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MATHEMATICS (041)
VOLUME II

कक्षा 12
CLASS XII
2025-26

सामग्री संवर्धन, मूल्यांकन और अध्ययन कैप्सूल का विकास
CONTENT ENRICHMENT, ASSESSMENT AND DEVELOPMENT OF STUDY CAPSULES

केन्द्रीय विद्यालय संगठन, रायपुर सम्भाग

KENDRIYA VIDYALAYA SANAGTHAN, RAIPUR REGION

संदेश

मुझे यह बताते हुए अपार हर्ष हो रहा है कि रायपुर संभाग के केंद्रीय विद्यालयों के स्नातकोत्तर गणित शिक्षकों द्वारा कक्षा 12 के छात्रों हेतु सामग्री संवर्धन, प्रभावी मूल्यांकन तकनीकों, और अध्ययन कैप्सूल के सफल विकास का कार्य किया गया है। यह पहल शिक्षण–अधिगम प्रक्रिया को और अधिक समृद्ध बनाते हुए विद्यार्थियों की जटिल गणितीय अवधारणाओं को सहज रूप में समझाने में सशक्त योगदान देगी।

आप सभी शिक्षकों ने सिद्ध किया है कि गुणवत्तापूर्ण शिक्षा तभी संभव है, जब शिक्षक गणिता की जटिलता को सरल, संरचित एवं प्रेरणादायक बनाते हैं। आपके द्वारा विकसित अध्ययन सामग्री में विषय–वस्तु की स्पष्टता, अभ्यास की विविधता एवं मूल्यांकन की सरलता व्यवसायिक शिक्षा का बीजारोपण करती है।

यह पहल केवल विद्यार्थियों के लिए लाभदायक नहीं, बल्कि अन्य शिक्षकों के लिए एक आदर्श मॉडल भी है। इससे यह प्रमाणित होता है – सहकार्य, नवाचार और ज्ञान–उन्मुख शिक्षाशैली से हम अपने छात्रों को विशेषज्ञता के साथ तैयारी करवा सकते हैं।

मैं समस्त गणित शिक्षकों को उनके इस समर्पण एवं उत्तम प्रयास के लिए हार्दिक शुभकामनाएँ देती हूँ। भविष्य में भी आपके द्वारा ऐसे प्रेरणादायक और शिक्षार्थी–केंद्रित कार्य की आशा करती हूँ।

"शिक्षा की असली शक्ति शिक्षक के नवोन्मेषी दृष्टिकोण और सहयोगी प्रयासों में निहित है।"

आभार एवं शुभकामनाओं सहित,

(पी.बी.एस. उषा)
उपायुक्त
के.वि.सं. क्षे.का. रायपुर

GUIDED BY

Shri Vivek Kumar Chauhan, Assistant Commissioner
Kendriya Vidyalaya Sangathan, Regional Office, Raipur

Shri Ravindra Kumar, Assistant Commissioner
Kendriya Vidyalaya Sangathan, Regional Office, Raipur

Course Director

Shri Sharad Kumar Kaiwart, Principal

PM SHRI Kendriya Vidyalaya, Bilaspur

Resource Person

1. **Shri Umesh Kumar Kurrey, PGT (Maths)**

   PM SHRI Kendriya Vidyalaya, Bilaspur

2. **Shri Shashank Shrivastav, PGT (Maths)**

   PM SHRI Kendriya Vidyalaya No. 2, Raipur

CONTRIBUTORS:

Sl. No.	KV Name	Name of teacher	Designation	Topic Allotted
1	CISF BHILAI	SRIJAN SINGH (GROUP LEADER)	PGT MATHS	LINEAR PROGRAMMING & PROBABLITY
2	AMBIKAPUR	TULSI PRASAD CHOURASIYA	PGT MATHS	
3	BACHELI	DEEPAK	PGT MATHS	
4	MAHASAMUND	V.N PRAHAN	PGT MATHS	
5	KORBA NO. IV	Mr ROSHAN LAKRA	PGT MATHS	
6	RAIGARH	TAPASHWINI PANIGRAHI (GROUP LEADER)	PGT MATHS	VECTORS & THREE DIMENSIONAL GEOMETRY
7	CRPF BILASPUR	NIRMALA SAHU	PGT MATHS	
8	JAGDALPUR	MONIKA GOYAL	PGT MATHS	
9	CHIRIMIRI	KHUSHBOO	PGT MATH	
10	JANJGIR	KHUSHBOO MEENA	PGT MATHS	
11	KURUD	KHEMLATA SAHU	PGT MATHS (CONTRACTUAL)	
12	DONGARGARH	DEEPAK KUMAR CHANDRAKAR (GROUP LEADER)	PGT MATHS	INTEGRALS, APPLICATION OF INTEGRALS & DIFFERENTIAL EQUATIONS
13	RAIPUR NO.1 (S-I)	ADITYA SHUKLA	PGT MATHS	
14	RAJNANDGAON	AMIT KUMAR SINGH	PGT MATHS	
15	KAWARDHA	KRISHNA PANDIT	PGT MATHS	
16	MANENDRAGARH	BRIJ MOHAN PRAJAPATI	PGT MATHS	
17	BMY BHILAI	JAI KUMAR CHOUDHARY (GROUP LEADER)	PGT MATHS	CONTINUITY AND DIFFERENTIABILITY & APPLICATION OF DERIVATIVES
18	DURG	M.A. NANDANWAR	PGT MATHS	
19	KHAIRAGARH	SANJAY KUMAR	PGT MATHS	
20	KIRANDUL	AMIT	PGT MATHS	
21	NARAYANPUR	PRINCE GUPTA	PGT MATHS	
22	JHAGRAKHAND	D.P CHAUBEY (GROUP LEADER)	PGT MATHS	RELATIONS AND FUNCTIONS, INVERSE TRIGONOMETRIC FUNCTIONS, MATRICES AND DETERMINANTS
23	NAYA RAIPUR	MANJEET KUMAR	PGT MATHS	
24	RAIPUR NO.1 (S-II)	KESHU JAIN	PGT MATHS	
25	KORBA NO. 2 (NTPC)	KASHYAP KUMAR MISHRA	PGT MATHS	
26	RAIPUR NO.2	RAMA SINGH	PGT MATHS	

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COURSE STRUCTURE
CLASS – XII
(2025-26)

One Paper **Max. Marks: 80**

No.	Units	Marks
I.	Relations and Functions	08
II.	Algebra	10
III.	Calculus	35
IV.	Vectors and Three - Dimensional Geometry	14
V.	Linear Programming	05
VI.	Probability	08
	Total	**80**
	Internal Assessment	**20**

Unit-I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability

Continuity and differentiability, chain rule, derivative of composite functions, derivatives of inverse trigonometric functions like $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real- life situations).

3. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \frac{px+q}{ax^2+bx+c}\, dx,$$

$$\int \frac{px+q}{\sqrt{ax^2+bx+c}}\, dx,\ \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx, \int \sqrt{ax^2 + bx + c}\, dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Application of the Integrals

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx} + py = q$, where p and q are functions of x or constants.

$\frac{dx}{dy} + px = q$, where p and q are functions of y or constants.

Unit-IV: Vectors and Three-dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three-dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming Problem

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem.

MATHEMATICS (Code No. – 041)
QUESTION PAPER DESIGN
CLASS – XII (2025-26)

Time: 3 hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	**80**	**100**

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper. However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note: For activities NCERT Lab Manual may be referred.

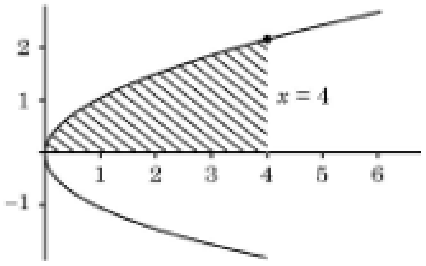
CBSE
PREVIOUS
YEARS
QUESTION
PAPERS

PAPER 1 (WITH SOLUTIONS)

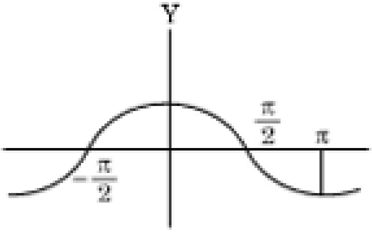
Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	If A = $\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, then A^{-1} is (A) $\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$ (B) $\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$ (C) $\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$ (D) $\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$	
Ans	(D) $\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$	1
Q2.	If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct ? (A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \perp \vec{b}$ (C) $\lvert \vec{b} \rvert > \lvert \vec{a} \rvert$ (D) $\lvert \vec{a} \rvert = \lvert \vec{b} \rvert$	
Ans	(B) $\vec{a} \perp \vec{b}$	1
Q3.	$\int_{-1}^{1} \frac{\lvert x \rvert}{x}\, dx, x \neq 0$ is equal to (A) -1 (B) 0 (C) 1 (D) 2	
Ans	(B) 0	1

Q4.	Which of the following is <u>not</u> a homogeneous function of x and y ? (A) $y^2 - xy$ (B) $x - 3y$ (C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$	
Ans	(D) $\tan x - \sec y$	1
Q5.	If $f(x) = \mid x \mid + \mid x - 1 \mid$, then which of the following is correct ? (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.	
Ans	(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.	1
Q6.	If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to : (A) 16 (B) 64 (C) 256 (D) 512	
Ans	(B) 64	1
Q7.	If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to : (A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{7}{9}$	
Ans	(C) $\frac{2}{3}$	1
Q8.	The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in [0, 2] is : (A) 0 (B) 2 (C) 4 (D) 5	
Ans	(C) 4	1

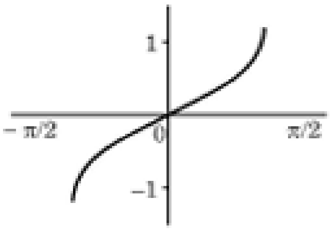
Q9.	Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9\ 8\ 7]$, which of the following is defined ? (A) Only AB (B) Only AC (C) Only BA (D) All AB, AC and BA	
Ans	(A) Only AB	1
Q10.	If $\int \frac{2^{\frac{1}{x}}}{x^2}\, dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to (A) $\frac{-1}{\log 2}$ (B) $-\log 2$ (C) -1 (D) $\frac{1}{2}$	
Ans	(A) $\frac{-1}{\log 2}$	1
Q11.	If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\vert \vec{a} \vert = \sqrt{37}$, $\vert \vec{b} \vert = 3$ and $\vert \vec{c} \vert = 4$, then angle between $\vec{b}$ and $\vec{c}$ is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	
Ans	(C) $\frac{\pi}{3}$	1
Q12.	The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is (A) $e^{\frac{y^2}{2}}$ (B) $\frac{1}{\sqrt{y}}$ (C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y^2}}$	
Ans	(B) $\frac{1}{\sqrt{y}}$	1

Q13.	If A = $\begin{bmatrix}7 & 0 & x\\0 & 7 & 0\\0 & 0 & y\end{bmatrix}$ is a scalar matrix, then y^x is equal to (A) 0 (B) 1 (C) 7 (D) ± 7	
Ans	(B) 1	1
Q14.	The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then (A) Z is maximum at (2, 72), minimum at (15, 20) (B) Z is maximum at (15, 20) minimum at (40, 15) (C) Z is maximum at (40, 15), minimum at (15, 20) (D) Z is maximum at (40, 15), minimum at (2, 72)	
Ans	(C) Z is maximum at (40, 15), minimum at (15, 20)	1
Q15.	If A and B are invertible matrices, then which of the following is <u>not</u> correct ? (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$ (C) adj (A) = $\mid A \mid A^{-1}$ (D) $\mid A \mid^{-1} = \mid A^{-1} \mid$	
Ans	(A) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
Q16.	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ? (A) It will only have a maximum value. (B) It will only have a minimum value. (C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.	
Ans	(C) It will have both maximum and minimum values.	1
Q17.	The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by  (A) $\int_0^4 x\,dx$ (B) $\int_0^2 y^2\,dy$ (C) $2\int_0^4 \sqrt{x}\,dx$ (D) $\int_0^4 \sqrt{x}\,dx$	
Ans	(D) $\int_0^4 \sqrt{x}\,dx$	1

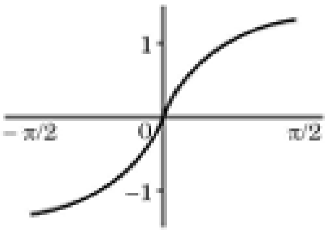
Q18. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?



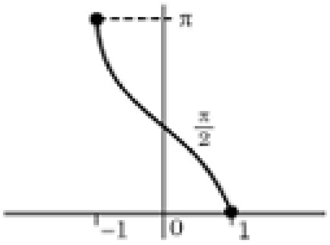
(A)



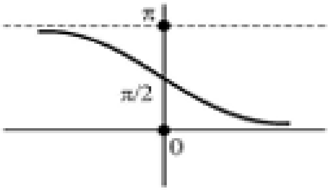
(B)



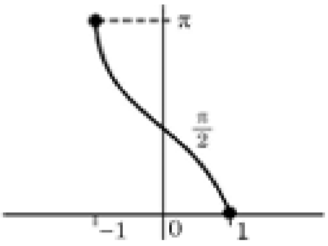
(C)



(D)



Ans (C) 1



Assertion – Reason Based Questions

Direction : Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19. **Assertion (A)** : Let Z be the set of integers. A function f : Z → Z defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

Ans (D) Assertion (A) is false, but Reason (R) is true. 1

Q20.	**Assertion (A)** : $f(x) = \begin{cases} 3x-8, & x \le 5 \\ 2k \ , & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$. **Reason (R)** : For a function f to be continuous at $x = a$, $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) = f(a)$.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1
	SECTION B This section comprises very short answer (VSA) type questions of **2 marks each**.	
Q21.	(a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. **OR** (b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.	
Ans(a)	**Let** $u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x}\left(-2\cos x \sin x\right)\log 2$	**1**
	Let $v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2\cos x \sin x$	½
	Now $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$	½
	OR	
Ans(b)	$\tan^{-1}\left(x^2 + y^2\right) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$	½
	Differentiate both sides wrt x,	
	$2x + 2y\frac{dy}{dx} = 0$	1
	$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$	½

Q22.	Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$	
Ans	$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$	
	$=\tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\frac{\pi}{3}\right]$	1
	$=\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$	1
Q23.	The diagonals of a parallelogram are given by $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$. Find the area of the parallelogram.	
Ans	$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$	1
	Area of parallelogram $=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$	
	$=\frac{1}{2}\sqrt{(-2)^2+3^2+7^2}=\frac{\sqrt{62}}{2}$	1
Q24.	Find the intervals in which function $f(x)=5x^{\frac{3}{2}}-3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.	
Ans	$f(x)=5x^{3/2}-3x^{5/2}\Rightarrow f'(x)=\frac{15}{2}\sqrt{x}(1-x)$	1
	For increasing / decreasing, put $f'(x)=0$	
	$\Rightarrow x=0,1$	
	(i) **When** $x\in[0,1]$, $f'(x)\geq 0$**. So,** f **is increasing when** $x\in[0,1]$	½
	(The intervals $(0,1),[0,1)$ **or** $(0,1]$ **can also be considered.)**	
	(ii) **When** $x\in[1,\infty)$, $f'(x)\leq 0$**. So,** f **is decreasing when** $x\in[1,\infty)$	½
	(The interval $(1,\infty)$ **can also be considered.)**	

Q25.	(a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings. **OR** (b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.	
Ans(a)	**Let the required angle between the kite strings be θ.** **Then,** $\cos\theta=\frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $\Rightarrow\cos\theta==\frac{\left(3\hat{i}+\hat{j}+2\hat{k}\right)\left(2\hat{i}-2\hat{j}+4\hat{k}\right)}{\sqrt{9+1+4}\sqrt{4+4+16}}=\frac{12}{\sqrt{336}}=\frac{3}{\sqrt{21}}$ $\Rightarrow\theta=\cos^{-1}\left(\frac{12}{\sqrt{336}}\right)$ **or** $\cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$	**1½** **½**
	OR	
Ans(b)	$\overline{BA}=-6\hat{i}+2\hat{j}+3\hat{k}$ **Required unit vector of magnitude 21** $=21\times\left(\frac{-6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{36+4+9}}\right)$ $=3\left(-6\hat{i}+2\hat{j}+3\hat{k}\right)$ **or** $-18\hat{i}+6\hat{j}+9\hat{k}$	**1** **½** **½**
	SECTION C This section comprises short answer (SA) type questions of **3 marks each**.	
Q26.	The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm ?	
Ans	**Let 'a' be the side of the triangle, so** $\frac{da}{dt}=3$ **cm/s** **Now area of an equilateral triangle,** $A=\frac{\sqrt{3}}{4}a^2$ $\Rightarrow\frac{dA}{dt}=\frac{\sqrt{3}a}{2}\times\frac{da}{dt}$ $\therefore\left.\frac{dA}{dt}\right]_{a=15\,\text{cm}}=\frac{\sqrt{3}\times15}{2}\times3=\frac{45\sqrt{3}}{2}$ **cm²/s**	**½** **1½** **1**

Q27. Solve the following linear programming problem graphically :

Maximise Z = $x + 2y$

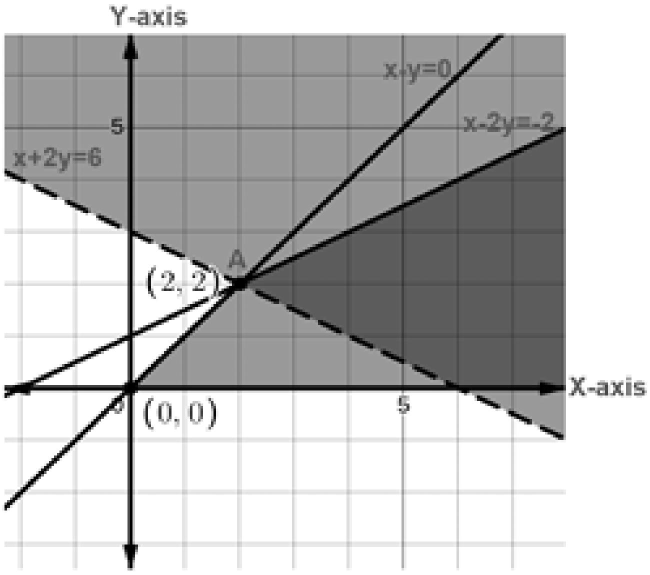
Subject to the constraints :

$x - y \geq 0$

$x - 2y \geq -2$

$x \geq 0, y \geq 0$

Ans



For correct graph and shading 1½

Corner Point	Value of $Z = x + 2y$
$O(0,0)$	0
$A(2,2)$	6

For correct table 1

Since feasible region is unbounded. Plot x + 2y > 6 which has common region with feasible region, thus Z has no maximum value. **½**

Ans

(a) Find : $\int \frac{x + \sin x}{1 + \cos x}\, dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$

Ans(a)	$\int \frac{x+\sin x}{1+\cos x}\,dx$	
	$=\int \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\,dx$	1
	$=\int x\left(\frac{1}{2}\sec^2\frac{x}{2}\right)dx+\int \tan\frac{x}{2}\,dx$	½
	$=x\tan\frac{x}{2}-\int \tan\frac{x}{2}\,dx+\int \tan\frac{x}{2}\,dx$	1
	$=x\tan\frac{x}{2}+C$	½
	OR	
Ans(b)	$\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$	
	$=\frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^4 x\sqrt{\tan x}}$	½
	$=\frac{1}{2}\int_0^{\pi/4} \frac{\left(1+\tan^2 x\right)\sec^2 x}{\sqrt{\tan x}}\,dx$	
	Put $\tan x=t \Rightarrow \sec^2 x\,dx=dt$	½
	$\therefore I=\frac{1}{2}\int_0^1 \frac{1+t^2}{\sqrt{t}}\,dt$	½
	$=\frac{1}{2}\int_0^1 \left(\frac{1}{\sqrt{t}}+t^{3/2}\right)dt$	
	$=\frac{1}{2}\left[2\sqrt{t}+\frac{2}{5}t^{5/2}\right]_0^1$	1
	$=\frac{6}{5}$	½

Q29.	(a) Verify that lines given by $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. **OR** (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.	
Ans(a)	**Rewriting the lines, we get** $\vec{r}=(\hat{i}-2\hat{j}+3\hat{k})+\lambda(-\hat{i}+\hat{j}-2\hat{k})$ **and** $\vec{r}=(\hat{i}-\hat{j}-\hat{k})+\mu(\hat{i}+2\hat{j}-2\hat{k})$	½
	Let $\vec{a}_1=\hat{i}-2\hat{j}+3\hat{k},\ \vec{a}_2=\hat{i}-\hat{j}-\hat{k},\ \vec{b}_1=-\hat{i}+\hat{j}-2\hat{k},\ \vec{b}_2=\hat{i}+2\hat{j}-2\hat{k}$ **Note that the dr's of given lines are not proportional so, they are not parallel lines.** **The lines will be skew if they do not intersect each other also.** **Here** $\vec{a}_2-\vec{a}_1=\hat{j}-4\hat{k},\ \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & -2\\ 1 & 2 & -2\end{vmatrix}=2\hat{i}-4\hat{j}-3\hat{k}$	½ + ½
	Consider $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)$ $=(\hat{j}-4\hat{k}).(2\hat{i}-4\hat{j}-3\hat{k})=8\neq 0$ **Hence lines will not intersect. So the lines are skew.**	½
	Shortest Distance $=\dfrac{\lvert(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\rvert}{\lvert\vec{b}_1\times\vec{b}_2\rvert}$ $=\dfrac{8}{\sqrt{4+16+9}}=\dfrac{8}{\sqrt{29}}$	1
OR		
Ans(b)	**Let the wicket keeper divides the line segment in ratio** $k:1$ $\therefore \vec{W}=\dfrac{k\vec{F}+1.\vec{B}}{k+1}$ 	1
	$\Rightarrow 6\hat{i}+12\hat{j}=\left(\dfrac{12k+2}{k+1}\right)\hat{i}+\left(\dfrac{18k+8}{k+1}\right)\hat{j}$	1
	$\Rightarrow k=\dfrac{2}{3}$ **Hence, the required ratio is** $2:3$	1

Q30. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows :

X	0	2	4	5
P(X)	p	2p	3p	p

Where X is the number of students absent.

(i) Calculate p. 1

(ii) Calculate the mean of the number of absent students on Saturday. 2

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

Ans(a)

(i) Since $\sum P(X)=1 \Rightarrow p+2p+3p+p=1$ ½

$\Rightarrow p=\frac{1}{7}$ ½

(ii) Mean $=\sum X.P(X)=0(p)+2(2p)+4(3p)+5(p)$ 1

$=21p=21\left(\frac{1}{7}\right)=3$ 1

OR

Ans(b)

Let E_1 : The applicant is a male

E_2 : The applicant is a female ½

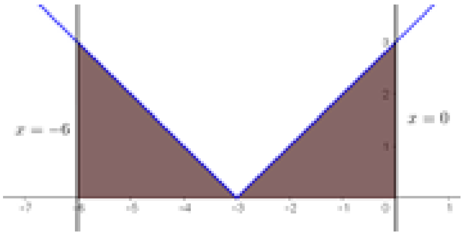
A : The candidate chosen will have distinction in the written test.

$P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A|E_1)=0.4, P(A|E_2)=0.35$ 1

$\therefore P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$

$=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$ 1

$=\frac{11}{30}$ ½

Q31.	Sketch the graph of y = | x + 3 | and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.	
Ans		**For correct graph: 1 mark**
	Required Area $=\int_{-6}^{0} y\,dx$	½
	$=2\int_{-3}^{0}(x+3)dx$	½
	$=2\left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$	½
	$=9$	½

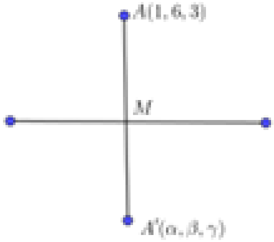
SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

Q32.	(a) If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$. **OR** (b) If $x = a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.	
Ans(a)	Let $x=\sin A$, $y=\sin B \Rightarrow A=\sin^{-1}x$, $B=\sin^{-1}y$ $\therefore \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ $\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$	1
	$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$	1
	$\Rightarrow \cot\left(\frac{A-B}{2}\right)=a \Rightarrow A-B=2\cot^{-1}a$	1
	$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$	½
	differentiate both sides wrt x, $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$ $\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$	1½

OR

Ans(b)	$x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$	
	$\Rightarrow\frac{dx}{d\theta}=a\left(-\sin\theta+\frac{1}{\tan\frac{\theta}{2}}\times\sec^2\frac{\theta}{2}\times\frac{1}{2}\right)$	½
	$=a\left(-\sin\theta+\frac{1}{\sin\theta}\right)=a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$	½
	$\frac{dx}{d\theta}=a\cot\theta\cos\theta$	½
	Also, $y=\sin\theta\Rightarrow\frac{dy}{d\theta}=\cos\theta$	½
	$\therefore\frac{dy}{dx}=\frac{\tan\theta}{a}$	1
	Differetiating wrt x,	
	$\frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{d\theta}{dx}$	
	$=\frac{\sec^3\theta\tan\theta}{a^2}$	1
	$\left.\frac{d^2y}{dx^2}\right]_{at\ \theta=\frac{\pi}{4}}=\frac{2\sqrt{2}}{a^2}$	1
Q33.	Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.	
Ans	$f(x)=2x^3-15x^2+36x+1$	
	$\Rightarrow f'(x)=6(x^2-5x+6)=6(x-2)(x-3)$	1
	$f'(x)=0\Rightarrow x=2,3\in[1,5]$	1
	Now $f(1)=24,\ f(2)=29,\ f(3)=28,\ f(5)=56$	2
	Hence, theabsolute maximum value is 56 and the absolute minimum value is 24.	1
Q34.	(a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, find the equation of the line joining A and A'. **OR** (b) Find a point P on the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.	

Ans(a)	The equation of given line is $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$ Any arbitrary point on the line is $M(\lambda, 2\lambda+1, 3\lambda+2)$ dr's of AM are $<\lambda-1, 2\lambda-5, 3\lambda-1>$ Here $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$ $\Rightarrow \lambda=1$ $\therefore M(1,3,5)$ is the foot perpendicular of the point A to the given line. Let image of point A in the line be $A'(\alpha,\beta,\gamma)$ Since M is the mid-point of AA', so $M\left(\frac{1+\alpha}{2},\frac{6+\beta}{2},\frac{3+\gamma}{2}\right)=M(1,3,5)$ $\Rightarrow A'(1,0,7)$ is the image of A. Also, Equation of AA' is $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$ 	1 1 ½ ½ 1 1
	OR	
Ans(b)	The given line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ and $Q(2,4,-1)$ Any random point on the line will be given by $P(\lambda-5, 4\lambda-3, -9\lambda+6)$ Since $PQ=7 \Rightarrow \sqrt{(\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2}=7$ $\Rightarrow 98(\lambda^2-2\lambda+1)=0 \Rightarrow \lambda=1$ Hence, the required point is $P(-4,1,-3)$ The equation of line PQ is $\frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$ or $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$	1 1 1 1 1
Q35.	A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions : • The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club. • The number of students in Music club should be 20 more than half the number of students in Sports club. • The total number of students to be allocated in all three clubs are 180. Find the number of students allocated to different clubs, using matrix method.	

Ans

Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively.

Here, $x = y + z,\ y = \frac{x}{2} + 20,\ x + y + z = 180$

$\Rightarrow x - y - z = 0,\ x - 2y = -40,\ x + y + z = 180$ **1½**

Given equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ **½**

$|A| = -4 \neq 0 \Rightarrow A^{-1}$ **exists.** **½**

$adjA = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$ **1**

$A^{-1} = \frac{1}{|A|} \times adjA = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$ **½**

$X = A^{-1}B$

$= \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$ **1**

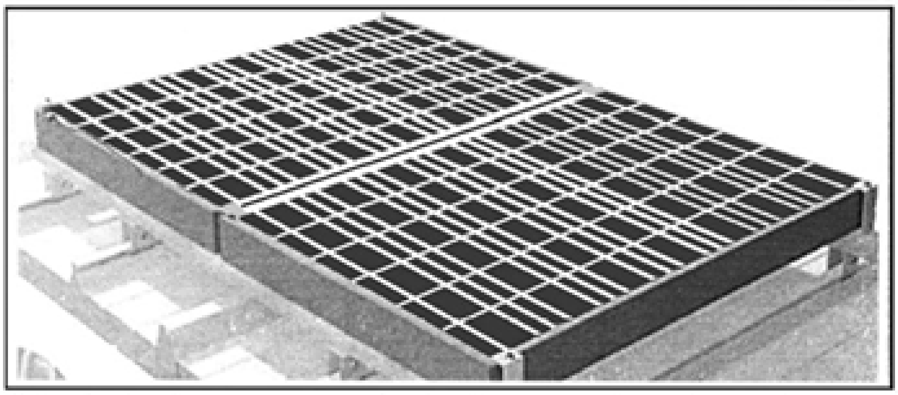
$\therefore x = 90,\ y = 65,\ z = 25$

Number of students allocated in sports, music and drama are 90 , 65 and 25 respectively .

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.



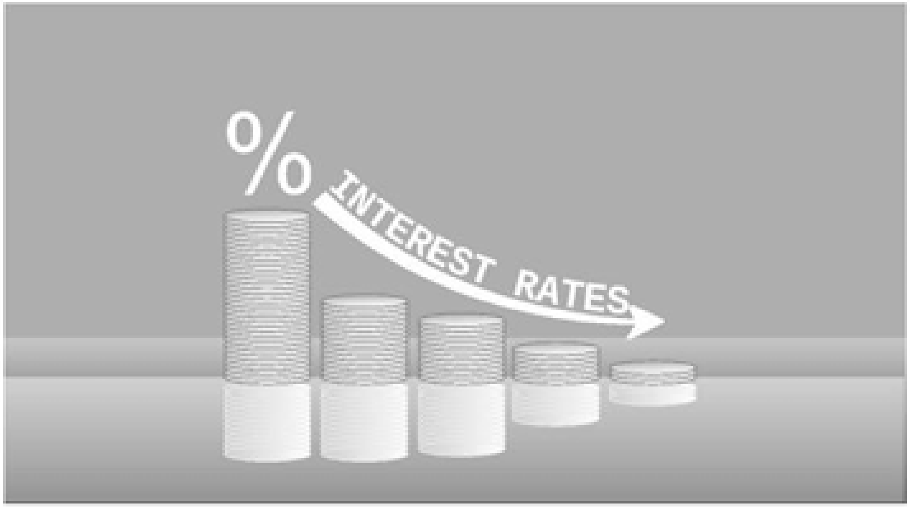
A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

	Based on this information, answer the following questions :	
	(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.	1
	(ii) Write the area of the solar panel as a function of x.	1
	(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.	2
	OR	
	(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.	2
Ans	$(i)\ 2x+3y=300$	1
	$(ii)\ A=xy=\frac{x}{3}(300-2x)$	1
	$(iii)(a)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$	½
	Also, $\frac{d^2A}{dx^2}=-\frac{4}{3}<0$. So, A is maximum at $x=75$	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\text{m}^2$	½
	OR	
	$(iii)(b)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$	½
	As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x=75$ from left to right, which means $x=75$ is the point of maximum.	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\text{m}^2$	½
	Note : Full credit to be given if the student takes equation as **2x + 2y = 300 or 2x + 4y = 300 or 4x + 4y = 300 or 4x + 3y = 300** **The solutions of sub-parts will differ and marks may be given accordingly.**	

Q37.	A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} : $R_1 = \{(2, 3), (3, 2)\}$ $R_2 = \{(1, 2), (1, 3), (3, 2)\}$ $R_3 = \{(1, 2), (2, 1), (1, 1)\}$ $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$ $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$ The students are asked to answer the following questions about the above relations : (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. **OR** (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?	
Ans	$(i)\ R_4$	**1**
	$(ii)\ R_5$	**1**
	$(iii)(a)\ R_1$ **and** R_3	**1+1**
	OR	
	$(iii)(b)$ **Required pairs to be added to make the relation R_2 as an equivalence relation are:**	
	$(1,1), (2,2), (3,3), (2,1), (3,1)$ **and** $(2,3)$	**2**

Q38.



A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following :

(i) What is the probability that a customer after availing the loan will default on the loan repayment ? **2**

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ? **2**

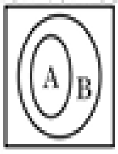
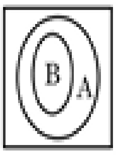
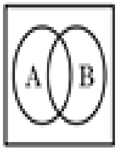
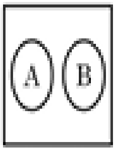
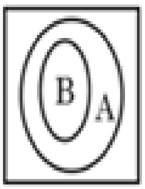
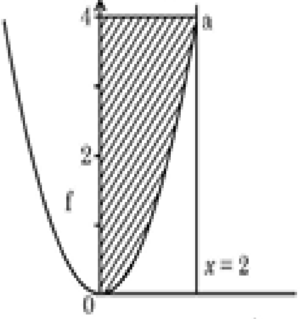
Ans	E_1 :customer avails loan on fixed rate	
	E_2 : customer avails loan on floating rate	
	E_3 : customer avails loan on variable rate	
	A:the person defaults on the loan	
	$\mathrm{P}(E_1)=\frac{1}{10},\mathrm{P}(E_2)=\frac{2}{10},\mathrm{P}(E_3)=\frac{7}{10}$	
	$\mathrm{P}(A\mid E_1)=\frac{5}{100},\mathrm{P}(A\mid E_2)=\frac{3}{100},\mathrm{P}(A\mid E_3)=\frac{1}{100}$	
	$(i)\mathrm{P}(A)=\mathrm{P}(E_1).\mathrm{P}(A\mid E_1)+\mathrm{P}(E_2).\mathrm{P}(A\mid E_2)+\mathrm{P}(E_3).\mathrm{P}(A\mid E_3)$	
	$=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$	1
	$=\frac{18}{1000}\text{or }\frac{9}{500}$	1
	$(ii)\mathrm{P}(E_3\mid A)=\frac{\mathrm{P}(E_3).\mathrm{P}(A\mid E_3)}{\mathrm{P}(E_1).\mathrm{P}(A\mid E_1)+\mathrm{P}(E_2).\mathrm{P}(A\mid E_2)+\mathrm{P}(E_3).\mathrm{P}(A\mid E_3)}$	
	$=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$	1
	$=\frac{7}{18}$	1

PAPER-2 (WITH SOLUTIONS)

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
SECTION-A This section comprises multiple choice questions (MCQs) of 1 mark each.		
1.	The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is (A) $\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$ (B) $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$ (C) $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert}$ (D) $\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert^2}\right)\vec{b}$	
Ans	(A) $\left(\frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$	1
2.	The function $f(x) = x^2 - 4x + 6$ is increasing in the interval (A) $(0, 2)$ (B) $(-\infty, 2]$ (C) $[1, 2]$ (D) $[2, \infty)$	
Ans	(D) $[2, \infty)$	1
3.	If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\,dx$ is (A) $\int_0^{2a} f\left(\frac{x}{2}\right) dx$ (B) $\int_0^{a} f(x)\,dx$ (C) $2\int_a^{0} f(x)\,dx$ (D) $2\int_0^{a} f(x)\,dx$	
Ans	(D) $2\int_0^a f(x)dx$	1

4.	If A = $\begin{bmatrix}1 & 12 & 4y\\ 6x & 5 & 2x\\ 8x & 4 & 6\end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is (A) –8 (B) 0 (C) 6 (D) 8	
Ans	(D) 8	1
5.	If $y = \sin^{-1}x, -1 \le x \le 0$, then the range of y is (A) $\left(\frac{-\pi}{2}, 0\right)$ (B) $\left[\frac{-\pi}{2}, 0\right]$ (C) $\left[\frac{-\pi}{2}, 0\right)$ (D) $\left(\frac{-\pi}{2}, 0\right]$	
Ans	(B) $[-\frac{\pi}{2}, 0]$	1
6.	If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is (A) $\frac{-\pi}{3}$ only (B) $\frac{\pi}{3}$ only (C) $\frac{\pi}{6}$ (D) $\pm\frac{\pi}{3}$	
Ans	No option is correct. Full marks may be awarded for attempting the question.	1
7.	If E and F are two events such that P(E) > 0 and P(F) ≠ 1, then $P(\overline{E}/\overline{F})$ is (A) $\frac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$ (C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$	
Ans	(D) $\frac{1-P(E\cup F)}{P(\bar{F})}$	1
8.	Which of the following can be both a symmetric and skew-symmetric matrix ? (A) Unit Matrix (B) Diagonal Matrix (C) Null Matrix (D) Row Matrix	
Ans	(C) Null Matrix	1

9.	The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is : (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$ (B) $x = 3t + 4, y = t + 3, z = 2t + 7$ (C) $x = 3t + 4, y = t - 3, z = 2t + 7$ (D) $x = 3t + 4, y = -t + 3, z = 2t + 7$	
Ans	(C) $x = 3t + 4, y = t - 3, z = 2t + 7$	1
10.	Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify 4 AB + 3(AB + BA) – 4 BA, where A and B are both matrices of order 2 × 2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as : Abhay : 6 AB Bina : 7 AB – BA Chhaya : 8 AB Devesh : 7 BA – AB Who answered it correctly ? (A) Abhay (B) Bina (C) Chhaya (D) Devesh	
Ans	(B) Bina	1
11.	A cylindrical tank of radius 10 cm is being filled with sugar at the rate of 100 π cm^3/s. The rate, at which the height of the sugar inside the tank is increasing, is : (A) 0.1 cm/s (B) 0.5 cm/s (C) 1 cm/s (D) 1.1 cm/s	
Ans	(C) 1 cm/s	1
12.	Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p} + \vec{q})$ will be a unit vector for what value of α ? (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$	
Ans	(D) $\frac{2\pi}{3}$	1

13.	The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point ? (A) (1, –5, 6) (B) (1, 5, 6) (C) (1, –5, –6) (D) (–1, –5, 6)	
Ans	(C) (1, -5, -6)	1
14.	If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ? (A)  (B)  (C)  (D) 	
Ans	 (B)	1
15.	The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \le x \le 2$ and y-axis is given by  (A) $\int_0^2 x^2\,dx$ (B) $\int_0^2 \sqrt{y}\,dy$ (C) $\int_0^4 x^2\,dx$ (D) $\int_0^4 \sqrt{y}\,dy$	
Ans	(D) $\int_0^4 \sqrt{y}dy$	1

16.	A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ? (A) The objective function maximizes the difference of the profit earned from products X and Y. (B) The objective function measures the total production of products X and Y. (C) The objective function maximizes the combined profit earned from selling X and Y. (D) The objective function ensures the company produces more of product X than product Y.	
Ans	(C) The objective function maximizes the combined profit earned from selling X and Y	1
17.	If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ? (A) A = B (B) AB = BA (C) A = 0 or B = 0 (D) A = I or B = I	
Ans	(B) AB = BA	1
18.	If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then (p – q) is (A) 0 (B) 1 (C) 2 (D) 3	
Ans	(B) 1	1
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	

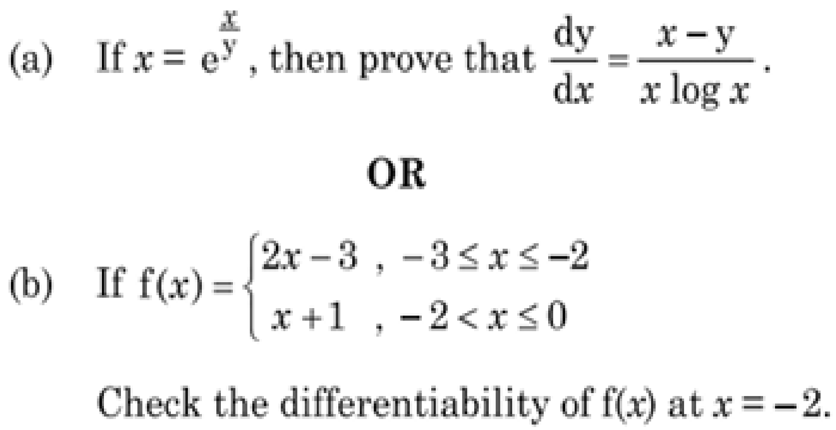
19.	**Assertion (A)** : A = diag [3 5 2] is a scalar matrix of order 3 × 3. **Reason (R)** : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.	
Ans	(D) Assertion (A) is false and Reason (R) is true.	1
20.	**Assertion (A)** : Every point of the feasible region of a Linear Programming Problem is an optimal solution. **Reason (R)** : The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.	
Ans	(D) Assertion (A) is false and Reason (R) is true.	1

SECTION-B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21	(a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$. **OR** (b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.	
21 (a) Ans	Let α be the angle which the vector $\vec{a}$ makes with all the three axes. Then $3\cos^2\alpha = 1$ $\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$ The unit vector along the vector $\vec{a} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$ $\vec{a} = 5(\hat{\imath} + \hat{\jmath} + \hat{k})$ OR	1 ½ ½
21 (b) Ans	 $\frac{QR}{QP} = \frac{3}{2}$	

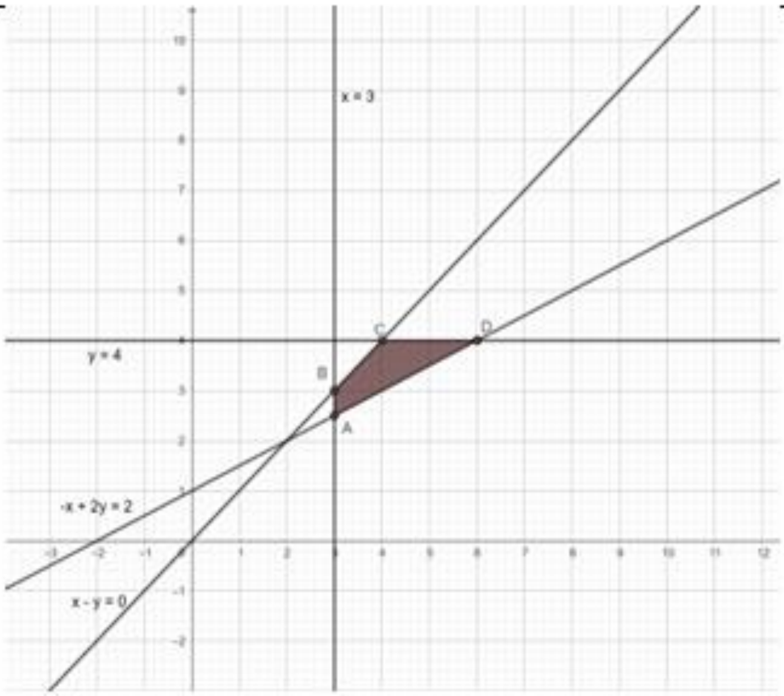
	Hence, R divides PQ, externally, in the ratio 1:3. The Position vector of R = $\vec{x} = \frac{\vec{\beta}-3\vec{\alpha}}{1-3} = \frac{3\vec{\alpha}-\vec{\beta}}{2}$	1 1
22.	Evaluate : $\int_0^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\ dx$	
Ans	Given definite integral = $\int_0^{\frac{\pi}{4}} \sqrt{(sinx+cosx)^2}dx$ $= \int_0^{\frac{\pi}{4}} (sinx+cosx)dx$ $= [-cosx+sinx]_0^{\frac{\pi}{4}}$ = 1	1 1
23.	Find the values of 'a' for which f(x) = sin x – ax + b is increasing on R.	
Ans	$f'(x) = cosx - a$ For f(x) to be increasing, $f'(x) \geq 0$ $i.e., cosx \geq a$ Since, $-1 \leq cosx \leq 1$ $\Rightarrow a \leq -1$ Hence, $a \in (-\infty, -1]$. (Also, accept $a \in (-\infty, -1)$)	 1 1
24.	If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.	
Ans	$\vec{\alpha}$ and $\vec{\beta}$ are collinear $\Rightarrow \frac{x-2}{3+2x} = \frac{1}{-2}$	1½

	$\Rightarrow x = \frac{1}{4}$	½
25	 (a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$. **OR** (b) If $f(x) = \begin{cases} 2x-3, & -3 \le x \le -2 \\ x+1, & -2 < x \le 0 \end{cases}$ Check the differentiability of f(x) at $x = -2$.	
25 (a) Ans	$x = e^{\frac{x}{y}}$ $\Rightarrow logx = \frac{x}{y}$ $\Rightarrow ylogx = x$ Differentiating both sides w.r.to x, we get $\frac{y}{x} + logx\frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx} = \frac{x-y}{xlogx}$ OR	½ 1 ½
25 (b) Ans	$Lf'(-2) = \lim_{h\to0}\frac{f(-2-h)-f(-2)}{-h} \quad (h > 0)$ $= \lim_{h\to0}\frac{2(-2-h)-3-(-7)}{-h}$ $= \lim_{h\to0} 2 = 2$ $Rf'(-2) = \lim_{h\to0}\frac{f(-2+h)-f(-2)}{h} \quad (h > 0)$ $= \lim_{h\to0}\frac{-2+h+1-(-7)}{h}$ $= \lim_{h\to0}\frac{6+h}{h}$, which does not exist, i.e., RHD does not exist.	1

	Therefore, the function is not differentiable at -2. Note: (1) If a student finds only RHD and concludes the result, full marks may be awarded. (2) If a student proves that the function is discontinuous at -2 and hence not differentiable at -2, full marks may be awarded.	1
SECTION-C This section comprises 6 Short Answer (SA) type questions of 3 marks each.		
26	(a) Solve the differential equation $2(y + 3) - xy\frac{dy}{dx} = 0$; given $y(1) = -2$. **OR** (b) Solve the following differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$.	
26(a) Ans	Given differential equation can be written as $\frac{y}{y+3}dy = \frac{2}{x}dx$ $\Rightarrow \int\left(1 - \frac{3}{y+3}\right)dy = 2\int\frac{1}{x}dx$ $\Rightarrow y - 3log\|y + 3\| = 2log\|x\| + C$ y = -2, when x = 1 $\Rightarrow C = -2$ Hence, the required particular solution is $\Rightarrow y - 3log\|y + 3\| = 2log\|x\| - 2$ OR	 1 1½ ½
26(b) Ans	Given differential equation can be written as $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$, which is linear in y. I.F. = $e^{\int\frac{2x}{1+x^2}dx} = e^{log(1+x^2)} = 1 + x^2$ The solution is given by $y(1 + x^2) = \int 4x^2dx$ $\Rightarrow y(1 + x^2) = \frac{4}{3}x^3 + C$ $or\ y = \frac{4x^3}{3(1+x^2)} + C\frac{1}{(1+x^2)}$, which is the required general solution	 1 1 1

27.	Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n $\in$ N." Find whether R is reflexive, symmetric and transitive or not.	
Ans	Let $x \in N$. Then we know that x is a multiple of itself. $\Rightarrow xRx$ Hence, R is reflexive. We have $2, 8 \in N$ such that 8 is a multiple of 2 $\Rightarrow 8R2$ But, 2 is not a multiple of 8. Hence, 2 is not R-related to 8. Therefore, R is not symmetric. Let $x, y, z \in N$ such that xRy, yRz Then $x = my, y = nz$ for some $\text{m}, n \in N$ $\Rightarrow x = mnz \Rightarrow x = pz,$ where $p = mn \in N.$ Hence, xRz Therefore, R is transitive.	1 1 1
28.	Solve the following linear programming problem graphically : Minimise $Z = x - 5y$ subject to the constraints : $x - y \geq 0$ $-x + 2y \geq 2$ $x \geq 3, y \leq 4, y \geq 0$	

Ans



Correct graph and shading 1½

Corner point	Value of Z = x – 5y
A (3, 2.5)	-9.5
B (3, 3)	-12
C (4, 4)	-16
D (6, 4)	-14

1

The minimum value of Z is -16, which is attained at x = 4, y = 4.

½

29

(a) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x + 1)^2 y_2 + (x + 1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

29(a)

Ans

The given function can be written as

$$y = 2\,log(x + 1) - logx$$

$$\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$$

1

$$\Rightarrow (x + 1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$$

	$\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$	1
	$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$	
	$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$	
	$\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$	1
	OR	
29(b)	$x\sqrt{1+y} + y\sqrt{1+x} = 0$	
Ans	$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$	
	$\Rightarrow x^2(1+y) = y^2(1+x)$	½
	$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$	
	$\Rightarrow (x-y)(x+y+xy) = 0$	1
	$x \neq y \Rightarrow x + y + xy = 0$	
	$\Rightarrow y = \frac{-x}{1+x}$	½
	$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$	1
30	(a) A die with number 1 to 6 is biased such that P(2) = $\frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice. **OR** (b) Two dice are thrown. Defined are the following two events A and B : A = {(*x*, y) : *x* + y = 9}, B = {(*x*, y) : *x* ≠ 3}, where (*x*, y) denote a point in the sample space. Check if events A and B are independent or mutually exclusive.	
30(a)	$P(2) = \frac{3}{10}, \text{P(any other number)} = 1 - \frac{3}{10} = \frac{7}{10}$	½
Ans	Let X represent the Random Variable "the number of 2's".	
	Then X = 0, 1, 2	½

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The probability distribution is
<table>
<tr><th>X</th><th>P(X)</th><th>XP(X)</th></tr>
<tr><td>0</td><td>$\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$</td><td>0</td></tr>
<tr><td>1</td><td>$\frac{3}{10}\times\frac{7}{10}\times 2=\frac{42}{100}$</td><td>$\frac{42}{100}$</td></tr>
<tr><td>2</td><td>$\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$</td><td>$\frac{18}{100}$</td></tr>
</table>
Mean = $\sum XP(X)=\frac{60}{100}=0.6$
OR
</td>
<td>1½
½</td>
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<td>30(b)
Ans</td>
<td>
$A=\{(3,6),(4,5),(5,4),(6,3)\}$
$P(A)=\frac{4}{36}=\frac{1}{9}, P(B)=\frac{30}{36}=\frac{5}{6}$
$P(A\cap B)=\frac{3}{36}=\frac{1}{12}$
$P(A)\times P(B)=\frac{5}{54}\neq P(A\cap B)$
Therefore, A and B are not independent.
A and B are not mutually exclusive as $A\cap B\neq\emptyset$
</td>
<td>1
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<td>31.</td>
<td>Find : $\int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\,dx.$</td>
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<td>Ans</td>
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$I=\int\frac{1}{x}\frac{x+a}{\sqrt{x^2-a^2}}dx=\int\frac{1}{\sqrt{x^2-a^2}}dx+a\int\frac{1}{x\sqrt{x^2-a^2}}dx$
$=\log\left|x+\sqrt{x^2-a^2}\right|+\sec^{-1}\left(\frac{x}{a}\right)+C$
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SECTION-D
This section comprises 4 Long Answer (LA) type questions of 5 marks each.
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<td>32.</td>
<td>Using integration, find the area of the region bounded by the line $y = 5x + 2$, the $x -$ axis and the ordinates $x = -2$ and $x = 2$.</td>
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<td>Ans</td>
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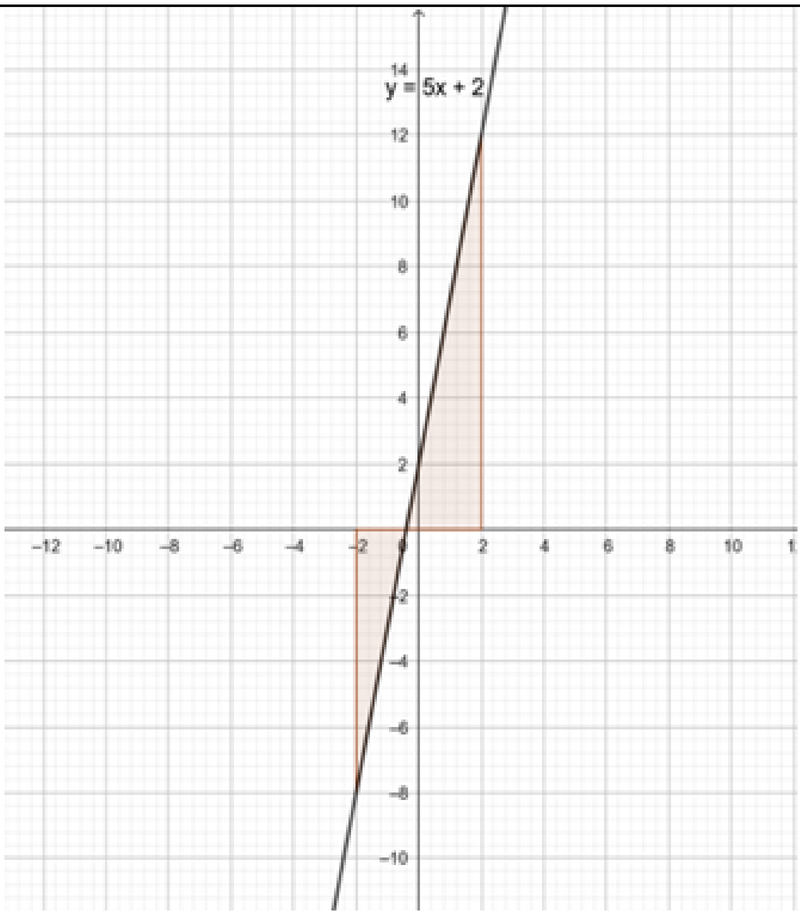
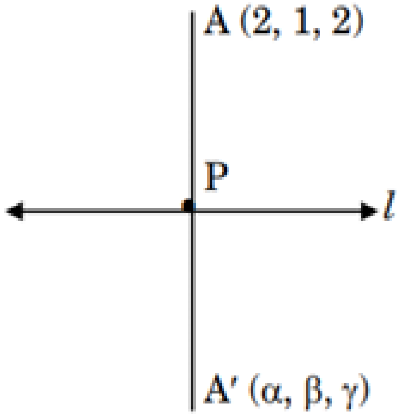
	 The required area $= \left\lvert\int_{-2}^{-\frac{2}{5}}(5x+2)dx\right\rvert + \int_{-\frac{2}{5}}^{2}(5x+2)dx$ $= \left\lvert\left[\frac{(5x+2)^2}{10}\right]_{-2}^{-\frac{2}{5}}\right\rvert + \left[\frac{(5x+2)^2}{10}\right]_{-\frac{2}{5}}^{2}$ $= \frac{64}{10} + \frac{144}{10} = \frac{104}{5}$	Correct sketch and shading 2 1 1 1
33.	Find : $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx.$	
Ans	$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+c}{x^2+1}$ Getting $A = \frac{3}{5}, B = \frac{2}{5}, C = \frac{1}{5}$ Given integral $= \frac{3}{5}\int\frac{1}{x+2}dx + \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{1}{x^2+1}dx$	2 1½

	$= \frac{3}{5}\log\lvert x+2\rvert + \frac{1}{5}\log(x^2+1) + \frac{1}{5}tan^{-1}x + C$	1½
34	(a) Find the shortest distance between the lines : $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ and $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$. **OR** (b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\,(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.	
34(a) Ans	The vector equations of the lines are $\vec{r} = -\hat{\imath} + \hat{\jmath} + 9\hat{k} + \lambda(2\hat{\imath} + \hat{\jmath} - 3\hat{k})$ $\vec{r} = 3\hat{\imath} - 15\hat{\jmath} + 9\hat{k} + \mu(2\hat{\imath} - 7\hat{\jmath} + 5\hat{k})$ $\overrightarrow{a_1} = -\hat{\imath} + \hat{\jmath} + 9\hat{k},\ \overrightarrow{a_2} = 3\hat{\imath} - 15\hat{\jmath} + 9\hat{k}$ $\overrightarrow{b_1} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}\ ,\ \overrightarrow{b_2} = 2\hat{\imath} - 7\hat{\jmath} + 5\hat{k}$ $\overrightarrow{a_2} - \overrightarrow{a_1} = 4\hat{\imath} - 16\hat{\jmath}$ $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = -16\hat{\imath} - 16\hat{\jmath} - 16\hat{k}$ S.D. $= \frac{\lvert(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})\rvert}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ OR	1 1 2 1

34(b

Ans



Let the image of A in the line be $A'(\alpha, \beta, \gamma)$

The point P, which is the point of intersection of the lines l and AA', will have coordinates $(\lambda + 4, -\lambda + 2, -\lambda + 2)$ for some λ. ½

Drs of AP are $< \lambda + 2, -\lambda + 1, -\lambda >$ ½

$AP \perp l$

$(\lambda + 2) - (-\lambda + 1) - (-\lambda) = 0$

$\Rightarrow \lambda = -\frac{1}{3}$ 1

Therefore, the coordinates of P are $(\frac{11}{3}, \frac{7}{3}, \frac{7}{3})$ ½

P is the mid-point of AA'

$\Rightarrow \frac{2+\alpha}{2} = \frac{11}{3}, \frac{1+\beta}{2} = \frac{7}{3}, \frac{2+\gamma}{2} = \frac{7}{3}$

$\Rightarrow \alpha = \frac{16}{3}, \beta = \frac{11}{3}, \gamma = \frac{8}{3}$ 1½

The coordinates of the image are $(\frac{16}{3}, \frac{11}{3}, \frac{8}{3})$

The equation of AA' is

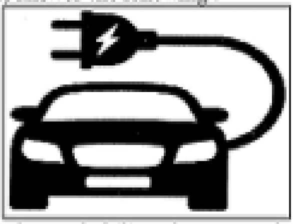
$$\frac{x-2}{\frac{10}{3}} = \frac{y-1}{\frac{8}{3}} = \frac{z-2}{\frac{2}{3}}$$ 1

or,

$$\frac{3(x-2)}{5} = \frac{3(y-1)}{4} = \frac{3(z-2)}{1}$$

35	(a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations : $x - y + z = 4$ $x - 2y - 2z = 9$ $2x + y + 3z = 1$ **OR** (b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of linear equations : $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$	
35(a) Ans	$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$ The system of equations is equivalent to the matrix equation: $BX = C$, where $C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = B^{-1}C$ $AB = 8I$ $\Rightarrow B^{-1} = \frac{1}{8}A$ $X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ $\therefore x = 3, y = -2, z = -1$ OR	2 ½ 1 1½
35(b) Ans	$\vert A\vert = 1 \neq 0 \Rightarrow A^{-1}$ exists. $adjA = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	1 1½

	$A^{-1} = \frac{1}{\lvert A\rvert}\text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ The given system of equations is equivalent to the matrix equation $A^T X = B$, where $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = (A^T)^{-1}B$ $\Rightarrow X = (A^{-1})^T B$ $\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\therefore x = 0, y = -5, z = -3$	½ ½ 1½
	SECTION-E This section comprises 3 case study based questions of 4 marks each	
36.	A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.  	

	Based on the above, answer the following : (i) How many relations can be there from S to J ? 1 (ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)} Check if it is bijective. 1 (iii) (a) How many one-one functions can be there from set S to set J ? 2 **OR** (iii) (b) Another student considers a relation R_1 = {(S_1, S_2), (S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. 2	
36 Ans (i)	The number of relations = $2^{4\times 3} = 2^{12}$	1
36 Ans (ii)	Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective.	1
36 (iii) (a)	There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0. OR	2
36 (iii) (b)	To make R_1 reflexive and not symmetric we need to add the following ordered pairs: $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$	2
37.	Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated). Based on the above, answer the following :  (i) (a) What is the probability that a randomly selected car is an electric car ? 2 **OR** (i) (b) What is the probability that a randomly selected car is a petrol car ? 2 (ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ? 1 (iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ? 1	

37(i) (a) Ans	Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is electric $P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$ $= \frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}$ $= \frac{155}{1000}\ or\ \frac{31}{200}$ OR	½ 1 ½
37(i)(b) Ans	Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is a petrol car $P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100} + \frac{10}{100} \times \frac{95}{100}$ $= \frac{845}{1000}\ or\ \frac{169}{200}$	½ 1 ½
37(ii) Ans	$P\left(\frac{C}{E}\right) = \frac{P(C) \times P(\frac{E}{C})}{P(E)}$ $= \frac{\frac{10}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}}$ $= \frac{\frac{50}{10000}}{\frac{1550}{10000}} = \frac{1}{31}$	1

37(iii) Ans	$P\left(\frac{\text{A or B}}{\text{E}}\right) = 1 - P\left(\frac{\text{C}}{\text{E}}\right) = 1 - \frac{1}{31} = \frac{30}{31}$	1
38.	 A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres. Based on the above, answer the following : (i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. 2 (ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. 2	
(i) Ans	$f'(x) = e^x(cosx + sinx)$ For critical points, $f'(x) = 0$ $\Rightarrow cosx + sinx = 0$ $\Rightarrow cosx = -sinx$	½
	For x to be a critical point $x \in (0, \pi)$, hence, $x = \frac{3\pi}{4}$	½
	For all $x \in \left[0, \frac{3\pi}{4}\right], f'(x) \geq 0$ Hence, *f* is increasing in $[0, \frac{3\pi}{4}]$	½
	Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $\left(0, \frac{3\pi}{4}\right)$ or $\left[0, \frac{3\pi}{4}\right)$ or $(0, \frac{3\pi}{4}]$ For all $x \in \left[\frac{3\pi}{4}, \pi\right], f'(x) \leq 0$ Hence, f is decreasing in $[\frac{3\pi}{4}, \pi]$	½
	Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $(\frac{3\pi}{4}, \pi)$ or $(\frac{3\pi}{4}, \pi]$ or $[\frac{3\pi}{4}, \pi)$	

(ii) Ans	$x = \frac{3\pi}{4}$ is a critical point $f''(x) = e^x(cosx - sinx) + e^x(cosx + sinx)$ $= 2e^x cosx$ $f''\left(\frac{3\pi}{4}\right) = -ve$ Hence, $\frac{3\pi}{4}$ is a point of local maximum.	 1 ½ ½

PAPER-3

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is :

(A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

2. If A and B are square matrices of same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to :

(A) A + B (B) BA

(C) 2 (A + B) (D) 2BA

3. For real x, let $f(x) = x^3 + 5x + 1$. Then :

(A) f is one-one but not onto on R

(B) f is onto on R but not one-one

(C) f is one-one and onto on R

(D) f is neither one-one nor onto on R

4. If $y = \sin^{-1} x$, then $(1 - x^2)\dfrac{d^2y}{dx^2}$ is equal to :

(A) $x\dfrac{dy}{dx}$ (B) $-x\dfrac{dy}{dx}$

(C) $x^2\dfrac{dy}{dx}$ (D) $-x^2\dfrac{dy}{dx}$

5. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :

(A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$

(C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$

6. If P is a point on the line segment joining (3, 6, –1) and (6, 2, – 2) and y-coordinate of P is 4, then its z-coordinate is :

(A) $-\dfrac{3}{2}$ (B) 0

(C) 1 (D) $\dfrac{3}{2}$

7. If M and N are square matrices of order 3 such that det (M) = m and MN = mI, then det (N) is equal to :

(A) – 1 (B) 1

(C) $-m^2$ (D) m^2

8. If $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to :

(A) – 4 (B) $-\dfrac{7}{2}$

(C) – 2 (D) – 1

9. If $f : N \rightarrow W$ is defined as

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases},$$

then f is :

(A)	injective only	(B)	surjective only
(C)	a bijection	(D)	neither surjective nor injective

10. The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a :

(A)	diagonal matrix	(B)	symmetric matrix
(C)	skew symmetric matrix	(D)	scalar matrix

11. If the sides AB and AC of Δ ABC are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is :

(A)	$2\sqrt{2}$ units	(B)	$\sqrt{18}$ units
(C)	$\frac{\sqrt{34}}{2}$ units	(D)	$\frac{\sqrt{48}}{2}$ units

12. The function f defined by

$$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$$

is ***not*** continuous at :

(A)	$x = 0$	(B)	$x = 1$
(C)	$x = 2$	(D)	$x = 5$

13. If $f(x) = 2x + \cos x$, then f(x) :

(A)	has a maxima at $x = \pi$	(B)	has a minima at $x = \pi$
(C)	is an increasing function	(D)	is a decreasing function

14. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ is equal to :

(A) $2(\sin x + x\cos\alpha) + C$

(B) $2(\sin x - x\cos\alpha) + C$

(C) $2(\sin x + 2x\cos\alpha) + C$

(D) $2(\sin x + \sin\alpha) + C$

15. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is :

(A) $-\frac{\pi}{4}$

(B) $\frac{\pi}{4}$

(C) $\tan^{-1} e - \frac{\pi}{4}$

(D) $\tan^{-1} e$

16. The order and degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$ are :

(A) order 2, degree 2

(B) order 2, degree 1

(C) order 2, degree not defined

(D) order 1, degree not defined

17. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is :

(A) $\frac{16}{9}$ sq. units

(B) $\frac{32}{9}$ sq. units

(C) $\frac{16}{3}$ sq. units

(D) $\frac{32}{3}$ sq. units

18. The corner points of the feasible region of a Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is :

(A) $a = b$

(B) $a = 3b$

(C) $b = 6a$

(D) $3a = 2b$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R) : Two events are independent if the occurrence of one does not effect the occurrence of the other.

20. *Assertion (A) :* In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.

Reason (R) : A feasible region is defined as the region that satisfies all the constraints.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Let A and B be two square matrices of order 3 such that det (A) = 3 and det (B) = – 4. Find the value of det (– 6AB).

22. (a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on [2, 4].

OR

(b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.

23. (a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

OR

(b) Find domain of $\sin^{-1}\sqrt{x-1}$.

24. Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x-axis using integration.

25. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) If $f : R^+ \rightarrow R$ is defined as $f(x) = \log_a x$ $(a > 0$ and $a \neq 1)$, prove that f is a bijection.

(R^+ is a set of all positive real numbers.)

OR

(b) Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) Write all elements of R.

(ii) Is R a function ? Justify.

(iii) Determine domain and range of R.

27. (a) Find k so that

$$f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at $x = -1$.

OR

(b) Check the differentiability of function $f(x) = x|x|$ at $x = 0$.

28. Evaluate :

$$\int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$$

29. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

30. Find the distance of the point (–1, –5, –10) from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

31. Solve the following Linear Programming Problem using graphical method :

Maximise $Z = 100x + 50y$

subject to the constraints

$3x + y \le 600$

$x + y \le 300$

$y \le x + 200$

$x \ge 0, y \ge 0$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. If A is a 3 × 3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

33. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight. *2*

(ii) In how many days will the plant attain its maximum height ? What is the maximum height ? *3*

34. (a) Find :

$$\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

35. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a} \times \vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

OR

(b) Find the equation of a line in vector and cartesian form which passes through the point (1, 2, – 4) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$, and

$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Some students are having a misconception while comparing decimals. For example, a student may mention that 78·56 > 78·9 as 7856 > 789. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47·7
Bijoy	47·07
Kartik	43·09
Dinesh	43·9
Devesh	45·2

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

(i) What is the probability of a student not having misconception but still answers Bijoy in the test ? *1*

(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? *1*

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? *2*

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? *2*

Case Study – 2

37. An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions :

(i) Find whether the two metro tracks are parallel. *1*

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$. *1*

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway. *2*

OR

(iii) (b) Find the shortest distance between Line A and Line B. *2*

Case Study – 3

38. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$,

where T(t) represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions :

(i) Find the expression for temperature of processor, T(t) given that T(0) = 85°C. *2*

(ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0 \cdot 03$, $\log_e 4 = 1 \cdot 3863$. *2*

PAPER-4

General Instructions :

Read the following instructions very carefully and strictly follow them :

- *(i)* *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*
- *(ii)* *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*
- *(iii)* *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*
- *(iv)* *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*
- *(v)* *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*
- *(vi)* *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*
- *(vii)* *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*
- *(viii)* *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*
- *(ix)* *Use of calculator is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is :

(A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

(C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$

(D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

2. If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is :

(A) 0·3

(B) 1

(C) 1·3

(D) 0·7

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is :

(A) Only AB is defined.

(B) Only BA is defined.

(C) AB and BA, both are defined.

(D) AB and BA, both are not defined.

4. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(A) 3 (B) 7

(C) ± 7 (D) ± 3

5. If $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

is continuous at x = 0, then the value of a is :

(A) 1 (B) −1

(C) ± 1 (D) 0

6. If $A = [a_{ij}]$ is a 3 × 3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is :

(A) 0 (B) −10

(C) 10 (D) 1

7. The principal value of $\cot^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$ is :

(A) $-\dfrac{\pi}{3}$ (B) $-\dfrac{2\pi}{3}$

(C) $\dfrac{\pi}{3}$ (D) $\dfrac{2\pi}{3}$

8. If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is :

(A) 0 (B) 1

(C) − 2 (D) − 4

9. If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is :

(A) f is continuous but not differentiable at $x = 2$.

(B) f is neither continuous nor differentiable at $x = 2$.

(C) f is continuous as well as differentiable at $x = 2$.

(D) f is not continuous but differentiable at $x = 2$.

10. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :

(A) $(1, -10)$ (B) $(1, 10)$

(C) $(10, 1)$ (D) $(-10, 1)$

11. $\int \sqrt{1+\sin x}\ dx$ is equal to :

(A) $2\left(-\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$ (B) $2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$

(C) $-2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$ (D) $2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$

12. $\int_0^{\pi/2} \cos x . e^{\sin x}\ dx$ is equal to :

(A) 0 (B) $1 - e$

(C) $e - 1$ (D) e

13. The area of the region enclosed between the curve $y = x|x|$, x-axis, $x = -2$ and $x = 2$ is :

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$

(C) 0 (D) 8

14. The integrating factor of the differential equation

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1 \text{ is :}$$

(A) $e^{-1/\sqrt{x}}$ (B) $e^{2/\sqrt{x}}$

(C) $e^{2\sqrt{x}}$ (D) $e^{-2\sqrt{x}}$

15. The sum of the order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2} \text{ is :}$$

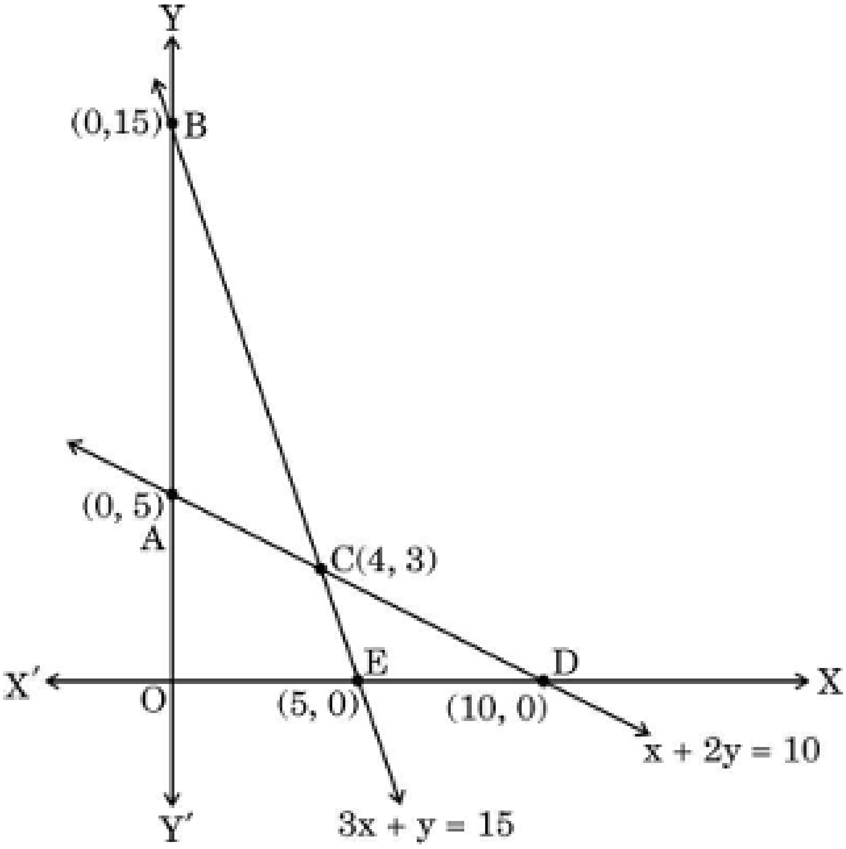
(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4

16. For a Linear Programming Problem (LPP), the given objective function Z = 3x + 2y is subject to constraints :

$x + 2y \leq 10$

$3x + y \leq 15$

$x, y \geq 0$



The correct feasible region is :

(A) ABC (B) AOEC

(C) CED (D) Open unbounded region BCD

17. Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are :

(A) (– 2, – 2) (B) (2, – 2) (C) (– 2, 2) (D) (2, 2)

18. The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are :

(A)	48 and 16	(B)	3 and 1
(C)	24 and 8	(D)	6 and 2

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

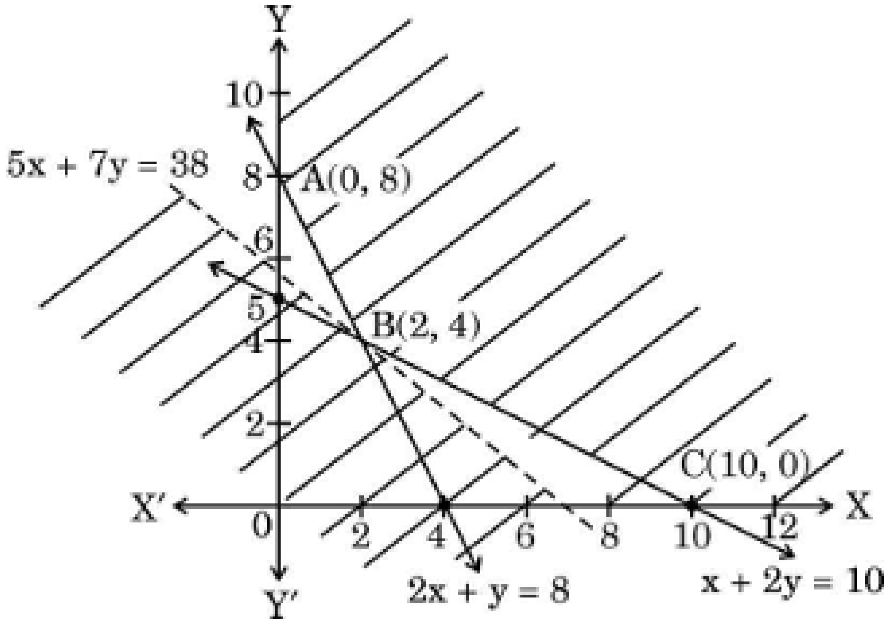
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min Z = 50x + 70y

subject to constraints

$2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0$

Z = 50x + 70y has a minimum value = 380 at B(2, 4).

Reason (R) : The region representing $50x + 70y < 380$ does not have any point common with the feasible region.

20. *Assertion (A) :* Let A = $\{x \in R : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm \sqrt{-1} \notin A$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.

22. Surface area of a balloon (spherical), when air is blown into it, increases at a rate of 5 mm^2/s. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing.

23. (a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x.

OR

(b) If $y = 5 \cos x - 3 \sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

24. (a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$.

OR

(b) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} . \vec{b} = \vec{a} . \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.

25. A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Find the value of 'a' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in R.

27. (a) Find :

$$\int \frac{2x}{(x^2+3)(x^2-5)}\,dx$$

OR

(b) Evaluate :

$$\int_1^4 \left(|x-2| + |x-4|\right)\,dx$$

28. Find the particular solution of the differential equation

$$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$$

given that $y = \frac{\pi}{4}$, when $x = 1$.

29. In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$

subject to constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x, y \geq 0$

30. (a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

31. (a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student :

(i) Buys both the colouring book and the box of colours.

(ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find :

(i) The probability distribution of the number of oranges he draws.

(ii) The expectation of the random variable (number of oranges).

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.

33. A furniture workshop produces three types of furniture – chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.

34. (a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

35. (a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

OR

(b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (–1, –1, 2).

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. *1*

(ii) Find $\frac{dS}{dx}$. *1*

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum. *2*

OR

(iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$. *2*

Case Study – 2

37. Let A be the set of 30 students of class XII in a school. Let $f : A \rightarrow N$, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following :

(i) Is f a bijective function ? *1*

(ii) Give reasons to support your answer to (i). *1*

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where

$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = 3x\}$.

List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. *2*

OR

(iii) (b) Let R be a relation defined by

$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = x^3\}$.

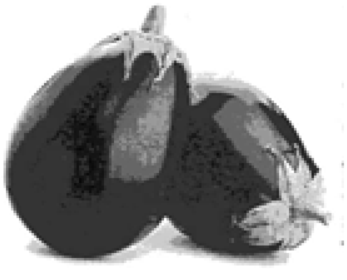
List the elements of R. Is R a function ? Justify your answer. *2*

Case Study – 3

38. A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Radish

Cabbage



Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate. *2*

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ? *2*

PAPER-5

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is :

(A) $n \times n$ (B) $n \times m$

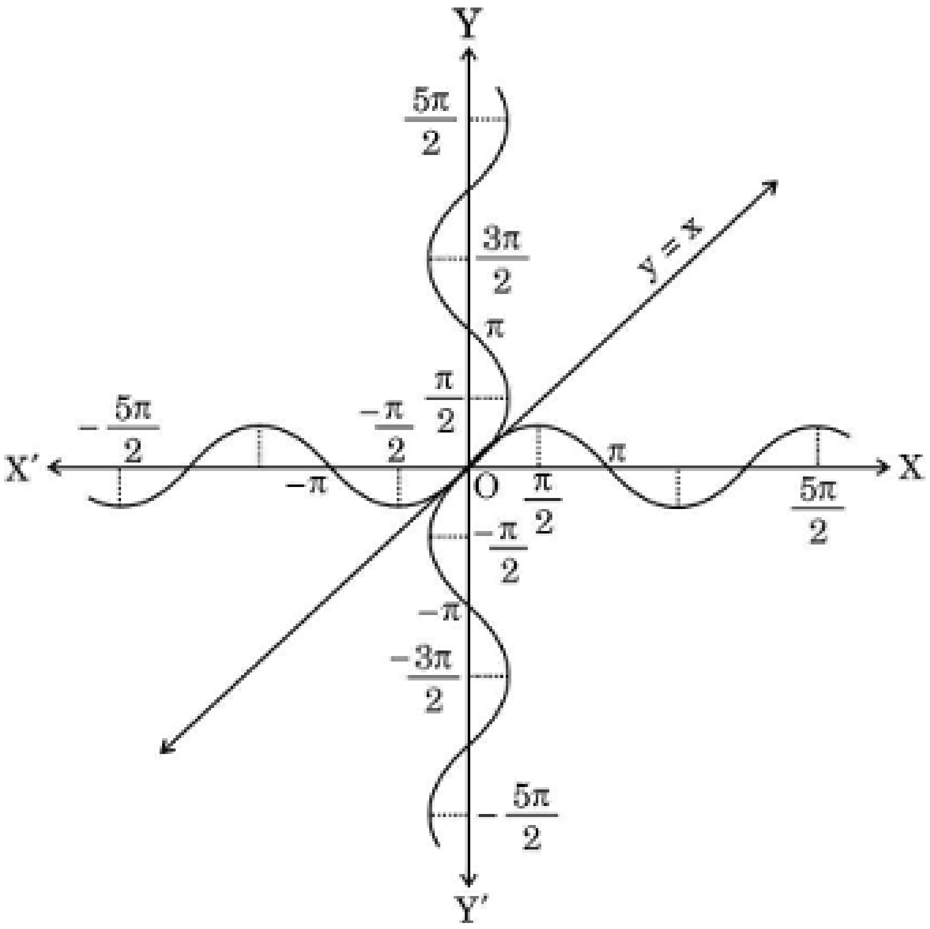
(C) $m \times m$ (D) $m \times n$

2. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an :

(A) scalar matrix (B) identity matrix

(C) symmetric matrix (D) skew-symmetric matrix

3. The following graph is a combination of :



(A) $y = \sin^{-1} x$ and $y = \cos^{-1} x$

(B) $y = \cos^{-1} x$ and $y = \cos x$

(C) $y = \sin^{-1} x$ and $y = \sin x$

(D) $y = \cos^{-1} x$ and $y = \sin x$

4. Sum of two skew-symmetric matrices of same order is always a/an :

(A) skew-symmetric matrix

(B) symmetric matrix

(C) null matrix

(D) identity matrix

5. $\left[\sec^{-1}(-\sqrt{2})-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

6. If $f(x) = \begin{cases} \dfrac{\log(1 + ax) + \log(1 - bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

is continuous at x = 0, then the value of k is :

(A) a
(B) a + b
(C) a – b
(D) b

7. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is :

(A) $\frac{x}{y}$
(B) $-\frac{x}{y}$
(C) $\frac{a}{x}$
(D) $\frac{a}{y}$

8. If y = a cos (log x) + b sin (log x), then $x^2y_2 + xy_1$ is :

(A) cot (log x)
(B) y
(C) – y
(D) tan (log x)

9. Let f(x) = |x|, x ∈ R. Then, which of the following statements is ***incorrect*** ?

(A) f has a minimum value at x = 0.
(B) f has no maximum value in R.
(C) f is continuous at x = 0.
(D) f is differentiable at x = 0.

10. Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, f(1) = 0. Then, f(x) is :

(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$
(B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$
(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$
(D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

11. $\int \frac{x + 5}{(x + 6)^2} e^x \, dx$ is equal to :

(A) log (x + 6) + C
(B) $e^x + C$
(C) $\frac{e^x}{x + 6} + C$
(D) $\frac{-1}{(x + 6)^2} + C$

12. The order and degree of the following differential equation are, respectively :

$$-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$$

(A) $-4, 1$

(B) 4, not defined

(C) 1, 1

(D) 4, 1

13. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is :

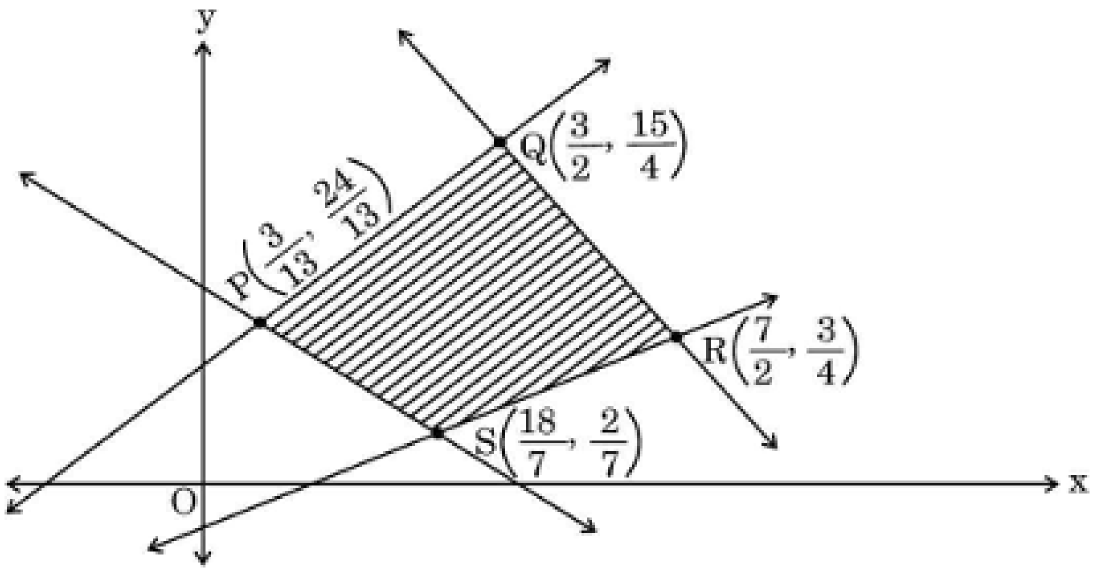
(A) $3e^{4y} + 4e^{-3x} + C = 0$

(B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$

(D) $3e^{-4y} + 4e^{3x} + 12C = 0$

14. For a Linear Programming Problem (LPP), the given objective function is Z = x + 2y. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



(Note : The figure is not to scale)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at S$\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at R$\left(\frac{7}{2}, \frac{3}{4}\right)$

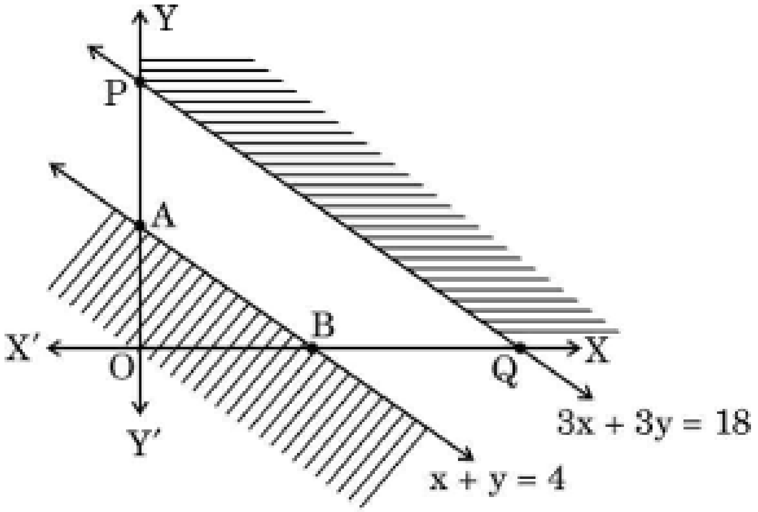
(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

15. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$x + y \leq 4, \; 3x + 3y \geq 18, \; x, y \geq 0$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

16. Let $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

17. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units

(C) 3 sq units (D) $\frac{4}{3}$ sq units

18. A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is :

(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2 \theta + \cos^2 \theta = 1$ and

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin \theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos \theta$.

20. *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

22. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

23. Determine the values of x for which $f(x) = \dfrac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function.

24. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

25. Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda\,(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu\,(2\hat{i} + 3\hat{k})$ intersect with each other.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Let $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ be two matrices. Then, find the matrix B if AB = C.

27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

28. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \rightarrow N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if n is even} \\ n+1, & \text{if n is odd} \end{cases}$ is a bijection.

29. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$$2x + y \geq 1000$$

$$x + 2y \geq 800$$

$$x, y \geq 0.$$

Draw a neat graph of the feasible region and find the minimum value of Z.

30. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

31. A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :

(i) in both committees

(ii) in only one committee

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x^2+1}{(x-1)^2\,(x+3)}\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

33. Draw a rough sketch for the curve $y = 2 + |x + 1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x + 1|$, $x = -4$, $x = 3$ and $y = 0$.

34. (a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

35. Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$.

A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1*

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}. *1*

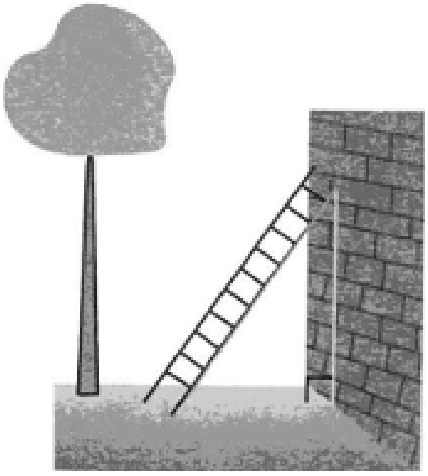
(iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2*

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*

Case Study – 2

37.



A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1*

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2*

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*

Case Study – 3

38. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. *2*

(ii) What is the probability that this defective smartphone was manufactured by company B ? *2*

PAPER 6

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

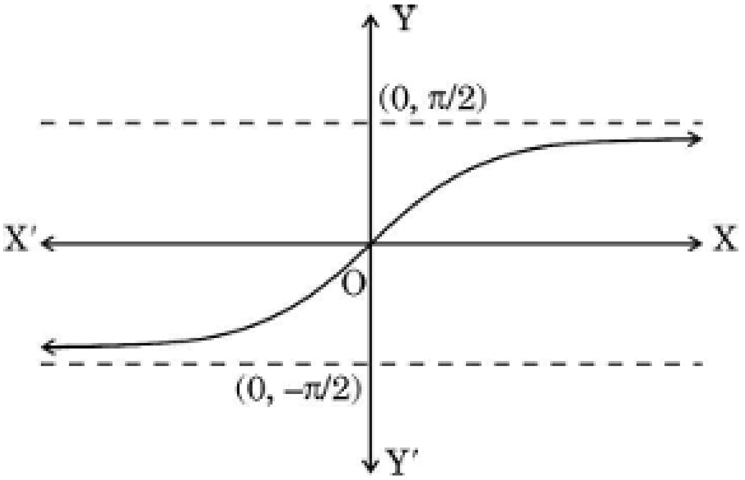
(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The given graph illustrates :



(A) $y = \tan^{-1} x$ (B) $y = \text{cosec}^{-1} x$

(C) $y = \cot^{-1} x$ (D) $y = \sec^{-1} x$

2. Domain of $f(x) = \cos^{-1} x + \sin x$ is :

(A) R (B) $(-1, 1)$

(C) $[-1, 1]$ (D) ϕ

3. What is the total number of possible matrices of order 3 × 3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(A) 9 (B) 512

(C) 615 (D) 64

4. The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an :

(A) scalar matrix (B) identity matrix

(C) null matrix (D) symmetric matrix

5. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is :

(A) 30 (B) 120

(C) 15 (D) 225

6. Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj } A|$ is :

(A) 5 (B) 125

(C) 25 (D) -5

7. If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x-y)$ is :

(A) 2 or 10 (B) -2 or 10

(C) 2 or -10 (D) -2 or -10

8. If $f(x) = \begin{cases} 1, & \text{if } x \le 3 \\ ax+b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \le x \end{cases}$ is continuous in R, then the values of a and b are :

(A) $a = 3,\ b = -8$ (B) $a = 3,\ b = 8$

(C) $a = -3,\ b = -8$ (D) $a = -3,\ b = 8$

9. If $f(x) = -2x^8$, then the correct statement is :

(A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

(C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

10. A spherical ball has a variable diameter $\frac{5}{2}(3x + 1)$. The rate of change of its volume w.r.t. x, when x = 1, is :

(A)	225π	(B)	300π
(C)	375π	(D)	125π

11. If $f : R \to R$ is defined as $f(x) = 2x - \sin x$, then f is :

(A)	a decreasing function	(B)	an increasing function
(C)	maximum at $x = \frac{\pi}{2}$	(D)	maximum at x = 0

12. $\int \frac{e^{9 \log x} - e^{8 \log x}}{e^{6 \log x} - e^{5 \log x}}$ dx is equal to :

(A)	$x + C$	(B)	$\frac{x^2}{2} + C$
(C)	$\frac{x^4}{4} + C$	(D)	$\frac{x^3}{3} + C$

13. For a function f(x), which of the following holds true ?

(A) $\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$

(B) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an even function

(C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function

(D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a + x)\,dx$

14. $\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$ is equal to :

(A) $\frac{1}{2}\cos^{-1}(e^x) + C$

(B) $\frac{1}{2}\sin^{-1}(e^x) + C$

(C) $\frac{e^x}{2} + C$

(D) $\sin^{-1}\left(\frac{e^x}{2}\right) + C$

15. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is :

(A) 6

(B) 1

(C) $\frac{1}{4}$

(D) 4

16. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are :

(A) orthogonal vectors

(B) parallel to each other

(C) unit vectors

(D) collinear vectors

17. If $P(A) = \frac{1}{7}$, $P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(\overline{A} \mid B)$ is :

(A) $\frac{6}{7}$

(B) $\frac{3}{4}$

(C) $\frac{4}{5}$

(D) $\frac{1}{5}$

18. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

(A) $\frac{2}{13}$

(B) $\frac{3}{26}$

(C) $\frac{19}{26}$

(D) $\frac{3}{13}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* $f(x) = \begin{cases} x \sin \frac{1}{x} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$ is continuous at x = 0.

Reason (R) : When $x \to 0$, $\sin \frac{1}{x}$ is a finite value between -1 and 1.

20. *Assertion (A) :* Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R) : $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$.

Discuss the bijectivity of the function.

22. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$.

23. (a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

OR

(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

24. In a Linear Programming Problem, the objective function $Z = 5x + 4y$ needs to be maximised under constraints $3x + y \leq 6$, $x \leq 1$, $x, y \geq 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.

25. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Show that the function $f : R \rightarrow R$ defined by $f(x) = 4x^3 - 5$, $\forall x \in R$ is one-one and onto.

OR

(b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy \text{ is a square of a natural number}, x, y \in N\}$. Determine if the relation R is an equivalence relation.

27. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?

28. Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.

29. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.

30. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

find the minimum value of Z.

31. (a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

OR

(b) Find the shortest distance between the lines :

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k}).$$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x^2+1}{(x^2+2)(2x^2+1)}\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$$

33. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

34. Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.

35. (a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).

OR

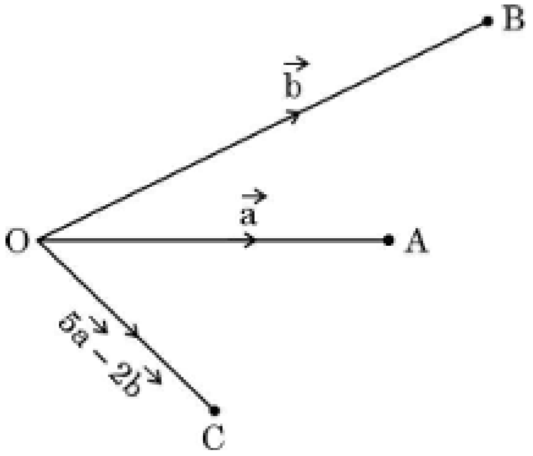
(b) Find the image of the point (−1, 5, 2) in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors. *1*

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. *1*

(iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$. *2*

OR

(iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. *2*

Case Study – 2

37. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.

(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation. *1*

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm. *1*

(iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm. *2*

OR

(iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm. *2*

Case Study – 3

38. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions :

(i) A person was tested randomly. What is the probability that he/she has contracted the disease ? *2*

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? *2*

PAPER 7

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B, 3** questions in Section – **C, 2** questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

This section consists of **20** multiple choice questions, each of **1** mark.

1. Which of the following functions from Z to Z is both one-one and onto ?

(A) $f(x) = 2x - 1$ (B) $f(x) = 3x^2 + 5$

(C) $f(x) = x + 5$ (D) $f(x) = 5x^3$

2. Value of $4 \cos \left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ is

(A) 3 (B) -3

(C) 1 (D) -1

3. If $A = \begin{bmatrix} x & 0 & m \\ y & z & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$, where I is a unit matrix, then $x + y + z + m$ is equal to

(A) 18 (B) 12

(C) 6 (D) 2

4. If $B = \begin{bmatrix} 23 & 41 & 57 \end{bmatrix} \begin{bmatrix} 31 & 42 \\ 53 & 64 \\ 75 & 86 \end{bmatrix}$, then the order of B is :

(A) 3×2 (B) 2×2

(C) 1×3 (D) 1×2

5. If A and B are square matrices of the same order, then $(A - B)^2 = ?$

(A) $A^2 - 2AB + B^2$ (B) $A^2 - AB - BA + B^2$

(C) $A^2 - 2BA + B^2$ (D) $A^2 - AB + BA + B^2$

6. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is

(A) 0 (B) 9

(C) -6 (D) 6

7. If $A^{-1} = \begin{bmatrix} 7 & 2 \\ 8 & 2 \end{bmatrix}$, then matrix A is

(A) $\begin{bmatrix} 2 & -2 \\ -8 & 7 \end{bmatrix}$

(B) $\begin{bmatrix} -7 & 8 \\ 2 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 1 \\ 4 & -\frac{7}{2} \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -1 \\ -4 & \frac{7}{2} \end{bmatrix}$

8. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is

(A) $\frac{-\sqrt{x}}{\sqrt{y}}$

(B) $-\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$

(C) $-\frac{\sqrt{y}}{\sqrt{x}}$

(D) $\frac{-2\sqrt{y}}{\sqrt{x}}$

9. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) -1

10. When x is positive, the minimum value of x^x is

(A) e^e

(B) $\frac{1}{e}$

(C) $e^{\frac{1}{e}}$

(D) $e^{\frac{-1}{e}}$

11. $\int \frac{2x^3}{4+x^8}\,\mathrm{d}x$ is equal to

(A) $\frac{1}{4}\tan^{-1}\frac{x^4}{2}+C$ (B) $\frac{1}{2}\tan^{-1}\frac{x^4}{2}+C$

(C) $\frac{1}{4}\tan^{-1}\frac{x^4}{4}+C$ (D) $\frac{1}{4}\tan^{-1}x^4+C$

12. $\int e^x\cdot\frac{x}{(1+x)^2}\,\mathrm{d}x$ is equal to

(A) $e^x\cdot\frac{x}{1+x}+C$ (B) $e^x\cdot\frac{1}{1+x}+C$

(C) $e^x\cdot\frac{1}{x}+C$ (D) $e^x\cdot\frac{1}{(1+x)^2}+C$

13. The area of the region bounded by the lines $y = x + 1$, $x = 1$, $x = 3$ and x-axis is

(A) 6 sq units (B) 8 sq units

(C) 7.5 sq units (D) 2 sq units

14. The integrating factor for solving the differential equation $x\cdot\frac{\mathrm{d}y}{\mathrm{d}x}-y=2x^2$ is

(A) x (B) $\frac{1}{x}$

(C) e^{-x} (D) $-\log x$

15. The number of vector(s) of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is (are) :

(A) one (B) two

(C) three (D) infinite

16. Of all the points of the feasible region, for maximum or minimum values of the objective function, the point lies :

(A) inside the feasible region

(B) at the boundary line of the feasible region

(C) at the corner points of the feasible region

(D) at the coordinate axes

17. The common region for the inequalities $x \geq 0$, $x + y \leq 1$ and $y \geq 0$, lies in

(A) IV Quadrant (B) II Quadrant

(C) III Quadrant (D) I Quadrant

18. A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{3}$. The probability that none of them is selected is :

(A) $\frac{11}{15}$ (B) $\frac{7}{15}$

(C) $\frac{8}{15}$ (D) $\frac{1}{5}$

Assertion – Reason Based Questions

Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true and Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A)** : The vectors $\vec{a} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$ are mutually perpendicular vectors.

Reason (R) : Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, if $\vec{a} \cdot \vec{b} = 0$.

20. **Assertion (A)** : $x^2dy = (2xy + y^2)dx$ is a homogeneous differential equation.

Reason (R) : A differential equation of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is a homogeneous differential equation.

SECTION – B

This section consists of **5** very short answer type questions, each of **2** marks.

21. Evaluate : $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

22. (a) Show that the function $f(x) = (x-1)^{\frac{1}{3}}$ is not differentiable at $x = 1$.

OR

(b) Differentiate $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ w.r.t. x.

23. If $y = 7x - x^3$ and x increases at the rate of 2 units per second, then how fast is the slope of the curve changing, when $x = 5$?

24. (a) If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then find $|\vec{a}|$.

OR

(b) Using vectors, find the value of K such that the points (K, –11, 2), (0, –2, 2) and (2, 4, 2) are collinear.

25. Find the angle between the two lines whose equations are $2x = 3y = -z$ and $6x = -y = -4z$.

SECTION – C

In this section there are **6** short answer type questions, each of **3** marks.

26. Find the intervals in which the function

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) strictly increasing

(b) strictly decreasing

27. (a) Find : $\int \frac{x^2 - x + 1}{(x-1)(x^2+1)} dx$

OR

(b) Evaluate : $\int_1^4 (|x| + |3-x|) dx$

28. (a) Find the particular solution of the differential equation, $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1 when $x = 0$.

OR

(b) Solve the differential equation : $2xy\frac{dy}{dx} = x^2 + 3y^2$.

29. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, then find a unit vector perpendicular to both the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

30. The corner points of the feasible region determined by some system of linear inequations, are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = ax + by$, where a, b > 0. Find the condition on a and b so that the maximum of Z occurs at both points (3, 4) and (0, 5).

31. (a) Find the probability distribution of the number of doublets in three throws of a pair of dice.

OR

(b) If E and F are two independent events with P(E) = p, P(F) = 2p and P(exactly one of E, F) = $\frac{5}{9}$, then find the value of p.

SECTION – D

This section consists of **4** long answer type questions, each of **5** marks.

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the system of equations :

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

33. (a) Differentiate $x^{\sin x} + (\sin x)^x$ w.r.t. x.

OR

(b) If $y = x + \tan x$, then prove that

$$\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$$

34. The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$. Find the value of a.

35. (a) Find the shortest distance between the lines given by

$\vec{r} = (4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$

OR

(b) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

SECTION – E

In this section there are **3** case-study based questions of **4** marks each.

36. An architect designs a building for a Company. The design of window on the ground floor is proposed to be different than at the other floors. The window is in the shape of a rectangle whose top length is surmounted by a semi-circular opening. This window has a perimeter of 10 m.

Based on the above information, answer the following :

- (i) If $2x$ and $2y$ represent the length and breadth of the rectangular portion of the window, then establish a relation between x and y.
- (ii) Find the total area of the window in terms of x.
- (iii) (a) Find the values of x and y for the maximum area of the window.

OR

- (iii) (b) If x and y represent the length and breadth of the rectangle, then establish the expression for the area of the window in terms of x only.

37. There are three categories of students in a class of 60 students : A : Very hardworking students, B : Regular but not so hard working, C : Careless and irregular students. It is known that 6 students in category A, 26 in category B and the rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination, is 0.002, of category B it is 0.02 and of category C, this probability is 0.20.

Based on the above information, answer the following :

- (i) Find the probability that a student selected at random is unable to get good marks in the final examination. **2**
- (ii) A student selected at random was found to be one who could not get good marks in the final examination. Find the probability, that this student is NOT of category A. **2**

38. Rajesh, a student of Class-XII, visited an exhibition with his family. There he saw a huge swing and found that it traced the path of a parabola $y = x^2$. The following questions came to his mind. Answer the questions :

(i) Let $f : R \rightarrow R$ be a function defined as $f(x) = x^2$. Find whether f is one-one function.

(ii) Let $f : R \rightarrow R$ be defined as $f(x) = x^2$. Find whether f is an onto function.

(iii) (a) Let $f : N \rightarrow N$ be defined as $f(x) = x^2$. Find whether f is one-one function. Also, find if it is an onto function.

OR

(iii) (b) Let $f : N \rightarrow \{1, 4, 9, 16, ...\}$ defined as $f(x) = x^2$, find where f is one-one function. Also, find if it is an onto function.

CHAPTER 1: RELATION AND FUNCTION

Q.N.	**QUESTIONS**	
1	Let f : N → Y be a function defined as f (x) = 4x + 3, where, Y = {y ∈ N: y = 4x + 3 for all x ∈ N}. Show that given function is one one and onto.	
2	If R = {(a, a^3): a is a prime number less than 5} be a relation. Find the range of R .	
3	Let A = R – {3}, B = R – {1}. Consider the function f: A → B defined by f(x)=$\frac{x-1}{x-3}$. Prove that f is one – one and onto.	
4	Let A = R – {3}, B = R – {2/3} . If f: A → B by f(x) = $\frac{2x-4}{3x-9}$, prove that f is a bijection	
5	If f : W→W defined as f(x)=$\begin{cases} x-1, if\ x\ is\ odd \\ x+1, if\ x\ is\ even \end{cases}$, show that f is one -one and onto.	
6	Prove that the modulus function f(x) = |x| is neither one – one nor onto.	
7	Show that f : R → R given by f(x)=$\frac{x}{x^2+1}$ ∀ x∈ R is neither one – one nor onto.	
8	Let $f: N \to N$ be defined by $(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$ Show that the function is not bijective.	
9	Let f : R → R be the function defined by f (x) = $\frac{1}{2-cosx}$ ∀x ∈R. Then, find the range of f.	
10	Case study questions: Students of Grade 12, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line y=x+4. Let L be the set of all lines which are parallel on the ground and R be a relation on L. Based on the above information, answer the following: a) Let R be a relation such that R= {($L1$, $L2$): $L1$ ∥ $L2$, $L1$, $L2$ ∈ L} .*Is R* an equivalence relation? Why? ‘ b) If $(x) = x + 4$, f: *N to N,hen check f is one to one or onto.* c) Write the range of the following functions: i) f(x) = $x_2 + 1$, $x \in R$ ii) $f(x) = \sqrt{4 - x_2}$, $x \in [-2, 2]$	
11	Let n be a fixed positive integer. Define a relation R in Z as follows: ∀a, b ∈Z, aRb if and only if a – b is divisible by n . Show that R is an equivalence relation.	

WORKSHEET

RELATIONS AND FUNCTIONS

SECTION –A(MCQ)		
Q.N.	**QUESTIONS**	
1	Let f : [2, ∞) → R be the function defined by $f(x) = x^2 – 4x + 5$, then the range of f is (a) R (b) [1, ∞) (c) [4, ∞) (d) [5, ∞)	
2	Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is (a) reflexive but not symmetric (b) reflexive but not transitive (c) symmetric and transitive (d) neither symmetric, nor transitive	
3	Let A = {1, 2, 3}. Then the number of relations containing (1, 2) and (1, 3), which are reflexive and symmetric but not transitive is (a) 1 (b) 2 (c) 3 (d) 4	
4	Let f : R → R be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and – 3, respectively, are (a) φ, {4, – 4} (b) {3, – 3}, φ (c) {4, –4}, φ (d) {4, – 4}, {2, – 2}	
5	Set A has 3 elements, and set B has 4 elements. Then the number of injective mappings that can be defined from A to B is (a) 144 (b) 12 (c) 24 (d) 64	
6	Let R be a relation in the set N given by R={(a,b): a+b=5, b>1}. Which of the following will satisfy the given relation? (a) (2,3) ∈ R (b) (4,2) ∈ R (c) (2,1) ∈ R (d) (5,0) ∈ R	
7	The function $f(x) =x^2+ 4x +4$ is-: (a) even (b) odd (c) neither even nor odd (d)none of these	
8	A function f:N→N is defined by $f(x)= x^2+12$. What is the type of function here? (a) bijective (b) surjective (c) injective (d) neither surjective nor injective	
SECTION –B(2/3 MARKS EACH)		
9	Let R be a relation described on the set of natural numbers N. Determine the domain and range of the relation. Also check whether R is symmetric, reflexive and/or transitive.	

	R = {x, y}: x ∈ N, y ∈ N, 2x+y = 41}	
10	Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.	
11	Let A = { 1 ,2 ,3} and define R = { (a , b) : a + b > 0 } . show that R is universal relation on set A .	
12	Let A = { a, b ,c } how many relation can be define in the set ? How many of these are reflexive ?	
13	Let A = { 2, 4 , 6 , 8} and R ={(a,b): a is greater than b,a,b$\in A$} on the set A . Write R as a set of order pairs , is the relation reflexive ?	
14	Let A = { 2, 4 , 6 , 8} and R = ={(a,b): a is greater than b ; a,b$\in A$ }on the set A . Write R as a set of order pairs , is the relation Symmetric?	
15	Let A = { 1 ,2 ,3} and define R = { (a , b) : a – b = 10 } . show that R is empty relation on set A .	
	LONG ANSWER	
16	Let A = R- {3} and B = R – {1}. Consider the function f: A →B defined by f (x) = (x- 2)/(x -3). Is f one-one and onto? Justify your answer.	
17	Consider a function f: $R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, show that f is bijective function.	
18	Show that the relation R: N X N → N X N defined by (a, b) R (c, d) => a + d = b + c is an equivalence relation.	
	SECTION –E (4 MARKS EACH)	
19	A school is organizing a debate competition with participants as speakers S {S_1 , S_2 , S_3 , S_4 } and these are judged by judges {J_1 , J_2 , J_3 } = . Each speaker can be assigned one judge. Let R be a relation from S to J defined as R {(x , y) : speaker x is judged by judge y; X∈S , y ∈J}. Based on the above, answer the following. (i) How many relations can be there from S to J? (ii) A student identifies a function from S to J as f ={(S_1 , J_1), (S_2 , J_2), (S_3 , J_3), (S_4 , J_4)} . Check if it is bijective. (iii) (a) How many one-one functions can be there from set S to set J? **OR** (b) Another student considers a relation R_1 ={(S_1 , S_2), (S_2 , S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.	4

ANSWERS
MCQ

1.(b) $[1, \infty)$ 2. (a) reflexive but not symmetric 3. **(a) 1**
4. (c) $\{4, -4\}, \varphi$ **5** (c) 24 **6**(a) $(2,3) \in R$
7.(c) neither even nor odd **8.** (c) injective

CASE STUDY

(i) Here n(S)4, n(J) =3 so, n(S XJ) 4 X3 =12 . Therefore, total number of relations from S to J are 2^{12} =4096 .

(ii) Note that f (S_2) =J_2 =f (S_3) . That is, S_2 and S_3 are both mapped to J_2 . Hence, f is not one-one. Also, every element of J have at least one pre-image in S. Hence, f is onto.

(iii) Since f is onto but not one-one, so f is not bijective.

(iv) (iii) (a) As n(S) =4, n(J) =3 = = i.e., n(S) > n(J) .
So, number of one-one functions from set S to set J is 0 (zero).

OR

(iii) (b) For reflexivity, we must add the ordered pairs : (1 ,1),(2, 2),(3, 3),(4, 4), (S_1 , S_1), (S_2 , S_2), (S_3 , S_3), (S_4 , S_4) . Since (S_1 , S_2) ∈R and (S_2 , S_4) ∈R_1 . So, we must not add the ordered pairs (S_2 , S_1) and (S_4 , S_2) in R_1 , otherwise it will become symmetric. Therefore, after adding minimum number of ordered pairs i.e., (S_1 , S_1), (S_2 , S_2), (S_3 , S_3), (S_4 , S_4) in R_1 so that it becomes reflexive but not symmetric, the new relation R_1 becomes R {(S_1 , S_2), (S_2 , S_4), (S_1 , S_1), (S_2 , S_2), (S_3 , S_3), (S_4, S_4)}

ASSERTION AND REASON

In the following question a statement of Assertion (A) is followed by a statement of reason (R). Pick the correct option: A. Both A and R are true and R is the correct explanation of A. B. Both A and R are true but R is not the correct explanation of A. C. A is true but R is false. D. A is false but R is true

1. Assertion (A): If n (A) =p and n (B) = q then the number of relations from A to B is 2pq Reason (R) : A relation from A to B is a subset of A x B

2 Assertion (A): The relation R in the set A = {1, 2, 3, 4, 5, 6} defined as R={ (x, y) :y is divisible by x} is not an equivalence relation. Reason (R) :The relation R will be an equivalence relation, if it is reflexive, symmetric and transitive.

3 Assertion (A) : A relation R ={ (1,1),(1,2),(2,2),(2,3)(3,3)}defined on the set A={1,2,3} is reflexive. Reason (R) : A relation R on the set A is reflexive if (a, a) for all a ∈ A.

4 Assertion (A): If R is the relation in the set A= {1, 2, 3 , 4, 5 } given by R={(a, b): |a - b| is even} R is an equivalence relation. Reason (R) : All elements of {1, 3, 5} are related to all elements of {2,4}

1.Answer: A Solution: A is true - No of elements of AXB = pxq, So the number of relations from A to B is 2pq R is true – every relation from A to B is a sub set of AXB
2.Answer : A Solution: A is true-R is reflexive and transitive but not symmetric ie (2,4)ϵR (4,2)ϵR R-true- Definition of an equivalence relation.

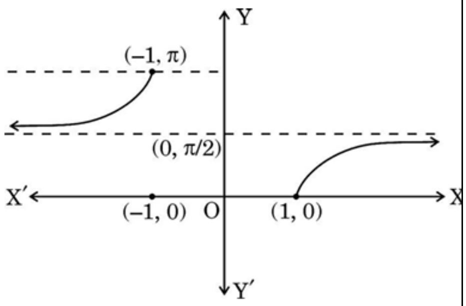
3.Answer: A Solution: A is true - (a,a) ,for all a A R is true – Correct explanation for reflexive relation.
4.Answer: C Solution: A is true- Since it an equivalence relation R is false – the modulus of difference between the two elements from each of these two subsets will not be even

CHAPTER 2: INVERSE TRIGONOMETRIC FUNCTIONS

SECTION A (MCQ)

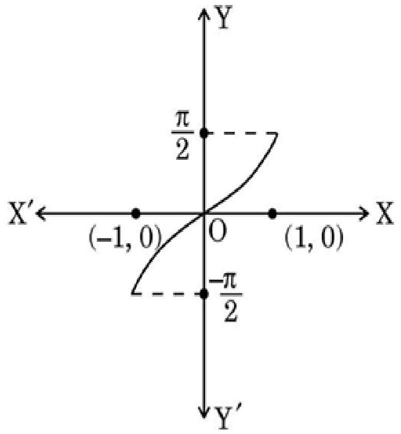
1.	Domain of $\sin^{-1}x + \cos x$ is			
	(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	(b) R	(c) $[-1, 1]$	(d) $(-1,1)$
2.	The value of $\sin^{-1}\left(cos\frac{\pi}{9}\right)$ is			
	(a) $\frac{\pi}{9}$	(b) $\frac{5\pi}{9}$	(c) $-\frac{5\pi}{9}$	(d) $\frac{7\pi}{18}$
3.	The value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is			
	(a) -1	(b) 1	(c) $\frac{\pi}{2}$	(d) 0
4.	The domain of $\sin^{-1}2x$ is			
	(a) [-1, 1]	(b) (-1, 1)	(c) $\left[-\frac{1}{2},\frac{1}{2}\right]$	(d) $\left(-\frac{1}{2},\frac{1}{2}\right)$
5.	Value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$ is equal to			
	(a) $\frac{2\pi}{3}$	(b) $\frac{3\pi}{4}$	(c) $\frac{\pi}{2}$	(d) $\frac{11\pi}{12}$
6.	The range of $\sin^{-1}x + \cos^{-1}x + \tan^{-1}x$			
	(a) $[0,\pi]$	(b) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$	(c) $(0,\pi)$	(d) $\left[0,\frac{\pi}{2}\right]$
7.	Find principal value of $\cos^{-1}\left(\cos\ \frac{5\pi}{4}\right)$			
	(a) $\frac{3\pi}{4}$	(b) $\frac{5\pi}{4}$	(c) $\frac{7\pi}{4}$	(d) $\frac{\pi}{2}$
8.	The domain of function $y = \cos^{-1}x$ is			
	(a) [-1, 1]	(b) $\left[-\frac{1}{2},\ \frac{1}{2}\right]$	(c) [-2, 2]	(d) None of these
9.	If $\tan^{-1}x = \frac{\pi}{10}$, for some $x \in R$, then the value of $\cot^{-1}x$ is			
	(a) $\frac{\pi}{5}$	(b) $\frac{2\pi}{5}$	(c) $\frac{3\pi}{5}$	(d) $\frac{4\pi}{5}$
10.	The value of $\tan^{-1}\left[tan\frac{9\pi}{8}\right]$			
	(a) $\frac{\pi}{8}$	(b) $\frac{\pi}{4}$	(c) $\frac{\pi}{2}$	(d) None of these
11.	One branch of $\cos^{-1}x$,other than the principal value branch corresponds to			
	(a) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$	(b)$[\pi, 2\pi] - \left[\frac{3\pi}{2}\right]$	(c) $[2\pi, 3\pi]$	(d) $(0, \pi)$
12.	The value of $\sin^{-1}\left[cos\frac{43\pi}{5}\right]$ is			
	(a) $\frac{3\pi}{5}$	(b) $-\frac{7\pi}{5}$	(c) $\frac{\pi}{10}$	(d) $-\frac{\pi}{10}$

13. The given graph illustrates :



(a) $\sec^{-1}x$ (b) $\cot^{-1}x$ (c) $\tan^{-1}x$ (d) $\text{cosec}^{-1}x$

14. The given graph illustrates :



(a) $\sec^{-1}x$ (b) $\cos^{-1}x$ (c) $\tan^{-1}x$ (d) $\sin^{-1}x$

In the following questions 13 and 14, a statement of assertion (A) is followed by a statement of reason (R).
Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

15. Assertion: If $2(\sin^{-1}x)^2 - 5(\sin^{-1}x) + 2 = 0$, then x has 2 solution.
Reason: $\sin^{-1}(sinx) = x$, if $-\frac{\pi}{2} \leq x \geq \frac{\pi}{2}$

16. Assertion : Domain of $y = \cos^{-1} x$ is [-1, 1]
Reason : The range of the principal value branch of $y = \cos^{-1} x$ is $[0, \pi] - \left[\frac{\pi}{2}\right]$

17. Assertion: The function y = $\tan^{-1}$x is always increasing.
Reason: Its derivative is always positive.

18. Assertion: $sin^{-1}x + cos^{-1}x = \frac{\pi}{2}$, $\forall\, x \in$ [-1, 1]
Reason: $sin^{-1}x$ and $cos^{-1}x$ are complementary.

SECTION-B (2 MARKS)

19.	Find the value of k if $\sin^{-1}\left[ktan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$
20.	Write $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ in simplest form.
21.	Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$
22.	Find the value of : $\tan^{-1}(1) + \cos^{-1}(-\frac{1}{2})$
23.	Express: $\tan^{-1}\left[\frac{\cos x}{1-\sin x}\right]$ in the simplest form , where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
24.	Evaluate **:** $\sec^2\left(tan^{-1}\frac{1}{2}\right) + \boldsymbol{cosec^2}\left(\cot^{-1}\frac{1}{3}\right)$

SECTION C (3Marks)

25.	Find the value of $\tan^{-1}\left(\frac{\mathbf{1}}{\sqrt{\mathbf{3}}}\right) + \cot^{-1}\left(\frac{\mathbf{1}}{\sqrt{\mathbf{3}}}\right) + \sin^{-1}(-\frac{1}{\sqrt{2}})$
26.	Find the domain and range of the function f(x) = $\sin^{-1}(x^2-4)$.
27.	Find value $of\ \ tan\ (cos^{-1}x\ )$ and hence evaluate $tan\ (cos^{-1}\frac{8}{17})$
28.	Solve for x : $\tan^{-1}\left(\frac{\mathbf{1-x}}{\mathbf{1+x}}\right) = \frac{1}{2}\tan^{-1}x, where\ x > 0$
29.	Find the values of x, which satisfies the equation $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

SECTION-D (5 Marks)

30.	Find the value of $\tan\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right)$, where |x| < 1, y > 0 and xy < 1
31.	(a) Find the value of $\sin^{-1}\left(sin\frac{2\pi}{3}\right) + \cos^{-1}\left(cos\frac{7\pi}{6}\right)$ (b) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$
32.	Show that $2\tan^{-1}\left\{tan\frac{a}{2}.tan\left(\frac{\pi}{4}-\frac{b}{2}\right)\right\} = \tan^{-1}\left(\frac{sin\ a.\ cos\ b}{cos\ a + sin\ b}\right)$
33.	Write the following functions in the simplest form: (a) $\tan^{-1}\left(\frac{\mathbf{\cos x - \sin x}}{\mathbf{\cos x + \sin x}}\right), -\frac{\pi}{4} < x < \frac{3\pi}{4}$ (b) $\tan^{-1}\left(\frac{\mathbf{3a^2x-x^3}}{\mathbf{a^3-3a^2x}}\right)$, $\boldsymbol{a} > 0,\ -\frac{a}{\sqrt{3}} < a < \frac{a}{\sqrt{3}}$

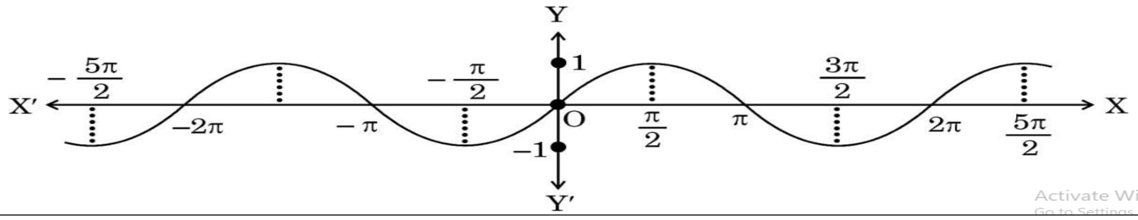
SECTION –E (COMPETENCY BASED QUESTIONS)

34.	If a function f : X → Y defined as f (x) = y is one-one and onto, then we can define a

unique function

g : Y → X such that g(y) = x , where x ∈ X and y = f (x), y ∈ Y . Function g is called the inverse of f. The domain of sine function is R and function sine: R → R is neither one-one nor onto. The following graph shows the sine function.

Let sine function be defined from set A to [–1, 1] such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from [–1, 1] to A. On the basis of above information, answer the following questions.



(i) If A is the interval other than principal value branch, give an example of one such interval.

(ii) If $\sin^{-1}x$ is defined from [–1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(-1)$

(iii) Draw the graph of $\sin^{-1}x$ from [–1, 1] to its principal value branch.

OR

Find the domain and range of f (x) = $2\sin^{-1}(1 - x)$.

35. In a physics experiment, the angle of inclination of a projectile is given by:

θ = $2\tan^{-1}x$ It is also observed that θ = $\sin^{-1}\left[\frac{2x}{1+x^2}\right]$

(i) Is the identity valid for x = 2 ? Justify your answer ?

(ii) Find the range of the of θ = $2\tan^{-1}x$

(iii) Find θ when x = $\frac{1}{\sqrt{3}}$, Convert θ into degrees.

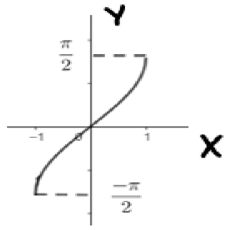
Or

Verify the equivalence of the two expressions.

……

Answer

1. (c) $[-1, 1]$	2. (d) $\frac{7\pi}{18}$	3. (b) 1	4. (c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
5. (d) $\frac{11\pi}{12}$	6. (c) (0,π)	7. (a) $\frac{3\pi}{4}$,	8. (a) [-1, 1]
9. (b) $\frac{2\pi}{5}$	10. (a) $\frac{\pi}{8}$	11. (c) $[2\pi, 3\pi]$	12. (d) $-\frac{\pi}{10}$
13. $(d)\operatorname{cosec}^{-1}x$	14. $(d)\sin^{-1}x$		
15. (c)	16. (c)	17. (a)	18. (a)
19 . $\frac{1}{2}$	20. $\frac{1}{2}\tan^{-1}x$	21. $\frac{\pi}{3}$	22. $\frac{11\pi}{12}$
23. $\frac{\pi}{4} + \frac{x}{2}$	24. $\frac{85}{36}$	25. $\frac{\pi}{4}$	26. domain $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{5}, \sqrt{3}]$ Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

<table>
<tr><td>27. $\frac{15}{8}$</td><td>28. $\frac{1}{\sqrt{3}}$</td><td>29. $\frac{1}{2}$</td><td>30. $\frac{x+y}{1-xy}$</td></tr>
<tr><td colspan="2">31. (a) $\frac{7\pi}{6}$, (b) $\frac{\pi}{4}$</td><td colspan="2">32. $to\ prove$</td></tr>
<tr><td colspan="2">33.(a) $\frac{\pi}{4} - x$, (b) 3$\tan^{-1}\frac{x}{a}$</td><td colspan="2"></td></tr>
<tr><td colspan="2">34. (i) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$or any other interval corresponding to the domain [-1,1]
(ii) $\frac{-2\pi}{3}$
(a)

(iii) b) $[-\pi, \pi]$</td><td colspan="2">35. (i) not valid
(ii) $(-\pi, \pi)$
(iii) $\frac{\pi}{3}$, 60^0</td></tr>
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WORKSHEET

INVERSE TRIGONOMETRIC FUNCTIONS

……

SECTION A (MCQ)

1.	The value of $\sin^{-1}\left(cos\frac{\pi}{9}\right)$ is			
	(a) $\frac{\pi}{9}$	(b) $\frac{5\pi}{9}$	(c) $-\frac{5\pi}{9}$	(d) $\frac{7\pi}{18}$
2.	If $\sec^{-1}x + \sec^{-1}y = 2\pi$, the value of $\text{cosec}^{-1}x + \text{cosec}^{-1}y\ is$			
	(a) π	(b) 2 π	(c) 32π	(d) −π
3.	The domain of the function $\cos^{-1}(2x-1)\ is$			
	(a) [0, 1]	(b) (−1, 1)	(c) [−1, 1]	(d) [0, π]
4.	One branch of $\cos^{-1}x$,other than the principal value branch corresponds to			
	(a) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$	(b)$[\pi, 2\pi] - \left[\frac{3\pi}{2}\right]$	(c) $[2\pi, 3\pi]$	(d) $(0, \pi)$
5.	The value of $\sin^{-1}\left[cos\frac{43\pi}{5}\right]$ is			
	(a) $\frac{3\pi}{5}$	(b) $-\frac{7\pi}{5}$	(c) $\frac{\pi}{10}$	(d) $-\frac{\pi}{10}$
6.	If $\sin^{-1}x = y$, then			
	(a) $0 \leq y \leq \pi$	(b) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	(c) $0 < y < \pi$	(d) $-\frac{\pi}{2} < x < \frac{\pi}{2}$
7.	$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to			
	(a) π	(b) $-\frac{\pi}{3}$	(c) $\frac{\pi}{3}$	(d) $\frac{2\pi}{3}$
8.	The domain of $\sin^{-1}2x$ is			
	(a) [-1, 1]	(b) (-1, 1)	(c) $\left[-\frac{1}{2},\frac{1}{2}\right]$	(d) $\left(-\frac{1}{2},\frac{1}{2}\right)$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R).

Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9.	Assertion (A): $\sin^{-1}\left(sin\frac{5\pi}{3}\right) = \frac{5\pi}{3}$ Reason (R): Inverse trigonometric functions are many-one.
10.	Assertion (A): All trigonometric functions have their inverses over their respective domains. Reason (R): The inverse of $\tan^{-1} x$ exists for some x ∈ R

SECTION-B (2 MARKS)

11.	Find the values of $\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$
12.	If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ then find the value of $\cot^{-1}x + \cot^{-1} y$
13.	Find the value of $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin(-\frac{\pi}{2})\right)$

SECTION C (3Marks)

14.	Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x$ where $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$
15.	Write the function in the simplest form : $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+cosx}}\right)$ where $0 < x < \pi$

SECTION-D (5 Marks)

16.	Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$
17.	Find the values of $\tan\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right)$, where |x| < 1, y > 0 and xy < 1

SECTION –E (COMPETENCY BASED QUESTIONS)

18.	Principal Value of Inverse Trigonometric Functions”. Teacher told that the value of an inverse trigonometric functions which lies in the range of principal branch is called the principal value of that inverse. Based on the given information, answer the following questions (i) Find the Principal value of : $\tan^{-1}[\sin\frac{\pi}{2}]$

(ii) The domain of the function $\cos^{-1}(x)\ is$

(iii) Find the value of $cos[tan^{-1}\left(\frac{3}{4}\right)]$

Or

Find the principal value of **sin** $\left\{\frac{\pi}{6} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$

19. A satellite communication system uses inverse trigonometric functions to calculate signal angles. The ground station needs to determine the elevation angle θ of the satellite above the horizon. If the satellite is at height h = 35,786 km above Earth's surface and the horizontal distance from the ground station is d km, then the elevation angle is given by:

$\theta = \tan^{-1}\left(\frac{h}{d}\right)$

The engineers also need to work with the relationship: $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$

(i) State the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$

(ii) If the horizontal distance d = 35,786 km, find the elevation angle θ.

(iii) The satellite system needs to calculate the phase difference between two signals. If the phase angles are given by $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$ and $\beta = \cos^{-1}\left(\frac{3}{5}\right)$, find the value of α + β.

Or

During signal transmission, the engineers encounter the equation: $2sin^{-1}(x) = cos^{-1}(2x^2 - 1)$. Find all possible values of x that satisfy this equation

CHAPTER 3 & 4: MATRICES ANDDETERMINANTS

S.N	QUESTIONS
	MCQ(1 MARK)
1	What is the total numbers of possible matrices of order 3x3 with each entry as 2,3,4? (a) 27 (b) 19683 (c) 19386 (d) 81
2	If matrix A is both symmetric and skew symmetric , then (a) A is diagonal matrix (b) A is square and zero matrix (c) A is square matrix (d) None of these
3	If A = $\begin{bmatrix}1 & 3\\3 & 4\end{bmatrix}$ and A^2 – k A – 5 I = 0 then the value of k is (a) 3 (b) 7 (c) 5 (d) 9
4	Suppose a 3 X 3 matrix A = $[a_{ij}]$, is formed using 0,1,2 as its elements .The number of such matrices which are skew -symmetric ,is (a) 27 (b) 3 (c) 729 (d) 81
5	If A and B are symmetric matrices of same order , then AB – BA is a (a) Skew symmetric matrix (b) Symmetric matrix (c) Zero matrix (d)Identity matrix
6	Choose the correct statement: (a) Every identity matrix is a scalar matrix . (b) Every scalar matrix is a identity matrix. (c) Each diagonal matrix is a identity matrix. (d) A square matrix with all the elements 1 is an identity matrix.
7	**.**If A is a symmetric matrix,thenA^3 is: **(a)** Symmetric Matrix **(b)** Skew Symmetric Matrix **(c)** Identity matrix **(d)**Row Matrix
8	If A and B are two square matrices each of order 3 with $\|A\|$ =4 and $\|B\|$ =5 Then $\|2AB\|$ = (a) 40 (b) 80 (c) 160 (d) 18
9	If A = $[2\ \ -3\ \ 4]$, B = $\begin{bmatrix}3\\2\\2\end{bmatrix}$, X = [1 2 3], Y = $\begin{bmatrix}2\\3\\4\end{bmatrix}$ AB + XY equals to (a) [28] (b) [24] (c) [12] (d) [-28]
10	If order of matrix X is $2 \times p$ and order of matrix Z is $n \times n$ and $n = p$, then the order of the matrix $(7X - 5Z)$ is (a) p x 2 (b) 2 x n (c) n x 3 (d) p x n
11	If $\begin{bmatrix}x-y & 2\\x & 5\end{bmatrix} = \begin{bmatrix}2 & 2\\3 & 5\end{bmatrix}$, then value of y is (a) 1 (b) 3 (c) 2 (d) 5
12	If A and B are two matrices such that AB =B and BA =A then $A^2 + B^2$ = ? (a) 2AB (b) 2 BA (c) A+B (d) AB
13	If e$\begin{bmatrix}e^x & e^y\\e^y & e^x\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$, then xy (a) 1 (b) 2 (c) 0 (d) -1

14 If A is square matrix such that A^2 = A , then $(I + A)^2$ - 3 A is

(a) I (b) 2 A (c) 3 I (d) A

15 For any 2x2 matrix ,if A(adjA) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$,then $|A|$ is equal to

(a) 20 (b) 100 (c) 10 (d) 0

16 If the system of equations , x+2y -3z =1 , (k+3) z=3 and (2k+1)x +z =0 is inconsistent , then value of k is

(a) -3 (b) $\frac{1}{2}$ (c) 0 (d) 2

17 .For what value of x, A is the skew symmetric matrix

$$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$$

(a) 1 (b) 3 (c) 2 (d) -2

18 .If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, B = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ then value of adj.(AB) is

(a) $\begin{bmatrix} d & b \\ a & c \end{bmatrix}$ (b) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

19 If a is a singular matrix ,then adjA is

(a) Non singular (b) singular (c) symmetric (d) not defined

20 The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

(a) Skew symmetric matrix (b) Symmetric matrix

(c) Scalar matrix (d) Diagonal matrix

ASSERTION AND REASON (1 MARK)

Each of these questions contains two statements, **Assertion and Reason**. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct

1 **Assertion**: Let A be a 2x2 matrix with non zero entries and let A^2=I,where I is 2x2 unit matrix, then A=A^{-1}

Reason: Determinant of A=1

2 **Assertion:** A null matrix of m x m is symmetric as well as skew symmetric matrix

Reason: If A=$A^{-1}, then$ A is a symmetric matrix and if A=$-A^{-1}$,then A is a skew symmetric matrix**.**

3 **Assertion:** Let A and B be 2 symmetric matrices of order 3 then A(BA) and (AB)A

Reason: AB is symmetric matrix ,if matrix multiplication of A with B is commutative

4 **Assertion:** Let A be square matrix of 2x2then adj(adjA)=A

Reason: $|adjA|$=$|A|$

<u>SHORT ANSWER QUESTIONS(2/3 MARKS)</u>

1 If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}$] = A + $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ then find the matrix A.

2 If matrix A = [1 2 3] , then write AA^T ,where A^T is the transpose of matrix A.

3	The matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, then find the values of a ,b and c.
4	If A =$\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix.
5	If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then find the value of k.
6	Write a square matrix of order 2, which is both symmetric and skew symmetric.
7	From the following matrix equation, find the value of x: $\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$
8	Ramesh and Suresh are throwing the balls and trying to hit others ball. If Ramesh throws the ball along 2x+5y=1 and Suresh throws the ball along 3x+2y = 7. Is it possible that the balls hit one another. If so find the point where the balls hit, using matrix method.
9	For a 2x2 matrix, A =$[a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$. Write the value of a_{12}.
10	Simplify: cos θ $\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ + sinθ $\begin{bmatrix} sin\theta & -cos\theta \\ cos\theta & sin\theta \end{bmatrix}$.
11	For what value of x, is the matrix A = $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew symmetric matrix.
12	Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.
13	If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, then write the value of x.
14	Find the value of x + y from the following equation : $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$
15	If matrix A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and A^2 = kA then write the value of k.
16	Show that A'A and AA' are both symmetric matrices for any matrix A.
17	If matrix A = If $\begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$,then write the value of λ.
18	If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.
19	Matrix A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find the value of a and b .
20	Find the value of x and y which makes the following pair of matrices equal. $\begin{bmatrix} 2x+y & 3y \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \end{bmatrix}$
21	If A and B are symmetric matrices, such that AB and BA are both defined, then prove that AB-BA is a skew symmetric matrix.
22	For the matrix A = $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, Find $A + A^T$ and verify that it is a symmetric matrix.
23	If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then prove that $A^2 - A + 2I = O$

LONG ANSWER TYPE QUESTIONS (5 MARKS)

1	If A = $\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$ and B = $\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and (A + B) ^{2} = A ^{2} + B ^{2} ,then find the values of a and b.
2	If A = $\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, B = $\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$ $and\ C = \begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$. Find a matrix D such that CD -AB =O.
3	Find the matrix A satisfying the matrix equation $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix} A \begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.
4	Determine the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$
5	Solve the following system of equations by matrix method. 3x – 2y + 3z = 8, 2x + y – z = 1, 4x – 3y + 2z = 4
6	The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.
7	If $A = \begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ find A^{-1}. Using A^{-1} solve the system of equations 2x – 3y + 5z = 11 , 3x + 2y – 4z = – 5 ,x + y – 2z = –3.
8	Solve the following system of equations by matrix method. $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$
9	Find the non -singular matrices A, if it is given that $adjA = \begin{bmatrix}-1 & -2 & 1\\ 3 & 0 & -3\\ 1 & -4 & 1\end{bmatrix}$

CASE BASED QUESTIONS (4 MARKS)

CASE BASED QUESTION

1. A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets A and B. Annual sales are indicated below

Market	**Products (in numbers)**		
	Pencil	**Eraser**	**Sharpener**
A	**10,000**	**2000**	**18,000**
B	**6000**	**20,000**	**8,000**

If the unit Sale price of Pencil, Eraser and Sharpener are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs.0.50 respectively, then,

Based on the above information answer the following:(Attempt any 4)

1) Total revenue of market A
    a. Rs.64,000
    b. Rs.60,400
    c. Rs.46,000
    d. Rs. 40600
2) Total revenue of market B
    a. Rs.35,000
    b. Rs.53,000
    c. Rs.50,300
    d. Rs.30,500
3) Cost incurred in market A
    a. Rs.13,000
    b. Rs.30,100
    c. Rs.10,300
    d. Rs.31,000
4) Profit in market A and B respectively are
    a. (Rs. 15,000, Rs.17,000)
    b. (Rs. 17,000, Rs. 15,000)
    c. (Rs. 51,000, Rs.71,000)
    d. (Rs. 10,000, Rs. 20,000)
5) Gross profit in both market
    a. Rs.23,000
    b. Rs.20,300
    c. Rs.32,000
    d. Rs.30,200

CASE STUDY : 2

Three schools DPS , CVC and KVS decided to organize a fair for collecting money for helping the food victims

They sold handmade fans , mats and plates from recycled material at a cost of ₹ 25 , ₹ 100 and ₹ 50 each respectively. The numbers of articles sold are given as

School / Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the information given above , answer the following questions.

1.What is the total money (in ₹) collected by the school DPS?

(a) 700 (b) 7000 (c) 6125 (d) 7875

2.What is the total amount of money (in ₹) collected by schools CVC and KVS?

(a) 14000 (b) 15725 (c) 21000 (d) 13125

3. What is the total amount of money (in ₹) collected by all three schools DPS , CVC and KVS ?

(a) 15775 (b) 14000 (c) 21000 (d) 17125

4. If the number of handmade fans and plates are interchanged for all the schools , then what is the total money (in ₹) collected by all the schools?

(a) 18000 (b) 6750 (c) 5000 (d) 21250

5. How many articles (in total) are sold by three schools?

(a) 230 (b) 130 (c) 430 (d) 330

CASE STUDY : 3

On her birthday , Seema decided to donate some money to children of an orphanage home.

If there were 8 children less , everyone would have got Rs 10 more. However , if there were 16 children more, everyone would have got Rs 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in ₹)

Based on the information given above , answer the following questions.

1. The equations in terms are

(b) $5x - 4y = 40$, $5x - 8y = - 80$
(c) $5x - 4y = 40$, $5x + 8y = 80$
(d) $5x - 4y = 40$, $5x + 8y = - 80$
(e) $5x + 4y = 40$, $5x - 8y = - 80$

2. Which of following matrix equations represent the information given above?

(a) $\begin{bmatrix} 5 & 4 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

3. The number of children who were given some money by Seema, is
(a) 30 (b) 40 (c) 23 (d) 32

4. How much amount (in ₹) is given to each child by Seema ?
(a) 32 (b) 30 (c) 62 (d) 26

5. How much amount Seema spends in distributing the money to all the students of the Orphanage?
(a) ₹609 (b) ₹ 960 (c) ₹906 (d) ₹ 690

Answer key

MCQ (MULTIPLE CHOICE QUESTIONS)

1.(b) 2.(b) 3.(c) 4.(a) 5.(a) 6.(a) 7.(a) 8. (c) 9. (a) 10.(b)

11. (a) 12.(c) 13.(a) 14.(a) 15.(c) 16.(a) 17.(c) 1 8. (d) 19. (b) 20.(a)

ANSWERS TO ASSERTION REASONING QUESTIONS	
1.	**(c)** Here correct reason is if A=A^{-1},then matrix is involuntary
2.	**(c)** If A=A^T, *then* A is a symmetric matrix and if A=$-A^T$,then A is a skew symmetric matrix
3.	**(b)** Both are correct ,but here the reason which explains the assertion is distributive law
4.	**(b)**Both are correct, but correct reason which explains the Assertion is $\lvert adjA \rvert = \lvert A \rvert^{n-1}$

SHORT ANSWER TYPE QUESTIONS

1 $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$ 2. 14 3. a= -2 , b=0 , c= -3 4. $\alpha = 0$

5 . k=17 6. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 7 x=1 , y=2 8 . Yes , x=3 and y = -1

9 . $a_{12} = \frac{1}{2}$ 10. . $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

11 . x=2 12 . $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

13. $\begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$ 14. x=2, y=9,x+y=1 15 . k=2

16. proof 17. λ =6 18. Proof 19. a= -2/3 , b= 3/2 20. x=3, y=0

21. proof 22. Proof 23. proof

LONG ANSWER TYPE QUESTIONS

1. a=1, b=4 2. D= . $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

3. A =$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ 4. Ans x = 3 , y = -2 & z = -1

5. Ans : x = 1, y = 2 and z = 3. 6. Ans : x = 1, y = 2, z = 3
7. Ans : x = 1, y = 2 , z = 3 8. Ans x = 2, y = 3 , z = 5

9. $A = \pm\frac{1}{\sqrt{3}}\begin{bmatrix} -6 & -1 & 3 \\ -3 & -1 & 0 \\ -6 & -3 & -3 \end{bmatrix}$

CASE STUDY QUESTIONS ANSWERS

Case study question 1 answers
1.(c) 2.(b) 3.(d) 4.(a) 5.(c)
Case study question 2 answers
1.(b) 2.(c) 3.(c) 4.(d) 5.(d)
Case study question 3 answers
1.(a) 2.(c) 3.(d) 4.(b) 5.(b)

WORKSHEET
MATRICES AND DETERMINANTS

Max Marks: 20 **Time: 40 Min**

1. If A is a 2×3 matrix such that AB and AB' both are defined, then find the order of the matrix B. **1**
2. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, find the values a, b and c. **1**
3. Prove that AA' is always a symmetric matrix for any square matrix of A. **1**
4. If A and B are square matrices, each of order 2 such that |A|=3 and |B|= - 2, then write the value of |3AB|. **1**
5. If A is a square matrix of order 3 such that |adj A| = 225, find |A′|. **1**
6. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then find the possible value(s) of x. **1**
7. Find the equation of the line joining A(1,3) and B(0,0) using determinants and find k if D(k,0) is a point such that area of triangle ABD is 3 sq units. **2**
8. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$ **2**
9. Given A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find BA and use it to find the values of x, y, z from given equations: **5**
$x - y = 3, \quad 2x + 3y + 4z = 17, \quad y + 2z = 17$

10. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that: $f(x)f(-y) = f(x-y)$ **5**

Work Sheet –I

1. 3×3 2. (a = - 2, b = 0, c = -3) 4. – 4, 5. ±15, 6. ±6, 7. 3x – y = 0; k = ±2,
8. $[\mathbf{-1} \quad \mathbf{2} \quad \mathbf{1}]$, **9. BA = 6 I; (x = - 14/3, y = - 23/3, z = 37/3).**

CHAPTER 5: CONTINUITY AND DIFFERENTIABILITY

MCQs (1 Mark)

Q.1 If $f(x) = |sinx|$, then

(a) f is everywhere differentiable

(b) f is everywhere continuous but not differentiable at $x = n\pi,\ n \in Z$

(c) f is everywhere continuous but not differentiable at $x = (2n + 1)\frac{\pi}{2}\ ,\ n \in Z$

(d) None of the above

Q.2 The function $f(x) = e^{|x|}$ is

(a) continuous everywhere but not differentiable at $x = 0$

(b) continuous and differentiable everywhere

(c) not continuous at $x = 0$

(d) None of the above

Q.3 If $y = \sqrt{\sin\,(sin\;x) + y}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{\cos\,(\sin x)\;cos\;x}{2y - 1}$ (b) $\frac{\cos\,(cos\;x)}{1 - 2y}$ (c) $\frac{\cos\,(sin\;x)}{1 - 2y}$ (d) $\frac{\cos\,(sin\;x)}{2y - 1}$

Q.4 The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1}x$ is:

(a) 2 (b) $\frac{-1}{2\sqrt{1-x^2}}$ (c) $\frac{2}{x}$ (d) $1 - x^2$

Q.5 The value of k for which the function $f(x) = \begin{cases} kx.\,\text{cosec}\ 3x, & x \neq 0 \\ 2, & x = 0 \end{cases}$ is continuous at x is equal to:

(a) 3 (b) 0 (c) 6 (d) $\frac{3}{2}$

Q.6 The value of k for which $f(x) = \begin{cases} \frac{1-\,sin^3x}{3cos^2x}, & if\ x \neq \frac{\pi}{2} \\ k, & if\ x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ is

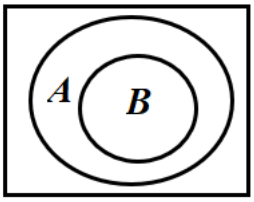
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 0 (d) can not be determined

Q.7 The value of x at which the function $f(x) = \frac{x + 1}{x^2 + x - 12}$ is discontinuous are

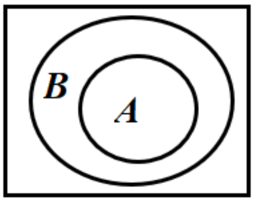
(a) $-3, 4$ (b) $3, -4$ (c) $-1, -3, 4$ (d) $-1, 3, 4$

Q.8 If A denotes the set of continuous functions and B denotes the set of differentiable functions, then which of the following depicts the correct relation between set A and B.

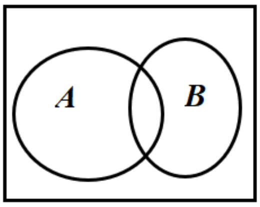
(a)



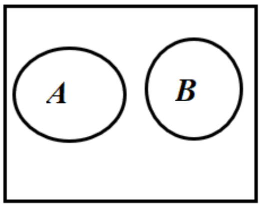
(b)



(c)



(d)



Q.9 The function $f(x) = x|x|,\ x \in R$ is differentiable:

(a) only at $x = 0$ (b) only at $x = 1$ (c) in R (d) in $R - \{0\}$

In the following question, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q.10 **Assertion:** $f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$ for $k = \frac{1}{4}$

Reason: For a function f to be continuous at $x = a$. If $\lim_{x \to a} f(x) = f(a)$.

SA - I (2 Marks)

Q.11 Examine the continuity of the function $f(x) = \begin{cases} \frac{1 - cos2x}{x^2}, & if\ x \neq 0 \\ 5, & if\ x = 0 \end{cases}$

Q.12 Examine the differentiability of the function $f(x) = \begin{cases} x\,[x], & if\ 0 \leq x < 2 \\ (x-1)x, & if\ 2 \leq x < 3 \end{cases}$ at $x = 2$.

Q.13 Find the derivative of the function: $\log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$.

Q.14 Differentiate w.r.t.x: $\cot^{-1}\{\sqrt{1+x^2} + x\}$

Q.15 If $x = \frac{1 + \log t}{t^2},\ y = \frac{3 + 2\log t}{t}$, find $\frac{dy}{dx}$.

Q.16 If $y = \sin x^{\cos^{-1}x}$, find $\frac{dy}{dx}$.

Q.17 If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, prove that:

$$y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$

SA - II (3 Marks)

Q.18 If $y = \sqrt{\frac{1-x}{1+x}}$, prove that $(1 - x^2)\frac{dy}{dx} + y = 0$

Q.19 If $x = a\ sin\ sin\ t\ - b\ cos\ cos\ t,\ y = a\ cos\ cos\ t\ + b\ sin\ sin\ t$, then prove that:

$$\frac{d^2y}{dx^2} = -\left(\frac{x^2+y^2}{y^3}\right).$$

Q.20 If $sin\ y = x\ sin\ (a + y)$, prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$.

Q.21 If $(a + bx)^{\frac{y}{x}} = x$, then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a + bx}\right)^2$

Q.22 If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$.

Q.23 If $x = a\sec^3\theta, y = a\tan^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

LA - (5 Marks)

Q.24 If $y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$, find $\frac{dy}{dx}$.

Q.25 Differentiate: $\tan^{-1}\left\{\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right\}$ w.r.t. $\cos^{-1}x^2$.

Q.26 If $(tan^{-1}x)^y + y^{cot\,x} = 1$, then find $\frac{dy}{dx}$.

Q.27 If $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$, evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$

Answer Keys

MCQs (1 Mark)

Q.1 (b) f is everywhere continuous but not differentiable at $x = n\pi,\ n \in Z$

Let $u(x) = \sin x$ and $v(x) = |x|$

$f(x) = vou(x) = v\{u(x)\} = v(sinx) = |\sin x\,|$

$u(x)$ and $v(x)$ both are continuous. Hence, $f(x) = vou(x)$ is also continuous but $v(x)$ is not differentiable at $x = 0$. So, $f(x)$ is not differentiable at $\sin x\ = 0 \Rightarrow x = n\pi, n \in Z$

Q.2 (a) continuous everywhere but not differentiable at $x = 0$

Let $u(x) =\ |x|$ and $v(x) = e^x$

$f(x) = vou(x) = v\{u(x)\} = v(|x|) = e^{|x|}$

$u(x)$ and $v(x)$ both are continuous. Hence, $f(x) = vou(x)$ is also continuous but $u(x)$ is not differentiable at $x = 0$. So, $f(x)$ is not differentiable at $x = 0$

Q.3 (a) $\frac{\cos(\sin x)\cos x}{2y-1}$

$y^2 = \sin(\sin x) + y \Rightarrow 2y\frac{dy}{dx} = \cos(\sin x)\cos x + \frac{dy}{dx} \Rightarrow (2y-1)\frac{dy}{dx} = \cos(\text{sinx})\cos x \Rightarrow$ $\frac{dy}{dx} = \frac{\cos(sinx)\cos x}{2y-1}$

Q.4 (a) 2

Let $u = \cos^{-1}(2x^2 - 1)$ and $v = \cos^{-1}x$

$\frac{du}{dx} = \frac{-1}{\sqrt{1-(2x^2-1)^2}} \cdot (4x) = \frac{-2}{\sqrt{1-x^2}}$ and $\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}}$

Now, $\frac{du}{dv} = \frac{du}{dx} \cdot \frac{dx}{dv} = \frac{-2}{\sqrt{1-x^2}} \cdot \left(-\sqrt{1-x^2}\right) = 2$

Q.5 (c) 6

$\frac{kx}{\sin 3x} = f(0) \Rightarrow \frac{k}{3}\frac{3x}{\sin 3x} = 2 \Rightarrow \frac{k}{3} = 2 \Rightarrow k = 6.$

Q.6 (b) $\frac{1}{2}$

$$\frac{1-sin^3x}{3cos^2x} = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{(1-sin\,x\,)(1+sin^2x + sin\,x\,)}{3(1-sin^2x)} = k \Rightarrow \frac{(1- sin\,x\,)(1 + sin^2x + sin\,x\,)}{3(1- sin\,x\,)(1 + sin\,x\,)} = k$$

$$\Rightarrow \frac{(1 + sin^2x + \ sin\,x\,)}{3(1 + sin\,x\,)} = k \Rightarrow k = \frac{1}{2}$$

Q.7 (b) $3, -4$

$$f(x) = \frac{x+1}{x^2+x-12} = \frac{x+1}{(x+4)(x-3)}$$

Q.8 (a)

Q.9 (d) in R – {0}

Q.10 (A) Both Assertion (A) and Reason (R) is true and the Reason (R) is the correct explanation of Assertion (A).

$$\frac{\sqrt{4+x}-2}{x} \times \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2} = \frac{4+x-4}{x\sqrt{4+x}+2} = \frac{1}{\sqrt{4+x}+2}$$

$$\frac{1}{\sqrt{4+x}+2} = \frac{1}{4} = f(0) \Rightarrow k = \frac{1}{4}$$

SA - I (2 Marks)

Q.11 $\frac{1- cos\,2x}{x^2} = \frac{2sin^2x}{x^2}$

$$LHL = \left\{\frac{sin\,(0-h)}{0-h}\right\}^2 = 2\left(\frac{sin\,h}{h}\right)^2 = 2$$

$$f(0) = 5$$

Since, $LHL \neq f(0)$. Hence, $f(x)$ is not continuous at $x = 0$

Q.12 $Lf'(2) = \frac{f(2-h)-f(2)}{-h} = \frac{(2-h)[2-h]-(2-1)\,2}{-h} = \frac{(2-h)\,(1)-2}{-h} = 1$

$$Rf'(2) = \frac{f(2+h)-f(2)}{h} = \frac{(2+h-1)(2+h)-(2-1)\,2}{h}$$

$$= \frac{(1+h)\,(2+h)-2}{h} = \frac{h^2+3h+2-2}{h} = (h+3) = 3$$

Since, $Lf'(2) \neq Rf'(2)$

So, $f(x)$ is not differentiable at $x = 2$

Q.13 Let $\frac{\pi}{4}+\frac{x}{2} = v, tan\ v\ = u, we\ get\ y = log\ u$

$\frac{dy}{du} = \frac{1}{u} = \frac{1}{tan\ v} = \frac{1}{tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}$

$\frac{du}{dv} = sec^2 v = sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$\frac{dv}{dx} = \frac{1}{2}$

Now, $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx} = \frac{1}{tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\cdot sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\cdot\frac{1}{2} = \frac{1}{2sin\left(\frac{\pi}{4}+\frac{x}{2}\right)cos\left(\frac{\pi}{4}+\frac{x}{2}\right)} = \frac{1}{sin\left(\frac{\pi}{2}+x\right)} = sec\left(\frac{\pi}{2}+\ x\right)$

Q.14 Let $x = cot\ \theta$, we get

$y = cot^{-1}(cosec\ \theta\ + \cot\theta) = cot^{-1}\left(\frac{1+ cos\theta}{sin\ \theta}\right) = cot^{-1}\left(\frac{2cos^2\left(\frac{\theta}{2}\right)}{2\ sin\frac{\theta}{2}\ cos\frac{\theta}{2}}\right)$

$$= cot^{-1}\left(cot\ \frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}cot^{-1}x$$

$\frac{dy}{dx} = -\frac{1}{2(1+x^2)}$

Q.15 $\frac{dx}{dt} = \frac{-1-2\ log\ t}{t^3}$ and $\frac{dy}{dt} = \frac{-1-2\ log\ t}{t^2}$

Hence, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = t$

Q.16 $log\ y\ = cos^{-1}x\ log\ sinx$

$\frac{1}{y}\frac{dy}{dx} = \frac{cos^{-1}x}{sin\ x}cos\ x\ - \frac{log\ sin\ x}{\sqrt{1-x^2}} \Rightarrow \frac{dy}{dx} = (sin\ x\ )^{cos^{-1}x}\left\{cos^{-1}x\ cot\ x\ - \frac{log\ sin\ x}{\sqrt{1-x^2}}\right\}$

Q.17 $\frac{dx}{d\theta} = -a\ sin\ \theta + b\ cos\ \theta\ = -y$ and $\frac{dy}{d\theta} = a\ cos\ \theta + b\ sin\ \theta\ \ = x$

$\frac{dy}{dx} = -\frac{x}{y}$

$\frac{d^2y}{dx^2} = -\frac{y\ -\ x\frac{dy}{dx}}{y^2} \Rightarrow y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

SA - II (3 Marks)

Q.18 $\frac{dy}{dx}=\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\sqrt{\frac{1+x}{1-x}}\ \frac{(1+x)\frac{d}{dx}(1-x)-(1-x)\frac{d}{dx}(1+x)}{(1+x)^2}=-\sqrt{\frac{1+x}{1-x}}\ \frac{1}{(1+x)^2}$

$(1-x^2)\frac{dy}{dx}=-\sqrt{\frac{1+x}{1-x}}\ \frac{1-x^2}{(1+x)^2}$

$$\Rightarrow(1-x^2)\frac{dy}{dx}=-\sqrt{\frac{1-x}{1+x}}\Rightarrow(1-x^2)\frac{dy}{dx}=-y\Rightarrow(1-x^2)\frac{dy}{dx}+y=0$$

Q.19 $x=a\,sin\,sin\,t\ -b\,cos\,cos\,t\,, y=a\,cos\,cos\,t\ +b\,sin\,sin\,t\,,$

$$\frac{dx}{dt}=a\,cos\,cos\,t\ +b\,sin\,sin\,t\ =y$$

$\frac{dy}{dt}=-a\,sin\,sin\,t\ +b\,cos\,cos\,t\ =-x$

$$\frac{dy}{dx}=-\frac{x}{y}$$

$\frac{d^2y}{dx^2}=-\frac{y\frac{d}{dx}(x)-x\frac{d}{dx}(y)}{y^2}=-\frac{y-x\frac{dy}{dx}}{y^2}=-\frac{y-x\left(-\frac{x}{y}\right)}{y^2}=-\left(\frac{x^2+y^2}{y^3}\right).$

Q.20 $x=\frac{sin\,y}{sin\,(a+y)}$

$\frac{dx}{dy}=\frac{sin\,(a+y)\frac{d}{dy}(sin\,y\,)-\,sin\,y\,\frac{d}{dy}(sin\,(a+y)\,)}{sin^2(a+y)}=\frac{sin\,(a+y)\,cos\,y-sin\,y\,cos\,(a+y)}{sin^2(a+y)}=\frac{sin\,(a+y-y)}{sin^2(a+y)}=\frac{sin\,a}{sin^2(a+y)}$

$\frac{dy}{dx}=\frac{sin^2(a+y)}{sina}$

Q.21 If $(a+bx)^{\frac{y}{x}}=x$, then prove that $x\ \frac{d^2y}{dx^2}=\left(\frac{a}{a\,+\,bx}\right)^2$

$log\,(a+bx)^{\frac{y}{x}}\ =log\,x\ \Rightarrow\frac{y}{x}=log\ \frac{x}{a+bx}$

$\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}=\frac{a+bx}{x}\left\{\frac{(a+bx)\frac{d}{dx}(x)-x\frac{d}{dx}(a+bx)}{(a+bx)^2}\right\}\Rightarrow x\frac{dy}{dx}-y=\frac{ax}{(a+bx)}$

Again, $x\ \frac{d^2y}{dx^2}+\frac{dy}{dx}-\frac{dy}{dx}=\frac{(a+bx)\frac{d}{dx}(ax)-ax\frac{d}{dx}(a+bx)}{(a+bx)^2}=\left(\frac{a}{a+bx}\right)^2$

Q.22 Given that, $x=tan\,\left(\frac{1}{a}\ log\,y\right)\ \Rightarrow a\,tan^{-1}x=log\,y$

Diff. $w.r.t.x$,

$$\frac{a}{1+x^2}=\frac{1}{y}\frac{dy}{dx}\Rightarrow(1+x^2)\frac{dy}{dx}=ay$$

Diff. $w.r.t.x$ again,

$$(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=a\frac{dy}{dx}\Rightarrow(1+x^2)\frac{d^2y}{dx^2}+(2x-a)\frac{dy}{dx}=0$$

Q.23 Here , $x = a\, sec^3\theta\ and\ y = a\, tan^3\theta$

$\frac{dy}{d\theta} = 3a\, tan^2\theta\, sec^2\theta$ and $\frac{dx}{d\theta} = 3a\, sec^3\theta\, tan\,\theta$

$\frac{dy}{dx} = sin\,\theta$

Now, $\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx} = \frac{d}{d\theta}(sin\,\theta\,)\ \cdot\frac{1}{3a\, sec^3\theta tan\theta} = \frac{cos^5\theta}{3a sin\theta}$

At $\theta = \frac{\pi}{4}, \frac{d^2y}{dx^2} = \frac{1}{12a}$

LA (5 Marks)

Q.24 Let $u = x^{cot\,x}$ and $v = \frac{2x^2-3}{x^2+x+2}$

$log\, u\ = cot\,x\, log\,x\ \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{cot\,x}{x} - log\,x\ cosec^2x \Rightarrow \frac{du}{dx} =\ x^{cot\,x}\left(\frac{cot\,x}{x} - log\,x\ cosec^2x\right)$

$\frac{dv}{dx} = \frac{(x^2+x+2)\frac{d}{dx}(2x^2-3)-(2x^2-3)\frac{d}{dx}(x^2+x+2)}{(x^2+x+2)^2} = \frac{2x^2+14x+3}{(x^2+x+2)^2}$

Now, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{cot\,x}\left(\frac{cot\,x}{x} - log\, log\,x\ cosec^2x\right) + \frac{2x^2+14x+3}{(x^2+x+2)^2}$

Q.25 Let $x^2\ = cos\,\theta$

Let, $u = tan^{-1}\left\{\frac{\sqrt{1+cos\,\theta}\ - \sqrt{1-cos\,\theta}}{\sqrt{1+cos\,\theta} - \sqrt{1-cos\,\theta}}\right\} = tan^{-1}\left\{tan\,\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right\} = \frac{\pi}{4}-\frac{1}{2}cos^{-1}x^2$

$\frac{du}{dx} = \frac{x}{\sqrt{1-x^4}}$

And let $v = cos^{-1}x^2$

$\frac{dv}{dx} = \frac{-2x}{\sqrt{1-x^4}}$

Now, $\frac{du}{dv} = -\frac{1}{2}$

Q.26 If $(tan^{-1}x)^y + y^{cot\,x}\ = 1$, to find $\frac{dy}{dx}$. Let u = $(tan^{-1}x)^y$ and v = $y^{cot\,x}$

u + v = 1. Implies $\frac{du}{dx} + \frac{dv}{dx} = 0$

now apply logarithmic derivative method

Q.27 If $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$, Evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$

WORKSHEET

CONTINUITY AND DIFFERENTIABILITY

MCQs (1 Mark)

Q.1 The function $f(x) = \frac{4-x^2}{4x-x^3}$ is

(a) discontinuous at only one point

(b) discontinuous at exactly two points

(c) discontinuous exactly three points

(d) None of the above

Q.2 The set of points where the function f given by $f(x) = |2x-1|\ sinx$ is differentiable is

(a) R

(b) $R - \left(\frac{1}{2}\right)$

(c) $(0,\ \infty)$

(d) None of the above

Q.3 If $y = log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{4x^3}{1-x^4}$

(b) $\frac{-4x}{1-x^4}$

(c) $\frac{1}{4-x^4}$

(d) $\frac{-4x^3}{1-x^4}$

Q.4 The set of all points where the function $f(x) = x + |x|$ is differentiable, is

(a) $(0,\ \infty)$ (b) $(-\infty,\ 0)$ (c) $(-\infty,\ 0) \cup (0,\ \infty)$ (d) $(-\infty,\ \infty)$

Q.5 The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function less than or equal to x is continuous at

(a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$

Q.6 If the function $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k, & x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is

(a) 0 (b) 1 (c) -1 (d) 2

Q.7 The value of b for which the function $f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^2 + 3bx, & 1 < x < 2 \end{cases}$ is continuous at every point of its domain is

(a) -1 (b) 0 (c) $\frac{13}{3}$ (d) 1

Q.8 $\lim\limits_{x \to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$ is equal to

(a) 1 (b) -1 (c) 0 (d) None of these

Q.9 If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is:

(a) -1 (b) 1 (c) $-e$ (d) $-\frac{1}{e}$

Q.10 **Assertion (A):** Let $y = t^{10} + 1$ and $x = t^8 + 1$, then $\frac{d^2y}{dx^2} = 20t^8$

Reason (R): $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

SA-I (2 Marks)

Q.11 Examine the continuity of the function $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & if\ x \neq 4 \\ 0, & if\ x = 4 \end{cases}$

Q.12 Check the differentiability of the function $f(x) = |x - 5|$, at the point $x = 5$.

Q.13 Find the derivative of the function: $log(\sec x + \tan x)$.

Q.14 Differentiate $.r.t.x$: $tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\},\ x \neq 0$

Q.15 If $y = sin^{-1}x + sin^{-1}\sqrt{1-x^2}$, $x \in (0,\ 1)$, find $\frac{dy}{dx}$.

Q.16 If $y = x^{cos^{-1}x}$, find $\frac{dy}{dx}$.

SA-II (3 Marks)

Q.17 Differentiate $w.r.t.x$: $e^{cos^{-1}\sqrt{1-x^2}}$

Q.18 If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

Q.19 If $cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = tan^{-1}a$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Q.20 If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

LSA (5 Marks)

Q.21 If $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$, find $\frac{dy}{dx}$.

Q.22 If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

Q.23 Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $w.r.t.tan^{-1}x.$

Q.24 If $y = A\cos(\log x) + B\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

Q.25 If $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$

Answer Keys

MCQs (1 Mark)

Q.1 (c) discontinuous exactly three points

We have $f(x) = \frac{4-x^2}{4x-x^3} = \frac{4-x^2}{x(2+x)(2-x)}$,

Clearly, $f(x)$ is discontinuous at exactly three points $x = 0, x = 2, x = -2.$

Q.2 (b) $R - \left(\frac{1}{2}\right)$

$Lf'(x) \neq Rf'(x)$ at $x = \frac{1}{2}$, $f(x)$ is not differentiable. Hence, $f(x)$ is differentiable in $R - \left\{\frac{1}{2}\right\}$

Q.3 (b) $\frac{-4x}{1-x^4}$

$y = \log\left(\frac{1-x^2}{1+x^2}\right) = \log(1-x^2) - \log(1+x^2)$

$\frac{dy}{dx} = \frac{-2x}{1-x^2} - \frac{2x}{1+x^2} = \frac{-4x}{1-x^4}$

Q.4 (c) $(-\infty,\ 0) \cup (0,\ \infty)$

$f(x) = \begin{cases} 0, & x < 0 \\ 2x, & x > 0 \end{cases}$

$Lf'(x) \neq Rf'(x)$ at $x = 0$, f is differentiable at $x \in R$ except 0.

Q.5 (b) $x = 1.5$

The function is continuous at all real numbers not equal to integers.

Q.6 (d) 2

$\lim_{x\to1} f(x) = \lim_{x\to1}\frac{x^2-1}{x-1} = f(1)$

$\lim_{x\to1}\frac{(x-1)(x+1)}{x-1} = k$

$\lim_{x\to1} x + 1 = k$

$1 + 1 = k$

$k = 2$

Q.7 (a) -1

$$\lim_{x\to 1^-}(5x-4)=f(1)=1,\ \lim_{x\to 1^+}4x^2+3bx=4+3b$$

$$4+3b=1$$

$$b=-1$$

Q.8 (a) 1

$$\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}=\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(2sin^2x)}}{x}=\lim_{x\to 0}\frac{\sin x}{x}=1$$

Q.9 (a) -1

Diff. $w.r.t.x$

$$e^y+xe^y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=1}=-1$$

Or

$$y=-\log x$$

$$\frac{dy}{dx}=-\frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=1}=-1$$

Q.10 **(**d) Assertion (A) is false but Reason (R) is true

$$\frac{dy}{dt}=10t^9,\frac{dx}{dt}=8t^7$$

$$\frac{dy}{dx}=\frac{5}{4}t^2$$

$$\frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}=\frac{d}{dt}\left(\frac{5}{4}t^2\right)\frac{dt}{dx}=\frac{10t}{4}\cdot\frac{1}{8t^7}=\frac{5}{16t^6}$$

SA-I (2 Marks)

Q.11 $$f(x)=\begin{cases}\frac{-(x-4)}{2(x-4)}=-\frac{1}{2}, & if\ x<4\\ =\frac{1}{2}, & if\ x>4\\ 0, & if\ x=4\end{cases}$$

$$f(4)=0$$

LHL= $\lim_{x\to 4^-}f(x)=-\frac{1}{2}$ and RHL = $\lim_{x\to 4^+}f(x)=\frac{1}{2}$

Clearly, $LHL\neq RHL$

$f(x)$ is not continuous at $x=4$

Q.12 $f(x) = \begin{cases} 5 - x, & if\ x < 5 \\ x - 5, & if\ x \geq 5 \end{cases}$

$$Lf'(x) = \lim_{h \to 0} \frac{f(5-h) - f(5)}{-h} = \lim_{h \to 0} \frac{5-(5-h)-0}{h} = -1$$

$$Rf'(x) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \to 0} \frac{5+h-5-0}{h} = 1$$

Clearly, $Lf'(x) \neq Rf'(x)$, $f(x)$ is not differentiable at $x = 5$.

Q.13 Let $u = \sec x + \tan x$, then $y = logu$

$\frac{du}{dx} = \sec x \tan x + sec^2 x$, $\quad \frac{dy}{du} = \frac{1}{u} = \frac{1}{(\sec x + \tan x)}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{(\sec x + \tan x)}(\sec x \tan x + sec^2 x) = \frac{secx\,(\sec x + \tan x)}{(\sec x + \tan x)} = \sec x$$

Q.14 Let $x = \tan\theta$

$$tan^{-1}\left\{\frac{\sqrt{1+tan^2\theta} - 1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sqrt{sec^2\theta} - 1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sec\theta - 1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$$

$$= tan^{-1}\left\{\frac{2sin^2\,\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\} = tan^{-1}\left\{\tan\frac{\theta}{2}\right\} = \frac{\theta}{2} = \frac{1}{2}tan^{-1}x$$

We have $y = \frac{1}{2}tan^{-1}x$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1+x^2}$$

Q.15 Let $x = \sin\theta$, we have $y = sin^{-1}(\sin\theta) + sin^{-1}(\cos\theta)$

Since, $0 < \sin\theta < 1 \Rightarrow 0 < \theta < \frac{\pi}{2} \Rightarrow 0 < \frac{\pi}{2} - \theta < \frac{\pi}{2}$

Therefore, $y = sin^{-1}(\sin\theta) + sin^{-1}\left\{\sin\left(\frac{\pi}{2} - \theta\right)\right\}$

$$y = \theta + \frac{\pi}{2} - \theta = \frac{\pi}{2}$$

Q.16 Taking log both sides

$\log y = cos^{-1}x \log x$

Using product rule

$$\frac{1}{y}\frac{dy}{dx} = \frac{cos^{-1}x}{x} + \frac{-\log x}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = x^{cos^{-1}x}\left(\frac{cos^{-1}x}{x} + \frac{-\log x}{\sqrt{1-x^2}}\right)$$

SA-II (3 Marks)

Q.17 $y = e^{cos^{-1}\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left\{e^{cos^{-1}\sqrt{1-x^2}}\right\} = e^{cos^{-1}\sqrt{1-x^2}}\frac{d}{dx}\left\{cos^{-1}\sqrt{1-x^2}\right\}$$

$$= e^{cos^{-1}\sqrt{1-x^2}} \cdot \frac{-1}{\sqrt{1-(1-x^2)}} \frac{d}{dx}\left\{\sqrt{1-x^2}\right\}$$

$$= e^{cos^{-1}\sqrt{1-x^2}} \cdot \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2\sqrt{1-x^2}} \frac{d}{dx}(1-x^2)$$

$$= e^{cos^{-1}\sqrt{1-x^2}} \cdot \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2\sqrt{1-x^2}} (-2x)$$

$$= e^{cos^{-1}\sqrt{1-x^2}} \frac{1}{\sqrt{1-x^2}}$$

Q.18 $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 = xy^2 - x^2y$

$\Rightarrow (x-y)(x+y) = -xy(x-y)$

$\Rightarrow x + y = -xy$

$\Rightarrow y + xy = -x$

$\Rightarrow y(1+x) = -x$

$\Rightarrow y = \frac{-x}{1+x}$

Now, $\frac{dy}{dx} = -\left\{\frac{(1+x)\cdot 1 - x(0+1)}{(1+x)^2}\right\} = -\frac{1}{(1+x)^2}$

Q.19 $cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = tan^{-1}a$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \cos(tan^{-1}a) = k, where\ k\ is\ a\ constant$

$\Rightarrow \frac{2x^2}{-2y^2} = \frac{k+1}{k-1}$

$\Rightarrow \frac{x^2}{y^2} = \frac{1+k}{1-k} = m,$ (another constant)

Diff $w.r.t.x$, $\frac{y^2\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(y^2)}{(y^2)^2} = 0$

$$\Rightarrow y^2(2x) - x^2(2y)\frac{dy}{dx} = 0$$

$\Rightarrow 2x^2y\frac{dy}{dx} = 2xy^2$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

Q.20 Taking log both sides

$y \log x = x - y$

$\Rightarrow y \log x + y = x$

$\Rightarrow y(1 + \log x) = x$

$\Rightarrow y = \frac{x}{1+\log x}$

Diff. $w.r.t.x$

$\frac{dy}{dx} = \frac{(1+\log x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\log x)}{(1+\log x)^2} = \frac{(1+\log x)\cdot 1 - x\left(0+\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$

LSA (5 Marks)

Q.21 Let $u = (\sin x)^{\tan x}$ and $v = (\cos x)^{\sec x}$

$\log u = \tan x \log(\sin x)$ and $\log v = \sec x \log(\cos x)$

$\frac{1}{u}\frac{du}{dx} = \tan x \frac{d}{dx}\{\log(\sin x)\} + \log(\sin x)\frac{d}{dx}(\tan x) = \tan x \ \frac{\cos x}{\sin x} + \log(\sin x) \ sec^2 x$

$\frac{du}{dx} = (\sin x)^{\tan x}\{1 + \log(\sin x) \ sec^2 x\}$

and $\frac{1}{v}\frac{dv}{dx} = \sec x \frac{d}{dx}\{\log(\cos x)\} + \log(\cos x)\frac{d}{dx}(\sec x) = \sec x \ \frac{-\sin x}{\cos x} + \sec x \tan x \log(\cos x)$

$\frac{dv}{dx} = (\cos x)^{\sec x}\{\sec x \tan x \log(\cos x) - \sec x \tan x\}$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= (\sin x)^{\tan x}\{1 + \log(\sin x) \ sec^2 x\} + (\cos x)^{\sec x}\{\sec x \tan x \log(\cos x) - \sec x \tan x\}$

Q.22 $\log x = \cos 2t$ and $\log y = \sin 2t$

$\frac{1}{x}\frac{dx}{dt} = -2\sin 2t$ and $\frac{1}{y}\frac{dy}{dt} = 2\cos 2t$

$\frac{dy}{dt} = 2y\cos 2t$ and $\frac{dx}{dt} = -2x \sin 2t$

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2y\cos 2t}{-2x\sin 2t} = -\frac{y\log x}{x \log y}$

Q.23 Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $w.r.t. tan^{-1}x.$

Let $x = \tan\theta$

$tan^{-1}\left\{\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sqrt{sec^2\theta}-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$

$$= tan^{-1}\left\{\frac{2sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\} = tan^{-1}\left\{\tan\frac{\theta}{2}\right\} = \frac{\theta}{2} = \frac{1}{2}tan^{-1}x$$

We have $y = \frac{1}{2}tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)}$

Let $t = tan^{-1}x \Rightarrow \frac{dt}{dx} = \frac{1}{1+x^2}$

$\frac{dy}{dt} = \frac{dy/dx}{dt/dx} = \frac{1/2(1+x^2)}{1/(1+x^2)} = \frac{1}{2}$

Q.24 $\frac{dy}{dx} = \frac{-A\sin(\log x)}{x} + \frac{B\cos(\log x)}{x} \Rightarrow x\frac{dy}{dx} = -A\sin(\log x) + B\cos(\log x)$

$\Rightarrow x\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 1 = -\frac{A\cos(\log x)}{x} - \frac{B\sin(\log x)}{x}$

$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\{A\cos(\log x) + B\sin(\log x)\} = -y$

$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Q.25 If $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$

$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\frac{d}{dx}(sin^{-1}x) - sin^{-1}x\frac{d}{dx}\sqrt{1-x^2}}{1-x^2}$

$(1-x^2)\frac{dy}{dx} = \sqrt{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}} - \frac{sin^{-1}x\,(-2x)}{2\sqrt{1-x^2}} = 1 + \frac{x\,sin^{-1}x}{\sqrt{1-x^2}} = 1 + xy$

$\Rightarrow (1-x^2)\frac{dy}{dx} = 1 + xy$

$\Rightarrow (1-x^2)\,\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x\frac{dy}{dx} + y$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$

CHAPTER 6: APPLICATION OF DERIVATIVES

Topics Covered:
- Rate of Change of Quantities
- Increasing/Decreasing Functions
- Maxima and Minima

Q1. If $y = x^2$, find the rate of change of y with respect to x when $x = 3$.
Q2. The radius of a circle increases at a constant rate of 2 cm/s. Find the rate of change of area when radius is 5 cm.
Q3. If $s = 4t^2 + 2t$, find velocity when $t = 2$.
Q4. A cube's edge increases at 1 cm/s. Find the rate of change of its volume when edge is 4 cm.

Q5. Radius of a sphere is increasing at 0.1 cm/s. Find rate of increase of volume when radius is 7 cm.

Q6. Check whether $f(x) = x^2$ is increasing or decreasing on $(-\infty, \infty)$.
Q7. $f(x) = x^3$, increasing or decreasing?

Q8. Determine intervals where $f(x) = -x^2 + 4x$ is increasing or decreasing.

Q9. Is $f(x) = \sin x$ increasing on $(0, \pi)$?
Q10. Find intervals of increase/decrease for $f(x) = x^3 - 3x$.

Q11. Find local maxima/minima of $f(x) = x^2 - 6x + 9$.

Q12. Maximum/minimum of $f(x) = -x^2 + 4x$.

Q13. Maximum/minimum of $f(x) = x^3 - 3x^2 + 4$.

Q14. Maximum area of a rectangle with perimeter 20 cm.
Q15. Find maximum value of $\sin x + \cos x$ on $[0, \pi/2]$.
Q16. Find minimum value of $x^2 + 9/x^2$, $x \neq 0$.
Q17. Find the maximum of $f(x) = x(10 - x)$.
Q18. A closed rectangular box with square base and volume 1000 cm³. Find minimum surface area.

Q19. $f'(x) = (1 - x^2)/(1 + x^2)^2 \Rightarrow$ Max at $x=1$, Value $= 1/2$

Q20. $h'(t) = 20 - 10t \Rightarrow t=2, h(2)=20$ m

Q.21 Find the altitude of a right circular cone of maximum curved surface of which can be inscribed in a sphere of Radius R.

Q.22 Find the shortest distance between the line $x-y+1 = 0$ and the curve $y^2 = x$.

Q.23 Answer the questions based on the given information.

Two metal rods, R_1 and R_2, of lengths 16 m and 12 m respectively, are insulated at both the ends. Rod R_1 is being heated from a specific point while rod R_2 is being cooled from a specific point. The temperature (T) in Celsius within both rods fluctuates based on the distance (x) measured from either end. The temperature at a particular point along the rod is determined by the equations $T = (16 - x)x$

and $T = (x - 12)x$ for rods R_1 and R_2 respectively, where the distance x is measured in meters from one of the ends.

(i) Find the rate of change of temperature at the mid point of the rod that is being heated.

(ii) Find the minimum temperature attained by the rod that is being cooled.

Q.24 A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.
A tap is connected to such a tank whose conical part is full of water.
Water is dripping out from a tap at the bottom at the uniform rate of $2\ cm^3/s$. The semi-vertical angle of the conical tank is 45° .
On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r.

(ii)Find rate of change of radius at an instant when $r = 2\sqrt{2}$cm.

(iii)Find the rate at which the wet surface of the conical tank is decreasing at an instant

when radius $r = 2\sqrt{2}$ cm.

Q.25 The sides of an equilateral triangle are increasing at the rate of 2 cm/ sec. Find the rate at which the area increases ,when the side is 10cm.

Q.26 The volume of a spherical balloon is increasing at the rate of $3\ cm^3$ / sec. Find the rate of increase of its surface area, when the radius is 2 cm.

SOLUTIONS:

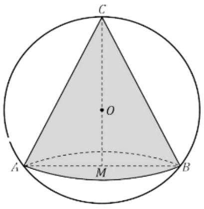
Q.1 $dy/dx = 2x \Rightarrow dy/dx$ at $x=3 = 2\times3 = 6$

Q.2 $A = \pi r^2 \Rightarrow dA/dt = 2\pi r(dr/dt) = 2\pi\times5\times2 = 20\pi\ cm^2/s$
Q.3 $v = ds/dt = 8t + 2 \Rightarrow v(2) = 18$
Q.4 $V = x^3 \Rightarrow dV/dt = 3x^2(dx/dt) = 3\times16\times1 = 48\ cm^3/s$
Q.5 $V = (4/3)\pi r^3 \Rightarrow dV/dt = 4\pi r^2(dr/dt) = 4\pi\times49\times0.1 = 19.6\pi$
Q.6 $f'(x) = 2x \Rightarrow$ Increasing for $x > 0$, Decreasing for $x < 0$
Q.7 $f'(x) = 3x^2 > 0 \Rightarrow$ Increasing everywhere
Q.8 $f'(x) = -2x + 4 \Rightarrow$ Increasing on $(-\infty, 2)$, Decreasing on $(2, \infty)$
Q9. $f'(x) = \cos x \Rightarrow$ Increasing on $(0, \pi/2)$, Decreasing on $(\pi/2, \pi)$
Q.10 $f'(x) = 3x^2 - 3 = 3(x+1)(x-1) \Rightarrow$ Increasing on $(-\infty,-1)\cup(1,\infty)$, Decreasing on $(-1,1)$
Q.11 $f'(x)=2x-6 \Rightarrow x=3$, $f''(x)=2>0 \Rightarrow$ Minima at $x=3$, Min value = 0
Q12 $f'(x) = -2x + 4 \Rightarrow x=2$, $f''(x)=-2<0 \Rightarrow$ Maxima at $x=2$, Max = 4
Q.13 $f'(x) = 3x(x-2) \Rightarrow x=0,2 \Rightarrow$ Max at $x=0$, Min at $x=2$
Q.14 $A = x(10 - x) = 10x - x^2 \Rightarrow$ Max at $x=5$, Area = $25\ cm^2$
Q.15 Max at $x=\pi/4 \Rightarrow$ Value = $\sqrt{2}$
Q16 $f'(x)=2x - 18/x^3 \Rightarrow x = \sqrt{3}$, Min = $2\sqrt{3}$
Q.17 Max at $x=5$, Value = 25
Q.18 $S= 2x^2+4xh$ find s' Minimum surface area 600 cm2

Q.19 $f'(x) = (1 - x^2)/(1 + x^2)^2 \Rightarrow$ Max at x=1, Value = ½

Q.20 $h'(t) = 20 - 10t \Rightarrow t=2, h(2)=20$

Q.21 Let O be the center of the sphere and given that R be radius of sphere.



Let r be radius of cone, h be height of cone and l be slant eight of the cone.

Therefore,

OA=OB=OC=R, CM=h ,AC = BC= l, AM=BM=r .

By right triangle AMO:

$OA^2 == AM^2 + OM^2$

Since OM = CM – OC= h - R

Therefore , $R^2 = r^2 + (h – R)^2 \Longrightarrow r^2 = 2hR- h^2$

Now

Slant height(l)= $\sqrt{\ h^2 + r^2} = \sqrt{2hR}$.

The curved surface area (CSA) of the cone is:

$S=\pi rl=\pi(\sqrt{\ 2hR - h^2})\ \sqrt{2hR}$

To simplify maxima analysis, define:

$Z=S^2=2\pi^2\ Rh(2Rh-h^2\ )$.

Differentiate Z w.r.t. h:

$\frac{dZ}{dh} =2\pi^2\ R(4Rh-3h^2)$.

Put $\frac{dZ}{dh}$ =0:

$4Rh-3h^2 =0 \Longrightarrow h(4R-3h)=0 \Longrightarrow\ h=0$ or $h=\frac{4R}{3}$.

Discard h= 0 (trivial case), so the meaningful critical value is $h =\frac{4R}{3}$.

For $h =\frac{4R}{3} ,\frac{d^2Z}{dh^2} = 2\pi^2\ R(4R-6h) = -8\ (\pi R)^2 <0$

Therefore, by second order derivative test, CSA of cone maximum for height $(h) =\frac{4R}{3}$.

Q.22 Let a point P on the curve be $P(t^2,t)$. The distance from P to the line x−y+1=0 is:

$D(t)=\frac{|t^2-t+1\ |}{\sqrt{\ (1)^2+(-1)^2}} = \frac{t^2-t+1}{\sqrt{2}}$.

since $t^2-t+1> 0$

let $f(t) = \frac{t^2-t+1}{\sqrt{2}}$

$f'(t) = \frac{2t-1}{\sqrt{2}}$

Put f'(t)=0 $\Rightarrow$ 2t−1=0 $\Rightarrow$ t=$\frac{1}{2}$

By using second order derivative test,

f''(t) = $\sqrt{2} > 0$

so t= $\frac{1}{2}$ is a **local minimum**.

At t= $\frac{1}{2}$

$D_{min} = \frac{3}{4\sqrt{2}}$.

Q.23

(i) Given that Rod R_1 length = 16 m

And Temperature function: T(x)=(16−x)x

Midpoint: x=$\frac{16}{2}$ = 8 m

Now $\frac{dT}{dx}$ = 16 – 2x ;

At x = 8m ; $\frac{dT}{dx}$ = 16-2(8) = 0°C/m

The rate of change of temperature at the midpoint of rod R_1 is 0 °C/m, indicating that the temperature is stationary at this point.

(ii) Given that Rod R_2 length = 12 m

And Temperature function: T(x)=(x - 12)x = x^2 – 12x;

$\frac{dT}{dx}$ = 2x - 12 ;

Now for find critical points put $\frac{dT}{dx}$ = 0;

$\Rightarrow$ 2x – 12 = 0 $\Rightarrow$ x = 6

Now Evaluate T(x) at endpoints and critical point:

$T(0)=0^2-12(0)=0$

$T(6)=6^2-12(6)=36-72=-36$

$T(12)=12^2-12(12)=144-144=0$

The minimum temperature attained by rod R_2 is **–36 °C** at **6 meters** from the measured end.

Q.24 (i) Let r be radius of cone and h be height of cone.
Since semi-vertical angle of the conical tank is 45° therefore height of cone (h) = radius of cone (r).

volume of water in the tank = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 r = \frac{1}{3}\pi r^3$.

(ii) Given that $\frac{dV}{dt}$ = -2 cm^3/sec

Volume (V) = $\frac{1}{3}\pi r^3$

Differentiate w.r.t. t, we have

$\frac{dV}{dt} = \frac{1}{3}\pi(3r^2)\frac{dr}{dt} \Longrightarrow -2 = \pi r^2\frac{dr}{dt} \quad \Longrightarrow \frac{dr}{dt} = \frac{-2}{\pi r^2}$

when r = $2\sqrt{2}$cm, $\frac{dr}{dt} = \frac{-2}{\pi(2\sqrt{2})^2} = \frac{-1}{4\pi}$ cm/sec

(iii) Let slant height of cone be l.

Now $l^2 = r^2 + h^2$

Since h = r , $l^2 = r^2 + r^2 = 2r^2 \Longrightarrow l = \sqrt{2}r$

Lateral area (S)=$\pi rl = \pi r \,.\, \sqrt{2}r = \pi\sqrt{2}r^2$

$\frac{dS}{dt} = \sqrt{2}\,\pi(2r)\,\frac{dr}{dt} = 2\sqrt{2}\,\pi r(\frac{-1}{4\pi})$ cm^2/sec

when radius r = $2\sqrt{2}$ cm,

$\frac{dS}{dt} = 2\sqrt{2}\,\pi(\,2\sqrt{2})(\frac{-1}{4\pi})$ cm^2/sec = -2 cm^2/sec

The wet surface area is shrinking at 2 cm²/s.

Q.25 Let x cm be the side and A be the area of the equilateral triangle at time t.

A =$\frac{\sqrt{3}}{4}x^2$

Rate of change (increase) of side x w.r.t. t = $\frac{dx}{dt}$ = 2cm/sec.

Rate of change of area w.r.t. t = $\frac{dA}{dt} = \frac{\sqrt{3}}{4}2x.\frac{dx}{dt} = \frac{\sqrt{3}}{2}x.2 = \sqrt{3}\,x$ cm^2/sec.

Q.26 Let r be the radius, V be the volume and S be the surface area of the spherical balloon at any time t.

V = $\frac{4}{3}\pi r^3$ and S= 4 πr^2

Rate of change (increase) of volume w.r.t. t = 3 cm^3/ sec.

$\frac{dV}{dt} = 3$

Now, V = $\frac{4}{3}\pi r^3$ => $\frac{dV}{dt} = \frac{4}{3}3\pi r^2\frac{dr}{dt}$ => 3= $4\pi r^2\frac{dr}{dt}$

=>$\frac{dr}{dt}$= $3/4\pi r^2$

Now , S= 4 πr^2 => $\frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi r(3/4\pi r^2)$ => $\frac{dS}{dt} = \frac{6}{3}$ =2cm^2/sec.

WORKSHEE:
APPLICATION OF DERIVATIVES

Concepts Covered

- 1. Derivative as Rate of Change
- 2. Increasing and Decreasing Functions
- 3. Tangent and Normal to a Curve
- 4. Maxima and Minima (Local/Global)
- 5. Simple Word Problems using Derivatives

Part A: Multiple Choice Questions (MCQs)

Q1. The slope of the tangent to the curve $y = x^3 - 3x + 2$ at $x = 1$ is:

A) 0 B) 1 C) -2 D) 3

Q2. For the function $f(x) = x^3 - 6x^2 + 9x + 15$, the function is increasing in the interval:

A) $(-\infty, 0)$ B) $(0, 2)$ C) $(2, \infty)$ D) $(0, 1)$

Q3. If the normal to the curve $y = x^2$ at a point (a, a^2) passes through the origin, then a is:

A) 0 B) 1 C) -1 D) ±1

Part B: Short Answer Type Questions

Q4. Find the equation of the tangent and normal to the curve $y = \sqrt{(3x - 2)}$ at the point where $x = 3$.

Q5. Find the point on the curve $y = x^2 + 7x + 10$ at which the tangent is horizontal.

Q6. Show that the function $f(x) = 3x^4 - 4x^3 + 6$ is increasing in $(-\infty, 0) \cup (1, \infty)$.

Q7. A spherical balloon is being inflated so that its volume increases at a rate of 100 cm^3/sec. Find the rate of increase of its radius when radius is 5 cm.

Part C: Long Answer Type Questions

Q8. Find the maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x + 1$ on the interval $[0, 4]$.

Q9. Find two positive numbers whose sum is 60 and whose product is maximum.

Q10. A closed cylindrical tank of volume 256π m^3 is to be made. Find the dimensions (radius and height) of the tank such that the surface area is minimum.

Q11. Find the point on the curve $y = \sqrt{x}$ which is closest to the point $(3, 0)$.

Part D: Previous Year-Based and Model Questions (CBSE 2023–2025 Style)

Q12. (CBSE 2024) The function $f(x) = x^4 - 4x^3 + 10$ has local minima at:

Find critical points and classify them using the second derivative test.

Q13. (CBSE 2023) If the slope of the tangent to the curve $y = ax^2 + bx + c$ at the point (1, 2) is 5, find the value of a and b given $a + b + c = 2$.

Q14. (CBSE 2024) A window is in the shape of a rectangle surmounted by a semicircular opening. The perimeter of the window is 10 m. Find the dimensions for which the area is maximum.

Q15. (Model 2025) Show that the function $f(x) = x/(x + 1)$ is increasing on $(-1, \infty)$.

Q16. The volume of a cube is increasing at the rate of 9 cm^3/sec. How fast is the surface area increasing when the edge is 3 cm?

Part E: Skill-Based Questions (Challenging)

Q17. Find the interval(s) in which the function $f(x) = x/(x^2 + 1)$ is increasing or decreasing.

Q18. Find the minimum distance between the point (0, 0) and the curve $y = x^2 + 1$.

Q19. Find two positive numbers whose product is 256 and whose sum is minimum.

Q20. A cone is being formed by folding a sector of a circle. Show that the cone of maximum volume is obtained when the radius of the sector is three times the slant height of the cone.

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CHAPTER 7: INTEGRAls

INTRODUCTION

IF f(x) is derivative of function g(x), then g(x) is known as antiderivative or integral of f(x)

i.e., $\frac{d}{dx}(g(x)) = f(x) \quad \Leftrightarrow \quad \int f(x)dx = g(x)$

STANDARD SET OF FORMULAS

*** Where c is an arbitrary constant.**

1. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$
2. $\int dx = x + c$
3. $\int \frac{1}{x} \, dx = \log|x| + c$
4. $\int \cos x \, dx = \sin x + c$
5. $\int \sin x \, dx = -\cos x + c$
6. $\int \sec^2 x \, dx = \tan x + c$
7. $\int cosec^2 x \, dx = -\cot x + c$
8. $\int \sec x \tan x \, dx = \sec x + c$
9. $\int cosec\, x \cot x \, dx = -cosec\, x + c$
10. $\int e^x \, dx = e^x + c$
11. $\int \tan x \, dx = \log|\sec x| + c$
12. $\int \cot x \, dx = \log|\sin x| + c$
13. $\int \sec x \, dx = \log|\sec x + \tan x| + c$
14. $\int cosec\, x \, dx = \log|cosec\, x - \cot x| + c$
15. $\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c$
16. $\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$
17. $\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + c$
18. $\int a^x \, dx = \frac{a^x}{\log a} + c$
19. $\int \frac{1}{\sqrt{x}} \, dx = 2\sqrt{x} + c$

INTEGRALS OF LINEAR FUNCTIONS

1. $\int (ax+b)^n \, dx \quad = \frac{(ax+b)^{n+1}}{(n+1)a} + c$

2. $\int \frac{1}{ax+b} \, dx \quad = \frac{\log (ax+b)}{a} + c$

3. $\int \sin (ax+b\,) dx \quad = \frac{-\cos (ax+b)}{a} + c$

In the same way if ax +b comes in the place of x, in the standard set of formulas, then divide the integral by a

SPECIAL INTEGRALS

1. $\int \frac{1}{x^2-a^2} \, dx \quad = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + \mathrm{c}$

2. $\int \frac{1}{a^2-x^2} \, dx \quad = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

3. $\int \frac{1}{x^2+a^2} \, dx \quad = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

4. $\int \frac{1}{\sqrt{x^2-a^2}} \, dx \quad = \log\left| x + \sqrt{x^2-a^2}\right| + c$

5. $\int \frac{1}{\sqrt{x^2+a^2}} \, dx \quad = \log |x + \sqrt{x^2+a^2}| + \mathrm{c}$

6. $\int \frac{1}{\sqrt{a^2-x^2}} \, dx \quad = \sin^{-1}\left(\frac{x}{a}\right) + c$

7. $\int \sqrt{x^2+a^2} \quad dx \quad = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \log\left| x + \sqrt{x^2+a^2}\right| + c$

8. $\int \sqrt{x^2-a^2} \, \mathrm{dx} \quad = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2} \log\left| x + \sqrt{x^2-a^2}\right| + c$

9. $\int \sqrt{a^2-x^2} \, \mathrm{dx} \quad = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$

INTEGRATION BY PARTS

1. $\int u.v \, dx \quad = u.\int v \, dx - \int\left(\frac{du}{dx}\int v \, dx\right) dx$

OR

The integral of product of two functions = (first function) x integral of the second function – integral of [(differential coefficient of the first function) × (integral of the second function)]

We can choose first and second function according to I L A T E where I → inverse trigonometric function $L \rightarrow logarithmic\ function,$ **A** $\rightarrow algebraic\ function$

T $\rightarrow trigonometric\ function$ **E** $\rightarrow exponential\ function$

2. $\int e^x(f(x) + f'(x)) \, dx = e^x f(x) + c$

Working Rule for different types of integrals

1. Integration of trigonometric function

Working Rule

(a) Express the given integrand as the algebraic sum of the functions of the following forms

(i) Sin kα , (ii) cos kα ,(iii) tan kα , (iv) cot kα , (v) sec kα ,(vi) cosec kα, (vii) sec^2 kα , (viii) cosec^2 kα , (ix) sec kα tan kα (x) cosec kα cot kα

For this use the following formulae whichever applicable

(i) $sin^2 x = \frac{1-cos\ 2x}{2}$ (ii) $cos^2 x = \frac{1+cos\ 2x}{2}$

(iii) $sin^3 x = \frac{3\ sin\ x-sin\ 3x}{4}$ (iv) $cos^3 x = \frac{3\ cos\ x+cos\ 3x}{4}$

(v) $\tan^2 x = \sec^2 x - 1$ (vi) $\cot^2 x = \text{cosec}^2 x - 1$

(vii) 2sin x sin y = cos (x – y) – cos (x + y)

(viii) 2 cos x cos y = cos (x + y) + cos (x – y)

(ix) 2 sin x cos y = sin (x + y) + sin (x – y)

(x) 2cos x sin y = sin (x + y) – sin (x – y)

2. Integration by substitution

(a) Consider I = $\int f(x)dx$

Put x = g(t) so that $\frac{dx}{dt} = g'(t)$

We write dx= $g'(t)dt$. Thus I= $\int f(x)dx = \int f(g(t)\ g'(t)\ dt$

(b) When the integrand is the product of two functions and one of them is a function g (x) and the other is k $g'(x)$, where k is a constant then Put g (x) = t

3. Integration of the types $\int \frac{dx}{ax^2+bx+c}$,$\int \frac{dx}{\sqrt{ax^2+bx+c}}$

In these forms change $ax^2 + bx + c$ in the form $A^2 + X^2$, $X^2 - A^2$, or $A^2 - X^2$

Where X is of the form x +k and A is a constant (by completing square method)

Then integral can be find by using any of the special integral formulae.

4. Integration of the types $\int \frac{px+q}{ax^2+bx+c}\ dx$,$\int \frac{px+q}{\sqrt{ax^2+bx+c}}\ dx$

In these forms split the linear px +q = $\lambda \frac{d}{dx}(ax^2 + bx + c) + \mu$

Then divide the integral into two integrals

The first integral can be find out by method of substitution and the second integral by completing square method as explained in 3

i.e. to evaluate $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\ dx = \int \frac{\lambda\ (2ax+b)+\mu}{ax^2+bx+c}$ dx

$$= \lambda \int \frac{2ax+b}{ax^2+bx+c}\ dx + \mu \int \frac{dx}{ax^2+bx+c}$$

↓ ↓

Find by substitution method + by completing square method

5. Integration of rational functions

In the case of rational function, if the degree of the numerator is equal or greater than degree of the denominator , then first divide the numerator by denominator and write it as

$\frac{Numerator}{Denominator} = \boldsymbol{Quotient} + \frac{Remainder}{Denominatior}$**, then integrate**

6. Integration by partial fractions

Integration by partial fraction is applicable for rational functions. There first we must check that degree of the numerator is less than degree of the denominator, if not, divide the numerator by denominator and write as $\frac{Numerator}{Denominator} = \boldsymbol{Quotient} + \frac{Remainder}{Denominatior}$ **and proceed for partial fraction of** $\frac{Remainder}{Denominatior}$

Sl. No.	Form of the rational functions	Form of the rational functions
1	$\frac{px+q}{(x-a)(x-b\,)}$	$\frac{A}{x-a}+\frac{B}{x-b}$
2	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a\,)(x-b\,)x-c\,)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4	$\frac{px^2+qx+r}{(x-a\,)^2(x-b\,)}$	$\frac{A}{x-a}+\frac{B}{(x-a\,)^2}+\frac{C}{x-b}$
5	$\frac{px^2+qx+r}{(x-a\,)(x^2+\ bx+c\,)}$ **Where x^2 + bx + c cannot be factorized further**	$\frac{A}{x-a}+\frac{Bx+C}{x^2+\ bx+c}$

DEFINITE INTEGRATION

Working Rule for different types of definite integrals

1. Problems in which integral can be found by direct use of standard formula or by transformation method

Working Rule

(i). Find the indefinite integral without constant c

(ii). Then put the upper limit b in the place of x and lower limit a in the place of x and subtract the second value from the first. This will be the required definite integral.

2. <u>Problems in which definite integral can be found by substitution method</u>

Working Rule

When definite integral is to be found by substitution then change the lower and upper limits of integration. If substitution is z = φ(x) and lower limit integration is a and upper limit is b Then new lower and upper limits will be φ(a) and φ(b) respectively.

<u>Properties of Definite integrals</u>

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$

2. $\int_a^b f(x)dx = \int_b^a f(x)dx$. **In particular,** $\int_a^a f(x)dx = 0$

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, **a < c < b**

4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

5. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

6. $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

7. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ **, if** $f(2a-x) = f(x)$ **and**

   =0 , **if** $f(2a-x) = -f(x)$

8. **(i)** $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, **if** f **is an even function, i.e., if** $f(-x) = f(x)$

   (ii) $\int_{-a}^a f(x)dx = 0$, **if** f **is an odd function, i.e., if** $f(-x) = -f(x)$

<u>Problem based on property</u>

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \text{ a < c < b}$$

Working Rule

This property should be used if the integrand is different in different parts of the interval [a,b] in which function is to be integrand. This property should also be used when the integrand (function which is to be integrated) is under modulus sign or is discontinuous at some points in interval [a,b]. In case integrand contains modulus then equate the functions whose modulus occur to zero and from this find those values of x which lie between lower and upper limits of definite integration and then use the property.

<u>Problem based on property</u>

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

Working Rule

Let $I = \int_0^a f(x)dx$

Then $\quad\quad I = \int_0^a f(a-x)dx$

(1) + (2) => $\quad\quad 2I = \int_0^a f(x)dx + \int_0^a f(a-x)dx$

$$I = \frac{1}{2}\int_0^a \{f(x) + f(a-x)\}dx$$

This property should be used when $f(x) + f(a-x)$ becomes an integral function of x.

Problem based on property

$\int_{-a}^{a} f(x)dx = 0$, if $f(x)$ is an odd function and $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$, if $f(x)$ is an even function.

Working Rule

This property should be used only when limits are equal and opposite and the function which is to be integrated is either odd/ even

SOLVED PROBLEMS

Evaluate the following integrals

1. $\int \frac{(1+log\,X)^2}{X}\,dx$

Solution : $\quad\quad$ **put 1+log x = t**

$$\frac{1}{x}\,dx = dt$$

$$\int \frac{(1+log\,X)^2}{X}\,dx = \int t^2\,dt$$

$$= \frac{t^3}{3} + c$$

$$= \frac{(1+logx)^3}{3} + c$$

2. $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx$

Solution

Put $e^x = t \;\; then\; e^x\,dx = dt$

$$\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}\,dx = \int \frac{dt}{\sqrt{5-4t-t^2}}$$

$$= \int \frac{dt}{\sqrt{-(t^2+4t-5)}}$$

$$= \int \frac{dt}{\sqrt{-(t^2+4t+4-4-5)}}$$

$$= \int \frac{dt}{\sqrt{-\{(t+2)^2-9\}}}$$

$$= \int \frac{dt}{\sqrt{3^2-(t+2)^2}}$$

$$= \sin^{-1}\frac{t+2}{3} + C = \sin^{-1}\left(\frac{e^x+2}{3}\right) + C$$

3. $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\, dx$

Solution

5x + 3 = A (2x + 4) + B => A = $\frac{5}{2}$ $and\ B = -7$

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}\, dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}\, dx$$

$$= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}\, dx + \int \frac{7}{\sqrt{x^2+4x+10}} dx$$

$$= \frac{5}{2}\int \frac{(2x+4)}{\sqrt{x^2+4x+10}}\, dx + 7\int \frac{1}{\sqrt{x^2+4x+10}} dx$$

$$= \frac{5}{2}\int \frac{dt}{\sqrt{t}} + 7\int \frac{1}{\sqrt{x^2+4x+4-4+10}} dx$$

$$= \frac{5}{2}\times 2\sqrt{t} + 7\int \frac{1}{\sqrt{(x+2)^2+6}} dx$$

$$= 5\sqrt{x^2+4x+10} + 7\log\left|x+2+\sqrt{x^2+4x+10}\right| + C$$

4. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\, dx$

Solution

$$I = \int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\, dx \quad \text{-------------------}(1)$$

Again $I = \int_0^{\pi} \frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\, dx$ **Using the property** $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^{\pi} \frac{(\pi-x)\tan x}{\sec x+\tan x}\, dx \quad \text{--------------------}\ (2)$$

Adding (1) and (2) we get

$$2I = \int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\, dx + \int_0^{\pi} \frac{(\pi-x)\tan x}{\sec x+\tan x}\, dx$$

$$= \pi\int_0^{\pi} \frac{\tan x}{\sec x+\tan x} dx = \pi\int_0^{\pi} \frac{\tan x(\sec x-\tan x)}{\sec^2 x-\tan^2 x}\, dx$$

$$= \pi\int_0^{\pi}(\tan x\sec x-\tan^2 x)\ \text{dx} = \pi\int_0^{\pi}(\tan x\sec x-\sec^2 x+1)dx$$

$$= \pi[\sec x-\tan x+x]_0^{\pi}$$

$$I = \pi\left(\frac{\pi}{2}-1\right)$$

5. $\int \sqrt{tanx}$ dx **[HOTS]**

Solution

Put tan x = t ^{2} then **sec^{2}x dx = 2t dt => dx =** $\frac{2t\,dt}{1+t^4}$

$$\int \sqrt{tanx}\text{ dx} = \int t\,\frac{2t\,dt}{1+t^4} = \int \frac{2t^2}{1+t^4}\,dt$$

$$= \int \frac{2}{\frac{1}{t^2}+t^2}\,dt \quad \text{(by dividing nr and dr by t}^2\text{)}$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)+\left(1-\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}}\,dt$$

$$= \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\,dt + \int \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\,dt$$

$$= \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2}\,dt + \int \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)^2-2}\,dt$$

$$= \int \frac{du}{u^2+2} + \int \frac{dv}{v^2-2}$$

{ 1st integral put $t-\frac{1}{t}$ **= u, then**$\left(1+\frac{1}{t^2}\right)dt = du,$

2nd integral put $t+\frac{1}{t} = v\ then\ \left(1-\frac{1}{t^2}\right)dt = dv$ **}**

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right) + \frac{1}{2\sqrt{2}}log\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right| + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}\,t}\right) + \frac{1}{2\sqrt{2}}log\left|\frac{t^2+1-\sqrt{2}\,t}{t^2+1+\sqrt{2}\,t}\right| + C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{tanx-1}{\sqrt{2\,tanx}}\right) + \frac{1}{2\sqrt{2}}log\left|\frac{tanx+1-\sqrt{2tanx}}{tanx+1-\sqrt{2tanx}}\right| + C$$

PRACTICE PROBLEMS

1 Mark Questions

1. Evaluate: $\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x}dx.$ **Ans:** $tan\,x + c$
2. Evaluate : $\int(\sin^2 x-\cos^2 x)\,dx$ **Ans:** $-\frac{1}{2}sin\,2x + c$
3. Evaluate : $\int_1^\infty \frac{1}{x^2+1}dx$ **Ans:** $\frac{\pi}{4}$
4. Evaluate:$\int \operatorname{cosec} x\,(\cot x-1)e^x dx$ **Ans :**$-e^x cosec\,x + c$
5. Evaluate:$\int \frac{1}{x+x\,\log x}dx$ **Ans:** $log(1+logx)+c$

2/3 Mark Questions

6. Evaluate:$\int \frac{10x^9+10^x log_e 10}{10^x+x^{10}}dx$ **Ans:** $log|10^x+x^{10}|+c$

7. Evaluate:$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$ **Ans:** $\boldsymbol{tan(e^x x)+c}$

8. Find :$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}$dx . **Ans:** $\boldsymbol{\log \sec x . \mathrm{cosec}\, x + c}$

9. Evaluate: Find $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}\, dx$ **Ans:** $\boldsymbol{\frac{2}{3} sin^{-1}\sqrt{\frac{x^3}{a^3}}+c}$

10. Evaluate:$\int_e^{e^2} \frac{dx}{x \log x}$ **Ans:** $\boldsymbol{log\ 2}$

11. Evaluate: $\int \frac{1+\sin 2x}{1+\cos 2x} e^{2x} dx$ **Ans:** $\boldsymbol{\frac{1}{2} e^{2x} tan\, x + c}$

12. Evaluate:$\int \frac{dx}{x(x^3+8)}$ **Ans:** $\boldsymbol{\frac{1}{24} log \left|\frac{x^3}{x^3+8}\right| + c}$

13. Find: $\int \frac{x^4}{(x-1)(x^2+1)} dx$

Ans: $\boldsymbol{\frac{x^2}{2} + x + \frac{1}{2} log\ |x-1| - \frac{1}{4}\ log|x^2+1| - \frac{1}{2} tan^{-1}x + c}$

14. Find: $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$ **Ans:** $\boldsymbol{x\ log(logx) - \frac{x}{logx} + c}$

15. Evaluate:$\int_{-1}^{1} |x \cos \pi\, x|\, dx$ **Ans:** $\boldsymbol{\frac{2}{\pi}}$

16. Evaluate: $\int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$ **Ans:**$\boldsymbol{-3\sqrt{5-2x-x^2} - 2sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c}$

17. Find $\int \frac{x^2+1}{x^4+1} dx$ **Ans:** $\boldsymbol{\frac{1}{\sqrt{2}} tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\, x}\right) + c}$

18. Find:$\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$ **Ans:** $\boldsymbol{-\frac{1}{30} tan^{-1}\left(\frac{sin\,\theta}{2}\right) + \frac{2}{15} tan^{-1}(2sin\theta) + c}$

19. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x . \mathrm{cosec}\, x} dx$ **Ans:** $\boldsymbol{\frac{\pi^2}{4}}$

20. Evaluate :$\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x\ + b^2 \sin^2 x}$ **Ans:** $\boldsymbol{\frac{\pi^2}{2ab}}$

21. Evaluate$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}} dx$ **Ans:** $\boldsymbol{\frac{\pi}{4}}$

22. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin\alpha . \sin x} dx$ **Ans:** $\boldsymbol{\frac{\pi(\pi-2\alpha)}{2\cos\alpha}}$

23. Evaluate: $\int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x})\quad dx$ **Ans :**$\boldsymbol{\sqrt{2}\pi}$

24. Evaluate$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16 \sin 2x} dx$ **Ans:** $\boldsymbol{\frac{1}{40} log 9}$

25. Evaluate:$\int_0^1 \cot^{-1}(1-x+x^2)\, dx$ **Ans:**$\boldsymbol{-\frac{\pi}{2} log\ 2}$

5 Marks Questions

26. Evaluate: $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x\, .\, \cos^2 x}\ dx$ **Ans:** $\boldsymbol{tan\, x - cot\, x - 3x + c}$

27. Find : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$ **Ans:** $\boldsymbol{\frac{6}{5}}$

28. Evaluate :$\int_0^{\frac{\pi}{2}} \log \sin x\, dx$ **Ans:** $\boldsymbol{-\frac{\pi}{2}\ log 2}$

29. Evaluate:$\int_1^4 [|x-1| + |x-2| + |x-3|]\, dx$ **Ans:** $\boldsymbol{\frac{19}{2}}$

30. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx,$ **Ans:** $\boldsymbol{\frac{\pi^2}{16}}$

WORK SHEET

INTEGRALS

INDEFINITE INTEGRAL

1.Given $\int 2^x dx = f(x)+c$ then f(x)=

(a) 2^x (b) $2^x \log e^2$ (c) $\frac{2^x}{\log 2}$ (d) $\frac{2^{x+1}}{x+1}$

2. Given $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is equal to

(a) $\sin^2 x - \cos^2 x + c$ (b) -1

(b) (c) tan x +cot x+ c (d) tan x- cot x +c

3. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$ is equal to

(a) 2(sin x+ x cos θ) +c (b) 2(sin x- x cos θ) + c

(c) 2(sin x +2x cos θ) +c (d) 2 (sin x- sin θ) +c

4. $\int \cot^2 x \, dx$ equals to

(a) Cot x – x +c (b) – cot x+ x +c

(c) cot x +x +c (d) – cot x-x +c

SHORT ANSWER TYPE QUESTIONS

1. Find $\int \frac{3+3\cos x}{x+\sin x} dx$
2. Find $\int \frac{dx}{\sqrt{5-4x-x^2}} dx$
3. Find $\int \frac{x^3-1}{x^2} dx$
4. Find $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$
5. Find $\int \frac{dx}{x^2+16} dx$
6. Find $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$
7. Find $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$
8. Find $\int \sqrt{1 - \sin 2x} \, dx$
9. Find $\int \frac{(x^2+\sin^2 x)\sec^2 x}{1+x^2} dx$
10. Find $\int e^x \frac{x-3}{(x-1)^3} dx$
11. Find $\int \sin^{-1}(2x) dx$
12. Find $\int \frac{3-5\sin x}{\cos^2 x} dx$
13. Find $\int \frac{\tan^2 x . \sec^2 x}{1-\tan^6 x} dx$
14. Find $\int \sin x \log(\cos x) \, dx$
15. Find $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x . \cos^2 x} dx$

LONG ANSWER TYPE QUESTIONS

1. Find $\int \frac{6x+8}{3x^2+6x+2}$ dx
2. Find $\int \frac{1-\tan^2 x}{1+\tan^2 x}$ dx
3. Find $\int \frac{x^4}{1+x^{10}}$ dx
4. Find $\int \frac{x^2 \tan^{-1} x^3}{1+x^6}$ dx
5. Find $\int \frac{\sin 8x}{\sqrt{1-\cos^4 4x}}$ dx

DEFINITE INTEGARL

MCQ's

Q.No	Question	Mark
1	$\int_{-\pi/4}^{\pi/4} \mathrm{Sec}^2 x dx$ (a) -1 (b) 0 (c) 1 (d) 2	1
2	$\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}$ dx is (a) 6 (b)0 (c) 1 (d) 4	1
3	$\int_0^{2/3} \frac{dx}{4+9x^2}$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c)$\pi/24$ (d)$\pi/4$	1
4	$\int_0^1 \frac{dx}{1+x^2}$ is (a) 0 (b) $\pi/4$ (c) $\pi/12$ (d) $\pi/6$	1
5	$\int_{-1}^{1} x^{17} + x^{71}$ dx is (a) 1 (b)0 (c)2 (d) 4	1

Problems for Practice

All the questions carry 3 marks

1 Evaluate $\int_0^1 \frac{\sin x}{1+\sin x} dx$

2 Evaluate $\int_0^1 \cot^{-1}(1-x-x^2) dx$

3 Evaluate $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x})\, dx$

4 Evaluate $\int_{-1}^{3/2} |x \sin \pi x| dx$

5 Evaluate $\int_0^1 \frac{x dx}{1+x^2}$

6 Evaluate $\int_1^3 |2x-1|$ dx

7 Evaluate $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}$ dx

8 Evaluate $\int_{-2}^{2} \frac{x^2 dx}{1+5^x}$

9 Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

10 Evaluate $\int_0^{\pi/2(} 2\log\cos x - \log\sin 2x) dx$

All the questions carry 5 marks

1 Evaluate $\int_{-1}^{2} |x^3 - x| dx$

2 Evaluate $\int_{-6}^{6} |x + 3|\ dx$

3 Evaluate $\int_{0}^{\pi/2} \frac{x\sin x\cos x}{\sin^4 x + \cos^{4x}} dx$

4 Evaluate $\int_{0}^{\pi} \frac{x dx}{a^2\cos^2 x + b^2\sin^2 x} dx$

5 Evaluate $\int_{0}^{\pi} \frac{x\tan x}{\tan x + \sec x} dx$

MCQ

1 $\int_0^2 (x^2+3)dx$ is

(a)8 (b) 25/3 (c)26/3 (d) 9

2 $\int_0^\pi \sin^2 x dx$ is (a)$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d)π

3 $\int_0^\pi \frac{dx}{1+\sin x}$ is (a)$\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c)$e^{\pi/2}$ (d) 2

4 $\int_0^1 \frac{1-x}{1+x} dx$

(a) $\frac{\log 2}{2}$ (b) $\frac{\log 2}{2}$-1 (c) 2log2-1 (d) 2log2 +1

5 $\int_0^{\pi/6} \cos x \cos 2x\ dx$

(a) ¼ (b) 5/12 (c) 1/3 (d)-1/12

6 $\int_0^1 \frac{dx}{e^x+e^{-x}}$

(a) 1-π/4 (b) $\tan^{-1} e$

(c)) $\tan^{-1} e + \pi/4$ (d) $\tan^{-1} e - \pi/4$

7 $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2} - 1$ (c)$\pi/2 + 1$ (d)0

8 $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$

(a) 2 (b) 3/4 (c) 0 (d) -2

9 $\int_0^1 \tan^{-1} \frac{2x-1}{1+x-x^2} dx$

(a) 1 (b) 0 (c) -1 (d)π/4

10 $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} dx$ is

(a)$\frac{\pi}{2}$ (b) π/3 (c) π/4 (d) π

11 $\int_0^{\pi/2} \frac{dx}{1+\tan x}$=

(a) $\frac{\pi}{2}$ (b) π/3 (c) π/4 (d) π

12 $\int_{-1}^{1} \sin^3 x \cos^2 x\ dx$

(a) 0 (b)1 (c)2 (d)3

ASSERTION AND REASONING BASED PROBLEMS

In the following questions a statement of assertion Statement 1 is followed by statement of reason Statement 2 Mark the correct choice as

(a) If statement 1 and statement 2 is true and statement 2 is the correct explanation of 1

(b) If statement 1 and statement 2 is true and statement 2 is not the correct explanation of 1

(c) If statement 1 is true and statement 2 is false

(d) If statement 1 is false and statement 2 is true

Now answer the following

1 Statement I $\int_0^{\pi/2} \sin^2 x dx = \pi/4$

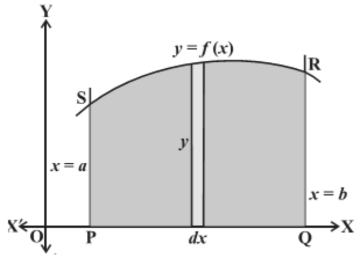
Statement II $\int_0^a f(x)dx = \int_0^a f(a-x)\ dx$

2Statement I $\int_2^3 \frac{\sqrt{x}dx}{\sqrt{x}+\sqrt{5-x}}$ =1/2

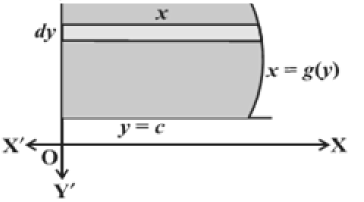
Statement II $\int_0^{2a} f(x)dx$=2$\int_0^a f(x)dx$ if f(x)=f(2a-x)

CHAPTER 8: APPLICATIONS OF INTEGRALS
SOME IMPORTANT RESULTS/CONCEPTS

** Area of the region PQRSP = $\int_a^b dA = \int_a^b y\ dx = \int_a^b f(x)\ dx$.



** The area A of the region bounded by the curve x = g (y), y-axis and the lines y = c, y = d is given by A= $\int_c^d x\ dy = \int_c^d g(y)\ dy$



MCQ

1	Area of region bounded by y= x^3 , x axis , x=1 and x= -2 (a) -9 sq units (b) -1/4 sq units (c) 15/4 sq units (d) 17/4 sq units
2	Area of region bounded by curve y=x and y = x^3 is (a) 1/2 sq units (b) 1 /4 sq units (c) 9/2 sq units (d) 9/4 sq units
3	The area of the region bounded by the parabola y = x^2 and y = |x| is (a) 3 (b) 1 / 2 (c) 1/3 (d) 2
4	The area of the region enclosed by the parabola x^2 = y, the line y = x + 2 and the x-axis, is (a) 5 / 9 (b) 9 / 5 (c) 5 /6 (d) 2 /3
5	The area enclosed by the circle $x^2+y^2 = 2$ is equal to: (a) 4л sq units (b) 2√2л sq units (c) 4л^2 sq units (d) 2л sq units

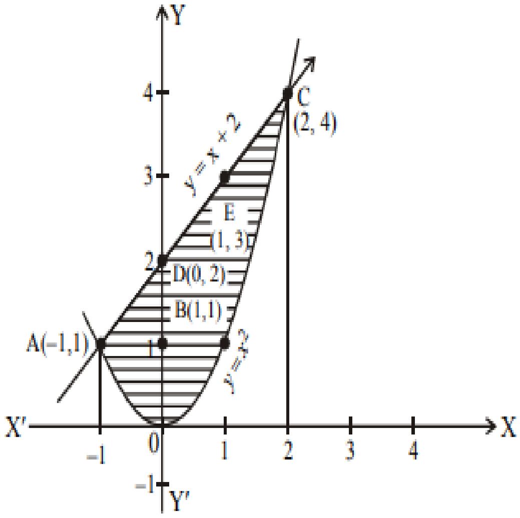
Short Answer type questions (Unsolved)

Q1	Find the area enclosed between curves y = x^2 + 2, y = x, x = 0, x = 3
2	Find the area enclosed between curves y = 4x – x^2 , 0 ≤ x ≤ 4, x-axis
3	Find the area enclosed between curves y = x^2 + 2, y = x, x = 0, x = 3
4	Find the area $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$ { (x , y) : $x^2 + y^2 < 1 < x+y$}
5	Find the area enclosed between curves y = x^3, x = - 2, x = 1, y = 0
6.	Find the area bounded by the curve y = cos x between x = 0 and x = 2π

Long Answer Type Questions :

Q. Find the area of the region enclosed by the parabola x^2 = y, the line y = x + 2 and the x-axis,

Sol. From the given equation
$x^2 = y$ and $y = x + 2$
$\Rightarrow x^2 = x + 2$
$\Rightarrow x^2 - x - 2 = 0$
$\Rightarrow (x-2)(x+1) = 0$
$\Rightarrow x = 2, \ x = -1$



For the parabola with vertex (0,0) and the axis of parabola is y-axis

	A	O	B	C
X	-1	0	1	2
Y	1	0	1	4

For the line $y = x+2$

	A	D	E	C
X	-1	0	1	2
Y	1	2	3	4

So the Required area $= \int_{-1}^{2}(x+2)dx - \int_{-1}^{2} x^2 \, dx$

$= \left[\frac{(x+2)^2}{2}\right]_{-1}^{2} - \left[\frac{x^3}{2}\right]_{-1}^{2}$

$= \frac{1}{2}[16-1]\left[\frac{8}{3} + \frac{1}{3}\right] = \frac{15}{3} - 3 \quad = \frac{9}{2}$

Long Answer Type Questions : (Unsolved)

Q1 .	Using the method of integration find the area bounded by the curve $\lvert x\rvert + \lvert y\rvert = 1$
Q2.	Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = –1 and x = 1.

ASSERTION - REASON TYPE QUESTIONS :

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct

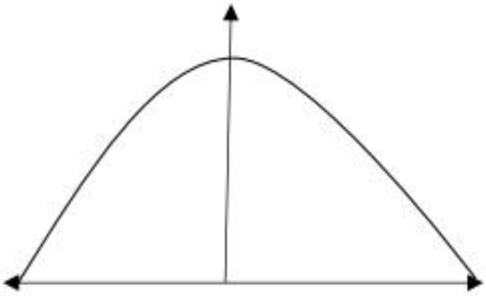
1. **Assertion :** The area bounded by the curve y = cos x in I quadrant x=0, and $x = \frac{л}{2}$ is 1 sq. unit.

Reason : $\int_0^{\frac{л}{2}} \cos x \, dx = 1$

2 **Assertion :** The area bounded by the curves y = sin x and y = –sin x from 0 to л is 3 sq. unit.
Reason : The area bounded by the curves is symmetric about x-axis.

CASE STUDY QUESTION :1

The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure.



Based on the information given above, answer the following questions:

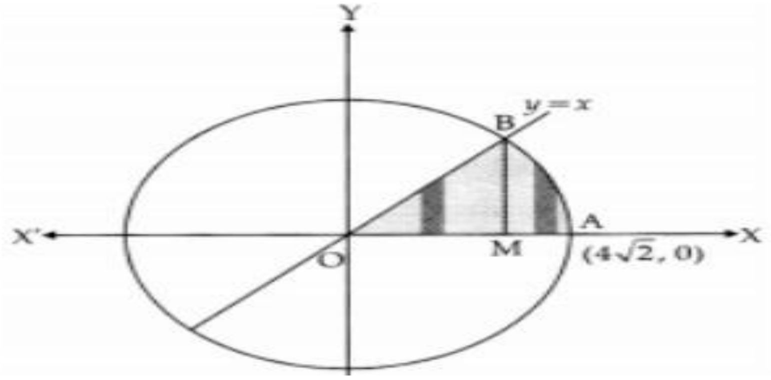
(i) The value of the integral $\int_{-50}^{50} \frac{x^2}{250}\, dx$ is __________.

(ii) The integrand of the integral $\int_{-50}^{50} x^2\, dx$ is _________ (even/odd) function

(iii) The area formed by the curve $x^2 = 250y$, x-axis , y = 0 and y = 10 is _____________ sq units.

CASE STUDY QUESTION : 2

In the figure given below O(0, 0) is the center of the circle. The line y = x meets the circle in the first quadrant at point B. Answer the following questions based on the given figure.



(i) The equation of the circle is ____________.

(ii) The co-ordinates of B are _______________.

(iii) Area of ΔOBM is ____________ sq. units.

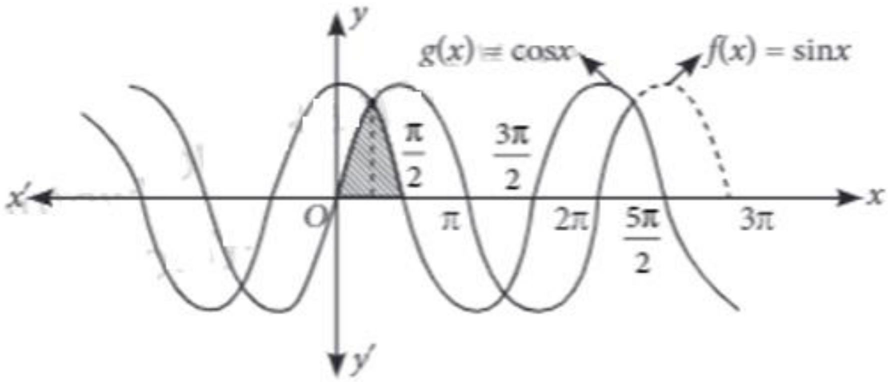
(iv) Area (BAMB) = ____________ sq. units.

(v) Area of the shaded region is ____________ sq. units.

CASE STUDY QUESTION : 3

The Graphs of two functions f(x) = sin x and g(x) = cos x are given below.

Based on the same, answer the following questions.



(i) In [0, π], the curves f(x) and g(x) intersect at x= _______.

(ii) Find the value of $\int_0^{\frac{\pi}{4}} \sin x \, dx$.

(iii) Find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$.

(iv) Find the value of $\int_0^{\pi} \sin x \, dx$.

ANSWER

MCQ

1. **Ans (c) 15 /4**	2. **Ans. (a) 1/2 sq units**	3. **Ans. (c) 1/3**
4. **Ans (c) 5 /6**	5. **Ans. (d) 2л sq units**	

Short Answer type questions (Unsolved)

1	**Ans. 21 /2 sq. units**
2	**Ans. 32 /3 sq. units**
3	**Ans. 21 /2 sq. units**
4	**Ans. π/4 - 1/ 2 sq. units.**
5	**Ans. 15 /4 sq. units**
6	**Ans. 4 sq. units**

Long Answer Type Questions : (Unsolved)

Q1 .	**2 sq. units**
Q2.	**13 /3 sq. units.**

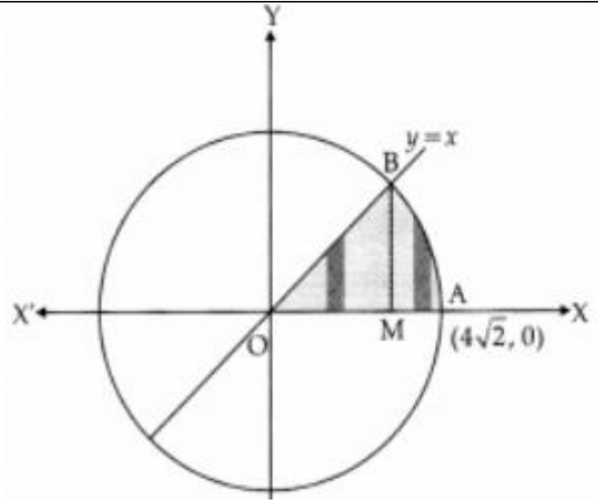
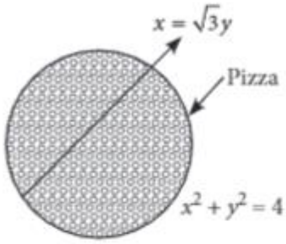
ASSERTION - REASON TYPE QUESTIONS :

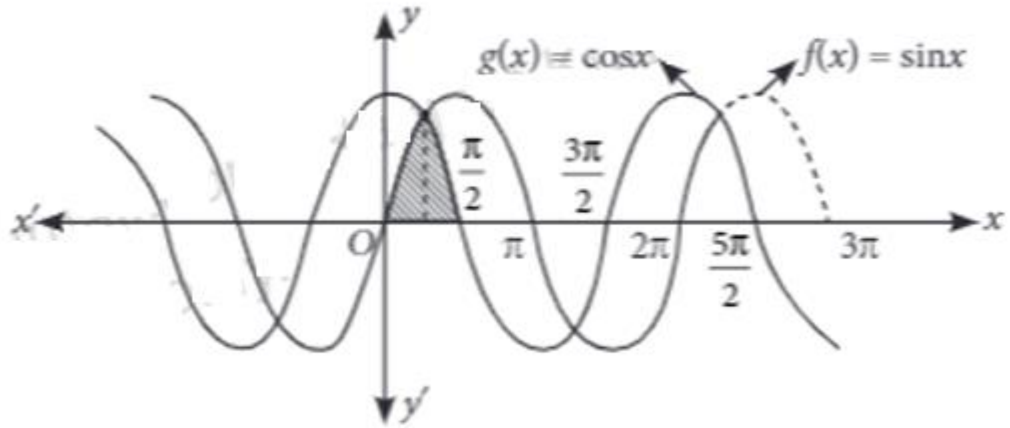
1. Ans (a) $\int_0^{\frac{л}{2}} \cos x\, dx = [\sin x]_0^{\pi/2} = \sin\frac{л}{2} - 0 = 1$
2. Ans. (d)

WORK SHEET

APPLICATIONS OF INTEGRAL

1.	Find the area enclosed by curve $4x^2 + 9y^2 = 36$ (a) 6л sq units (b) 4лsq units (c) 9л sq units (d) 36л sq units
2.	The area enclosed between the graph of $y = x^3$ and the lines $x = 0, y = 1, y = 8$ is (a) 7 (b) 14 (c) 45/4 (d)None of these
3.	The area of the region bounded by the curve $y^2 = x$, the y-axis and between y = 2 and y = 4 is (s) 52/3 sq. units (b) 54/3 sq. units (c) 56/3 sq. units (d) None of these
4.	The Area of region bounded by the curve $y^2 = 4x$, and its latus rectum above x axis (a)0 sq units (b) 4/3sq units (c) 3/3 sq units(d) 2/3 sq units
5.	The Area of region bounded by curve y=x and $y = x^3$ is (a)1/2 sq units (b) 1 /4 sq units (c) 9/2 sq units (d) 9/4 sq units
6.	The area enclosed by the circle $x^2+y^2 = 2$ is equal to: (a)4л sq units (b) $2\sqrt{2}$л sq units (c) $4л^2$ sq units (d) 2л sq units
7.	The area of the region bounded by the parabola $y = x^2$ and $y = \|x\|$ is (a) 3 (b) 1 / 2 (c) 1/3 (d) 2
8.	Find the area enclosed between curves $y = x^2 + 2$, y = x, x = 0, x = 3
9.	Find the area of the region bounded by the curve y=sinx between the lines x=0 , x=π/2 and the x-axis.
10.	Find the area enclosed between curves $y = 4x - x^2$, $0 \le x \le 4$, x-axis
11.	Find the area $\{(x,y): x^2 + y^2 \le 1 \le x + y\}$ { (x , y) : $x^2 + y^2 < 1 < x+y$}
12.	Find the area enclosed between $y^2 = 4ax$ and its latus rectum
13.	Find the area bounded by the curve y = cos x between x = 0 and x = 2π
14.	Find the area of the region bounded by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
15.	Using the method of integration find the area bounded by the curve $\|x\| + \|y\| = 1$
16.	Find the area enclosed between curves $y = x^3$, x = - 2, x = 1, y = 0
17.	Using integration find the area of the region $x^2 + y^2 = 4$ *and* $x = \sqrt{3}y$ with x-axis in first quadrant.
18.	In the figure given below O(0, 0) is the center of the circle. The line y = x meets the circle in the first quadrant at point B. Answer the following questions based on the given figure.

	 (i) The equation of the circle is ____________. (ii) The co-ordinates of B are _______________. (iii) Area of ΔOBM is ____________ sq. units. (iv) Ar (BAMB) = ____________ sq. units. (v) Area of the shaded region is ____________ sq. units.
19.	  A child cuts a pizza with a knife. Pizza is circular in shape which is represented by $\mathrm{x}^2 + \mathrm{y}^2 = 4$ and sharp edge of the knife is represented by $x = \sqrt{3}y$. Based on this information, answer the following questions. (i) The points of intersection of the edge of the knife and the pizza as shown in the figure are _________ and _________. (ii) Which of the following shaded portions represents the smaller area bounded by pizza and edge of knife in the first quadrant?
20.	Graphs of two functions f(x) = sin x and g(x) = cos x are given below. Based on the same, answer the following questions.



(i) In [0, π], the curves f(x) and g(x) intersect at x= _______.

(ii) Find the value of $\int_0^{\frac{\pi}{4}} \sin x \, dx$.

(iii) Find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$.

(iv) Find the value of $\int_0^{\pi} \sin x \, dx$.

CHAPTER 9: DIFFERENTIAL EQUATIONS

EQUATION CONTAINING d/dx ,d^2y/dx^2 d^3y/dx^3 etc.with variables and constants is called differential equation.

Ex- $\frac{dy}{dx} = 3x, \frac{dy}{dx} = x + y, \frac{dy}{dx} = \frac{y}{x}, \frac{d^2y}{dx^2} + y = 0, and\ \frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

$$\frac{d^3y}{dx^3} - \frac{3d^2y}{dx^2} + \frac{5xdy}{dx} = 7x^2$$

*ORDER AND DEGREE-MAX NO OF DIFFERENTIATION DONE IN DIFF EQUATION IS CALLED ORDER AND MAX POWER OF THAT DERIVATIVE IS CALLED DEGREE.

EX: (1) $\frac{d^2y}{dx^2} - \frac{7xdy}{dx} = r^2\sqrt{1 + \frac{d^2y}{dx^2}}$ (2) $(\frac{d^2y}{dx^2})^{\frac{1}{3}} = \left(\frac{d^3y}{dx^3}\right)^{\frac{1}{5}}$;degree and order of diff eq are m and n then $m + n =$?

(3) $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + sin(\frac{dy}{dx}) + 1 = 0$

(4) $\frac{d^2y}{dx^2} + logy = cos3x$ (5) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = xsin(\frac{dy}{dx})$

(5) if p and q are the order and degree of the differential equation

$\left(\frac{dy}{dx}\right)^5 + \frac{4\left(\frac{dy}{dx}\right)^3}{\left(\frac{dy}{dx}\right)^2} + \left(\frac{dy}{dx}\right)^2 = x$ then value of p+q is -----------

(6)The sum of degree and order of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + 1 + x = 0$ is ---

(7)Do the given differential equation have same order degree $6\left(\frac{d^3y}{dx^3}\right)^4 + 9\left(1 + \frac{dy}{dx}\right)^3 + 5y = x^8$ YES/NO?

MAKING OF DIFFERENTIAL EQUATION:

(1)Count no of arbitrary equation and do differentiation as many times as no of arbitrary const.

(2)by using given and derivatives done eliminate arbitrary and make a final perfect differential eq without any arbitrary.

EX: (1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (2) $(x-h)^2 + (y-k)^2 = r^2$ (3) $y = mx + c$

(4) $(y-k)^2 = 4a(x-h)$

SOLUTION OF DIFFERENTIAL EQUATION:*Solution is the relation between variables and constant(arbitrary or fixed) which satisfy given differential equation :there are two types (a)general solution (includes as many arbitrary as the order of diff equation) (b)particular solution (arbitrary const holds particular value for special condition)

EX: $\frac{d^2y}{dx^2} + y = 0$ gen sol :- $y = acosx + bsinx$ particular solution y= $3sinx + 4cosx$

*# Solution depends on type of differential eq and finally all types converts in separate variable by using different techniques so understand properly separate variable.

$f(x)f(y)+\frac{dy}{dx}=0\,,f(x)\,dx\;+f(y)dy\;=0$ etc are sap var .

*# $f(x+y)\frac{dy}{dx}\;+k=0$ type when x and y cant saperate then put $x+y=v$ then find dy/dx in terms of dv/dx then solve by using sap var.EX:(1) $\frac{dy}{dx}\;+cos(x+y)\;+1=0$ (hint $x+y=v$)

(2) $\frac{dy}{dx}\;+(4x+y+1)^2=\;0$ (hint $4x+y+1=v$) (3)$\frac{dy}{dx}\;+xtan(y-x)=1$ (hint y-x=v).

*# Homogeneous differential equation dy/dx =x^nf(y/x) (put y=vx then get dy/dx in terms of dv/dx) and for type dx/dy put x=vy .

$x^2dy+(xy+y^2)dx=0$;ans $x^2y=c(2x+y)$.

#The slope of the tangent at (x ,y) to a curve passing through the point (1 ,$\pi/4$) is given by $\frac{y}{x}=\cos\left(\frac{2x}{y}\right)$,then the equation of the curve is ----------(hint put y=vx).

Assertion(A): Equation $\frac{dy}{dx}\;=\frac{siny}{sinx}$ is a homogeneous differential equation of order 0 .

Reason:To solve a differential equation of the form $\frac{dy}{dx}\;=f(\frac{y}{x})$,we put $y=vx$.

(a) (b) (c) (d)

LINEAR DIFFERENTIAL EQUATION

$dy/dx\;+py=Q$;where P and Q function of x or constant(sol: y(i.f.)=$\int Q(i.f.)dx+c$ i.f.=$e^{\int pdx}$.# $dx/dy\;+Px\;=Q$;where P and Q are function of y or constant.then its solution x(i.f.) =$\int Q(i.f.)dy+c$;where i.f.= $e^{\int pdy}$

EX: (1) $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$ (hint –divide by (1+x^2) both side).

(2)$\frac{xdy}{dx}\;+logx\,.y=xe^xx^{-\frac{1}{2}logx}$ (x>0)at x=2 .

(3) if y(x) satisfies the differential equation $\frac{dy}{dx}\;-ytanx=2xsecx$ and y(0) $=0$ then

(a) $y(\frac{\pi}{3})=\frac{\pi^2}{9}$ (b) $y(\frac{\pi}{3})=\frac{4\pi}{3}+2\frac{\pi^2}{2\sqrt{3}}$

(c)) y($\frac{\pi}{4}$)=$\frac{\pi^2}{8\sqrt{2}}$ (d)) y”($\frac{\pi}{4}$)=$\frac{\pi^2}{18}$

(4)Case based

If an equation is of the form dy/dx +Py =Q ;where P and Q function of x or constant then such equation is known as lineaer differential equation.its solution is : y(i.f.)=$\int Q(i.f.)dx+c$ where i.f.=$e^{\int pdx}$

Now suppose (1+sinx)dy/dx +ycosx +x =0

Based on above information answer the following

(i) The value of P and Q resp are-----

(ii) Find I.F.

(iii) Find the solution of DE.

(iv) If y(0)=1 then y equals

CASE BASED

Covid 19 vaccines are delivered to 90k senior citizens in a state the rate at which covid -19 vaccines are given is directly propotional to the no of senior citizen who have not been administered the vaccines.by the end of 3rd week ¾ th number of senior citizen have been given the covid 19 vaccines .how many will have been given the vaccines by the end of 4th week can be estimated using the solution to the differential equation

$\frac{dy}{dx} = k(90 - y)$,where x denotes the number of weeks and y the number of senior citizens who have been given the vaccines .based on above information solve following question .

Q1.The order and degree of the given differential equation are:

a.1 and 1   b.2 and not defined   c.1 and 0   d.0 and 1

Q2.Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(90 - y)$.

a.Variable saparable   b.Homogeneous differential eq.

c.Solving linear   d.All of the above

Q3.The general solution of the differential equation dy/dx=k(90-y),

a. $log(50 - y) = kx + c$ b. $-log(90 - y) = kx + c$

c. $log(90 - y) = log(kx) + c$ d. $50 - y = kx + c$

Q4.The value of C in the particular solution given that $y(0) = 10$ and

$k = .025$ is:

$a. log10 \quad b. log(\frac{1}{80}) \quad c. log80 \quad d. 80$

Q5.Which of the following solutions may be used to find the number of senior citizens who have been given covid -19 vaccines ?

$a. y = 90 - e^{kx} \quad b. y = 90 - e^{-kx} \quad c. y = 90(1 - e^{-kx}) \quad d. y = 90(1 - e^{kx})$

END

SOLUTION:-

Detailed Solutions of Case based 2

Q1. The order and degree of the given differential equation are:

Given the differential equation: $\frac{dy}{dx} = k(90 - y)$

This is a first-order differential equation as the highest derivative is$\frac{dy}{dx}$.

Degree is 1 since dy/dx is raised to the power 1 and is not under any root or function.

✅ Answer: (a) Order = 1, Degree = 1

Q2. Which method of solving a differential equation can be used to solve dy/dx = k(90 - y)?

This equation is linear in the form $\frac{dy}{dx} + ky = 90k$ or $\frac{dy}{dx} = k(90 - y)$, and can also be separated.

Hence, all three methods apply:
- Variable separable
- Homogeneous (though less direct)
- Linear differential equation

✅ Answer: (d) All of the above

Q3. The general solution of the differential equation dy/dx = k(90 - y) is:

Separate variables:

$$\frac{dy}{(90 - y)} = k\,dx$$

Integrating both sides:

$$\int \frac{1}{90 - y}\,dy = \int k\,dx$$

$\Rightarrow -ln|90 - y| = kx + C$

$\Rightarrow ln|90 - y| = -kx + C$

$\Rightarrow 90 - y = Ce^{-kx}$

$\Rightarrow y = 90 - Ce^{-kx}$

Q4. The value of C in the particular solution given a condition (say y(3) = 67.5):

From the general solution: $y = 90 - Ce^{-kx}$

Use $y(3) = 67.5 \Rightarrow 67.5 = 90 - Ce^{-3k}$

$\Rightarrow Ce^{-3k} = 90 - 67.5 = 22.5$

✅ Therefore, $C = 22.5\,e^{3k}$

Q5. Which solution helps estimate how many citizens have been vaccinated by end of 4th week?

The differential equation is $\frac{dy}{dx} = k(90 - y)$.

From the general solution: $y = 90 - Ce^{-kx}$, and using data like $y(3) = 67.5$,
we can compute C and predict y(4) by substituting x = 4.

✅ So, solution $y = 90 - Ce^{-kx}$ is used to estimate future vaccination count.

Solutions of Linear Differential Equations

Q6. Identify P and Q in the equation $(1 + sinx)\frac{dy}{dx} + y\,cosx + x = 0$

Rewrite in standard linear form: $\frac{dy}{dx} + P(x)y = Q(x)$

$\Rightarrow \frac{dy}{dx} + \left(\frac{cosx}{1+sinx}\right)y = \frac{-x}{1+sinx}$

✅ Therefore, $P = \frac{cosx}{1+sinx}, Q = -\frac{x}{1+sinx}$

Q7. Find the Integrating Factor (IF) for $\frac{dy}{dx} + P(x)y = Q(x)$ with $P(x) = \frac{cosx}{1+sinx}$

Integrating Factor, $IF = e^{\int P(x)dx}$

Let's integrate P(x):

$\int \frac{cosx}{1+sinx}dx \rightarrow Let\, u = 1 + sinx \Rightarrow du = cosx\, dx$

$\Rightarrow \int \frac{du}{u} = ln|u| = ln|1 + sinx|$

$\Rightarrow IF = e^{ln|1+sinx|} = |1 + sinx|$

✅ So, Integrating Factor (IF) = $1 + sinx$

Q8. Solve the differential equation: $\frac{dy}{dx} + (\frac{cosx}{1+sinx})\, y = -\frac{x}{1+sinx}$

Using $IF = 1 + sinx$:

General solution: $y(IF) = \int Q(IF)\, dx + C$

$\Rightarrow y(1 + sinx) = \int(-x)dx + C = -\frac{x^2}{2} + C$

✅ Therefore, the solution is: $y(1 + sinx) = -\frac{x^2}{2} + C$

Q9. Find particular solution if $y(0) = 1$

From previous: $y(1 + sinx) = -\frac{x^2}{2} + C$

At $x = 0, sin(0) = 0, y = 1$:

$\Rightarrow 1(1 + 0) = -0 + C \Rightarrow C = 1$

✅ Final solution: $y(1 + sinx) = -\frac{x^2}{2} + 1$

WORKSHEET

DIFFERENTIAL EQUATIONS

1. What is the degree of the differential equation $y\left(\frac{d^2y}{d^2x}\right)^3 + x\left(\frac{dy}{dx}\right)^4 + y^5 = 0$

   (a) 6 (b) 4 (c) 5 (d) 3

2. The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + 4\frac{d^2y}{d^2x} + 5 = 0$ is

   (a) order 1 and degree 2 (b) order 2 and degree 2

   (c) order 2 and degree 1 (d) order 1 and degree 1

3. The Integrating Factor of the differential equation $\frac{dy}{dx} - \frac{y}{x} = 2x^2$is

   (a) x^2 (b) x (c) $-\frac{1}{x}$ (d) $\frac{1}{x}$

4. Find the particular solution of the differential equation $\frac{dy}{dx} + sec^2x.y = \tan x\,.sec^2x$, given that y(0) = 0. (3)

5. Solve the differential equation given by $xdy - ydx - \sqrt{x^2 + y^2}dx = 0$. (3)
6. Find the general solution of the differential equation :$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$.
7. Solve the following differential equation :$xe^{\frac{x}{y}} - y + x\frac{dy}{dx} = 0$.
8. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}, y(1) = 0$.
9. Find the general solution of the differential equation
10. $e^x \tan y\, dx + (1 - e^x)sec^2 ydy = 0$.
11. Find the particular solution of the differential equation $xdx - ye^y\sqrt{1 + x^2}dy = 0$, given that $y = 1$, when $x = 0$.
12. Solve the differential equation $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$.

CHAPTER 10: VECTOR ALGEBRA

1	In $\Delta ABC, \overrightarrow{AB} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{I} - \hat{J} + 4\hat{K}$. If D is midpoint of BC, then $\overrightarrow{AD}$ = $a)$ $4\hat{\imath} + 6\hat{k}$ b) $2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ c) $\hat{\imath} - \hat{\jmath} + \hat{k}$ d) $2\hat{\imath} + 3\hat{k}$
2	Which of the following vectors is equally inclined to axes a) $\hat{\imath} + \hat{\jmath} + \hat{k}$ b) $\hat{\imath} - \hat{\jmath} + \hat{k}$ c) $\hat{\imath} - \hat{\jmath} - \hat{k}$ d) $-\hat{\imath} + \hat{\jmath} - \hat{k}$
3	The cosine of the angle which the vector $\sqrt{2}\,\hat{\imath} + \hat{\jmath} + \hat{k}$ makes with y-axis is a) 1 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{3}$
4	The area of a parallelogram whose diagonal is $2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and one side is $3\hat{\imath} + \hat{\jmath} - \hat{k}$ is a) $3\sqrt{2}$ units b) $4\sqrt{2}$ units c) $6\sqrt{2}$ units d) 6 units
5	If $\lvert\vec{a}\rvert = 8$, $\lvert\vec{b}\rvert = 3$ and $\lvert\vec{a}.\vec{b}\rvert = 12\sqrt{3}$ then the value of $\lvert\vec{a}\ X\ \vec{b}\rvert$ is a) 12 b) $12\sqrt{3}$ c) 6 d) $4\sqrt{3}$
6	If $\vec{a},\ \vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors then the value of $\lvert 2\hat{a} + \hat{b} + \hat{c}\rvert$ is a) $\sqrt{5}$ b) $\sqrt{3}$ c) $\sqrt{2}$ d) $\sqrt{6}$
7	The cosine of the angle which the vector $\sqrt{2}\,\hat{\imath} + \hat{\jmath} + \hat{k}$ makes with y-axis is a) 1 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{3}$
8	If for non zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\ X\ \vec{b}$ is a unit vector and $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \sqrt{2}$, then the angle between $\vec{a}$ and $\vec{b}$ is a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $-\frac{\pi}{2}$
9	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\lvert\vec{a}\rvert=3$, $\lvert\vec{b}\rvert=4$, $\lvert\vec{c}\rvert=5$, and each of them is perpendicular to the sum of the other two, find $\lvert\vec{a}+\vec{b}+\vec{c}\rvert$.
10	Show that the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $\hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ form the sides of a right-angled triangle.
11	Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=3\hat{\imath}+\hat{\jmath}+4\hat{k}$ and $\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$.
12	If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$
13	The two vectors $\hat{\jmath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. Find the length of the median through A.
14	If $\lvert\vec{a}\rvert = 8, \lvert\vec{b}\rvert = 3$ and $\lvert\vec{a} \times \vec{b}\rvert = 12$ then find the value of $\vec{a}.\vec{b}$.
15	Find the value of λ for which the two vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are perpendicular.
16	Find the position vector of the point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio 1:2.

17	Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $-\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$
18	Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of π/4 and π/2 with y and z - axes, respectively.
19	Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 2) and (–1, 0, 8), respectively.
20	A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of vector $\vec{r}$ is $2\sqrt{3}$ units, find vector $\vec{r}$.
21	A vector $\vec{r}$ has magnitude 14 and direction ratios 2, 3, – 6. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x-axis.
22	If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.
23	Find the value of λ for which the two vectors $2\hat{\imath} - \widehat{3\jmath} + 2\hat{k}$ and $2\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are parallel.
24	What will be the value of $(\vec{a} \times \hat{\imath})^2 + (\vec{a} \times \hat{\jmath})^2 + (\vec{a} \times \hat{k})^2$, for any vector $\vec{a}$?
25	If $\lvert\vec{a}\rvert = 10, \lvert\vec{b}\rvert = 2$ and $\vec{a}.\vec{b} = 12$ then find the value of $\lvert\vec{a} \times \vec{b}\rvert$.
26	If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\lvert\vec{a}\rvert = 2, \lvert\vec{b}\rvert = 3, \lvert\vec{c}\rvert = 5$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.
27	If $\vec{a}$ is any non-zero vector, then find the value of $(\vec{a}.\hat{\imath})\hat{\imath} + (\vec{a}.\hat{\jmath})\hat{\jmath} + (\vec{a}.\hat{k})\hat{k}$.
28	If $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = 144$ and $\vec{a} = 4$, then find the value of $\vec{b}$.

VECTORS
ANSWER KEY

1	d) $2\hat{\imath} + 3\hat{k}$
2	a) $\hat{\imath} + \hat{\jmath} + \hat{k}$
3	b) $\frac{1}{2}$
4	a) $3\sqrt{2}$ units
5	a) 12
6	d) $\sqrt{6}$
7	b) $\frac{1}{2}$
8	c) $\frac{\pi}{6}$
9	$\left(\vec{a} + \vec{b} + \vec{c}\right)^2 = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = 50; \quad \lvert\vec{a} + \vec{b} + \vec{c}\rvert = 5\sqrt{2}$
10	*Find the angle between each pair of vectors.*
11	$\vec{a} \times \vec{b} = 5\hat{\imath} + \hat{\jmath} - 4\hat{k}$ $\lvert\vec{a} \times \vec{b}\rvert = \sqrt{42}$
12	$\vec{a}.\left(\lambda\vec{b} + \vec{c}\right) = 0 \implies \lambda = -2$
13	Median $\lvert\overrightarrow{AD}\rvert = \frac{1}{2}\lvert 3\hat{\imath} + 5\hat{k}\rvert = \frac{\sqrt{34}}{2}$
14	$\theta = \pm\frac{\pi}{6}, \vec{a}.\vec{b} = 12\sqrt{3}$
15	$\lambda = 8$
16	$\frac{4\vec{a} + \vec{b}}{3}$
17	$\pm 10\left(\hat{\imath} - \hat{\jmath} + \hat{k}\right)$
18	$\pm 3\hat{\imath} + 3\hat{\jmath}$
19	$\frac{22}{7}\hat{\imath} + \frac{33}{7}\hat{\jmath} - \frac{66}{7}\hat{k}$
20	Using direction cosines, $\vec{r} = \pm 2\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right)$
21	Direction Cosines: $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$, Components of $\vec{r}$ = 4, 6, -12
22	$\vec{a}.\vec{c} = 3 \implies x + y + z = 3$ $\vec{a} \times \vec{c} = \vec{b} \Rightarrow x = \frac{5}{3}, y = \frac{2}{3}, z = \frac{2}{3}$ $\vec{c} = \frac{5}{3}\hat{\imath} + \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$
23	No value of λ
24	$2\lvert\vec{a}\rvert^2$
25	$\lvert\vec{a} \times \vec{b}\rvert = 16$
26	$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -19$
27	$\vec{a}$
28	Use $sin^2\theta + cos^2\theta = 1, \vec{b} = 3$

WORKSHEET

VECTOR ALGEBRA

1. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$ then the value of $\vec{a}.\vec{b}$ IS

$a)$ $12\sqrt{3}$ b) 12 c) -12 d) $-12\sqrt{3}$

2. Area of a parallelogram whose diagonals are along vectors $\hat{\imath} + 2\hat{k}$ and $2\hat{\jmath} - 3\hat{k}$ is

a) $\sqrt{29}$ b) $\frac{1}{2}\sqrt{29}$ c) $-4\,\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ d) None of these

3. If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors then the value of $|2\hat{a} + \hat{b} + \hat{c}|$ is

$a)$ $\sqrt{5}$ b) $\sqrt{3}$ c) $\sqrt{2}$ d) $\sqrt{6}$

4. For what value of p , is $(\hat{\imath} + \hat{\jmath} + \hat{k})$ p a unit vector ?

a) $\pm\frac{1}{\sqrt{3}}$ b) ± 1 c) $\pm\frac{1}{3}$ d) $\pm\sqrt{3}$

5. Which of the following vectors is equally inclined to axes

a) $\hat{\imath} + \hat{\jmath} + \hat{k}$ b) $\hat{\imath} - \hat{\jmath} + \hat{k}$ c) $\hat{\imath} - \hat{\jmath} - \hat{k}$ d) $-\hat{\imath} + \hat{\jmath} - \hat{k}$

6. Show that the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $\hat{\imath}-3\hat{\jmath}-5\hat{k}$ and $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ form the sides of a right-angled triangle.

7. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$.

8. Find a unit vector perpendicular to both **a** = 3**i** + 2**j** + 2**k** and **b** = **i** + 2**j** + 3**k**.

9. Find the value of λ for which the two vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are perpendicular.

10. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$ then find the value of $|\vec{a} - \vec{b}|$.

11. If the sum of two unit vectors is a unit vector , prove that the magnitude of their difference is $\sqrt{3}$.

12. .Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|$ =3 and $|\vec{b}| = \frac{\sqrt{3}}{2}$ and $\vec{a}$ x$\vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ and $\vec{b}$?

13. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

CASE BASED QUESTIONS

14.A man is watching an aeroplane which is at the coordinate A(4,-1,3) assuming that the man is at O(0,0,0) .At the same time he saw a bird at the coordinate point B(2,0,4).

Based on the above information answer the following:

(a) Find the vector $\overrightarrow{AB}$.

(b) Find the distance between aeroplane and bird.

(c) Find the unit vector along $\overrightarrow{AB}$.

OR

Find the direction cosines of $\overrightarrow{AB}$.

15.A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let If $\vec{a}, \vec{b}$ and $\vec{c}$ be three nonzero vectors .

(a) If $|\vec{a} + \vec{b}\ | = |\vec{a} - \vec{b}|$ then find the relation between $\vec{a}$ and $\vec{b}$.

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between then find $|\vec{a} - \vec{b}|$.

(c) If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ an angle between $\vec{b}$ and $\vec{c}$ is 30° then find k if $\vec{a} = k(\vec{b} \times \vec{c})$.

OR

Find the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ as diiagonals.

CHAPTER 11: THREE DIMENSIONAL GEOMETRY

1. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$,the value of a is

a) 3 b) ± 3 c) $-\pm\sqrt{3}$ d) $\sqrt{3}$

2. The point (x,y,z) on the xy-plane divides the line segment joining the points (1,2,3) and (3,2,1) in the ratio

a) 3:1 externally b) 3: internally c) 2:1 externally d) 2:1 internally

3. The lines $\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k} + \alpha\left(2\hat{\imath} + 3\hat{\jmath} - \widehat{6k}\right)$ *and* $\vec{r}$=$2\hat{\imath} - \hat{\jmath} - \hat{l} + \beta\left(6\hat{\imath} + 9\hat{\jmath} - 18\hat{k}\right)$; $(where\ \alpha\ and\ \beta are\ scalar)$ are

a) coincident b)skew c)intersecting d) parallel

4. The acute angle between the line $\vec{r} = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \mu(\hat{\imath} - \hat{\jmath})$ and the X-axis

a) $\frac{\pi}{4}$ $b)\frac{\pi}{3}$ $c)\frac{\pi}{6}$ $d)\frac{\pi}{2}$ $\frac{\pi}{4}$

5. The two lines x=ay +b , z=cy +d and x= a’y + b’ , z= c’y + d’ are perpendicular to each other if

a) $\frac{a}{a\prime} + \frac{c}{c'}$ =1 b)$\frac{a}{a\prime} + \frac{c}{c\prime} = -1$ c)aa’+cc’=1 d) aa’+cc’=-1

6.If a line makes angles α, β, γ *t*hen write the value of

cos 2α +cos 2β+cos 2γ.

7.The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, – 2) is 4. Finds its Z coordinate

8.Find the cartesian and vector equation of the line which passes through the point $(-2\,,4,-5\ )$ and parallel to the line given by

$\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$ Ans- $r = -2\hat{\imath} + 4\hat{\jmath} - 5\hat{k} + \mu\left(3\hat{\imath} + 5\hat{\jmath} + 6\hat{k}\right)$

9. Find the coordinates of the foot of perpendicular drawn from the point A (- 1, 8, 4) to the line joining the points B(0, – 1, 3) and C(2, – 3, – 1). Hence, find the image of the point A in the line BC.

10. That the lines $\frac{X+1}{3} = \frac{y+3}{5} = \frac{Z+5}{7}$ and $\frac{X-2}{1} = \frac{y-4}{3} = \frac{Z-6}{5}$ intersect. Also, find their point of intersection.

11.The cartesian equation of a line is 6x – 2 = 3y + 1 = 2z – 2. Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, – 1, – 1) which are parallel to the given line.

12.Find the points on the line $\frac{X+2}{3} = \frac{Y+1}{2} = \frac{Z-3}{2}$ a distance of 5 units from the point P(1, 3, 3).

13. $\overrightarrow{AB} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$ *and* $\overrightarrow{CD} = -3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ are two vector. The position vector of the point A and C are $6\hat{\imath} + 7\hat{\jmath} + 4k$ *and* $-9\hat{\jmath} + 2\hat{k}$, respectively. Find the position vector of point Pon the line AB and a point Q on the line CD such that PQ is perpendicular to vector AB and CD

14. Find the angle between the lines whose direction cosines are given by by the equation l+m+n=0, $m^2 + n^2 - l^2 = 0$

15. Find the angle between the lines

$$\vec{r} = 2\hat{\imath} - 5\hat{\jmath} + \hat{k} + \lambda(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}) \quad \text{and} \quad \vec{r} = 7\hat{\imath} - 6\hat{\jmath} - 6\hat{k} + \vartheta(\hat{\imath} + \widehat{2\jmath} + 2\hat{k})$$

16. Define skew lines. Using only vector approach , find the shortest distance between the following two skew lines .

$$\vec{r} = (8 + 3s)\hat{\imath} - (9 + 16s)\hat{\jmath} + (10 + 7s)\hat{k}$$

and

$$\vec{r} = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k} + t(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$$

17. Find the coordinate of the foot of the perpendicular drawn from the point P(0,2,3) to the lines

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$$

SOLUTIONS

1. $\pm\sqrt{3}$ 2. 3:1 externally 3. Parallel 4. $\frac{\pi}{4}$ 5. d) aa'+cc'=-1

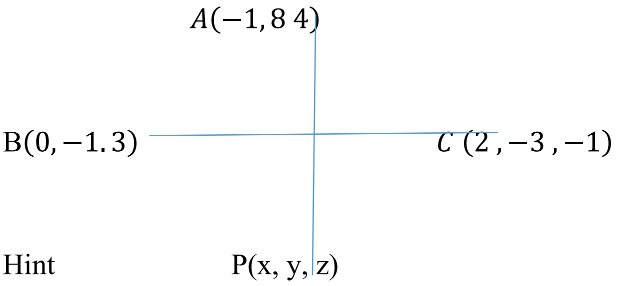
6. Ans-(-1)

7. Hint- z coordinate (0 ,0, 1) Ans- (-1)

8. Ans-Vector form: $\vec{r} = -2\hat{\imath} + 4\hat{\jmath} - 5\hat{k} + \mu(3\hat{\imath} + 5\hat{\jmath} + 6\hat{k})$

Cartesian form: $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

9. Hint- direction cosines of AP and direction cosines BC is perpendicular Ans- (-3 ,-6 10)



Hint

10. Ans-P($\frac{1}{2}$,$\frac{-1}{2}$,$\frac{-3}{2}$)

11. Ans-$\vec{r} = 2\hat{\imath} - \hat{\jmath} - \hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + \widehat{3k})$

12. Ans-the required points as $(-2,-1,3)$ or $(4,3,7)$

13. Ans-$3\hat{\imath} + 8\hat{\jmath}$+$3\hat{k}$, $-3\,\hat{\imath} - 7\hat{\jmath} + 6\hat{k}$

14. Hint- l = - (m + n) Ans-60°

15. Hint- use formula angle between two line Ans- $\cos\theta = \frac{19}{21}$

16. Ans-14 units

17. Ans-(2,3,-1)

WORKSHEET

THREE DIMENSIONAL GEOMETRY

1. If the direction cosines of a line are $\left(\frac{1}{a},\frac{1}{a},\frac{1}{a}\right)$,the value of a is

a) 3 b) ±3 c) -±√3 d) √3

2. Vector equation of a line is $\vec{r} = (4\hat{\imath} - 2\hat{\jmath} + 5\hat{k}) + \mu(\hat{\imath} + \widehat{3\jmath} - 2\hat{k})$,the Cartesian form of a line is:

(a) $\frac{x-4}{1} = \frac{y+2}{3} = \frac{z-5}{-2}$ (b) $\frac{x+4}{1} = \frac{y-2}{3} = \frac{z+5}{-2}$

(c) $\frac{x-4}{1} = \frac{y+2}{3} = \frac{z-5}{2}$ (d) $\frac{x+4}{-1} = \frac{y-2}{3} = \frac{z+5}{2}$

3. If a line makes angles $\frac{\pi}{2},\frac{\pi}{3}$ and θ with the positive x, y and z axes respectively, then θ is

(a) $\pm\frac{\pi}{6}$ (b) $\pm\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ only (d) $\frac{\pi}{3}$ only

4. The acute angle between the line $\vec{r} = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \mu(\hat{\imath} - \hat{\jmath})$ and the X-axis

a) $\frac{\pi}{4}$ $b)\frac{\pi}{3}$ $c)\frac{\pi}{6}$ $d)\frac{\pi}{2}$

5. The equation of a line passing through the point (3,-1,5) and parallel to vector $(\hat{\imath} + \widehat{2\jmath} - \hat{k})$ is;

(a) $x = t + 3$, $y = 2t - 1$, $z = -t + 5$

(b) $x = t + 3$, $y = -2t - 1$, $z = -t + 5$

(c) $x = t + 3$, $y = 2t - 1$, $z = t + 5$

(d) $x = t - 3$, $y = 2t - 1$, $z = -t + 5$

6. . Write the vector equation of the line whose Cartesian equations is

$\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$

7. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

8. Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and $\frac{x-1}{0}=\frac{y}{-3}=\frac{z-7}{2}$ and perpendicular to these given lines.

9. Find the point Q on the line $\frac{2x+4}{6}=\frac{y+1}{2}=\frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1,2,3).

10. . Find the coordinates of a point where the line $\frac{x+2}{1}=\frac{y-5}{3}=\frac{z+1}{5}$ cuts YZ- plane.

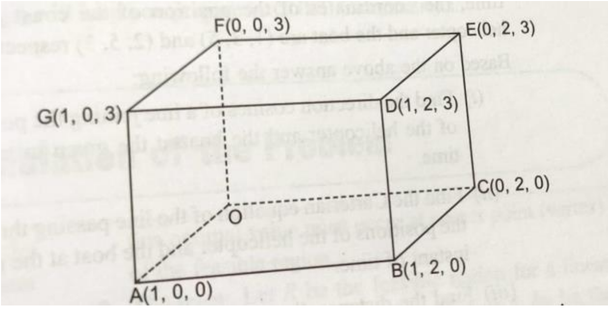
11. The cartesian equations of a line are 3x + 1 =6y – 2 = 1 – z. Find the fixed point through which it passes , its direction ratios and also its vector equation.

12. Show that the lines $\vec{r}=3\hat{\imath}+2\hat{\jmath}-4\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\vec{r}=5\hat{\imath}-2\hat{\jmath}+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$ are intersecting. Hence find their point of intersection.

13. Find the shortest distance of the following lines:

$\vec{r}=\hat{\imath}+2\hat{\jmath}+\hat{k}+\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})$ and $\vec{r}=2\hat{\imath}-\hat{\jmath}-\hat{k}+\mu(2\hat{\imath}+\hat{\jmath}+2\hat{k})$.

14. Anu made a cuboidal fish tank having coordinates O(0,0,0) , A(1,0,0) , (1,2,0) , C(0,2,0) ,D(1,2,3) ,E(0,2,3),F(0,0,3) and G(1,0,3)



a) Find the the direction cosines of $\overrightarrow{AB}$.
b) Write cartesian equation of the diagonal $\overrightarrow{OD}$.
c) Find the direction ratios of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

OR

Show that the line $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are perpendicular to each other.

15.Read the following passage and answer the questions given below:

Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

Two such wires lie along the lines $l_1: \frac{1-x}{3}=\frac{7y-14}{p}=\frac{z-3}{2}$

$l_2: \frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$

(i) Write the direction ratios of the line l_1.
(ii) If $\cos\alpha, cos\beta\ and\ cos\gamma$ are the direction ratios of the line l_2, then find the value of $sin^2\alpha+sin^2\beta+sin^2\gamma$.
(iii) Find the value of p if the lines l_1 and l_2 are perpendicular to each other.

OR

If the lines l_1 and l_2 are perpendicular to each other, find the vector equation of a line passing through the point (1,2,3) and parallel to the line l_2.

CHAPTER 12: LINEAR PROGRAMMING

BASIC CONCEPTS

What is LPP: LPP or Linear Programming Problem, is a **mathematical optimization technique** used to find the best outcome (maximum or minimum) of a linear function, subject to linear constraints and non-negative restrictions on the variables.

1. **Objective function:** Linear function Z = ax + by, where a, b are constants, which has to be maximized or minimized is called a linear objective function.
2. **Constraints:** The linear inequalities or equations or **restrictions** which are imposed on the variables of a linear programming problem are called constraints.
3. **Optimization problem:** A problem which seeks to maximize or minimize a linear function (say of two variables x and y) subject to certain constraints as determined by a set of linear inequalities is called an optimization problem.
4. **Feasible region:** The common region determined by all the constraints including non-negative constraints x, y ≥ 0 of a linear programming problem is called the feasible region.
5. **Feasible solutions:** Points within and on the boundary of the feasible region represent feasible solutions of the constraints.
6. **Optimal (feasible) solution:** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
7. **Corner point method:** The method comprises of the following steps:

1\. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.

2\. Evaluate the objective function Z = ax + by at each corner point. Let M and m, respectively denote the largest and smallest values of these points.

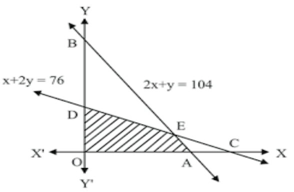
3\. (i) When the feasible region is bounded, M and m are the maximum and minimum values of Z.

(ii) In case, the feasible region is unbounded, we have:

(a) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

(b) Similarly, m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has **no minimum value.**

S.N.	Questions
1	If any objective function(Z) have same maximum value at two corner points in a feasible region, then in how many points between these two points Z will be maximum? a) 5 b) 3 c) infinite d) only those 2
2	The corner points of the feasible region in the graphical representation of a linear programming problem are (2 , 72), (15, 20) and (40 , 15).If Z =18x + 9y be the objective function, then a) Z is maximum at (2 , 72), minimum at (15 , 20). b) Z is maximum at (15 , 20), minimum at (40 , 15). c) Z is maximum at (40 , 15), minimum at (15 , 20). d) Z is maximum at (40 , 15), minimum at (2 , 72).
3	The objective function Z = ax + by of an LPP if its maximum value 42 at (4 , 6) and minimum value 19 at (3 , 2).Which of the following is true? a) a =9 and b = 1 b) a = 3 and b = 5 c) a = 5 and b = 2 d) a = 5 and b = 3
4	Shown below is a linear programming problem (LPP). Maximize Z = x + y

	Subject to the constraints: $x + y \leq 1$ $-3x + y \geq 3$ $x \geq 0$ $y \geq 0$ Which of the following is true about the feasible region of the above LPP? a) It is bounded. b) It is unbounded c) There is no feasible region for the given LPP. d) Cannot conclude anything from the given LPP.
5	A linear programming problem deals with the optimization of a/an: a) logarithmic function b) linear function c) quadratic function d) exponential function
6	The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called: a) feasible solutions b) infeasible solutions c) optimal solutions d) constraints
7	The maximum value of the objective function $Z = 5x + 10y$ subject to the constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$ is a) 300 b) 800 c) 600 d) 400
8	The optimal value of the objective function is attained at the points a) which are the corner points of the feasible region b) on x-axis c) on y-axis d) none of these
9	Of the following, which group of constraints represents the feasible region given below ? a) $x + 2y \leq 76$, $2x + y \geq 104$, $x,y \geq 0$ b) $x + 2y \leq 76$, $2x + y \leq 104$, $x,y \geq 0$ c) $x + 2y \geq 76$, $2x + y \geq 104$, $x,y \geq 0$ d) $x + 2y \geq 76$, $2x + y \geq 104$, $x,y \geq 0$ 
10	$Z = 8x + 10y$, subject to $2x + y \geq 1$, $2x + 3y \geq 15$, $y \geq 2$, $x \geq 0$, $y \geq 0$. The minimum value of Z occurs at a) (4.5 , 2) b) (1.5, 4) c) 0 , 6) d) (0 , 5)
11	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct? a) It will only have a maximum value. b) It will only have a minimum value. c) It will have both maximum and minimum values. d) It will have neither maximum nor minimum values.
12	The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is: a) 2 b) 3 c) 4 d) 5
13	The objective function of an LPP is a) a quadratic function b) a constant c) a linear function to be optimized d) none of these
14	The feasible region is the set of points which satisfy a) the objective functions b) some of the given constraints c) all of the given constraints d) none of these

15 A point out of the following points lie in plane represented by $2x + 3y < 12$ is

a) (0 , 3) b) (3 , 3) c) (4 , 3) d) (0 , 5)

16 Feasible region formed by the constraints $x + y \leq 4$,, $3x + 3y \geq 18$, $x \geq 0, y \geq 0$ is_____

(a) bounded
(b) unbounded
(c) lies first and second quadrant
(d) does not exist

17 Which of the following statement is correct?

(a) Every linear programming problem has at least one optimal solution.
(b) Every linear programming problem has a unique optimal solution.
(c) If a linear programming problem has two optimal solutions, then it has infinitely many solutions.
(d) If a feasible region is unbounded, then linear programming problem has no solution.

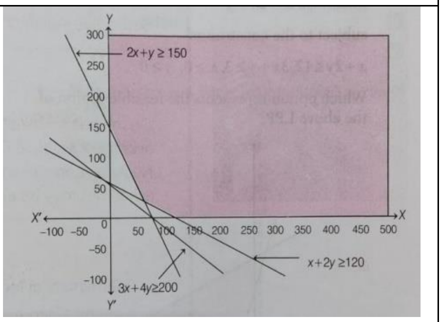
18 The position of points O (0, 0) and P (2, -3) in the region of graph of inequation $2x - 3y < 5$ will be _____ .

(a) O inside and P outside
(b) O and P both inside
(c) O and P both outside
(d) O outside and P inside

19 In the LPP, $x \geq 0$ and $y \geq 0$ are called:

a) Additional equations
b) Non-negative constraints
c) Inverse conditions
d) Elimination rules

20 The objective function of a linear programming problem (LPP), $Z = 4x + 3y$ has to be minimised.
The feasible region of this LPP, along with its constraints is shown in the adjacent graph.
Which constraint, if removed will not affect the feasible region?



a) $x + 2y \geq 120$
b) $2x + y \geq 150$
c) $3x + 4y \geq 200$
d) any of the given constraints, if removed will affect the feasible region.

21 Minimise $Z = 50x + 70y$

Subject to the constraints:

$2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

22 Maximise and minimise $Z = x + 2y$ subject to the constraints

$x + 2y \geq 100$
$2x - y \leq 0$
$2x + y \leq 200$
$x, y \geq 0$

23 Maximise $Z = 15x + 10y$, Subject to the constraints

$2x + y \leq 40$
$2x + 3y \leq 80$ and $x, y \geq 0$

24 Maximise : $P = 40x + 50y$, Subject to the constraints

$3x + y \le 9$
$x + 2y \le 8$
and $x \ge 0, y \ge 0$

25 Maximize $Z = 0.7x + y$, Subject to the constraints,
$2x + 3y \le 120$ …(i)
$2x + y \le 80$ …(ii)
and $x, y \ge 0$ …(iii)

26 Maximise $Z = 7x + 4y$, Subject to the constraints,
$3x + 2y \le 12$, $3x + y \le 9$ and $x \ge 0, y \ge 0$

27 Maximise Z = 10% of x + 9% of y or $Z = 0.1x + 0.09y$
Subject to constraints, $x + y = 50000$
$x \ge y$ or $x - y \ge 0$
$x \ge 20000$ and $y \ge 10000$

ANSWERS

1 c) infinite

2 c) Z is maximum at (40 , 15), minimum at (15 , 20).
Hint:

Corner points	$Z = 18x + 9y$
(2 , 72)	684
(15 , 20)	450 (Minimum)
(40 , 15)	855 (Maximum)

3 b) a = 3, b = 5
Hint:
$Z = ax + by$
As Z has maximum value 42 at (4 , 6), minimum value 19 at (3 , 2).
$\therefore 4a + 6b = 42$ --------(1)
$3a + 2b = 19$ ---------(2)
On solving eq^n.(1) and eq^n.(2), we get a = 3, b = 5

4 c) There is no feasible region for the given LPP.

5 b) Linear function

6 d) Constraints

7 c) 600
Hint:
Draw the graph of equalities and obtain the feasible region.
The corner points so obtained are (60 , 30), (40 , 20), (60 , 0) and (120 , 0).
Evaluate the value of Z on the above corner points and we get maximum value of Z is 600 at (60 , 30) and (120 , 0).

8 a) which are the corner points of the feasible region

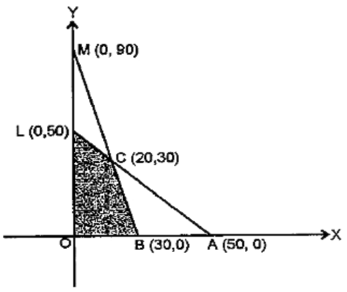
9 b) $x + 2y \le 76$, $2x + y \le 104$, $x,y \ge 0$
Hint:
As the shading of equality is towards the origin.

10 d) (0 , 5)
Hint:
Draw the graph of equalities and obtain the feasible region(which is unbounded).
The corner points so obtained are (0 , 5) and (4.5 , 2).

11	c) It will have both maximum and minimum values.
12	a) 2
13	c) a linear function to be optimized
14	c) all of the given constraints
15	a) (0, 3)
16	d) does not exist
17	(c) If an linear programming problem has two optimal solutions, then it has infinitely many solutions. **Explanation:** If a linear programming problem has two optimal solutions, it means the objective function is constant along a line segment connecting those two solutions, and every point on that line is also optimal.
18	(a) O inside and P outside **Hint:** Put O(0 , 0) and P(2 , -3) in the given inequation 2x - 3y < 5
19	b) Non-negative constraints
20	c) $3x + 4y \geq 200$
21	The minimum value of Z is 380 obtained at the point (2, 4).
22	The maximum value of Z is 400 at 0(0, 200) and the minimum value of Z is 100 at all the points on the line segment joining (0, 50) and (20, 40).
23	The maximum value of Z is 350 at (10, 20)
24	P is maximum at x = 2 and y = 3 and maximum value of P is 230
25	Maximum value of Z is 41 at (30, 20).
26	Maximum value of Z is 26 at (2, 3).
27	The maximum value of Z is 4900 at (40000,10000)

WORKSHEET

LINEAR PROGRAMMING

S.N.	MCQs	MARKS
1	The maximum or minimum value of the objective function occurs at: a) Origin b) Boundary line c) Corner points of the feasible region d) Centre of feasible region	1
2	The maximum value of $Z = 4x + y$ for a L.P.P., whose feasible region is shown below is: a) 50 b) 110 c) 120 d) 170 	1

3	In the LPP, $x \geq 0$ and $y \geq 0$ are called: a) Additional equations b) Non-negative constraints c) Inverse conditions d) Elimination rules	1
4	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct? a) It will only have a maximum value. b) It will only have a minimum value. c) It will have both maximum and minimum values. d) It will have neither maximum nor minimum values.	1
	SA (2 MARKS EACH)	
5	Minimize $Z = 50x + 70y$ Subject to the constraints: $2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$	2
6	Solve the following LPP graphically: Maximize: $Z = 2x + 3y$, subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$	2
7	Maximize : $P = 40x + 50y$, Subject to the constraints $3x + y \leq 9$ $x + 2y \leq 8$ and $x \geq 0$, $y \geq 0$	2
	ANSWERS	
1	c) Corner points of the feasible region	
2	b) (2, 3)	
3	b) 110	
4	b) Non-negative constraints	
5	c) It will have both maximum and minimum values.	
6	The minimum value of Z is 380 obtained at the point (2, 4).	
7	The maximum value of Z is 12 at the point (0, 4).	
8	P is maximum at $x = 2$ and $y = 3$ and maximum value of P is 230	

CHAPTER 13: PROBABILITY

ASSERTION-REASONING

Options:

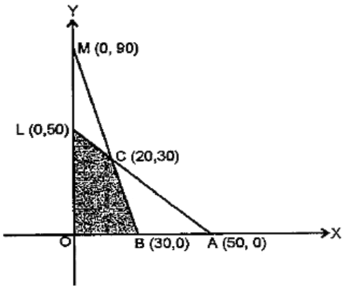
(A) Both A and R are true, and R is the correct explanation of A.
(B) Both A and R are true, but R is not the correct explanation of A.
(C) A is true, but R is false.
(D) A is false, but R is true.

1	**Assertion (A):** If events A and B are mutually exclusive, then $P(A \cap B) = 0$. **Reason (R):** Two mutually exclusive events can occur simultaneously.
2	**Assertion (A):** For any two independent events A and B, $P(A \cap B) = P(A) \times P(B)$. **Reason (R):** If A and B are independent, then the occurrence of one does not affect the probability of the other.
3	**Assertion (A):** If $P(A \mid B) = P(A)$, then A and B are independent events. **Reason (R):** In independent events, conditional probability equals the unconditional probability.
4	**Assertion (A):** If two events are independent, they cannot be mutually exclusive. **Reason (R):** Mutually exclusive events imply $P(A \cap B) = 0$, while independent events imply $P(A \cap B) = P(A) \times P(B)$.
5	**Assertion (A):** The conditional probability $P(B \mid A)$ is defined only when $P(A) \neq 0$. **Reason (R):** Division by zero is not defined.
6	**Assertion (A):** If A and B are independent events, then A and B′ are also independent. **Reason (R):** The independence of events is not affected by taking complements.
7	**Assertion (A):** If A is a subset of B, then $P(A \cap B) = P(A)$. **Reason (R):** The intersection of two events is the set of outcomes common to both.
8	**Assertion (A):** If $P(A \cup B) = P(A) + P(B)$, then A and B are mutually exclusive. **Reason (R):** For mutually exclusive events, $P(A \cap B) = 0$.
9	**Assertion (A):** If $P(A) = 0$, then $P(A \cup B) = P(B)$. **Reason (R):** A null event does not affect the probability of union.
10	**Assertion (A):** The total probability of all elementary outcomes of a random experiment is 1. **Reason (R):** The sample space of a random experiment is a finite non-empty set.
11	An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone. Based on the given information, answer the following questions: (i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy? (ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?
12	A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%,60%,35%. Based on the given information, answer the following questions: (i) find the probability of a randomly chosen seed to germinate. (ii) find the probability that seed will not germinate given that it is of the type A_3
13	An item is manufactured by three machines A, B and C. Out of the total numbers of items manufactured during a specified period,50% are manufactured on A, 30% are manufactured on B, 20% are manufactured on C. 2% of items produced on A, 2% of items produced on B and 3%

	produced on C are defective. All the items are stored at one storeroom. (i) One item is drawn at random and is found to be defective. What is the probability that it is manufactured on machine A? (ii) One item is drawn at random and is found to be defective. What is the probability that it is manufactured on machine B?
14	The reliability of a COVID PCR test is specified as follows: Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive. (i) A person is selected at random and tested. What is the probability that he is tested positive? (ii) What is the probability that the ‘person is actually having COVID given that ‘he is tested as COVID positive’?
15	An electronic assembly consists of two sub-systems A and B. From previous testing procedures, the following probabilities are assumed to be known: P (A fails) = 0.2 P (B fails alone) = 0.15 P (A and B fail) = 0.15 Based on the given information, answer the following questions: (i) P (A fails/ B has failed) (ii) P (A fails alone)
16	40 % students of a college reside in the hostel and the remaining resides out. At the end of the year, 50 % of the hostellers got A grade while from outside students, only 30 % got A grade in the examination. At the end of the year, a student of the college was chosen at random and was found to have gotten A grade. What is the probability that the selected student was a hosteller?
17	An insurance company insured 3000 scooterists, 4000 car drivers and 5000 motorbike drivers. The probabilities of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with accident. What is the probability that he is car driver?
	Answer
1	**Correct Answer:** (C) **Explanation:** Mutually exclusive events cannot happen at the same time, so P(A ∩ B) = 0. However, the reason is false because it incorrectly states that such events can occur together.
2	**Correct Answer:** (A) **Explanation:** This is the definition of independence. Both A and R are true and R correctly explains A.
3	**Correct Answer:** (A) **Explanation:** When P(A|B) = P(A), it means B has no influence on A → independence. Both A and R are true and related.
4	**Correct Answer:** (A) **Explanation:** If P(A) and P(B) are both positive and A ∩ B = 0 (mutually exclusive), then they can’t also be independent. Hence, both A and R are true, and R is the correct explanation.
5	**Correct Answer:** (A) **Explanation:** Since P(B|A) = P(A ∩ B) / P(A), P(A) must be non-zero to avoid division by zero. R explains A correctly.
6	**Correct Answer:** (A) **Explanation:** If A and B are independent, then A and not-B (B′) are also independent, by probability properties. Both A and R are correct and related.
7	**Correct Answer:** (A) **Explanation:** If A is entirely within B, then the intersection is just A. So their probabilities are

	equal. R supports A well
8	**Correct Answer:** (A) **Explanation:** Normally, P(A ∪ B) = P(A) + P(B) − P(A ∩ B). So if the sum equals union, the intersection must be 0 → mutually exclusive. R explains A correctly.
9	**Correct Answer:** (A) **Explanation:** Since A has zero probability, the union with B remains the same as P(B). R supports A correctly.
10	**Correct Answer:** (B) **Explanation:** The total probability is always 1, but the sample space need not be finite (e.g., countably infinite). So R is not the correct explanation.
11	(i) $\frac{7}{25}$(ii) $\frac{3}{7}$
12	(i) 0.49 (ii) 0.65
13	(i) $\frac{5}{11}$(ii) $\frac{3}{11}$
14	(i)0.01089 (ii) $\frac{10}{121}$
15	(i) 1 (ii) 0.05
16	Ans: $\frac{10}{19}$
17	Ans : 2/15

WORKSHEET LINEAR PROGRAMMING

S.N.	MCQs	MARKS
1	The maximum or minimum value of the objective function occurs at: a) Origin b) Boundary line c) Corner points of the feasible region d) Centre of feasible region	1
2	The maximum value of Z = 4x + y for a L.P.P., whose feasible region is shown below is: a) 50 b) 110 c) 120 d) 170 	1
3	In the LPP, x ≥ 0 and y ≥ 0 are called: a) Additional equations b) Non-negative constraints	1

	c) Inverse conditions d) Elimination rules	
4	If the feasible region of a linear programming problem with objective function Z = ax + by, is bounded, then which of the following is correct? a) It will only have a maximum value. b) It will only have a minimum value. c) It will have both maximum and minimum values. d) It will have neither maximum nor minimum values.	1
	SA (2 MARKS EACH)	
5	Minimize $Z = 50x + 70y$ Subject to the constraints: $2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$	2
6	Solve the following LPP graphically: Maximize: $Z = 2x + 3y$, subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$	2
7	Maximize : $P = 40x + 50y$, Subject to the constraints $3x + y \leq 9$ $x + 2y \leq 8$ and $x \geq 0, y \geq 0$	2
	ANSWERS	
1	c) Corner points of the feasible region	
2	b) (2, 3)	
3	b) 110	
4	b) Non-negative constraints	
5	c) It will have both maximum and minimum values.	
6	The minimum value of Z is 380 obtained at the point (2, 4).	
7	The maximum value of Z is 12 at the point (0, 4).	
8	P is maximum at x = 2 and y = 3 and maximum value of P is 230	

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