तत् त्वं पूषन अपावृणु
केंन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN (KVS)

ZIET CHANDIGARH

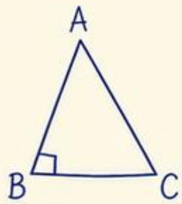
$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

$3x - 2 = 10$

$2x + 5 = 15$

SUPPORT MATERIAL

MATHEMATICS

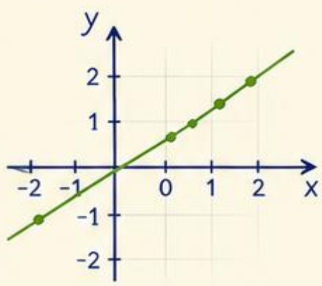


$a^2 + b^2 = (a+b)^2 - 2ab$

FOR

CLASS VIII

(PART I)



$-8 + 12 = 4$

CONCEPTUAL UNDERSTANDING | PROBLEM SOLVING | REAL LIFE APPLICATIONS | PRACTICE & MASTERY

LEARN WITH UNDERSTANDING | PRACTICE REGULARLY | THINK LOGICALLY | SUCCEED CONFIDENTLY

संरक्षक

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श्रीमती श्रुति भार्गव
उपायुक्त एवं निदेशक
आंचलिक शिक्षा एवं प्रशिक्षण संस्थान, चंडीगढ़

श्री अजीत कुमार यादव
सह-कोर्स निदेशक
(प्राचार्य पीएम् श्री केंद्रीय विद्यालय -1 अम्बाला कैंट)

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FOREWORD

It gives me immense pleasure to present the **"Support Material for Class VIII Mathematics (Part I) based on the New NCERT Textbook – Ganita Prakash."** This resource has been thoughtfully developed to support teachers in implementing the vision of the **National Education Policy (NEP) 2020**, the National Curriculum Framework for School Education (NCFSE) 2023, and the competency-based approach envisaged by CBSE and Kendriya Vidyalaya Sangathan.

The new NCERT textbook introduces mathematics through meaningful contexts, hands-on learning, logical reasoning, and problem-solving. It encourages students to explore concepts through activities, discussions, and real-life situations rather than relying solely on procedural learning. This support material has been prepared with the objective of assisting teachers in translating these pedagogical principles into effective classroom practices.

The material includes chapter-wise learning objectives, concept summaries, competency-based questions, higher-order thinking tasks. It is designed to strengthen conceptual understanding, promote mathematical thinking, and encourage joyful learning among students.

I appreciate the sincere efforts of the Associate Course Director, Resource Persons and Mathematics teachers who have contributed their expertise and dedication to the preparation of this publication. Their commitment towards enhancing the quality of mathematics education is truly commendable.

I am confident that this support material will serve as a valuable companion for teachers and will contribute significantly to improving students' mathematical understanding, confidence, and achievement.

I extend my best wishes to all teachers and students for a rewarding and enriching learning journey.

Deputy Commissioner & Director
Kendriya Vidyalaya Sangathan
Zonal Institute of Education & Training, Chandigarh

CONTENT PREPARED BY:

TOPIC	NAME OF TEACHER (TGT Maths)	NAME OF KV	REGION
A SQUARE AND A CUBE	DEEPAK KUMAR	SECTOR-22 ROHINI	DELHI
POWER PLAY	YOGITA KHANNA	SHAKURBASTI	DELHI
A STORY OF NUMBERS	NITESH KUMAR	DR. RAJENDRA PRASAD	DELHI
QUADRILATERALS	ANNU	NO.1 DELHI CANTT.	DELHI
NUMBER PLAY	NEERU RANI	GOLE MARKET	DELHI
WE DISTRIBUTE, YET THINGS MULTIPLY	OM PARKASH SINGH	BHIKHIWIND	CHANDIGARH
PROPORTIONAL REASONING-1			

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POWER PLAY			
A STORY OF NUMBERS	NEELAM YADAV	PALWAL	GURUGRAM
QUADRILATERALS			
NUMBER PLAY	PRIYANKA SHARMA	DHARAMSHALA	GURUGRAM
WE DISTRIBUTE, YET THINGS MULTIPLY			
PROPORTIONAL REASONING-1	GARIMA BHATT	ITBP BHANU	GURUGRAM

1 A SQUARE AND A CUBE

Numbers reveal beautiful patterns. Squares and cubes are two such special patterns.

IN THIS CHAPTER YOU WILL LEARN

- Square numbers and their properties
- Square roots
- Cube numbers and their properties
- Cube roots
- Patterns, estimates and number sense
- Historical connections and puzzles

THE LOCKER PUZZLE (Chapter Opening)

100 lockers numbered 1 to 100 are toggled in rounds.

- Person 1 opens every locker.
- Person 2 toggles every 2nd locker.
- Person 3 toggles every 3rd locker … and so on.

1	2	…	10
11	12	…	20
…	…	4	…
91	92	…	100

A locker is toggled as many times as the number of factors of its number. Only lockers with an odd number of factors remain open — i.e., the square numbers.

Open lockers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.
The passcode was hidden in the lockers touched exactly twice — the prime numbers: 2 – 3 – 5 – 7 – 11.

KEY DEFINITIONS

- **Square Number:** A number obtained by multiplying a number by itself.
  $n \times n = n^2$ (Read as 'n squared')
- **Perfect Square:** The squares of natural numbers.
- **Square Root:** If $y = x^2$, then x is the square root of y.
  Denoted by $\sqrt{y}$ (positive square root).
  Every perfect square has two integral square roots: $+\sqrt{y}$ and $-\sqrt{y}$.
- **Cube (or Cube Number):** A number obtained by multiplying a number by itself three times.
  $n \times n \times n = n^3$ (Read as 'n cubed')
- **Perfect Cube:** The cubes of natural numbers.
- **Cube Root:** If $y = x^3$, then x is the cube root of y.
  Denoted by $\sqrt[3]{y}$.

SQUARE NUMBERS

Squares of first 10 natural numbers

n	n^2	Value	n	n^2	Value
1	1^2	1	6	6^2	36
2	2^2	4	7	7^2	49
3	3^2	9	8	8^2	64
4	4^2	16	9	9^2	81
5	5^2	25	10	10^2	100

Properties & Patterns

- Perfect squares end with 0, 1, 4, 5, 6 or 9.
- Squares can have only an even number of zeros at the end.
- A number ends in 2, 3, 7 or 8 → Not a square.
- If a number has 1 or 9 in the units place, its square ends in 1.
- If a number has 6 in the units place, its square also ends in 6.
- Squares increase by consecutive odd numbers.

SQUARES AS SUM OF ODD NUMBERS

$1 = 1$
$4 = 1 + 3$
$9 = 1 + 3 + 5$
$16 = 1 + 3 + 5 + 7$
$25 = 1 + 3 + 5 + 7 + 9$
$36 = 1 + 3 + 5 + 7 + 9 + 11$

In general: n^2 = sum of first n odd numbers.

FACTORS AND SQUARES

- A locker is toggled as many times as the number of factors of its number.
- Numbers with an odd number of factors are exactly the square numbers.
- Every factor (except in a square number) has a partner factor. In a square number, one factor pairs with itself.

6 : 1×6, 2×3	4 : 1×4, 2×2	9 : 1×9, 3×3	1 : 1×1
Factors: 1, 2, 3, 6	Factors: 1, 2, 4	Factors: 1, 3, 9	Factors: 1
(Even)	(Odd)	(Odd)	(Odd)
Closed	Open	Open	Open

Only square numbers remain open.
Hence, open lockers are the squares.

FINDING SQUARE ROOTS

Methods

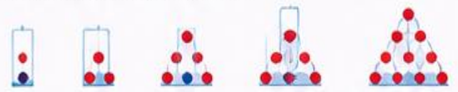
1. List squares in order.
2. Use: n^2 = sum of first n odd numbers. Subtract successive odd numbers.
3. Prime factorisation: factors in pairs.
4. Estimation:
   - Find nearest perfect squares.
   - Check units digit (√ ends in 0,1,2,3,4,5,6,7,8,9).

Examples

- $\sqrt{1936} = 44$ (since $44^2 = 1936$)
- $\sqrt{576} = 24$ (since $576 = (2^2)(3^2)(4^2) = (2\times3\times4)^2$)
- $40 < \sqrt{1936} < 50$, last digit 4 or 6
  $45^2 = 2025 > 1936 \Rightarrow \sqrt{1936} = 44$

MORE ABOUT SQUARES

- The difference between consecutive squares is an odd number.
- Numbers between n^2 and $(n+1)^2$ are:
  $n^2+1, n^2+2, \dots, (n+1)^2-1$ → Total $2n$ numbers.
- There are n perfect squares between $(n-1)^2$ and n^2.
  e.g., between 0 – 100: 1–10 squares, 100 – 200: 11–14 squares, 200 – 300: 15–17 squares, …
- Relation with triangular numbers:



1	1+3=4	1+3+5=9	1+3+5+7=16	1+3+5+7+9=25
$1 = 1^2$	$=2^2$	$=3^2$	$=4^2$	$=5^2$

CUBE NUMBERS

Cubes of first 10 natural numbers

n	n^3	Value	n	n^3	Value
1	1^3	1	6	6^3	216
2	2^3	8	7	7^3	343
3	3^3	27	8	8^3	512
4	4^3	64	9	9^3	729
5	5^3	125	10	10^3	1000

Properties & Patterns

- Possible last digits: 0, 1, 8, 9, or even number of zeros.
- A cube cannot end with exactly two zeros.
- Number of digits in cubes:
  1–9 (1 digit), 10–21 (2 digits), 22–46 (3 digits), …
- n^3 increases by adding $(3n^2 + 3n + 1)$ each time.

CUBES AS SUM OF CONSECUTIVE ODD NUMBERS

$1 = 1$
$8 = 1 + 3 + 5$
$27 = 1 + 3 + 5 + 7 + 9$
$64 = 1 + 3 + 5 + 7 + 9 + 11 + 13$
$125 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$

In general: n^3 = sum of first n^2 odd numbers.

EXAMPLES OF PATTERNS

1. Units digit pattern (Squares)
   Possible last digits: 0, 1, 4, 5, 6, 9.
   Not possible: 2, 3, 7, 8.
2. Zeros at the end
   If a number ends with k zeros, its square ends with $2k$ zeros.
3. Parity
   Even^2 = Even, Odd^2 = Odd.
4. Example sets
   $1^2, 9^2, 11^2, 19^2, 21^2, 29^2, \dots$ end in 1.
   $4^2, 6^2, 16^2, 26^2, \dots$ end in 6.

FINDING CUBE ROOTS

Methods

1. Prime factorisation: factors in triplets.
2. List cubes in order.
3. Using pattern & estimation.

Example

- $3375 = 3\times3\times3\times5\times5\times5 = (3\times5)^3 = 15^3$
  $\Rightarrow \sqrt[3]{3375} = 15$
- Estimation:
  $15^3 = 3375 < x < 16^3 = 4096 \Rightarrow \sqrt[3]{3500} \approx 15.$

SQUARE & CUBE COMPARISON

Aspect	Square (n^2)	Cube (n^3)
Definition	$n \times n$	$n \times n \times n$
Notation	n^2	n^3
Roots	$\sqrt{n}$	$\sqrt[3]{n}$
Factors (for perfect)	In pairs	In triplets
Unit digit pattern	0,1,4,5,6,9	0,1,8,9 or even 0's
Zeros rule	k zeros → $2k$ zeros	k zeros → $3k$ zeros
Sum of odd numbers	First n odd numbers	First n^2 odd numbers

HISTORICAL CONNECTION

- **Babylonians** (1700 BCE) compiled lists of squares and cubes for practical use in land measurement and construction.
- In ancient India, squares were called "varga" and cubes were called "ghana".
- **Aryabhata** (499 CE) defined varga as both the square figure and the square of a number.
- The word *mula* (root) from Sanskrit (meaning root of a plant, basis, origin) gave rise to the words *jidhr* (Arabic) and *radix* (Latin) — from which we get "root" in mathematics.
- **Ramanujan** (1910s) astonished Hardy by identifying 1729 as the smallest number that is the sum of two cubes in two different ways: $1^3 + 12^3 = 9^3 + 10^3 = 1729$ (Taxicab number).

PUZZLE CORNER

1. **Square Pairs!**
   Arrange numbers 1 to 17 in a row so that the sum of every adjacent pair is a square.
   (**Hint:** unique arrangement exists.)
2. Arrange numbers 1 to 32 in a circle so that adjacent numbers add up to a square.

IMPORTANT FORMULAS

- $n^2 = n \times n$
- $\sqrt{n^2} = n$ (for positive n)
- $n^3 = n \times n \times n$
- $\sqrt[3]{n^3} = n$
- $n^2 - (n-1)^2 = 2n - 1$ (odd number)
- $n^3 - (n-1)^3 = 3n^2 - 3n + 1$
- Sum of first n odd numbers $= n^2$
- Sum of first n^2 odd numbers $= n^3$

KEY TAKEAWAYS

- ✓ Squares and cubes have beautiful patterns.
- ✓ Square numbers are sums of first n odd numbers.
- ✓ Cube numbers are sums of first n^2 odd numbers.
- ✓ Prime factorisation helps identify perfect squares (pairs) and perfect cubes (triplets).
- ✓ Square root (√) and cube root ($\sqrt[3]{y}$) are inverse operations.
- ✓ Patterns help us estimate and solve large problems.

REMEMBER

Look for patterns, factorise smartly, estimate wisely, and numbers will reveal their secrets!

A SQUARE AND A CUBE

Learning Outcomes:

LO1: Identify square and cube numbers

LO2: Determine square roots and cube roots

LO3: Recognize numerical patterns

LO4: Apply concepts to real-life situations.

Competencies:

C1: Number sense

C2: Pattern recognition

C3: Logical reasoning

C4: Computational fluency

C5: Problem solving

Q.No.	QUESTION	LO	Competency
	MCQ's (One Mark Each)		
1	Which one of the following numbers has an odd number of factors? A. 24 B. 36 C. 45 D. 50	LO3	C1
2	Which of the following numbers cannot be a perfect square? A. 529 B. 784 C. 578 D. 961	LO1	C2
3	A number has **4 zeros** at the end. How many zeros will its square have at the end? A. 4 B. 6 C. 8 D. 10	LO3	C1
4	Which of the following is a perfect cube but not a perfect square? A. 64 B. 125 C. 729 D. 4096	LO1	C1
5	If the square of a natural number is **2025**, then the number is A. 40 B. 44 C. 45 D. 50	LO2	C3
6	Which one of the following numbers is **not** a perfect cube? A. 512 B. 729 C. 1331 D. 1458	LO1	C4
7	The prime factorisation of a number is $2^4 \times 3^2 \times 5^2$ Its square root is A. 60 B. 120 C. 180 D. 240	LO2	C4
8	What is the sum of the first 12 odd natural numbers? A. 121 B. 132 C. 144 D. 169	LO3	C2 C4
9	Which lockers remain open after the locker puzzle? A. Prime-numbered lockers B. Even-numbered lockers C. Square-numbered lockers D. Odd-numbered lockers	LO3 LO4	C5

10	Which statement is always true? A. Every perfect square is a perfect cube. B. Every perfect cube has an odd number of factors. C. Every perfect square has an even number of factors. D. Every perfect square has two integral square roots.	LO1	C1
Short Answer Type (Two Marks Each)			
11	A number has the prime factorisation: $2^4 \times 3^2$ Is it a perfect square? Find its square root.	LO1 LO2	C4
12	Explain why 225 is both a perfect square and not a perfect cube.	LO1	C3 C4
13	Find the smallest number that should be multiplied by **72** to make it a perfect square.	LO1	C1 C4
14	Without actual multiplication, state whether 6400 is a perfect square. Give a reason.	LO1	C3 C4
15	Write any four perfect cubes between 1 and 1000.	LO1	C1 C3 C4
16	Find the value of: (a) $\sqrt{324}$ (b) $\sqrt[3]{3375}$	LO2	C4
17	Find the difference between the squares of 26 and 25. What pattern do you observe?	LO1	C2 C4
18	A square field has an area of 625 m^2. Find its perimeter.	LO1 LO4	C4 C5
19	Write the first five perfect squares and the first five perfect cubes.	LO1	C3 C4
20	Explain why only square-numbered lockers remain open in the locker puzzle.	LO1 LO4	C3 C5
Short Answer Type (Three Marks Each)			
21	A number has the prime factorization $2^3 \times 3^2 \times 5$ (i) Is the number a perfect square? Give a reason. (ii) Find the smallest number by which it should be multiplied to make it a perfect square.	LO1	C3 C4
22	Without performing long multiplication, determine whether each of the following can be a perfect square. Give reasons. (i) 578 (ii) 4900 (iii) 1764	LO1	C3 C4
23	A square garden has an area of 784 m^2. (i) Find its side. (ii) Find its perimeter. (iii) If fencing costs ₹75 per metre, find the total fencing cost.	LO1 LO3	C3 C4 C5

24	Using prime factorisation, determine whether 1156 is a perfect square. If yes, find its square root.	LO1 LO2	C4 C5
25	Find the cube root of 10648 using prime factorisation.		
Long Answer Type (Five Marks Each)			
26	A number has the prime factorization $N = 2^3 \times 3^2 \times 5^4$ Answer the following: (i) Is Na perfect square? Give a reason. (ii) Find the smallest number by which Nshould be multiplied to make it a perfect square. (iii) Find the square root of the resulting perfect square.	LO1 LO2	C4 C5
27	A square park has an area of 1296 m². (i) Find the length of one side of the park. (ii) Find its perimeter. (iii) If a pathway 2 m wide is built all around the outside of the park, what will be the outer side length of the park? (iv) Find the perimeter of the outer boundary. (v) Find the increase in perimeter due to the pathway.	LO1 LO2 LO4	C3 C4 C5
28	Using prime factorisation, answer the following: (i) Show whether 9408 is a perfect square. (ii) Find the smallest number by which it should be multiplied to make it a perfect square. (iii) Find the square root of the resulting perfect square	LO1 LO2	C3 C4 C5
Case Study Based			
29	**(Sports Ground Design)** The Sports Club of a school is developing a square practice ground for students. The area available for the ground is 1600 m². Around the ground, a walking track of 3 m width is to be constructed. The Physical Education teacher also wants to place one lamp post at each corner of the outer boundary. Answer the following questions: (i) Find the side of the square practice ground. (ii) Find the perimeter of the original ground. (iii) Find the side length of the outer boundary after constructing the track. (iv) Find the perimeter of the outer boundary.	LO1 LO2 LO4	C3 C4 C5
30	**(Packing Wooden Cubes)** A carpenter manufactures wooden cubes of side 6 cm. These cubes are packed inside a large cubical box whose side is 24 cm. Answer the following questions: (i) How many small cubes fit along one edge of the large cube? (ii) How many small cubes can be packed in the large cube? (iii) If each small cube weighs 250 g, what is the total weight of all cubes? (iv) 64 is both a perfect square and a perfect cube (True/False)	LO1 LO2 LO4	C3 C4 C5

Answers:

Q.	1	2	3	4	5
Ans.	B	C	C	B	C
Q.	6	7	8	9	10
Ans.	D	A	C	C	D
Q.	11	12	13	14	15
Ans.	Perfect square; $\sqrt{144} = 12$	$225 = 15^2$; not a perfect cube	2	es; $6400 = 80^2$	8, 27, 64, 125 (or any four correct cubes)
Q.	16	17	18	19	20
Ans.	(a) 18 (b) 15	**51**; difference of consecutive squares is an odd number	100 m	Squares: 1, 4, 9, 16, 25; Cubes: 1, 8, 27, 64, 125	Square-numbered lockers remain open because only square numbers have an odd number of factors.
Q.	21	22	23	24	25
Ans.	i)Not a perfect square; ii)multiply by 10	(i) Not a square (ii) Perfect square ($70^2 = 4900$) (iii) Perfect square ($42^2 = 1764$)	ide = 28 m, Perimeter = 112 m, Cost = ₹8400	Perfect square; $\sqrt{1156} = 34$	$\sqrt[3]{10648} = 22$
Q.	26	27	28	29	30
Ans.	Not a perfect square; multiply by **2**; square root = **300**	Side = 36 m, Perimeter = 144 m, Outer side = 40 m, Outer perimeter = 160 m, Increase = 16 m	Not a perfect square; multiply by 6; square root = 168	40m 160m 46m 184m	4 64 16kg True

Challenging Questions (Stage 1)

1.A number has exactly 15 factors. Can it be a perfect square? Justify your answer with an example.

2. Without calculating the square, determine which number has the larger square: 98 or 101. Explain your reasoning.

3. A number is divisible by 4 and 9 but not by 25. Can it be a perfect square? Explain.

4. Find two consecutive natural numbers between which the square root of 170 lies.

5. A cube has 729 unit cubes. Find the number of unit cubes along one edge.

Challenging Questions (Stage 2)

1. Find the smallest perfect square that is divisible by 18, 24 and 30.

2. Find the least number that should be divided by 5400 to obtain a perfect cube.

3. A square floor has **2025 tiles**, one tile occupying one square unit.Without multiplication, determine the number of tiles along one side.

4. Find the value of $51^2 - 49^2$ without finding the individual squares.

5. he product of two consecutive natural numbers is **never** a perfect square.
Explain why.

Challenging Questions (Stage 2)

1. Find the smallest number greater than **500** that is both a perfect square and a perfect cube.

2. If $x^2 = 1764$ and $y^3 = 1728$, find $x + y$.

3. How many perfect squares are there between 1000 and 2000?

4. The volume of a cube is 13824 cm^3. Find the length of one edge.

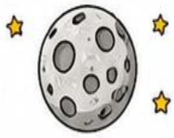
5. A number is a perfect square and a perfect cube. It has exactly 49 factors. Find the number.

Answers (Challenging Questions)

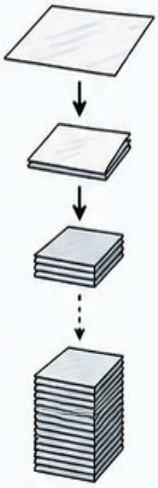
	Q1	Q2	Q3	Q4	Q5
Stage1	Yes; Example: **144**	101^2 is larger	Yes; Example: **36**	13 and 14	9
Stage2	3600	25	45	200	Product of consecutive natural numbers cannot be a perfect square
Stage3	729	54	13 perfect squares	24cm	2^{48}

2 POWER PLAY

Just 46 folds of a paper can reach the Moon!



1. THE IDEA



- Fold a sheet of paper. Each time you fold, its thickness doubles.
- This is an example of exponential (multiplicative) growth.
- Amazing fact: After just 46 folds, the thickness is more than 7,00,000 km → enough to reach the Moon!

2. THICKNESS AFTER FOLDS

Initial thickness of paper = 0.001 cm

Thickness doubles after each fold.

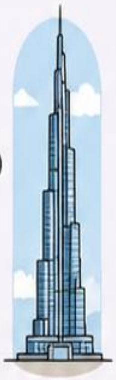
Fold	Thickness	Fold	Thickness	Fold	Thickness
1	0.002 cm	7	0.128 cm	13	8.192 cm
2	0.004 cm	8	0.256 cm	14	16.384 cm
3	0.008 cm	9	0.512 cm	15	32.768 cm
4	0.016 cm	10	1.024 cm	16	65.536 cm
5	0.032 cm	11	2.048 cm	17	≈ 131 cm
6	0.064 cm	12	4.096 cm		

(We use '≈' to mean approximately equal to.)

After 10 folds ≈ **1.024 cm** | After 17 folds ≈ **131 cm** (a little more than 4 feet)

3. AMAZING GROWTH!

Folds	Approx. Thickness	
After 26 folds	≈ 670 m	(Burj Khalifa is 830 m tall)
After 30 folds	≈ 10.7 km	(Height of planes fly)
After 46 folds	> 7,00,000 km	(Can reach the Moon!)



4. THICKNESS DOUBLE EVERY TIME!

From any point, the thickness after 10 more folds increases by 10^{24} times.

Range of Folds	Thickness (approx.)	Times Increased By
0 to 10	1.023 cm	10^{24}
10 to 20	10.474 m	10^{24}
20 to 30	10.726 km	10^{24}
30 to 40	10984.2 km	10^{24}

5. EXPONENTIAL NOTATION – KEY CONCEPTS

General Form

n^a means n multiplied by itself a times.

Example: $5^4 = 5 \times 5 \times 5 \times 5 = 625$

Base = 5 Exponent (power) = 4

Common Powers

$n^2 = n \times n$ (read as 'n squared')

$n^3 = n \times n \times n$ (read as 'n **cubed**')

$n^4 = n \times n \times n \times n$ (read as 'n to the power 4')

$n^7 = n \times n \times n \times n \times n \times n \times n$

(read as 'n to the power 7')

and so on...

Examples

- $4^3 = 4 \times 4 \times 4 = 64$
- $(-4)^3 = (-4) \times (-4) \times (-4) = -64$
- $a \times a \times a \times b \times b = a^3b^2$
- $a \times a \times b \times b \times b \times b = a^2b^4$

Remember!

$4 + 4 + 4 = 3 \times 4 = 12$

but

$4 \times 4 \times 4 = 4^3 = 64$

Express as Product of Prime Factors

Example: Express 32400 in exponential form.

$32400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3$

$= 2^4 \times 5^2 \times 3^4$

Try It!

- What is $(-1)^5$? → -1 (negative)
- What is $(-1)^{56}$? → $+1$ (positive)
- Is $(-2)^4 = 16$? → Yes

6. QUICK FORMULA

If the initial thickness of paper is v cm, then thickness after n folds = $v \times 2^n$

Example: If $v = 0.001$ cm and $n = 10$, then thickness $= 0.001 \times 2^{10} = 1.024$ cm

Key Takeaway

Small steps of doubling **create huge results** – that's the **Power Play**!

POWER PLAY

Learning Outcomes:

LO1: Understand powers and exponents.

LO2: Apply laws of exponents in calculations.

LO3: Represent large and small numbers using powers of 10.

LO4: Solve real-life problems involving exponents.

LO5: Develop logical reasoning and problem-solving skills.

Competencies:

C1: Conceptual understanding

C2: Numerical computation

C3: Logical reasoning

C4: Real-life application

Q.No.	QUESTION	LO	Competency
	MCQ's (One Mark Each)		
1	81 can be expressed as: (a) 9^4 (b) 3^4 (c) 27^3 (d) 3^3	LO1	C1
2	The value of 5^2 x 5^{-2} is (a) 1 (b) 5 (c) 5^4 (d) 5^{-4}	LO2	C2
3	What is the value of $7^3 \div 7$? (a) 7 (b) 49 (c) 343 (d) 497	LO2	C2
4	The exponential form of 625 is (a) 5^3 (b) 5^2 (c) 5^4 (d) 5^5	LO1	C1
5	The standard form of 0.000072 is (a) 72×10^4 (b) 72×10^{-4} (c) 7.2×10^5 (d) 7.2×10^{-5}	LO3	C2
6	Which of the following is equal to $2^5 \times 3^2$? (a) 96 (b) 144 (c) 288 (d) 1	LO2	C2
7	What is the value of 7^1 x 7^0x 7^1? (a) 7 (b) 101 (c) 7^{101} (d) 49	LO2	C2
8	How many 4-digit codes can be formed using digits 0-9 (repetition allowed)? (a) 1000 (b) 10,000 (c) 9,000 (d) 100	LO4 LO5	C2 C3 C4
9	$(-2)^4 \times (-3)^2$ is equal to: (a) 144 (b) -144 (c) 36 (d) -36	LO2	C2

<table>
<tr><td>10</td><td>The value of : 2^0 x 3^0 + 5^0
(a) 3 (b) 2 (c) 5 (d) 11</td><td>LO1</td><td>C1
C2</td></tr>
<tr><td colspan="4">Short Answer Type (Two Marks Each)</td></tr>
<tr><td>11</td><td>Write one crore in powers of 10.</td><td>LO3</td><td>C2</td></tr>
<tr><td>12</td><td>Find the value of 5^0</td><td>LO1</td><td>C1</td></tr>
<tr><td>13</td><td>Every square number is also a cube.TRUE /FALSE</td><td>LO5</td><td>C3</td></tr>
<tr><td>14</td><td>Product of two cube numbers is a cube.TRUE/FALSE</td><td>LO5</td><td>C3</td></tr>
<tr><td>15</td><td>Write exponential form of 648 .</td><td>LO1
LO2</td><td>C1
C2</td></tr>
<tr><td>16</td><td>Simplify using laws of exponents: $5^2 \times 5^3$</td><td>LO2</td><td>C2</td></tr>
<tr><td>17</td><td>Write 256 as a product of powers of prime factors.</td><td>LO1</td><td>C2</td></tr>
<tr><td>18</td><td>Which is greater: 3^6 or 2^9?</td><td>LO5</td><td>C3</td></tr>
<tr><td>19</td><td>Express 10000 as both a square and a cube power if possible.</td><td>LO1</td><td>C1</td></tr>
<tr><td>20</td><td>Find the fourth power of 6.</td><td></td><td></td></tr>
<tr><td colspan="4">Short Answer Type (Three Marks Each)</td></tr>
<tr><td>21</td><td>Express the height of a bundle of 500 papers placed on each other if the thickness of one paper is 0.0016 cm, in standard form.</td><td>LO4
LO5</td><td>C2
C3
C4</td></tr>
<tr><td>22</td><td>Find the unit digit in the value of 3^{100}.</td><td>LO5</td><td>C3</td></tr>
<tr><td>23</td><td>Express the product of 1.5×10^3 and 2.5×10^{-5} in the standard form.</td><td>LO3</td><td>C2</td></tr>
<tr><td>24</td><td>Convert the following to exponential form:
(a) $8 \times 8 \times 8 \times 8$ (b) $z \times z$
(c) $4 \times 4 \times 6 \times 6 \times 6$</td><td>LO1</td><td>C2</td></tr>
<tr><td>25</td><td>Write the following as a product of their prime factors using powers.
(a) 924 (b) 875 (c) 1080</td><td>LO1</td><td>C1
C2</td></tr>
<tr><td colspan="4">Long Answer Type (Five Marks Each)</td></tr>
<tr><td>26</td><td>Fill in the blanks
<table>
<tr><td></td><td>Expression</td><td>Simplified Form</td></tr>
<tr><td>(a)</td><td>7^{-2}</td><td>…………</td></tr>
<tr><td>(b)</td><td>……………</td><td>4500</td></tr>
<tr><td>(c)</td><td>10°</td><td>…………</td></tr>
<tr><td>(d)</td><td>$(3^{-1})^2$</td><td>…………</td></tr>
<tr><td>(e)</td><td>………………..</td><td>400</td></tr>
</table></td><td>LO1
LO2
LO3
LO5</td><td>C1
C2
C3</td></tr>
<tr><td>27</td><td>Express the number appearing in the following statements in standard form.
(i) The population of a city is 650000.
(ii) The distance between Earth and Sun is about 93000000 miles.
(iii) A petri dish contains 0.000045 g of bacteria.</td><td>LO3
LO4</td><td>C2
C4</td></tr>
</table>

	(iv) The mass of a hydrogen atom is approximately 0.0167 g.		
28	Simplify each expression to its simplest form. (a) $(2^3 \times 5^{-2})^2$ (b) $3^0 + 4^0$ (c) $(10^4 \div 10^{-3})^2$ (d) $3^0 - 4^0$ (e) $(2^{-3})^{-2}$	LO2	C2
Case Study Based			
29	A company manufactures tiny microchips with dimensions 5×10^3 m in length, 2×10^{-3} m in width and 1×10^3 m in height  (a) Calculate the volume of one microchip.Express your answer in cubic metres using exponents. (b) If the company produces 106 microchips then what would be the total volume of all the microchips produced? Express the total volume in standard form.	LO1 LO2 LO3, LO4 LO5	C2 C3 C4
30	A warehouse has 10 aisles, each aisle has 10 shelves and each shelf stores 10 boxes.  (a) Express total boxes using powers. (b) Find total boxes.	LO1 LO2 LO3 LO4 LO5	C1 C2 C3 C4 C5

Answers:

Q.	1	2	3	4	5	6	7	8
Ans.	b	a	b	c	d	c	b	
Q.	9	10	11	12	13	14	15	16
Ans.	(a) 144	(c)2	10^7	1	False	False	$2^3 \times 3^4$	3125
Q.	17	18	19	20	21	22	23	24
Ans.	2^8	2^9	100^2	6^4 =1296	8×10^{-1} cm	1	3.75×10^{-2}	8^4,z^2, 4^2 x 6^3
Q.	25	26	27	28	29	30		
Ans.	(a)2^2x3x7x11 (b)5^3 x7 (c) 2^3 x 3^3 x5	(a)1/49 (b)45 x 10^2 (c)1 (d)1/9 (e)4 x10^2	(i) 6.5 $\times 10^5$ (ii) 9.3 $\times 10^7$ miles (iii) 4.5 $\times 10^{-5}$ g (iv)1.67 X10^{-3}	(a) 64/625 (b) 2 (c)10^{14} (d)0 (e)64	(a) 10^4 cubic metres (b) 10^{10} cubic metres	(a) 10^3 (b)1000		

Challenging Questions (Stage 1)

Q1. Find the value of 2^{10} .

Q2. Express one billion using powers of 10(in standard form)

Q3. Simplify and write the answer in the exponential form.

$(2^5 \div 2^8)^5 \times 2^{-5}$

Q4. Express 4^{-3} as a power with the base 2.

Q5 Simplify and write the answer in the exponential form.

$(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

Challenging Questions (Stage 2)

Q1. Find the value of $256^{0.16} \times 16^{0.18}$.

Q2. Find x if $3^x = 243$.

Q3. What would you put in the place of '?' to balance the equation ?

$6.807 \times 10^6 \times ? = 6.807 \times 10^{10}$

Q4. If a virus triples every hour, represent population after 8 hours.

Q5. What should 69^{-3} be multiplied by to get 1 ?

Challenging Questions (Stage 3)

Q1. Find smallest n such that $2^n > 5000$.

Q2. Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$

Q3. What is 0^n, where n is a positive integer?

Q4. A city population doubles every decade. Represent population after 4 decades

Q5. A stadium has 10 sections, each with 10 rows and each row with 10 seats. Express total seats using powers of 10.

Answers (Challenging Questions)

	Q1	Q2	Q3	Q4	Q5
Stage1 /Ans	1024	$\underline{10^9}$	2^{-20}	2^{-6}	$[100]^{-3}$
Stage2 /Ans	4	5	$\underline{10^4}$	3^8	69^{3}
Stage3 /Ans	13	m=1	0	16 times	10^3

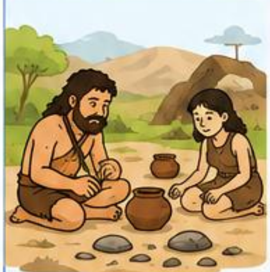
3 A STORY OF NUMBERS

From ancient symbols to modern numbers – a journey through time!

1. REEMA'S CURIOSITY

- Reema found a paper with strange symbols.
- Her father explained that around 4000 years ago, the Mesopotamians used such symbols to represent numbers.
- Reema's curiosity led to many questions.

2. WHY DID HUMANS COUNT?



- Even in the Stone Age, humans needed to count.
- They counted food, animals, goods in trade, offerings in rituals, and the passing of days.
- They wanted to predict events like new moon, full moon, and seasons.
- But their ways of writing numbers were different from what we use today.

3. ORIGIN OF OUR MODERN NUMBER SYSTEM

Origin in India
- Ancient Indian texts like the *Yajurrveda Samhita* used names of numbers based on powers of 10.
- Example: *eka* (1), *dasha* (10), *shata* (100), *sahasra* (1,000), *āyuta* (10,000)… up to 10^{12} and beyond.

Early use of 0 and ten digits
- The first known use of numbers with ten digits (including 0 as a dot) is in the Bakhshali manuscript (c. 3rd century CE).
- Aryabhata (c. 499 CE) explained and used this system in his works.

Spread to the Arab world
- Around 800 CE, the Indian system reached the Arab world.
- Popularised by Al-Khwārizmī in *On the Calculation with Hindu Numerals* (c. 825) and Al-Kindi in *On the Use of the Hindu Numerals* (c. 830).

Reach to Europe and the world
- Reached Europe and parts of Africa by around 1100 CE.
- Fibonacci (c. 1200) encouraged Europe to adopt Indian numerals.
- Widely accepted in Europe by the 17th century.
- Now used in every corner of the world!

Why 'Arabic' or 'Hindu' numerals?
- Europeans called them 'Arabic numerals' as they learned them from the Arab world.
- Arabs called them 'Hindu numerals'.
- Today we use terms like 'Hindu numerals', 'Indian numerals', or 'Hindu-Arabic numerals'.

KEY TAKEAWAY

Our modern number system has its root in India and travelled across the world to become the most efficient way to write and work with numbers.

4. EVOLUTION OF DIGITS (0 TO 9)

Stage/Period	0	1	2	3	4	5	6	7	8	9
Brahmi (c. 3rd century BCE)	•	|	ꟸ	3	ⴸ	፶	ε	ɀ	8	९
Hindu (Gwalior) (c. 6th century)	o	1	२	3	ε	5	6	7	8	9
Sanskrit–Devanagari (c. 8th–10th century)	०	१	२	३	४	५	६	७	८	९
West Arabic (Gubar) (c. 10th century)	•	١	٢	٣	٤	٥	٦	٧	/	٩
East Arabic (c. 10th century)	ʌ	١	٢	٣	٤	٥	٦	٧	٨	٩
11th century (Apices)	ˆ	ı	ƺ	ʒ	४	ƽ	५	ʏ	8	9
15th century	o	1	2	3	4	5	6	7	8	9
16th century (Dürer)	0	1	2	3	4	5	6	7	8	9

Over centuries, these shapes evolved into the digits we use today.

5. IMPORTANCE

- The Indian numeral system made calculations simpler and faster.
- It laid the foundation for mathematics, science and technology.
- Its global adoption shows the power of human knowledge and sharing.

6. QUICK FACTS

- Counting began: Stone Age
- Mesopotamian symbols used: Around 4000 years ago
- First use of 0 in Bakhshali manuscript: c. 3rd century CE
- Indian system reached Arabs: Around 800 CE
- Reached Europe: Around 1100 CE (Popularised by Fibonacci c. 1200)
- Widely accepted in Europe: By the 17th century

Numbers tell the story of human thinking, curiosity and innovation across time and civilisations!

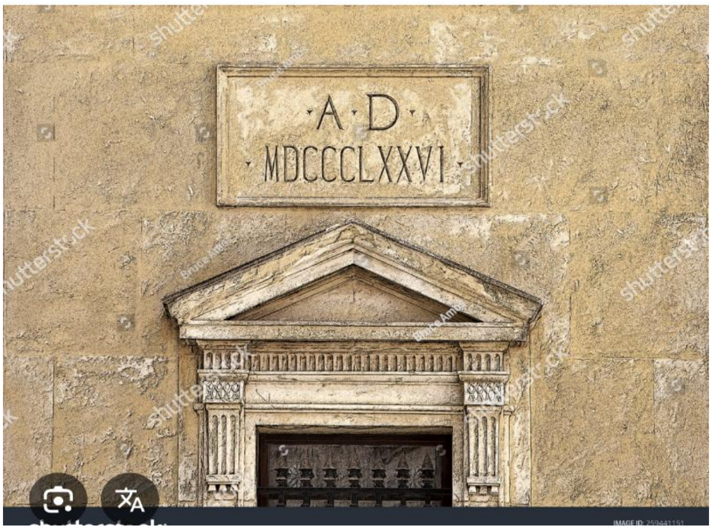
STORY OF NUMBERS

Learning Outcomes:
LO1: Understand evolution of number systems.
LO2: Compare Hindu-Arabic and Roman numerals.
LO3: Appreciate place value and large numbers.
Competencies:
C1: Historical awareness.
C2: Conceptual understanding.
C3: Numeracy.
C4: Cultural appreciation.

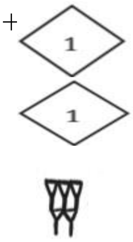
Q.No.	QUESTION	LO	Competency
	MCQ's (One Mark Each)		
1	The Lebombo bone, discovered in South Africa and estimated to be around 44,000 years old, is an important example of: A) Roman numerals B) Early tally marks C) Body part counting D) Place value system	LO1	C1 C4
2	A teacher is introducing the base 5 number system. She asked the students to identify the third landmark number (power of the base) of base 5. (a) 5 (b) 10 (c) 25 (d) 125	LO3	C2 C3
3	Which two number systems are still in use in modern times? (a) Roman and Mayan (b) Babylonian and Egyptian (c) Roman and Mesopotamian (d) Hindu and Binary	LO1	C1 C4
4	A group of indigenous people in Australia called Gumulgals counts a group of animals as "ukasar-urapon". Then they found another group with same number of animals that is counted as "ukasar-urapon." What is the total count of the animals of both groups? (a) ukasar-ukasar-urapon (b) ukasar-ukasar (c) ukasar-ukasar-ukasar(d) ukasar-urapon	LO1	C1 C4
5	A Roman number inscribed on a tombstone is MCMLXXXII. What is this number in Hindu-Arabic numerals? (a) 1982 (b) 1972 (c) 1882 (d) 1872	LO2	C3
6	In the Egyptian number system, landmark numbers are powers of 10. How would 324 be represented? (a) Three 100-symbols, two 10-symbols, four 1-symbols (b) Three 100-symbols and 24 ones (c) 324 repeated symbols (d) One 300-symbol and one 24-symbol	LO1 LO3	C2 C3
7	The Mayan civilisation used a symbol that looked like a seashell to represent: (a) 10 (b) 20 (c) 0 (d) 100	LO1	C1 C4
8	The number 10101 in a binary (base-2) system is equivalent to what number in the decimal system? (a) 10 (b) 12 (c) 21 (d) 16	LO3	C2 C3

9	Which ancient civilization used a base-60 system? (a) Chinese (b) Indian (c) Mesopotamian (d) Egyptian	LO1	C1
10	Which mathematician popularized Hindu numerals in Europe? (a) Aryabhata b) Fibonacci (c) Brahmagupta (d) Al-Kindi	LO1	C1
Short Answer Type (Two Marks Each)			
11	A monument was built in the year MCML in Rome. (a) Write your answer in Hindu-Arabic numerals. (b) How old is the monument today (as on date)? Write your answer in Roman numerals.	LO2	C3
12	Represent 10458 in the Egyptian number system (landmark powers of 10).	LO1 LO3	C2 C3
13	Compare Mayan and Mesopotamian numeral systems with respect to base.	LO1	C1 C2
14	A merchant records the price of silk using rod numerals as: 8 (Zong), 5 (Heng) and 3 (Zong). Convert it into Hindu-Arabic numbers.	LO1 LO3	C3
15	The Gumulgal people count in groups of two. Express 9 and 10 in their number system (using ukasar for 2 and urapon for 1).	LO1	C1 C4
16	In the Mesopotamian (base-60) system, represent the number 125. Show the grouping (how many 60s and 1s).	LO1, LO3	C2 C3
17	Convert the number 27 into Roman numerals. Show the grouping of landmark numbers used.	LO2	C3
18	What is a base-n number system? Explain with the example of the base-5 system and its landmark numbers.	LO3	C2
19	The Mayans used dots for 1 and bars for 5. How many symbols used to represent the number 13?	LO1	C2 C3
20	Explain how the Hindu number system uses place value to represent any number using only ten symbols (0–9). Give one advantage over the Egyptian system for writing very large numbers.	LO3	C2 C3
Short Answer Type (Three Marks Each)			
21	Convert the Roman numeral CLXIV to Hindu-Arabic numerals. Then subtract XLV from it and express the result in both Roman and Hindu-Arabic numerals. Show all steps.	LO2	C3
22	Zero is considered one of the greatest inventions in mathematics. Explain any three ways in which zero makes the Hindu number system more efficient.	LO1 LO3	C1 C2
23	The Gumulgal people of Australia counted in groups of two. Express the numbers 13 and 17 in their system (ukasar = 2, urapon = 1). Compare their system with the Hindu-Arabic system in terms of efficiency for large numbers.	LO1	C1 C2 C4

24	Compare the Mayan number system (base nearly 20) with the Mesopotamian number system (base-60) with respect to their base, use of symbols, and place value. Give one advantage of each system.	LO1	C1 C2
25	Convert the following rod numeral representations into Hindu-Arabic numerals: (a) 2 (Heng), 6 (Zong), 3 (Heng), 4 (Zong) (b) 4 (Heng), 1 (Zong), 8 (Heng), 2(Zong)	LO1 LO3	C3
	Long Answer Type (Five Marks Each)		
26	"The invention of zero and the positional (place value) system revolutionized mathematics." Justify this statement by describing the contributions of Indian mathematicians Aryabhata and Brahmagupta. Explain how these ideas helped in representing large numbers and performing calculations easily. Compare this with the limitations of the Egyptian or Mesopotamian systems using at least one numerical example.	LO1 LO3	C1 C2
27	Compare the Roman numeral system with the Hindu-Arabic numeral system in detail. Explain with examples why the Hindu-Arabic system is more suitable for performing arithmetic operations like addition, subtraction, and multiplication. Use the numbers 456 and 289 to illustrate addition and subtraction in both systems.	LO2	C2 C3
28	"Different civilizations developed number systems suited to their needs." Discuss this statement with reference to the Egyptian, Roman, and Gumulgal (Australian indigenous) number systems. Explain the strengths and limitations of each. Why did the Hindu-Arabic system eventually become the global standard? Support your answer with suitable examples.	LO1	C1 C2 C4
	Case Study Based		
29	During a school project on "Our Mathematical Heritage", students of Class VIII visited a museum and learned about different ancient number systems. They were given the task to represent the number 284 using Egyptian, Roman, and Mayan systems. The teacher explained that the Egyptian system uses repeated symbols for powers of 10, Roman uses additive and subtractive notation, while the Mayan system is vigesimal (base-20) with dots and bars. 	LO1 LO3	C1 C2 C3 C4

	(a) Represent 284 in the Egyptian number system and count the total number of symbols used. (b) Convert 284 to the Mayan number system (show grouping by 20s and units). (c) Which of these three systems is closest to the modern Hindu-Arabic system and why?		
30	While exploring a historic fort, Aarav noticed a date engraved on an old building as MDCCCLXXVI. Curious about its meaning, he asked his history teacher. The teacher explained that ancient monuments often use Roman numerals to record important years and events.  Questions: (a)Convert MDCCCLXXVI into the Hindu-Arabic numeral system. (b)Write the Hindu-Arabic numeral 1947 in Roman numerals. (c)State one limitation of the Roman numeral system that makes calculations difficult.	LO2	C1 C3 C4

Answers:

Q.	1	2	3	4	5
Ans.	b	d	d	c	a
Q.	6	7	8	9	10
Ans.	a	c	c	c	b
Q.	11	12	13	14	15
Ans.	(a) 1950 (b) CCLXXVI (approx. 76 years as on 2026)	1×10000 + 0×1000  +	Mayan → nearly base-20 (vigesimal); Mesopotamian → base-60 (sexagesimal). Both use place value and zero placeholder.	8×100 + 5×10 + 3×1 = 853	9 = ukasar-ukasar-ukasar-ukasar-urapon 10 = ukasar-ukasar-ukasar-ukasar-ukasar
Q.	16	17	18	19	20
Ans.	125 = 2×60 + 	XXVII	A base-n system has landmark numbers as powers of n. Example (base-5): 1, 5, 25, 125, 625	2 bars (10) + 3 dots (3) = 13 (total 5 symbols)	Hindu system uses place value — digit position determines value (e.g., 3, 30, 300). Advantage: Only 10 symbols needed for any large number vs Egyptian's repeated symbols.
Q.	21	22	23	24	25
Ans.	CLXIV = 164 164 – 45 = 119 Roman: CXIX	1.Acts as placeholder (distinguishes 3, 30, 300). 2.Enables positional system for large numbers3 Treated as a number (Brahmagupta) — allows operations like 0 + a = a, a × 0 = 0.	13 = ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-urapon 17 = ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-urapon Hindu-Arabic is far more efficient due to place value and zero.	Mayan: base ~20, dots & bars, vertical placement. Mesopotamian: base-60, wedge symbols. Both use place value. Advantage Mayan: good for calendar; Mesopotamian: time measurement.	(a) 2634 (b) 4182

Q.	26	27	28	29	30
30 Ans.	Aryabhata used zero as placeholder in place value. Brahmagupta treated zero as a number and gave arithmetic rules. These enabled compact representation of large numbers and easy calculations. Egyptian system needs hundreds of repeated symbols for large numbers.	Roman: No zero, non-positional, difficult for arithmetic. Hindu-Arabic: Positional, zero, very efficient. 456 + 289 = 745 (easy in Hindu-Arabic; tedious in Roman).	Egyptian: Additive, repetitive symbols. Roman: Additive + subtractive. Gumulgal: Grouping by 2s. Hindu-Arabic became global due to zero + place value — compact and excellent for calculations.	(a)Egyptian: 2×100 + 8×10 + 4×1 → 14 symbols  = (b) Mayan: 14×20 + 4 → 14 in 20s place (2 bars + 4 dots), 4 in units. (c) Hindu-Arabic is closest due to place value and zero.	(a) 1876 (b) MCMXLVII (c) No zero and non-positional → very difficult for arithmetic operations.

Challenging Questions (Stage 1)

Q.1.Convert the Roman numeral LXXV to Hindu-Arabic numerals.

Q.2.Which ancient civilization used the base-60 (sexagesimal) number system that still influences our time measurement?

Q.3.What is the base of the Egyptian number system?

Q.4.Write the first three landmark numbers in the base-5 system.

Q.5.The Gumulgal people counted in groups of two (ukasar = 2, urapon = 1). Express the numbers 7 and 11 in their system. Why is this system more efficient than pure tally marks?

Challenging Questions (Stage 2)

Q.1.Convert MCDXLVI to Hindu-Arabic numerals. How old would a monument built in this year be in 2026?

Q.2.Represent 236 in the Egyptian number system. How many symbols are needed?

Q.3. Represent 100 using the Mayan Number system system

Q.4.Compare the Roman and Hindu-Arabic systems for performing the subtraction CLXX – LXV. Solve in both systems and explain why one is easier.

Q.5.Using the ideas of base systems discussed in the chapter, convert the number 365 (days in a year) into a hypothetical base-12 system (like some ancient systems). Represent it using powers of 12 and discuss its advantages for calendar-related counting

Challenging Questions (Stage 3)

Q.1.Using the concept of base-n systems, explain why base-10 feels "natural" yet base-60 (Mesopotamian) or base-20 (Mayan) suited certain cultures.

Q.2.Why did the Chinese alternate between vertical (Zong) and horizontal (Heng) rod symbols in their numeral system?

Q.3.Discuss with examples how the transmission of Hindu-Arabic numerals from India to Europe via the Arab world impacted scientific progress and trade.

Q.4.Explain with an example how the place value system in the Hindu-Arabic numerals makes it easier to represent and add large numbers compared to tally marks or Roman numerals.

Q.5.In the Mayan system, represent 142 (show grouping by 20s). Why did the Mayans use 360 instead of 400 in their calendar place value?

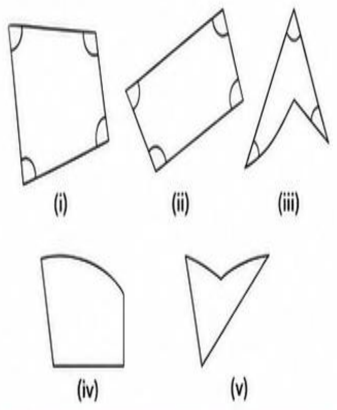
Answers (Challenging Questions)

	Q.1	Q.2	Q.3	Q.4	Q.5
Stage 1	75	Mesopotamian	10	1, 5, 25	7 = ukasar-ukasar-ukasar-ukasar-urapon-ukasar-urapon; 11 = ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-urapon; More efficient due to grouping.
Stage 2	1446; Age in 2026 = 580 years	2×100 + 3×10 + 6×1 → 11 symbols	5×20 + 0 → 5 dots in 20s place	Roman: difficult; Hindu-Arabic: easy (170 – 65 = 105)	365 in base-12: 2×144 + 5×12 + 5 = 2,5,5 (base-12)
Stage 3	Base-10 natural (10 fingers); base-60 good for time/division; base-20 for Mayan calendar.	To avoid confusion in writing direction / for different place values.	Enabled faster trade calculations and scientific progress in Europe.	Example: 234 + 156 easy in place value vs Roman or tally.	7×20 + 2; Used 360 for calendar (approx. days in a year).

4 QUADRILATERALS — SUMMARY OF CONCEPTS & DEFINITIONS

1. QUADRILATERAL – BASICS

- A quadrilateral is a four-sided polygon.
- The angles of a quadrilateral are the angles between its sides.
- Figs. (i), (ii) and (iii) are quadrilaterals. The others are not.



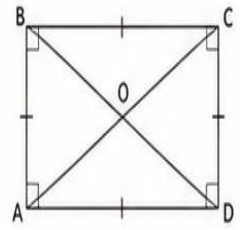
Quadrilateral = 4 sides, 4 vertices, 4 angles

2. RECTANGLE

Definition: A rectangle is a quadrilateral in which–

(i) All angles are right angles (90°).

(ii) Opposite sides are equal.



Properties (using diagonals)

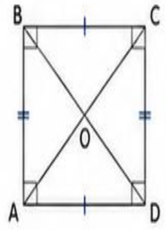
- Diagonals are equal. (AC = BD)
- Diagonals bisect each other. (OA = OC and OB = OD)
- Diagonals make any angle with these properties.
- Opposite sides are equal.
- All angles are 90°.

3. SQUARE

Definition: A square is a quadrilateral in which–

(i) All angles are right angles (90°), and

(ii) All sides are of equal length.

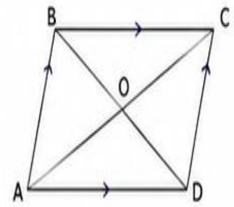


Properties

- All properties of a rectangle.
- All sides are equal (AB = BC = CD = DA).
- Diagonals are equal and bisect each other at right angles.
- Each diagonal bisects the angles of the square.

4. PARALLELOGRAM

Definition: A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

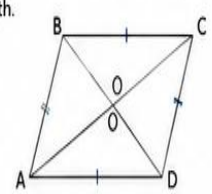


Properties (using diagonals)

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- Consecutive (adjacent) angles are supplementary (add up to 180°).
- Diagonals bisect each other. (OA = OC, OB = OD)
- Diagonals are not necessarily equal.

5. RHOMBUS

Definition: A rhombus is a quadrilateral in which all four sides are of equal length.

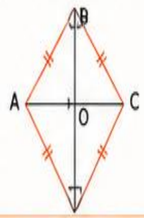


Properties (using diagonals)

- All sides are equal (AB = BC = CD = DA).
- Opposite sides are parallel.
- Opposite angles are equal.
- Consecutive angles are supplementary.
- Diagonals bisect each other.
- Diagonals are perpendicular to each other.
- Each diagonal bisects the angles of the rhombus.

6. KITE

Definition: A kite is a quadrilateral in which two pairs of adjacent sides are equal.

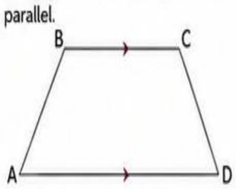


Properties (using diagonals)

- Two pairs of adjacent sides are equal.
- One pair of opposite angles is equal ($\angle A = \angle C$).
- The diagonals are perpendicular to each other.
- One diagonal bisects the other.
- The diagonal along the axis of symmetry (BD) bisects the angles at B and D.

7. TRAPEZIUM (Trapezoid)

Definition: A trapezium is a quadrilateral in which only one pair of opposite sides are parallel.



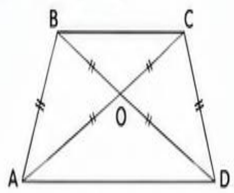
Properties

- Exactly one pair of opposite sides is parallel.
- The other two sides are non-parallel.
- No special relation between angles and diagonals.

Note: If the non-parallel sides are equal, it is called an **isosceles trapezium**. Then, the base angles are equal and diagonals are equal.

8. ISOSCELES TRAPEZIUM

Definition: A trapezium in which the non-parallel sides are equal.



Properties

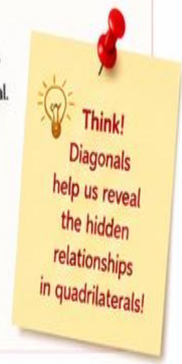
- One pair of opposite sides is parallel.
- Non-parallel sides are equal.
- Base angles are equal ($\angle A = \angle D$, $\angle B = \angle C$).
- Diagonals are equal (AC = BD).
- Consecutive angles on the same leg are supplementary.

9. QUICK COMPARISON TABLE

Figure	Definition (Key Condition)	Opposite Sides	Opposite Angles	Consecutive Angles	Diagonals	Special Property
Rectangle	4 right angles; opposite sides equal	Equal	Equal	Supp.	Equal; bisect each other	All angles 90°
Square	4 right angles; all sides equal	Equal	Equal	Supp.	Equal; bisect at 90°	All sides equal
Parallelogram	Both pairs opposite sides parallel	Equal & //	Equal	Supp.	Bisect each other	Diagonals not necessarily equal
Rhombus	All sides equal	Equal & //	Equal	Supp.	Bisect; ⊥ to each other	Diagonals bisect angles
Kite	Two pairs adjacent sides equal	–	One pair equal	–	⊥ to each other; one bisects the other	One diagonal bisects angles
Trapezium	One pair of opposite sides parallel	–	–	–	No special relation	–
Isosceles Trapezium	One pair parallel; non-parallel sides equal	–	Base angles equal	Same leg supp.	Diagonals are equal	–

10. KEY TAKEAWAYS

- A quadrilateral has 4 sides, 4 vertices and 4 angles.
- Rectangles have 4 right angles and opposite sides equal.
- Squares have all properties of rectangles + all sides equal.
- Parallelograms have both pairs of opposite sides parallel.
- Rhombuses have all sides equal.
- Kites have two pairs of adjacent sides equal.
- Trapeziums have one pair of opposite sides parallel.
- Isosceles trapeziums have equal non-parallel sides and equal base angles.
- Diagonals help in identifying and proving properties.



QUADRILATERALS

Learning Outcomes

LO1: Classify Quadrilaterals

LO2: Investigate properties of sides and angles

LO3: Angles and Diagonals

LO4: Solve the geometric problems using these properties

Competencies

C1: Spatial Reasoning

C2: Visualisation

C3: Deductive thinking

C4: Geometric construction

Q.No	QUESTION	LO	Competency
MCQ's (One Mark Each)			
1	A quadrilateral has _____ sides.	LO1	C1
2	Sum of interior angles of a quadrilateral is: (a) 180° (b) 360° (c) 270° (d) 90°	LO2	C3
3	A rectangle has all angles equal to: (a) 60° (b) 90° (c) 120° (d) 45°	LO2	C2
4	A square is also a: (a) Triangle (b) Rectangle (c) Pentagon (d) None	LO2	C4
5	A parallelogram has opposite sides: (a) Unequal (b) Equal and parallel (c) Perpendicular (d) Curved	LO2	C2
6	A quadrilateral with one pair of parallel sides is a: (a) Kite (b) Trapezium (c) Square (d) Rhombus	LO1	C1
7	A rhombus has all sides: (a) Equal (b) Unequal (c) Parallel only (d) None	LO2	C2
8	Number of diagonals in a quadrilateral is: (a) 1 (b) 2 (c) 3 (d) 4	LO1	C1
9	If three angles of a quadrilateral are 90°, 80°, and 100°, the fourth angle is: (a) 90° (b) 80° (c) 70° (d) 100°	LO3	C3
10	Which quadrilateral has all sides equal and all angles 90°? (a) Rhombus (b) Kite (c) Square (d) Trapezium	LO4	C4
Short Answer Type (Two Marks Each)			
11	Define a quadrilateral.	LO1	C1
12	Write any two properties of a rectangle.	LO2	C2
13	Find the fourth angle if three angles are 80°, 90°, and 100°.	LO3	C3
14	Write any two properties of a square.	LO2	C2
15	How is a rhombus different from a square?	LO4	C4
16	Name the quadrilateral having exactly one pair of parallel sides.	LO1	C1
17	Draw and label a parallelogram ABCD.	LO2	C2

18	State the angle-sum property of a quadrilateral.	LO2	C3
19	Name any two quadrilaterals with opposite sides parallel.	LO1	C4
20	How many diagonals does a quadrilateral have? Draw them.	LO2	C2
	Short Answer Type (Three Marks Each)		
21	Construct a KITE whose diagonals are of lengths 6cm and 8 cm.	LO3	C2
22	Explain any three properties of a parallelogram.	LO2	C2
23	Compare a square and a rectangle (any three points).	LO4	C4
24	Explain why a square is also a rectangle.	LO4	C4
25	A quadrilateral has angles 90°, 80°, 70°, and x°. Find x.	LO3	C3
	Long Answer Type (Five Marks Each)		
26	Explain the properties of a square with a neat diagram.	LO4	C4
27	Compare parallelogram, rectangle, and square on the basis of sides and angles.	LO4	C4
28	ABCD is a parallelogram where ∠A= (2x + 50°) and ∠C = (3x + 40°). (i) Find the value of x (ii) find the measure of each angle.	LO4	C4
	Case Study Based		
29	Riya is designing a square garden. Each side of the garden is equal and all angles are 90°. a) Name the quadrilateral formed. b) Write two properties of this quadrilateral.	LO4	C4
30	A window frame has one pair of opposite sides parallel and the other pair not parallel. a) Name the quadrilateral b) Write one property of it.	LO4	C4
31	The Shri Yantra is a sacred geometric diagram composed of nine interlocking triangles that form 43 smaller, subsidiary triangles around a central point (the *bindu*). It is a treasure trove of sacred geometry, featuring the Golden Ratio (Φ ≈ 1.618), circle-line intersections, and trigonometric ratios.  Question 1: The Shri Yantra is primarily built from interlocking triangles. There are 4 upward-pointing triangles (representing Shiva) and 5 downward-pointing triangles (representing Shakti). If all 9 of these primary triangles intersect to create smaller regions, how many total smaller triangles (known as *konas*) are formed in the center?	LO1 LO2 LO3 LO4	C1 C2 C3 C4

	Question 2: The outer part of the Shri Yantra consists of a large square known as the *Bhupura*. If the side length of this square is 20 cm, what is its total area, and what is the maximum area of a circle that can be perfectly inscribed inside it? Question 3: The outermost lotus ring of the Shri Yantra features 16 petals. Assuming each petal is an identical, symmetrical curved shape that spans an equal angle at the center of the diagram, what is the central angle in degrees that defines the arc of each petal? Question 4: Within the 9 interlocking triangles, many are isosceles triangles. Suppose one of these primary downward-pointing triangles has a base of 12 cm and two equal legs of 10 cm each. (a) What is the height of the triangle? (b) What is its total area?		

ANSWERS

Q.	1	2	3	4	5
Ans.	4 sides	360°	90°	Rectangle	Equal and parallel
Q.	6	7	8	9	10
Ans.	Trapezium	Equal	2	90°	Square
Q.	11	12	13	14	15
Ans.	A polygon with four sides	Opposite sides equal; all angles 90°.	90°.	All sides equal; all angles 90°.	Square has all angles 90°, rhombus need not
Q.	16	17	18	19	20
Ans.	Trapezium.	Diagram.	Sum of interior angles = 360°.	Parallelogram, Rectangle.	Two diagonals.
Q.	21	22	23	24	25
Ans.	Correct construction	Opposite sides equal, opposite sides parallel, opposite angles equal	Any three valid comparison points.	Because it has opposite sides equal and all angles 90°.	x = 120°.
Q.	26	27	28	29	30
Ans.	Properties of square with diagram.	Comparison table of sides and angles.	Missing angle = 70°, property used: angle-sum of quadrilateral = 360	a) Square (b) Any two properties	a) Trapezium (b) One pair of parallel sides.
Q.	31				
Ans.	43 small triangles., Area = 400 cm^2. Circle Area = 100π $cm^2 \approx 314.16$ cm^2., 22.5°. a)8 cm b)48 cm^2				

Challenging questions (Stage 1)

1.One angle of a quadrilateral is 120° and the remaining three angles are equal. Find the measure of each of

the three equal angles.

2. Construct a KITE whose diagonals are of lengths 6cm and 8 cm.

3. One angle of a parallelogram is of measure 70°. Find the measure of the remaining angles of the

parallelogram.

4. The angles of a quadrilateral are x,2x,4x,5x. Find the value of x and also all the angles.

5. Draw a quadrilateral whose one pair of opposite sides are parallel and the other two non parallel

Challenging questions (Stage 2)

1. A quadrilateral has angles 80°, 100°, 70° and x°. What is the value of x?

(a) 100° (b) 110° (c) 120° (d) 90°

2. If ABCD is a rectangle, then

(a) ∠B=90 (b) AC=BD (c) AB=CD (d) All are true

3. Which of the following quadrilaterals has both pairs of opposite sides parallel and all angles equal?

(a) Rhombus (b) Parallelogram (c) Squar (d) Trapezium

4. Which of the following statement is FALSE about a Rhombus?

a) Opposite sides are equal b) Opposite angles are equal

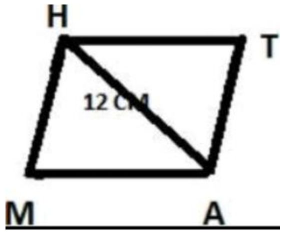
c) All angles are equal d) Opposite sides are parallel

5.Consider the parallelogram MATH shown below in which AH=12 cm

For the parallelogram to be a rectangle, what should be the length of TM?

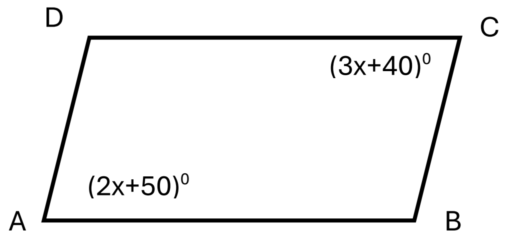
a) TM= 12 cm b)TM is less than 12 cm

c) TM is greater than 12 cm d)none of these

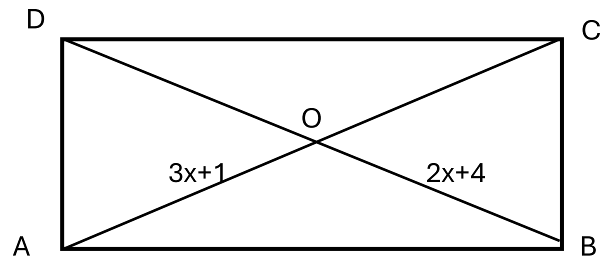


Challenging questions (Stage 2)

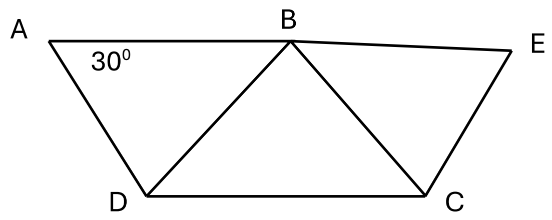
1. ABCD is a parallelogram where ∠A = (2x + 50°) and ∠C = (3x + 40°).
   (i) Find the value of x.
   (ii) Find the measure of each angle.



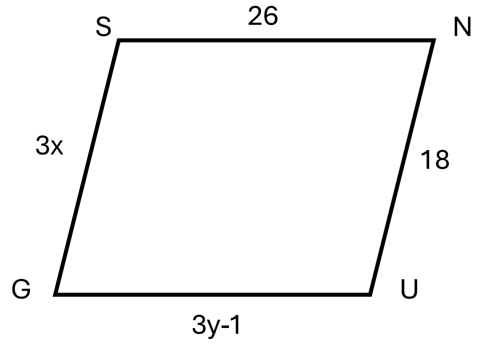
2. ABCD is a rectangle. Its diagonals meet at O.
Find x, if OA = 3x + 1 and OB = 2x + 4.



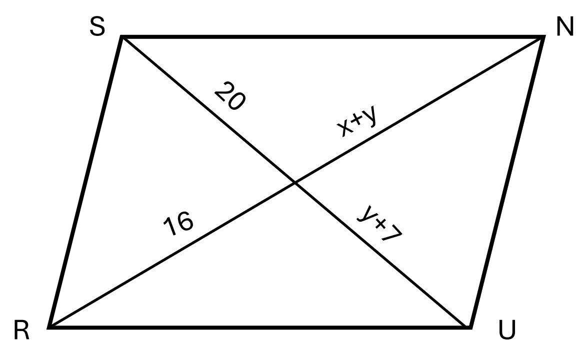
3. In the given figure, ABCD and BDCE are parallelograms with common base DC. If BC ⊥ BD, then find ∠BEC.



4. GUNS is a parallelogram. Find x and y.



5. RUNS is a parallelogram. Find x and y.



Answers (Challenging Questions)

Q.	1	2	3	4	5
Stage 1	(b) 110°	(d) All are true	(c) Square	(c) All angles are equal	(a) TM= 12 cm
Stage 1	80°	Correct construction by drawing perpendicular bisector	∠D=110°	30°,60°,120°,150°	Isosceles trapezium
Stage 1	(i) x=10° (ii) ∠D=110°	X=3	∠BEC =60°	x= 6,y =9	x=13,y=3

CHAPTER 5: NUMBER PLAY

POSTER 1 – NUMBER PLAY BASICS

5.1 IS THIS A MULTIPLE OF?

1. SUM OF CONSECUTIVE NUMBERS

Many natural numbers can be written as a sum of consecutive numbers in one or more ways.

Examples:

$7 = 3 + 4$

$10 = 1 + 2 + 3 + 4$

$12 = 3 + 4 + 5$

$15 = 7 + 8 = 4 + 5 + 6$

$= 1 + 2 + 3 + 4 + 5$

Key Ideas

- All odd numbers can be written as a sum of two consecutive numbers.
- All even numbers can be written as a sum of one or more consecutive numbers.
- 0 can be written as a sum of consecutive numbers using negative numbers.

2. FOUR CONSECUTIVE NUMBERS & SIGNS

Take 4 consecutive numbers (e.g., 3, 4, 5, 6). Place '+' and '–' between them. There are 8 possible expressions.

Numbers	Sign Pattern (+ for plus, – for minus)	Expression	Result
3 4 5 6	+ + +	3 + 4 + 5 + 6	18
3 4 5 6	+ + –	3 + 4 + 5 – 6	6
3 4 5 6	+ – +	3 + 4 – 5 + 6	8
3 4 5 6	+ – –	3 + 4 – 5 – 6	–4
3 4 5 6	– + +	3 – 4 + 5 + 6	10
3 4 5 6	– + –	3 – 4 + 5 – 6	–2
3 4 5 6	– – +	3 – 4 – 5 + 6	0
3 4 5 6	– – –	3 – 4 – 5 – 6	–12

No matter how the '+' and '–' signs are placed between 4 consecutive numbers, the result is always an even number.

3. PARITY – WHY DO ALL 8 EXPRESSIONS HAVE THE SAME PARITY?

For any numbers a, b, c, d (not only consecutive), all expressions $a \pm b \pm c \pm d$ have the same parity (all even or all odd).

(1) Sign Switching

Switching the sign of any term changes the value by an even number.

Example:

$(a + b - c - d) - (a - b - c - d)$

$= 2b$ (even number)

Numbers that differ by an even number have the same parity.

(2) Parity Rules

odd ± odd = even

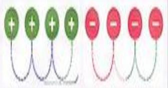
even ± even = even

odd ± even = odd

So, $a \pm b$ has the same parity. Extending this, all $a \pm b \pm c \pm d$ have the same parity.

(3) Integer (Token) Model

Using positive (+) and negative (–) tokens: equal numbers of + and – tokens cancel or form pairs, leaving an even result.



Net result is always even.

4. BREAKING EVEN – QUICK GUIDE

Identify which expressions are always even (for any integers).

Always Even	Not Always Even
✓ $2a + 2b$	✗ $3g + 5h$
✓ $4m + 2n$	✗ $13k - 5k$
✓ $2u - 4v$	✗ b^2
✓ $6m - 3n$	✗ $x + 1$
✓ $4k \times 3j$ (product of even and anything)	
✓ 2×2 (i.e., 4)	
✓ $\frac{2}{3} + \frac{4}{5}$ (sum of two even numbers)	

5. KEY DEFINITIONS

- **Parity:** Whether a number is even or odd.
- **Even Number:** Divisible by 2.
- **Odd Number:** Not divisible by 2.
- **Consecutive Numbers:** Numbers that follow one after another in natural order (e.g., 3, 4, 5, 6).

Big Idea: Patterns in numbers help us make general conclusions without checking every case. Reasoning makes mathematics powerful and beautiful!

POSTER 2 – DIVISIBILITY MASTER CHART

5.2 CHECKING DIVISIBILITY QUICKLY

Divisibility Rules at a Glance

Divisible by	Rule / Trick	Example	Why it Works
2	Last digit is even (0, 2, 4, 6, 8).	4826 ✓ 1573 ✗	Even numbers end with an even digit.
3	Sum of digits is divisible by 3.	354 (3+5+4=12) ✓ 458 (4+5+8=17) ✗	Multiples of 3 have digit sum multiple of 3.
4	Last two digits form a number divisible by 4.	3168 (68 ✓) 5233 (33 ✗)	Based on place value (100 = 4 x 25).
5	Last digit is 0 or 5.	750 ✓ 8425 ✓ 731 ✗	Numbers ending in 0 or 5 are multiples of 5.
6	Divisible by both 2 and 3.	132 (✓ and ✓) 245 (✗ or ✗)	6 = 2 x 3
8	Last three digits form a number divisible by 8.	12056 (056=56 ✓) 35124 (124 ✗)	Based on place value (1000 = 8 x 125).
9	Sum of digits is divisible by 9.	729 (7+2+9=18 ✓) 456 (4+5+6=15 ✗)	Multiples of 9 have digit sum multiple of 9.
10	Last digit is 0.	640 ✓ 1230 ✓ 987 ✗	All multiples of 10 end with 0.
11	Difference of the sums of alternating digits is 0 or a multiple of 11.	5720 (5–7+2–0=0) ✓ 86134 (8–6+1–3+4=4) ✗	Based on alternating place values.
24	Divisible by both 3 and 8.	1248 (✓ and ✓) 2504 (✓ and ✓) 3766 (✗ or ✗)	24 = 3 x 8

Remainders and Divisibility

- If a number leaves remainder 0 when divided by n, it is divisible by n.
- If remainder ≠ 0, it is not divisible by n.
- Divisibility helps us check quickly without doing full division.

Always / Sometimes / Never

When we study a statement about numbers:

Always – True for all cases.

Sometimes – True for some cases.

Never – True for no cases.

Example: Statement: "The sum of two even numbers is odd."

Answer: Never (because even + even = even)

Factors & Multiples

- **Factor of a number:** a number that divides it exactly.
- **Multiple of a number:** obtained by multiplying it by whole numbers.
- Every number has infinitely many multiples but limited factors.

Example: Factors of 12 = 1, 2, 3, 4, 6, 12

Multiples of 12 = 12, 24, 36, 48, ...

Using LCM for Divisibility

If a number is divisible by two or more numbers, it is divisible by their LCM.

Example: If a number is divisible by 4 and 6, it is divisible by LCM(4,6) = 12.

- Many divisibility rules come from place value.
- Look at last digit(s) or digit sum to decide.
- Reasoning helps us understand WHY the rule works.

POSTER 3 – DIGITAL ROOTS & DIGITS IN DISGUISE

DIGITAL ROOTS

Digital Root of a number: Repeatedly add the digits until you get a single-digit number. (For 0, the digital root is 0.)

How to Find

1. Add all the digits.
2. If the sum has more than one digit, add its digits again.
3. Repeat until you get a single digit.

Examples

- 9875 → 9+8+7+5=29 → 2+9=11 → 1+1=2
  Digital root = 2
- 642 → 6+4+2=12 → 1+2=3 Digital root = 3
- 1001 → 1+0+0+1=2 Digital root = 2

Key Properties

- Numbers with the same digital root leave the same remainder when divided by 9.
- Digital root is 0 if the number is a multiple of 9.
- It is 3 if the number is not a multiple of 9 but is a multiple of 3.

Aryabhata's Method (for Divisibility by 9)

Subtract 1 from the number and add the digits of the result. Repeat the process. If you get 0 (or a multiple of 9), the number is divisible by 9.

Example: 8736

8736 – 1 = 8735 → 8+7+3+5 = 23

23 – 1 = 22 → 2+2 = 4

4 is not 0 or a multiple of 9 → 8736 is not divisible by 9.

5.3 DIGITS IN DISGUISE (CRYPTARITHMS)

A cryptarithm is a math puzzle where letters stand for digits (0–9). Each letter has a fixed digit. Different letters represent different digits.

General Rules

- Each letter = one digit (0–9).
- Different letters = different digits.
- Leading letter of a number ≠ 0.
- Use logic, place value, and elimination.

Strategies to Solve

✓ Look for the leading letter (cannot be 0).

✓ Check the units column first.

✓ Find letters that produce a carry.

✓ Use uniqueness of digits to eliminate possibilities.

Addition Cryptarithm – Example

```
  S E N D
+ M O R E
---------
M O N E Y
```

Idea: Work column by column (from right to left).

- Y = (D + E) unit digit
- Find carry if D + E ≥ 10
- Continue to thousands place.

Multiplication Cryptarithm – Example

```
    T W O
  X T W O
  -------
    T W O
  T W O
T W O
---------
T H R E E
```

Idea: Use place value and partial products carefully.

Practice Ideas

✓ Try simple puzzles first (2–3 letters).

✓ Use trial and error smartly.

✓ Cross out impossible options.

✓ Check your answer in the end.

Remember

Cryptarithms build logic, patience and number sense. Practice makes you a better problem solver!

Mathematics is full of patterns, rules and puzzles. Observe, reason and enjoy the beauty of numbers!

NUMBER PLAY

Learning Outcomes:
LO1: Apply divisibility rules
LO2: Find factors and multiples
LO3: Identify prime and composite numbers
LO4: Solve number-based puzzles
Competencies:
C1: Critical thinking
C2: Analytical ability
C3: Number reasoning
C4: Conjecturing

Q.No	Question	LO	Competency
	MCQs (One Mark Each)		
1	Which of the following is divisible by 9? (a) 234 (b) 235 (c) 236 (d) 237	LO1 LO2	C2 C3
2	Which number is a prime number? (a) 21 (b) 29 (c) 35 (d) 49	LO3	C2
3	LCM of 4 and 6 is: (a) 12 (b) 24 (c) 6 (d) 18	LO2	C2 C3
4	Which number is divisible by both 2 and 3? (a) 15 (b) 18 (c) 25 (d) 27	LO1	C2
5	HCF of 12 and 18 is: (a) 6 (b) 3 (c) 12 (d) 9	LO2	C2
6	Which is a composite number? (a) 13 (b) 17 (c) 21 (d) 19	LO3	C2
7	A number divisible by 10 must end with: (a) 5 (b) 0 (c) 2 (d) 1	LO1	C3
8	Which is the smallest prime number? (a) 0 (b) 1 (c) 2 (d) 3	LO3	C2
9	Which number is a multiple of 7? (a) 48 (b) 49 (c) 50 (d) 51	LO2	C2
10	Which rule checks divisibility by 3? (a) Last digit even (b) Sum of digits divisible by 3 (c) Ends with 0 (d) Multiply digits	LO1	C3
	2 Marks Questions		
11	Check whether 456 is divisible by 3 and 9.	LO1	C2
12	Find the HCF of 24 and 36.	LO2	C2
13	Write all factors of 28.	LO2	C3
14	Is 51 a prime number? Justify.	LO3	C1 C2
15	Find LCM of 5 and 10.	LO2	C2
16	Check if 121 is prime or composite.	LO3	C2
17	Find the smallest number divisible by both 4 and 6.	LO2	C3

18	Write first 5 multiples of 8.	LO2	C3
19	Test divisibility of 145 by 5 and 9.	LO1	C2
20	Find two prime numbers between 10 and 20.	LO3	C2
3 Marks Questions			
21	Find HCF and LCM of 12 and 15.	LO2	C2 C3
22	Explain divisibility rule of 11 with example.	LO1	C3
23	Find all prime numbers less than 30.	LO3	C2
24	Find smallest number divisible by 3, 4, and 5.	LO2	C3 C4
25	Check whether 221 is prime using factor method.	LO3	C2
5 Marks Questions			
26	Find LCM of 12, 15 and 20 using prime factorization.	LO2	C2 C3
27	Explain all divisibility rules (2, 3, 5, 9, 10) with examples.	LO1	C3
28	Solve a number puzzle:- A number is divisible by 2, 3, and 5. What is the smallest such number?	LO4	C1 C4
Case Study Based Questions			
29	A shopkeeper arranges chocolates in packets of 6 and 8. Find the smallest number of chocolates needed so that no chocolate is left.	LO2 LO4	C2 C4
30	A number when divided by 3, 4, and 5 leaves no remainder. Find such smallest number and explain the method.	LO4	C2 C4

Answers:

Q.	1	2	3	4	5
Ans.	A	B	A	B	A
Q.	6	7	8	9	10
Ans.	C	B	C	B	B
Q.	11	12	13	14	15
Ans.	Divisible by 3, Not Divisible by 9	12	1, 2, 4, 7, 14, 28	No, composite (3 × 17)	10
Q.	16	17	18	19	20
Ans.	Composite (11 × 11)	12	8, 16, 24, 32, 40	Divisible by 5	11, 13, 17, 19 (any two)
Q.	21	22	23	24	25
Ans.	HCF = 3, LCM = 60	Example: 121 → (1+1) − 2 = 0 ⇒ divisible by 11	2, 3, 5, 7, 11, 13, 17, 19, 23, 29	60	Not prime (13 × 17)
Q.	26	27	28	29	30
Ans.	60	2 → last digit even 3 → sum divisible by 3 5 → ends with 0 or 5 9 → sum divisible by 9 10 → ends with 0	30	24	60

Challenging Questions (Stage 1)

Q.1 Find a number divisible by 4 and 6 but not by 8.

Q.2 Find smallest number divisible by 7 and 9.

Q.3 Check if 143 is prime.

Q.4 Find HCF of 45 and 75.

Q.5 Write 3 composite numbers between 10–20.

Challenging Questions (Stage 2)

Q.1 Find LCM of 8, 12, 20.

Q.2 Explain why 1 is neither prime nor composite.

Q.3 Find all factors of 36.

Q.4 Solve puzzle: A mystery number has the following properties:

- It is divisible by 3
- It is divisible by 5
- It is not divisible by 2

Find the smallest such number and explain your reasoning.

Q.5 Find prime numbers between 30–50.

Challenging Questions (Stage 3)

Q.1 Find smallest number divisible by 2,3,4,5,6.

Q.2 Prove 37 is prime.

Q.3 Find HCF of 3 numbers (12,18,24).

Q.4 Find the smallest number which is divisible by 4, 6 and 9.

Q.5 Find a number which is divisible by 2, 3 and 5 but not by 4.

Answers (Challenging Questions)

	Q.1	**Q.2**	**Q.3**	**Q.4**	**Q.5**
Stage 1	12	63	Not prime (11 × 13)	15	12, 14, 15
Stage 2	120	1 has only one factor	1, 2, 3, 4, 6, 9, 12, 18, 36	15	31, 37, 41, 43, 47
Stage 3	60	Prime (only divisible by 1 and 37)	6	36	30

6 CHAPTER 6: WE DISTRIBUTE, YET THINGS MULTIPLY

SUMMARY OF CONCEPTS & DEFINITIONS

1. BIG IDEA

Algebra uses letters to write general statements about patterns and relations.

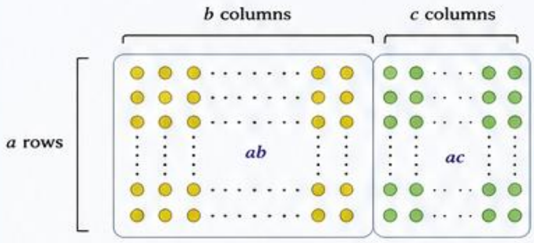
Distributivity is a property that relates multiplication and addition. It helps us prove statements, discover patterns and solve problems.

2. DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION

$$\boxed{a(b + c) = ab + ac}$$

This means multiplying a by $(b + c)$ is the same as multiplying a by b and a by c, then adding.

Diagram (Area Model)



Also,

$$(a + b)c = ca + cb = ac + bc$$

(Distributivity + Commutativity)

Remember: $a(b + c)$ and $23(27 + 1)$ mean $a \times (b + c)$ and $23 \times (27 + 1)$ respectively.

3. INCREMENTS IN PRODUCTS

Let the initial two numbers be a and b (product $= ab$).

1 If second number b is increased by 1:

$$\boxed{a(b + 1) = ab + a}$$

Increase $= a$

Example: $23(27 + 1) = 23 \times 27 + 23$

Increase $= 23$

2 If first number a is increased by 1:

$$\boxed{(a + 1)b = ab + b}$$

Increase $= b$

Example: $(23 + 1)27 = 23 \times 27 + 27$

Increase $= 27$

3 If both numbers are increased by 1:

$$\boxed{(a + 1)(b + 1) = ab + (a + b + 1)}$$

Increase $= a + b + 1$

Example: $(23 + 1)(27 + 1) = 23 \times 27 + (23 + 27 + 1)$

Increase $= 51$

4 If one number is increased by 1 and the other decreased by 1:

$$\boxed{(a + 1)(b - 1) = ab + b - a - 1}$$

Increase $= b - a - 1$

Example: $(23 + 1)(27 - 1) = 23 \times 27 + 27 - 23 - 1$

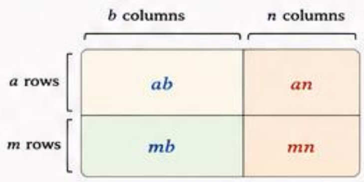
Increase $= 3$

General Case (Increase by m and n)

If a is increased by m and b by n:

$$\boxed{(a + m)(b + n) = ab + mb + an + mn}$$

Increase $= an + bm + mn$



$$(a + m)(b + n) = ab + mb + an + mn \quad \text{(Identity 1)}$$

4. KEY DEFINITIONS

- **Distributive Property:** $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$.
- **Product:** The result of multiplying two numbers.
- **Increase in Product:** The amount by which the product becomes greater after a change.
- **Identity:** An equality that is true for all values of the letter-numbers (or all numbers in a certain set).
- **Commutative Property:** $ab = ba$.

5. WHAT TO REMEMBER (AT A GLANCE)

Situation	New Product	Increase in Product
Only b increased by 1	$a(b + 1)$	a
Only a increased by 1	$(a + 1)b$	b
Both a and b increased by 1	$(a + 1)(b + 1)$	$a + b + 1$
a increased by 1, b decreased by 1	$(a + 1)(b - 1)$	$b - a - 1$
a increased by m, b increased by n	$(a + m)(b + n)$	$an + bm + mn$

6. WORKS FOR ALL INTEGERS

These results hold even when a and b are negative integers.

Example:

Let $a = -5$, $b = 8$

Both increased by 1:

$(a + 1)(b + 1) = (-4)(9) = -36$

$ab + (a + b + 1) = (-5)(8) + (-5 + 8 + 1) = -40 + 4 = -36$ ✓

One increased, one decreased:

$(a + 1)(b - 1) = (-4)(7) = -28$

$ab + b - a - 1 = (-5)(8) + 8 - (-5) - 1 = -40 + 8 + 5 - 1 = -28$ ✓

Integers also satisfy the distributive property: $x(y + z) = xy + xz$ (for any integers x, y, z). Hence all the above formulas are valid for integers.

7. VISUAL SUMMARY

Distributivity Makes It Easy!

- Write compact expressions
- See patterns clearly
- Prove identities and properties
- Solve problems smartly

Think: Break a product into smaller parts using distributivity, then combine.

8. QUICK CHECK

Find the increase in the product in each case (in terms of a and b).

1. $a(b + 1)$
2. $(a + 1)b$
3. $(a + 1)(b + 1)$
4. $(a + 1)(b - 1)$
5. $(a + m)(b + n)$

CORE MESSAGE: Distribute, simplify, and patterns appear. Distributivity turns complex expressions into simple truths!

WE DISTRIBUTE YET THINGS MULTIPLY

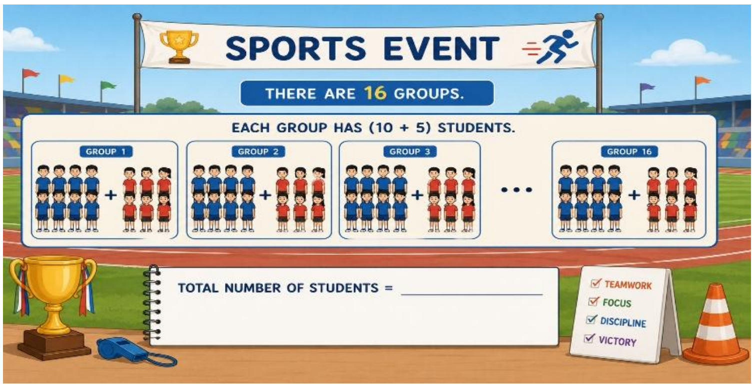
Learning Outcomes:

LO1: Understand the distributive property of multiplication over addition and subtraction.
LO2: Apply distributive property to simplify calculations.
LO3: Use distributive property in algebraic expressions.
LO4: Solve real-life problems using distributive property.
LO5: Develop logical reasoning and computational skills.

Competencies:

C1: Conceptual understanding
C2: Application of mathematical operations
C3: Algebraic reasoning
C4: Problem-solving

Q.No	Question	LO	Competency
MCQs (One Mark Each)			
1	Which property is used in $7(5+3)=7\times5+7\times3$? a) Commutative b) Associative c) Distributive d) Identity	LO1	C1
2	Find $12\times(10+2)$. a) 120 b) 122 c) 132 d) 144	LO2	C2
3	Simplify: $6(a+4)$ a) $6a+4$ b) $6a+24$ c) $10a$ d) $24a$	LO3	C3
4	The value of $8(15-5)$ is: a) 80 b) 100 c) 120 d) 160	LO2	C2
5	Which expression is equivalent to 9×19? a) $9(20-1)$ b) $9(10+9)$ c) $19+9$ d) $9(20+1)$	LO2	C2
6	Expand: $4(2x+3)$ a) $8x+3$ b) $8x+12$ c) $6x+12$ d) $8x+7$	LO3	C3
7	Find 25×14 using distributive property. a) 300 b) 325 c) 350 d) 375	LO2	C2
8	Simplify: $5(m-2)$ a) $5m-2$ b) $5m-10$ c) $3m$ d) $10m$	LO3	C3
9	The distributive property helps in: a) Division only b) Mental multiplication c) Finding factors only d) Subtraction only	LO5	C4
10	Find 11×21. a) 221 b) 231 c) 241 d) 252	LO2	C2
2 Marks Questions			
11	Use distributive property to find 14×12.	LO2	C2
12	Expand $7(x+5)$.	LO3	C3
13	Find 18×9 using distributive property.	LO2	C2
14	Simplify $3(4a+2)$.	LO3	C3
15	Evaluate $12(20-3)$.	LO2	C2

16	Expand 5(2m+7).	LO3	C3
17	Find 16×13 using distributive property.	LO2	C2
18	Simplify 9(p−4).	LO3	C3
19	Evaluate 25×18 using distributive property.	LO2	C2
20	Expand 8(3x+5).	LO3	C3
3 Marks Questions			
21	Use distributive property to find 23×14.	LO2	C2
22	Simplify 4(3x+5)+2x.	LO3	C3
23	A library has 18 shelves. Each shelf contains (20+4) books. Find the total number of books.	LO4	C4
24	Expand and simplify 6(2a+3)−4a.	LO3	C3
25	Find 27×19 using distributive property.	LO2	C2
5 Marks Questions			
26	A school buys 25 packets of notebooks. Each packet contains (30+2) notebooks. Find the total number of notebooks.	LO4	C4
27	Simplify 7(4x+3)+5(2x−1).	LO3	C3
28	A farmer plants 24 rows of trees. Each row contains (15+5) trees. Find the total number of trees.	LO4	C4
Case Study Based Questions			
29	A shopkeeper sells 18 cartons of juice. Each carton contains (12+3) juice boxes. Find the total number of juice boxes using distributive property.  	LO4	C4
30	There are 16 groups in a sports event. Each group has (10+5) students. Find the total number of students.  	LO4	C4

Answers

Q	1	2	3	4	5	6	7	8
Ans	C	144	6a+24	80	9(20−1)	8x+12	350	5m−10
Q	9	10	11	12	13	14	15	16
Ans	Mental multiplication	231	168	7x+35	162	12a+6	204	10m+35
Q	17	18	19	20	21	22	23	24
Ans	208	9p−36	450	24x+40	322	14x+20	432	8a+18
Q	25	26	27	28	29	30		
Ans	513	800	38x+16	480	270	240		

Challenging Questions (Stage 1)

Q.1. Find 39×12 using distributive property.
Q.2. Simplify 8(2x+5)−3x.
Q.3. Find 48×19 mentally.
Q.4. Expand 6(3a−4).
Q.5. A box contains (25+8) balls. Find balls in 15 boxes.

Challenging Questions (Stage 2)

Q.1. Simplify 5(3x+4)+2(2x−1).
Q.2. Find 67×18 using distributive property.
Q.3. Expand and simplify 7(2a+5)−3(a−2).
Q.4. A school has 32 classes with (25+4) students each. Find total students.
Q.5. Find 99×37 mentally.

Challenging Questions (Stage 3)

Q.1. Simplify 8(4x−3)+5(2x+7).
Q.2. Find 125×28 using distributive property.
Q.3. Expand and simplify 9(3a−2)−4(a+5).
Q.4. A warehouse has 48 shelves with (18+7) boxes each. Find total boxes.
Q.5. Find 999×24 using distributive property.

Answers (Challenging Questions)

	1	2	3	4	5
Stage 1	468	13x+40	912	18a−24	495
Stage 2	19x+18	1206	11a+41	928	3663
Stage 3	42x+11	3500	23a−38	1200	23976

CHAPTER 7: PROPORTIONAL REASONING – 1

SUMMARY OF CONCEPTS & DEFINITIONS

1. OBSERVING SIMILARITY IN CHANGE

Digital images can be resized and oriented.
We observe how changes in width and height affect the look of an image.

Set of Images


Image A


Image B


Image C


Image D


Image E

Measurements (in mm)

Image	Width	Height
Image A	60	40
Image B	40	20
Image C	30	20
Image D	90	60
Image E	60	60

- **Images A, C and D look similar.** Their widths and heights change by the same factor (through multiplication).
- **Images B and E look different.** Their widths and heights do not change by the same factor.

If width and height change by the same factor, the images look similar. Such changes are proportional.

2. RATIOS

A ratio compares two quantities of the same kind.
It is written as $a : b$ and read as "a is to b".

Key Points

- a and b are the terms of the ratio.
- In $a : b$, for every 'a' units of the first quantity, there are 'b' units of the second quantity.
- Ratio of width : height
  - Image A = 60 : 40
  - Image C = 30 : 20
  - Image D = 90 : 60

The ratios of width to height of images A, C and D are proportional. (Their terms change by the same factor.)

3. RATIOS IN THEIR SIMPLEST FORM

Two ratios are in proportion if their simplest forms are the same.
To get the simplest form → divide both terms by their HCF.

Image	Ratio (W : H)	HCF	Simplest Form
A	60 : 40	20	3 : 2
B	40 : 20	20	2 : 1
C	30 : 20	10	3 : 2
D	90 : 60	30	3 : 2
E	60 : 60	60	1 : 1

Images A, C and D have the same simplest form (3 : 2). So, their ratios are proportional.

Images B and E have different simplest forms, so their ratios are not proportional to A, C and D.

We use $a : b :: c : d$ to show that the ratios $a : b$ and $c : d$ are proportional.

Examples: • 60 : 40 :: 30 : 20 • 60 : 40 :: 90 : 60

4. INCREMENTS IN PRODUCTS (USING DISTRIBUTIVITY)

Let the initial two numbers be a and b (product = ab).
Using distributive property:

$$a(b+c) = ab + ac$$
$$(a+b)c = ac + bc$$

1. If second number b is increased by 1:
   $a(b+1) = ab + a$
   Increase = a
2. If first number a is increased by 1:
   $(a+1)b = ab + b$
   Increase = b
3. If both a and b are increased by 1:
   $(a+1)(b+1)$
   $= ab + (a + b + 1)$
   Increase = $a + b + 1$
4. If a is increased by 1 and b is decreased by 1:
   $(a+1)(b-1)$
   $= ab + b - a - 1$
   Increase = $b - a - 1$

General Case

If a is increased by m and b by n:

$(a+m)(b+n)$
$= ab + mb + an + mn$

Increase = $an + bm + mn$

(Identity 1)

	b	n
a	ab	an
m	mb	mn

Product = sum of products of each term of $(a+m)$ with each term of $(b+n)$.

5. PROBLEM SOLVING WITH PROPORTIONAL REASONING

Example 1

Are 3 : 4 and 72 : 96 proportional?

3 : 4 is in simplest form.
HCF of 72 and 96 = 24.
72 : 96 = 3 : 4

➡ **Both ratios are same in simplest form. Hence, they are proportional.** ✓

Example 2

6 glasses of lemonade need 10 spoons of sugar. How many spoons for 18 more glasses (same sweetness)?

Ratio of glasses : spoons = 6 : 10
For 18 more glasses → 18 : x
Since ratios are proportional:

$$6 : 10 = 18 : x \Rightarrow x = \frac{18 \times 10}{6} = 30$$

➡ **She should add 30 spoons of sugar.**

KEY TAKEAWAYS ★

- ✓ Similarity comes from proportional change in dimensions.
- ✓ Ratio compares two quantities.
- ✓ Simplify ratios using HCF.
- ✓ Equal simplest forms ⇒ Ratios are proportional.
- ✓ Distributive property helps find changes in products.
- ✓ Proportional reasoning helps solve real-life problems.

CORE MESSAGE: When quantities change in the same proportion, relationships stay the same. Ratios and proportional reasoning help us understand patterns and solve everyday problems!

PROPORTIONAL REASONING–1

Learning Outcomes:
LO1: Compare quantities using ratios
LO2: Identify proportional relationships
LO3: Solve unit-rate and scale problems

Competencies:
C1: Quantitative reasoning
C2: Estimation
C3: Real-life application
C4: Data interpretation

Q.No	Question	LO	Competency
MCQs (One Mark Each)			
1	Ratio of 24 to 36 in simplest form. a)3:2 b)9:4 c)2:3 d)4:9	LO1	C1
2	Equivalent ratios 4:6 is a) 10:15 b)15:10 c)6:4 d)1:1	LO2	C1
3	If 5:8 = x:24 then value of x is a)10 b)15 c)5 d)20	LO2	C1
4	If cost of 4 notebooks ₹120 then unit cost is: a) ₹480 b) ₹30 c) ₹360 d) ₹120	LO3	C2
5	If a:b = 3:5, and b = 20 the value of a is: a)12 b)10 c)3 d)5	LO1	C1
6	Is 12, 18, 24 in proportion? a)Yes b)No c)May be d) Can't say	LO2	C1
7	Fraction notation of the ratio 2:3 is: a)2.3 b)0.66 c)3/2 d)2/3	LO1	C1
8	If 7:x = 21:42 the value of x: a)14 b)10 c)7/2 d)126	LO2	C1
9	Identify if directly proportional: cost and number of items. a) Directly proportional b) Inversely proportional c)Both d)Data insufficient	LO2	C3
10	Which of the following is not equivalent to others: 4:6, 14:21, 25:75, 2:3 a) 2:3 b) 25:75 c) 4:6 d) 14:21	LO2	C3
2 Marks Questions			
11	Simplify 18:24 and check if equal to 3:4.	LO1	C1
12	Find x if 6:x = 9:15.	LO2	C1
13	A car travels 120 km in 3 hours. Find unit rate.	LO3	C3
14	Are 8:12 and 14:21 proportional? Justify.	LO2	C1

15	Divide ₹200 in ratio 2:3.	LO1	C3
16	Find missing term: 9:12 = x:20.	LO2	C1
17	Check if 5,10,15,30 are proportional.	LO2	C1
18	Find value of x: 4:5 = 16:x.	LO2	C1
19	If 3 pens cost ₹45, find cost of 1 pen.	LO3	C3
20	Identify if speed and time are inversely proportional.	LO2	C3
3 Marks Questions			
21	Divide 360 in ratio 3:5:4.	LO1	C3
22	If 8 workers complete work in 6 days, how many days for 4 workers?	LO3	C3
23	Find x and y if x:y = 2:3 and x+y = 50.	LO1	C1
24	Check if 12, 18, 20, 30 are proportional and justify.	LO2	C1
25	A map scale is 1:1000. Find actual distance for 5 cm.	LO3	C4
5 Marks Questions			
26	A recipe uses flour and sugar in ratio 3:2. If 600g flour is used, find sugar required and total mixture.	LO1 LO3	C3
27	A train covers 180 km in 3 hours. Find distance in 5 hours assuming constant speed.	LO3	C3
28	The ratio of boys to girls is 4:5. Total students = 180. Find number of boys and girls.	LO1	C3
Case Study Based Questions			
29	A shop sells 5 books for ₹200. (a) Cost of 1 book (b) Cost of 8 books (c) Is relation proportional? (d) Find cost of 12 books	LO2 LO3	C3 C4
30	In a class, boys: girls = 3:2. Total = 50. (a) Number of boys (b) Number of girls (c) New ratio if 10 girls join (d) Is relation proportional?	LO1 LO3	C3 C4

Answers

Q	1	2	3	4	5	6	7	8
Ans	c	a	b	b	a	a	d	a
Q	9	10	11	12	13	14	15	16
Ans	a	b	3:4 Yes	10	40 km/h	Yes	₹80, ₹120	15
Q	17	18	19	20	21	22	23	24
Ans	Yes	20	₹15	Yes	90,150,120	12 days	20,30	Yes
Q	25	26	27	28	29	30		
	5000 m	400g sugar 1000g total	300 km	80 boys, 100 girls	₹40,₹320,Yes,₹480	30,20, 3:3,Yes		

Challenging Questions(Stage 1)

1. Find x: 3:5 = x:10

2. Find the ratio of a weight of 4 kg to weight of 350 g.

3. Write as ratios and reduce ratios to their simplest forms:

(a) 112and 208
(b) 104 kg and 221 kg

4. 18:24:: 20: __________

5. Give two ratios that are proportional to 4:9

(a) ___________________________________
(b) ___________________________________

Challenging Questions (Stage 2)

1. If 12 notebooks cost ₹180, find the cost of 20 notebooks. Are the quantities in direct proportion? Justify your answer.

2. A car travels 150 km using 10 litres of petrol. How much distance will it travel using 16 litres of petrol, assuming constant mileage? Identify the type of proportion.

3. The cost of 5 pens is ₹75.
(i) Find the cost of 8 pens.
(ii) Find how many pens can be bought for ₹150.
(iii) Verify whether the relationship is proportional.

4. In a school, the ratio of boys to girls is 3 : 5. If total students = 120, find the number of boys and girls. Also check for total = 200.

5. If 18 meters of cloth cost ₹540, find:
(i) the cost of 25 meters of cloth
(ii) how many meters of cloth can be bought for ₹900
Also state the type of proportion.

Challenging Questions (Stage 3)

1. A contractor plans to build a wall.

- 12 workers can complete the wall in 10 days.
- After 4 days, the contractor increases the number of workers to 18.

Answer the following:
(i) How much work is completed in the first 4 days?
(ii) How much work remains?
(iii) How many days will the remaining work take with 18 workers?
(iv) Total number of days required to complete the wall.

2. The ratio of incomes of A and B is 4 : 5, and the ratio of their expenditures is 3 : 4. If both save ₹2000 each, find their incomes.

3. The cost of apples varies directly with their weight. If 3.5 kg of apples cost ₹280:
(i) Find the cost of 5 kg apples
(ii) How many kg of apples can be bought for ₹400
(iii) If the price increases by ₹10 per kg, find the new cost of 5 kg apples

4. A map is drawn to a scale of 1 : 50,000. The distance between two cities on the map is 6 cm.
(i) Find the actual distance in km.
(ii) If another map uses a scale of 1 : 25,000, what will be the map distance for the same actual distance?

5. 15 workers can complete a piece of work in 24 days. After 8 days, 5 workers leave the job. How many more days will the remaining workers take to complete the work?

Answers (Challenging Questions)

	Q1	Q2	Q3	Q4	Q5
Stage 1	6	80:7	7:13 8:17	80/3	8/18, 12/27
Stage 2	300 yes	240 Direct	120 10 Direct	45, 75 75,125	750 30 Direct
Stage 3	2/5 3/5 4Days 8Days	8000 10000	400 5kg ₹450	3km 12cm	24days