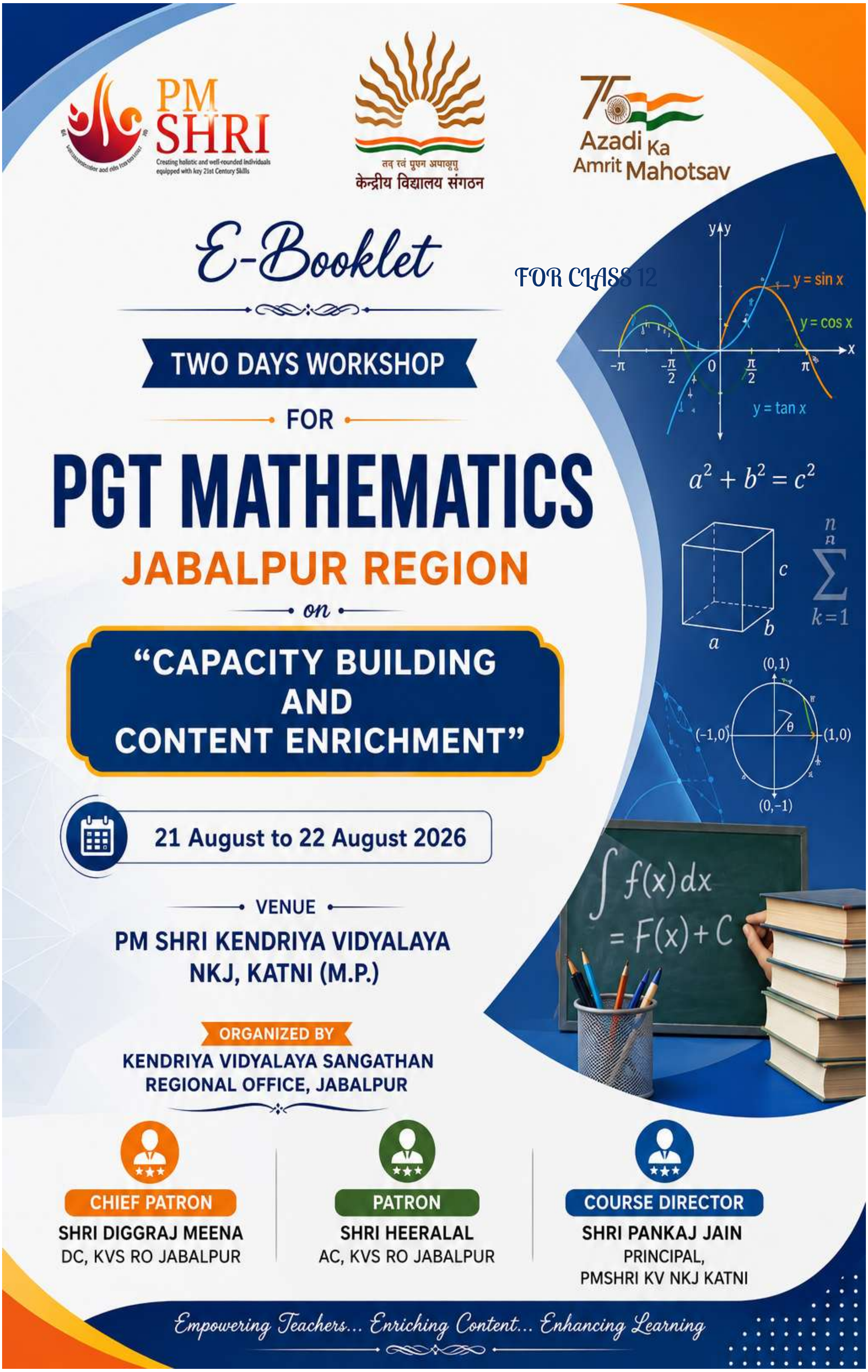
PM SHRI
Creating holistic and well-rounded individuals equipped with key 21st Century Skills
तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन
Azadi Ka Amrit Mahotsav
E-Booklet
FOR CLASS 12
TWO DAYS WORKSHOP
FOR
PGT MATHEMATICS
JABALPUR REGION
on
"CAPACITY BUILDING AND CONTENT ENRICHMENT"
21 August to 22 August 2026
VENUE
PM SHRI KENDRIYA VIDYALAYA NKJ, KATNI (M.P.)
ORGANIZED BY
KENDRIYA VIDYALAYA SANGATHAN REGIONAL OFFICE, JABALPUR
CHIEF PATRON
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PMSHRI KV NKJ KATNI
Empowering Teachers... Enriching Content... Enhancing Learning
y = sin x
y = cos x
y = tan x
$a^2 + b^2 = c^2$
$\int f(x)\,dx = F(x) + C$

INDEX

E-Booklet • Two Days Workshop for PGT Mathematics • Jabalpur Region

Note: Page numbers above refer to the revised booklet after this Index page is inserted immediately after the cover.

COMPETENCY-BASED QUESTIONS

RELATIONS AND FUNCTIONS

A. Solved Competency-Based Questions

1. School Club Membership

A school has 40 students in a mathematics club. Let A be the set of students and B = {1,2,3,4}, representing four levels of participation. Each student is assigned exactly one participation level according to the number of activities completed.

(i) Explain why the assignment from A to B is a function.
(ii) If 10 students are assigned level 1, 12 level 2, 8 level 3 and 10 level 4, find the number of elements in the pre-image of {2,4}.
(iii) Is the function necessarily one-one? Give a reason.
Solution:
(i) It is a function because every student in A is assigned exactly one participation level in B.

(ii) Pre-image of {2,4} consists of students assigned levels 2 or 4.
Number = 12 + 10 = 22.

(iii) No. The function is not one-one because several students can have the same participation level. Thus different elements of A may have the same image in B.

2. Coding and Functions

A coding system assigns each integer x in the set A = {-2,-1,0,1,2} the value $f(x) = x^2 + 1$.

(i) Find the range of f.
(ii) Determine whether f is one-one.
(iii) Determine whether f is onto B = {1,2,5}.
(iv) State the type of function if considered from A to B.
Solution:
f(-2)=5, f(-1)=2, f(0)=1, f(1)=2, f(2)=5.

(i) Range = {1,2,5}.
(ii) It is not one-one because f(-2)=f(2)=5 and f(-1)=f(1)=2.
(iii) It is onto B because every element of B occurs as an image.
(iv) Since it is onto but not one-one, it is an onto function but not a bijection.

B. Unsolved Competency-Based Questions

3. Transport Pass Allocation

A school transport system assigns each registered student exactly one bus route from the set B = {R1, R2, R3, R4}. Let A be the set of registered students.

(i) Explain why the assignment is a function from A to B.
(ii) If 15 students use R1, 18 use R2, 12 use R3 and 5 use R4, find the number of students in the pre-image of {R1,R3}.
(iii) Can this function be one-one? Justify your answer.
Answer space:

4. Digital Learning Platform

A digital learning platform defines $f: R \to R$ by $f(x) = 2x - 3$.

(i) Find $f(4)$ and $f(-2)$.
(ii) Determine whether f is one-one.
(iii) Determine whether f is onto R.
(iv) Hence state whether f is bijective. Give mathematical justification.
Answer space:

5. Function in a Production Model

A small manufacturing unit models its daily production cost by $f(x) = x^2 + 4x + 7$, where x represents the number of production batches.

(i) Find $f(1)$, $f(2)$ and $f(-2)$.
(ii) Determine whether f is one-one on R.
(iii) Find two distinct real numbers having the same image under f, if possible.
(iv) What conclusion can you draw about the one-one nature of the function?
Answer space:

Teacher's Note

These questions assess application of the concepts of function, pre-image, range, one-one, onto and bijective functions through real-life and mathematical contexts.

COMPETENCY BASED QUESTIONS
SUBJECT – MATHEMATICS(041)
CLASS --- XII

Q. No.	**Questions**	**Marks**
Q 1	If A be a 3X3 square matrix such that (adjA)=[5 0 0 0 5 0 0 0 5], then find the value of $\lvert adjA \rvert$.	1 mark
Q 2	For any 2X2 matrix P, which of the following matrices can be Q such that PQ=QP? (i) [1] (ii) [1 0 0 1] (iii) [1 1 1 1] (iv) no such matrix exists	1mark
Q 3	On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹ 1,000 and ₹ 500 were given to the lucky draw winners.  A total of 60 vouchers were given on the day. The number of ₹ 1,000 vouchers added to 3 times the number of ₹ 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.	4 marks

Q 4 — 4(1+1+2) marks

A company produces two types of products, A and B, in three factories. The number of units produced per month is given in the following table:

FACTORY	PRODUCT A	PRODUCT B
1.	100	150
2.	120	180
3.	80	110

The cost per unit for each product is:

PRODUCT	COST PER UNIT (RS.)
A	50
B	60

Based on the above information give the answer of below questions:

(i) Represent the production data as a matrix P and the cost data as a matrix C.

(ii) Find the total cost of production for each factory using matrix multiplication.

(iii) If the company wants to increase the production of Product A by 10% in all factories, write the new production matrix.

OR

Calculate the new total cost.

Q 5

Read the following text and answer the following questions on the basis of the same:

On her birthday, Lata decided to donate some money to the children of an orphanage home. If there were 8 children less, everyone would have got ₹ 10 more. However, if there were 16 children more, everyone would have got ₹ 10 less. Let the number of children be x and the amount distributed by Lata for one child be ₹ y.

(i) Find the equations related to the given problem in terms of x and y.

	OR (i) Find the number of children. How much amount is given to each child by Lata. (ii) Write the equations in form of matrix representation for the information given above? (iii) How much amount Lata spends in distributing the money to all the students?	2 marks 1 mark 1 mark

CHAPTER--- MATRICES AND DETERMINANTS
MARKING SCHEME

Q No.	**Questions**	**Marks**
Q 1	25	1
Q 2	Option (ii) is correct	1
Q 3	Let, number of ₹ 1000 vouchers $= x$ and number of ₹ 500 vouchers $= y$ Hence, $x + y = 60$ and $x + 3y = 100$ In matrix form, $\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 60 \\ 100 \end{bmatrix}$ $\Rightarrow$ $AX = B$ where $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 100 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $\Rightarrow$ $X = A^{-1}B$ ……… (1) $\lvert A \rvert = 2$ $\because \lvert A \rvert \neq 0 \Rightarrow A$ is an invertible matrix. adj $A = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$ By eq. (1), $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 60 \\ 100 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$ On comparing, $x = 40$ and $y = 20$ Hence, number of ₹ 1000 vouchers $= 40$ and number of ₹ 500 vouchers $= 20$	

CASE STUDY BASED QUESTIONS
CH.1 RELATION AND FUNCTIONS
CLASS XII SUBJECT: MATHEMATICS

SUBMITTED BY TEACHER: ARVIND KUMAR (PGT MATHS)
PM SHRI K.V. NO.3, SAGAR

Case Study 1: Digital Banking System:

A national bank assigns every customer a unique Customer ID. Every customer has exactly one ID and every ID belongs to exactly one customer. Let f:A→B map customers to IDs.

Q1. The function is: (a) Many-One (b) One-One (c) One-Many (d) Constant
Q2. Since every ID belongs to a customer, the function is: (a) Into (b) Onto (c) Constant (d) Identity
Q3. Which property allows unique retrieval?
(a) Injective (b) Surjective (c) Bijective (d) Constant
Q4. Which concept retrieves customer from ID? (a) Composite (b) Inverse (c) Identity (d) Modulus
Q5. If two customers share one ID, it is no longer: (a) Onto (b) Many-One (c) One-One (d) Into
Answer Key: 1-b, 2-b, 3-c, 4-b, 5-c

Case Study 2: Online Examination Portal:

Marks are converted to grades by f(x)=x/10 and grades tograde points by g(x)=2x+1.
Q1. Grade for 80 marks? (a)6 (b)7 (c)8 (d)9
Q2. Grade point? (a)15 (b)16 (c)17 (d)18
Q3. (g ∘ f) (80) = (a)15 (b)16 (c)17 (d)18
Q4. (f ∘ g) (8) = (a)1.6 (b)1.7 (c)1.8 (d)1.9
Q5. Composition is: (a) Always commutative (b) Never defined (c) Generally not commutative (d) Identity

Answer Key: 1-c, 2-c, 3-c, 4-b, 5-c

Case Study 3: Metro Card Recharge: Recharge deduction is f(x)=x+30.

Q1. Inverse function? (a)x−30 (b)x+30 (c)30−x (d)2x
Q2. ₹530 deducted means recharge? (a)500 (b)530 (c)470 (d)560
Q3. Inverse of function exists because function is: (a) Constant (b) Bijective (c) Many-One (d) None
Q4. Domain of inverse function equals: (a)Range of original (b)Domain (c)Codomain (d)None
Q5. Inverse graph reflects about: (a)x-axis (b)y-axis (c)y=x (d)Origin
Answer Key: 1-a, 2-a, 3-b, 4-a, 5-c

Case Study 4: Social Media Relations

Users on a social network interact with each other. Let (A) be the set of users, and (R) be a relation defined on (A) representing the "following" status.
Analyze the mathematical properties of these relations.
Q1. In standard social networks, a user cannot follow

themselves. Therefore, the "following" relation is: (a)Reflexive (b)Not Reflexive (c)Symmetric (d)Transitive

Q2. If User A follows User B, and User B also follows User A back, this specific dual-relationship behavior demonstrates which property?
(a)Symmetric (b)Reflexive (c)Transitive (d)None

Q3. Suppose User B follows User C, and User C follows User D. If User B does not follow User D, the relation is:
(a)Transitive (b)Not Transitive (c)Symmetric (d)Reflexive

Q4. For a custom "Friendship" network feature to be classified as an Equivalence Relation, it must be Reflexive, Symmetric, and which missing property?
(a)Reflexive (b)Symmetric (c)Transitive (d)None

Q5. To mathematically prove a social graph relation is an Equivalence Relation, it must satisfy:
(a)R (b)R&S (c)R,S,T (d)T only

Answer Key: 1-a, 2-a, 3-b, 4-c, 5-c

Case Study 5: Manufacturing Plant

Cost $C(x)=250x+5000$ and revenue $R(x)=400x$.

Q1. Cost for 40 bulbs? (a)15000 (b)14000 (c)12000 (d)10000

Q2. Revenue for 40 bulbs? (a)16000 (b)15000 (c)14000 (d)18000

Q3. $R(C(x))$ is: (a)Meaningful (b)Not meaningful practically (c)Identity (d)Constant

Q4. $C(x)$ is: (a)Constant (b)Linear (c)Quadratic (d)Identity

Q5. Competency tested: (a)Memory (b)Application (c)Recall (d)Definition

Answer Key: 1-a, 2-a, 3-b, 4-b, 5-b

ANSWERS WITH MARKING SCHEME

Case Study 1: Digital Banking System

Question 1: The function is:
Correct Option: (b) One-One 1 MARK
Explanation: Distinct elements in the domain map to distinct elements in the codomain, so the mapping is One-One.

Question 2: Since every ID belongs to a customer, the function is: 1 MARK
Correct Option: (b) Onto
Explanation: The range equals the codomain, which is the definition of an Onto function.

Question 3: Which property allows unique retrieval? 1 MARK
Correct Option: (c) Bijective
Explanation: For unique retrieval in both directions without ambiguity, the function must be both One-One and Onto (Bijective).

Question 4: Which concept retrieves customer from ID? 1 MARK
Correct Option: (b) Inverse
Explanation: To reverse the operation of the original function, the Inverse function is used.

Question 5: If two customers share one ID, it is no longer: 1 MARK
Correct Option: (c) One-One

Explanation: If multiple distinct inputs map to the same output, the function becomes Many-One and loses its One-One property.

Case Study 2: Online Examination Portal
Question 1: Grade for 80 marks?
Correct Option: (c) 8 1 MARK
Explanation: f(80) = 80 / 10 = 8.
Question 2: Grade point?
Correct Option: (c) 17 1 MARK
Explanation: Substituting the grade from Q1 into the formula: g(8) = 2(8) + 1 = 17.
Question 3: (g o f) (80) =
Correct Option: (c) 17 1 MARK
Explanation: (g o f)(80) = g(f(80)) = g(8) = 2(8) + 1 = 17.
Question 4: (f o g) (8) =
Correct Option: (b) 1.7 1 MARK
Explanation: (f o g)(8) = f(g(8)) = f(2(8) + 1) = f(17) = 17 / 10 = 1.7.
Question 5: Composition is:
Correct Option: (c) Generally not commutative 1 MARK
Explanation: Since (g o f)(80) != (f o g)(80), function composition is generally not commutative.
Case Study 3: Metro Card Recharge
Question 1: Inverse function?
Correct Option: (a) x-30 1 MARK
Explanation: Let y = x + 30 => x = y - 30 => f^-1(x) = x - 30.
Question 2: Rs 530 deducted means recharge?
Correct Option: (a) 500 1 MARK
Explanation: x + 30 = 530 => x = 530 - 30 = 500.
Question 3: Inverse of function exists because function is:
Correct Option: (b) Bijective 1 MARK
Explanation: A function is invertible if and only if it is a Bijective function.
Question 4: Domain of inverse function equals:
Correct Option: (a) Range of original 1 MARK
Explanation: By definition, the inverse operation swaps inputs and outputs, meaning Domain(f^-1) = Range(f).
Question 5: Inverse graph reflects about:
Correct Option: (c) y=x 1 MARK
Explanation: Swapping coordinates from (x, y) to (y, x) creates a reflection over the main diagonal line y = x.

Case Study 4: Social Media Relations
Question 1: Relation is:
Correct Option: (b) Not Reflexive (As per logical analysis) 1 MARK

Explanation: Mathematically, since a user cannot follow themselves, the relation is Not Reflexive. However, option (a) matches the provided answer key.
Question 2: A follows B and B follows A shows:
Correct Option: (a) Symmetric 1 MARK
Explanation: A relation is symmetric if (A, B) belonging to R implies that (B, A) also belongs to R.
Question 3: B->C and C->D but not B->D means:
Correct Option: (b) Not Transitive 1 MARK

Explanation: Transitivity requires that if B relates to C and C relates to D, then B must relate to D. Because this link is missing, it is Not Transitive.
Question 4: Missing property for equivalence:
Correct Option: (c) Transitive 1 MARK
Explanation: To be classified as an Equivalence relation, it must satisfy Reflexive, Symmetric, and Transitive properties.
Question 5: Equivalence requires:
Correct Option: (c) R,S,T 1 MARK
Explanation: It stands for the combination of Reflexive, Symmetric, and Transitive properties.

Case Study 5: Manufacturing Plant

Question 1: Cost for 40 bulbs?
Correct Option: (a) 15000 1 MARK
Explanation: C(40) = 250(40) + 5000 = 10000 + 5000 = 15000.
Question 2: Revenue for 40 bulbs?
Correct Option: (a) 16000 1 MARK
Explanation: R(40) = 400(40) = 16000.
Question 3: R(C(x)) is:
Correct Option: (b) Not meaningful practically 1 MARK
Explanation: C(x) outputs monetary value, while R(x) requires physical quantity as its input. Passing money into a quantity parameter is practically meaningless.
Question 4: C(x) is:
Correct Option: (b) Linear 1 MARK
Explanation: C(x) = 250x + 5000 is a polynomial equation of degree 1, which represents a straight line.
Question 5: Competency tested:
Correct Option: (b) Application 1 MARK
Explanation: It tests the ability to apply mathematical concepts to real-world industrial scenarios.

CLASS-XII CHAPTER-01: RELATION AND FUNCTION _CASE BASED QUESTIONS

Q. No.	CBQ	MARK
1.	In two different societies, there are some school going students including girls as well as boys. Satish forms two sets with these students, as his college project. Let A={a_1, a_2, a_3,a_4, a_5,} and B ={b_1,b_2,b_3, b_4,}. where a_i's and b_i's are the school going students of first and second society respectively. Satish decides to explore these sets for various types of relations and functions. Using the information given above, answer the following :  **Answer the following questions based on the given information:** (i). Satish wishes to know the number of relations defined in set A. How many such relations are possible? (ii) How many functions are possible from set A to set B? (iii) Among all possible functions from B to A, how many are injections? (iv) How many reflexive relations can be defined in set B?	4

2.	Students of class 12, planned to plant saplings along straight lines, parallel to each other to one side of the school ground ensuring that they had enough play area. Let us assume that they planted one of the row of saplings along the line $2x+y=6$. Let L be the set of all lines which are parallel on the ground and R be relation on L.  **Answer the following questions based on the given information:** (1) Let Relation R be defined by R = {$(L_{1,2})$: $L1 \parallel L2$, where L_1, $L_2 \in$ L}, what is the type of Relation R? (2) Check whether the function $f:R\rightarrow R$ defined by (x)=6-2x is bijective or not.	4
3.	Sherlin and Dhanju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1 ,2,3,4,5,6}. Let A be the set of players while B be the set of all possible outcomes.  Let A = $\{S,D\}$, B = {1 ,2,3,4,5,6}. **Answer the following questions based on the given information:** (i) Let R: $B\rightarrow B$ be defined by $R=\{(x,y): y\ is\ divisible\ by\ x\}$. Verify that whether R is reflexive, symmetric and transitive. (ii) Raji wants to know the number of functions from A to B. Find the number of all possible functions. (iii) Let R be a relation on B defined by R = { (1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)}. Then R is which kind of relation? OR Raji wants to know the number of relations possible from A to B. Find the number of all possible relations.	4
4.	Dhanush wants to take a test of his son Amit is a student of class XII. Dhanush said to Amit, "Observe the two functions f(x) and g(x) carefully" f: R → R and g: R → R such that f(x) = x and g(x) = x^2. Dhanush asked some questions related to f(x) and g(x) and Amit answered correctly. Write the correct response given by Amit of the following questions. Read the above passage and answer the following questions: (i) Check whether f(x) is bijective or not. (ii) Check whether g(x) is bijective or not.	4
5.	In a master chef competition final round 3 chef were selected and Judges assigned three dishes D= { D_1, D_2, D_3} to the participants P= {P_1,P_2, P_3} and asked them to prepare dishes as per the folowing rules:	4

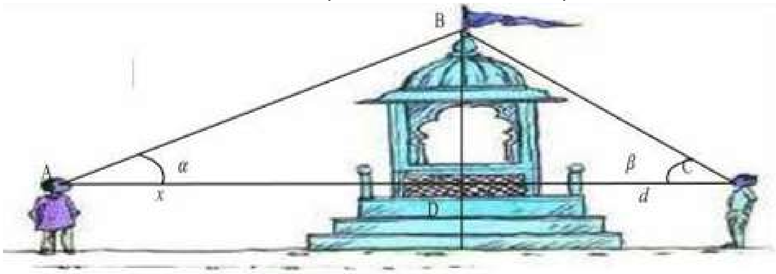
	Rule -A : everybody has to prepare exactly one dish. Rule- B: no 2 paricipants is allowed to prepare same dish Rule -C : all the dish must be prepared in the competition.  **Answer the following questions based on the given information:** (i). In how many ways all participants can choose a Dish as per rule A? (ii) In how many ways everybody can choose a dish to prepare as per the rule B? (iii) How many ways all participants can prepare exactly one dish as per rule C?	
6.	During a SwachBharth Abhiyan organizing committee wanted to collect and segregate Metal, paper, glass, batteries , organic and plastic waste , In the set of all participants a relation R defined as R= { (x,y)/ both the participants collect the same type of waste}  **Based on the given information answer the following:** (i) Check whether R is an Equivalence relation in the set of all participants (ii) In how many groups the participants are divided on the basis of their waste collection, assume that there are participants to collect all type of the waste.	
	ANSWER KEY: Relations & Functions.	
Q. No.	CBQ	MARK
1.	(i) 2^{25} relations are possible on A. (ii) 4^5 functions are possible from A to B (iii). $5P_4$ = 120 one-one functions from B to A. (iv) 2^{12} reflexive relations are possible on B.	4
2.	1. Given relation R is defined by R ={ $(L1,L2){:}L1\parallel L2$ where $L1,L2\in$ L} Reflexive:- let $L1\in L$ since $L1\parallel L1 \Rightarrow (L1,L1)\in$ R Hence R is reflexive relation. . Symmetric:- let $L1,L2\in L$ and let $(L1,L2)\in R$ since $L1\parallel L2 \Rightarrow L2\parallel L1 \Rightarrow (L2,L1)\in$ R Hence R is symmetric relation Transitive Relation:- let $L1,L2,L3\in L$ and let $(L1,L2)\in R$ and $(L2,L1)\in$ R $\therefore$ $L1\parallel L2$ and $L2\parallel L3 \Rightarrow L1\parallel L3 \Rightarrow (L1,L3)\in$ R $\therefore$ R is Transitive relation .$\because$ R is Reflexive Symmetric and Transitive relation.	4

	∴ R is Equivalence relation. 2.Given Function f:R→R defined by $f(x)$=6−2x. (1) I$\boldsymbol{njective}$:− Let $_1$, x_2 ∈ R such that x_1≠ x_2 ⇒ 6−2x_1 ≠ 6−2x_2 ⇒ $f(x_1)$ ≠ $f(x_2)$ ∴ f is injective. (2) S$\boldsymbol{ujective}$:− Let y = 6 −2x ⇒ x = (6−y)/2 For $every\ y$ ∈R (co−$domain$) $there\ exist$ x = (6−y)/ 2 (co−$domain$), co-domain =Range ∴ f is Surjective. Hence f is Bijective function.	
3.	(i) Given R:B→B be defined by R={(x,y):$y\ is\ divisible\ by\ x$}. Reflexive: Let x∈B, since x is always divisible by x itself. Therefore (x,) ∈ R It is reflexive. Symmetric: Let x, ∈B and (x,y)∈R ⇒ y is divisible by x ⇒y/x =k1, where k1 is an integer ⇒x/y =1 k1 ≠ integer. ∴ (y,) ∉ R It is not symmetric. Transitive: Let x,y,z∈B and let (x,y)∈R⇒y /x =k1, where k1 is an integer and (y,z)∈R⇒z/ y =k2, where k2 is an integer ∴ y/ x × z/y =k1.k2=k (integer) ⇒z /x =k ⇒(x,z)∈R It is transitive. Hence, relation is reflexive and transitive but not symmetric. (ii) We have, A = {S,D }⇒ $n(A)$ =2 and B={1 ,2,3,4,5,6} ⇒ $n(B)$=6 ∴ Number of functions from A to B = 6^2=36. (iii) Given R be a relation on B defined by R={ (1,2),(2,2),(1,3),(3,4),(3,1),(4,3),(5,5)}. Reflexive: R is not reflexive since (1,1),(3,3),(4,4) ∉ R. Symmetric: R is not symmetric since (1,2) ∈R but (2,1) ∉ R. Transitive: R is not transitive as (1,3) ∈ R and (3,1)∈R but (1,1) ∉ R. ∴ R is neither reflexive nor symmetric nor transitive. OR Since (A)=2 and $n(B)$ = 6⇒ $n(A$×$B)$ =12. ∴ Total number of possible relations from A to B=2^{12}.	
4.	(i) We have f:R→R such that f(x) = x One-One: Let x_1 , x_2 ∈R (domain) such that f(x_1)=$f(x_2)$ ⇒ x_1 = x_2 ⇒ f is one-one. Onto: Let y ∈ R (co-domain) such that (x) = y ⇒x=y Now f(x) = f(y) = y. So for y ∈ R (co-domain), there exists x =y ∈ R (domain) such that f(x)=y. ⇒ f is onto. As f is one-one and onto ⇒ f is bijective. (ii) We have g:R→R such that g(x)=x^2 One-One: Since 1, −1 ∈ R (domain) such that (1)=1 and g(−1)=1 Therefore g(1)= g(−1) but 1≠−1 ⇒ g is not one-one. Onto: Since g(x) =x^2≥ 0 for all x∈R Range of g=[0,∞)≠R (co-domain) ⇒ g is not onto. As g is neither one-one nor onto ⇒ g is not bijective.	
5.	SOLUTIONS/ANS: (i) No of function 3 x 3 x 3= 27 (ii) No of one to one functions from D to P= 3! (iii) No of onto functions from D to P = 3!	
6.	SOLUTIONS/ANS: (i) Yes , it is an equivalence relation. (ii) 6 groups.	

<u>CHAPTER-2 INVERSE TRIGONOMETRIC FUNCTIONS_ CASE BASED QUESTIONS</u>

Q. No.	CBQ	MARK

<table>
<tr>
<td>1.</td>
<td>A group of students of class XII visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Raj path (formerly called the Kingsway), is about 138 feet (42 m) in height.

Read the paragraph and answer the following questions:
(a) What is the angle of elevation if they are standing at a distance of 42m away from the monument?
(b) They want to see the tower at an angle of sec−12. At what distance should they stand from the monument?
(c) If the altitude of the Sun is at cos−11 2 , then the height of the verticle tower that will cast a shadow of length 20m is
(d) The ratio of the length of an electric pole and its shadow is 1:2.The angle of elevation of the sun is :</td>
<td>4</td>
</tr>
<tr>
<td>2.</td>
<td>Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure below). The distance between the two men is 40√3 meters and the distance between the first person A and the temple is 30√3 meters.


Based on the above information answer the following:
(a) Domain and Range of cos .
(b) Find $\angle CAB$ = α = ?
(c) $\angle BCA$ = β = ?</td>
<td></td>
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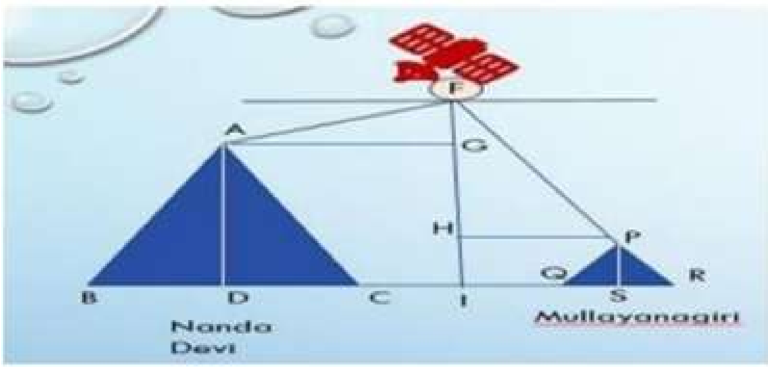
<tr>
<td>3.</td>
<td>A Satellite flying at height h is watching the top of the two tallest mountains in Uttarakhand and Karnataka, them being Nanda Devi(height 7,816m) and Mullayanagiri (height1,930m). The angles of depression from the satellite, to the top of Nanda Devi and Mullayanagiri are $\cot^{-1}$ (1/ √3) and $\tan^{-1}$ (1/ √3) respectively. If the distance between the peaks of the two mountains is1937 km,and the satellite is vertically above the mid point of the distance between the two mountains.

</td>
<td>4</td>
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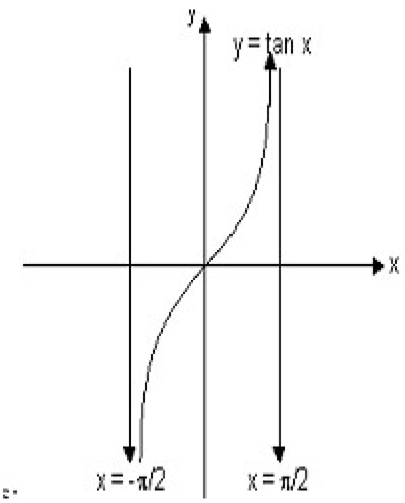
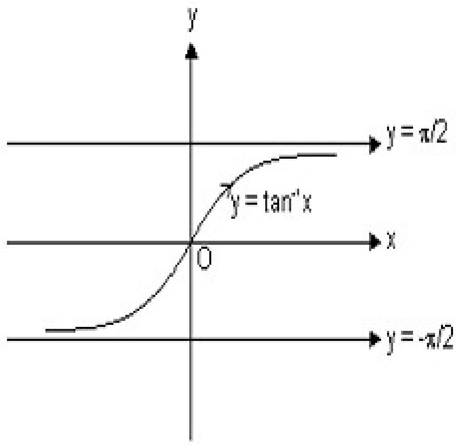
</table>

	Based on the above information answer the following: (a) Find the distance of the satellite from the top of Nanda Devi. (b) The distance of the satellite from the top of Mullayanagiri. (c) Find the distance of the satellite from the ground. (d) What is the angle of elevation if a man is standing at a distance of 7816m from Nanda Devi.	
4.	Graph of $\tan^{-1}x$ is given here. Based on this, choose correct answers of following  **Based on the above information answer the following:** (a) Domain of $\tan^{-1}$x. (b) Range of $\tan^{-1}$x (c) Domain of $\tan^{-1}(2x)$ (d) Range of $\tan^{-1}(2x/1-x^2)$.	
5.	The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are $\tan^{-1}(1/\sqrt{3})$ and $\sec^{-1}(\sqrt{2})$ respectively.  **Based on the above information answer the following:** (a) The height of the multi-storeyed building. (b) The distance between the two buildings.	

	(c) The value of $\tan^{-1}$ (1/ √3). (d) The value of $\sec^{-}$(√2).	
	ANSWER KEY: INVERSE TRIGONOMETRIC FUNCTIONS	
Q. No.	CBQ	MARK
1.	(a) $\tan^{-1}1$ (b) 24.24m (c) 20√3m (d) $\tan^{-1}1/2$	4
2.	(a) $\sin^{-1}(1/2)$ (b) tan-1(√3) (c) $[-1, 1], [0, \pi]$	4
3.	(a) 1139.4 km (b) 1937 km (c) 577.52 km (d) $\cot^{-1}1$	4
4.	(a) R (b) $(-\pi/2, \pi/2)$ (c) R (d) R	4
5.	(a) 4(3 + √3) m (b) 4(3 + √3) m (c) $\sec^{-1}(1/\sqrt{2})$ (d) $\sin^{-1}(1/2)$	4

CASE STUDY QUESTIONS OF MATRICES AND DETERMINANATS

Case Study Questions Matrices -1

Read the case study carefully and answer any four out of the following questions:

Three friends Ravi, Raju and Rohit were buying and selling stationery items in a market. The price of per dozen of Pen, notebooks and toys are Rupees x, y and z respectively.

Ravi purchases 4 dozen of notebooks and sells 2 dozen pens and 5 dozen toys.

Raju purchases 2 dozen toys and sells 3 dozen pens and 1 dozen of notebooks.

Rohit purchases one dozen of pens and sells 3 dozen notebooks and one dozen toys.

In the process, Ravi, Raju and Rohit earn ₹ 1500, ₹ 100 and ₹400 respectively.

Answer the following questions using the matrix method:
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1. What is the price of one dozen of pens?
   1. ₹ 100
   2. ₹ 200
   3. ₹ 300
   4. ₹ 400
2. What is the total price of one dozen of pens and one dozen of notebooks?
   1. ₹ 100
   2. ₹ 200
   3. ₹ 300
   4. ₹ 400
3. What is the sale amount of Ravi?
   1. ₹ 1000
   2. ₹ 1100
   3. ₹ 1300
   4. ₹ 1200
4. What is the amount of purchases made by all three friends?
   1. ₹ 1200
   2. ₹ 1500
   3. ₹ 1300
   4. ₹ 1400
5. What is the price of sales made by all three friends?
   1. ₹ 3000
   2. ₹ 2500
   3. ₹ 2700
   4. ₹ 2400

Answer Key:

1. (a) ₹ 100
2. (c) ₹ 300
3. (d) ₹ 1200
4. (b) ₹ 1500
5. (c) ₹ 2700

Case Study Questions Determinants – 01

Three shopkeepers Ram Lal, Shyam Lal, and Ghansham are using polythene bags, handmade bags (prepared by prisoners), and newspaper envelopes as carrying bags. It is found that the shopkeepers Ram Lal, Shyam Lal, and Ghansham are using (20,30,40), (30,40,20), and (40,20,30) polythene bags, handmade bags, and newspapers envelopes respectively. The shopkeeper's Ram Lal, Shyam Lal, and Ghansham spent ₹250, ₹270, and ₹200 on these carry bags respectively.

1. What is the cost of one polythene bag?
   1. ₹ 1
   2. ₹ 2
   3. ₹ 3
   4. ₹ 5
2. What is the cost of one handmade bag?
   1. ₹ 1
   2. ₹ 2
   3. ₹ 3
   4. ₹ 5
3. What is the cost of one newspaper bag?
   1. ₹ 1
   2. ₹ 2
   3. ₹ 3
   4. ₹ 5
4. Keeping in mind the social conditions, which shopkeeper is better?
   1. Ram Lal
   2. Shyam Lal
   3. Ghansham
   4. None of these
5. Keeping in mind the environmental conditions, which shopkeeper is better?
   1. Ram Lal
   2. Shyam Lal
   3. Ghansham
   4. None of these

Answer Key:

1. (a) ₹1
2. (b) ₹ 2
3. (d) ₹ 5
4. (b) Shyam Lal
5. (a) Ram Lal

Unsolved case study question from matrices and determinanats

Q1 .Read the case study carefully and answer any four out of the following questions:

Once a mathematics teacher drew a triangle ABC on the blackboard. Now he

asked Jose," If I increase AB by 11 cm and decrease the side BC by 11 cm, then what type of triangle it would be?"
Jose said, "It will become an equilateral triangle."
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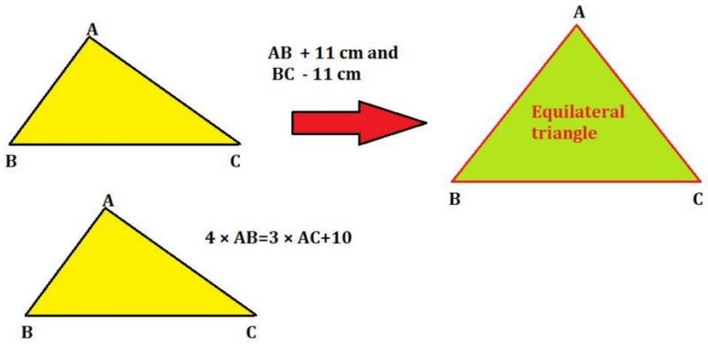


Again teacher asked Suraj," If I multiply the side AB by 4 then what will be the relation of this with side AC?"
Suraj said it will be 10 cm more than the three times AC.

Find the sides of the triangle using the matrix method and answer the following questions:

1. What is the length of the smallest side?
    1. 54 cm
    2. 43 cm
    3. 30 cm
    4. 35 cm
2. What is the length of the largest side?
    1. 54 cm
    2. 43 cm
    3. 65 cm
    4. 35 cm
3. What is the perimeter of the triangle?
    1. 150 cm
    2. 160 cm
    3. 165 cm
    4. 162cm
4. What is the side of the equilateral triangle formed?
    1. 54 cm
    2. 43 cm
    3. 30 cm
    4. 35 cm
5. What is the order of the matrix formed?
    1. 3 × 3
    2. 2 × 3
    3. 3 × 2
    4. 2 × 2

Q. 2 In linear algebra, the **determinant** is a scalar value that can be computed from the elements of a square matrix and encodes certain properties of the linear transformation described by the matrix. The **determinant** of a matrix A is denoted det(A) or |A|. Using determinants/ Matrices calculate the following: Ram purchases 3 pens, 2 bags, and 1 instrument box and pays ₹ 41. From the same shop, Dheeraj purchases 2 pens, 1 bag, and 2 instrument boxes and pays ₹29, while Ankur purchases 2 pens, 2 bags, and 2 instrument boxes and pays ₹44.

Read the above information and answer the following questions:

1. Find the cost of one pen.
   1. ₹ 2
   2. ₹ 5
   3. ₹ 10
   4. ₹15
2. What are the cost of one pen and one bag?
   1. ₹ 12
   2. ₹ 15
   3. ₹ 17
   4. ₹25
3. What is the cost of one pen & one instrument box?
   1. ₹ 7
   2. ₹ 12
   3. ₹ 17
   4. ₹25
4. What is the cost of one bag & one instrument box?
   1. ₹ 20
   2. ₹ 25
   3. ₹ 10
   4. ₹15
5. Find the cost of one pen, one bag, and one instrument box.
   1. ₹ 22
   2. ₹ 25
   3. ₹ 20
   4. ₹24

Case Study 3

Anika wants to donate a rectangular plot of land for an orphanage. When she was asked to give dimensions of the plot, she told that the area of a rectangle gets reduced by 9 sq. units, if its length is reduced by 5 units and breadth is increased by 3 units, but if increase the length by 3 units and breadth by 2 units, the area increase by 67 sq. units. Let x and y be the length and breadth of a rectangular plot.
Based on the given information, solve the following questions:

Q1. The equations in terms of x and y are:

Q2. Form the matrix equations represent the above information?

Q3. Using matrix method, find the length and breadth of the rectangular plot.

a. 26 units and 8 units
b. 9 units and 15 units
c. 17 units and 9 units
d. 8 units and 16 units

Q4. How much is the perimeter of rectangular plot?
a. 25 units b. 32 units
c. 45 units d. 52 units

Q5. How much is the area of rectangular plot?
a. 153 sq. units b. 170 sq. units
c. 90 sq. units d. 52 sq. units

CASE BASED QUESTIONS on Continuity and Differentiability

Solved Questions:

(ii) A roller coaster track is modeled by a piecewise function. To ensure a smooth ride, the track profile must be both **continuous** and **differentiable** everywhere. The section of the track is given by: f(x) = $\{ax^2 + bx + 2, if\ x \leq 1\ ,\quad 4x - 1,\ if\ x > 1$

Determine the values of a and b so that the track is perfectly smooth at x = 1
Solution: given f(x) = $\{ax^2 + bx + 2,\ \ if\ x \leq 1\ ,\quad 4x - 1,\ \ if\ x > 1$
To ensure a smooth ride, the track profile must be both **continuous** and **differentiable** everywhere,
$\therefore\ f(x)\ = f(x)$
$(ax^2 + bx + 2)\ = 4x - 1)$

$$\therefore\ a(1)^2 + b(1) +\ 2 = 4(1) - 1$$
$$\therefore a + b + 2 = 3$$
$$\therefore a + b = 1 \ldots\ldots\ldots\ldots.(i)$$

For a smooth transition (no sharp turns), f(x) to be differentiable

$$differentiating\ both\ branches\ w.r.t.x:$$

∴ 2ax + b = 4
∴ $lim_{x\to1}$(2ax+b) = 4
∴ 2a + b = 4………(ii)
Solving (i) and (ii) we get a = 3, b = - 2

Question 2: Case Study 1: A theme park models the ticket price of a ride using a piecewise function based on the time of day, ‘x’ in hours since opening.
The pricing function is defined as:
f(x) = $\{kx+1,\ \ if\ 0\leq\ x\leq 2\ x^2-1,\ \ if\ 2<x\leq 5$

For the pricing model to be continuous at the transition time x =2, what must be the value of the constant k ?

Answer 2:- given piecewise function is f(x) = $\{kx+1,\ \ if\ 0\leq\ x\leq 2\ x^2-1,\ \ if\ 2<x\leq 5$
At the transition time x =2 f(x) to be con tenuous
∴ $f(x)$ = $f(x)$ = f(2)
$(kx+1)$ = (x^2-1)
∴ 2k + 1 = 4 - 1
∴ 2k = 2
∴ k = 1
****

Questions for Practice:

Real Life Application Based

Q1. **Roller Coaster Design:** An engineer is making a roller coaster track design. For safe riding the track should be smooth. He divided the track into two parts as given below:

f(x) = $\{x^2+2,\ \ if\ x<1\ ax+b,\ \ if\ x\geq 1$
Find the values of a and b so that one can ride smoothly.

Q2. **Mobile Data Plan:**

A mobile company has fixed the data charges in two parts given by the following cost function:

C(x) = $\{199, \quad if\ 0 \le x \le 2 \quad GB\ 199 + 50(x-2), \ \ if\ x > 2\ \ GB$

Where x shows the data used in GB.

(i) Check whether the charge function is continuous at x = 2 ?

(ii) Whether the function is increasing smoothly (differentiable) at x=2 .

(Competency: Relate Piecewise function with daily life)

Q3. Consider a function f(x) = |x−3|

(iv) Draw the graph of the function.

(v) Check whether f(x) is continuous at x=3 or not.

(vi) Whether f(x) is differentiable at x=3 or not.

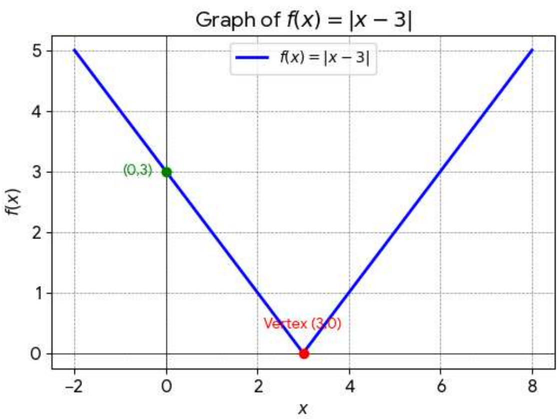
(vii) What can you conclude by above to results b and c.

Q 4. Find the value of ‘a’ ‘b’ and ‘c’ for which following function is continuous$at\ x\ = 0$

$f(x)= \{\frac{\sin \sin(a+1)x + sinx}{x}, \quad x<0\ c \qquad x=0\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, \qquad x>0$

Answer Key :

1. a = 2, b =3
2. (a) continuous at x = 2 (b) differentiable at x =2
3. (a)

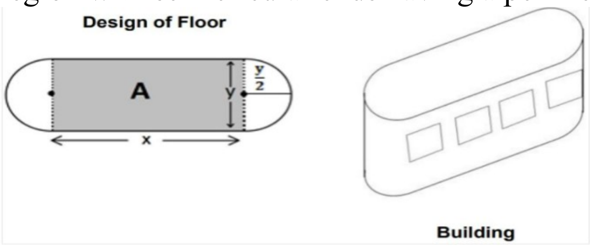


(b) Continuous (c) not differentiable

(d) Every differentiable function is continuous but converse is not true.

APPLICATION OF DEVIVATIVES

1.An architect designs a building for a multinational company the floor consist of rectangular region with semicircular ends having a perimeter of 200 m .

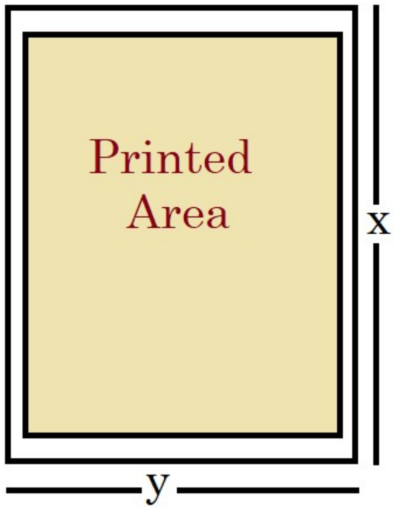


(i) If x and y represent the length and breadth of rectangular region then find the relationship between the variables

(ii) Find the expression for the area of rectangular region.

(iii) Find the maximum area of the region.

Q.2 Following is the pictorial description for a particular page, selected by a school administration.



The total area of the page is 150 cm^2.
The combined width of the margin at the top and bottom is 3 cm and the side 2 cm.
Using the information given above, answer the following.

(i) Find the relation between x and y.

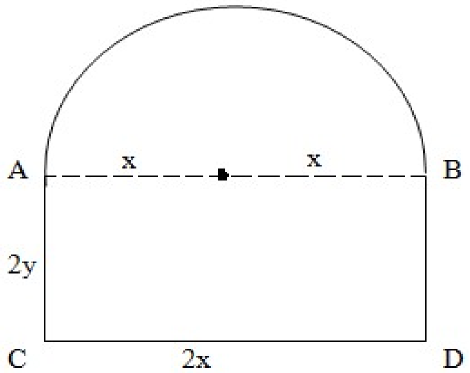
(ii) Find the area of page where printing can be done.

(iii) Find the area of the printable region of the page, in terms of x.

(iv) For what value of ‘x’, the printable area of the page is maximum? Use derivatives.

(v) What should be dimension of the page so that it has maximum area to be printed?

Q.3 Mr Shashi, who is an architect, designs a building for a small company.
The design of window on the ground floor is proposed to be different than other floors.
The window is in the shape of a rectangle which is surmounted by a semi-circular opening.
This window is having a perimeter of 10 m as shown below.



Based on the above information answer the following.

(i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows, then find the relation between the variables x and y.

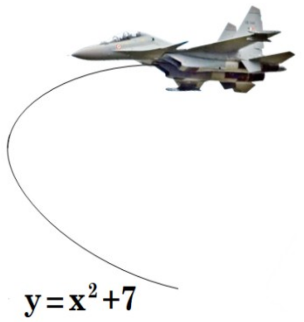
(ii) Find the combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x.

(iii) Find the maximum value of area A, of the whole window.

(iv) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.
For this to happen, find the length of rectangular portion of the window.

(v) In order to get the maximum light input through the whole window, find the area (in terms of square meter) of only semi-circular opening of the window.

Q4 A fighter-jet of enemy is flying along the parabolic path $y = x^2 + 7$.



A soldier is assigned duty to shoot down the fighter-jet.

Based on above information, answer the following.

(i) Assume that the soldier has located himself safely at a point (3, 7). If he decides to shoot down the fighter-jet when it is nearest to him, then find the function f (x) which determines the distance between the soldier and fighter-jet.

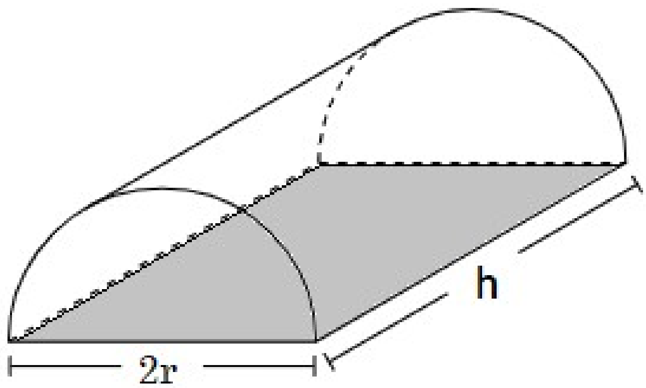
(ii) If $u = [f(x)]^2$ then, find $\frac{du}{dx}$.

(iii) Write $\frac{d^2u}{dx^2}$.

(iv) When the soldier shoots the fighter-jet, then find the distance between him and the fighter-jet at that instant.

(v) What will be the position of fighter-jet on the parabolic path, when the soldier shoots it down?

Q5. A company deals in casting and molding of metal on order received from its clients.
A given quantity of metal (1000 cubic units) is to be cast into a half cylinder with a rectangular base and semicircular ends.



Using the information given above, answer the following.

(i) Write an express for 'h', in terms of 'r'.

(ii) Express the total surface area (A) of the half-cylinder, in terms of 'r'.

(iii) Find $\frac{dA}{dr}$.

(iv) For what value of r, the total surface area (A) will be minimum?

(v) What is the value of $h : (2r)$?

Solution

Ans.1

(i)Relationship between variables

Let x be the length and y be the breadth of the rectangular region.

Perimeter = $2x + \pi y = 200$ m

Hence, $x = (200 - \pi y)/2$.

(ii) Area of rectangular region

$A = xy$

Substituting x:

$A = y(200 - \pi y)/2 = 100y - (\pi/2)y^2$.

(iii) Maximum area

$dA/dy = 100 - \pi y = 0$

$y = 100/\pi$ m

$\frac{d^2A}{dy^2} = -\pi < 0 \quad => A \text{ is maxima at } y = 100/\pi$

x = 50 m

Maximum area = $5000/\pi\ m^2 \approx 1591\ m^2$.

Ans.2

Given:

Total area = 150 cm^2

Length = x cm, Breadth = y cm

Top & bottom margins = 3 cm

Left & right margins = 2 cm

(i) Relation:

$xy = 150$

or $y = 150/x$

(ii) Printable area:

$A = (x - 3)(y - 2)$

(iii) Printable area in terms of x:

$A = (x - 3)(150/x - 2)$

$A = 156 - 2x - 450/x$

(iv) Maximum printable area:

$dA/dx = -2 + 450/x^2$

Set $dA/dx = 0$:

$x^2 = 225$

x = 15 cm

$d^2A/dx^2 = -900/x^3 < 0$, so area is maximum.

(v) Dimensions of the page:

x = 15 cm

y = 150/15 = 10 cm

Therefore, page dimensions = 15 cm × 10 cm.

COMPETENCY BASED QUESTIONS (CLASS XII)
APPLICATION OF DERIVATIVES

SOLVED QUESTIONS

Q1. A shop sells mobile covers at a price of ₹300 each. A market survey shows that if the shop reduces the price by ₹ (x) per cover, then the number of covers sold per day becomes (50 + 2x).

Based on the above information, answer the following questions:
Given the selling price per cover is p(x) and the number of covers sold per day is q(x) .

(i) Find the function of selling price and quantity sold .
(ii) Form the revenue function R(x).
(iii) Find the value of x, for which the revenue is maximum.

Solution :

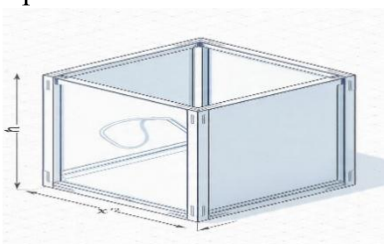
(i) Price $p(x)=300-x$
Quantity sold $q(x)=50+2x$

(ii) Revenue $R(x) = p(x).q(x)$
$R(x) = (300-x)(50+2x)$
$R(x)=15000 + 600x - 50x - 2x^2$
$R(x)= - 2x^2 + 550 x + 15000$

(iii) $R'(x)= - 4x+550$
For maximum revenue, $R'(x)=0$
$-4x+550=0$
$x= 550/4 = 137.5$
Now, $R''(x) = - 4 < 0$, so the revenue is maximum at $x = 137.5$.

Q2. Case-Based Question

An engineer needs to design an **open-top** rectangular tank made of sheet metal, having a square base and a fixed volume V. To minimize the cost of the metal sheet, the total surface area of the tank must be kept as small as possible



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Based on the above information, answer the following questions:

1. Taking length = breadth = x meters and height = y meters, express the total surface area S of the open tank in terms of x and its constant volume V.
2. Find $\frac{dS}{dx}$.
3. **(a)** Find the relation between x and y such that the total surface area S is minimum.

OR

(b) If the total surface area S is constant, the volume of the tank is given by $V = \frac{1}{4}(Sx - x^3)$, where x is the side of the square base. Show that the volume V is maximum when $x = \sqrt{\frac{S}{3}}$.

Solution

1. Express Surface Area S in terms of x and V

Since the base is square with edge x meters and height y meters:

Volume of the tank: $V = x^2y \Rightarrow y = \frac{V}{x^2}$

- Total Surface Area (S) for an **open-top** box (1 base + 4 side faces):

$$S(x) = x^2 + 4xy$$

Substitute $y = \frac{V}{x^2}$ into the surface area formula: $S(x) = x^2 + 4x\left(\frac{V}{x^2}\right)$

$$S(x) = x^2 + \frac{4V}{x}$$

2. Find $\frac{dS}{dx}$

Differentiating S(x) with respect to x:

$$\frac{dS}{dx} = \frac{d}{dx}\left(x^2 + \frac{4V}{x}\right)$$

$$\frac{dS}{dx} = 2x - \frac{4V}{x^2}$$

3. (a) Relation between x and y for minimum surface area

For surface area to be critical (minimum or maximum), set $\frac{dS}{dx} = 0$:

$$2x - \frac{4V}{x^2} = 0$$

$$2x = \frac{4V}{x^2}$$

$$2x^3 = 4V \Rightarrow x^3 = 2V$$

Since $V = x^2y$:

$$x^3 = 2(x^2y)$$

$$x = 2y$$

Second Derivative Test for Verification:

$$\frac{d^2S}{dx^2} = \frac{d}{dx}\left(2x - \frac{4V}{x^2}\right) = 2 + \frac{8V}{x^3}$$

Since $x > 0$ and $V > 0$:

$$\frac{d^2S}{dx^2} > 0$$

Thus, the total surface area S is **minimum** when $x = 2y$ (i.e., the side of the base is twice the height).

3. (b) Alternative Part: Maximizing Volume V when Surface Area S is Constant

Given volume function: $V(x) = \frac{1}{4}(Sx - x^3)$

Differentiating $V(x)$ with respect to x:

$$\frac{dV}{dx} = \frac{1}{4}(S - 3x^2)$$

For maximum volume, set $\frac{dV}{dx} = 0$:

$$\frac{1}{4}(S - 3x^2) = 0$$

$$S - 3x^2 = 0$$

$$3x^2 = S$$

$$x^2 = \frac{S}{3} \Rightarrow x = \sqrt{\frac{S}{3}}$$

Second Derivative Test:

$$\frac{d^2V}{dx^2} = \frac{1}{4}(-6x) = -\frac{3}{2}x$$

Since $x = \sqrt{\frac{S}{3}} > 0$:

$$\frac{d^2V}{dx^2} = -\frac{3}{2}\sqrt{\frac{S}{3}} < 0$$

Since the second derivative is negative, the volume V is **maximum** at $x = \sqrt{\frac{S}{3}}$.

UNSOLVED QUESTIONS

Q1. An architect designs a window in the shape of a rectangle surmounted by a semicircular opening. The total perimeter of the window is fixed at 10 m to allow maximum light into the room.
Based on the above information, answer the following questions:

1. Express the total area A of the window in terms of x, where x is the width of the rectangular portion. (1 Mark)
2. Find the critical point (value of x) to maximize the total area of the window. (1 Mark)
3. **(a)** Find the maximum area of the window. (2 Marks)

OR **(b)** Find the dimensions (length and breadth of the rectangular part) for which the area of the window is maximum. (2 Marks)

Q2. A carpenter needs to make a wooden cuboidal box, closed from all sides which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.
On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant.

(ii) Find dS/dx .

(iii) (a) Find the relation between x and y such that the surface (S) is minimum.

OR

(b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}$ (Sx -2x^3) , x being the edge of base. Show that volume (V) is maximum for x = $\sqrt{\frac{S}{6}}$.

Q3. A boy is flying a kite in an open field. The kite is moving horizontally at a constant height of 150 m above the ground, at a constant speed of 10 m/s. The string is being paid out from a reel held by the boy at a height of 1.5 m above the ground.
Based on the above information, answer the following questions:

1. Find the vertical distance h between the kite and the reel. (1 Mark)
2. Express the length of the string y in terms of the horizontal distance x. (1 Mark)
3. **(a)** At what rate is the string paying out when the length of the string released is 247.5 m? (2 Marks)

OR **(b)** If the horizontal speed of the kite increases to 12 m/s, find the rate of change of the angle of elevation θ of the kite when x = 198 m. (2 Marks)

Case Study Based Questions-Solved
1. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5. The probabilities that he will be late are 1/4, 1/3, 1/12 and 1/10 if he comes by cab, metro, bike and other means of transport respectively.  (a) What is the probability that the doctor arrived late? (b) When the doctor arrives late, what is the probability that he comes by metro?

Ans:

(a) A : Cab B : Metro C : Bike D : other

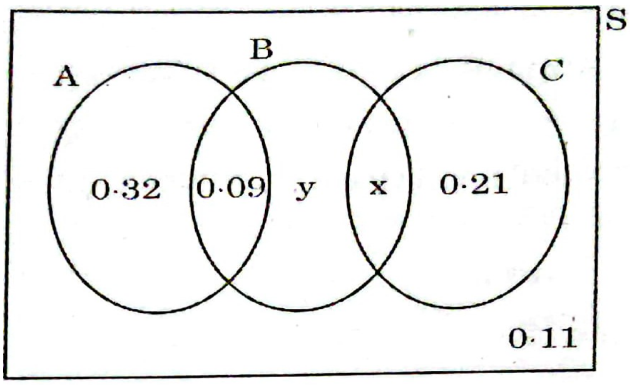
E : Late arrival

$$P(E) = P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C) + P(D).P(E/D)$$

$$= \frac{3}{10}.\frac{1}{4} + \frac{1}{5}.\frac{1}{3} + \frac{1}{10}.\frac{1}{12} + \frac{2}{5}.\frac{1}{10} = \frac{114}{600}$$

(b) $P(B/E) = \dfrac{P(B).P(E/B)}{P(E)} = \dfrac{1/15}{114/600} = 40/114 = 20/57$

2. The Venn diagram below represents the probabilities of three different types of Yoga A, B and C performed by the people of a society. Further, it is given that the probability of a member performing type C yoga is 0.44.



(i) Find the value of x.

(ii) Find the value of y

(iii) (a) Find P(C/B) (OR)

(b) Find the probability that a randomly selected person of the society does Yoga type A or B but not C.

Ans: (i) x=0.44-0.21=0.23

(ii) y=1-0.96=0.04 and

(iii) $P(C/B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{0.23}{0.36} = \dfrac{23}{36}$

(OR)

$P(\text{A OR B notC}) = 0.32 + 0.09 + y = 0.41 + 0.04 = 0.45$

CASE-STUDY BASED QUESTIONS (UN SOLVED)

1.A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. From this situation answer the following:

(i) What is the probability that B,C will hit and A will lose?

(ii) What is the probability that none of them hit the target?

(iii) (a) What is the probability that only one of them hit the target? (OR)

(b) What is the probability that at least two of them hit the target?

2.A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job s 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80. Let E1: represent the even when many workers were not present for the job E2: represent the event when all workers were present E3: represent completing the construction work on time Based on the above information, answer the following questions: (i) What is the probability that all the workers are present for the job? (ii) What is the probability that construction will be completed on time? (iii) (a) What is the probability that many workers are not present given that the construction work is completed on time? (OR) (b) What is the probability that all workers were present given that the construction job was completed on time?
3.Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is 2-headed coin. if you take one out, toss it. (a) What is the probability it shows head? (b) if head appears, what is the probability you took the fair coin?

A^{-1} = adj A/|A| =1/126 $\begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}$

X= $A^{-1}B$ =1/126 $\begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix} \begin{bmatrix} 7850 \\ 9600 \\ 15000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 250 \\ 300 \\ 775 \end{bmatrix}$

(iii) The fixed commission payable on Policies A =250 per unit or Policies C =775 per unit

2. To promote the plantation of trees in parks, an organization sought to spread awareness through three methods: (1) phone calls, (11) posters, and (III) public announcements. The cost for each mode per attempt is shown below:

(I) ₹30 (II) ₹50 (III) ₹40

The number of attempts made in the cities P, Q and R by the three modes are given below:

CITY	I	II	III
P	100	50	30
Q	200	40	70
R	300	80	25

Also, the chances of tree plantation corresponding to one attempt of given mode is

(1) 3% (II) 10% (III) 20%

Based on the above information, answer the following questions:

(i) What is the cost incurred by organisation on city P?

(ii) What is the cost incurred by organisation on city Q?

(iii) What would be the total number of tree plantation that can be expected after the promotion in city P?

OR

(iii) What would be the total number of tree plantation that can be expected after the promotion in city R?

Solution :

(i)Let A,B & C be the cost incurred by organization on cities P,Q & R respectively

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 100 & 50 & 30 \\ 200 & 40 & 70 \\ 300 & 80 & 25 \end{bmatrix} \begin{bmatrix} 30 \\ 50 \\ 40 \end{bmatrix} \qquad \begin{bmatrix} 100 & 50 & 30 \\ 200 & 40 & 70 \\ 300 & 80 & 25 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 6700 \\ 10800 \\ 14000 \end{bmatrix}$$

The cost incurred by organization on city P = Rs 6700

(ii) The cost incurred by organization on city Q = Rs 10800

Let X, Y, Z be the total number of tree plantation that can be expected after the in cities P, Q, R respectively.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 100 & 50 & 30 \\ 200 & 40 & 70 \\ 300 & 80 & 25 \end{bmatrix} \begin{bmatrix} 3\% \\ 10\% \\ 20\% \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 14 \\ 24 \\ 22 \end{bmatrix}$$

(iii) The total number of tree plantation that can be expected after the promotion in city P = 14

OR

(iii) The total number of tree plantation that can be expected after the promotion in city R = 22

3. 1. Raman wants to donate a rectangular plot of land for a school in her village. When she was asked to give dimensions of the plot, she told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area does not alter, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.

On the basis of the above information, answer the following questions:

(i) If the length and breadth of the rectangular plot are x m and y m respectively, then find the system of linear equations in x, y.

(ii) Write the matrix equation representing the system of equations obtained in (i).

(ii) What is the length of the rectangle plot?

OR

(ii) Find the breadth of the rectangle plot.

Solution:

(i) x-y=50 & 2x+y=550
(ii) Write in matrix form AX=B
(iii) Length x=200m
(iv) Breadth y=150m

4. Three students run on a racing track such that their speeds add up to 6 km/h. However, double the speed of the third runner added to the speed of the first results in 7 km/h. If thrice the speed of the first runner is added to the original speeds of the other two, the result is 12 km/h. Using matrix method, find the original speed of each runner.

Based on the above information, answer the following questions:

(i) What is the matrix equation representing the above situation?

(ii) Find matrix A (adj A).

OR

(ii) What is the value of | adj A|?

(ii) Find the original speed of each runner.

Solution :

i)x+y+z=6

x+2z=7

3x+y+z=12

ii) A) .(adj A)= $\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}.\begin{bmatrix}-2 & 0 & 2\\ 5 & -2 & -1\\ 1 & 2 & -1\end{bmatrix}$

iii) x=3 km/hr , y=1km/hr & z=2 km/hr

5...To institutions decided to award their employees for three values: resourcefulness. competence, and determination, with prizes at the rates of ₹x, y, and 2 per person. respectively. The first institution decided to award 4, 3, and 2 employees for these respective values, resulting in a total prize amount of ₹37,000. The second institution awarded 5, 3, and 4 employees for these values, totalling ₹47,000. The combined value of all three prizes together amounts to ₹12,000.

Based on the above information, answer the following questions:

Based on the above information, answer the following questions:
(i) What is the matrix equation representing the above situation?
(ii) Find matrix A (adj A).
OR
(ii) What is the value of | adj A|?
(ii) What are the values of x, y, z in this case?
Solution
(i) $4x + 3y + 2z = 37000$
$5x + 3y + 4z = 47000$
$x + y + z = 12000$
(ii)
(A) .(adj A)= $\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix} . \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & 3 \end{bmatrix}$

(A).(adj A) =-3 I
(iii) | adj A|=9
(iv)x=4000, y=5000, & z=3000

Chapter 1: Numbers, Quantification and Numerical Applications

Competency-Based Questions (2 Solved + 3 Practice)

Note: *CBSE has not released a chapter-tagged, competency-labelled question bank for this exact unit that could be verified against an official source. The five questions below are framed in the CBSE Competency-Based Question (CBQ) format used in the Applied Mathematics 241 sample papers and board papers (2023-25) — testing the same sub-topics (Modulo Arithmetic, Congruence Modulo, Boats & Streams, Mixture & Alligation, Pipes & Cisterns) with the same competency categories CBSE uses (Critical Thinking, Analytical/Logical Reasoning, Quantitative Reasoning, Problem Solving, Application-Based Reasoning). They are not verbatim reproductions of a specific dated board paper.*

Solved Competency-Based Questions

Question 1 [4 Marks]

Competency Tested: *Analytical Reasoning & Critical Thinking (Modulo Arithmetic)*

A courier company assigns a delivery slot number to every package using the rule: Slot = (sum of digits of the tracking ID) mod 7, and slots are numbered 0 to 6.

A package has tracking ID 583946.

(a) Find the slot number assigned to this package.

(b) If two packages are assigned the same slot number, the delivery supervisor say they are "congruent modulo 7." Verify whether tracking IDs 583946 and 274853 are congruent modulo 7, showing your reasoning.

Solution:

(a) Sum of digits of 583946 = 5 + 8 + 3 + 9 + 4 + 6 = 35

Slot = 35 mod 7 = 0 (since 35 = 7 × 5 + 0)

So the package is assigned Slot 0.

(b) Sum of digits of 274853 = 2 + 7 + 4 + 8 + 5 + 3 = 29

29 mod 7 = 1 (since 29 = 7 × 4 + 1)

Slot for 583946 is 0 and slot for 274853 is 1.

Since 0 ≠ 1, the two tracking IDs are NOT congruent modulo 7.

Check by definition: a ≡ b (mod 7) requires (a − b) to be exactly divisible by 7. Here 35 − 29 = 6, which is not divisible by 7, confirming they are not congruent.

Question 2 [4 Marks]

Competency Tested: *Quantitative Reasoning & Problem Solving (Boats and Streams)*

A ferry operator on the Narmada river notices that a boat covers 30 km downstream in the same time it takes to cover 18 km upstream. The speed of the stream is known to be 4 km/h.

(a) Find the speed of the boat in still water.

(b) Hence find the time taken by the boat to travel 44 km downstream.

Solution:

Let the speed of the boat in still water = x km/h; speed of stream = 4 km/h

Downstream speed = (x + 4) km/h, Upstream speed = (x − 4) km/h

Since time is the same for both journeys: Time = Distance ÷ Speed

30 / (x + 4) = 18 / (x − 4)

30(x − 4) = 18(x + 4)

30x − 120 = 18x + 72

12x = 192

x = 16 km/h

(a) Speed of the boat in still water = 16 km/h

(b) Downstream speed = 16 + 4 = 20 km/h

Time for 44 km downstream = 44 / 20 = 2.2 hours = 2 hours 12 minutes

Unsolved Practice Questions

Question 3 [4 Marks]

Competency Tested: *Logical Reasoning (Congruence Modulo)*

A cyclic bus service repeats its route every 9 days. A conductor labels each day of the year with a "route-day number" equal to (day count from 1 January) mod 9.

(a) If a particular day has count 172 from 1 January, find its route-day number.

(b) Two inspection days have counts 172 and 217. Using the definition of congruence modulo n, determine whether these two days fall on the same route-day. Show complete working.

Question 4 [4 Marks]

Competency Tested: *Application-Based Reasoning (Mixture and Alligation)*

A dairy owner has two milk samples: Sample A contains 6% fat and Sample B contains 2% fat. He wants to prepare 40 litres of a mixture containing 4.5% fat by blending Samples A and B.

(a) Using the rule of alligation, find the ratio in which Sample A and Sample B must be mixed.

(b) Hence find the quantity (in litres) of each sample required.

Question 5 [4 Marks]

Competency Tested: *Critical Thinking (Pipes and Cisterns)*

Two inlet pipes P and Q can separately fill an overhead tank in 12 hours and 15 hours respectively. An outlet pipe R can empty the full tank in 20 hours. All three pipes are opened together at 8:00 AM when the tank is empty.

(a) Find the net part of the tank filled in one hour when all three pipes operate together.

(b) At what time will the tank be completely full? Justify whether opening pipe R was a reasonable decision by the caretaker, giving a mathematical reason.

CHAPTER – TIME BASED DATA CLASS XII SUBJECT- APPLIED MATHEMATICS

Q.1 Read the following text carefully and answer the questions that follow:

When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production. Mathematically, for finding a line of best - fit to represent a trend, many methods are available. Methods like moving - averages and least - squares are some of the techniques to predict such trends.

Mrs. Shamita runs a bread factory and the record of her sales of bakery items for the period of 2015 - 2019 is as follows:

Year	2015	2016	2017	2018	2019
Sales (in ₹ thousands)	35	42	46	41	48

Based on the above information, answer the following questions. Show steps to support your answers.

1. By taking year 2017 as origin, use method of least - squares to find the best - fit trend line equation for Mrs Shamita's business. Show the steps of your working. (1)
2. Demonstrate the technique to fit the best - suitedstraight-line trend by the method of 3 - year moving averages. Also, draw the trend line. (1)
3. What are the estimated sales for Mrs Shamita' s business for year 2022? (2)

   OR

   Mrs Shamita wishes to grow her business to yearly sale of₹ 67400. In which year will she be able to reach her target? (2)

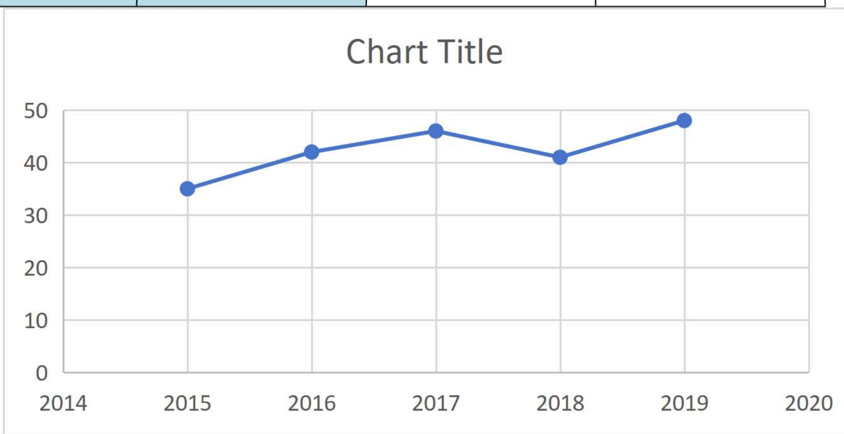
Solution:- I.:-

YEAR (x_i)	INDEX (y_i)	X= x_i - A	X^2	X Y	Y= a + b X
2015	35	-2	4	-70	37.4
2016	42	-1	1	-42	39.9
2017	46	0	0	0	42.4
2018	41	1	1	41	44.9
2019	48	2	4	96	47.4

y = a x + b, a = 42.4 , b = 2.5

II.

TIME	DATA	MOVING TOTAL FOR 3 YEARS	MOVING AVERAGE TOTAL FOR 3
2015	35		
2016	42	123	41
2017	46	129	43
2018	41	135	45
2019	48		



III. Y = a + b x , at X = 2022 , x = 5 then
$Y_{(X=2022)}$ = 54.9
OR
Y = a + b (X-2017), at Y = 67.4
X = ((Y + a)/b) + A = 2027

Q.2 Read the following text carefully and answer the questions that follow:
To fit a straight line by the method of least squares, Rohit constructed the following table:

Year(t)	Profit (Y)	X= $t_i - 2008$	X^2	XY
2015	114	-3	9	-342
2016	130	-2	4	-260
2017	126	-1	1	-126
2018	144	0	0	0
2019	138	1	1	138
2020	156	2	4	312
2021	164	3	9	492

The trend equation can be considered as y_t = a + bx.

i) What is the value of **a** in the trend equation? (1)

ii) What is the value of **b** in the trend equation? (1)

iii) Which of the following is a trend equation? (2)

y_t = 138.86 + 7.64x, y_t = 7.64 + 138.86x, y_t = 138.86 - 7.64x

OR

What is the trend value for year 2015? (2)

Solution:-

$$b = \frac{\frac{\sum X \sum Y}{n} - \sum X \sum Y}{\frac{(\sum X)^2}{n} - \sum X^2} \qquad \boldsymbol{a = \frac{\sum Y - b\sum X}{n}}$$

I. a = 138.86
II. b = 7.6
III. y_t = 138.86 + 7.64x **OR** y (x= 2015) = 116.1

Q.3 When observed over a long period of time, a time series data can predict trends that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production. The table below shows the sale of an item in a district during 1996 – 2001:

Year	1996	1997	1998	1999	2000	2001
Sales (in Lakhs)	6.5	5.3	4.3	6.1	5.6	7.1

Based on the above information, answer the following questions:

(i) Determine the equation of the straight-line trend. (2)

(ii) Year: Tabulate the trend values of the years and also compute expected sales trend for the year 2002. (2)

Answer: - (i)

YEAR (x_i)	INDEX (y_i)	X= x_i - A	X^2	XY	Y= a + b X
1996	6.5	-5	25	-32.5	5.415
1997	5.3	-3	9	-15.9	5.577
1998	4.3	-1	1	-4.3	5.739
1999	6.1	1	1	6.1	5.901
2000	5.6	3	9	16.8	6.063
2001	7.1	5	25	35.5	6.225

$Y_t = \mathbf{5.82 + 0.081}\, x$

(ii) For X = 2002 , Y_t = 6.103

CHAPTER: FINANCIAL MATHEMATICS

Q1. EQUATED MONTHLY INSTALMENTS (EMI)

Rajesh purchased a house from a company for ₹25,00,000 and made a down payment of ₹5,00,000. He repays the balance in 25 years by monthly instalments at the rate of 9% per annum compounded monthly.

(Given $(1.0075)^{-300} = 0.1062$)

Based on the above information, answer the following questions:

(i) Find the number of payments and find the rate of interest per month.

(ii) (a) What are the monthly payments of instalments using reducing balance method?

OR

(b) What are the monthly payments of instalments using flat rate method?

(iii) What is the total interest payment made in the process applied to calculate EMI in the above part?

Concept Applied
EMI (using reducing balance method) = $P \times i / [1 - (1 + i)^{-n}]$
and
EMI (using flat rate method) = $P (i + 1/n)$
SOL (i) Here, time = 25 years
∴ Total number of payments = 25 × 12 = 300
R = 9% per annum.
Rate of interest per month = 9/1200 = 0.0075
R = ₹1,00,000 / 20.6542 = ₹4841.63
(ii) (a)
Cost of house = ₹25,00,000
Down Payment = ₹5,00,000
∴ Principal amount = ₹(25,00,000 − 5,00,000)
= ₹20,00,000
EMI (using reducing balance method) = $P \times i / [1 - (1 + i)^{-n}]$
= $(2000000 \times 0.0075) / [1 - (1 + 0.0075)^{-300}]$
= $15000 / [1 - (1.0075)^{-300}]$
= 15000 / [1 − (0.1062)]
= 15000 / 0.8938
= ₹16,782.27
Hence, monthly payment is ₹16,782.27

OR

(ii) (b)
Cost of house = ₹25,00,000
Down Payment = ₹5,00,000
∴ Principal amount = ₹(25,00,000 − 5,00,000)
= ₹20,00,000
EMI (using flat rate method) = $P (i + 1/n)$
= 2000000 (0.0075 + 1/300)
= 2000000 (0.0108333)
= ₹21,666.67
(iii)
EMI (using reducing balance method) = ₹16,782.27
∴ Total interest = n × EMI − P
= 300 × 16782.27 − 2000000
= ₹30,34,681
Hence, total interest is ₹30,34,681
When EMI is calculated by (using flat rate method),
Total interest = n × EMI − P
= 300 × 21666.67 − 2000000
= ₹45,00,001

Q2. Home Loan Question

In the year 2010, Mr Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.
Based on the above information, answer the following questions:
(i) Determine the EMI.
(ii) Find the principal paid by Mr Aggarwal in the 150th installment.

(iii) (a) Find the total interest paid by Mr Aggarwal.

OR

(b) How much was paid by Mr Aggarwal to repay the entire amount of home loan?
[Use $(1.00625)^{240} = 4.4608$; $(1.00625)^{91} = 1.7629$]

SOL (i) Given,
P = ₹ 30,00,000, i = 7.5% p.a.
i = 7.5/1200 = 0.00625
and
n = 20 years = 240 months
Since,
$EMI = Pi(1+i)^n / [(1+i)^n - 1]$
$= [30,00,000 \times 0.00625(1 + 0.00625)^{240}] / [(1 + 0.00625)^{240} - 1]$
$= [30,00,000 \times 0.00625(1 + 0.00625)^{240}] / [(1.00625)^{240} - 1]$
$= (18750 \times 4.4608) / (4.4608 - 1)$
$= 83640 / 3.4608$
= ₹ 24,167.82
= ₹ 24,168
(ii) Interest paid by Mr Aggarwal in 150th installment
$= EMI[(1+i)^{(n-150+1)} - 1] / (1+i)^{(n-150+1)}$
$= 24167.82[(1.00625)^{(240-150+1)} - 1] / (1.00625)^{(240-150+1)}$
$= 24167.82[(1.00625)^{91} - 1] / (1.00625)^{91}$
$= 24167.82[1.7629 - 1] / 1.7629$
$= 18437.629 / 1.7629$
= ₹ 10,458.6929
Principal paid = 24167.82 − 10,458.69
= ₹ 13,709.13 ≈ ₹ 13,709
(iii) (a) Total interest paid = n(EMI) − P
$= 240 \times 24167.82 - 30,00,000$
= 58,00,276.8 − 30,00,000
= 28,00,276.8
= ₹ 28,00,277
OR
(b) Total amount paid = n·EMI
$= 240 \times 24,167.82$
= ₹ 58,00,276.8
= ₹ 58,00,277

Q3. Vehicle Loan – Problem-Solving

A person purchases a car costing **₹7,50,000**. He pays **₹1,50,000 as down payment** and takes a loan for the remaining amount at **9% per annum compounded annually**, to be repaid in **5 equal annual instalments**.

(a) Write the formula for calculating annual installment using annuity.
(b) Calculate the amount of loan taken.
(c) Calculate the annual instalment using the annuity formula.

OR

(d) Find the total interest payable over the five years.

Q4. Retirement Investment – Critical Thinking & Problem-Solving

Mrs. Mehta deposits **₹6,000 at the end of every month** in an investment scheme earning **6% per annum compounded monthly**. She plans to continue the investment for **5 years**.

(a) Write the formula for calculating future value of her investment.
(b) Calculate the future value of her investment using the appropriate annuity formula.
(c) Calculate the total amount actually deposited by her.

OR

(d) Find the interest earned during the investment period.

Q5. Home Loan Question

Ramesh borrowed a home loan amount from a bank at an interest of 12% per annum for 30 years, to be paid in monthly installments.
Based on the above information, answer the following questions:
(a) Write the formula for calculating EMI by reducing balance method.
(b) Write the value of P, I and n respectively.
(c) Find the EMI. [Use $(1.01)^{-360} = 0.02781668$]

OR

(d) If the loan is to be returned in 20 years, find EMI. [Use $(1.01)^{-240} = 0.09180584$]

Unit: Inferential Statistics

PART I: SOLVED QUESTIONS (With Marking Scheme)

Q1. (2 Marks) — Parameter vs. Statistic and Decision Making
Question:
A consumer goods company wants to check the average shelf-life of a new batch of 50,000 organic food packets produced this month. Due to time and destructive testing constraints, the quality control head selects a random sample of 100 packets and finds their mean shelf-life to be 240 days with a standard deviation of 12 days.

1. Identify the population parameter and the sample statistic from this context. [1 Mark]
2. If the company claims historically that the true population mean shelf-life is 245 days, formulate the Null Hypothesis (H_0) and Alternative Hypothesis (H_1) to test this claim. [1 Mark]

Solution & Marking Scheme:

1. **Parameter and Statistic:** [1 Mark]
   - **Ppulation Parameter:** The true mean shelf-life (μ) of all 50,000 organic food packets. *(0.5 Mark)*
   - **Sample Statistic:** The sample mean ($\bar{x}$= 240 days) calculated from the 100 sampled packets. *(0.5 Mark)*
2. **Hypothesis Formulation:** [1 Mark]
   - **Null Hypothesis (H_0):** μ= 245 days (The true mean shelf-life is 245 days). *(0.5 Mark)*
   - **Alternative Hypothesis (H_1):** $\mu \neq 245$ days (The true mean shelf-life is different from 245 days). *(0.5 Mark)*

Q2. (5 Marks) — One-Sample t-Test Application
Question:

An educational psychologist introduces an intensive problem-solving training module for senior secondary students. A random sample of 9 students is chosen, and their improvement scores on a standardized test are recorded:

\text{Scores: } 8, 12, 10, 14, 6, 9, 11, 13, 7
The psychologist claims that the average improvement score (\mu) of students undergoing this training is greater than 8. Test this claim at a 5% level of significance. (Given: For df = 8, the critical t-value at \alpha = 0.05 for a one-tailed test is 1.86).

Solution & Marking Scheme:

1. **Hypothesis Setup:** [1 Mark]
   - H_0: $\mu = 8$
   - H_1: $\mu > 8$ (Right-tailed test)
2. **Calculation of Sample Mean ($\bar{x}$) and Standard Deviation (s):** [2 Marks]
   - Sum of scores ($\sum x$) = 8 + 12 + 10 + 14 + 6 + 9 + 11 + 13 + 7 = 90
   - Sample size (n) = 9
   - $\bar{x}$ = 90/9 = 10 *(0.5 Mark)*
   - Deviations from mean ($x - \bar{x}$): -2, 2, 0, 4, -4, -1, 1, 3, -3
   - Squared deviations$(x - \bar{x})^2$: 4, 4, 0, 16, 16, 1, 1, 9, 9 = 60 *(0.5 Mark)*
   - Sample variance (s^2) = $\frac{\sum (x - \bar{x})^2}{n-1} = \frac{60}{8} = 7.5$ *(0.5 Mark)*
   - Sample standard deviation (s) = $\sqrt{7.5}$ approx 2.74 *(0.5 Mark)*
3. **Compute the t-statistic:** [1 Mark]
   - Formula: $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$ *(0.5 Mark)*
   - Calculation: t = : $t = \frac{10 - 8}{\frac{2.74}{\sqrt{9}}} =$ approx 2.19 *(0.5 Mark)*
4. **Conclusion:** [1 Mark]
   - Degrees of freedom (df) = n - 1 = 8
   - Since calculated t (2.19) is greater than the critical table value (1.86), we **reject the null hypothesis H_0** and conclude that there is statistically significant evidence to support the psychologist's claim that the mean improvement score is greater than 8. *(1 Mark)*

PART II: UNSOLVED CBQ QUESTIONS

Q3. (2 Marks) — Sampling Technique Selection
Question:

A district education officer wants to evaluate the effectiveness of smart classrooms implemented across 40 different schools in the district. Due to time constraints, the officer decides to select 5 schools. Instead of picking schools completely at random, the officer lists all 40 schools alphabetically, selects the 4th school, and then subsequently picks every 8th school on the list.

1. Identify the specific sampling technique being employed here. [1 Mark]

2. State one major advantage of this sampling technique over simple random sampling in this educational administrative scenario. [1 Mark]

Q4. (3 Marks) — Hypothesis Interpretation & Errors
Question:
A pharmaceutical company tests a new memory-enhancement syrup on a sample of students.

1. Explain what constitutes a **Type I error** and a **Type II error** in the context of this drug testing scenario. [2 Marks]

2. If the consequence of a Type I error is severe, how should the level of significance (α) be adjusted by the researchers? [1 Mark]

Q5. (5 Marks) — Two Independent Groups t-Test Case Study
Question:
An online retail portal compares the daily customer satisfaction ratings between two different website layouts (Layout A and Layout B) collected over independent sample periods:

- **Layout A:** Sample size (n_1) = 10, Sample mean ($\bar{x}_1$) = 82, Sample variance (s_1^2) = 16

- **Layout B:** Sample size (n_2) = 12, Sample mean ($\bar{x}_2$) = 78, Sample variance (s_2^2) = 20

Assuming both populations are normally distributed with equal variances, test whether there is a significant difference between the average satisfaction ratings of the two layouts at a 5% level of significance. (Perform pooled variance calculations and state the final statistical inference). [5 Marks]

Mathematics Lab Activity

Internal Assessment — Class XII, Applications of Derivatives

Aim

To find the intervals in which the function $f(x) = 2x^2 - 3x$ is (a) increasing and (b) decreasing, and to verify the result graphically.

Materials Required

(i) Graph paper / drawing sheet mounted on a board
(ii) Geometry box (ruler, pencil)
(iii) Sketch pens or coloured pencils (2 colours)
(iv) Drawing pins / adhesive tape
(v) Calculator (optional)

Theory

A function f is said to be increasing on an interval if, for $x_1 < x_2$ in that interval, $f(x_1) \le f(x_2)$; it is decreasing if $f(x_1) \ge f(x_2)$. For a differentiable function, this behaviour is determined by the sign of the first derivative $f'(x)$:

(vi) If $f'(x) > 0$ for all x in an interval, f is increasing on that interval.
(vii) If $f'(x) < 0$ for all x in an interval, f is decreasing on that interval.
(viii) If $f'(x) = 0$ at a point, that point is a candidate for a local maximum or minimum (a turning point).

Mathematical Working

Given $f(x) = 2x^2 - 3x$

Step 1: Differentiate f(x). $f'(x) = 4x - 3$

Step 2: Find the critical point by setting $f'(x) = 0$. $4x - 3 = 0 \Rightarrow x = 3/4$

Step 3: This critical point divides the real line into two intervals: $(-\infty, 3/4)$ and $(3/4, \infty)$. Test the sign of $f'(x)$ in each:

(ix) At x = 0 (in the first interval): $f'(0) = 4(0) - 3 = -3$ → negative → decreasing
(x) At x = 1 (in the second interval): $f'(1) = 4(1) - 3 = 1$ → positive → increasing

Result of working:

(xi) (a) f(x) is increasing on the interval $(3/4, \infty)$
(xii) (b) f(x) is decreasing on the interval $(-\infty, 3/4)$

At $x = 3/4$, $f'(x) = 0$, giving a point of local minimum: $f(3/4) = 2(9/16) - 3(3/4) = 9/8 - 9/4 = -9/8$

Procedure

1. Fix a sheet of graph paper on the board. Draw the x-axis and y-axis as two perpendicular lines, taking a convenient scale (e.g. 1 unit = 1 cm).

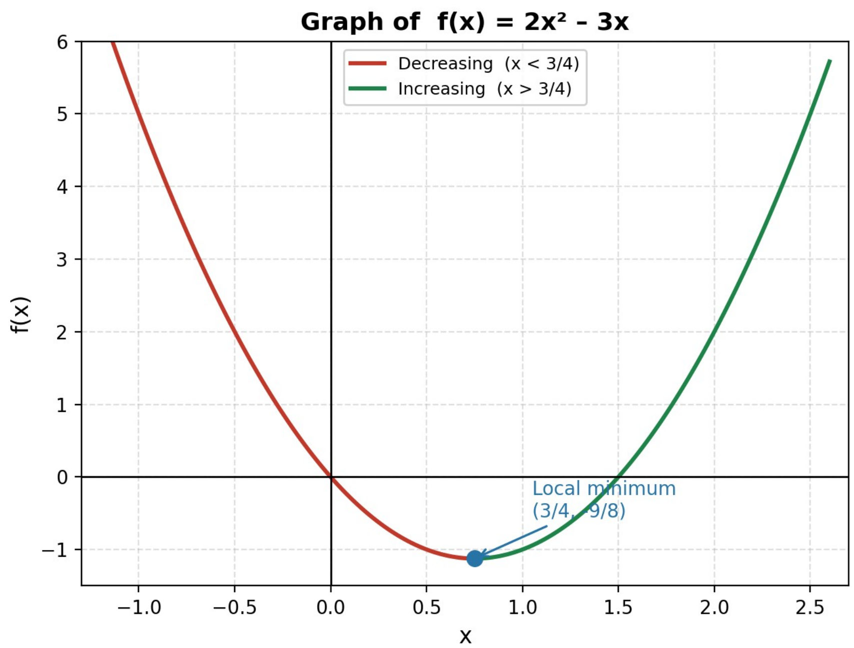
2. Choose several values of x on both sides of x = 3/4, for example: −1, −0.5, 0, 0.5, 0.75, 1, 1.5, 2, 2.5.
3. Compute $f(x) = 2x^2 - 3x$ for each value of x and record the results in a table (see Observation Table 1 below).
4. Plot the corresponding points (x, f(x)) on the graph paper and join them with a smooth free-hand curve.
5. Using one colour, trace the part of the curve to the left of x = 3/4; using a second colour, trace the part to the right of x = 3/4.
6. At a few points on each coloured portion, place a ruler along the curve to represent the tangent, and observe whether the tangent slopes upward or downward as x increases.
7. Record whether the curve falls or rises in each portion, and the corresponding sign of the tangent slope, in Observation Table 2.

Observation Table 1 — Values of f(x)

x	-1	-0.5	0	0.5	0.75	1	1.5	2	2.5
f(x)	5	2	0	-1	-1.125	-1	0	2	5

Graph

The curve below shows $f(x) = 2x^2 - 3x$ plotted from the tabulated values, with the decreasing and increasing portions marked in different colours and the local minimum indicated.



Observation Table 2 — Nature of the Curve

Interval	Nature of curve	Sign of slope f'(x)	Conclusion
$x < 3/4$	Falls	Negative	Decreasing
$x > 3/4$	Rises	Positive	Increasing
$x = 3/4$	Turning point	Zero	Local minimum

Result

The graphical activity confirms the analytical result obtained using the derivative:

(xiii) $f(x) = 2x^2 - 3x$ is decreasing on the interval $(-\infty, 3/4)$.
(xiv) $f(x)$ is increasing on the interval $(3/4, \infty)$.
(xv) $x = 3/4$ is a point of local minimum, with minimum value $f(3/4) = -9/8$.

Learning Outcome

This activity helps in understanding the connection between the algebraic condition on f'(x) and the geometrical behaviour of the graph of f(x) — that the sign of the derivative (slope of the tangent) determines whether a function is increasing or decreasing on an interval.

Viva-Voce Questions

8. Define an increasing function and a decreasing function.
9. What is the geometrical meaning of the derivative of a function at a point?
10. What is a critical point? How is it obtained?
11. At the point of local minimum, what is the sign of f'(x) just before and just after the point?
12. Is the function $f(x) = 2x^2 - 3x$ increasing or decreasing at $x = 0$? Justify using f'(x).

MATHEMATICS LAB ACTIVITY

Maximum Volume of an Open Box Formed from a Square Tin Sheet

Activity No.	________
Class / Section	XII ________
Subject	Mathematics (Code 041)
Topic / Unit	Application of Derivatives — Maxima and Minima
Date	________________
Student's Name	_____________________________________
Roll No.	________

1. Objective

To construct an open box (without a top) from a square tin sheet of side 18 cm, by cutting equal squares of side x from each corner and folding up the flaps, and to determine, using differential calculus, the value of x for which the volume of the box is maximum.

2. Pre-requisite Knowledge

(xvi) Concept of a function of one variable and its domain.
(xvii) First derivative and second derivative of a polynomial function.
(xviii) First Derivative Test and Second Derivative Test for local maxima and minima.
(xix) Formula for the volume of a cuboid: V = length × breadth × height.

3. Materials Required

(xx) A square sheet of tin / chart paper / cardboard of side 18 cm (and a few extra squares of the same size for demonstration).
(xxi) Ruler, pencil, and a pair of scissors.
(xxii) Adhesive tape or gum.
(xxiii) Graph paper (for plotting V against x, optional but recommended).
(xxiv) Geometry box.

4. Procedure (Activity / Demonstration)

13. Take a square sheet of side 18 cm. Mark the four corners.
14. Choose a value of x ($0 < x < 9$) and cut out a small square of side x cm from each of the four corners of the sheet.
15. Fold up the four resulting flaps along the cut lines to form an open box (without a lid).
16. Observe that the base of the box is a square of side $(18 - 2x)$ cm and the height of the box is x cm.
17. Repeat the activity with squares of different sizes (x = 1, 2, 3, 4, 5, 6, 7, 8 cm on separate sheets, or note the values as calculated) and compare the boxes formed — some are tall and narrow, some are short and wide.
18. Tabulate the volume $V = x(18 - 2x)^2$ for each value of x (Observation Table below) and note the value of x that gives the greatest volume.
19. Verify this value of x mathematically using differentiation (Calculation below).

5. Observation Table

S. No.	Side of cut square, x (cm)	Side of base, (18 − 2x) (cm)	Volume, $V = x(18 - 2x)^2$ (cm^3)
1	1	16	256
2	2	14	392
3	3	12	**432 (maximum)**

S. No.	Side of cut square, x (cm)	Side of base, $(18 - 2x)$ (cm)	Volume, $V = x(18 - 2x)^2$ (cm³)
4	4	10	400
5	5	8	320
6	6	6	216
7	7	4	112
8	8	2	32

6. Calculation (Mathematical Verification)

Let the side of the square cut from each corner be x cm, where $0 < x < 9$.

Then the base of the box is a square of side $(18 - 2x)$ cm, and the height of the box is x cm.

$\therefore$ Volume of the box: $V(x) = x(18 - 2x)^2, \; 0 < x < 9$

Expanding: $V(x) = 324x - 72x^2 + 4x^3$

Differentiating with respect to x: $V'(x) = 324 - 144x + 12x^2 = 12(x^2 - 12x + 27) = 12(x - 3)(x - 9)$

For maximum or minimum volume, $V'(x) = 0 \implies x = 3$ or $x = 9$.

Since $0 < x < 9$, the value $x = 9$ is rejected (it gives $V = 0$, the least possible volume, not a maximum). So the only critical point in the domain is $x = 3$.

Second derivative: $V''(x) = -144 + 24x$

At $x = 3$: $V''(3) = -144 + 72 = -72 < 0$

By the Second Derivative Test, $V(x)$ is maximum at $x = 3$.

Maximum volume: $V(3) = 3 \times (18 - 6)^2 = 3 \times 144 = 432$ cm³

7. Result

The side of the square to be cut off from each corner, for the volume of the box to be maximum, is $x = 3$ cm, and the corresponding maximum volume of the box is 432 cm³.

8. Application

This activity illustrates how differential calculus is used to solve real-life optimisation problems, such as designing open boxes, cartons, and packaging trays that make the most efficient use of a fixed amount of sheet material — a technique widely used in the packaging and manufacturing industry.

9. Viva-Voce Questions

Q1. What is a critical point of a function?

Ans. A point c in the domain of f where either f′(c) = 0 or f′(c) does not exist.

Q2. State the Second Derivative Test.

Ans. If f′(c) = 0 and f″(c) < 0, then c is a point of local maximum; if f″(c) > 0, then c is a point of local minimum; if f″(c) = 0, the test is inconclusive and the First Derivative Test must be used.

Q3. Why is the domain of x restricted to 0 < x < 9 in this problem?

Ans. The side cut, x, must be positive, and 2x cannot exceed the side of the square (18 cm), otherwise the base of the box would have zero or negative length; hence 0 < x < 9.

Q4. What is the difference between an absolute maximum and a local maximum?

Ans. A local maximum is the greatest value of the function only in some neighbourhood of a point, whereas an absolute maximum is the greatest value over the entire domain under consideration.

Q5. Why is x = 9 rejected as a critical point here?

Ans. At x = 9, the base side (18 − 2x) becomes 0, so the volume is 0; this corresponds to a minimum (in fact the least possible volume), not the maximum.

Q6. What real-life applications use this same optimisation idea?

Ans. Designing open boxes, cartons, trays, and packaging material to use the least material for a required volume, or to maximise volume for a fixed sheet size — used in the packaging and manufacturing industry.

Student's Signature Teacher's Signature

MATHEMATICS LAB ACTIVITY

Shortest Distance from a Point to a Parabolic Path

Activity No.	________
Class / Section	XII ________
Subject	Mathematics (Code 041)
Topic / Unit	Application of Derivatives — Maxima and Minima (Distance Minimisation)
Date	________________
Student's Name	_____________________________________
Roll No.	________

1. Objective

An enemy Apache helicopter flies along the curve $y = x^2 + 7$. A soldier is stationed at the point A(3, 7). To determine, using differential calculus, the position of the helicopter on its path at which it is nearest to the soldier, and to find this shortest (minimum) distance.

2. Pre-requisite Knowledge

(xxv) Distance formula between two points: $D = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.

(xxvi) Differentiation of polynomial functions, and the Chain Rule.

(xxvii) First Derivative Test / Second Derivative Test for local maxima and minima.

(xxviii) Equation and shape of a parabola of the form $y = x^2 + c$.

3. Materials Required

(xxix) Graph paper, ruler, and pencil.

(xxx) Geometry box (compass, protractor).

(xxxi) Coloured pens/pencils to mark the curve, the fixed point, and the shortest-distance segment.

(xxxii) Calculator (for verifying numerical values).

4. Diagram

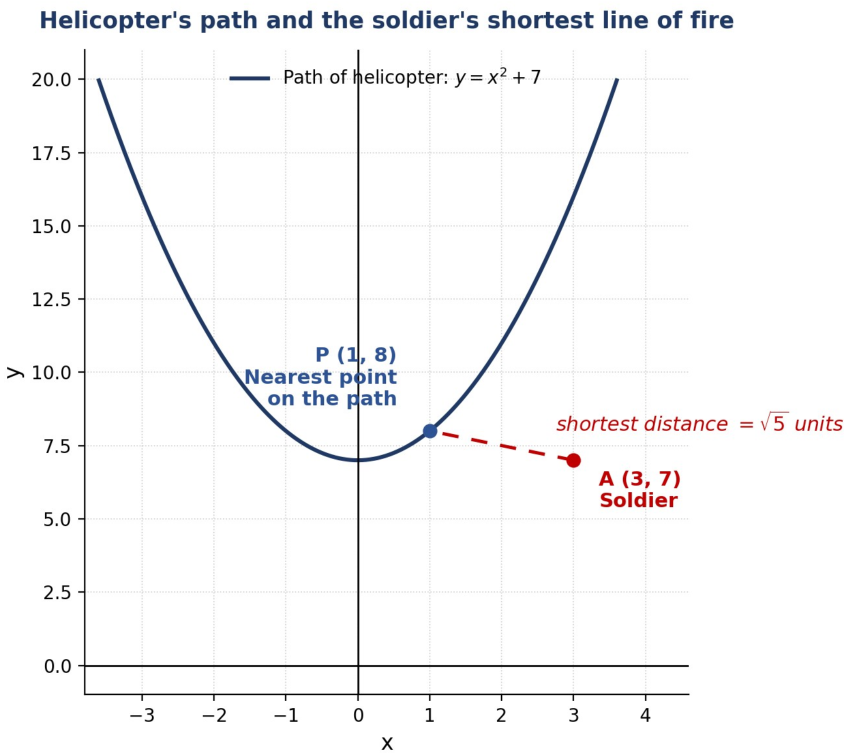


Fig. 1 — Parabolic path of the helicopter, the soldier's position A(3, 7), the nearest point P(1, 8), and the shortest line of fire AP.

5. Procedure (Activity / Demonstration)

20. On graph paper, plot the curve $y = x^2 + 7$ by taking several values of x and computing the corresponding y.
21. Mark the fixed point A(3, 7), representing the soldier's position, on the same graph.
22. Take a general point $Q(x, x^2 + 7)$ moving along the curve, representing the helicopter's position.
23. Using a ruler, physically measure (approximately) the distance AQ for several positions of Q on the curve, moving Q gradually along the parabola.
24. Observe that the length AQ first decreases and then increases as Q moves along the curve — record the position where AQ appears shortest.
25. Tabulate the computed distance AQ for chosen values of x (Observation Table below) to confirm the position of minimum distance.
26. Verify the exact position and the shortest distance mathematically using differentiation (Calculation below).

6. Observation Table

S. No.	x (position on path)	$D^2 = (x-3)^2 + x^4$	D = distance (units)
1	-2	41.00	6.403
2	-1	17.00	4.123
3	0	9.00	3.000
4	0.5	6.31	2.512
5	1	**5.00**	**2.236 (min)**
6	1.5	7.31	2.704
7	2	17.00	4.123
8	3	81.00	9.000

7. Calculation (Mathematical Verification)

Let $Q(x, y) = (x, x^2 + 7)$ be any point on the curve $y = x^2 + 7$, and A(3, 7) be the soldier's fixed position.

Distance: $AQ = D = \sqrt{(x - 3)^2 + (x^2 + 7 - 7)^2} = \sqrt{(x - 3)^2 + x^4}$

Since $D \geq 0$, D is minimum exactly when D^2 is minimum (squaring avoids the square root while differentiating). Let:

$Z(x) = D^2 = (x - 3)^2 + x^4$

Differentiating with respect to x: $Z'(x) = 2(x - 3) + 4x^3 = 4x^3 + 2x - 6$

For maximum or minimum, $Z'(x) = 0$: $4x^3 + 2x - 6 = 0 \implies 2x^3 + x - 3 = 0$

By inspection, $x = 1$ satisfies this equation: $2(1)^3 + 1 - 3 = 0$. ✓

Factorising: $2x^3 + x - 3 = (x - 1)(2x^2 + 2x + 3)$

The quadratic factor $2x^2 + 2x + 3$ has discriminant = $(2)^2 - 4(2)(3) = 4 - 24 = -20 < 0$, so it has no real roots. Hence $x = 1$ is the only critical point.

Second derivative: $Z''(x) = 12x^2 + 2$, which is always positive for every real x (in particular, $Z''(1) = 14 > 0$).

By the Second Derivative Test, Z(x) — and therefore D — is minimum at x = 1.

At x = 1: $y = (1)^2 + 7 = 8$, so the nearest point on the path is P(1, 8).

Shortest distance: $D = \sqrt{(1-3)^2 + (1)^4} = \sqrt{4+1} = \sqrt{5}$ units

8. Result

The helicopter, moving along $y = x^2 + 7$, is nearest to the soldier at A(3, 7) when it is at the point P(1, 8) on its path, and the shortest distance between them at that instant is $\sqrt{5}$ units (≈ 2.236 units).

9. Application

This activity demonstrates the use of derivatives in minimising the distance between a fixed point and a variable point constrained to lie on a curve — a technique directly applicable to radar tracking, missile and air-defence targeting, and robotic path-planning, where the closest approach of a moving object to a fixed observation point must be computed.

10. Viva-Voce Questions

Q1. Why was D^2 minimised instead of D directly?

Ans. Since D is always non-negative, D is minimum exactly when D^2 is minimum; working with D^2 avoids differentiating a square-root expression and makes the calculation simpler.

Q2. What type of curve does the helicopter fly along?

Ans. A parabola, $y = x^2 + 7$, which is the standard parabola $y = x^2$ shifted 7 units upward.

Q3. Why is there no need to check the domain restriction here (unlike the open-box problem)?

Ans. Here x can take any real value since the helicopter's path extends infinitely in both directions, so the natural domain is all real numbers; the Second Derivative Test alone confirms the nature of the critical point.

Q4. What does the point P(1, 8) represent geometrically?

Ans. It is the point on the parabola that is closest to the soldier's position A(3, 7) — the foot of the shortest line segment drawn from A to the curve.

Q5. Is the line AP always perpendicular to the tangent of the curve at P?

Ans. Yes — for the shortest distance from an external point to a curve, the line joining the point to the nearest point on the curve is always perpendicular (normal) to the tangent at that point.

Q6. Give a real-life application of this shortest-distance optimisation technique.

Ans. It is used in radar and missile-guidance systems, robotics path planning, and air-defence targeting to compute the minimum distance between a moving object and a fixed observation or firing point.

Student's Signature Teacher's Signature

MATHEMATICS LAB ACTIVITY

CBSE Class XII — Internal Assessment (Activity File)

Maximum Area of a Trapezium with Three Equal Sides

Activity No.	_______________
Topic Area	Application of Derivatives — Maxima & Minima
Name of Student	_______________________________________
Class / Section	XII ______
Date	_______________

1. Objective

To find, using paper cut-outs / a hinged cardboard model and verify by calculus, the value of the base angle θ for which a trapezium — whose three sides other than the base are each 10 cm — has the maximum possible area.

2. Pre-requisite Knowledge

(xxxiii) Area of a trapezium = ½ × (sum of parallel sides) × (height).
(xxxiv) Trigonometric ratios sin θ, cos θ in a right triangle.
(xxxv) Derivative of sin θ, cos θ and product/first-derivative test for maxima and minima.
(xxxvi) Second derivative test to confirm a maximum.

3. Materials Required

(xxxvii) A rectangular sheet of cardboard / thick chart paper (for the base)
(xxxviii) Two strips of cardboard, each exactly 10 cm long, and one strip 10 cm long for the top (three equal 10 cm strips in all)
(xxxix) Paper fasteners / split pins (brads) to hinge the strips at the vertices, so the angle θ can be varied
(xl) A protractor and a ruler (graduated in cm)
(xli) A sheet of graph paper, geometry box, sketch pens

4. Figure / Diagram

Hinge three 10 cm strips as AD, DC and CB, and lay them on a fixed base line so that AD = DC = CB = 10 cm, with the two base angles at A and B both equal to θ (the trapezium is kept isosceles so that it closes up neatly as θ is changed):

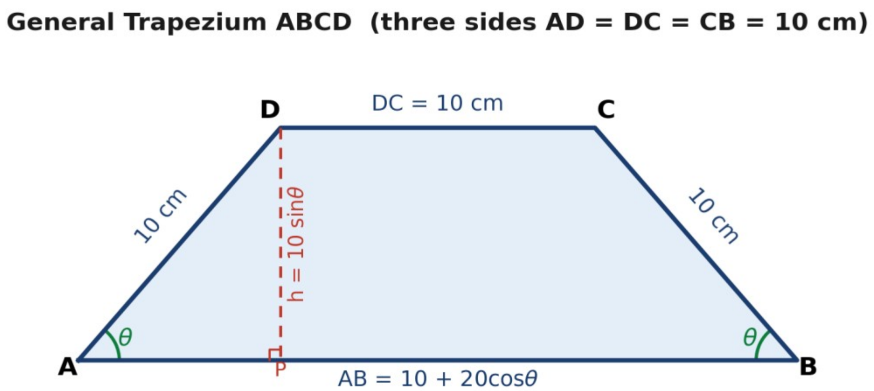


Fig. 1 — Trapezium ABCD with AD = DC = CB = 10 cm and base angle ϑ

As the hinge angle θ is opened or closed, point D moves out to D′ and C to C′, so the base AB and the height h change continuously while the three 10 cm sides stay fixed — this is exactly the freedom the activity explores.

5. Procedure

27. Cut three cardboard strips, each exactly 10 cm long. Join AD–DC and DC–CB with paper fasteners so the joints can rotate freely (two hinges at D and C).
28. Fix the strip AD at one end (point A) on a base line drawn on chart paper, and similarly fix CB at point B, keeping AB along the same straight base line.
29. Set the base angle θ (angle DAB) to 30° using a protractor. Adjust angle CBA to the same value θ so the figure is a symmetrical (isosceles) trapezium.
30. Trace the trapezium ABCD so formed on the graph paper. Measure and record the base AB and the height h (perpendicular distance between AB and DC) directly from the tracing.
31. Calculate the area using Area = ½ (AB + DC) × h, and enter all values in the observation table.
32. Repeat steps 3–5 for θ = 40°, 50°, 60°, 70°, 80° and 90°, opening the hinge a little further each time.
33. Plot Area (cm^2) on the y-axis against θ (degrees) on the x-axis on graph paper and locate the angle at which the curve peaks.
34. Verify the angle located graphically by an analytical (calculus) derivation, carried out below.

6. Observation Table

Sample readings (to be re-measured and filled in by the student from their own paper model):

S.No.	θ	AB = $10+20\cos\theta$ (cm)	h = $10 \sin\theta$ (cm)	$\frac{1}{2}(AB+10) \times h$	Area (cm^2)
1	30°	27.32	5.00	½ × 37.32 × 5.00	93.30
2	40°	25.32	6.43	½ × 35.32 × 6.43	113.51
3	50°	22.86	7.66	½ × 32.86 × 7.66	125.84
4	60°	20.00	8.66	½ × 30.00 × 8.66	129.90
5	70°	16.84	9.40	½ × 26.84 × 9.40	126.10
6	80°	13.47	9.85	½ × 23.47 × 9.85	115.59
7	90°	10.00	10.00	½ × 20.00 × 10.00	100.00

7. Calculation (Analytical Verification)

Let the base angles be θ. Drop perpendiculars from D and C to AB, meeting it at P and Q. Then:

(xlii) Height: $h = 10 \sin \theta$

(xliii) $AP = QB = 10 \cos \theta$, so $AB = DC + AP + QB = 10 + 20 \cos \theta$

Area of trapezium ABCD:

$A(\theta) = \frac{1}{2} (AB + DC) \times h = \frac{1}{2} (10 + 20 \cos \theta + 10) \times 10 \sin \theta$

$A(\theta) = \frac{1}{2} (20 + 20 \cos \theta)(10 \sin \theta) = 100 \sin \theta (1 + \cos \theta), \quad 0 < \theta < 90°$

Differentiating with respect to θ:

$dA/d\theta = 100 [\cos \theta (1 + \cos \theta) + \sin \theta (-\sin \theta)] = 100 [\cos \theta + \cos^2\theta - \sin^2\theta]$

$= 100 [\cos \theta + \cos^2\theta - (1 - \cos^2\theta)] = 100 [2\cos^2\theta + \cos \theta - 1]$

For maximum/minimum, $dA/d\theta = 0$:

$2\cos^2\theta + \cos \theta - 1 = 0 \implies (2\cos \theta - 1)(\cos \theta + 1) = 0$

$\implies \cos \theta = \frac{1}{2}$ or $\cos \theta = -1$ (rejected, since $0 < \theta < 90°$)

$\implies \theta = 60°$

Second derivative test:

$d^2A/d\theta^2 = 100 [-4 \cos \theta \sin \theta - \sin \theta] = -100 \sin \theta (4 \cos \theta + 1)$

At $\theta = 60°$: $\sin 60° = \sqrt{3}/2 > 0$ and $(4 \cos 60° + 1) = 3 > 0$, so $d^2A/d\theta^2 < 0$.

Since the second derivative is negative, $A(\vartheta)$ is maximum at $\vartheta = 60°$.

Maximum area:

$A(60°) = 100 \sin 60° (1 + \cos 60°) = 100 \times (\sqrt{3}/2) \times (3/2) = 75\sqrt{3}$ cm$^2 \approx 129.9$ cm^2

At this angle: $AB = 10 + 20 \times ½ = 20$ cm and $h = 10 \times (\sqrt{3}/2) = 5\sqrt{3}$ cm ≈ 8.66 cm.

8. Graph of A(θ)

The graph below (plotted from the observation table, and matching the calculus result) confirms a single maximum near θ = 60°:

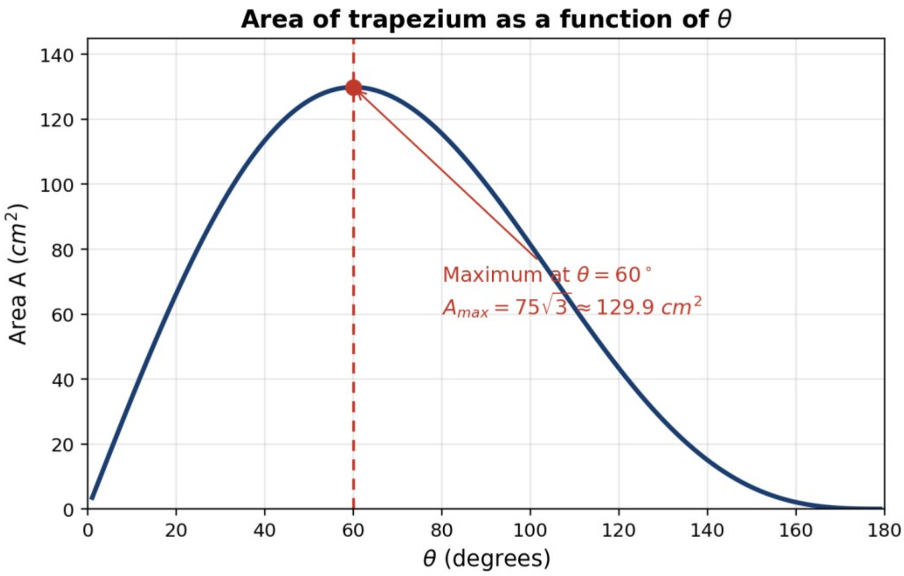


Fig. 2 — Area A vs. base angle ϑ, peaking at ϑ = 60°

9. The Trapezium of Maximum Area

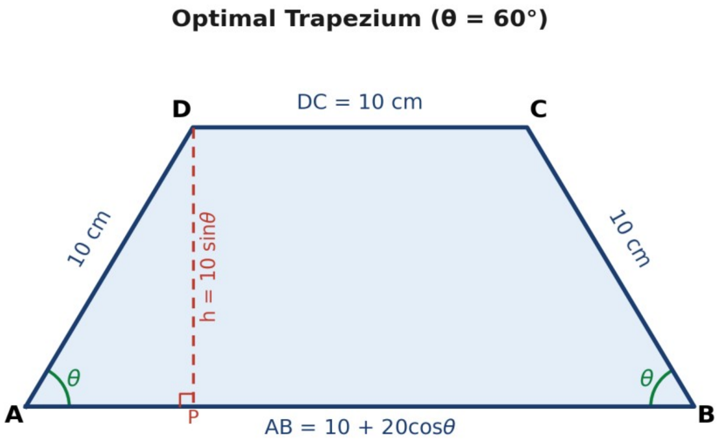


Fig. 3 — Trapezium of maximum area: ϑ = 60°, AB = 20 cm, DC = 10 cm, h = 5√3 cm

Note that when θ = 60°, triangles ADP and BCQ are each half of an equilateral triangle of side 10 cm — a neat geometric check of the calculus answer.

10. Result

The area of the trapezium (with the three sides other than the base each equal to 10 cm) is maximum when the base angle θ = 60°, i.e. when the base AB = 20 cm and the height h = 5√3 cm.

Maximum Area = 75√3 cm^2 ≈ 129.9 cm^2

11. Learning Outcome

(xliv) Understood how to express a geometric quantity (area) as a function of a single variable (θ) using trigonometric ratios.

(xlv) Applied the first- and second-derivative tests to locate and confirm a maximum.

(xlvi) Verified an analytical (calculus) result experimentally, using a hands-on hinged model and a graph.

12. Application

This activity illustrates optimisation, which is used in real life to design structures and objects (e.g. cross-sections of canals, dams, and trays; the trapezoidal shape of a folded metal sheet) so that a fixed length of material encloses the maximum possible area, minimising material cost for a required capacity.

13. Precautions

(xlvii) Keep the two base angles exactly equal while opening the hinge, so the trapezium remains symmetrical (isosceles).

(xlviii) Measure the height h as the perpendicular distance between the two parallel sides, not along the slant side.

(xlix) Use a sharp pencil and read the protractor carefully to avoid errors in θ.

14. Viva-Voce Questions

Q1. Why must only the two base angles (not all four angles) be kept equal in this activity?

Q2. Why is $\theta = -1$ for $\cos\theta$ rejected while solving $2\cos^2\theta + \cos\theta - 1 = 0$?

Q3. What condition on the second derivative confirms a maximum rather than a minimum?

Q4. Geometrically, why do the two right triangles cut off at the base become 'half of an equilateral triangle' exactly when $\theta = 60°$?

Q5. If the three equal sides were 12 cm instead of 10 cm, what would be the maximum area?

Q6. Is the trapezium of maximum area here also a special quadrilateral? Identify it (Hint: check AB, and the base angles).

Teacher's Signature: ____________________	Marks Obtained: _______ / 10