective, optimal value, non-negative

9 Maximum value of the objective function Z = ax+by in a LPP always occurs at only one corner point of the feasible region.

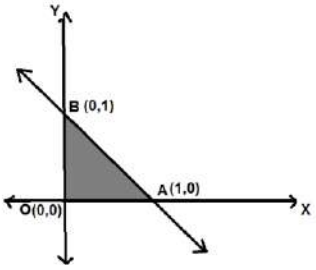
(a) true	(b) false
(c) can’t say	(d) partially true

10 Region represented by $x \geq 0, y \geq 0$ is:

(a) First quadrant	(b) Second quadrant
(c) Third quadrant	(d) Fourth quadrant

11 Z =3x + 4y, Subject to the constraints x+y 1, $x, y \geq 0$.

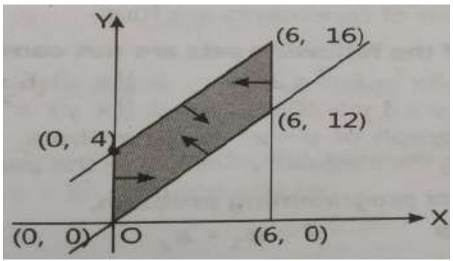
the shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0,0),(1,0) and (0,1), respectively.



The maximum value of Z is 2.

(a) true	(b) false
(c) can’t say	(d) partially true

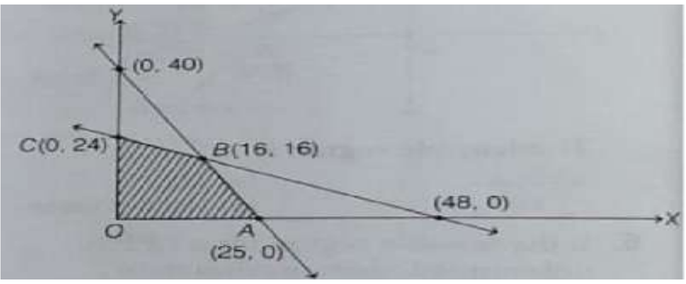
12 The feasible region for an LPP is shown shaded in the figure. Let Z = 3x-4y be objective function. Maximum value of Z is:



(a) 0	(b) 8

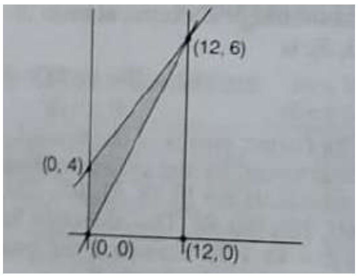
(c) 12	(d) -18

13 The maximum value of Z = 4x+3y, if the feasible region for an LPP is as shown below, is



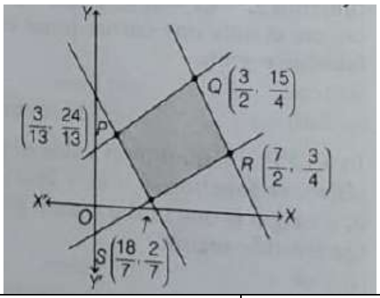
(a) 112	(b) 100
(c) 72	(d) 110

14 The feasible region for an LPP is shown shaded in the figure. Let Z = 4x-3y be objective function. Maximum value of Z is:



(a) 0	(b) 8
(c) 30	(d) -18

15 In the given figure, the feasible region for a LPP is shown. Find the maximum and minimum value of Z = x+2y.



(a) 8, 3.2	(b) 9, 3.14
(c) 9, 4	(d) none of these

16 The linear programming problem minimize $Z= 3x+2y$, subject to constraints

(a) One solution	(b) No feasible solution
(c) Two solutions	(d) Infinitely many solutions

$x+y8$, $3x+5y\ \ 15$, $x,y\geq0$, has

17 The graph of the inequality $2x+3y > 6$ is:

(a) half plane that contains the origin	(b) half plane that neither contains the origin nor the points of the line $2x+3y =6$
(c) whole XOY-plane excluding the points on the line $2x+3y =6$	(d) entire XOY-plane

18 Of all the points of the feasible region for maximum or minimum of objective function the points

(a) Inside the feasible region	(b) At the boundary line of the feasible region
(c) Vertex point of the boundary of the feasible region	(d) None of these

19 The maximum value of the object function $Z = 5x + 10y$ subject to the constraints $x + 2y \leq 120$, $x + y \geq 60$, $x – 2y \geq 0$, $x \geq 0$, $y \geq 0$ is

(a) 300	(b) 600
(c) 400	(d) 800

20 $Z = 6x + 21y$, subject to $x + 2y \geq 3$, $x + 4y \geq 4$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at

(a) (4, 0)	(b) (28, 8)
(c) (2,2/7)	(d) (0, 3)

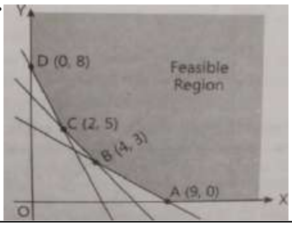
21 Shape of the feasible region formed by the following constraints $x + y \leq 2$, $x + y \geq 5$, $x \geq 0$, $y \geq 0$

(a) No feasible region	(b) Triangular region
(c) Unbounded solution	(d) Trapezium

22 Maximize $Z = 4x + 6y$, subject to $3x + 2y \leq 12$, $x + y \geq 4$, $x, y \geq 0$.

(a) 16 at (4, 0)	(b) 24 at (0, 4)
(c) 24 at (6, 0)	(d) 36 at (0, 6)

23 Feasible region for an LPP shown shaded in the following figure. Minimum of $Z = 4x+3y$ occurs at the point:



(a) (0,8)	(b) (2,5)
(c) (4,3)	(d) (9,0)

24 The region represented by the inequalities $x \geq 6, y \geq 2, 2x + y \leq 0, x \geq 0, y \geq 0$ is

(a) unbounded	(b) a polygon
(c) exterior of a triangle	(d) None of these

25 Minimize $Z = 13x - 15y$ subject to the constraints : $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$.

(a) -23	(b) -32
(c) -30	(d) -34

26 The solution set of the inequality $3x + 5y < 4$ is

(a) an open half-plane not containing the origin.	(b) an open half-plane containing the origin.
(c) the whole XY-plane not containing the line $3x + 5y = 4$.	(d) a closed half plane containing the origin.

27 The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function Z = 4x + 6y occurs at

(a) (0.6, 1.6) *only*	(b) (3, 0) only
(c)(0.6, 1.6) and (3, 0) only	(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)

28 The value of objective function is maximum under linear constraints-------

(a) at the centre of feasible region	(b) at (0,0)
(c) at a vertex of feasible region	(d) the vertex which is of maximum distance from (0,0)

29 Which of the following is correct

(a) Every LPP has an optimal solution	(b) A LPP has unique optimal solution
(c) If LPP has two optimal solutions then it has infinite number of optimal solutions.	(d) the vertex which is of maximum distance from (0,0)

30 Objective function of LPP is

(a) a constraint	(b) A function to be maximized or minimised
(c) a relation between the decision variables	(d) equation of a straight line

3 MARKS QUESTIONS:

1. Maximise Z = 3x + 4y subject to the constraints: x + y ≤ 4, x ≥ 0, y ≥ 0.

   Solution: Objective function is Z = 3x + 4 y ……(1).
   The given constraints are : x + y ≤ 4, x ≥ 0, y ≥ 0.
   The corner points obtained by constructing the line x+ y= 4, are (0,0),(0,4) and (4,0).

Corner points	**Z = 3x +4y**
O (0 ,0)	Z = 3(0)+4(0) = 0
A (4 , 0)	Z = 3(4) + 4 (0) = 12
B (0 , 4)	Z = 3(0) + 4 (4) = 16 …(Max.)

Therefore Z = 16 is maximum at (0 , 4).

2. Maximise Z = x + y subject to x + 4y ≤ 8, 2x + 3y ≤ 12, 3x + y ≤ 9, x ≥ 0, y ≥ 0.

   Solution:

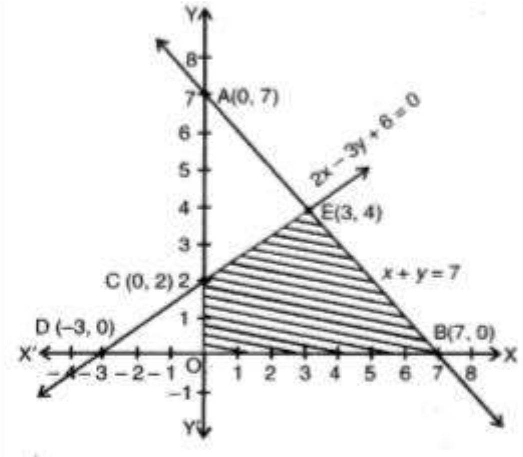
   Here , Maximise Z = x + y subject to x + 4y ≤ 8, 2x + 3y ≤ 12, 3x + y ≤ 9, x ≥ 0, y ≥ 0.

Corner points	**Z = x + y**
P(0 , 0)	0
Q(3 , 0)	3
R(0, 2)	2
S(28/11 , 15/11)	43/11.(Max.)

Hence the maximum value is 43/11

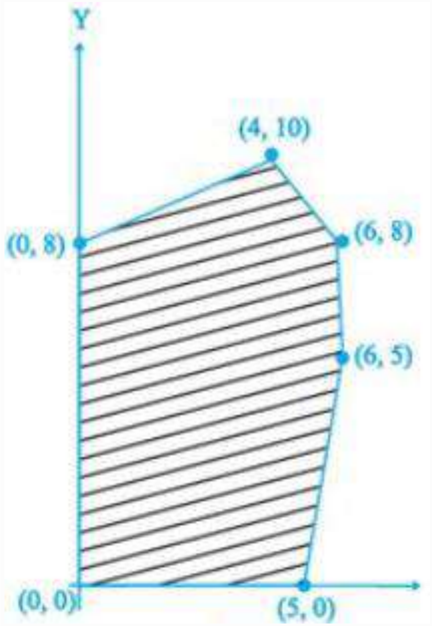
3. Minimise Z = 13x – 15y, subject to the constraints: x+y≤7 ,2x–3y+6≥0,x≥0,y≥0

   Solution:
   Considerx+y=7
   When x=0, then y=7x=0, then y=7 and
   when y=0, then x=7y=0, then x=7
   So, A(0, 7) and B(7, 0) are the points on line
   x+y=7



Consider 2x–3y+6=02x–3y+6=0
When x = 0, then y = 2 and when y = 0, then x = – 3, So C(0, 2) and D(-3, 0) are the points on line 2x–3y+6=02x–3y+6=0
Also, we have x > 0 and v > 0.
The feasible region OBEC is bounded, so, minimum value will obtain at a comer point of this feasible region.
Corner points are O(0, 0), B(7, 0), E(3, 4) and C(0, 2)
Z=13x–15y
At O(0,0),Z=0O(0,0),Z=0
At B(7,0),Z=13(7)–15(0)=91B(7,0),Z=13(7)–15(0)=91
At E(3,4),Z=13(3)–15(4)=−21E(3,4),Z=13(3)–15(4)=−21
At C(0,2),Z=13(0)–15(2)C(0,2),Z=13(0)–15(2)
= -30 (minimum)
Hence , the minimum value is -30 at the point (0, 2).

4. The feasible solution for an LPP is shown in Figure. Let Z = 3x – 4y be the objective function. Minimum of Z occurs at

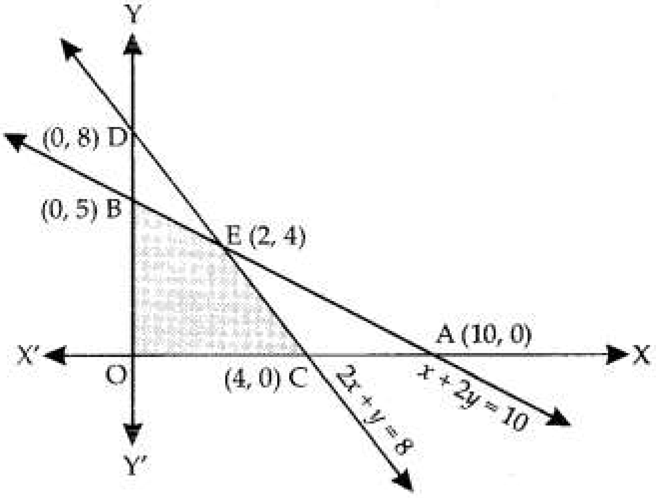


Solution:

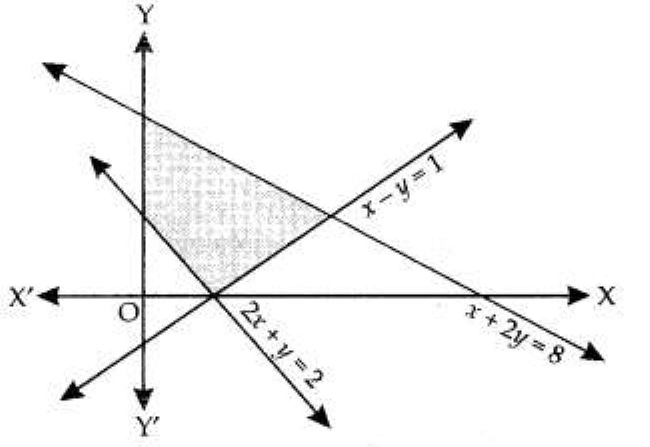
Corner points	**$Z = 3x – 4y$**
(0, 0)	0
(5,0)	15
(6,8)	-14
(6 ,5)	-2
(4,10)	-28
(0,8)	-32……………..(Min.)

The minimum value occurs at (0,8)

5. Solve the system of linear inequations: $x + 2y \leq 10$; $2x + y \leq 8$.
Solution:
Draw the st. lines $x + 2y = 10$ and $2x + y = 8$.
These lines meet at E (2,4).
Hence, the solution of the given linear inequations is shown as shaded in the following figure :



6. Find the linear constraints for which the shaded area in the figure below is the solution set:



Solution:

From the above shaded portion, the linear constraints are :

$2x + y \geq 2, x - y \leq 1,$

$x + 2y \leq 8,\ x \geq 0,\ y \geq 0$

7. Minimize $Z = 50x+70y$ …(Z is cost , x and y Food Type1 and Type 2) (1)

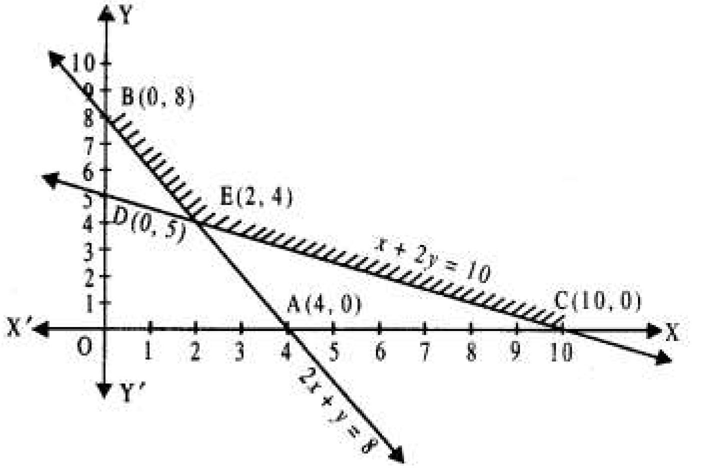
Subject to $2x + y \geq 8$ …(2)

$x + 2y \geq 10$ ….(3)

and $x \geq 0, y \geq 0$ ……..(4)

For the solution, we draw the lines :

$x = 0, y = 0, 2x + y = 8$ and $x + 2y = 10$



The feasible regin is as shown with vertices C(10,0), E(2,4) and B(0,8).

Applying Corner Point Method, we have

Corner Point	$Z = 50x + 70y$
C: (10,0)	500

E:(2,4)	380 (Minimum)
B: (0,8)	560

Thus minimum cost is ₹380 when 2 kg of Food I and 4 kg of Food II are mixed.

8. maximize when p is profit , x and y are Toy Type 1 and Toy Type 2

$P = 50x + 60y$ …(1)

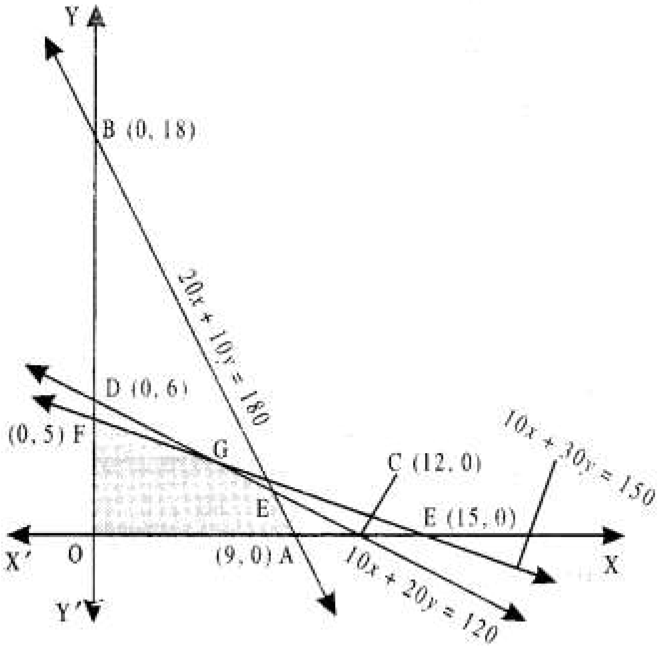
Subject to constraints :

$20x + 10v \leq 180$ …(2)

$10x + 20y \leq 120$ …….(3)

$10x + 30y \leq 150$ ……(4)

and $x \geq 0, y \geq 0$ …(5)



Applying Corner Point Method we have :

Corner point	P = 50jc + 60y
O: (0,0)	0
F:(0,5)	300
G: (6,3)	480
H: (8,2)	520 (Maximize)

A: (9,0)	450

Hence, maximum profit is ₹520 when x = 8 and y = 2. i.e., when 8 toys of type A and 2 toys of type B are made.

<u>Practice Questions:</u>

No.	LPP	Z	No.	LPP	Answer
01	Min Z = 10(x – 7y + 190) x + y ≥4 ;x + y ≤8, x ≤5 ; y ≤5	(0,5)	08	Max Z = 400x + 300 y x +y ≤ 200; x ≥ 20,y ≥ 4x	(40,160)
02	Max z = 30 x + 25 y 3x+3y ≤18, 3x+2y ≤15	(3,3)	09	Min Z = 6x + 10y x + 2y ≥ 10; 2x + 2y ≥ 12; 3x + y ≥ 8	(2,4)
03	Max z = 11 x + 9y 180x + 120 y ≤1500 x + y ≤ 10	(5,5)	10	Min Z = 4x + 3y 200x+100y ≥ 4000;2x + y ≥ 40 40x + 40 y ≥ 1400	(5,30)
04	Max Z= 220x + 180 y 3600 x + 2400 y ≤57600 x + y ≤ 20	(8,12)	11	Min z = 5x + 7y 2x + y ≥8 ; x + 2y ≥ 10	(2,4)
05	Min Z = 5x + 3y $\frac{10x}{100}+\frac{6y}{100} \geq 14$; $\frac{5x}{100}+\frac{10y}{100} \geq 14$	(80,100)			
06	Max z = 60x + 40 y 1000 x + 1200 y ≤ 9000 12x + 8y ≤ 72	$\left(\frac{9}{4}, \frac{45}{8}\right)$			
07	Min z = 400x + 300 y 100x + 200 y ≥ 13000 300x + 400 y ≥ 20000 200x + 100 y ≥ 15000	$\left(\frac{170}{3}, \frac{110}{3}\right)$			

<u>Practice MCQ's</u>

1. The maximum value of $P = 3x + 5y,$ subject to $x \leq 2, y \leq 3, x + y \leq 4, x \geq 0 \; and \; y \geq 0$ is

   a) 15 b) 16 c) 18 d) 20

2. The maximum value of $P = 4x + 2y,$ subject to $4x + 2y \leq 46,\ x + 3y \leq 24,\ x \geq 0,$ and $y \geq 0,$ occurs at

   a) exactly one point b) two points

   c) three points d) and infinite number of points

3. A firm manufacturing two types of electrical items A and B can make a profit of Rs 160 per unit of 'A' and Rs. $240/-$ per unit of B both A and B make use of two essential components a motor and transformer each unit of A requires 3 motors and 2 transformers and each unit of B requires 2 motors and 4 transformers the total supply of components per month is restricted to 210 motors and 300 transformers if x and y denoted. The number of items A and B which the firm should produce in order to maximize the profit then the objective function is

a) $z=3x+2y$ b) $z=2x+4y$ c) $z=210x+300y$ d) $z=160x+240y$

4. The maximum value $P=2x+5y$ subject to $x+2y\le 10, x\ge 12, x, y\ge 0$ is $=$

a) 10 b) 12 c) 18 d) no solution

Hint: Feasible region existing in negative direction

5. The minimum value of $z=2x+y$ subject $5x+10y\le 50, x+y\ge 1, y\le 4 \, and \, x, y\ge 0$ is $=$

a) $\frac{1}{3}$ b) 1 c) $\frac{1}{2}$ d) $\frac{5}{2}$

6. The corner points of the feasible region determined by the following system of linear inequalities.

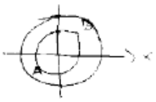
$2x+y\le 10,\ x+3y\le 15,\ x, y\ge 0$ are $(0,0),(5,0),(3,4)$ and $(0,5)$ let $z=px+qy$ where $p, q>0.$ condition $p \, and \, q$ so, that The maximum of z occurs at both $(3,4)$ and $(0,5)$ is

a) $p=q$ b) $p=2q$ c) $p=3q$ d) $q=3p$

Hint: $z \, at (3,4) = z \, at (0,5)$

7. Which of the following set is not convex set

a) $\{(x,y): x+y\le 1\}$ b) $\{(x,y): x^2+y^2\le 1\}$ c) $\{(x,y): 1\le x^2+y^2\le 3\}$ d) $\{(x,y): 2x^2+3y^2\le 6\}$



Hint: $x^2+y^2=1 \, and \, x^2+y^2=3$

Clearly the line joining A and B doesn't it lie in the set.

8. The optional value of the objective function in attained at the points

a) on $x-axis$ b) on $y-axis$

c) which are at the corner-points of the feasible region

d) which are at the points of intersection of the in equation with $y-axis$

9. The solution set of inequality $x\ge 0$ is

a) Half plane on the left of $y-axis$

b) Half plane on the right of $y-axis$ including $y-axis$

c) Half plane on the right of $y-axis$ excluding $y-axis$

d) Half plane on the right of $y-axis$ and above $x-axis$

10. Solution of the equality $y\le 0$ is

a) Half plane below $x-axis$, excluding the point on $x-axis$

b) Half plane above $x-axis$

c) Half plane above $x-axis$, including the point on $x-axis$

d) Half plane below $x-axis$, including the point on $x-axis$

11. The solution set of the inequality's $x \geq 0 \, and \; y \leq 0$ is

a) First Quadrant b) Second Quadrant c) Third Quadrant d) Fourth Quadrant

12. The region represents by the inequalities $x \geq 6,\; y \geq 3,\; 2x+y \geq 10,\; x \geq 0, y \geq 0$ is

a) unbounded b) A polygon c) Bounded Region d) Exterior of a triangle

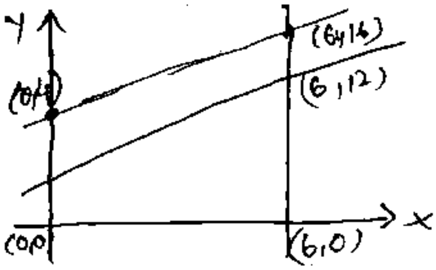
13. The feasible solution for a L.P.P is shown in the figure let $z = 3x - 4y$ be the objective function minimum of z occurs at

a) $(0,0)$ b) $(0,8)$ c) $(5,0)$ d) $(4,10)$

14. The feasible region for an L.P.P is shown in the following figure, let $F = 3x - 4y$ be the objective function minimum value of F is

a) 0 b) -16

c) 12 d) -46



15. The feasible region of an L.P.P is always

a) A closed set b) An unbounded set c) A bounded set d) A convex set

16. If a L.P.P admits optimal solution at two consecutive vertices of the feasible region, then

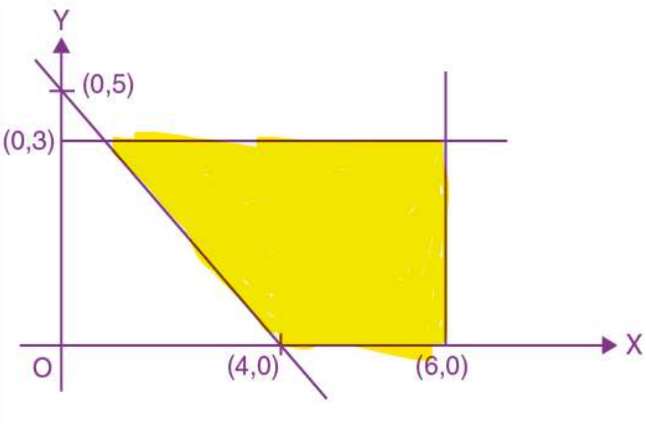
a) The required optimal solution in at the midpoint of line joining these two points

b) The optimal solution occurs at every point on the line joining these two points

c) The L.P.P under consideration is not solution

d) The L.P.P under consider must be reconstructed.

17. The shaded region in the following figure is the solution set of the inequations



a) $5x+4y \geq 20, x \leq 6, y \geq 4, x \geq 0, y \geq 0$ b) $5x+4y \geq 20, x \leq 6, y \leq 4, x \geq 0, y \geq 0$

c) $5x+4y\le 20, x\le 6, y\le 4, x\ge 0, y\ge 0$ c) $5x+4y\ge 20, x\ge 6, y\le 4, x\ge 0, y\ge 0$

18. The region represented by the in equation system $x, y\ge 0$ $y\le 6,$ $x+y\le 3$ is

a) Unbounded in first quadrant

b) Unbounded in first and second quadrant

c) Bounded in the first quadrant

d) Bounded in the second quadrant

19. In a LPP, The linear function which has to be maximized (or) minimized is called a linear function

a) Objective b) Feasible c) Subjective d) contraints

20. Corner points of the feasible region for and LPP are $(0,2)$ $(3,0)$ $(6,0)$ $(6,8)$ *and* $(0,5)$

$F=4x+6y$ be the objective function

The minimum value of F occurs at

a) $(0,2)$ only b) $(3,0)$ *only*

c) The midpoint of the line segment joining the points $(0,2)$ *and* $(3,0)$ *only*

d) Any point on the line segement joining the points $(0,2)$ *and* $(3,0)$

21. In a LPP, The objective function is always

a) Linear b) Quadratic c) inequality d) cubic

22. A feasible region of a system of linear in equality is said to be bounded if it can be enclosed with in a

a) Triangle b) polygon c) circle d) tow vertical lines

23. If an LPP admits an optimal solution at two consecutive vertices of a feasible region, then -

a. The required optimal solution is at the midpoint of the line joining two points

b. The optimal solution occurs at every point on the line joining these two points

c. The LPP under consideration is not solvable

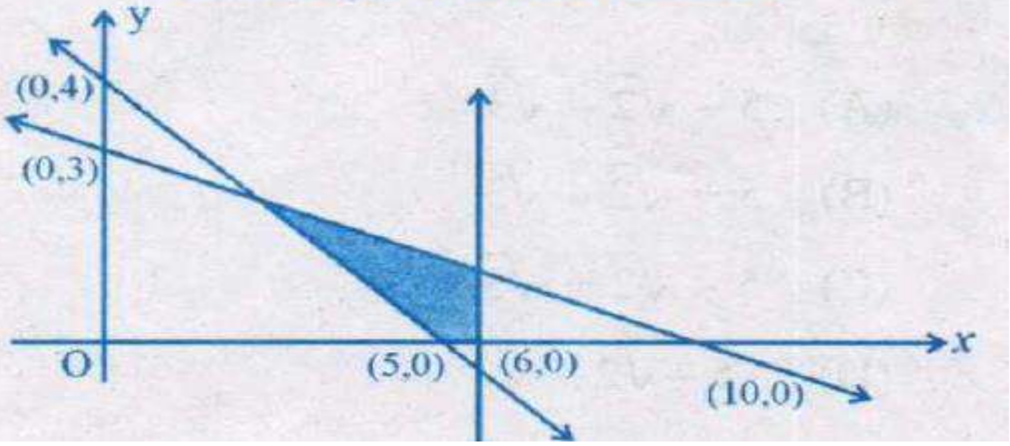
d. The LPP under consideration must be reconstructed

24. Corner points of the feasible region determined by the system of linear constraints are (0,3),(1,1) and (3,0). Let z = px+qy, where p,q>0. Condition on p and q so that the minimum of z occurs at (3,0) and (1,1) is

a. p = q b. p = 2q c. p = q/2 d. p = 3q

25. The shaded region in the figure is the solution set of the inequations

solution set of the inequations



a. $4x+5y\geq 20$, $3x+10y\leq 30$, $x\leq 6$, $x, y\geq 0$.

b. $4x+5y\geq 20$, $3x+10y\leq 30$, $x\geq 6$, $x, y\geq 0$.

c. $4x+5y\leq 20$, $3x+10y\leq 30$, $x\leq 6$, $x, y\geq 0$.

d. $4x+5y\leq 20$, $3x+10y\leq 30$, $x\geq 6$, $x, y\geq 0$.

26. For the LPP; maximize $z = x+4y$ subject to the constraints $x + 2y \leq 2$, $x + 2y \geq 8$, $x, y \geq 0$

a. $z_{max} = 4$ b. $z_{max} = 18$ c. $z_{max} = 16$ d. Has no feasible solution

27. Feasible region (shaded) for a LPP is shown in the Fig. 14.6. Minimum of $Z = 4x + 3y$ occurs at the point

(A) (0, 8) (B) (2, 5) (C) (4, 3) (D) (9, 0)

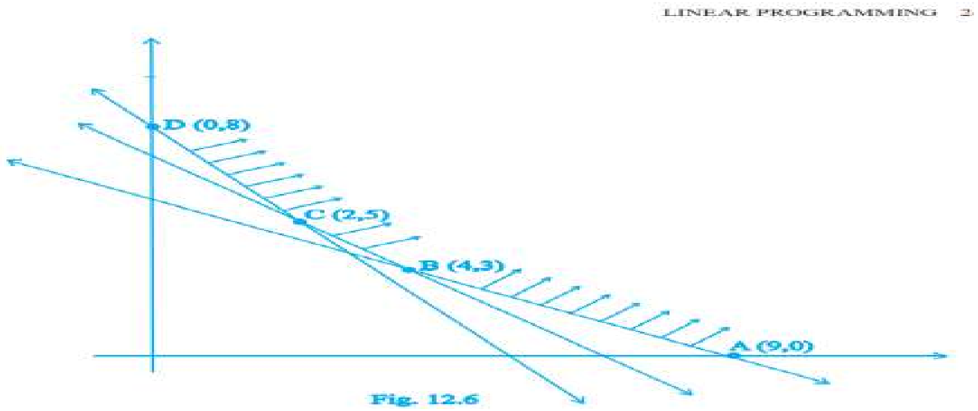


Fig. 12.6

Q.No 28 to 30

27. The feasible solution for a LPP is shown in Fig. 12.12. Let $Z = 3x - 4y$ be the

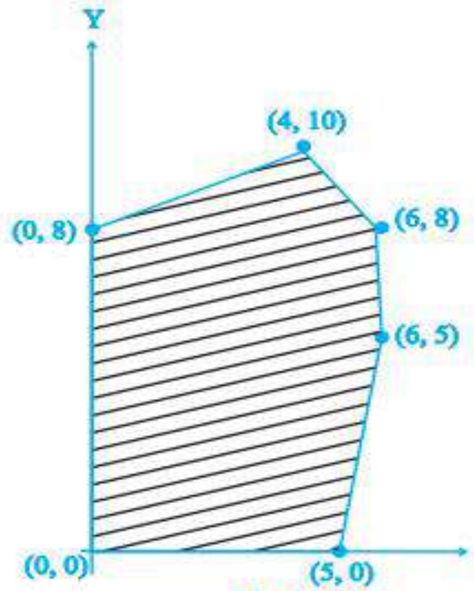


Fig. 12.12

objective function. Minimum of Z occurs at

(A) (0, 0) (B) (0, 8) (C) (5, 0) (D) (4, 10)

28. Refer to Exercise 27. Maximum of Z occurs at

(A) (5, 0) (B) (6, 5) (C) (6, 8) (D) (4, 10)

29. Refer to Exercise 27. (Maximum value of Z + Minimum value of Z) is equal to

(A) 13 (B) 1 (C) – 13 (D) – 17

31. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $F = 4x + 6y$ be the objective function. The Minimum value of F occurs at

(A) (0, 2) only (B) (3, 0) only

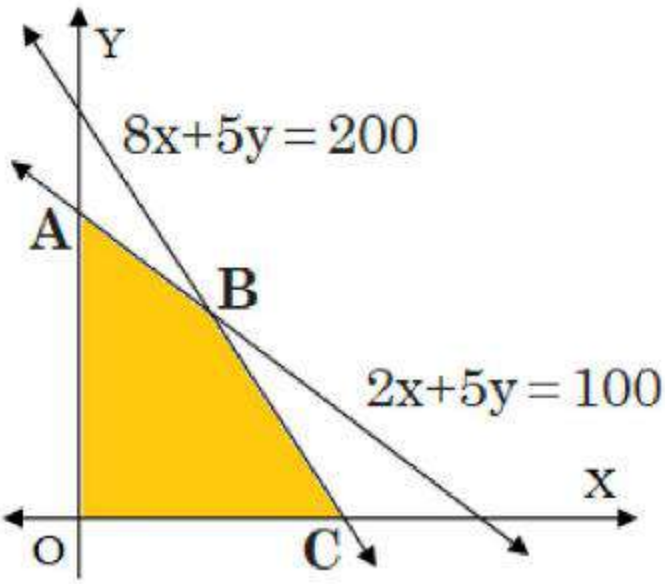
(C) the mid point of the line segment joining the points (0, 2) and (3, 0) only

(D) any point on the line segment joining the points (0, 2) and (3, 0).

<u>CASE BASED QUESTIONS:</u>

i) Sony rides his bike at 25 kmph, She has to spend `2 per km on diesel and if she rides it at a faster speed of 40 kmph, the diesel cost increases to `5 per km. She has `100 to spend on diesel. Let she travels x km with speed 25 kmph and y km with speed 40 kmph.

The feasible region for the LPP is shown below.



Based on the given information, answer the following questions.

(i) What is the point of intersection of lines as shown in the figure?

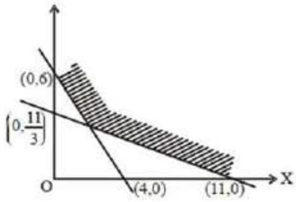
(ii) Write all the corner points of the feasible region shown in above graph.

(iii) If Z=x+y be the objective function, then find the point at which the maximum value

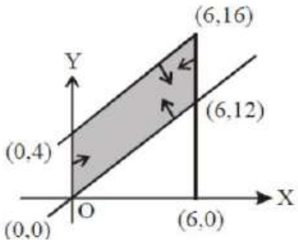
of Z occurs.

PRACTICE QUESTIONS :

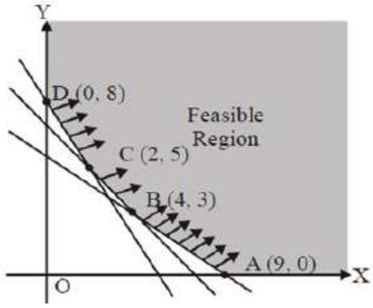
1. Find the maximum value of the objective function Z = 5x + 10 y subject to the constraints
   $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$.
2. Find the maximum value of Z = 3x + 4y subjected to constraints $x + y \leq 40$, $x + 2y \leq 60$, $x \geq 0$ and
   $y \geq 0$
3. Find the points where the minimum value of Z occurs:
   Z = 6x + 21 y, subject to $x + 2y \geq 3$, $x + 4y \geq 4$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$.
4. For the following feasible region, write the linear constraints.



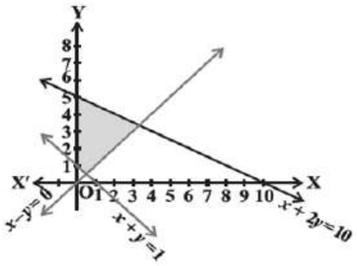
5. The feasible region for LPP is shown shaded in the figure.
Let Z = 3 x – 4 y be the objective function, then write the maximumvalue of Z.



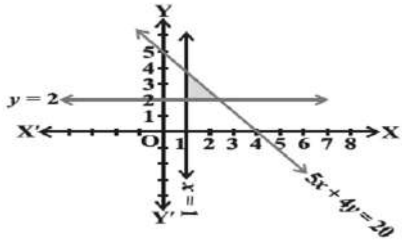
6. Feasible region for an LPP is shown shaded in the following figure. Find the point where minimum of $Z = 4x + 3y$ occurs.



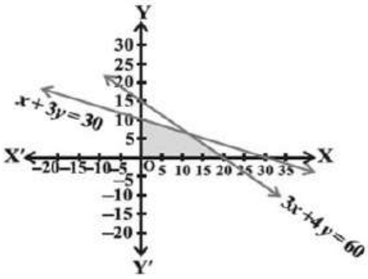
7. Write the linear inequations for which the shaded area in the following figure is the solution set.



8. Write the linear inequations for which the shaded area in the following figure is the solution set.



9. Write the linear inequations for which the shaded area in the following figure is the solution set.



10. Solve the following Linear Programming Problems graphically: Maximise $Z = 5x + 3y$ subject to $3x + 5y \le 15$, $5x + 2y \le 10$, $x \ge 0$, $y \ge 0$.

ANSWERS

1. 600 **2.140**

3. $(2, \frac{1}{2})$

4. x ≥ 0, y ≥ 0, 3x+ 2y ≥12, x +3y ≥ 11

5. 0

6. (2,5)

7. x + 2y ≤ 10, x + y ≥ 1, x – y ≤ 0, x ,y≥ 0

9. 3x + 4y ≤ 60, x +3y ≤ 30, x ≥0, y≥0

8. 5x + 4y ≤ 20, x ≥ 1, y ≥ 2

10. Maximum Z = $\frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$

PROBABILITY

Important Points

1. **Conditional Probability** :If E and F are two events associated with the same sample space of a randomexperiment, then the conditional probability of the event E under the condition that theevent F has occurred, written as$P(E/F)$ is given by
   $P(E/F) = \frac{P(E\cap F)}{P(F)}$,$P(F) \neq 0$.
2. **Multiplication Theorem on Probability**:
   a)Let E and F be two events associated with a sample space of an experiment. Then
   $P(E \cap F) = P(E)\, P(F/E)$, provided $P(E) \neq 0$
   $P(E \cap F) = P(F)P(E/F)$, provided $P(F) \neq 0$.
   b) If E, F and G are three events associated with a sample space, then
   $P(E \cap F \cap G) = P(E)\, P(F/E)P(G/F \cap E)$.
3. **Independent Events:** Let E and F be two events associated with a sample space S. If the probability ofoccurrence of one of them is not affected by the occurrence of the other, then we saythat the two events are independent. Thus, two events E and F will be independent, if
   a) $P(E/F) = P(E)$, $P(E) \neq 0$.
   b) $P(F/F) = P(F)$, $P(F) \neq 0$.
4. **Theorem of Total Probability :**Let $\{E_1, E_2, ..., E_n\}$ be a partition of the sample space S. Let A be any event associatedwith S, then
   $P(A) = \sum_{j=1}^{n} P(E_j)P(A/E_j)$.
5. **Bayes' Theorem :** If $E_1, E_2, ..., E_n$are mutually exclusive and exhaustive events associated with a samplespace, and A is any event of non zero probability, then
   $$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}.$$
6. **Random Variable and its Probability Distribution :**A random variable is a real valued function whose domain is the sample space of arandom experiment.
   The probability distribution of a random variable X is the system of numbers

X	x_1	x_2	x_3	…	x_n
P(X)	p_1	p_2	p_3	…	p_n

Where $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$

7. **Mean of a Random Variable:** Let X be a random variable assuming values $x_1, x_2, ... x_n$with probabilities $p_1, p, ... p_n$, respectively such that $p_i \geq 0$,$\sum_{i=1}^{n} p_i = 1$
   Mean of X, denoted by μ [orexpected value of X denoted by E (X)] is defined as
   $$\mu = E(X) = \sum_{i=1}^{n} x_i p_i$$

SOLVED QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. If $P(A/B\ ) = P(A'/B)$, then which of the following statement is true ?
   a) $P(A) = P(A')$ b) $P(A) = 2P(B)$
   c) $P(A \cap B) = \frac{1}{2}P(B)$ d) $P(A \cap B) = 2\ P(B)$

Ans : c

2. Let E be an event of a sample space S of a random experiment , then $P(S/E) =$
   a) $P(S \cap E)$ b) $P(E)$ c) 0 d) 1

Ans: d

3. Let E and F be two events such that ,$P(E) = 0.1$, $P(F) = 0.3, P(E \cup F) = 0.4$ then $P(F/E) =$
   a) 0.6 b)0.5 c)0.4 d) 0

Ans:d

4. The probability distribution of a random variable X is

X	0	1	2	3	4
P(X)	0.1	k	2k	k	0.1

Where k is some unknown constant .
The probability that the random variable takes the value 2 is :
a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{4}{5}$ d) 1

Ans : b

5. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :
   a) $A \subset B$, but $A \neq B$ b) $A = B$
   c) $A \cap B = \phi$ d) $P(A) = P(B)$

Ans : d

6. Two events A and B will be independent if
   a) A and B are mutually exclusive events b) $P(A) = P(B)$
   c) $P(A'B') = (1 - P(A))(1 - P(B))$ d) $P(A) + P(B) = 1$

Ans: c

7. If the sum of numbers obtained on throwing a pair of dice is 9 , then the probability that number obtained on one of the dice is 4 , is:
   a) $\frac{1}{9}$ b)$\frac{4}{9}$ c)$\frac{1}{18}$ d) $\frac{1}{2}$

Ans : d

8. If for two events A and B ,$P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P(B/A) =$
   a) $\frac{1}{2}$ b) $\frac{3}{5}$ c) $\frac{2}{5}$ d)$\frac{2}{3}$

Ans: d

9. The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is :
   a) $\frac{7}{20}$ b) $\frac{1}{5}$ c)$\frac{3}{20}$ d)$\frac{4}{5}$

Ans: a

10. The events E and F are independent . If $P(E) = 0.3$, $P(E \cup F) = 0.5$, then $P(E/F) - P(F/E) =$
    a) $\frac{1}{7}$ b)$\frac{2}{7}$ c)$\frac{3}{35}$ d)$\frac{1}{70}$

Ans: d

ASSERTION REASONING QUESTIONS

.Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

11. **Assertion (A)** : Given two independent events A and B such that $P(A) = 0.3$ and $P(B) = 0.6$, then P(A and not B)= 0.12.
Reason(R) : For two independent events A and B , P (A and B)=P(A).P(B).

Ans: a

12. **Assertion (A)** : For an event A of the sample space S of a random experiment , $P(S/A) = P(A/A) = 1$.
Reason(R) : For three events A ,B and C of the sample space S of a random experiment
$P(A \cup B/C) = P(A/C) + P(B/C) - P(A \cap B/C)$.

Ans:b

13. **Assertion (A)** : The mean number of tails in three tosses of a coin is 1.5.
Reason(R) : The mean of the probability distribution

X	x_1	x_2	x_3	…	x_n
P(X)	p_1	p_2	p_3	…	p_n

is $\sum_{i=1}^{n} x_i p_i$

Ans : a

14. **Assertion (A)** : Probability of drawing a red ball from any of the bag I or II , when bag I contains 3 red and 5 blue balls whereas bag II contains 4 red and 4 blue balls is $\frac{7}{16}$.
Reason(R) : There are two bags I and II , bag I contains 3 red and 5 blue balls whereas bag II contains 4 red and 4 blue balls . A ball is drawn from either of bag and is found red . Probability of it to be from bag I is $\frac{3}{7}$.

Ans: b

15. **Assertion (A)** : Two dice are rolled and it is given that the sum of the number on both the dice is greater than 6 . The probability of getting a doublet is $\frac{3}{16}$.
Reason(R) : Probability of E when F is given is $P(E/F) = \frac{P(E \cap F)}{P(F)}$.

Ans:d

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)

16. Find $[\,P(B/A) + P(A/B)]$, if $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$.
Soln: Given $P(A \cup B) = \frac{3}{5} \Rightarrow P(A) + P(B) - P(A \cap B) = \frac{3}{5}$

$$\Rightarrow \frac{3}{10} + \frac{2}{5} - P(A \cap B) = \frac{3}{5} \Rightarrow P(A \cap B) = \frac{1}{10}$$

$$[\,P(B/A) + P(A/B)] = \frac{P(A \cap B)}{P(A)} + \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{7}{12}$$

17. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$ respectively , if both of them try to solve the problem independently , then find the probability that the problem is solved .

Soln: P(problem is solved) = 1−P(not solved) $= 1 - P(A' \cap B')$

$$= 1 - P(A')P(B')$$

$$= 1 - \frac{1}{3}.\frac{2}{5} = \frac{13}{15}$$

18. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, withoutreplacement. Find the probability that the number on each card is a multiple of 7.

Soln: Multiples of 7 from 1 to 25 are 7, 14, 21

P (number on each card is a multiple of 7)= $\frac{3}{25} \times \frac{2}{24} = \frac{1}{100}$

19. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball,drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at randomfrom the second bag. Find the probability that the ball drawn is white.

Soln: Case I: White ball is transferred from bag I to bag II

P(white ball from bag II)= $\frac{4}{9} \times \frac{7}{14}$

Case II: Black ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{5}{9} \times \frac{6}{14}$

Total Probability = $\frac{4}{9} \times \frac{7}{14} + \frac{5}{9} \times \frac{6}{14} = \frac{29}{63}$

20. In a toss of three different coins , find the probability of coming up of three heads , if it is known that at least one head comes up .

Soln: Let E_1:Getting three heads

E_2: Getting at least one head

$P(E_2) = \frac{7}{8}$, $P(E_1 \cap E_2) = \frac{1}{8}$

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{\frac{1}{8}}{\frac{7}{8}} = \frac{1}{7}$$

21. Two rotten apple are mixed with 8 fresh apples . Find the probability distribution of number of rotten apples , if two apples are drawn at random , one-by-one without replacement .

Soln:

X	0	1	2
P(X)	$\frac{8}{10} \times \frac{7}{9} = \frac{56}{90} = \frac{28}{45}$	$2\left(\frac{8}{10} \times \frac{2}{9}\right) = 2 \times \frac{16}{90} = \frac{16}{45}$	$\frac{2}{10} \times \frac{1}{9} = \frac{2}{90} = \frac{1}{45}$

22. Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that $2\,P(X = x_1) = 3\,P(X = x_2) = P(X = x_3) = 5\,P(X = x_4)$

Find the probability distribution of X.

Soln: Let $P(X = x_3) = k$

Then $P(X = x_1) = \frac{k}{2}$, $P(X = x_2) = \frac{k}{3}$, $P(X = x_4) = \frac{k}{5}$

Now $\frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$

X	x_1	x_2	x_3	x_4

P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

23. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\bar{A} \cup \bar{B}) = \frac{1}{4}$. Find whether the events A and B are independent or not .

Soln:$P(\bar{A} \cup \bar{B}) = \frac{1}{4} \Longrightarrow P(\overline{A \cap B}) = \frac{1}{4} \Longrightarrow 1 - P(A \cap B) = \frac{1}{4}$

$$\Longrightarrow P(A \cap B) = \frac{3}{4}$$

$$P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

Since $P(A \cap B) \neq P(A) \times P(B)$, A and B are not independent .

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

24. Find the mean of number of tails in two tosses of a coin .

Soln: Let X = number of tails

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$
X.P(X)	0	$\frac{2}{4}$	$\frac{2}{4}$

Mean= $0 + \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$

25. Out of two bags , bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls .One ball is drawn at random from one of the bags and is found to be red.Find the probability that it was drawn from bag B.

Soln: Let E_1:Event of choosing bag A

E_2: Event of choosing bag B

A: red ball is found

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$, $P(A/E_1) = \frac{3}{5}$, $P(A/E_2) = \frac{5}{9}$

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} = \frac{25}{52}$$

26. Out of a group of 50 people , 20 always speak the truth . Two persons are selected at random from the group (without replacement) . Find the probability distribution of number of selected persons who always speak the truth .

Soln:

X	0	1	2
P(X)	$\frac{30}{50} \times \frac{29}{49} = \frac{87}{245}$	$2\left(\frac{30}{50} \times \frac{20}{49}\right) = \frac{120}{245}$	$\frac{20}{50} \times \frac{19}{49} = \frac{38}{245}$

27. A man is known to speak the truth 3 out of 5 times . He throws a pair of different coins and reports that he got a pair of heads . Find the probability that a pair of heads actually occurs.

Soln:: Let E_1:Getting both heads

E_2: Not getting both heads

A:Reporting two heads

$P(E_1)=\frac{1}{4}$, $P(E_2)=\frac{3}{4}$, $P(A/E_1)=\frac{3}{5}$, $P(A/E_2)=\frac{2}{5}$

$$P(E_1/A)=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}=\frac{1}{3}$$

28. A and B are independent events such that $P(A\cap\bar{B})=\frac{1}{4}$ and $P(\bar{A}\cap B)=\frac{1}{6}$. Find $P(A)$ and $P(B)$.

Soln: $P(A\cap\bar{B})=\frac{1}{4}\Longrightarrow P(A).P(\bar{B})=\frac{1}{4}$

$$P(\bar{A}\cap B)=\frac{1}{6}\Longrightarrow P(\bar{A}).P(B)=\frac{1}{6}$$

Let $P(A)=x$, $P(B)=y$

$$P(A).P(\bar{B})=\frac{1}{4}\Longrightarrow x(1-y)=\frac{1}{4}$$

$$P(\bar{A}).P(B)=\frac{1}{6}\Longrightarrow (1-x)y=\frac{1}{6}$$

$$\Longrightarrow x-y=\frac{1}{12}$$

Eliminating y , we get $12x^2-13x+3=0\Longrightarrow x=\frac{3}{4},x=\frac{1}{3}$

$$P(A)=\frac{1}{3}\ ,P(B)=\frac{1}{4}$$

$P(A)=\frac{3}{4}, P(B)=\frac{2}{3}$

29. A and B throw a die alternately till one of them gets a ‘6’ and wins the game. Find their respective probabilities of winning , if A starts the game first.

Soln: P(getting a six) $=\frac{1}{6}$, $P($ not getting a six $)=\frac{5}{6}$

$P($A wins$)=\frac{1}{6}+\left(\frac{5}{6}\right)^2\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\cdots=\frac{\frac{1}{6}}{1-\frac{25}{36}}=\frac{6}{11}$

P(B wins)$=1-P(A\ wins)=1-\frac{6}{11}=\frac{5}{11}$

30. A pair of dice is thrown simultaneously . If X denotes the absolute difference of numbers obtained on the pair of dice , then find the probability distribution of X.

Soln:

X	0	1	2	3	4	5
P(X)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

31. Two numbers are selected from first six even natural numbers at random without replacement . If X denotes the greater of two numbers selected , find the probability distribution of X .

Soln:

X	4	6	8	10	12
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

LONG ANSWER QUESTIONS (5 MARKS)

32. In answering a question on a multiple choice test , a student either knows the answer or guesses . Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses . Assuming that a student guesses at the answer will be

correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer given that he answered correctly ?

Soln: Let E_1:Student knows the answer

E_2: Student guesses the answer

A:Student answered correctly .

$P(E_1) = \frac{3}{5}$, $P(E_2) = \frac{2}{5}$, $P(A/E_1) = 1$, $P(A/E_2) = \frac{1}{3}$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)} = \frac{9}{11}$$

33. A box contains 10 tickets , 2 of which carry a prize of ₹ 8 each , 5 of which carry ₹ 4 each , and remaining 3 carry a prize of ₹ 2 each . If one ticket is drawn at random , find the mean value of the prize.

Soln: Let X denote the prize value.

Here X can take values of 8, 4 and 2.

$$P(X = 8) = \frac{2}{10} = \frac{1}{5}$$

$P(X = 4) = \frac{5}{10} = \frac{1}{2}$

$P(X = 2) = \frac{3}{10}$

X	8	4	2
P(X)	$\frac{1}{5}$	$\frac{1}{2}$	$\frac{3}{10}$
X.P(X)	$\frac{8}{5}$	2	$\frac{6}{10} = \frac{3}{5}$

Mean $= \frac{8}{5} + 2 + \frac{3}{5} = \frac{21}{5}$

CASE-STUDY BASED QUESTIONS (4 MARKS)

34. In a group activity class , there are 10 students whose ages are 16,17,15,14,19,17,16,19,16 and 15 years . One student is selected at random such that each has equal chance of being chosen and age of the student is recorded .

On the basis of above information answer the following questions

(i) Find the probability that the age of the selected student is a composite number .

(ii) Let X be the age of the selected student , what can be the value of X.

(iii) a) Find the probability distribution of random variable X and hence find the mean age .

OR

(iii) b) A student was selected at random and his age was found to be greater than 15 years . Find the probability that his age is a prime number .

Soln: (i) P(Age of selected student is a composite number)$= P(14,15,16) = \frac{6}{10} = \frac{3}{5}$

(ii) X can be 16,17,15,14,19

(iii) $a)$

X	14	15	16	17	19
P(X)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$
X.P(X)	$\frac{14}{10}$	$\frac{30}{10}$	$\frac{48}{10}$	$\frac{34}{10}$	$\frac{38}{10}$

Mean $= \frac{164}{10} = 16.4$ years

OR

(iii) b) P(Age is a prime number / Age is greater than 15) $= \frac{2}{3}$

35. A Shopkeeper sells three types of flower seeds A_1, A_2 and A_3 . They are sold as a mixture where the proportions are $4:4:2$respectively . The germination rates of the three types of seeds are $45\%, 60\%$ and 35%.

Based on the above information , answer the following questions :

(i) What is the probability of a randomly selected seed to germinate .

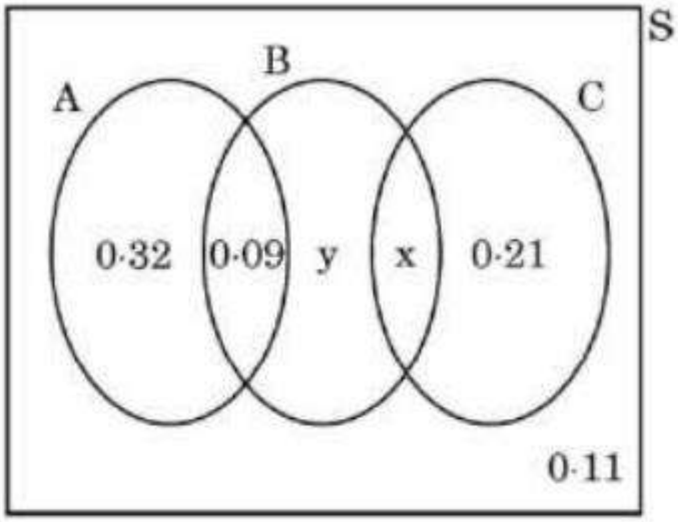
(ii) What is the probability that the randomly selected seed is of type A_1 , given that it germinates ?

Soln: (i)P(randomly selected seed to germinate)

$$=\frac{4}{10}*\frac{45}{100}+\frac{4}{10}*\frac{60}{100}+\frac{2}{10}*\frac{35}{100}=\frac{490}{1000}=\frac{49}{100}$$

(ii) P(selected seed is of type A_1/ Seed germinates) $=\frac{\frac{4}{10}*\frac{45}{100}}{\frac{49}{100}}=\frac{18}{49}$

36. The Venn diagram below represents the probabilities of three different types of yoga A , B, and C performed by people of a society . Further it is given that probability of a member performing type C yoga is 0.44.



On the basis of above information , answer the following questions

(i) Find the value of x

(ii) Find the value of y

(iii) a) Find $P(C/B)$

OR

b) Find the probability that a randomly selected person of the society does yoga of type A or B , but not C.

Soln:(i)$x + 0\cdot 21 = 0\cdot 44 \Rightarrow x = 0\cdot 23$

(ii)$0\cdot 41 + y + 0\cdot 44 + 0.11 = 1 \Rightarrow y = 0\cdot 04$

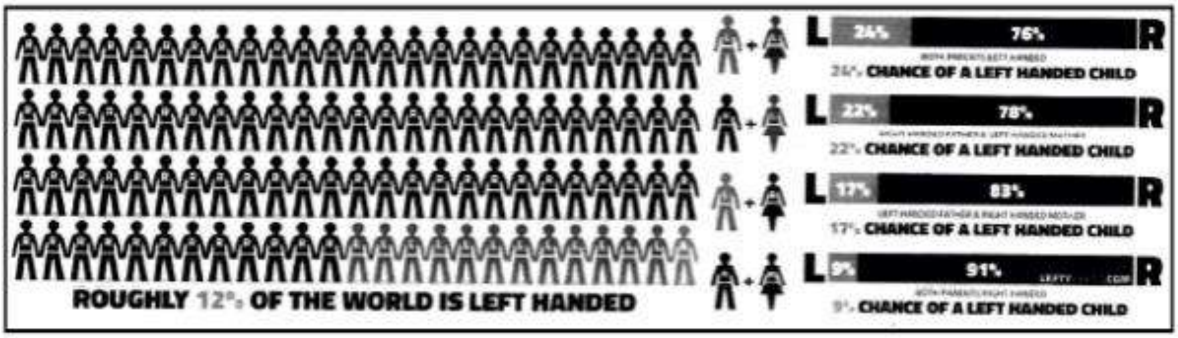
(iii) a) $P(C/B)=\frac{P(C\cap B)}{P(B)}=\frac{0.23}{0.36}=\frac{23}{36}$

OR

b) P(A OR B but not C) = 0·32 + 0.09 + 0.04

= 0·45

37. Recent studies suggest that roughly 12% of the world population is left handed



Depending upon the parents , the chances of having a left handed children are as follows:

A : When both father and mother are left handed :

Chances of left handed child is 24%.

B: When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C: When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D: When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that $P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$ and L denotes the event that child is left handed . Based on the above information answer the following questions

(i) Find $P(L/C)$
(ii) Find $P(\bar{L}/A)$
(iii) a) Find $P(A/L)$

OR

b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed .

Soln: (i) $P(L/C) = \frac{17}{100}$

(ii) $P(\bar{L}/A) = 1 - P(L/A) = 1 - \frac{24}{100} = \frac{76}{100} = \frac{19}{25}$

(iii) a) $P(A/L) = \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}} = \frac{24}{72} = \frac{1}{3}$

OR

b) probability that a randomly selected child is left handed given that exactly one of the parents is left handed = $P(L/B \cup C) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$.

38. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the given information, answer the following questions.

(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

Soln : Let E1 = The policy holder is accident prone.
E2 = The policy holder is not accident prone.
E = The new policy holder has an accident within a year of purchasing a policy.

(i) $P(E) = P(\text{E1})\times P(E/\text{E1}) + P(E) \times P(E/\text{E2})$

$$= \frac{20}{100} \times \frac{6}{10} + \frac{80}{100} \times \frac{2}{10} = \frac{7}{25}$$

(ii) By Bayes' theorem $P(\text{E1/E}) = \frac{P(\text{E1})\times \text{P}(E/\text{E1})}{P(E)} = \frac{\frac{20}{100}\times\frac{6}{10}}{\frac{280}{1000}} = \frac{3}{7}$

PRACTICE QUESTIONS

1. From the set $\{1,2,3,4,5\}$ two numbers a and b $(a \neq b)$ are chosen at random . The probability that $\frac{a}{b}$ is an integer is :

   a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) $\frac{3}{5}$

2. A bag contains 3 white , 4 black and 2 red balls . If 2 balls are drawn at random without replacement , then the probability that both the balls are white is

   a) $\frac{1}{18}$ b) $\frac{1}{36}$ $c)$ $\frac{1}{12}$ d) $\frac{1}{24}$

3. Three dice are thrown simultaneously . The probability of obtaining a total score of 5 is

   a) $\frac{5}{216}$ b) $\frac{1}{6}$ c) $\frac{1}{36}$ d) $\frac{1}{49}$

4. A card is picked at random from a pack of 52 playing cards . Given that the picked card is queen , the probability of this card to be a card of spade is

   a) $\frac{1}{3}$ b) $\frac{4}{13}$ c) $\frac{1}{4}$ d) $\frac{1}{2}$

5. A die is thrown once . Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5 . Then $P(A \cup B)$ is

   a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) 0 d) 1

6. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$ then $P(B'/A\ )$ is

   a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{3}{4}$ d) 1

7. Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women , find the probability of choosing a good orator.

8. A bag contains two coins , one biased and the other unbiased . When tossed , the biased coin has a 60% chance of showing heads . One of the coins is selected at random and on tossing it shows tails. What is the probability it was an unbiased coin ?

9. The probability distribution of a random variable X , where k is a constant is given below

$$P(X = x) = \begin{cases} 0.1 & if\ \ x = 0 \\ kx^2 & if\ \ x = 1 \\ kx & if\ \ x = 2or3 \\ 0 & otherwise \end{cases}$$

   Determine

   a) the value of k b) $P(X \leq 2)$

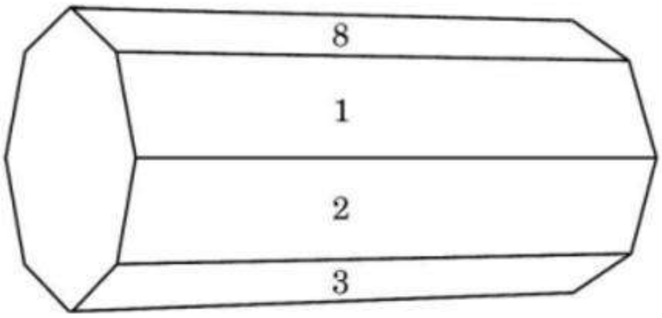
10. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of number of rotten apples , if three apples are drawn one by one with replacement .

11. In a shop X , 30 tins of ghee of type A and 40tins of ghee of type B which look alike , are kept for sale .While in shop Y , similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y .

12. There are two bags , I and II . Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a Ball is drawn randomly from Bag II . If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

13. An urn contains 5 red , 2 white and 3 black balls . Three balls are drawn one-by-one, at random without replacement . Find the probability distribution of the number of white balls .

14. An octagonal prism is a three dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces . If it has 24 edges and 16 vertices .



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted . Let X denote the number obtained on the bottom face and the following table gives the probability distribution of X.

X	1	2	3	4	5	6	7	8
P(X)	p	$2p$	$2p$	p	$2p$	p^2	$2p^2$	$7p^2 + p$

Based on above information , answer the following questions :

(i) Find the value of p

(ii) Find $P(X > 6)$

(iii) a) Find $P(X = 3m)$, where m is a natural number .

OR

b) Find the mean $E(X)$

15. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on times is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let :E_1 represent the event when many workers were not present for the job;
E_2 represent the event when all workers were present; and
E: represent completing the construction work on time.

Based on the above information. answer the following questions:

(i) What is the probability that all the workers are present for the job?

(ii) What is the probability that construction will be completed on time?

(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time?

-OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time.

SAMPLE QUESTION PAPER -1

Class:-XII

Session 2024-25

Mathematics (Code-041)

Time: 3 hours **Maximum marks: 80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.

2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.

3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.

4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.

5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.

6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub – parts.

Section –A

(Multiple Choice Questions)

Each question carries 1 mark

Q:1	Given A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = 3I$ then (a) $1 + \alpha^2 + \beta\gamma = 0$. (b) $1 - \alpha^2 - \beta\gamma = 0$. (c) $3 - \alpha^2 - \beta\gamma = 0$ (d) $3 + \alpha^2 + \beta\gamma = 0$.	1
Q.2	If f(x) = $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x=b & x+c & 0 \end{vmatrix}$, then (a) f(a) = 0 (b) f(b) = 0 (c) f(0) = 0 (d) f(1) = 0	1
Q.3	If x= 1 is root of the $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}$ = 0 then sum of the two roots is (a)4 (b)-1 (c)-3 (d) 1	1
Q.4	if A = $\begin{bmatrix} p & 8 \\ 2 & 2p \end{bmatrix}$ is a singular matrix, then the value of p is (a)$2\sqrt{2}$ (b)$\pm 2\sqrt{2}$ (c)8 (d) $-\sqrt{2}$	1
Q.5	If A = $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ then the value of $\lvert adj\ A \rvert$ is (a)-8 (b)0 (c)16 (d)64	1
Q.6	$\int 2^{x+2}dx$ =	1

	(a)2^{x+2} +C (b)2^{x+2} log2 +C (c) $\frac{2^{x+2}}{log2}$+C (d) 2.$\frac{2^{x}}{log2}$ +C	
Q.7	The function f(x) = cot x is discontinuous on the set (a){ x=nπ : n∈ Z} (b) { x=2nπ : n∈ Z} (c) { x=(2n+1)$\pi/2$: n∈ Z} (d){ x=n$\pi/2$: n∈ Z}	1
Q.8	The function f: R →R is given by f(x) = -|x-1| is (a)continuous as well as differentiable at x = 1 (b) not continuous but differentiable at x = 1 (c) continuous but not differentiable at x = 1 (d) Neither continuous nor differentiable at x = 1	1
Q.9	The order and degree of the differential equation $\frac{d^2y}{dx^2}$ + sin($\frac{dy}{dx}$)$^{1/4}$ + $x^{1/5}$ = 0 respectively are (a)2 and 3 (b) 3 and 3 (c) 2 and 2 (d) 2 and not defined	1
Q.10	Integrating factor of differential equation (1-y^2) $\frac{dx}{dy}$ +yx = ay, (-1$< y < 1$) is (a) $\frac{1}{-1+y^2}$ (b) $\frac{1}{\sqrt{y^2-1}}$ (c) $\frac{1}{1-y^2}$ (d) $\frac{1}{\sqrt{-y^2+1}}$	1
Q.11	If $\vec{a}+\vec{b}=\hat{\imath}$ and $\vec{a}=2(\hat{\imath}-\hat{\jmath}+\hat{k})$ then $\|\vec{b}\|$ equals to (a) $\sqrt{14}$ (b) 3 (c)$\sqrt{12}$ (d)$\sqrt{17}$	1
Q.12	The value of λ for which the vectors 3$\hat{\imath}$ -6 $\hat{\jmath}$ +$\hat{k}$ and 2$\hat{\imath}$ -4 $\hat{\jmath}$ +λ$\hat{k}$ are parallel is (a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$	1
Q.13	For any vector $\vec{a}$,the value of $(\vec{a}\ X\ \hat{\imath})^2$+ $(\vec{a}\ X\ \hat{\jmath})^2$ + $(\vec{a}\ X\ \hat{k})^2$ = (a) $2\vec{a}^2$ (b) $4\vec{a}^2$ (c) $3\vec{a}^2$ (d) $\vec{a}^2$	1
Q.14	If $\vec{a}$is any non-zero vector, then $(\vec{a}.\hat{\imath})\ i$+$(\vec{a}.\hat{\jmath})\hat{\jmath}$+ $(\vec{a}.\hat{k})\hat{k}$ is equal to (a) $\vec{a}.\vec{b}$ (b) $\vec{a}$ (c) 0 (d) none of these	1
Q.15	Direction cosines of the line $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$are : (a) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ (b) $\frac{-2}{7},\frac{3}{7},\frac{6}{7}$ (c) $\frac{2}{7},\frac{-3}{7},\frac{6}{7}$ (d) $\frac{-2}{7},\frac{-3}{7},\frac{-6}{7}$	1
Q.16	The corner points of the feasible region determine by the system of linear constraints are (0,3) , (1,1) and (3,0) .Let Z = px + qy , when p,q>0 condition on p and q so that Minimum of the Z occur at (3,0) and (1,1) is (a) p = q (b) p = 3q (c) p = q/2 (d) p= 2q	1
Q.17	The number of solution(s) of the system of Inequations x + 2y ≤3 , 3x + 4y≥ 12 , x≥ 0,y ≥ 0 is (a) 0 (b) 2 (c) 3 (d) infinite	1
Q.18	Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is (a) $\frac{1}{2}$ (b) $\frac{1}{3}$(c) $\frac{2}{3}$ (d) $\frac{4}{7}$	1
	ASSERTION-REASON BASED QUESTIONS **In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.** **(a) Both (A) and (R) are true and (R) is the correct explanation of (A).**	

	(b) Both (A) and (R) are true but (R) is not the correct explanation of (A). **(c) (A) is true but (R) is false.** **(d) (A) is false but (R) is true.**	
Q.19	Assertion (A) : The number of all onto functions from the set {1,2,3,4,5} to itself is 5! Reason (R) : Total number of all onto functions from the set {1,2,3,4,5,……n} to itself is n!	1
Q.20	Assertion (A) : The function f(x) = $x^2 - x$ is increasing in the interval $(\frac{1}{2}, \infty)$ Reason (R) : For above function $f'(x) = 2x+1$.	1
	SECTION – B	
Q.21	Find the value of $\sin^{-1}[\cos(\frac{43\pi}{5})]$ OR Find the principal value of $\cos^{-1}[\cos(-680^0)]$.	2
Q.22	Find the value of a for which the function f(x) = sin x – ax + b is increasing on R.	2
Q.23	Find the rate of change of volume of sphere with respect to its surface area when radius is 2 cm. OR The amount of pollution content added in air in a city due to x diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added.	2
Q.24	Find the intervals in which the function f(x) = $\frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is strictly decreasing.	2
Q.25	$Evalute: \int \frac{x^2+2}{x+1}$ dx.	2
	SECTION – C	
Q.26	If y = $(cosx)^{(cosx)^{(cosx)\ldots\ldots\infty}}$ then show that $\frac{dy}{dx} = \frac{y^2\, tanx}{y\log(\cos x)-1}$.	3
Q.27	Integrate the function $\frac{x^2}{1-x^4}\ w.r.t.x.$ OR Integrate the function $\frac{2x}{(x^2+1)(x^2+2)} w.r.t.x.$	3
Q.28	Evaluate : $\int_{-1}^{2} \lvert x^3 - x \rvert$ dx.	3
Q.29	Solve the differential equation: $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}\ where$ $x \in (-\infty, -1) \cup (1, \infty)$ OR If a curve y= f(x), passing through the point (1,2) is the solution of the differential equation $2x^2dy = (2xy + y^2)dx$. Find the value of $f(\frac{1}{2})$.	3
Q.30	Maximize Z=300x+190y subjected to constraints $x+y \le 24$, $x+\frac{1}{2}y \le 16$, x, y ≥0 OR Minimize Z= 10x+4y subjected to constraints $4x+y \ge 80$, $2x+y \ge 60$, x, y ≥0	3
Q.31	The random variable X can take only the values 0,1,2,3 given that P(X=0)=P(X=1)=p and P(X=2)=P(X=3)=a such that $\sum p_i x_i^2 = 2\sum p_i x_i$, find the value of p.	
	SECTION - D	

Q.32 A function f:[-4,4]→ $[0,4]$ *is given by* $f(x)=\sqrt{16-x^2}$. Show that f is an onto function but not one-one function. Further, find all possible values of a for which f(a)=$\sqrt{7}$.

OR

Determine the relation R defined on the set of all real numbers R = {(a,b) : a,b∈ R and a -b +$\sqrt{3}$ ∈ S, where S is the set of all irrational numbers.} is reflexive, symmetric and transitive.

Q.33 Use product $\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$ to solve the system of equations.

x-y+2z = 1 ,2y – 3z = 1, 3x-2y +4z = 2.

Q.34 Using integration find the area of the region {(x,y):$x^2+y^2\leq 9$,x+y ≥3 } .

Q.35 Find the value of $a^2+b^2+c^2$ where point (a,b,c) is the image of (1,2,-3) on the line $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}$

OR

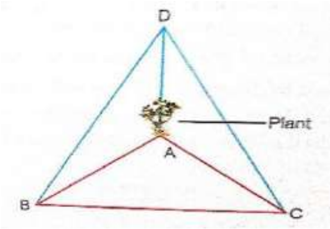
Find the coordinate of the foot of the perpendicular drawn from point A(5,4,2) to the line $\vec{r}$ = $-\hat{\imath}+3\hat{\jmath}+\hat{k}+\lambda(2\hat{\imath}+3\hat{\jmath}-\hat{k})$ also find image of A in this line.

SECTION – E (case – study)

Q.36

Aryan purchased an air plant holder which is in shape of tetrahedron. Let A,B,C and D be the coordinate of the air plant holder where A=(1,2,3) B=(3,2,1)

C=(2,1,2) D=(3,4,3)



i)Find the vector $\overrightarrow{AB}$.

ii) Find the vector $\overrightarrow{CD}$.

iii) Find the unit vector along $\overrightarrow{BC}$

OR

Find the area triangle BCD.

Q.37

Read the following and answer the questions:

Relation between the height of the plant (y in cm) with respect to exposure to sun light is governed by the following equation, Y = 4x $-\frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

	 i) Find the rate of growth of plant w.r.t sunlight. ii) What is the maximum height of the plant? iii) What will be the height of the plant after two days? OR (iii) If the height of the plant is $\frac{7}{2}$ cm, the number of days it has been exposed to the sunlight.	
Q.38	After observing attendance of class 12, class teacher Sh.Mool Singh comes on conclusion that 30% students have 100% attendance and 70 % students are irregular to attend class When he observed previous year result he found that 70% of all students who have 100% attendance attain distinction marks while 10% irregular students attain distinction marks.at the end of the year one student is selected at random from the class.  (i) Find the total probability of the selected students having distinction marks from the class. (ii)if in random selection chosen student has distinction marks. Find the probability that the student has 100% attendance.	

Marking Scheme

Sample paper no 1 (2024-2025)

Section A

1.(c)	2.(c)	3.(b)	4.(b)	5.(a)	6. (c)
7. (a)	8. (c)	9. (d)	10. (d)	11. (b)	12. (d)
13. (a)	14. (b)	15. (c)	16. (c)	17. (a)	18. (d)
19. (a)	20. (c)				

Q no	Answer	Marks
21	$\frac{-\pi}{10} \in [\frac{-\pi}{2},\frac{\pi}{2}]$ OR $\frac{2\pi}{9} \in [0,\pi]$	2
22	$f'(x) = cosx - a$ $\{f'(x) > 0\}$ a $\in (-\infty, -1)$	1 ½ 1/2
23	$V = \frac{4\pi r^3}{3}$, $S = 4\pi r^2$ $\frac{dV}{dS} = \frac{\frac{dV}{dr}}{\frac{dS}{dr}}$ $\frac{dV}{dS} = 1\ {cm^3}/{cm^2}$ OR $P'(x) = 0.015x^2 + 0.04x + 30$ $P'(3) = 30.255$	1 ½ ½ 1 1
24	$f'(x) = 6x^3 - 12x^2 - 90x$ $f'(x) = 0\ at\ x = 1,3,-3$ Strictly decreasing $(-\infty,-3) \cup (0,3)$	1 1
25	$\frac{x^2+2}{x+1} = x - 1 + \frac{3}{x+1}$ I = $\frac{x^2}{2} - x + 3log\|x+1\| + C$	1 1
26	Let y = $(cosx)^y$ Logy = log$(cosx)^y$ Differentiate with respect to x $\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\left(log(cosx)\right) + log(cosx)\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{-y^2 tanx}{ylog(cosx) - 1}$	1 1 1
27	I = $\int \frac{x^2}{(x-1)(x+1)(x^2+1)}dx$ I = $\int (\frac{1}{2(x^2+1)} - \frac{1}{4(x+1)} - \frac{1}{4(x-1)})dx$ I = $\int I = \frac{1}{4}log\left\|\frac{1+x}{1-x}\right\| - \frac{1}{2}tan^{-1}x + C$ OR Substitute $x^2 = t$	1 1 1 ½

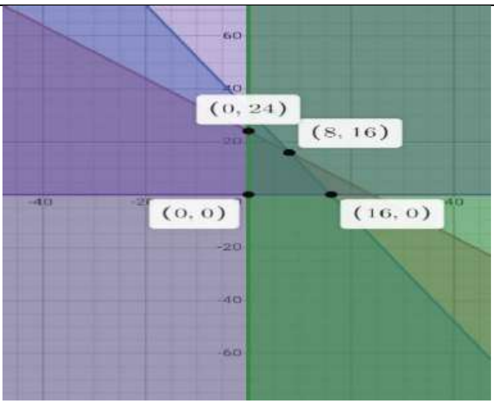
<table>
<tr><td></td><td>I = $\int \frac{1}{(t+1)(t+2)}dt$,
Using partial fraction A =1 and B = -1
I = $\log(\frac{x^2+1}{x^2+2})$+C</td><td>½

1
1</td></tr>
<tr><td>28</td><td>I = $\int_{-1}^{0}(x^3-x)dx+\int_{0}^{1}(x-x^3)dx+\int_{1}^{2}(x^3-x)dx$
I = $\frac{11}{4}$</td><td>2

1</td></tr>
<tr><td>29</td><td>P = $\frac{2x}{x^2-1}, Q = \frac{2x}{(x^2-1)^2}$
$IF = \ x^2-1$
y$(x^2-1) = log\left|\frac{x-1}{x+1}\right| + C$
OR
$\frac{dy}{dx} = \frac{(2xy+y^2)}{2x^2}$
Put y = vx
$\frac{dv}{v^2}$=$\frac{dx}{2x}$
After integration,
$\frac{-2x}{y} = log|x| + C$
C = -1
f$(\frac{1}{2}) = \frac{1}{1+log2}$</td><td>1
1
1

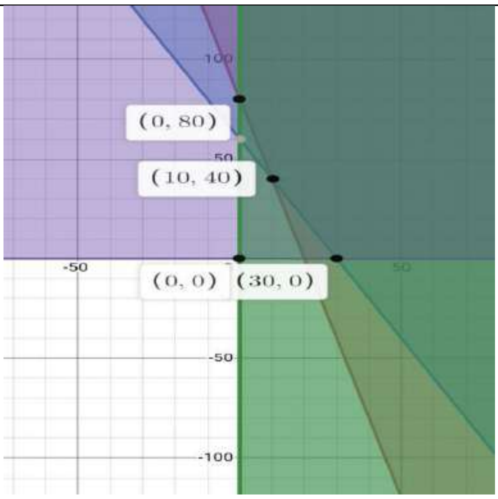
1

1

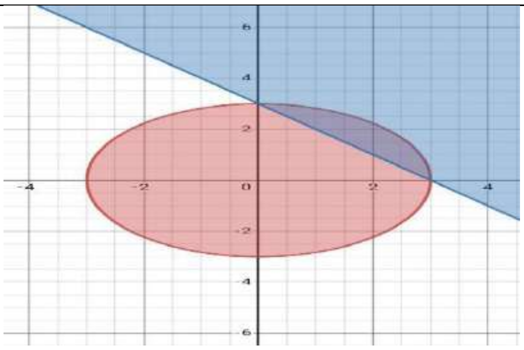
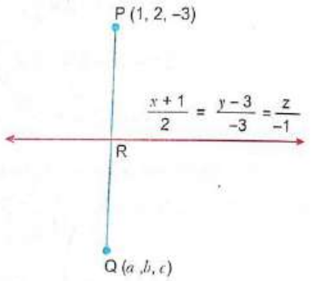
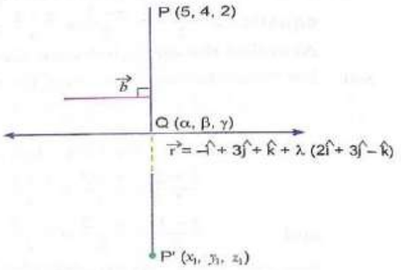
1</td></tr>
<tr><td>30</td><td>

Max value 5440 at (8, 16).
<table>
<tr><td>Corner point</td><td>Z = 300x+190y</td><td></td></tr>
<tr><td>(0,0)</td><td>0</td><td></td></tr>
<tr><td>(0,24)</td><td>4500</td><td></td></tr>
<tr><td>(8,16)</td><td>5440</td><td>Max</td></tr>
<tr><td>(16,0)</td><td>4800</td><td></td></tr>
</table>
OR</td><td>2

1</td></tr>
</table>

<table>
<tr>
<td></td>
<td>


Feasible region is unbounded. We will draw the graph of 10x+4y<260.
<table>
<tr><td>Corner point</td><td>Z = 10x+4y</td><td></td></tr>
<tr><td>(30,0)</td><td>300</td><td></td></tr>
<tr><td>(10,40)</td><td>260</td><td>min</td></tr>
<tr><td>(0,80)</td><td>320</td><td></td></tr>
</table>
Min value 260 at (10, 40).
</td>
<td>2
1</td>
</tr>
<tr>
<td>31</td>
<td>p + 13a=2(p+5a)
Also 2a = 1-2p
p = $\frac{3}{8}$</td>
<td>1
1
1</td>
</tr>
<tr>
<td>32</td>
<td>Let y = $\sqrt{16-x^2}$, y≥0
x = $\pm\sqrt{16-y^2}$
Hence function is onto.
For one-one,
f(-1) = f(1)
hence f is not one one function.
a = ± 3
Or
(a,a)ϵR
R is reflexive.
If (a,b) ϵR but (b,a) $does\ not\ belongs\ to\ R$.
If (a,b) ϵR, (b,c) $\epsilon R\ but\ (a,c) does\ not\ belongs\ to\ R$.
Hence given function is reflexive but neither symmetric nor transitive.</td>
<td>1
1
1
1
1
1
2
2</td>
</tr>
<tr>
<td>33</td>
<td>AC= I and $A^{-1} = C$
x= 0, y= 5 and z = 3</td>
<td>3
2</td>
</tr>
</table>

34	 Area of shaded region = $\int_0^3[\sqrt{9-x^2}-(3-x)]dx$ = $\frac{9(\pi-2)}{4}$ sq unit	2 3
35	-λ).  Midpoint of (1,2,-3) and (a,b,c) is (1,1,-1). $a^2+b^2+c^2$ = 1+0+1=2 OR  $\overrightarrow{PQ}=(\alpha-5)\hat{\imath}+(\beta-4)\hat{\jmath}+(\gamma-2)\hat{k}$ Using $\overrightarrow{PQ}$ *is perpendicular to* $\vec{b}$. Length of perpendicular = $2\sqrt{6}$ Required image = (-3,8,-2).	1 2 2 1 1 1 2
36	(i) $\overrightarrow{AB}$ = $2\hat{\imath}-2\hat{k}$ $(ii)\overrightarrow{CD}$ = $\hat{\imath}++3\hat{\jmath}+\hat{k}$ (iii) $\frac{1}{\sqrt{3}}(-\hat{\imath}-\hat{\jmath}+\hat{k})$ OR Area = $\frac{1}{2}[\overrightarrow{BC}$ X $\overrightarrow{BD}]$ = $\sqrt{6}\ sq\ unit.$	1 1 2
37	(i) $\frac{dy}{dx}=4-x$ (ii) Height of plant = 8 cm (iii) Height of plant after 2 days = 6 cm OR x = 1cm.	1 1 2

38	(i) Total probability = $\frac{3}{10} X \frac{7}{10} + \frac{1}{10} X \frac{7}{10}$ = $\frac{7}{25}$(ii) P = $\frac{\frac{3}{10}X\frac{7}{10}}{\frac{3}{10}X\frac{7}{10}+\frac{1}{10}X\frac{7}{10}}$ = $\frac{3}{4}$	2 2 2

SAMPLE QUESTION PAPER-2
(2024-25)

CLASS:XII MAX MARKS:80
SUBJECT : MATHEMATICS TIME : 3 HRS

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1 Mark

1. The number of all matrices of order 2×2 with each entry 2 or 3 is
   a) 4 b) 8 c)16 d)32
2. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value of α is
   $a) \pm 1$ b) ± 2 c)$\pm\sqrt{5}$ d) $\pm\sqrt{7}$
3. The angle between $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$ is
   a) $\frac{\pi}{3}$ b)$\frac{2\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{5\pi}{6}$
4. If the function $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous , then the value of k is
   a) $\frac{2}{7}$ b)$\frac{7}{2}$ c) $\frac{3}{7}$ d) $\frac{4}{7}$
5. $\int e^x \left(\log\sqrt{x} + \frac{1}{2x}\right) dx =$
   $a) e^x logx + c$ b) $e^x \log\sqrt{x} + c$ c) $\frac{e^x}{2x} + c$
   d)$e^x \log x^2 + c$
6. The order and the degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^3 + 5x = 0$ are
   a) 3 ; 6 b) 3 ; 3 c) 3;9 d) 6;3

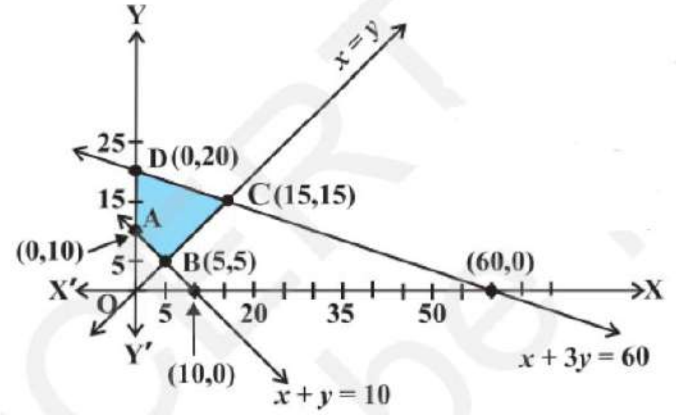
7. The graph of the inequality $2x + 3y > 6$ is
   a) Half plane that contains the origin
   b) Half plane that neither contains the origin nor the points of the line $2x + 3y = 6$
   c) Whole XOY-plane excluding the points on the line $2x + 3y = 6$
   d) Entire XOY-plane
8. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the points $A(2,3,-4)$, $B(3,-4,-5)$ and $C(3,2,-3)$ respectively then $|\vec{a} + \vec{b} + \vec{c}|$=
   a) $\sqrt{113}$ b) $\sqrt{185}$ c) $\sqrt{209}$ d) $\sqrt{203}$
9. $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\cot x}} dx =$

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

10. For what value of x the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is a singular matrix

a) 1 b) -1 c)2 d)-2

11. Based on the given shaded region as the feasible region in the graph , at which point(s) is the objective function $z = 3x + 9y$ maximum



a) Point B b) Point C c) Point D
   d) every point on the line segment CD

12. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then $x =$

a) ± 1 b) ± 2 c)$\pm\sqrt{2}$ d) $\pm\sqrt{3}$

13. If $A = \begin{bmatrix} 3 & 1 \\ 2 & -3 \end{bmatrix}$, then the value of $|adj\ A|$ is

a) 11 b)-11 c)9 d)-9

14. If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$, Find $P(A'/B')$
   a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{5}{6}$

15. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2 cosx$ is

a) $\log x$ b) $-\log x$ c) x d) $\frac{1}{x}$

16.If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx} =$

a) e^{y-x} b) e^{x+y} c) $-e^{y-x}$ d) $2e^{x-y}$

17. The value of p for which $p(\hat{\imath} + \hat{\jmath} + \hat{k})$ is a unit vector is

a) 0 b) $\frac{1}{\sqrt{3}}$ c) 1 d) $\sqrt{3}$

18.The coordinates of the foot of the perpendicular drawn from the point $(2,-3,4)$ on the y –axis is

a) $(2,3,4)$ b)$(-2,-3,-4)$ c)$(0,-3,0)$ d)$(2,0,4)$

ASSERTION-REASON BASED QUSETIONS

In the following questions , a statement of assertion (A) is followed by as statement of Reason (R) . Choose the correct answer out of the following choices .

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true .

19.**Assertion(A):** $\sin^{-1}(0.76)$ is defined

Reason (R): $\sin^{-1}(0.76)$ is defined because it is defined for all real numbers.

20.**Assertion(A):** $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ is perpendicular to $\vec{b} = -\hat{\imath} + \hat{\jmath}$

Reason (R): Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other if $\vec{a}.\vec{b} = 0$ is

SECTION B

This section comprises of very short answer type questions (VSA) of 2 marks each

21. Show that the signum function $f: R \to R$ given by

$$f(x) = \begin{cases} 1 \ , if \ x > 0 \\ 0 \ , if \ x = 0 \\ -1 \ , if \ x < 0 \end{cases}$$

Is neither one-one nor onto

OR

Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$

22. If $x = at^2$, $y = 2at$ then find $\frac{d^2y}{dx^2}$

23. The radius of a right circular cylinder is increasing at the rate of 2cm/s and its height is decreasing at the rate of 8cm/s. Find the rate of change of its volume, when the radius is 3 cm and height is 6cm.

24. Using vectors, find the area of triangle ABC with vertices $A(1,1,1)\ B(1,2,3)\ and\ C(2,3,1)$.

25. Find the angle between the lines $\vec{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$

OR

Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find $\int \sin^{-1} x\, dx$.

27. Evaluate $\int_{-4}^{4} |x + 2|\, dx$.

OR

Evaluate $\int_1^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}} dx$

28. Find $\int \frac{2x}{x^2+3x+2}\, dx$

29. Find the general solution of the following differential equation
$2xe^{\frac{y}{x}}dy + \left(x - 2ye^{\frac{y}{x}}\right)dx = 0$

OR

Find the particular solution of the differential equation $(2x^2 + y)\frac{dx}{dy} = x$ given that $when\ x = 1\,, y = 2$

30. Solve the following linear programming problem graphically
Maximize $z = 3x + 5y$ subject to $x + y \leq 5\,;\ x \geq 3\,;\ x \leq 4\,; y \geq 0$

31. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and thenanother ticket is drawn without replacing the first one in the bag. Find theprobability distribution of the number of even numbers on the ticket. Also find the mean of the probability distribution.

OR

Find the probability distribution of the number of successes in two tosses of a die,when a success is defined as “number greater than 5”.Also find the mean of the probability distribution.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$; Find A^{-1}

Hence , solve the following system of equations

$$3x + 4y + 2z = 8$$
$$2y - 3z = 3$$
$$x - 2y + 6z = -2$$

33.Let N be the set of natural numbers and R be the relation on N × N defined by(a, b) R (c, d) iff ad = bc for all a, b, c, d ∈ N. Show that R is an equivalence relation.

OR

Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Rightarrow (x - y)$ *is divisible by* 3 is an equivalence relation.

34. If the area between the curves $x = y^2$ and $x = 4$ divided into two equal parts by the line $x = a$, then find the value of a using integration .

35.Prove that the line through A (0, –1, –1) and B(4, 5, 1) intersects the line throughC(3, 9, 4) and D(– 4, 4, 4).

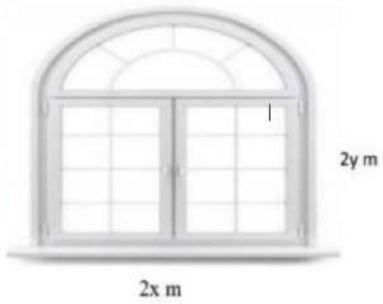
OR

Find the vector and Cartesian equations of the line which is perpendicular to the lineswith equations$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}and\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$and passes through the point(1, 1, 1).

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub parts (i) , (ii) ,(iii) of marks 1,1,2 respectively . The third case study question has two sub parts of 2 marks each)

36.



Case-study 1 : Read the following passage and answer the questions given below ($2x$ and $2y$ are length and breadth of rectangular part)
The windows of a newly constructed building are in the form of a rectangle surmounted by a semi circle . The perimeter of each window is 40m.

(i) Find the relation between x and y

(ii) What is the area of the window in terms of x

(iii) Find the value of x for which area of window will be maximum ?

OR

Find the value of y for which area of window will be maximum ?

37. **Case Study -2** : The total profit function of a company is given by $P(x) = -5x^2 + 125x + 37500$ where x is the production of the company

(i) Find the critical point of the function ?

(ii) Find the interval in which the function is strictly increasing ?

(iii) If $P(x) = -5x^2 + mx + 37500$ and 14 is the critical point , then find the value of m

OR

Find the absolute maximum for this value of m in $[0,16]$

38. Case Study -3

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the given information, answer the following questions.

(i)what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

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