

केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN

आंचलिक शिक्षा एवं प्रशिक्षण संस्थान, चंडीगढ़

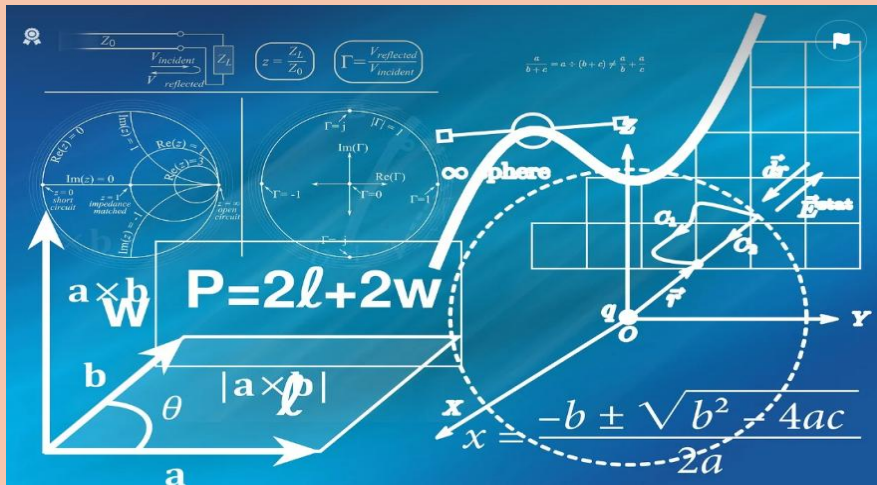
ZONAL INSTITUTE OF EDUCATION AND TRAINING, CHANDIGARH



MATERIAL ON COMPETENCY BASED QUESTIONS:
CLASS XII

MATHEMATICS (041)

(5 DAY WORKSHOP FROM 18.08.2025-22.08.2025)



संरक्षक

श्रीमती प्राची पांडे, आई.ए.एवं ए.एस.

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उपायुक्त एवं निदेशक,

केन्द्रीय विद्यालय संगठन

आंचलिक शिक्षा एवं प्रशिक्षण संस्थान

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Foreword

It gives me immense pleasure to present this *Competency-Based Question Bank* developed during the workshop organized by **ZIET Chandigarh** for **PGT (Mathematics)** teachers. This initiative aligns with the vision of the **National Education Policy (NEP) 2020**, which emphasizes a shift from rote learning to competency-based and application-oriented education.

The resource has been meticulously curated by a team of experienced and dedicated teachers, who have collaboratively prepared question bank that promote conceptual understanding, analytical thinking, and real-life application of mathematical ideas. Each question reflects an effort to align classroom assessment with clearly defined learning outcomes and competencies, ensuring that our students not only “know” mathematics but also “understand” and “apply” it meaningfully.

This Question Bank will serve as a valuable resource for teachers in planning assessments that truly reflect student competencies and support continuous, comprehensive, and joyful learning. It is also a testimony to the professional commitment and creative spirit of our teachers who continue to innovate and contribute to improving the quality of education in Kendriya Vidyalayas.

I extend my sincere appreciation to all the participants for their dedication and teamwork in bringing this important academic resource to fruition.

I also extend my sincere gratitude to Mr. Mohan Chandra Satyawali, Principal, Kendriya Vidyalaya SAC Vastrapur, Ahmedabad for the meticulous vetting of the Question Bank developed for the students of Mathematics. . The time and effort devoted by him in ensuring the pedagogical soundness and alignment with the CBSE curriculum are truly commendable.

I am confident that this compilation will serve the purpose of assessment of students to ensure attainment of learning outcomes.

(SHRUTI BHARGAVA)

Deputy Commissioner and Director
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2	RAM MURTI	PGT Maths	BHUNGA	Inverse Trigonometric Function
3	KANWAL	PGT Maths	HALWARA NO. 2	Matrices and Determinats
4	VIVEK GYAD	PGT Maths	JALALABAD	Continuity and Differentiability
5	SURAJ SINGH	PGT Maths	PATHANKOT NO. 1	Application of Derivatives
6	GARIMA JAIN	PGT Maths	PATHANKOT NO. 4	Integral
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13	LEEJA P.N.	PGT MATHS	NARELA

RELATION AND FUNCTION

Learning Outcomes

1. Understand types of relations and functions

2. Find domain, range, and inverse of functions

3. Apply composition of functions

Q.No.	QUESTION	LOs	COMPETENCIES COVERED
1.	If $A = \{1,2\}$, $B = \{3,4,5\}$ then the number of functions from A to B is: a) 6 b) 8 c) 9 d) 12	LO1	Understanding Problem Solving
2.	Let $R = \{(a, b) \in Z \times Z: a-b \in Z\}$ Then R is: a) Reflexive only b) Symmetric only c) An equivalence relation d) Transitive only	LO1	Analyzing Problem Solving
3.	The function $f(x) = x $ is: a) One-one and onto b) One-one but not onto c) Many-one and onto d) Many-one but not onto	LO1 LO2	Logical Reasoning Understanding
4.	The domain of the function $f(x) = \sqrt{4 - x^2}$ is: a) $(-\infty, \infty)$ b) $(-2,2)$ c) $[-2,2]$ d) $(2,\infty)$	LO1 LO2	Logical Reasoning Critical Thinking
5.	If $f(x) = 2x + 1$, then $f^{-1}(x)$ is: a) $\frac{x+1}{2}$ b) $\frac{x-1}{2}$ c) $2x-1$ d) $2x+1$	LO1 LO2	Analyzing Problem Solving
6.	6. If $f(x) = x^2$, $g(x) = \sqrt{x}$, then $(g \circ f)(x)$ is: a. x b. $ x $ c. x^2 d. Undefined	LO1 LO3	Logical Reasoning Problem Solving
7.	Which of the following is a function? a. $\{(1,2),(1,3)\}$ b. $\{(2,1),(3,1),(4,1)\}$ c. $\{(1,2),(2,3),(3,2),(1,4)\}$ d. $\{(5,5),(6,6),(5,6)\}$	LO1	Logical Reasoning Critical Thinking
8.	If a function $f: R \rightarrow R$ is defined by $f(x) = x^3$, then f is: a. One-one and onto b. One-one but not onto c. Many-one and onto d. Many-one but not onto	LO1 LO2	Analysing Logical Reasoning Critical Thinking

9.	The range of the function $f(x) = \frac{1}{x^2+1}$ is: a. (0, 1) b. (0,1] c. [1,∞) d. (1,∞)	LO1 LO2	Logical Reasoning Critical Thinking
10.	A function f is invertible if and only if: a. f is one-one b. f is onto c. f is bijective d. f is many-one	LO1 LO2	Logical Reasoning Critical Thinking
11.	Define Relation R on $A = \{1, 2, 3\}$ by $R = \{(a, b) \in A \times A : a \leq b\}$. Write all the elements of R.	LO1	Logical Reasoning Problem Solving
12.	Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x-4$. Show that f is one-one.	LO1 LO2	Logical Reasoning Critical Thinking
13.	Find the domain and range of the function $f(x) = \frac{1}{x+2}$.	LO1 LO2	Logical Reasoning Problem Solving
14.	Determine whether the relation $R = \{(a, b) : a, b \in \mathbb{Z}, a-b \text{ divisible by } 5\}$ is an equivalence relation.	LO1	Logical Reasoning Problem Solving Computation
15.	Find the composition $(f \circ g)(x)$, where $f(x) = x^2$, $g(x) = 2x+1$.	LO1 LO2 LO3	Logical Reasoning Problem Solving Conceptual clarity
16.	Let $A = \{1,2,3\}$ and $B = \{4,5\}$. Define a relation R from A to B such that the sum of the elements is even. Write R and determine whether it is a function.	LO1 LO2	Logical Reasoning Problem Solving
17.	Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Is f one-one and onto? Justify your answer.	LO1 LO2	Analyzing Computation
18.	Let $f(x) = 3x+4$, and $g(x) = x^2$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$. Are they equal?	LO1 LO2 LO3	Logical Reasoning Problem Solving
19.	Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = 2x+3$, is invertible. Find f^{-1}	LO1 LO2	Logical Reasoning Problem Solving computation

20.	If a function $f:A \rightarrow B$ is bijective, and A and B are finite sets with $ A = B $, prove that $f^{-1}:B \rightarrow A$ is also a bijection.	LO1 LO2	Understanding Mathematical Formulation
21.	Let the function $f:\mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x)=x^2 + 2x + 1$. a) Is f one-one? Justify. b) Is f onto? Justify. c) Find the range of f.	LO1 LO2	Analyzing Conceptual Understanding
22.	Let $f(x)=2x+3$ and $g(x)=x^2$. Find: a) $(f \circ g)(x)$ b) $(g \circ f)$ c) Verify whether $(f \circ g)(x)=(g \circ f)(x)$	LO1 LO2 LO3	Logical Reasoning Problem Solving
23.	Let $f:\mathbb{R} \setminus \{2\} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{3x+1}{x-2}$ a) Find the domain and range of the function. b) Is the function one-one? Justify your answer.	LO1 LO2	Problem Solving Analyzing
24.	Case Study 1: Student Database in a School Context: A school maintains a database of students' roll numbers and their names. Each student is identified by a unique roll number, and no two students have the same roll number. Let the set of roll numbers be: $A = \{101, 102, 103, 104\}$ and the corresponding set of student names be: $B = \{\text{Anita, Bharat, Chetna, Deepak}\}$ A relation R is defined from set A to set B as: $R = \{(101, \text{Anita}), (102, \text{Bharat}), (103, \text{Chetna}), (104, \text{Deepak})\}$ Q1. Is R a function? Justify your answer. Q2. Write the relation R in roster form. Q3. Is the function one-one and onto? Give reasons.	LO1 LO2	Real Life Application Decision making Conceptual Understanding
25.	Case Study 2: Online Food Ordering System Context: An online food delivery platform assigns each order number to a restaurant. Let the order numbers be: $O = \{o_1, o_2, o_3, o_4\}$ and restaurants be : $R = \{\text{Zomato, Swiggy, UberEats}\}$ The function $f: O \rightarrow R$ is defined as: $f(o_1) = \text{Zomato}, f(o_2) = \text{Swiggy}, f(o_3) = \text{Zomato},$ $f(o_4) = \text{UberEats}$ Q1. Write the function f in roster form. Q2. Determine the domain, co-domain, and range. Q3. Is the function one-one? Why or why not? Q4. Is the function onto? Justify.	LO1 LO2	Real Life Application Decision making Conceptual Understanding

ANSWER KEY

1.	(c)
2.	(c)
3.	(d)
4.	(c)
5.	(b)
6.	(b)
7.	(b)
8.	(a)
9.	(b)
10.	(c)
11.	$R = \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$
12.	Proof
13.	Domain: $\mathbb{R} - \{-2\}$ Range: $\mathbb{R} - \{0\}$
14.	yes
15.	$(f \circ g)(x) = f(g(x)) = f(2x+1) = (2x+1)^2 = 4x^2 + 4x + 1$
16.	$R = \{(1,5), (2,4), (3,5)\}$ Yes, it's a function.
17.	$f(x) = x^2$ is neither one-one nor onto.
18.	$(f \circ g)(x) \neq (g \circ f)(x)$
19.	$f^{-1}(x) = \frac{x-3}{2}$
20.	proof
21.	a) Is f one-one? Let's check whether two different inputs can give the same output: Suppose $f(x_1) = f(x_2)$ $(x_1 + 1)^2 = (x_2 + 1)^2 \Rightarrow x_1 + 1 = \pm(x_2 + 1)$ This gives two cases: $x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$ $x_1 + 1 = -(x_2 + 1) \Rightarrow x_1 + 1 = -x_2 - 1 \Rightarrow x_1 + x_2 = -2$ So for example, $x_1 = -3, x_2 = 1 \Rightarrow f(-3) = f(1) = 4$ Hence, f is not one-one. b) Is f onto from $\mathbb{R} \rightarrow \mathbb{R}$? Since the function outputs values of the form $(x + 1)^2$, the output (range) is always ≥ 0 So, not every real number is covered (e.g., -1 is not in the range). Therefore, f is not onto. c) Range of f: $f(x) = (x + 1)^2 \geq 0$, minimum value is 0 at $x = -1$
22.	a) $(f \circ g)(x) = f(g(x)) = f(x^2) = 2(x^2) + 3 = 2x^2 + 3$ b) $(g \circ f)(x) = g(f(x)) = g(2x + 3) = (2x + 3)^2 = 4x^2 + 12x + 9$ c) Are they equal? Compare: $f \circ g = 2x^2 + 3, \quad g \circ f = 4x^2 + 12x + 9$ Clearly, $f \circ g \neq g \circ f$ Conclusion: Composition of functions is not commutative.
23.	a) Domain: The function is undefined when the denominator is 0, i.e., $x - 2 = 0 \Rightarrow x = 2$

	• So, Domain = $\mathbb{R} - \{2\}$ • Range: Let $\frac{3x+1}{x-2}$ Solve for x in terms of y: $y(x-2) = 3x+1 \Rightarrow yx-2y = 3x+1 \Rightarrow yx-3x = 2y+1 \Rightarrow x(y-3) = 2y+1 \Rightarrow x = \frac{2y+1}{y-3}$ • This expression is undefined when $y=3$ • So, the function cannot take the value 3 Range = $\mathbb{R} - \{3\}$ b) Is the function one-one? Assume: $f(x_1) = f(x_2) \Rightarrow \frac{3x_1+1}{x_1-2} = \frac{3x_2+1}{x_2-2}$ Cross-multiply: $(3x_1+1)(x_2-2) = (3x_2+1)(x_1-2)$ Simplify both sides: LHS: $3x_1x_2 - 6x_1 + x_2 - 2$ RHS: $3x_1x_2 - 6x_2 + x_1 - 2$ Set them equal: $3x_1x_2 - 6x_1 + x_2 - 2 = 3x_1x_2 - 6x_2 + x_1 - 2$ Cancel common terms: $-6x_1 + x_2 = -6x_2 + x_1 \Rightarrow -6x_1 - x_1 = -6x_2 + x_2 \Rightarrow -7x_1 = -7x_2 \Rightarrow x_1 = x_2$ Therefore, function is one-one.
24.	1. Yes, R is a function because each element of set A (domain) is associated with exactly one element of set B (co-domain), and no roll number is repeated. 2. $R = \{(101, Anita), (102, Bharat), (103, Chetna), (104, Deepak)\}$ 3. • One-one (Injective): Yes, because each student is assigned a unique roll number, and no names are repeated. • Onto (Surjective): Yes, because every element in the codomain B has a pre-image in the domain A. So, the range = co-domain.
25.	1. $f = \{(o_1, Zomato), (o_2, Swiggy), (o_3, Zomato), (o_4, UberEats)\}$ 2. • Domain: $\{o_1, o_2, o_3, o_4\}$ • Co-domain: $\{Zomato, Swiggy, UberEats\}$ • Range: $\{Zomato, Swiggy, UberEats\}$ 3. No, the function is not one-one because Zomato is assigned to both o_1 and o_3. So, the same output is linked to multiple inputs. 4. Yes, it is onto because every restaurant in the co-domain has at least one order mapped to it. So, the range = co-domain.

Inverse Trigonometric Function

Learning Outcomes:

1. Define and graph inverse trigonometric functions
2. Apply properties and restrictions in problem-solving

Q.No	MCQ (One Mark Each)	LOs	Competencies
1	Which of the following is the principal value branch of $\sec^{-1} x$? (a) $(-\frac{\pi}{2}, \frac{\pi}{2})$ (b) $[0, \pi] - \{\frac{\pi}{2}\}$ (c) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (d) $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$	LO1 LO2	Conceptual Understanding Decision making, Conceptual clarity
2	The domain of $\sin^{-1} 2x$ is (a) $[-1, 1]$ (b) $(-1, 1)$ (c) $[-\frac{1}{2}, \frac{1}{2}]$ (d) $(-\frac{1}{2}, \frac{1}{2})$	LO1 LO2	Understanding ,Problem Solving, Analytical Skill
3	If $\sin^{-1} x = y$ then (a) $0 \leq x \leq \pi$ (b) $-\pi/2 \leq y \leq \pi/2$ (c) $-1 < x < 1$ (d) $-1 < y < 1$	LO1 LO2	Understanding ,Critical Thinking, ,Logical Reasoning ,
4	The value of $\cos^{-1} [\cos(\frac{4\pi}{3})]$ is (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $-\frac{\pi}{3}$	LO1 LO2	Conceptual Understanding, Problem Solving, Deductive Reasoning
5	The value of $\sin(\frac{\pi}{3} - \sin^{-1}(-\frac{1}{2}))$ is (a) -1 (b) 1 (b) (c) $\pi/2$ (d) 0	LO1 LO2	Basic Recall/Conceptual, Critical Thinking ,Precision and Accuracy
6	The principal values of $\tan^{-1}(-\sqrt{3})$ is (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $-\frac{\pi}{3}$	LO1 LO2	Reasoning and Conceptual Understanding ,Problem Solving, Computational Skill
7	The domain of $y = \cos^{-1}(x^2 - 4)$ is (a) $[3, 5]$ (b) $[0, \pi]$ (c) $[-\sqrt{5}, -\sqrt{3}] \cap [-\sqrt{5}, \sqrt{3}]$ (d) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$	LO1 LO2	Understanding, Higher Order Thinking /Analytical, Algebraically handling
8	The value of $\sec^{-1}(-\frac{2}{\sqrt{3}}) + 3\cot^{-1} \sqrt{3}$ (a) $\frac{5\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{-4\pi}{3}$ (d) $\frac{4\pi}{3}$	LO1 LO2	Application of Formulae, Problem solving, Attention,
9	Value of $\sin \left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) \right]$ (a) 1 (b) -1 (c) 0 (d) 2	LO1 LO2	Conceptual understanding , Critical thinking Application of Formulae

	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true but R is not the correct explanation of A. (C) A is true but R is false. (D) A is false but R is true.		
10	Assertion: (A) The principal value of $\cos^{-1}(\cos 7\pi/6)$ is $5\pi/6$ Reason: (R) $\cos^{-1}(\cos x) = x$, for all $x \in [0, \pi]$ (a) A (b) B (c) C (d) D	LO1 LO2	Understanding, Critical Thinking ,Justification Skill
11	Find the value of $\cos(\tan^{-1} \{ \cot(\sin^{-1} 1/2) \})$	LO1	Understanding, Problem solving ,Precision and logical sequencing
12	Write the value of $\tan^{-1}[2 \sin(2 \cos^{-1} \frac{\sqrt{3}}{2})]$	LO1	Conceptual Understanding, critical thinking Procedural Fluency
13	Express in the simplest form: $\tan^{-1} \frac{x}{\sqrt{a^2-x^2}}$, $ x < a$	LO1 LO2	Understanding ,Algebraic Manipulation & Reasoning, Conceptual clarity
14	Evaluate: $\sec^2(\tan^{-1} \frac{1}{2}) + \operatorname{cosec}^2(\cot^{-1} \frac{1}{3})$	LO1	Conceptual Understanding ,critical thinking, Logical consistency
15	If $\sin(\sin^{-1} \frac{1}{5} + \cos^{-1} x) = 1$, then find the value of x.	LO1	Basic recall & Understanding ,problem solving, logical deduction

ANSWER

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	b	2	c	3	b	4	b	5	b
6	d	7	d	8	d	9	a	10	a
11	$\frac{1}{2}$	12	$\frac{\pi}{3}$	13	$\sin^{-1} \frac{x}{a}$	14	$\frac{85}{36}$	15	$\frac{1}{5}$

MATRICES AND DETERMINANTS

Learning Outcomes:

1. Perform matrix operations
2. Find transpose, inverse of matrices
3. Apply matrices to solve systems of equations

Q. NO.	SECTION A (1 MARKS EACH)	LOs	Competencies
1.	$ A = \begin{vmatrix} \sin x & \square & \sin 3x \\ 3 \cos x & 6 \sin 2x & 9 \cos 3x \\ 2 \sin x & 4 \cos 2x & 2 \sin 3x \end{vmatrix}$ If $ A = 0$, which of the following could be $\square$ (A) $\sin 2x$ (B) $\cos 2x$ (C) $2 \cos 2x$ (D) $4 \cos 2x$	LO1	Analytical/ Problem Solving, critical thinking
2.	Which of the following matrices is non singular ? a) $\begin{bmatrix} -6 & 3 & 5 \\ 0 & 0 & 0 \\ 4 & 9 & -5 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 2 & 5 \\ 2 & 0 & 7 \\ 5 & 7 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 6 \\ 4 & 7 & 8 \end{bmatrix}$	LO1	Conceptual + Application
3.	The value of k, for which the matrix $\begin{bmatrix} 0 & 2 & -1 \\ -2 & k & 5 \\ 1 & -5 & 2k \end{bmatrix}$ is skew symmetric is a) -1 b) 0 c) 2 d) 3	LO1 LO2	Conceptual + Computation
4.	Let $A = \begin{bmatrix} 1 & \sin \alpha & 1 \\ -\sin \alpha & 1 & \sin \alpha \\ -1 & -\sin \alpha & 1 \end{bmatrix}$, where $0 \leq \alpha \leq 2\pi$. Then , a) $\text{Det}(A) = 0$ (b) $\text{Det}(A) \in (2,3)$ (c) $\text{Det}(A) \in (2,4)$ (d) $\text{Det}(A) \in [2,4]$	LO1	Analytical/ Problem Solving
5.	If A is a square matrix of order 3 and $ A = -4$, then $ A^2 $ is a) -4 b) 16 c) -16 d) -8	LO1	Conceptual + Problem solving
6.	The equation of a line joining the points A(3, 1) and B(9,3) is a) $x = y + 6$ b) $x = y + 2$ c) $x = 3y$ d) $y = 3x$	LO1	Understanding, Application
7.	The number of possible matrices of order 2×2 with each entry 1, 2 or 3 is a) 27 b) 9 c) 81 d) 3	LO1	Logical, Fundamental theorem of Counting
8.	If A and B are symmetric matrices of same order, then $(AB^T - 2BA^T)$ is a a) Skew symmetric matrix b) Symmetric matrix c) Neither symmetric matrix nor skew symmetric matrix d) Null matrix	LO1 LO2	Conceptual, Analytical

9.	Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that $\text{adj } A = A$. Then, $(a + b + c + d)$ is equal to : (a) $2a$ (b) $2b$ (c) $2c$ (d) 0 (b)	LO1 LO2	Conceptual, Problem solving
10.	If area of a triangle with vertices $(3,2)$, $(-1,4)$ and $(6,k)$ is 7 sq units, then possible values of k is/are : a) 3 b) -4 c) $-3, 4$ d) $3, -4$	LO1	Application , computational
SECTION B (2 MARKS EACH)			
11.	If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$	LO1	Conceptual, Problem Solving
12.	If for any 2×2 square matrix A , $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $ A $	LO1 LO2	Conceptual, Problem Solving
13.	Find the value of x , such that the points $(0,2)$, $(1, x)$ and $(3, 1)$ are collinear.	LO1	Application, Problem solving
14.	Examine the consistency of the system of equations : $3x + y = 5$ $2x - y = 6$	LO1 LO2 LO3	Analytical reasoning
15.	If matrix $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + x^4 + x^8 + x^{16}$, find $f(A)$	LO1	Understanding , application
SECTION C (3 MARKS EACH)			
16.	Achal wants to purchase 2 kg of sugar, 10 kg of wheat and 5 kg of rice. In a general store near his house, these groceries were priced at Rs. 50, Rs. 35 and Rs. 40 per kg whereas in a supermarket, these groceries were priced at Rs. 44, Rs. 30 and Rs. 38 per kg respectively. The cost of travelling to the supermarket is Rs. 20. Using matrix multiplication, find Achal's total savings if he buys the groceries from supermarket. Show your work.	LO1 LO2 LO3	Application, problem solving
17.	Two distributors of a chips company distribute two varieties of chips packets- Spicy chips (SC) and Cheesy Chips (CC). At the beginning of a certain month, the number of chips packets available with the distributors is shown in a matrix M . The number of chips packets distributed during that month by them is shown in matrix N . $M = \begin{matrix} & \begin{matrix} SC & CC \end{matrix} \\ \begin{matrix} \rightarrow \text{distributor 1} \\ \rightarrow \text{distributor 2} \end{matrix} & \begin{bmatrix} 965 & 498 \\ 872 & 689 \end{bmatrix} \end{matrix}$ $N = \begin{matrix} & \begin{matrix} SC & CC \end{matrix} \\ \begin{matrix} \rightarrow \text{distributor 1} \\ \rightarrow \text{distributor 2} \end{matrix} & \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix} \end{matrix}$ i) Find the number of chips packets remaining with the distributors at the end of that month. ii) A packet of Spicy Chips and Cheesy Chips costs Rs. 10 and Rs. 20 respectively. Find the total cost of the chips distributed by each distributor that month using matrix multiplication. Show your work and give your answer in the matrix form.	LO1 LO2 LO3	Application, Problem solving, Analytical skill

18.	A theatre has two categories of tickets, one for adults and one for children. Bindu's family paid Rs.400 to buy 6 tickets for adults and 4 tickets for children. Nalini's family paid Rs. 325 to buy 5 tickets for adults and 3 tickets for children. Find the cost of each category of the ticket by matrix method. Show your steps.	LO1 LO2 LO3	Application , Problem solving																				
19.	Solve the matrix equation $A \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 2 & 3 \end{bmatrix}$ by using the concept of inverse of a matrix.	LO1 LO2	Problem solving, conceptual, computational																				
20.	Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = 90$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$.	LO1 LO2 LO3	Knowledge , Analytical reasoning																				
SECTION D (5 MARKS EACH)																							
21.	Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold are given : <table border="1"> <tr> <th>School</th><th>A</th><th>B</th><th>C</th></tr> <tr> <th>Articles</th><td></td><td></td><td></td></tr> <tr> <td>Handmade fans</td><td>40</td><td>25</td><td>35</td></tr> <tr> <td>Mats</td><td>50</td><td>40</td><td>50</td></tr> <tr> <td>Plates</td><td>20</td><td>30</td><td>40</td></tr> </table> Find the funds collected by each school separately by selling the above articles. Also find the total funds collected for the purpose.	School	A	B	C	Articles				Handmade fans	40	25	35	Mats	50	40	50	Plates	20	30	40	LO1 LO2 LO3	Application Problem solving, critical thinking
School	A	B	C																				
Articles																							
Handmade fans	40	25	35																				
Mats	50	40	50																				
Plates	20	30	40																				
22.	The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$; (where $0 \leq x \leq 14$ and $a, b, c \in \mathbb{R}$ and $a \neq 0$) with respect to a XY – coordinate system in the vertical plane. The ball passes through the points (2, 15), (4,25) and (14,15). Determine the values of a , b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball.	LO1 LO2 LO3	Analytical + Reasoning + Attention to detail, Precision																				
23.	A body is launched from the ground and it follows a parabolic path given by $y = ax^2 + bx + c$; (where $a, b, c \in \mathbb{R}$ and $a \neq 0$) with respect to XY- coordinate system in the vertical plane. The body passes through the points (1 , 6) , (-1, 4) and (0, 3) . Determine the values of a , b and c by solving the system of linear equations in a, b and c using matrix method.	LO1 LO2 LO3	Problem Solving + computation, Accuracy																				

	SECTION E (4 MARKS EACH)		
24.	On her birthday, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹ 10 more. However, if there were 16 children more, everyone would have got ₹ 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹) Based on the information given above, answer the following questions: (i) Write matrix equations to represent the information given above. (ii) Find the number of children who were given some money by Seema. (iii) How much amount is given to each child by Seema ? OR How much amount Seema spends in distributing the money to all the students of the Orphanage ?	LO1 LO2 LO3	Application + Real life Problem solving, communication
25.	Two Schools A and B want to award their selected students on the values of sincerity, Truthfulness and Helpfulness. The school A wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of ₹ 1600. School B wants to spend ₹ 2300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the 3 values as before). The total amount of award for one prize on each values is ₹ 900. Based on above information, answer the following questions : (i) Write the matrix equation of the form of $AX = B$ for the above given situation. (ii) Find A^{-1}. (iii) Find the award money for each value.	LO1 LO2 LO3	Application + Real life Problem solving

ANSWER

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	c	2	c	3	b	4	d	5	b
6	c	7	c	8	c	9	a	10	c
11	k = -1	12	$ A = 8$	13	$x = \frac{5}{3}$	14	Consistent with unique solution.	15	$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
16	52	17	$\begin{bmatrix} 17540 \\ 16720 \end{bmatrix} \rightarrow \text{Distributor 1}$ $\begin{bmatrix} 16720 \end{bmatrix} \rightarrow \text{Distributor 2}$	18	50 25	19	$\begin{bmatrix} -7/2 & 3/2 \\ 0 & 0 \\ 1/2 & 1/2 \end{bmatrix}$	20	$x = 0,$ $y = 5$ $z = 3$
21	21000	22	$y = -\frac{1}{2}x^2 + 8x + 1$	23	a = 2; b = 1 c = 3				

24	(i) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix};$ (ii) $\begin{bmatrix} 5x - 4y \\ 5x - 8y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix} \Rightarrow 5x - 4y = 40 \text{ and } 5x - 8y = -80$ <i>On solving above two equations, we get $x = 32$, $y = 30$</i> Hence, number of children $x = 32$ (iii) ₹ 30 ; OR ₹ $32 \times 30 = ₹ 960$	
25	(i) ₹ x , ₹ y , ₹ z are awarded to the selected students on the values of sincerity, Truthfulness and Helpfulness. According to given condition, $3x + 2y + z = 1600$ $4x + y + 3z = 2300$ $x + y + z = 900$ (ii) $\text{Adj}(A) = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$ $A^{-1} = \frac{\text{Adj}(A)}{ A } = \frac{-1}{5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$ (iii) $x = 200$, $y = 300$, $z = 400$	

Continuity and Differentiability

Learning Outcomes:

1. Test continuity and differentiability of functions
2. Apply derivative rules (chain, product, quotient)
3. Differentiate implicit and composite functions

Q.N.	QUESTIONS	LOs	Competency
	One Marks Questions		
1	The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = - x-1 $ is (a) continuous as well as differentiable at $x = 1$ (b) not continuous but differentiable at $x = 1$ (c) continuous but not differentiable at $x = 1$ (d) neither continuous nor differentiable at $x = 1$	LO1	Understanding Logical accuracy and analytical reasoning
2	The value of k , for which the function $f(x) = \begin{cases} kx^2 + 5, & \text{if } x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$. (a) 1 (b) -3 (c) 3 (d) 5	LO1	Precision and systematic approach in solving.
3	$\frac{d}{dx}(3^x + x^3 + 3^3) =$ (a) $3^{x-1} \log 3 + 3x^2$ (b) $3^x \log 3 + 3x^2 + 9$ (c) $3^x \log 3 + 3x^2$ (d) $\frac{3^x}{\log 3} + 3x^2$	LO1 LO2	Carefulness and attention to form of expression
4	The value of k for which the function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at $x = 0$ is : (a) 1 (b) 2 (c) Any real number (d) 0	LO1	Persistence in step-by-step problem solving.
5	If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is : (a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$ (c) $\ln \left \sec\left(\frac{\pi}{4} - x\right) \right $ (d) $-\ln \left \sec\left(\frac{\pi}{4} - x\right) \right $	LO1 LO2	Consistency in applying mathematical rules.
6	If the function $f(x) = x+1 + x+2 $ is not differentiable at p and q then the value of $p+q$ is (a) 3 (b) -3 (c) ± 3 (d) 0	LO1	Identification of points
7	The value of k which makes the function $f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{if } x \neq 0 \\ k-1, & \text{if } x = 0 \end{cases}$, is continuous at $x = 0$ (a) 1 (b) 0 (c) ± 1 (d) -1	LO1	Apply limit process to verify continuity.
8	The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ where $y = e^{\sin x}$ is (a) 1 (b) 0 (c) ± 1 (d) -1	LO1 LO2	substitution

9	For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $(\frac{1}{4}, \frac{1}{4})$ is ... (a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 2	LO2 LO3	Relation between derivative and slope of tangent
10	$\frac{d}{dx} \left(e^{\sin^2 x} \right) =$ (a) $e^{\cos^2 x}$ (b) $e^{\sin^2 x} \cdot \cos^2 x$ (c) $\sin 2x \cdot e^{\sin^2 x}$ (d) $\cos 2x \cdot e^{\sin^2 x}$	LO2	Differentiation rules and simplification
Two Marks Questions			
11	If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$	LO2 LO3	Methodical verification
12	Find the value of k for which $f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 3$	LO1	Precision and correctness.
13	Find $\frac{d}{dx} \left[\tan^{-1} \left(\frac{\sqrt{1+x^2} - 1}{x} \right) \right]$	LO2	Clarity and focus in solving
14	If $x^{13}y^7 = (x+y)^{20}$ then prove that $\frac{dy}{dx} = \frac{y}{x}$	LO2 LO3	Creativity and systematic approach.
15	Show that $f(x) = x - 1 $ is not differentiable at $x = 1$	LO1	Accuracy and perseverance
Three Marks Questions			
16	If $y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$	LO2 LO3	Advanced application of derivatives
17	If $y = \left(x + \sqrt{x^2 + a^2} \right)^n$, prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$.	LO2 LO3	Apply rules of differentiation to combine and simplify expressions
18	If $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$, continuous at $x = 1$, find the values of a and b.	LO1	Use conditions to solve for constants a and b.
19	If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.	LO2 LO3	Second order derivative of parametric function
20	Find a and b if the function given by $f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 2x + 1, & \text{if } x \geq 1 \end{cases}$ is differentiable at $x=1$	LO1	Differentiability conditions
Five Marks Questions			
21	If $y = \left(x + \sqrt{1 + x^2} \right)^n$ then show that $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2y$	LO2 LO3	Persistence in solving stepwise

22	Find $\frac{dy}{dx}$ when $y = x^{\log x} + (\log x)^x$	LO2 LO3	Complex derivatives with logarithms
23	If the function $f(x) = \begin{cases} x^2 + ax + b, & \text{if } 0 \leq x < 2 \\ 3x + 2, & \text{if } 2 \leq x \leq 4 \\ 2ax + 5b, & \text{if } 4 < x \leq 8 \end{cases}$ is continuous on $[0, 8]$, find the value of a & b.	LO1	Solve for constants ensuring continuity on a closed interval
Four Marks Questions			
24	A manufacturing company tracks its production cost using mathematical models. When production is below 3 units, the cost is modeled by a quadratic function because of fixed setup costs combined with variable expenses. For production at or above 3 units, a linear model is assumed as bulk production reduces per-unit cost. The management wants to ensure the cost curve is realistic, smooth, and without sudden jumps at the transition. This highlights knowledge of continuity, skills in solving algebraic relations, and the value of precision in financial analysis. Consider $f(x) = \{x^2 + 1, x < 3; ax + b, x \geq 3\}$. (i) Find the relation between a and b for continuity at $x=3$. (ii) Find a and b if the function is differentiable at $x=3$. (iii) Modify the function so that it is both continuous and differentiable at $x=3$	LO1 LO2 LO3	real-life application and financial modeling
25	The temperature T (in °C) of a body placed in a room with surrounding constant temperature of 25°C decreases according to the function $T(t) = 80e^{(-0.1t)} + 25$, where t is time in minutes. The derivative of this function represents the rate of change of temperature at any instant of time. This model helps to understand Newton's Law of Cooling, which states that the rate of change of temperature is proportional to the difference between body temperature and surrounding temperature. 1. Find the rate of change of temperature function dT/dt. 2. Calculate the rate of cooling of the body at $t=0$. 3. After how many minutes will the body reach 45°C?	LO1 LO2 LO3	Co-relation between mathematics and science.

ANSWER

Q	Ans
1	(c)
2	(b)
3	(c)
4	(c)
5	(a)
6	(b)
7	(b)
8	(b)
9	(c)
10	(c)
11	Verify
12	12
13	$\frac{1}{2(1+x^2)}$
14	Verify
15	Verify
16	Verify
17	Verify
18	$a = 3, b = 2$
19	$\frac{1}{a}$
20	$a=1, b=2$
21	Verify
22	$(\log x)^{x-1}[1+\log x \cdot \log(\log x)] + 2x^{\log x-1} \log x$
23	$a = 3, b = -2$
24	(i) $3a+b=10$ (ii) $a=6, b=-8$ $f(x)=\{x^2+1, x<3; 6x-8, x\geq 3\}$
25	1) $dT/dt = -8e^{(-0.1t)} \text{ } ^\circ\text{C/min.}$ 2) At $t=0 \Rightarrow dT/dt = -8 \text{ } ^\circ\text{C/min.}$ 3) $80e^{(-0.1t)}+25=45 \Rightarrow e^{(-0.1t)}=1/4 \Rightarrow t=10\ln(4) \approx 13.86 \text{ minutes.}$

Application of Derivatives

Learning Outcomes:

1. Interpret and apply derivatives in rate of change
2. Identify increasing/decreasing functions
3. Solve maxima/minima problems

Q.No	Question	LOs	Competency
1	Diameter of a sphere $\frac{3}{2}(2x + 5)$, the rate of change of its surface area with respect to x is: a) $18\pi(2x + 5)$ b) $9\pi(2x + 5)$ c) $\frac{3}{2}$ d) $\frac{3}{4}$	LO1	Understanding Problem Solving Critical Thinking Computation
2	If the side of a square are decreasing at the rate of 1.5 cm/s, the rate of decrease of its parameter is: a) 1.5 cm/s b) 6 cm/s c) 3 cm/s d) 2.25 cm/s	LO1	Understanding Problem Solving Computation Critical Thinking
3	The total revenue in Rs received from the sale of x unit of an article is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue when x = 15 is: a) 126 b) 116 c) 96 d) 90	LO3	Understanding Problem Solving Application Computation
4	The rate of change of area of circle with respect to its radius r at r = 6 cm is: a) 10π b) 12π c) 8π d) 11π	LO1	Understanding Critical thinking Problem Solving Computation
5	The maximum value of $f(x) = x^2 + \frac{250}{x}$, $x > 0$ is: a) None b) 25 c) 50 d) 75	LO3	Understanding Critical thinking Problem Solving Computation
6	Maximum value of $y = \sin x \cdot \cos x$ when $x \in [0, \frac{\pi}{2}]$ is: a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{4}{\sqrt{2}}$ d) $\sqrt{2}$	LO3	Understanding Analyze Constructing Critical thinking Problem Solving
7	If at x = 1, the function $f(x) = x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0,2], then the value of a is: a) 124 b) -124 c) 120 d) -120	LO3	Critical thinking Problem Solving Computation Accuracy
8	The absolute maximum value of $y = x^3 - 3x + 2$ in $0 \leq x \leq 2$ is: a) 4 b) 6 c) 2 d) 0s	LO3	Understanding Critical thinking Problem Solving Computation
9	In which of these intervals is the function $f(x) = 3x^2 - 4x$ is strictly decreasing? a) $(-\infty, \frac{2}{3})$ b) (0,2) c) $(\frac{2}{3}, \infty)$ d) $(-\infty, \infty)$	LO2	Understanding Critical thinking Problem Solving Computation
10	The function $f(x) = 4 - 3x + 3x^2 - x^3$ $x \in R$ is: a) decreasing function b) increasing function c) strictly increasing on R d) neither increasing not decreasing on R	LO2	Understanding Critical thinking Problem Solving Computation

	TWO MARKS EACH		
11	Find an angle θ , where $0 < \theta < \frac{\pi}{2}$, which increases twice as fast as its sine.	LO1	Understanding Interpret Critical thinking Problem Solving Computation
12	The length x of a rectangle is decreasing at the rate of 2 cm/sec and the width y is increasing at the rate 2 cm/sec. When $x = 12$ cm and $y = 5$ cm, find the rate of change of (i) Perimeter (ii) the area of the rectangle.	LO1	Understanding Construct Critical thinking Problem Solving Computation Accuracy
13	Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval $[1, 2]$.	LO3	Understanding Critical thinking Problem Solving Computation Accuracy
14	Find the interval in which the functions are increasing or decreasing $f(x) = 5x^3 - 15x^2 - 120x + 3$.	LO2	Understanding Critical thinking Problem Solving Computation Accuracy
15	A man 2 m high, walks at a uniform speed of 6 m per minute away from a lamp post, 5 meters high. Find the rate at which the length of his shadow increases.	LO1	Understanding Critical thinking Problem Solving Computation Accuracy
	THREE MARKS EACH		
16	A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meters per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.	LO1	Understanding Critical thinking Logical Thinking Visualization Problem Solving Computation
17	Find the intervals in which the function $f(x) = \log(1 + x) - \frac{2x}{2+x}$ is increasing or decreasing.	LO2	Identifying Critical thinking Problem Solving Computation Accuracy
18	A kite is moving horizontally at the height of 151.5 m. If the speed of kite is 10 m per second, how fast is the string being let out, when the kite is 250 m away from the boy who is flying the kite? The height of the boy is 1.5 m.	LO1	Understanding Critical thinking Problem Solving Analyzing Computation
19	Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$, is strictly increasing or decreasing.	LO2	Understanding Critical thinking Problem Solving Computation Accuracy

20	Determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is decreasing or increasing.	LO2	Understanding Critical thinking Problem Solving Computation Accuracy
FIVE MARKS EACH			
21	An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.	LO3	Understanding Identifying Application Critical thinking Problem Solving Analyzing Computation Accuracy
22	Show that the semi vertical angle of a cone of maximum volume and given slant height is $\tan^{-1} \sqrt{2}$ or $\cos^{-1} \frac{1}{\sqrt{3}}$.	LO3	Understanding Identifying Critical thinking Problem Solving Analyzing Computation Accuracy
23	Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27} \pi h^3 \tan^2 \alpha$. Also, show that height of the cylinder is $\frac{h}{3}$.	LO3	Understanding Identifying Application Critical thinking Problem Solving Analyzing
CASE STUDY BASED(FOUR MARKS EACH)			
24	A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such depth that its capacity is 250 cubic metres. Cost of land is Rs. 500 per square metres. The cost of digging increases with the depth and cost for whole pool is Rs. $4000(\text{depth})^2$. Suppose the side of the square plot is x metres and depth is h metres. 	LO3	Understanding Identifying Application Critical thinking Problem Solving Analyzing Logical Thinking Computation Accuracy

	On the basis of the above information, answer the following questions: - Write cost $C(h)$ as function in terms of h. - Find critical point. - Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool? OR Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find relation between x and h for minimum cost.		
25	A technical company is designing a rectangular solar panel installation on a roof using 300m of boundary materials. The design includes a partition running parallel to one of the sides dividing the area (roof) into 2 sections.  Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres. Based on this information answer the following questions: - Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. - Write the area of the solar panel as the function of x. - Find the critical point of the area function. Use second derivative test to determine the critical points at the maximum area. Also find the maximum area. or Using first derivative test calculate the maximum area, the company can enclose with the 300m of boundary material considering the parallel partition.	LO3	Understanding Identifying Application Critical thinking Problem Solving Analyzing Logical Thinking Computation Accuracy

ANSWERS

1. b	2. b	3. a	4. b	5. a	6. b	7. c	8. a	9. a	10. a
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11. $\theta = \frac{\pi}{3}$

12. i) 0 cm/sec ii) 14 cm²/sec

13. Absolute Maximum value = $\frac{5}{2}$, Absolute Minimum value = 2

14. Increasing $(-\infty, -2] \cup [4, \infty)$, Decreasing $[-2, 4]$

15. 4 m/min

16. $\frac{35}{88}$ m/hour

17. Increasing $(-1, 0) \cup (0, \infty)$

18. 8m/sec

19. Strictly Increasing $(0, \frac{\pi}{4}) \cup (\frac{7\pi}{12}, \frac{11\pi}{12})$, Strictly Decreasing $(\frac{\pi}{4}, \frac{7\pi}{12}) \cup (\frac{11\pi}{12}, \pi)$

20. Increasing $[1, 2] \cup [3, \infty)$, Decreasing $(-\infty, 1] \cup [2, 3]$

24. **a.** $C(h) = 500(\frac{250}{h}) + 4000(h)^2$, **b.** $\frac{dC}{dh} = 0 \Rightarrow h = 2.5\text{m}$, **c.** Total Cost = Rs 7500 or $h =$

$2.5 \Rightarrow x = 10 \Rightarrow x = 4h$.

25. **a.** $2x + 3y = 300$, **b.** $x.(100 - \frac{2x}{3})$, **c.** $x = 75 \Rightarrow y = 50 \Rightarrow \text{Area} = 3750 \text{ sq. unit}$ or $x =$

$75 \Rightarrow \text{Area} = 3750 \text{ sq. unit}$

INTEGRALS

Learning Outcomes:

1. Understand and apply indefinite integrals
2. Use integration techniques (substitution, by parts, partial fractions)

<u>Q.N</u>	<u>MULTIPLE CHOICE QUESTIONS</u>	<u>LOs</u>	<u>COMPETENCIE</u> <u>S</u>
1.	$\int e^{5 \log x} dx$ is equal to (A): $\frac{x^5}{5} + C$ (B): $\frac{x^6}{6} + C$ (C): $5x^4 + C$ (D): $6x^5 + C$	LO1	K: Concepts & Definitions S: Identification, Understanding V: Logical clarity, accuracy
2.	Evaluate $\int \frac{dx}{\sin^2 x \cos^2 x}$ (A): $\tan x + \cot x + C$ (B): $\tan x - \cot x + C$ (C): $(\tan x + \cot x)^2 + C$ (D): $(\tan x - \cot x)^2 + C$	LO1	K: Concepts & Definitions S: Identification, Understanding V: Logical clarity, accuracy
3.	If $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = ax + b \log 4e^x + 5e^{-x} + C$ then values of a & b: (A): $a = -1/8, b = 7/8$ (B): $a = -\frac{1}{8}, b = -7/8$ (C): $a = 1/8, b = 7/8$ (D): $a = \frac{1}{8}, b = -7/8$	LO1 LO2	Conceptual Understanding Identification, Computational Logical reasoning clarity, accuracy
4.	The value of $\int (\cos^2 2x - \sin^2 2x) dx$ is (A): $-\frac{\sin 4x}{4} + C$ (B): $\frac{\cos 4x}{4} + C$ (C): $\frac{\sin 4x}{4} + C$ (D): $-\frac{\cos 4x}{4} + C$	LO1	Concepts & Definitions Identification, Computation Understanding Logical clarity, accuracy
5.	Q5: The value of $\int \frac{3x}{3x-1} dx$ is (A): $\frac{1}{3} \log 3x - 1 + C$ (B): $x + \log 3x - 1 + C$ (C): $x + \frac{1}{3} \log 3x - 1 + C$ (D): None of above	LO1	K: Concepts & Definitions S: Identification, Understanding V: Logical clarity, accuracy
6.	: The value of $\int \frac{1}{x-\sqrt{x}} dx$ is (A): $2 \log (\sqrt{x} - 1) + C$ (B): $x + \frac{2}{3} x^{\frac{3}{2}} + C$ (C): $\frac{1}{2} \log (\sqrt{x} - 1) + C$ (D): None of above	LO1 LO2	K: Concepts & Definitions S: Identification, Understanding, Manipulation V: Logical, Analytical clarity, accuracy

7.	$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to (A): $2(\sin x + x \cos \alpha) + C$ (B): $2(\sin x - x \cos \alpha) + C$ (C): $2(\sin x + 2x \cos \alpha) + C$ (D): $2(\sin x - 2x \cos \alpha) + C$	LO1 LO2	K: Concepts & Definitions S: Identification, Manipulative V: Logical clarity, accuracy
8.	$\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ is equal to (A): $\tan(xe^x) + C$ (B): $\cot(xe^x) + C$ (C): $\cot(e^x) + C$ (D): $\tan(e^x(1+x)) + C$	LO1 LO2	K: Concepts & Definitions S: Identification, Manipulation V: Logical Attention to detail, accuracy
9.	ASSERTION- REASON BASED QUESTIONS Each of the following questions contains two statements: Assertion (A) and Reason (R). Each of the questions has four alternative choice, only one of which is correct statement (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true. Assertion (A): $\int \frac{1}{x^2+2x+3} dx = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + C$ Reason (R): $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$	LO1 LO2	Concepts & Definitions Identification, Understanding Clarity, accuracy
10.	Assertion (A): $\int e^x \left[\frac{1}{x} - \frac{1}{x^2} \right] dx = \frac{e^x}{x} + C$ Reason (R): $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$	LO1 LO2	Concepts & Definitions Identification, Understanding Clarity, accuracy
<u>VERY SHORT ANSWER TYPE QUESTIONS</u>			
11.	Evaluate $\int \frac{\sin x}{\sin(x+a)} dx$	LO1 LO2	K: Concepts & Definitions S: Identification, Understanding V: Logical
12.	Evaluate $\int x^2 \log x dx$	LO1 LO2	K: Concepts & Definitions S: Identification, Understanding V: Logical
13.	Evaluate $\int \frac{dx}{\sin(x-a)\sin(x-b)}$	LO1 LO2	K: Conceptual understanding S: Identification, Analytical V: Logical, Decision making, Critical thinking

14.	Evaluate $\int x(1-x)^n dx$	LO1 LO2	K: Concepts S: Identification, Understanding clarity, accuracy
15.	$\int \frac{\sin^6 x}{\cos^8 x} dx$	LO1 LO2	Concepts Identification, Manipulation Logical clarity, accuracy
	SHORT ANSWER TYPE QUESTIONS (3 marks)		
16.	Evaluate $\int \sqrt{10-4x-x^2} dx$	LO1 LO2	K: Concepts & Definitions S: Identification, Manipulative V: Logical, Computational
17.	Evaluate $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}} dx$	LO1 LO2	Conceptual Understanding S: Identification, Analytical, Manipulative, Logical , Decision making, Critical thinking
18.	Evaluate $\int [\log(\log x) + 1/(\log x)^2] dx$	LO1 LO2	Concepts & Definitions Identification, Manipulative, Logical
19.	Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$	LO1 LO2	Concepts & Definitions Identification, understanding Logical
20.	Evaluate $\int \frac{(3\sin x - 2)\cos x}{5 - \cos 2x - 4\sin x} dx$	LO1 LO2	K: Conceptual Understanding S: Identification, Logical Reasoning, Critical thinking
	<u>LONG ANSWER TYPE QUESTIONS</u>		
21.	Find $\int \frac{dx}{\sin x + \sin 2x}$	LO1 LO2	K: Concepts & Definitions S: Identification, Understanding V: Logical clarity, accuracy

22.	Evaluate $\int [\sqrt{\tan x} + \sqrt{\cot x}] dx$	LO1 LO2	K: Concepts & Definitions S: Identification, Understanding V: Logical
23.	Evaluate $\int \sqrt{\tan x} dx$	LO1 LO2	Concepts & Definitions Identification, Understanding Logical

ANSWERS

1.	B	6.	A
2.	B	7.	A
3.	B	8.	A
4.	C	9.	A
5.	C	10.	A

11.	$(x+a) \cos a - \sin a \cdot \log \sin(x+a) + C$
12.	$x^3 \log x/3 - 1/9x^3 + C$
13.	$\operatorname{cosec}(b-a) \log \left \frac{\sin(x-b)}{\sin(x-a)} \right + C$
14.	$\frac{(x)^{n+1}}{n+1} - \frac{(x)^{n+2}}{n+2} + C$
15.	$\tan^7 x / 7 + C$
16.	$\frac{1}{4}(2x-1)\sqrt{10-4x-x^2} + 9/4 \log (2x-1) + \sqrt{10-4x-x^2} + C$
17.	$\sin^{-1} \frac{(e^x+2)}{3} + C$
18.	$x \log(\log x) - x/\log x + C$
19.	$-2\sqrt{1-x} + \sqrt{x}\sqrt{1-x} + \cos^{-1} \sqrt{x} + C$
20.	$3 \log(2 - \sin x) + 4/(2 - \sin x) + C$
21.	$\frac{1}{2} \log 1 + \cos x + 1/6 \log 1 - \cos x - 2/3 \log 1 + 2\cos x + C$
22.	$\sqrt{2} \sin^{-1}(\sin x - \cos x) + C$
23.	$\frac{\tan^{-1}(\frac{\tan x - 1}{\sqrt{2} \tan x})}{\sqrt{2}} + 1/2\sqrt{2} \log \left \frac{\tan x + 1 - \sqrt{2} \tan x}{\tan x + 1 + \sqrt{2} \tan x} \right + C$

DEFINITE INTEGRAL

Learning Outcomes:

1. Understand and apply indefinite and definite integrals
2. Use integration techniques (substitution, by parts, partial fractions)

S.N.	<u>1 MARKS QUESTIONS</u>	LOs	Competency
1	$\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$ is a) $\frac{\pi}{2}$ b) π c) 0 d) 1	LO1 LO2	analytical thinking
2	Value of $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx$ is a). $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$	LO1 LO2	understanding the property of definite integral application , analysing , problem solving analytical thinking, critical thinking
3	Value of $\int_{-1}^2 \frac{ x }{x} dx$ is	LO1 LO2	understanding the value of modulus x application , analysing , problem solving analytical thinking, critical thinking
	<u>2 MARKS QUESTIONS</u>		
4	Evaluate $\int_0^{\pi/2} \cos^4 x \, dx$	LO1 LO2	understanding the concepts of T functions problem solving, analyzing , integration skill , computational skill , analytical thinking
5	Evaluate $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$	LO1 LO2	understanding the property, application, analysing , problem solving, analytical thinking, critical thinking
6	Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$	LO1 LO2	understanding the problem Understanding the property, problem solving, analytic thinking , critical thinking , calculation ability

<u>3 MARKS QUESTIONS</u>			
7	Evaluate $\int_0^{\pi/4} \log(1 + \tan x) dx$	LO1 LO2	understanding the property, trigonometric identities and log rules ,application of property , analysing , problem solving , computational skill , analytical thinking
8	Evaluate $\int_0^{\pi/2} \log \sin x dx$	LO1 LO2	understanding the properties, trigonometric identities and log rules,application of property , analysing , problem solving, computational skill , analytical thinking
9	$\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$	LO1 LO2	understanding the value of $\sin 2x$ as co-related with numerator value , substitution method of integration, application ,analysing , problem solving, computational skill , analytical thinking
<u>5 MARKS QUESTIONS</u>			
10	Evaluate the integral $\int_1^4 x - 1 + x - 2 + x - 3 dx$	LO1 LO2	understanding the value of modulus function, application , analyzing , problem solving, analytical thinking, critical thinking
11	Evaluate $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$	LO1 LO2	understanding the problem Understanding the properties. application, analyzing , , problem solving, analytic thinking , critical thinking ,
12	Evaluate the integral $\int_{-1}^{3/2} x \sin \pi x dx$	LO1 LO2	understanding the concept of modulus function and quadrant system,application , problem solving , analytical thinking

ANSWER

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	b	2	d	3	c	4	$\frac{3\pi}{16}$	5	0	6	$\pi^2/4$
7	$\frac{\pi}{8} \log 2$	8	$-\frac{\pi}{2} \log 2$	9	$\frac{1}{40} \log 9$	10	$\frac{19}{2}$	11	$\frac{\pi^2}{2ab}$	12	$\frac{3\pi+1}{\pi^2}$

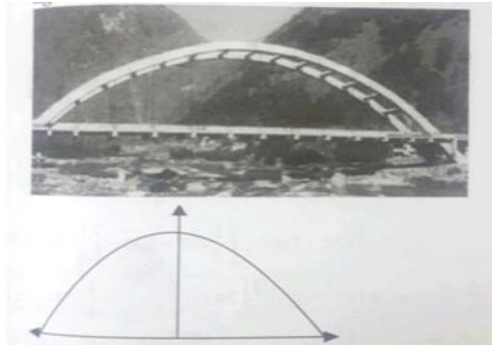
APPLICATION OF INTEGRATION

Learning Outcomes:

1. Find area under curve using definite integrals
2. Solve problems on area bounded by curves

Q,NO	QUESTION	LOs	COMPETENCY
MCQ's (1 Mark)			
1	Area of region bounded by $y = x$ and $y = x^2$ is: A) 1/6 B) 1/3 C) 1/2 D) 2/3	LO1 LO2	Conceptual understanding, Problem Solving,
2	A car's velocity over time is $v(t)=3t$ for $0 \leq t \leq 4$ (m/s). Distance traveled in this period is: A) 6m B) 12m C) 24m D) 48m	LO1	Understanding of concepts, computational skill, Analytical thinking
3	A particle moves on a line with velocity $v(t)=\sin t$, $t \in [0, 2\pi]$. Net displacement equals: A) 0 B) 2 C) 1 D) $4/\pi$	LO1	Conceptual understanding, Critical thinking,
4	Assertion (A): Area between $y = \sin x$ and x-axis over $[0, 2\pi]$ is 4. Reason (R): $\int_0^{2\pi} \sin x \, dx = 4$. A) A true, R true, R explains A B) A true, R true, R does not explain A C) A true, R false D) A false, R true	LO1	Understanding of area under curves, Logical analysis, attention to detail, decision-making
5	Assertion (A): The area of the region bounded by the curve $y = x^2$ and the lines $x = 0$, $x = 2$, and $y = 0$ is $8/3$ square units. Reason (R): The area is calculated by integrating the function y with respect to x from 0 to 2, i.e., $\int_0^2 x^2 \, dx$.	LO1 LO2	Conceptual understanding, Analytical skills, Decision-making
2 MARKS QUESTIONS			
6	If $y = x^2$ and $y = 4x$ intersect the x-axis bounded region in the first quadrant, the area enclosed between the curves $y = 4x$ and $y = x^2$ is:	LO1 LO2	Conceptual understanding, Graphical interpretation, Problem Solving,
7	Find Area enclosed by $y = \sqrt{x}$, x-axis, and $x=4$.	LO1 LO2	Conceptual understanding, Graphical interpretation, Problem Solving,
8	Find the area bounded by the line $y = x$, x-axis and lines $x = -1$ to $x = 2$.	LO1 LO2	Conceptual understanding, Graphical interpretation, Problem Solving, Analytical skills and thinking

	3 MARKS QUESTIONS		
9	Given the graph of $x^2+y^2=16$, for $0 \leq x \leq 4$, find the area enclosed between the curve and the x-axis over this interval.	LO1 LO2	Graphical interpretation, Integration skills, visualization
10	Using symmetry, find the area bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$	LO1	Analyze symmetry, Logical reasoning, Problem solving
11	Find the area bounded by the curve $y=x^2$, the x-axis, and the vertical lines $x=1$ and $x=3$	LO1 LO2	Understanding area under parabola, Application, Definite integration,
	5 MARKS QUESTIONS		
12	A water trough has a cross-section in the shape of the area between the curve $y=4-x^2$ and the x-axis. If the trough is 2 meters long, find the volume of water it can hold.	LO1 LO2	Conceptual understanding, Mathematical modeling of real-life problems, Problem solving, analytical thinking
13	A company wants to build a glass wall in the shape enclosed between the parabola $y=x^2$ and the line $y=4$. The wall extends from the point where the parabola intersects the line on the left to the point where it intersects on the right. Find the total area of glass required to construct the wall. Explain each step clearly, including finding points of intersection, setting up the integral, and evaluating it.	LO1 LO2	Conceptual understanding, Mathematical modeling of real-life problems, application of integrals in daily life, Problem-Solving, logical reasoning, Visualization
	CASE STUDY BASED QUESTIONS		
14	A park is in the shape of a region bounded between two curves: The upper boundary is the parabola $y=4-x^2$ and the lower boundary is the line $y=x$, for $x \in [-2, 1]$. The municipal corporation wants to lay grass only in the region enclosed between these two curves. As a student volunteer, you are asked to calculate: (a) The area of the region that will be covered with grass. (b) If the cost of laying grass is ₹50 per square metre, calculate the total cost.	LO1 LO2	Knowledge of curves and their graphs, Problem Solving, Application in real-life context, Mathematical communication

15	The bridge connects two hills 100 feet apart. The arch of the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure  Based on the information given above, answer the following questions: (i) Find the equation of the parabola designed on the bridge. (ii) Find the value of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$ (iii) what is the type of The integrand of the integral $\int_{-50}^{50} x^2 dx$. (iv) Find the The area formed by the curve $x^2 = 250y$, x – axis, $y=0$ and $y=10$ (v) What is the area formed by the curve $x^2 = 250y$, y axis , $y=2$ and $y=4$ is	LO1 LO2	Understanding of concepts, Problem Solving, Real-life application, logical reasoning, decision- making
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Answers

Q.no	Answer
1	$A=1/6$
2	$C=24$ M
3	$A=0$
4	C
5	A
6	$32/3$ sq, units
7	$8/3$
8	$5/2$
9	4π
10	6π sq. units
11	$26/3$ sq, units
12	$64/3$ m ³
13	The area of the glass wall required is $32/3$ square units.
14	Area covered with grass ≈ 9.89 m ² Total cost $\approx ₹494.50$
15	(i) $x^2 = -250y$ (ii) $1000/3$ (iii) Even (iv) $1000/3$ (v) $\frac{80\sqrt{10}-40\sqrt{5}}{3}$

DIFFERENTIAL EQUATIONS

Learning Outcomes:

1. Find order and Degree of given Differential Equation
2. Solve first-order, first-degree differential equations
3. Interpret solutions in real-world contexts

MULTIPLE CHOISE QUESTIONS (1 marks each)			
Q.No.	Questions	LOs	Competencies
1	The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$: a)4 b)3/2 c)2 d) Not defined	LO1	Analytical/ Problem Solving
2	Solution of $\frac{dy}{dx}y = 1$, $y(0)= 1$ is given by a) $xy=-e^x$ b) $xy=-e^{-x}$ c) $xy=-1$ d) $y= 2e^x - 1$	LO2	Conceptual, Problem solving
3	The number of arbitrary constants in the general solution of a differential equation of fourth order are: a) 0 b) 2 c)3 d)4	LO1	Understanding, Application
4	The integrating factor of the differential equation $\frac{dy}{dx}(x \log x) + y = 2 \log x$ is: a) e^x b) $\log x$ c) $\log(\log x)$ d) x	LO1 LO2	Logical, Fundamental theorem of Counting
5	Assertion: Homogeneous system of linear equations is always consistent. Reason: $x = 0$, $y = 0$ is always a solution of the homogeneous system of equation with unknown x and y .	LO2	Conceptual, Analytical
VERY SHORT TYPE QUESTIONS (2 MARKS EACH)			
6	Solve the differential equation: $y \, dx + (x-y^2)dy = 0$	LO2	Conceptual, Problem Solving
7	Solve the differential equation: $\cos x \frac{dy}{dx} = \cos 3x - \cos 2x$	LO2	Conceptual, Problem Solving
8	Verify that the function $x+y = \tan^{-1} y$ is a solution of the differential equation $y^2y' + y^2 + 1 = 0$.	LO2	Application, Problem solving
SHORT TYPE QUESTIONS (3 MARKS EACH)			
9	Solve the differential equation $(1 + y^2) \tan^{-1}x \, dx + 2y(1 + x^2)dy = 0$	LO2	Understanding ,Critical Thinking, ,Logical Reasoning
10	Find the general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$	LO2	Conceptual Understanding, Problem Solving, Deductive Reasoning

11	Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}$; $y(0) = 5$.	LO2	Conceptual, Problem Solving
LONG TYPE QUESTIONS (5 MARKS EACH)			
12	Find the particular solution of the differential equation: $xe^{\frac{y}{x}} - y\sin\frac{y}{x} + x\frac{dy}{dx}\sin\frac{y}{x} = 0$, for $x = 1, y = 0$.	LO2	Logical Thinking, Problem Solving, Computational Logical Conclusion
13	Solve the differential equation $\frac{dy}{dx} = y \tan x$; $y = 1$ when $x = 0$.	LO2	Critical Thinking, Problem Solving, Computational
14	Solve the differential equation $(x + y - 3)\frac{dy}{dx} = 5$.	LO2	Analyses Of Problem, Problem Solving
12	Find the particular solution of the differential equation: $xe^{\frac{y}{x}} - y\sin\frac{y}{x} + x\frac{dy}{dx}\sin\frac{y}{x} = 0$, for $x = 1, y = 0$.	LO2	Computational Analyses Of Problem, Problem Solving
CASE STUDY BASED QUESTION (4 MARKS EACH)			
15	Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered by the drops. By the end of the 2 nd week half the children have been given the polio drops. How many will be given the drops by the end of 3 rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50-y)$ where x denotes the number of weeks and y the number of children who have been given the drops. (i) State the order of the given differential equation. (ii) The solution of the given differential equation is: (iii) Which of the following solution may be used to find the number of children who have been given the polio drops?	LO2 LO3	Conceptual, Critical Thinking, Problem Solving
16	In a college hostel accommodating 1000 students, one of the hosteller came in carrying corona virus and the hostel was isolated. The rate at which the virus spreads is assumed to be proportional to the product of the number of infected students and remaining students. There are 50 infected students after 4 days. (i) If $n(t)$ denotes the number of infected student by corona virus at any time t then find the maximum value of $n(t)$: (ii) Write the differential equation to find $n(t)$ (iii) Find the value of $n(4)$.	LO2 LO3	Conceptual, Problem Solving

ANSWER

Q No.	ANSWER	Q No.	ANSWER
1	option-c	2	option-d
3	option-d	4	option-b
5	Both A and R are true and R is the correct explanation of A	6	$xy = y^3/3 + C$
7	$y = \sin 2x - x - 2\sin x - \log \sec x + \tan x + C$	8	It is a solution
9	$\frac{1}{2}(\tan^{-1} x)^2 + \log(1 + y^2) = C$	10	$\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = C$
11	$y = e^{x^2}(x^3 + 5)$	12	$\left[\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right) \right] e^{\frac{-y}{x}} = \log x^2 + 1$
13	$y \cos x = 1$ or $y = \sec x$	14	$y - 5 \log x + y + 2 = C_1$
15	(i) order 1 (ii) $(-\log 50 - y = kx + C)$ (iii) $y = 50(1 - e^{-kx})$	16	(i) 1000 (ii) $\frac{dn}{dt} = n(1000 - n)$ (iii) 50

VECTOR AND THREE DIMENSIONAL GEOMETRY

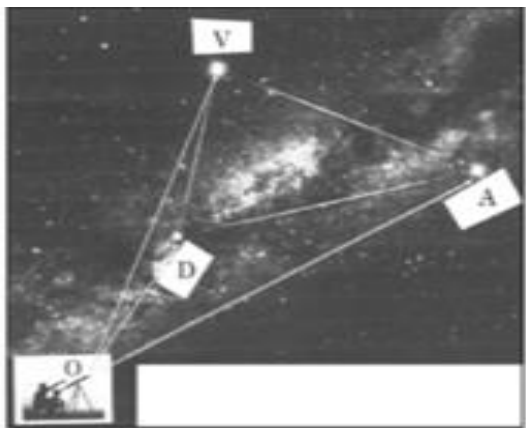
Learning Outcomes:

1. Perform operations on vectors
2. Find dot and cross product
3. Apply vectors to solve geometry problems
4. Find direction cosines, direction ratios
5. Solve problems involving line in 3D
6. Find angle between two lines.

Q.No.	Question	LOs	Competency
	MCQ (One Mark Each)		
1	The vector of the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ (A). $\hat{i} - 2\hat{j} + 2\hat{k}$ (B). $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ (C). $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (D). $9(\hat{i} - 2\hat{j} + 2\hat{k})$	LO1	Conceptual understanding, Rational Thinking, Accuracy, Precision
2	The magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $9/2$, is (A). 2 (B). 3 (C). 4 (D). 5	LO1 LO2	1. Understanding 2. Co-relation 3. Analytical reasoning 4. problem solving
3	The projection of the vectors $2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} + \hat{k}$ is (A). $\frac{10}{\sqrt{6}}$ (B). $\frac{10}{\sqrt{3}}$ (C). $\frac{5}{\sqrt{6}}$ (D). $\frac{5}{\sqrt{3}}$	LO1 LO2 LO3	1. Conceptual understanding 2. knowledge 3. Interpretation .
4	The angle between the vectors $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ (A). $\cos^{-1}(\frac{-1}{2})$ (B). 60° (C). $\cos^{-1}(\frac{-1}{3})$ (D). $\cos^{-1}(\frac{-2}{3})$	LO2	1. Understanding 2. Co-relation 3. analytical reasoning 4. problem solving
5	Direction ratios of the line $\frac{4-x}{2} = \frac{y-1}{6} = \frac{z-7}{7}$ (A). 2, 6, 3 (B). -2, 6, 3 (C). 2, -6, 3 (D). None of above	LO4	1. Conceptual understanding 2. analytical reasoning 3. problem solving
6	The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$ is (A) parallel to x - axis (B). Parallel to y - axis (C) Parallel to z-axis (D). Perpendicular to z-axis	LO4	1. Conceptual understanding 2. application 3. precision
7	If a line makes angles α, β, γ with the positive direction of co-ordinates axes, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$. (A). 1 (B) 2 (C). 3 (D). 4	LO4	1. Understanding 2. knowledge 3. accuracy 4. interpretation

8	The vector equation of the line joining the points (3, -2, -5) and (3, -2, 6) is: (A). $(4\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(12\hat{k})$ (B). $(4\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(12\hat{k})$ (C). $(4\hat{i} - 4\hat{j} - 5\hat{k}) + \lambda(12\hat{k})$ (D). $(-4\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(12\hat{k})$	LO5	1. Understanding 2. conceptual knowledge 3. computation ability 4 precision
9	The unit perpendicular to the vector $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ (A). $\hat{k}$ (B). $\hat{j} - \hat{k}$, (C). $\hat{i}$ (D). $\hat{j}$	LO2	1. intetpetation 2. analyze 3. application 4 precision
10	If the direction cosines of a line are λ, λ and λ, then λ is equal to (A). $-\frac{1}{\sqrt{3}}$ (B). 1 (C). $\frac{1}{\sqrt{3}}$ (D). $\frac{\pm 1}{\sqrt{3}}$	LO4	1. Understanding 2. analyze 3. application 4 precision
Two Marks Each			
11	If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ are the position vectors of the points A, B, C and D, find the angle between $\vec{AB}$ and $\vec{CD}$. Deduce that $\vec{AB}$ and $\vec{CD}$ are collinear.	LO2	1. Understanding 2. analyze 3. accuracy 4 computational skill
12	If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} = 5, \vec{b} = 12$ and $\vec{c} = 13, \vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.	LO2 LO3	1. Understanding 2. analyze 3. accuracy 4 computational skill
13	Find a vector of magnitude 6, perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.	LO2 LO3	1. Understanding 2. application 3. accuracy 4 problem solving
14	If a line makes α, β and γ with x- axis, y- axis and z- axis respectively, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.	LO2 LO3	1. Understanding 2. analyze 3. critical thinking 4 precision

15	Find the coordinate of points on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ which are at a distance $\sqrt{11}$ units from the origin.	LO5	1. Understanding 2. analytical skills 3. accuracy 4 critical thinking
	THREE MARKS EACH		
16	The magnitude of the vector product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 2. Find the value of λ .	LO2	1. Understanding 2. knowledge 3. accuracy 4 application
17	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $ \vec{a} = 3$, $ \vec{b} = 4$, $ \vec{c} = 5$ and each one of them being perpendicular to the sum of the other two, find $ \vec{a} + \vec{b} + \vec{c} $.	LO2	1. knowledge 2. application 3. accuracy 4 problem solving
18	Find the shortest distance between the lines given by $\vec{r} = 4\hat{i} - \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$	LO5 LO6	1. Understanding 2. computational skill 3. analyzing 4 precision
19	Find the angle between the lines given by $2x = 3y = -z$ and $6x = -y = -4z$	LO6	1. Understanding 2. computational skill 3. accuracy 4 application
20	Find the distance of the point $(-1, -5, -10)$ from the intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$.	LO5 LO6	1. Understanding 2. recall 3. computational 4 precision
	FIVE MARKS EACH		
21	Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 18$.	LO2	1. knowledge 2. analyze 3. accuracy 4 application
22	Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$	LO5 LO6	1. Understanding 2. recall 3. computational skill
23	Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{1} = \frac{y+1}{2} = \frac{-2z+6}{-4}$. Find the length of the line segment joining the points(given point and image point)	LO5 LO6	1. computational 2. analyze 3. accuracy

	FOUR MARKS EACH (CASE STUDY BASED)		
24	An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O(0,0)$ and the three stars have their locations at the points D, A and V having position vectors $2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, $7\mathbf{i}+5\mathbf{j}+8\mathbf{k}$ and $-3\mathbf{i}+7\mathbf{j}+11\mathbf{k}$ respectively.  Based on the above information, answer the following questions: i) How far is the star V from star A? ii) Find the unit vector in the direction of $\overrightarrow{DA}$ iii) Find the measure of $\angle VDA$ OR - iii). What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$	LO1 LO2 LO3	1. Understanding 2. analyze 3. accuracy 4 precision 5.recalling 6.interpretation
25	Let the polished side of the mirror be along the line $\frac{x}{1} - \frac{1-y}{-2} = \frac{2z-4}{6}$. A point $P(1,6,3)$, some distance away from the mirror, has its image formed behind the mirror. i) Find the coordinates of the image point. ii) Find the distance between point and image.	LO4 LO5 LO6	1. Understanding 2. analyze 3. application Problem solving

ANSWER

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	c	2	b	3	a	4	c	5	c
6	c	7	b	8	a	9	a	10	d

Q	Ans	Q	Ans	Q	Ans
11	0	12	-169	13	$\pm\sqrt{6}(-\hat{i} + 2\hat{j} - \hat{k})$
14	proof	15	(1,3,1) and (-1,-3,-1)	16	1
17	$5\sqrt{2}$	18	$\frac{2}{\sqrt{5}}$	19	$\pi/2$
20	$\sqrt{97}$	21	$64\hat{i} - 2\hat{j} - 28\hat{k}$	22	$(1,6,0), 2\sqrt{6}$
23	$(6,-3,-1), \sqrt{122}$	24	i) $\sqrt{113}$ (ii) $\frac{1}{3\sqrt{5}}(5\hat{i} + 2\hat{j} + 4\hat{k})$ (iii) $\cos^{-1}\left(\frac{11}{45\sqrt{2}}\right)$ or $\frac{11}{3\sqrt{5}}$	25	i)(1,0,7) (ii) $2\sqrt{13}$

LINEAR PROGRAMMING PROBLEM

Learning Outcomes:

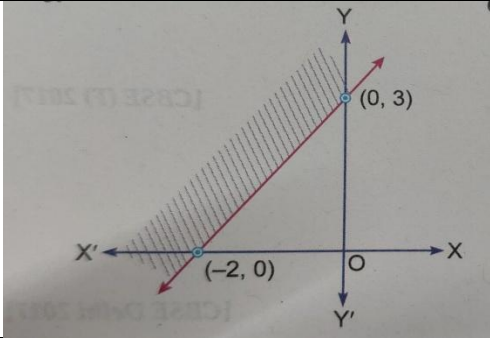
1. Solve graphically
2. Interpret feasible region and optimal solution

S.R	QUESTION	LOs	COMPETENCIES
	MCQ(ONE MARK EACH)		
1.	The number of feasible solutions of the linear programming problem given as Maximize $Z = 15x + 30y$ subject to constraints $3x + y \leq 12$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$ is : b) 1 b) 2 c) 3 d) infinite	LO1 LO2	1. Conceptual understanding 2. computation 3. analytic approach 4. problem solving 5. decision making
2.	The objective function $Z = Ax + By$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true? b) $A=9, B=1$ b) $A=9, B=2$ c) $A=3, B=5$ d) $A=5, B=3$	LO2	1. interpretation skill 2. mathematical formulation 3. ability to calculate 4. logical thinking 5. critical thinking
3.	If the corner point of the feasible region of an LPP are (0,3), (3,2), and (0,5), then the minimum value of $Z = 11x + 7y$ is: b) 21 (c) 33 c) 14 (d) 35 d)	LO2	1. ability to find 2. computation 3. Problem solving 4. Application 5. performing operations with accuracy
4.	The maximum value of $Z = 3x + 4y$ subject to the constraints $x \geq 0, y \geq 0$ and $x + y \leq 1$ is : a) 7 b) 4 c) 3 d) 10	LO1 LO2	1. Conceptual understanding 2. Logical thinking 3. Analytical approach
5.	If the feasible region of a linear programming problem with objective function $Z = ax + by$ is bounded, then which of the following is correct? (a) It will only have a maximum value. (b) It will only have a minimum value. (c) It will have both maximum and minimum value. (d) It will neither maximum nor minimum value.	LO2	1. optimization the problem 2. Analytical skill 3. Problem solving attitude 4. Logical thinking 5. Abstract thinking

	TWO MARKS EACH		
6.	Solve the linear equation $-3x+2y \geq 6$ graphically.	LO1 LO2	1. Recall 2. Concept understanding 3. Visualization and graphical representation 4. Applying
7.	Let $Z=ax+by$ has optimal value of same type at two points (2, 3), (5, 7), then drive the relationship between a and b.	LO2	1. Attention to detail 2. Application 3. Computation Skill 4. Problem solving 5. Logical thinking
	THREE MARKS EACH		
8.	Solve the following linear programming problem graphically : Maximize: $Z= x+2y$ Subject to the constraints : $x-y \geq 0$ $x-2y \geq -2$ $x \geq 0$ $y \geq 0$	LO1 LO2	1. Recall 2. Visualization and graphical representation 3. Mathematical computation 4. Logical thinking 5. Analytical approach
9.	Solve the following linear programming problem graphically : Minimize : $Z= x- 5y$ Subject to the constraints : $x-y \geq 0$ $-x+2y \geq 2$ $x \geq 3$ $y \leq 4$ $y \geq 0$	LO1 LO2	1. Conceptual understanding 2. Ability to compute 3. Mathematical computation 4. .Visualization of problem 5. abstract thinking
10.	Maximize $Z = 15x+ 10 y$ Subject to the constraints : $2x+y \leq 40$ $2x+3y \leq 80$ $x \geq 0, y \geq 0$	LO1 LO2	1. critical thinking 2. computation 3. problem solving 4. application of optimization 5. evaluation of solution

	FOUR MARKS EACH (CASE STUDY BASED)		
11.	Read the following passage and answer the following questions : A dealer Ramprakash residing in a rural area opens a shop and start his business with Rs 5760. He wishes to purchase a number of ceiling fans and table fans. A ceiling fan costs him Rs 360 and table fan coasts Rs 240.  (i) If Ramprakash purchase x ceiling fans, y table fans. he has space in his store for at most 20 items, then write its constraints. (ii) If he expects to sell ceiling fan at profit Rs 22 and table fan for Rs 18, then express the profit. (iii) What is the maximum profit of selling all the fans.	LO1 LO2	1. Conceptual understanding 2. Real life application and decision making 3. Problem formulation 4. Abstract thinking 5. Performing operations
12.	Mrs. Nidhi wants to invest at most Rs 12,000 in two types of shares A and B. According to the rules, she has to invest at least Rs 2000 in share A and at least Rs 4000 in share B. If the rate of interest on share A is 8% per annum and on share B is 10% per annum. (i) If Mrs Nidhi invest x in share A and invest y in share B. If the total interest received by Mrs. Nidhi from both type of shares is represented by Z, formulate the LPP. (ii) To maximize the interest on both type of shares, find the invested amount on both shares A and B.	LO1 LO2	1. Conceptual understanding 2. Problem formulation and computation 3. Real life application and decision making 4. abstract thinking 5. Critical thinking 6. Financial literacy

ANSWER

Q.No.	ANSWER
1	Solution - d
2	Solution – c
3	Solution – a
4	Solution – b
5	Solution – c
6	
7	Solution $-3a+4b=0$
8	Solution : No maximum value of Z
9	Solution : Minimum value of Z is -16 at (4,4)
10	Solution : Z= 350 at (10,20)
11	Solution : (i) $x+y \leq 20$ (ii) $Z = 22x+18 y$ (iii)Rs 392
12	(i)Max $Z = -\frac{2x}{25} + \frac{y}{10}$ Rs subject to the constraints $X \geq 2000$, $y \geq 4000$, $x+y \leq 12000$ $x, y \geq 0$ (ii) $x = 2000$ Rs , $y = 10,000$ Rs

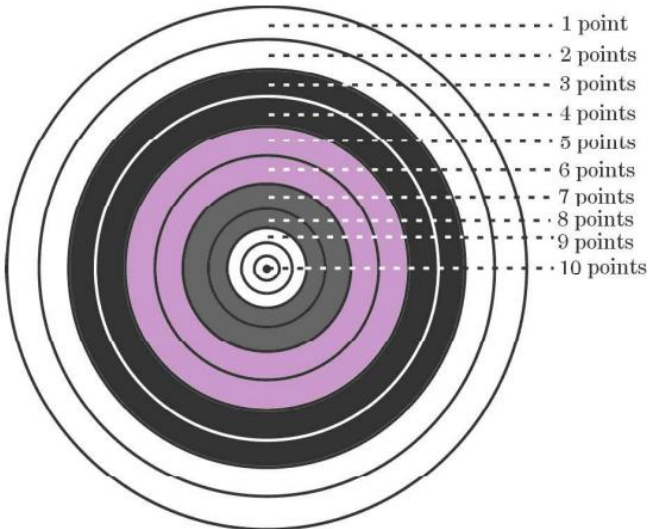
PROBABILITY

Learning Outcomes:

1. Use conditional probability and Bayes' theorem
2. Solve problems on independent/dependent events
3. Apply properties of random variables and distribution

Q.No.	Question	LOs	Competency
MCQ(1 MARKS QUESTION)			
1	A card is drawn from a well-shuffled pack of 52 cards. Let A = "the card is red", B = "the card is a king". Then, $P(A \cap B) = ?$ (a) $1/26$ (b) $1/13$ (c) $1/52$ (d) $1/2$	LO1 LO2	1.Knowledge 2.Recalling 3. Understanding.
2	If A and B are two independent events such that $P(A) = 0.5$ and $P(B) = 0.4$, then $P(A \cup B)$ is: (a) 0.9 (b) 0.7 (c) 0.6 (d) 0.8	LO2	1. Conceptual Knowledge 2.co-relation
3	The probability of solving a problem by A is $1/3$ and by B is $1/4$. If both try independently, then the probability that the problem is solved is: (a) $1/2$ (b) $7/12$ (c) $5/12$ (d) $1/6$	LO2	1.Knowledge 2.Understanding 3.Computatuon ability
VERY SHORT ANSWER (2 MARKS QUESTION)			
4	Two cards are drawn from a pack of 52 cards. Find the probability that both are black.	LO2	1.Critical thinking 2.Application 3.Knowlege 4.Computational ability
5	A die is thrown twice. Find the probability that the sum is 10.	LO2	1.Knowlege 2.Understanding 3. Application 4.Computational ability
6	A bag contains 5 red and 4 blue balls. Two balls are drawn without replacement. Find the probability that one is red and one is blue.	LO2	1.Knowledge 2.Application 3.Computational ability

SHORT ANSWER (3 MARKS QUESTION)			
7	A family has 2 children. Find the probability that both are boys, if it is known that (i) at least one of the children is a boy (ii) the elder child is a boy	LO1	1.Understanding 2.Knowledge 3.Thinking Ability
8	Given $P(A) = 1/4$, $P(B) = 2/3$ and $P(A \cup B) = 3/4$. Are the events independent? Justify your answer.	LO2	1.Knowledge 2.Understanding 3.Recalling concept
9	A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?	LO1	1.Conceptual Knowledge 2.Analysing 3.Differentiating 4.Recall 5.Computational 6.Application
LONG ANSWER (5 MARKS QUESTION)			
10	An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?	LO1	1.Data analyzing 2. Differentiating 3.Recall 4.Computational 5.logical reasoning
11	Two cards are drawn simultaneously without replacement from a well shuffled pack of 52 cards. Find the mean and variance of the number of aces.	LO1	1.Analysing 2. Imagination 3.Recall 4. Computational 5. Critical thinking
12	A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?	LO1	1.Conceptual Knowledge 2.Imagination 3.Recall 4.Critical Thinking

CASE STUDY BASED QUESTION(FOUR MARKS)			
13	Read the following passage and then answer the questions given below. There are two anti craft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time. (i)What is the probability that the shell fired from exactly one of them hit the plane? (ii)If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?	LO1 LO2	1.Imagination 2.understanding 3.knowledge 4.visualization 5.computational 6.Application
14	In a game of Archery, each ring of the Archery target is valued. The centre-most ring is worth 10 points and rest of the rings are allotted points 9 to 1 in sequential order moving outwards.  1 point 2 points 3 points 4 points 5 points 6 points 7 points 8 points 9 points 10 points Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely to earn 10 points with a probability of 0.9. Based on the above information, answer the following. (i)Write the probability that archer A does not earn 10 points. (ii)Write the probability that archer B does not earn 10 points. (iii)If both of them hit the Archery target, then find the probability that exactly one of them earns 10 points. OR If both of them try to hit the Archery target, then find the probability that it was hit by archer A.	LO1 LO2	1.Imagination 2.understanding 3.knowledge 4.visualization 5.computational 6.Application

ANSWER

Question	Answer	Question	Answer
1	$1/52$	2	0.70
3	$1/2$	4	$25/102$
5	$1/12$	6	$5/9$
7	(i) $1/3$ (ii) $1/2$	8	Yes
9	$1/3$	10	$1/52$
11	Mean- $36/221$ Variance- $400/2873$	12	$2/5$
13	(i) 0.38 (ii) $1/3$	14	(i) 0.2 (ii) 0.1 (iii) 0.26 or $8/17$