PUMDET–2017

Subject : Mathematics

Time Allowed: 1Hour 30 Minutes | *Maximum Marks*: 100

Booklet No. 21000479

INSTRUCTIONS

Candidates should read the following instructions carefully before answering the questions:

1. This question paper contains **50** MCQ type objective questions. Each question has four answer options given, viz. A, B, C and D.
2. Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combinations of more than one answer will fetch – ½ mark. No answer will fetch 0 mark.
3. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
4. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.
5. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
6. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.
7. Write your name (in block letters), name of the examination centre and put your full signature in appropriate boxes in the OMR.
8. The **OMR**s will be processed by electronic means. Hence it is liable to become invalid if there is any mistake in the question booklet number or roll number entered or if there is any mistake in filling corresponding bubbles. Also it may become invalid if there is any discrepancy in the name of the candidate, name of the examination centre or signature of the candidate vis-a-vis what is given in the candidate's admit card. The **OMR** may also become invalid due to folding or putting stray marks on it or any damage to it. the consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
10. Hand over the OMR to the invigilator before leaving the Examination Hall.

$\mathbb{N}, \mathbb{Q}, \mathbb{R}$ respectively denotes the set of natural numbers, the set of rational numbers and the set of real numbers.

1. Define $f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ by

   $f(x, y, z) = (x + 1, y - 1, z)$ for all

   $(x, y, z) \in \mathbb{R}^3$.

   Then the map f is

   (A) open and closed

   (B) open but not closed

   (C) closed but not open

   (D) neither open nor closed

2. Define $f: (0,1) \longrightarrow \mathbb{R}$ by

   $f(t) = \frac{2t-1}{2\sqrt{t(1-t)}}$ for all $t \in (0,1)$.

   Then the map f is

   (A) bijective

   (B) injective but not surjective

   (C) surjective but not injective

   (D) neither injective nor surjective

3. Let G be a group of order 7×17×19. Then G is

   (A) cyclic

   (B) abelian but not necessarily cyclic

   (C) nonabelian

   (D) not necessarily abelian

4. Let G be a group of order 175. Then G is

   (A) cyclic

   (B) abelian but not necessarily cyclic

   (C) nonabelian

   (D) not necessarily abelian

5. Let V be a vector space with a basis $\{v_1, v_2, \ldots v_n\}$. Define a linear transformation $T: V \longrightarrow V$ by

   $T(v_i) = v_{i+1}$ for all $1 \leq i \leq n - 1$, and
   $T(v_n) = 0$.

   Then T is

   (A) diagonalizable

   (B) $T^k = 0$ for some $k \in \mathbb{N}$

   (C) both

   (D) neither

6. Let $A \in M_{n \times n}(F)$, where F is a field. Define $T: M_{n \times n}(F) \to M_{n \times n}(F)$ by

   $T(B) = AB$ for all $B \in M_{n \times n}(F)$.

   If A is diagonalizable, then T is

   (A) diagonalizable

   (B) $T^k = 0$ for some $k \in \mathbb{N}$

   (C) both

   (D) neither

7. Let $\mathbb{R}[x]$ be the space of all real polynomials in x and $D: \mathbb{R}[x] \longrightarrow \mathbb{R}[x]$ be the differentiation map i.e., $D(p)(x) = p'(x)$ for all $p \in \mathbb{R}[x]$. Then

   (A) D is invertible

   (B) D has a right inverse

   (C) D has a left inverse

   (D) D is diagonalizable

8. Let V be a finite dimensional vector space and $T: V \longrightarrow V$ be a linear operator. Define N = null space of T, and R= range space of T. If rank (T^2) =rank(T), then

   (A) N and R are disjoint

   (B) $N \subset R$

   (C) $N = R$

   (D) $\dim(N) = \dim(R)$

9. Let $M, N \in M_{n\times n}(\mathbb{R})$ be such that $M^n = N^n = 0$ but $M^{n-1} \neq 0 \neq N^{n-1}$. Then

(A) M and N represent the same linear transformation on $\mathbb{R}^n$.

(B) M and N may represent different linear transformations on $\mathbb{R}^n$.

(C) M and N represent surjective linear transformations on $\mathbb{R}^n$.

(D) M and N represent injective linear transformations on $\mathbb{R}^n$.

10. Let t_1, t_2, t_3 be distinct real numbers and c_1, c_2, c_3 be any real numbers. Then which one of the following is the most appropriate?

(A) There is exactly one polynomial function $p: \mathbb{R} \longrightarrow \mathbb{R}$ such that $p(t_i) = c_i$ for all $i = 1,2,3$.

(B) There is exactly one polynomial function $p: \mathbb{R} \longrightarrow \mathbb{R}$ of degree at most 2 such that $p(t_i) = c_i$ for all $i = 1,2,3$.

(C) There is a continuous function $f: \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(t_i) = c_i$ for all $i = 1,2,3$.

(D) There is exactly one polynomial function $p: \mathbb{R} \longrightarrow \mathbb{R}$ of degree at most 3 such that $p(t_i) = c_i$ for all $i = 1,2,3$.

11. The initial value problem $\dot{x}(t) = 3x^{2/3}$, $x(0) = 0$ in an interval around $t = 0$ has

(A) no solution

(B) unique solution

(C) finitely many solutions

(D) infinitely many solutions

12. For the system of ODE $\frac{d}{dt}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$

(A) every solution is bounded.

(B) every solution is bounded and periodic.

(C) every solution is not bounded but periodic.

(D) every solution is only periodic.

13. Which of the following polynomials represent the given data:

x	1	$\frac{1}{2}$	3
y	3	-10	2

(A) $3 + 26(x-1) + \frac{53}{6}x^2$

(B) $\frac{2}{5}(x-1)(x-1/2) - 8(x-1)(x-3) + 3(x-3)(x-1/2)$

(C) $(x-3)(x+10) + \frac{1}{2}(x+10)(x-2) + 3(x-2)(x-3)$

(D) $-\frac{3}{2}\left(x-\frac{1}{2}\right)(x-3) - 8(x-1)(x-3) + \frac{2}{3}(x-1)(x-1/2)$

14. Let $(x(t), y(t))$ satisfy the system of ODEs $\begin{matrix} \frac{dx}{dt} = -x + ty \\ \frac{dy}{dt} = tx - y \end{matrix}$, If (x_1, y_1) and (x_2, y_2) are two solutions and $\phi = x_1y_2 - x_2y_1$ then $\frac{d\phi}{dt} =?$

(A) -2ϕ

(B) 2ϕ

(C) $-\phi$

(D) ϕ

15. The BVP $x^2y'' - 2xy' + 2y =$ subject to the boundary conditions $\begin{matrix} y(1) + \alpha y'(1) = 1 \\ y(2) + \beta y'(2) = 2 \end{matrix}$ has a unique solution if:

(A) $\alpha = -1, \beta = 2$

(B) $\alpha = -1, \beta = -2$

(C) $\alpha = -2, \beta = 2$

(D) $\alpha = -3, \beta = 2/3$

16. The iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$, $n > 0$ for a given $x_0 \neq 0$ is an instance of

(A) a fixed point iteration for $f(x) = x^2 - 2$.

(B) Newton's Method for $f(x) = x^2 - 2$.

(C) fixed point iteration for $f(x) = \frac{x^2+2}{2x}$.

(D) Newton's Method for $f(x) = x^2 + 2$.

17. Suppose G be a graph with 12 edges. If G has six vertices each of degree 3 and the rest have degree less than 3, then the minimum number of vertices that G can have is

(A) 8

(B) 9

(C) 10

(D) 11

18. In a two-person-zero-sum symmetric game,

(A) the pay off is always zero.

(B) Maximizing player's pay-off is always greater than minimizing player's pay-off.

(C) Minimizing player's pay-off is always greater than maximizing player's pay-off.

(D) pay-off is always non-zero number.

19. A tosses a coin twice. He receives Rs.2 if he tosses 2 heads. He looses Re.1 if the toss is a head and a tail. He wins Rs.3 if he tosses 2 tails. Then $A's$ average win in the game is

(A) 5/7

(B) 7/5

(C) 7/12

(D) 5/12

20. The number of real solutions of the equation $x = 99 \sin(\pi x)$ is

(A) 1

(B) 19

(C) 99

(D) 199

21. The limiting value of $\frac{1.2+2.3+\cdots+n.(n+1)}{n^3}$ as $n \to \infty$ is

(A) 0

(B) 1

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

22. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$, then $A^{100} + A^5$ is

(A) $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(B) $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$

(C) $A = \begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$

(D) $A = \begin{pmatrix} -1 & -1 \\ 2 & -2 \end{pmatrix}$

23. A function f is defined on $(-1,1)$ by $f(x) = x^\alpha \sin\frac{1}{x^\beta}, x \neq 0$ and $f(0) = 0$. Then f' is continuous if

(A) $0 < \beta < \alpha - 1$

(B) $0 < \alpha < \beta + 1$

(C) $\beta = \alpha$

(D) $0 < \alpha < \beta - 1$

24. $x^2 + x + 1$ is a factor of $(x + 1)^n - x^n - 1$, whenever

(A) n is odd

(B) n is odd and multiple of 3

(C) n is even multiple of 3

(D) n is odd and not a multiple of 3

25. If $a^2 + b^2 + c^2 = 1$ then $ab + ac + bc$ lies in

(A) $\left[-\frac{1}{2}, 1\right]$

(B) $\left[\frac{1}{2}, 1\right]$

(C) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(D) $[-1,1]$

26. If α and β are two complex numbers, then the maximum value of $\frac{\alpha\bar{\beta}+\bar{\alpha}\beta}{|\alpha\beta|}$ is

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

27. The value of a for which the sum of squares of the roots of equation $x^2 - (a-2)x - a - 1 = 0$ assumes the least value is

(A) 0

(B) 1

(C) 2

(D) 3

28. Which of the following statement is true?

(A) $|\sin z| \leq 1$ for all $z \in \mathbb{C}$

(B) $\sin^2 z + \cos^2 z = 1$, for all $z \in \mathbb{C}$

(C) $\sin 2z = 2 \sin z \cos z$ for all $z \in \mathbb{C}$

(D) $\sin 3z = 3 \sin z - 4 \sin^3 z$ for all $z \in \mathbb{C}$

29. Which of the following sequence of functions $\{f_n\}$ is uniformly convergent?

(A) $f_n(x) = nxe^{-nx^2}, x \in [0,1]$

(B) $f_n(x) = n^2x^2e^{-nx}, x \in [0, \infty)$

(C) $f_n(x) = xe^{-nx}, x \in [0, \infty)$

(D) $f_n(x) = nx(1 - x^2)^n, x \in [0,1]$

30. Which one is the true statement?

(A) Any group of order 8 is cyclic.

(B) Any group of order 5 is abelian.

(C) Any group of order 11 has a nontrivial subgroup.

(D) There are no nontrivial homomorphisms from the cyclic group of order n to S_n.

31. If a function $f: \mathbb{C} \to \mathbb{C}$ is defined by $f(z) = |z|^2, \forall z \in \mathbb{C}$, then f is differentiable at

(A) $z = 0$

(B) $z = 1$

(C) $z = 2$

(D) $z = 3$

32. If a and b are two elements of an abelian group such that $0(a)m$ and $0(b) = n$ then $0(ab)$ is

(A) max $\{m,n\}$

(B) mn

(C) $g.c.d.\{m,n\}$

(D) $l.c.m.\{m,n\}$

33. Let X be a compact metric space and consider the space $C(X) = \{f: X \to \mathbb{R}: f \text{ is continuous}\}$ with the metric $d(f,g) = \sup_X |f(x) - g(x)|$. Then in which case the set of functions $\{f_n: X \to \mathbb{R}\}_{n\geq 0}$ is compact?

(A) $f_n(x) = n \; \forall x \in X$

(B) $f_n(x) = \frac{n}{n+1} \forall x \in X$

(C) $f_n(x) = \frac{1}{n} \; \forall x \in X$ and if $n \neq 0, f_0(x) = 0 \; \forall x \in X$

(D) $f_n(x) = \frac{1}{n} \; \forall x \in X$ and if $n \neq 0, f_0(x) = 1 \; \forall x \in X$

34. Let $I = \int_1^\infty e^{ax^2+bx+c} dx$, where a, b, c are constants. For which values of a, b, c the integral I will be convergent?

(A) $a = -1, b = 0, c = 100$

(B) $a = 1, b = 10, c = 0$

(C) $a = 0, b = 1, c = 0$

(D) $a = 0, b = 0, c = -10$

35. Which of the following function is uniformly continuous on its domain $(0, \infty)$,?

(A) $p(x) = x^2$

(B) $f(x) = e^x$

(C) $g(x) = e^{-x}$

(D) $h(x) = x^2 \sin x$

36. Which of the following sets is dense in the plane with respect to the usual Euclidean metric?

(A) The lines through origin with integer slopes.

(B) The lines through origin with rational slopes.

(C) The concentric circles centered at origin with integer radius.

(D) The upper half plane containing points (x, y) such that $y > 0$ union the integral points i.e. $\mathbb{Z} \times \mathbb{Z}$.

37. Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{bmatrix}$. Which of the following is true?

(A) A is invertible for every $t \in \mathbb{R}$.

(B) A is invertible for all $t > 0$.

(C) A is invertible for all $t < 0$.

(D) A is invertible for all $t \in \mathbb{Z}$.

38. Which of the following functions is Riemann integrable on $[0,1]$?

(A) $D(x) = \begin{cases} 0 & if\ x \in \mathbb{Q} \\ 1 & if\ x \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$

(B) $g(x) = \begin{cases} \frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$

(C) $h(x) = \begin{cases} x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ x & if\ x = 0 \end{cases}$

(D) $p(x) = \begin{cases} x & if\ x \in \mathbb{Q} \\ \frac{1}{x} & if\ x \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$

39. The interval of convergence of $\sum(\log n)^2 x^n$ is

(A) $(-1,1)$

(B) $(-2,2)$

(C) $(-1/2,1/2)$

(D) $\mathbb{R}$

40. The remainder when $f(x) = 4x^4 + 3x^3 + x^2 + 5x + 1$ is divided by $x^2 + 1$ is

(A) $2x + 4$

(B) $x + 2$

(C) $0.x + 2$

(D) $2x + 3$

41. Let $A: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with eigen-values $\frac{2}{3}$ and $\frac{9}{5}$. Then there is a non-zero vector $v \in \mathbb{R}^2$ such that

(A) $\|Av\| > 2\|v\|$

(B) $\|Av\| < \frac{1}{2}\|v\|$

(C) $\|Av\| = \|v\|$

(D) $Av = 0$

42. The determinant of the matrix

$M = \begin{bmatrix} 1 & 10 & 1 \\ 12 & 22 & 11 \\ 23 & 34 & 21 \end{bmatrix}$ is

(A) -11

(B) 0

(C) 11

(D) 110

43. The closure of the subset $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1, x \in \mathbb{Q}, y \in \mathbb{Q}\}$ inside the unit disk $\mathbb{D} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ is

(A) A proper subset of the unit disk $\mathbb{D}$.

(B) $\mathbb{D}\setminus\{(x,y) \in \mathbb{D}: x \notin \mathbb{Q}$ or $y \notin \mathbb{Q}\}$.

(C) All of $\mathbb{D}$

(D) $\mathbb{D} \cup \{(x,y): x^2 + y^2 = 1\}$.

44. The number of real roots of the equation $x^3 - 3x + 1 = 0$ is

(A) 0

(B) 1

(C) 2

(D) 3

45. Which of the following is an injective function on $(0,\infty)$?

(A) $f(x) = \exp(2x + 5)$

(B) $f(x) = x^2 - x$

(C) $f(x) = \sin(\exp(2x + 5))$

(D) $f(x) = \frac{x+2}{x^2+1}$

46. If $2n$ boys are divided into two equal subgroup, the probability that the two tallest boys will be in the same subgroup is

(A) $\frac{1}{2n-1}$

(B) $\frac{n-1}{2n-1}$

(C) $\frac{1}{2}$

(D) $\frac{1}{2n}$

47. The series $\sum_{n=1}^{\infty} \frac{n}{n^4+n^2+1}$

(A) converges to $\frac{1}{2}$

(B) converges to $\frac{1}{4}$

(C) does not converge

(D) converges to $\frac{2}{3}$

48. Each face of a cube is to be painted with one of the 6 colours such that every two adjacent faces have different colours. The number of ways that the faces can be painted is

(A) 180

(B) 230

(C) 720

(D) 90

49. Let $f: \mathbb{R} \to \mathbb{R}$ be a non-zero smooth function such that $f(x)f(y) = f(\sqrt{x^2+y^2})$ for all $x, y \in \mathbb{R}$ and $\lim_{|x| \to \infty} f(x) = 0$. Then

(A) $f(x)$ is an odd function.

(B) $f(x)$ is an even function and $f(0) \neq 1$.

(C) $f(x)$ is an even function and $f(0) = 1$.

(D) neither even nor odd.

50. The volume enclosed by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$ is

(A) πabc

(B) $\frac{1}{3}\pi abc$

(C) $\frac{4}{3}\pi abc$

(D) $\frac{2}{3}\pi abc$

Space for rough work

Space for rough work

Space for rough work