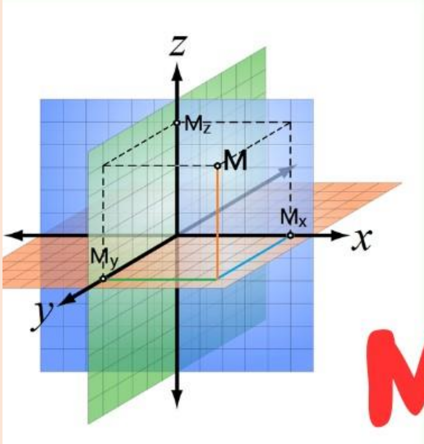
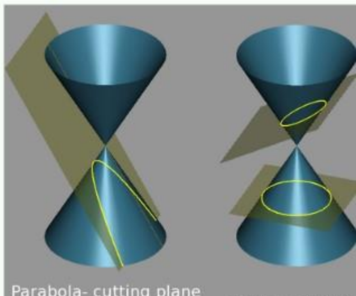
KENDRIYA VIDYALAY SANGATHAN

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन




# MATHEMATICS LAB MANUAL




## CLASS 12

ZONAL INSTITUTE OF EDUCATION AND TRAINING
MYSURU

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# DIRECTOR'S MESSAGE……

It gives me immense pleasure that we present the Manual of Mathematics Lab activities for **CLASS 12** prepared by the PGTs (Mathematics) of the feeder regions during the 3-day online workshop on **"Enhancement of Mathematics Lab Activities and Resource Material**". This collective vision will enable the students and teachers with the tools for success and intellectual growth through learning by doing.

The PGTs (Mathematics) from the feeder Regions namely Bangalore, Chennai, Ernakulam and Hyderabad, evincing their interest and involvement, have invested their knowledge and expertise in preparation of the Manual of Mathematics Lab activities.

I extend my sincere appreciation for the commitment and dedication of the team of PGT(Mathematics) from the four feeder Regions, Mr. Agimon A Chellamcott, Principal, KV Idukki, Ernakulam Region & Associate Course Director, the Resource persons Mr. M. Srinivasan, PGT(Maths), K V Ashok Nagar, Chennai Region & Mr. Jaseer K P, PGT(Maths), No2, Mangalore, Bengaluru Region and Mr. D. Sreenivasulu, Training Associate (Mathematics) from ZIET Mysore who has been the Coordinator of this assignment.

I hope that the knowledge shared here will inspire others also to make meaningful changes in the classroom and to achieve excellence in Mathematics.

With Best wishes.

**MENAXI JAIN**
**DIRECTOR**
**ZIET MYSURU**

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# ACTIVITY - 1

**Mr. SREEKUMAR R**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA SHIFT 2**
**ADOOR**
**ERNAKULAM REGION**

**NAME OF THE ACTIVITY: TO VERIFY THAT THE RELATION R IN THE SET L OF ALL LINES IN A PLANE, DEFINED BY $R = \{(l, m) : l \perp m\}$ IS SYMMETRIC BUT NEITHER REFLEXIVE NOR TRANSITIVE.**

**OBJECTIVE:**

- To apply geometric principles and properties of lines in a plane to reinforce understanding of the relation between perpendicular lines.
- To define and understand the concepts of symmetry, reflexivity, and transitivity in the context of relations.
- To verify that the relation R defined on the set L of all lines in a plane by $R = \{(l, m): l \perp m\}$ is symmetric, but neither reflexive nor transitive.

**PRE-REQUISITE KNOWLEDGE:**

Ordered pair, Cross product of two sets. Definition of reflexive, symmetric and transitive relations.

**MATERIAL REQUIRED:**

- Paper
- Colour thread
- Scissors
- Gum
- Scale etc..

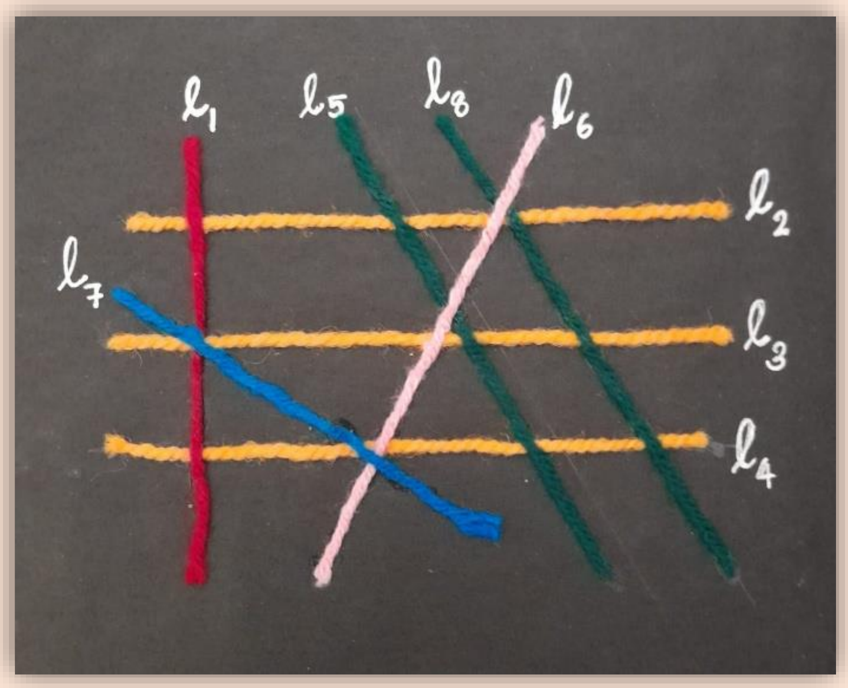
**METHOD OF CONSTRUCTION:**

1. Take a piece of white paper. Cut the coloured threads in suitable length.

2. Fix these pieces of threads in the paper randomly so that some of them are parallel to each other, some are perpendicular to each other and some are inclined.

3. Name these lines as $l_1$, $l_2$, $l_3$, $l_4$, $l_5$, $l_6$, $l_7$ and $l_8$ (these numbers can be increased if required).

4. The threads are to be fixed on the paper using gum in such a way that it will not damage the white paper on which it is affixed and the threads should appear as straight lines as shown.




**DEMONSTRATION:**

- Let the threads represent $l_1$, $l_2$... $l_8$
- Students identify that some of the lines are parallel, some are perpendicular.
- They understand that $l_1 \perp$ to each of the lines $l_2$, $l_3$ and $l_4$

- They understand $l_6$ is ⊥ to $l_7$
- $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in R$
- $l_2 \parallel l_3, l_3 \parallel l_4$ and $l_5 \parallel l_8$

**OBSERVATION:**

- ✓ In figure, students observe that no line is ⊥ to itself. Hence the relation is not reflexive
- ✓ In figure $l_1$ is ⊥ to $l_2$, i.e. $(l_1, l_2) \in R$. Also, $l_2 \perp l_1 \therefore (l_2, l_1) \in R$

  Hence, $(l_1, l_2) \in R \Rightarrow (l_2, l_1) \in R$

  Similarly, $l_1 \perp l_3$. Is $l_3$ perpendicular to $l_1$?

  Student gives the answer: YES

  Hence $(l_3, l_1) \in R$

- ✓ Also, $l_6$ ⊥ to $l_7$. $\therefore (l_6, l_7) \in R$. Is $l_7$ ⊥ to $l_6$? YES, $\therefore (l_7, l_6) \in R$

  Hence $(l_6, l_7) \in R \Rightarrow (l_7, l_6) \in R$

  $\therefore$ the relation R is verified to be symmetric

- ✓ In figure, $l_2$ ⊥ to $l_1$ and $l_1$⊥ to $l_3$. i.e., $(l_2, l_1) \in R$ and $(l_1, l_3) \in R$.

  Is $l_2$ ⊥to $l_3$?

  NO

  Hence $(l_2, l_3) \notin R$

  $\therefore$ R is not transitive

**APPLICATION:**

This activity helps in demonstrating **perpendicularity of lines** is symmetric but not a reflexive and transitive relation.

**CONCLUSION:** The relation perpendicularity of lines in a plane is not an equivalence relation.

**LEARNING OUTCOMES:**

The student analyses the three types of relations and they also understand that perpendicularity of lines is not an equivalence relation.

**SOURCES OF ERROR AND PRECAUTIONS:**

- Labelling the diagram is to be properly done
- Utmost care is to be taken while handling scissors.

## VIVA QUESTIONS AND ANSWERS

**Question 1**: What is a reflexive relation?

Answer: A relation R on a set A is said to be reflexive if

$(a, a) \in R$ for all $a \in A$.

**Question 2:** What is a symmetric relation?

Answer: A relation R on a set A is said to be symmetric if

$(a, b) \in R \Rightarrow (b, a) \in R$.

**Question 3:** What is a transitive relation?

Answer: A Relation R on a set A is said to be a transitive relation if

$(a, b), (b, c) \in R \Rightarrow (a, c) \in R$

**Question 4:** What is an equivalence relation?

Answer: A Relation R on a set A is said to be an equivalence relation if it is reflexive, symmetric and transitive.

**Question 5:** Whether parallelism of lines on the set L of lines on a plane is an equivalence relation?

Answer: Yes, It is an equivalence relation.

**Question 6:** Whether similarity of triangles defined on the set A of all triangles in a plane is an equivalence relation.

Answer: Yes, It is an equivalence relation.

**Question 7:** Whether congruence of triangles defined on the set A of all triangles in a plane is an equivalence relation.

Answer: Yes, It is an equivalence relation.

**Question 8:** Let A be the set of books in a library. Let the relation R defined on A by $R = \{(x, y): x \text{ and } y \text{ have the same pages}, x, y \in A\}$ is an equivalence relation.

Answer: Yes, It is an equivalence relation.

# ACTIVITY - 2

**Ms. SHAILJA NIRANJAN**
**PGT(MATHS)**
**KENDRIYA VIDYALAYA**
**KARAIKUDI**
**CHENNAI REGION**

**NAME OF THE ACTIVITY: EQUIVALENCE RELATION**

**OBJECTIVE:**

- To apply geometric principles and properties of lines in a plane to reinforce understanding of the relation between perpendicular lines.
- To define and understand the concepts of symmetry, reflexivity, and transitivity in the context of relations.
- To verify that the relation R in the set L of all lines in a plane, defined by

  $R = \{(l, m) : l \,||\, m\}$ is an equivalence Relation

**PRE-REQUISITE KNOWLEDGE:**

- Definition of Relation
- Types of relation
  1. Reflexive Relation
  2. Symmetric Relation
  3. Transitive Relation
  4. Equivalence Relation

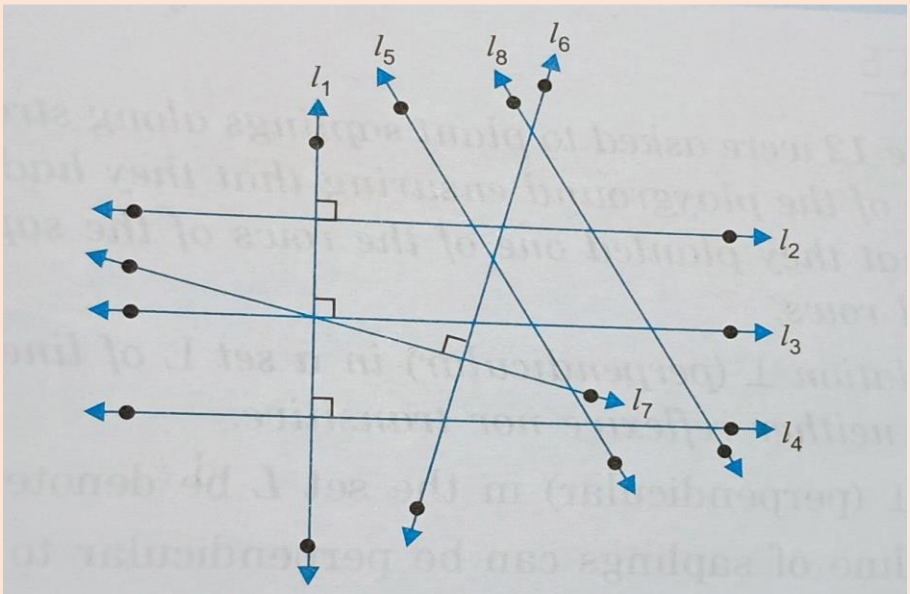
**MATERIAL REQUIRED:**

- A Cardboard
- White Paper
- Drawing Pins or Nails
- Glue

- Protractor
- Pencil/Pen
- Coloured Threads

**METHOD OF CONSTRUCTION:**

- Take a cardboard of convenient size
- Paste a white sheet of paper on it.
- Fix some nails randomly on the white sheet of paper.
- Now, take some colourful threads. Cut them into small pieces and tie them securely around the nails. Make sure that the threads are pulled tight.
- Put some of the threads parallel to each other, some perpendicular to each other (using scale and protractor), and some neither be parallel nor perpendicular as shown in Figure.




**DEMONSTRATION:**

Let the threads represent the lines $l_1$, $l_2$, $l_3$…….$l_8$.

1. Line $l_1$ is perpendicular to each of the lines $l_2$, $l_3$ and $l_4$.
2. Line $l_6$ is perpendicular to $l_7$.
3. Line $l_2$ is parallel to $l_3$, $I_3$ is parallel to $l_4$ and $l_5$ is parallel to $l_8$.

**OBSERVATIONS:**

1. We know that every line is parallel to itself. So, the relation $R=\{(l, m) : l \parallel m\}$ ……… a reflexive relation (is/is not).
2. In the Fig., observe that $l_2 \parallel l_3$ . Therefore, $l_3$ ………..…. $l_4$ ( $\parallel$, $\nparallel$ )

   So, $(l_2, l_3) \in R \Rightarrow (l_3, l_2)$ ………….. R ($\notin$, $\in$)

   Similarly, $l_3 \parallel l_4$ Therefore, $l_4$ ………..…. $l_3$. ( $\parallel$, $\nparallel$ )

   So, $(l_3, l_4) \in R \Rightarrow (l_4, l_3)$ ………….. R ( $\notin$, $\in$)

   Also $l_5 \parallel l_8$ . Therefore, $l_8$ ……….. $l_5$ ( $\parallel$, $\nparallel$ )

   and $(l_5, l_8) \in R \Rightarrow (l_8, l_5)$ ………….. R ( $\notin$, $\in$)

   Thus, the relation R ………………. a symmetric relation. (is/is not)
3. In the Fig., we observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Therefore, $l_2$ ……………… $l_4$. ( $\parallel$, $\nparallel$ )

   So, $(l_2, l_3) \in R$ and $(l_3 , l_4) \in R \Rightarrow (l_2 ,l_4 )$ ……………….. R. ($\notin$, $\in$)

   Similarly, $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Therefore, $l_3$………..…. $l_2$. ( $\parallel$, $\nparallel$ )

   So, $(l_3, l_4) \in R$ and $(l_4 , l_2) \in R \Rightarrow (l_3,l_2 )$ ……………….. R. ($\notin$, $\in$) .

   Thus, the relation R ………………. a transitive relation. (is/is not)

**APPLICATION:**

This activity is useful in understanding the concept of an equivalence relation.

**CONCLUSION:**

We have verified the given relation R is **reflexive**, **symmetric** and **transitive**.

So, it is an **equivalence** relation.

**LEARNING OUTCOMES:**

After completion this activity students able to:

1. Identify equivalence relation.
2. Distinguish among reflexive, symmetric and transitive relation.

**SOURCES OF ERROR AND PRECAUTIONS:**

Precaution should be taken while using nails.

## VIVA QUESTIONS WITH ANSWERS

Questions:1. Define a relation between two sets. Provide an example.

Answer: A relation between two sets (A) and (B) is a subset of the Cartesian product (A × B).

Questions:2. What is the difference between a relation and a function?

Answer: The difference between a relation and a function is that a relation can have many outputs for a single input, but a function has a single input for a single output.

Questions:3. What are the types of relation?

Answer: Empty relation, Universal relation, Identity relation, reflexive relation, symmetric relation, transitive relation and equivalence relation.

Questions:4. What is an empty relation?

Answer: An empty relation (or void relation) is one in which there is no relation between any elements of a set.

Questions:5. What is an identity relation?

Answer: In an identity relation, every element of a set is related to itself only

Questions:6. What is an equivalence relation?

Answer: A relation which is reflexive, symmetric and transitive is an equivalence relation.

Questions:7. Define Reflexive relation.

Answer: In a reflexive relation, every element map to itself.

Questions:8. What is a symmetric relation?

Answer: A relation R is symmetric if and only if (b, a) ∈ R is true when (a, b) ∈ R.

Questions:9. What is a transitive relation?

Answer: For transitive relation, if (x, y) ∈ R, (y, z) ∈ R, then (x, z) ∈ R.

Questions:10. Give an example of a relation which is symmetric but neither reflexive nor transitive.

Answer: The relation on the set of lines in the plane defined by

$R = \{(I, m): l \text{ } is \text{ } perpendicular \text{ } to \text{ } m\}$ is symmetric, but it is neither reflexive nor transitive

# ACTIVITY - 3

**Ms. PREETI DHAMA**
**PGT(MATHEMATICS)**
**P M SHRI KENDRIYA VIDYALAYA**
**MEG AND CENTRE**
**BENGALURU REGION**

**NAME OF THE ACTIVITY**: **TO DEMONSTRATE A FUNCTION WHICH IS NOT ONE -ONE BUT IS ONTO.**

**OBJECTIVE:**

- To define and clarify the concepts of injective (one-to-one) and surjective (onto) functions.
- To work with specific examples of functions that are onto but not one-to-one, fostering a deeper understanding through practical applications.
- To understand that there are some functions which are not one-one but onto.

**PRE-REQUISITE KNOWLEDGE:**

Student must be familiar with the meaning of functions, domain of function, range of function, codomain of function, one-one and onto function.

**MATERIAL REQUIRED:**

- Wooden board of suitable measurement
- Board Pins
- Scissors
- String
- Adhesive
- Coloured Papers
- Paper Strips

**METHOD OF CONSTRUCTION:**

Cut the card board into two equal pieces with the help of scissors. On the two pieces of wooden boards paste coloured paper with the help of adhesive. On one card board fix board pins as shown in figure-1. On the other piece of card board fix the board pins as shown in figure -2.

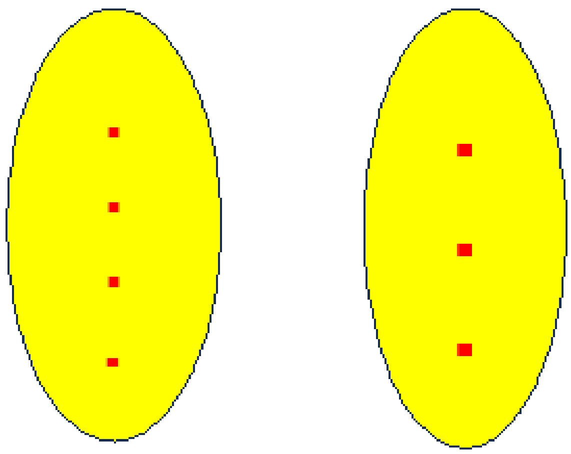


Figure-1 Figure-2

On the first piece of card board assign name to the board pins as a, b, c, d respectively, as shown in figure-3.

On the second piece of card board assign name to the board pins as x, y, z respectively, as shown in figure-4.

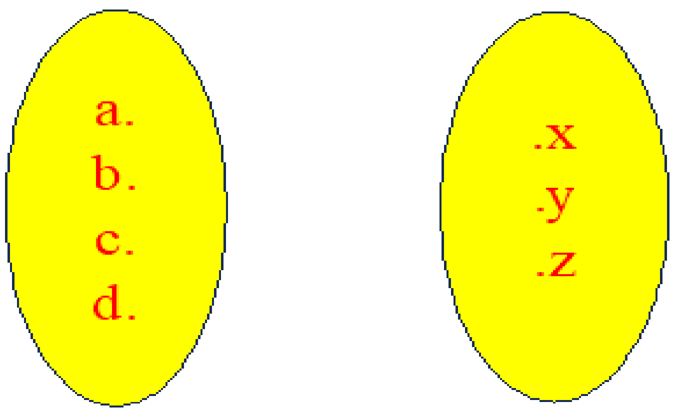
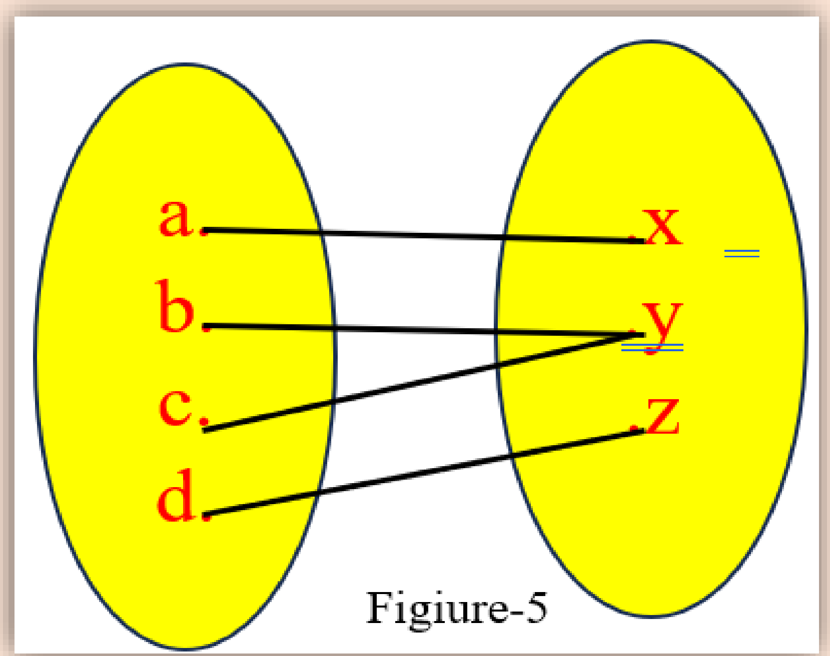



Figure-3 Figure-4

Join the board pins of the first card board with the board pins of the second piece of car board as shown in figure-5.



Figiure-5

**DEMONSTRATION:**

- Take a set $A = \{a, b, c, d\}$ and a set $B = \{x, y, z\}$.
- Join the elements of set A to the elements of set B as shown in figure-5.
- Using string join '$a$' *with* '$x$', join '$b$' *with* '$y$', join '$c$' *with* '$y$', join '$d$' *with* '$z$'.
- So, in this way we will get a function $\{(a, x), (b, y), (c, y), (d, z)\}$ with domain $\{a, b, c, d\}$ and codomain of the function as $\{x, y, z\}$.

**OBSERVATION:**

We observe that:

- The element 'x' of B is the image of the element _______ of A.
- The element 'y' of B is the image of the element _______ of A.
- The element 'y' of B is the image of the element _______ of A.
- The element 'z' of B is the image of the element ________of A.

- All the elements of A __________ (have/does not have) images in B.
- All the elements of B _____________ (have /does not have) pre-images in A.
- The function is _______________________ (one-one/not one-one).
- The function is _____________________ (onto/not onto).

**APPLICATION**:

- ✓ This activity helps in understanding of concept of one-one and onto functions. Understanding functions is fundamental in computer science, particularly in areas like database management and algorithm design.
- ✓ Functions are used in various fields such as economics, engineering, and natural sciences to model relationships between variables.

**CONCLUSION:**

1) Not One-One: The function is not one-one because there are two distinct elements in the domain that map to the same element in the codomain.
2) Onto: The function is onto because every element in the codomain has at least one pre-image in the domain.
3) Understanding the Concepts: This activity helps in understanding the difference between one-one and onto functions. It shows that a function can be surjective without being injective.

**LEARNING OUTCOMES:**

1) Understanding the Concept: Students will be able to define and distinguish between one-one and onto functions.
2) Identification Skills: Students will identify examples of functions that are not one-one but are onto from a given set of functions.
3) Application of Knowledge: Students will apply their understanding to construct a function that is not one-one but is onto.

4) Analytical Skills: Students will analyses why a given function is not one-one but is onto, using appropriate mathematical reasoning.
5) Problem-Solving: Students will solve problems involving functions, demonstrating their ability to determine whether a function is one-one, onto, both, or neither.
6) Communication: Students will effectively communicate their reasoning and solutions, both in written and verbal forms.

**SOURCES OF ERROR AND PRECAUTIONS:**

Sources of Error: Incorrect Mapping, Misinterpretation of Results, Errors in recording or interpreting data.

Precautions: Board pins should not be very pointed, Boar pins must have proper holdable area, Clear Instructions should be given to the students, encourage students to double-check their work and mappings, Peer Review can be done.

## VIVA QUESTIONS WITH SUGGESTIVE ANSWERS

Question 1: What is the definition of a function that is not one-one (injective)?

Answer: A function is not one-one if there exist at least two distinct elements in the domain that map to the same element in the codomain. In other words, the function does not have a unique mapping for each element in the domain.

Question 2: What does it mean for a function to be onto (surjective)?

element in the domain. This means that the function covers the entire codomain.

Question 3: Provide an example of a function that is not one-one but is onto?

Answer: Whatever student will tell, teacher may evaluate accordingly.

Question 4: How can you verify that a function is not one-one?

Answer: To verify that a function is not one-one, we need to find at least two distinct elements in the domain that map to the same element in the codomain. If such elements exist, the function is not one-one.

Question 5: How can you verify that a function is onto?

Answer: To verify that a function is onto, we need to check that every element in the codomain has at least one pre-image in the domain. If every element in the codomain is mapped by some element in the domain, the function is onto.

Question 6: Why is it important to understand the difference between one-one and onto functions?

Answer: Understanding the difference between one-one and onto functions is important because these properties have different implications in various Mathematical contexts. For example, in linear algebra, injective functions are related to the concept of linear independence, while surjective functions are related to the concept of spanning sets.

Question 7: What are some real-life applications of functions that are not one-one but onto?

Answer: Whatever student will tell, teacher may evaluate accordingly.

Question 8: What precautions should be taken to avoid errors when demonstrating a function that is not one-one but onto?

Answer: Double-checking mappings, using examples, and peer review to ensure accuracy and understanding. It's also important to verify that every element in the codomain is covered and that there are distinct elements in the domain mapping to the same element in the codomain.

Question 9: What is the significance of the codomain in determining whether a function is onto?

Answer: The codomain is crucial in determining whether a function is onto because a function is onto only if every element in the codomain is mapped by at least one element in the domain. The codomain defines the set of possible outputs that need to be covered by the function.

Question 10: How can you modify a function that is not one-one but onto to make it one-one?

Answer: To make a function one-one, you need to ensure that each element in the domain maps to a unique element in the codomain. This might involve redefining the function or changing the domain or codomain to eliminate duplicate mappings.

# ACTIVITY - 4

**Ms. SHANVIA STANLEY V**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA 2**
**NAVAL BASE KOCHI**
**ERNAKULAM REGION**

**NAME OF THE ACTIVITY**: **ONE - ONE FUNCTION BUT NOT ONTO**

**OBJECTIVE:**

- To define and clarify the concepts of injective (one-to-one) and surjective (onto) functions.
- To work with specific examples of functions that are one-to-one but not onto, fostering a deeper understanding through practical applications.
- To understand that there are some functions which are onto but not one-one.

**PRE-REQUISITE KNOWLEDGE**:

Definition of a function, one-one function and onto function

**MATERIALS REQUIRED:**

- Chart Paper
- Colour Papers
- Sketch Pen
- Scale
- Scissors
- Glue

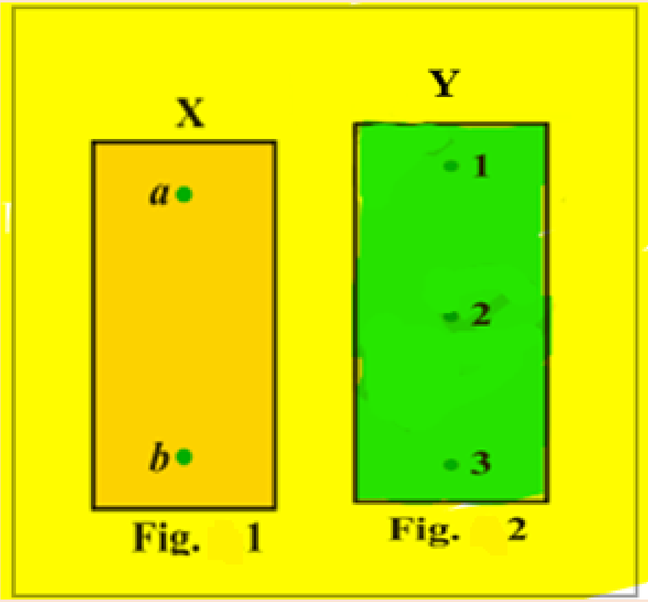
**METHOD OF CONSTRUCTION**

- Paste a colour strip on the left-hand side of the chart paper and mark two points a and b as shown in figure1
- Paste another strip on the right-hand side of the chart paper and mark three points say 1, 2 and 3 on it as shown in the figure 2.

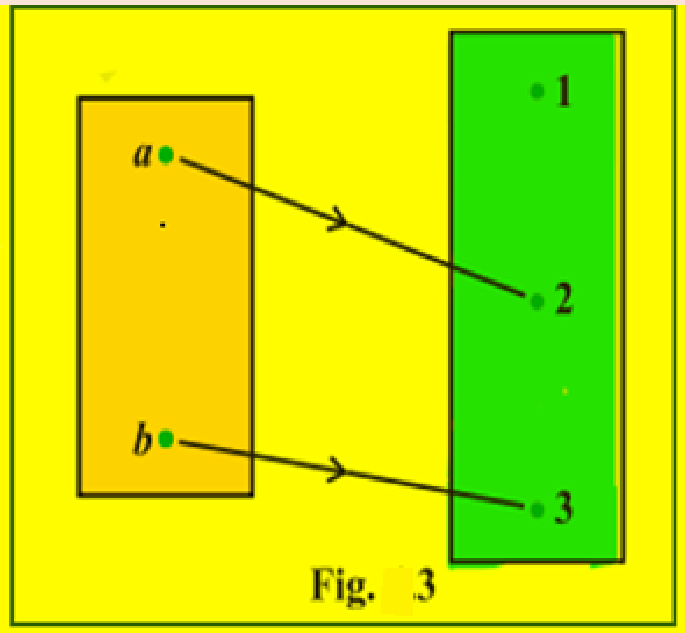
- Join the points on the left strip to the points on the right strip as shown in the figure 3.

**DEMONSTRATION**:

1. Take the set X = {a, b}
2. Take the set Y = {1, 2, 3}.




3. Join elements of X to the elements of Y as shown in figure 3



**OBSERVATION:**

- The image of the element a of X in Y is _______________.

  The image of the element b of X in Y is _______________.

  So, the Figure 3 represents a ________________.
- Every element in X has a _____________ image in Y.

  So, the function is _____________ (one-one/not one-one).
- The pre-image of the element 1 of Y in X ___________ (exists/does not exist).

  So, the function is __________ (onto/not onto).

  Thus, Figure 3 represents a function which is __________ but not onto.

**APPLICATION**:

This activity can be used to demonstrate the concept of one-one but not onto function.

**CONCLUSION**:

A function which is one-one but not onto is demonstrated through the activity

**LEARNING OUTCOMES:**

- ✓ To identify a function which is one -one
- ✓ To identify a function which is onto
- ✓ To analyse a function which is one -one but not onto

**SOURCES OF ERROR AND PRECAUTIONS:**

Be careful to take number of elements in X such that it should be less than number of elements in Y. (i e, 2 elements in X and 3 elements in Y)

Error may occur due to incorrect mapping and misinterpretation of results.

## VIVA QUESTIONS AND ANSWERS:

1. A function $f: \{1,2,3\} \rightarrow \{a, b, c\}$ given by $f = \{(1, a), (2, c), (3, c)\}$ is one-one or onto?

   Answer: Neither one-one nor onto
2. A function f: A →B is an onto function then range of f is ___________

Answer: B

3. Is g (x) = | x – 2 | onto, where g: R→R?

Answer: Not onto as range = $[0,\infty)$

4. A function $f : [0, \pi/2] \to R$ given by f(x) = sin x is one-one or not?

Answer: One-one

5. Let A= {a, b}. Then number of one-one functions from A to A possible are___

Answer: .2! =2

6. Let the function $f : N \to N$ be defined by f(x) = 2 x + 3,$\forall x \in N$.Is f onto?

Answer: Not onto

7. Let A = {1,2,3,4} and B= {a, b, c}. Then number of one-one functions from A to B?

Answer: 0, as n(A) $> n(B)$

8. Let the function $f : R \to R$ given by f(x) = $x^3 + 1$,$\forall x \in R$.Is f one-one?

Answer: One-one

9. Let the function f: $R_+ \to [4,\infty)$ given by f(x) = $x^2 + 4$ .Is f bijective ?

Answer: Bijective function (one-one and onto)

10. Let the function $f : R \to R$ given by f(x) = [x], greatest integer function. Is f one -one and onto?

Answer: Neither one-one nor onto

# ACTIVITY - 5

**Mr. KENDRAPAL SINGH**
**PGT(MATHS)**
**KENDRIYA VIDYALAYA**
**THAKKOLAM**
**CHENNAI REGION**

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**NAME OF THE ACTIVITY: TO DRAW THE GRAPH OF $sin^{-1}x$ USING THE GRAPH OF $Sin\ x$ AND DEMONSTRATE THE CONCEPT OF MIRROR REFLECTION (ABOUT THE LINE $y = x$).**

**OBJECTIVE:**

- To identify and describe the domain and range of the inverse sine function, highlighting that the domain is $[-1,1]$ and the range is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
- To understand the relationship between the graph of sin x and its inverse $sin^{-1}x$ by demonstrating the concept of mirror reflection about the line $y = x$.

**PRE-REQUISITE KNOWLEDGE:**

- Basic understanding of trigonometric functions and their inverses.
- Graphical representation of $sin\ x$.
- Concept of symmetry and reflection about a line.

**MATERIAL REQUIRED:**

- Graph paper
- Pencil
- Ruler
- Geometry set
- Coloured pens
- Adhesive
- Cutter
- Eraser

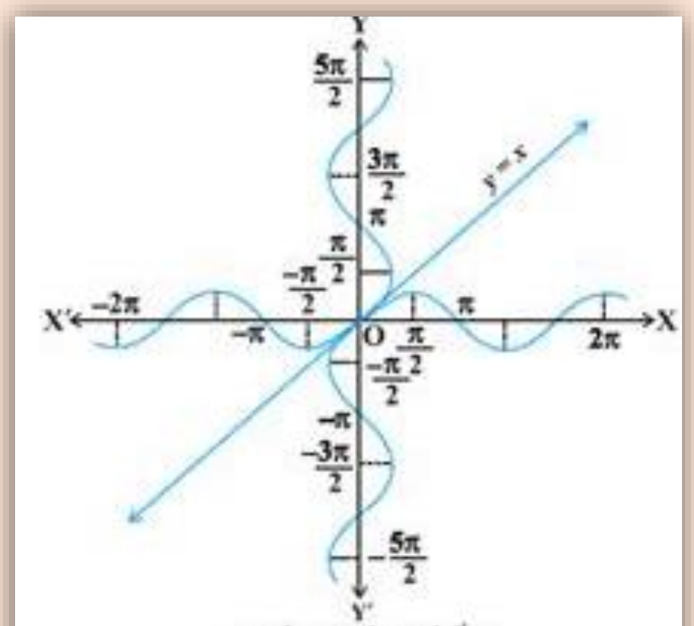
- Card board

**METHOD OF CONSTRUCTION:**

- Take a card board of suitable measurement.
- Paste a graph paper on it.
- On the graph paper draw two perpendicular lines and name them X′OX and YOY′.
- Draw the graph of sin x on the paper.
- Mark the line y = x on the same plane.
- Reflect the graph of sin x about the line y = x.
- The reflected graph represents the inverse function $\sin^{-1}x$.

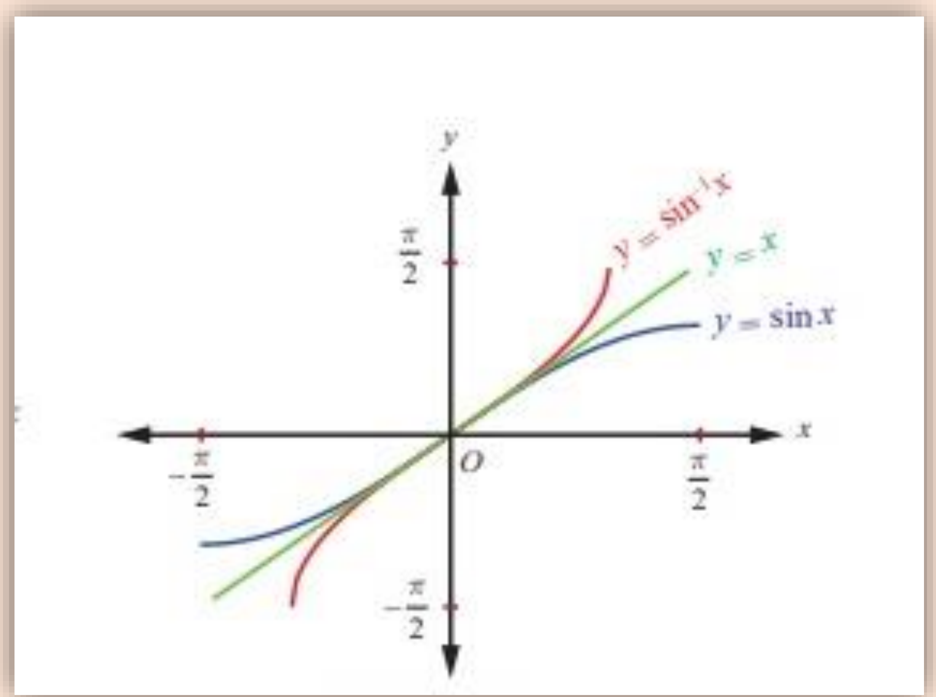
**DEMONSTRATION:**

1. **Graphing sin x**:
   - Plot points for sin x over the interval [−1,1].
   - Connect the points smoothly to form the sine wave.
2. **Marking the Line y = x**:
   - Draw a straight line passing through the origin with a slope of 1.
3. **Reflecting the Graph**:
   - Reflect each point of the graph of sin x across the line y = x.




$y = \sin x$ and $y = \sin^{-1} x$

- The resulting points will form the graph of $\sin^{-1}x$.




**OBSERVATION:**

The graph of $\sin^{-1}x$ reflects the graph of sin x across the line y = x. The range and domain of the functions are interchanged.

**APPLICATION:**

This concept is useful in understanding inverse trigonometric functions and their properties, particularly in calculus and mathematical analysis.

**CONCLUSION:**

The graph of an inverse function can be obtained by reflecting the graph of the original function about the line y = x. This helps in visualizing the relationship between a function and its inverse.

**LEARNING OUTCOMES:**

- Ability to graphically represent trigonometric functions and their inverses.

- Understanding of the concept of mirror reflection about the line $y = x$.
- Enhanced comprehension of the relationship between functions and their inverses.

**SOURCES OF ERROR AND PRECAUTIONS:**

- Ensure accurate plotting of points to avoid distortion in the reflection.
- Use a sharp pencil and a ruler for precise graphing.
- Double-check the reflection across the line $y = x$ for accuracy.

## VIVA QUESTIONS AND ANSWERS

1. What is the relationship between sin x and $\sin^{-1}x$?

Answer: The relationship between sin x and $\sin^{-1} x$ is that they are inverse functions. The function $\sin^{-1} x$ undoes the action of sin x. If $y = \sin x$, then $x = \sin^{-1} y$. This means that $\sin^{-1}(\sin x) = x$ within the appropriate domain and range.

2. How do you derive the graph of $\sin^{-1}x$ from sin x?

Answer: The graph of $\sin^{-1}x$ is derived by reflecting the graph of sin x across the line $y = x$. Since inverse functions swap their x- and y-values, reflecting the graph across this line visually represents this reversal.

3. What is the significance of the line $y = x$ in graphing inverse functions?

Answer: The line $y = x$ is significant because it serves as the axis of symmetry for a function and its inverse. Reflecting a function's graph across this line gives the graph of its inverse function, demonstrating the interchange of the domain and range.

4. Can the graph of any function be reflected across the line y = x to obtain its inverse? Why or why not?

Answer: Not every function has an inverse that is also a function. For a function to have an inverse, it must be one-to-one (bijective), meaning it passes both the vertical and horizontal line tests. If a function is not one-to-one, reflecting its graph across the line y = x will not produce a valid function because the inverse would fail the vertical line test.

5. What are the domain and range of $\sin^{-1}x$?

Answer: The domain of $\sin^{-1}x$ is $[-1,1]$ and the range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

6. Explain the concept of symmetry in the context of this activity.

Answer: In this activity, symmetry refers to the mirror-like property between the graph of a function and its inverse when reflected across the line y = x. This symmetry is a visual representation of how the roles of the input (x) and output (y) values are interchanged in inverse functions.

7. What happens to the points on the line y = x during the reflection process?

Answer: Points on the line y = x remains unchanged during the reflection process because they are equidistant from both axes, meaning their x and y coordinates are the same.

8. How does the reflection affect the shape of the original function's graph?

Answer: The reflection changes the orientation of the graph, swapping the roles of the x- and y-coordinates. This transformation may alter the overall appearance of the graph, but the essential shape remains consistent with the behavior of the inverse function.

9. Why is it important to understand inverse trigonometric functions?

Answer: Understanding inverse trigonometric functions is crucial because they allow us to solve equations involving trigonometric functions, especially in contexts like calculus, physics, and engineering, where determining angles from ratios is necessary.

10. How does this activity help in visualizing the relationship between a function and its inverse?

Answer: This activity helps visualize the relationship by providing a concrete method to see how a function and its inverse relate graphically. By reflecting the graph across the line $y = x$, students can clearly see how the domain and range are swapped, reinforcing the concept of inverse functions.

# ACTIVITY – 6

**Mr. RAMESH KUMAR**
**PGT(MATHS)**
**KENDRIYA VIDYALAYA**
**IISC**
**BENGALURU REGION**

Name of the Activity: **PRINCIPAL VALUE OF THE FUNCTION $sin^{-1} x$ USING A UNIT CIRCLE**

**OBJECTIVE**:

- To understand the graph of trigonometric and inverse trigonometric functions
- To explore the principal value of the function $sin^{-1} x$

**PRE-REQUISITE KNOWLEDGE:**

Basic knowledge of trigonometric ratios and properties of inverse trigonometric functions

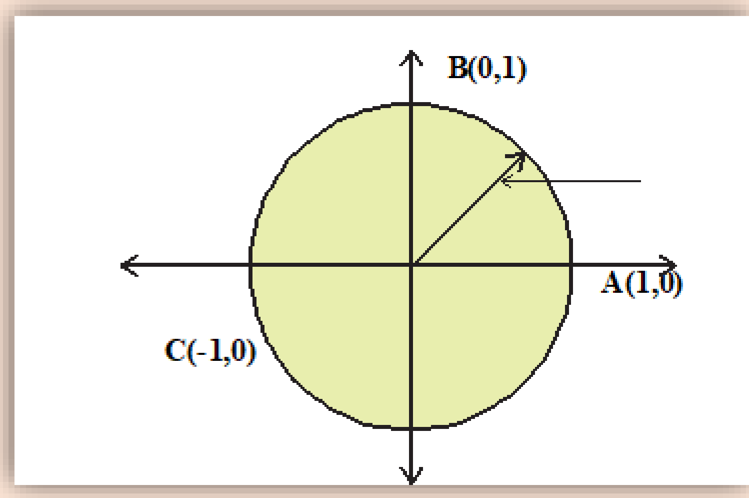
**MATERIALS REQUIRED:**

- Cardboard,
- White Chart Paper
- Nails
- Ruler
- Adhesive
- Steel Wires
- Needle.

**METHOD OF CONSTRUCTION:**

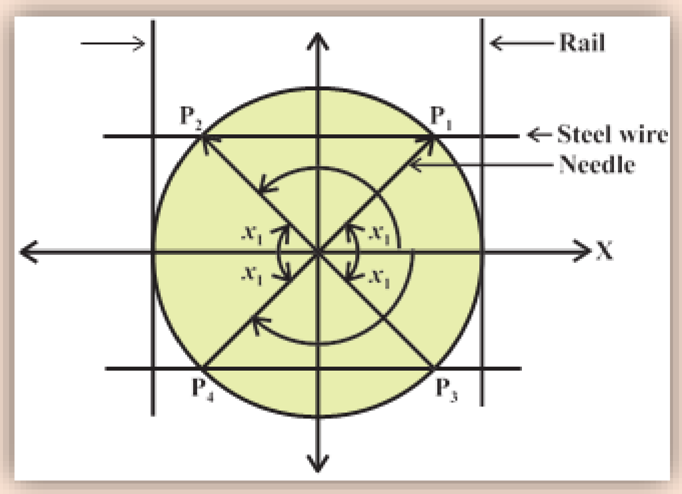
- Take a cardboard of a convenient size and paste a white chart paper on it
- Draw a unit circle with centre O on it.
- Through the centre of the circle, draw two perpendicular lines $X'OX$ and $YOY'$ representing x-axis and y-axis.

- Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis.
- Fix two rails on opposite sides of the cardboard which are parallel to y-axis. Fix one steel wire between the rails such that the wire can be moved parallel to x-axis.
- Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle.




**DEMONSTRATION:**

- Keep the needles at an arbitrary angle, say x, with the positive direction of x-axis. Measure of angle in angle in radian is equal to the length of intercepted arc of the unit circle.
- Slide the steel wire between the rails, parallel to x-axis such that the wire meets with free end of the needles (say P1)
- Denote the y-coordinates of the point p1 as y1, where y1 is the perpendicular distance of steels wire from the x- axis of the unit circle giving y1=sin x1.
- Rotate the needles further anticlockwise and keep it at the angle $\pi$-x1, Find the value of y-coordinates of intersecting point p2 with the help of sliding steel wire. Value of y-coordinates for the points p1 and p2 are same for the different value of angles, yc =sinx1 and y1 =sin($\pi$-x1).This demonstration that sine function is not one-to- one for angle considered in first and second quadrants.




**OBSERVATION:**

- Sine function is non-negative in ………and………. quadrants.
- For the quadrants 3rd and 4th, Sine function is…….
- $\theta$ =arc sin y ⇒y=………. $\theta$ where-$\frac{\pi}{2} \leq \theta \leq$…….
- The other domain of sine function on which it is one- one and onto provides………..for arc sine function.
- The principal value range of $sin^{-1}$ x is ……….

## VIVA QUESTIONS AND ANSWERS

Question1.What is the principal value of $sin^{-1}$ x?

Answer: -$\frac{\pi}{2} \leq \theta \leq.\frac{\pi}{2}$

Question 2. What is the domain of $sin^{-1}$ x?

Answer: [-1,1]

Question 3. What is the value of $sin^{-1}$ x+$cos^{-1}$ x?

Answer: $\frac{\pi}{2}$

Question 4. What is the value of $sin^{-1}$ $(\frac{1}{2})$ ?

Answer: $\frac{\pi}{6}$

Question 5. What is the value of domain $cos^{-1}$ x?

Answer: [-1,1]

Question 6. What is the domain of $sin^{-1}$ 4x?

Answer: [-$\frac{1}{4}$, $\frac{1}{4}$,]

Question 7. What is the value of $sin^{-1}$ ($sin\frac{4\pi}{5}$) ?

Answer: $\frac{\pi}{5}$

Question 8. What is the value of $tan^{-1}$ x+$cot^{-1}$ x?

Answer: $\frac{\pi}{2}$

Question 9. What is the value of sin($sin^{-1}$ x) if -1≤x≤1.

Answer: $x$

Question 10. What is the principal value of $cos^{-1}$ x?

Answer: [0, $\pi$]

# ACTIVITY – 7

**Mr. L. SURYA CHANDRA**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA**
**AFS BEGUMPET**
**HYDERABAD REGION**

**NAME OF THE ACTIVITY: Graph of $a^x$ and $log_a x$**

**OBJECTIVE:**

- To sketch the graphs of $a^x$ and $log_a x, a > 0, a \neq 1$, and to examine that they are mirror images of each other.
- To explore and understand the relationship between these two functions, their properties, and their graphical behaviors
- To Understand the Inverse Relationship and recognize their significance in various fields.

**PRE-REQUISITE KNOWLEDGE:**

- Definition of $a^x$ and $log_a x$ If $log_a x = y$ then $x=a^y$
- Concept of coordinate axes and plotting the points on it.

**MATERIAL REQUIRED:**

- Sketch Pens
- Geometrical Instruments
- Graph Paper
- Pencil
- A plain mirror

**PROCEDURE:**

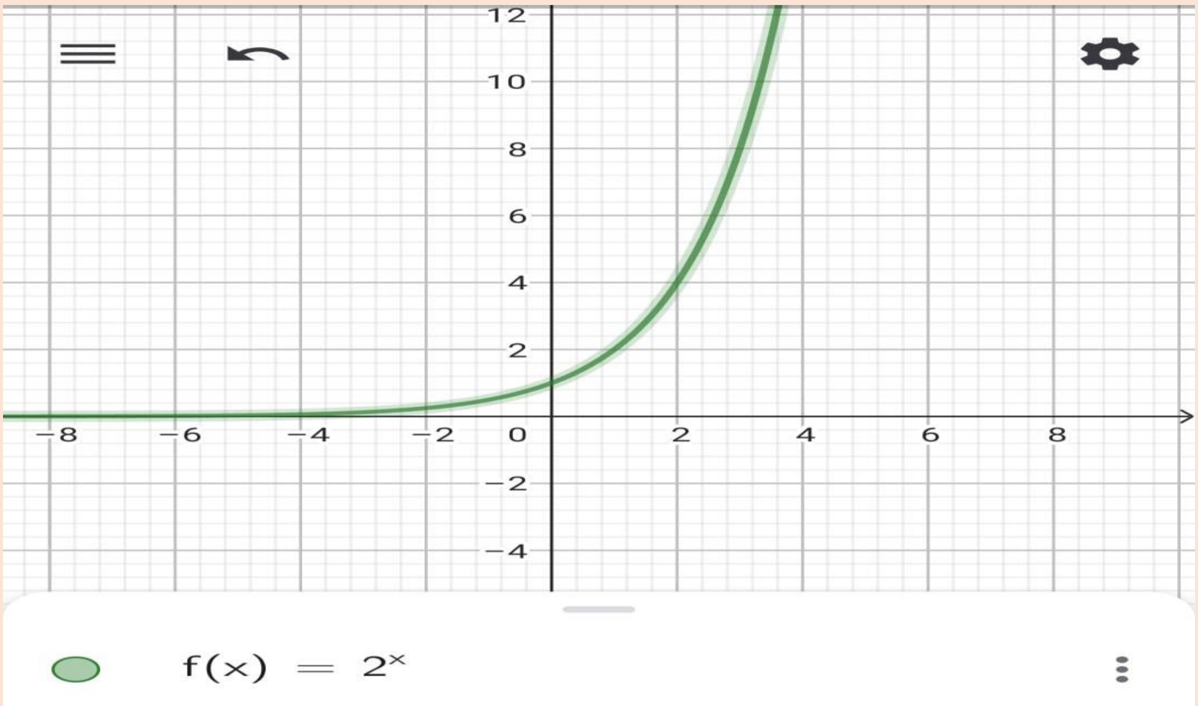
- Take a drawing board and paste a graph paper of convenient size on it with adhesive.

- Draw two perpendicular lines $XOX'$ and $YOY'$ depicting coordinate axis.
- Mark graduations on the two axes as shown in figure
- Find some ordered pairs satisfying $y = a^x$ and $y = log_a x$. Plot these points corresponding to the ordered pairs and join them by free hand. Fix thin wires along these curves using drawing pins
- Draw the graph of $y = x$, and fix a wire along the graph, using drawing pins.

1. For $y = a^x$, take $a = 2$ (Say), and find ordered pairs satisfying it as

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x | 0 | 1 | -1 | 2 | -2 | 3 | -3 | ½ | -½ | 4 |
| $2^x$ | 1 | 2 | 0.5 | 4 | ¼ | 8 | ⅛ | 1.4 | 0.7 | 16 |

And plot these points on the graph paper and fix a drawing pin at each point.

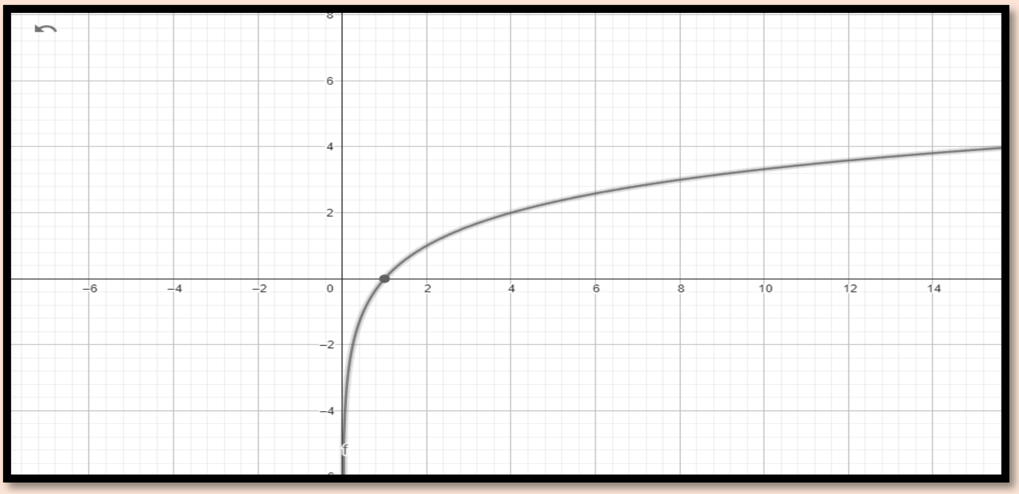



2. Join the bases of drawing pins with a thin wire. This will represent graph of $2^x$
3. Now $log_2 x = y$ gives $x = 2^y$.

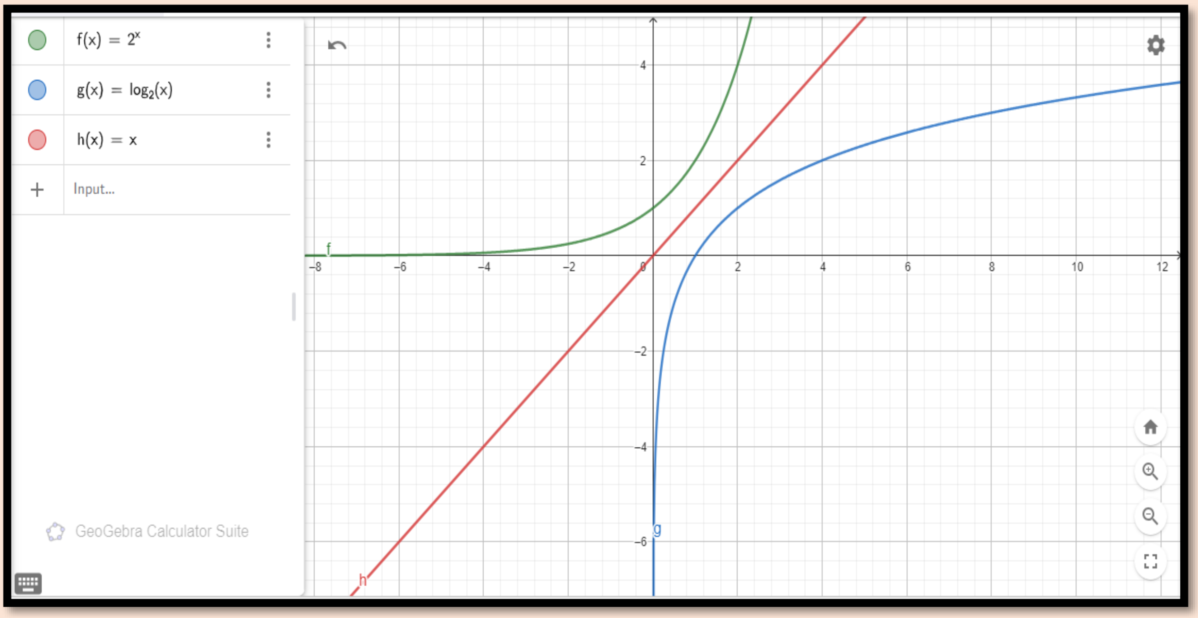
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x = 2^y$ | 1 | 2 | ½ | 4 | ¼ | 8 | ⅛ |
| $y = log_2 x$ | 0 | 1 | -1 | 2 | -2 | 3 | -3 |

Plot these points on the graph paper and fix a drawing pin at each plotted point.

Join the bases of the drawing pins with a thin wire. This will represent the graph of $log_2 x$.



4. Draw the graph of line $y = x$ on the graph paper.
5. Now, place a mirror along the wire representing $y = x$. It can be seen that the two graphs of the given functions are mirror images of each other in the line $y = x$.



**OBSERVATION:**

- ✓ Image of ordered pair (2, 4) on the graph of $y = 2^x$ in $y = x$ is (4, 2) It lies on the graph of $y = log_2 x$ .
- ✓ Image of the point (8, 3) on the graph $log_2 x$ , in y = x is (3, 8) which lies on the graph of $y = 2^x$. Repeat this process for some more points lying on the two graphs.

**APPLICATION:**

Application of logarithmic function is to find the compound interest, exponential growth, and to find the PH level of substance, to know the magnitude of an Earthquake.

**CONCLUSION:**

We have sketched the graphs of $a^x$ and $log_a x, a > 0, a \neq 1$ and examined their mirror images.

**LEARNING OUTCOMES:**

- To understand the definition of logarithmic function and exponential function.
- To observe the domain and range of logarithmic function and exponential function.
- To draw the graph of logarithmic function and exponential function.
- To observe that the graphs of $a^x$ and $log_a x$ , $a > 0$, $a \neq 1$ are mirror images of each other.

**SOURCES OF ERROR**:

- Committing mistakes while plotting the points

**PRECAUTIONS:**

- Choose proper scale on both x and y- axis.
- Mark and join the points carefully.

**VIVA QUESTIONS AND ANSWERS:**

1. If $log_b x = y$ then what is the value of $x$?

Answer: $x = b^x$

2. what are common logarithms?

Answer: Logarithms with base 10 are called common logarithms.

3. what are Natural logarithms?

Answer: Logarithms with base ‘e’ are called Natural logarithms.

4. what is the domain of log function?

Answer: Set of all positive Real Numbers.

5. what is the range of log function?

Answer: Set of all Real Numbers.

6. What is exponential function?

Answer: The exponential function with positive base b > 1 is the function $y = f(x) = b^x$ .

7. What is natural exponential function?

Answer: Exponential Function with base ‘e’ is called natural exponential function

8. What is common exponential function?

Answer: Exponential Function with base 10 is called common exponential function.

9.What is the domain of exponential function?

Answer: Set of all Real Numbers.

10. What is the range of exponential function?

Answer: Set of all positive Real Numbers

11. what is the shape of graph of $y = x$?

Answer: Straight Line.

# ACTIVITY – 8

**Ms. SABITHA A K**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA**
**PAYYANUR**
**ERNAKULAM REGION**

**NAME OF THE ACTIVITY: TO FIND ANALYTICALLY THE LIMIT OF A FUNCTION F (X) AT $x = c$ AND ALSO TO CHECK THE CONTINUITY OF THE FUNCTION AT THAT POINT.**

**OBJECTIVE:**

- To define and compute the limit of a function f(x) as x approaches a given point c.
- To compute the left-hand limit and the right-hand limit
- To find analytically the limit of a function $f(x)$ at $x = c$ and also to check the continuity of the function at that point

**PRE-REQUISITE KNOWLEDGE:**

Limit of a function, value of f(x) at certain points

**MATERIAL REQUIRED:**

- Paper
- Pencil
- Calculator

**METHOD OF CONSTRUCTION:**

➢ Consider the function given by $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ 6, & x = 2 \end{cases}$

- Take some points on the left and some points on the right of $c(= 2)$ which are very near to c. i.e $x = 2.1$ ,2.01,2.001 … … (on the rightof $c = (2)$ *and* $x = 1.9,1.99,1.999 \dots \dots$ (on the left of $c = (2)$
- Find the corresponding values of $f(x)$ for each of the values considered in step 2.
- Record the values of points on the left and right side of $c$ as $x$ and the corresponding values of $f\ (x)$ in a form of a table.

**DEMONSTRATION:**

1.The values of $x$ and $f(x)$ are recorded as follows.

For points on the left of $c(= 2)$

$i)\ x = 1.9 \quad f(x) = 3.9$

$ii. x = 1.99$ , $\quad f(x) =$ ____________

$iii. x = 1.999$ , $\quad f(x) =$ _____________

$iv. x = 1.9999$ , $\quad f(x) =$ ____________

$v. x = 1.99999$ , $\quad f(x) =$ _____________

$vi. x = 1.999999$ , $f(x) =$ ______________

For points on the right of $c(= 2)$

$i)\ x = 2.1 \quad f(x) = 4.1$

$ii). x = 2.01 \quad f(x) =$ ______________

$iii). x = 2.001$ , $\quad f(x) =$ ______________

$iv). x = 2.0001$ , $\quad f(x) =$ _______________

$v). x = 2.00001$ , $\quad f(x) =$ ________________

$vi).\, x = 2.000001$ , $f(x) =$ ________________

**OBSERVATION:**

- ✓ The value of $f\ (x)$ is approaching to ________, as $x \to 2$ from the left.
- ✓ The value of $f\ (x)$ is approaching to ________, as $x \to 2$ from the right.
- ✓ So, $\lim_{x \to 2^-} f(x) =$________ and ( $\lim_{x \to 2^+} f(x) =$ ________.
- ✓ Therefore, $\lim_{x \to 2} f(x) =$________ and $f(2) =$________.
- ✓ Is $\lim_{x \to 2} f(x) = f(2)$ ? Yes /No
- ✓ Since $\lim_{x \to c} f(x) \neq f(c)$ , so, the function is ________ at $x = 2$ (continuous/ not continuous).

**APPLICATION:**

This activity is useful in understanding the concept of limit and continuity of a function at a point.

**CONCLUSION:**

A real function $f$ on a subset of the real numbers and $c$ be a point in the domain of $f$. Then $f$ is continuous at $c$ if $\lim_{x \to c} f(x) = f(c)$

**LEARNING OUTCOMES:**

- Define a function and a continuous function.
- Understand the importance of limits in relation to continuity.
- Define and understand continuity at a point and over intervals.
- Accurately calculate limits and assess continuity.
- Analyze, and interpret functions graphically and in real-world contexts.
- Use appropriate techniques to solve continuity problems and verify results.

**SOURCES OF ERROR**:

1. Calculation Errors:

Mistakes in algebraic manipulation or limit calculations,

especially when dealing with complex functions.

2.Boundary Issues:

Neglecting to check continuity at the boundaries of intervals, particularly in piecewise functions

3.Failing to recognize where the function is not defined.

**PRECAUTIONS:**

Double-check limit evaluations and use proper limit properties. Thoroughly analyze each piece of a piecewise function and its continuity at transition points. Simplify expressions accurately and ensure no steps are skipped. Ensure the function is defined at the point where continuity is being checked.

## VIVA QUESTIONS AND ANSWERS

Question 1. Let $f$ be a real function on a subset of the real numbers and $c$ be a point in the domain of $f$.What will be the condition for $f$ to be continuous at $x = c$?

Answer: $\lim_{x \to c^-} f(x) = (\lim_{x \to c^+} f(x) = f(c)$

Question 2. What is the value of $\lim_{x \to 0} \frac{sinx}{x}$ ?

Answer:1

Question 3. If $f(x) = \begin{cases} \frac{x}{|x|}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$ then what is the value of $\lim_{x \to 0^-} f(x)$? Is f continuous at $x = 0$?

Answer: $\lim_{x \to 0^-} f(x) = -1$

F is not continuous at $x = 0$ since $\lim_{x \to 0+} f(x) = 1$ .Limit of f at $x = 0$ does not exist.

Question 4. For the value of k $f(x) = \begin{cases} kx^2, if x \leq 2 \\ 3\ if\ x > 2 \end{cases}$ is continuous at $x = 2$

Answer:$k = \frac{3}{4}$

Question 5. What can you say about the continuity of greatest integer function?

Answer: The greatest integer function is continuous at all points except at integer points.

Question 6. Is the function defined by $f\ (x)\ =\ tan\ x$ is a continuous function? Justify your answer.

Answer: The function $f\ (x) = \tan x = \frac{sinx}{\cos x}$ . This is defined for all real numbers such that $cos\ x\ \neq\ 0, i.e., x\ \neq\ (2n\ + 1)\ \pi/2$ . We know that both sine and cosine functions are continuous. Thus $tan\ x$ being a quotient of two continuous functions is continuous wherever it is defined.

Question 7. How do you determine if a function is continuous using its graph?

Answer: On a graph, a function is continuous if you can draw it without lifting your pen, meaning there are no breaks, jumps, or holes.

Question 8. Give an example of a function that is continuous everywhere but not differentiable everywhere.

Answer: The function $f(x) =| x |$ is continuous everywhere but not differentiable at $x = 0$

Question 9. What is the difference between continuity at a point and continuity on an interval?

Answer: Continuity at a point refers to the function being continuous at a specific point $x = c$. Continuity on an interval means the function is continuous at every point within that interval.

Question 10. What is the relationship between continuity and differentiability?

Answer: Differentiability implies continuity; if a function is differentiable at a point, it must be continuous at that point. However, continuity alone does not guarantee differentiability.

Question 11. What is a piecewise function, and how can you determine its continuity?

Answer: A piecewise function is defined by different expressions in different intervals. To determine its continuity, check that each piece is continuous within its interval and ensure that the function values match at the boundaries between intervals.

Question 12. Find the limit of the function $f(x) = \frac{x^3-8}{x-2}$ as $x$ approaches 2, and determine if the function is continuous at $x = 2$

Answer: $\lim_{x\to 2} f(x) = 12$

For $f(x)$ to be continuous at $x = 2$ it must be defined at $x = 2$ and equal to the limit as $x$ approaches 2. Since $f(x)$ is not defined $at\ x = 2$, it cannot be continuous at that point.

# ACTIVITY - 9

**Mr. KAILASH CHANDER C**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA**
**KADAPA (ANDHRA PRADESH)**
**HYDERABAD REGION**

**NAME OF THE ACTIVITY:** A FUNCTION IS CONTINUOUS AT GIVEN POINT $\Delta y = |f(x_0 + \Delta x) - f(x_0)|$ IS ARBITRARILY SMALL PROVIDED. ΔX IS SUFFICIENTLY SMALL

**OBJECTIVE:**

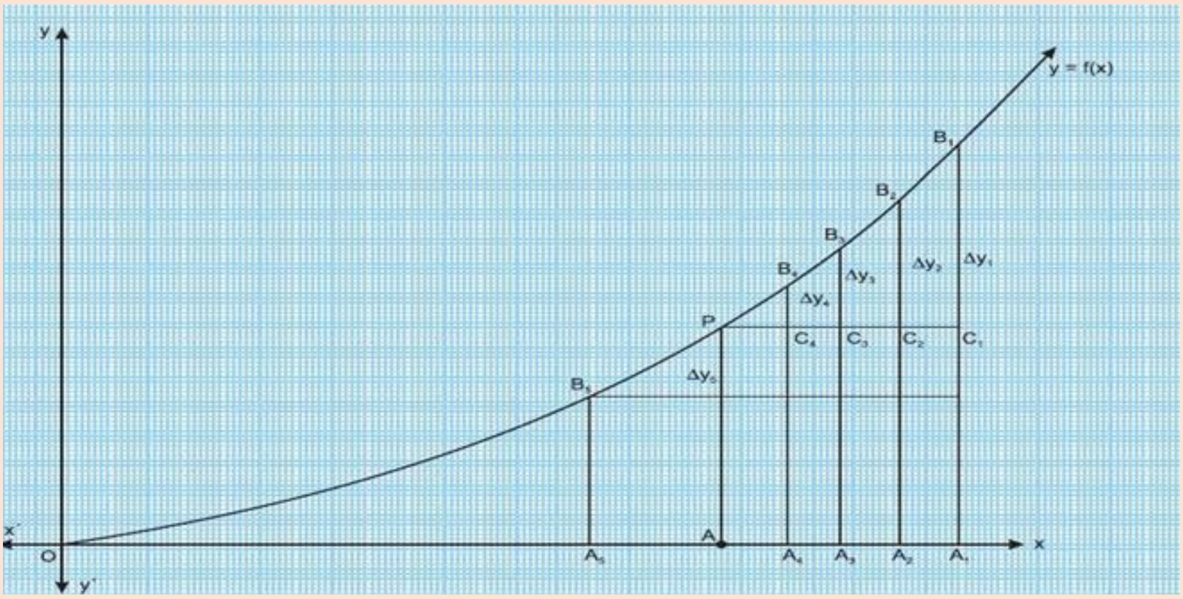
- To verify that for a function f to be continuous at given point $x_0$, $\Delta y = |f((x_0 +, \Delta x) - f(x_0))|$ is arbitrarily small provided. Δx is sufficiently small.
- To explore and understand the concept of continuity at a point in a function and how small changes in $x$ lead to small changes in the function's output.

**MATERIAL REQUIRED:**

- Pencil
- Scale
- Calculator
- Adhesive
- Hardboard
- White Sheets

**METHOD OF CONSTRUCTION:**

- Paste a white sheet on the hardboard.
- Draw the curve of the given continuous function as represented in the Figure.




- Take any point A $(x_0, 0)$ on the positive side of x-axis and corresponding to this point, mark the point $P(x_0, y_0)$ on the curve.

**DEMONSTRATION:**

- Take one more point $A_1$ $(x_0 + \Delta x_1, 0)$ to the right of A, where $\Delta x_1$ is an increment in x.
- Draw the perpendicular from $A_1$ to meet the curve at $B_1$. Let the coordinates of be $B_1(x_0 + \Delta x_1, y_0 + \Delta y_1)$
- Draw a perpendicular from the point $P(x_0, y_0)$ to meet $A_1B_1$ at $C_1$
- Now measure $AA_1 = \Delta x_1$ (say) and record it and also measure $B_1C_1 = \Delta y_1$ and record it.
- Reduce the increment in x to $\Delta x_2$ ( i.e $\Delta x_2 < \Delta x_1$ ) to get another point $A_2(x_0 + \Delta x_2, 0)$. Get the corresponding point $B_2$ on the curve
- Let the perpendicular $PC_1$ intersects $A_2B_2$ at $C_2$
- Again, measure $AA_2 = \Delta x_2$ and record it. Measure $B_2C_2 = = \Delta y_2$ and record it.
- Repeat the above steps for some more points so that $\Delta x$ becomes smaller and smaller.

**OBSERVATION:**

| S.No. | Value of increment in $x_0$ | Corresponding increment in y |
|---|---|---|
| 1. | $\|\Delta x_1\|$ =0.1 | $\|\Delta y_1\|$ =0.2100 |
| 2. | $\|\Delta x_2\|$ =0.09 | $\|\Delta y_2\|$ =0.1881 |
| 3. | $\|\Delta x_3\|$ =0.08 | $\|\Delta y_3\|$ =0.1664 |
| 4. | $\|\Delta x_4\|$ =0.07 | $\|\Delta y_4\|$ =0.1449 |
| 5. | $\|\Delta x_5\|$ =0.06 | $\|\Delta y_5\|$ =0.1236 |
| 6. | $\|\Delta x_6\|$ =0.05 | $\|\Delta y_6\|$ =0.1025 |
| 7. | $\|\Delta x_7\|$ =0.04 | $\|\Delta y_7\|$ =0.0816 |

1. So, Δy becomes to zero when Δx becomes smaller.

2. Thus $\lim_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

**APPLICATION:**

This activity is helpful in explaining the concept of derivative (left hand or right hand) at any point on the curve corresponding to a function.

**VIVA QUESTIONS AND ANSWERS**

Question 1: Explain the continuity of the function f = |x| at x = 0.

**Solution:** From the given function, we define that, f(x) = {-x, if x<0 and x, if x≥0

It is clearly mentioned that the function is defined at 0 and f(0) = 0. Then the left-hand limit of f at 0 is Limx→0- f(x) = limx→0- (-x) = 0

Similarly, for the right-hand side, Limx→0+ f(x) = limx→0+ (x) = 0

Therefore, for the both left hand and the right-hand limit, the value of the function coincide at the point x = 0.

Therefore, the function f is continuous at the point $x = 0$.

Question 2: Explain the continuity of the function $f(x) = sin\ x\ .cos\ x$

**Solution:** We know that $sin\ x\ and\ cos\ x$ are continuous functions. It is known that the product of two continuous functions is also a continuous function.

Hence, the function $f(x) = sin\ x\ cos\ x$ is a continuous function.

Question 3: The function $f(x) = [x], where\ [x]$ denotes the greatest integer function is continuous at:

**Solution:** The greatest integer function is discontinuous at all integer so f(x) is discontinuous at real number other than integers.

Question 4: Let f(x) = |sin x|. Then

**Solution:** f is everywhere continuous but not differentiable at x = nπ, n ∈ Z.

Question 5: Can a function be continuous but not differentiable?

**Solution:** Yes, a function can be continuous but not differentiable. An example is the absolute value function f(x)=∣x∣, which is continuous everywhere but not differentiable at x=0 due to the sharp corner at that point.

Question 6: What is the geometric interpretation of a derivative?

**Solution:** The derivative of a function at a point represents the slope of the tangent line to the graph of the function at that point, indicating the rate of change of the function.

Question 7: How is differentiability different from continuity?

**Solution:** Differentiability requires that a function has a defined derivative at a point, meaning it has a well-defined tangent at that point. A differentiable function is always continuous, but a continuous function is not necessarily differentiable.

Question 8: What does it mean for a function to be continuous?

**Solution:** A function is continuous at a point a if f(a) is defined, $\lim_{x \to a} f(x)$ exists, and $\lim_{x \to a} f(x) = f(a)$. This means there are no breaks, jumps, or holes at a.

Question 9: What is a limit in calculus?

**Solution:** A limit is the value that a function f(x) approaches as x approaches a specific point aaa. It describes the behavior of the function near that point.

Question 10: The number of points at which the function f (x) $= \frac{1}{x - [x]}$ is not continuous is

(A) 1 (B) 2 (C) 3 (D) infinite Ans: (D)

# ACTIVITY - 10

**Mr. J. JAGAN MOHAN**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA**
**AFS, HAKIMPET**
**HYDERABAD REGION**

**NAME OF THE ACTIVITY**: INCREASING AND DECREASING FUNCTIONS

**OBJECTIVE:**

- To understanding the concept of increasing and decreasing functions
- TO understand the behavior of functions by analyzing their growth or decline over an interval.
- TO Explore Graphical Interpretation.

**PRE-REQUISITE KNOWLEDGE**:

Knowledge of a function, Derivative of a function, Definition of increasing and decreasing functions, Graph of a function, Tangent to a curve, Angle made by a line with the positive x-axis.

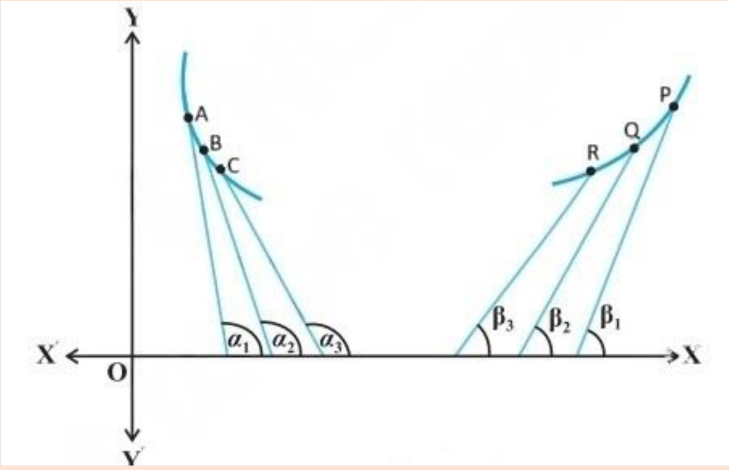
**MATERIAL REQUIRED:**

- Pieces of wire of different lengths
- piece of plywood of suitable size
- white paper
- adhesive
- geometry box
- trigonometric tables.

**METHOD OF CONSTRUCTION:**

- Take a piece of plywood of a convenient size and paste a white paper on it.

- Take two pieces of wires of length say 20 cm each and fix them on the white paper to represent $x - axis\ and\ y - axis$.
- Take two more pieces of wire each of suitable length and bend them in the shape of curves representing two functions and fix them on the paper as shown in the Figure.
- Take two straight wires each of suitable length for the purpose of showing tangents to the curves at different points on them.




**DEMONSTRATION:**

- ✓ Take one straight wire and place it on the curve (on the left) such that it is Mathematics 133 tangent to the curve at the point say A and making an angle α1 with the positive direction of x-axis.
- ✓ α1 is an obtuse angle, so tanα1 is negative, i.e., the slope of the tangent at A (derivative of the function at A) is negative.
- ✓ Take another two points say B and C on the same curve, and make tangents, using the same wire, at B and C making angles α2 and α3, respectively with the positive direction of x-axis.
- ✓ Here again α2 and α3 are obtuse angles and therefore slopes of the tangents tan α2 and tan α3 are both negative, i.e., derivatives of the function at B and C are negative.

- ✓ The function given by the curve (on the left) is a decreasing function.
- ✓ On the curve (on the right), take three-point P, Q, R, and using the other straight wires, form tangents at each of these points making angles β1, β2, $\beta_3$ respectively with the positive direction of x-axis, as shown in the figure. β1, β2, β3 are all acute angles. So, the derivatives of the function at these points are positive. Thus, the function given by this curve (on the right) is an increasing function.

**OBSERVATION:**

1. α1 = _______ , > 90°

   α2 = _______ > _______,

   α3 = _______> _______,

   tan α1 = _______, (negative)

   tan α2 = _______, ( _______ ),

   tan α3 = _______, ( _______).

   Thus the function is _______.

2. β1 = _______< 90°,

   β2 = _______, < _______,

   β3 = _______ , < _______

   tan β1 = _______ , (positive),

   tan β2 = _______, ( _______ ),

   tan β3 = _______( _______ ).

   Thus, the function is _______.

**<u>APPLICATION</u>**:

This activity may be useful in explaining the concepts of decreasing and increasing functions and also in understanding the concept of first derivative test for maxima and minima.

**CONCLUSION:** Clarity in the concept of increasing and decreasing functions is achieved.

**LEARNING OUTCOMES:**

- Draws the tangents to the curve at different points.
- Finds the angle made by the lines and finds the slope.
- Connects the concept of derivatives with slope of tangent.
- Concludes that if for increasing function the derivative is positive and for decreasing function it is negative.

**SOURCES OF ERROR AND PRECAUTIONS:**

Care to be taken while measuring the angle made by the tangents with x-axis

## VIVA QUESTIONS AND ANSWERS

1. Define an increasing function.

Answer: A function f is said to be increasing on a open interval I, If $a < b$ then

$$f(a) \leq f(b)\ for\ \ all\ a, b\ in\ I$$

2. How can you determine if a function is increasing or decreasing on an interval using its derivative?

Answer: In an interval I f is said to be increasing if $f^{l}(x) > 0, \forall x\ in\ I$

In an interval I f is said to be decreasing if $f^{l}(x) < 0, \forall x\ in\ I$

3. What does it mean if the derivative of a function is zero at a point?

Answer: f is neither increasing nor decreasing at that point.

4. Can a function be increasing in one interval and decreasing in another?

Answer: yes, for example $sinx$ function is increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in the interval $\left(\frac{\pi}{2}, \pi\right)$

5. Explain how the first derivative test is used to identify local maxima.

Answer: A point c is said to be point of local maxima if the derivative of f(x) changes its sign from positive to negative as we move from left to right of c.

6. Can we say that modulus function is increasing or decreasing without using the derivative?

Answer: By observing the graph we could say the modulus function is increasing in $1^{st}$ quadrant and decreasing in the $2^{nd}$ quadrant.

7. If a function's $2^{nd}$ order derivative is positive at a point x=3 then what can you say about the point?

Answer: The point 3 is called as point of local minimum.

8. If a function's $2^{nd}$ order derivative is negative at a point x=3 then what can you say about the point?

Answer: The point 3 is called as point of local maxima.

9. Define a strictly decreasing function?

Answer: A function f is said to be strictly decreasing if $a < b \text{ then } f(a) > f(b)$

10. Can a function have a constant derivative and still be considered increasing or decreasing?

Answer: If the derivative of a function is a positive constant, the function is strictly increasing because the slope of the tangent to the curve is positive at every point. Conversely, if the derivative is a negative constant, the function is strictly decreasing.

# ACTIVITY - 11

**Mr. P P RAMA MURTY**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA**
**ELURU**
**HYDERABAD REGION**

**NAME OF THE ACTIVITY:** LOCAL MAXIMA, LOCAL MINIMA AND POINT OF INFLECTION

**OBJECTIVE:**

- To understand the concept of critical points, where the derivative of the function is zero or undefined.
- To identify critical points and recognize how they relate to the local behavior of the function.
- To determine whether a function has a local maximum or minimum at a critical point using the First Derivative Test
- To understand the concept of Local Maxima, Local Minima and point of inflection

**PRE-REQUISITE KNOWLEDGE:**

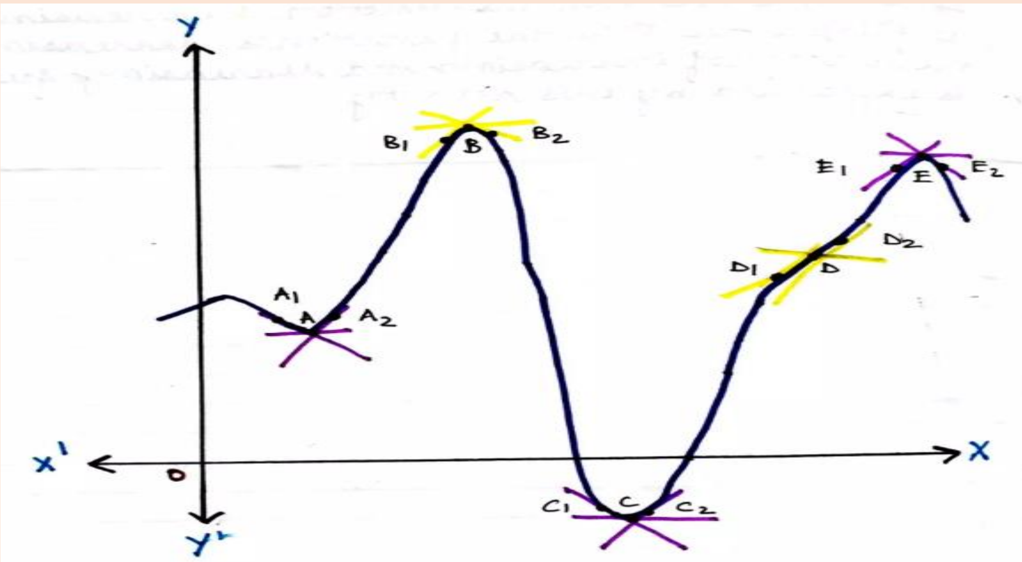
Definition of a Function, Graphing Functions, Concept of a Derivative and familiarity with the concept of testing critical points to determine local maxima or minima.

**MATERIALS REQUIRED:**

- A piece of plywood
- Wires
- Adhesive
- White paper.

**METHOD OF CONSTRUCTION:**

- Take a drawing board of convenient size and paste a white paper on it.
- Take two pieces of wires of convenient lengths and fix them on the drawing board which are bisected at point O. One is horizontal ($XOX'$) and is called x-axis and the other one vertical ($YOY'$) is called y-axis.
- Take another wire of suitable length and bend it in the shape of curve. Fix this curved wire on the white paper pasted on the drawing sheet as shown in the figure.
- Take wires of suitable lengths and fix them at the points A, B, C, D and E. These represent the tangents of the curves at points A, B, C, E which are parallel to the x-axis.
- The slopes of tangents at these points are zero i.e., the value of first order derivative of a function is zero at all these points and the tangent at point D intersects the curve.
- Now, take two points, one to the immediate left of A and one to the immediate right of A. Name these points A1 and A2.
- Repeat the above process for points B, C, D and E. Let the respective points be (B1, B2), (C1, C2), (D1, D2) and (E1, E2)
- Now, draw the tangents at these points (A1, A2,…)



**DEMONSTRATION:**

- In the figure, wires at the points A, B, C and E represent tangents to the curve and are parallel to the axis. The slopes of tangents at these points are zero, i.e., the value of the first derivative at these points is zero. The tangent at E intersects the curve.
- Sign of the slope of the tangent (first derivative) at point immediate left of A(i. e A1) is negative and Sign of the slope of the tangent (first derivative) at point immediate right of A( i. e A2) is positive.
- The same happens at point C.
- At the points A and C, sign of the first derivative changes from negative to positive. So, they are the points of local minima.
- At the point B and E, sign of the first derivative changes from positive to negative. So, they are the points of local maxima.
- At the point D, sign of first derivative does not change. So, it is a point of inflection.

**OBSERVATION:**

- ✓ Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is negative.
- ✓ Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is positive.
- ✓ Sign of the first derivative at a point on the curve to immediate left of C is negative.
- ✓ Sign of the first derivative at a point on the curve to immediate right of C is positive.
- ✓ Sign of the first derivative at a point on the curve to immediate left of B is positive.

- ✓ Sign of the first derivative at a point on the curve to immediate right of B is negative.
- ✓ Sign of the first derivative at a point on the curve to immediate left of E is positive.
- ✓ Sign of the first derivative at a point on the curve to immediate right of E is negative.
- ✓ Sign of the first derivative at a point immediate left of D and immediate right of D does not change

**APPLICATION:**

- This activity may help in explaining the concepts of points of local maxima, local minima and inflection.
- The concepts of maxima/minima are useful in problems of daily life such as making of maximum revenue at minimum cost.

**Example:**

A telephone company in a town has 500 subscribers and collects fixed charges of Rs 300 per subscriber. The company proposes to increase the annual subscription and it is believed that for every increase of Re1, one subscriber will discontinue the service. Find what increase in subscription will bring the maximum revenue?

**Solution:**

Let the total revenue be ‘R’

Let the proposed increase in annual subscription be Rs ‘x’

So, no. Of subscribers believed to be discontinued = ‘x’

Now, no. Of subscribers using the service = 500 - x

Annual subscription of each subscriber = Rs (300 + x)

Total revenue, R = (500 - x) (300 + x)

$= 150000 + 200x - x^2$

$dR/dx = 200 - 2x$ and $d^2R/dx^2 = -2$

By equating, $dR/dx$ to ‘0’, we get $x=100$ and $d^2R/dx^2 = -2 < 0$ for all ‘x’

Hence, R is maximum when ‘x’ = 100

Thus, total revenue will be maximum if annual subscription is increased by Rs 100

**CONCLUSION:** From the above discussion we conclude that point A and B are the points of local minima and the points C and D are the points of local maxima and the point P is neither minima nor maxima and it is a point of inflection.

**LEARNING OUTCOMES:**

1. Explain the difference between local maxima and local minima.

2. Describe how slope of the tangent changes near different points.

**SOURCES OF ERROR AND PRECAUTIONS:**

Care to be taken while finding sign of the slope of tangent at different points.

## VIVA QUESTIONS AND ANSWERS

Question1.Define the point of local maxima of a function y= f(x)

Ans: If the value of the derivative of a function at a point A changes from positive to negative when we move from left to right of point A, then the point A is called the point of local maxima

Question 2. Define the point of local minima of a function y= f(x)

Ans: If the value of the derivative of a function at a point A changes from negative to positive when we move from left to right of point A, then the point A is called the point of local minima

Question 3. What do you mean by maximum value of a function y= f(x)?

Ans: For a continuous function f(x), the value of f(a) is said to be the maximum if f(a) is the greatest of all its values for ‘x’ lying in some neighborhood of ‘a’

Question 4. What do you mean by minimum value of a function y= f(x)?

Ans: For a continuous function f(x), the value of f(a) is said to be the minimum if f(a) is the smallest of all its values for 'x' lying in some neighborhood of 'a'

Question 5. If a point on the curve is neither a point of local maxima nor a local minima, then what is that point called?

Ans: point of inflection

Question 6. What do you mean by point of inflection on a curve?

Ans: If there is no change in the sign of slope of tangent at a point on the curve, then that is called a point of inflection.

Question 7. The sum of two non-zero numbers is 8. What is the minimum value of the sum of their reciprocals?

Ans: 1/2

Question 8. Find the point of local minima of the function f given by $f(x) = 3 + |x|$, $x \in R$.

Ans: x = 0 is a point of local minima of 'f'

Question 9. Find local maximum value of the function f given by

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

Ans: x = 0 is a point of local maxima and local maximum value of 'f' at x = 0 is f (0) = 12

Question 10. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

Ans: The sum of squares of numbers is minimum when the numbers are 15/2 and 15/2

# ACTIVITY - 12

**Ms. PRAJISHA**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA**
**KANNUR**
**ERNAKULAM REGION**

**NAME OF THE ACTIVITY:** **RATE OF CHANGE OF QUANTITIES**

**OBJECTIVE:**

- To understand the concept of **rate of change** as the ratio of the change in one quantity to the change in another.
- To find the time when the area of the rectangle of given dimensions becomes maximum, if the length is decreasing and the breadth is increasing at given rates.

**PRE-REQUISITE KNOWLEDGE:**

If $y = f(x)$, then $\frac{dy}{dx}$ = rate of change of y with respect to x

Also , $\frac{dy}{dx}$ at x = $x_0$ is the rate of change of y with respect to x at $x_0$.

Rate of change is usually defined by change of quantity with respect to time.

That is, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

**MATERIAL REQUIRED:**

- Colour papers
- Scissors
- Glue
- Geometry box

**METHOD OF CONSTRUCTION:**

Take a rectangle of a particular dimension and apply the change in dimension as length is decreasing at the rate of a cm/sec and breadth is increasing at the rate of b cm/sec.

Find at what time the area will be maximum by manually changing dimensions and finding corresponding area.

**DEMONSTRATION:**

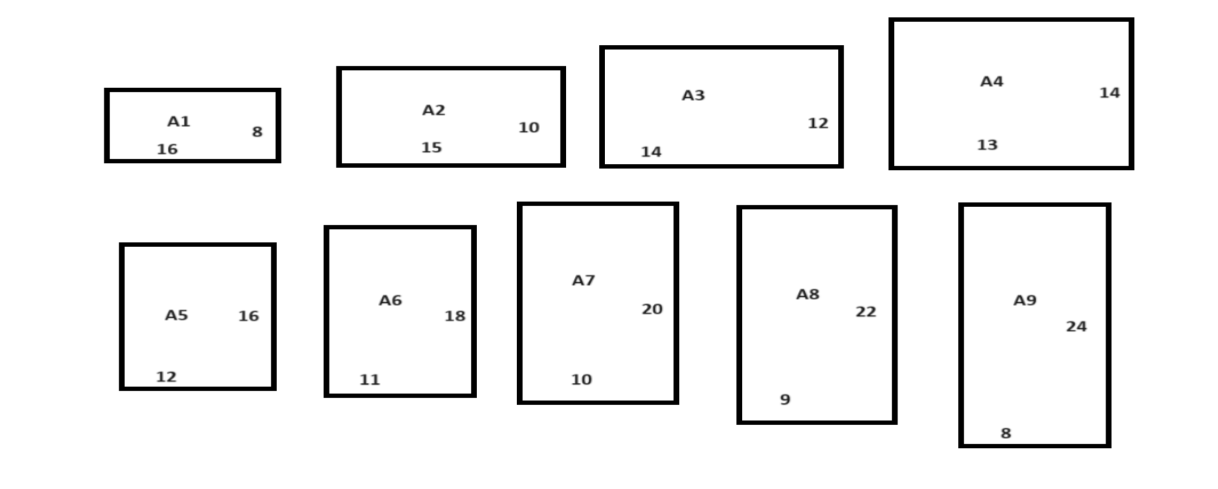
**Step 1:** Take a rectangle A1, say of dimension 16 cm × 8 cm.

**Step 2:** Suppose that the length of the rectangle is decreasing at the rate of 1 cm per second and the breadth is increasing at the rate of 2 cm per second.

**Step 3:** Cut out rectangles A2, A3, A4, A5, A6, A7, A8 and A9 of dimensions 15 cm × 10 cm , 14 cm × 12 cm, 13 cm × 14 cm, 12 cm × 16 cm, 11 cm × 18 cm, 10cm × 20 cm, 9 cm × 22 cm and 8 cm × 24 cm.

**Step 4:** Paste the rectangles in record book.

**Step 5:** Length of the rectangle is decreasing at the rate of 1 cm per second and the breadth is increasing at the rate of 2 cm per second, find the areas of each rectangle.




**OBSERVATION:**

Area of Rectangle A1 = 16 × 8 = 128 cm2

Area of Rectangle A2 = 15 × 10 = 150 cm2 (after 1 sec)

Area of Rectangle A3 = 14 × 12 = 168 cm2 (after 2 sec)

Area of Rectangle A4 = 13 × 14 =182 cm2 (after 3 sec)

Area of Rectangle A5 = 12 ×16 = 192 cm2 (after 4 sec)

Area of Rectangle A6 = 11 × 18 = 198 cm2 (after 5 sec)

Area of Rectangle A7 = 10 × 20 = 200 cm2 (after 6 sec)

Area of Rectangle A8 = 9 × 22 = 198 cm2 (after 7 sec)

Area of Rectangle A9 = 8 × 24 = 192 cm2 (after 8 sec)

Thus, the area of the rectangle is maximum after 6 secs.

Alternatively,

Length = a units and breadth = b units

Length after 1 sec = a – t units

Breadth after 1 sec = b + 2b units

Area of the rectangle = $(a - t)(b + 2t)$

$= ab + 2at - bt - 2t^2$

$\frac{dA}{dt} = 2a - b - 4t$

$\frac{dA}{dt} = 0 \Longrightarrow 2a - b - 4t = 0$

$t = \frac{2a-b}{4}$ (Critical point)

$\frac{d^2t}{dt^2} = -4 < 0$

Hence Area is maximum at $t = \frac{2a-b}{4}$

Now a= 16 cm and b = 8 cm

$t = \frac{2\times16-8}{4} = 6$ sec.

**APPLICATION:**

This activity can be used in explaining the concept of rate of change and optimization of a function.

**CONCLUSION:**

We have learned to find the time when the area of rectangle of given dimensions become maximum.

**LEARNING OUTCOMES:**

Finds extrema or points of extrema for any real-life relation modelled or a mathematical problem applying the tests of derivatives.

**SOURCES OF ERROR AND PRECAUTIONS**:

Care to be taken while cutting and pasting the diagrams

## VIVA QUESTIONS AND ANSWERS

1. Define the rate of change of quantity.

   Answer: If a quantity y changes with respect to a variable x, the rate of change of y with respect to x is given by the derivative $\frac{dy}{dx}$. This represents how much y changes for a small change in x.

2. What is the rate of change of distance with respect to time called?

   Answer: The rate of change of distance with respect to time is called speed, which is represented as $\frac{ds}{dt}$.

3. Find the rate of change of the area of a circle with respect to its radius r, at r = 6cm.

   Answer: Area = $\pi$r2

   $\frac{dA}{dr} = 2\pi r$

   r = 6 cm $\frac{dA}{dr} = 12\pi$

4. If the rate of increase of $x^3-5x^2+5x+8$ is twice the rate of increase of x, find the value of x.

   Answer: $y = x^3-5x^2+5x+8$

   $\frac{dy}{dx} = 3x^2-10x+5$

   $3x^2-10x+5 = 2\frac{dx}{dx}$

   Solving $x = \frac{1}{3}, 3$

5. The diameter of a circle is increasing at the rate of 1 cm/sec. when its radius is $\pi$ cm, find the rate of increase of its area.

   Answer: D=2r, $\frac{dD}{dt} = 2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{2}$

A= $\pi$r2 $\Longrightarrow \frac{dA}{dt}$ = $2\pi r \frac{dr}{dt}$ = π2 cm2/sec

6. Rate of change of velocity with respect to time is called as ……………….

   Answer: Acceleration

7. The rate of change of area of a square is 40 cm2/sec. What will be the rate of change of side if the side is 10 cm.

   Answer: $\frac{dA}{dt}$ = 40 cm2/sec = 2 a $\frac{da}{dt}$ $\Longrightarrow \frac{da}{dt}$ $at$ a = 10 cm is = 2 cm/sec

8. The length of the rectangle is changing at a rate of 4 cm/sec and the area is changing at the rate of 8 cm/sec. What will be the rate of change of width if the length is 4 cm and the width is 1 cm.

   Answer: $\frac{dA}{dt} = l\,\frac{db}{dt} + b\,\frac{db}{dt}$ $\Longrightarrow \frac{db}{dt}$ = 1 cm/sec

9. For which of the values of x, the rate of increase of the function y=3x$^2$-2x+7 is 4 times the rate of increase of x.

   Answer: $\frac{dy}{dt}$=4. $\frac{dx}{dt}$

   $y = 3x^2 - 2x + 7$

   $\frac{dy}{dt} = (6x - 2)\frac{dx}{dt}$

   4. $\frac{dx}{dt}$ = $(6x - 2)\frac{dx}{dt}$

   $4 = 6x - 2$

   $6x = 6, x = 1$

10. If the circumference of the circle is changing at the rate of 5 cm/s then what will be rate of change of area of the circle if the radius is 6cm.

    Answer: 30 cm2/s

# ACTIVITY - 13

**Mr. SUBHASH A B**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAY NO.1**
**AFS JALAHALLI WEST**
**BENGALURU REGION**

**NAME OF THE ACTIVITY:** MAXIMISE THE VOLUME OF A RECTANGULAR BOX FORMED FROM A GIVEN RECTANGULAR SHEET

**OBJECTIVE:**

- To develop an understanding of how to apply calculus to solve real-world optimization problems.
- To learn how to maximize the volume of a three-dimensional object (the open box) given constraints from a two-dimensional material (the rectangular sheet).
- Understand and apply optimization techniques using calculus.

**PRE-REQUISITE KNOWLEDGE:**

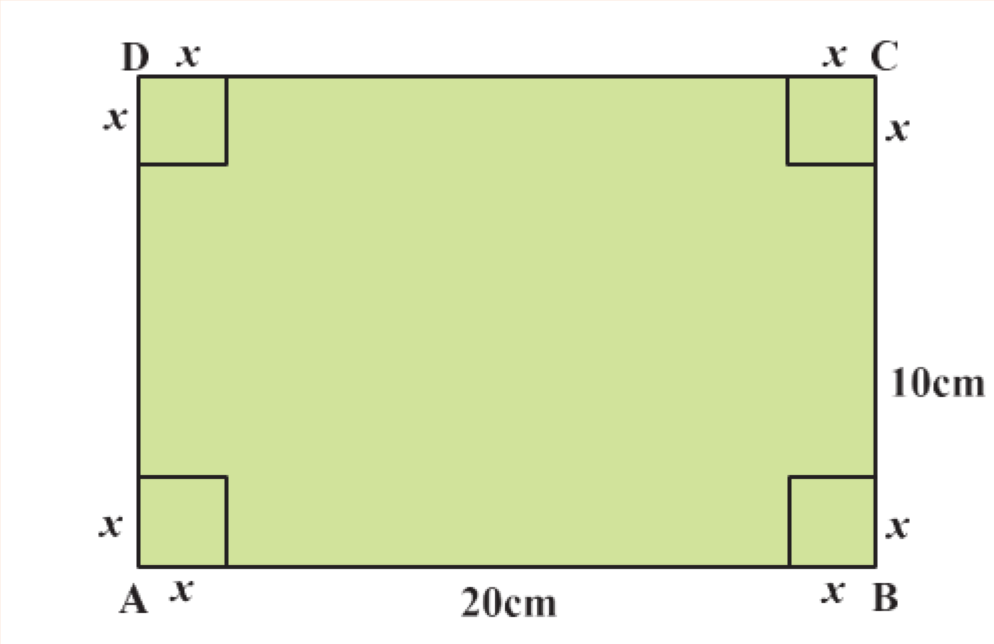
- Volume of a cuboid $V = lbh$.
- Derivative of a function
- $2^{nd}$ derivative test to find the maxima.

**MATERIAL REQUIRED:**

- Chart paper
- Scissors
- Cello tape
- Calculator

**METHOD OF CONSTRUCTION:**

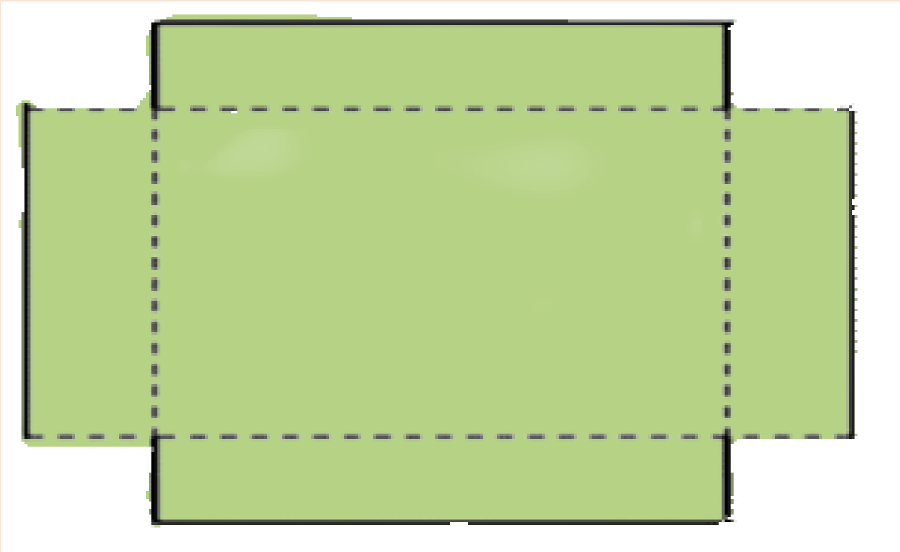
- Take a rectangular chart paper of size 20 cm × 10 cm and name it as ABCD.
- Cut four equal squares each of side $x$ cm from each corner A, B, C and D.
- Make an open box by folding its flaps using cello tape/adhesive.

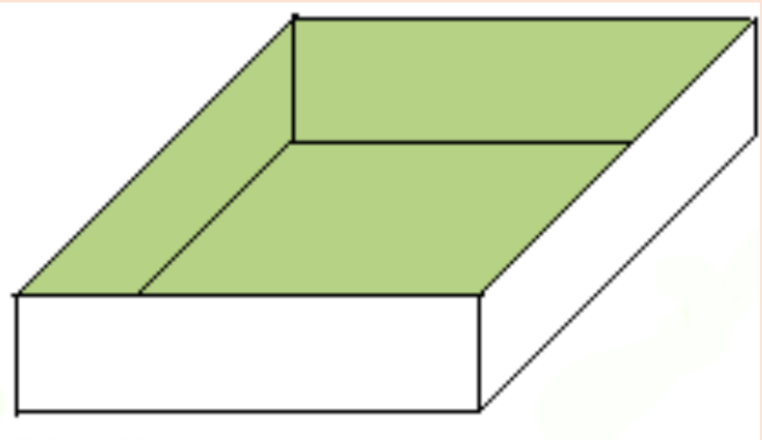



**DEMONSTRATION:**

- Start by gathering all materials: a rectangular chart paper (20 cm × 10 cm), scissors, and cello tape.
- Identify the points A, B, C, and D at the four corners of the rectangular chart paper.
- Using a ruler, mark and measure squares of side $x$ cm at each corner of the rectangle.
- Cut out the marked squares from each corner using scissors.
- Fold up the flaps formed by the cut squares to shape the rectangular chart paper into an open box.
- Secure the flaps using cello tape or adhesive to ensure the box retains its shape.

- Repeat the process with different values of *x* to determine which configuration creates the maximum volume.





**OBSERVATION:**

1. When x = 1, Volume of the box = 144 cm3
2. When x = 1.5, Volume of the box = 178.5 cm3

3. When x = 1.8, Volume of the box = 188.9 cm3.
4. When x = 2, Volume of the box = 192 cm3.
5. When x = 2.1, Volume of the box = 192.4 cm3.
6. When x = 2.2, Volume of the box = 192.2 cm3.
7. When x = 2.5, Volume of the box = 187.5 cm3.
8. When x = 3, Volume of the box = 168 cm3.

Clearly, volume of the box is maximum when x = 2.1.

**APPLICATION:**

This activity is useful in explaining the concepts of maxima/minima of functions. It is also useful in making packages of maximum volume with minimum cost.

Let V denote the volume of the box.

Now $V = (20 - 2x)(10 - 2x)x$

or $V = 200x - 60x^2 + 4x^3$

$\frac{dV}{dx} = 200 - 120x + 12x^2$. For maxima or minima, we have,

$\frac{dV}{dx} = 0$, i.e., $3x^2 - 30x + 50 = 0$

i.e., $x = \frac{30 \pm \sqrt{900 - 600}}{6} = 7.9$ or $2.1$

Reject $x = 7.9$.

$\frac{d^2V}{dx^2} = -120 + 24x$

When $x = 2.1$, $\frac{d^2V}{dx^2}$ is negative.

Hence, V should be maximum at $x = 2.1$.

**CONCLUSION:** The maximum volume of the box is achieved when the side length of the cut squares is 2.1 cm.

**LEARNING OUTCOMES:**

- ✓ Understanding the process of optimizing dimensions to achieve maximum volume.
- ✓ Applying the second derivative test to identify maxima in practical scenarios.
- ✓ Gaining hands-on experience with geometric constructions and measurements.
- ✓ Developing skills to use mathematical principles in real-world applications.
- ✓ Enhancing problem-solving abilities through iterative experimentation and observation.

**SOURCES OF ERROR AND PRECAUTIONS:**

- Ensure precise measurement of the sides and squares to avoid inaccuracies in dimensions.
- Carefully align and fold the flaps to ensure they meet perfectly at the edges for a sturdy box structure.
- Use appropriate amount of adhesive to prevent excess spillage or insufficient bonding that could weaken the box joints.

## VIVA QUESTIONS AND ANSWERS

1. What is the objective of this activity?

Answer: The objective of this activity is to construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner.

2. Explain how the volume of the box is calculated.

Answer: The volume of the box is calculated using the formula for the volume of a cuboid $V=l\times b\times h$, where l, b, and h are the length, breadth, and height of the box after cutting squares from the corners and folding the flaps.

3. Why is it important to consider the maxima in this activity?

Answer: It is important to consider the maxima in this activity to determine the value of x (side of the cut squares) that maximizes the volume of the box. This helps in finding the optimal dimensions for maximum efficiency.

4. What is the role of the second derivative test in this activity?

Answer: The second derivative test is used to confirm that the value of x found corresponds to a maximum volume by checking the concavity of the function. If the second derivative is negative at a point, the function has a local maximum there.

5. What happens to the volume if the side of the square cut is too large?

Answer:If the side of the square cut is too large, the resulting box will have a smaller volume or might not be feasible, as it would reduce the base area significantly

6. How did you find the value of x that gives the maximum volume?

Answer: The value of x that gives the maximum volume is found by experimenting with different values of x and calculating the corresponding volume. In this activity, the maximum volume was achieved when $x=2.1$ cm.

7. What are the practical applications of finding maxima and minima in real-world scenarios?

Answer: Finding maxima and minima has practical applications in optimization problems, such as designing containers with maximum volume and minimum material cost, and in various fields like economics, engineering, and logistics.

8. How does the size of the original sheet affect the maximum volume of the box?

Answer: The size of the original rectangular sheet affects the maximum volume, as a larger sheet can potentially create a box with a greater volume, while a smaller sheet limits the possible dimensions.

9. Can there be more than one value of x that maximizes the volume? Why or why not?

Answer: There is generally only one value of x that maximizes the volume in this scenario because the volume function is a continuous and differentiable function of x with a single maximum within the feasible range

10. What is the slope of the tangent at the point of maxima?

Answer: At the point of maxima, the slope of the tangent to the volume function is zero, indicating a critical point where the volume either reaches a maximum or minimum

# ACTIVITY - 14

**Mr. A M VIJAYAN**
**PGT(MATHS)**
**PM SHRI KENDRIYA VIDYALAYA**
**DGQA, CHENNAI**
**CHENNAI REGION**

NAME OF THE ACTIVITY: - VERIFY THAT ANGLE IN A SEMI -CIRCLE IS A RIGHT ANGLE, USING VECTOR METHOD.

**OBJECTIVE:**

- To use vector algebra to solve problems in geometry
- To understand of how vectors can be applied to prove geometrical results, in contrast to traditional Euclidean methods.
- To demonstrate that, angle in a semi-circle is a right angle, using vector method.

**PRE- REQUISITE KNOWLEDGE:**

- Knowledge of circle and its properties.
- Knowledge of vectors.

**MATERIALS REQUIRED:**

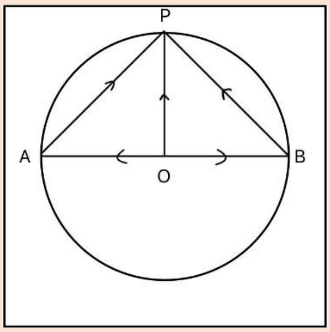
- Cardboard,
- Chart Paper,
- Adhesive,
- Pens,
- Pencils,
- Geometry Box,
- Eraser,
- Thread,
- Paper arrow heads,
- Drawing Pins,

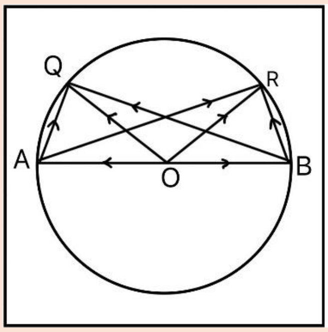
- Scissor.

**METHOD OF CONSTRUCTION:**

- Take a thick card board of 30cmx30cm.
- On the cardboard, paste white chart paper of the same size using an adhesive.
- On this chart paper draw a circle with centre O and radius 5cm.
- Draw the diameter AB.
- Fix drawing pins at the points O, A, B, P and Q
- Join OP, OA, OB, AP, AQ, BQ, OQ and BP using threads.
- Put arrows on OA, OB, OP, AP, BP, OQ, AQ and BQ to show them as

  vectors, using paper arrow heads, as shown in the figure.




**DEMONSTRATION:**

- Using protector, measure the angle between the vector $\overrightarrow{AP}$ and $\overrightarrow{BP}$

  i.e. $\angle APB = 90^0$

- Similarly, the angle between the vectors $\overrightarrow{AQ}$ and $\overrightarrow{BQ}$ i.e. $\angle AQB = 90^0$
- Repeat the above process by taking some more points R, S, T …. On the semi-circles, forming vectors AR, BR; AS, BS; AT, BT; …. etc., i.e. angle formed between two vectors in a semi – circle is a right angle.

**OBSERVATION:**

By actual measurement.

$|\overrightarrow{OP}| = |\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OQ}|$ = r = a = p = …………

$|\overrightarrow{AP}|$ =------------------,$|\overrightarrow{BP}|$ = -------------,$|\overrightarrow{AB}|$ -----------------,

$|\overrightarrow{AQ}|$ = ----------------------- , $|\overrightarrow{BQ}|$ = -----------------------

$|\overrightarrow{AP}|^2 + |\overrightarrow{BP}|^2$ = ------------------ , $|\overrightarrow{AQ}|^2 + |\overrightarrow{BQ}|^2$ = ------------------

So, $\angle APB$ = ------------, $\overrightarrow{AP} \cdot \overrightarrow{BP}$ =-------------, $\angle AQB$ = --------- and $\overrightarrow{AQ} \cdot \overrightarrow{BP}$ = -------------

Similarly, for points R, S, T

$\angle ARB$ =------------------,$\angle ASB$ = -----------------,$\angle ATB$ =------------------.

$i.e$ angle in a semi-circle is a right angle.

**APPLICATION:**

This activity can be used to explain the concepts of

(i) Opposite vectors
(ii) Vectors of equal magnitude.
(iii) Perpendicular vectors
(iv) Dot product of two vectors.

**VERIFICATION:**

Let OA = OB = a = OP =p

$\overrightarrow{OA} = -\vec{a}$ , $\overrightarrow{OB} = \vec{a}$ , $\overrightarrow{OP} = \vec{p}$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP} = \vec{a} + \vec{p}$., $\overrightarrow{AP} = \vec{p} - \vec{a}$.

$\overrightarrow{AP} \cdot \overrightarrow{BP} = (\vec{p} + \vec{a}).(\vec{p} - \vec{a}) = |\vec{p}|^2 - |\vec{a}|^2 = 0$, since $|\vec{p}|^2 = |\vec{a}|^2$.

So, the angle APB between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ is a right angle.

Similarly, $\overrightarrow{AQ} \cdot \overrightarrow{BQ} = 0$, so, $\angle AQB = 90^0$ and so on

**CONCLUSION:**

The conclusion is that THE ANGLE IN A SEMI-CIRCLE IS A RIGHT ANGLE is a fundamental concept in geometry, specifically in the study of circle and angles through vectors.

**LEARNING OUTCOMES:**

The circle theorem that states that an angle in a semi-circle is a right angle is that the angle formed by the arc that makes a semi-circle is 90 degrees, no matter where the line touches the semi – circle. this is because a semi-circle is a half of a circle, which measures 180 degrees, and an inscribed angle that intercepts an arc of 180 degrees must be half of a circle or 90 degrees.

**SOURCES OF ERROR AND PRECATIONS:**

While using scissor care to be taken.

Tying the thread to be tighten.

Fixation of drawing pins to be taken care.

**VIVA-VOCE QUESTIONS AND ANSWERS**

1. What is the scalar product of vectors $\vec{a}$ and $\vec{b}$

   Answer: the scalar product of vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a} \cdot \vec{b}$ is defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ , where $\theta$ ($0 \le \theta \le \pi$) is the angle between them.
2. What is dot product of two like vectors?

   Answer: If the vectors $\vec{a}$ and $\vec{b}$ are like then angle between them is $0^0$.

   so, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos 0^0 = \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$.
3. If $\vec{a} \cdot \vec{b} = 0$, what are the possibilities of $\vec{a}$ and $\vec{b}$ ?

   Answer:

i) $\vec{a}$ =0 and $\vec{b} \neq 0$

ii) $\vec{a} \neq 0$ and $\vec{b}$ = 0

iii) $\vec{a}$ and $\vec{b}$ are perpendicular.

4. What are the values of $\hat{\imath}.\hat{\imath}$, $\hat{\jmath}.\hat{\jmath}$, and $\hat{k}.\hat{k}$.

Answer: $\hat{\imath}.\hat{\imath}$ = $\hat{\jmath}.\hat{\jmath}$ = $\hat{k}.\hat{k}$ =1.

Justification $\hat{\imath}.\hat{\imath}$ =$|\hat{\imath}||\hat{\imath}|.\cos 0^0 = |x||x| = 1.$

5. What are the values of $\hat{\imath}.\hat{\jmath}$, $\hat{\jmath}.\hat{k}$, and $\hat{k}.\hat{\imath}$,

Answer: $\hat{\imath}.\hat{\jmath}$= $\hat{\jmath}.\hat{k}$= $\hat{k}.\hat{\imath}$ =0

Justification $\hat{\imath}.\hat{\jmath}$ =$|\hat{\imath}||\hat{\jmath}|$ cos$9^0$= 0

6. If $\vec{a}$ = $2\hat{\imath} - \hat{\jmath}$ , $\vec{b}$ = $\hat{\imath} + 2\hat{\jmath} - \hat{k}$, find $\vec{a}.\ \vec{b}$. Comment on the answer.

Answer: $\vec{a}.\ \vec{b}$ = 2-2 = 0. Here $\vec{a}$ is perpendicular to $\vec{b}$.

7. If $\vec{a}$ = $2\hat{\imath} + \hat{\jmath}$ , $\vec{b}$ = $\hat{\imath} + 2\hat{\jmath} - \hat{k}$, find $\vec{a}.\ \vec{b}$. Comment on the answer

Answer: $\vec{a}.\ \vec{b}$ = 2 + 2 = 4 ≠ 0, $\vec{a}$ and $\vec{b}$ are not perpendicular.

8. If $\vec{a}$ and $\vec{b}$ are any two vectors, then what is the value of ( $\vec{a}$). (- $\vec{b}$)

and (- $\vec{a}$). ($\vec{b}$)

Answer: ( $\vec{a}$). (- $\vec{b}$) = ( $-\vec{a}$). ( $\vec{b}$) = -( $\vec{a}$). ($\vec{b}$).

9. If $\vec{a} \times \vec{b} = \vec{0}$ what are the possibilities of $\vec{a}$ and $\vec{b}$ ?

Answer:

$\vec{a}$ =0 and $\vec{b} \neq 0$

$\vec{a} \neq 0$ and $\vec{b}$ = 0

$\vec{a}$ and $\vec{b}$ are parallel.

10. Which of the following is correct?

i) $\vec{a}.\ \vec{b} = \vec{b}.\ \vec{a}$

ii) $\vec{a}$ x $\vec{b} = \vec{b}$ x $\vec{a}$

Answer (i)

# ACTIVITY - 15

**Mr. T. SELVAM**
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**SIVAGANGA**
**CHENNAI REGION**

NAME OF THE ACTIVITY: **SHORTEST DISTANCE BETWEEN TWO SKEW LINES**

**OBJECTIVE:**

- To understand the concept of **skew lines** —two lines in three-dimensional space that are neither parallel nor intersecting.
- To measure the shortest distance between two skew lines and verify it analytically.

**PRE-REQUISITE KNOWLEDGE:**

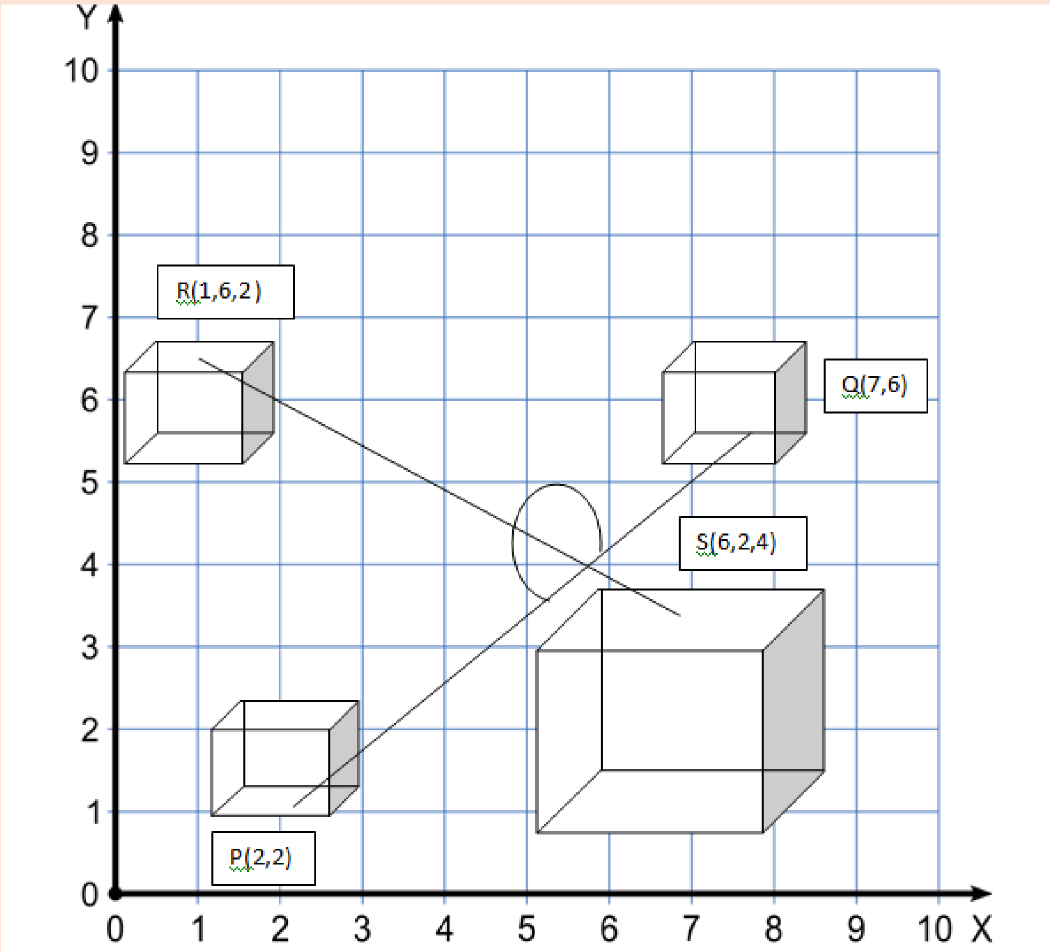
- ✶ Formula of vector equation of a line joining two points in space.
- ✶ Concept of Skew Lines.
- ✶ Formula to find the shortest distance between two skew lines.
- ✶ Cross and dot product of two vectors

**MATERIALS REQUIRED:**

- A piece of plywood of size 30 cm X 20 cm
- A squared paper (Graph paper)
- Three wooden blocks of sizes 2 cm X 2 cm X 2 cm each and one wooden block of size 2 cm X 2 cm X 4 cm
- Wires of different lengths
- Set squares
- Adhesive
- Pen
- Pencil etc.

**METHOD OF CONSTRUCTION:**

- Paste a graph paper on a piece of plywood.
- On the graph paper, draw two lines OA and OB to represent X-axis and Y-axis respectively.
- Name the three blocks of sizes 2 cm X 2 cm X 2 cm as I, II & III.
- Name the other block of size 2 cm X 2 cm X 4 cm as IV.
- Place the blocks I, II, & III such a way that their base centres are at the points (2,2), (1,6) and (7,6) respectively.
- Place the block IV to coincide its base centre a (6,2).
- Name the points (2,2), (7,6) as P,Q respectively.
- Name the centres of tops of the blocks II and IV as R & S respectively.
- Place a wire to join the points P and Q. It is one of the skew lines.
- Place another wire to join the points R and S. It is the other skew line.



**DEMONSTRATION:**

- A set-square is placed in such a way that its one perpendicular side is along the wire PQ.
- Move the set-square along PQ till its other perpendicular side touches the other wire.
- Measure the distance between the two lines in this position using set-square. This is the shortest distance between two skew lines.
- Analytically, find the equation of line joining P(2,2,0) and Q(7,6,0) and other line joining R(1,6,2) and S(6,2,4).
- Find the shortest distance by using the formula $\left|\frac{(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right|$
- The distance obtained in two cases will be same.

**OBSERVATION:**

- ✓ Coordinates of point P are __ ,__,__.
- ✓ Coordinates of point Q are __ ,__,__.
- ✓ Coordinates of point R are __ ,__,__.
- ✓ Coordinates of point S are __ ,__,__.
- ✓ The vector equation of the line $\overrightarrow{PQ}$ is $\vec{r} = (\quad) + \alpha(\quad)$
- ✓ The vector equation of the line $\overrightarrow{RS}$ is $\vec{r} = (\quad) + \beta(\quad)$
- ✓ Shortest distance between $\overrightarrow{PQ}$ & $\overrightarrow{RS}$ = ________.
- ✓ Shortest distance by actual measurement = ________.
- ✓ The above two distances are ___________.

**CONCLUSION:**

Obtained the Shortest Distance between two skew lines by using the formula and verified it physically.

**APPLICATION:**

This activity can be used to explain the concept of skew lines and the shortest distance between two lines in space.

**LEARNING OUTCOME:**

- ♦ Able to find the coordinates of the points.
- ♦ Learning to construct lines in space by using two points.
- ♦ Able to verify the given lines as skew lines or not.
- ♦ Learning the fact that the two lines are skew lines then they do not intersect.
- ♦ Able to find the shortest distance between two skew lines by using formula.

**SOURCES OF ERRROR AND PRECAUTIONS:**

- ❈ Select the two pairs of points properly to get two skew lines.
- ❈ Use the thin sticks instead of wires.
- ❈ Do the cross and dot product of vectors carefully.

## VIVA QUESTIONS AND ANSWERS

Question1. What are skew lines?

Answer: The lines that are neither intersecting nor parallel are called skew lines. They are non-coplanar.

Question 2. Can skew lines intersect? If not why?

Answer: Skew lines do not intersect as they are non-coplanar.

Question 3. What can you decide about the vectors_if the cross product of two vectors is zero?

Answer: The vectors are parallel.

Question 4. What can you decide about the vectors_if the dot product of two vectors is zero?

Answer: The vectors are perpendicular.

Question 5. What do you conclude if the shortest distance is zero?

Answer: Either the two vectors are parallel or intersecting.

Question 6. Suppose two pairs of vector lines are made by using the 4 points of the top of any of the cubes. Then what will be your answer? Why?

Answer: Since all the 4 points are coplanar the two lines thus made must be parallel or intersecting. So the shortest distance is zero.

Question 7. Find the direction ratios of the lines $\overrightarrow{PQ}$ and $\overrightarrow{RS}$.

Answer: $\overrightarrow{PQ} = (2\hat{\imath} + 2\hat{\jmath}) + \alpha(5\hat{\imath} + 4\hat{\jmath})$ Its DRs are 5, 4, and 0

$\overrightarrow{RS} = (\hat{\imath} + 6\hat{\jmath} + 2\hat{k}) + \beta(5\hat{\imath} - 4\hat{\jmath} + 2\hat{k})$ Its DRs are 5,-4, and 2

Question 8. What is the condition to be satisfied by the direction ratios of parallel lines?

Answer: They are proportional.

Question 9. Find the direction cosines of the lines $\overrightarrow{PQ}$ and $\overrightarrow{RS}$.

Answer: Direction Cosines of $\overrightarrow{PQ}$ are $\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}}$ $and$ 0

Direction Cosines of $\overrightarrow{RS}$ are $\frac{5}{\sqrt{45}}, \frac{-4}{\sqrt{45}}$ $and$ $\frac{2}{\sqrt{45}}$

Question 10. How will you check the lines to be parallel in terms of direction cosines?

Answer: They are equal.

# ACTIVITY – 16

**Mrs. SREELATHA**
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**NAL BANGALORE**
**BANGALORE REGION**

**NAME OF THE ACTIVITY:** TO MAXIMISE THE AREA OF A RECTANGLE WHEN ITS PERIMETER IS GIVEN

**OBJECTIVE:**

- To verify through an activity that of all rectangles of given perimeter, the square has the maximum area.

**PRE-REQUISITE KNOWLEDGE:**

Area and perimeter of a rectangle, Differentiation and the concept of maxima and minima.

**MATERIAL REQUIRED:**

- Drawing Board, Colour Papers,
- White Paper,
- Scale,
- Pencil,
- Paper Cutter
- Glue Stick.

**METHOD OF CONSTRUCTION:**

- Take a drawing board and paste a white paper on it.
- Construct rectangles of same perimeter (48cm) with different length and breadth.

  $R_1$ with length = 16cm breadth =8cm;

$R_2$ with length = 15cm breadth =9cm;

$R_3$ with length = 14cm breadth =10cm;

$R_4$ with length = 13cm breadth =11cm;

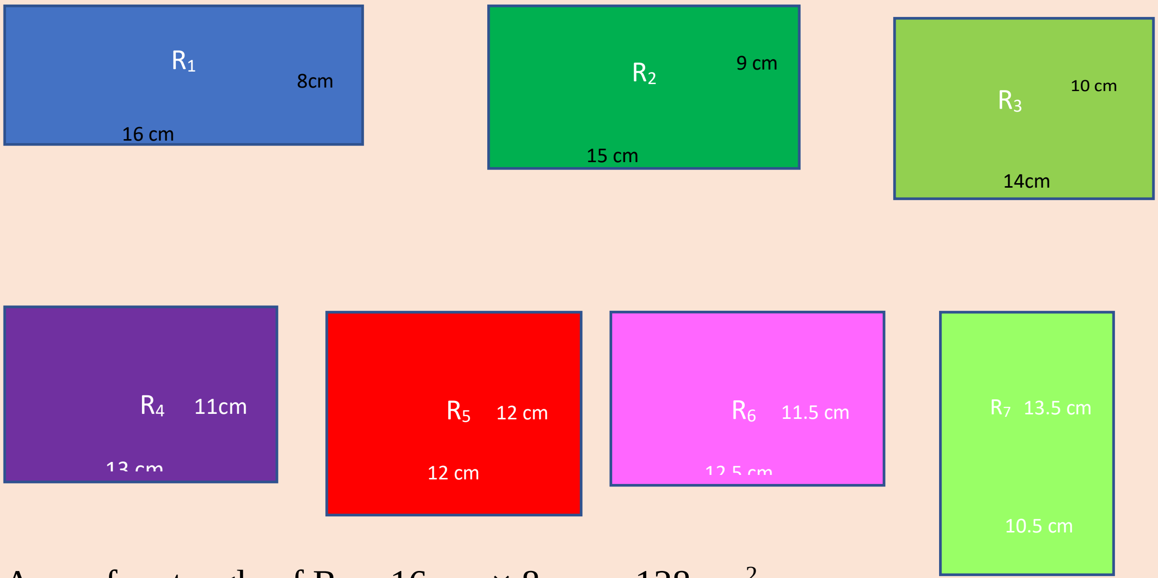
$R_5$ with length = 12cm breadth =12cm;

$R_6$ with length = 12.5cm breadth =11.5cm;

$R_7$ with length = 10.5cm breadth =13.5cm;

**DEMONSTRATION:**

- Cut out the rectangles and paste them on the white paper on the drawing board.




- Area of rectangle of $R_1$ = 16 cm × 8 cm = 128 cm$^2$

  Area of rectangle $R_2$ = 15 cm × 9 cm = 135 cm$^2$

  Area of $R_3$ = 140 cm$^2$

  Area of $R_4$ = 143 cm$^2$

  Area of $R_5$ = 144 cm$^2$

  Area of $R_6$ = 143.75 cm$^2$

  Area of $R_7$ = 141.75 cm$^2$

- Perimeter of each rectangle is same but their area are different. Area of rectangle $R_5$ is the maximum. It is a square of side 12 cm.

## OBSERVATION:

- ✓ Perimeter of each rectangle $R_1$, $R_2$, $R_3$, $R_4$ , $R_5$ , $R_6$ , $R_7$ is __________.
- ✓ Area of the rectangle $R_3$ is _________ than the area of rectangle $R_5$ .
- ✓ Area of the rectangle $R_6$ is ________ than the area of rectangle $R_5$ .
- ✓ The rectangle $R_5$ has the dimensions _______ × _______ and hence it is a _________.
- ✓ Of all the rectangles with same perimeter, the _________ has the maximum area.

### PROOF OF THE RESULT USING DERIVATIVES

Let the length and breadth of rectangle be x and y.

The perimeter of the rectangle P = 48 cm.

$2(x + y) = 48$ or $x + y = 24$ $y = 24 - x$

Let A (x) be the area of rectangle, then $A(x) = xy = x(24 - x) = 24x - x^2$

$A'(x) = 24 - 2x$ $A'(x) = \Rightarrow 24 - 2x = 0 \Rightarrow x = 12$

$A''(x) = -2$ $A''(12) = -2$, which is negative.

Therefore, area is maximum when $x = 12, y = 24 - x = 24 - 12 = 12$

So, $x = y = 12$

Hence, amongst all rectangles, the square has the maximum area.

## APPLICATION:

This activity is useful in explaining the concept of maximum of a function. It can be extended to real life problems on construction, painting the roof of a building etc.

## CONCLUSION:

Amongst all rectangles of given perimeter, the square has the maximum area.

**LEARNING OUTCOMES:**

- Students are able to understand the concept of maximum of a function.
- They are able to apply the conditions of maximum and minimum in solving problems of real-life situation.

**SOURCES OF ERROR AND PRECAUTION:**

Students must be cautioned while using sharp paper cutters.

## VIVA QUESTIONS AND ANSWERS

1 What will be the maximum value of area if Perimeter of the rectangle is 32cm?

Answer: 64cm$^2$

2 Can you get another result regarding the area and perimeter of a rectangle?

Answer: Yes, amongst all rectangles of given area, the square has least perimeter.

3 What is the maximum and minimum value of sinx +1 ?

Answer: Maximum value= 2, Minimum value= 0

4 What is the maximum and minimum value of f(x) = x in (0, 1)?

Answer: f(x) has no maximum and no minimum value in (0,1)

5 What is the maximum and minimum value of the function $f(x) = x \ in \ [0,1]$?

Answer: Maximum value = 1 and minimum value= 0

6 What is first derivative test?

Answer: If $f'(a) = 0 \ f'(a-h) < 0 \ and \ f'(a+h) > 0$, then f(x) has local minimum at x=a.

If $f'(a) = 0 \ f'(a-h) > 0 \ and \ f'(a+h) < 0$, then f(x) has local minimum at x=a.

7 What is second derivative test?

Answer: If $f'(a) = 0 \ f''(a) < 0$ then f(x) has local maximum at $x = a$

If $f'(a) = 0$ $f''(a) > 0$ then f(x) has local minimum at $x = a$

8 If for a function f(x), f'(a)=0 f''(a)= 0, what can you say about the point a.

Answer: a is a point of inflection

9 What is the slope of tangent at the point of local maximum?

Answer: Slope of tangent = 0

10 Can the local maximum be absolute maximum?

Answer: Local maximum may or may not be equal to absolute maximum.