गणित

MATHEMATICS

VOLUME I

कक्षा 12

CLASS XII

2025-26

सामग्री संवर्धन, मूल्यांकन और अध्ययन कैप्सूल का विकास

CONTENT ENRICHMENT, ASSESSMENT AND DEVELOPMENT OF STUDY CAPSULES

केन्द्रीय विद्यालय संगठन, रायपुर सम्भाग

KENDRIYA VIDYALAYA SANAGTHAN, RAIPUR REGION

संदेश

मुझे यह बताते हुए अपार हर्ष हो रहा है कि रायपुर संभाग के केंद्रीय विद्यालयों के स्नातकोत्तर गणित शिक्षकों द्वारा कक्षा 12 के छात्रों हेतु सामग्री संवर्धन, प्रभावी मूल्यांकन तकनीकों, और अध्ययन कैप्सूल के सफल विकास का कार्य किया गया है। यह पहल शिक्षण–अधिगम प्रक्रिया को और अधिक समृद्ध बनाते हुए विद्यार्थियों की जटिल गणितीय अवधारणाओं को सहज रूप में समझाने में सशक्त योगदान देगी।

आप सभी शिक्षकों ने सिद्ध किया है कि गुणवत्तापूर्ण शिक्षा तभी संभव है, जब शिक्षक गणिता की जटिलता को सरल, संरचित एवं प्रेरणादायक बनाते हैं। आपके द्वारा विकसित अध्ययन सामग्री में विषय–वस्तु की स्पष्टता, अभ्यास की विविधता एवं मूल्यांकन की सरलता व्यवसायिक शिक्षा का बीजारोपण करती है।

यह पहल केवल विद्यार्थियों के लिए लाभदायक नहीं, बल्कि अन्य शिक्षकों के लिए एक आदर्श मॉडल भी है। इससे यह प्रमाणित होता है – सहकार्य, नवाचार और ज्ञान–उन्मुख शिक्षाशैली से हम अपने छात्रों को विशेषज्ञता के साथ तैयारी करवा सकते हैं।

मैं समस्त गणित शिक्षकों को उनके इस समर्पण एवं उत्तम प्रयास के लिए हार्दिक शुभकामनाएँ देती हूँ। भविष्य में भी आपके द्वारा ऐसे प्रेरणादायक और शिक्षार्थी–केंद्रित कार्य की आशा करती हूँ।

"शिक्षा की असली शक्ति शिक्षक के नवोन्मेषी दृष्टिकोण और सहयोगी प्रयासों में निहित है।"

आभार एवं शुभकामनाओं सहित,

(पी.बी.एस. उषा)
उपायुक्त
के.वि.सं. क्षे.का. रायपुर

GUIDED BY

Shri Vivek Kumar Chauhan, Assistant Commissioner

Kendriya Vidyalaya Sangathan, Regional Office, Raipur

Shri Ravindra Kumar, Assistant Commissioner

Kendriya Vidyalaya Sangathan, Regional Office, Raipur

Course Director

Shri Sharad Kumar Kaiwart, Principal

PM SHRI Kendriya Vidyalaya, Bilaspur

Resource Person

1. **Shri Umesh Kumar Kurrey, PGT (Maths)**

   PM SHRI Kendriya Vidyalaya, Bilaspur

2. **Shri Shashank Shrivastav, PGT (Maths)**

   PM SHRI Kendriya Vidyalaya No. 2, Raipur

CONTRIBUTORS:

Sl. No.	KV Name	Name of teacher	Designation	Topic Allotted
1	CISF BHILAI	SRIJAN SINGH (GROUP LEADER)	PGT MATHS	LINEAR PROGRAMMING & PROBABLITY
2	AMBIKAPUR	TULSI PRASAD CHOURASIYA	PGT MATHS	
3	BACHELI	DEEPAK	PGT MATHS	
4	MAHASAMUND	V.N PRAHAN	PGT MATHS	
5	KORBA NO. IV	Mr ROSHAN LAKRA	PGT MATHS	
6	RAIGARH	TAPASHWINI PANIGRAHI (GROUP LEADER)	PGT MATHS	VECTORS & THREE DIMENSIONAL GEOMETRY
7	CRPF BILASPUR	NIRMALA SAHU	PGT MATHS	
8	JAGDALPUR	MONIKA GOYAL	PGT MATHS	
9	CHIRIMIRI	KHUSHBOO	PGT MATH	
10	JANJGIR	KHUSHBOO MEENA	PGT MATHS	
11	KURUD	KHEMLATA SAHU	PGT MATHS (CONTRACTUAL)	
12	DONGARGARH	DEEPAK KUMAR CHANDRAKAR (GROUP LEADER)	PGT MATHS	INTEGRALS, APPLICATION OF INTEGRALS & DIFFERENTIAL EQUATIONS
13	RAIPUR NO.1 (S-I)	ADITYA SHUKLA	PGT MATHS	
14	RAJNANDGAON	AMIT KUMAR SINGH	PGT MATHS	
15	KAWARDHA	KRISHNA PANDIT	PGT MATHS	
16	MANENDRAGARH	BRIJ MOHAN PRAJAPATI	PGT MATHS	
17	BMY BHILAI	JAI KUMAR CHOUDHARY (GROUP LEADER)	PGT MATHS	CONTINUITY AND DIFFERENTIABILITY & APPLICATION OF DERIVATIVES
18	DURG	M.A. NANDANWAR	PGT MATHS	
19	KHAIRAGARH	SANJAY KUMAR	PGT MATHS	
20	KIRANDUL	AMIT	PGT MATHS	
21	NARAYANPUR	PRINCE GUPTA	PGT MATHS	
22	JHAGRAKHAND	D.P CHAUBEY (GROUP LEADER)	PGT MATHS	RELATIONS AND FUNCTIONS, INVERSE TRIGONOMETRIC FUNCTIONS, MATRICES AND DETERMINANTS
23	NAYA RAIPUR	MANJEET KUMAR	PGT MATHS	
24	RAIPUR NO.1 (S-II)	KESHU JAIN	PGT MATHS	
25	KORBA NO. 2 (NTPC)	KASHYAP KUMAR MISHRA	PGT MATHS	
26	RAIPUR NO.2	RAMA SINGH	PGT MATHS	

INDEX

COURSE STRUCTURE
CLASS – XII
(2025-26)

One Paper **Max. Marks: 80**

No.	Units	Marks
I.	Relations and Functions	08
II.	Algebra	10
III.	Calculus	35
IV.	Vectors and Three - Dimensional Geometry	14
V.	Linear Programming	05
VI.	Probability	08
	Total	**80**
	Internal Assessment	**20**

Unit-I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operations on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non- commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability

Continuity and differentiability, chain rule, derivative of composite functions, derivatives of inverse trigonometric functions like $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$, derivative of implicit functions. Concept of exponential and logarithmic functions. Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives

Applications of derivatives: rate of change of quantities, increasing/decreasing functions, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real- life situations).

3. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \frac{px+q}{ax^2+bx+c}\,dx,$$
$$\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx, \int \sqrt{a^2 \pm x^2}\,dx, \int \sqrt{x^2 - a^2}\,dx, \int \sqrt{ax^2+bx+c}\,dx$$

Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Application of the Integrals

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only)

5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$\frac{dy}{dx} + py = q$, where p and q are functions of x or constants.

$\frac{dx}{dy} + px = q$, where p and q are functions of y or constants.

Unit-IV: Vectors and Three-dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three-dimensional Geometry

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, skew lines, shortest distance between two lines. Angle between two lines.

Unit-V: Linear Programming Problem

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem.

MATHEMATICS (Code No. – 041)

QUESTION PAPER DESIGN

CLASS – XII (2025-26)

Time: 3 hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	**80**	**100**

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper. However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note: For activities NCERT Lab Manual may be referred.

1. RELATION AND FUNCTION

A relation can be mathematically defined as the linking or connection between two different objects or quantities.

Examples of relations:

- {(a, b) ∈ A × B: a is the brother of b},
- {(a, b) ∈ A × B: a is the sister of b},
- {(a, b) ∈ A × B: age of a is greater than the age of b},
- {(a, b) ∈ A × B: total marks obtained by a in the final examination is less than the total marks obtained by b in the final examination},
- {(a, b) ∈ A × B: a lives in the same locality as b}. However, abstracting from this, we define mathematically a relation R from A to B as an arbitrary subset of A*xB*

Types of Relations

- Empty Relation
- Universal Relation
- Reflexive Relation
- Symmetric relation
- Transitive relation
- Equivalence relation

Empty Relation: A relation R in a set A is called empty relation if no element of A is related to any element of A, i.e., R = φ ⊂ A × A.

Universal Relation: A relation R in a set A is called universal relation if each element of A is related to every element of A, i.e., R = A × A.

Reflexive Relation: A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Symmetric relation R in X is a relation satisfying (a, b) ∈ R implies (b, a) ∈ R.

Transitive relation R in X is a relation satisfying (a, b) ∈ R and (b, c) ∈ R implies that (a, c) ∈ R.

Equivalence relation R in X is a relation which is reflexive, symmetric and transitive.

Functions

Functions are defined as a special kind of relations.

Types of Functions

1) One-one Function

A function $f : X \to Y$ is one-one (or injective) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in X$.

2) Onto Function

A function f : X → Y is onto (or surjective) if given any y ∈ Y, ∃ x ∈ X such that f(x) = y.

3) One-One and Onto Function

A function f : X → Y is one-one and onto (or bijective), if f is both one-one and onto.

Composition of functions

The composition of functions f : A → B and g : B → C is the function gof : A → C given by gof(x) = g(f(x)) ∀ x ∈ A.

Invertible Function

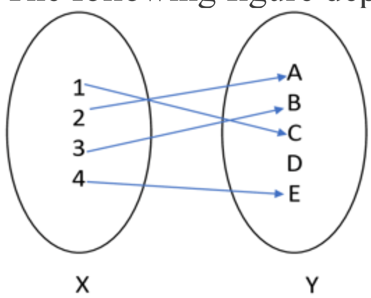
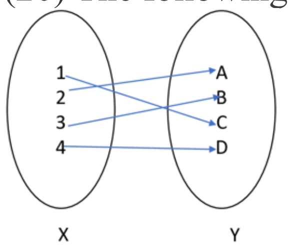
A function f : X → Y is invertible if ∃ g : Y → X such that gof = IX and fog = IY.

Condition- A function f : X → Y is invertible if and only if f is one-one and onto

---

SECTION –A(1 MARK EACH)		
Q.N.	**QUESTIONS**	**MARKS**
1	**A relation R in a set A is called ________, if (a_1, a_2) ∈ R implies (a_2, a_1) ∈ R, for all a_1, a_2 ∈ A.** (a) symmetric (b) transitive (c) equivalence (d) non-symmetric	**1**
2	**Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is** (a) Reflexive and symmetric(b) Transitive and symmetric (c) Equivalence (d) Reflexive, transitive but not symmetric	**1**
3	**The maximum number of equivalence relations on the set A = {1, 2, 3} are** (a) 1 (b) 2 (c) 3 (d) 5	**1**
4	**If set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is** (a) 720 (b) 120 (c) 0 (d) none of these	**1**

5	**Let R be the relation in the set N given by R={(a,b):a=b–2,b>6}. Choose the correct answer:** **(a) (2,4)∈R (b) (3,8)∈R (c)(6,8)∈R (d)(8,7)∈R**	1
6	**Let f : R → R be defined by f(x) = 1/x ∀ x ∈ R. Then f is** (a) one-one (b) onto (c) bijective (d) f is not defined	1
7	**Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is** (a) reflexive but not symmetric (b) reflexive but not transitive (c) symmetric and transitive (d) neither symmetric, nor transitive	1
8	**Let A = {1, 2, 3}. Then the number of relations containing (1, 2) and (1, 3), which are reflexive and symmetric but not transitive is** **(a) 1 (b) 2 (c) 3 (d) 4**	1
9	**Let f : R → R be defined by f(x) = x^2 + 1. Then, pre-images of 17 and – 3, respectively, are** (a) φ, {4, – 4} (b) {3, – 3}, φ (c) {4, –4}, φ (d) {4, – 4}, {2, – 2}	1
10	**Set A has 3 elements, and set B has 4 elements. Then the number of injective mappings that can be defined from A to B is** (a) 144 (b) 12 (c) 24 (d) 64	1
11	**Let f : [2, ∞) → R be the function defined by f(x) = x^2 – 4x + 5, then the range of f is** (a) R (b) [1, ∞) (c) [4, ∞) (d) [5, ∞)	1
12.	Which of the following relations is transitive but not reflexive for the set S={3, 4, 6}? (a) R = {(3, 4), (4, 6), (3, 6)} (b) R = {(1, 2), (1, 3), (1, 4)} (c) R = {(3, 3), (4, 4), (6, 6)} (d) R = {(3, 4), (4, 3)}	1
13.	Which of the following relations is symmetric and transitive but not reflexive for the set I = {4, 5}? (a) R = {(4, 4), (5, 4), (5, 5)} (b) R = {(4, 4), (5, 5)} (c) R = {(4, 5), (5, 4)} (d) R = {(4, 5), (5, 4), (4, 4)}	1

14	Let R be a relation in the set N given by R={(a,b): a+b=5, b>1}. Which of the following will satisfy the given relation? (a) (2,3) ∈ R (b) (4,2) ∈ R (c) (2,1) ∈ R (d) (5,0) ∈ R	**1**
15	**The function f(x)** =x^2+ 4x +4 is-: (a) even (b) odd (c) neither even nor odd (d)none of these	**1**
16	A function f:N→N is defined by f(x)= x^2+12. What is the type of function here? (a) bijective (b) surjective (c) injective (d) neither surjective nor injective	**1**
17	Let A={1,2,3} and B={4,5,6}. Which one of the following functions is bijective? (a) f={(2,4),(2,5),(2,6)} (b) f={(1,5),(2,4),(3,4)} (c) f={(1,4),(1,5),(1,6)} (d) f={(1,4),(2,5),(3,6)}	**1**
18	Let M={5,6,7,8} and N={3,4,9,10}. Which one of the following functions is neither one-one nor onto? (a) f={(5,3),(7,4),(6,4),(8,9)} (b) f={(5,3),(6,4),(7,9),(8,10)} (c) f={(5,4),(5,9),(6,3),(7,10),(8,10)} (d) f={(6,4),(7,3),(7,9),(8,10)}	**1**
19	The following figure depicts which type of function?  (a) one-one (b) onto (c) many-one (d) both one-one and onto	**1**
20	(20) The following figure depicts which type of function?  (a) injective (b) bijective (c) surjective (d) neither injective nor surjective	**1**

SECTION –B(2/3 MARKS EACH)		
21	1. Is f: N → N given by $f(x) = x^2$, one - one ? Give Reason .	
22	**Let R be a relation described on the set of natural numbers N. Determine the domain and range of the relation. Also check whether R is symmetric, reflexive and/or transitive.** **R = {x, y}: x ∈ N, y ∈ N, 2x+y = 41}**	
23	What is the range of the function. $f(x) = \frac{\lvert x-1 \rvert}{x-1}$, $x \neq 1$?	
24	Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.	
25	Let A = { 1 ,2 ,3} and define R = { (a , b) : a + b > 0 } . Show that R is universal relation on set A .	
26	Let A = { a, b ,c } how many relations can be defined in the set ? How many of these are reflexive ?	
27	**Prove that the relation R on the set N×N defined by (a ,b) R (c ,d) ⇒ad(b+c) = bc(a+d), for all** **(a, b),(c ,d)∈ N×N is an equivalence relation.**	
28	Let A = { 2, 4 , 6 , 8} and R ={(a,b): a is greater than b and a,b∈ A} on the set A . Write R as a set of order pairs , is the relation reflexive ?	
29	Let A = { 2, 4 , 6 , 8} and R ={(a,b): a is greater than band a,b∈ A} on the set A . Write R as a set of order pairs , is the relation Symemtric ?	
30	Let A = { 2, 4 , 6 , 8} and R = {(a,b): a is greater than b and a,b∈ A} on the set A . Write R as a set of order pairs , is the relation Transitive ?	
31	Let A = { 1 ,2 ,3} and define R = { (a , b) : a – b = 12 } . show that R is empty relation on set A .	

LONG ANSWER		
32	Let A = R-{3} and B = R – {1}. Consider the function f: A →B defined by f (x) = (x- 2)/(x -3). Is f one-one and onto? Justify your answer.	5
33	Show that the relation R in the set A $=\{1,2,3,4,5\}$ given by R $=\{(a,b): \lvert a-b\rvert \text{ is divisible by } 2\}$ is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,\ 4\}$ are related to each other, but no element of $\{1,3,5\}$ is related to any element of $\{2,\ 4\}$.	5
34	Consider a function f: $R_+ \to [-5,\infty)$ given by f$(x)=9x^2+6x-5$, show that f is bijective function.	5
35	**Show that the relation R defined by (a, b) R (c, d) => a + d = b + c on the set N XN is an equivalence relation.**	5
	SECTION –E (4 MARKS EACH)	
36	Amit and Vivek are students of class XII . Their maths teacher told them to collect the names of 5 students of class X and 4 students of class IX , Amit collected the names of students is denoted by A = { Anshul , Garima , Aditi , Shravan, Nitin } and Vivek collected the names of students denoted by B = { Rajat , Jagriti ,Ankush , Avi } . Since discussion of Relation and function was given in the class . From the above information give the answer of following question . (i)How many functions exist from A to B? (ii)If you want to know no. of relations exist from A to B . How many such relations are possible ? (iii)Let R: A→ A defined by R={ (x , y) : total marks obtained by x is less then the total marks obtained by y the R is a. Reflexive and Symmetric b.Symmetric and Transitive c.Equivalence Relation d.None of these (iv)How many Symmetric relations exist on Set A ?	4
37	In two different societies, there are some school going students - including girls as well as boys. Satish forms two sets with these students, as his college project.	

Let A={ a_1 , a_2 , a_3 , a_4 , a_5 } and B ={ b_1 , b_2 , b_3 , b_4 }
where a_i 's and b_i 's are the school going students of first and second society respectively.

Satish decides to explore these sets for various types of relations and functions.

Using the information given above, answer the following :

(i) Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

(ii) Let R : A→ A, R ={ (x, y) : x and y are students of same sex }. Then relation R is

a . reflexive only b.reflexive and symmetric but not transitive
c .reflexive and transitive but not symmetric d.an equivalence relation

(iii)Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. What is difference between their results?

(iv)Let R : A → B , R = { (a_1 , b_1), (a_2 , b_1), (a_3 , b_3), (a_4 , b_2),
1. (a_5 , b_2)} , then R is

a. neither one-one nor onto b.one-one but, not onto
c.only onto, but not one-one d.one-one and onto both

ANSWERS
MCQ

(1) (a) symmetric (2) (d) Reflexive, transitive but not symmetric

(3) (d) 5 (4) (c) 0 (5) **(c)(6,8)∈R** (6) (d) f is not defined

(7) (a) reflexive but not symmetric (8) **(a) 1**

(9) (c) {4, –4}, φ (10) (c) 24

(11) (b) [1, ∞) (12)(a) R = {(3, 4), (4, 6), (3, 6)}
(13)(d) R = {(4, 5), (5, 4), (4, 4)} (14) (a) (2,3) ∈ R

(15) (c) neither even nor odd

(16)(d) neither surjective nor injective

(17)d)f={(1,4),(2,5),(3,6)} (18)(a)f={(5,3),(7,4),(6,4),(8,9)}
19)(a)One-one
20)b)bijective

21) Yes, f is one-one $\because \forall x_1, x_2 \in N \Rightarrow x_1^2 = x_2^2$.

22) The range of the relation is {1,3, 5, 7,….., 39}

The domain of the relation R is {1, 2, 3, 4, 5, 6, 7 …., 20}

R is neither symmetric nor reflexive and not even transitive.

23)Firstly, redefine the function by using the definition of modulus function, i.e by using

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Further, simplify it to get the range

Given, function is f(x) = |x−1|/x−1, x ≠ 1

The above function can be written as

$$f(x) = \begin{cases} \frac{x-1}{x-1}, & \text{if } x > 1 \\ -\frac{(x-1)}{x-1}, & \text{if } x < 1 \end{cases}$$

$$\Rightarrow \quad f(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } x < 1 \end{cases}$$

24)Answer:

The relation R on set A = {I, 2, 3, 4, 5, 6} is defined as (a, b) ∈ R iff

b = a + 1Therefore, R = {(1, 2),(2, 3), (3, 4), (4,5), (5, 6)}

Clearly, (a, a) ∉ R for any as a ∈ A. So, R is not reflexive on A.

We observe that (1, 2) ∈ R but (2,1) ∉ R.

So, R is not symmetric.

We also observe that (1, 2) ∈ R and (2, 3) ∈ R but (1, 3) ∉ R. So, R is not transitive.

25)

26.To show that the relation R defined on the set A={1,2,3} is a universal relation, we need to demonstrate that R is equal to A×A.

1. **Define the Set** A:

A={1,2,3}

2. **Define the Relation** R:

The relation R is defined as:

R={(a,b):a+b>0}

3. **Find** A×A:
The Cartesian product A×A consists of all ordered pairs (a,b) where a and b are elements of A. Thus:
A×A={(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}

4. **Check Each Pair in** A×A:
We need to check if each pair (a,b) in A×A satisfies the condition a+b>0.

- For (1,1): 1+1=2>0
- For (1,2): 1+2=3>0
- For (1,3): 1+3=4>0
- For (2,1): 2+1=3>0
- For (2,2): 2+2=4>0
- For (2,3): 2+3=5>0
- For (3,1): 3+1=4>0
- For (3,2): 3+2=5>0
- For (3,3): 3+3=6>0

1. **Conclusion**:
   Since every pair (a,b) in A×A satisfies the condition a+b>0, we conclude that:
   R=A×A
   Therefore, R is a universal relation on the set A.

26)Step 1: Determine the number of relations on set A

1. **Identify the elements of the set**: The set A has 3 elements: a,b,c.
2. **Find the Cartesian product** A×A: The Cartesian product A×A consists of all ordered pairs formed by taking one element from A and pairing it with another element from A. Thus, we have:
A×A={(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)}
This gives us a total of 3×3=9 ordered pairs.
3. **Calculate the number of relations**: A relation on set A is any subset of A×A. The number of subsets of a set with n elements is given by 2n. Therefore, the number of relations is:
2|A×A|=29=512

Step 2: Determine the number of reflexive relations

1. **Understand reflexive relations**: A relation is reflexive if every element is related to itself. For our set A, this means that the pairs (a,a),(b,b),(c,c) must be included in any reflexive relation.
   2. **Identify mandatory pairs**: The mandatory pairs for reflexivity are (a,a),(b,b),(c,c). This accounts for 3 pairs.
   3. **Count remaining pairs**: The remaining pairs that can either be included or excluded from the relation are:
   (a,b),(a,c),(b,a),(b,c),(c,a),(c,b)

There are 6 such pairs.

4. **Calculate the number of reflexive relations**: Each of the remaining 6 pairs can either be included or excluded independently, which gives us: 2^6 =64

Final Answer

- The total number of relations that can be defined in the set A is **512**.
- The number of reflexive relations is **64**.

27)Here, (a,b)R(c,d)⇔ad(b+c)=bc(a+d) for all (a,b),(c,d)∈N×N.
First we will check R for reflexive.
For, (a,b)R(a,b),
⇒ab(b+a)=ba(a+b), which is true.
So, R is reflexive.
Now, we will check R for symmetric.
For, (a,b)R(c,d),
⇒ad(b+c)=bc(a+d)
⇒bc(a+d)=ad(b+c)
⇒cb(d+a)=da(c+b)
⇒(c,d)R(a,b) is true.
So, R is symmetric.
Now, we will check R for transtivity.
For, (a,b)R(c,d) and (c,d)R(e,f)
⇒ad(b+c)=bc(a+d) and cf(d+e)=de(c+f)
⇒abd+adc=abc+bcd and cfd+cef=ced+def
⇒abd−abc=bcd−acdandcfd−ced=def−cef
⇒ab(d−c)=cd(b−a)andcd(f−e)=ef(a−c)
⇒abb−a=cdd−candcdd−c=eff−e
⇒abb−a=eff−e
⇒abf−abe=efb−efa
⇒abf+efa=efb+abe
⇒af(b+e)=be(f+a)
So, (a,b)R(e,f) is true.
∴R is transitive.
As R is reflexive, symmetric and transitive, R is an equivalence relation.

28).R = {(8, 6), (8, 4), (8, 2) (6, 4), (6, 2), (4, 2)} Not reflexive

(29)R = {(8, 6), (8, 4), (8, 2) (6, 4), (6, 2), (4, 2) Not symmetric

(30) R = {(8, 6), (8, 4), (8, 2) (6, 4), (6, 2), (4, 2)   Not Transitive

31)To show that the relation R is an empty relation on the set A={1,2,3}, we need to analyze the condition defined for the relation R.

1. **Define the Set and Relation**:
- Let A={1,2,3}.
- The relation R is defined as R={(a,b)∈A×A:a−b=12}.

2. **Find the Cartesian Product** A×A:
- The Cartesian product A×A consists of all ordered pairs where the first element is from A and the second element is also from A.
- Thus, A×A={(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)}.

3. **Check Each Ordered Pair Against the Condition**:
- We need to check each ordered pair (a,b) in A×A to see if it satisfies the condition a−b=12.

- For (1,1):
1−1=0(not 12)
- For (1,2):
1−2=−1(not 12)
- For (1,3):
1−3=−2(not 12)
- For (2,1):
2−1=1(not 12)
- For (2,2):
2−2=0(not 12)
- For (2,3):
2−3=−1(not 12)
- For (3,1):
3−1=2(not 12)
- For (3,2):
3−2=1(not 12)
- For (3,3):
3−3=0(not 12)

4. **Conclusion**:
- After checking all possible pairs, we find that none of the pairs satisfy the condition a−b=12.
- Therefore, R=∅ (the empty set).
- Since R contains no elements, we conclude that R is an empty relation on the set A.

Final Result:

Thus, we have shown that R is an empty relation on the set A.

Long answer

32)Given function:

$f(x) = (x- 2)/(x -3)$

Checking for one-one function:

$f(x_1) = (x_1 - 2)/ (x_1 - 3)$

$f(x_2) = (x_2-2)/ (x_2-3)$

Putting $f(x_1) = f(x_2)$

$(x_1-2)/(x_1-3)= (x_2-2)/(x_2 -3)$

$(x_1-2) (x_2 - 3) = (x_1 - 3) (x_2-2)$

$x_1 (x_2 - 3)- 2 (x_2 -3) = x_1 (x_2 - 2) - 3 (x_2 - 2)$

$x_1 x_2 -3x_1 -2x_2 + 6 = x_1 x_2 - 2x_1 -3x_2 + 6$

$-3x_1 - 2x_2 =- 2x_1 -3x_2$

$3x_2 -2x_2 = - 2x_1 + 3x_1$

$x_1= x_2$

Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$

Thus, the function f is one-one function.

Checking for onto function:

$f(x) = (x-2)/(x-3)$

Let $f(x) = y$ such that y B i.e. $y \in R - \{1\}$

So, $y = (x -2)/(x- 3)$

$y(x -3) = x- 2$

$xy -3y = x -2$

$xy - x = 3y-2$

$x (y -1) = 3y- 2$

$x = (3y - 2)/(y-1)$

For $y = 1$, x is not defined But it is given that. $y \in R - \{1\}$

Hence, $x = (3y- 2)/(y- 1) \in R - \{3\}$ Hence, f is onto.

33.Since $|a–a|$ is even,
$\therefore (a, a) \in R$
$\therefore$ R is reflexive.
(ii) Let $(a, b) \in R$ Then $|a – b|$ is even
$\therefore |b – a|$ is even
$\therefore (b, a) \in R$ and R is symmetric.
(iii) Let $(a, b), (b, c) \in R$
Then $a – b = \pm 2m$, $b – c = \pm 2n$
$\therefore a – c = \pm 2(m + n)$, where m, n are integers.
$\therefore (a, c) \in R$ and hence R is transitive
Thus, R is an equivalence relation.

34). $R+ \rightarrow [- 5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$
Let y be an arbitrary element of $[- 5, \infty)$.
Let $y = 9x^2 + 6x - 5$
$\Rightarrow y = (3x + 1)^2 - 1 - 5$
$\Rightarrow y = (3x + 1)^2 - 6$
$\Rightarrow (3x + 1)^2 = y + 6$
$\Rightarrow 3x + 1 = \sqrt{y} + 6$ [as $y \geq - 5 \Rightarrow y + 6 > 0$]
$\Rightarrow x = [(\sqrt{y} + 6) - 1]/3$
$\therefore$ f is onto, there by range $f = [- 5, \infty)$.

	35.Let (a, b) in N × N We know that a+b=b+a .(a,b)R(a,b) ⇒ R is reflexive. (ii) Let (a,b),(c,d)∈N×N and(a,b)R(c,d) ⇒a+d=b+c ⇒b+c=a+d ⇒c+b=d+a ⇒(c,d)R(a,b) ∴ R is symmetric. (iii) Let ⇒a+d=b+c ⇒b+c=a+d ⇒c+b=d+a ⇒(c,d)R(a,b) ⇒a+d=b+candc+f=d+e ⇒a+d+c+f=b+c+d+e ⇒a+f=b+e ⇒(a,b)R(e,f) ⇒ R is transitive. ∴ R is reflexive, symmetric and transitive. ⇒ R is an equivalence relation . Hence Proved . **Case based** **36 1.(i)C (ii) B (iii)D (iv) A** **37(i)4 (ii)4 (iii) 3 (iv) 1**	
	ASSERSSION REASONING **In the following question a statement of Assertion (A) is followed by a statement of reason (R). Pick the correct option: A. Both A and R are true and R is the correct explanation of A. B. Both A and R are true but R is not the correct explanation of A. C. A is true but R is false. D. A is false but R is true**	
1.	**Assertion (A): If n (A) =p and n (B) = q then the number of relations from A to B is 2pq** **Reason (R) : A relation from A to B is a subset of A x B**	
2	**Assertion (A): The relation R in the set A = {1, 2, 3, 4, 5, 6} defined as R={ (x, y) :y is divisible by x} is not an equivalence relation.** **Reason (R) :The relation R will be an equivalence relation, if it is reflexive, symmetric and transitive.**	

3	**Assertion (A): If R is the relation defined in set {1, 2, 3, 4, 5, 6} as R = {(a, b) :b = a + 1} , then R is reflexive** **Reason (R) : The relation R in the set A is reflexive if aRa for every a ∈ A.**	
4	**Assertion (A) : A relation R ={ (1,1),(1,2),(2,2),(2,3)(3,3)}defined on the set A={1,2,3} is reflexive. Reason (R) : A relation R on the set A is reflexive if (a,a) ,for all a**	
5.	**Assertion (A): If R is the relation in the set A= {1, 2, 3 , 4, 5 } given by R={(a, b): |a - b| is even} R is an equivalence relation.** **Reason (R) : All elements of {1, 3, 5} are related to all elements of {2, 4}**	
	1.Answer: A Solution: A is true - No of elements of AXB = pxq, So the number of relations from A to B is 2pq R is true – every relation from A to B is a sub set of AXB **2.Answer : A Solution: A is true-R is reflexive and transitive but not symmetric ie (2,4)ϵR (4,2)ϵR R-true- Definition of an equivalence relation.** **3.Answer :D Solution : A is false – (x,x) R R is true - (1, 2) R (2, 1) R** **4.Answer: A Solution: A is true - (a,a) ,for all a A R is true – Correct explanation for reflexive relation.** **5.Answer: C Solution: A is true- Since it an equivalence relation R is false – the modulus of difference between the two elements from each of these two subsets will not be even**	

INVERSE TRIGONOMETRIC FUNCTIONS

The inverse trigonometric functions are the inverse functions of the trigonometric functions written as
$\sin^{-1} x$ **,** $\cos^{-1} x$, $\tan^{-1} x$, $\text{cosce}^{-1} x$, $\sec^{-1} x$, $\cot^{-1} x$
If $= \sin x$ **, then** $x = \sin^{-1} y$
If $= \cos x$ **, then** $x = \cos^{-1} y$
Similarly, for other trigonometric functions, inverse trigonometric functions are written.

Domain & Range of Inverse Trigonometric Functions

Functions	**Domain**	**Range** (Principal Value Branches)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\text{cosec}^{-1} x$	$R-(-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\sec^{-1} x$	$R-(-1,1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$
$\cot^{-1} x$	R	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Within the domain	$\sin^{-1}(\sin \theta) = \theta$	$\cos^{-1}(\cos \theta) = \theta$
	$\tan^{-1}(\tan \theta) = \theta$	$\cot^{-1}(\cot \theta) = \theta$
	$\sec^{-1}(\sec \theta) = \theta$	$\text{cosec}^{-1}(\text{cosec}\, \theta) = \theta$

SECTION A (MCQ)

Que 1.	The domain of $\sin^{-1} 2x$ is			
	(a) [-1, 1]	(b) (-1, 1)	(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	(d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$
Que 2.	Which of the following is the principal value branch of cosec^{-1} x?			
	(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	(b) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$	(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	(d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
Que 3.	Find principal value of $\cos^{-1}\left(\cos \frac{5\pi}{4}\right)$			
	(a) $\frac{3\pi}{4}$	(b) $\frac{5\pi}{4}$	(c) $\frac{7\pi}{4}$	(d) $\frac{\pi}{2}$

Que 4.	The domain of function $y = \cos^{-1}x$ is			
	(a) [-1, 1]	(b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$	(c) [-2, 2]	(d) None of these
Que 5.	The value of $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$ is equal to			
	(a) $-\frac{\pi}{3}$	(b) $\frac{\pi}{3}$	(c) $\frac{4\pi}{3}$	(d) $\frac{2\pi}{3}$
Que 6.	Value of $\cos\left[\frac{\pi}{6} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$ *is equal to*			
	(a) $-\frac{\sqrt{3}}{2}$	(b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$	(c) $\frac{\sqrt{5}-1}{4}$	(d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$
Que 7.	The value of $\tan^{-1}\left[tan\,\frac{9\pi}{8}\right]$			
	(a) $\frac{\pi}{8}$	(b) $\frac{\pi}{4}$	(c) $\frac{\pi}{2}$	(d) None of these
Que 8	Domain of $\sin^{-1}x + \cos x$ is			
	(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	(b) R	(c) $[-1, 1]$	(d) $(-1,1)$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Que 9.	Assertion : Domain of $y = \cos^{-1}x$ is $[-1, 1]$ Reason : The range of the principal value branch of $y = \cos^{-1}x$ is $[0, \pi] - \left[\frac{\pi}{2}\right]$
Que 10.	Assertion (A): We can write $\sin^{-1}x = (\sin x)^{-1}$ Reason (R) : Any value in the range of the principal value branch is called the principal Value of that inverse trigonometric function.

SECTION-B (2 MARKS)

Que 11.	Find the principal value of $\sin^{-1}\frac{1}{\sqrt{2}}$
Que 12.	Find the value of k if $\sin^{-1}\left[k\,tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$
Que 13.	Find the value of : $\tan^{-1}(1) + \cos^{-1}(-\frac{1}{2})$

SECTION C **(3Marks)**

Que 14.	Find the value of $\sin^{-1}\left(sin\,\frac{2\pi}{3}\right) + \cos^{-1}\left(cos\,\frac{7\pi}{6}\right)$

Que 15.	Epress the functions $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$ in the simplest form:

………

Solution

1.	(c) $\left[-\frac{1}{2},\frac{1}{2}\right]$
2.	(d) $[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$
3.	(a) $\frac{3\pi}{4}$
4	(a) [-1, 1]
5.	(d) $\frac{2\pi}{3}$
6.	(a) $-\frac{\sqrt{3}}{2}$
7	(a) $\frac{\pi}{8}$
8	(c) $[-1,1]$
9	(c) Assertion (A) is true but reason (R) is false.
10	(d) Assertion (A) is false but reason (R) is true.
11	$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\sin^{-1}\left(\sin\frac{\pi}{4}\right)=\frac{\pi}{4}$
12	$\sin^{-1}\left[k\,tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]=\frac{\pi}{3}$ Or, $k\,tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{2}$ Or, $k\,tan\left(2\,\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $k\,tan\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$ $k\sqrt{3}=\frac{\sqrt{3}}{2}$ or k= $\frac{1}{2}$
13	let $\tan^{-1}(1)=y$ Then tan y = 1 y = $\frac{\pi}{4}$ $\cos^{-1}(-\frac{1}{2})$ = x Cos x = $-\frac{1}{2}$ x = $\frac{2\pi}{3}$ as domain (0, π) $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)=$ $=\frac{\pi}{4}+\frac{2\pi}{3}$ $=\frac{11\pi}{12}$
14	$sin^{-1}(\sin 2\pi/3)+\cos^{-1}(\cos 2\pi/3)$ $=sin^{-1}(\sin(\pi-\pi/3)+2\pi/3$ $=sin^{-1}(\sin(\pi/3)+2\pi/3$ $=sin^{-1}(\sin(\pi/3)+2\pi/3$ $=\frac{\pi}{3}+\frac{2\pi}{3}=\pi$
15	Let x= sec θ, then $\sqrt{x^2-1}=\sqrt{sec^2\theta-1}$ =tan θ Therfore $\cot^{-1}\frac{1}{\sqrt{x^2-1}}=\cot^{-1}(cot\,\theta)=\theta=sec^{-1}\,\theta$

3/4. MATRICES & DETERMINANTS

IMPORTANT CONCEPTS/RESULTS

- A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.
- A matrix having m rows and n columns is called a matrix of order m×n.
- An m×n matrix is a square matrix if m = n.
- If A = B then both matrices have the same order and corresponding elements of both matrices wiil be equal.
- **Operations on Matrices:** **(i)Addition of matrices:** If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are two matrices of the same order. Then, $A + B = [a_{ij} + b_{ij}]_{m \times n}$.

(ii) Multiplication of a matrix by a scalar: If $A = [a_{ij}]_{m\times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k i.e. $kA = [ka_{ij}]_{m\times n}$

(iii) Difference of matrices: If $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times n}$are two matrices then

$A - B = [a_{ij} - b_{ij}]_{m\times n}$, for all value of i and j. In other $A - B = A + (-1) B$

(iv) Multiplication of matrices: The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B.Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$. Then $AB = C = [c_{ik}]_{m \times p}$, where $c_{ik} = a_{i1} b_{1k} + a_{i2} b_{2k} + a_{i3} b_{3k} + ... + a_{in}b_{nk} = \sum_{j=1}^{n} a_{ij}b_{jk}$

Transpose of a Matrix: If A = [*aij*] be an *m* × *n* matrix, then the matrix obtained by interchanging

the rows and columns of A is called the *transpose* of A. Transpose of the matrix A is

denoted by A' or A^T.

Properties of transpose of the Matrices: For any matrices A and B of suitable orders, we have

(i) $(A^{T})^T = A$ $(ii)(KA)^T = KA^T (iii)(A + B)^T = A^T + B^T (iv)(AB)^T = B^T A^T$

Symmetric Matrix: A square matrix M is said to be symmetric if $A^T = A$

e.g.$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$, $\begin{bmatrix} x & y & z \\ y & u & v \\ z & v & w \end{bmatrix}$

Note: **there will be symmetry about the principal diagonal in Symmetric Matrix.**

Skew symmetric Matrix: A square matrix M is said to be symmetric if $A^T = -A$

e.g.$\begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix}$

Note**: All the principal diagonal element of a skew symmetric Matrix are zero.**

Determinant: For every Square Matrix we can associate a number which is called the Determinant of the square Matrix.

Determinant of a matrix of order one

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a.

Determinant of a matrix of order two

Let A= $\begin{bmatrix} a & b \\ x & y \end{bmatrix}$ be an Square Matrix of order2× 2 then the determinant of A is denoted by $|A|$ and defined by $|A| = \begin{vmatrix} a & b \\ x & y \end{vmatrix}$= ay-bx

Determinant of a matrix of order 3× 3: Let us consider the determinant of a square matrix of order 3, $|A|=$ $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Expansion along first row $|A| = a(qz - yr) - b(pz - xr) + c(py - qx)$

We can expand the determinant with respect to any row or any column.

Minors and cofactors:

Minor of an element $a_{ij}$of a determinant is the determinant obtained by deleting its *i*th row and *j*th column in which elementa_{ij} lies. Minor of an elemen$a_{ij}$is denoted by M_{ij}.

Cofactors: cofactors of an elementa_{ij} $denoted$ by A_{ij} and is defined by $A_{ij} = (-1)^{i+j}M_{ij}$ where M_{ij} is the minor of a_{ij}.

Adjoint of a Matrix: The adjoint of a square matrix A = $[a_{ij}]_{n\times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n\times n}$ where A_{ij} is the cofactor of the element a_{ij}.

Let A = $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ be a Matrix of order 2 × 2, Then **adj(A)** = $\begin{bmatrix} \boldsymbol{\delta} & -\boldsymbol{\beta} \\ -\boldsymbol{\gamma} & \boldsymbol{\alpha} \end{bmatrix}$

Inverse of a Matrix: Inverse of a Square Matrix A is defined as $\boldsymbol{C} = \frac{\boldsymbol{adj(A)}}{|\boldsymbol{A}|}$

Note: If A and B are Square Matrix of order n then

(i) **A(adj A) = (adj A)A=**$|\boldsymbol{A}|\boldsymbol{I}$ **,** where I is the Identity Matrix of order n.

(ii) A square Matrix A is said to be **singular and non-singular** according as $|\boldsymbol{A}| = \boldsymbol{0}$ $\boldsymbol{and}$ $|\boldsymbol{A}| \neq \boldsymbol{0}$

(iii) $|\boldsymbol{adj}(\boldsymbol{A})|= |\boldsymbol{A}|^{\boldsymbol{n}-\boldsymbol{1}}$ $(\boldsymbol{For\ a\ square\ Matrix\ of\ order}\ \boldsymbol{3} \times \boldsymbol{3}\ |\boldsymbol{adj}(\boldsymbol{A})| = |\boldsymbol{A}|^{\boldsymbol{2}})$

(iv) $|\boldsymbol{AB}| = |\boldsymbol{A}||\boldsymbol{B}|$

(v) $|\boldsymbol{kA}| = \boldsymbol{k}^{\boldsymbol{n}}|\boldsymbol{A}|$

(vi) **If AB = BA = I , then A^{-1} = B and B^{-1} = $(A^{-1})^{-1}$ = A**

(vii) $|\boldsymbol{A}'| = |\boldsymbol{A}|$

(viii) $(\boldsymbol{A}')^{-\boldsymbol{1}} = (\boldsymbol{A}^{-\boldsymbol{1}})'$

(ix) $(\boldsymbol{A}')^{\boldsymbol{n}} = (\boldsymbol{A}^{\boldsymbol{n}})'$

SOME ILLUSTRATIONS/EXAMPLES (WITH SOLUTION)

(i) Multiple Choice Questions:

Q.1. If A is 2 × 3 matrix such that AB and AB′ both are defined, then find the order of the matrix B

(a) 2×3 (b) 3×3 (c) 2×2 (d) Not defined

Solution: Let order of B be $m \times n$

$\because$ AB is defined, $\therefore$ No. of columns in A = No. of rows in B

$\Rightarrow 3 = m$

Order of $B' = n \times m$

Again $\because$ AB′ is defined, $\therefore$ No. of columns in A = No. of rows in B′

$\Rightarrow 3 = n$

So, order of B = $m \times n = 3 \times 3$. $\therefore$ Correct option is (b).

Q.2. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) of x is/are

(a) 1 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\pm\sqrt{3}$

Solution: Since $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 - 20 = 2x^2 - 24 \Rightarrow -18 + 24 = 2x^2 \Rightarrow 2x^2 = 6 \Rightarrow x = \pm\sqrt{3}$

$\therefore$ Correct option is (d).

Q.3. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is

(a) 1 (b) -1 (c) 2 (d) 0

Solution: $|A| = |kA|$ and n=2

$|A| = k^2 |A|$ ($\because |kA| = k^n |A|$)

$\Rightarrow k^2 = 1$ $\Rightarrow k = \pm 1$ $\Rightarrow$ Sum of all values of k = + 1 – 1 = 0

$\therefore$ Correct option is (d).

(ii) Case Based Study Question:

Q.4. To promote the usage of house toilets in villages especially for women, an organisation tried to generate awareness among the villagers through (i) house calls (ii) letters and (iii) announcements. The cost for each mode per attempt is (i) Rs 50 (ii) Rs 20 (iii)Rs40 respectively. The number of attempts made in the villages X, Y and Z are given below:

	(i)	**(ii)**	**(iii)**
X	400	300	100
Y	300	250	75
Z	500	400	150

Also the chance of making of toilets corresponding to one attempt of given modes is:

(i) 2% (ii) 4% (iii) 20%

Let A, B, C be the cost incurred by organisation in three villages respectively.
Based on the above information answer the following questions

(A) Form a required matrix on the basis of the given information.
(B) Form a matrix, related to the number of toilets expected in villagers X, Y, Z after the promotion campaign.
(C) What is total amount spent by the organisation in all three villages X, Y and Z

OR

What are the total number of toilets expected after promotion campaign?

Solution:(A) Rs A, Rs B and Rs C are the cost incurred by the organisation for villages X, Y, Z respectively, therefore matrix equation will be

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

(B) Let number of toilets expected in villagers X, Y, Z be x, y, z respectively

Therefore required matrix is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

(C). $\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} 20000 + 6000 + 4000 \\ 15000 + 5000 + 3000 \\ 25000 + 8000 + 6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$

Total money spent = 30000 + 23000 + 39000 = 92000 Rs

OR

From part (B) the required matrix for the expected number of toilets is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix} = \begin{bmatrix} 8 + 12 + 20 \\ 6 + 10 + 15 \\ 10 + 16 + 30 \end{bmatrix} = \begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

So, total number of toilets expec ted in 3 villages are = 40 + 31 + 56 = 127

(iii) Short Answer Type Questions:

Q.5. If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ then find the value of x and y.

Solution: Given $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ or $\begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ So 2x – y = 10 and 3x + y = 5

On solving we get x = 3 and y = -4

Q.6. If A is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.

Solution: L.H.S. $= (I + A)^3 = I^3 + A^3 + 3I^2A + 3IA^2$

$= I + A^2.A + 3IA + 3IA^2 = I + A.A + 3A + 3IA$

$= I + A^2 + 3A + 3A = I + A + 3A + 3A = I + 7A =$ R.H.S.

(iv) Long Answer Type Questions:

Q.7. If $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, then find BA and use this to solve the system of equations: y + 2z = 7, x – y = 3 and 2x + 3y + 4z = 17.

Solution: $BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$

$$B\left(\frac{1}{6}A\right) = I \qquad \Rightarrow \quad B^{-1} = \frac{1}{6}A = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

The given equations can be re-written as, x – y = 3, 2x + 3y + 4z = 17, and y + 2z = 7

$$\therefore BX = C \; i.e. \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = B^{-1}C \; i.e. \; \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1 \; and \; z = 4$

QUESTIONS FOR PRACTICE WITH SOLUTION

QN	QUESTIONS
	MCQ (1 MARK)
1	If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to **(a)** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
2	If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x and y is **(a)**x = 3, y = 1 **(b)** x = 2, y = 3 **(c)** x = 2, y = 4 **(d)** x = 3, y = 3
3	Which one is not correct **(a)** (AB)′ = B′A′ **(b)** A′B′ = (BA)′ **(c)** (**k**A)′ = kA′ **(d)** A′ = A
4	If $\begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix}^2 = \begin{vmatrix} 3 & 2 \\ 1 & x \end{vmatrix} - \begin{vmatrix} x & 3 \\ -2 & 1 \end{vmatrix}$, then the value of x is: **(a)** 6 **(b)** 3 **(c)** 7 **(d)** 1
5	If A is a square matrix of order 3 and |A| = 5, then |adj A| is **(a)** 5 **(b)** 25 **(c)** 125 **(d)** $\frac{1}{5}$
6	If A is a symmetric matrix,thenA^3 is: **(a)** Symmetric Matrix **(b)** Skew Symmetric Matrix **(c)** Identity matrix **(d)**Row Matrix
7	If the area of a triangle with vertices (-3, 0), (3, 0) and (0, k) is 9 sq units. Then the value of k will be (a)9 (b)3 (c)-9 (d)6
8	If A = $\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists, if (a) $\lambda = 2$ (b) $\lambda \neq 2$ (c) $\lambda \neq -2$ (d) None of these
Assertion Reason Based Question:	

(a) Both A & R are true & R is the correct explanation of A
(b) Both A & R are true but R is not the correct explanation of A
(c)A is true but R is false
(d)A is false but R is true

9 For A and B square matrices of same order, choose appropriate option

Assertion (A): $(A + B)^2 \neq A^2 + 2AB + B^2$

Reason (R): Generally, $AB \neq BA$

Assertion: If the matrix $A = \begin{bmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then $\lambda = 4$.

Reason[A]: If A is a singular matrix, then $|A| = 0$.

Short Answer type Questions (2/3 MARKS)

11 If x = - 4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then find the sum of other two roots.

12 If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then find (A – 2I) (A – 3I).

13 If $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then find the value of $(X^2 - X)$.

14 Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.
Order of X is 2 x 2 (why)

15 A matrix A of order 3x3 is such that $|A| = 4$ .Find the value of $|2A|$.

16 Evaluate $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$

17. Express the following matrix as the sum of a symmetric and skew-symmetric matrix, and verify the result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$.

Long Answer type Questions (5 ,MARKS)

18 Solve the following system of equations by matrix method.

$3x - 2y + 3z = 8$, $2x + y - z = 1$, $4x - 3y + 2z = 4$

19 If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, Find A^{-1}. Hence, solve the system of equations :

$3x + 3y + 2z = 1$, $x + 2y = 4$, $2x - 3y - z = 5$

20 If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations: $x - 2y = 3$, $2x - y - z = 2$, $-2y + z = 3$.

ANSWER

S.N	ANSWER
1	ANS (d)
2	ANS (b)
3	ANS (d)
4	ANS (a)
5	ANS (b)
6	ANS (a)
7	ANS (b)
8	ANS (d)
9	ANS (a)
10	ANS (a)
11	Solution : Expanding along first Row $f(x) = x(x^2 - 2) - 2(x - 3) + 3(2 - 3x)$ $= x^3 - 2x - 2x + 6 + 6 - 9x$ $= x^3 - 13x + 12$ Use the fact that x = -4 is a root and factorize $f(x) = (x + 4)(x^2 - 4x + 3)$ $f(x) = (x + 4)(x - 1)(x - 3)$ Roots are x = -4, 1, and 3 The other two roots are 1 and 3. Their sum is 1 + 3 = 4
12	Solution : $(A - 2I)(A - 3I) = \{ \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \} \{ \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \}$ $= \{ \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \} \{ \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} - \}$ $= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
13	Do yourself:
14	Let $X = \begin{bmatrix} x & y \\ u & v \end{bmatrix}$ Now $\begin{bmatrix} x & y \\ u & v \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$. Multiplying and comparing the elements we get X+4y =-7 , 2x+5y = -8 , 3x+6y = -9 , X+4y =-7 , 2x+5y = -8 , 3x+6y = -9 Solving we get $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$
15	$\lvert 2A \rvert = 2^3 \times \lvert A \rvert = 8 \times 4 = 32$ Ans 32
16	ANS : 0
17	ANS : $\begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$

18	Let A = $\begin{bmatrix} 3 & -2 & 03 \\ 2 & 1 & -1 \\ 04 & 3 & 2 \end{bmatrix}$ X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ B= $\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ Then Above system of equations can be expressed as AX =B Or , X = $A^{-1}B$ |A| = -17 **adj (A) =** $\begin{bmatrix} \mathbf{-1} & \mathbf{-5} & \mathbf{-1} \\ \mathbf{-8} & \mathbf{-6} & \mathbf{9} \\ \mathbf{-10} & \mathbf{1} & \mathbf{7} \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-17} \begin{bmatrix} -1(8) + (-5)(1) + (-1)4 \\ -8(8) + (-6)(1) + 9(4) \\ -10(8) + 1(1) + 7(4) \end{bmatrix}$ Ans : x = 1, y = 2 and z = 3.
19	Let A = A = $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$ Find A^{-1} (as usual) Let C = $\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 2 & -3 & -1 \end{bmatrix}$ X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ B= $\begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$ Then Above system of equations can be expressed as CX =B Or , X = $C^{-1}B$ Using The property $(\boldsymbol{A}')^{-1} = (\boldsymbol{A}^{-1})'$ **Solve the Equations.**
20	Do as illustrated example 7.

5. COUNTINUITY AND DIFFERENTIABILITY

KEY POINTS:

Continuity at a Point: A function f(x) is said to be continuous at a point x = a, if

Left hand limit of f(x) at (x = a) = Right hand limit of f(x) at (x = a) = Value of f(x) at (x = a)

i.e. if at x = a, LHL = RHL = f(a)

where, LHL = limit of f(x) and RHL = limit of f(x) at x = a

Note: To evaluate LHL of a function f(x) at (x = a), put x = a – h and to find RHL, put x = a + h.

f is continuous at c if $\lim_{x \to c} f(x) = f(c)$ or

$$\lim_{h \to 0} f(c-h) = f(c) = \lim_{h \to 0} f(c+h)$$

Differentiability: A function f(x) is said to be differentiable at a point x = a, if

Left hand derivative at (x = a) = Right hand derivative at (x = a)

i.e. LHD at (x = a) = RHD (at x = a), where Right hand derivative, where

Right hand derivative, $Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Left hand derivative, $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

Product Rule: Let y = f(x) g(x). Then, by using product rule, it's derivative is written as

$$\frac{dy}{dx} = \left[\frac{d}{dx}(f(x))\right] g(x) + \left[\frac{d}{dx}(g(x))\right] f(x).$$

Quotient Rule: Let y = f(x)g(x); g(x) ≠ 0, then by using quotient rule, it's derivative is written as

$$\frac{dy}{dx} = \frac{g(x) \times \frac{d}{dx}[f(x)] - f(x) \times \frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

Chain Rule: Let y = f(u) and u = f(x), then by using chain rule, we may write

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \text{ when } \frac{dy}{du} \text{ and } \frac{du}{dx} \text{ both exist.}$$

Rules of logarithmic function

log mn = log m+log n

$log\ log\ \left(\frac{m}{n}\right)$ = log m−log n

log (mn) =n log mn

Change of base rule, $\log_a b = \frac{b}{a}$

loge = 1, log1 = 0, $e^{logf(x)}=f(x)$

Differentiation of Functions in Parametric Form: A relation expressed between two variables x and y in the form x = f(t), y = g(t) is said to be parametric form with t as a parameter, when

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

(whenever dxdt≠0)
Note: dy/dx is expressed in terms of parameter only without directly involving the main variables x and y.

Second order Derivative: It is the derivative of the first order derivative.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Some Standard Derivatives;

(i) $\frac{d}{dx}(\sin x) = \cos x$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(v) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

(vi) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(vii) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(viii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

(ix) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(x) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

(xi) $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

(xii) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

(xiii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(xiv) $\frac{d}{dx}(\text{constant}) = 0$

(xv) $\frac{d}{dx}(e^x) = e^x$

(xvi) $\frac{d}{dx}(\log_e x) = \frac{1}{x}, x > 0$

(xvii) $\frac{d}{dx}(a^x) = a^x \log_e a, a > 0$

<u>Logarithmic Differentiation</u>: Let y = $[f(x)]^{g(x)}$ ..(i)

So by taking log (to base e) we can write Eq. (i) as log y = g(x) log f(x). Then, by using chain

$$\frac{dy}{dx} = [f(x)]^{g(x)}\left[\frac{g(x)}{f(x)} f'(x) + g'(x)\log f(x)\right]$$

MULTIPLE CHOICE QUESTIONS

1. The function f(x) = |x| at x = 0 is

(a) Continuous but not differentiable (b) differentiable but not continuous

(c) Continuous and differentiable (d) discontinuous and differentiable

2. If $f(x) = \begin{cases} \frac{\sin \pi x}{5x}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at x = 0 , then k is equal to

(a) $\frac{5}{\pi}$ (b) $\frac{\pi}{5}$ (c) 1 (d) 0

3. Differentiate $\cos^2(x^3)$ with respect to x^3 is equal to

(a) – $\cos(2x^3)$ (b) - $\sin(2x^3)$ (c) $\sin(2x^3)$ **(d) $\cos(2x^3)$**

4. The derivative of cos x with respect to sin x is

(a) cot x (b) tan x (c) - cot x (d) – tan x

5. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is

(a) 3/2 (b) 3/4t (c) 3/2t (d) – 3/2t

6. The derivative of $\cot^{-1}(e^x)$ with respect to x at x = 0 is

(a) 0 (b) 1 (c) ½ (d) -1/2

7.If y = $a \sin \sin mx + b \cos \cos mx$ then $\frac{d^2y}{dx^2}$ is

(a) m^2 y (b) - m^2 y (c) m y (d) - my

8. The derivative of $\log \log \left(x + \sqrt{x^2+1}\right)$ with respect to x is

(a) $\sqrt{x^2+1}$ (b) x $\sqrt{x^2+1}$ (c) $\frac{x}{\sqrt{x^2+1}}$ (d) $\frac{1}{\sqrt{x^2+1}}$

9. If f(x) = $\{ax^2 + 1,\ x > 1\ x + a,\ \ x \leq 1$ is derivable at x = 1, then the value of a is

(a) 0 (b) 1 (c) ½ (d) 2

10. The function f(x) = $\begin{cases} \frac{e^{3x}-e^{-5x}}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at x = 0 for the value of k as

(a) 3 (b) 5 (c) 2 (d) 8

ASSERTION – REASON QUESTION

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

1.Assertion : The function f(x) = |x| is everywhere continuous

Reason : Every differential function is continuous .

2. **Assertion :** If f(x) and g(x) are two continuous functions such that f(0) = 3 , g(0) = 2 and $[f(x) + g(x)] = 5$

Reason : If f(x) and g(x) are two continuous functions at x = a , then $[f(x) + g(x)] = [f(x)] + [f(x)]$

3. **Assertion (A):** Every differentiable function is continuous but converse is not true.

Reason (R): Function f(x) = |x| is continuous.

4. **Assertion : If** a function f is discontinuous at c, then c is called a point of discontinuity.

Reason : A function is continuous at x = c, if the function is defined at x = c and the value of the function at x = c equals the limit of the function at x = c

5. **Assertion :** f (x) $=x^n \sin\sin\left(\frac{1}{x}\right)$ is differentiable for all real values of x (n ≥ 2).

Reason : For n ≥ 2, $[f(x)] = 0$

SECTION – B (2 MARKS)

1. **Differentiate the function w. r. t. x if f(x) =** $sec\left[tan\left(\sqrt{x}\right)\right]$

2 **If the function f(x) =** $\begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ p, & x = 0 \end{cases}$ **is continues at x = 0 , then find p**

3. **If** $-2x^2 - 5xy + y^2 = 76$ **, then find** $\frac{dy}{dx}$

4. **Differentiate** $\left(\frac{5^x}{x^5}\right)$ **with respect to x**

5. **Find** $\frac{dy}{dx}$ **at** $\theta = \frac{\pi}{3}$ **if** $x = a(\theta - sin\theta)$ *and* $y = a(1 + cos\theta)$.

6. **If the function f(x) given by: f(x) =** $\begin{cases} 3ax + b & if\ x > 1 \\ 11 & if\ x = 11 \\ 5ax - 2b & if\ x < 1 \end{cases}$ **is continuous at** $x = 1$**, find the value of** ***a*** **and** ***b*****.**

7. **Find the value of k ,if the function f(x) =** $\begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2} & if\ x \neq 2 \\ k & if\ x = 2 \end{cases}$ **is continuous at x =2**

8. **Check the differentiability of the function f defined by f(x) = |x - 5| at x = 5**

9. **If f(x) =** $\begin{cases} ax + b, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x < 2 \end{cases}$ **is differentiable function in (0, 2) , then find a and b**

10. **Differentiate** sin^2x **with respect to** e^{cosx}.

SECTION - C (3 MARKS)

1. **If y = A** e^{mx} **+ B** e^{nx} **, show that** $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

2. **If y = 3 cos(logx) + 4 sin(logx) , show that** $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

3 **If** $(cosx)^y = (cosy)^x$ **, find** $\frac{dy}{dx}$

4. **If** $x = 3cost - 2t,\ y = 3sint - 2t$ **then show that** $\frac{dy}{dx}$ $=cot\, cot\, t$ **.**

5. If $y = (x)^{\log x} + (\log x)^{x}$, find $\frac{dy}{dx}$.

6. Find $\frac{dy}{dx}$, $if\ y + cos(xy) = k$

7. If $xsin(a+y) + sina\ cos(a+y) = 0$, prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{sin\ a}$

8. If $x = \sqrt{a^{sin^{-1}t}}$ and $y = \sqrt{a^{cos^{-1}t}}$, show that $x\frac{dy}{dx} + y = 0$

9. If $xy = e^{x-y}$, show that $\frac{dy}{dx} = \frac{xy-y}{x+xy}$

10 . If $y = \tan x + \sec x$, prove that $\frac{d^2y}{dx^2} = \frac{cosx}{(1-sinx)^2}$

SECTION – D (5 MARKS)

1. If $y = \left(x + \sqrt{1+x^2}\right)^n$, prove that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$

2. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

3. If $x = a(cost + log\ log\ tan\ tan\ \frac{t}{2})$ and $y = asint$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. at $t = \frac{\pi}{4}$

4. If $(e)^{\frac{x}{x-y}}(x-y) = a$, prove that $y\frac{dy}{dx} + x = 2y$

5 If $x = \cos t\,(3 - 2\cos^2 t)$ and $y = \sin t\,(3 - 2\sin^2 t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

6 If $y = log\left(x + \sqrt{x^2+a^2}\right)$, show that $(a^2+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

SECTION – E (Case Study Based Questions) (4 MARKS)

1. A function f(x) is said to be differentiable at a point x = a, if
   Left hand derivative at (x = a) = Right hand derivative at (x = a)
   i.e. LHD at (x = a) = RHD (at x = a), where Right hand derivative and Left hand derivatives are;

LHD $= \frac{f(a-h)-f(a)}{-h}$ **and RHD** $= \frac{f(a+h)-f(a)}{h}$

For a function **f(x)** $= \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{4} + \frac{13}{4}, x < 1 \end{cases}$

On the basis above information, answer the following questions;

(i) What is RHD of f(x) at x = 1 ? [1]

(ii) What is LHD of f(x) at x = 1? [1]

(iii) (a) check the function f(x) is differentiable at x = 1 [2]

OR

(b) Find f ’(2) and f ‘(-1)

2. A potter made a mud vessel, where the shape of the pot is based on f(x) = |x – 3| + | x – 2|, where f(x) represents the height of the pot.

(i) When x > 4 What will be the height in terms of x? (2)

(ii) When the x value lies between (2, 3) then find function f(x) [2]

3. Sumit has a doubt in the continuity and differentiability problem, but due to COVID-19 he is unable to meet with his teachers or friends. So he decided to ask his doubt with his friends Sunita and Vikram with the help of video call. Sunita said that the given function is continuous for all the real value of x while Vikram said that the function is continuous for all the real value of x except at x = 3.

The given function is f (x) = $\frac{x^2-9}{x-3}$

Based on the above information, answer the following questions:

(i) Whose answer is correct? (1)

(ii) Find the derivative of the given function with respect to x. (1)

(iii) Find the value of f ′(3). (1)

(iv) Find the second differentiation of the given function with respect to x. [1]

ANSWER KEY

MULTIPLE CHOICE QUESTIONS

1.**Option (a)**

2. **Option (b)**

As f(x) is continuous at x = 0

$[f(x)]$ = f(x) at x = 0 $\Rightarrow \lim_{x\to 0} \frac{\sin \pi x}{5x}$ = f(0)

$\Rightarrow \frac{1}{5} \lim_{x\to 0} \frac{\sin \pi x}{\pi x} \pi = k \Rightarrow \frac{1}{5}(1)\pi = k$

K = $\frac{\pi}{5}$

3.Option (b) Let u = $\cos^2(x^3)$ and v = x^3

$\frac{du}{dx} = -2\cos(x^3)\sin(x^3)\,3x^2$ and $\frac{dv}{dx}$ =**3 x²** $\Rightarrow$ $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = -$ **sin (2x³)**

4. **Option (d)** Let u = cos(x) and v = sin x

$\frac{du}{dx}$ = **-- sin x** , $\frac{dv}{dx}$ = **cos x** $\Rightarrow$ $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$ = **- tan x**

5. **Option (b)** $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{2t} \quad \Rightarrow \frac{d^2y}{dx^2} = \frac{3}{2}\frac{1}{2t} = \frac{3}{4t}$

6. **Option (d)** $\frac{d}{dx}[(e^x)] = \frac{-1}{1+e^{2x}}$ **at x = 0**

$= \frac{-1}{1+1} = \frac{-1}{2}$

7. Option (b) **y =** $a \sin \sin mx + b \cos \cos mx$

$\frac{dy}{dx} = am \cos mx - bm \sin mx \quad \Rightarrow \frac{d^2y}{dx^2} = -a\,m^2 \sin mx$ **-** $b\,m^2 \cos mx$

$= -m^2(a \sin mx + b \cos mx)$

$= -my^2$

8. Option (d) $\frac{dy}{dx} = \frac{1}{x+\sqrt{x^{2+1}}}\left[1 + \frac{2x}{2\sqrt{x^{2+1}}}\right] = \frac{1}{\sqrt{x^{2+1}}}$

9. Option (c) f(x) is differentiable if LHD = RHD

LHD $= \frac{d}{dx}(x + a) = 1$ **and RHD (at x = 1) =** $\frac{d}{dx}(ax^2 + 1)$

$= 2xa = 2a$

2a = 1 ⇒ $a = \frac{1}{2}$

10. **Option (d)** **f(x) is continuous if** $[f(x)]$ **= f(a)**

LHL = $3\left[\frac{e^{-3h}-1}{-3h}\right] + 5\left[\frac{e^{5h}-1}{5h}\right] + \quad = 3$

= 3 (1) + 5(1) = 8

ASSERTION – REASON QUESTION

1.Option (b)

2. Option (a)

By algebra of limits

$[f(x) + g(x)] = \ [f(x)] + \ [f(x)] \qquad \Rightarrow f(0) + g(0) = 3 + 2 = 5$

3.Option (b)

4. Option (b)

5. Option (d)

$[f(x)] = \lim_{x\to 0} \ x^n \sin\sin\left(\frac{1}{x}\right)$ **= 0 for positive integer n**

F(0) does not exits

F is not continuous

SECTION – B (2 MARKS)

1. **y =** $sec\ sec\left[tan(\sqrt{x})\right]$

$\frac{dy}{dx} = \ sec\ sec\left[tan(\sqrt{x})\right] \quad tan\ tan\left[tan(\sqrt{x})\right] \ \frac{d}{dx}\left(\ tan(\sqrt{x})\ \right)$

$= \ sec\ sec\left[tan(\sqrt{x})\right] \quad tan\ tan\left[tan(\sqrt{x})\right] \ sec^2\left(\sqrt{x}\right) \quad \frac{1}{2\sqrt{x}}$

2.As f(x) is continuous at x = 0

$[f(x)]$ **= f(x) at x = 0** $\qquad \Rightarrow \lim_{x\to 0} \frac{1-cos4x}{8x^2},$ **= p**

P = $\lim_{x\to 0} \frac{2sin^2 2x}{8x^2} = \lim_{x\to 0}\left[\frac{sin2x}{2x}\right]^2$

P = 1

3. $\frac{d}{dx}(-2x^2 - 5xy + y^2) = \frac{d}{dx}(76) \qquad \Rightarrow$ **- 4x – 5**$\left[x\frac{dy}{dx} + y + 2y\frac{dy}{dx}\right] = 0$

⇒**-4x - 5** $x\frac{dy}{dx} - 5y - 10y\frac{dy}{dx}$ **= 0**

$\Rightarrow \frac{dy}{dx}(-10y - 5x) = 4x + 5y$

$$\frac{dy}{dx} = \frac{-(4x+5y)}{10y+5x}$$

4. y = $\left(\frac{5^x}{x^5}\right)$

$\frac{dy}{dx} = \frac{x^5 5^x log5 - 5^x 5x^4}{x^{10}} \quad \Rightarrow \frac{x^4 5^x (xlog5-5)}{x^{10}} \quad \Rightarrow \frac{5^x(xlog5-5)}{x^6}$

5. $x = a(\theta - sin\theta) and\ y = a(1 + cos\theta)$**.**

$\frac{dx}{d\theta} = a(1 - cos\theta)$, $\quad \frac{dy}{d\theta} = -asin\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-asin\theta}{a(1-cos\theta)} = \frac{-2sin\left(\frac{\theta}{2}\right)cos\left(\frac{\theta}{2}\right)}{2sin^2\frac{\theta}{2}} = -\frac{\theta}{2}$

$$\frac{dy}{dx} at\ \theta = \frac{\pi}{3} = -\frac{\pi}{6} = -\sqrt{3}$$

6. As f(x) is continuous at x = 1

11

LHL = $[5ax - 2b] = 11$

5a- 2b = 11 ………………..(i)

RHL = $[3ax + b] = 11$

3a + b = 11……………………..(ii)

Solving (i) and (ii) a = 3 and b = 2

7 As f(x) is continuous at x = 1

$[f(x)]$ = **f(x) at x = 2**

$[f(x)] = f(2) = k \quad \Rightarrow \lim_{x\to 0}\left[\frac{x^3+x^2-16x+20}{(x-2)^2}\right]$

K = $\lim_{x\to 0}\left[\frac{(x^2+3x-10)(x-2)}{(x-2)^2}\right] \Rightarrow \lim_{x\to 0}(x+5)$

K = 2+5 = 7

8. As **f(x) = |x - 5|**

f(x) = $\{(x-5),\ x \geq 5\ -(x-5),\quad x < 5$

f(x) is differential if LHD = RHD

LHD = $\frac{d}{dx}[-(x-5)]$ = **-1** **RHD** = $\frac{d}{dx}[(x-5)] = 1$

LHD ≠ RHD

F(x) is not differential at x = 5

9. **f(x) is differentiable** at x = 1 $\in (0,1) \Rightarrow$ LHD = RHD at x = 1

LHD = $\frac{d}{dx}(ax+b) = a$ **and RHD** = $\frac{d}{dx}(2x^2 - x) = 4x - 1 = (4-1) = 3$

a = 3

since differentiable function is continues

f(x) is continues

a + b = 1 $\Rightarrow 3 + b = 1 \Rightarrow b = -2$

10 **Let u** = $sin^2 x$ **and** **v** = e^{cosx}

$\frac{du}{dx}$ = **2 sin x cos x** , $\frac{dv}{dx} = -\, e^{cosx} sinx \Rightarrow$ $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-2cosx}{e^{cosx}}$

SECTION -C (3 marks)

1.If y = A e^{mx} + B e^{nx} ,…………(i)

$\frac{dy}{dx} = Am$ e^{mx} + **Bn** e^{nx}

$$\frac{d^2y}{dx^2} = A\,m^2 e^{mx} + Bn^2 e^{nx}$$

LHS = $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$ **, After putting value of** $\frac{d^2y}{dx^2}, \frac{dy}{dx}$ $and\ \ y, we\ get$

= 0

2.y = 3 cos(log x) + 4 sin(log x)

$\frac{dy}{dx} = -3sin(logx)\frac{1}{x} + 4cos(logx)\frac{1}{x}$

$$x\frac{dy}{dx} = -3sin(logx) + 4cos(logx)$$

X $\frac{d^2y}{dx^2} + \frac{dy}{dx} = -3\,cos(logx)\frac{1}{x}$ **−4 sin(log x)** $\frac{1}{x}$

$x^2 \ \frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = -y$

x^2 $\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} + y = 0$

3. $(cosx)^y = (cosy)^x$

Taking log of both sides ; y log (cos x) = x log(cos y)

- $y\frac{1}{cosx}\ sinx + log(cosx)\frac{dy}{dx} = -x\frac{1}{cosy}siny\ \frac{dy}{dx}$ **+ log(cos y)**

- y tan x $+ log(cosx)\frac{dy}{dx} = -x\ tany\frac{dy}{dx}$ **+ log(cos y)**

$$\frac{dy}{dx} = \frac{log(\cos y)+y\tan x)}{x\tan y+ log(cosx)}$$

4. $x = 3cost - 2t,\ \ y = 3sint - 2t$

$\frac{dx}{dt}$ = **-3 sin t + 6 cos²t sin t ,** $\frac{dy}{dt} = 3cos\ t - 6\ sin^2$**t cos t**

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{3cos\ t-6\ sin2t\ cos\ t}{3\ sin\ t + 6\ cos2t\ sin\ t} = \textbf{cot t}$$

5. $y = (x)^{\log x} + (\log x)^x$

y = u + v $\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ **……………….(i)**

u = $(x)^{\log x} \Rightarrow log\ u =$ **logx (log x)= (logx)²**

$$\frac{1}{u}\frac{du}{dx} = \frac{2logx}{x}$$

$\frac{du}{dx} = u\left[\frac{2logx}{x}\right] = (x)^{logx}\left[\frac{2logx}{x}\right]$,

v = $(\log x)^x \Rightarrow log\ v = x$ **log(log x)**

$\frac{1}{v}\frac{dv}{dx} = x\frac{1}{x\ logx} +$ **log(log x)** $\Rightarrow \frac{dv}{dx} = (\log x)^x\left[\frac{1}{logx} + log(log\ x)\right]$

$$\frac{dy}{dx} = (x)^{logx}\left[\frac{2logx}{x}\right] + (\log x)^x\left[\frac{1}{logx} + log(log\ x)\right]$$

6. $y + cos(xy) = k$

2 sin y cosy $\frac{dy}{dx} - sin(xy)\left[x\frac{dy}{dx} + y\right] = 0$

Sin2 y $\frac{dy}{dx} - xsin(xy)\frac{dy}{dx} - ysin(xy) = 0 \quad \Rightarrow \frac{dy}{dx} = \frac{ysin(xy)}{sin2y-xsin(xy)}$

7. $xsin(a + y) + sina\ cos(a + y) = 0$

$xin(a + y) = -\ sina\ cos(a + y) \ \Rightarrow x = \frac{-sina\ cos(a+y)}{x\ sin(a+y)} = -sina\ cot(a + y)$

$\frac{dx}{dy} = sina\ cosec^2(a + y) = \frac{sina}{sin^2(a+y)} \ \Rightarrow \frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$

8. $x = \sqrt{a^t}$ **and** $y = \sqrt{a^t}$

$x = \sqrt{a^t}$ ……………………….(i) $, y = \sqrt{a^t}$ ……………(ii)

multiplying (i) and (ii) ; $x\ y = \sqrt{a^t}\ \sqrt{a^t}$

$xy = \sqrt{a^{t+cos^{-1}t}} = \sqrt{a^{\frac{\pi}{2}}}$

$x\frac{dy}{dx} + y = 0$

9. **x y = e $^{x-y}$**

Taking log of both sides; log(xy) = x – y

$$\frac{1}{xy}\left[x\frac{dy}{dx}+y\right]=1-\frac{dy}{dx} \quad \Rightarrow \frac{dy}{dx}(x+xy)=(xy-y)$$

$$\frac{dy}{dx}=\frac{(xy-y)}{(x+xy)}$$

10. **y = tan x + sec x** $\Rightarrow y=\frac{1+sinx}{cosx}$ $\quad \Rightarrow \frac{dy}{dx}=\frac{cosx(cosx)-(1+sinx)(-sinx}{cos^2x}$

$$\frac{dy}{dx}=\frac{1+sinx}{cos^2x}=\frac{1+sinx}{1-sin^2x}=\frac{1}{1-sinx}$$

$$\frac{d^2y}{dx^2}=\frac{cosx}{(1-sinx)^2}$$

SECTION – D (5 MARKS) [HOTS]

1 **y =** $\left(x+\sqrt{1+x^2}\right)^n \dots\dots\dots\dots\dots\dots\dots\dots (i)$

$$\frac{dy}{dx}=\frac{n\left(x+\sqrt{1+x^2}\right)^n}{\sqrt{1+x^2}} \quad \Rightarrow \quad \frac{dy}{dx}=\frac{ny}{\sqrt{1+x^2}}$$

$\sqrt{1+x^2}\ \frac{dy}{dx}$ = **ny**

Again differentiate; $\sqrt{1+x^2}\ \frac{d^2y}{dx^2}+\frac{2x}{2\sqrt{1+x^2}}\frac{dy}{dx}=n\frac{dy}{dx}$

$(1+x^2)\frac{dy}{dx}$**+x** $\frac{dy}{dx}$ = **n**$\sqrt{1+x^2}\,\frac{ny}{\sqrt{1+x^2}}$ $\quad \Rightarrow$ **(1+ x^2)** $\frac{d^2y}{dx^2}+x\frac{dy}{dx}-n^2y=0$

2. **If x** $\sqrt{1+y}$ **+ y**$\sqrt{1+x}$ **= 0**

x $\sqrt{1+y}$ = **y**$\sqrt{1+x}$

squaring both sides ; $\Rightarrow x^2(1+y)=y^2(1+x)$

$$x^2-y^2=y^2x-x^2y$$

(x – y) (x + y) + x y (x – y) = 0

(x – y) + x y = 0 $\Rightarrow y=\frac{-x}{1+x}$ **on differentiation;**

$$\frac{dy}{dx}=\frac{-1}{(1+x)^2}$$

3. **If** $x=a(cost+log\ log\ \ tan\ tan\ \frac{t}{2})$ **and** $y=asint$

Y = a sin t $\Rightarrow \frac{dy}{dt}=a\ cost$; $x=a(cost+log\ log\ \ tan\ tan\ \frac{t}{2})$

$$\frac{dx}{dt}=-sint+cosect$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{cost}{-sint+cosec\, t}$ **= tan t**

$\frac{d^2y}{dx^2} = sec^2$**t** $\frac{dt}{dx} = sec^3$**t tan t**

At **x =** $\frac{\pi}{4}$, $\frac{d^2y}{dx^2} = 2\sqrt{2}$

4. $(e)^{\frac{x}{x-y}}$ **(x – y) = a**

taking log of both sides ; log (x – y) + $\frac{x}{x-y}$ **= log a**

on differentiation;

$$\frac{1}{x-y}\left(1-\frac{dy}{dx}\right)+x-y-x\left(1-\frac{dy}{dx}\right)=0$$

y $\frac{dy}{dx}$ **+ x = 2y**

5. **x = cos t (3 – 2 cos²t) and y = sin t (3 – 2 sin²t)**

$\frac{dx}{dt} = -3sin\, t + 6$ **cos²t sin t** , $\frac{dy}{dt} = 3cost - 6$ **sin²t cost**

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3cost-6\,sin^2t\,cost}{-3sin\,t+6\,cos^2t\,sin\,t} = \frac{cost(1-2sin^2t)}{sint(2cos^2t-1)} = \frac{cost\,cos\,2t}{sint\,cos2t}$ **= cot t**

$\frac{dy}{dx}\left(at\ t=\frac{\pi}{4}\right) = cot\ \frac{\pi}{4}$ **= 1**

6. y = $log\left(x+\sqrt{x^2+a^2}\right)$**…………………..(i)**

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{a^2+x^2}}\left[1+\frac{2x}{2\sqrt{a^2+x^2}}\right] = \frac{1}{\sqrt{a^2+x^2}}$$

Again differentiate;

$$\frac{d^2y}{dx^2} = \frac{-x}{(x^2+a^2)^{\frac{3}{2}}}$$

(a²+ x²) $\frac{d^2y}{dx^2} = \frac{-x}{\sqrt{a^2+x^2}}$ $\Rightarrow$ **(a²+ x²)** $\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = 0$

SECTION – E (Case Study Based Questions) (4 MARKS)

1. (i) RHD = $\left[\frac{f(1+h)-f(1)}{h}\right] = \lim_{h\to 0}\frac{|1+h-3|-|-2|}{h} = -1$

(ii) LHD = $\left[\frac{f(1-h)-f(1)}{-h}\right] = \left[\frac{h^2+4h}{-4h}\right]$ **= −1**

(iii)(a) since LHD = RHD = - 1 at x = 1

F(x) is differential at x = 1

OR

(b) f (x) = $\{3-x \quad ,1 \le x < 3\ \ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}\ \ ,x<1$

F ' (x) = $\{-1 \quad ,1 \le x < 3 \; \frac{x}{2} - \frac{3}{2} \;\; ,x < 1$

F ' (2) = - 1 and F ' (-1) = $\frac{-1}{2} - \frac{3}{2} = -2$

2. f(x) = |x – 3| + | x – 2|,

$f(x) = \{5 - 2x \,, x < 2 \; 1 \quad , \quad 2 \le x < 3 \; 2x - 5 \;\; , x \ge 3$

(i) **when x** $> 4 \,, f(x) = 2x - 5$

(ii) **When the x value lies between (2, 3) ,then**

F(x) = 1 , x $\in (2,3)$

3. (i) Vkram's answer is correct .

(ii) f (x) = $\frac{x^2-9}{x-3}$ **= x + 3** $\Rightarrow f'(x) = 1$

(iii) **f(x) = x + 3** $\Rightarrow f'(3) = 1$

(iv) **f(x) = x + 3** $\Rightarrow f'(x) = 1$

$f''(x) = 0$

Application of Derivatives

(Volume I)

- Rate of change of Quantities

Previous knowledge:-

1. Formulae of area and volume of 2D and 3D objects:

Perimeter of square = 4 sides, Perimeter of rectangle = 2(l + b), Perimeter circle = $2\pi r$,

Perimeter semi circle= πr, Area of rectangle = length x breadth, Area of parallelogram = base $\times$ height

Area of triangle = $\frac{1}{2}$ base $\times$ height, Area of circle = $\pi\, r^2$, Area of semi circle = $\frac{\pi\, r^2}{2}$

Slant height of a cone $l = \sqrt{h^2 + r^2}$, Curved surface area of cone = $\pi r l$,

Total surface area of cone = $\pi r(r + l)$, Volume of cone = $\frac{\pi\, r^2 h}{3}$, Curved surface area of cylinder = $2\pi r h$,

Total surface area of cylinder = $2\pi r\ (r + h)$, Volume of cylinder = $\pi r^2 h$, Curved(Total) surface area of sphere = $4\ \pi\ r^2$, Volume of sphere = $\frac{4}{3}\ \pi r^3$, Curved surface area of hemi sphere = $2\ \pi\ r^2$, Total surface area of hemisphere = $3\ \pi\ r^2$, Volume of hemisphere = $\frac{2}{3}\ \pi r^3$,

Meaning of rate of change of quantities:

1. The derivative $\frac{ds}{dt}$ represents the rate of change of distance s w.r.t. time t. In the similar way whenever one quantity y is changing with respect to another quantity x, satisfying some rule, then $\frac{dy}{dx}$ represents the rate of change of y w.r.t x. To find the instantaneous change in y with respect to x, when x = x_0 then it is $\left[\frac{dy}{dx}\right]_{x=\ x_0}$

2. If two variables x and y are varying w.r.t. another variable t, that is x and y both are functions of t then by chain rule $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}, if\ \frac{dx}{dt} \neq 0$

3. The function C(x) is called the total cost function, where x is the production of units of items and marginal cost is the instantaneous rate of change of total cost at any level of output.

4. The function R(x) is called the total revenue function, where x is the production of units of items and marginal revenue is the instantaneous rate of change of total cost function w.r.t to the production of x units.

Increasing and Decreasing Functions:-

Previous knowledge:-

Let I be an interval contained in the domain of a real valued function f. Then f is said to be

(i) increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in$ I.

(ii) strictly increasing on I, if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ I.

(iii) constant on I, if $f(x) = c$ for all $x \in$ I, where c is a constant.

(iv) decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in$ I.

(v) strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ I.

Q.1 The Maximum value of $[x(x-1)+1]^{\frac{1}{3}}$, $0 \leq x \leq 1$ is:

(a) 1 (b) $\frac{1}{2}$ (c) 1 (d) 0

Q.2 For which interval is $f(x)=3x^2-4x$ strictly decreasing?

(a) $(-\infty, \frac{2}{3})$ (b) (0,2) (c) $(\frac{2}{3}, \infty)$ (d) $(-\infty, \infty)$

Q.3 The function $y=x^2e^{-x}$ is decreasing in interval

(a) (0,2) (b) $(-\infty,0)$ (c) $(2,\infty)$ (d) $(-\infty,0)\cup(2,\infty)$

Q.4 **Assertion (A):** f(x)=sin x and f(x)=cos x are both decreasing in the interval $[\frac{\pi}{2},\pi]$.

Reason (R): The derivative of a decreasing function is also decreasing in the same interval.

(a) Both Assertion (A) and Reason (R) are true, and R explains A.

(b) Both A and R are true, but R does **not** explain A.

(c) A is true; R is false.

(d) A is false; R is true.

Q.5 **Assertion:** The maximum value of f(x)=sin x+cos x for $0<x<\frac{\pi}{2}$ is at $x=\frac{\pi}{4}$.

Reason: $f''(x)<0$ at $x=\frac{\pi}{4}$.

(a) Both Assertion (A) and Reason (R) are true, and R explains A.

(b) Both A and R are true, but R does **not** explain A.

(c) A is true; R is false.

(d) A is false; R is true.

Q.6 For the curve $y = 5x - 2x^3$, if x is increses at the rate of 2 units/sec.,then find the rate of change of slope of the cuirve when x=3.

Q.7 The amount of pollution content added in air in a city due to x- diesel vehicles is given by $P(x)=0.005x^3+0.02x^2+30x$. Find the marginal increases in pollution content when 3 diesel vehicles are added.

Q.8 Show that the function f defined by $f(x) = (x-1)e^x +1$ is an increasing function for all $x>0$.

Q.9 The sides of an equilateral triangle are increasing at the rate of 2 cm/ sec. Find the rate at which the area increases ,when the side is 10cm.

Q.10 The volume of a spherical balloon is increasing at the rate of 3 cm^3 / sec. Find the rate of increase of its surface area, when the radius is 2 cm.

SOLUTIONS:

Q.1 $f'(x)=\frac{2x-13}{3[x^2-x+1]^{\frac{2}{3}}}$; critical point at $x=\frac{1}{2}$

At endpoints: f(0)=f(1)=1 ; at critical point: $f(\frac{1}{2}) = \frac{3^{\frac{1}{3}}}{4} < 1$

Maximum value is 1, occurring at x=0 or x=1

Answer: (c) 1

Q.2 $f'(x)=6x-4$.

Decreasing means $f'(x)<0 \implies 6x-4<0 \implies x<2/3$

Answer: (a) $(-\infty,\frac{2}{3})$.

Q.3 $f'(x)=e^{-x}(2x-x^2)$

Decreasing means $f'(x)<0 \implies x(2-x)<0 \implies x<0$ or $x>2$.

Answer: (d) $(-\infty,0)\cup(2,\infty)$.

Q.4 (Assertion):

On $(\frac{\pi}{2},\pi)$,
sin x: derivative cos x < 0⟹sin x is decreasing.
Also,cos x:derivative −sin x < 0 ⟹cos x is decreasing on (0,π).

Assertion is true.

R (Reason):

The statement "a differentiable function that is decreasing has a derivative that is also decreasing" is incorrect.

Counterexample: Take f(x)=sin x . On $(0, \frac{\pi}{2})$,f is increasing, but its derivative cos x is decreasing. This shows the derivative need not follow the same monotonicity direction as the original function.
Reason is false.

Answer: (c)

Q.5 Assertion:

$f'(x)=\cos x-\sin x$, Put $f'(x)=0 \Rightarrow \tan x=1 \Rightarrow x=\frac{\pi}{4}$.

Second derivative test:
$f''(x)=-(\sin x+\cos x) \Rightarrow f''(\frac{\pi}{4})=-\sqrt{2}<0$.

confirming a local maximum at $x=\frac{\pi}{4}$.

Since sinx+cosx is continuous and differentiable on $(0,\frac{\pi}{2})$, the critical point gives the absolute maximum within that open interval.

Therefore, Assertion (A) is TRUE.

Reason (R):

$f''(x)<0$ at $x=\frac{\pi}{4}$.

This statement follows from:
$f''(x)=-(\sin x+\cos x) \Rightarrow f''(\frac{\pi}{4})=-\sqrt{2}<0$.

A negative second derivative indicates concavity downward, which confirms that $x=\frac{\pi}{4}$ is indeed a maximum.

Therefore, Reason (R) is TRUE

Answer: (a)

Q.6 Slope of the curve

$y'=5-6x^2$,

Since dx/dt=2 units/sec., slope's rate wrt time:

$\frac{dy'}{dt}=\frac{dy'}{dx}.\frac{dx}{dt}$ = -12x.2 = -24x units/sec.

When x=3, slope's rate wrt time = -72 units/sec.

Q.7 $P(x)=0.005x^3+0.02x^2+30x$

$P'(x)=0.015x^2+0.04x+30$

$\Rightarrow P'(3)=0.135+0.12+30=30.255$

Q.8 $f'(x)=(x-1)e^x+e^x=xe^x$

For x>0:

Since e^x is always positive and x is positive,

so

$f'(x)=xe^x>0$ for all x>0.

Because the derivative f′(x) is strictly positive for all x>0, f(x) is strictly increasing over that domain.

Q.9 Let x cm be the side and A be the area of the equilateral triangle at time t.

$A=\frac{\sqrt{3}}{4}x^2$

Rate of change (increase) of side x w.r.t. t = $\frac{dx}{dt}$ = 2cm/sec.

Rate of change of area w.r.t. t = $\frac{dA}{dt} = \frac{\sqrt{3}}{4}2x.\frac{dx}{dt} = \frac{\sqrt{3}}{2}x.2 = \sqrt{3}\ x\ cm^2/sec.$

Q.10 Let r be the radius, V be the volume and S be the surface area of the spherical balloon at any time t.

$V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

Rate of change (increase) of volume w.r.t. t = 3 cm^3/ sec.

$\frac{dV}{dt} = 3$

Now, $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}3\pi r^2\frac{dr}{dt} \Rightarrow 3 = 4\pi r^2\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = 3/4\pi r^2$

Now , $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi r(3/4\pi r^2) \Rightarrow \frac{dS}{dt} = \frac{6}{3} = 2cm^2/sec.$

Practice Questions on rate of change:-

(1 Mark)

Q 1. Find the rate of change of the area of circle with respect to the radius r, when radius r = 5 cm.

Q 2. The total cost C(x) in rupees, associated with the production of x units of an item is given by

$C(x) = 0.007\ x^3 - 0.003\ x^2 + 15\ x + 4000$. Find the marginal cost when 17 units are produced.

Q 3. The total revenue R(x) in rupees received from the sale of x units of a product is given by

$R(x) = 13\ x^2 + 26\ x + 15$. Find the marginal revenue when 7 units are sold.

(2 Marks)

Q 4.A stone is dropped into a quiet lake and waves move in circles at the speed of 5cm/sec. At the instant when the radius of circular wave is 8 cm,how fast is the enclosed area increasing?

Q 5.A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

Q 6.A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground Away from the wall at the rate of 2cm/sec.How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

Q 7.Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.How fast is the height of the sand cone increasing when the height is 4 cm?

(3 Marks)

Q 8. A man of height 2 meters walks at a uniform speed of 5 km/s away from a lamp post which is 6 meters high. Find the rate at which the length of his shadow increases.

Q 9. A car starts from a point P at time t = 0 seconds and stops at point Q. The distance x, in meters, covered by it, in t seconds given by :

$x = t^2\left(2 - \frac{t}{3}\right).$

Find the timetaken by it to reach Q and also find distance between P and Q.

Q 10. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. its semi vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meters per hour. Find the rate at which the level of the water is raising at the instant when the depth of water in the tank is 4 m.

Answers:	1. 10 π cm^2/cm	2. Rs. 20.967	3.Rs. 208	4.80 π cm^2/s
	5.$\frac{1}{\pi}$cm/s	6. 8/3cm	7.$\frac{1}{48\pi}$ cm/s	8. 2.5 km/s
	9. t = 4 sec. Distance = 32/3 m.		10. 35/88 m/h	

7. INTEGRATION

	MCQ:- (1MARK)
1	If $\frac{d}{dx}$(f(x)) = log x, then f(x) equals: (a) $\frac{-1}{x}+c$ (b)x(logx-1)+c (c)x(logx+x)+c (d) $\frac{1}{x}+c$
2	$\int_0^{\frac{\pi}{6}} sec^2(x-\frac{\pi}{6})\, dx$ is equal to:
	(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$
3	If $f'(x) = x+\frac{1}{x}$, then $f(x)$ is (a) $x^2+log\|x\|+c$ (b) $\frac{x^2}{2}+log\|x\|+c$ (c) $\frac{x}{2}+log\|x\|+c$ (d) $\frac{x}{2}-log\|x\|+c$
4	$\int_0^2 \sqrt{4-x^2}\, dx\ equals$ (a) 2log2 (b)-2log2 (c) $\frac{\pi}{2}$ (d) π
5	$\int_a^b f(x)\, dx\ is\ equal\ to$: (a) $\int_a^b f(a-x)\, dx$ (b) $\int_a^b f(a+b-x)\, dx$ (c) $\int_a^b f(x-(a+b))\, dx$ (d) $\int_a^b f((a-x)+(b-x))\, dx$
6	$\int_0^{\frac{\pi}{2}} \frac{sinx-cosx}{1+cosxsinx} dx$ is equal to: (a) π (b) Zero(0) (c) $\int_0^{\frac{\pi}{2}} \frac{2sinx}{1+cosxsinx} dx$ (d) $\frac{\pi^2}{4}$
7	$\int_a^b f(x)\, dx = 0. if$: (a) $f(-x) = f(x)$ (b) $f(-x) = -f(x)$ (c) $f(a-x) = f(x)$ (d) $f(a-x) = -f(x)$
8	$\int e^x(cosx - sinx)dx\ is\ equal\ to$ (a) $e^x cosx + c$ (b) $e^x sinx + c$ (c) $-e^x cosx + c$ (d) $-e^x sinx + c$
9	The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}\ is$: (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{18}$
10	The value of $\int_{-1}^{1} x\|x\|dx$ is: (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $-\frac{1}{6}$ (d) 0
	Short Answer Type Questions (2/3 Marks)
11	$Evaluate: \int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right) dx$
12	Evaluate: $\int e^x(tanx + logsec\, x\,)dx$

13	$Evaluate \int \frac{1}{4x^2-4x+3}dx$
14	Find: $\int \frac{x^2}{(x^2+4)((x^2+9)}dx$
15	Find: $\int \frac{2x}{(x^2+1)(x^2+2)}dx$
16	Find: $\int \frac{dx}{\sqrt{9-4x^2}}$
17	$Find\ the\ value\ of \int_1^4 \lvert x-5\rvert\, dx$
18	Evaluate: $\int_1^3[\lvert x-1\rvert+\lvert x-2\rvert+\lvert x-3\rvert]$dx
	Long Answer type Question
19	Evaluate: $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$
20	Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{sinx}}dx$
21	Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{sin^{100}x}{sin^{100}x+cos^{100}x}dx$
22	Evaluate: $\int_{\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}}$
23	Evaluate: $\int_0^{\pi} \frac{xtanx}{secx.cosecx}dx$
	Answers
1	x(logx-1)+c
2	$\frac{1}{\sqrt{3}}$
3	$\frac{x^2}{2}+log\lvert x\rvert+c$
4	π
5	$\int_a^b f(a+b-x)\,dx$
6	Zero(0)
7	$f(-x)=-f(x)$
8	$e^x cosx+c$
9	$\frac{\pi}{2}$
10	0
11	Using property $\int e^x\big(f(x)-f'(x)\big)dx$ =$e^x f(x)+c$ =$\frac{1}{x}e^x+c$
12	Using property $\int e^x\big(f(x)-f'(x)\big)dx$ =$e^x f(x)+c$ =e^x.log $secx+c$
13	$=\frac{1}{2\sqrt{2}}tan^{-1}(\frac{2x-1}{\sqrt{2}})$+c
14	Put$x^2=t$ $\frac{t}{(t+4)(t+9)}=\frac{A}{t+4}+\frac{B}{t+9}$ A=-4/5

	B=9/5 $=\frac{-2}{5}tan^{-1}\left(\frac{x}{2}\right)+\frac{3}{5}tan^{-1}\left(\frac{x}{3}\right)+c$
15	Put$x^2=t$ 2xdx=dt =log$\left(\frac{x^2+1}{x^2+2}\right)+c$
16	$\frac{1}{2}sin^{-1}(\frac{2x}{3})$+c
17	15/2
18	5
19	$5x+3=A\frac{d}{dx}(x^2+4x+10)$+B A=5/2 , B=-7 =5$\sqrt{x^2+4x+10}$ -log[(x+2)+ $\sqrt{x^2+4x+10}$] $+c$
20	apply property $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ I= π
21	Using property $\int_{-a}^{a} f(x)\,dx=2\int_0^a f(x)\,dx$, when f(x) is even. I=$\frac{\pi}{2}$
22	Use property $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$ 2I= $\frac{\pi}{6}$ I=$\frac{\pi}{12}$
23	Convert the problem in sinx and cosx Then apply property $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ =$\frac{\pi^2}{4}$
	Work Sheet
1.	$\int\frac{secx}{secx-tanx}dx\ equals$: (a) $secx-tanx+c$ (b) $secx+tanx$+c (c) $tanx-secx+c$ (d) $-(secx+tanx)+c$
2	Antiderivative of $\sqrt{1+sin2x}$, $x\in\left[0,\frac{\pi}{4}\right]$ is: (a)$cosx+sinx$ (b) $-cosx+sinx$ (c) $cosx-sinx$ (d) $-cosx-sinx$
3	$\int_{-1}^{1}\frac{x^3+\|x\|+1}{x^2+2\|x\|+1}dx$ equals to (a)log2 (b)2log2 (c)logx (d)2
4	The value of$\int_{-1}^{1}\|x\|dx$ is: (a) *-2* (b)-1 (c) *1* (d) 2
5	The primitive of $\frac{2}{1+cos2x}$ is (a)sec^2x (b)2sec^2xtanx (c) tanx (d)-cotx
6	Find : $\int\frac{1}{\sqrt{x(\sqrt{x}+1)(\sqrt{x}+2)}}dx$
7	Evaluate: $\int\frac{x+2}{\sqrt{x^2-4x-5}}dx$
8	Evaluate: $\int_0^{\pi}\frac{x}{1+sinx}dx$

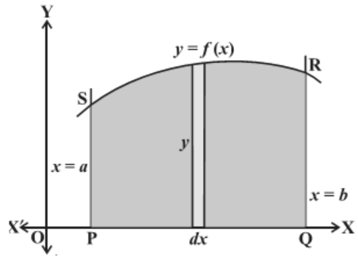
9	Evaluate: $\int_1^3[\|x-1\|+\|x-2\|]$dx
10	Find: $\int \frac{x^2}{(x^2+1)(1-x)}dx$

Hints and Answers

1	$\frac{3\pi}{20}-\frac{1}{10}log3$ Hint: Proceed with $cosx = K(3cosx+sinx)+L\frac{d}{dx}(3cosx+sinx)$
2	$\frac{1}{2}\left(\frac{\pi}{2}-log2\right)$ Hint: Use identity $cos2x = cos^2x - sin^2x$ the proceed
3	2-$\sqrt{2}$
4	$\frac{1}{40}log9$
5	$\frac{\pi^2}{16}-\frac{\pi}{4}+\frac{1}{2}log2$ Hint: Use integration by Parts
6	$\frac{\pi}{4}+\frac{1}{2}$ Hint put x^2=t then rationalize
7	Use Property P4
8	π^2 Hint: break the problem in I1+I2
9	2 log2
10	$\frac{3\pi+1}{\pi^2}$ Hint: Use additive property and break the problem in two parts by finding critical points.
11	$\log(xe^x)-\log(1+xe^x)+\frac{1}{1+xe^x}$+ C Hint: put xe^x=t and proceed with substitution.
12	$(1-x)(\sqrt{x}-2)-sin^{-1}\sqrt{x}+c$ Hint: Put x=t^2 proceed with substitution.
13	$=\frac{1}{2\sqrt{3}}log\left\|\frac{\sqrt{3}+(sinx-cosx)}{\sqrt{3}-(sinx-cosx)}\right\|+tan^{-1}(sinx+cosx)$ Hint; convert the numerator in the form $(sinx+cosx)+(cosx-sinx)$
14	$\frac{1}{4}log\|x^4+3x^2+2\|-\frac{3}{4}log\left\|\frac{x^2+1}{x^2+2}\right\|+c$ Hint:- $x^2 = t$ *then* proceed with substitution.
15	$-\frac{1}{3}logcos3x+\frac{1}{2}logcos2x+logcosx+c$ Hint: *exapand using* $\tan(2x+x)=3x$
	.

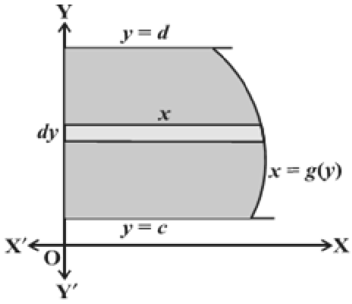
8. APPLICATIONS OF THE INTEGRALS

** Area of the region PQRSP $= \int_a^b dA = \int_a^b y\, dx = \int_a^b f(x)\, dx$.



** The area A of the region bounded by the curve x = g (y), y-axis and the lines y = c, y = d is given by

$$A = \int_c^d x\, dy = \int_c^d g(y)\, dy$$



MCQ

1	Find the area enclosed by curve $4x^2 + 9y^2 = 36$ (a) 6л sq units (b) 4л sq units (c) 9л sq units (d) 36л sq units
2	The area enclosed between the graph of $y = x^3$ and the lines x = 0, y = 1, y = 8 is (a) 7 (b) 14 (c) 45/4 (d)None of these
3	The area of the region bounded by the curve $y^2 = x$, the y-axis and between y = 2 and y = 4 is (a) 52/3 sq. units (b)54/3 sq. units (c) 56/3 sq. units (d)None of these
4	The area of the region bounded by the curve $x^2 = 4y$ and the straight line x = 4y – 2 is (a) 3/8 sq. units (b) 5/8 sq. units (c)7/8 sq. units (d) 9/8 sq. units
5	Area of region bounded by the curve $y^2 = 4x$, and its latus rectum above x axis (a) 0 sq units (b) 4/3 sq units (c) 3/3 sq units (d) 2/3 sq units

Short Answer Type Question

1.	Find the area bounded by the line y = x, x-axis and lines x = – 1 to x = 2.
2	Find the area between the curves y = x and $y = x^3$.

<u>Short Answer type questions (Unsolved)</u>

<u>**1.**</u>	Find the area of the region bounded by the curve $y = \sin x$ between the lines x=0 , x=π/2 and the x-axis.
<u>**2.**</u>	Find the area enclosed between curves $y = 4x – x^2$, $0 \le x \le 4$, x-axis

3.	Find the area enclosed between $y^2 = 4ax$ and its latus rectum.
4	Find the area bounded by the curve y = cos x between x = 0 and x = 2π

ASSERTION - REASON TYPE QUESTIONS:

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct

1. **Assertion :** The area bounded by the curve y = cos x in I quadrant x=0, and $x=\frac{л}{2}$ is 1 sq. unit.

Reason : $\int_0^{\frac{л}{2}} \cos x \, dx = 1$

2. Assertion : The area bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant is given by $\int_0^a \sqrt{a^2 - x^2}\, dx$
Reason : The same area can also be found by

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left|x + \sqrt{a^2 + x^2}\right| + c$$

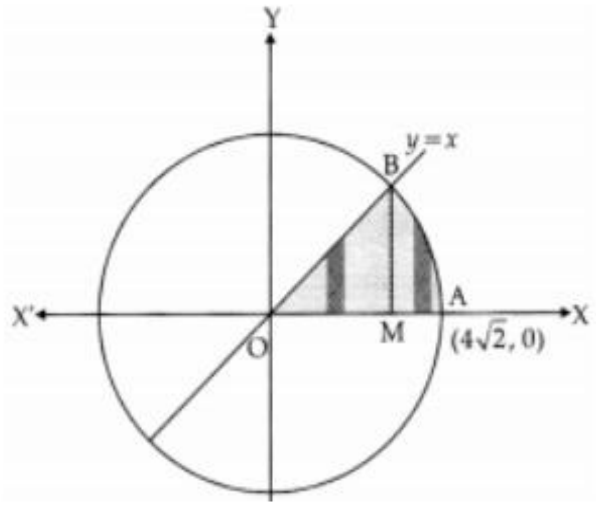
3.Assertion : The area bounded by the circle y = sin x and y = –sin x from 0 to л is 3 sq. unit.
Reason : The area bounded by the curves is symmetric about x-axis.

Long Answer Type Questions : (Unsolved)

Q1.	Find the area of the region bounded by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
Q2	Using the method of integration find the area bounded by the curve $\lvert x\rvert + \lvert y\rvert = 1$

CASE STUDY QUESTION

1 In the figure given below O(0, 0) is the center of the circle. The line y = x meets the circle in the first quadrant at point B. Answer the following questions based on the given figure.



(i) The equation of the circle is ____________.

(ii) The co-ordinates of B are _______________.

(iii) Area of ΔOBM is ____________ sq. units.

(iv) Ar (BAMB) = ____________ sq. units.

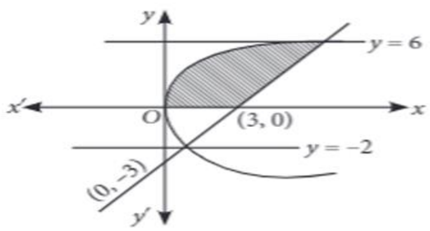
(v) Area of the shaded region is ____________ sq. units.

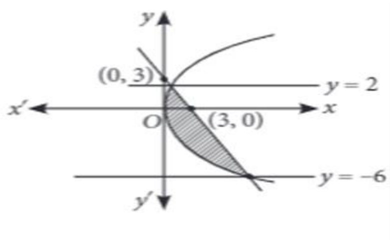
2. Consider the curve $y^2 = 4x$ and straight line $x + y = 3$ and answer the following questions based on the same.

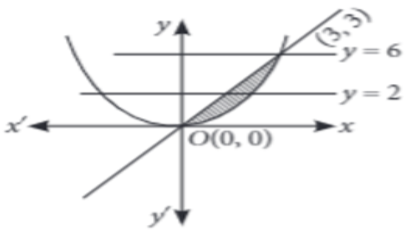
(i) The line $x + y = 3$ intersects x-axis at _____ and y-axis at ______.

(ii) The point(s) of intersection of the two curves is(are) ___________.

(iii) The area bounded by the given two curves can be represented as

(a) 

(b) 

(c) 

(d) None of these

(iv) Value of the integral $\int_{-6}^{2}(3 - y)dy$ is _________.

Answer

1	**Answer: (a)** 6л sq units

Explanation:

$4x^2 + 9y^2 = 36$

$\frac{4X^2}{36} + \frac{9y^2}{36} = 1$

$\frac{X^2}{9} + \frac{y^2}{4} = 1$

a = 3 , b = 2

Area of ellipse = л ab = л.3.2 = 6 л sq. units.

2 **Answer: (c) 45/4**

Explanation:

Given curve, $y=x^3$ or $x = y^{1/3}$.

Hence, the required area, $A = \int_1^8 y^{\frac{1}{3}}\,dy$

$A = [(y^{4/3})/(4/3)]_1^8$

Now, apply the limits, we get

A = (¾)(16-1)

A = (¾)(15) = 45/4.

Hence, option (c) 45/4 is the correct answer.

3 **Answer: (c) 56/3**

Explanation:

Given: $y^2 = x$

Hence, the required area, $A = {}_2\int^4 y^2\,dy$

$A = [y^3/3]_2^4$

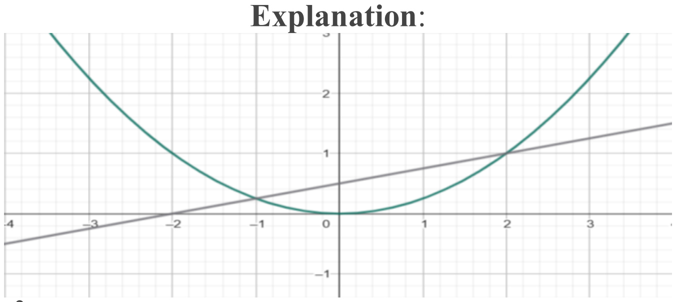
$A = (4^3/3) – (2^3/3)$

A = (64/3) – (8/3)

A = 56/3 sq. units.

4 **Answer: (d) 9/8 sq. units**

Explanation:



For the curves $x^2 = y$ and x = 4y-2, the points of intersection are x = -1 and x = 2.

Hence, the required area, $A = \int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4}\right) dx$

Now, integrate the function and apply the limits, we get

A = (¼)[(10/3)-(-7/6)]

A = (¼)(9/2) = 9/8 sq. units

Hence, the correct answer is option (d) 9/8 sq. units

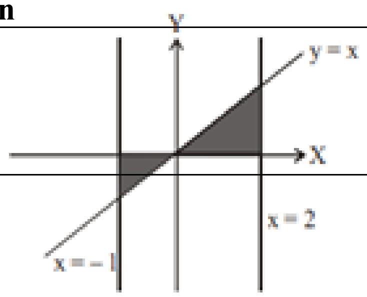
5 **Ans (b) 4/3 sq units**

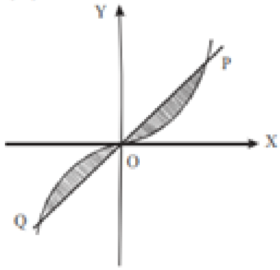
Short Answer Type Question

1 **Sol.** We have, y = x, a line

Required Area = Area of shaded region

$= \left|\int_{-1}^{0} x\,dx\right| + \left|\int_{0}^{2} x\,dx\right| = \left|\frac{x^2}{2}\right|_{-1}^{0} + \left|\frac{x^2}{2}\right|_{0}^{2}$



	$= \left\|-\frac{1}{2}\right\| + \left\|\frac{2}{1}\right\| = 2 + \frac{1}{2} = \frac{5}{2}$ sq. units
2	The required area is symmetrical about the origin as shown in the diagram, So Required Area $= 2\int_0^1 (x - x^3)\, dx$ $= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]$ $= 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2}$. 
	<u>**Short Answer type questions (Unsolved)**</u>
1	**Ans 4 sq units**
2	**Ans. 32 /3 sq. units**
3	**Ans. 8 a^2/3 sq. units.**
4	**Ans. 4 sq. units**
	ASSERTION - REASON TYPE QUESTIONS:
1	**Ans (a)** $\int_0^{\frac{\pi}{2}} \cos x\, dx = [\sin x]_0^{\pi/2} = \sin\frac{\pi}{2} - 0 = 1$
2	**Ans. (b)**
3	**Ans. (d)**
	Long Answer Type Questions : (Unsolved)
1	**Ans. πab**.
2	**Ans. 4 sq. units**

9. DIFFERENTIAL EQUATIONS

- ✓ **Definition**: An equation involving derivatives of the dependent variable w. r. t independent variable/s is known as a differential equation.

  Examples of differential equations are: $\frac{dy}{dx} = e^x, \frac{d^2y}{dx^2} + y = 0, \frac{d^3y}{dx^3} + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0.$

- ✓ **Order of a differential equation**: Order of a differential equation is the order of the highest order derivative. The differential equations given above involve the highest derivative of first, second and third order respectively. Therefore, the order of these equations are 1, 2 and 3 respectively.

- ✓ **Degree of a differential equation**: Degree of a differential equation is the highest power of the highest order derivative in it and it is defined if it is a polynomial equation in its derivatives.
  Consider the following differential equations:

  $$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}, \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y, (y'')^2 + (y')^3 = x\,sin\,(y')$$

  The degree of the first two differential equations given above are 1 and 3 respectively whereas the degree of the third equation is not defined as it is not a polynomial.

- ✓ Order and degree (if defined) of a differential equation are always positive integers.
- ✓ **Solution of a differential equation**: A function which satisfies the given differential equation is called its solution.
- ✓ **General Solution of a Differential Equation**: The solution which contains arbitrary constants is called the general solution (primitive) of the differential equation.
- ✓ **Particular Solution of a Differential Equation**: The solution free from arbitrary constants i.e., the solution obtained from the general solution by giving particular values to the arbitrary constants is called a particular solution of the differential equation.
- ✓ **Methods of Solving First Order, First Degree Differential Equations:**

  Differential equations with variables separable: A first order-first degree differential equation is of the form $\frac{dy}{dx} = F(x, y)$……………(1)

  If F(x, y) can be expressed as a product g (x) h(y), where, g(x) is a function of x and h(y) is a function of y, then the differential equation (1) is said to be of variable separable type.

  The differential equation (1) then has the form $\frac{dy}{dx} = h(y).\, g(x).$ ………………..(2)

  If $h(y) \neq 0$, separating the variables , the equation (1) can be written as

  $\frac{1}{h(y)} dy = g(x)dx$ ……………….(3)

  Integrating both sides of (3), we get

  $\int \frac{1}{h(y)} dy = \int g(x)dx$ …………………(4)

  Thus, (4) provides the solutions of given differential equation in the form

  $H(y) = G(x) + C$ ……………(5)

  Here, H (y) and G (x) are the anti-derivatives of $\frac{1}{h(y)}$ and g(x) respectively and C is the arbitrary constant.

Homogeneous differential equations: A differential equation of the form $\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{F(x, y)}$ is said to be homogenous if F(x, y) is a homogenous function of degree zero.

To solve a homogeneous differential equation of the type

$\frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{F(x, y)} = \mathrm{g}\left(\frac{y}{x}\right)$................(1)

We make the substitution $y = v.x$ .... (2)

Differentiating equation (2) with respect to x, we get

$\frac{\mathrm{dy}}{\mathrm{dx}} = v + x\frac{dv}{dx}$...................(3)

Substituting the value of $\frac{\mathrm{dy}}{\mathrm{dx}}$ from equation (3) in equation (1), we get

$v + x\frac{dv}{dx} = g(v)$

Or

$x\frac{dv}{dx} = g(v) - v$ ....(4)

Separating the variables in equation (4), we get

$\frac{dv}{g(v)-v} = \frac{dx}{x}$ ....(5)

Integrating both sides of equation (5), we get

$\int\frac{dv}{g(v)-v} = \int\frac{dx}{x} + C$ ....(6)

Equation (6) gives general solution (primitive) of the differential equation (1) when we replace v by $\frac{y}{x}$.

- ✓ If the homogeneous differential equation is in the form $\frac{dx}{dy} = F(x, y)$ where, $F(x, y)$ is homogenous function of degree zero, then we make substitution $\frac{x}{y} = v$ i.e., $x = vy$ and we proceed further to find the general solution as discussed above by writing $\frac{dx}{dy} = F(x, y) = h\left(\frac{x}{y}\right)$.

Linear differential equations: A differential equation of the form $\frac{dy}{dx} + Py = Q$ where, P and Q are constants or functions of x only, is known as a first order linear differential equation.

To solve the first order linear differential equation of the type

$\frac{dy}{dx} + Py = Q$ …………………(1)

Multiply both sides of the equation by a function of x say $g\ (x)$ to get

$g(x)\frac{dy}{dx} + P\big(g(x)\big)y = Q.g(x)$ ………………..(2)

Choose $g(x)$ in such a way that R.H.S. becomes a derivative of $y\ .\ g\ (x)$.

i.e. $g(x)\frac{dy}{dx} + P.g(x).y = \frac{d}{dx}[y.g(x)]$

or $g(x)\frac{dy}{dx} + P.g(x).y = g(x)\frac{dy}{dx} + y.g'(x)$

$\Rightarrow P.g(x) = g'(x)$

Or $P = \frac{g'(x)}{g(x)}$

Integrating both sides with respect to x, we get

$\int P\,dx = \int \frac{g'(x)}{g(x)}$dx

Or $\int P\,dx = \log(g(x))$

Or $g(x) = e^{\int Pdx}$

On multiplying the equation (1) by $g(x) = e^{\int Pdx}$, the L.H.S. becomes the derivative of some function of x and y. This function $g(x) = e^{\int Pdx}$ is called *Integrating Factor* (I.F.) of the given differential equation.

Substituting the value of $g\ (x)$ in equation (2), we get

$e^{\int Pdx}\frac{dy}{dx} + Pe^{\int Pdx}y = Q.e^{\int Pdx}$

Or $\frac{d}{dx}\left(ye^{\int Pdx}\right) = Q.e^{\int Pdx}$

Integrating both sides with respect to x, we get

$y.e^{\int Pdx} = \int\left(Q.e^{\int Pdx}\right)$dx
Or $y = e^{-\int Pdx}.\int\left(Q.e^{\int Pdx}\right)dx + C$

which is the general solution of the differential equation.

EXERCISES

MULTIPLE CHOICE QUESTIONS

1) The order and degree of the differential of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 - 5\frac{dy}{dx} + 6 = 0$ is

(A) 2, 3 (B) 3, 2 (C) 1, 1 (D) 0, 3

2) The order and degree of the differential of the differential equation $y = px + \sqrt{1+p^2}$, where $p = \frac{dy}{dx}$ is

(A) 1, ½ (B) 1, 2 (C) 2, 1 (D) 2, 2

3) The order and degree of the differential of the differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$ is

(A) 2, 1 (B) 1, 1 (C) 2, degree is not defined (D) 1, 2

4) The solution of the differential equation $\cos\left(\frac{dy}{dx}\right) = a, a \in R$ is

(A) $y = \sin^{-1} a + c$ (B) $y = x\sin^{-1} x + c$ (C) $y = x\cos^{-1} a + c$ (D) None of these

5) The integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ is

(A) 1 (B) $e^{-2\sqrt{x}}$ (C) $e^{2\sqrt{x}}$ (D) e^{x^2}

6) The integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} + xy = ax, -1 < x < 1$, is

(A) $\frac{1}{x^2-1}$ (B) $\frac{1}{\sqrt{x^2-1}}$ (C) $\frac{1}{1-x^2}$ (D) $\frac{1}{\sqrt{1-x^2}}$

7) The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are:

(A) 1, 2 (B) 2, 3 (C) 2, 1 (D) 2, 6

8) The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^4 - 3x$ is:

(A) x (B) $\frac{1}{x}$ (C) x–1 (D) log (x–1)

9) The solution of the differential equation $\frac{dy}{dx} = \frac{1}{log\, y}$ is:

(A) $log\, y = x + c$ (B) $y\, log\, y - y = x + c$

(C) $log\, y - y = x + c$ (D) $y\, log\, y + y = x + c$

10) The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is:

(A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$ (C) $\frac{x^2 + y^2}{xy}$ (D) $cos^2\left(\frac{x}{y}\right)$

11) The order of the differential equation $(y'')^2 + (y')^3 = x\, sin\,(y')$ is :

(A) 1 (B) 2 (C) 3 (D) not defined

12) The number of solutions of differential equation $\frac{dy}{dx} - y = 1$, given that y(0) = 1, is :

(A) 0 (B) 1 (C) 2 (D) infinitely many

13) The degree and order of differential equation $(y'')^2 + log\,(y') = x^5$ respectively are:

(A) not defined, 5 (B) 5, not defined (C) 2, 2 (D) not defined, 2

14) $x\, log\, x\frac{dy}{dx} + y = 2\, log\, x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined

15) The general solution of the differential equation $xdy + ydx = 0$ is:

(A) $xy = c$ (B) $x + y = c$ (C) $x^2 + y^2 = c^2$ (D) $logy = logx + c$

16) The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y(y > 0)$ is:

(A) $\frac{1}{x}$ (B) x (C) y (D) $\frac{1}{y}$

17) The number of arbitrary constants in the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x +; y(0) = 0$ is/are

(A) 2 (B) 1 (C) 0 (D) 3

18) The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0$

(A) $\frac{2}{x}$ (B) x^2 (C) $e^{\frac{2}{x}}$ (D) $e^{\log(2x)}$

19) The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is:

(A) 3 (B) 2 (C) 5 (D) 4

20) What is the product of the order and degree of the differential equation

$\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(A) 3 (B) 4 (C) 6 (D) 2

ASSERTION AND REASON

In the following question, a statement of assertion(A) is followed by a statement of reason(R). Choose the correct answer out of the following choices:

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

1) Assertion (A): The general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is given by $y = \frac{x^2}{4} + cx^{-2}$

Reason (R): The general solution of linear differential equation is given by $y(I.F.) = \int\{(I.F.) \times Q\}dx + c$

2) **Assertion (A):** The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + 4\frac{d^2y}{d^2x} + 5\sin\left(\frac{dy}{dx}\right) = 0$ is 2 and 1 respectively.

Reason (R): Order of a differential equation is the order of the highest order derivative and degree of a differential equation is the highest power of the highest order derivative in it and it is defined if it is a polynomial equation in its derivatives.

DESCRIPTIVE QUESTIONS

1) Solve the Differential equation: $e^{y-x}\frac{dy}{dx} = 1$.

2) Solve the differential equation: $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$.

3) Solve the differential equation: $x(x^2 - 1)\frac{dy}{dx} = 1; y(2) = 0$.

4) Solve the differential equation $(x^2 - y^2)\,dx + 2xydy = 0$.

5) Find the particular solution of the differential equation given by $x^2\frac{dy}{dx} - xy = x^2cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1, y = \frac{\pi}{2}$.

6) Solve: $\frac{dy}{dx} = \frac{y-x}{y+x}$.

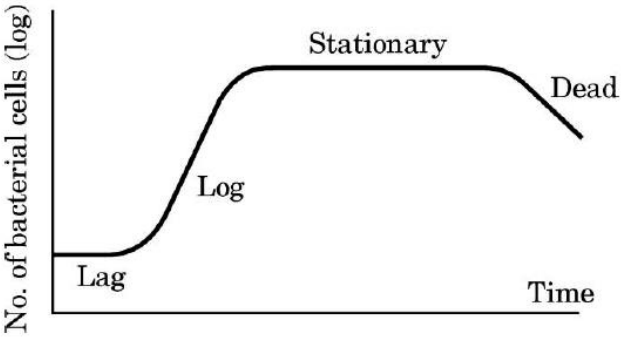
7) Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ given that $y = 0$ when $x = 1$.

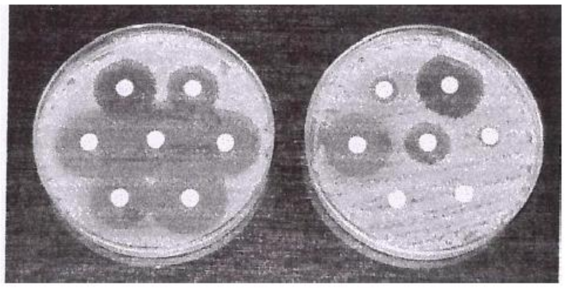
8) Solve: $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

9) Find the general solution of the differential equation : $ydx = (x + 2y^2)dy$.

CASE STUDY

1) A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as: $\frac{dP}{dt} = kP$, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions:

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'.

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k.

10. VECTOR ALGEBRA

1.For what value of p , is $(\hat{\imath} + \hat{\jmath} + \hat{k})$ p a unit vector ?

a) $\pm\frac{1}{\sqrt{3}}$ b) ± 1 c) $\pm\frac{1}{3}$ d) $\pm\sqrt{3}$

2. The magnitude of vector $\vec{a}= 3\hat{\imath} - 4j$
   a) 5 b) -5 c) $\sqrt{5}$ d) $-\sqrt{5}$
3. If $|\vec{a} + \vec{b}\,| = |\vec{a} - \vec{b}|$, Then angle between $\vec{a}$ and $\vec{b}$ is

   a) 0° b) 90° c) . 180° d) 60°

4. The magnitude of projection of $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ on $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is a) $\frac{4}{81}$
   b) $\frac{4}{9}$ c) $\frac{8}{9}$ $d)\,\frac{4}{3}$
5. If $\vec{a}, \vec{b}$ and $\vec{a} + \vec{b}$ are three unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$ then the value of θ is
   a) 120° b) 150° c) . 60° d) 30°
6. If $(2\hat{\imath} + 6\hat{\jmath} + 14\hat{k})$ X $(\hat{\imath} - \lambda\hat{\jmath} + 7\hat{k}) = \vec{0}$ then the value of λ is
   a) $\frac{27}{2}$ b) $-\frac{27}{2}$ c) 3 d) -3
7. If ABCD is a parallelogram and AC and BD are its diagonal ,then $\overrightarrow{AC} + \overrightarrow{BD}$ is a) $2\overrightarrow{DA}$
   b) $2\overrightarrow{AB}$ c) $2\overrightarrow{BC}$ d) $2\overrightarrow{CD}$
8. The area of a parallelogram whose adjacent sides are given by the vectors $\vec{a}= 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ is (in sqm) a) 7 b) $\sqrt{26}$
   c) $\sqrt{42}$ d) $\frac{\sqrt{42}}{2}$
9. Find the unit vector in the direction of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$.
10. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, find $\vec{a}\cdot\vec{b}$.
11. Find a vector perpendicular to both $\vec{a}$ = $\vec{a}= 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$
12. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}= 3\hat{\imath} + 4\hat{\jmath}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath}$ find the angle between them.
13. If $\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$, find the value of $(\vec{r}\,X\hat{\jmath}).(\vec{r}\,X\hat{k}) - 12$.

14.Find the scalar projection of the vector $\vec{a}= 3\hat{\imath} - 4\hat{\jmath} + \hat{k}$ on $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.

15. Find the value of x if the vectors $\vec{a}= \hat{\imath} + 2\hat{\jmath} + x\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ are perpendicular.

16.Find a unit vector perpendicular to both $\vec{a}= 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

17.Find the angle between the vectors $\vec{a}= 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - 4\hat{k}$. Also, state whether they are acute, obtuse, or perpendicular.

18.Show that the three points A(1, 2, 3), B(2, 4, 5), and C(3, 6, 7) are collinear using vectors.

19. If $\vec{a}= \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ on $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find a vector $\vec{c}$ such that it is perpendicular to both $\vec{a}$ and $\vec{b}$.

20.A drone company is testing delivery routes using vector algebra. The drone flies from a warehouse located at point A, then to a checkpoint B, and finally to the delivery point C. The position vectors of these points with respect to the origin (O) are:

$\overrightarrow{OA}$= 3 $\hat{\imath}$+ 2 $\hat{\jmath}$ + $\hat{k}$, $\overrightarrow{OB}$= 3 $\hat{\imath}$- 2 $\hat{\jmath}$ + 4$\hat{k}$, $\overrightarrow{OC}$ = 5 $\hat{\imath}$- 4$\hat{\jmath}$ + 3$\hat{k}$.

The company wants to analyse the drone's path and performance using vector operations.

Answer the following questions:

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$

(b) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

(c) Is the path taken by the drone a straight line? Justify.

SOLUTIONS

1. a) $\pm\frac{1}{\sqrt{3}}$ 2. c) $\sqrt{5}$ 3. b) 90° 4. $d)\frac{4}{3}$ 5) a) 120°

6. d) -3 7. c) $2\overrightarrow{BC}$ 8. c) $\sqrt{42}$

9. unit vector= $\frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}}{7}$

10. $\vec{a}\cdot\vec{b}$ =($\hat{\imath}$ – 2$\hat{\jmath}$ + $\hat{k}$).(2$\hat{\imath}$ + $\hat{\jmath}$ -3$\hat{k}$) = $1.2 + (-2).1 + 1.(-3)$=2-2-3= -3

12. cosθ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{(3\hat{\imath}+4\hat{\jmath}).(\hat{\imath}-2\hat{\jmath})}{5.\sqrt{5}} = \frac{-5}{5\sqrt{5}}$ $\theta = \cos^{-1}(\frac{-1}{\sqrt{5}})$

13. $(\vec{r}\, X\hat{\jmath})$ = $(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})$ X $\hat{\jmath}$ = $3\hat{k} - 6\hat{\imath}$

$(\vec{r}\, X\hat{k})$ = -3$\hat{\jmath}$ – 2$\hat{\imath}$

$(\vec{r}\, X\hat{\jmath}).(\vec{r}\, X\hat{k})$ – 12= $(3\hat{k} - 6\hat{\imath})$.(-3$\hat{\jmath}$ – 2$\hat{\imath}$) -12 = 12-12=0

14. Scalar projection= $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{-7}{3}$

15. Vectors are perpendicular so $\vec{a}\cdot\vec{b} = 0$

3-4+x=0 ⇒ x=1

16. Required vector = $\frac{\vec{a}\, X\vec{b}}{|\vec{a}X\vec{b}|} = \frac{2\hat{\imath}-7\hat{\jmath}+4\hat{k}}{\sqrt{69}}$

$\vec{a}\, X\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & 2 \\ 1 & 2 & 3 \end{vmatrix}$ = 2 $\hat{\imath}$-7 $\hat{\jmath}$ + 4$\hat{k}$, $|\vec{a}\, X\vec{b}| = \sqrt{69}$

17. Using cosθ = $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$ find θ.

18. $\overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, $\overrightarrow{BC} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

Since $\overrightarrow{AB}$ *and* $\overrightarrow{BC}$ are parallel and B is a common point so A,B and C are collinear.

19. $\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & -2 \end{vmatrix} = \hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

20. *i)* $\overrightarrow{AB} = -4\hat{\jmath} + 3\hat{k}$, $\overrightarrow{BC} = 2\hat{\imath} - 2\hat{\jmath} - \hat{k}$

ii) $\cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{BC}}{|\overrightarrow{AB}||\overrightarrow{BC}|} = \frac{-11}{15}$

iii) No . Reason: Check for collinearity(whether $\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC}$ $or\ not$)

11. THREE DIMENSIONAL GEOMETRY

Q.1 If α, β and γ are the angles made by a line with the positive directions of the x, y, and z axes respectively, then which of the following is correct?

(a) $cos^2\alpha - cos^2\beta + cos^2\gamma = 0$

(b) $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$

(c) $cos^2\alpha - cos^2\beta - cos^2\gamma = 0$

(d) $\cos\alpha + \cos\beta + \cos\gamma = 0$

Q.2 Vector equation of a line is $\vec{r} = (4\hat{\imath} - 2\hat{\jmath} + 5\hat{k}) + \mu(\hat{\imath} + \hat{3\jmath} - 2\hat{k})$,the Cartesian form of a line is:

(a) $\frac{x-4}{1} = \frac{y+2}{3} = \frac{z-5}{-2}$

(b) $\frac{x+4}{1} = \frac{y-2}{3} = \frac{z+5}{-2}$

(c) $\frac{x-4}{1} = \frac{y+2}{3} = \frac{z-5}{2}$

(d) $\frac{x+4}{-1} = \frac{y-2}{3} = \frac{z+5}{2}$

Q.3 The vector equation of a line is $\vec{r} = (5-s)\hat{\imath} + (3+2s)\hat{\jmath} + (1-s)\hat{k}$, the Cartesian equation of the line passing through point (0,1,-2) and parallel to it is,

(a) $\frac{x}{-1} = \frac{y-1}{2} = \frac{z+2}{-1}$

(b) $\frac{x}{1} = \frac{y-1}{-2} = \frac{z+2}{1}$

(c) $\frac{x}{-1} = \frac{y-1}{2} = \frac{z+2}{1}$

(d) $\frac{x}{1} = \frac{y-1}{2} = \frac{z+2}{1}$

Q.4 If a line makes angles $\frac{\pi}{2}, \frac{\pi}{3}$ and θ with the positive x, y and z axes respectively, then θ is

(a) $\pm\frac{\pi}{6}$ (b) $\pm\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ only (d) $\frac{\pi}{3}$ only

Q.5 The equation of a line passing through the point (3,-1,5) and parallel to vector $(\hat{\imath} + \hat{2\jmath} - \hat{k})$ is;

(a) $x = t + 3$, $y = 2t - 1$, $z = -t + 5$

(b) $x = t + 3$, $y = -2t - 1$, $z = -t + 5$

(c) $x = t + 3$, $y = 2t - 1$, $z = t + 5$

(d) $x = t - 3$, $y = 2t - 1$, $z = -t + 5$

Q.6 Determine the value of p so that the lines $\vec{r} = (1+t)\hat{\imath} + (2-t)\hat{\jmath} + (3+2t)\hat{k}$ and

$\vec{r} = 2\hat{\imath} + ps\hat{\jmath} + (4-s)\hat{k}$ are perpendicular.

Q. 7 Find the distance of the point (5,4,-2) from the point of intersection of the lines:

$$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-5}{1} \text{ and } \frac{x-5}{1} = \frac{y+3}{2} = \frac{z-6}{-2}$$

Q.8 Find the vector equation of the line passing through the point (3,0,-1) and perpendicular to both the lines:

$$\frac{x-4}{3} = \frac{y+2}{2} = \frac{z+1}{-1} \text{ and } \frac{x+1}{1} = \frac{y-1}{-2} = \frac{z-2}{2}$$

Q.9 Consider the lines:

$$\frac{x+1}{3} = \frac{y-3}{2} = \frac{z+4}{-1} \text{ and } \frac{x-3}{2} = \frac{y+1}{-1} = \frac{z-5}{2}$$

Find the shortest distance between them.

Q.10 The line L_1 passes through the point (1,2,−1) and has direction ratios proportional to (2,−1,2)
Another line L_2 passes through (3,0,4) and has direction ratios proportional to (1,2,−1).
Find the shortest distance between these two lines.

Q.11 Find the coordinates of the foot of the perpendicular drawn from the point (5,0,1) to the line

$$\frac{x}{2} = \frac{y-3}{-1} = \frac{z+4}{4}$$

Also, find the perpendicular distance of the point from the line.

Q.12 For the parallelogram EFGH with vertices
E(1,2,3), F(4,5,6), G(3,8,7), H(0,5,4)
(i) Find the cartesian equations of all the sides.
(ii) Find the coordinates of the foot of the perpendicular from point E to line GH.

Q.13 The point P(3,0,−2) is reflected in the line

$$\frac{x-2}{4} = \frac{y+1}{1} = \frac{z}{5}$$

Find the coordinates of the image of the point .

Q.14 Find the image of the point (2,-1,5) in the line,

$\vec{r} = (11\hat{\imath} - 2\hat{\jmath} - 8\hat{k}) + t(10\hat{\imath} - 4\hat{\jmath} - 11\hat{k})$

Q.15 Find the coordinates of the point Q on the line

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1}$$

such that the distance between Q and the point P(5,1,0) is 4 units.

SOLUTIONS

Q.1(b) ,**Q.2** (a) ,**Q.3** (a) ,**Q.4** (c) ,**Q.5** (a)

Answer 6. Rearranging both equations

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + t(\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\vec{r} = (2\hat{\imath} + 0\hat{\jmath} + 4\hat{k}) + s(0\hat{\imath} + p\hat{\jmath} - \hat{k})$

If both lines are perpendicular then,

$$(1)(0) + (-1)(p) + (2)(-1) = 0$$

$$p = -2$$

Answer 7. Line 1: $x = 3t + 2\,, y = -2t - 1\,, z = t + 5$

Line 2 : $x = u + 5\,, y = 2u - 3\,, z = -2u + 6$

Now comparing the value of x , y and z ,we find the solution for ‘t’ and ‘u’ as6

t = 1 and u = 0

Then for line 1 : $x = 5\,, y = -3\,, z = 6$

For line 2 : $x = 5\,, y = -3\,, z = 6$

Hence point of intersection is $(5,-3,6)$

Distance of point $(5,-3,6)$ from point (5,4,-2) is $\sqrt{113}$.

Answer 8. Equation of line passing through point $(3,0,-1)$

L1 : $\frac{x-3}{a} = \frac{y-0}{b} = \frac{z+1}{c} = k$ ………….(i)

L2 : $\frac{x-4}{3} = \frac{y+2}{2} = \frac{z+1}{-1}$

L3 : $\frac{x+1}{1} = \frac{y-1}{-2} = \frac{z-2}{2}$

If line L1 and L2 are perpendicular and line L1 and L3 are perpendicular then,

$$3a + 2b - c = 0$$

and $$a - 2b + 2c = 0$$

Solving both equations

$c = -4a\ ,\quad b = \frac{-7a}{2}$

Substituting these values in equation (i) ,we get

$$\frac{x-3}{2} = \frac{y-0}{-7} = \frac{z+1}{-8} = k$$

Answer 9. Vector form of both lines,

$\vec{r} = (-\hat{\imath} + 3\hat{\jmath} - 4\hat{k}) + t(3\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$\vec{r} = (3\hat{\imath} - \hat{\jmath} + 5\hat{k}) + u(2\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$$\vec{a}_2 - \vec{a}_1 = (4\hat{\imath} - 4\hat{\jmath} + 9\hat{k})$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & -1 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= (3\hat{\imath} - 8\hat{\jmath} - 7\hat{k})$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{122}$$

$$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = -19$$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$d = \frac{19}{\sqrt{122}}\ \ units.$$

Answer 10. Equations of given lines respectively,

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + t(2\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\vec{r} = (3\hat{\imath} + 0\hat{\jmath} + 4\hat{k}) + u(\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$$\vec{a}_2 - \vec{a}_1 = (2\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= (-3\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{50}$$

$$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 11$$

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$d = \frac{11}{\sqrt{50}} \ units$$

Answer 11. Let $P(x, y, z)$ be the foot of perpendicular drawn from point $A(5,0,1)$ to the given line.

Coordinates of point P= $(2k, -k + 3, 4k - 4)$

Direction ratios of line AP = $(2k - 5, -k + 3\ , 4k - 5)$

Direction ratios of given line = $(2, -1, 4)$

Since both lines are perpendicular then,

$$2(2k - 5) + (-1)(-k + 3) + 4(4k - 5) = 0$$

$$k = \frac{11}{7}$$

Foot of perpendicular =P$(\frac{22}{7}, \frac{10}{7}, \frac{16}{7})$

Perpendicular distance of point (5,0,1) from line = $\frac{\sqrt{350}}{7}$

Answer 12. (i) Equation of line EF:

$$\frac{x - 1}{3} = \frac{y - 2}{3} = \frac{z - 3}{3}$$

Equation of line FG:

$$\frac{x-4}{-1}=\frac{y-5}{3}=\frac{z-6}{1}$$

Equation of line GH:

$$\frac{x-3}{-3}=\frac{y-8}{-3}=\frac{z-7}{-3}$$

Equation of line HE:

$$\frac{x-0}{1}=\frac{y-5}{-3}=\frac{z-4}{-1}$$

(ii) Let $P(x, y, z,)$ be the foot of perpendicular drawn from point $E(1,2,3)$ to the given line GH.

Coordinates of point P= $(-3t+3, -3t+8, -3t+7)$

Direction ratios of line AP = $(-3t+2, -3t+6, -3t+4)$

Direction ratios of given line = $(-3, -3, -3)$

Since both lines are perpendicular then,

$$-3(-3t+2)+(-3)(-3t+6)+(-3)(-3t+4)=0$$

$$t=\frac{4}{3}$$

Foot of perpendicular = P$(-1,4,3)$

Answer 13. Let $Q(x, y, z,)$ be the foot of perpendicular drawn from point $P(3,0,-2)$ to the given line and R$(a, b, c,)$ be the coordinates of the image of point P.

Coordinates of point Q= $(4t+2, t-1, 5t)$

Direction ratios of line PQ = $(4t-1, t-1, 5t+2)$

Direction ratios of given line = $(4,1,5)$

Since both lines are perpendicular then,

$$4(4t+2)+(1)(t-1)+5(5t+2)=0$$

$$t=\frac{-5}{42}$$

Foot of perpendicular =Q$\left(\frac{32}{21}, \frac{-47}{42}, \frac{-25}{42}\right)$

Since Q is the mid point of points P and R,

$$x = \frac{a+3}{2}, y = \frac{b}{2}, z = \frac{c-2}{2}$$

Substituting values of x,y and z ,we get

Coordinates of point R(a, b, c) = $\left(\frac{1}{21}, \frac{-47}{21}, \frac{17}{21}\right)$

Answer 14. Let $Q(x, y, z,)$ be the foot of perpendicular drawn from point $P(2, -1, 5)$ to the given line and R$(a, b, c,)$ be the coordinates of the image of point P.

Let $x\hat{\imath} + y\hat{\jmath} + z\hat{k} = (11\hat{\imath} - 2\hat{\jmath} - 8\hat{k}) + t(10\hat{\imath} - 4\hat{\jmath} - 11\hat{k})$

Comparing on both sides,

$$x = 11 + 10t, y = -2 - 4t, z = -8 - 11t$$

Coordinates of point Q= $(11 + 10t, -2 - 4t, -8 - 11t)$

Direction ratios of line PQ = $(10t + 9, -4t - 1, -11t - 13)$

Direction ratios of given line = $(10, -4, -11)$

Since both lines are perpendicular then,

$$10(10t + 9) + (-4)(-4t - 1) + (-11)(-11t - 13) = 0$$

$$t = -1$$

Foot of perpendicular =Q$(1,2,3)$

Since Q is the mid point of points P and R,

$$x = \frac{a+2}{2}, y = \frac{b-1}{2}, z = \frac{c+5}{2}$$

Substituting values of x,y and z ,we get

Coordinates of point R(a, b, c) = $(0,5,1)$

Answer 15. Let the coordinates of point Q be (x, y, z).

Then $Q(x, y, z) = (t + 2, 2t - 1, -t + 3)$

Given that distance between point P and Q is 4 units,

Thus, $(t + 2 - 5)^2 + (2t - 1 - 1)^2 + (-t + 3 - 0)^2 = (4)^2$

$$3t^2 - 10t + 3 = 0$$

Using Quadratic formula ,we get two values for t such as,

$$t = 3 \ \ and \ t = \frac{1}{3}$$

Substituting the values of t , we get two points $Q(x, y, z) = (5,5,0) \ \ and \ \left(\frac{7}{3}, \frac{-1}{3}, \frac{8}{3}\right)$

13. Linear Programming

BASIC CONCEPTS

What is LPP: LPP, or Linear Programming Problem, is a **mathematical optimization technique** used to find the best outcome (maximum or minimum) of a linear function, subject to linear constraints and non-negative restrictions on the variables.

1. **Objective function:** Linear function Z = ax + by, where a, b are constants, which has to be maximized or minimized is called a linear objective function.
2. **Constraints:** The linear inequalities or equations or **restrictions** which are imposed on the variables of a linear programming problem are called constraints.
3. **Optimization problem:** A problem which seeks to maximize or minimize a linear function (say of two variables x and y) subject to certain constraints as determined by a set of linear inequalities is called an optimization problem.
4. **Feasible region:** The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of a linear programming problem is called the feasible region.
5. **Feasible solutions:** Points within and on the boundary of the feasible region represent feasible solutions of the constraints.
6. **Optimal (feasible) solution:** Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.
7. **Corner point method:** The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function Z = ax + by at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is bounded, M and m are the maximum and minimum values of Z.
   (ii) In case, the feasible region is unbounded, we have:
4. (a) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.
   (b) Similarly, m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has **no minimum value.**

MCQs

S.N. **Questions**

1 What is the objective function in a linear programming problem?

a) A condition to be satisfied

b) A function to be maximized or minimized

c) A type of constraint

d) A type of inequality

2 What are constraints in LPP?

a) Goals of the problem

b) Unwanted equations

c) Conditions in the form of inequalities

d) Values of the objective function

3 Which quadrant is considered in LPP for real-life problems?

a) First

b) Second

c) Third

d) Fourth

4 The maximum or minimum value of the objective function occurs at:

a) Origin

b) Boundary line

c) Corner points of the feasible region

d) Centre of feasible region

5 In the LPP, $x \geq 0$ and $y \geq 0$ are called:

a) Additional equations

b) Non-negative constraints

c) Inverse conditions

d) Elimination rules

6 If one constraint is $x + y \leq 6$, which point satisfies this?

a) (5, 3)

b) (2, 3)

c) (4, 4)

d) (6, 6)

7 If the feasible region is a triangle, how many corner points does it have?

a) 1

b) 2

c) 3

d) Infinite

8 Which of these is a correct objective function?

a) $x^2 + y$

b) $3x + 4y$

c) $x\sqrt{y} + y$

d) $\sqrt{x} + y$

9 The common region determined by all the constraints of a linear programming problem is called:

a) an unbounded region b) an optimal region c) a bounded region d) a feasible region

10 The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called:

a) feasible solutions b) constraints c) optimal solutions d) infeasible solutions

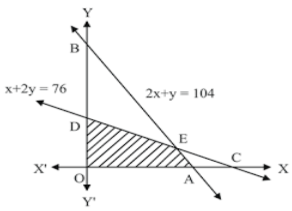
11 Of the following, which group of constraints represents the feasible region given below ?

a) $x + 2y \leq 76$, $2x + y \geq 104$, $x,y \geq 0$

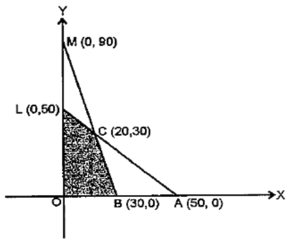
b) $x + 2y \leq 76$, $2x + y \leq 104$, $x,y \geq 0$

c) $x + 2y \geq 76$, $2x + y \geq 104$, $x,y \geq 0$

d) $x + 2y \geq 76$, $2x + y \geq 104$, $x,y \geq 0$



12 The maximum value of $Z = 4x + y$ for a L.P.P., whose feasible region is shown below is:



a) 50

b) 110

c) 120

d) 170

13 The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0$, $x + y \geq 4$ is:

a) 0 b) 1 c) 2 d) 3

14 A linear programming problem is a follows:

Minimize $Z = 30x + 50y$

Subject to the constraints,

$3x + 5y \geq 15$

$2x + 3y \leq 18$

$x \geq 0, y \geq 0$

In the feasible region, the minimum value of Z occurs at

a) A unique point

b) No point

c) Infinitely many points

d) Two points only

15 If a feasible region is a polygon, the optimal value of the objective function must occur at:

(a) The centre of the polygon

(b) The centroid of the polygon

(c) One of the vertices of the polygon

(d) The origin

16 The solution set of the inequation $3x + 2y > 3$ is

a) half plane containing the origin

b) half plane not containing the origin

c) the point being on the line $3x + 2y = 3$

d) none of these

17 If a feasible region is empty, then the LPP has:

(a) No solution

(b) An unbounded solution

(c) A unique solution

(d) Infinitely many solutions

18 A set of values of decision variables that satisfies the linear constraints and non-negativity conditions of an LPP is called its:

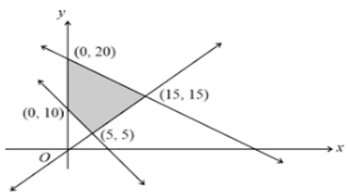
a) feasible region b) corner points c) unbounded solutions d) none of the above

19 The corner points of a feasible region determined by the system of linear constraints are (0 , 19), (5 , 5), (15 , 15) and(0 , 20).Let Z=px + qy, where p, q > 0, condition on p and q so that the maximum of Z at both points (15, 15) and (0, 20) is

a) p = q b) p = 2q c) q = 2p d) q = 3p

20 Observe the graph of the feasible region of the cost optimisation LPP shaded below.

Which of the following inequalities is one of the constraints of the LPP ?



a) $5x + y \geq 10$

b) $x + y \geq 12$

c) $3x – 2y \leq 6$

d) $x + y \geq 10$

21 Solve the following LPP graphically:

Maximise: $Z = 2x + 3y$, subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$

22 Solve the following linear programming problem graphically:

Minimise: $Z = 200 x + 500 y$ subject to the constraints:

$x + 2y \geq 10$, $3x + 4y \leq 24$, $x \geq 0, y \geq 0$

23 Solve the following problem graphically:

Minimise and Maximise Z = 3x + 9y

Subject to the constraints: $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$ $x \geq 0, y \geq 0$

24 Solve the following LPP graphically:

Minimise $Z = 5x + 10y$ subject to the constraints

$x + 2y \leq 120,$ $x + y \geq 60,$

$x - 2y > 0$ and $x, y \geq 0$

SOLUTIONS

1 b) A function to be maximized or minimized

Explanation:

The objective function is a mathematical expression that represents the quantity we aim to optimize (either maximize or minimize)

2 c) Conditions in the form of inequalities

Explanation:

Constraints are limitations or restrictions placed on the decision variables in the form of inequalities.

3 a) First

Explanation:

This is because the decision variables, which represent quantities, are usually non-negative in practical applications.

4 c) Corner points of the feasible region

Explanation:

This is because the feasible region, defined by the constraints, is a convex polygon, and the objective function, being linear, will either increase or decrease as you move along its edges.

5 b) Non-negative constraints

Explanation:

Non-negative means greater than or equal to zero.So, $x \geq 0$, $y \geq 0$ are called non-negative constraints.

6 b) (2, 3)

Explanation:

As $x + y \leq 6$

Put $x = 2$, $y = 3$ in LHS

∴ 2 + 3 = 5 ≤ 6 which is true.

7 c) 3

Explanation:

This is because a triangle has 3 vertices.

8 b) 3x + 4y

Explanation:

Objective functions are in linear form.

9 d) a feasible region

10 b) constraints

Explanation:

Constraints are the restrictions imposed on decision variables involved in an objective function of a linear programming problem

11 b) $x + 2y \leq 76$, $2x + y \leq 104$, $x,y \geq 0$

12 c) 120

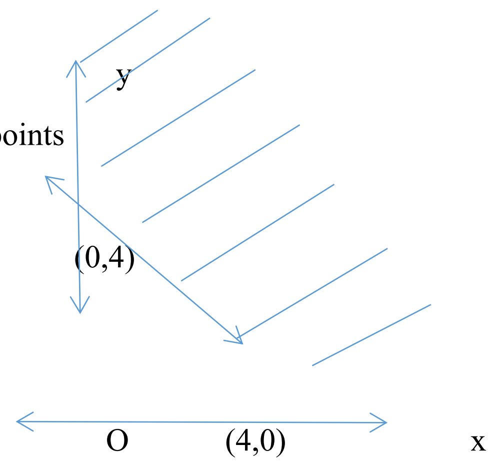
Explanation:

Corner point	Value of Z = 4x + y
L(0 , 50)	Z = 4(0) + 50 = 50
C(20 , 30)	Z = 4(20) + 30 = 110
B(30 , 0)	**Z = 4(30) + 0 = 120→MAXIMUM**
O(0 , 0)	Z = 4(0) + 0 = 0

13 c) 2

Explanation:

From the graph, it is clear that number of corner points are 2.



14 c) Infinitely many points

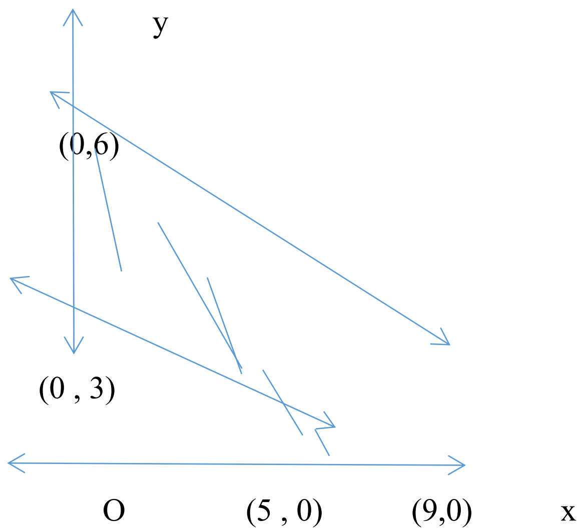
Explanation:

Let the equation of constraints be

$3x + 5y = 15$

$2x + 3y = 18$

$x = 0, y = 0$



Corner point	Z = 30x + 50y
(5 , 0)	150
(9 , 0)	270
(0 , 3)	150
(0 , 6)	300

15 (c) One of the vertices of the polygon

Explanation:

As a polygon is bounded and by corner point method, maximum/minimum value of Z occurs at corner point.

16 b) half plane not containing the origin

Explanation:

$3x + 2y > 3$

Put $x = 0, y = 0$

$3(0) + 2(0) = 0 < 3$

So, the half plane does not contain the origin.

17 (a) No solution

Explanation:

As the feasible region is solution set and if it is empty then there is no solution for the given LPP.

18 a) feasible region

Explanation:

A feasible region is defined as an area bounded by a set or collection of coordinates that satisfy a system of given inequalities.

19 d) q = 3p

Explanation:

Z = px + qy

Z is maximum at (15 , 15) and (0 , 20).

∴ 15p + 15q = p(0) + 20q

⇒15p = 3q

⇒ 3p = q

⇒ q = 3p

20 d) x + y ≥ 10

Explanation:

Equation of line joining the points (0 , 10) and (5 , 5) is

$y-10 = \frac{10-5}{0-5}(x-0)$

$\Rightarrow y-10 = \frac{5}{-5}x$

⇒y-10 = -x

⇒x + y = 10

Also the shading the away from the origin.

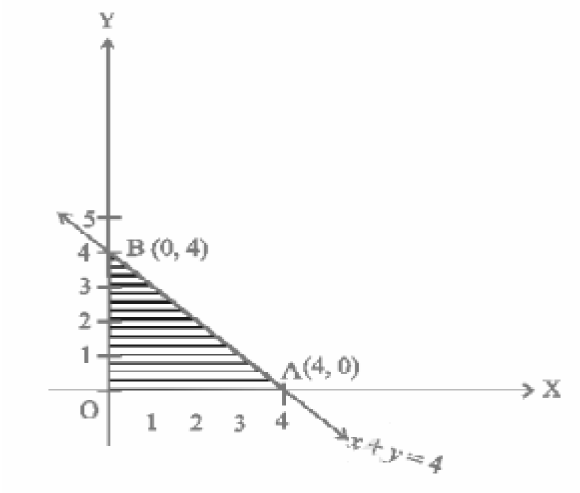
So, the correct inequality is x + y ≥ 10.

21 **Solution:** Let x + y = 4

If x = 0, y = 4 and y = 0, x = 4

x	0	4
y	4	0

Draw the graph of x + y =4 as below.



The shaded region (OAB) in the above figure is the feasible region determined by the system of constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 4$.

The feasible region OAB is bounded and the maximum value will occur at a corner point of the feasible region.

Corner Points are O(0, 0), A (4, 0) and B (0, 4).

Evaluate Z at each of these corner points.

Corner Point	Value of Z
O (0, 0)	2 x 0 + 3 x 0 = 0
A (4, 0)	2 x 4 + 3 x 0 = 8
B (0, 4)	2 x 0 + 3 x 4 = 12 ← maximum

Hence, the maximum value of Z is 12 at the point (0, 4).

22 Let $x + 2y = 10$

Put $x = 0$ then $y = 5$ and $y = 0$ then $x = 10$

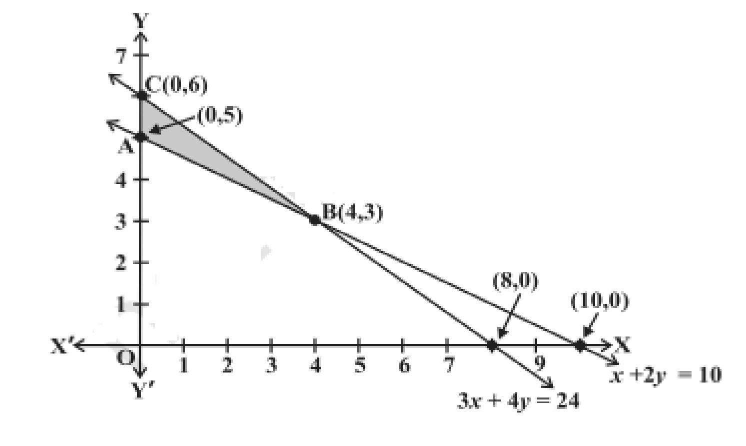
x	0	10
y	5	0

Now $3x + 4y = 24$

Put $x = 0$ then $y = 6$ and $y = 0$ then $x = 8$

x	0	8
y	6	0

Let us draw the graph of $x + 2y = 10$, $3x + 4y = 24$ and $x \geq 0$, $y \geq 0$



The shaded region in the above figure is the feasible region ABC determined by the

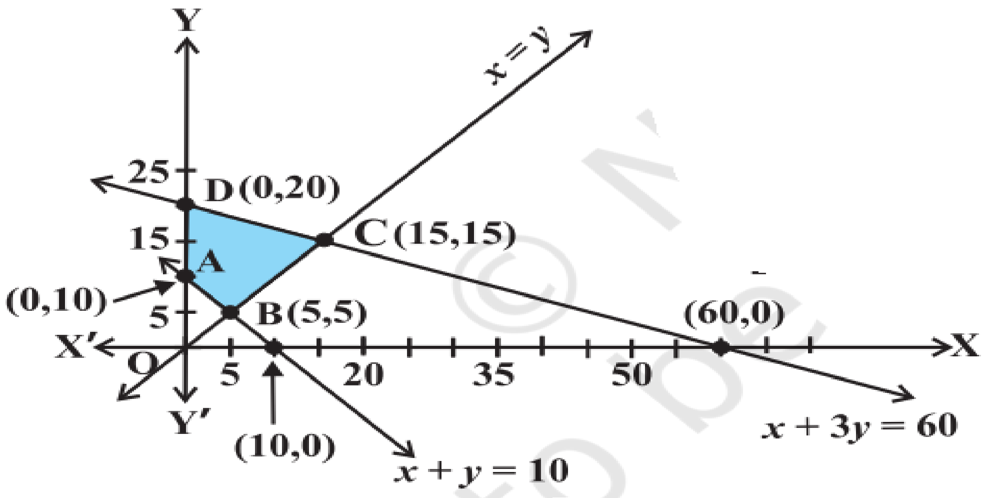
system of constraints, which is bounded. The coordinates of corner point A, B and C are (0,5), (4,3) and (0,6) respectively.

Calculation of Z = 200x + 500y at these points.

Corner Point	Value of Z
(0, 5)	200 x 0 + 500 x 5 = 2500
(4, 3)	200 x 4 + 500 x 3 = 2300 ← Minimum
(0, 6)	200 x 0 + 500 x 6 = 3000

Hence, the minimum value of Z is 2300 is at the point (4, 3).

23 Draw the graph of the inequalities (2), (3), (4) and (5)



The feasible region ABCD is bounded and the coordinates of corner points are A(0,10), B(5,5), C(15,15) and D(0,20).

Corner Points	Corresponding value of Z = 3x + 9y
A (0, 10	90

B (5, 5)	60←Minimum
C (15, 15)	180←Maximum (Multiple optimal solutions)
D (0, 20)	180 ←Maximum

From the table, we find that the minimum value of Z is 60 at the point B (5, 5) of the feasible region. The maximum value of Z on the feasible region occurs at the two corner points C (15, 15) and D (0, 20) and it is 180 in each case.

24 Our problem is to minimise

$Z = 5x + 10y$ … (i)

Subject to constraints

$x + 2y \leq 120$ …(ii)

$x + y \geq 60$ …(iii) $x - 2y \geq 0$ … (iv) and $x \geq 0, y \geq 0$

Table for line $x + 2y = 120$ is

x	0	120
y	60	0

Put (0, 0) in the inequality $x + 2y \leq 120$, we get

$0 + 2x \leq 120$

$\Rightarrow 0 \leq 120$ (which is true)

So, the half plane is towards the origin.

Now, draw the graph of the line $x + y = 60$

x	0	60
y	60	0

On putting (0, 0) in the inequality $x + y \geq 60$, we get

$0 + 0 \geq 60 \Rightarrow 0 \geq 60$ (which is false)

So, the half plane is away from the origin.

Thirdly, draw the graph of the line $x - 2y = 0$.

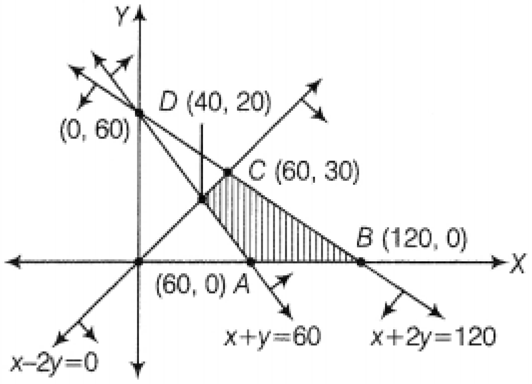
x	0	10
y	0	5

On putting (5, 0) in the inequality $x - 2y \geq 0$, we get

$5 - 2 \times 0 \geq 0 \Rightarrow 5 \geq 0$ (which is true)

Thus, the half plane is towards the X-axis. Since, $x, y \geq 0$

∴The feasible region lies in the first quadrant.



Clearly, feasible region is ABCDA.
On solving equations x – 2y = 0 and x + y = 60,
we get D(40,20) and on solving equations
x -2y = 0 and x + 2y = 120, we get C (60, 30). The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30) and D (40, 20). The values of Z at these points are as follows

Corner point	Z = 5x + 10y
A (60, 0)	300 (minimum)
8(120,0)	600
C (60, 30)	600
D (40, 20)	400

The minimum value of Z is 300 at the points (60, 0)

14. Probability

SOME IMPORTANT RESULTS/CONCEPTS

** Sample Space and Events:
The set of all possible outcomes of an experiment is called the sample space of that experiment. It is usually denoted by S. The elements of S are called events and a subset of S is called an event.
ϕ ($\subset$ S) is called an impossible event and
S($\subset$ S) is called a sure event.
** Probability of an Event.
(i) If E be the event associated with an experiment, then probability of E, denoted by P(E) is defined as $P(E) = \dfrac{\text{number of outcomes in E}}{\text{number of total outcomes in sample space S}}$
it being assumed that the outcomes of the experiment in reference are equally likely.
(ii) P(sure event or sample space) = P(S) = 1 and P(impossible event) = P(ϕ) = 0.
(iii) If $E_1, E_2, E_3, \ldots, E_k$ are mutually exclusive and exhaustive events associated with an experiment (i.e. if $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_k) = S$ and $E_i \cap E_j = \phi$ for $i, j \in \{1, 2, 3, \ldots, k\}$ $i \neq j$), then
$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_k) = 1$.
(iv) $P(E) + P(E^C) = 1$
** If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by
$P(E|F) = \dfrac{P(E \cap F)}{P(F)}$ provided $P(F) \neq 0$
** Multiplication rule of probability : $P(E \cap F) = P(E)\, P(F|E)$
$= P(F)\, P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.
** Independent Events :E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.
Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) . P(F)$.
** Bayes' Theorem :If $E_1, E_2, \ldots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and A is any event of nonzero probability, then
$P(E_i|A) = \dfrac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j)P(A|E_j)}$ for any i = 1, 2, 3, ..., n

1 In a class of 100 students, 60 passed in Mathematics, 70 passed in Physics, and 50 passed in both. If a student is selected at random, what is the probability that the student passed at least one subject?

(A) 0.80 (B) 0.60 (C) 0.90 (D) 0.70

2 A committee of 3 is formed from 3 boys and 2 girls. What is the probability that the committee includes at least one girl?

(A) 9/10
(B) 4/5
(C) 3/5
(D) ½

3 The probability that a student solves a problem is 3/5. If 4 students attempt independently, what is the probability that at least one of them solves it?

(A) 256/625
(B) 369/625

(C) 625/625
(D) 609/625

4 In a test, the probability that student A passes is 0.9 and B passes is 0.8. The probability that both pass is 0.72. What is the probability that at least one of them passes?

(A) 0.98
(B) 0.85
(C) 0.90
(D) 0.80

5 An unbiased die is thrown three times. What is the probability that the number 6 appears at most once?

(A) 200/216
(B) 91/216
(C) 36/216
(D) 1/6

6 Let A and B be independent events with P(A) = 1/3 and P(B) = 1/4. What is the probability of A ∪ B?

(A) 1/2
(B) 7/12
(C) 5/12
(D) 1/6

7 A and B are events such that P(A) = 0.6, P(B) = 0.5, and P(A ∪ B) = 0.9. What is P(A ∩ B)?

(A) 0.2
(B) 0.1
(C) 0.4
(D) 0.3

8 If the odds in favour of event A are 3:2, what is P(A)?

(A) 2/5
(B) 3/5
(C) 1/2
(D) 5/8

9 If P(E) = 3/4, P(F) = 1/2, and P(E ∩ F) = 1/4, what is P(E | F)?

(A) 1/2
(B) 2/3
(C) 3/4
(D) 1/4

10 In a bag with 4 red and 6 blue balls, 2 balls are drawn without replacement. What is the probability both are of the same colour?

(A) 1/2
(B) 5/9
(C) 21/45
(D) 14/45

11 Two dice are thrown. What is the probability that their sum is a prime number?

(A) 5/18
(B) 15/36
(C) 10/36
(D) 12/36

12 A random 2-digit number is selected. What is the probability it is divisible by 7?

(A) 13/90
(B) 12/90
(C) 11/90
(D) 14/90

13 Two cards are drawn without replacement. What is the probability both are face cards?

(A) 3/221
(B) 11/221
(C) 12/221
(D) 9/221

14 A man tells the truth 3 out of 4 times. He reports a 6 after rolling a die. What is the probability it was actually a 6?

(A) 3/22
(B) 1/4
(C) 3/8
(D) 3/10

15 A can solve a problem with probability 1/3, B with 1/4. What is the probability the problem is solved if both try independently?

(A) 1/2
(B) 7/12
(C) 5/12
(D) 3/4

16 A student solves part A of a paper with probability 0.7 and part B with 0.4. If P(at least one part) = 0.8, what is P(both parts)?

(A) 0.3
(B) 0.2
(C) 0.4
(D) 0.1

17 A and B are independent events. P(A) = 0.6, P(B) = 0.5. What is $P(A \cup B')$?

(A) 0.8
(B) 0.7
(C) 0.9
(D) 0.3

18 P (E) = 0.4 and $P(E \cup F) = 0.64$. If E and F are independent, what is P(F)?

(A) 0.4
(B) 0.5

(C) 0.6
(D) 0.3

19 A fair coin is tossed 5 times. What is the probability of getting exactly 3 heads?

(A) 5/16
(B) 10/32
(C) 3/8
(D) 5/32

20 If P(E) = 0.6, P(F) = 0.5, and P(E ∩ F) = 0.3, then events E and F are:

(A) Mutually exclusive
(B) Independent
(C) Dependent
(D) Complementary

21 Assume that each born child is equally likely to be a boy or a girl. If a family has two children, then what is the conditional probability that both are girls? Given that
(i) the youngest is a girl?
(ii) atleast one is a girl?

22 If P(not A) = 0.7, P(B) = 0.7 and P(B/A) = 0.5, then find P(A/B).

23 A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'number is even' and B be the event 'number is marked red'. Find whether the events A and B are independent or not.

24 A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

25 Evaluate P(A ∪ B), if 2P (A) = P(B) = 5/13 and P(A/ B) = 2/5.

26 Prove that if E and F are independent events, then the events E and F' are also independent.

SOLUTIONS

1 **Correct Answer:** (A) 0.80

Explanation:
Let A = passed in Math, B = passed in Physics.
P(A) = 60/100 = 0.6
P(B) = 70/100 = 0.7
P(A ∩ B) = 50/100 = 0.5
P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.6 + 0.7 − 0.5 = 0.8

2 **Correct Answer:** (A) 9/10

Explanation:
Total ways to choose 3 from 5 = C(5, 3) = 10
Ways with no girl = C(3, 3) = 1 (all boys)
So, at least one girl = 10 − 1 = 9
Required probability = 9/10

3 **Correct Answer:** (D) 609/625

Explanation:
P(not solving) = 2/5

$P(\text{all 4 fail}) = (2/5)^4 = 16/625$
So, $P(\text{at least one solves}) = 1 - 16/625 = 609/625$

4 **Correct Answer:** (A) 0.98

Explanation:
$P(\text{at least one}) = P(A) + P(B) - P(A \cap B)$
$= 0.9 + 0.8 - 0.72 = 0.98$

5 **Correct Answer:** (A)

Explanation:
$P(6 \text{ appears 0 times}) = (5/6)^3 = 125/216$
$P(6 \text{ appears exactly once}) = C(3,1) \times (1/6) \times (5/6)^2 = 3 \times (1/6) \times (25/36) = 75/216$
So, total $= 125/216 + 75/216 = 200/216$

6 **Correct Answer:** (B) 7/12

Explanation:
$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$
$= 1/3 + 1/4 - 1/12 = 7/12$

7 **Correct Answer:** (A) 0.2

Explanation:
$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.5 - 0.9 = 0.2$

8 **Correct Answer:** (B) 3/5

Explanation:
$P(A) = a / (a + b) = 3 / (3 + 2) = 3/5$

9 **Correct Answer:** (A) 1/2

Explanation:
$P(E \mid F) = P(E \cap F) / P(F) = (1/4) / (1/2) = ½$

10 **Correct Answer:** (C) 21/45

Explanation:
$P(\text{both red}) = (4/10) \times (3/9) = 12/90$
$P(\text{both blue}) = (6/10) \times (5/9) = 30/90$
Sum $= 42/90 = 21/15$

11 **Correct Answer:** (B) 15/36

Explanation:
Prime sums possible: 2, 3, 5, 7, 11
Number of favorable outcomes = 15
$P = 15/36$

12 **Correct Answer:** (A) 13/90

Explanation:
Total 2-digit numbers = 90
Multiples of 7 from 14 to 98 = 13
$P = 13/90$

13 **Correct Answer:** (C) 12/221

Explanation:
Face cards = 12
P = (12/52) × (11/51) = 132/2652 = 12/241

14 **Correct Answer:** (C) 3/8

Explanation:
Using Bayes' Theorem,
P(actual 6 | reported 6) = (1/6 × 3/4) / [1/6 × 3/4 + 5/6 × 1/4] = 3/8

15 **Correct Answer:** (A) 1/2

Explanation:
P(not solved) = (2/3)(3/4) = 1/2
So, P(solved) = 1 − 1/2 = 1/2

16 **Correct Answer:** (B) 0.3

Explanation:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.8 = 0.7 + 0.4 - x \Rightarrow x = 0.3$

17 **Correct Answer:** (A) 0.8

Explanation:
$P(B') = 0.5$
$P(A \cap B') = 0.6 \times 0.5 = 0.3$
$P(A \cup B') = 0.6 + 0.5 - 0.3 = 0.8$

18 **Correct Answer:** (B) 0.5

Explanation:
$P(E \cup F) = P(E) + P(F) - P(E) \times P(F)$
$0.64 = 0.4 + x - 0.4x$
$0.24 = 0.6x \Rightarrow x = 0.4$

19 **Correct Answer:** (A) 5/16

Explanation:
C(5,3) × (1/2)^5 = 10 × (1/32) = 10/32 = 5/16

20 **Correct Answer:** (B) Independent

Explanation:
$P(E) \times P(F) = 0.6 \times 0.5 = 0.3 = P(E \cap F)$
⇒ Events are independent

21 Let B and b represent elder and younger boy child. Also, G and g represent elder and younger girl child. If a family has two children, then all possible cases are
S = {Bb, Bg, Gg, Gb}
∴ n(S) = 4
Let us define event A : Both children are girls, then A = {Gg} ⇒ n(A) = 1
(i) Let E_1 : The event that youngest child is a girl.

Then, $E_1 = \{Bg, Gg\}$ and $n(E_1) = 2$

so $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

and $A \cap E_1 = \{Gg\} \Rightarrow n(A \cap E_1) = 1$

so $P(A \cap E_1) = \frac{n(A \cap E_1)}{n(S)} = \frac{1}{4}$

Now, $P\left(\frac{A}{E_1}\right) = \frac{P(A \cap E_1)}{P(E_1)} = \frac{1/4}{1/2} = \frac{1}{2}$

$\therefore$ Required probability $= \frac{1}{2}$

(ii) Let E: The event that atleast one is girl.
Then, $E = \{Eg, Gg, Gb\} \Rightarrow n(E) = 3$,

$$P(A/E) = \frac{P(A \cap E)}{P(E)} = \frac{1}{3}$$

22 Given, $P(A') = 0.7$, $P(B) = 0.7$ and $P\left(\frac{B}{A}\right) = 0.5$

Clearly, $P(A) = 1 - P(A') = 1 - 0.7 = 0.3$

Now, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow \quad 0.5 = \frac{P(A \cap B)}{0.3}$

$\Rightarrow \quad P(A \cap B) = 0.15$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} \Rightarrow P\left(\frac{A}{B}\right) = \frac{3}{14}$

23 When a die is thrown, the sample space is
$S = \{1, 2, 3, 4, 5, 6\}$
$\Rightarrow n(S) = 6$
Also, A: number is even and B: number is red.
$\therefore A = \{2, 4, 6\}$ and $B = \{1, 2, 3\}$ and $A \cap B = \{2\}$
$\Rightarrow n(A) = 3$, $n(B) = 3$ and $n(A \cap B) = 1$

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$

Now, $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6} = P(A \cap B)$

$\therefore \quad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events.

24 Let us denote the numbers on black die by $B_1, B_2, \ldots\ldots, B_6$ and the numbers on red die by $R_1, R_2, \ldots\ldots, R_6$.
Then, we get the following sample space.
$s = \{(B_1, R_1), (B_1, R_2), \ldots\ldots, (B_1, R_6), (B_2, R_2), \ldots\ldots, (B_6, B_1), (B_6, B_2), \ldots\ldots, (B_6, R_6)$
Clearly, $n(S) = 36$
Now, let A be the event that sum of number obtained on the die is 8 and B be the event that red die shows a

number less than 4.

Then, A = {(B_2, R_6), (B_6,R_2), (B_3,R_5), (B_5,R_3), (B_4,R_4)}

and B = {(B_1, R_1), (B_1,R_2), (B_1,R_3), (B_2,R_1), (B_2,R_2), (B_2,R_3) ,…….., (B_6,R_1), (B_6,R_2), (B_6,R_3)}

⇒ A ∩ B = {(B_6, R_2), (B_5, R_3)}

Now, required probability,

$$P(A/B) = P(A\cap B)/P(B) = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$

25 We have, $2P(A) = P(B) = \frac{5}{13}$

$$\Rightarrow P(A) = \frac{5}{26} \text{ and } P(B) = \frac{5}{13} \text{ and } P(A/B) = \frac{2}{5}$$

$$\because P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$$

$$\therefore \quad \frac{2}{5} = \frac{P(A\cap B)}{5/13}$$

$$\Rightarrow \quad P(A\cap B) = \frac{2}{5}\times\frac{5}{13} = \frac{2}{13}$$

$$\because P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$= \frac{5+10-4}{26} = \frac{11}{26}$$

26 Given, E and F are independent events, therefore

⇒P(E ∩ F) = P(E) P(F) …….. (i)

Now, we have,

P(E ∩ F') + P(E ∩ F) = P(E)

P(E ∩ F') = P(E) – P(E ∩ F)

P(E ∩ F') = P(E) – P(E) P(F) [using Eq. (i))

P(E ∩ F') = P(E) [1 – P(F)]

P (E ∩ F') = P(E) P(F')

∴ E and F 'are also independent events.

Hence proved.

WORKSHEET

RELATION AND FUNCTION

SECTION –A(MCQ)		
Q. N.	**QUESTIONS**	
1	**Let f : [2, ∞) → R be the function defined by f(x) = $x^2 – 4x + 5$, then the range of f is** (a) R (b) [1, ∞) (c) [4, ∞) (d) [5, ∞)	
2	**Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is** (a) reflexive but not symmetric (b) reflexive but not transitive (c) symmetric and transitive (d) neither symmetric, nor transitive	
3	**Let A = {1, 2, 3}. Then the number of relations containing (1, 2) and (1, 3), which are reflexive and symmetric but not transitive is** **(a) 1 (b) 2 (c) 3 (d) 4**	
4	**Let f : R → R be defined by f(x) = x^2 + 1. Then, pre-images of 17 and – 3, respectively, are** (a) ϕ, {4, – 4} (b) {3, – 3}, ϕ (c) {4, –4}, ϕ (d) {4, – 4}, {2, – 2}	
5	**Set A has 3 elements, and set B has 4 elements. Then the number of injective mappings that can be defined from A to B is** (a) 144 (b) 12 (c) 24 (d) 64	
6	Let R be a relation in the set N given by R={(a,b): a+b=5, b>1}. Which of the following will satisfy the given relation? (a) (2,3) ∈ R (b) (4,2) ∈ R (c) (2,1) ∈ R (d) (5,0) ∈ R	
7	**The function f(x) =**x^2+ 4x +4 is-: (a) even (b) odd (c) neither even nor odd (d)none of these	
8	A function f:N→N is defined by f(x)= x^2+12. What is the type of function here? (a) bijective (b) surjective (c) injective (d) neither surjective nor injective	
SECTION –B(2/3 MARKS EACH)		

9	**Let R be a relation described on the set of natural numbers N. Determine the domain and range of the relation. Also check whether R is symmetric, reflexive and/or transitive.** **R = {x, y}: x ∈ N, y ∈ N, 2x+y = 41}**	
10	Check whether the relation R defined on the set A = {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.	
11	Let A = { 1 ,2 ,3} and define R = { (a , b) : a + b > 0 } . show that R is universal relation on set A .	
12	Let A = { a, b ,c } how many relation can be define in the set ? How many of these are reflexive ?	
13	Let A = { 2, 4 , 6 , 8} and R ={(a,b): a is greater than b,a,b∈ A} on the set A . Write R as a set of order pairs , is the relation reflexive ?	
14	Let A = { 2, 4 , 6 , 8} and R = ={(a,b): a is greater than b ; a,b∈ A }on the set A . Write R as a set of order pairs , is the relation Symmetric?	
15	Let A = { 1 ,2 ,3} and define R = { (a , b) : a – b = 10 } . show that R is empty relation on set A .	
	LONG ANSWER	
16	Let A = R- {3} and B = R – {1}. Consider the function f: A →B defined by f (x) = (x- 2)/(x -3). Is f one-one and onto? Justify your answer.	
17	Consider a function f: $R_+ \to [-5, \infty)$ given by f$(x) = 9x^2 + 6x - 5$, show that f is bijective function.	
18	**Show that the relation R defined by (a, b) R (c, d) => a + d = b + c on the set N XN is an equivalence relation.**	
	SECTION –E (4 MARKS EACH)	
19	**A school is organizing a debate competition with participants as speakers S {S_1 , S_2 , S_3 , S_4 } and these are judged by judges {J_1 , J_2 , J_3 } = . Each speaker can be assigned one judge. Let R be a**	**4**

relation from S to J defined as R {(x , y) : speaker x is judged by judge y; $X \in S$, $y \in J$}. Based on the above, answer the following.

(i) How many relations can be there from S to J?

(ii) A student identifies a function from S to J as f ={(S_1 , J_1), (S_2 , J_2), (S_3 , J_3), (S_4 , J_4)} . Check if it is bijective.

(iii) (a) How many one-one functions can be there from set S to set J?

(iv) OR

(b) Another student considers a relation R_1 ={(S_1 , S_2), (S_2 , S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

ANSWERS
MCQ

1.(b) $[1, \infty)$ 2. (a) reflexive but not symmetric 3. **(a) 1**

4. (c) $\{4, -4\}, \varphi$ **5** (c) 24 **6**(a) $(2,3) \in R$

7.(c) neither even nor odd **8.** (c) injective

Case study

(i) Here n(S)4, n(J) =3 so, n(S XJ) 4 X3 =12 . Therefore, total number of relations from S to J are 2^{12} =4096 .

(ii) Note that f (S_2) =J_2 =f (S_3) . That is, S_2 and S_3 are both mapped to J_2 . Hence, f is not one-one. Also, every element of J have at least one pre-image in S. Hence, f is onto.

(iii) Since f is onto but not one-one, so f is not bijective.

(iv) (iii) (a) As n(S) =4, n(J) =3 = = i.e., n(S) > n(J) .

So, number of one-one functions from set S to set J is 0 (zero).

OR

	(iii) (b) For reflexivity, we must add the ordered pairs : (1 ,1),(2, 2),(3, 3),(4, 4), (S_1 , S_1), (S_2 , S_2), (S_3 , S_3), (S_4 , S_4) . Since (S_1 , S_2) $\in$R and (S_2 , S_4) $\in R_1$. So, we must not add the ordered pairs (S_2 , S_1) and (S_4 , S_2) in R_1 , otherwise it will become symmetric. Therefore, after adding minimum number of ordered pairs i.e., (S_1 , S_1), (S_2 , S_2), (S_3 , S_3), (S_4 , S_4) in R_1 so that it becomes reflexive but not symmetric, the new relation R_1 becomes R {(S_1 , S_2), (S_2 , S_4), (S_1 , S_1), (S_2 , S_2), (S_3 , S_3), (S_4, S_4)}	
	ASSERSSION REASONING **In the following question a statement of Assertion (A) is followed by a statement of reason (R). Pick the correct option: A. Both A and R are true and R is the correct explanation of A. B. Both A and R are true but R is not the correct explanation of A. C. A is true but R is false. D. A is false but R is true**	
1.	**Assertion (A): If n (A) =p and n (B) = q then the number of relations from A to B is 2pq Reason (R) : A relation from A to B is a subset of A x B**	
2	**Assertion (A): The relation R in the set A = {1, 2, 3, 4, 5, 6} defined as R={ (x, y) :y is divisible by x} is not an equivalence relation. Reason (R) :The relation R will be an equivalence relation, if it is reflexive, symmetric and**	

3	**Assertion (A) : A relation R ={ (1,1),(1,2),(2,2),(2,3)(3,3)}defined on the set A={1,2,3} is reflexive. Reason (R) : A relation R on the set A is reflexive if (a,a) ,for all a**	
4	**Assertion (A): If R is the relation in the set A= {1, 2, 3 , 4, 5 } given by R={(a, b): |a - b| is even} R is an equivalence relation. Reason (R) : All elements of {1, 3, 5} are related to all elements of $\{2,4\}$**	
	1.Answer: A Solution: A is true - No of elements of AXB = pxq, So the number of relations from A to B is 2pq R is true – every relation from A to B is a sub set of AXB 2.Answer : A Solution: A is true-R is reflexive and transitive but not symmetric ie (2,4)ϵR (4,2)ϵR R-true- Definition of an equivalence relation. 3.Answer: A Solution: A is true - (a,a) ,for all a A R is true – Correct explanation for reflexive relation. 4.Answer: C Solution: A is true- Since it an equivalence relation R is false – the modulus of difference between the two elements from each of these two subsets will not be even	

INVERSE TRIGONOMETRIC FUNCTIONS

WORKSHEET

……

:

SECTION A (MCQ)

Que 1.	The value of $\sin^{-1}\left(\cos\frac{\pi}{9}\right)$ is			
	(a) $\frac{\pi}{9}$	(b) $\frac{5\pi}{9}$	(c) $-\frac{5\pi}{9}$	(d) $\frac{7\pi}{18}$
Que 2.	If $\sec^{-1}x + \sec^{-1}y = 2\pi$, the value of $\text{cosec}^{-1}x + \text{cosec}^{-1}y$ *is*			
	(a) π	(b) 2π	(c) 32π	(d) $-\pi$
Que 3.	The domain of the function $\cos^{-1}(2x-1)$ *is*			
	(a) [0, 1]	(b) (–1, 1)	(c) [–1, 1]	(d) [0, π]
Que 4.	One branch of $\cos^{-1}x$,other than the principal value branch corresponds to			
	(a) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$	(b) $[\pi, 2\pi]-\left[\frac{3\pi}{2}\right]$	(c) $[2\pi, 3\pi]$	(d) $(0, \pi)$
Que 5.	The value of $\sin^{-1}\left[\cos\frac{43\pi}{5}\right]$ is			
	(a) $\frac{3\pi}{5}$	(b) $-\frac{7\pi}{5}$	(c) $\frac{\pi}{10}$	(d) $-\frac{\pi}{10}$
Que 6.	If $\sin^{-1}x = y$, then			
	(a) $0 \le y \le \pi$	(b) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	(c) $0 < y < \pi$	(d) $-\frac{\pi}{2} < x < \frac{\pi}{2}$
Que 7.	$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to			
	(a) π	(b) $-\frac{\pi}{3}$	(c) $\frac{\pi}{3}$	(d) $\frac{2\pi}{3}$
Que 8	The domain of $\sin^{-1}2x$ is			
	(a) [-1, 1]	(b) (-1, 1)	(c) $\left[-\frac{1}{2},\frac{1}{2}\right]$	(d) $\left(-\frac{1}{2},\frac{1}{2}\right)$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R).
Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Que 9.	Assertion (A): $\sin^{-1}\left(sin\frac{5\pi}{3}\right) = \frac{5\pi}{3}$ Reason (R): Inverse trigonometric functions are many-one.
Que 10.	Assertion (A): All trigonometric functions have their inverses over their respective domains. Reason (R): The inverse of $\tan^{-1} x$ exists for some x ∈ R

SECTION-B (2 MARKS)

Que 11.	Find the values of $\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$
Que 12.	If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ then find the value of $\cot^{-1}x + \cot^{-1} y$
Que 13.	Find the value of $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin(-\frac{\pi}{2})\right)$

SECTION C (3Marks)

Que 14.	Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$ where $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$
Que 15.	Write the function in the simplest form : $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+cosx}}\right)$ where $0 < x < \pi$

SECTION-D (5 Marks)

Que 16.	Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$
Que 17.	Find the values of $\tan\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right)$, where $\|x\| < 1$, $y > 0$ and $xy < 1$

SECTION –E (COMPETENCY BASED QUESTIONS)

Que 18.	Principal Value of Inverse Trigonometric Functions". Teacher told that the value of an inverse trigonometric functions which lies in the range of principal branch is called the principal value of that inverse. Based on the given information, answer the following questions (i) Find the Principal value of : $\tan^{-1}[\sin\frac{\pi}{2}]$ (ii) The domain of the function $\cos^{-1}(x)$ *is*

	(iii) Find the value of $cos[tan^{-1}\left(\frac{3}{4}\right)]$ Or Find the principal value of **sin** $\left\{\frac{\pi}{6} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$
Que 19.	A satellite communication system uses inverse trigonometric functions to calculate signal angles. The ground station needs to determine the elevation angle θ of the satellite above the horizon. If the satellite is at height h = 35,786 km above Earth's surface and the horizontal distance from the ground station is d km, then the elevation angle is given by: $\theta = \tan^{-1}\left(\frac{h}{d}\right)$ The engineers also need to work with the relationship: $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$ (i) State the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ (ii) If the horizontal distance d = 35,786 km, find the elevation angle θ. (iii) The satellite system needs to calculate the phase difference between two signals. If the phase angles are given by $\alpha = \sin^{-1}\left(\frac{3}{5}\right)$ and $\beta = \cos^{-1}\left(\frac{3}{5}\right)$, find the value of α + β. Or During signal transmission, the engineers encounter the equation: $2sin^{-1}(x) = cos^{-1}(2x^2 - 1)$. Find all possible values of x that satisfy this equation

MATRICES Work Sheet

Max Marks: 20 **Time: 40 Min**

1. If A is a 2×3 matrix such that AB and AB' both are defined, then find the order of the matrix B. **1**
2. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew symmetric matrix, find the values a, b and c. **1**
3. Prove that AA' is always a symmetric matrix for any square matrix of A. **1**
4. If A and B are square matrices, each of order 2 such that |A|=3 and |B|= - 2, then write the value of |3AB|. **1**
5. If A is a square matrix of order 3 such that |adj A| = 225, find |A′|. **1**
6. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then find the possible value(s) of x. **1**
7. Find the equation of the line joining A(1,3) and B(0,0) using determinants and find k if D(k,0) is a point such that area of triangle ABD is 3 sq units. **2**
8. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$ **2**
9. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find BA and use it to find the values of x, y, z from given equations: **5**
   $x - y = 3, \quad 2x + 3y + 4z = 17, \quad y + 2z = 17$
10. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that: $f(x)f(-y) = f(x-y)$ **5**

Work Sheet –I

1. 3×3 2. (a = - 2, b = 0, c = -3) 4. – 4, 5. ±15, 6. ±6, 7. 3x – y = 0; k = ±2, 8. $[-1 \quad 2 \quad 1]$, 9. BA = 6 I; (x = - 14/3, y = - 23/3, z = 37/3).

Work Sheet -II

1	If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then x will be: (a) 3 (b) -3 (c) ±3 (d) any number
2	If x, y ∈ R, then the determinant $\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval (A) $[-\sqrt{2}, \sqrt{2}]$ (b) $[-1, 1]$ (c) $[-\sqrt{2}, 1]$ (d) $[-1, -\sqrt{2}]$
3	If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$ = kabc, then the value of k is: (a) 0 (b) 1 (c) 2 (d)4
4	If A= $\begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$ then the value of $\lvert A\,(adj\,A)\rvert$ is: (a) 100 I (b) 10 I (c) 10 (d)1000
5	If the area of a triangle with vertices (2,-6), (5,4) and (k,4) is 35 sq units, then k is (a) 12 (b) -2 (c) -12,-2 (d) 12, -2
6	Assertion(A) : For any square matrix B with real number entries , B +B^T is skew symmetric matrix and B – B^T is symmetric matrix . Reason(R) : A square matrix B can be expressed as the sum of symmetric and skew symmetric matrix

7	If A is a invertible matrix, show that for any scalar k≠0, $(Ka)^{-1} = \frac{1}{k} A^{-1}$. hence calculate $(3A)^{-1}$, where A= $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
8	If A= $\begin{bmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations: 2x + y – 3z= 13 3x + 2y +z =4 X + 2y -z =8
9	Use the product of matrices $\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$ to solve the following equations: x + 2y -3z =6 3x + 2y -2z = 3 2x – y +z =2
10	The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.
11	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Using A^{-1} solve the system of equations 2x – 3y + 5z = 11 , 3x + 2y – 4z = – 5 ,x + y – 2z = –3.
12	Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$,Find BA and use of this to solve the system of equations: $y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17$
13	Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$
14	Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = -9, -x + 2y - 2z = 4, 2x - 3y + 4z = -3$
15	Three students Ram, Mohan and Ankit go to a shop to buy stationary. Ram purchases 2 dozen note books, 1 dozen pens and 4 pencils. Mohan purchases 1 dozen note book , 6 pens and 8 pencils. Ankit purchases 6 note books , 4 pens and 6 pencils. A note book costs ₹ 15, a pen costs ₹4.50 and a pencil costs ₹ 1.50. Let A and B be the matrices representing the number of items purchased by the three students and the prices of items respectively. Based on the above information answer the following questions. (i) What is the order of the matrix B representing the prices of the items (ii) What is the order of the matrix A representing items purchased by the three students (iii) What is the order of the matrix AB. OR Find the total amount of bill by all the three students.

CONTINUITY AND DIFFERENTIABILITY

(WORKSHEET)

MCQs (1 Mark)

Q.1 The function $f(x) = \frac{4-x^2}{4x-x^3}$ is

(a) discontinuous at only one point

(b) discontinuous at exactly two points

(c) discontinuous exactly three points

(d) None of the above

Q.2 The set of points where the function f given by $f(x) = |2x-1|\, sinx$ is differentiable is

(a) R

(b) $R - \left(\frac{1}{2}\right)$

(c) $(0, \infty)$

(d) None of the above

Q.3 If $y = log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{4x^3}{1-x^4}$

(b) $\frac{-4x}{1-x^4}$

(c) $\frac{1}{4-x^4}$

(d) $\frac{-4x^3}{1-x^4}$

Q.4 The set of all points where the function $f(x) = x + |x|$ is differentiable, is

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

Q.5 The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function less than or equal to x is continuous at

(a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$

Q.6 If the function $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k, & x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is

(a) 0 (b) 1 (c) -1 (d) 2

Q.7 The value of b for which the function $f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^2 + 3bx, & 1 < x < 2 \end{cases}$ is continuous at every point of its domain is

(a) -1 (b) 0 (c) $\frac{13}{3}$ (d) 1

Q.8 $\lim_{x \to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$ is equal to

(a) 1 (b) -1 (c) 0 (d) None of these

Q.9 If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is:

(a) -1 (b) 1 (c) $-e$ (d) $-\frac{1}{e}$

Q.10 **Assertion (A):** Let $y = t^{10} + 1$ and $x = t^8 + 1$, then $\frac{d^2y}{dx^2} = 20t^8$

Reason (R): $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

SA-I (2 Marks)

Q.11 Examine the continuity of the function $f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & if\ x \neq 4 \\ 0, & if\ x = 4 \end{cases}$

Q.12 Check the differentiability of the function $f(x) = |x - 5|$, at the point $x = 5$.

Q.13 Find the derivative of the function: $log(\sec x + \tan x)$.

Q.14 Differentiate $.r.t.x$: $tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$, $x \neq 0$

Q.15 If $y = sin^{-1}x + sin^{-1}\sqrt{1-x^2}$, $x \in (0,\ 1)$, find $\frac{dy}{dx}$.

Q.16 If $y = x^{cos^{-1}x}$, find $\frac{dy}{dx}$.

SA-II (3 Marks)

Q.17 Differentiate $w.r.t.x$: $e^{cos^{-1}\sqrt{1-x^2}}$

Q.18 If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

Q.19 If $cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = tan^{-1}a$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Q.20 If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

LSA (5 Marks)

Q.21 If $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$, find $\frac{dy}{dx}$.

Q.22 If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

Q.23 Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $w.r.t. tan^{-1}x$.

Q.24 If $y = A\cos(\log x) + B\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

Q.25 If $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

Answer Keys

MCQs (1 Mark)

Q.1 (c) discontinuous exactly three points

We have $f(x) = \frac{4-x^2}{4x-x^3} = \frac{4-x^2}{x(2+x)(2-x)}$,

Clearly, $f(x)$ is discontinuous at exactly three points $x = 0, x = 2, x = -2$.

Q.2 (b) $R - \left(\frac{1}{2}\right)$

$Lf'(x) \neq Rf'(x)$ at $x = \frac{1}{2}$, $f(x)$ is not differentiable. Hence, $f(x)$ is differentiable in $R - \left\{\frac{1}{2}\right\}$

Q.3 (b) $\frac{-4x}{1-x^4}$

$y = \log\left(\frac{1-x^2}{1+x^2}\right) = \log(1-x^2) - \log(1+x^2)$

$\frac{dy}{dx} = \frac{-2x}{1-x^2} - \frac{2x}{1+x^2} = \frac{-4x}{1-x^4}$

Q.4 (c) $(-\infty, 0) \cup (0, \infty)$

$f(x) = \begin{cases} 0, & x < 0 \\ 2x, & x > 0 \end{cases}$

$Lf'(x) \neq Rf'(x)$ at $x = 0$, f is differentiable at $x \in R$ except 0.

Q.5 (b) $x = 1.5$

The function is continuous at all real numbers not equal to integers.

Q.6 (d) 2

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2-1}{x-1} = f(1)$$

$$\lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = k$$

$$\lim_{x\to 1} x+1 = k$$

$$1+1 = k$$

$$k = 2$$

Q.7 (a) -1

$$\lim_{x\to 1^-} (5x-4) = f(1) = 1, \ \lim_{x\to 1^+} 4x^2 + 3bx = 4 + 3b$$

$$4 + 3b = 1$$

$$b = -1$$

Q.8 (a) 1

$$\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x} = \lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(2sin^2x)}}{x} = \lim_{x\to 0} \frac{\sin x}{x} = 1$$

Q.9 (a) -1

Diff. $w.r.t.x$

$$e^y + xe^y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -1$$

Or

$$y = -\log x$$

$$\frac{dy}{dx} = -\frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -1$$

Q.10 **(**d) Assertion (A) is false but Reason (R) is true

$$\frac{dy}{dt} = 10t^9, \frac{dx}{dt} = 8t^7$$

$$\frac{dy}{dx} = \frac{5}{4}t^2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = \frac{d}{dt}\left(\frac{5}{4}t^2\right)\frac{dt}{dx} = \frac{10t}{4}\cdot\frac{1}{8t^7} = \frac{5}{16t^6}$$

SA-I (2 Marks)

Q.11 $f(x) = \begin{cases} \frac{-(x-4)}{2(x-4)} = -\frac{1}{2}, \ if\ x < 4 \\ = \frac{1}{2}, \ if\ x > 4 \\ 0, \ if\ x = 4 \end{cases}$

$f(4) = 0$

LHL= $\lim_{x \to 4^-} f(x) = -\frac{1}{2}$ and RHL = $\lim_{x \to 4^+} f(x) = \frac{1}{2}$

Clearly, $LHL \neq RHL$

$f(x)$ is not continuous at $x = 4$

Q.12 $f(x) = \begin{cases} 5 - x, \ if\ x < 5 \\ x - 5, \ if\ x \geq 5 \end{cases}$

$Lf'(x) = \lim_{h \to 0} \frac{f(5-h)-f(5)}{-h} = \lim_{h \to 0} \frac{5-(5-h)-0}{h} = -1$

$Rf'(x) = \lim_{h \to 0} \frac{f(5+h)-f(5)}{h} = \lim_{h \to 0} \frac{5+h-5-0}{h} = 1$

Clearly, $Lf'(x) \neq Rf'(x)$, $f(x)$ is not differentiable at $x = 5$.

Q.13 Let $u = \sec x + \tan x$, then $y = logu$

$\frac{du}{dx} = \sec x \tan x + sec^2 x$, $\quad \frac{dy}{du} = \frac{1}{u} = \frac{1}{(\sec x + \tan x)}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{(\sec x + \tan x)} (\sec x \tan x + sec^2 x) = \frac{secx\ (\sec x + \tan x)}{(\sec x + \tan x)} = \sec x$

Q.14 Let $x = \tan\theta$

$tan^{-1}\left\{\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sqrt{sec^2\theta}-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$

$$= tan^{-1}\left\{\frac{2sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\} = tan^{-1}\left\{\tan\frac{\theta}{2}\right\} = \frac{\theta}{2} = \frac{1}{2}tan^{-1}x$$

We have $y = \frac{1}{2}tan^{-1}x$

$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1+x^2}$

Q.15 Let $x = \sin\theta$, we have $y = sin^{-1}(\sin\theta) + sin^{-1}(\cos\theta)$

Since, $0 < \sin\theta < 1 \Rightarrow 0 < \theta < \frac{\pi}{2} \Rightarrow 0 < \frac{\pi}{2} - \theta < \frac{\pi}{2}$

Therefore, $y = sin^{-1}(\sin\theta) + sin^{-1}\left\{\sin\left(\frac{\pi}{2} - \theta\right)\right\}$

$$y = \theta + \frac{\pi}{2} - \theta = \frac{\pi}{2}$$

Q.16 Taking log both sides

$$\log y = cos^{-1}x \log x$$

Using product rule

$$\frac{1}{y}\frac{dy}{dx} = \frac{cos^{-1}x}{x} + \frac{-\log x}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = x^{cos^{-1}x}\left(\frac{cos^{-1}x}{x} + \frac{-\log x}{\sqrt{1-x^2}}\right)$$

SA-II (3 Marks)

Q.17 $y = e^{cos^{-1}\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left\{e^{cos^{-1}\sqrt{1-x^2}}\right\} = e^{cos^{-1}\sqrt{1-x^2}}\frac{d}{dx}\left\{cos^{-1}\sqrt{1-x^2}\right\}$$

$$= e^{cos^{-1}\sqrt{1-x^2}} \cdot \frac{-1}{\sqrt{1-(1-x^2)}}\frac{d}{dx}\left\{\sqrt{1-x^2}\right\}$$

$$= e^{cos^{-1}\sqrt{1-x^2}} \cdot \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}(1-x^2)$$

$$= e^{cos^{-1}\sqrt{1-x^2}} \cdot \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$= e^{cos^{-1}\sqrt{1-x^2}}\frac{1}{\sqrt{1-x^2}}$$

Q.18 $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 = xy^2 - x^2y$

$\Rightarrow (x-y)(x+y) = -xy(x-y)$

$\Rightarrow x + y = -xy$

$\Rightarrow y + xy = -x$

$\Rightarrow y(1+x) = -x$

$\Rightarrow y = \frac{-x}{1+x}$

Now, $\frac{dy}{dx} = -\left\{\frac{(1+x)\cdot 1 - x(0+1)}{(1+x)^2}\right\} = -\frac{1}{(1+x)^2}$

Q.19 $cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = tan^{-1}a$

$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \cos(tan^{-1}a) = k, where\ k\ is\ a\ constant$

$\Rightarrow \frac{2x^2}{-2y^2} = \frac{k+1}{k-1}$

$\Rightarrow \frac{x^2}{y^2} = \frac{1+k}{1-k} = m,$ (another constant)

Diff $w.r.t.x,$ $\frac{y^2\frac{d}{dx}(x^2)-x^2\frac{d}{dx}(y^2)}{(y^2)^2} = 0$

$\Rightarrow y^2(2x) - x^2(2y)\frac{dy}{dx} = 0$

$\Rightarrow 2x^2y\frac{dy}{dx} = 2xy^2$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

Q.20 Taking log both sides

$y\log x = x - y$

$\Rightarrow y\log x + y = x$

$\Rightarrow y(1+\log x) = x$

$\Rightarrow y = \frac{x}{1+\log x}$

Diff. $w.r.t.x$

$$\frac{dy}{dx} = \frac{(1+\log x)\frac{d}{dx}(x)-x\frac{d}{dx}(1+\log x)}{(1+\log x)^2} = \frac{(1+\log x)\cdot 1-x\left(0+\frac{1}{x}\right)}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

LSA (5 Marks)

Q.21 Let $u = (\sin x)^{\tan x}$ and $v = (\cos x)^{\sec x}$

$\log u = \tan x \log(\sin x)$ and $\log v = \sec x \log(\cos x)$

$\frac{1}{u}\frac{du}{dx} = \tan x\frac{d}{dx}\{\log(\sin x)\} + \log(\sin x)\frac{d}{dx}(\tan x) = \tan x\ \frac{\cos x}{\sin x} + \log(\sin x)\ sec^2 x$

$\frac{du}{dx} = (\sin x)^{\tan x}\{1 + \log(\sin x)\ sec^2 x\}$

and $\frac{1}{v}\frac{dv}{dx} = \sec x\frac{d}{dx}\{\log(\cos x)\} + \log(\cos x)\frac{d}{dx}(\sec x) = \sec x\ \frac{-\sin x}{\cos x} + \sec x \tan x \log(\cos x)$

$\frac{dv}{dx} = (\cos x)^{\sec x}\{\sec x \tan x \log(\cos x) - \sec x \tan x\}$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= (\sin x)^{\tan x}\{1 + \log(\sin x)\ sec^2 x\} + (\cos x)^{\sec x}\{\sec x \tan x \log(\cos x) - \sec x \tan x\}$

Q.22 $\log x = \cos 2t$ and $\log y = \sin 2t$

$\frac{1}{x}\frac{dx}{dt} = -2\sin 2t$ and $\frac{1}{y}\frac{dy}{dt} = 2\cos 2t$

$\frac{dy}{dt} = 2y\cos 2t$ and $\frac{dx}{dt} = -2x\sin 2t$

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2y\cos 2t}{-2x\sin 2t} = -\frac{y\log x}{x\log y}$

Q.23 Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ $w.r.t. tan^{-1}x.$

Let $x = \tan\theta$

$$tan^{-1}\left\{\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sqrt{sec^2\theta}-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{\sec\theta-1}{\tan\theta}\right\} = tan^{-1}\left\{\frac{1-\cos\theta}{\sin\theta}\right\}$$

$$= tan^{-1}\left\{\frac{2sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right\} = tan^{-1}\left\{\tan\frac{\theta}{2}\right\} = \frac{\theta}{2} = \frac{1}{2}tan^{-1}x$$

We have $y = \frac{1}{2}tan^{-1}x \Rightarrow \frac{dy}{dx} = \frac{1}{2(1+x^2)}$

Let $t = tan^{-1}x \Rightarrow \frac{dt}{dx} = \frac{1}{1+x^2}$

$\frac{dy}{dt} = \frac{dy/dx}{dt/dx} = \frac{1/2(1+x^2)}{1/(1+x^2)} = \frac{1}{2}$

Q.24 $\frac{dy}{dx} = \frac{-A\sin(\log x)}{x} + \frac{B\cos(\log x)}{x} \Rightarrow x\frac{dy}{dx} = -A\sin(\log x) + B\cos(\log x)$

$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 1 = -\frac{A\cos(\log x)}{x} - \frac{B\sin(\log x)}{x}$

$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\{A\cos(\log x) + B\sin(\log x)\} = -y$

$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Q.25 If $y = \frac{sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$

$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\frac{d}{dx}(sin^{-1}x) - sin^{-1}x\frac{d}{dx}\sqrt{1-x^2}}{1-x^2}$

$(1-x^2)\frac{dy}{dx} = \sqrt{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}} - \frac{sin^{-1}x\,(-2x)}{2\sqrt{1-x^2}} = 1 + \frac{x\,sin^{-1}x}{\sqrt{1-x^2}} = 1 + xy$

$\Rightarrow (1-x^2)\frac{dy}{dx} = 1 + xy$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = x\frac{dy}{dx} + y$

$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$

Worksheet: Application of Derivatives

Concepts Covered

- 1. Derivative as Rate of Change
- 2. Increasing and Decreasing Functions
- 3. Tangent and Normal to a Curve
- 4. Maxima and Minima (Local/Global)
- 5. Simple Word Problems using Derivatives

Part A: Multiple Choice Questions (MCQs)

Q1. The slope of the tangent to the curve $y = x^3 - 3x + 2$ at $x = 1$ is:

A) 0 B) 1 C) -2 D) 3

Q2. For the function $f(x) = x^3 - 6x^2 + 9x + 15$, the function is increasing in the interval:

A) $(-\infty, 0)$ B) $(0, 2)$ C) $(2, \infty)$ D) $(0, 1)$

Q3. If the normal to the curve $y = x^2$ at a point (a, a^2) passes through the origin, then a is:

A) 0 B) 1 C) -1 D) ±1

Part B: Short Answer Type Questions

Q4. Find the equation of the tangent and normal to the curve $y = \sqrt{(3x - 2)}$ at the point where $x = 3$.

Q5. Find the point on the curve $y = x^2 + 7x + 10$ at which the tangent is horizontal.

Q6. Show that the function $f(x) = 3x^4 - 4x^3 + 6$ is increasing in $(-\infty, 0) \cup (1, \infty)$.

Q7. A spherical balloon is being inflated so that its volume increases at a rate of 100 cm^3/sec. Find the rate of increase of its radius when radius is 5 cm.

Part C: Long Answer Type Questions

Q8. Find the maximum and minimum values of the function $f(x) = x^3 - 6x^2 + 9x + 1$ on the interval $[0, 4]$.

Q9. Find two positive numbers whose sum is 60 and whose product is maximum.

Q10. A closed cylindrical tank of volume 256π m^3 is to be made. Find the dimensions (radius and height) of the tank such that the surface area is minimum.

Q11. Find the point on the curve $y = \sqrt{x}$ which is closest to the point $(3, 0)$.

Part D: Previous Year-Based and Model Questions (CBSE 2023–2025 Style)

Q12. (CBSE 2024) The function $f(x) = x^4 - 4x^3 + 10$ has local minima at:

Find critical points and classify them using the second derivative test.

Q13. (CBSE 2023) If the slope of the tangent to the curve $y = ax^2 + bx + c$ at the point (1, 2) is 5, find the value of a and b given $a + b + c = 2$.

Q14. (CBSE 2024) A window is in the shape of a rectangle surmounted by a semicircular opening. The perimeter of the window is 10 m. Find the dimensions for which the area is maximum.

Q15. (Model 2025) Show that the function $f(x) = x/(x + 1)$ is increasing on $(-1, \infty)$.

Q16. The volume of a cube is increasing at the rate of 9 cm^3/sec. How fast is the surface area increasing when the edge is 3 cm?

Part E: Skill-Based Questions (Challenging)

Q17. Find the interval(s) in which the function $f(x) = x/(x^2 + 1)$ is increasing or decreasing.

Q18. Find the minimum distance between the point (0, 0) and the curve $y = x^2 + 1$.

Q19. Find two positive numbers whose product is 256 and whose sum is minimum.

Q20. A cone is being formed by folding a sector of a circle. Show that the cone of maximum volume is obtained when the radius of the sector is three times the slant height of the cone.

WORK SHEET INTEGRAL

INDEFINITE INTEGRAL

1.Given $\int 2^x dx$= f(x)+c then f(x)=

(a) 2^x (b) 2^x $\log e^2$ (c) $\frac{2^x}{\log 2}$ (d) $\frac{2^{x+1}}{x+1}$

2. Given $\int \frac{1}{\sin^2 x\cos^2 x}$ dx is equal to

(a) $\sin^2x - \cos^2x + c$ (b) -1

(b) (c) tan x +cot x+ c (d) tan x- cot x +c

3. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}$ dx is equal to

(a) 2(sin x+ x cos θ) +c (b) 2(sin x- x cos θ) + c

(c) 2(sin x +2x cos θ) +c (d) 2 (sin x- sin θ) +c

4. $\int \cot^2 x$ dx equals to

(a) Cot x – x +c (b) – cot x+ x +c

(c) cot x +x +c (d) – cot x-x +c

SHORT ANSWER TYPE QUESTIONS

1. Find $\int \frac{3+3\cos x}{x+\sin x}$ dx
2. Find $\int \frac{dx}{\sqrt{5-4x-x^2}}$ dx
3. Find $\int \frac{x^3-1}{x^2}$ dx
4. Find $\int \frac{\sin^2x-\cos^2x}{\sin x \cos x}$ dx
5. Find $\int \frac{dx}{x^2+16}$ dx
6. Find $\int \frac{\cos 2x+2\sin^2x}{\cos^2x}$ dx
7. Find $\int \frac{\sec^2x}{\sqrt{\tan^2x+4}}$ dx
8. Find $\int \sqrt{1-\sin 2x}$ dx
9. Find $\int \frac{(x^2+\sin^2x)\sec^2x}{1+x^2}$ dx
10. Find $\int e^x \frac{x-3}{(x-1)^3}$ dx
11. Find $\int \sin^{-1}(2x)dx$
12. Find $\int \frac{3-5\sin x}{\cos^2x}$ dx
13. Find $\int \frac{\tan^2x.\sec^2x}{1-\tan^6x}$ dx
14. Find $\int \sin x \log(\cos x)$ dx
15. Find $\int \frac{\sin^6x+\cos^6x}{\sin^2x.\cos^2x}$ dx

LONG ANSWER TYPE QUESTIONS

1. Find $\int \frac{6x+8}{3x^2+6x+2}$ dx
2. Find $\int \frac{1-\tan^2x}{1+\tan^2x}$ dx

3. Find $\int \frac{x^4}{1+x^{10}}\,dx$

4. Find $\int \frac{x^2\tan^{-1}x^3}{1+x^6}\,dx$

5. Find $\int \frac{\sin 8x}{\sqrt{1-\cos^4 4x}}\,dx$

DEFINITE INTEGARL

MCQ's

Q.No	Question	Mark
1	$\int_{-\pi/4}^{\pi/4} \text{Sec}^2 x dx$ (a) -1 (b) 0 (c) 1 (d) 2	1
2	$\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} dx$ is (a) 6 (b)0 (c) 1 (d) 4	1
3	$\int_{0}^{2/3} \frac{dx}{4+9x^2}$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c)$\pi/24$ (d)$\pi/4$	1
4	$\int_{0}^{1} \frac{dx}{1+x^2}$ is (a) 0 (b) $\pi/4$ (c) $\pi/12$ (d) $\pi/6$	1
5	$\int_{-1}^{1} x^{17} + x^{71} dx$ is (a) 1 (b)0 (c)2 (d) 4	1

Problems for Practice

All the questions carry 3 marks

1 Evaluate $\int_0^1 \frac{\sin x}{1+\sin x} dx$

2 Evaluate $\int_0^1 \cot^{-1}(1-x-x^2)dx$

3 Evaluate $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x})\, dx$

4 Evaluate $\int_{-1}^{3/2} |x\sin\pi x| dx$

5 Evaluate $\int_0^1 \frac{x dx}{1+x^2}$

6 Evaluate $\int_1^3 |2x-1|\, dx$

7 Evaluate $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\, dx$

8 Evaluate $\int_{-2}^{2} \frac{x^2 dx}{1+5^x}$

9 Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

10 Evaluate $\int_0^{\pi/2} (2\log\cos x - \log\sin 2x) dx$

All the questions carry 5 marks

1 Evaluate $\int_{-1}^{2} |x^3 - x| dx$

2 Evaluate $\int_{-6}^{6} |x+3|\, dx$

3 Evaluate $\int_0^{\pi/2} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x} dx$

4 Evaluate $\int_0^{\pi}\frac{xdx}{a^2\cos^2x+b^2\sin^2x}dx$

5 Evaluate $\int_0^{\pi}\frac{x\tan x}{\tan x+\sec x}dx$

MCQ

1 $\int_0^2(x^2+3)dx$ is

(a)8 (b) 25/3 (c)26/3 (d) 9

2 $\int_0^{\pi}\sin^2 xdx$ is (a)$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d)π

3 $\int_0^{\pi}\frac{dx}{1+\sin x}$ is (a)$\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c)$e^{\pi/2}$ (d) 2

4 $\int_0^1\frac{1-x}{1+x}dx$

(a) $\frac{\log 2}{2}$ (b) $\frac{\log 2}{2}$-1 (c) 2log2-1 (d) 2log2 +1

5 $\int_0^{\pi/6}\cos x\cos 2x\, dx$

(a) ¼ (b) 5/12 (c) 1/3 (d)-1/12

6 $\int_0^1\frac{dx}{e^x+e^{-x}}$

(a) $1-\pi/4$ (b) $\tan^{-1}e$

(c)) $\tan^{-1}e+\pi/4$ (d) $\tan^{-1}e-\pi/4$

7 $\int_0^1\sqrt{\frac{1-x}{1+x}}dx$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2}-1$ (c)$\pi/2+1$ (d)0

8 $\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

(a) 2 (b) 3/4 (c) 0 (d) -2

9 $\int_0^1\tan^{-1}\frac{2x-1}{1+x-x^2}dx$

(a) 1 (b) 0 (c) -1 (d)$\pi/4$

10 $\int_0^{\pi/2}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$ is

(a)$\frac{\pi}{2}$ (b) $\pi/3$ (c) $\pi/4$ (d) π

11 $\int_0^{\pi/2}\frac{dx}{1+\tan x}=$

(a) $\frac{\pi}{2}$ (b) $\pi/3$ (c) $\pi/4$ (d) π

12 $\int_{-1}^{1}\sin^3x\cos^2x\,dx$

(a) 0 (b)1 (c)2 (d)3

ASSERTION AND REASONING BASED PROBLEMS

In the following questions a statement of assertion Statement 1 is followed by statement of reason Statement 2 Mark the correct choice as

(a) If statement 1 and statement 2 is true and statement 2 is the correct explanation of 1

(b) If statement 1 and statement 2 is true and statement 2 is not the correct explanation of 1

(c) If statement 1 is true and statement 2 is false

(d) If statement 1 is false and statement 2 is true

Now answer the following

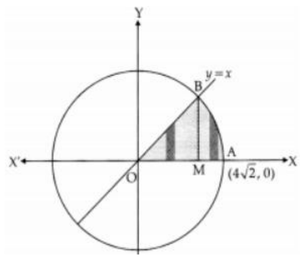
1 Statement I $\int_0^{\pi/2} \sin^2 x dx = \pi/4$

Statement II $\int_0^a f(x)dx = \int_0^a f(a-x)\, dx$

2Statement I $\int_2^3 \frac{\sqrt{x}dx}{\sqrt{x}+\sqrt{5-x}}$ =1/2

Statement II $\int_0^{2a} f(x)dx=2\int_0^a f(x)dx$ if f(x)=f(2a-x)

WORK SHEET APPLICATION OF INTEGRAL

1.	Find the area enclosed by curve $4x^2 + 9y^2 = 36$ (b) 6л sq units (b) 4лsq units (c) 9л sq units (d) 36л sq units
2.	The area enclosed between the graph of $y = x^3$ and the lines $x = 0, y = 1, y = 8$ is (a) 7 (b) 14 (c) 45/4 (d)None of these
3.	The area of the region bounded by the curve $y^2 = x$, the y-axis and between $y = 2$ and $y = 4$ is (s) 52/3 sq. units (b) 54/3 sq. units (c) 56/3 sq. units (d) None of these
4.	The Area of region bounded by the curve $y^2 = 4x$, and its latus rectum above x axis (a)0 sq units (b) 4/3sq units (c) 3/3 sq units(d) 2/3 sq units
5.	The Area of region bounded by curve y=x and $y = x^3$ is (a)1/2 sq units (b) 1 /4 sq units (c) 9/2 sq units (d) 9/4 sq units
6.	The area enclosed by the circle $x^2+y^2 = 2$ is equal to: (a)4л sq units (b) 2√2л sq units (c) $4л^2$ sq units (d) 2л sq units
7.	The area of the region bounded by the parabola $y = x^2$ and $y = \|x\|$ is (a) 3 (b) 1 / 2 (c) 1/3 (d) 2
8.	Find the area enclosed between curves $y = x^2 + 2$, $y = x$, $x = 0$, $x = 3$
9.	Find the area of the region bounded by the curve y=sinx between the lines x=0 , x=π/2 and the x-axis.
10.	Find the area enclosed between curves $y = 4x – x^2$, $0 \leq x \leq 4$, x-axis
11.	Find the area $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$ $\{ (x , y) : x^2 + y^2 < 1 < x+y\}$
12.	Find the area enclosed between $y^2 = 4ax$ and its latus rectum
13.	Find the area bounded by the curve y = cos x between x = 0 and x = 2π
14.	Find the area of the region bounded by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
15.	Using the method of integration find the area bounded by the curve $\|x\| + \|y\| = 1$
16.	Find the area enclosed between curves $y = x^3$, $x = - 2$, $x = 1$, $y = 0$
17.	Using integration find the area of the region $x^2 + y^2 = 4$ and $x = \sqrt{3}y$ with x-axis in first quadrant.
18.	In the figure given below O(0, 0) is the center of the circle. The line y = x meets the circle in the first quadrant at point B. Answer the following questions based on the given figure. 

(i) The equation of the circle is ____________.

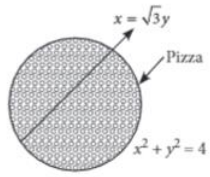
(ii) The co-ordinates of B are _______________.

(iii) Area of ΔOBM is ____________ sq. units.

(iv) Ar (BAMB) = ____________ sq. units.

(v) Area of the shaded region is ____________ sq. units.

19.



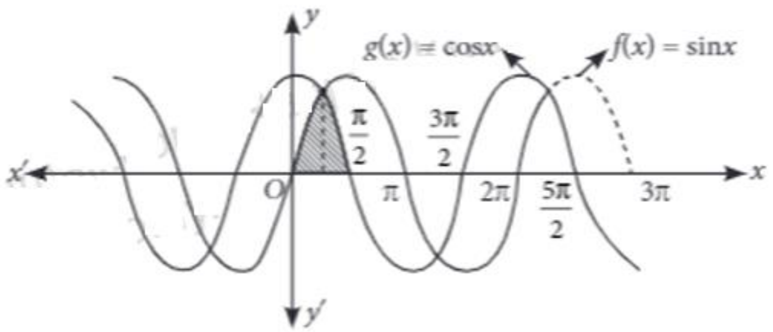
A child cuts a pizza with a knife. Pizza is circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of the knife is represented by $x = \sqrt{3}y$. Based on this information, answer the following questions.

(i) The points of intersection of the edge of the knife and the pizza as shown in the figure are _________ and _________.

(ii) Which of the following shaded portions represents the smaller area bounded by pizza and edge of knife in the first quadrant?

20. Graphs of two functions f(x) = sin x and g(x) = cos x are given below.

Based on the same, answer the following questions.



(i) In [0, π], the curves f(x) and g(x) intersect at x= _______.

(ii) Find the value of $\int_0^{\frac{\pi}{4}} \sin x \, dx$.

(iii) Find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$.

(iv) Find the value of $\int_0^{\pi} \sin x \, dx$.

Worksheet

Differential Equations

1. What is the degree of the differential equation $y\left(\frac{d^2y}{d^2x}\right)^3 + x\left(\frac{dy}{dx}\right)^4 + y^5 = 0$

   (a) 6 (b) 4 (c) 5 (d) 3

2. The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + 4\frac{d^2y}{d^2x} + 5 = 0$ is

   (a) order 1 and degree 2 (b) order 2 and degree 2

   (c) order 2 and degree 1 (d) order 1 and degree 1

3. The Integrating Factor of the differential equation $\frac{dy}{dx} - \frac{y}{x} = 2x^2$is

   (a) x^2 (b) x (c) $-\frac{1}{x}$ (d) $\frac{1}{x}$

4. Find the particular solution of the differential equation $\frac{dy}{dx} + sec^2x.y = \tan x\,.sec^2x$, given that y(0) = 0. (3)
5. Solve the differential equation given by $xdy - ydx - \sqrt{x^2 + y^2}dx = 0$. (3)
6. Find the general solution of the differential equation :$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$.
7. Solve the following differential equation :$xe^{\frac{x}{y}} - y + x\frac{dy}{dx} = 0$.
8. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}, y(1) = 0$.
9. Find the general solution of the differential equation $e^x \tan y\, dx + (1 - e^x)sec^2ydy = 0$.
10. Find the particular solution of the differential equation $xdx - ye^y\sqrt{1 + x^2}dy = 0$, given that $y = 1$, when $x = 0$.
11. Solve the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$.

WORKSHEET

VECTOR ALGEBRA

1. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$ then the value of $\vec{a}.\vec{b}$ IS

$a)$ $12\sqrt{3}$ b) 12 c) -12 d) $-12\sqrt{3}$

2. Area of a parallelogram whose diagonals are along vectors $\hat{\imath} + 2\hat{k}$ and $2\hat{\jmath} - 3\hat{k}$ is

a) $\sqrt{29}$ b) $\frac{1}{2}\sqrt{29}$ c) $-4\,\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ d) None of these

3. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors then the value of $|2\hat{a} + \hat{b} + \hat{c}|$ is

$a)$ $\sqrt{5}$ b) $\sqrt{3}$ c) $\sqrt{2}$ d) $\sqrt{6}$

4. For what value of p , is $(\hat{\imath} + \hat{\jmath} + \hat{k})$ p a unit vector ?

a) $\pm\frac{1}{\sqrt{3}}$ b) ± 1 c) $\pm\frac{1}{3}$ d) $\pm\sqrt{3}$

5. Which of the following vectors is equally inclined to axes

a) $\hat{\imath} + \hat{\jmath} + \hat{k}$ b) $\hat{\imath} - \hat{\jmath} + \hat{k}$ c) $\hat{\imath} - \hat{\jmath} - \hat{k}$ d) $-\hat{\imath} + \hat{\jmath} - \hat{k}$

6. Show that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ form the sides of a right-angled triangle.

7. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$.

8. Find a unit vector perpendicular to both **a** = 3**i** + 2**j** + 2**k** and **b** = **i** + 2**j** + 3**k**.

9. Find the value of λ for which the two vectors $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ are perpendicular.

10. If $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$ then find the value of $|\vec{a} - \vec{b}|$.

11. If the sum of two unit vectors is a unit vector , prove that the magnitude of their difference is $\sqrt{3}$.

12. .Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|$ =3 and $|\vec{b}| = \frac{\sqrt{3}}{2}$ and $\vec{a}$ x$\vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ and $\vec{b}$?

13. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

CASE BASED QUESTIONS

14.A man is watching an aeroplane which is at the coordinate A(4,-1,3) assuming that the man is at O(0,0,0) .At the same time he saw a bird at the coordinate point B(2,0,4).

Based on the above information answer the following:

(a) Find the vector $\overrightarrow{AB}$.

(b) Find the distance between aeroplane and bird.

(c) Find the unit vector along $\overrightarrow{AB}$.

OR

Find the direction cosines of $\overrightarrow{AB}$.

15.A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let If $\vec{a}, \vec{b}$ and $\vec{c}$ be three nonzero vectors .

(a) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then find the relation between $\vec{a}$ and $\vec{b}$.

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between then find $|\vec{a} - \vec{b}|$.

(c) If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ an angle between $\vec{b}$ and $\vec{c}$ is 30° then find k if $\vec{a} = k(\vec{b} \times \vec{c})$.

OR

Find the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ as diagonals.

WORKSHEET THREE DIMENSIONAL GEOMETRY

1. If the direction cosines of a line are $\left(\frac{1}{a},\frac{1}{a},\frac{1}{a}\right)$,the value of a is

   a) 3 b) ± 3 c) -$\pm\sqrt{3}$ d) $\sqrt{3}$

2. Vector equation of a line is $\vec{r} = (4\hat{\imath} - 2\hat{\jmath} + 5\hat{k}) + \mu(\hat{\imath} + \hat{3\jmath} - 2\hat{k})$,the Cartesian form of a line is:

   (a) $\frac{x-4}{1} = \frac{y+2}{3} = \frac{z-5}{-2}$ (b) $\frac{x+4}{1} = \frac{y-2}{3} = \frac{z+5}{-2}$

   (c) $\frac{x-4}{1} = \frac{y+2}{3} = \frac{z-5}{2}$ (d) $\frac{x+4}{-1} = \frac{y-2}{3} = \frac{z+5}{2}$

3. If a line makes angles $\frac{\pi}{2},\frac{\pi}{3}$ and θ with the positive x, y and z axes respectively, then θ is

   (a) $\pm\frac{\pi}{6}$ (b) $\pm\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ only (d) $\frac{\pi}{3}$ only

4. The acute angle between the line $\vec{r} = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \mu(\hat{\imath} - \hat{\jmath})$ and the X-axis

   a) $\frac{\pi}{4}$ $b)\frac{\pi}{3}$ $c)\frac{\pi}{6}$ $d)\frac{\pi}{2}$

5. The equation of a line passing through the point (3,-1,5) and parallel to vector $(\hat{\imath} + \hat{2\jmath} - \hat{k})$ is;

   (a) $x = t + 3$, $y = 2t - 1$, $z = -t + 5$

   (b) $x = t + 3$, $y = -2t - 1$, $z = -t + 5$

   (c) $x = t + 3$, $y = 2t - 1$, $z = t + 5$

   (d) $x = t - 3$, $y = 2t - 1$, $z = -t + 5$

6. . Write the vector equation of the line whose Cartesian equations is

$\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$

7. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

8. Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

9. Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1,2,3).

10. . Find the coordinates of a point where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts YZ- plane.

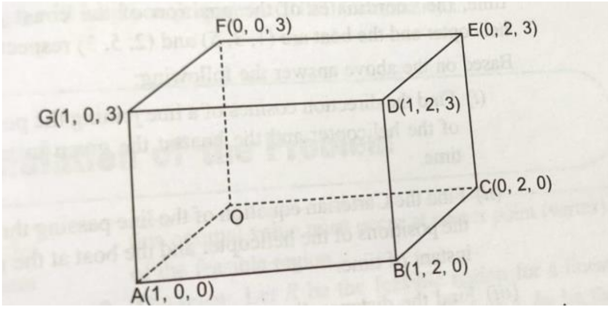
11. The cartesian equations of a line are 3x + 1 =6y – 2 = 1 – z. Find the fixed point through which it passes , its direction ratios and also its vector equation.

12. Show that the lines $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ are intersecting. Hence find their point of intersection.

13. Find the shortest distance of the following lines:

   $\vec{r} = \hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} - \hat{\jmath} - \hat{k} + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$.

14. Anu made a cuboidal fish tank having coordinates O(0,0,0) , A(1,0,0) , (1,2,0) , C(0,2,0) ,D(1,2,3) ,E(0,2,3),F(0,0,3) and G(1,0,3)



a) Find the the direction cosines of $\overrightarrow{AB}$.
b) Write cartesian equation of the diagonal $\overrightarrow{OD}$.
c) Find the direction ratios of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

OR

Show that the line $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are perpendicular to each other.

15.Read the following passage and answer the questions given below:

Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

Two such wires lie along the lines $l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$

$l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$

(i) Write the direction ratios of the line l_1.

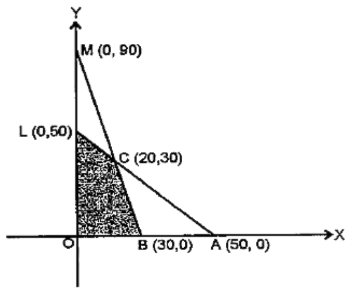
(ii) If $\cos\alpha, \cos\beta$ and $\cos\gamma$ are the direction ratios of the line l_2, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

(iii) Find the value of p if the lines l_1 and l_2 are perpendicular to each other.

OR

If the lines l_1 and l_2 are perpendicular to each other, find the vector equation of a line passing through the point (1,2,3) and parallel to the line l_2.

WORKSHEET LINEAR PROGRAMMING

S.N.	MCQs	MARKS
1	The maximum or minimum value of the objective function occurs at: a) Origin b) Boundary line c) Corner points of the feasible region d) Centre of feasible region	1
2	The maximum value of Z = 4x + y for a L.P.P., whose feasible region is shown below is:  a) 50 b) 110 c) 120 d) 170	1
3	In the LPP, $x \geq 0$ and $y \geq 0$ are called: a) Additional equations b) Non-negative constraints c) Inverse conditions d) Elimination rules	1
4	If the feasible region of a linear programming problem with objective function Z = ax + by, is bounded, then which of the following is correct? a) It will only have a maximum value. b) It will only have a minimum value. c) It will have both maximum and minimum values. d) It will have neither maximum nor minimum values.	1
	SA (2 MARKS EACH)	
5	Minimize Z = 50x + 70y Subject to the constraints: $2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$	2
6	Solve the following LPP graphically: Maximize: Z = 2x + 3y, subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$	2

7 Maximize : $P = 40x + 50y$, Subject to the constraints

$3x + y \leq 9$

$x + 2y \leq 8$ 2

and $x \geq 0, y \geq 0$

ANSWERS

1 c) Corner points of the feasible region

2 b) (2, 3)

3 b) 110

4 b) Non-negative constraints

5 The minimum value of Z is 380 obtained at the point (2, 4).

6 The maximum value of Z is 12 at the point (0, 4).

7 P is maximum at $x = 2$ and $y = 3$ and maximum value of P is 230

PROBABILITY WORKSHEET

S.N.	MCQs	Marks
1	A committee of 3 is formed from 3 boys and 2 girls. What is the probability that the committee includes at least one girl? (A) 9/10 (B) 4/5 (C) 3/5 (D) ½	1
2	A and B are events such that P(A) = 0.6, P(B) = 0.5, and P(A ∪ B) = 0.9. What is P(A ∩ B)? (A) 0.2 (B) 0.1 (C) 0.4 (D) 0.3	1
3	If P(E) = 0.6, P(F) = 0.5, and P(E ∩ F) = 0.3, then events E and F are: (A) Mutually exclusive (B) Independent (C) Dependent (D) Complementary	1
	ASSERTION/REASON TYPE QUESTIONS	1
	Choose the correct options for questions 4 and 5: **Options:** (A) Both A and R are true, and R is the correct explanation of A. (B) Both A and R are true, but R is not the correct explanation of A. (C) A is true, but R is false. (D) A is false, but R is true.	
4	**Assertion (A):** If two events are independent, they cannot be mutually exclusive. **Reason (R):** Mutually exclusive events imply P(A ∩ B) = 0, while independent events imply P(A ∩ B) = P(A) × P(B).	
5	**Assertion (A):** If P(A) = 0, then P(A ∪ B) = P(B). **Reason (R):** A null event does not affect the probability of union.	
6	A die is thrown three times. Events A and B are defined as below: A : 4 on the third throw B : 6 on the first and 5 on the second throw Find the probability of A given that B has already occurred.	
7	An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?	
8	A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even' then find whether E and F are independent ?	
9	A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.	
10	A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.	

ANSWERS

1	(A) $\frac{9}{10}$
2	(A) 0.2

3 (B) Independent

4 (A)

5 (A)

6 $\frac{1}{6}$

7 $\frac{3}{7}$

8 E and F are independent events.

9 $\frac{3}{8}$

10 $\frac{5}{11}$

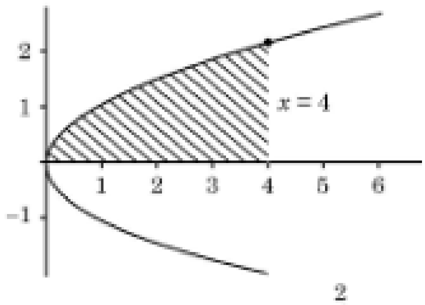
CBSE PREVIOUS YEARS QUESTION PAPERS

PAPER 1 (WITH SOLUTION)

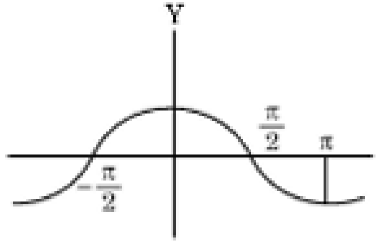
Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is (A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	
Ans	(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	1
Q2.	If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct ? (A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \perp \vec{b}$ (C) $\lvert \vec{b} \rvert > \lvert \vec{a} \rvert$ (D) $\lvert \vec{a} \rvert = \lvert \vec{b} \rvert$	
Ans	(B) $\vec{a} \perp \vec{b}$	1
Q3.	$\int_{-1}^{1} \frac{\lvert x \rvert}{x}\, dx, x \neq 0$ is equal to (A) –1 (B) 0 (C) 1 (D) 2	
Ans	(B) 0	1

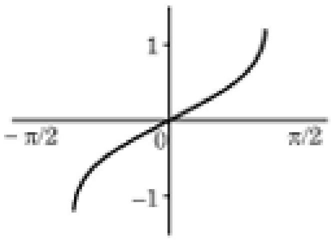
Q4.	Which of the following is <u>not</u> a homogeneous function of x and y ? (A) $y^2 - xy$ (B) $x - 3y$ (C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$	
Ans	(D) $\tan x - \sec y$	1
Q5.	If $f(x) = \lvert x \rvert + \lvert x - 1 \rvert$, then which of the following is correct ? (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$. (B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$. (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.	
Ans	(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.	1
Q6.	If A is a square matrix of order 2 such that det (A) = 4, then det (4 adj A) is equal to : (A) 16 (B) 64 (C) 256 (D) 512	
Ans	(B) 64	1
Q7.	If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to : (A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{7}{9}$	
Ans	(C) $\frac{2}{3}$	1
Q8.	The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in [0, 2] is : (A) 0 (B) 2 (C) 4 (D) 5	
Ans	(C) 4	1

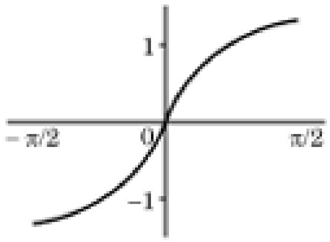
Q9.	Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9\ 8\ 7]$, which of the following is defined ? (A) Only AB (B) Only AC (C) Only BA (D) All AB, AC and BA	
Ans	(A) Only AB	1
Q10.	If $\int \frac{2^{\frac{1}{x}}}{x^2}\, dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to (A) $\frac{-1}{\log 2}$ (B) $-\log 2$ (C) -1 (D) $\frac{1}{2}$	
Ans	(A) $\frac{-1}{\log 2}$	1
Q11.	If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\lvert \vec{a} \rvert = \sqrt{37}$, $\lvert \vec{b} \rvert = 3$ and $\lvert \vec{c} \rvert = 4$, then angle between $\vec{b}$ and $\vec{c}$ is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	
Ans	(C) $\frac{\pi}{3}$	1
Q12.	The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is (A) $e^{\frac{y^2}{2}}$ (B) $\frac{1}{\sqrt{y}}$ (C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y^2}}$	
Ans	(B) $\frac{1}{\sqrt{y}}$	1

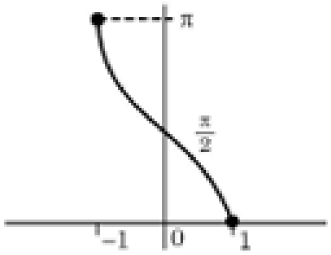
Q13.	If A = $\begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to (A) 0 (B) 1 (C) 7 (D) ± 7	
Ans	(B) 1	1
Q14.	The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then (A) Z is maximum at (2, 72), minimum at (15, 20) (B) Z is maximum at (15, 20) minimum at (40, 15) (C) Z is maximum at (40, 15), minimum at (15, 20) (D) Z is maximum at (40, 15), minimum at (2, 72)	
Ans	(C) Z is maximum at (40, 15), minimum at (15, 20)	1
Q15.	If A and B are invertible matrices, then which of the following is <u>not</u> correct ? (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$ (C) adj (A) = $\vert A \vert A^{-1}$ (D) $\vert A \vert^{-1} = \vert A^{-1} \vert$	
Ans	(A) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
Q16.	If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ? (A) It will only have a maximum value. (B) It will only have a minimum value. (C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.	
Ans	(C) It will have both maximum and minimum values.	1
Q17.	The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by  (A) $\int_0^4 x\,dx$ (B) $\int_0^2 y^2\,dy$ (C) $2\int_0^4 \sqrt{x}\,dx$ (D) $\int_0^4 \sqrt{x}\,dx$	
Ans	(D) $\int_0^4 \sqrt{x}\,dx$	1

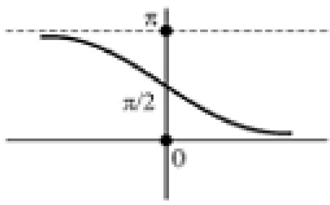
Q18. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse ?



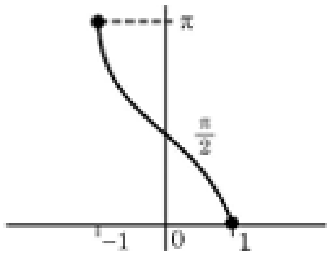
(A)


(B)


(C)


(D)


Ans (C) — 1



Assertion – Reason Based Questions

Direction : Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19. **Assertion (A) :** Let Z be the set of integers. A function f : Z → Z defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

Ans (D) Assertion (A) is false, but Reason (R) is true. — 1

Q20.	**Assertion (A)** : $f(x) = \begin{cases} 3x - 8, & x \le 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$. **Reason (R)** : For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.	(a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. **OR** (b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.	
Ans(a)	Let $u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x}(-2\cos x \sin x)\log 2$	1
	Let $v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2\cos x \sin x$	½
	Now $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$	½
	OR	
Ans(b)	$\tan^{-1}(x^2 + y^2) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$	½
	Differentiate both sides wrt x,	
	$2x + 2y\frac{dy}{dx} = 0$	1
	$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$	½

Q22.	Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$	
Ans	$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ $=\tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\frac{\pi}{3}\right]$ $=\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$	 1 1
Q23.	The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.	
Ans	$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$ Area of parallelogram $=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$ $=\frac{1}{2}\sqrt{(-2)^2+3^2+7^2}=\frac{\sqrt{62}}{2}$	1 1
Q24.	Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.	
Ans	$f(x)=5x^{3/2}-3x^{5/2}\Rightarrow f'(x)=\frac{15}{2}\sqrt{x}(1-x)$ For increasing/decreasing, put $f'(x)=0$ $\Rightarrow x=0,1$ (*i*) When $x\in[0,1]$, $f'(x)\geq 0$. So, f is increasing when $x\in[0,1]$ (The intervals $(0,1)$, $[0,1)$ or $(0,1]$ can also be considered.) (*ii*) When $x\in[1,\infty)$, $f'(x)\leq 0$. So, f is decreasing when $x\in[1,\infty)$ (The interval $(1,\infty)$ can also be considered.)	1 ½ ½

Q25.	(a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings. **OR** (b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.	
Ans(a)	**Let the required angle between the kite strings be θ.** **Then,** $\cos\theta = \frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$ $\Rightarrow \cos\theta == \frac{(3\hat{i}+\hat{j}+2\hat{k})(2\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{12}{\sqrt{336}}\right)$ or $\cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$	 1½ ½
OR		
Ans(b)	$\overrightarrow{BA} = -6\hat{i} + 2\hat{j} + 3\hat{k}$ **Required unit vector of magnitude 21** $= 21 \times \left(\frac{-6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{36+4+9}}\right)$ $= 3(-6\hat{i}+2\hat{j}+3\hat{k})$ or $-18\hat{i}+6\hat{j}+9\hat{k}$	1 ½ ½
SECTION C This section comprises short answer (SA) type questions of **3 marks each**.		
Q26.	The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm ?	
Ans	**Let 'a' be the side of the triangle, so** $\frac{da}{dt} = 3$ **cm/s** **Now area of an equilateral triangle,** $A = \frac{\sqrt{3}}{4}a^2$ $\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}a}{2} \times \frac{da}{dt}$ $\therefore \left.\frac{dA}{dt}\right]_{a=15\,cm} = \frac{\sqrt{3}\times 15}{2} \times 3 = \frac{45\sqrt{3}}{2}$ cm²/s	½ 1½ 1

Q27. Solve the following linear programming problem graphically :

Maximise $Z = x + 2y$

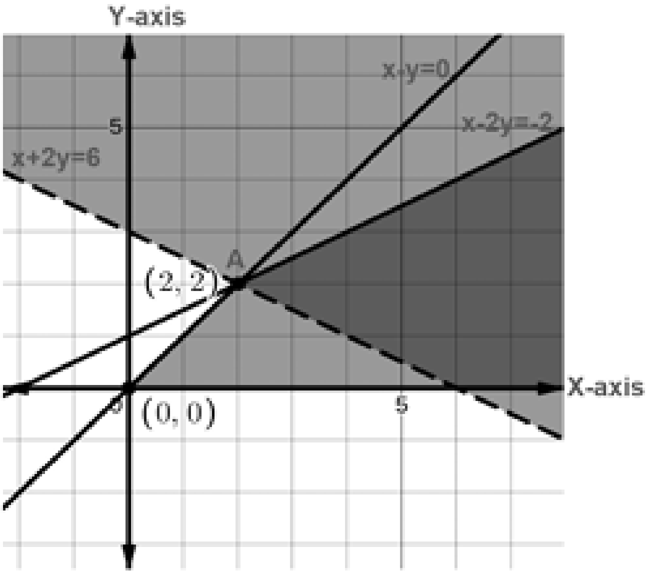
Subject to the constraints :

$x - y \geq 0$

$x - 2y \geq -2$

$x \geq 0,\ y \geq 0$

Ans



For correct graph and shading 1½

Corner Point	Value of $Z = x + 2y$
$O(0,0)$	0
$A(2,2)$	6

For correct table 1

Since feasible region is unbounded. Plot $x + 2y > 6$ which has common region with feasible region, thus Z has no maximum value. ½

Ans

(a) Find : $\int \frac{x + \sin x}{1 + \cos x}\, dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}}$

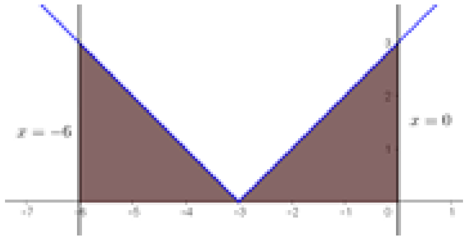
Ans(a)	$\int \frac{x+\sin x}{1+\cos x}dx$ $=\int \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$ $=\int x\left(\frac{1}{2}\sec^2\frac{x}{2}\right)dx+\int \tan\frac{x}{2}dx$ $=x\tan\frac{x}{2}-\int \tan\frac{x}{2}dx+\int \tan\frac{x}{2}dx$ $=x\tan\frac{x}{2}+C$	1 ½ 1 ½
OR		
Ans(b)	$\int_0^{\pi/4}\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$ $=\frac{1}{2}\int_0^{\pi/4}\frac{dx}{\cos^4 x\sqrt{\tan x}}$ $=\frac{1}{2}\int_0^{\pi/4}\frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}}dx$ Put $\tan x=t \Rightarrow \sec^2 x\,dx=dt$ $\therefore I=\frac{1}{2}\int_0^1\frac{1+t^2}{\sqrt{t}}dt$ $=\frac{1}{2}\int_0^1\left(\frac{1}{\sqrt{t}}+t^{3/2}\right)dt$ $=\frac{1}{2}\left[2\sqrt{t}+\frac{2}{5}t^{5/2}\right]_0^1$ $=\frac{6}{5}$	½ ½ ½ 1 ½

Q29.	(a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. **OR** (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.	
Ans(a)	**Rewriting the lines, we get** $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$ **and** $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$	½
	Let $\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$, $\vec{b}_1 = -\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$ **Note that the dr's of given lines are not proportional so, they are not parallel lines.** **The lines will be skew if they do not intersect each other also.** **Here** $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$	½ + ½
	Consider $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)$ $= (\hat{j} - 4\hat{k}).(2\hat{i} - 4\hat{j} - 3\hat{k}) = 8 \neq 0$ **Hence lines will not intersect. So the lines are skew.**	½
	Shortest Distance $= \dfrac{\lvert(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)\rvert}{\lvert\vec{b}_1 \times \vec{b}_2\rvert}$ $= \dfrac{8}{\sqrt{4+16+9}} = \dfrac{8}{\sqrt{29}}$	1
	OR	
Ans(b)	**Let the wicket keeper divides the line segment in ratio** $k:1$ $\therefore \vec{W} = \dfrac{k\vec{F} + 1.\vec{B}}{k+1}$ 	1
	$\Rightarrow 6\hat{i} + 12\hat{j} = \left(\dfrac{12k+2}{k+1}\right)\hat{i} + \left(\dfrac{18k+8}{k+1}\right)\hat{j}$	1
	$\Rightarrow k = \dfrac{2}{3}$ **Hence, the required ratio is** $2:3$	1

<table>
<tr><td>Q30.</td><td>(a) The probability distribution for the number of students being absent in a class on a Saturday is as follows :

X	0	2	4	5
P(X)	p	2p	3p	p

Where X is the number of students absent.
(i) Calculate p. — 1
(ii) Calculate the mean of the number of absent students on Saturday. — 2
OR
(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.</td><td></td></tr>
<tr><td>Ans(a)</td><td>(i) Since $\sum P(X)=1 \Rightarrow p+2p+3p+p=1$
$\Rightarrow p=\frac{1}{7}$
(ii) Mean $=\sum X.P(X)=0(p)+2(2p)+4(3p)+5(p)$
$=21p=21\left(\frac{1}{7}\right)=3$</td><td>½
½
1
1</td></tr>
<tr><td colspan="3">OR</td></tr>
<tr><td>Ans(b)</td><td>Let E_1 : The applicant is a male
E_2 : The applicant is a female
A : The candidate chosen will have distinction in the written test.
$P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A|E_1)=0.4, P(A|E_2)=0.35$
$\therefore P(A)=P(E_1)P(A|E_1)+P(E_2)P(A|E_2)$
$=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$
$=\frac{11}{30}$</td><td>½
1
1
½</td></tr>
</table>

Q31.	Sketch the graph of $y = \| x + 3 \|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.	
Ans	 **Required Area** $= \int_{-6}^{0} y\,dx$ $= 2\int_{-3}^{0} (x+3)\,dx$ $= 2\left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$ $= 9$	For correct graph: 1 mark ½ ½ ½ ½

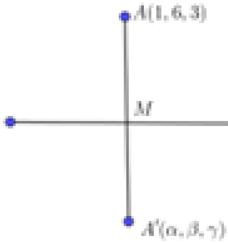
SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

Q32.	(a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **OR** (b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.	
Ans(a)	Let $x = \sin A,\ y = \sin B \Rightarrow A = \sin^{-1} x,\ B = \sin^{-1} y$ $\therefore \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ $\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = 2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$ $\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow A - B = 2\cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ differentiate both sides wrt x, $\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	1 1 1 ½ 1½

OR

Ans(b)	$x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ $\Rightarrow\frac{dx}{d\theta}=a\left(-\sin\theta+\frac{1}{\tan\frac{\theta}{2}}\times\sec^2\frac{\theta}{2}\times\frac{1}{2}\right)$ $=a\left(-\sin\theta+\frac{1}{\sin\theta}\right)=a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$ $\frac{dx}{d\theta}=a\cot\theta\cos\theta$ Also, $y=\sin\theta\Rightarrow\frac{dy}{d\theta}=\cos\theta$ $\therefore\frac{dy}{dx}=\frac{\tan\theta}{a}$ Differetiating wrt x, $\frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{d\theta}{dx}$ $=\frac{\sec^3\theta\tan\theta}{a^2}$ $\left.\frac{d^2y}{dx^2}\right]_{at\ \theta=\frac{\pi}{4}}=\frac{2\sqrt{2}}{a^2}$	 ½ ½ ½ ½ 1 1 1
Q33.	Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on [1, 5].	
Ans	$f(x)=2x^3-15x^2+36x+1$ $\Rightarrow f'(x)=6(x^2-5x+6)=6(x-2)(x-3)$ $f'(x)=0\Rightarrow x=2,3\in[1,5]$ Now $f(1)=24,\ f(2)=29,\ f(3)=28,\ f(5)=56$ Hence, theabsolute maximum value is 56 and the absolute minimum value is 24.	 1 1 2 1
Q34.	(a) Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, find the equation of the line joining A and A'. **OR** (b) Find a point P on the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$ such that its distance from point Q(2, 4, –1) is 7 units. Also, find the equation of line joining P and Q.	

Ans(a)	The equation of given line is $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$ 	
	Any arbitrary point on the line is $M(\lambda,2\lambda+1,3\lambda+2)$	1
	dr's of AM are $<\lambda-1,2\lambda-5,3\lambda-1>$ Here $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$	1
	$\Rightarrow \lambda=1$	½
	$\therefore M(1,3,5)$ is the foot perpendicular of the point A to the given line. Let image of point A in the line be $A'(\alpha,\beta,\gamma)$	
	Since M is the mid-point of AA', so $M\left(\frac{1+\alpha}{2},\frac{6+\beta}{2},\frac{3+\gamma}{2}\right)=M(1,3,5)$	½
	$\Rightarrow A'(1,0,7)$ is the image of A.	1
	Also, Equation of AA' is $\frac{x-1}{0}=\frac{y-6}{-3}=\frac{z-3}{2}$	1
OR		
Ans(b)	The given line is $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda$ and $Q(2,4,-1)$	
	Any random point on the line will be given by P $(\lambda-5,4\lambda-3,-9\lambda+6)$	1
	Since $PQ=7\Rightarrow\sqrt{(\lambda-7)^2+(4\lambda-7)^2+(-9\lambda+7)^2}=7$	1
	$\Rightarrow 98(\lambda^2-2\lambda+1)=0\Rightarrow\lambda=1$	1
	Hence, the required point is $P(-4,1,-3)$	1
	The equation of line PQ is $\frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}$ or $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$	1
Q35.	A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions : • The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club. • The number of students in Music club should be 20 more than half the number of students in Sports club. • The total number of students to be allocated in all three clubs are 180. Find the number of students allocated to different clubs, using matrix method.	

Ans

Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively.

Here, $x = y + z,\ y = \frac{x}{2} + 20,\ x + y + z = 180$

$\Rightarrow x - y - z = 0,\ x - 2y = -40,\ x + y + z = 180$ **1½**

Given equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ **½**

$|A| = -4 \neq 0 \Rightarrow A^{-1}$ **exists.** **½**

$adjA = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$ **1**

$A^{-1} = \frac{1}{|A|} \times adjA = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$ **½**

$X = A^{-1}B$

$= \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$ **1**

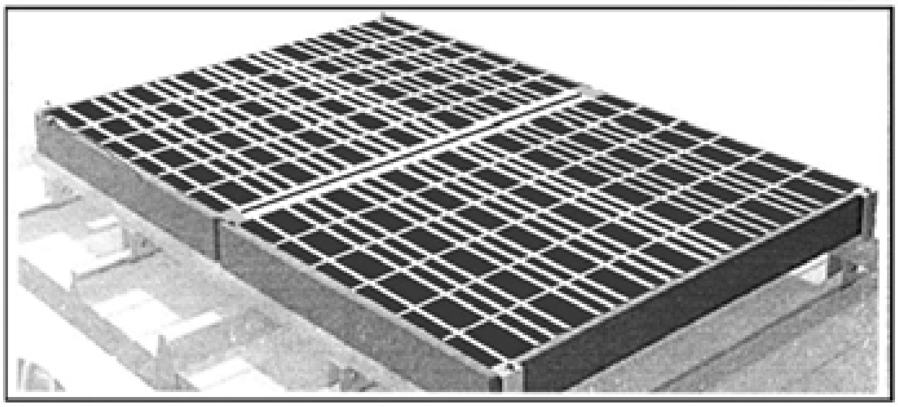
$\therefore x = 90,\ y = 65,\ z = 25$

Number of students allocated in sports, music and drama are 90 , 65 and 25 respectively .

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

	Based on this information, answer the following questions : (i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. **1** (ii) Write the area of the solar panel as a function of x. **1** (iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. **2** **OR** (iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition. **2**	
Ans	$(i)\ 2x+3y=300$	1
	$(ii)\ A=xy=\frac{x}{3}(300-2x)$	1
	$(iii)(a)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$	½
	Also, $\frac{d^2A}{dx^2}=-\frac{4}{3}<0$. So, A is maximum at $x=75$	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\text{m}^2$	½
	OR	
	$(iii)(b)\ A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$	
	$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$	½
	For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$	½
	As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x=75$ from left to right, which means $x=75$ is the point of maximum.	½
	Also, maximum area is $A=\frac{75}{3}(300-150)=3750\text{m}^2$	½
	Note : Full credit to be given if the student takes equation as **$2x+2y=300$ or $2x+4y=300$ or $4x+4y=300$ or $4x+3y=300$** **The solutions of sub-parts will differ and marks may be given accordingly.**	

Q37.	A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} : $R_1 = \{(2, 3), (3, 2)\}$ $R_2 = \{(1, 2), (1, 3), (3, 2)\}$ $R_3 = \{(1, 2), (2, 1), (1, 1)\}$ $R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$ $R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$ The students are asked to answer the following questions about the above relations : (i) Identify the relation which is reflexive, transitive but not symmetric. (ii) Identify the relation which is reflexive and symmetric but not transitive. (iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive. **OR** (iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?	
Ans	$(i)\ R_4$ $(ii)\ R_5$ $(iii)(a)\ R_1$ **and** R_3 **OR** $(iii)(b)$ **Required pairs to be added to make the relation** R_2 **as an equivalence relation are:** $(1,1),(2,2),(3,3),(2,1),(3,1)$ **and** $(2,3)$	**1** **1** **1+1** **2**

Q38.	 A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively. Based on the above information, answer the following : (i) What is the probability that a customer after availing the loan will default on the loan repayment ? **2** (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ? **2**

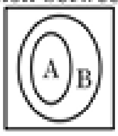
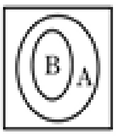
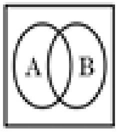
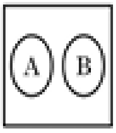
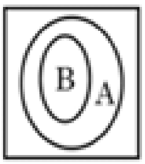
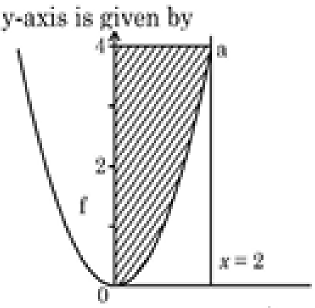
Ans	E_1 :customer avails loan on fixed rate E_2 :customer avails loan on floating rate E_3 :customer avails loan on variable rate A:the person defaults on the loan $P(E_1)=\frac{1}{10}, P(E_2)=\frac{2}{10}, P(E_3)=\frac{7}{10}$ $P(A\mid E_1)=\frac{5}{100}, P(A\mid E_2)=\frac{3}{100}, P(A\mid E_3)=\frac{1}{100}$ $(i) P(A)=P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)$ $=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$ $=\frac{18}{1000}$ or $\frac{9}{500}$ $(ii) P(E_3\mid A)=\frac{P(E_3).P(A\mid E_3)}{P(E_1).P(A\mid E_1)+P(E_2).P(A\mid E_2)+P(E_3).P(A\mid E_3)}$ $=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$ $=\frac{7}{18}$	 1 1 1 1

PAPER-2 (WITH SOLUTIONS)

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
SECTION-A This section comprises multiple choice questions (MCQs) of 1 mark each.		
1.	The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is (A) $\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$ (B) $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$ (C) $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert}$ (D) $\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert^2}\right)\vec{b}$	
Ans	(A) $\left(\frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert^2}\right)\vec{b}$	1
2.	The function $f(x) = x^2 - 4x + 6$ is increasing in the interval (A) $(0, 2)$ (B) $(-\infty, 2]$ (C) $[1, 2]$ (D) $[2, \infty)$	
Ans	(D) $[2, \infty)$	1
3.	If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\,dx$ is (A) $\int_0^{2a} f\left(\frac{x}{2}\right) dx$ (B) $\int_0^{a} f(x)\,dx$ (C) $2\int_a^{0} f(x)\,dx$ (D) $2\int_0^{a} f(x)\,dx$	
Ans	(D) $2\int_0^a f(x)dx$	1

4.	If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is (A) -8 (B) 0 (C) 6 (D) 8	
Ans	(D) 8	1
5.	If $y = \sin^{-1}x, -1 \le x \le 0$, then the range of y is (A) $\left(\frac{-\pi}{2}, 0\right)$ (B) $\left[\frac{-\pi}{2}, 0\right]$ (C) $\left[\frac{-\pi}{2}, 0\right)$ (D) $\left(\frac{-\pi}{2}, 0\right]$	
Ans	(B) $[-\frac{\pi}{2}, 0]$	1
6.	If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is (A) $\frac{-\pi}{3}$ only (B) $\frac{\pi}{3}$ only (C) $\frac{\pi}{6}$ (D) $\pm\frac{\pi}{3}$	
Ans	No option is correct. Full marks may be awarded for attempting the question.	1
7.	If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is (A) $\frac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$ (C) $1 - P(E/F)$ (D) $\frac{1 - P(E \cup F)}{P(\overline{F})}$	
Ans	(D) $\frac{1-P(E\cup F)}{P(\bar{F})}$	1
8.	Which of the following can be both a symmetric and skew-symmetric matrix ? (A) Unit Matrix (B) Diagonal Matrix (C) Null Matrix (D) Row Matrix	
Ans	(C) Null Matrix	1

9.	The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is : (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$ (B) $x = 3t + 4, y = t + 3, z = 2t + 7$ (C) $x = 3t + 4, y = t - 3, z = 2t + 7$ (D) $x = 3t + 4, y = -t + 3, z = 2t + 7$	
Ans	(C) $x = 3t + 4, y = t - 3, z = 2t + 7$	1
10.	Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify 4 AB + 3(AB + BA) – 4 BA, where A and B are both matrices of order 2 × 2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as : Abhay : 6 AB Bina : 7 AB – BA Chhaya : 8 AB Devesh : 7 BA – AB Who answered it correctly ? (A) Abhay (B) Bina (C) Chhaya (D) Devesh	
Ans	(B) Bina	1
11.	A cylindrical tank of radius 10 cm is being filled with sugar at the rate of 100π cm^3/s. The rate, at which the height of the sugar inside the tank is increasing, is : (A) 0.1 cm/s (B) 0.5 cm/s (C) 1 cm/s (D) 1.1 cm/s	
Ans	(C) 1 cm/s	1
12.	Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p} + \vec{q})$ will be a unit vector for what value of α ? (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$	
Ans	(D) $\frac{2\pi}{3}$	1

13.	The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point ? (A) (1, –5, 6) (B) (1, 5, 6) (C) (1, –5, –6) (D) (–1, –5, 6)	
Ans	(C) (1, -5, -6)	1
14.	If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ? (A)  (B)  (C)  (D) 	
Ans	 (B)	1
15.	The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \le x \le 2$ and y-axis is given by  (A) $\int_0^2 x^2\,dx$ (B) $\int_0^2 \sqrt{y}\,dy$ (C) $\int_0^4 x^2\,dx$ (D) $\int_0^4 \sqrt{y}\,dy$	
Ans	(D) $\int_0^4 \sqrt{y}dy$	1

16.	A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ? (A) The objective function maximizes the difference of the profit earned from products X and Y. (B) The objective function measures the total production of products X and Y. (C) The objective function maximizes the combined profit earned from selling X and Y. (D) The objective function ensures the company produces more of product X than product Y.	
Ans	(C) The objective function maximizes the combined profit earned from selling X and Y	1
17.	If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ? (A) A = B (B) AB = BA (C) A = 0 or B = 0 (D) A = I or B = I	
Ans	(B) AB = BA	1
18.	If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then (p − q) is (A) 0 (B) 1 (C) 2 (D) 3	
Ans	(B) 1	1
	Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	

19.	**Assertion (A)** : A = diag [3 5 2] is a scalar matrix of order 3 × 3. **Reason (R)** : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.	
Ans	(D) Assertion (A) is false and Reason (R) is true.	1
20.	**Assertion (A)** : Every point of the feasible region of a Linear Programming Problem is an optimal solution. **Reason (R)** : The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.	
Ans	(D) Assertion (A) is false and Reason (R) is true.	1
	SECTION-B This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.	
21	(a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$. **OR** (b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.	
21 (a) Ans	Let α be the angle which the vector $\vec{a}$ makes with all the three axes. Then $3\cos^2\alpha = 1$ $\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$ The unit vector along the vector $\vec{a} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$ $\vec{a} = 5(\hat{\imath} + \hat{\jmath} + \hat{k})$ OR	1 ½ ½
21 (b) Ans	 $\frac{QR}{QP} = \frac{3}{2}$	

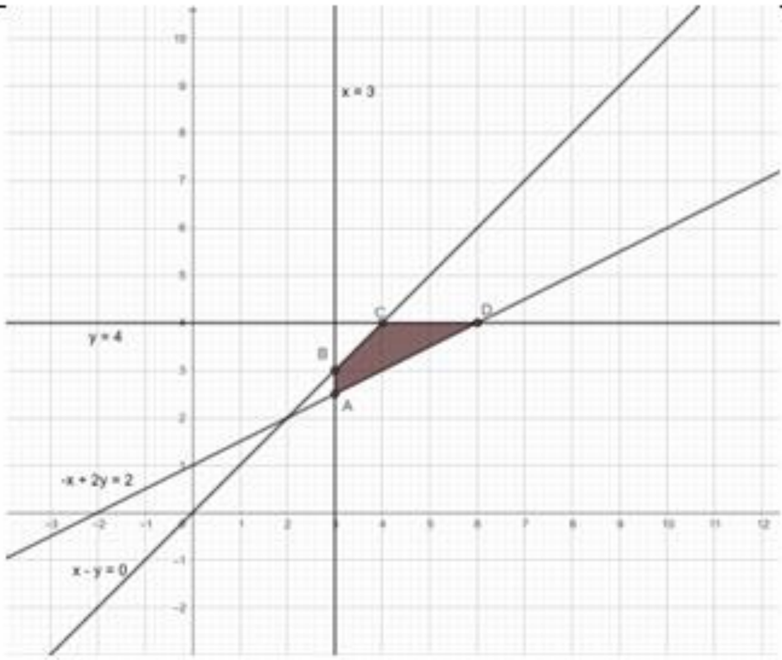
	Hence, R divides PQ, externally, in the ratio 1:3. The Position vector of R = $\vec{x} = \frac{\vec{\beta}-3\vec{\alpha}}{1-3} = \frac{3\vec{\alpha}-\vec{\beta}}{2}$	1 1
22.	Evaluate : $\int_0^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\ dx$	
Ans	Given definite integral = $\int_0^{\frac{\pi}{4}} \sqrt{(sinx+cosx)^2}dx$ $= \int_0^{\frac{\pi}{4}} (sinx+cosx)dx$ $= [-cosx+sinx]_0^{\frac{\pi}{4}}$ = 1	1 1
23.	Find the values of ‘a’ for which f(x) = sin x – ax + b is increasing on R.	
Ans	$f'(x) = cosx - a$ For f(x) to be increasing, $f'(x) \geq 0$ $i.e., cosx \geq a$ Since, $-1 \leq cosx \leq 1$ $\Rightarrow a \leq -1$ Hence, $a \in (-\infty, -1]$. (Also, accept $a \in (-\infty, -1)$)	 1 1
24.	If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x-2)$ $\vec{a}+\vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.	
Ans	$\vec{\alpha}$ and $\vec{\beta}$ are collinear $\Rightarrow \frac{x-2}{3+2x} = \frac{1}{-2}$	1½

	$\Rightarrow x = \frac{1}{4}$	½
25	(a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$. **OR** (b) If $f(x) = \begin{cases} 2x-3, & -3 \le x \le -2 \\ x+1, & -2 < x \le 0 \end{cases}$ Check the differentiability of f(x) at $x = -2$.	
25 (a) Ans	$x = e^{\frac{x}{y}}$ $\Rightarrow logx = \frac{x}{y}$ $\Rightarrow ylogx = x$ Differentiating both sides w.r.to x, we get $\frac{y}{x} + logx\frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx} = \frac{x-y}{xlogx}$ OR	 ½ 1 ½
25 (b) Ans	$Lf'(-2) = \lim_{h\to 0}\frac{f(-2-h)-f(-2)}{-h}$ $(h > 0)$ $= \lim_{h\to 0}\frac{2(-2-h)-3-(-7)}{-h}$ $= \lim_{h\to 0} 2 = 2$ $Rf'(-2) = \lim_{h\to 0}\frac{f(-2+h)-f(-2)}{h}$ $(h > 0)$ $= \lim_{h\to 0}\frac{-2+h+1-(-7)}{h}$ $= \lim_{h\to 0}\frac{6+h}{h}$, which does not exist, i.e., RHD does not exist.	 1

	Therefore, the function is not differentiable at -2. Note: (1) If a student finds only RHD and concludes the result, full marks may be awarded. (2) If a student proves that the function is discontinuous at -2 and hence not differentiable at -2, full marks may be awarded.	1
	SECTION-C This section comprises 6 Short Answer (SA) type questions of 3 marks each.	
26	(a) Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$. **OR** (b) Solve the following differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$.	
26(a) Ans	Given differential equation can be written as $\frac{y}{y+3} dy = \frac{2}{x} dx$ $\Rightarrow \int \left(1 - \frac{3}{y+3}\right) dy = 2 \int \frac{1}{x} dx$ $\Rightarrow y - 3\log\lvert y + 3\rvert = 2\log\lvert x\rvert + C$ y = -2, when x = 1 $\Rightarrow C = -2$ Hence, the required particular solution is $\Rightarrow y - 3\log\lvert y + 3\rvert = 2\log\lvert x\rvert - 2$ OR	 1 1½ ½
26(b) Ans	Given differential equation can be written as $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$, which is linear in y. I.F. = $e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$ The solution is given by $y(1 + x^2) = \int 4x^2 dx$ $\Rightarrow y(1 + x^2) = \frac{4}{3}x^3 + C$ $or\ y = \frac{4x^3}{3(1+x^2)} + C\frac{1}{(1+x^2)}$, which is the required general solution	 1 1 1

27.	Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n $\in$ N." Find whether R is reflexive, symmetric and transitive or not.	
Ans	Let $x \in N$. Then we know that x is a multiple of itself. $\Rightarrow xRx$ Hence, R is reflexive. We have $2, 8 \in N$ such that 8 is a multiple of 2 $\Rightarrow 8R2$ But, 2 is not a multiple of 8. Hence, 2 is not R-related to 8. Therefore, R is not symmetric. Let $x, y, z \in N$ such that xRy, yRz Then $x = my, y = nz$ for some $m, n \in N$ $\Rightarrow x = mnz \Rightarrow x = pz$, where $p = mn \in N$. Hence, xRz Therefore, R is transitive.	1 1 1
28.	Solve the following linear programming problem graphically : Minimise $Z = x - 5y$ subject to the constraints : $x - y \geq 0$ $-x + 2y \geq 2$ $x \geq 3, y \leq 4, y \geq 0$	

Ans



Correct graph and shading 1½

Corner point	Value of Z = x – 5y
A (3, 2.5)	-9.5
B (3, 3)	-12
C (4, 4)	-16
D (6, 4)	-14

1

The minimum value of Z is -16, which is attained at x = 4, y = 4. ½

29

(a) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

29(a) Ans

The given function can be written as

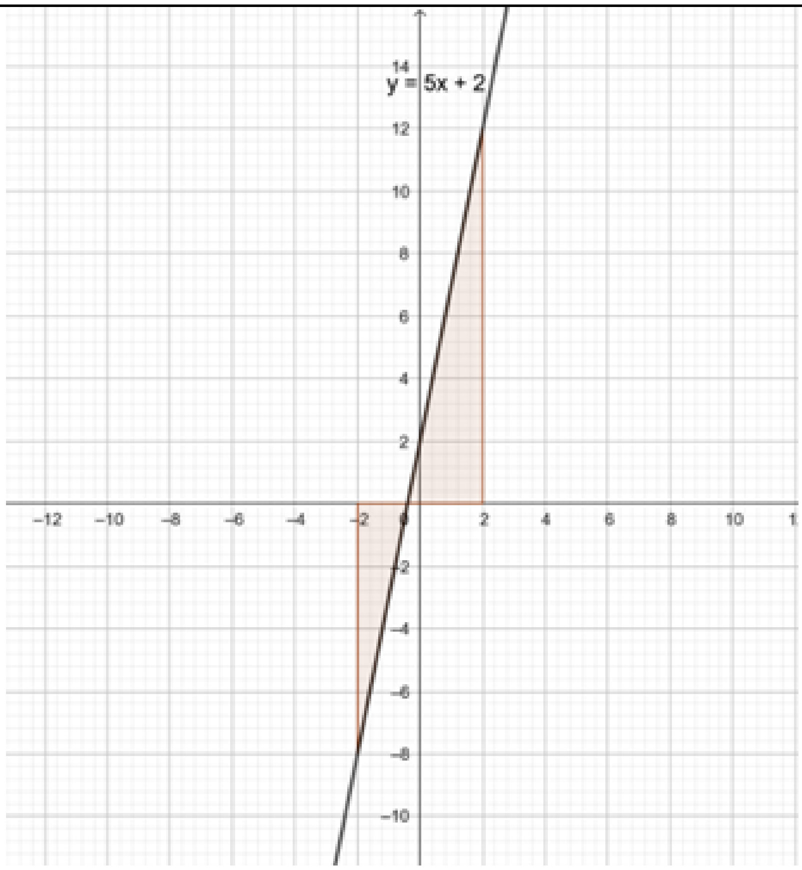
$y = 2\,log(x+1) - logx$

$\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$ 1

$\Rightarrow (x+1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$

	$\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$ $\Rightarrow x(x+1)^2y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$ $\Rightarrow x(x+1)^2y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$ $\Rightarrow x(x+1)^2y_2 + (x+1)^2y_1 = 2$ OR	1 1
29(b) Ans	$x\sqrt{1+y} + y\sqrt{1+x} = 0$ $\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ $\Rightarrow x^2(1+y) = y^2(1+x)$ $\Rightarrow (x-y)(x+y) + xy(x-y) = 0$ $\Rightarrow (x-y)(x+y+xy) = 0$ $x \neq y \Rightarrow x + y + xy = 0$ $\Rightarrow y = \frac{-x}{1+x}$ $\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$	 ½ 1 ½ 1
30	(a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice. **OR** (b) Two dice are thrown. Defined are the following two events A and B : A = {(x, y) : x + y = 9}, B = {(x, y) : x ≠ 3}, where (x, y) denote a point in the sample space. Check if events A and B are independent or mutually exclusive.	
30(a) Ans	$P(2) = \frac{3}{10}$, $P(\text{any other number}) = 1 - \frac{3}{10} = \frac{7}{10}$ Let X represent the Random Variable "the number of 2's". Then X = 0, 1, 2	½ ½

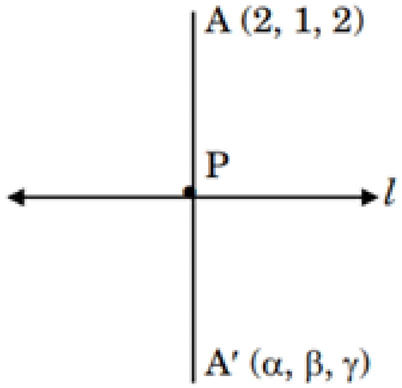
	The probability distribution is <table><tr><th>X</th><th>P(X)</th><th>XP(X)</th></tr><tr><td>0</td><td>$\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$</td><td>0</td></tr><tr><td>1</td><td>$\frac{3}{10}\times\frac{7}{10}\times 2=\frac{42}{100}$</td><td>$\frac{42}{100}$</td></tr><tr><td>2</td><td>$\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$</td><td>$\frac{18}{100}$</td></tr></table> Mean = $\sum XP(X) = \frac{60}{100} = 0.6$ OR	1½ ½
30(b) Ans	$A = \{(3,6),(4,5),(5,4),(6,3)\}$ $P(A) = \frac{4}{36} = \frac{1}{9}, P(B) = \frac{30}{36} = \frac{5}{6}$ $P(A\cap B) = \frac{3}{36} = \frac{1}{12}$ $P(A)\times P(B) = \frac{5}{54} \neq P(A\cap B)$ Therefore, A and B are not independent. A and B are not mutually exclusive as $A \cap B \neq \emptyset$	1 ½ 1 ½
31.	Find : $\int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\,dx.$	
Ans	$I = \int \frac{1}{x}\frac{x+a}{\sqrt{x^2-a^2}}dx = \int \frac{1}{\sqrt{x^2-a^2}}dx + a\int \frac{1}{x\sqrt{x^2-a^2}}dx$ $= \log\left\|x+\sqrt{x^2-a^2}\right\| + \sec^{-1}\left(\frac{x}{a}\right) + C$	1 1+1
	SECTION-D This section comprises 4 Long Answer (LA) type questions of 5 marks each.	
32.	Using integration, find the area of the region bounded by the line $y = 5x + 2$, the $x-$axis and the ordinates $x = -2$ and $x = 2$.	
Ans		

	 The required area $= \left\|\int_{-2}^{-\frac{2}{5}}(5x+2)dx\right\| + \int_{-\frac{2}{5}}^{2}(5x+2)dx$ $= \left\|\left[\frac{(5x+2)^2}{10}\right]_{-2}^{-\frac{2}{5}}\right\| + \left[\frac{(5x+2)^2}{10}\right]_{-\frac{2}{5}}^{2}$ $= \frac{64}{10} + \frac{144}{10} = \frac{104}{5}$	Correct sketch and shading 2 1 1 1
33.	Find : $\int \frac{x^2+x+1}{(x+2)(x^2+1)}$ dx.	
Ans	$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+c}{x^2+1}$ Getting $A = \frac{3}{5}, B = \frac{2}{5}, C = \frac{1}{5}$ Given integral $= \frac{3}{5}\int \frac{1}{x+2}dx + \frac{1}{5}\int \frac{2x}{x^2+1}dx + \frac{1}{5}\int \frac{1}{x^2+1}dx$	2 1½

	$=\frac{3}{5}\log\|x+2\|+\frac{1}{5}\log(x^2+1)+\frac{1}{5}tan^{-1}x+C$	1½
34	(a) Find the shortest distance between the lines : $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ and $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$. **OR** (b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.	
34(a) Ans	The vector equations of the lines are $\vec{r}=-\hat{\imath}+\hat{\jmath}+9\hat{k}+\lambda(2\hat{\imath}+\hat{\jmath}-3\hat{k})$ $\vec{r}=3\hat{\imath}-15\hat{\jmath}+9\hat{k}+\mu(2\hat{\imath}-7\hat{\jmath}+5\hat{k})$ $\vec{a_1}=-\hat{\imath}+\hat{\jmath}+9\hat{k},\ \vec{a_2}=3\hat{\imath}-15\hat{\jmath}+9\hat{k}$ $\vec{b_1}=2\hat{\imath}+\hat{\jmath}-3\hat{k}\ ,\ \vec{b_2}=2\hat{\imath}-7\hat{\jmath}+5\hat{k}$	1
	$\vec{a_2}-\vec{a_1}=4\hat{\imath}-16\hat{\jmath}$	1
	$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 1 & -3\\ 2 & -7 & 5\end{vmatrix}=-16\hat{\imath}-16\hat{\jmath}-16\hat{k}$	2
	S.D. $=\frac{\|(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})\|}{\|\vec{b_1}\times\vec{b_2}\|}=\frac{12}{\sqrt{3}}=4\sqrt{3}$	1
	OR	

34(b

Ans



Let the image of A in the line be $A'(\alpha, \beta, \gamma)$

The point P, which is the point of intersection of the lines l and AA', will have coordinates $(\lambda + 4, -\lambda + 2, -\lambda + 2)$ for some λ. ½

Drs of AP are $< \lambda + 2, -\lambda + 1, -\lambda >$ ½

$AP \perp l$

$(\lambda + 2) - (-\lambda + 1) - (-\lambda) = 0$

$\Rightarrow \lambda = -\frac{1}{3}$ 1

Therefore, the coordinates of P are $(\frac{11}{3}, \frac{7}{3}, \frac{7}{3})$ ½

P is the mid-point of AA'

$\Rightarrow \frac{2 + \alpha}{2} = \frac{11}{3}, \frac{1 + \beta}{2} = \frac{7}{3}, \frac{2 + \gamma}{2} = \frac{7}{3}$

$\Rightarrow \alpha = \frac{16}{3}, \beta = \frac{11}{3}, \gamma = \frac{8}{3}$ 1½

The coordinates of the image are $(\frac{16}{3}, \frac{11}{3}, \frac{8}{3})$

The equation of AA' is

$\frac{x - 2}{\frac{10}{3}} = \frac{y - 1}{\frac{8}{3}} = \frac{z - 2}{\frac{2}{3}}$ 1

or,

$\frac{3(x - 2)}{5} = \frac{3(y - 1)}{4} = \frac{3(z - 2)}{1}$

35	(a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations : $x - y + z = 4$ $x - 2y - 2z = 9$ $2x + y + 3z = 1$ **OR** (b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of linear equations : $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$	
35(a) Ans	$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$ The system of equations is equivalent to the matrix equation: $BX = C$, where $C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\Rightarrow X = B^{-1}C$ $AB = 8I$ $\Rightarrow B^{-1} = \frac{1}{8}A$ $X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ $\therefore x = 3, y = -2, z = -1$ OR	2 ½ 1 1½
35(b) Ans	$\|A\| = 1 \neq 0 \Rightarrow A^{-1}$ exists. $adjA = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	1 1½

$$A^{-1} = \frac{1}{|A|}\text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

The given system of equations is equivalent to the matrix equation

$A^T X = B$, where $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ½

$\Rightarrow X = (A^T)^{-1}B$

$\Rightarrow X = (A^{-1})^T B$ ½

$$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\therefore x = 0, y = -5, z = -3$ 1½

SECTION-E

This section comprises 3 case study based questions of 4 marks each

36. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

	Based on the above, answer the following : (i) How many relations can be there from S to J ? **1** (ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)} Check if it is bijective. **1** (iii) (a) How many one-one functions can be there from set S to set J ? **2** **OR** (iii) (b) Another student considers a relation R_1 = {(S_1, S_2), (S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**	
36 Ans (i)	The number of relations = $2^{4\times3} = 2^{12}$	1
36 Ans (ii)	Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective.	1
36 (iii) (a)	There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0. OR	2
36 (iii) (b)	To make R_1 reflexive and not symmetric we need to add the following ordered pairs: $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$	2
37.	Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated). Based on the above, answer the following :  (i) (a) What is the probability that a randomly selected car is an electric car ? **2** **OR** (i) (b) What is the probability that a randomly selected car is a petrol car ? **2** (ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ? **1** (iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ? **1**	

37(i) (a) Ans	Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is electric $P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$ $= \frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}$ $= \frac{155}{1000}\ or\ \frac{31}{200}$ OR	½ 1 ½
37(i)(b) Ans	Let A = Amber manufactures the car B = Bonzi manufactures the car C = Comet manufactures the car E = The selected car is a petrol car $P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$ $P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P(\frac{E}{C})$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100} + \frac{10}{100} \times \frac{95}{100}$ $= \frac{845}{1000}\ or\ \frac{169}{200}$	½ 1 ½
37(ii) Ans	$P\left(\frac{C}{E}\right) = \frac{P(C) \times P(\frac{E}{C})}{P(E)}$ $= \frac{\frac{10}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}}$ $= \frac{\frac{50}{10000}}{\frac{1550}{10000}} = \frac{1}{31}$	1

37(iii) Ans	$P\left(\frac{A \text{ or } B}{E}\right) = 1 - P\left(\frac{C}{E}\right) = 1 - \frac{1}{31} = \frac{30}{31}$	1
38.	 A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres. Based on the above, answer the following : (i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. 2 (ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. 2	
(i) Ans	$f'(x) = e^x(cosx + sinx)$ For critical points, $f'(x) = 0$ $\Rightarrow cosx + sinx = 0$ $\Rightarrow cosx = -sinx$ For x to be a critical point $x \in (0, \pi)$, hence, $x = \frac{3\pi}{4}$ For all $x \in \left[0, \frac{3\pi}{4}\right], f'(x) \geq 0$ Hence, f is increasing in $[0, \frac{3\pi}{4}]$ Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $\left(0, \frac{3\pi}{4}\right)$ or $\left[0, \frac{3\pi}{4}\right)$ or $(0, \frac{3\pi}{4}]$ For all $x \in \left[\frac{3\pi}{4}, \pi\right], f'(x) \leq 0$ Hence, f is decreasing in $[\frac{3\pi}{4}, \pi]$ Note: If a student concludes the answer in any of the following intervals, full marks may be awarded: $(\frac{3\pi}{4}, \pi)$ or $(\frac{3\pi}{4}, \pi]$ or $[\frac{3\pi}{4}, \pi)$	 ½ ½ ½ ½

(ii) Ans	$x = \frac{3\pi}{4}$ is a critical point $f''(x) = e^x(cosx - sinx) + e^x(cosx + sinx)$ $= 2e^x cosx$ $f''\left(\frac{3\pi}{4}\right) = -ve$ Hence, $\frac{3\pi}{4}$ is a point of local maximum.	 1 ½ ½

PAPER-3

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is :

(A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

2. If A and B are square matrices of same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to :

(A) A + B (B) BA

(C) 2 (A + B) (D) 2BA

3. For real x, let $f(x) = x^3 + 5x + 1$. Then :

(A) f is one-one but not onto on R

(B) f is onto on R but not one-one

(C) f is one-one and onto on R

(D) f is neither one-one nor onto on R

4. If $y = \sin^{-1} x$, then $(1 - x^2)\dfrac{d^2y}{dx^2}$ is equal to :

(A) $x\dfrac{dy}{dx}$ (B) $-x\dfrac{dy}{dx}$

(C) $x^2\dfrac{dy}{dx}$ (D) $-x^2\dfrac{dy}{dx}$

5. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :

(A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$

(C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$

6. If P is a point on the line segment joining (3, 6, –1) and (6, 2, – 2) and y-coordinate of P is 4, then its z-coordinate is :

(A) $-\dfrac{3}{2}$ (B) 0

(C) 1 (D) $\dfrac{3}{2}$

7. If M and N are square matrices of order 3 such that det (M) = m and MN = mI, then det (N) is equal to :

(A) – 1 (B) 1

(C) $-m^2$ (D) m^2

8. If $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to :

(A) – 4 (B) $-\dfrac{7}{2}$

(C) – 2 (D) – 1

9. If $f : N \to W$ is defined as

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases},$$

then f is :

(A) injective only

(B) surjective only

(C) a bijection

(D) neither surjective nor injective

10. The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a :

(A) diagonal matrix

(B) symmetric matrix

(C) skew symmetric matrix

(D) scalar matrix

11. If the sides AB and AC of Δ ABC are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is :

(A) $2\sqrt{2}$ units

(B) $\sqrt{18}$ units

(C) $\frac{\sqrt{34}}{2}$ units

(D) $\frac{\sqrt{48}}{2}$ units

12. The function f defined by

$$f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$$

is ***not*** continuous at :

(A) $x = 0$

(B) $x = 1$

(C) $x = 2$

(D) $x = 5$

13. If $f(x) = 2x + \cos x$, then f(x) :

(A) has a maxima at $x = \pi$

(B) has a minima at $x = \pi$

(C) is an increasing function

(D) is a decreasing function

14. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ is equal to :

(A) $2(\sin x + x\cos\alpha) + C$
(B) $2(\sin x - x\cos\alpha) + C$
(C) $2(\sin x + 2x\cos\alpha) + C$
(D) $2(\sin x + \sin\alpha) + C$

15. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is :

(A) $-\frac{\pi}{4}$
(B) $\frac{\pi}{4}$
(C) $\tan^{-1} e - \frac{\pi}{4}$
(D) $\tan^{-1} e$

16. The order and degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$ are :

(A) order 2, degree 2
(B) order 2, degree 1
(C) order 2, degree not defined
(D) order 1, degree not defined

17. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is :

(A) $\frac{16}{9}$ sq. units
(B) $\frac{32}{9}$ sq. units
(C) $\frac{16}{3}$ sq. units
(D) $\frac{32}{3}$ sq. units

18. The corner points of the feasible region of a Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is :

(A) $a = b$
(B) $a = 3b$
(C) $b = 6a$
(D) $3a = 2b$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R) : Two events are independent if the occurrence of one does not effect the occurrence of the other.

20. *Assertion (A) :* In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.

Reason (R) : A feasible region is defined as the region that satisfies all the constraints.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Let A and B be two square matrices of order 3 such that det (A) = 3 and det (B) = – 4. Find the value of det (– 6AB).

22. (a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on [2, 4].

OR

(b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.

23. (a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1 + x^2}}\right)$.

OR

(b) Find domain of $\sin^{-1}\sqrt{x-1}$.

24. Calculate the area of the region bounded by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the x-axis using integration.

25. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) If $f : R^+ \to R$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection.

(R^+ is a set of all positive real numbers.)

OR

(b) Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) Write all elements of R.

(ii) Is R a function ? Justify.

(iii) Determine domain and range of R.

27. (a) Find k so that

$$f(x) = \begin{cases} \dfrac{x^2 - 2x - 3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at $x = -1$.

OR

(b) Check the differentiability of function $f(x) = x|x|$ at $x = 0$.

28. Evaluate :

$$\int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right)dx$$

29. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

30. Find the distance of the point (–1, –5, –10) from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

31. Solve the following Linear Programming Problem using graphical method :

Maximise $Z = 100x + 50y$

subject to the constraints

$3x + y \leq 600$

$x + y \leq 300$

$y \leq x + 200$

$x \geq 0, y \geq 0$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. If A is a 3 × 3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

33. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight. *2*

(ii) In how many days will the plant attain its maximum height ? What is the maximum height ? *3*

34. (a) Find :

$$\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$$

35. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a} \times \vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

OR

(b) Find the equation of a line in vector and cartesian form which passes through the point (1, 2, – 4) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$, and

$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Some students are having a misconception while comparing decimals. For example, a student may mention that 78·56 > 78·9 as 7856 > 789. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47·7
Bijoy	47·07
Kartik	43·09
Dinesh	43·9
Devesh	45·2

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

(i) What is the probability of a student not having misconception but still answers Bijoy in the test ? *1*

(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? *1*

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? *2*

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? *2*

Case Study – 2

37. An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions :

(i) Find whether the two metro tracks are parallel. *1*

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$. *1*

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway. *2*

OR

(iii) (b) Find the shortest distance between Line A and Line B. *2*

Case Study – 3

38. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$,

where T(t) represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions :

(i) Find the expression for temperature of processor, T(t) given that $T(0) = 85°C$. *2*

(ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0{\cdot}03$, $\log_e 4 = 1{\cdot}3863$. *2*

PAPER-4

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is :

(A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

(C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$

(D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

2. If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is :

(A) $0{\cdot}3$

(B) 1

(C) $1{\cdot}3$

(D) $0{\cdot}7$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is :

(A) Only AB is defined.
(B) Only BA is defined.
(C) AB and BA, both are defined.
(D) AB and BA, both are not defined.

4. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(A) 3 (B) 7
(C) ± 7 (D) ± 3

5. If $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$
is continuous at $x = 0$, then the value of a is :

(A) 1 (B) -1
(C) ± 1 (D) 0

6. If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is :

(A) 0 (B) -10
(C) 10 (D) 1

7. The principal value of $\cot^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$ is :

(A) $-\dfrac{\pi}{3}$ (B) $-\dfrac{2\pi}{3}$
(C) $\dfrac{\pi}{3}$ (D) $\dfrac{2\pi}{3}$

8. If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is :

(A) 0 (B) 1
(C) -2 (D) -4

9. If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is :

(A) f is continuous but not differentiable at $x = 2$.

(B) f is neither continuous nor differentiable at $x = 2$.

(C) f is continuous as well as differentiable at $x = 2$.

(D) f is not continuous but differentiable at $x = 2$.

10. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :

(A) $(1, -10)$ (B) $(1, 10)$

(C) $(10, 1)$ (D) $(-10, 1)$

11. $\int \sqrt{1+\sin x}\, dx$ is equal to :

(A) $2\left(-\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$ (B) $2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$

(C) $-2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$ (D) $2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$

12. $\int_0^{\pi/2} \cos x . e^{\sin x}\, dx$ is equal to :

(A) 0 (B) $1 - e$

(C) $e - 1$ (D) e

13. The area of the region enclosed between the curve $y = x|x|$, x-axis, $x = -2$ and $x = 2$ is :

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$

(C) 0 (D) 8

14. The integrating factor of the differential equation

$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ is :

(A) $e^{-1/\sqrt{x}}$ (B) $e^{2/\sqrt{x}}$

(C) $e^{2\sqrt{x}}$ (D) $e^{-2\sqrt{x}}$

15. The sum of the order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2} \text{ is :}$$

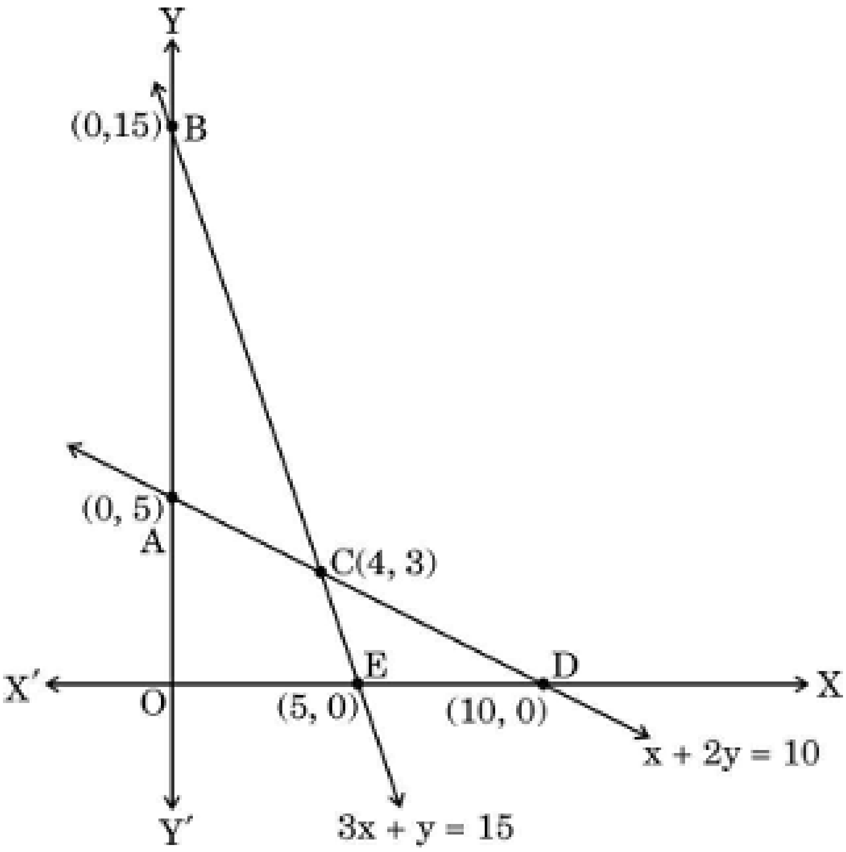
(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4

16. For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints :

$x + 2y \le 10$

$3x + y \le 15$

$x, y \ge 0$



The correct feasible region is :

(A) ABC (B) AOEC

(C) CED (D) Open unbounded region BCD

17. Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are :

(A) (– 2, – 2) (B) (2, – 2) (C) (– 2, 2) (D) (2, 2)

18. The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are :

(A) 48 and 16 (B) 3 and 1

(C) 24 and 8 (D) 6 and 2

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

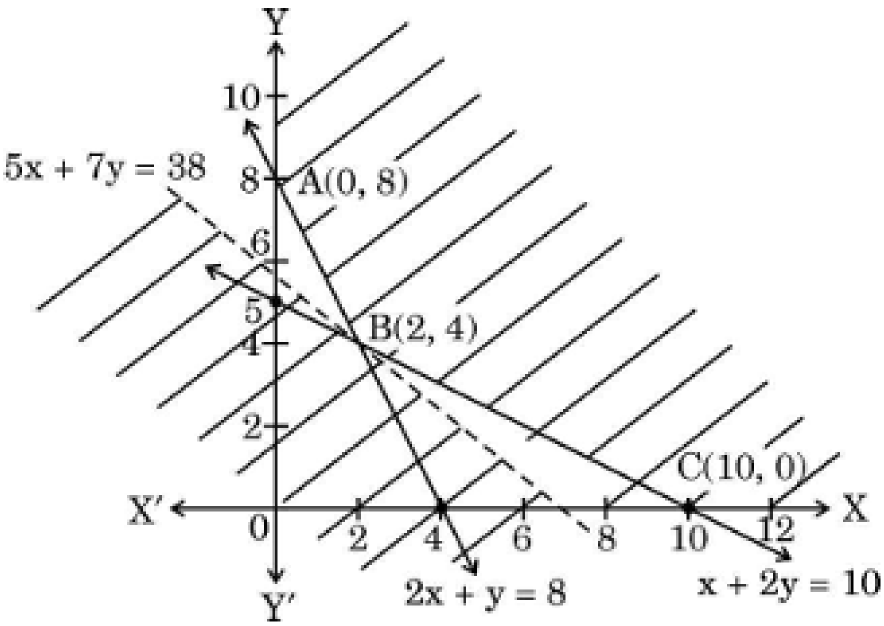
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min $Z = 50x + 70y$

subject to constraints

$2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

$Z = 50x + 70y$ has a minimum value = 380 at B(2, 4).

Reason (R) : The region representing $50x + 70y < 380$ does not have any point common with the feasible region.

20. *Assertion (A) :* Let $A = \{x \in R : -1 \le x \le 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.

22. Surface area of a balloon (spherical), when air is blown into it, increases at a rate of 5 mm^2/s. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing.

23. (a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x.

OR

(b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

24. (a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$.

OR

(b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.

25. A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Find the value of 'a' for which f(x) = $\sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in R.

27. (a) Find :

$$\int \frac{2x}{(x^2+3)(x^2-5)}\,dx$$

OR

(b) Evaluate :

$$\int_1^4 \left(|x-2| + |x-4|\right)\,dx$$

28. Find the particular solution of the differential equation

$$\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x\,dy = 0$$

given that $y = \frac{\pi}{4}$, when x = 1.

29. In the Linear Programming Problem (LPP), find the point/points giving maximum value for Z = 5x + 10y

subject to constraints

$x + 2y \le 120$

$x + y \ge 60$

$x - 2y \ge 0$

$x, y \ge 0$

30. (a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

31. (a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student :

(i) Buys both the colouring book and the box of colours.

(ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find :

(i) The probability distribution of the number of oranges he draws.

(ii) The expectation of the random variable (number of oranges).

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.

33. A furniture workshop produces three types of furniture – chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.

34. (a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

35. (a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

OR

(b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (–1, –1, 2).

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. *1*

(ii) Find $\frac{dS}{dx}$. *1*

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum. *2*

OR

(iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$. *2*

Case Study – 2

37. Let A be the set of 30 students of class XII in a school. Let $f : A \rightarrow N$, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following :

(i) Is f a bijective function ? *1*

(ii) Give reasons to support your answer to (i). *1*

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where

R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}.

List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. *2*

OR

(iii) (b) Let R be a relation defined by

R = {(x, y) : x, y are Roll Numbers of students such that $y = x^3$}.

List the elements of R. Is R a function ? Justify your answer. *2*

Case Study – 3

38. A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Radish Cabbage Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate. *2*

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ? *2*

PAPER-5

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is :

(A) $n \times n$ (B) $n \times m$

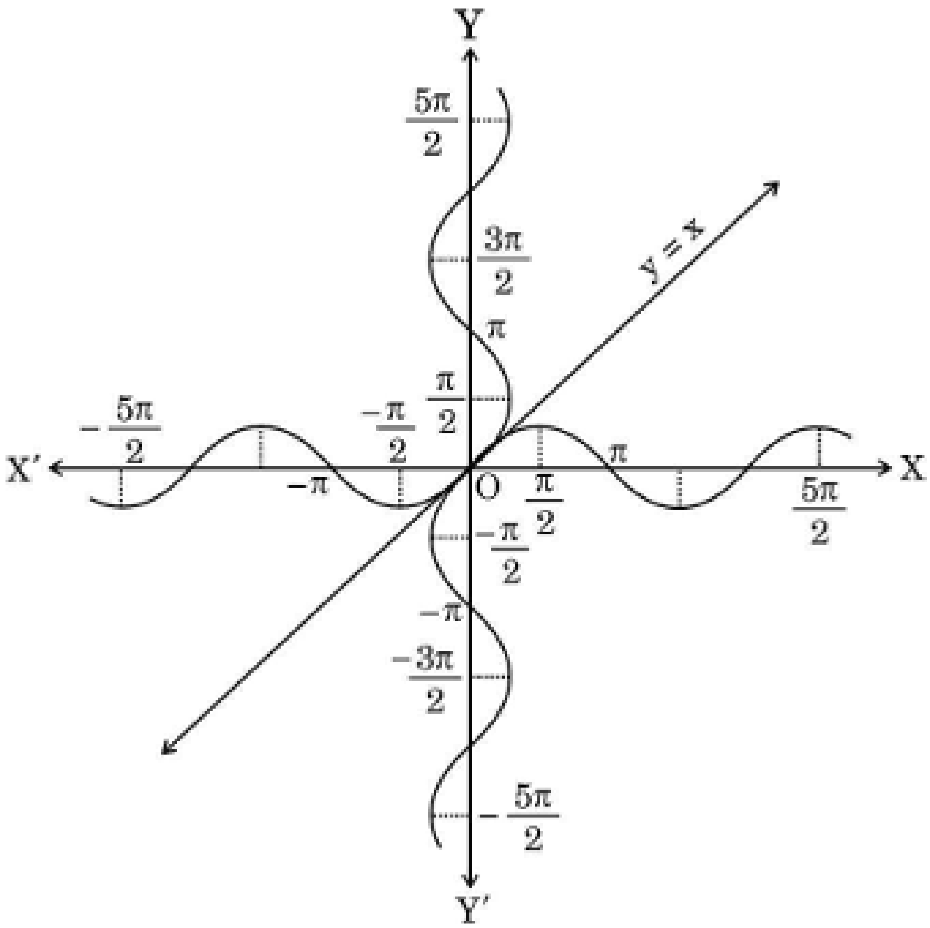
(C) $m \times m$ (D) $m \times n$

2. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an :

(A) scalar matrix (B) identity matrix

(C) symmetric matrix (D) skew-symmetric matrix

3. The following graph is a combination of :



(A) $y = \sin^{-1} x$ and $y = \cos^{-1} x$

(B) $y = \cos^{-1} x$ and $y = \cos x$

(C) $y = \sin^{-1} x$ and $y = \sin x$

(D) $y = \cos^{-1} x$ and $y = \sin x$

4. Sum of two skew-symmetric matrices of same order is always a/an :

(A) skew-symmetric matrix

(B) symmetric matrix

(C) null matrix

(D) identity matrix

5. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

6. If $f(x) = \begin{cases} \dfrac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

is continuous at x = 0, then the value of k is :

(A) a (B) a + b

(C) a − b (D) b

7. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\dfrac{dy}{dx}$ is :

(A) $\dfrac{x}{y}$ (B) $-\dfrac{x}{y}$

(C) $\dfrac{a}{x}$ (D) $\dfrac{a}{y}$

8. If y = a cos (log x) + b sin (log x), then $x^2y_2 + xy_1$ is :

(A) cot (log x) (B) y

(C) − y (D) tan (log x)

9. Let f(x) = |x|, x ∈ R. Then, which of the following statements is ***incorrect*** ?

(A) f has a minimum value at x = 0.

(B) f has no maximum value in R.

(C) f is continuous at x = 0.

(D) f is differentiable at x = 0.

10. Let $f'(x) = 3(x^2 + 2x) - \dfrac{4}{x^3} + 5$, f(1) = 0. Then, f(x) is :

(A) $x^3 + 3x^2 + \dfrac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \dfrac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \dfrac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \dfrac{2}{x^2} + 5x - 11$

11. $\displaystyle\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to :

(A) log (x + 6) + C (B) $e^x + C$

(C) $\dfrac{e^x}{x+6} + C$ (D) $\dfrac{-1}{(x+6)^2} + C$

12. The order and degree of the following differential equation are, respectively :

$$-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$$

(A) $-4, 1$ (B) 4, not defined

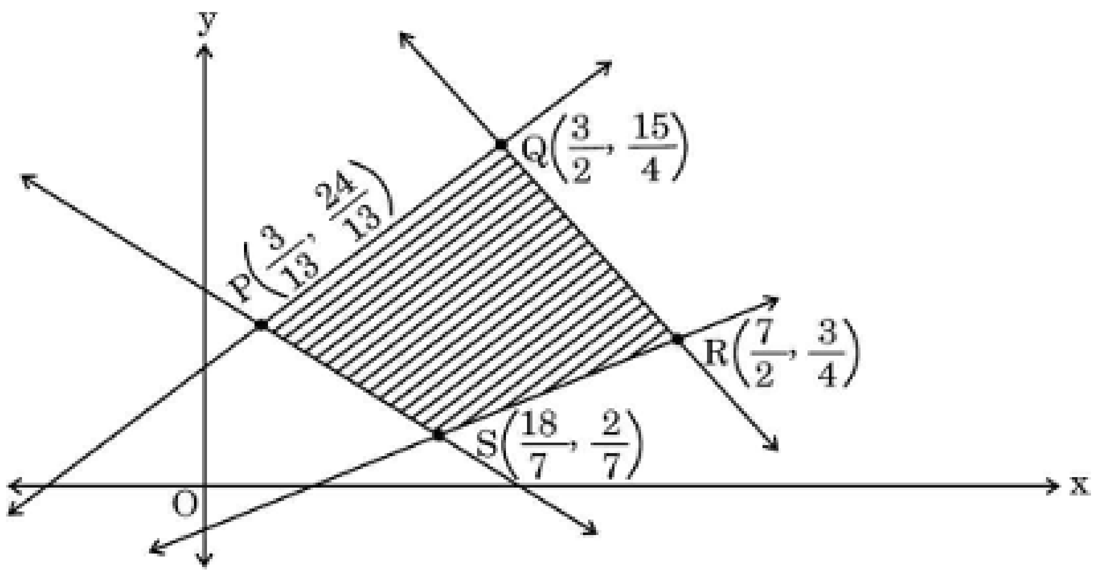
(C) 1, 1 (D) 4, 1

13. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

14. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



(Note : The figure is not to scale)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

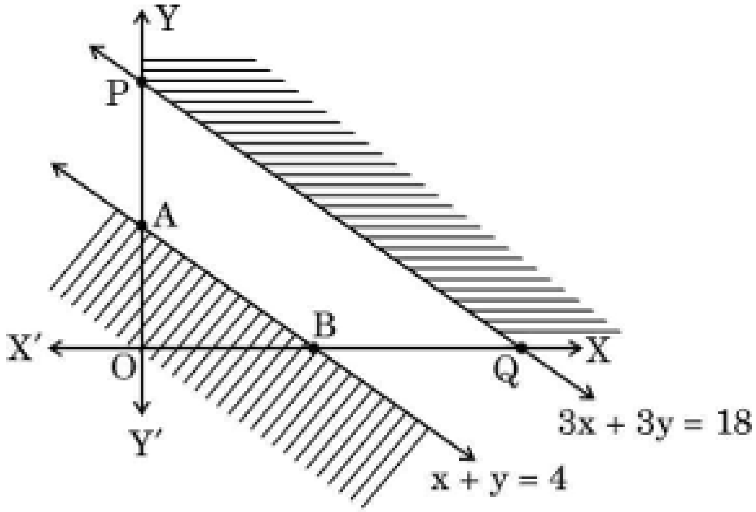
(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

15. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$$x + y \leq 4, \;\; 3x + 3y \geq 18, \;\; x, y \geq 0$$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

16. Let $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

17. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units

(C) 3 sq units (D) $\frac{4}{3}$ sq units

18. A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is :

(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2 \theta + \cos^2 \theta = 1$ and

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$.

20. *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

22. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

23. Determine the values of x for which $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function.

24. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

25. Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Let $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ be two matrices. Then, find the matrix B if AB = C.

27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

28. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \rightarrow N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if n is even} \\ n+1, & \text{if n is odd} \end{cases}$ is a bijection.

29. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$2x + y \geq 1000$

$x + 2y \geq 800$

$x, y \geq 0.$

Draw a neat graph of the feasible region and find the minimum value of Z.

30. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

31. A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :

(i) in both committees

(ii) in only one committee

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x^2 + 1}{(x-1)^2 (x+3)} \, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x} \, dx$$

33. Draw a rough sketch for the curve $y = 2 + |x + 1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x + 1|$, $x = -4$, $x = 3$ and $y = 0$.

34. (a) Solve the differential equation : $x^2y \, dx - (x^3 + y^3) \, dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

35. Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$.

A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1*

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}. *1*

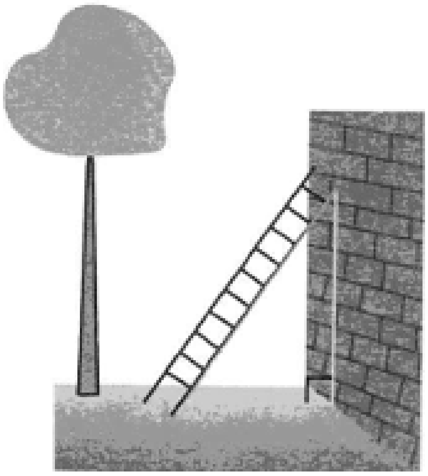
(iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2*

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*

Case Study – 2

37.



A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1*

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2*

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*

Case Study – 3

38. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. *2*

(ii) What is the probability that this defective smartphone was manufactured by company B ? *2*

PAPER 6

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

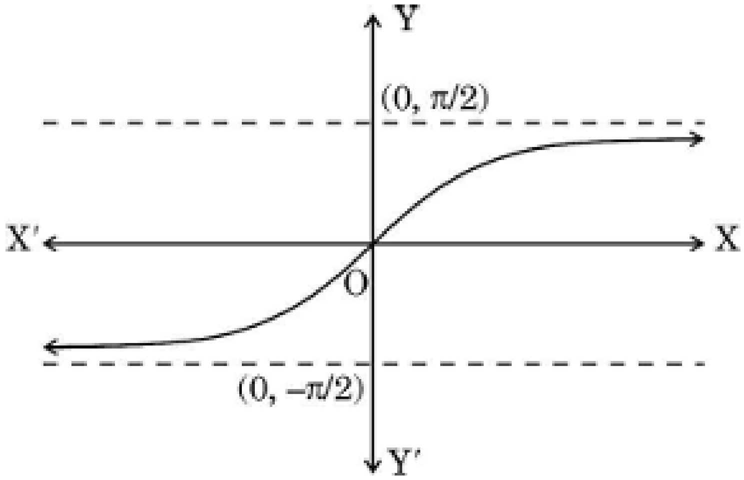
(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The given graph illustrates :



(A) $y = \tan^{-1} x$ (B) $y = \text{cosec}^{-1} x$

(C) $y = \cot^{-1} x$ (D) $y = \sec^{-1} x$

2. Domain of $f(x) = \cos^{-1} x + \sin x$ is :

(A) R (B) $(-1, 1)$

(C) $[-1, 1]$ (D) ϕ

3. What is the total number of possible matrices of order 3 × 3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(A) 9 (B) 512

(C) 615 (D) 64

4. The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an :

(A) scalar matrix (B) identity matrix

(C) null matrix (D) symmetric matrix

5. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is :

(A) 30 (B) 120

(C) 15 (D) 225

6. Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj } A|$ is :

(A) 5 (B) 125

(C) 25 (D) – 5

7. If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x - y)$ is :

(A) 2 or 10 (B) – 2 or 10

(C) 2 or – 10 (D) – 2 or – 10

8. If $f(x) = \begin{cases} 1, & \text{if } x \le 3 \\ ax + b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \le x \end{cases}$ is continuous in R, then the values of a and b are :

(A) $a = 3,\ b = -8$ (B) $a = 3,\ b = 8$

(C) $a = -3,\ b = -8$ (D) $a = -3,\ b = 8$

9. If $f(x) = -2x^8$, then the correct statement is :

(A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

(C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

10. A spherical ball has a variable diameter $\frac{5}{2}(3x + 1)$. The rate of change of its volume w.r.t. x, when x = 1, is :

(A) 225π (B) 300π

(C) 375π (D) 125π

11. If $f : R \to R$ is defined as $f(x) = 2x - \sin x$, then f is :

(A) a decreasing function (B) an increasing function

(C) maximum at $x = \frac{\pi}{2}$ (D) maximum at x = 0

12. $\int \frac{e^{9 \log x} - e^{8 \log x}}{e^{6 \log x} - e^{5 \log x}} dx$ is equal to :

(A) $x + C$ (B) $\frac{x^2}{2} + C$

(C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$

13. For a function f(x), which of the following holds true ?

(A) $\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$

(B) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an even function

(C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function

(D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a + x)\,dx$

14. $\int \frac{e^x}{\sqrt{4-e^{2x}}}\, dx$ is equal to :

(A) $\frac{1}{2} \cos^{-1}(e^x) + C$

(B) $\frac{1}{2} \sin^{-1}(e^x) + C$

(C) $\frac{e^x}{2} + C$

(D) $\sin^{-1}\left(\frac{e^x}{2}\right) + C$

15. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is :

(A) 6

(B) 1

(C) $\frac{1}{4}$

(D) 4

16. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are :

(A) orthogonal vectors

(B) parallel to each other

(C) unit vectors

(D) collinear vectors

17. If $P(A) = \frac{1}{7}$, $P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(\bar{A} \mid B)$ is :

(A) $\frac{6}{7}$

(B) $\frac{3}{4}$

(C) $\frac{4}{5}$

(D) $\frac{1}{5}$

18. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

(A) $\frac{2}{13}$

(B) $\frac{3}{26}$

(C) $\frac{19}{26}$

(D) $\frac{3}{13}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* $f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at $x = 0$.

Reason (R) : When $x \to 0$, $\sin \frac{1}{x}$ is a finite value between -1 and 1.

20. *Assertion (A) :* Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R) : $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$.

Discuss the bijectivity of the function.

22. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$.

23. (a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

OR

(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

24. In a Linear Programming Problem, the objective function $Z = 5x + 4y$ needs to be maximised under constraints $3x + y \leq 6$, $x \leq 1$, $x, y \geq 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.

25. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Show that the function $f : R \rightarrow R$ defined by $f(x) = 4x^3 - 5$, $\forall\ x \in R$ is one-one and onto.

OR

(b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy \text{ is a square of a natural number}, x, y \in N\}$. Determine if the relation R is an equivalence relation.

27. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?

28. Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.

29. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.

30. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

find the minimum value of Z.

31. (a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

OR

(b) Find the shortest distance between the lines :

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k}).$$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x^2+1}{(x^2+2)(2x^2+1)}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}\, dx$$

33. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

34. Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.

35. (a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).

OR

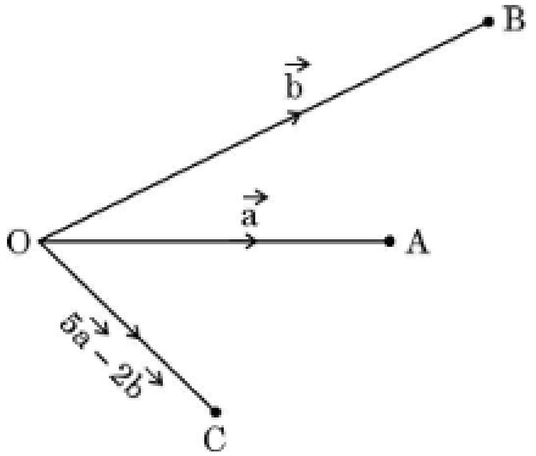
(b) Find the image of the point (−1, 5, 2) in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors. *1*

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. *1*

(iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$. *2*

OR

(iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. *2*

Case Study – 2

37. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.

(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation. *1*

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm. *1*

(iii) (a) If it is given that r = 3 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*

OR

(iii) (b) If it is given that r = 1 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*

Case Study – 3

38. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions :

(i) A person was tested randomly. What is the probability that he/she has contracted the disease ? *2*

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? *2*

PAPER 7

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

This section consists of **20** multiple choice questions, each of **1** mark.

1. Which of the following functions from Z to Z is both one-one and onto ?

(A) $f(x) = 2x - 1$ (B) $f(x) = 3x^2 + 5$

(C) $f(x) = x + 5$ (D) $f(x) = 5x^3$

2. Value of $4\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ is

(A) 3 (B) − 3

(C) 1 (D) − 1

3. If $A = \begin{bmatrix} x & 0 & m \\ y & z & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$, where I is a unit matrix, then $x + y + z + m$ is equal to

(A) 18 (B) 12

(C) 6 (D) 2

4. If $B = \begin{bmatrix} 23 & 41 & 57 \end{bmatrix} \begin{bmatrix} 31 & 42 \\ 53 & 64 \\ 75 & 86 \end{bmatrix}$, then the order of B is :

(A) 3×2 (B) 2×2

(C) 1×3 (D) 1×2

5. If A and B are square matrices of the same order, then $(A - B)^2 = ?$

(A) $A^2 - 2AB + B^2$ (B) $A^2 - AB - BA + B^2$

(C) $A^2 - 2BA + B^2$ (D) $A^2 - AB + BA + B^2$

6. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is

(A) 0 (B) 9

(C) − 6 (D) 6

7. If $A^{-1} = \begin{bmatrix} 7 & 2 \\ 8 & 2 \end{bmatrix}$, then matrix A is

(A) $\begin{bmatrix} 2 & -2 \\ -8 & 7 \end{bmatrix}$

(B) $\begin{bmatrix} -7 & 8 \\ 2 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 1 \\ 4 & -\frac{7}{2} \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -1 \\ -4 & \frac{7}{2} \end{bmatrix}$

8. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is

(A) $\frac{-\sqrt{x}}{\sqrt{y}}$

(B) $-\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$

(C) $-\frac{\sqrt{y}}{\sqrt{x}}$

(D) $\frac{-2\sqrt{y}}{\sqrt{x}}$

9. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) -1

10. When x is positive, the minimum value of x^x is

(A) e^e

(B) $\frac{1}{e}$

(C) $e^{\frac{1}{e}}$

(D) $e^{\frac{-1}{e}}$

11. $\int \frac{2x^3}{4+x^8} dx$ is equal to

(A) $\frac{1}{4}\tan^{-1}\frac{x^4}{2}+C$ (B) $\frac{1}{2}\tan^{-1}\frac{x^4}{2}+C$

(C) $\frac{1}{4}\tan^{-1}\frac{x^4}{4}+C$ (D) $\frac{1}{4}\tan^{-1}x^4+C$

12. $\int e^x \cdot \frac{x}{(1+x)^2} dx$ is equal to

(A) $e^x \cdot \frac{x}{1+x}+C$ (B) $e^x \cdot \frac{1}{1+x}+C$

(C) $e^x \cdot \frac{1}{x}+C$ (D) $e^x \cdot \frac{1}{(1+x)^2}+C$

13. The area of the region bounded by the lines $y = x + 1$, $x = 1$, $x = 3$ and x-axis is

(A) 6 sq units (B) 8 sq units

(C) 7.5 sq units (D) 2 sq units

14. The integrating factor for solving the differential equation $x \cdot \frac{dy}{dx} - y = 2x^2$ is

(A) x (B) $\frac{1}{x}$

(C) e^{-x} (D) $-\log x$

15. The number of vector(s) of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is (are) :

(A) one
(B) two
(C) three
(D) infinite

16. Of all the points of the feasible region, for maximum or minimum values of the objective function, the point lies :

(A) inside the feasible region
(B) at the boundary line of the feasible region
(C) at the corner points of the feasible region
(D) at the coordinate axes

17. The common region for the inequalities $x \geq 0$, $x + y \leq 1$ and $y \geq 0$, lies in

(A) IV Quadrant
(B) II Quadrant
(C) III Quadrant
(D) I Quadrant

18. A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{3}$. The probability that none of them is selected is :

(A) $\frac{11}{15}$
(B) $\frac{7}{15}$
(C) $\frac{8}{15}$
(D) $\frac{1}{5}$

Assertion – Reason Based Questions

Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true and Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A)** : The vectors $\vec{a} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$ are mutually perpendicular vectors.

    Reason (R) : Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, if $\vec{a} \cdot \vec{b} = 0$.

20. **Assertion (A)** : $x^2dy = (2xy + y^2)dx$ is a homogeneous differential equation.

    Reason (R) : A differential equation of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is a homogeneous differential equation.

SECTION – B

This section consists of **5** very short answer type questions, each of **2** marks.

21. Evaluate : $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

22. (a) Show that the function $f(x) = (x-1)^{\frac{1}{3}}$ is not differentiable at $x = 1$.

OR

(b) Differentiate $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ w.r.t. x.

23. If $y = 7x - x^3$ and x increases at the rate of 2 units per second, then how fast is the slope of the curve changing, when $x = 5$?

24. (a) If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then find $|\vec{a}|$.

OR

(b) Using vectors, find the value of K such that the points (K, –11, 2), (0, –2, 2) and (2, 4, 2) are collinear.

25. Find the angle between the two lines whose equations are $2x = 3y = -z$ and $6x = -y = -4z$.

SECTION – C

In this section there are **6** short answer type questions, each of **3** marks.

26. Find the intervals in which the function

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) strictly increasing

(b) strictly decreasing

27. (a) Find : $\int \frac{x^2 - x + 1}{(x-1)(x^2+1)} dx$

OR

(b) Evaluate : $\int_1^4 (|x| + |3-x|) dx$

28. (a) Find the particular solution of the differential equation, $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1 when $x = 0$.

OR

(b) Solve the differential equation : $2xy\frac{dy}{dx} = x^2 + 3y^2$.

29. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, then find a unit vector perpendicular to both the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

30. The corner points of the feasible region determined by some system of linear inequations, are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = ax + by$, where a, b > 0. Find the condition on a and b so that the maximum of Z occurs at both points (3, 4) and (0, 5).

31. (a) Find the probability distribution of the number of doublets in three throws of a pair of dice.

OR

(b) If E and F are two independent events with P(E) = p, P(F) = 2p and P(exactly one of E, F) = $\frac{5}{9}$, then find the value of p.

SECTION – D

This section consists of **4** long answer type questions, each of **5** marks.

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the system of equations :

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

33. (a) Differentiate $x^{\sin x} + (\sin x)^x$ w.r.t. x.

OR

(b) If $y = x + \tan x$, then prove that

$$\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$$

34. The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$. Find the value of a.

35. (a) Find the shortest distance between the lines given by

$\vec{r} = (4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$

OR

(b) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

SECTION – E

In this section there are **3** case-study based questions of **4** marks each.

36. An architect designs a building for a Company. The design of window on the ground floor is proposed to be different than at the other floors. The window is in the shape of a rectangle whose top length is surmounted by a semi-circular opening. This window has a perimeter of 10 m.

Based on the above information, answer the following :

(i) If $2x$ and $2y$ represent the length and breadth of the rectangular portion of the window, then establish a relation between x and y.

(ii) Find the total area of the window in terms of x.

(iii) (a) Find the values of x and y for the maximum area of the window.

OR

(iii) (b) If x and y represent the length and breadth of the rectangle, then establish the expression for the area of the window in terms of x only.

37. There are three categories of students in a class of 60 students : A : Very hardworking students, B : Regular but not so hard working, C : Careless and irregular students. It is known that 6 students in category A, 26 in category B and the rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination, is 0.002, of category B it is 0.02 and of category C, this probability is 0.20.

Based on the above information, answer the following :

(i) Find the probability that a student selected at random is unable to get good marks in the final examination. **2**

(ii) A student selected at random was found to be one who could not get good marks in the final examination. Find the probability, that this student is NOT of category A. **2**

38. Rajesh, a student of Class-XII, visited an exhibition with his family. There he saw a huge swing and found that it traced the path of a parabola $y = x^2$. The following questions came to his mind. Answer the questions :

(i) Let $f : R \to R$ be a function defined as $f(x) = x^2$. Find whether f is one-one function.

(ii) Let $f : R \to R$ be defined as $f(x) = x^2$. Find whether f is an onto function.

(iii) (a) Let $f : N \to N$ be defined as $f(x) = x^2$. Find whether f is one-one function. Also, find if it is an onto function.

OR

(iii) (b) Let $f : N \to \{1, 4, 9, 16, ...\}$ defined as $f(x) = x^2$, find where f is one-one function. Also, find if it is an onto function.