केन्द्रीय विद्‌यालय संगठन

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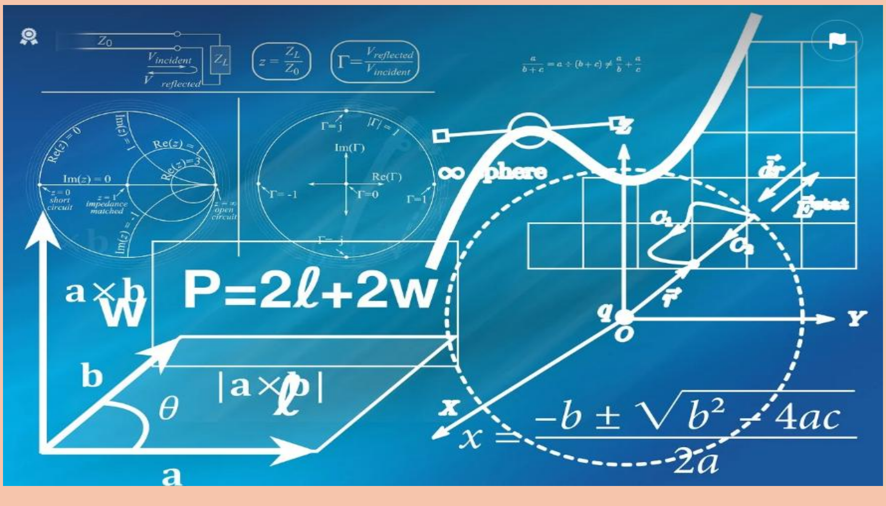
केन्द्रीय विद्यालय संगठन

REVISION NOTES FOR MATHEMATICS: CLASS XII

(BASED ON CBSE CURRICULUM)



संरक्षक

श्रीमती प्राची पांडे, आई.ए.एवं ए.एस.

आयुक्त

केन्द्रीय विद्यालय संगठन,नई दिल्ली

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श्रीमती श्रुति भार्गव

उपायुक्त एवं निदेशक,

केन्द्रीय विद्यालय संगठन

आंचलिक शिक्षा एवं प्रशिक्षण संस्थान

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REVISION NOTES FOR MATHEMATICS :CLASS XII

RELATION AND FUNCTION

Cartesian Product

A×B= Set of all possible ordered pairs from A to B

$$A = \{a, b, c\}, B = (1,2,3)$$

$$Then\ A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2, (b, 3), ((c, 1), (c, 2), (c, 3)\}$$

RELATION

It is a subset of $A \times B$ under a particular given condition

A={1,5,7,10}, B=(0,2,6,9} and relation from A to B defined as

$R: A \to B, R = \{(x, y): x < y\}$ (Set Builder form)

R={(1,2),(1,6),(1,9),(5,6),(5,9),(7,9)} (Roster Form)

Domain of R=set of elements of A which are related to any element of B={1,5,7}

Co-domain of R=set B

Range of R= set of elements of B which are related to any element of A={2,6,9}

Empty Relation – A relation is said to be an empty relation, if it has no elements, i.e. no element of set A is mapped or linked to any element of A. It is denoted by = ∅.	**Universal Relation –** A relation R in a set A is actually a universal relation if each element of A is related to every element of A, i.e., R = A × A. It is called the full relation.	**Identity Relation –** A relation R on A is said to be an identity relation if each and every element of A is related to itself, that is, R = {(a, a): for all a ∈ A}

Types of Relation ($R: A \to A$)

Reflexive

$if\ (a,a) \in R\ \forall\ a$

Symmetric

$if\ (a,b) \in R \to (b,a) \in R$

Transitive

$if\ (a,b), ((b,c) \in R\ \to (a,c) \in R$

Reflexive +Symmetric +Transitive

= Equivalence Relation

<u>EQUIVALENCE CLASS</u> : For an equivalence relation in a set X, the equivalence class of 'a' is denoted by [a],is the subset of X containing all elements b related to a.

<u>Number of relation R in a set A</u>, if n(A)= n	
Number of relations from A to A = 2^{n^2}	Number of reflexive relations from A to A = 2^{n^2-n}
Number of symmetric relations from A to A = $2^{\frac{n(n+1)}{2}}$	Number of relations from A to A which are not symmetric = $2^{(n^2-n)/2}$

Function:

A relation f: A →B is said to be a function if every element of A is related to a one & only one (unique) element of B.

Domain: Set A

Co-domain: Set B

Range: Set of elements of B which are related to any element of A

Range ⊂ Codomain B

Types of function

One - one (injective)

if distinct elements of set A have distinct images in B, i.e.

$x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ or

$f(x_1) = f(x_2)$ then $x_1 = x_2$

Onto function (surjective)

if each element in B has at least one pre image in A.

Co-domain=range

Bijective function (Invertible Function)

One - one (injective)+ Onto function (surjective)

INVERSE TRIGONOMETRIC FUNCTIONS

Number of possible functions

If a set A has p elements and set B has q elements, then

Total number of possible function from A to B	q^p		
One-one functions	p!	$\frac{q!}{(q-p)!}$	0
Onto functions	p!	0	$q^p - q_{c_1}(q-1)^p + q_{c_2}(q-2)^p + \cdots$
Bijective functions	p!	0	0

IF sinx =y → x=$\sin^{-1}$y ($\sin^{-1}y \neq \frac{1}{sin} y$)

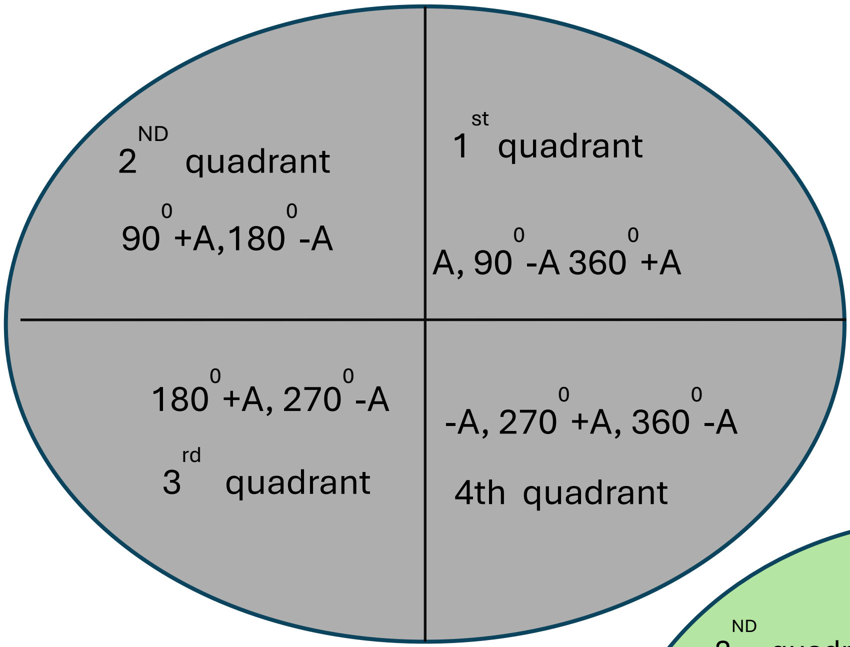
Function	Domain(x)	Range(y)	Function	Domain(x)	Range(y)
y=$\sin^{-1}$x	[-1,1]	[-π/2,π/2]	y=cosec^{-1}x	R-(-1,1)	[-π/2,π/2]-{0}
y=$\cos^{-1}$x	[-1,1]	[0,π]	y=$\sec^{-1}$x	R-(-1,1)	[0,π]-{π/2}
y=$\tan^{-1}$x	R →(-∞,∞)	(-π/2,π/2)	y=$\cot^{-1}$x	R	(0,π])

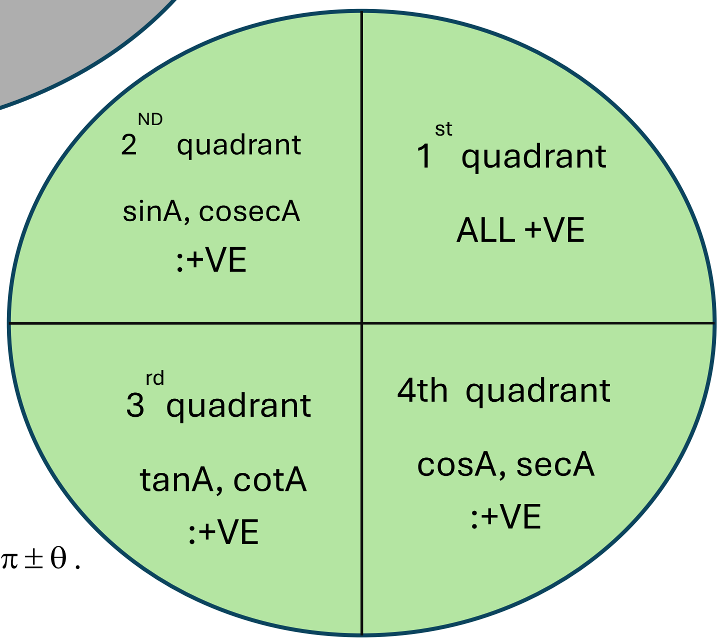
Properties of Inverse Trigonometric Functions

Properties	Condition	Properties	Condition
$\sin^{-1}$(sinx)=x	x ε[-π/2,π/2]	sin($\sin^{-1}$x)=x	x ε [-1,1]
$\cos^{-1}$(cosx)=x	x ε [0,π]	cos($\cos^{-1}$x)=x	x ε [-1,1]

$\tan^{-1}(\tan x)=x$	$x \varepsilon (-\pi/2,\pi/2)$	$\tan(\tan^{-1}x)=x$	$x \varepsilon R$
$\mathrm{cosec}^{-1}x= \sin^{-1}(1/x)$		$\sin 1(-x) =- \sin^{-1}x$	
$\sec^{-1}x =\cos^{-1}(1/x)$		$\cos^{-1}(-x) =\pi- \cos^{-1}x$	
$\cot^{-1}(1/x) = \tan^{-1}x$		$\tan^{-1}(-x) =- \tan^{-1}x$	

Other Important Formulas of Trigonometric Function





All T-ratio changes iN $\frac{\pi}{2}\pm\theta$ and $\frac{3\pi}{2}\pm\theta$

sin	$\Longleftrightarrow$	cos
tan	$\Longleftrightarrow$	cot
sec	$\Longleftrightarrow$	cosec

while remains unchanged in $\pi\pm\theta$ and $2\pi\pm\theta$.

$\sin\left(\frac{\pi}{2}\pm\theta\right) = \cos\theta$	$\sin(\pi\pm\theta) = \mp\sin\theta$
$\cos\left(\frac{\pi}{2}\pm\theta\right) = \mp\sin\theta$	$\cos(\pi\pm\theta) = -\cos\theta$
$\tan\left(\frac{\pi}{2}\pm\theta\right) = \mp\cot\theta$	$\tan(\pi\pm\theta) = \pm\tan\theta$
$\cot\left(\frac{\pi}{2}\pm\theta\right) = \mp\tan\theta$	$\cot(\pi\pm\theta) = \pm\cot\theta$

$\sec\left(\frac{\pi}{2} \pm \theta\right) = \mp\text{cosec}\theta$	$\sec(\pi \pm \theta) = -\sec\theta$
$\text{cosec}\left(\frac{\pi}{2} \pm \theta\right) = \sec\theta$	$\text{cosec}(\pi \pm \theta) = \mp\text{cosec}\theta$
$\sin\left(\frac{3\pi}{2} \pm \theta\right) = -\cos\theta$	$\sin(2\pi \pm \theta) = \pm\sin\theta$
$\cos\left(\frac{3\pi}{2} \pm \theta\right) = \sin\theta$	$\cos(2\pi \pm \theta) = \cos\theta$
$\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$	$\tan(2\pi \pm \theta) = \pm\tan\theta$
$\cot\left(\frac{3\pi}{2} \pm \theta\right) = \mp\tan\theta$	$\cot(2\pi \pm \theta) = \pm\cot\theta$
$\sec\left(\frac{3\pi}{2} \pm \theta\right) = \text{cosec}\theta$	$\sec(2\pi \pm \theta) = \sec\theta$
$\text{cosec}\left(\frac{3\pi}{2} \pm \theta\right) = -\sec\theta$	$\text{cosec}(2\pi \pm \theta) = \pm\text{cosec}\theta$

** Sum and Difference formule:

$\sin(A + B) = \sin A \cos B + \cos A \sin B$	$\sin(A - B) = \sin A \cos B - \cos A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$	$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
$\cot(A + B) = \frac{\cot A.\cot B - 1}{\cot B + \cot A}$	$\cot(A - B) = \frac{\cot A.\cot B + 1}{\cot B - \cot A}$

$\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$, $\quad$ $an\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$

$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$
$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

****Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions** :

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$	$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$
$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$	$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

Formulae for the transformation of a sum or difference of two circular functions into product of two circular functions :

$\sin C + \sin D = 2 \sin \frac{C + D}{2} \cos \frac{C - D}{2}$	$\sin C - \sin D = 2 \cos \frac{C + D}{2} \sin \frac{C - D}{2}$
$\cos C + \cos D = 2 \cos \frac{C + D}{2} \cos \frac{C - D}{2}$	$\cos C - \cos D = -2 \sin \frac{C + D}{2} \sin \frac{C - D}{2}$

**** Formulae for t-ratios of múltiple and sub-múltiple angles :**

$\sin 2A = 2 \sin A \cos A = \dfrac{2\tan A}{1+\tan^2 A}$.

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$1 + \cos 2A = 2\cos^2 A$, $1 - \cos 2A = 2\sin^2 A$,

$1 + \cos A = 2\cos^2 \dfrac{A}{2}$ $1 - \cos A = 2\sin^2 \dfrac{A}{2}$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$, $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$.

$\sin 3A = 3\sin A - 4\sin^3 A$, $\cos 3A = 4\cos^3 A - 3\cos A$

**** Some important trigonometric substitutions :**

$\sqrt{a^2+x^2}$	Put $x = a\tan\theta$ or $a\cot\theta$
$\sqrt{x^2-a^2}$	Put $x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$
$\sqrt{a+x}$ or $\sqrt{a-x}$ or both	Put $x = a\cos 2\theta$
$\sqrt{a^n+x^n}$ or $\sqrt{a^n-x^n}$ or both	Put $x^n = a^n\cos 2\theta$
$\sqrt{1+\sin 2\theta}$	$= \sin\theta + \cos\theta$
$\sqrt{1-\sin 2\theta}$	$= \cos\theta - \sin\theta,\ 0 < \theta < \dfrac{\pi}{4}$
	$= \sin\theta - \cos\theta,\ \dfrac{\pi}{4} < \theta < \dfrac{\pi}{2}$

Other useful Formulae:

$2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$, $2\cos^{-1}x = \cos^{-1}(2x^2-1)$

$$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} = \cos^{-1}\frac{1-x^2}{1+x^2} = \sin^{-1}\frac{2x}{1+x^2}$$

$3\sin^{-1}x = \sin^{-1}(3x-4x^3)$, $3\cos^{-1}x = \cos^{-1}(4x^3-3x)$,

$$3\tan^{-1}x = \tan^{-1}\frac{3x-x^3}{1-3x^2}$$

MATRICES & DETERMINANTS

A matrix is a rectangular array of $m\times n$ numbers arranged in m rows and n columns.

$$A=\begin{bmatrix} a_{11} & a_{12}\ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22}\ldots\ldots\ldots a_{2n} \\ & \\ a_{m1} & a_{m2}\ldots\ldots\ldots a_{mn}\end{bmatrix}_{m\times n} \quad \begin{array}{l}\rightarrow \textbf{ROWS} \\ \downarrow \textbf{COLUMNS}\end{array} \qquad A=\begin{array}{c|ccc} & & & \\ R_1 & a_{11} & a_{12} & a_{13} \\ R_2 & a_{21} & a_{22} & a_{23} \\ R_3 & a_{31} & a_{32} & a_{33} \\ & C_1 & C_2 & C_3\end{array}$$

OR

$A=[a_{ij}]_{m\times n}$, where **i** = 1, 2,…., m ; **j** = 1, 2,….,n.

* **Row Matrix** : A matrix which has only one row is called row matrix.
$A=[a_{ij}]_{1\times n}$

* **Column Matrix** : A matrix which has only one column is called column matrix.
$A=[a_{ij}]_{m\times 1}$.

* **Square Matrix**: A matrix in which number of rows are equal to number of columns, is called a square matrix $A=[a_{ij}]_{m\times m}$

$$A=\begin{bmatrix} \boxed{a_{11}} & a_{12} & a_{13} \\ a_{21} & \boxed{a_{22}} & a_{23} \\ a_{31} & a_{32} & \boxed{a_{33}}\end{bmatrix}$$ In square matrix elements a_{ii} are called diagonal elements

In above matrix a_{11}, a_{22}, a_{33} are diagonal elements.

* **Diagonal Matrix** : A square matrix is called a Diagonal Matrix if all non-diagonal elements are zero.the elements, $a_{ij}=0$,$i\neq j$

(There is no condition on diagonal elements.)

* **Scalar Matrix**: A square matrix is called scalar matrix it all the elements, except diagonal elements are zero and diagonal elements are same non-zero quantity.
$A=[a_{ij}]_{n\times n}$, where $a_{ij}=0$ if $i\neq j$. and $a_{ij}=k$, $i=j$.

* **Identity or Unit Matrix** : A square matrix in which all the non-diagonal elements are zero and diagonal elements are unity is called identity or unit matrix.
$A=[a_{ij}]_{n\times n}$, where $a_{ij}=0$ if $i\neq j$. and $a_{ij}=1$, $i=j$.

* **Null Matrix** : A matrices in which all element are zero.

* **Equal Matrices** : Two matrices are said to be equal if they have same order and all their corresponding elements are equal.

* **Multiplication by a scalar with a matrix :**

$kA=k\,[a_{ij}]_{m\times n}=[k\,(a_{ij})]_{m\times n}$

***Properties of matrix addition :**

(i) **Commutative Law :**If A = and B are matrices of the same order, then A + B = B + A.

(ii)**Associative Law :**For any three matrices A, B &C of the same order,
(A + B) + C = A + (B + C).

(iii)**Existence of identity:** Let A be an $m\times n$ matrix and O be an $m\times n$ zero matrix, then A + O = O + A = A. i.e. O is the additive identity for matrix addition.

$$O=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix} of\ order\ 3\times 3, \qquad O=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix} of\ order\ 3\times 2$$

(iv) **Additive inverse :** Let A be any matrix, then we have another matrix as – A such that

A + (– A) = (– A) + A= O. So – A is the additive inverse of A or negative of A.

* **Properties of scalar multiplication of a matrix :** If A and B be two matrices of the same order, and k and l are scalars, then k(A +B) = k A + kB,

***Product of matrices:** If A& B are two matrices, then product AB is defined, if number of column of A = number of rows of B.

i.e $A=\left[a_{ij}\right]_{m\times n}$ $and\ B=\left[b_{jk}\right]_{n\times p}$ $then\ AB=\left[c_{ik}\right]_{m\times p},\ where\ c_{ik}=\sum_{j=1}^{n}a_{ij}b_{jk}$

$$A=\begin{bmatrix}\boxed{a_{11} & a_{12}}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{bmatrix}_{3\times 2}\quad and\ B=\begin{bmatrix}\boxed{\begin{matrix}b_{11}\\ b_{21}\end{matrix}} & \begin{matrix}b_{12}\\ b_{22}\end{matrix} & \begin{matrix}b_{13}\\ b_{23}\end{matrix}\end{bmatrix}_{2\times 3}$$ then order of ab=3 × 3

$$AB=C=\begin{bmatrix}a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} & a_{11}b_{13}+a_{12}b_{23}\\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} & a_{21}b_{13}+a_{22}b_{23}\\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} & a_{31}b_{13}+a_{32}b_{23}\end{bmatrix}$$

***Properties of multiplication of matrices :**

(i) Product of matrices is not commutative. i.e. AB ≠ BA.

(ii) Product of matrices is associative. i.e. A(BC) = (AB)C

(iii) Product of matrices is distributive over addition i.e.(A+B) C = AC + BC

(iv) For every square matrix A, there exist an identity matrix of same order such that IA = AI = A.

* **Transpose of matrix** : If A is the given matrix, then the matrix obtained by interchanging the rows and columns is called the transpose of a matrix.

$$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\\ a_{31} & a_{32}\end{bmatrix}_{3\times 2}\quad then\ A^{T}(OR\ A')=\begin{bmatrix}a_{11} & a_{21} & a_{31}\\ a_{12} & a_{22} & a_{32}\end{bmatrix}_{2\times 3}$$

* **Properties of Transpose** :

If A & B are matrices such that their sum & product are defined, then

(i). $(A^T)^T = A$ (ii). $(A+B)^T = A^T + B^T$ (iii). $(KA^T) = K.A^T$ where K is a scalar.

(iv). $(AB)^T = B^T A^T$ (v). $(ABC)^T = C^T B^T A^T$.

* **Symmetric Matrix** : A square matrix is said to be symmetric if $A = A^T$

i.e. If $A=[a_{ij}]_{m\times m}$, then $a_{ij}=a_{ji}$ for all i, j.

Also elements of the symmetric matrix are symmetric about the main diagonal

* **Skew symmetric Matrix** : A square matrix is said to be skew symmetric if $A^T = -A$.

If $A=[a_{ij}]_{m\times m}$, then $a_{ij}=-a_{ji}$ for all i, j.

* Every square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

$$A=\frac{1}{2}(A+A^{T})+\ \frac{1}{2}(A-A^{T})$$

(Symmetric) **(Skew Symmetric)**

* **Determinant** :To every square matrix we can assign a number called determinant
If $A = [a_{11}]$, det. $A = |A| = a_{11}$.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $|A| = a_{11}a_{22} - a_{21}a_{12}$.

* **Properties** :

(i) The determinant of the square matrix A is unchanged when its rows and columns are interchanged. $|A| = |A'|$

(ii) The determinant of a square matrix obtained by interchanging two rows (or two columns) is negative of given determinant.

(iii) If two rows or two columns of a determinant are identical, value of the determinant is zero.

(iv) If all elements of a row or column of a square matrix A are multiplied by a non-zero number k, then determinant of the new matrix is k times the determinant of A.

(v) If elements of any one column (or row) are expressed as sum of two elements each, then determinant can be written as sum of two determinants.

(vi) Any two or more rows (or column) can be added or subtracted proportionally.

(vii) If A & B are square matrices of same order, then $|AB| = |A|\,|B|$

***Singular matrix**:

A square matrix 'A' of order 'n' is said to be singular, if $|A| = 0$.

* **Non -Singular matrix** :

A square matrix 'A' of order 'n' is said to be non-singular, if $|A| \neq 0$.

* If A and B are non-singular matrices of the same order, then AB and BA are also non-singular matrices of the same order.

* Let A be a square matrix of order $n \times n$, then $|kA| = k^n|A|$.

***Minor** of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.

Minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order $n - 1$.

* **Cofactor** of an element a_{ij}, denoted by A_{ij} is defined by $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is minor of a_{ij}.

* If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

$$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ \cancel{a_{21}} & a_{22} & a_{23} \\ \cancel{a_{31}} & a_{32} & a_{33} \end{bmatrix}$$

cofactor of $a_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = +(a_{22}a_{33} - a_{32}a_{23}) = c_{11}$

$$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & \cancel{a_{22}} & a_{23} \\ a_{31} & \cancel{a_{32}} & a_{33} \end{bmatrix}$$

cofactor of $a_{12} = (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = -(a_{21}a_{33} - a_{31}a_{23}) = c_{12}$

Then cofactor matrix $C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$

***Adjoint of matrix** :

If $A=[a_{ij}]$ be a square matrix then transpose of a matrix $C=[c_{ij}]$, where c_{ij} is the cofactor of a_{ij} element of matrix A, is called the adjoint of A.

Adjoint of A = Adj. A $= C^T = \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$.

A(Adj.A) = (Adj. A)A = | A| I.

* If A be any given square matrix of order n, then A(adjA) = (adjA) A = |A| I,

* If A is a square matrix of order n, then. $|adjA| = |A|^{n-1}$

***Inverse of a matrix** : Inverse of a square matrix A exists, if A is non-singular or square matrix A is said to be invertible and $A^{-1} = \frac{1}{|A|} adjA$

***System of Linear Equations** :

I) Solve the system of equations

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$.

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow AX = B \Rightarrow X = A^{-1}B \quad ; \{\,|A| \neq 0\}.$$

II) If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ Find A^{-1} and hence solve the system of equation

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$.

Sol. First find the A^{-1} by cofactor method then

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow AX = B \Rightarrow X = A^{-1}B$$

III) If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_2 & c_2 & c_3 \end{bmatrix}$ Find A^{-1} and hence solve the system of equation

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$

Sol. First find the A^{-1} by cofactor method then

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow A'X = B \qquad \Rightarrow X = (A')^{-1}B \;\; \Rightarrow X = (A^{-1})'B$$

IV) If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $and\ B = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix}$

Find AB and hence solve the system of equation

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$

Sol. AB$= \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = kI$ then $A^{-1} = \frac{1}{k}B\ and\ B^{-1} = \frac{1}{k}A$

Then $\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow AX = C \;\; \Rightarrow X = A^{-1}C = \frac{1}{k}BC$

V) If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_2 & c_2 & c_3 \end{bmatrix}$ and $B = \begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{bmatrix}$

Find AB and hence solve the system of equation

$a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$

AB= $\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \lambda I$ then $A^{-1} = \frac{1}{\lambda}B$ and $B^{-1} = \frac{1}{\lambda}A$

Then $\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow A'X = C \Rightarrow X = (A')^{-1}C = \frac{1}{\lambda}(A^{-1})'C = \frac{1}{\lambda}B'C$

***Criteria of Consistency**.

(i) If $|A| \neq 0$, then the system of equations is said to be consistent & has a unique solution.

(ii) If $|A| = 0$ and (adj. A)B = 0, then the system of equations is consistent and has infinitely many solutions.

(iii) If $|A| = 0$ and (adj. A)B$\neq$ 0, then the system of equations is inconsistent and has no solution.

$A(\text{adj}A) = \|A\|I = (\text{adj}A)A$	Adj(AB)=(adjB)(adjA)	$\text{adj}A^T = (\text{adj}A)^T$	
$\text{adj}(\text{adj}A) = \|A\|^{n-2}A$	$\|\text{adj}A\| = \|A\|^{n-1}$	$\|\text{adj}(\text{adj}A)\| = \|A\|^{(n-1)^2}$	
$\|A(\text{adj}A)\| = \|\|A\|I\| = \|A\|^n$	$\|A^{-1}\| = \frac{1}{\|A\|}$	$(A^T)^{-1} = (A^{-1})^T$	

CONTINUITY& DIFFERENTIABILITY

Important concepts

* A function f is said to be continuous at x = a if

Left hand limit = Right hand limit = value of the function at x = a

i.e. $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$ LHL=RHL=f(a)

i.e. $\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+h) = f(a)$

* A function is said to be differentiable at x = a

if $\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ (LHD=RHD)

$\frac{d}{dx}(x^n) = n\,x^{n-1}$	$\frac{d}{dx}(x) = 1$	$\frac{d}{dx}(c) = 0, \forall\, c \in R$
$\frac{d}{dx}(a^x) = a^x \log a, a > 0, a \neq 1.$	$\frac{d}{dx}(e^x) = e^x.$	$\frac{d}{dx}(\|x\|) = \frac{x}{\|x\|}, x \neq 0$
$\frac{d}{dx}(\log_a\|x\|) = \frac{1}{x \log a},$	$\frac{d}{dx}(\log x) = \frac{1}{x}, x > 0$	$\frac{d}{dx}(ku) = k\frac{du}{dx}$

$a > 0, a \neq 1, x \neq 0$		
$\frac{d}{dx}(\sin x) = \cos x, \forall x \in R.$	$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}.$	$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$
$\frac{d}{dx}(\cos x) = -\sin x, \forall x \in R.$	$\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}.$	$\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$
$\frac{d}{dx}(\tan x) = \sec^2 x, \forall x \in R.$	$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$, $\forall x \in R$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x, \forall x \in R.$	$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$, $\forall x \in R.$	$f(x)^{g(x)} = e^{g(x)\log_e f(x)}$
$\frac{d}{dx}(\sec x) = \sec x \tan x, \forall x \in R.$	$\frac{d}{dx}(\sec^{-1}x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}.$	
$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$, $\forall x \in R.$	$\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	Differentiation by Chain Rule: $\frac{d}{dx}f(\phi(x)) = f'(\phi(x))\phi'(x)$

Differentiation of parametric form: $x=f_1(t)$, $y=f_2(t)$

then $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dx}} = \phi(t)\ (let)$ $\quad$ $then\ \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\phi(t)}{\frac{dx}{dt}}$

Differentiation by using logarithm:

If $y = f(x)^{\phi(x)}$ $then\ we\ can\ write\ y = e^{\phi(x)\log(f(x))}$ $now\ we\ can\ find\ \frac{dy}{dx} by\ chain\ rule$

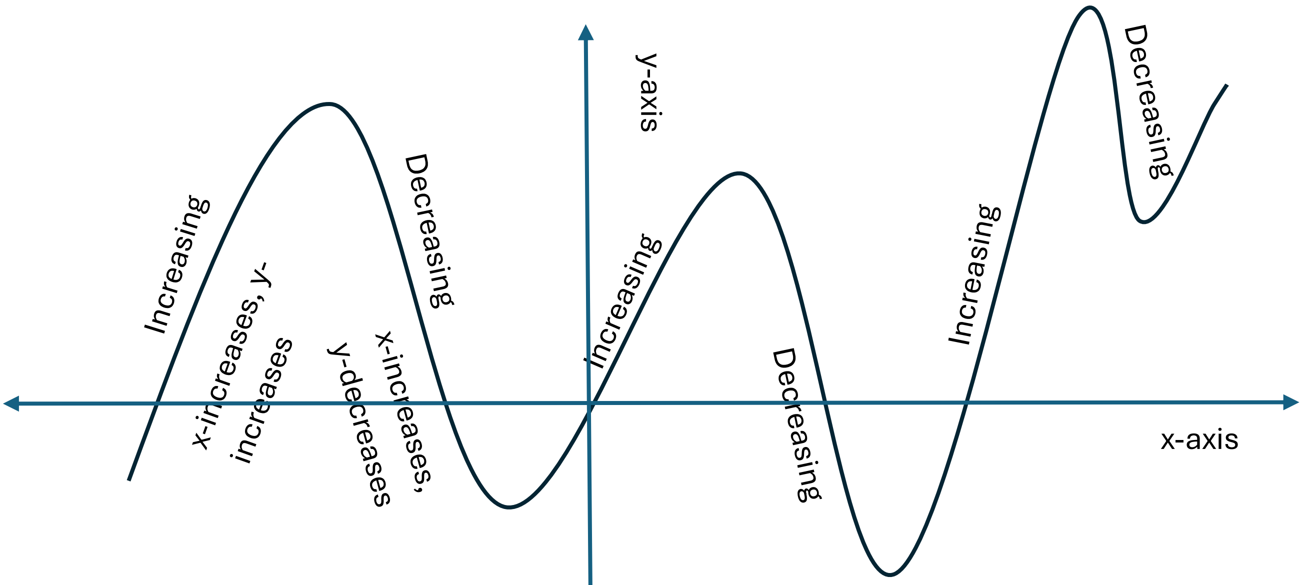
APPLICATION OF DERIVATIVE

RATE OF CHANGE OF QUANTITIES

Rate of change: Let y=f(x) be a relation between two variables x and y. Then dy/dx represents the rate of change of y w.r.t. x and $(dy/dx)_{x=a}$ represents rate of change of y w.r.t. x at x=a.

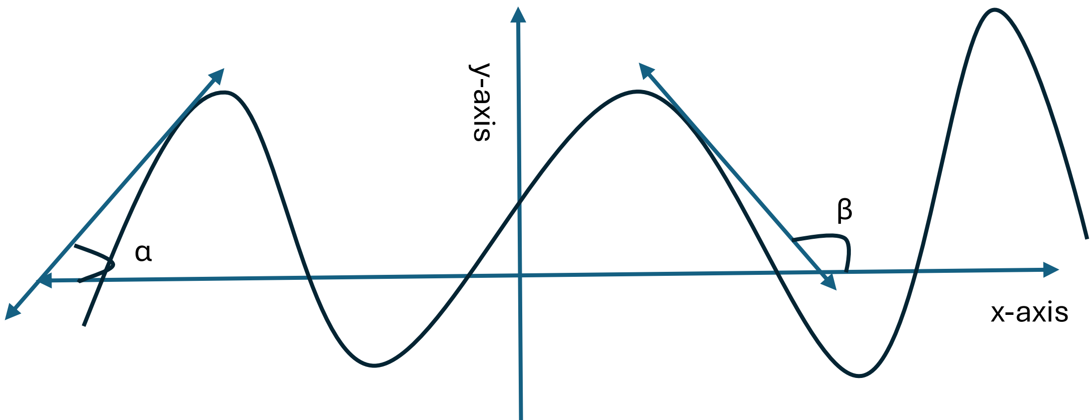
$(dy/dx)_{x=a}$ also represents the slope of tangent to the curve y=f(x) at x=a.

Increasing and Decreasing Function



If in the graph of the curve:

Condition	Nature of function
x-increases, y-increases	→Increasing function
x- increases, y-decreases	→ Decreasing Function
x-decreases, y-decreases	→ Increasing function
x-decreases, y-increases	→ decreasing function



In the above figure:

α→acute angle →tanα=+ve →slope of tangent=+ve →f'(x)=+ve and function is increasing

β→obtuse angle →tanβ=-ve →slope of tangent=-ve →f'(x)=-ve and function is decreasing

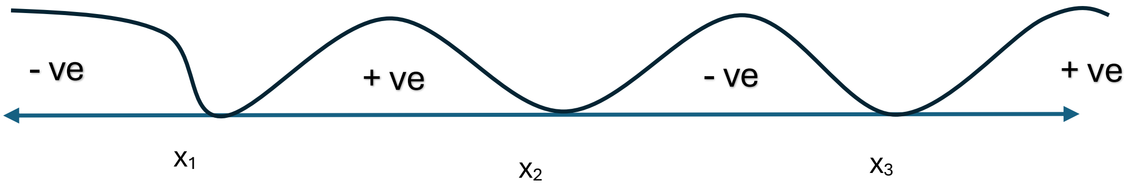
it means: For increasing function f'(x)=+ve or f'(x)>0

For decreasing Function f'(x)=-ve or f'(x)<0

For critical points: f'(x)=0

For finding the interval where y=f(x) is increasing or decreasing

y=f(x) , for critical points f'(x)=0→ $x=x_1, x_2, x_3$ mark the values on number line



Take any value of x=c other than x_1, x_2, x_3 and check the sign of f'(x)

Let f'(c)= +ve and c ε (x_1, x_2) → between x_1, x_2 there will be +ve sign others in alternate pattern

Now for Increasing Function: f'(x)>0 → +ve parts → $x \in (x_1, x_2) \cup (x_3, \infty)$

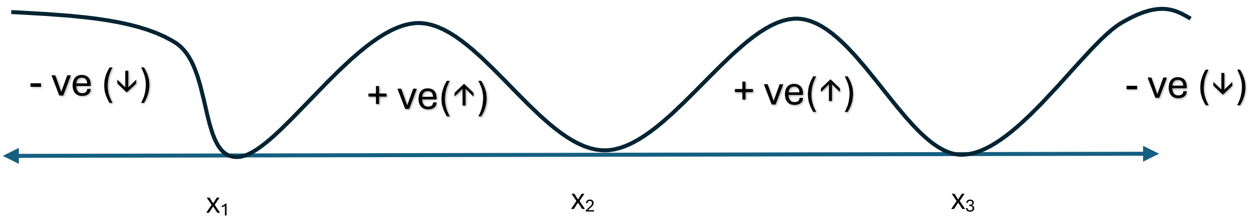
Now for Decreasing Function: f'(x)<0 → -ve parts → $x \in (-\infty, x_1) \cup (x_2, x_3)$

If y=f(x) and for critical points f'(x)=0

→$(x-x_1)(x-x_2)^{2(}x-x_3)^3=0$→ $x=x_1, x_2, x_2, x_3, x_3, x_3$

If value of x=a is repeated (even number of times)	Same sign on the both sides of x=a
If value of x=a is repeated (even number of times)	opposite signs on the both sides of x=a

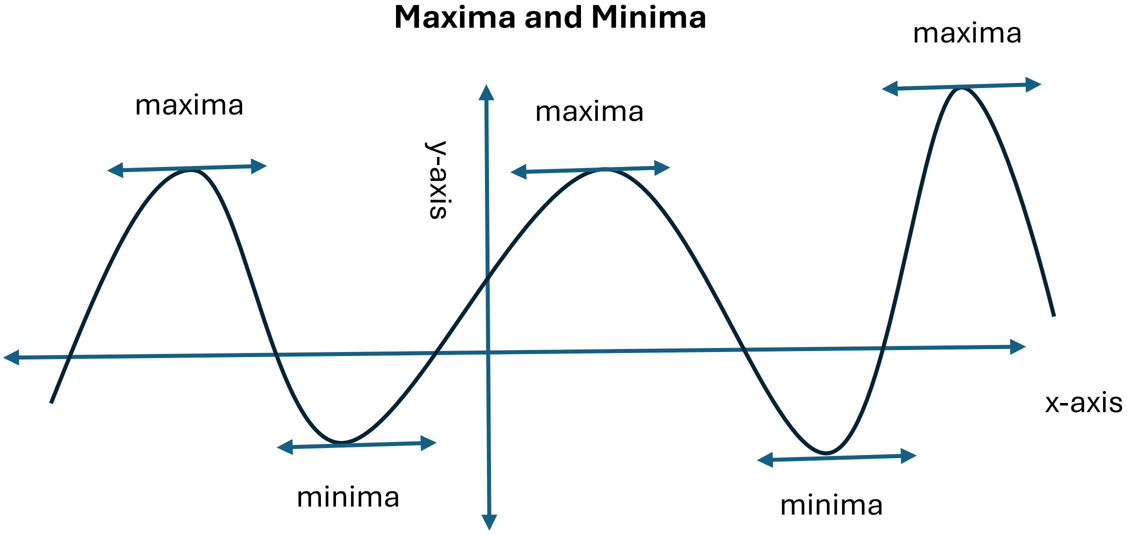
Let f'(c)= +ve and c ε (x_1, x_2) → between x_1, x_2 there will be +ve sign



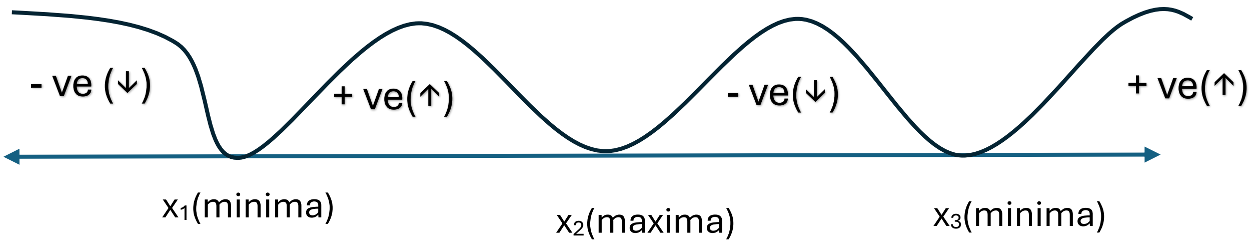
Now for Increasing Function: f'(x)>0 → +ve parts → $x \in (x_1, x_2) \cup (x_2, x_3)$

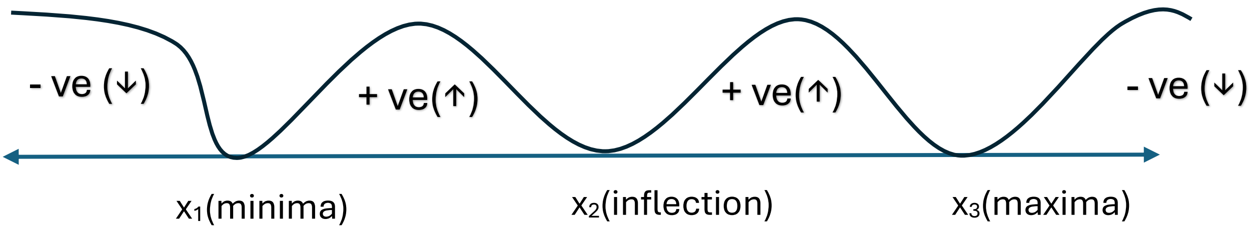
Now for Decreasing Function: f'(x)<0 → -ve parts → $x \in (-\infty, x_1) \cup (x_3, \infty)$

Maxima and Minima



ON NUMBER LINE

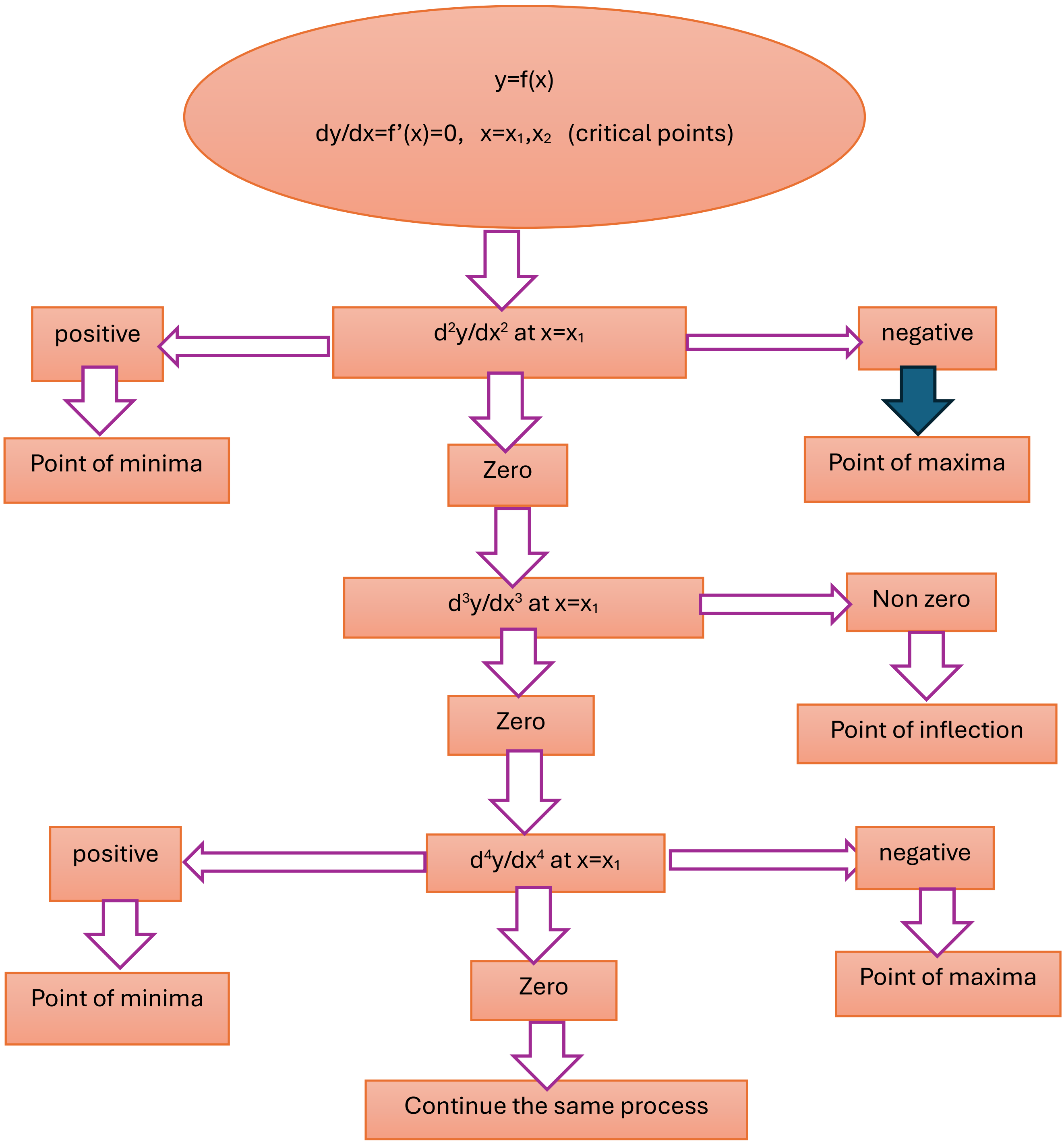




The critical points where

the curve changes it's nature from increasing to decreasing	**→ point of maxima**
the curve changes it's nature from decreasing to increasing	**→ point of minima**
the curve changes it's nature from increasing to increasing OR decreasing to decreasing	**→ point of inflection**

HIGHER DERIVATIVE TEST FOR LOCAL MAXIMA AND MINIMA



INDEFINITE INTEGRAL

1. If $\frac{d}{dx}f(x) = F(x)$,Then we write $\int f\ (x)dx\ =\ F(x) +\ c$ These integrals are called indefinite integrals; C is called a constant of integration.

Derivatives	Integrals (Anti derivatives)
$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$	$\int x^n\ dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$
$\frac{d}{dx}(sinx) = cosx$	∫cos x dx = sin x +C
$\frac{d}{dx}(-cosx) = sinx$	∫sin x dx=- cos x +C
$\frac{d}{dx}(tanx) = sec^2x$	$\int sec^2x\,dx = tanx + C$
$\frac{d}{dx}(-cotx) = cosec^2x$	$\int cosec^2x\,dx = -cotx + C$
$\frac{d}{dx}(secx) = secx\ tanx$	$\int secx\ tanx\,dx = secx + C$
$\frac{d}{dx}(-cosecx) = cosecx\ cotx$	$\int -cosecx\ cotx\,dx = cosecx + C$
$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}dx = sin^{-1}x + C$
$\frac{d}{dx}(-cos^{-1}x) = \frac{1}{\sqrt{1-x^2}}$	$\int -\frac{1}{\sqrt{1-x^2}}dx = cos^{-1}x + C$
$\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}dx = tan^{-1}x + C$
$\frac{d}{dx}(-cot^{-1}x) = \frac{1}{1+x^2}$	$\int -\frac{1}{1+x^2}dx = cot^{-1}x + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x\ dx = e^x + C$
$\frac{d}{dx}(log\|x\|) = \frac{1}{x}$	$\int \frac{1}{x}\ dx = log\|x\| + C$
$\frac{d}{dx}(a^x) = a^x loga$	$\int a^x loga\,dx = a^x + C$
$\frac{d}{dx}$(log |sec x|) =$tanx$	$\int tanxdx$ = log |sec x|+C
$\frac{d}{dx}$(log sin |x|) =cot$x$	∫ cot x dx = log |sin x|+C
$\frac{d}{dx}$(log |sec x + tan x |) =sec x	∫ sec x dx = log |sec x+ tan x|+C
$\frac{d}{dx}$(log |cosec x - cot x |) =cosec x	∫ cosec x dx =log |cosec x - cot x |+C

2. Methods of integrations.

(a) Integration by substitution

Consider I = ∫ f (x) dx

$$Put\ x = g(t) so\ that\ \frac{dx}{dt} = g'(t). we\ write\ dx = g'(t)dt$$

$$I = \int f(x)dx = \int f\left(g(t)\right)g'(t)\,dt$$

(b) Integration using partial fractions

Numerator and denominator should be polynomial

Degree of numerator < degree of denominator

Form	Partial Fraction
$\frac{f(x)}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
$\frac{f(x)}{(x-a)(x-b)^3}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{(x-b)^2}+\frac{D}{(x-b)^3}$
$\frac{f(x)}{(x-a)(x^2+b)}$	$\frac{A}{(x-a)}+\frac{Bx+C}{(x^2+b)}$

(c) Integration by parts:

$$\int u.vdx = u\int vdx - \int\left(\frac{d}{dx}u\int vdx\right)dx$$

Selection of u and v : The function which become first in ILATE may be taken as u

Letters of **ILATE** Rule	Abbreviation	Examples
I	Inverse trigonometric functions	$sin^{-1}x, \cos^{-1}x, \tan^{-1}x\ etc.$
L	Logarithmic functions	log x, lnx
A	Algebraic functions	ax^2+bx+c
T	Trigonometric functions	sin x, cos x etc.
E	Exponential functions	e^x ,2^x

1. Integrals of Some Particular Functions

$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}log\left\|\frac{x-a}{x+a}\right\| + C$	$\int e^x[f(x)+f'(x)] = e^x f(x) + C$
$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}log\left\|\frac{a+x}{a-x}\right\| + C$	
$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$	
$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} + C$	$\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
$\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left\|x+\sqrt{x^2-a^2}\right\| + C$	$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left\|x+\sqrt{x^2-a^2}\right\| + C$
$\int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left\|x+\sqrt{x^2+a^2}\right\| + C$	$\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left\|x+\sqrt{x^2+a^2}\right\| + C$

Completing the square:

$$\mathbf{a}\boldsymbol{x}^2 + \boldsymbol{bx} + \boldsymbol{c} = \boldsymbol{a}\left(\boldsymbol{x}^2 \pm \frac{\boldsymbol{b}}{\boldsymbol{a}} \pm \frac{\boldsymbol{c}}{\boldsymbol{a}}\right) = \boldsymbol{a}\left(\left(\boldsymbol{x} \pm \frac{\boldsymbol{b}}{\mathbf{2}\boldsymbol{a}}\right)^{\mathbf{2}} - \left(\frac{\boldsymbol{b}}{\mathbf{2}\boldsymbol{a}}\right)^{\mathbf{2}} \pm \frac{\boldsymbol{c}}{\boldsymbol{a}}\right)$$

$$\int \frac{px+q}{\mathbf{ax^2+bx+c}}dx$$

$$\int \frac{px+q}{\sqrt{\mathbf{ax^2+bx+c}}}dx$$

$$\int (px+q)\sqrt{\mathbf{ax^2+bx+c}}\,dx$$

px+q= $\boldsymbol{A\frac{d}{dx}(ax^2+bx+c)+B}$

=A(2ax+b)+B

px+q =2Aax+(Ab+B)

Now finding the values of A and

$$\int \frac{A(2ax+b)+B}{\mathbf{ax^2+bx+c}}dx$$

$$\int \frac{A(2ax+b)+B}{\sqrt{\mathbf{ax^2+bx+c}}}dx$$

$$\int (A(2ax+b)+B)\sqrt{\mathbf{ax^2+bx+c}}\,dx$$

Let $\mathbf{ax^2+bx+c} =$ $\boldsymbol{t}$

$$\int \frac{A(2ax+b)+B}{\mathbf{ax^2+bx+c}}dx$$

$$= A\int \frac{(2ax++b)}{\mathbf{ax^2+bx+c}}dx + B\int \frac{1}{\mathbf{ax^2+bx+c}}dx$$

$$=A\int \frac{1}{\mathbf{t}}dt + B\int \frac{1}{\textbf{completing the square}}dx$$

$$Alogt + B(formula\ of\ \int \frac{1}{(x^2 \pm a^2)}dx\ OR \int \frac{1}{(a^2-x^2)}dx\ \ )$$

$$\int \frac{A(2ax+b)+B}{\sqrt{\mathbf{ax^2+bx+c}}}dx$$

$$=A\int \frac{(2ax++b)}{\sqrt{\mathbf{ax^2+bx+c}}}dx + B\int \frac{1}{\sqrt{\mathbf{ax^2+bx+c}}}dx$$

$$=A\int \frac{1}{\sqrt{\mathbf{t}}}dt + B\int \frac{1}{\sqrt{\textbf{completing the square}}}dx$$

$$= 2A\sqrt{t} + B(Formula\ of\ \int \frac{1}{\sqrt{x^2 \pm a^2}}dx\ OR\ \int \frac{1}{\sqrt{a^2-x^2}}dx\ \ )$$

$$\int (A(2ax+b) + B)\sqrt{\mathbf{ax^2+bx+c}}\,dx$$

$$= \int A(2ax+b)\sqrt{\mathbf{ax^2+bx+c}}\,dx + B\int \sqrt{\mathbf{ax^2+bx+c}}\,dx$$

$$= A\int \sqrt{t}dx + B\int \sqrt{completing\ the\ square}dx$$

$$= \frac{2A}{3}(t)^{3/2} + B(formula\ of\ \int \sqrt{x^2 \pm a^2}dx\ OR\ \int \sqrt{a^2 \pm x^2}dx)$$

DEFINITE INTEGRAL

- Let f be a continuous function on [a, b].
- If F is any antiderivative of f i.e $\int f(x)dx = F(x) + c$ defined on [a, b],

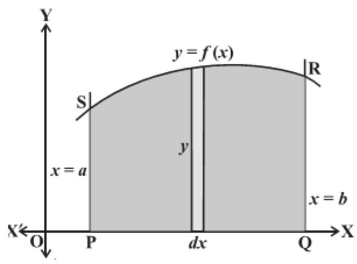
then the definite integral of f from a to b is defined by $\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$

GENERAL PROPERTIES OF DEFINITE INTEGRAL

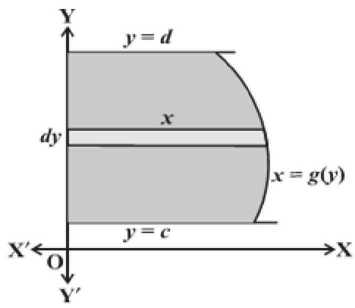
$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$	$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$	$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$
$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$	$\int_{-a}^a f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$
$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x). \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$	

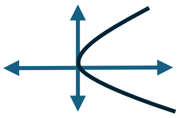
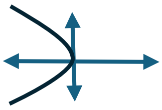
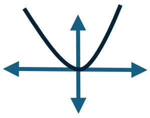
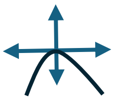
APPLICATIONS OF THE INTEGRALS
(Area of bounded Region)

** Area of the region PQRSP = $\int_a^b dA = \int_a^b y\,dx = \int_a^b f(x)\,dx$.



** The area A of the region bounded by the curve x = g (y), y-axis a
the lines y = c, y = d is given by A= $\int_c^d x\,dy = \int_c^d g(y)\,dy$



Equation and Shape	
	$y^2=4ax$
	$y^2= - 4ax$
	$x^2=4ay$
	$x^2= - 4ay$

DIFFERENTIAL EQUATIONS

Equation in terms of derivatives is known as differential equation

Order of Differential Equation: Order of highest order derivative

Degree of Differential equation: Highest power of highest order derivative (When equation is in polynomial form in terms of derivative)

Homogeneous Differential equation:

$\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}, where\ f_1(x,y) and\ f_2(x,y)$ are homogeneous functions of same degree

(For solution let y=vx, dy/dx=v +x dv/dx)

Linear Differential Equation:

1. $\frac{dy}{dx} + py = q$, where p & q are function of x or constant(independent of y)

   $Sol: y.e^{\int pdx} = \int (qe^{\int pdx})dx, where\ e^{\int pdx}\ is\ integrating\ factor$
2. $\frac{dx}{dy} + px = q$, where p & q are function of y or constant(independent of x)

   $Sol: x.e^{\int pdy} = \int (qe^{\int pdy})dy, where\ e^{\int pdy}\ is\ integrating\ factor$

VECTOR ALGEBRA

(xiii) SOME IMPORTANT RESULTS :

Let $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$

(a) $\vec{a} + \vec{b} = (x_1 + x_2)\hat{\imath} + (y_1 + y_2)\hat{\jmath} + (z_1 + z_2)\hat{k}$

(b) If $\vec{a} = \vec{b}$, then $x_1 = x_2,\ y_1 = y_2\ ,\ z_1 = z_2$

(c) $\lambda(\overrightarrow{a}) = \lambda\{x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}\} = \lambda x_1\hat{\imath} + \lambda y_1\hat{\jmath} + \lambda z_1\hat{k}$

(d) If $\vec{a}$ and $\vec{b}$ are collinear or parallel then $\vec{a} = \lambda\vec{b}$, $\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2} = \lambda$

By triangle law of addition: $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ = Position vector of P - Position vector of Q $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(xv) SECTION FORMULA:

Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$

R is a point which divide the line segment

(a) In the ratio m:n internally. O

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n} = \frac{(mx_2+nx_1)}{m+n}\hat{\imath} + \frac{(my_2+ny_1)}{m+n}\hat{\jmath} + \frac{(mz_2+nz_1)}{m+n}\hat{k}$$

(b) In the ratio m:n externally.

$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n} = \frac{(mx_2-nx_1)}{m-n}\hat{\imath} + \frac{(my_2-ny_1)}{m-n}\hat{\jmath} + \frac{(mz_2-nz_1)}{m-n}\hat{k}$$

(c) Mid-point formula.(ratio is 1:1)

$$\vec{r} = \frac{\vec{a}+\vec{b}}{2} = \frac{(x_2+x_1)}{m+n}\hat{\imath} + \frac{(y_2+y_1)}{2}\hat{\jmath} + \frac{(z_2+z_1)}{2}\hat{k}$$

Centriod:

(a) Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ are the position vectors of vertices of

Then $\overrightarrow{OG} = \frac{\vec{a}+\vec{b}+\vec{c}}{3} = \frac{(x_1+x_2+x_3)}{3}\hat{\imath} + \frac{(y_1+y_2+y_3)}{3}\hat{\jmath} + \frac{(z_1+z_2+z_3)}{3}\hat{k}$

(xvi) PRODUCT OF TWO VECTORS:

(1) Scalar Product(Dot product):

Let $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$ and θ is an angle between them. Then $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

<u>Some important results :</u>

- $\vec{a}.\vec{b}$ = is a real number.

$\vec{a}.\vec{b} = \vec{b}.\vec{a} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$	$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$	$\vec{a}.\vec{a} = \lvert\vec{a}\rvert\lvert\vec{a}\rvert = (\lvert\vec{a}\rvert)^2 = a^2$
If $\theta = 90^0$, then $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos 90^0 = 0$	$\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$	$\vec{a}.\vec{b} = \vec{b}.\vec{a}$
If $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$ **then** $\vec{a}.\vec{b} = x_1x_2 + y_1y_2 + z_1z_2$	Projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert}$	$\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$
$\vec{a}.\vec{a} = \lvert\vec{a}\rvert^2$	$(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = \lvert\vec{a}\rvert^2 - \lvert\vec{b}\rvert^2$	$\lvert\vec{a} + \vec{b}\rvert^2 = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\vec{a}.\vec{b}$

Vector Product(Cross product):

Let $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$

and $\boldsymbol{\theta}$ is an angle between them.

$\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}|\sin\theta)\hat{n}$ = is a vector quantity perpendicular to the plane containing vectors $\vec{a}$ and $\vec{b}$. Where $\hat{n}$ = is unit normal vectors.($\hat{n}$ is perndicular to both $\vec{a}$ and $\vec{b}$ it means $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta$	$\vec{a}\times\vec{b} = -(\vec{b} \times \vec{a})$	$\sin\theta = \frac{\lvert\vec{a}\times\vec{b}\rvert}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$
$\hat{n} = \frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert}$	**If** $\vec{a}\parallel\vec{b}$, then $\vec{a}\times\vec{b} = 0$	$\vec{\imath}\times\vec{\imath} = \vec{\jmath}\times\vec{\jmath} = \vec{k}\times\vec{k} = 0$
$\vec{\imath}\times\vec{\jmath} = \hat{k}$	$\vec{\jmath}\times\vec{k} = \hat{\imath}$	$\vec{k}\times\vec{\imath} = \hat{\jmath}$
If $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{b} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$ $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$		

- **<u>Application of cross product:</u>**

(1). If $\vec{a}$ and $\vec{b}$ are two adjacent sides of a triangle then area of triangle $= \frac{1}{2}|(\vec{a} \times \vec{b})|$

(2). If $\vec{a}$ and $\vec{b}$ are two adjacent sides of a parallelogram then area of parallelogram $=|(\vec{a} \times \vec{b})|$

(3). If $\vec{a}$ and $\vec{b}$ are two Diagonals of a Quadrilateral then area of Quadrilateral $=\frac{1}{2}|(\vec{a} \times \vec{b})|$

PROPERTIES OF CROSS PRODUCT:

1). Distributive property of vector product over addition(Distributive Law)

(i) $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$ (ii) $(\vec{b} + \vec{c}) \times \vec{a} = (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a})$

2). If $\boldsymbol{m}$ is a scalar, then $\boldsymbol{m}(\vec{a} \times \vec{b}) = (\overrightarrow{\boldsymbol{m}a}) \times \vec{b} = \vec{a} \times (\boldsymbol{m}\vec{b})$

<u>Langrange's Identity:</u> $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

<u>Some more important results of vector product:</u>

1). If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices A,B,C of a triangle ABC, then Area of triangle ABC = $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

LINEAR PROGRAMMING

** **An Optimisation Problem** : A problem which seeks to maximise or minimise a function is called an optimisation problem.

**** Objective Function Linear function** : Z = ax + by, where a and b are constants, which has to be maximised or minimised is called a linear objective function.

**** Constraints** : The linear inequalities or restrictions on the variables of an LPP are called constraints. The conditions $x \geq 0$, $y \geq 0$ are called non-negative constraints.

**** Feasible Region** : The common region determined by all the constraints including non-negative constraints $x \geq 0$, $y \geq 0$ of an LPP is called the feasible region for the problem.

**** Feasible Solutions**: Points within and on the boundary of the feasible region for an LPP represent feasible solutions.

**** Infeasible Solutions** : Any Point outside feasible region is called an infeasible solution.

**** Optimal (feasible) Solution :** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution. **Solving linear programming problem using Corner Point Method.**

The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points(vertices) either by inspection or by solving the two equations of the lines intersecting at the point.

2. Evaluate the objective function Z=ax+ by at each corner point. Let M and m, respectively denote the largest and smallest values of these points.

3. (i) When the feasible region is bounded, M and m are the maximum and minimum value of Z.

(ii) In case, the feasible region is unbounded, we have:

(a) M is the maximum value of Z , if the open half plane determined by ax+ by >M has no point in common with the feasible region.

Otherwise, Z has no maximum value.

(b) Similarly, m is the minimum value of Z, if the open half plane determined by ax+ by<m has no point in common with the feasible region.

Otherwise, Z has no minimum value.

Three Dimensional Geometry

Direction Ratios of line passing through two points (x_1,y_1,z_1) and (x_2,y_2,z_2)

$x_2-x_1 : y_2-y_1 : z_2-z_1 \rightarrow a,b,c$

Direction cosines of line

Line makes an angle α,β and γ with x, y, z axes respectively the direction cosines are cosα,cosβ cosγ→ l,m,n

$(l^2+m^2+n^2=1)$

a,b,c are direction ratios of line then

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}$

$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$

Angle between two lines

Dc's are l_1,m_1,n_1 and l_2,m_2,n_2 then

$\cos\Theta = l_1l_2+m_1m_2+n_1n_2$

Dr's are a_1,b_1,c_1 and a_2,b_2,c_2 then

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

If lines are perpendicular then $cos\theta = 0$ → $l_1l_2+m_1m_2+n_1n_2 = 0$ OR $a_1a_2+b_1b_2+c_1c_2 = 0$

Equation of line

(Passing through a point (x_1,y_1,z_1) with Dr's a,b,c

Cartesian Form

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Vector Form

$$\vec{r} = \left(x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}\right) + \lambda(a\hat{\imath} + b\hat{\jmath} + c\hat{k})$$

Equation of line

(Passing through two point (x_1,y_1,z_1) and (x_2,y_2,z_2)

Cartesian Form

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Vector Form

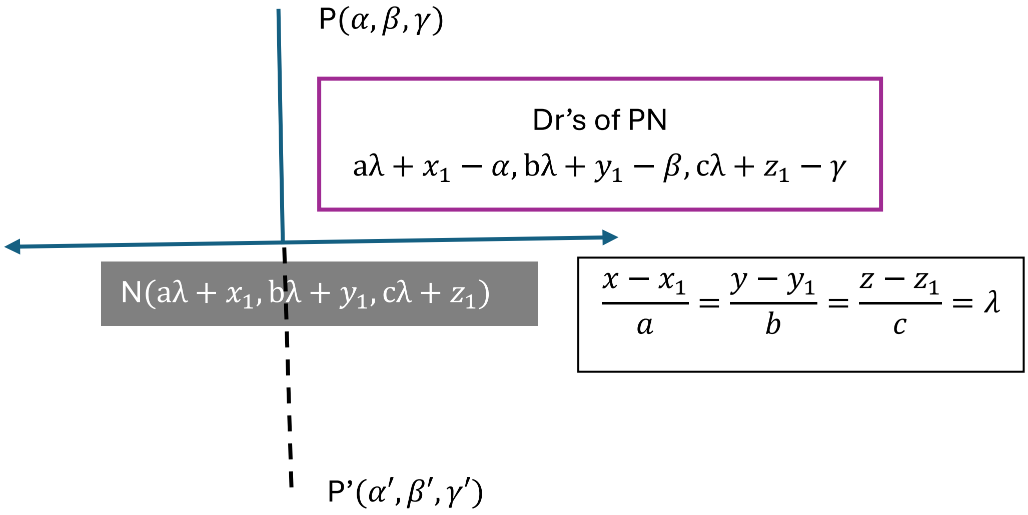
$$\vec{r} = \left(x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}\right) + \lambda[(x_2-x_1)\hat{\imath} + (y_2-y_1)\hat{\jmath} + (z_2-z_1)\hat{k}]$$

Equation of line passing through (x_1,y_1,z_1) and parallel to a line with Dr's a,b,c	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
Equation of line passing through (x_1,y_1,z_1) And perpendicular to two lines whose Dr's are a_1,b_1,c_1 and a_2,b_2,c_2	Let a,b,c are Dr's of required line which is perpendicular two given lines then by perpendicular condition $aa_1+bb_1+cc_1=0$ and $aa_2+bb_2+cc_2=0$ then $\frac{a}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \frac{b}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \frac{c}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ OR $\frac{a}{b_1c_2-b_2c_1} = \frac{b}{a_1c_2-a_2c_1} = \frac{c}{a_1b_2-a_2b_1}$ then denominator will be the values of a,b,c and equation of line will $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

Point of intersection of Two Lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$	$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}=\lambda$ Any point on the line $P \equiv (a_1\lambda + x_1, b_1\lambda + y_1, c_1\lambda + z_1)$
	$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}=\mu$ Any point on the line Q$\equiv (a_2\mu + x_2, b_2\mu + y_2, c_2\mu + z_2)$
	For intersection $P \equiv Q$ $a_1\lambda + x_1 = a_2\mu + x_2 \quad (1)$ $b_1\lambda + y_1 = b_2\mu + y_2 \quad (2)$ $c_1\lambda + z_1 = c_2\mu + z_2 \quad (3)$
	Find the value of λ & μ $by\ using\ equation (1) and\ (2)$
	If these values of λ & μ satisfy the equation(3) then lines intersect and point of intersection is P OR Q with the value of λ & μ If these values of λ & μ do not satisfy the equation(3) then lines do not intersect

Shortest Distance between Two Lines	
Equations of lines in vector form $\vec{r} = (x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}) + \lambda(a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}) = \overrightarrow{u_1} + \lambda\overrightarrow{v_1}$ $\vec{r} = (x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}) + \mu(a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}) = \overrightarrow{u_2} + \lambda\overrightarrow{v_2}$	
$\overrightarrow{u_1} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}, \quad \overrightarrow{u_2} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$ $\overrightarrow{v_1} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}, \quad \overrightarrow{v_1} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$	
$\overrightarrow{u_1} - \overrightarrow{u_2} = (x_1 - x_2)\hat{\imath} + (y_1 - y_2)\hat{\jmath} + (y_1 - y_2)\ k$ $\overrightarrow{v_1} \times \overrightarrow{v_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = (b_1c_2 - b_2c_1)\hat{\imath} - (a_1c_2 - a_2c_1)\hat{\jmath} + (a_1b_2 - a_2b_1)\hat{k}$	
If $\overrightarrow{v_1} \times \overrightarrow{v_2} \neq 0$	$S.D. = \left\| \frac{(\overrightarrow{u_1} - \overrightarrow{u_2}).(\overrightarrow{v_1} \times \overrightarrow{v_2})}{\|\overrightarrow{v_1} \times \overrightarrow{v_2}\|} \right\|$
If $\overrightarrow{v_1} \times \overrightarrow{v_2} = 0$	$S.D. = \frac{\|(\overrightarrow{u_1} - \overrightarrow{u_2}) \times \overrightarrow{v_1}\|}{\|\overrightarrow{v_1}\|}$
If S.D =0 it means lines intersect each other	

Foot of Perpendicular from a point (α, β, γ) to a line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$, Length of Perpendicular From P to the line and Image of P w.r.t the line



Any point on the line N$(a\lambda + x_1, b\lambda + y_1, c\lambda + z_1)$

Dr's of PN=$a\lambda + x_1 - \alpha, b\lambda + y_1 - \beta, c\lambda + z_1 - \gamma$
If N is the foot of perpendicular then
PN is perpendicular to the line →$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$
$a(a\lambda + x_1 - \alpha) + b(b\lambda + y_1 - \beta) + c(c\lambda + z_1 - \gamma) = 0$
now find the value of λ and N$(a\lambda + x_1, b\lambda + y_1, c\lambda + z_1)$ is the foot of perpendicular.

Length of perpendicular from P to the line= distance PN by distance formula

Let the Image of P(α, β, γ) is P'$(\alpha', \beta', \gamma')$

Then N should be the mid point of PP'→ N= $(a\lambda + x_1, b\lambda + y_1, c\lambda + z_1) = \left(\frac{\alpha+\alpha\prime}{2}, \frac{\beta+\beta\prime}{2}, \frac{\gamma+\gamma\prime}{2}\right)$

Now we can find the values of α', β', γ' and P' (image of P)

PROBABILITY

Key Terms:

Trial: A single performance of an experiment.

Sample Space (S): The set of all possible outcomes of a random experiment.

Event: A subset of the sample space.

Mutually Exclusive Events: Two events A and B are said to be **mutually exclusive** if **they cannot occur at the same time**. i.e:

$A \cap B = \emptyset$ OR $P(A \cup B) = P(A) + P(B)$

Exhaustive Events: When the union of events covers the entire sample space.

If $E_1, E_2, \ldots, E_n$ are exhaustive events, then:

$P(E_1 \cup E_2 \cup \cdots \cup E_n) = 1$ OR $E_1 \cup E_2 \cup \cdots \cup E_n = S$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Important Concepts

Conditional Probability: Probabilty of A when B is given

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

Multiplication Theorem of Probability

If A and B are two events, then:

$P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$

Independent Events: $P(A \cap B) = P(A)P(B)$

Bayes' Theorem: Used to find the probability of a cause given an observed event.

Let $E_1, E_2, \ldots, E_n$ be mutually exclusive and exhaustive events and A be an event such that $P(A) > 0$, then:

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{j=1}^{n} P(E_j)P(A/E_j)}$$

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \cdots \ldots \ldots . P(E_n)P(A/E_n)}$$