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TARGET-40

Class-12

Mathematics

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Long Answers Type Questions

Class-12 Mathematics (20 marks)

Relations & Functions

1. Show that the relation R in the set $A = \{1, 2, 3, 4, 5, \dots \dots \dots \dots 12\}$ given by $R = \{(a, b) : |a - b| \text{is divisible by } 4\}$ is an equivalence relation.
2. Let $A = \{1,2,3, \dots, 9\}$ and R be the relation in A X A defined by (a, b) R (c , d) iff $a + d = b + c$ for all a, b, c ,d $\in$ A .Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)]

Matrices & Determinants

3. Find the product AB, where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$,$B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations: $x - y = 3,\quad 2x + 3y + 4z = 17, y + 2z = 7$
4. If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find $A^{-1}$and hence solve the given equations: $2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3$
5. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find $A^{-1}$and hence solve the given equations: $2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3$

Applications of Integrals

6. Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.
7. Find the area of the region bounded by the parabola $y = x^2$ and the lines $y = |x|$.
8. Using integration find the area of region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1).
9. Sketch the graph of $y = |x + 3|$ and evaluate $\int_{-6}^{0} |x + 3| dx$
10. Using the method of integration, find the area of the region bounded by the line $2x + y = 4, 3x - 2y = 6$ and $x - 3y + 5 = 0$.

3 D Geometry

11. Find the image of the point P $(3,\ 5,\ 3)$in the line $\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$
12. Find the shortest distance between the lines whose vector equations are: $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$

13. Find the shortest distance between the following pair of lines:

$$\frac{x-1}{2}=\frac{y+1}{3}=z\ ;\qquad \frac{x+1}{5}=\frac{y-2}{1}, z=2$$

14. Find the equation of the perpendicular drawn from the point P (2,4, -1) to the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{9}$. Also write down the coordinate of foot of the perpendicular and image of P.

15. Find the equation of a line passing through $(2,-3,5)$ and perpendicular to the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$; $\frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$

16. Find the vector and Cartesian equations of the line passing through the point $(2,1,3)$ and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

17. Find whether the lines $\vec{r}=\left(\hat{\imath}-\hat{\jmath}-\hat{k}\right)+\lambda(2\hat{\imath}+\hat{\jmath})$ and $\vec{r}=(2\hat{\imath}-\hat{\jmath})+\mu\left(\hat{\imath}+\hat{\jmath}-\hat{k}\right)$ intersect or not. If intersecting, find their point of intersection.

Linear Programming

18. Minimise and Maximise $Z=3x+9y$. Subject to the constraints:
$x+3y\leq 60$; $x+y\geq 10$; $x\leq y$; $x\geq 0$; $y\geq 0$

19. Minimise $Z=50x+70y$. Subject to the constraints:
$2x+y\geq 8$; $x+2y\geq 10$; $x,y\geq 0$. (Unbounded)

Probability (Only Bayes' Theorem Based)

20. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time

21. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

22. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Short Answers Type Questions

Class-12 Mathematics (20 marks)

Relation & Functions

1. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.
   (i) f : R → R defined by $f\ (x) = 3 - 4x$ (ii) f : R → R defined by $f\ (x) = 1 + x^2$
2. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one.
3. If $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$ for all $x \in A$. Then show that f is bijective.
4. Show that the function $f : R \to R$ defined as $f(x) = x^2$ is neither one-one nor onto.
5. Check whether the relation $\boldsymbol{R}$ in R defined by $\boldsymbol{R} = \{(\boldsymbol{a}, \boldsymbol{b}) : \boldsymbol{a} \leq \boldsymbol{b}^3\}$ is reflexive, symmetric or transitive.

Inverse Trigonometric Functions

6. Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$
7. Simplify: $\tan^{-1}\left[2\cos\left\{2\sin^{-1}\left(\frac{1}{2}\right)\right\}\right]$
8. Prove that: $\tan\left\{\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right\} = \frac{3-\sqrt{5}}{2}$
9. Prove that: $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

Matrices & Determinants

10. If A is a square matrix of order 3, |A'| = – 3, then |AA'| =?
11. For what value of x, the matri$x$: $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?
12. If A is a square matrix of order 3 such that | adj A | = 64, find |3A|
13. If A is a square matrix of order 3 and $|3\ \mathrm{A}| = \mathrm{K}|\mathrm{A}|$, then write the value of K
14. If A is a square matrix of order 3 such that $|\mathrm{adj}\ \mathrm{A}| = 225$, find $|\mathrm{A}'|$
15. Express A= $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$ as sum of symmetric and skew symmetric matrices.
16. If A= $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$,then find the value of $A^2 - 3A + 2I$.
17. If A= $\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ and BA= (b_{ij}), find $\mathrm{b}_{21} + \mathrm{b}_{32}$.

18. If Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 0 & -3 \\ 3a & 3 & 0 \end{bmatrix}$ is skew-symmetric, find the values of a and b.

Continuity and Differentiability

19. (a) If f(x)=$\begin{cases} \frac{1-\cos 4x}{x^2}, & \text{if} x < 0 \\ a\ , & \text{if} x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if} x > 0 \end{cases}$ is continuous at $x = 0$,Find the value of 'a'

(b) Find value of 'k' for which f(x) =$\begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ x \neq 0 \\ k & if\ x = 0 \end{cases}$ is continuous at x=0.

(c) Find the value of 'k' for which f(x) =$\begin{cases} \frac{1-\cos kx}{x \sin x}, & if\ x \neq 0 \\ \frac{1}{2} & if\ x = 0 \end{cases}$ is continuous at x=0.

(d) Find the value of k so that $f(x) = \begin{cases} \frac{\text{k} \cos x}{\pi - 2x}, & \text{if} x \neq \frac{\pi}{2} \\ 5, & \text{if} x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$

20. The function f (x) is defined as f (x) = $\begin{cases} x^2 + ax + b & ,0 \leq x < 2 \\ 3x + 2 & ,2 \leq x \leq 4 \\ 2ax + 5b & ,4 < x \leq 8 \end{cases}$

If f(x) is continuous on[0 ,8] , find the values of 'a' and 'b'

21. (a) Find the derivative of $\sin x$ with respect to $\cos x$.

(b) Find the derivative of e^x with respect to $\cos x$.

22. If $x = a\left(\cos\theta + \log \tan\frac{\theta}{2}\right)$ $and\ y = a \sin\theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$

23. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$

24. Find, $\frac{dy}{dx}$ of the function $y = (\log x)^x + x^{\sin x}$

25. If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log\ x}{\{\log(xe)\}^2}$

26. If $y = (\tan^{-1}x)^2$, then prove that $(1 + x^2)^2 y_2 + 2x\,(1 + x^2)y_1 = 2$

Application of Derivatives

27. Find the intervals in which the function f $(x) = -2x^3 - 6x^2 + 18x + 11$ is (a) strictly increasing (b) strictly decreasing.

28. Prove that the function f$(x) = x^3 - 6x^2 + 12x - 18$ is increasing on R

29. Find the points of local maxima, local minima and the points of inflection of the function f $(x) = x^5 - 5x^4 + 5x^3 - 1$. Also, find the corresponding local maximum and local minimum values.

Differential Equations

30.(a) Find the sum of the degree and the order for the following differential equation: $\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right] = 0$

(b) Find the product of the order and degree of the differential equation: $x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + y^2 = 0$

(c) Find the sum of order and degree of differential equation $(y'')^2 + (y''')^3 + (y')^4 + y^5 = 0.$

(d) Write the degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + x^3 = 0.$

31.(a) Find the integrating factor of the following differential equation: $x \log x \frac{dy}{dx} + y = 2\log x$

(b) Find the integrating factor of the differential equation $\frac{dy}{dx} + xy = x^2$.

(c) Write the integrating factor of the differential equation:

$$(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$$

32.Solve the differential equation:$x^2\frac{dy}{dx} = 2xy + y^2$. when $y = 1$ and $x = 1$

33.Solve: $\frac{dy}{dx} + y\cot x = 4\,x \operatorname{cosec} x$, given $y = 0$ when $x = \frac{\pi}{2}$

34.Solve the differential equation: $\frac{dy}{dx} - 3y\cot x = \sin 2x$, given $y = 2$ when $x = \frac{\pi}{2}$

35.Write solution of the differential equation: $\frac{dy}{dx} = e^x + 2x$.

36.Find the particular solution of the differential equation $\frac{dy}{dx}$ = y tanx, y=1 when x=0.

37.Find the general solution of $\frac{dy}{dx}$ + 2 tanx y = sinx.

38.Solve $\frac{dy}{dx}$ = 1 + x + y + xy.

39.Show that the diff. equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is homogeneous and solve it.

40.Solve the differential equation:$(1+x^2)\frac{dy}{dx} + y = e^{tan^{-1}x}$

Vectors

41.Write the value of

(a) $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{k} \times \hat{\imath}) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$.

(b) $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$.

42.Find the projection of $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ on $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$.

43.Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$

44.Find the value of 'p' for which vectors $3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$ are parallel.

45. For what values of μ are the vectors $\vec{a} = 2\hat{\imath} + \mu\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ perpendicular to each other.?

46. Find angle θ between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} - \hat{k}$

47. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} . \vec{b}|^2 = 400$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$.

48. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k}$, find a unit vector in the direction of $\vec{a} - 2\vec{b}$

49. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

50. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 5$, $|\vec{b}| = 12$ and $|\vec{c}| = 13$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of $\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}$

51. Find the area of the triangle with vertices A $(1, 1, 2)$, B $(2, 3, 5)$ and C $(1, 5, 5)$

52. If θ is the angle between two vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, find $\sin\theta$.

53. Find the area of parallelogram, whose adjacent sides are determined by the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$.

54. Show that the points with position vectors $2\hat{\imath} + \hat{\jmath} - \hat{k}$, $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ are collinear.

55. Find the area of a parallelogram whose diagonals are determined by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{b} = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

56. Find a vector in the direction of vector $5\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ which has magnitude 8 units.

Three-Dimensional Geometry

57. Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$

58. (a) Write the direction ratios of the line: $3x + 1 = 6y - 2 = 1 - z$

(b) The equation of a line is $5x - 3 = 15y + 7 = 3 - 10z$ write the direction cosines of the line.

(c) Write the direction ratios of the following line: $x = -3, \frac{y-4}{3} = \frac{2-z}{1}$

(d) The equation of a line is given by $\frac{4-x}{2} = \frac{y+3}{3} = \frac{z+2}{6}$. Write the direction cosines of a line parallel to given line.

59. If the Cartesian equations of a line are, $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line.

60. (a) Find the value of p, so that the lines $\frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other.

(b) Find the values of p so that the lines: $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles

61.(a) Find the value of λ so that the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular to each other.

(b) Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = z$ and $x = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

62.Find the vector and cartesian equation of the line that passes through the points $(3, -2, -5)$ *and* $(3, -2, 6)$

Probabilities

63.Find the probability distribution of

(a) number of heads in two tosses of a coin.

(b) number of tails in the simultaneous tosses of three coins.

(c) number of heads in four tosses of a coin.

64.Two cards are drawn successively with replacement from a well- shuffled pack of 52 playing cards. Find the probability distribution of number of kings and hence find the mean of the distribution.

65.From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

66.(a) A random variable X has the following distribution table:

X	0	1	2	3	4	5	6	7
P (X)	0	k	2 k	2 k	3 k	k^2	$2k^2$	$7k^2 + k$

Determine (i) k (ii) $P(X < 3)$ (III) $P(X > 6)$ (iv) $P(X \leq 3)$

(b) The following probability is distribution of random variable X. Find x.

X	0	1	2	3
P(X)	$\frac{1}{10}$	x	$\frac{3}{10}$	$\frac{4}{10}$

(c) A random variable X has the following distribution table:

X	1	2	3	4
P (X)	k	2k	3k	4k

Determine $P(X < 3)$

67.Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

68.Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

69.Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

70.If $P(A) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$, then find P(B)

71.If $P(A \cap \overline{B}) = 0.15, P(B) = 0.10$, then find the value of $P(A/\overline{B})$

72.If $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$, and $P(A \cup B) = \frac{3}{4}$ then find the value of $P(A/B). P(A'/B)$.

73. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then find the value of $P(A' \cap B')$

74. Given $P(E) = 0.8, (F) = 0.7, (E \cap F) = 0.6$. Find $P(\bar{E}/\bar{F})$

75. If A and B are two independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then find $P(B'/A)$

76. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

77. If A and B are two independent events with $\mathbf{P(A) = 0.3}$ and $\mathbf{P(B) = 0.4}$, then find the value of (i) $\mathbf{P(A \cap B)}$ (ii) $\mathbf{P(A \cup B)}$ Ans: (i) 0.12 (ii) 0.58

78. If A and B are two independent events such that $P(\bar{\mathbf{A}} \cap \mathbf{B}) = \frac{\mathbf{2}}{\mathbf{15}}$ and $P(\mathbf{A} \cap \bar{\mathbf{B}}) = \frac{\mathbf{1}}{\mathbf{6}}$, then find $P(\mathbf{A})$ and $P(\mathbf{B})$

Integrals

79. (a) $\int_{-2}^{3} |x-1| dx$ (b) $\int_{0}^{2} |2x-1| dx$ (c) $\int_{0}^{2} |5-x| dx$
    (d) $\int_{0}^{2} |x+5| dx$ (e) $\int_{-1}^{2} |x^3 - x|\, dx$
    (f) $\int_{-1}^{2} (|x+1| + |x| + |x-1|) dx$

80. $\int logx\, dx$

81. $\int_0^{\frac{\pi}{2}} \log sinx\, dx$

82. (a) $\int \frac{x^3+3x+4}{\sqrt{x}} dx$ (b) $\int sec\,x\,(sec\,x + tan\,x) dx$

83. $\frac{tan^4 \sqrt{x}\, sec^2 \sqrt{x}}{\sqrt{x}}$

84. (i) $\int \frac{e^{2x}-1}{e^{2x}+1} dx$ (ii) $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} dx$

85. $\int tan^2(2x-3)\, dx$

86. (i) $\int cos^2 x\, dx$ (ii) $\int sin^3 x\, dx$
    (iii) $\int sin^3 x\, cos^3 x dx$ (iv) $\int sin^3 x\, cos^2 x\, dx$ (v) $\int sin^2 x\, dx$

87. (i) $\int \frac{1}{\sqrt{7-6x-x^2}} dx$ (ii) $\int \frac{x+2}{\sqrt{x^2+2x+3}} dx$
    (iii) $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$ (iv) $\int \frac{x^2+1}{x^2-5x+6} dx$ (v) $\int \frac{1}{3x^2+13x-10} dx$

88. (i) $\int e^x \left(tan^{-1} x + \frac{1}{1+x^2}\right) dx$ (ii) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) \mathrm{dx}$

89. (i) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sin^2 x\, dx$ (ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$

90. (i) $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$ (ii) $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{tanx}} dx$
    (iii) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{cot\,x}}{\sqrt{cot\,x}+\sqrt{tan\,x}} dx$ (iv) $\int_0^{a} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$

*****BEST WISHES*****