PUMDET-2019

Subject : STATISTICS

(Booklet Number)

Duration: 90 Minutes

Full Marks: 100

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks 2. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C, or D.

3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.

4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.

5. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.

6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.

7. The OMR is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/ signature of the candidate, name of the examination centre. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.

8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** and his/her candidature will be summarily cancelled.

9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.

10. Hand over the OMR to the invigilator before leaving the Examination Hall.

CONFIDENTIAL

STATISTICS

1. The budget expenditures of several countries incurred on various heads (e.g. defence, health, human resource development, infrastructure development etc.) for a particular year are available. Suppose one wishes to compare the proportional expenditure on different heads.

   The diagram which will best give a relevant representation of this comparison, is

   (A) multiple horizontal bar diagram

   (B) multiple line diagram

   (C) multiple divided bar diagram

   (D) multiple column diagram

2. In a simple regression model with response y and regressor x, the slope coefficient β is most efficiently estimated if

   (A) the x-values are closely clustered

   (B) the y-values are widely dispersed

   (C) the x-values are widely dispersed

   (D) the errors are widely dispersed

3. Total number of decennial censuses conducted in independent India till 2018 is

   (A) 5 (B) 6

   (C) 7 (D) 8

4. Consider a variable taking k+1 non-negative integer value 0, 1, ..., k respectively with frequencies $f_0, f_1, ..., f_k$. Then the mean is given by $\sum_{n=0}^{k} F_n$ where F_n's are the

   (A) cumulative frequencies of the less than type

   (B) relative cumulative frequencies of the greater than type

   (C) cumulative frequencies of the greater than type

   (D) relative cumulative frequencies of the less than type

8. Which of the following is the value of the Goodman - Kruskal Gamma measure from the following data on preference for Old Indian Cinema over various age groups ?

Preference → Age (in years) ↓	Like	Indifferent	Dislike
≤ 18	5	15	20
> 18 but < 45	15	10	7
≥ 45	10	10	8

(A) 0.3377 (B) –0.4951

(C) 0.4951 (D) –0.3377

9. The Laspeyre's and Paasche's *quantity* indices for a commodity are 180 and 170 respectively. If the Edgeworth-Marshall *price* index is 190, then Laspeyre's *price* index is approximately

(A) 200.67 (B) 197.04

(C) 202.81 (D) 203.52

10. An AR(1) process, $X_t = \alpha X_{t-1} + \varepsilon_t$, where ε_t's are i.i.d. with mean zero and constant variance, is stationary and causal

(A) if and only if $|\alpha| < 1$

(B) if and only if $|\alpha| > 1$

(C) if and only if $|\alpha| = 1$

(D) for any α

11. A plot of the time series giving GDP of a country between 1960 to 2000 showed high peaks in the years 1963, 1970, 1977, 1984, 1991 and 1998 and troughs in 1966, 1973, 1980, 1987 and 1994. This indicates

(A) a seventh-degree polynomial trend (B) absence of any trend

(C) seasonal variations (D) a cyclical fluctuation.

17. What does the phrase "in control" mean with respect to processes ?

(A) An in-control process is one in which the proportion of output that is defective falls within the agreed-upon range

(B) An in-control process is one in which the process width (i.e., 6σ) is substantially wider than the specification width

(C) An in-control process is statistically stable; it is free of assignable or non-random variation

(D) An in-control process is statistically stable; it is free of unassignable or random variation

18. The two regression lines between x and y are $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$. If the variance of x is 81, the variance of y is

(A) 81 (B) 20

(C) 9 (D) 144

19. Suppose the variable x takes values in [a, b] (a <b). Then the maximum value of the variance (with divisor 20 NOT 19) of 20 observations on x will be

(A) $(b - a)^2 / 4$ (B) $(b - a)^2 / 20$

(C) $(b + a)^2 / 4$ (D) $(b + a)^2 / 20$

20. Two independent random variables X and Y have variances 3 and 5 respectively. The correlation coefficient between the variables X + Y and 3 X – Y is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{4}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{\sqrt{7}}$

24. The determinant of the matrix $A = \begin{pmatrix} 1 & 0 & c & c \\ 0 & 1 & 1 & 1 \\ 1 & c & 0 & c \\ 1 & c & c & 0 \end{pmatrix}$ is

(A) c
(B) 3c
(C) $3c^2$
(D) $3c^3$

25. The mean of a finite population of 100 units is to be estimated by a simple random sample mean with a coefficient of variation 10%. If the size of the sample required is 11 for with replacement sampling, the required sample size for without replacement sampling is

(A) 8
(B)
(C) 10
(D) 11

26. $\lim_{n \to \infty} \left(\sqrt{n+5} - \sqrt{n}\right)$

(A) is 0
(B) is 5
(C) is 1/5
(D) does not exist

27. The power series $\sum_{n=0}^{\infty} 3^{-n} x^{2n}$ is convergent if

(A) $|x| \leq \sqrt{3}$
(B) $-\sqrt{3} \leq x < \sqrt{2}$
(C) $|x| < 3$
(D) $|x| < \sqrt{3}$

28. Suppose $P(A \cap B) > 0$ and $P(A|B) = P(B|A)$. Then which of the following is necessarily true ?

(A) $P(A \cap B) = \{P(A)\}^2$
(B) $P(A \cup B) = P(A \cap B)$
(C) $P(A \cap B) + P(A \cup B) = 2.P(A)$
(D) $P(A \cup B).P(A \cap B) = 3.\{P(B)\}^2$

33. A bag contains N cards numbered 1, 2, 3, ...N. A simple random sample of size n is drawn without replacement. Let $X_1, X_2, \ldots X_n$ be the numbers of the cards drawn at the first, second, ..., n^{th} draw respectively. Which of the following is an unbiased estimator for N, where M is the sample mean of $X_1, X_2, \ldots X_n$?

(A) $M - \frac{1}{2}$ (B) $2M + 1$

(C) $M + \frac{1}{2}$ (D) $2M - 1$

34. Let $X_1, X_2, \ldots X_n$ be independent and identically distributed standard normal random variables and let $T_n \frac{\sum_{i=1}^{n} X_i^2}{n}$, $n \geq 1$. Then the limiting distribution of which of the following statistics is standard normal ?

(A) $\sqrt{\frac{n}{2}}(T_n - 1)$ (B) $\sqrt{n}(T_n - 2)$

(C) $\sqrt{\frac{n}{2}}(T_n - 2)$ (D) $\sqrt{n}(T_n - 1)$

35. Let X be a Poisson random variable with parameter 2.5. At which value of r, $P(X = r)$ is maximized ?

(A) 1 (B) 2

(C) 3 (D) 4

36. Suppose the distribution of X is Cauchy (0, 1). What is the distribution of $\frac{1}{X}$?

(A) Cauchy (0, 1) (B) Logistic (0, 1)

(C) Normal (0, 1) (D) Double Exponential (0, 1)

41. The distribution of a random variable X is given by the probability density function f, where

$$f(x) = \begin{cases} \frac{x-\theta}{8}, \text{if } \theta \le x \le \theta + 4 \\ 0, \text{othrewise} \end{cases}$$

For what value of d, [X – d, X +1] is a confidence interval for θ with confidence coefficient 0.95 ?

(A) 4.0249 (B) 3.8987

(C) 2.6245 (D) 1.4363

42. The probability distribution of a random variable X is given by the density function

$$f_\theta(x) = \begin{cases} (x-\theta)e^{-(x-\theta)}, \text{if } x \ge \theta \\ 0, \text{othrewise} \end{cases}$$, where θ $(-\infty < \theta < \infty)$ is an unknown parameter. Let M_n be the arithmetic mean of a random sample of size n from the above population, n ≥ 1. Which of the following estimators is consistent for θ ?

(A) $M_n - \frac{2}{n}$ (B) $M_n + 4 - \frac{3}{n}$

(C) $M_n - 2 - \frac{4}{n}$ (D) $M_n + 2 - \frac{5}{n}$

43. A number N is chosen at random and with equal probability from the set {2, 3, 4}. Then a number U is chosen at random and with equal probability from the set {1, 2, …, N}. What is the value of E (N|U = 2) ?

(A) $\frac{36}{13}$ (B) 3

(C) $\frac{41}{15}$ (D) $\frac{41}{14}$

48. Let $A=(a_{ij})$ be a 10×10 matrix where $a_{ij} = i + j$, $i, j = 1, 2, \ldots 10$. Then what is the value of trace $(A11^T)$? [$1=1^{10\times 1} = (1, 1, 1, \ldots 1)^T$ and 1^T is the transpose of 1)

(A) 110

(B) 1100

(C) 605

(D) 1605

49. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed Poisson (λ) $(\lambda > 0)$ random variables. Then as $n \to \infty$, P (min $\{X_1, X_2, \ldots X_n\} > 1$) tends to

(A) 0

(B) 1

(C) $e^{-\lambda}$

(D) $e^{e^{-\lambda}}$

50. Let X_1, X_2 and X_3 be independent and identically distributed random variables with variance 1. Then the variance – covariance matrix of the random vector

$Y = (X_1, X_1 + X_2, X_1 + X_2 + X_3)^T$ is

(A) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 3 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$