PUMDET-2024

Subject : PHYSICS/ASTROPHYSICS

4032300798
(Booklet Number)

Duration : 90 Minutes   No. of Questions : 50   Full Marks : 100

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks 2. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
3. Use only **Black/Blue ink ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR sheet.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR Sheet.** Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your signature (as is appeared in Admit Card) in appropriate boxes in the OMR Sheet.
7. The OMR Sheet is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/signature of the candidate, name of the examination centre. The OMR Sheet may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones, bluetooth devices etc. inside the examination hall. Any candidate found with such prohibited items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough work must be done on the question booklet itself. Additional blank pages are given in the question booklet for rough work.
10. Hand over the OMR Sheet to the invigilator before leaving the Examination Hall.
11. Candidates are allowed to take the Question Booklet after examination is over.

Signature of the Candidate : ____________________
(as in Admit Card)

Signature of the Invigilator : ____________________

SPACE FOR ROUGH WORK

Physics

1. Consider a classical ideal gas on N molecules in equilibrium at temperature T. Each molecule has two energy levels $-\varepsilon$ and $+\varepsilon$. The mean energy of the gas is

(A) 0

(B) $N\varepsilon \tanh\left(\frac{\varepsilon}{K_B T}\right)$

(C) $-N\varepsilon \tanh\left(\frac{\varepsilon}{K_B T}\right)$

(D) $\frac{\varepsilon}{2}$

2. According to the single particle nuclear shell model, the spin parity of the ground state of ${}^{17}_{8}O$ is

(A) $\frac{1}{2}^-$

(B) $\frac{3}{2}^-$

(C) $\frac{3}{2}^+$

(D) $\frac{5}{2}^+$

3. In the band structure defined in the tight binding model, the energy dispersion (E vs. K) relation for electrons in one dimensional array of atoms having lattice constant a and total length L is

$$E = E_o - \beta - 2r\cos(Ka) \text{ [Symbols have their usual meanings]}$$

Now, the effective mass of electrons in the band is given by

(A) $\frac{\hbar^2}{2ra^2 \sin(Ka)}$

(B) $\frac{\hbar^2}{ra^2 \sin(Ka)}$

(C) $\frac{\hbar^2}{ra^2\cos(Ka)}$

(D) $\frac{\hbar^2}{2ra^2\cos(Ka)}$

4. At room temperature T, electrons in a metal do not have the same specific heat as normal gas molecules. This is because

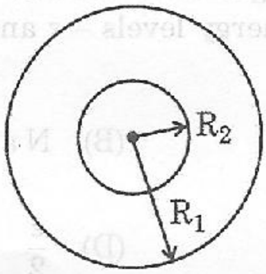
(A) the Fermi level is much higher than $K_B T$.

(B) electrons repel each other unlike gas molecules.

(C) electrons are paired.

(D) electrons are strongly bound in a metal.

5. Consider two, single turn, co-planar, concentric coils of radii R_1 and R_2 with $R_1 >> R_2$. The mutual inductance between the two coils is proportional to



(A) $\frac{R_1}{R_2}$ (B) $\frac{R_2}{R_1}$

(C) $\frac{R_2^2}{R_1}$ (D) $\frac{R_1^2}{R_2}$

6. The principal quantum number of the Bohr orbit in a hydrogen atom having radius of 0.01 mm is :

(A) 250 (B) 350

(C) 475 (D) 435

7. A one-dimensional harmonic oscillator is in the state

$$\psi(x) = \frac{1}{\sqrt{14}} \left[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)\right],$$

where, $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is

(A) zero (B) $\frac{3}{\sqrt{14}}$

(C) $\frac{9}{14}$ (D) 1

8. The Lagrangian of a system is given by

$$L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$$

The above Lagrangian describes motion of a/an

(A) Harmonic Oscillator (B) Damped Harmonic Oscillator

(C) Anharmonic Oscillator (D) System with unbound motion

9. A measurement establishes the position of a proton with an accuracy of $\pm 1.00 \times 10^{-11}$ m. Assuming v << c the uncertainty in the proton's position after 1 sec will be

(A) 2.24×10^2 m (B) 6.48×10^4 m

(C) 3.15×10^3 m (D) 9.69×10^5 m

[It is given that mass of proton is 1.672×10^{-27} kg.]

10. An electron with energy E_e and a proton with energy E_p have the same de Broglie wavelength. Then

(A) $E_e > E_p$ (B) $E_e < E_p$

(C) $E_e \geq E_p$ (D) $E_e \leq E_p$

11. If the motion of a particle is described by $x = 5 \cos (8\pi t)$, $y = 5 \sin (8\pi t)$ and $z = 5t$, then the trajectory of the particle is

(A) Circular (B) Elliptical

(C) Helical (D) Spiral

12. Which one of the following decay is permissible ?

(A) $n \rightarrow p + \beta^- + \bar{\nu}$ (B) $n \rightarrow p + \beta^+ + \bar{\nu}$

(C) $p \rightarrow n + \beta^- + \bar{\nu}$ (D) $p \rightarrow n + \beta^+ + \bar{\nu}$

13. Consider a doped semiconductor having the electron and hole mobilities μ_n and μ_h, respectively. Its intrinsic carrier density is n_i. The hole concentration p for which the conductivity is minimum at a given temperature is

(A) $n_i \sqrt{\frac{\mu_n}{\mu_h}}$ (B) $n_i \sqrt{\frac{\mu_h}{\mu_n}}$

(C) $n_i \frac{\mu_h}{\mu_n}$ (D) $n_i \frac{\mu_n}{\mu_h}$

14. Metallic monovalent sodium crystallizes in body centered cubic structure. If the length of the unit cell is 4×10^{-8} cm, the concentration of conduction electrons in metallic sodium is

(A) 6.022×10^{23} cm^{-3} (B) 3.125×10^{22} cm^{-3}

(C) 2.562×10^{21} cm^{-3} (D) 1.250×10^{20} cm^{-3}

15. A student writes for the wave function of a free particle in one dimension as

$\psi(x) = N \exp [i(Kx^2 - \omega t)]$. N is a normalization constant

This is not correct because

(A) It is not normalizable.

(B) It does not satisfy the Schrödinger wave equation.

(C) The sign of the first term in the exponent is wrong.

(D) It does not satisfy the required boundary conditions.

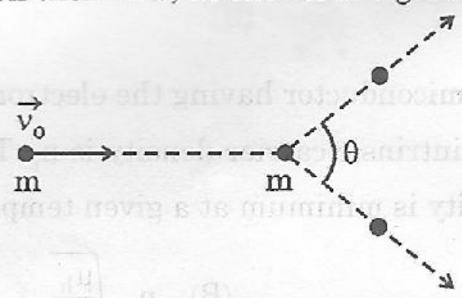
16. A particle of mass m is confined in a 2-D infinite square well potential of side a. The eigen-energy of the particle in a given energy state is $E = \frac{25\pi^2\hbar^2}{ma^2}$.

The state is

(A) 4-fold degenerate (B) 3-fold degenerate

(C) 2-fold degenerate (D) non-degenerate

17. A particle of mass m moving with velocity $\vec{v}_0$ collides another particle of same mass (m) at rest elastically. After collision the particles move in such a way that the angle between them is θ, as shown in figure. The value of θ is :



(A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{2}$

(C) $\theta = \pi$ (D) $\theta = \frac{\pi}{6}$

18. The physical dimension of the wave function $\psi(\vec{r})$ of a particle moving in three dimensional space is

(A) $[L]^2$ (B) $[L]^3$

(C) $[L]^{\frac{1}{2}}$ (D) $[L]^{\frac{-3}{2}}$

19. If L_x, L_y and L_z are respectively the x, y and z components of angular momentum operator L, the commutation relation of $[L_x L_y, L_z]$ is equal to

(A) $i\hbar\ (L_x^2 + L_y^2)$ (B) $2i\hbar\ L_z$

(C) $i\hbar\ (L_x^2 - L_y^2)$ (D) zero

20. Suppose P_1 and P_2 be the probabilities that a 1s electron be at distance a_0 and $\frac{a_0}{2}$ from the nucleus. Then, the value of $\frac{P_1}{P_2}$ will be

(A) $2e^{-1}$ (B) $4e^{-1}$

(C) $3e^{-1}$ (D) $2e^{-2}$

21. X-ray lasers are very difficult to construct as the ratio of Einstein's "A" and "B" co-efficients is

(A) 1 (B) Exceedingly large

(C) Extremely small (D) zero

22. The shortest wavelength present in the radiation from an X-ray machine with an accelerating potential of 50 kV is

(A) 0.056 nm (B) 0.025 nm

(C) 0.146 nm (D) 0.245 nm

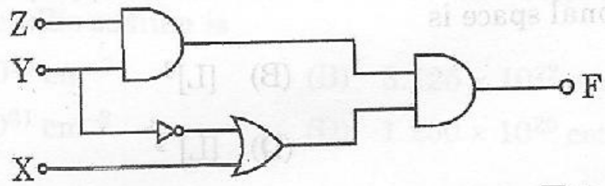
23. The spin and total linear momentum of a Cooper pair are respectively

(A) $1(\hbar)$, 0 (B) $0(\hbar)$, $0(\hbar\vec{K})$

(C) $2\hbar$, $2\hbar\vec{K}$ (D) $-2\hbar$, $-2\hbar\vec{K}$

($\hbar\vec{K}$ is the linear momentum of a single electron)

24. Identify the function F generated by the logic network shown below.



(A) $F = (X + Y)Z$ (B) $F = Z + Y + \overline{Y}X$

(C) $F = ZY(Y + X)$ (D) $F = XYZ$

25. The Coulomb potential that leads to atomic wave functions of H atom is spherically symmetric. This implies that

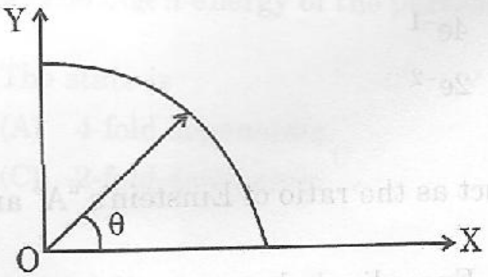
(A) the wave functions are all spherically symmetric.

(B) the probabilities are all spherically symmetric.

(C) the energies cannot depend on the magnetic quantum number m.

(D) the energies cannot depend on the orbital quantum number l.

26.



A segment of circular wire of radius R, extending from $\theta = 0$ to $\frac{\pi}{2}$, carries a constant linear charge density λ. The electric field at the origin O is

(A) $\frac{\lambda}{4\pi \epsilon_0 R}(-\hat{x} - \hat{y})$ (B) $\frac{\lambda}{4\pi \epsilon_0 R}\left(-\frac{1}{\sqrt{2}}\hat{x} - \frac{1}{\sqrt{2}}\hat{y}\right)$

(C) $\frac{\lambda}{4\pi \epsilon_0 R}\left(-\frac{1}{2}\hat{x} - \frac{1}{2}\hat{y}\right)$ (D) 0

27. If electrons had spin $\frac{3}{2}$, then

(A) the Bohr levels would change (B) the size of atoms will change

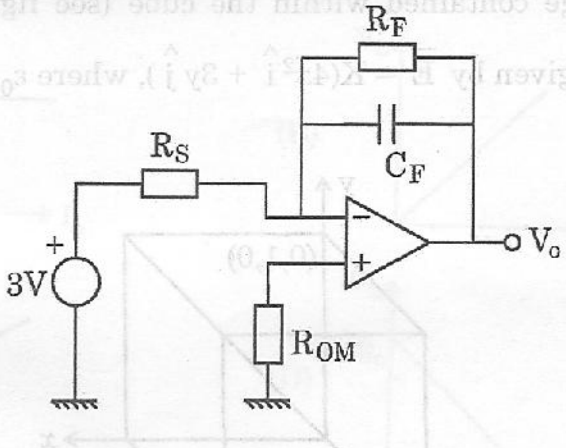
(C) chemical properties will change (D) the atoms will not be stable

28. Six simple harmonic oscillations each of same frequency and equal amplitude are superposed. The phase difference between any two consecutive oscillations i.e. $\phi_n - \phi_{n-1} = \Delta\phi$ is constant, where ϕ_n is the phase of the n-th oscillation. If the resultant amplitude of the superposition is zero, the phase difference $\Delta\phi$ will be

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) 2π

29. Figure given below shows a practical integrator with $R_S = 30\ M\Omega$, $R_F = 20\ M\Omega$ and $C_F = 0.1\ \mu F$. If a step (DC) voltage of + 3V is applied as input for $0 \le t \le 4$ (t is in seconds), the output voltage is



(A) a ramp function of –6 V

(B) a step function of –12 V

(C) a ramp function of –15 V

(D) a ramp function of –4 V

30. In Young's double slit experiment if the widths of the slits are in the ratio 9 : 1, then the ratio of the intensities between the central maxima and first minimum in the interference pattern is

(A) 1 : 3

(B) 3 : 1

(C) 4 : 1

(D) 1 : 4

31. Two bodies A and B contain an equal number of molecules of the same gas. If the volumes are V_A and V_B, respectively and λ_A and λ_B denote the respective mean free paths, then

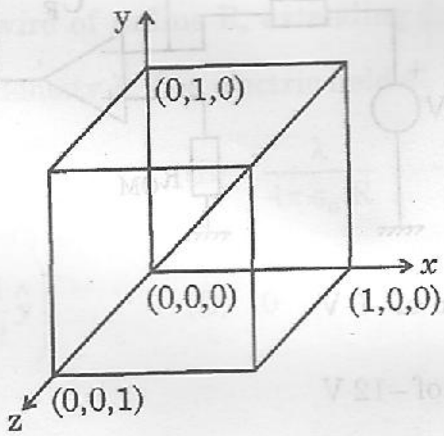
(A) $\lambda_A = \lambda_B$

(B) $\frac{\lambda_A}{V_A} = \frac{\lambda_B}{V_B}$

(C) $\frac{\lambda_A}{V_A^{\frac{1}{2}}} = \frac{\lambda_B}{V_B^{\frac{1}{2}}}$

(D) $V_A\lambda_A = V_B\lambda_B$

32. The total charge contained within the cube (see figure below) in which the electric field is given by $\vec{E} = K(4x^2\hat{i} + 3y\hat{j})$, where ε_0 is the permittivity of the free space, is



(A) $7K\varepsilon_0$

(B) $5K\varepsilon_0$

(C) $3K\varepsilon_0$

(D) Zero

33. Certain radioactive nuclei emit α-particles which are the nuclei of helium. The reason why helium nuclei rather than nuclei of other elements is emitted because

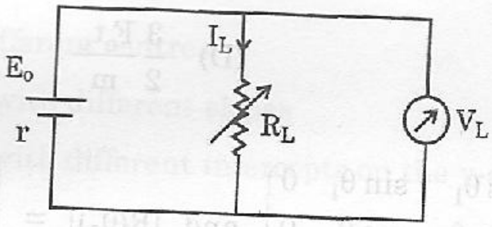
(A) helium nucleus has zero angular momentum.

(B) helium is a noble element.

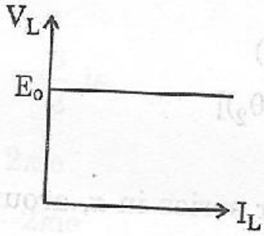
(C) helium has the same number of neutrons and protons.

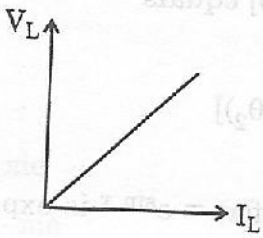
(D) helium nucleus is very tightly bound.

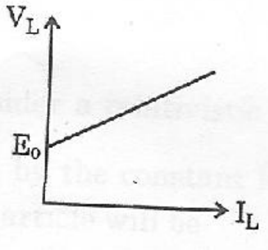
34. A variable resistance R_L is connected across a voltage source of emf E_o and internal resistance r as shown in figure :

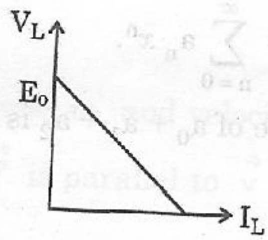


The current through the resistance R_L is I_L and the voltage drop across R_L measured by a voltmeter is V_L. The V_L vs. I_L curve will look like

(A) 

(B) 

(C) 

(D) 

35. The Fermi energy of two dimensional non-relativistic free electrons with areal density n will be proportional to

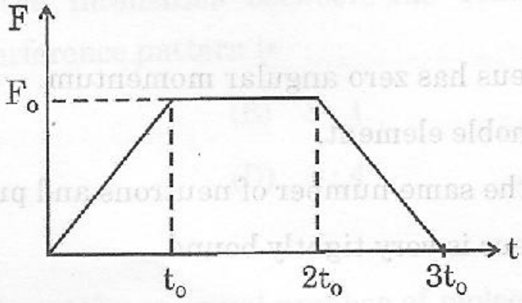
(A) n

(B) $n^{\frac{2}{3}}$

(C) n^2

(D) $n^{\frac{3}{2}}$

36. The following figure shows the variation of the force (F) acting on a particle of mass m at different time (t) :



If the velocity of the particle at t = 0 is zero, then its velocity at $t = 3t_o$ will be

(A) $\frac{F_o t_o}{m}$ (B) $\frac{F_o t_o}{2m}$

(C) $\frac{2F_o t_o}{m}$ (D) $\frac{3}{2}\frac{F_o t_o}{m}$

37. If $[R(\theta_1)] = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $[R(\theta_2)] = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then $[R(\theta_1)] \times [R(\theta_2)]$ equals

(A) $R(\theta_1\theta_2)$ (B) $R(\theta_1 + \theta_2)$

(C) $R[2(\theta_1 + \theta_2)]$ (D) $R[2(\theta_1 - \theta_2)]$

38. The function $f(x) = e^{\sin x}$ is expanded as a Taylor series in x, around $x = 0$, in the form $f(x) = \sum_{n=0}^{\infty} a_n x^n$.

Now, the value of $a_0 + a_1 + a_2$ is

(A) 0 (B) $\frac{3}{2}$

(C) $\frac{5}{2}$ (D) 5

39. The unit normal to the surface $-x^2yz^2 + 2xy^2z = 1$ at the point P(1, 1, 1) will be

(A) $\hat{i}$ (B) $\hat{j} + \hat{k}$

(C) $\hat{j}$ (D) $\hat{k}$

40. A Carnot engine whose low temperature reservoir is at 7 °C has an efficiency of 40%. It is desired to increase the efficiency to 50%. By how many degrees should the temperature of the source be increased ?

(A) 560 °C (B) 840 °C

(C) 280 °C (D) $\left(\frac{280}{3}\right)$ °C

41. The solutions to the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ are a family of

(A) Circles with different radii

(B) Circles with different centres

(C) Straight lines with different slopes

(D) Straight lines with different intercepts on the y-axis.

42. The value of the integral $\int_c \frac{e^z}{z^2 - 3z + 2}\, dz$, where the contour is the circle $|z| = \frac{3}{2}$ is

(A) $2\pi ie$ (B) πie

(C) $-2\pi ie$ (D) $-\pi ie$

43. Consider a relativistic particle of rest mass m_o and velocity $\vec{v}$ which is acted upon by the constant force $\vec{F}$, where $\vec{F}$ is parallel to $\vec{v}$. The acceleration of the particle will be

(A) $\frac{\vec{F}}{m_o}\left(1-\frac{v^2}{c^2}\right)^{3/2}$ (B) $\frac{\vec{F}}{m_o}\left(1-\frac{v^2}{c^2}\right)^{1/2}$

(C) $\frac{\vec{F}}{m_o}\left(1-\frac{v^2}{c^2}\right)^{-3/2}$ (D) $\frac{\vec{F}}{m_o}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$

44. A sphere of radius R and mass M rolls without slipping on a surface with speed v. The ratio of translational K.E. (Kinetic Energy) and rotational K.E. is

(A) $\frac{1}{5}$ (B) $\frac{5}{2}$

(C) $\frac{2}{5}$ (D) 5

45. A space ship is approaching a source of light with a speed equal to 0.5 c (c is the speed of light). Light coming from the source of light as seen by a person in the space ship travels with speed equal to

(A) 0.5 c (B) c

(C) 1.5 c (D) 2.0 c

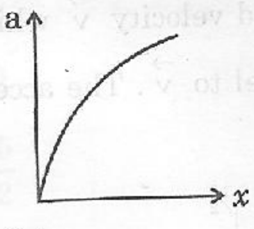
46. Let M be 2 × 2 matrix. Its trace is 6 and its determinant has value 8. Its eigen values are,

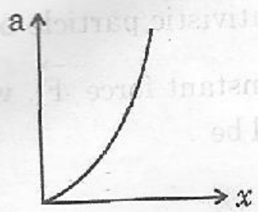
(A) 2 and 4 (B) 2 and 8

(C) 2 and 6 (D) –2 and –3

47. The velocity (v) vs. distance (x) curve of a particle is

The acceleration (a) vs. distance (x) curve will be

(A)


(B)


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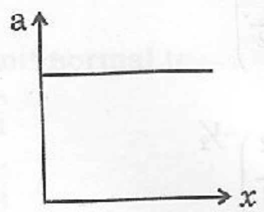
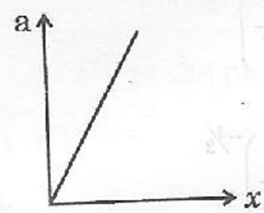


(D)


48. A 20 cm column of a certain solution causes right-hand rotation of 38° and 30 cm column of a second solution causes left-hand rotation of 24°. Now the amount of rotation to be produced by 30 cm column of a mixture of the above two solutions in the volume ratio 1 : 2 will be

(A) 5° (B) 4°

(C) 2° (D) 3°

49. Experimental measurements of heat capacity per mole of Aluminium at low temperature indicates that the data can be fitted to the formula, $c_v = aT + bT^3$, where $a = 0.00135\ JK^{-2}\ mole^{-1}$, $b = 2.48 \times 10^{-5}\ JK^{-4}\ mole^{-1}$ and T is the temperature in Kelvin. The entropy of a mole of Aluminum at such temperatures is given by the formula

(A) $aT + \frac{b}{3}T^3 + c$, where $c > 0$ is a constant

(B) $\frac{aT}{2} + \frac{b}{4}T^3 + c$, where $c > 0$ is a constant

(C) $aT + \frac{b}{3}T^3$

(D) $\frac{aT}{2} + \frac{b}{4}T^3$

50. The electric field $(\vec{E})$ associated with the linearly polarized electromagnetic wave moving along + z direction with angular frequency ω and making an angle 60° with x-axis is represented as

(A) $\vec{E}(z, t) = \hat{i}\frac{E_o}{2}\cos(kz - \omega t) + \hat{j}\frac{\sqrt{3}}{2}E_o\cos(kz - \omega t)$

(B) $\vec{E}(z, t) = \hat{i}\frac{E_o}{2}\sin(kz - \omega t) + \hat{j}\frac{E_o}{2}\cos(kz - \omega t)$

(C) $\vec{E}(z, t) = \hat{i}\frac{E_o}{2}\sin(kz - \omega t) - \hat{j}\frac{\sqrt{3}}{2}E_o\cos(kz - \omega t)$

(D) $\vec{E}(z, t) = \hat{k}\frac{\sqrt{3}}{2}E_o\cos(kz - \omega t)$

[Here E_o is the amplitude of the wave]