PUMDET-2024

Subject : MATHEMATICS

4032100581
(Booklet Number)

Duration : **90** Minutes | No. of Questions : **50** | Full Marks : **100**

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks **2.** In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
3. Use only **Black/Blue ink ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR sheet.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR Sheet**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your signature (as is appeared in Admit Card) in appropriate boxes in the OMR Sheet.
7. The OMR Sheet is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/signature of the candidate, name of the examination centre. The OMR Sheet may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones, bluetooth devices etc. inside the examination hall. Any candidate found with such prohibited items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough work must be done on the question booklet itself. Additional blank pages are given in the question booklet for rough work.
10. Hand over the OMR Sheet to the invigilator before leaving the Examination Hall.
11. Candidates are allowed to take the Question Booklet after examination is over.

Signature of the Candidate : ____________________
(as in Admit Card)

Signature of the Invigilator : ____________________

SPACE FOR ROUGH WORK

Mathematics

1. Which of the following are respectively the supremum and infimum (if exist) of the set ?

$\{ x \in \mathbb{R} : x \sin \frac{1}{x} = 0 \}$

(A) $\pi, 0$ (B) $\frac{1}{\pi}, 0$

(C) $\frac{1}{\pi}, -\frac{1}{\pi}$ (D) $\pi, -\pi$

2. Which of the following is the derived set of $\{ x \in \mathbb{R} : \big| \, |3 - x| - |x + 2| \, \big| = 5\}$

(A) $[3, \infty)$ (B) $(-\infty, -2]$

(C) $(-\infty, -3] \cup [2, \infty)$ (D) $(-\infty, -2] \cup [3, \infty)$

3. $\underset{n \to \infty}{\text{Lt}} [\log (6 + n^2) - 3 \log n]$ is

(A) 0 (B) $-\infty$

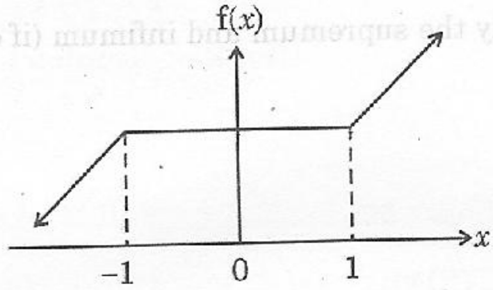
(C) ∞ (D) 1

4. The function $f(x) = \begin{cases} x, x \in Q \\ \frac{1}{x^2}, x \in \mathbb{R} \setminus Q \end{cases}$ is continuous

(A) nowhere (B) at exactly two points

(C) at exactly one point (D) at infinitely many points

5. The graph of a function f is given as follows :



Then f is differentiable

(A) everywhere except the points 1, −1

(B) everywhere except the point 1

(C) everywhere except the point −1

(D) nowhere

6. The integral $\int\limits_{|z|=2} \frac{\cos z}{z^3}\, dz$ equals

(A) πi (B) $-\pi i$

(C) $2\pi i$ (D) $-2\pi i$

7. For the function $f(z) = z \sin\left(\frac{1}{z-1}\right)$, the point $z = 1$ is

(A) an essential singularity (B) a pole of order one

(C) a removable singularity (D) a non-singular point

8. The sum of the residues at the poles of the function $f(z) = \frac{z^2+1}{z^2-2z}$ is

(A) 2 (B) 3

(C) 4 (D) 6

9. The Mobius transformation which fixes 1, 3, 5 is

(A) $w = \frac{1-z}{1+5z}$ (B) $w = \bar{z}$

(C) $w = \text{Re}(z)$ (D) the identity transformation

10. On $\mathbb{R}^2$ define a metric as follows :

$\rho((x_1, y_1), (x_2, y_2)) = \text{Max.} \{|x_1 - x_2|, |y_1 - y_2|\}$, for $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2$.

Then the set $S = \{ (x, y) \in \mathbb{R}^2 : \rho((x, y),(0, 0)) = 1 \}$ is

(A) a circle (B) an ellipse

(C) a square (D) a rhombus

11. On $\mathbb{N}$ define a metric as follows :

$$D(m, n) = \begin{cases} |m-n|, \text{ if both m, n are either even or odd} \\ 1+|m-n|, \text{otherwise} \end{cases}$$

then

(A) $(\mathbb{N}, d)$ is complete

(B) $(\mathbb{N}, d)$ is incomplete

(C) any open set in $(\mathbb{N}, d)$ is infinite

(D) $(\mathbb{N}, d)$ is compact

12. Let G be a group and $a \in G$ with order of a is 24. Then order of a^{16} is

(A) 8 (B) 3

(C) 16 (D) 12

13. The group $\mathbb{Z}_2 \times \mathbb{Z}_2$ is

(A) cyclic

(B) isomorphic to Kleine's 4-group

(C) isomorphic to $\mathbb{Z}_4$

(D) none of these

14. The order of the center of the symmetric group S_3 is

(A) 1 (B) 2

(C) 6 (D) 3

15. The ring of all 3×3 matrices of real numbers is a

(A) Commutative ring without zero divisors

(B) Commutative ring with zero divisors

(C) Non-Commutative ring without zero divisors

(D) Non-Commutative ring with zero divisors

16. Let $f(x)$ be an n^{th} degree irreducible polynomial over $\mathbb{Z}_p$, p being a prime number. Then the number of elements of the field $\mathbb{Z}_p[x]/\langle f(x)\rangle$ is

(A) p (B) n

(C) p^n (D) n^p

17. Let V be the set of all 4 × 4 real matrices $A = (a_{ij})$ such that $a_{11} + a_{22} + a_{33} + a_{44} = 0$. Then V is a real vector space of dimension

(A) 4 (B) 16

(C) 15 (D) V is not a vector space

18. A linear transformation T rotates each vector in $\mathbb{R}^2$ clockwise through 90°. The matrix corresponding to T with respect to the standard ordered basis $\left(\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right)$ is

(A) $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$ (B) $\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$

(C) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ (D) $\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$

19. The characteristic polynomial of the matrix $A = \begin{bmatrix}2 & 1\\3 & -1\end{bmatrix}$ is

(A) $x^2 - x - 5$ (B) $x^2 - 2x + 5$

(C) $x^2 - x + 5$ (D) $x^2 - 2x - 5$

20. The sum of all positive integers less than 101 and prime to 101 is

(A) 4040 (B) 2020

(C) 5050 (D) 4141

21. Let $n \in \mathbb{N}$. Then the complex number $\left(\frac{\sqrt{3}+i}{2}\right)^n$ is purely imaginary if and only if

(A) $n = 0 \pmod 6$ (B) $n = 1 \pmod 6$

(C) $n = 2 \pmod 6$ (D) $n = 3 \pmod 6$

22. If one of the roots of the equation $x^3 + ax^2 + bx + c = 0$ is equal to the sum of the other two then

(A) $a^3 + 8c - 8ab = 0$ (B) $a^3 - 8c - 4ab = 0$

(C) $a^3 - 8c + 4ab = 0$ (D) $a^3 + 8c - 4ab = 0$

23. If $z = \cos\frac{8\pi}{11} + i\sin\frac{8\pi}{11}$ then $\mathrm{Re}\,(z + z^2 + z^3 + z^4 + z^5)$ is equal to

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

24. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d > 0. Then the eigen values of A are

(A) complex
(B) real but equal
(C) real and distinct
(D) all complex and equal

25. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to

(A) $\pm\frac{1}{3} |\vec{a}| \cdot |\vec{b}| \cdot |\vec{c}|$
(B) $\pm |\vec{a}| \cdot |\vec{b}| \cdot |\vec{c}|$
(C) 0
(D) $\pm \frac{1}{|\vec{a}| \cdot |\vec{b}| \cdot |\vec{c}|}$

26. The directional derivative of $\phi = x^2 + z^2 - y^2$ at the point (1, 3, 2) is maximum in the direction

(A) $\hat{i} - 2\hat{j} + 2\hat{k}$
(B) $2\hat{i} - 6\hat{j} + 4\hat{k}$
(C) $2\hat{i} + 6\hat{j} - 4\hat{k}$
(D) $2\hat{i} - 6\hat{j} - 4\hat{k}$

27. The area bounded by the bisectors of the angle between the lines $x^2 - y^2 + 2y = 1$ and the lines $x - y = a$ is

(A) $\frac{a(a-1)}{2}$
(B) $\frac{a^2-1}{2}$
(C) $\frac{1}{2}(a+1)^2$
(D) $\frac{(a-1)(a-2)}{2}$

28. The locus of a point $p(\alpha, \beta)$ such that the lines $y = \alpha x + \beta$ are tangents to the hyperbola $a^2x^2 - b^2y^2 = a^2b^2$ is

(A) an ellipse (B) a circle

(C) a parabola (D) a hyperbola

29. The surface generated by a straight line which intersects the lines $y = 0, z = 1$; $x = 0, z = -1$ and the curve $z = 0, xy + 1 = 0$ is

(A) $z^2 + 1 = xy$ (B) $z^2 + xy = 1$

(C) $z^2 - 1 = xy$ (D) $z - 1 = xy$

30. The singular solution to the differential equation $y = px + p^3$, where $p = \frac{dy}{dx}$ is

(A) $4y^3 + 27x^2 = 0$ (B) $4x^2 + 27y^3 = 0$

(C) $4y^2 + 27x^3 = 0$ (D) $4x^3 + 27y^2 = 0$

31. The differential equation $\left|\frac{dy}{dx}\right| + |y| = 0, y(0) = 1$ has

(A) no solution (B) a unique solution

(C) finitely many solutions (D) infinitely many solutions

32. A solution curve of the equation $xy' = 2y$ passing through (1, 2) also passes through

(A) (2, 1) (B) (0, 0)

(C) (4, 24) (D) (1, 1)

33. The complete integral of the partial differential equation $\frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} = 2xy$ is

(A) $z = x^2 + y^2 + c$

(B) $z = \frac{1}{2}x^2 + y^2 + c$

(C) $z = x^2 - y^2 + c$

(D) $z = x^2 y^2 + c$

34. The number of extreme points of the convex set

$S = \{(x, y) \in \mathbb{R}^2 : |x| \leq 1, |y| \leq 1\}$ is

(A) 0

(B) 2

(C) 4

(D) infinite

35. The value of the objective function at an optimal solution of the L.P.P.

Max. $Z = 10x + 15y$

Subject to $x + y \geq 2$

$3x + 2y \leq 6$

$x, y \geq 0$ will be

(A) 30

(B) 45

(C) 20

(D) 25

36. In 2 × 2 pay-off matrix $\begin{bmatrix} 6 & 1 \\ x & 5 \end{bmatrix}$ the value of the game is 5, then the value of x is

(A) 0

(B) 1

(C) 3

(D) 5

37. Given $f(x) = \frac{1}{x}$ for x = a, b, c. The second order divided difference f[a, b, c] is equal to

(A) 1

(B) $\frac{1}{a} - \frac{2}{b} + \frac{1}{c}$

(C) $\frac{1}{abc}$

(D) $\frac{-1}{abc}$

38. The Newton – Raphson iteration formula for the square root of the real number (A – 5), where A > 5, is

(A) $x_{n+1} = \frac{x_n^{\ 2} - A + 5}{2x_n}$

(B) $x_{n+1} = \frac{x_n^{\ 2} + A - 5}{2x_n}$

(C) $x_{n+1} = x_n - \frac{x_n}{\sqrt{A-5}}$

(D) $x_{n+1} = x_n - \frac{1}{\sqrt{A-5}}$

39. The Runge – Kutta method of order two is used to solve the differential equation $\frac{dy}{dx} = f(x)$, y(0) = 0 with step size h. Then the solution at x = h is given by

(A) $y(h) = \frac{h}{4}\left[f(0) + 2.f\left(\frac{h}{2}\right) + f(h)\right]$

(B) $y(h) = \frac{h}{2}\left[2f(0) + f\left(\frac{h}{2}\right) + 2f(h)\right]$

(C) $y(h) = \frac{h}{6}\left[f(0) + 4.f\left(\frac{h}{2}\right) + f(h)\right]$

(D) $y(h) = \frac{h}{2}\left[f(0) + f(h)\right]$

40. Consider the function $f(x) = \sqrt{2+x}$, for $x \geq -2$ and the iteration $x_{n+1} \doteq f(x_n)$, $n \geq 0$, $x_0 = 1$. What are the possible limits of the iteration ?

(A) 0
(B) –1
(C) 2
(D) 1

41. A particle moves in a straight line from a fixed point O with velocity v under a force which produces an acceleration $\mu^2 x$, x being its distance from O. Then the distance for the velocity to be increased to 2v is

(A) $\sqrt{2}\,\mu\,V$
(B) $\frac{\sqrt{2}}{\mu}V$
(C) $\frac{\sqrt{3}}{\mu}V$
(D) $\frac{\mu}{\sqrt{3}}V$

42. If two bodies A, B at rest having masses in the ratio 2:1 are subjected to forces in the ratio 1:2 for equal interval of time, then the ratio of their momentum will be

(A) 1:1
(B) 1:2
(C) 2:1
(D) 1:3

43. If the radial velocity is proportional to the transverse velocity, then the path is

(A) a conic
(B) an equiangular spiral
(C) a cardiode
(D) a straight line

44. A heavy particle slides down a smooth cycloid $s = 4a\sin\psi$ whose axis is vertical and vertex downwards starting from rest at the cusp. The velocity of the particle when it reaches the lowest point is

(A) $2\sqrt{ag}$ (B) $\sqrt{ag}$

(C) $4\sqrt{ag}$ (D) $\sqrt{2ag}$

45. Let $f(x) = \begin{cases} c.|x|, -1 < x < 1 \\ 0 \quad ,\text{elsewhere} \end{cases}$

Then f is a possible probability density function if

(A) $c = 0$ (B) $c = 1$

(C) $c = \frac{1}{2}$ (D) c is any real number

46. Mean and Variance are same for

(A) Poisson distribution (B) Uniform distribution

(C) Log normal distribution (D) Exponential distribution

47. Let $f(x, y) = x^5 y^2 \tan^{-1}\left(\frac{y}{x}\right)$

Then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to

(A) $2f$ (B) $3f$

(C) $5f$ (D) $7f$

48. Let $\vec{F} = ay\ \hat{i} + z\hat{j} + x\hat{k}$ and c be the positively oriented closed curve given by

$x^2 + y^2 = 1, z = 0$. If $\oint_{V} \vec{F} . d\vec{r} = \pi$, then the value of a is

(A) -1 (B) 0

(C) $\frac{1}{2}$ (D) 1

49. If X is a Poisson random variable with mean 2, then $P(|X - 3| < 1)$ will be

(A) $\frac{3}{2}e^{-2}$ (B) $3\,e^{-2}$

(C) $\frac{4}{3}e^{-2}$ (D) $\frac{3}{4}e^{-2}$

50. The number of basic solutions of the following linear equations :

$4\,x_1 + 2\,x_2 + 3\,x_3 - 8\,x_4 = 6$

$3\,x_1 + 5\,x_2 + 4\,x_3 - 6\,x_4 = 8$

are

(A) 6 (B) 3

(C) 4 (D) 5