**PUMDET-2023**

**Subject : MATHEMATICS**

**(Booklet Number)**

Duration : **90** Minutes | No. of Questions : **50** | Full Marks : **100**

## INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks **2**. In case of incorrect answer or any combination of more than one answer, **½** mark will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
3. Use only **Black/Blue ink ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR Sheet**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your signature (as is appeared in Admit Card) in appropriate boxes in the OMR Sheet.
7. The OMR Sheet is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/signature of the candidate, name of the examination centre. The OMR Sheet may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones, bluetooth devices etc. inside the examination hall. Any candidate found with such prohibited items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough work must be done on the question booklet itself. Additional blank pages are given in the question booklet for rough work.
10. Hand over the OMR Sheet to the invigilator before leaving the Examination Hall.
11. Candidates are allowed to take the Question Booklet after examination is over.

Signature of the Candidate : ______________________________
(as in Admit Card)

Signature of the Invigilator : ______________________________

SPACE FOR ROUGH WORK

**1.** Consider the polynomial

$$p(z) = z^5 + z^3 + 5z^2 + 2$$

In the annular region $1 < |z| < 2$, $p(z)$ has

(A) 5 (B) 3

(C) 2 (D) 1

zero or zeros

**2.** Let $f(z) = \dfrac{z+1}{z \sin z}$, then f has

(A) No singularity (B) Removable discontinuity at z = 0

(C) Pole of order 2 at z = 0 (D) Simple pole at z = 0

**3.** Let $f(z) = |z-a|^2$, 'a' is fixed point in $\mathbb{C}$. Then

(A) f is discontinuous function

(B) f is differentiable everywhere in $\mathbb{C}$

(C) f is differentiable only at a point in $\mathbb{C}$ but nowhere else

(D) f is not differentiable anywhere in $\mathbb{C}$

**4.** Let $f(z) = x^2y^2 + i.2x^2y^2$. Then

(A) f is discontinuous everywhere in $\mathbb{C}$

(B) f is analytic everywhere in $\mathbb{C}$

(C) f is nowhere analytic in $\mathbb{C}$

(D) Cauchy-Riemann equations do not hold anywhere in $\mathbb{C}$

**5.** $\displaystyle\int_1^{\infty} \frac{\cos ax - \cos bx}{x} dx$, $a, b \in \mathbb{R}^+$, then

(A) The integral is proper

(B) The integral is divergent

(C) The integral is convergent

(D) $\displaystyle\int_1^{X} f(x)dx$ is unbounded $\left( f(x) = \dfrac{\cos ax - \cos bx}{x} \right)$

**6.** The solution of $xy\dfrac{\partial^2 z}{\partial x \partial y} - \dfrac{\partial z}{\partial y} y = x^2$ is given by

(A) $z = xy\log(xy) + f(x, y)$ (B) $z = x\log y + y\log x + f(x)$

(C) $z = y^2 \log x + yf(x) + f(y)$ (D) $z = x^2 \log y + xf(y) + g(x)$

Here x > 0, y > 0 and f, g are arbitrary functions.

**7.** Solution of $(y+z)p - (x+z)q = x - y$, (p, q have their usual meaning) is given by

(A) $f(y+z, -x-z) = 0$ (B) $f(zx + y - z, x^2 + y^2 + z^2) = 0$

(C) $f(x+y+z, x^2 + y^2 - z^2) = 0$ (D) $f(x+y+z, x^2 + y^2 + z^2) = 0$

**8.** Let (X, $d_1$) and (Y, $d_2$) be metric spaces and (X×Y, d) denote the product space with the product metric d. Then

(A) (X×Y, d) is never complete

(B) (X×Y, d) is complete if any one of (X, $d_1$) and (Y, $d_2$) be complete

(C) (X×Y, d) is complete iff (X, $d_1$) and (Y, $d_2$) both are complete

(D) (X×Y, d) is complete if $d_1(x,y) < d_2(x,y)$

**9.** Let $X = \mathbb{N}$ and $d : X \times X \to \mathbb{R}$ be given by

$$d(m,n) = \frac{|m-n|}{mn} \text{ for all } m, n \in \mathbb{N}.$$

Then

(A) (X, d) is not a metric space

(B) (X, d) is pseudo metric space

(C) (X, d) is complete metric space

(D) (X, d) is incomplete metric space

**10.** Let $X = \{(x, y) \in \mathbb{R} : x^2 + y^2 < 1\}$ and metric d be defined by

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]}$$

(A) (X, d) is a complete metric space

(B) (X, d) is incomplete metric space

(C) No sequence in (X, d) is a Cauchy sequence

(D) All sequences in (X, d) are Cauchy

**11.** Let S be the sample space of the random experiment of throwing simultaneously two unbiased dice and $E_k = \{(a, b) \in S : ab = k\}$ . If $p_k = P(E_k)$, there the correct among the following is

(A) $p_9 < p_{19} < p_2$

(B) $p_5 < p_{21} < p_1$

(C) $p_9 < p_{18} < p_6$

(D) $p_4 < p_{13} < p_1$

**12.** $\oint_C (\cos x dx + 2y^2 dy + z dz)$, where C is the curve $x^2 + y^2 = 1,\ z = 1$, is given by

(A) 0

(B) 1

(C) 2

(D) $\frac{1}{2}$

**13.** S.H.M. in a resisting medium is

(A) Forced Oscillation

(B) Damped Oscillation

(C) Damped Forced Oscillation

(D) Not Oscillatory motion

**14.** If a particle describes a curve $r = ae^{b\theta}, (a, b > 0)$ with a constant angular velocity, then the cross-radial acceleration

(A) varies as the distance from the pole

(B) varies as the square of the distance from the pole

(C) varies inversely as the distance from the pole

(D) varies inversely as the square of the distance from the pole

**15.** The value of $\Delta^n(a^x)$ is

(A) $(a^h - 1)^n a^x$ (B) $(a^h + 1)^n a^x$

(C) $(a^{nh} - 1)^n a^x$ (D) $(a^{nh} + 1)^n a^x$

**16.** For the fixed point iteration $x_{k+1} = g(x_k), k = 0, 1, 2, .....$, consider the following statements P and Q:

P: If $g(x) = 1 + \frac{2}{x}$, then fixed point iteration converges to 2 for all $x_0 \in [1, 100]$

Q: If $g(x) = \sqrt{2 + x}$, then fixed point iteration converges to 2 for all $x_0 \in [0, 100]$

Then

(A) both P and Q are true (B) only P is true

(C) only Q is true (D) neither P nor Q is true

**17.** If the primal has no feasible solution, then its dual has

(A) bounded solution

(B) feasible solution

(C) either unbounded or no feasible solution

(D) feasible solution, but no optimal solution

**18.** In a transportation problem with m origin and n destination, if the number of allocations is equal to $m + n - 1$, then the solution is called

(A) degenerate solution
(B) non-feasible solution
(C) unbounded solution
(D) non-degenerate solution

**19.** Let f(x) be continuously differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t\to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $\int_1^2 f(x)dx$ is

(A) $\frac{1}{3}\log 2 + \frac{14}{9}$
(B) $\frac{1}{3}\log 2 - \frac{14}{9}$
(C) $\frac{1}{2}\log 3 + \frac{14}{9}$
(D) $\frac{1}{2}\log 3 - \frac{14}{9}$

**20.** The solution of the Ordinary Differential Equation $\frac{dy}{dx} + P(x)y = Q(x)$ may be expressed in the form

(A) $y = e^{\int Qdx} + C$
(B) $y = e^{-\int Pdx} + C$
(C) $y = e^{\int (P+Q)dx} + C$
(D) $y = \frac{Q}{P} - e^{-\int Pdx}.\left\{C + e^{\int Pdx} d\left(\frac{Q}{P}\right)\right\}$

Where C is the integrating constant

**21.** The equation of the right circular cone whose vertex is at the origin, axis is the X-axis and the semi-vertical angle is $\frac{\pi}{3}$, is

(A) $x^2 + y^2 = 3z^2$
(B) $z^2 + x^2 = 3y^2$
(C) $y^2 + z^2 = 3x^2$
(D) $y^2 + x^2 = 9z^2$

**22.** If the polar of a point with respect to the parabola $y^2 = 4ax, (a > 0)$ touches the parabola $x^2 = 4by, (b > 0)$, then the locus of the point is the

(A) $x^2 + y^2 = 4a^2$

(B) $y^2(x + 2b) + 4a^2 = 0$

(C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(D) $xy + 2ab = 0$

**23.** $\sin z = 2$ is

(A) solvable neither in $\mathbb{R}$ nor in $\mathbb{C}$

(B) solvable in $\mathbb{R}$ and $\mathbb{C}$

(C) solvable in $\mathbb{C}$ only and then one of the expressions is $z = \left(2n + \frac{1}{2}\right)\pi - i\log\left(2 + \sqrt{3}\right)$ where n is integer

(D) Solvable in $\mathbb{C}$ and $z = \frac{n\pi}{2} + \left(\frac{\pi}{2} - 2n\pi\right)i$, n is integer

**24.** Which one of the following is correct?

(A) If the quotient group G/H is cyclic, then G must be cyclic.

(B) $\exists$ a non-cyclic group H for which quotient group G/H is cyclic.

(C) Every group G is isomorphic to a permutation group.

(D) An infinite cyclic group has infinitely many generators.

**25.** Let $R \equiv M_2(z)$, $I = \left\{\begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix} : a, b \in z\right\}$, then

(A) I is not a subring of R

(B) I is an Ideal of R

(C) I is a subring but not any Ideal of R

(D) I is a left Ideal of R only

**26.** Let $f(x) = x^4 - 4x^3 + 4x^2 + C,\ C \in \mathbb{R}$, then in (1, 2)

(A) f(x) has atmost one zero for $-1 < C < 0$

(B) f(x) cannot have any zero in (1, 2)

(C) f(x) has infinitely many zeros for all $C \in \mathbb{R}$

(D) f(x) has two zeros in (1, 2) for all $C \in \mathbb{R}$

**27.** If a prime integer p divides the order of a Group G, then G contains

(A) at least one normal subgroup of order p

(B) no normal subgroup of order p

(C) infinitely many normal subgroups

(D) exactly one normal subgroup

**28.** $\det A = \begin{vmatrix} 2021 & 2020 & 2020 & 2020 \\ 2021 & 2021 & 2020 & 2020 \\ 2021 & 2021 & 2021 & 2020 \\ 2021 & 2021 & 2021 & 2021 \end{vmatrix}$ is

(A) 1 (B) 0

(C) 2021 (D) 2020

**29.** The last digit of $(2004)^5$ is

(A) 4 (B) 8

(C) 6 (D) 2

**30.** Let G be a group of order 28. Then

(A) G contains elements of order 7

(B) G may not contain any elements of order 7

(C) G contains elements of order 12

(D) G contains elements of order 5

**31.** Which one is true ?

(A) Groups of order pq, p and q are prime, are simple groups

(B) Group of order pq, p and q are prime, is cyclic

(C) Let G be a finite group of order 2n, where n is odd integer and greater than 1. Then G is not simple

(D) Groups of order 56 are simple groups

**32.** A subset W ($\subset \mathbb{R}$) is called a Ring if it contains 1 and if $\forall a, b \in W$, the numbers a – b and ab are also in W. Let S = $\left\{ \frac{m}{2^n} : \text{m, n are integers} \right\}$ and T = $\left\{ \frac{p}{q} : \text{p, q are integers and q odd} \right\}$, Then

(A) neither S nor T is a Ring

(B) S is a Ring but T is not a Ring

(C) T is a Ring but S is not a ring

(D) Both S and T are Rings

**33.** Let $A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{pmatrix}$. Then

(A) Eigen values of A are real but not distinct

(B) Eigen vectors of A are linearly dependent

(C) A is diagonalizable

(D) Eigen values of A are not all real

**34.** Consider the linear transformation (T):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Then

(A) T is one-to-one

(B) Ker T consists of elements of form (a, b, c) where $a^2 + b^2 = c^2$

(C) Ker T consists of elements of form (a, b, c) where $a^2 + b^2 = 3c^2$

(D) Ker T consists of elements of form (a, b, c) where $a^2 + b^2 = 2c^2$

**35.** Consider the curve represented by $y = 2 - |x^5 - 1|$ . Then

(A) Curve has no point of inflexion

(B) (0, 1) is a point of inflexion

(C) (1, 0) is a point of inflexion

(D) (0, 0) is a point of inflexion

**36.** The volume of the catenoid formed by the revolution about the x-axis, of the area bounded by the catenary $y = \frac{a}{2}(e^{x/a} + e^{-x/a})$, the y-axis, the x-axis and ordinate is

(A) $\frac{\pi a}{2}(sy + ax)$ (B) $\pi(sy + ax)$

(C) $\frac{\pi a}{2}(sy - ax)$ (D) $\pi[(sy + ax) - a]$

where s is the length of the arc between (0, a) and (x, y)

**37.** Consider the curve $\Gamma$ represented by

$$f(x, y) = y^2 - 2x^2y - x^4y - x^4 = 0$$

Then

(A) $\Gamma$ has no double point

(B) $\Gamma$ has a node at (0, 0)

(C) $\Gamma$ has a cusp of second species

(D) $\Gamma$ has a cusp of first species

**38.** Let $\Gamma$ denote the graph of f defined by

$$y = f(x) = \log\frac{2x-1}{x+2} \quad \text{on } (-\infty, -2) \cup \left(\frac{1}{2}, \infty\right)$$

Then

(A) the curve has no asymptote

(B) the curve has only horizontal asymptote

(C) the curve has only vertical asymptote

(D) $x = \frac{1}{2}$, $x = -2$ and $y = \log 2$ are asymptotes

**39.** Let $f(x, y) = \left(x^3 + y^3\right)^{1/3}$. Then

(A) f is not continuous at (0, 0)

(B) neither $\frac{\partial f}{\partial x}$ nor $\frac{\partial f}{\partial y}$ exists at (0, 0)

(C) f is differentiable at (0, 0)

(D) f is not differentiable at (0, 0)

**40.** The equation $f\left(\frac{y}{x}, \frac{z}{x}\right) = 0$ defines z implicitly as a function of x and y, say

$z = g(x, y)$. Given $g_x$, $g_y$ are continuous and $D_2 f\left(\frac{y}{x}, \frac{z(x,y)}{x}\right) \neq 0$. Then

(A) g(x, y) is not homogeneous function of x and y

(B) g(x, y) is a homogeneous function of x and y

(C) $g_x g_y = 0$

(D) $g_x^{\,2} + g_y^{\,2} = 1$

[$D_2 f$ means partial derivative of f with respective to second argument]

**41.** Let $I(a,b) = (a+b)^p$, $J(a,b) = a^p + b^p$, $0 \le p \le 1$. Then

(A) $I(a,b) \ge J(a,b)$

(B) $I(a,b) \le J(a,b)$

(C) no specific order relation exists between $I(a,b)$ and $J(a,b)$

(D) $I(a,b) \ge J(a,b)$ in $0 \le p \le \frac{1}{2}$ and $I(a,b) \le J(a,b)$ in $\frac{1}{2} \le p \le 1$

**42.** Let $f:[a,b] \to \mathbb{R}$ be of Bounded Variation (BV) in [a, b] and there exists a function $F:[a,b] \to \mathbb{R}$ such that $F' = f$ in [a, b]. Then

(A) f has discontinuity of both kind

(B) f has discontinuity of first kind only

(C) f has discontinuity of second kind only

(D) f is continuous in [a, b]

**43.** Let $f_n(x) = x^n - x^{2n}$ in $[0,1]$. Then

(A) $\{f_n(x)\}_n$ is not pointwise convergent in [0, 1]

(B) $\{f_n(x)\}_n$ is pointwise convergent but not uniformly convergent in [0, 1]

(C) $\{f_n(x)\}_n$ is uniformly convergent in [0, 1]

(D) $\lim\limits_{n\to\infty} f_n(x)$ is not bounded

**44.** Let $u(x) = 2 + 2x + \sin 2x,\ v(x) = (2x + \sin 2x)e^{\sin x}$. Then

(A) $\lim\limits_{x\to\infty} \dfrac{u(x)}{v(x)} = 0,\ \lim\limits_{x\to\infty} \dfrac{u'(x)}{v'(x)} = 0$

(B) $\lim\limits_{x\to\infty} \dfrac{u(x)}{v(x)} = 0$ but $\lim\limits_{|x|\to\infty} \dfrac{u'(x)}{v'(x)}$ does not exist

(C) $\lim\limits_{x\to\infty} \dfrac{u(x)}{v(x)} = \infty = \lim\limits_{x\to\infty} \dfrac{u'(x)}{v'(x)}$

(D) $\lim\limits_{x\to\infty} \dfrac{u(x)}{v(x)}$ does not exist but $\lim\limits_{x\to\infty} \dfrac{u'(x)}{v'(x)} = 0$

**45.** Let $u_n(x) = \dfrac{\sin 2^n \pi x}{2^n},\ x \in \mathbb{R}$, then

(A) $\sum\limits_n u_n(x)$ is not uniformly convergent on $\mathbb{R}$

(B) $\sum\limits_n u_n(x)$ can be differentiated term-by-term

(C) $\sum\limits_n u_n'(x)$ is convergent in $\mathbb{R}$

(D) $\sum\limits_n u_n(x)$ cannot be differentiated term-by-term

**46.** Let $I(R) = \int\limits_0^{\pi/2} e^{-R\sin x}dx,\ J(R) = \dfrac{\pi}{2R}\left(1 - e^{-R}\right),\ R > 0$. Then

(A) I(R) ≤ J(R)

(B) I(R) > J(R)

(C) I(R) = J(R)

(D) no specific order relation exists

**47.** Let $I_n = \int_0^1 e^{x^2} \sin(nx)\, dx, (n \in \mathbb{N})$. Then $\lim_{n\to\infty} I_n$

(A) does not exist

(B) is 0

(C) is 1

(D) is e

**48.** Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function and let $\int_0^{2x} f(3t)\,dt = \frac{x}{\pi}\sin(\pi x)\ \forall x \in \mathbb{R}$. Then $f\left(\frac{1}{2}\right)$ is

(A) $\frac{1}{2}$

(B) $\frac{3+2\sqrt{3}}{16}$

(C) $\frac{6+\pi\sqrt{3}}{24\pi}$

(D) $\frac{6-\pi\sqrt{3}}{12\pi}$

**49.** The series $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(A) is divergent

(B) is convergent and represents 2 in the number scale

(C) represents e

(D) represents $\log_e 2$

**50.** The series $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

(A) consists of terms which are increasing

(B) the $n^{th}$ term does not tend to zero

(C) is convergent

(D) $n^{th}$ term $\to \infty$ and so the series is divergent

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## SPACE FOR ROUGH WORK