PUMDET-2023

Subject : PHYSICS/ASTROPHYSICS

(Booklet Number)

Duration : **90** Minutes | No. of Questions : **50** | Full Marks : **100**

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks **2**. In case of incorrect answer or any combination of more than one answer, **½** mark will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
3. Use only **Black/Blue ink ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR Sheet**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your signature (as is appeared in Admit Card) in appropriate boxes in the OMR Sheet.
7. The OMR Sheet is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/signature of the candidate, name of the examination centre. The OMR Sheet may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones, bluetooth devices etc. inside the examination hall. Any candidate found with such prohibited items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough work must be done on the question booklet itself. Additional blank pages are given in the question booklet for rough work.
10. Hand over the OMR Sheet to the invigilator before leaving the Examination Hall.
11. Candidates are allowed to take the Question Booklet after examination is over.

Signature of the Candidate : ________________________________
(as in Admit Card)

Signature of the Invigilator : ________________________________

SPACE FOR ROUGH WORK

Physics

1. In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that the number doubles in every 4 hours, how many may be expected at the end of the 12 hours?

(A) 8 times the original number
(B) 4 times the original number
(C) 12 times the original number
(D) 10 times the original number

2. Consider the integral $\oint_C (\sin z + 2\bar{z})dz$ where C is a closed circle of unit radius. The value of the integral will be

(A) 0
(B) $2\pi i$
(C) $4\pi i$
(D) $\sin 1$

3. The closed line integral of the vector $\vec{A} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ around a square of side b which has a corner at the origin, one side on the x-axis and the other side on the y-axis will be

(A) $2b^3$
(B) $4b^3$
(C) b^3
(D) $-\frac{b^3}{4}$

4. Which of the following are eigen values of the matrix $\begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix}$?

(A) 4, 0
(B) 2, 2
(C) i, –i
(D) 1, 3

5. Consider the following 2nd order differential equation

$$(x+2)^2(x-1)y''+3(x-1)y'+2y=0$$

In this equation

(A) x = 1 is a regular singular point (B) x = – 2 is a regular singular point

(C) x = 0 is a irregular singular point (D) x = 1 is a irregular singular point

6. Consider the series

$$x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$$

The series converges provided

(A) $|x| > 1$ (B) $|x| \ge 1$

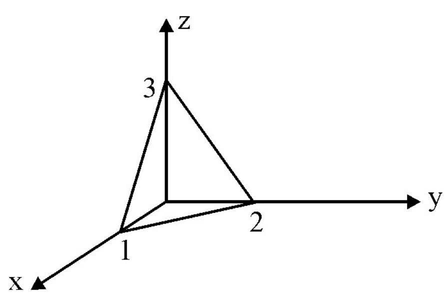
(C) $|x| < 1$ (D) $|x| \le 1$

7. The value of the integral $\iint_R xy^2 dxdy$, where $R: x^2+y^2 \le 4;\ x \ge 0$ is

(A) 0 (B) $\frac{64}{15}$

(C) $\frac{32}{15}$ (D) $\frac{4}{15}$

8.



The component of the unit vector $\hat{n}$ perpendicular to the plane shown in the figure above is

(A) $\frac{6\hat{i}+3\hat{j}+2\hat{k}}{7}$ (B) $\frac{3\hat{i}+6\hat{j}+2\hat{k}}{7}$

(C) $\frac{6\hat{i}+2\hat{j}+3\hat{k}}{7}$ (D) $\frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}$

9. Consider the polynomial $B_n(x)$ defined by

$$\frac{te^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} \frac{B_n(x)}{n!} t^n$$

Then $B_n'(x)$ is [$B_n'(x)$ denotes differentiation of $B_n(x)$ with respect to x]

(A) $nB_n(x)$

(B) $nB_{n-1}(x)$

(C) $(n-1)B_n(x)$

(D) $(n+1)B_n(x)$

10. The classical probability of finding a particle is the region $0 \le x \le L$ is $P(x) = \frac{A}{L}$, where A and L are constants. The variance $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ of the position of the particle is

(A) $\frac{L}{2}$

(B) 0

(C) $\frac{L}{2\sqrt{3}}$

(D) L

11. A particle is dropped freely from a height H. When it reaches to the ground, it will be deflected due to the Coriolis force towards

(A) east in northern hemisphere, but west in southern hemisphere

(B) east in southern hemisphere, but west in northern hemisphere

(C) east in both hemisphere

(D) west in both hemisphere

12. The degrees of freedom of a system of three particles on a line with fixed inter-particle separations and constrained to move in space is

(A) 6 (B) 4

(C) 7 (D) 5

13. A particle of mass m with speed v approaches a stationary target M. The masses bounce of each other without any loss in total energy. After collision, the angle between the particles in the CM frame is

(A) 180° (B) 90°

(C) 45° (D) 0°

14. Which of the following operators is invariant under Lorentz transformation along $x - x'$ axis?

(A) $\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}$ (B) $\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}$

(C) $\left(\frac{1}{c}\frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right)^2$ (D) $\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right)^2$

15. A particle is oscillating inside the potential $V(x) = Ax^4,\ (A > 0)$. If the amplitude of the oscillation is a_0 , then time period T of oscillation is

(A) proportional to $\frac{1}{a_0}$ (B) proportional to $\frac{1}{\sqrt{a_0}}$

(C) proportional to $a_0^{\ 2}$ (D) independent of a_0

16. A planet of mass m moves in a circular orbit of radius R in the gravitational potential $V(r) = -\frac{k}{r}, (k > 0)$. The orbital angular momentum of the planet is

(A) $\sqrt{2mkR}$ (B) mkR

(C) $\sqrt{mkR}$ (D) $2mkR$

17. Lagrangian of a system is given by

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}\alpha x^2 - \alpha x\dot{x}t$$

The equation of motion corresponding to the Lagrangian is

(A) $m\ddot{x} = 0$ (B) $m\ddot{x} + \alpha x = 0$

(C) $m\ddot{x} + \alpha\dot{x} + \alpha x = 0$ (D) $m\ddot{x} + \alpha\dot{x}xt = 0$

18. An insect flies on a spiral trajectory such that its polar co-ordinates at time t are given by $r = be^{\omega t}$, $\theta = \omega t$, where b and ω are positive constants. The angle between velocity and acceleration at time t is

(A) 30° (B) 60°

(C) 45° (D) 90°

19. Consider a unit mass oscillator whose equation of motion is

$$\ddot{x} + 10\dot{x} + 16x = 0$$

This corresponds to

(A) a harmonic oscillator

(B) overdamped harmonic oscillator

(C) underdamped harmonic oscillator

(D) critically damped harmonic oscillator

20. Which of the following functions represent a travelling wave?

(A) $f\left(x^2 - 2xvt + v^2t^2\right)$ (B) $f\left(x - vt\right)$

(C) $f\left(x^2 - v^2t^2\right)$ (D) $f\left(x^2 + v^2t^2\right)$

21. The number of cyclic coordinates for a particle falling freely under gravity is

(A) 1 (B) 2

(C) 3 (D) 0

22. A rod of length 10 m, making an angle 30° with x' - axis is at rest in S' frame. S' frame is moving with uniform velocity 0.6 c along $x - x'$ axis with respect to S frame. The length of the rod measured from S frame will be (S and S' both are inertial frames)

(A) 9.86 m (B) 7.86 m

(C) 8.54 m (D) 6.84 m

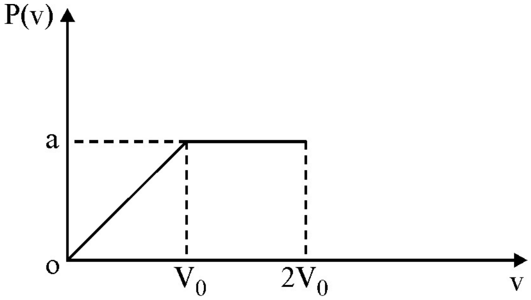
23. Photon gas has an entropy S varying with volume (V) and temperature (T)

[Note energy density $\rho = aT^4$] as

(A) $S = \frac{4}{3}aV^2T^3$ (B) $S = \frac{1}{3}aVT^3$

(C) $S = \frac{4}{3}aVT^3$ (D) $S = \frac{2}{3}aVT^3$

24. A hypothetical speed distribution for a sample of N gas molecules is shown below $P(v) = 0$ for $v > 2V_0$

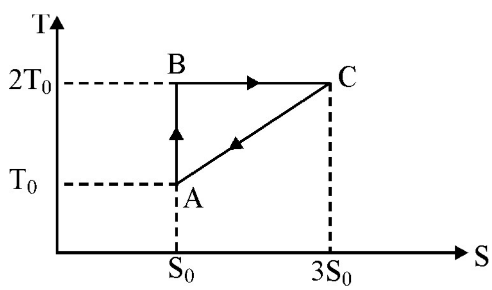


How many particles have speeds between $1.2\ V_0$ and $1.9\ V_0$?

(A) $\frac{N}{5}$ (B) $\frac{7N}{15}$

(C) $\frac{2N}{21}$ (D) $\frac{N}{8}$

25. Consider the following reversible cycles represented by right-angled triangle in a T-S diagram.



The efficiency of the above cycle is

(A) 66% (B) 50%

(C) 33% (D) 25%

26. Consider a system whose three energy levels are given by 0, ε, 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The average energy of a particle for $K_BT >> \varepsilon$ is

(A) ε (B) 0

(C) 2.5ε (D) 2ε

27. In a certain process, the entropy of a substance is known to be equal to S_0 at temperature T_0. If in that process the heat capacity of the substance is C (Constant), then temperature as a function of entropy S can be expressed as

(A) $T = T_0\left(\frac{S-S_0}{C}\right)$ (B) $T = T_0 e^{-\left(\frac{S-S_0}{C}\right)}$

(C) $T = T_0 e^{\frac{S-S_0}{C}}$ (D) $T = T_0\left(\frac{S_0-S}{C}\right)$

28. Consider 1 mole of an ideal gas with internal energy U_1 and 1 mole of Van der Waals gas with internal energy U_2. The Van der Waals parameters a and b are positive. Then

(A) $\left(\frac{\partial U_1}{\partial V}\right)_T = 0; \left(\frac{\partial U_2}{\partial V}\right)_T = 0$ (B) $\left(\frac{\partial U_1}{\partial V}\right)_T = 0; \left(\frac{\partial U_2}{\partial V}\right)_T > 0$

(C) $\left(\frac{\partial U_1}{\partial V}\right)_T > 0; \left(\frac{\partial U_2}{\partial V}\right)_T > 0$ (D) $\left(\frac{\partial U_1}{\partial V}\right)_T = 0; \left(\frac{\partial U_2}{\partial V}\right)_T < 0$

29. A gas of molecules, each of mass m is in thermal equilibrium at temperature T. The velocity components of the molecules along the Cartesian axes are ν_x, ν_y and ν_z. The mean value of $\left(\nu_x + b\nu_y\right)^2$ is

(A) $\left(1+b^2\right)\frac{3k_BT}{m}$ (B) $\frac{2k_BT}{m}$

(C) $\left(1+b^2\right)\frac{k_BT}{m}$ (D) $\left(1+b\right)^2\frac{k_BT}{m}$

30. An EM wave is to pass through an interface separated by two media having dielectric constants ε_1 and ε_2 . If $\varepsilon_1 = 2\,\varepsilon_2$, the wave will be totally reflected if the angle of incidence is

(A) 0° (B) 60°

(C) 30° (D) 45°

31. A cube of side a has its edges parallel to x, y and z axis of a rectangular co-ordinate system. A uniform $\vec{E}$ field is parallel to y axis and a uniform magnetic field $\vec{B}$ is parallel to z axis. If $\vec{S}$ denotes the Poynting vector and U the energy density stored in the cube, then

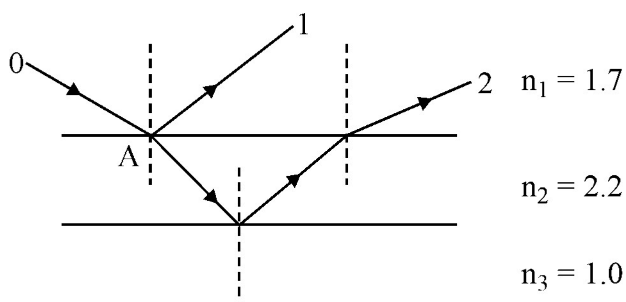
(A) $\vec{S} \neq 0;\ U = 0$

(B) $\vec{S} \neq 0;\ \frac{dU}{dt} \neq 0$

(C) $\vec{S} = 0;\ \frac{dU}{dt} = 0$

(D) $\vec{\nabla}.\vec{S} = 0;\ \frac{dU}{dt} = 0$

32.



Consider the following ray diagram with OA as incident ray. Which of the following statement is correct, regarding phase change on reflection only.

(A) ray 1 and 2 suffer a phase change of π.

(B) ray 1 suffers a phase change of π and ray 2 does not suffer any phase change.

(C) ray 1 and 2 both do not suffer any phase change.

(D) ray 1 suffers no phase change but ray 2 suffers phase change of π.

33. The components of the electric field of an electromagnetic plane wave are given by

$$E_x = E_0 \sin(kz - \omega t)$$
$$E_y = E_0 \cos(kz - \omega t)$$
$$E_z = 0$$

Then, the propagation direction of wave and the polarization state are respectively

(A) + z direction, Left circularly polarized

(B) – z direction, Left elliptically polarized

(C) + z direction, Right elliptically polarized

(D) – z direction, Right circularly polarized

34. The electric field of any region is given by

$$\vec{E} = 4x\hat{i} - 8y\hat{j}$$

The equation of the lines of force is

(A) $x^2y = \text{constant}$ (B) $xy^2 = \text{constant}$

(C) $\frac{x^2}{y} = \text{constant}$ (D) $\frac{x}{y^2} = \text{constant}$

35. Consider a double slit interference pattern where central maximum contains exactly 9 fringes. How many fringes lie between the 1st and 2nd minima in one side of the central maximum ?

(A) 4 (B) 3

(C) 5 (D) 8

36. Given uniform magnetic field $\vec{B} = B_0\hat{k}$ (B_0 = constant), a possible choice for the magnetic vector potential $\vec{A}$ is

(A) $\vec{A} = B_0 y\hat{i}$ (B) $\vec{A} = B_0\left(y\hat{i} + x\hat{j}\right)$

(C) $\vec{A} = B_0\left(x\hat{i} + y\hat{j}\right)$ (D) $\vec{A} = -B_0 y\hat{i}$

37. A disc of radius R rotates with a constant angular velocity ω about its own axis. The surface charge density of this disc varies as $\sigma = \alpha r^2$ (α is a constant), where r is the distance from the centre of the disc. The magnetic field intensity at the centre of the disc will be

(A) $\mu_0\alpha\omega R^3$ (B) $\frac{\mu_0\alpha\omega R^3}{6}$

(C) $\frac{\mu_0\alpha\omega R^4}{8}$ (D) $\frac{\mu_0\alpha\omega R^3}{3}$

38. Consider a free particle Hamiltonian operator $\hat{H} = \frac{\hat{p}^2}{2m}$. Then $\frac{d}{dt}\left\langle\frac{\hat{x}\hat{p} + \hat{p}\hat{x}}{2}\right\rangle$ will be equal to

(A) 0 (B) $\frac{\langle\hat{p}\rangle}{m}$

(C) $\frac{\langle\hat{p}^2\rangle}{m}$ (D) $\frac{\langle\hat{x}^2\rangle}{m}$

39. Any eigen function regardless of its energy in an infinite square well is periodic in time with a period of

(A) $\frac{mL}{\hbar\pi}$ (B) $\frac{4mL^2}{\hbar\pi}$

(C) $\frac{2mL^2}{\hbar\pi}$ (D) $\frac{mL}{2\hbar\pi}$

(m = mass of the particle; L = Length of the well)

40. Eigen functions of the Hamiltonian $\hat{H} = \hat{T} + \hat{V}$ of a harmonic oscillators are

(A) eigen functions of $\hat{T}$ as well as $\hat{V}$ (B) eigen function of $\hat{T}$ but not of $\hat{V}$

(C) eigen function of $\hat{V}$ but not of $\hat{T}$ (D) eigen function of neither $\hat{T}$ nor $\hat{V}$

41. Which of the following wave functions is/are not accepted in quantum mechanics?

I. $\Psi(x) = \tan x$; II. $\Psi(x) = e^{-|x|}$

III. $\Psi(r, \theta, \phi) = e^{i\phi/3}$ IV. $\Psi(r, \theta, \phi) = e^{i\phi}$

(A) I only (B) I and III

(C) II and III (D) I, II, III and IV

42. Let $\psi_{n\ell m}$ denote the eigen state of hydrogen atom in the usual notation. Consider the state

$$\frac{1}{5}\left[2\Psi_{200} - 3\Psi_{211} + \sqrt{7}\Psi_{210} - \sqrt{5}\Psi_{21-1}\right]$$

The above state is an eigen state of

(A) L^2 and L_z (B) H and L_z

(C) H, L^2 and L_z (D) H only

43. The band energy of an electron in a crystal for a particular k direction has the form $E(k) = A - B\cos ka$, where A > 0 and B > 0, and $-\pi < ka < \pi$. The electron has a hole-like behaviour over the following range of k :

(A) $\frac{\pi}{2} < ka < \frac{3\pi}{2}$ (B) $\frac{\pi}{2} < ka < \pi$

(C) $0 < ka < \frac{\pi}{4}$ (D) $0 < ka < \pi$

44. Consider a rectangular lattice in two dimensions with primitive lattice vectors $\vec{a_1} = a\hat{i}$ and $\vec{a_2} = 2a\hat{j}$. Which of the following represents reciprocal Lattice Vectors $(\vec{b_1}, \vec{b_2})$ for this lattice ?

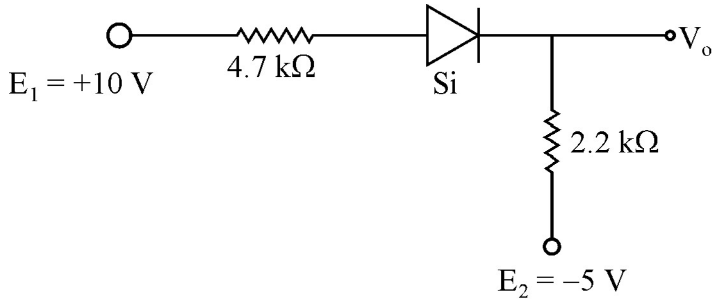
(A) $\vec{b_1} = \frac{\pi}{a}\hat{i},\ \vec{b_2} = \frac{\pi}{2a}\hat{j}$ (B) $\vec{b_1} = \frac{\pi}{a}\hat{i},\ \vec{b_2} = \frac{2\pi}{a}\hat{j}$

(C) $\vec{b_1} = 0\hat{i},\ \vec{b_2} = \frac{2\pi}{a}\hat{j}$ (D) $\vec{b_1} = \frac{\pi}{2a}\hat{i},\ \vec{b_2} = \frac{\pi}{a}\hat{j}$

45. The term symbol for Fe^{+2} having last shell configuration $3d^6$ is

(A) $^3F_2$ (B) $^5D_4$

(C) $^5D_0$ (D) $^2D_{3/2}$

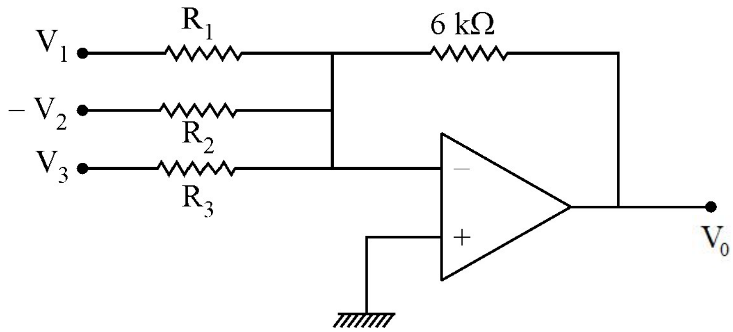
46. 

Consider the above circuit with a Si diode. The current I in the above circuit is

(A) 1.97 mA (B) 3.02 mA

(C) 2.64 mA (D) 2.07 mA

47. For the circuit shown below, the output is $V_0 = -V_1 + 2V_2 - 3V_3$



The values of resistances R_1, R_2 and R_3 are respectively

(A) 6 kΩ, 2 kΩ and 3 kΩ
(B) 2 kΩ, 6 kΩ and 3 kΩ
(C) 6 kΩ, 3 kΩ and 2 kΩ
(D) 6 kΩ, 3 kΩ and 3 kΩ

48. On the basis of extreme single particle shell model, the ground state spin parity of ${}_{21}Sc^{45}$ will be

(A) $\left(\frac{3}{2}\right)^{+}$
(B) $\left(\frac{5}{2}\right)^{-}$
(C) $\left(\frac{7}{2}\right)^{-}$
(D) $\left(\frac{5}{2}\right)^{+}$

49. Consider a strong interaction: $\Pi^{+} + p \rightarrow \Sigma^{+} + X$. The z component of Isospin (I_3) of X is

(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) +1
(D) –1

50. In the quark model, the Δ^{++} , a spin –3/2 resonance of charge +2, consists of which of the following combinations of quarks?

(A) uuu
(B) uud
(C) ddd
(D) ud

SPACE FOR ROUGH WORK