केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN

आंचलिक शिक्षा एवं प्रशिक्षण संस्थान, चंडीगढ़

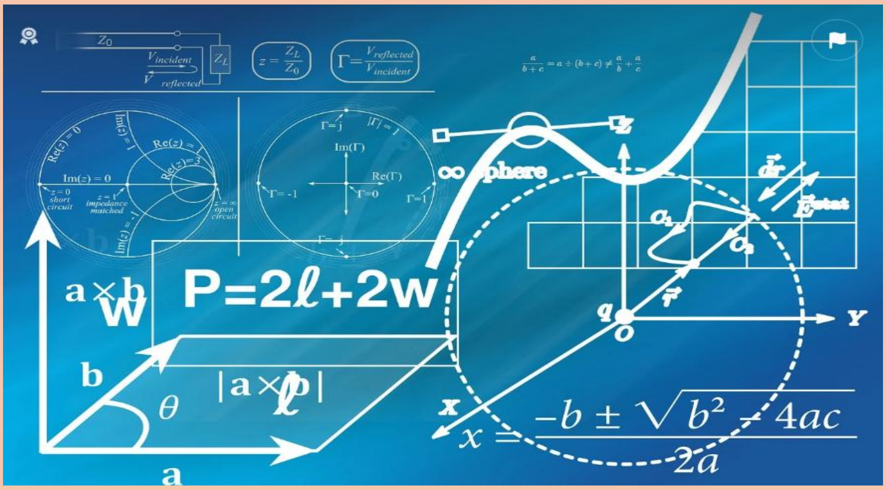
ZONAL INSTITUTE OF EDUCATION AND TRAINING, CHANDIGARH

तत् त्वं पूषन् अपावृणु

केन्द्रीय विद्यालय संगठन

REVISION NOTES FOR MATHEMATICS: CLASS XI

(BASED ON CBSE CURRICULUM)



संरक्षक

श्रीमती प्राची पांडे, आई.ए.एवं ए.एस.

आयुक्त

केन्द्रीय विद्यालय संगठन,नई दिल्ली

Mrs. Prachi Pandey, IA&AS
Commissioner
Kendriya Vidyalaya Sangathan,New Delhi

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श्रीमती श्रुति भार्गव

उपायुक्त एवं निदेशक,

केन्द्रीय विद्यालय संगठन

आंचलिक शिक्षा एवं प्रशिक्षण संस्थान

PREPARED BY
SANJAY KUMAR
TA MATHS
ZIET CHANDIGARH

REVISION NOTES FOR MATHEMATICS (CLASS XI)

SETS

Set

(a well-defined collection of objects)

Roster or tabular form

$A=\{2,4,6,8,10\}$

$B=\{a, e, i, o, u\}$

Set-builder form

$A=\{x: x=2n, n \in N, n\leq 5\}$

$B = \{x:$ x is a vowel in English alphabet$\}$

Types of Set

Empty (null) set

(No element in the set)

$A=\{\ \}$

$\phi=\{x: x \in R, x^2+4=0\}$

Finite set → Number of elements is finite

Infinite set → Number of elements is infinite

Equal Sets : Two sets A and B are said to be equal if they have exactly the same elements and we write A=B

Singleton set : If a set A has only one element, we call it a singleton set. Thus, { a } is a singleton set.

Subsets :

A set A is said to be a subset of a set B if every element of A is also an element of B.

In other words, $A\subset B$ if whenever $a \in A$, then $a \in B$. Thus $A \subset B$ if $a \in A \Rightarrow a \in B$

If A is not a subset of B, we write $A \not\subset B$.

Two sets A and B are said to be equal if they have exactly the same elements and we write A=B

Every set A is a subset of itself, i.e., $A \subset A$.

**** φ is a subset of every set.**

** If $A\subset B$ and $A \neq B$, then A is called a **Proper subset** of B and B is called superset of A.

INTERVALS

Closed Interval :
$x \in [a, b] \rightarrow \{x: a \leq x \leq b\}$

Open Interval :
$x \in (a, b) \rightarrow \{x: a < x < b\}$

Closed open Interval :
$x \in [a, b) \rightarrow \{x: a \leq x < b\}$

Open closed Interval :
$x \in (a, b] \rightarrow \{x: a < x \leq b\}$

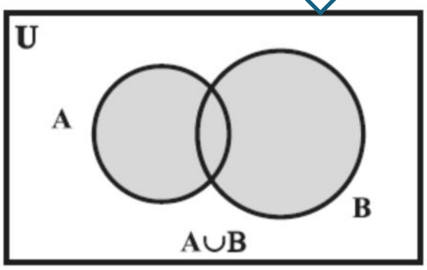
Universal Set :
The largest set under consideration is called Universal set.

Union of sets

The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both)

$A \cup B = \{ x : x \in A \text{ or } x \in B \}$.
$x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$

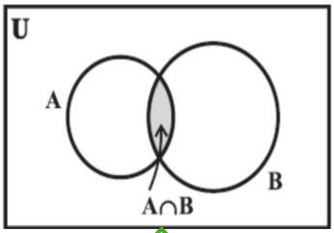
$x \notin A \cup B \Rightarrow x \notin A \text{ and } x \notin B$



Properties of the Operation of Union

(i) $A \cup B = B \cup A$ (Commutative law)
(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)
(iii) $A \cup \varphi = A$ (Law of identity element, φ is the identity of $\cup$)
(iv) $A \cup A = A$ (Idempotent law)
(v) $U \cup A = U$ (Law of U)

Intersection of sets



The intersection of two sets A and B is the set of all those elements which belong to both A and B.

Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$

Some Properties of Operation of Intersection

(i) $A \cap B = B \cap A$ (Commutative law).

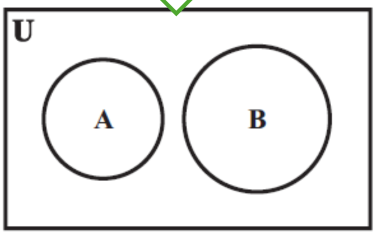
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

(iii) $\varphi \cap A = \varphi$, $U \cap A = A$ (Law of φ and U).

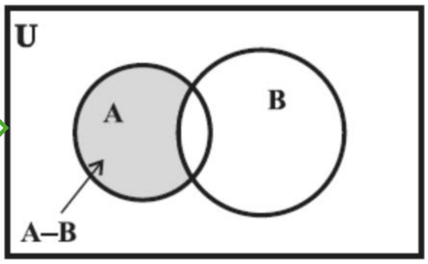
(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i. e., $\cap$ distributes over $\cup$

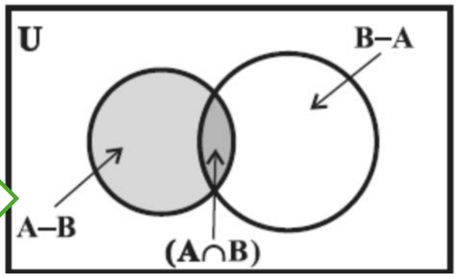
Disjoint sets : If A and B are two sets such that $A \cap B = \varphi$, then A and B are called disjoint sets.



Difference of sets : The difference of the sets A and B in this order is the set of elements which belong to A but not to B. Symbolically, we write $A - B$ and read as " A minus B".
$A - B = \{ x : x \in A \text{ and } x \notin B \}$.



The sets $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets, i.e., the intersection of any of these two sets is the null set.

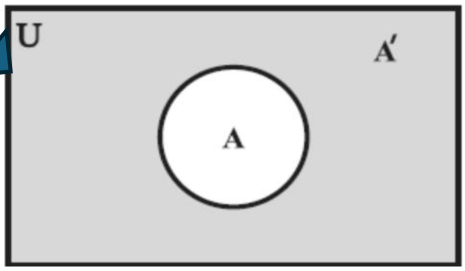


Complement of a Set

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U.

Thus, $A' = \{x : x \in U \text{ and } x \notin A\}$.

Obviously $A' = U - A$



Some Properties of Complement Sets

1. Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \varphi$

2. De Morgan's law: (i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

3. Law of double complementation : $(A')' = A$

4. Laws of empty set and universal set $\varphi' = U$ and $U' = \varphi$.

IMPORTANT PROPERTIES

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$, if $A \cap B = \varphi$.

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap$

RELATION AND FUNCTION

Cartesian Product

The cartesian product $A \times B$ is the set of all ordered pairs of elements from A and B,

i.e., $A \times B = \{(a, b) : a \in A, b \in B\}$

If $(a, b) = (c, d) \rightarrow a = c, b = d$

If there are m elements in A and n elements in B, then there will be pq elements in $A \times B$, i.e. if $n(A) = m$ and $n(B) = n$, then $n(A \times B) = m\,n$.

$A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet

RELATION

A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product $A \times B$ under a particular given condition

Domain: set of all first element of ordered pair in R (i.e. the set of all elements of A which are related to any element of B)

Range $\subseteq$ co-domain.

Codomain=B

Range: set of all second element of ordered pair in R (i.e. the set of all elements of B which are related to any element of A)

The total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total number of relations is 2^{pq}.

A relation R from A to A is also stated as a relation on A.

Function

A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

Domain: Set A

Co-domain: Set B

Range: set of all second element of ordered pair in R (i.e. the set of all elements of B which are related to any element of A)

Algebra of real functions

** **Addition of two real functions :** Let $f : X \to \mathbf{R}$ and $g : X \to \mathbf{R}$ be any two real functions, where $X \subset \mathbf{R}$.

Then, we define $(f + g): X \to \mathbf{R}$ by $(f + g)(x) = f(x) + g(x)$, for all $x \in X$.

** **Subtraction of a real function from another :** Let $f : X \to \mathbf{R}$ and $g : X \to \mathbf{R}$ be any two real functions,

where $X \subset \mathbf{R}$. Then, we define $(f - g) : X \to \mathbf{R}$ by $(f - g)(x) = f(x) - g(x)$, for all $x \in X$.

** **Multiplication by a scalar :** Let $f : X \to \mathbf{R}$ be a real valued function and α be a scalar. Here by scalar, we mean a real number. Then the product αf is a function from X to **R** defined by

$$(\alpha f)(x) = \alpha f(x),\ x \in X.$$

** **Multiplication of two real functions :** The product (or multiplication) of two real functions $f : X \to \mathbf{R}$ and $g : X \to \mathbf{R}$ is a function $fg : X \to \mathbf{R}$ defined by $(fg)(x) = f(x)\, g(x)$, for all $x \in X$.

** **Quotient of two real functions** Let f and g be two real functions defined from $X \to \mathbf{R}$ where $X \subset \mathbf{R}$. The quotient of f by g denoted by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$, $x \in X$

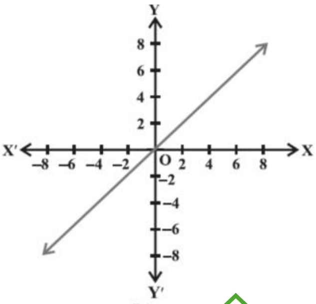
Types of Function

Identity function

f : **R** →**R** by y = f(x) = x

Domain=R

Range=R



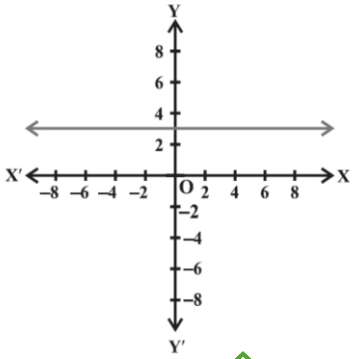
$f(x) = x$

Constant function

f: **R** → **R** by y = f (x) = c, x∈**R**

Domain=R

Range={c}



$f(x) = 3$

Greatest integer function :

The function f : **R** → **R** defined by f(x) = [x], x∈**R** assumes the value of the greatest integer, less than or equal to x.

[x] = –1 for –1 ≤ x < 0

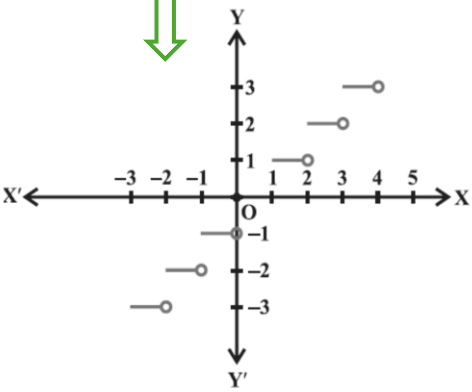
[x] = 0 for 0 ≤ x < 1

[x] = 1 for 1 ≤ x < 2

[x] = 2 for 2 ≤ x < 3 and so on.

Domain=R

Range= Z



$f(x) = [x]$

Modulus function

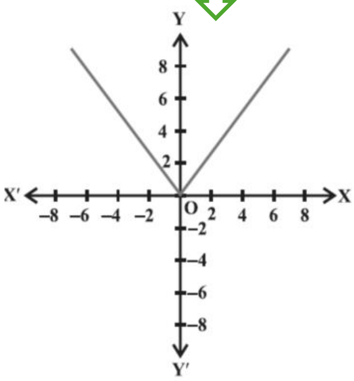
The function f: **R**→**R** defined by

f(x) = |x| for each x∈ **R**

$$f(x) = \begin{Bmatrix} x, & x \geq 0 \\ -x, & x \leq 0 \end{Bmatrix}$$

Domain=R

Range= {-1,0,1}



$f(x) = |x|$

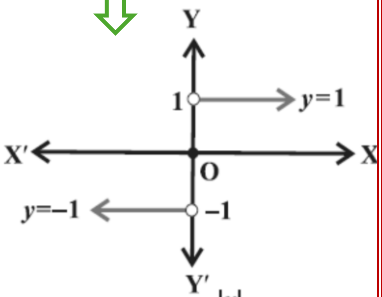
Signum function

The function f : **R**→**R** defined by

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

Domain=R

Range= {-1,0,1}



$f(x) = \frac{|x|}{x}$

TRIGONOMETRIC FUNCTIONS

Angles

Angle is a measure of rotation of a given ray about

English System (Sexagesimal system)

(i)1 right angle = 90 degrees = 90^o.

(ii) 1^o = 60 minutes = 60'.

(iii) 1' = 60 second = 60''.

French System (Centesimal system)

(i) 1 right angle = 100 grades = 100 g.

(ii) 1 g = 100 minutes = 100'

(iii) 1' = 100 seconds = 100''

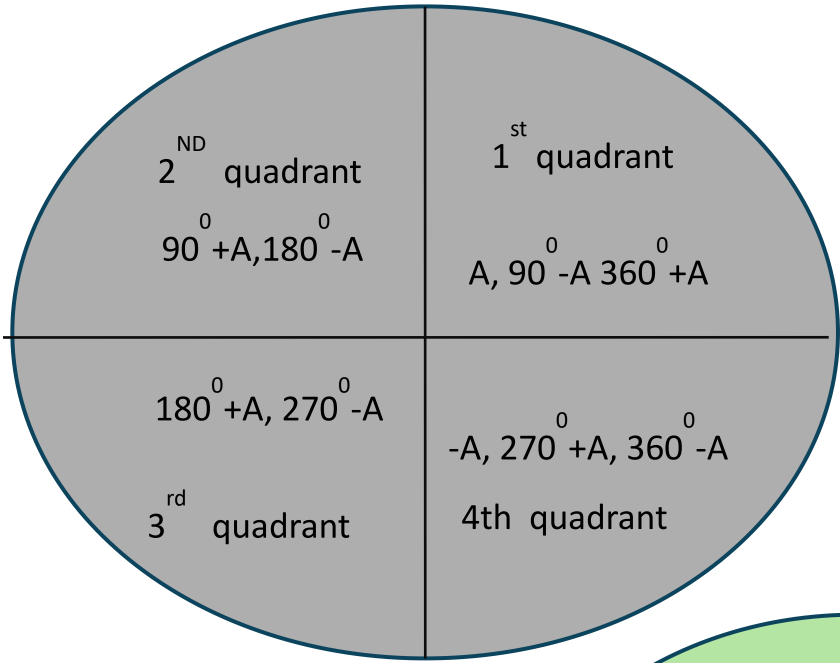
Circular System.

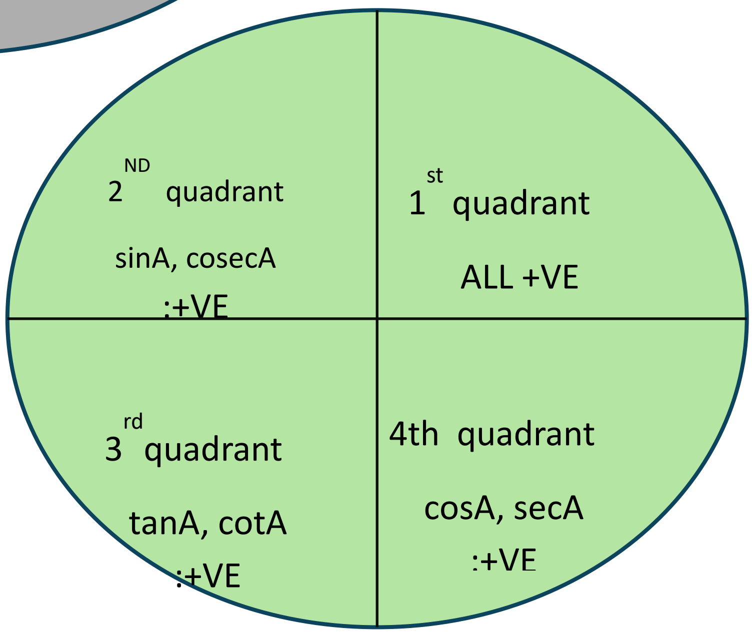
180^o = 200 g = π radians = 2 right angles, where a radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.

The circular measure θ of an angle subtended at the centre of a circle by an arc of length **l** is equal to the ratio of the length **l** to the radius r of the circle.

Each interior angle of a regular polygon of n sides is equal to $\frac{2n-4}{n}$ right angles.

T-ratios	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	n.d	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0





All T-ratio changes in

$\frac{\pi}{2} \pm \theta$ and $\frac{3\pi}{2} \pm \theta$

sin	$\longleftrightarrow$	cos
tan	$\longleftrightarrow$	cot
sec	$\longleftrightarrow$	cosec

while remains unchanged in $\pi \pm \theta$ and $2\pi \pm \theta$.

$\sin\left(\frac{\pi}{2} \pm \theta\right) = \cos\theta$	$\sin(\pi \pm \theta) = \mp\sin\theta$
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$	$\cos(\pi \pm \theta) = -\cos\theta$
$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$	$\tan(\pi \pm \theta) = \pm\tan\theta$
$\cot\left(\frac{\pi}{2} \pm \theta\right) = \mp\tan\theta$	$\cot(\pi \pm \theta) = \pm\cot\theta$
$\sec\left(\frac{\pi}{2} \pm \theta\right) = \mp\mathrm{cosec}\theta$	$\sec(\pi \pm \theta) = -\sec\theta$
$\mathrm{cosec}\left(\frac{\pi}{2} \pm \theta\right) = \sec\theta$	$\mathrm{cosec}(\pi \pm \theta) = \mp\mathrm{cosec}\theta$
$\sin\left(\frac{3\pi}{2} \pm \theta\right) = -\cos\theta$	$\sin(2\pi \pm \theta) = \pm\sin\theta$
$\cos\left(\frac{3\pi}{2} \pm \theta\right) = \sin\theta$	$\cos(2\pi \pm \theta) = \cos\theta$
$\tan\left(\frac{3\pi}{2} \pm \theta\right) = \mp\cot\theta$	$\tan(2\pi \pm \theta) = \pm\tan\theta$
$\cot\left(\frac{3\pi}{2} \pm \theta\right) = \mp\tan\theta$	$\cot(2\pi \pm \theta) = \pm\cot\theta$
$\sec\left(\frac{3\pi}{2} \pm \theta\right) = \mathrm{cosec}\theta$	$\sec(2\pi \pm \theta) = \sec\theta$
$\mathrm{cosec}\left(\frac{3\pi}{2} \pm \theta\right) = -\sec\theta$	$\mathrm{cosec}(2\pi \pm \theta) = \pm\mathrm{cosec}\theta$

Sum and Difference formule:

$\sin(A + B) = \sin A \cos B + \cos A \sin B$	$\sin(A - B) = \sin A \cos B - \cos A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$	$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
$\cot(A + B) = \frac{\cot A . \cot B - 1}{\cot B + \cot A}$	$\cot(A - B) = \frac{\cot A . \cot B + 1}{\cot B - \cot A}$

$\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$, $\tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$

$\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions :

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$	$2\cos A\sin B = \sin(A+B) - \sin(A-B)$
$2\cos A\cos B = \cos(A+B) + \cos(A-B)$	$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Formulae for the transformation of a sum or difference of two circular functions into product of two circular functions :

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$	$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$	$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

**** Formulae for t-ratios of múltiple and sub-múltiple angles :**

$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$			
$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A}$			
$1 + \cos 2A = 2\cos^2 A$	$1 - \cos 2A = 2\sin^2 A$	$1 + \cos A = 2\cos^2\frac{A}{2}$	$1 - \cos A = 2\sin^2\frac{A}{2}$
$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$,	$\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$		
$\sin 3A = 3\sin A - 4\sin^3 A$	$\cos 3A = 4\cos^3 A - 3\cos A$		
$\sin 18^\circ = \frac{\sqrt{5}-1}{4} = \cos 72^\circ$	$\cos 36^\circ = \frac{\sqrt{5}+1}{4} = \sin 54^\circ$		
$\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ$	$\text{Cos}18^0 = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^\circ$.		
$\tan\left(22\frac{1}{2}\right)^\circ = \sqrt{2} - 1$	$\cot\left(22\frac{1}{2}\right)^\circ = \sqrt{2} + 1$		

COMPLEX NUMBER

REAL NUMBER

a

Imaginary Number

ib where $i = \sqrt{-1}$

COMPLEX NUMBER

Combination of Real and Imaginary

z=a+ib

Real part of z=a

Imaginary part of z=b

Properties of imaginary number

$i^2=-1$, $i^3=-I$, $i^4=1$

$i^{(4n+1)}=I$, $i^{(4n+2)}=-1$, $i^{(4n+3)}=-i$

Properties of Complex number

$z_1 = a + ib$ and $z_2 = c + id$

If $z_1 = z_2 \rightarrow a=b,\ c = d$

If z=a+ib

$\text{Modulus of } z = |z| = \sqrt{(a^2 + b^2}$

Argument of z

$$\theta = tan^{-1}\left|\frac{y}{x}\right|$$

i) x→ +ve, y→+ve (1st quardrant) then Arg z = θ

ii) x→-ve, y→+ve (2nd quardrant) then Arg z = $\pi - \theta$

iii) x→-ve, y→-ve (3rd quardrant) then Arg z = $(\pi + \theta)$ OR $(-\pi + \theta)$

iv) x→+ve, y→-ve (4th quardrant) then Arg z = $(2\pi - \theta) OR\ (-\theta)$

Other Properties of Complex number

If $z_1 = a + ib$ and $z_2 = c + id$

$z_1 \pm z_2 = (a \pm c) + i(b \pm d)$

$z_1 + z_2 = z_2 + z_1$

$z_1 z_2 = z_2 z_1$

$z = a + ib$

$Conjugate\ of\ z = \bar{z} = a - ib$

$|z| = |\bar{z}|$

$z\bar{z} = |z|^2$

Other Properties of modulus of complex number

If z, z_2 are two complex numbers then

$|z| = 0 \Leftrightarrow z = 0$ i.e., real part and imaginary part are zeroes.

$|z| = |\bar{z}| = |-z|$, $z.\ \bar{z} = |z|^2$

$|z_1.z_2| = |z_1|.|z_2|$, $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \neq 0$

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2\mathrm{Re}(z_1\ \bar{z}_2)$

$|z_1-z_2|^2 = |z_1|^2 + |z_2|^2 - 2\mathrm{Re}(z_1\ \bar{z}_2)$

$|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

Triangle inequalities :

(i) $|z_1 + z_2| \leq |z_1| + |z_2|$

(ii) $|z_1 - z_2| \leq |z_1| + |z_2|$

(iii) $|z_1 + z_2| \geq |z_1| - |z_2|$

(iv) $|z_1 - z_2| \geq ||z_1| - |z_2||$

Other Properties of conjugate of complex number

(i) $\overline{(\bar{z})} = z$, (ii) $z + \bar{z} = 2\mathrm{Re}(z)$

(iii) $z - \bar{z} = 2i\mathrm{Im}(z)$ (iv) $z + \bar{z} = 0 \Rightarrow$ z is purely real.

(v) $z.\ \bar{z} = [\mathrm{Re}(z)]^2 + [\mathrm{Im}(z)]^2$. (vi) $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

(vii) $\overline{z_1.z_2} = \bar{z}_1.\bar{z}_2$ (viii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$

Cube roots of unity

Let $\omega = \sqrt[3]{1}$ $\therefore$ $\omega^3 = 1$

$\Rightarrow \omega^3 - 1 = 0$ $\Rightarrow$ $(\omega - 1)(\omega^2 + \omega + 1) = 0$

$\Rightarrow \omega = 1$, $\omega = \frac{-1 \pm i\sqrt{3}}{2}$.

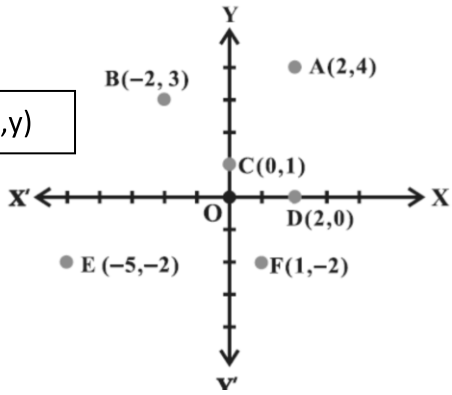
Cube roots of 1 are 1, ω, ω^2, and $\omega^3=1$, and $\omega^2 + \omega + 1 = 0$

then cube roots of -1 are $-1, -\omega, -\omega^2$.

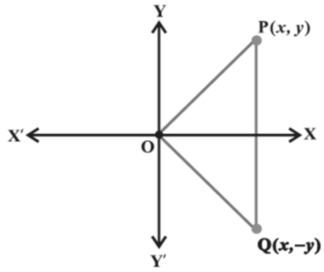
Argand Plane and Polar Representation

$z = x + iy = (x, y)$

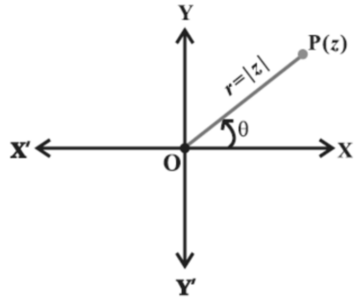
Representation of z as (x,y)



Representation of conjugate



Representation of modulus and argument



LINEAR INEQUALITIES

Symbols

'<' , x<y means x is less than y
'>', x>y means x is more than y
'≤', x≤y means x is less or equal to y
'≥', x≥ y means x is more than or equal to y

Types of Inequalities

Numerical inequalities : $3 < 5;\ \ 7 > 5$
* **Literal inequalities :** $x < 5; y > 2;\quad x \geq 3, y \leq 4$
* **Double inequalities :** $3 < 5 < 7\ \ , 2 < y < 4$
* **Linear inequalities :** $ax + b < 0,\ cx + d \geq 0$
* **Quadratic inequalities :** $ax^2 + bx + c > 0,\ ax^2 + bx + c \leq 0$

Properties of inequalities

$i) x < y \rightarrow\ -x > -y$

$ii) x < y \rightarrow \dfrac{1}{x} > \dfrac{1}{y}$

$iii) x^2 > a^2 \rightarrow \pm x > a$

$\rightarrow x > a\ OR - x > a$

$\rightarrow x > a\ OR\ x < -a$

x is a variable and a is constant

PERMUTATIONS AND COMBINATIONS

Factorial Notation

$n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n.$ $[0! = 1! = 1]$

$n! = n(n-1)! = n(n-1)(n-2)!$ [provided $(n \geq 2)$]

$= n(n-1)(n-2)(n-3)!$ [provided $(n \geq 3)$]

PERMUTATION

A permutation is an arrangement in a definite order of a number of objects

The number of permutations of n different objects taken r at a time, where $0 < r \leq n$

$P(n, r)$ OR $^nP_r = \dfrac{n!}{(n-r)!}, 0 \leq r \leq n$ $[\,^nP_0 = 1 = {}^nP_n\,]$

The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r.

The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is $\dfrac{n!}{p_1!p_2!\ldots p_k!}$.

*The number of permutations of a dissimilar thing taken all at a time along a circle is $(n-1)!$.

**The number of ways of arranging a distinct object along a circle when clockwise and anticlockwise arrangements are considered alike is $\dfrac{1}{2}(n-1)!$.

** The number of ways in which $(m + n)$ different things can be divided into two groups containing m and n things is $\dfrac{(m+n)!}{m!\,n!}$.

COMBINATION

A combination is the way of selection of r objects out of n objects

No. of ways (No. of combinations)= $^nC_r = \dfrac{n!}{r!(n-r)!}$.

Properties of ^nC_r and ^nP_r

** $^nP_r = {}^nC_r\, r!\,,\ 0 \leq r \leq n$

** $^nC_0 = 1 = {}^nC_n$

$^nC_1 = n = {}^nC_{n-1}$

$^nC_2 = \dfrac{n(n-1)}{2!} = {}^nC_2$

$^nC_3 = \dfrac{n(n-1)(n-3)}{3!} = {}^nC_{n-3}$

** $^nC_r = {}^nC_s \Rightarrow r = s$ or $r + s = n$

BINOMIAL THEOREM

$(a+b)^1=a+b={}^1c_0a^1b^0+{}^1c_1a^0b^1$

$(a+b)^2=a^2+2ab+b^2={}^2c_0a^2b^0+{}^2c_1a^1b^1+{}^2c_2a^0b^2$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3={}^3c_0a^3b^0+{}^3c_1a^2b^1+{}^3c_2a^1b^2+{}^3c_3a^0b^3$

$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4={}^4c_0a^4b^0+{}^4c_1a^3b^1+{}^4c_2a^2b^2+{}^4c_3a^1b^3+{}^4c_4a^0b^4$

$(a+b)^5=a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5={}^5c_0a^5b^0+{}^5c_1a^4b^1+{}^5c_2a^3b^2+{}^5c_3a^2b^3+{}^5c_4a^1b^4+{}^5c_5a^0b^5$

Similarly

$$\mathbf{(a+b)^n = {}^nc_0a^n+{}^nc_1a^{n-1}b+{}^nc_2a^{n-2}b^2+{}^nc_3a^{n-3}b^3+\ldots\ldots\ldots\ldots+{}^nc_ra^{n-r}b^r+\ldots\ldots\ldots+{}^nc_{n-1}ab^{n-1}+{}^nc_nb^n}$$

It is Binomial Theorem

Total number of terms in the expansion of $(a+b)^n=n+1$

When a=1, b=x

$$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots+{}^nC_{n-1}x^{n-1}+{}^nC_nx^n$$

When a=1, b= -x

$$(1-x)^n={}^nC_0-{}^nC_1x+{}^nC_2x^2-\ldots+(-1)^n\ {}^nC_nx^n$$

When x=1 in $(1+x)^n$

$${}^nC_0+{}^nC_1+{}^nC_2+{}^nC_3+\ldots+{}^nC_n=2^n$$

When x=1 in $(1-x)^n$

$${}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+\ldots+(-1)^{n-1}\ {}^nC_n=0$$

General term in the expansion of $\mathbf{(a+b)^n}$= $\mathbf{T_{r+1}={}^nc_r\ a^{n-r}\ b^r}$

Middle term in the expansion of $(a+b)^n$

(i) $\left(\frac{n}{2}+1\right)^{th}$ term if n is even (ii) $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+1}{2}+1\right)^{th}$ terms if n is odd.

Pascal's Triangle

Index	Coefficients
0	${}^0C_0$ (=1)
1	${}^1C_0$ (=1) ${}^1C_1$ (=1)
2	${}^2C_0$ (=1) ${}^2C_1$ (=2) ${}^2C_2$ (=1)
3	${}^3C_0$ (=1) ${}^3C_1$ (=3) ${}^3C_2$ (=3) ${}^3C_3$ (=1)
4	${}^4C_0$ (=1) ${}^4C_1$ (=4) ${}^4C_2$ (=6) ${}^4C_3$ (=4) ${}^4C_4$ (=1)
5	${}^5C_0$ (=1) ${}^5C_1$ (=5) ${}^5C_2$ (=10) ${}^5C_3$ (=10) ${}^5C_4$ (=5) ${}^5C_5$ (=1)

SEQUENCE AND SERIES

Sequence : is an arrangement of numbers in a definite order according to some rule.

Series : If $a_1, a_2, a_3, \ldots, a_n$, be a given sequence. Then, the expression $a_1 + a_2 + a_3 + .. + a_n$ is a series

Arithmetic Progression (A.P.) : is a sequence in which terms increase or decrease regularly by the same constant d (common difference)If a is the first tem and d is the common difference the A.P. a, a + d, a + 2d, ... $\mathbf{a_n \ldots}$ $\boldsymbol{l}$

nth term from beginning $= a_n = a + (n - 1)\,d$. **nth term from end** $= l - (n - 1)\,d$

$$\textit{Sum to n terms of AP} = S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + a_n]$$

Arithmetic mean (A.M.) between two numbers a and b is $\frac{a+b}{2}$.

** **n arithmetic means** between two numbers a and b are

$$a + \frac{(b-a)}{n+1},\ a + \frac{2(b-a)}{n+1},\ a + \frac{3(b-a)}{n+1},\ \ldots,\ a + \frac{n(b-a)}{n+1}.$$

** Sum of n A.M.^{S} $= n\left(\frac{a+b}{2}\right)$

** Three consecutive terms in A.P. are $a - d,\ a,\ a + d$.
Five consecutive terms in A.P. are $a - 2d,\ a - d,\ a,\ a + d,\ a + 2d$.

Common Diff.=d

Four consecutive terms in A.P. are $a - 3d,\ a - d,\ a + d,\ a + 3d$.
Six consecutive terms in A.P. are $a-5d,\ a - 3d,\ a - d,\ a + d,\ a + 3d,\ a+5d$

Common Diff=2d

Other Properties of AP

** If a, b, c are in A.P. and $k(\neq 0)$ is any constant, then

(i) $a + k,\ b + k,\ c + k$ are also in A.P.
(ii) $a - k,\ b - k,\ c - k$ are also in A.P.
(iii) ak, bk, ck are also in A.P
(iv) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P.

Geometric Progression (G . P.)

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called geometric progression, if each term is non-zero and

$\frac{a_{k+1}}{a_k} = r(\text{constant})$, for $k \geq 1$.

By taking a is the first term and r is the common ratio Then GP is $a, ar, ar^2, ar^3, \ldots, ar^{n-1}$

General term of a G .P. $= a_n = ar^{n-1}$.

Sum to n terms of a G .P.$= S_n = \frac{a(r^n-1)}{r-1}$

Sum of terms of an infinite G.P. $= S_\infty = \frac{a}{1-r} \quad if\ |r| <$

Geometric Mean (G .M.): of two positive numbers a and b is the number is $\sqrt{ab}$.

** **n geometric mean** between two numbers a and b are

$a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, \ldots a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

Three consecutive terms in G.P. are $\frac{a}{r}, a, ar$.

Four consecutive terms in G.P. are $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

Five consecutive terms in G.P. are $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

Six consecutive terms in G.P. are $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

COORDINATE GEOMETRY

COORDINATE GEOMETRY

Distance between two points $A(x_1, y_1)$ & $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Section formula

i)Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$internally in the ratio m:n is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

ii)Coordinates of a point dividing the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ externally in the ratio m : n is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

iii)Mid point of the line joining $A(x_1, y_1)$ & $B(x_2, y_2)$ is $\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$

Centroid of a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

In centre of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ is $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

where a = BC, b = AC, c = AB.

Area of ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ $= \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

STRAIGHT LINE

Equation of any line parallel to X-axis is y = a & equation of X-axis is y = 0.

Equation of any line parallel to Y-axis is x = b equation of Y axis is x = 0.

General Equation of line

Linear equation of two variable

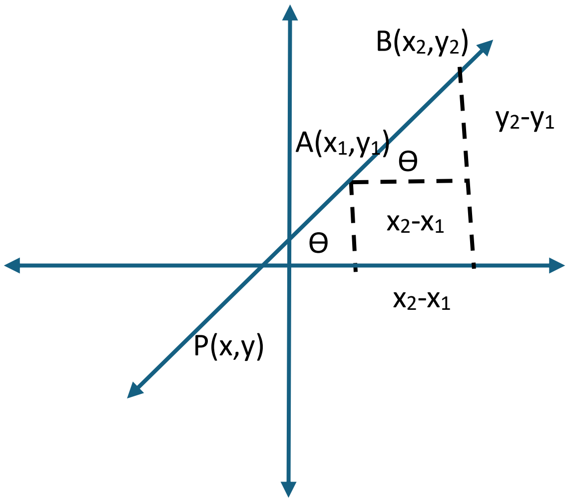
ax + by +c=0

Slope of a line

If line makes an angle θ with positive direction of x-axis then slope of line =m= $\tan\theta$

If line a pssing through two points $A(x_1,y_1)$ and $B(x_2,y_2)$ then according to the given figure

$\tan\theta = \frac{y_2-y_1}{x_2-x_1}$= m (slope of AB)



For x-axis (or parallel to x-axis) $\theta=0 \to \tan\theta=0 \to m=0$

For y-axis (or parallel to y-axis) $\theta=\pi/2 \to \tan\theta=\infty \to m=\infty$

For acute θ, $\tan\theta$=+ve → m=+ve

For obtuse θ, $\tan\theta$=-ve → m=-ve

Equation of Line

If line a pssing through two points $A(x_1,y_1)$ and $B(x_2,y_2)$ and P(x,y) is any point on the line then according to the given figure

PA parallel to AB

→slope of PA=slope of AB

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\to \boldsymbol{y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)}$ **(Two point formula)**

Now we know that $\frac{y_2-y_1}{x_2-x_1} = slope = m$, Hence

$\boldsymbol{y - y_1 = m(x - x_1)}$ **(Slope -point form)**

$y - y_1 = mx - mx_1 \to y = mx - mx_1 + y_1$

$y = mx + (y_1 - mx_1)$

$\boldsymbol{y = mx + c}$ **(Slope- intercept form)**

If line cuts the X and Y axes at (a,0) and (0,b) then by two point formula equation of line

$y - 0 = \frac{b-0}{0-a}(x-a) \to y = \frac{-b}{a}(x-a)$

$\boldsymbol{\frac{y}{b} = -\frac{x}{a} + 1 \to \frac{x}{a} + \frac{y}{b} = 1}$ ***(Intercept form)***

(a & b are intercepts on axes)

Angle between two lines

Let two lines which make angles θ_1 and θ_2 and α is the angle between these two lines then $(tan\theta_1 = m_1, tan\theta_2 = m_2 (slopes))$

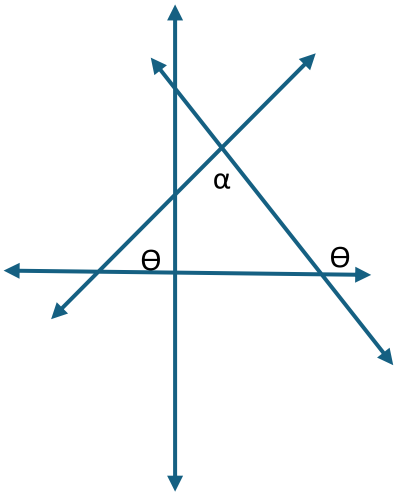
$\theta_1 = \theta_2 + \alpha$ (Exterior angle)

$\alpha = \theta_1 - \theta_2 \rightarrow tan\alpha = \tan(\theta_1 - \theta_2)$

$$tan\alpha = \frac{tan\theta_1 - tan\theta_2}{1 + tan\theta_1 tan\theta_2} \rightarrow tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2}$$

For parallel lines $m_1 = m_2$

For perpendicular lines $m_1 m_2 = -1$



Equation of the line parallel to $ax + by + c = 0$ is $ax + by + \lambda = 0$.

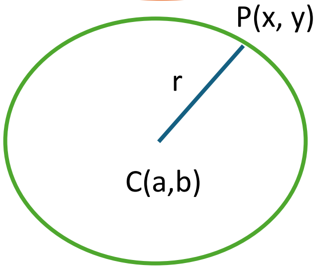
Equation of the line perpendicular to $ax + by + c = 0$ is $bx - ay + \lambda = 0$.

Distance of a point $P(x_1, y_1)$ from the line $ax + by + c = 0$ is $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

Distance between two parallel lines $ax + by + c_1 = 0$ & $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

CONIC SECTIONS

Circle



It is a locus of a point P(x, y) which moves such that it's distance from a fixed point (a,b) remains constant.

That fixed point (a,b) is called Centre and that constant distance is known as Radius r of circle.

Equation of Circle

$(x - a)^2 + (y - b)^2 = r^2$

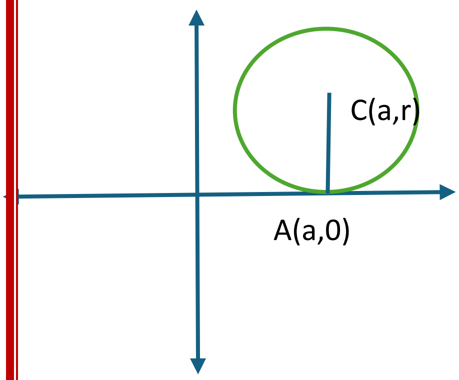
(By distance formula)

After solving the above squares the equation of circle (General Form)

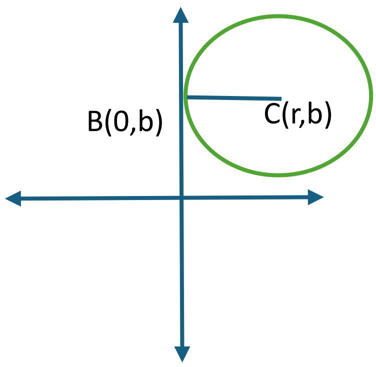
$x^2 + y^2 + 2gx + 2fy + c = 0$

whose Centre is $(-g, -f)$ and Radius is $\sqrt{g^2 + f^2 - c}$

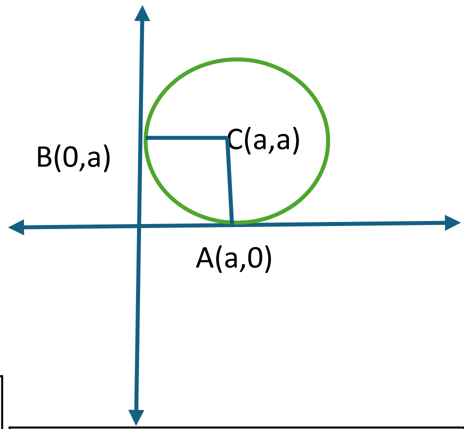
Equation of a circle when end points of diameter as $A(x_1, y_1)$, $B(x_2, y_2)$ is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.



Circle touches x-axis at A(a,0) then Centre C(a, r) and radius r



Circle touches x-axis at B(0,b) then Centre C(r, b) and radius r



Circle touches both axes at A(a,0) and B(0,a)then Centre C(a, a) and radius a

CONIC

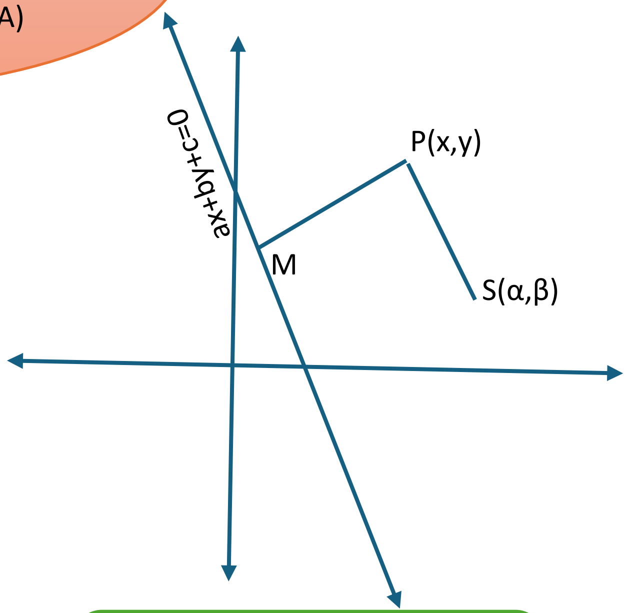
(PARABOLA, ELLIPSE, HYPERBOLA)

It is a locus of a point P(x,y) which moves such that the ratio of it's distance from a fixed point S(α,β) to it's perpendicular distance from a fixed line ax+by+c=0 remains constant.

Fixed point S(α,β)→Focus

fixed line ax+ by +c =0→Directrix

Constant ratio →eccentricity



PS/PM=e

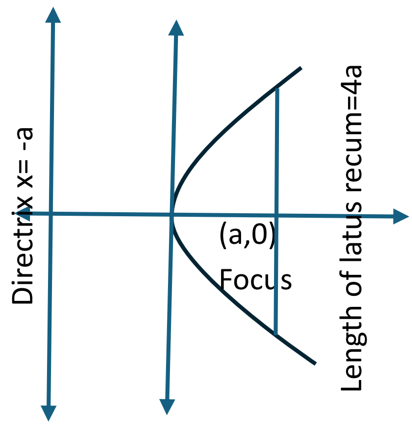
→PS=e.PM

For parabola : e = 1,
For ellipse : e < 1,
For hyperbola: e > 1
For circle : e = 0

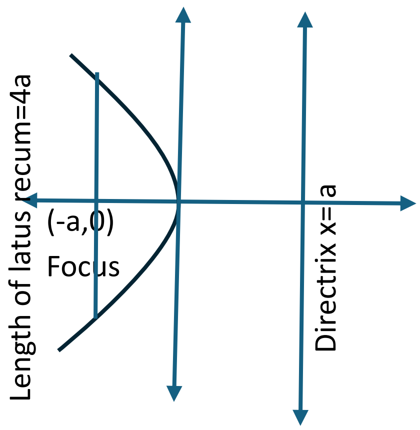
STANDARD FORMS OF CONIC

PARABOLA

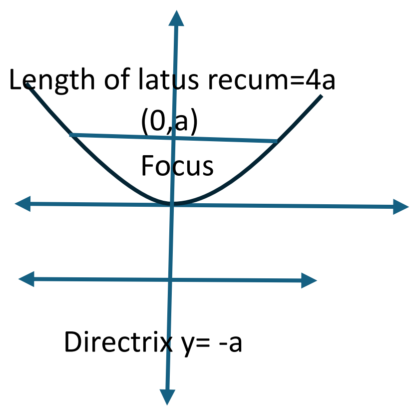
$y^2 = 4ax$



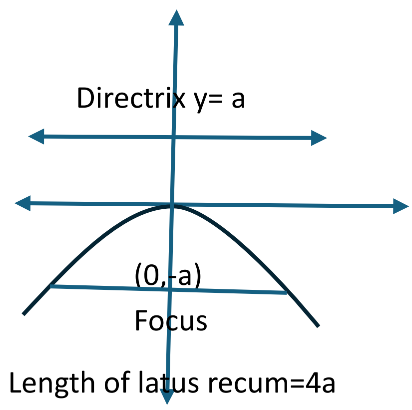
$y^2 = -4ax$

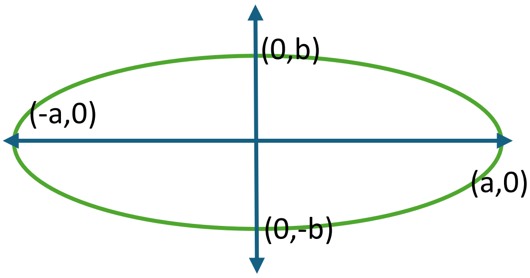
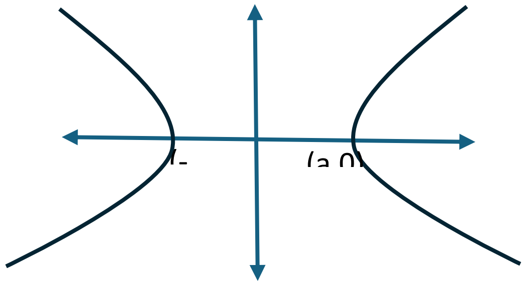
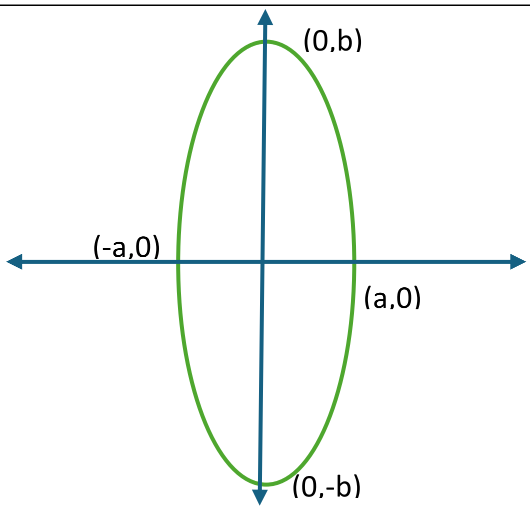
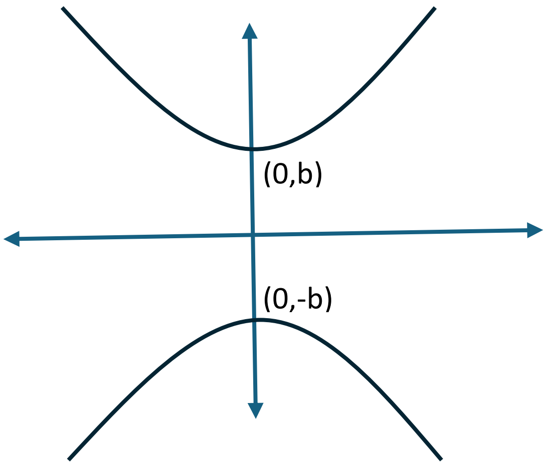


$x^2 = 4ay$



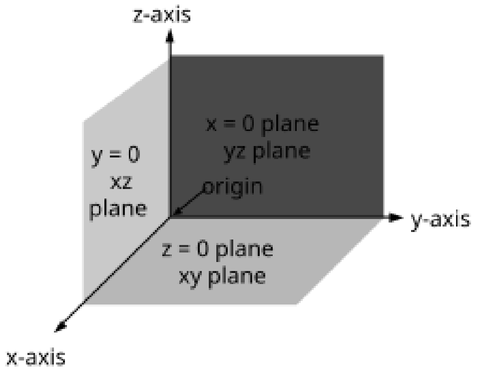
$x^2 = -4ay$

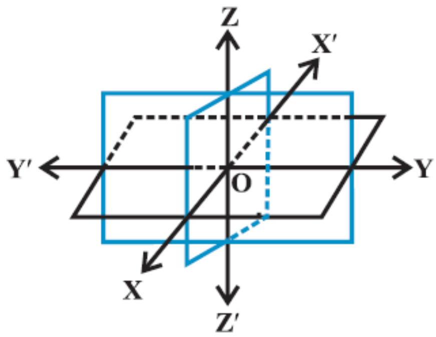


ELLIPSE	HYPERBOLA
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ (a > b)$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
	
Centre(0,0), Vertices $(\pm a, 0)$ Length of major axis=2a Length of minor axis=2b For Eccentricity: $b^2 = a^2(1 - e^2)$ Focus =$(\pm ae, 0)$, Directrices: $x = \pm\frac{a}{e}$ $lenght\ of\ latus\ rectum = \frac{2b^2}{a}$	Centre(0,0), Vertices $(\pm a, 0)$ Length of Transverse axis axis=2a Length of conjugate axis=2b For Eccentricity: $b^2 = a^2(e^2 - 1)$ Focus =$(\pm ae, 0)$, Directrices: $x = \pm\frac{a}{e}$ $lenght\ of\ latus\ rectum = \frac{2b^2}{a}$
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ ,(a < b)$	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
	
Centre(0,0), Vertices $(0, \pm b)$ Length of major axis=2b Length of minor axis=2a For Eccentricity: $a^2 = b^2(1 - e^2)$ Focus =$(0, \pm be)$, Directrices: y= $\pm\frac{b}{e}$ $lenght\ of\ latus\ rectum = \frac{2a^2}{b}$	Centre(0,0), Vertices $(0, \pm b)$ Length of transverse axis=2b Length of conjugate axis=2a For Eccentricity: $a^2 = b^2(e^2 - 1)$ Focus =$(0, \pm be)$, Directrices: y= $\pm\frac{b}{e}$ $lenght\ of\ latus\ rectum = \frac{2a^2}{b}$

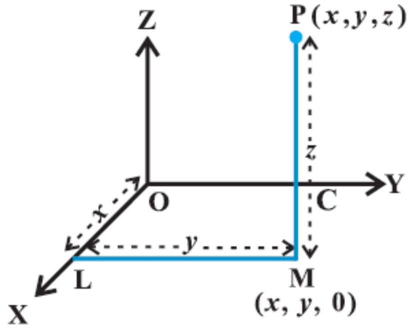
INTRODUCTION TO THREE-DIMENSIONAL GEOMETRY

Coordinate Axes and Coordinate Planes in Three Dimensional Space : In three dimensions, the coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called the x, y and z-axes.The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.





Coordinates of a Point in Space :



The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y, z).

Here x, y and z are the distances from the YZ, ZX and XY-planes.

Any point (i) on x-axis is of the form (x, 0, 0)

(ii) on y-axis is of the form (0, y, 0)

(iii) on z-axis is of the form (0, 0, z).

Any point (i) in XY-plane is of the form (x, y, 0)

(ii) in YZ-plane is of the form (0, y, z)

(iii) on ZX-plnane is of the form (x, 0, z).

Octants → Coordinates ↓	I XOYZ	II X′OYZ	III X′OY′Z	IV XOY′Z	V XOYZ′	VI X′OYZ′	VII X′OY′Z′	VIII XOY′Z′
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

*Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

** **Section Formula** : The coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_1, y_1, z_1) internally and externally in the ratio m : n are given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$, $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$ respect.

** The coordinates of the mid point of the line joining P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is

$$\left(\frac{x_1+y_1+z_1}{2}, \frac{x_2+y_2+z_2}{2}\right)$$

** The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) & (x_3, y_3, z_3) are $\left(\frac{x_1+y_1+z_1}{3}, \frac{x_2+y_2+z_2}{3}, \frac{x_3+y_3+z_3}{3}\right)$.

LIMITS AND DERIVATIVES
Indeterminate forms
(Can not be Determined)

$\frac{0}{0}$	$\frac{\infty}{\infty}$	$0.\infty$	$\infty-\infty$	0^{∞}	∞^{0}	1^{∞}
$0,1,\infty$	$0,1,\infty$	$0,\infty$	$0,-\infty,\infty$	$0,\infty$	$1,\infty$	$1,\infty$
Not Unique	Not Unique	Not Unique	Not Unique	Not Unique	Not Unique	Not Unique

$\lim\limits_{x \to a} f(x) = l$, if to a given $\in > 0$, there exists a +ve number S such that

$| f(x) - l| < \in$ for $| x - a| < \delta$. i.e LHL=RHL

$\lim\limits_{h\to 0} f(a-h) = \lim\limits_{h\to 0} f(a+h)$

** Some Standard Results on Limits :

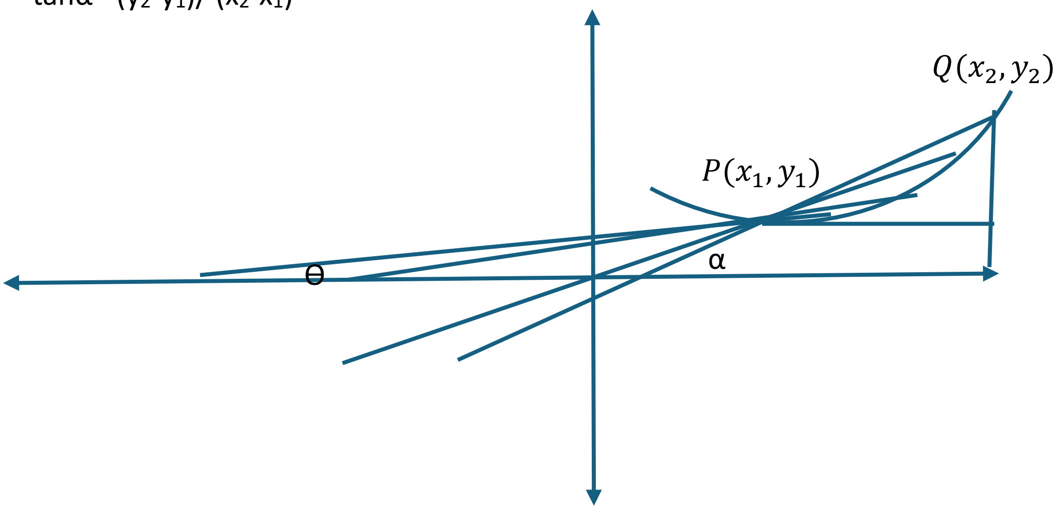
If f(x) = K, a constant function, then $\lim\limits_{x\to a} f(x) = K$.		$\lim\limits_{x\to a} k.f(x) = k \lim\limits_{x\to a} f(x)$
$\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$		$\lim\limits_{x\to a} f(x).\, g(x) = \lim\limits_{x\to a} f(x).$ $\lim\limits_{x\to a} g(x)$
$\lim\limits_{x\to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$		$\lim\limits_{x\to a} \log f(x) = \log\left(\lim\limits_{x\to a} f(x)\right)$
$\lim\limits_{x\to a} [f(x)]^{1/n}. = \left[\lim\limits_{x\to a} f(x)\right]^{1/n}$ Provided $\left[\lim\limits_{x\to a} f(x)\right]^{1/n}$ is a real number.		
Sandwich Theorem (or squeeze principle). If f, g, h are functions such that $F(x) \le g(x) \le h(x)$ as $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} h(x) = l$, then $\lim\limits_{x\to a} g(x) = l$		
$\lim\limits_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}$	$\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1,$	$\lim\limits_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$
$\lim\limits_{x\to 0} \frac{e^x - 1}{x} = \log e = 1$	$\lim\limits_{x\to 0} \frac{a^x - 1}{x} = \log a$	$\lim\limits_{x\to 0} (1 + x)^{1/x} = e$
$\lim\limits_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = C$	$\lim\limits_{x\to 0} \frac{\sin^{-1} x}{x} = 1$	$\lim\limits_{x\to 0} \frac{\tan^{-1} x}{x} = 1$

Differentiation

Let P(x_1, y_1) and Q $(x_2, y_2)$are any two points on the curve

- When Q→P $y_2 \to y_1$ and $x_2 \to x_1$ $x_2 - x_1 \to 0$
- then α→θ and $x_2 - x_1 = \Delta x \to 0$ it means tanα→tanθ
- and tanα = perpendicular/base

tanα= $(y_2-y_1)/(x_2-x_1)$



It means tanθ $=\lim_{Q \to P} tan\alpha = \lim_{y_2 \to y_1} \frac{y_2-y_1}{x_2-x_1} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = slope\ of\ tangent\ at\ P$

First Principle of Differentiation

Let y=f(x) equation of any curve

y+Δy= f(x+Δx)

Δy= f(x+Δx)-y

Δy= f(x+Δx)-f(x)

$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ is known as First principle of differentiation

Some Standard Results of differentiation

$\frac{d}{dx}(x^n) = nx^{n-1}(x \in R, n \in R, x > 0)$	$\frac{d}{dx}(x) = 1$	$\frac{d}{dx}(c) = 0$(where c is a constant)
$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = a^x \log_e a(a \in R, a > 0)$	$\frac{d}{dx}(\log_e x) = \frac{1}{x}(x > 0)$

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\cos x) = -\sin x$	$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$	$\frac{d}{dx}u.v = u\frac{d}{dx}u + \frac{d}{dx}v$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}$

PROBABILITY

Random Experiments : An experiment is called random experiment if it satisfies the following two conditions:

(i) It has more than one possible outcome.

(ii) It is not possible to predict the outcome in advance.

** **Outcomes and sample space :** A possible result of a random experiment is called its outcome. The set of all possible outcomes of a random experiment is called the sample space associated with the experiment. Each element of the sample space is called a sample point. Any subset E of a sample space S is called an event.

** **Impossible and Sure Events :** The empty set φ and the sample space S describe events. φ is called an impossible event and S, i.e., the whole sample space is called the sure event.

** **Compound Event :** If an event has more than one sample point, it is called a Compound event.

** **Complementary Event** : For every event A, there corresponds another event A' called the

complementary event to A. It is also called the event 'not A'.

** **The Event 'A or B' :** When the sets A and B are two events associated with a sample space, then '$A \cup B$' is the event 'either A or B or both'. This event '$A \cup B$' is also called 'A or B'.

The Event 'A and B' : If A and B are two events, then the set $A \cap B$ denotes the event 'A and B'.

** **The Event 'A but not B' :** the set A – B denotes the event 'A but not B'. $A - B = A \cap B'$

** **Mutually exclusive events :** two events A and B are called mutually exclusive events if the occurrence of any one of them excludes the occurrence of the other event, i.e., if they can not occur simultaneously. In this case the sets A and B are disjoint i.e. $A \cap B = \varphi$.

** **Exhaustive events :** if $E_1, E_2, ..., E_n$ are n events of a sample space S and if

$E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = \bigcup_{i=1}^{n} E_i = S$, then $E_1, E_2, ..., E_n$ are called exhaustive events.

if $E_i \cap E_j = \varphi$ for $i \neq j$ i.e., events Ei and Ej are pairwise disjoint and $\bigcup_{i=1}^{n} E_i = S$, then

events $E_1, E_2, ..., E_n$ are called **mutually exclusive and exhaustive events.**

** **Axiomatic Approach to Probability :** Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval [0,1] satisfying the following axioms

(i) For any event E, $P(E) \geq 0$

(ii) $P(S) = 1$

(iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

From the axiomatic definition of probability it follows that

(i) $0 \leq P(\omega i) \leq 1$ for each $\omega i \in S$

(ii) $P(\omega 1) + P(\omega 2) + \ldots + P(\omega n) = 1$

(iii) For any event A, $P(A) = \Sigma P(\omega i)$, $\omega i \in A$.

** **Equally likely outcomes** : All outcomes with equal probability.

** **Probability of an event**: For a finite sample space with equally likely outcomes

Probability of an event $P(A) = \dfrac{n(A)}{n(S)}$, where n(A) = number of elements in the set A,

n(S) = number of elements in the set S.

** **Probability of the event 'A or B' :** $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

For mutually exclusive events A and B, we have $P(A \cup B) = P(A) + P(B)$

** **Probability of event 'not A' =** $P(A') = P(\text{not } A) = 1 - P(A)$.

STATISTICS

Mean	$\dfrac{\sum x_i}{n}$	$\dfrac{\sum f_i x_i}{\sum f_i}$	$A + \dfrac{\sum f_i d_i}{\sum f_i}$ Where A=Assumed mean, $d_i = x_i - A$	$A + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right) h$ Where A=Assumed mean, $u_i = \dfrac{x_i - A}{h}$
Median	If n is total number of observations and the observations are arranged in ascending order i)n is odd: then median=$\frac{n+1}{2}$ *th term* (ii) n is even: then median=$\dfrac{\frac{n}{2} th\ term + \left(\frac{n}{2}+1\right) the\ term}{2}$		$Median = l + \left(\dfrac{\frac{n}{2} - c}{f}\right) h$ *l=lower limit of median class* *n=total number of observations=*$\sum f_i$ *c= cumulative frequency of class proceeding of median class* *f= frequency of median class* *h=length of class interval (class size)*	
Mode	Value of observation (x) which has the highest frequency	$Mode = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$ *l= lower limit of modal class* $f_1 = frequency\ of\ modal\ class$ $f_0 = Frequency\ of\ preceeding\ class$ $f_2 = frequency\ of\ suceeding\ class$ *h= class size*		

Mean deviation

(i) For ungrouped data

$$\text{M.D.}(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } \bar{x} = \text{Mean}$$

$$\text{M.D.}(M) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } M = \text{Median}$$

(ii) For grouped data

(a) Discrete frequency distribution

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{n} f_i|x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$$

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n}|x_i - M|$$

(b) Continuous frequency distribution

$$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|, \text{ using } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)h$$

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n}|x_i - M|, \text{ using } M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

(i) For ungrouped data

$$\text{M.D.}(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } \bar{x} = \text{Mean}$$

$$\text{M.D.}(M) = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|, \text{ where } M = \text{Median}$$

(ii) For grouped data

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