PUMDET-2018 **82240001**

Subject: Mathematics **(Booklet Number)**

Duration: 90 minutes **Full Marks: 100**

Instructions

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks 2. In case of incorrect answer or any combination of more than one answer, ½ marks will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C, or D.
3. Use only Black/Blue ball point pen to mark the answer by complete filling up of the respective bubbles.
4. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the OMR. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.
7. The OMRs will be processed by electronic means. Hence it is liable to become invalid if there is any mistake in the question booklet number or roll number entered or if there is any mistake in filling corresponding bubbles. Also it may become invalid if there is any discrepancy in the name of the candidate, name of the examination centre or signature of the candidate vis-a-vis what is given in the candidate's admit card. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be reported against & his/her candidature will be summarily cancelled.
9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
10. Hand over the OMR to the invigilator before leaving the Examination Hall.

ROUGH WORK ONLY

1.	$x^4 - 14x^2 + 24x - K = 0$ has 4 real and unequal roots. Then (A) $11 < K < 117$ (B) $-11 < K < -8$ (C) $8 < K < 11$ (D) $0 < K < 8$
2.	Let a, b, c denote the lengths of the sides of a triangle and let p, q, r are real numbers in descending order. Also let $I = a^2(p-q)(p-r) + b^2(q-r)(q-p) + c^2(r-p)(r-q)$. Then (A) $I \le 0$ (B) $I \ge 0$ (C) $I = 0$ (D) No specific relation can be ascertained between I and 0
3.	Let the mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x - y + 2z,\ x - 2y + 2z,\ 2x + y)$. Then (A) T is only homomorphism (B) T is an isomorphism (C) T is one-one but not onto (D) T is onto but not one-one
4.	Let $A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. Given that 5 is an eigen value of A with multiplicity 2, the basis for the corresponding eigen space is given by, (A) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \& \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (B) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \& \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (C) $\begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} \& \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ (D) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \& \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$
5.	Let A be a square matrix with entries. Let an integer 'n' be the eigen value of A. Then, (A) n > det A (B) n cannot divide det A (C) n is a divisor of det A (D) n > 2 det A
6.	Let H and K be two finite normal subgroups of co-prime order of a group $(G, \cdot)$. Then for all $h \in H$ and $k \in K$, hk equals to, (A) 1 (B) h^2 (C) k^2 (D) kh

7.	Choose the correct answer: (A) Two finite groups of same order are always isomorphic to each other. (B) There does not exist a group $(G,\cdot)$ with a normal subgroup $(H,\cdot)$ such that the factor group G/H is not isomorphic to any subgroup of G. (C) The set $I=\left\{a+b\sqrt{3}\in Z\left[\sqrt{3}\right]\right\}$ (a-b is an even integer) is an Ideal of the Ring $Z\left[\sqrt{3}\right]$ (where the symbols have their usual meaning) (D) There exists an Integral domain with six elements.
8.	Choose the correct one: (A) There does not exist any non-abelian group having a proper normal subgroup (B) All groups of order 77 are cyclic. (C) A cyclic group of order 28 has 14 generators. (D) Consider the symmetric group S_3 and the set T = {-1, 1}. Consider the mapping $f:S_3\rightarrow T$ defined by $f(\sigma)=\begin{cases}1, & \text{if } \sigma \text{ be even permutation}\\ -1, & \text{if } \sigma \text{ be odd permutation}\end{cases}$ Then f is a monomorphism.
9.	Let $(G,*)$ be a group and let $a,b\in G$. Then (A) if $(a*b)^K=a^K*b^K$ holds for just two consecutive integers (K), the group is abelian. (B) if $(a*b)^K=a^K*b^K$ holds for three consecutive integers (K), the group is abelian. (C) $(a*b)^K=a^K*b^K$ is not any criteria for the commutativity of the group. (D) if $(a*b)^K=a^K*b^K$ holds for four consecutive integers (K), only then the group will be abelian.
10.	Let $(R,+,\cdot)$ be a non zero Ring with the property that for each $a\neq 0$, there exists a unique $b\in R$ such that $a.b.a=a$. Then (A) R has no divisor of zero (B) R has divisor of zero (C) $b.a.b\neq b$ (D) R has no unity

11.	Let $\{f_n(x)\}_n$ be monotone in [a, b] for each n and $\sum_n f_n(a)$, $\sum_n f_n(b)$ converge absolutely. Then (A) $\sum_n f_n(x)$ is only pointwise convergent in [a, b] (B) $\sum_n f_n(x)$ is uniformly convergent in [a, b] (C) $\sum_n f_n(x)$ is bounded but not convergent (D) $\sum_n f_n(x)$ is divergent series.
12.	Let $I_n = \int_0^{\pi/2} f(x)\sin(nx)dx$. Then (A) $\lim_{n\to\infty} I_n$ does not exist (B) $\lim_{n\to\infty} I_n = 0$ (C) $\lim_{n\to\infty} I_n$ is oscillating (D) $\lim_{n\to\infty} I_n$ cannot be deduced
13.	Let f be uniformly continuous on $\mathbb{R}$ and $\{a_n\}_n$ converge to $a(\in \mathbb{R})$. Let $f_n(x) = f(x + a_n)$ for all $x \in \mathbb{R}, n \in \mathbb{N}$. Then (A) $\{f_n(x)\}_n$ is only pointwise convergent (B) $\{f_n(x)\}_n$ is uniformly convergent in $\mathbb{R}$ (C) $\{f_n(x)\}_n$ is divergent sequence (D) $\{f_n(x)\}_n$ is only bounded but not pointwise convergent
14.	Let $f:[a,b] \to \mathbb{R}$ be Riemann integrable in [a, b] and let $x, y \in [a,b]$. It is defined that $F(x,y) = \int_x^y f(t)\, dt$. Then (A) F is discontinuous with respect to both x and y (B) F is continuous in y for each x and is continuous in x for each y (C) F is continuous in y for each x but is discontinuous in x for each y (D) F is continuous in x for each y but discontinuous in y for each x

15.	Let f be continuous and non-zero in $\mathbb{R}$. Let a_0 be arbitrary and $a_n = a_{n-1} + f(a_{n-1})$ for all $n \geq 1$. Then (A) $\{a_n\}_n$ converges to zero (B) $\{a_n\}_n$ converges to a non-zero limit (C) $\{a_n\}_n$ is divergent (D) $\{a_n\}_n$ is bounded
16.	Let $\{a_n\}_n$ be a sequence in $\mathbb{R}$ and $b_n = \frac{1}{n^2}\sum_{K=1}^{n} a_K$. Then (A) $\{a_n\}_n$ is unbounded $\Rightarrow \{b_n\}_n$ is unbounded (B) $\{a_n\}_n$ is oscillatory $\Rightarrow \{b_n\}_n$ is oscillatory (C) $\{a_n\}_n$ converges to a non-zero limit $\Rightarrow \{b_n\}_n$ converges to a non-zero limit (D) $a_n \to 0 \Rightarrow b_n \to 0$
17.	Let $f(x) = \begin{cases} a_n + \sin \pi x, \text{ if } x \in (2n, 2n+1] \\ b_n + \cos \pi x, \text{ if } x \in (2n-1, 2n] \end{cases}$. Then (A) f is continuous only when both $\{a_n\}_n$ and $\{b_n\}_n$ are convergent (B) f is continuous only when both $\{a_n\}_n$ and $\{b_n\}_n$ converge to same limit (C) f is continuous only when both $\{a_n\}_n$ and $\{b_n\}_n$ are not convergent (D) f is never continuous
18.	Let $\int_0^1 (1+\cos^8 x)(ax^2+bx+c)\,dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)\,dx$. Then the equation $ax^2+bx+c=0$ has (A) no root in [1, 2] (B) double roots in [0, 2] (C) at least one root in [1, 2] (D) two imaginary roots
19.	Let $f(x) = \int_{\sin x}^{\cos x} e^{-t^2}$. Then $f'\left(\frac{\pi}{4}\right)$ (A) may not exists (B) exists and is $-\sqrt{\frac{1}{e}}$ (C) exists and is $-\sqrt{\frac{2}{e}}$ (D) exists and is $\sqrt{\frac{2}{e}}$

20.	$\lim\limits_{n\to\infty}\frac{1}{n^2}\sum\limits_{K=1}^{n}[Kx]$ is equal to (A) x (B) $\frac{x}{2}$ (C) 0 (D) 1 ([x] denotes the greatest integer less than or equal to the real number x)
21.	Let f and g be differentiable in an open interval $I(\subset \mathbb{R})$ and let $a \in I$. Let $h: I \to \mathbb{R}$ be defined as, $h(x)=\begin{cases} f(x), & \text{for } x<a \\ g(x), & \text{for } x \ge a \end{cases}$. Then (A) h is not differentiable at x=a under any condition (B) h is differentiable at 'a' if $f(a)=g(a)$ (C) h is differentiable at 'a' if $f'(a)=g'(a)$ (D) h is differentiable at 'a' if $f(a)=g(a)$ and $f'(a)=g'(a)$
22.	Let $0<\alpha<\beta<1$. Then $\sum\limits_{K=1}^{\infty}\int\limits_{1/(K+\beta)}^{1/(K+\alpha)}\frac{dx}{1+x}$ is equal to (A) $\log_e\frac{\beta}{\alpha}$ (B) $\log_e\frac{1+\beta}{1+\alpha}$ (C) $\log_e\frac{1+\alpha}{1+\beta}$ (D) ∞
23.	Let $\sum\limits_{n=0}^{\infty}a_n x^n$ represent $f(x)$ in the interval I. Let $\{f(x)\}^2 = f(2x)$ in some neighbourhood of $x=0$ and $f(0)=1$. Then (A) $a_0=1,\ a_2=a_1^2,\ a_3=a_1^3$ (B) $a_0=1,\ a_2=\frac{1}{2}a_1^2,\ a_3=\frac{1}{6}a_1^3$ (C) $a_0=1,\ a_2=\frac{1}{6}a_1^2,\ a_3=\frac{1}{2}a_1^3$ (D) $a_0=1,\ a_2=6a_1^3,\ a_3=2a_1^2$

24.	Let $f:[a,b]\to\mathbb{R}$ be differentiable function and $f'(x)\neq 0$ in (a, b). Then (A) there exists a point d in (a, b) such that f(a), f(d), f(b) are in A.P. (B) there does not exist any point d in (a, b) for which f(a), f(d), f(b) are not in A.P. (C) there exists a point d in (a, b) such that f(a), f(d), f(b) are in G.P. (D) there does not exist any such point d in (a, b) for which f(a), f(d), f(b) have any specific relation
25.	The domain of convergence of the series $\sum_{n=0}^{\infty}\frac{1,3,5,.....(2n-1)}{n!}\left(\frac{1-z}{z}\right)^{n}$ is (A) $\lvert z-1\rvert<\frac{1}{2}$ (B) $\lvert z-1\rvert<\frac{3}{2}$ (C) $\left\lvert z-\frac{4}{3}\right\rvert<\frac{2}{3}$ (D) $\left\lvert z-\frac{2}{3}\right\rvert<\frac{1}{2}$
26.	Let $p(x,y)$ and $q(x,y)$ be harmonic functions in domain D. Then $\left(\frac{\partial p}{\partial y}-\frac{\partial q}{\partial x}\right)+i\left(\frac{\partial p}{\partial x}+\frac{\partial q}{\partial y}\right)$ is (A) not analytic in D (B) analytic in D (C) not continuous in D (D) not differentiable at any point in D
27.	Let $f(z)=\begin{cases}\frac{(\bar{z})^{2}}{z}, & z\neq 0\\ 0, & z=0\end{cases}$. Then (A) f' exists at z = 0 (B) f is analytic at z = 0 (C) f' does not exist at z = 0 (D) f' exists everywhere
28.	The real part of $f(z)\big(=u(x,y)+i\ v(x,y)\big)$ is given by $x(x+2y)+3$. Then (A) $f(z)$ is necessarily analytic in its domain D (B) $u(x,y)$ is harmonic function (C) $f(z)$ is not analytic in its domain D (D) Second order partial derivatives of u are not continuous
29.	Let a_n be the n^{th} digit in the decimal representation of $\frac{1}{7}$. Then the radius of convergence of $\sum a_n^n z^n$ (A) can not be ascertained (B) is 1 (C) is $1/8$ (D) is ∞

30.	For a vector $\vec{\alpha} = (2x - yz)\hat{i} + (2y - zx)\hat{j} + (2z - xy)\hat{k}$ such that $\vec{\alpha} = \nabla\phi$ and $\phi(1,-2,1) = 4$, the function ϕ will be (A) $x^2 + y^2 + z^2 + xyz + 4$ (B) $x^2 + y^2 + z^2 - xyz - 4$ (C) $x^2 + y^2 + z^2 + xyz - 4$ (D) $x^2 + y^2 + z^2 - xyz + 4$
31.	Given two orthogonal vectors $\vec{A}, \vec{B}$ in $\mathbb{R}^3$, each of length 1 unit. Let $\vec{P}$ be a vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. Then (A) $\vec{P}$ is orthogonal to $\vec{B}$ (B) $\vec{P}$ is not orthogonal to $\vec{B}$ (C) $\vec{P}$ is orthogonal to $\vec{A}$ (D) $\vec{P}$ is not orthogonal to $\vec{A}$
32.	The equation of the right circular cone, whose vertex is at the origin, axis is the x-axis and the semi-vertical angle is $\pi/6$, is given by (A) $3x^2 = y^2 + z^2$ (B) $3y^2 = z^2 + x^2$ (C) $3z^2 = x^2 + y^2$ (D) $x^2 = 3(y^2 + z^2)$
33.	The equation of the sphere, on which the intersection of the plane $5x - 2y + 4z - 7 = 0$ and the given sphere $x^2 + y^2 + z^2 - 3x + 4y - 2z = 5$ is a great circle is (A) $x^2 + y^2 + z^2 + 2x + 2y + 2z - 12 = 0$ (B) $x^2 + y^2 + z^2 + 2x + 2y + 2z + 12 = 0$ (C) $x^2 + y^2 + z^2 - 2x - 2y - 2z + 12 = 0$ (D) $x^2 + y^2 + z^2 - 2x - 2y - 2z - 12 = 0$
34.	If a number is correct up to n significant figures and the first significant digit of the number is K then the relative error is (A) less than $\frac{1}{K \times 10^n}$ (B) less than $\frac{1}{K \times 10^{n+1}}$ (C) less than $\frac{1}{K \times 10^{n-1}}$ (D) greater than $\frac{1}{K \times 10^{n-1}}$
35.	The third order divided difference of the function $f(x) = \frac{1}{x}$ with arguments a, b, c, d is (A) $\frac{1}{abcd}$ (B) $-\frac{1}{abd}$ (C) $-\frac{1}{abc}$ (D) $-\frac{1}{abcd}$

36.	The Newton-Raphson's scheme for finding k^th^ root of a number 'a' is (A) $x_{n+1}=\frac{1}{K}\left((K-1)x_n+\frac{a}{x_n^{K-1}}\right)$ (B) $x_{n+1}=\frac{1}{K}\left((K-1)x_n-\frac{a}{x_n^{K-1}}\right)$ (C) $x_{n+1}=\frac{1}{K-1}\left((K-2)x_n-\frac{a}{x_n^{n-1}}\right)$ (D) $x_{n+1}=\frac{1}{K}\left((K-1)x_n+\frac{a}{x_n^{K}}\right)$
37.	Which of the following is not a Simple Harmonic motion with given centre, if the displacement 'x' and velocity 'v' of the moving particle at any time 't' is given by the law (A) $x = a\cos nt + b\sin nt$ with centre at x=0 (B) $v^2 = a + bx - cx^2$ with centre at $x=\frac{b}{2c}$ (C) $v^2 = 16\left(ax - 2x^2 - 4a^2\right)$ with centre at x=2a (D) $v^2 = 64\left(8ax - x^2 - 12a^2\right)$ with centre at x=4a
38.	A particle describes a curve $y=\frac{c}{2}\left[e^{x/c}+e^{-x/c}\right]$ under a force, which is always parallel to the positive direction of y-axis. Then the law of force F is (A) $F \propto \frac{1}{x^2}$ (B) $F \propto \frac{1}{y^2}$ (C) $F \propto y$ (D) $F \propto x$
39.	A particle starts from rest from infinity will reach the earth's surface with a velocity (radius of earth=a) (A) $\sqrt{ga}$ (B) $\sqrt{2ga}$ (C) $\sqrt{3ga}$ (D) $2\sqrt{ga}$
40.	An urn contains 3 red and 6 black balls. Balls are drawn at random one by one without replacement. The probability that the second red ball appears at the fifth draw is (A) $\frac{1}{9!}$ (B) $\frac{4!}{9!}$ (C) $4\left(\frac{6!4!}{9!}\right)$ (D) $\frac{6!4!}{9!}$

41.	The distribution function of the probability density function $f(x)=\begin{cases}\lvert x\rvert & \text{when } -1<x<1\\ 0 & \text{else where}\end{cases}$ is (A) $F(x)=\begin{cases}1 & \text{when } -1<x<1\\ 0 & \text{else where}\end{cases}$ (B) $F(x)=\begin{cases}\frac{1+x^2}{2} & \text{when } -1<x<1\\ 0 & \text{else where}\end{cases}$ (C) $F(x)=\begin{cases}0 & \text{when } -\infty<x<-1\\ \frac{1+x^2}{2} & \text{when } -1\le x\le 0\\ \frac{1-x^2}{2} & \text{when } 0<x\le 1\\ 1 & \text{when } 1<x<\infty\end{cases}$ (D) $F(x)=\begin{cases}0 & \text{when } -\infty<x<-1\\ \frac{1-x^2}{2} & \text{when } -1\le x\le 0\\ \frac{1+x^2}{2} & \text{when } 0<x\le 1\\ 1 & \text{when } 1<x<\infty\end{cases}$
42.	Let (X, d) be a metric space. Define the mapping $e: X, X \to \mathbb{R}$ sucht that $(x,y)\to \frac{d(x,y)}{1+d(x,y)}$. Then, (A) e is not a metric (B) e is a metric (C) e is only pseudo metric (D) triangle inequality does not hold
43.	Let (X, d) be a metric space and A, B are non-empty sets in X. Then (A) $d(A,B)\neq 0 \Rightarrow A\cap B=\Phi$ (B) $A\cap B=\Phi \Rightarrow d(A,B)\neq 0$ (C) $A\cap B=\Phi \Leftrightarrow d(A,B)=0$ (D) there is no mathematical connection between the results $A\cap B=\Phi$ and $d(A,B)\neq 0$
44.	Let X be the set of all infinite sequences of complex numbers, not necessarily convergent, not even bounded. Let $\{K_n\}_n$ be arbitrary fixed sequence of positive real numbers such that $\sum_n K_n$ converges. For $z=\{z_n\}_n$ & $\omega=\{\omega_n\}_n$ in X, let $d(z,\omega)=\sum \frac{\lvert z_n-\omega_n\rvert}{1+\lvert z_n-\omega_n\rvert} K_n$. Then (A) (X, d) is a bounded metric space (B) (X, d) is not a metric space (C) (X, d) is not bounded (D) (X, d) is pseudo metric space.

45.	Consider the set $C^2_{[a,b]}$ of all functions continuous on the closed and bounded interval [a, b], where the distance function $P(x,y)=\left(\int_a^b \lvert x(t)-y(t)\rvert^2 dt\right)^{1/2}$. Then (A) $C^2_{[a,b]}$ is a complete metric space (B) $C^2_{[a,b]}$ is an incomplete metric space (C) $C^2_{[a,b]}$ is not a metric space (D) In $C^2_{[a,b]}$, the triangle inequality does not hold.
46.	Let A be non-empty set of $\mathbb{R}$. Define $d(x,a)=\inf\{\lvert x-a\rvert : a\in A\}$ for every $x\in\mathbb{R}$. Then let g(x) = d(x, A) for every $x\in\mathbb{R}$. (A) g(x) is continuous everywhere (B) g(x) is discontinuous on $\mathbb{R}$ (C) g(x) is unbounded function (D) g(x) is only bounded on $\mathbb{R}$
47.	Let $f:[0,4]\to[3,9]$ be a continuous function. Then (A) there must be an x such that f(x) = 4x + 1 (B) there must be an x such that 3f(x) = 2x + 6 (C) there must be an x such that 2f(x) = 3x + 6 (D) there must be an x such that f(x) = x
48.	If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel straight lines, the distance between them is (A) $2\sqrt{\frac{g^2-ac}{a(a+b)}}$ (B) $2\sqrt{\frac{f^2-ab}{b(a+b)}}$ (C) $2\sqrt{\frac{h^2-ac}{a(a+b)}}$ (D) $\frac{2c}{a+b}$
49.	The locus of a point, whose polar with respect to the parabola $y^2=4ax$ is parallel to the line $lx+my=1$ is the (A) straight line $lx-my+4=0$ (B) diameter of the given parabola (C) straight line $y+2a=0$ (D) $y-\frac{l}{m}=0$

50.	The equation of the cylinder, whose guiding curve is f(y, z) = 0, x = 0 and whose generators are parallel to a fixed line with direction vector (l, m, n) is (A) $f\left(x-\frac{l}{n}z,\ y-\frac{m}{n}z\right)=0$ (B) $f\left(y-\frac{m}{l}x,\ z-\frac{n}{l}x\right)=0$ (C) $f\left(x+\frac{l}{n}z,\ y+\frac{m}{n}z\right)=0$ (D) $f\left(y+\frac{m}{l}x,\ z+\frac{n}{l}x\right)=0$