JELET-2021

For B.Sc. candidates

1093000505

(Booklet Number)

Duration: **2** Hours

Full Marks: **100**

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks **1.** In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C, or D.

3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.

4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.

5. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.

6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.

7. The OMR is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/ signature of the candidate, name of the examination centre. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.

8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** and his/her candidature will be summarily cancelled.

9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.

10. Hand over the OMR to the invigilator before leaving the Examination Hall.

JELET-2021

SPACE FOR ROUGH WORK

1. The value of $\begin{vmatrix} 1+x & y & z \\ x & 1+y & z \\ x & y & 1+z \end{vmatrix}$ is

(A) $1+x+y+z$
(B) $xy+yz+zx$
(C) xyz
(D) $x+y+z$

2. Which one of the following is not an associative operation ?

(A) on $(\mathbb{Z}, *)$, $x * y = x + y + 1$ for all $x, y \in \mathbb{Z}$

(B) on $(\mathbb{R}, *)$, $x * y = |x + y|$ for all $x, y \in \mathbb{R}$

(C) on $(\mathbb{R}, *)$, $x * y = |x + y + xy|$, for all $x, y \in \mathbb{R}$

(D) on $(\mathbb{R}, *)$, $x * y = x + y - xy$

3. The value of the determinant $\begin{vmatrix} 1986 & 1987 & 1988 \\ 1989 & 1990 & 1991 \\ 1992 & 1993 & 1994 \end{vmatrix}$ is

(A) 0
(B) 1
(C) -1
(D) 3

4. Let G be a non-void set and operation $*$ be defined on G as follows :

$a * b = a$, then

(A) $(G, *)$ is a group

(B) $(G, *)$ is not a group

(C) $(G, *)$ is not a semigroup

(D) $(G, *)$ is a quasigroup

5. Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R},\ ad \neq 0 \right\}$ be a group under matrix multiplication.

Let $N = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}$, then

(A) (N, ∗) is a subgroup of (G, ∗)
(B) (N, ∗) is not a subgroup of (G, ∗)
(C) (N, ∗) is only a semigroup
(D) (N, ∗) is not even groupoid

6. Let $M = \left\{ \begin{pmatrix} x & y \\ z & u \end{pmatrix} : x, y, z, u \in Z \right\}$. Let ∗ denote usual matrix multiplication and + denote usual matrix addition. Then

(A) (M, +) has no additive identity
(B) (M, ∗) has no multiplicative identity
(C) (M, +, ∗) is a ring
(D) (M, +, ∗) is not a ring

7. The system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have a unique solution if

(A) $\lambda = 3, \mu \neq 10$
(B) $\lambda \neq 3$, μ may have any value
(C) $\lambda = 3$ and $\mu = 10$
(D) $\lambda \neq 3, \mu \neq 10$

8. The equation $2x^3 - 9x^2 + 12x + \alpha = 0$ has two equal roots. Then the value of α is

(A) 4
(B) 5
(C) – 5
(D) 6

9. If $x^4 - x^3 + 2x^2 - 3x + 1 = (x-3)^4 + a(x-3)^3 + b(x-3)^2 + c(x-3) + d$, then c is equal to

(A) 11
(B) 47
(C) 90
(D) 64

10. To remove the 2nd term of the equation $x^3 - 12x^2 - 6x - 10 = 0$, the roots of the equation should be increased by

(A) – 4
(B) 4
(C) 3
(D) – 3

11. If x_1, x_2, x_3 are the roots of the equation $x^3 + x + 1 = 0$, then the value of $(1 + x_1^2)(1 + x_2^2)(1 + x_3^2)$ is

(A) -1 (B) 1

(C) 2 (D) -2

12. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then the value of $\Sigma(\alpha + \beta - \gamma)^2$ is

(A) $24r + q$ (B) $-24r$

(C) $-8q$ (D) $24r$

13. If the roots of the equation $x^3 - 12x + k = 0$ lie in $[-4, -3)$, $(0, 1)$ and $(2, 3)$, then the range of k is

(A) $9 < k < 16$ (B) $9 < k < 11$

(C) $0 < k < 11$ (D) $-9 < k < 16$

14. If one root of $x^3 - rx^2 + rx - 4 = 0$ be the reciprocal of the other, then the value of r is

(A) 1 (B) 2

(C) 4 (D) 5

15. Consider the equation $x^n - nx + n - 1 = 0$, $(n > 1)$. The equation

(A) has no multiple root (B) has multiple root of order 2

(C) has multiple root of order 3 (D) has multiple root of order 4

16. In the set of integers, which of the following relations is not an equivalence relation ?

(A) xRy : if $x \leq y$ (B) xRy : if $x = y$

(C) xRy : if $x - y$ is an even integer (D) xRy : if $x \equiv y \pmod 3$

17. On the set N of natural numbers, define the relation R by aRb iff the G.C.D of a and b is 2. Then R is

(A) Reflexive but not symmetric (B) Symmetric only

(C) Reflexive and transitive (D) An equivalence relation

18. Which of the following statement is true ?

(A) $f : \mathbb{R} \to \mathbb{R}^+$ defined by $f(x) = x^2 + 2$ is injective

(B) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x + 4$ is surjective

(C) $f : [0, \pi] \to [-1, 1]$ defined by $f(x) = \cos x$ not is injective

(D) $f : \mathbb{R} \to [-1, 1]$ defined by $f(x) = \sin x$ is injective

19. On $\mathbb{R}$, a relation ρ is defined as, $x\rho y$ if and only if $x - y$ is zero or irrational. Then

(A) ρ is equivalence relation

(B) ρ is only reflexive relation

(C) ρ is reflexive and symmetric but not transitive

(D) ρ is reflexive and transitive but not symmetric

20. For a square matrix A of order 3, which one is correct ?

(A) $A + A^T$ is skew-symmetric

(B) $A - A^T$ is symmetric

(C) $(A - A^T)^{-1}$ exists

(D) $(A + A^T)^{-1}$ exists

21. For $x^3 = 1$, if $(a + bx + cx^2)\begin{vmatrix} 1 & b & c \\ x^2 & c & a \\ x & a & b \end{vmatrix} = \begin{vmatrix} \alpha & b & c \\ \beta & c & a \\ \gamma & a & b \end{vmatrix}$, then $(\alpha, \beta, \gamma) =$

(A) (a, b, c)

(B) (a, c, b)

(C) (c, a, b)

(D) (b, a, c)

22. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then $\det(P^2 + Q^2) =$

(A) 1

(B) 0

(C) -1

(D) -2

23. Let $A = \begin{pmatrix} 1 & 1 \\ x & y \\ x^2 & y^2 \end{pmatrix}$, then AA^T is

(A) Skew-symmetric (B) Orthogonal

(C) Singular (D) Symmetric

24. If A is a real matrix of order 3, then the rank of the matrix $A - A^T$ is

(A) 3 (B) < 3

(C) > 3 (D) 4

25. $\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$ is equal to

(A) $a+b$ (B) $2a+3b$

(C) $3a+4b$ (D) 0

26. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$. Then the matrix equation $AX = B$ holds if X is

(A) $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ (B) $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

(C) $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

27. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ if

(A) a, b, c are in A.P.

(B) a, b, c are in G.P. or $x-\alpha$ is factor of $ax^2+2bx+c$

(C) a, b, c are in H.P.

(D) only if α is a root of the equation $ax^2+2bx+c=0$

28. $\begin{vmatrix} a & 3+i & -1 \\ 3-i & 0 & -1+i \\ -1 & -1-i & 1 \end{vmatrix}$ is

(A) a real number

(B) purely imaginary number

(C) of the form A + iB ($AB \neq 0$)

(D) 0

[for all real a]

29. Let $(\cos\theta + i\sin\theta)^7 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^2$. The general value of θ is

(A) $\frac{2k\pi}{7} - \frac{\pi}{21}$

(B) $\frac{2k\pi}{7} + \frac{2\pi}{21}$

(C) $\frac{3k\pi}{7} + \frac{2\pi}{21}$

(D) $k\pi - \frac{\pi}{21}$

(k is integer)

30. The real part of z where $\cos z = -2$ is

(A) $(n+1)\pi$

(B) $2n\pi$

(C) $(4n+1)\pi$

(D) $(2n+1)\pi$

31. For any positive integer n (> 1), all the solutions z of $z^n = (z+1)^n$ lie on the line

(A) parallel to real axis

(B) parallel to imaginary axis

(C) through the origin

(D) $y = x + 1$

32. If $z_r = \cos\dfrac{\pi}{3^r} + i\sin\dfrac{\pi}{3^r}$, r = 1, 2, ... then $\lim\limits_{n\to\infty} (z_1, z_2, ..., z_n) =$

(A) $-i$ (B) i

(C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{2}$

33. If $e^z = i$ then z lies on the line

(A) $x = 0$ (B) $y = 0$

(C) $y = x$ (D) $y = -x$

34. If $i^{\alpha + i\beta} = z$ then $|z|^2 =$

(A) $e^{(4n+1)\pi\beta}$ (B) $e^{(2n+1)\pi\beta}$

(C) $e^{-(2n+1)\pi\beta}$ (D) $e^{-(4n+1)\pi\beta}$

(where n is integer)

35. If $z = \left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right)^5 + \left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i\right)^5$, then

(A) $Re(z) > 0$ (B) $Im(z) > 0, Re(z) > 0$

(C) $Re(z) > 0, Im(z) < 0$ (D) $Im(z) = 0$

36. S and T are the foci of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and B is end of the minor axis. If Δ STB is an equilateral triangle, the eccentricity of the ellipse is

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{\sqrt{2}}$

(C) $\dfrac{1}{3}$ (D) $\dfrac{1}{\sqrt{3}}$

37. The chord $lx + my + n = 0$ of the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ will subtend a right angle at the origin if

(A) $\frac{1}{b^2}-\frac{1}{a^2}=\frac{n^2}{l^2+m^2}$

(B) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{n^2}{l^2+m^2}$

(C) $\frac{1}{b^2}-\frac{1}{a^2}=\frac{l^2+m^2}{n^2}$

(D) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{l^2+m^2}{n^2}$

38. The locus of the mid-point of the chord of the circle $x^2 + y^2 - 2x - 2y - 2 = 0$, which makes an angle 120° at the centre is

(A) $x^2 + y^2 - 2x - 2y + 1 = 0$

(B) $x^2 + y^2 - 2x - 2y - 1 = 0$

(C) $x^2 + y^2 + 2x + 2y + 1 = 0$

(D) $x^2 + y^2 + 2x + 2y - 1 = 0$

39. The line $x - y = 5$ meets the curve $2(x^2 + y^2) - 5x + 5y - 25 = 0$ at P and Q. The angle between the pair of lines OP and OQ (O : origin) is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

40. $x^2 + y^2 + 2x - 8y + 2 = 0$ represents

(A) a non-central conic

(B) a pair of straight lines

(C) a central conic with its centre at (–1, 2)

(D) a central conic with its centre at (1, –2)

41. The straight lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ and $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$

(A) intersect at (5, –6, 7)

(B) are parallel

(C) intersect at (5, –7, 6)

(D) are not co-planar

42. If the axes are rotated through an angle α, then the lines $x \cos \alpha + y \sin \alpha = p$ transforms to a line which is

(A) parallel to new x-axis

(B) parallel to the new y-axis

(C) a line through the origin

(D) a line equally inclined to the coordinate axes

43. If the straight lines represented by the equation $x^2 (\tan^2 \varphi + \cos^2 \varphi) - 2xy \tan \varphi + y^2 \sin^2 \varphi = 0$ makes angles α and β with the axis of x, then $| \tan \alpha - \tan \beta | =$

(A) 1

(B) $\frac{1}{2}$

(C) 3

(D) 2

44. The angle between the pair of lines joining the origin to the points of intersection of the conic $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ with the line $3x - 2y = 1$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

45. If a point P (α, β) moves along a fixed straight line and (γ, δ) be the pole of that fixed line with respect to a given circle, then the polar of P with respect to the fixed circle always passes through the fixed point

(A) (α, γ)

(B) (α, δ)

(C) (β, γ)

(D) (γ, δ)

46. The polar of $(-a, 2a)$ with respect to $x^2 + y^2 - 2ax - 3a^2 = 0$ always touches the parabola

(A) $y^2 = 4ax$

(B) $y^2 = -4ax$

(C) $x^2 = 4ay$

(D) $x^2 = -4ay$

47. The sum of the reciprocals of two perpendicular focal chords of the conic $\frac{l}{r} = 1 + e\cos\theta$ is

(A) $\frac{2+e^2}{l}$ (B) $\frac{2+e^2}{2l}$

(C) $\frac{2-e^2}{l}$ (D) $\frac{2-e^2}{2l}$

48. The straight line $x = pz + q$, $y = p'z + q'$ intersects the conic $z = 0$, $ax^2 + by^2 = 1$ if

(A) $aq^2 + bq'^2 = 1$ (B) $aq'^2 + bq^2 = 1$

(C) $aq^2 - bq'^2 = 1$ (D) $aq'^2 - bq^2 = 1$

49. When the origin shifted to the point (a, b) without rotation of the axes, the equation $\frac{x}{a} + \frac{y}{b} = 1$ transforms to

(A) $\frac{x}{a} + \frac{y}{b} = 0$ (B) $\frac{x}{a} + \frac{y}{b} + 1 = 0$

(C) $\frac{x}{b} + \frac{y}{a} = 0$ (D) $\frac{x}{b} + \frac{y}{a} + 1 = 0$

50. Polar co-ordinate of centre of the circle $r = g\cos\theta + f\sin\theta$ is

(A) $\left(\frac{1}{2}\sqrt{(g^2+f^2)}, \tan^{-1}\frac{f}{g}\right)$ (B) $\left(\frac{g}{f}, \sin^{-1}\frac{f}{g}\right)$

(C) $\left(\frac{f}{g}, \cos^{-1}\frac{f}{g}\right)$ (D) $\left(f+g, \frac{f}{g}\right)$

51. The equation of the plane containing the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z = 12$ is

(A) $9x - 2y + 5z + 4 = 0$ (B) $9x - 2y - 5z + 4 = 0$

(C) $9x + 2y - 5z + 4 = 0$ (D) $9x + 2y + 5z = 4$

52. Let ℓ be the line joining the origin to the point of intersection of the lines represented by $2x^2 - 3xy - 2y^2 + 10x + 5y = 0$. If ℓ is perpendicular to the line $kx + y + 3 = 0$, then k is equal to

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$

(C) 1 (D) 2

53. A variable plane is at a distance 3 units from the origin and meets the axes at A, B, C. The locus of the centroid of the triangle ABC is $x^{-2} + y^{-2} + z^{-2} = \alpha$ where α is equal to

(A) 1 (B) -1

(C) $\frac{1}{3}$ (D) 3

54. If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three non-zero vectors such that $\vec{\alpha} \times \vec{\beta} = \vec{\beta} \times \vec{\gamma} = \vec{\gamma} \times \vec{\alpha}$, then

(A) $\vec{\alpha} = \frac{1}{3}(\vec{\beta} + \vec{\gamma})$ (B) $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are unit vectors

(C) $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0}$ (D) $\vec{\alpha} = (\vec{\beta} - \vec{\gamma})$

55. If $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{0}$ and $|\vec{\alpha}| = 7, |\vec{\beta}| = 3, |\vec{\gamma}| = 5$, the angle between $\vec{\beta}$ and $\vec{\gamma}$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) π

56. A unit vector in the xy-plane which is perpendicular to the vector $\vec{\alpha} = \hat{i} - \hat{j} + \hat{k}$ is

(A) $\hat{i} + \hat{j}$ (B) $\sqrt{2}(\hat{i} + \hat{j})$

(C) $\pm\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ (D) $\sqrt{3}(\hat{i} + \hat{j})$

57. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are defined by the relations

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]},\ \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]} \text{ and } \vec{r} = \frac{\vec{b} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

then $\vec{a} \cdot \vec{p} + \vec{b} \cdot \vec{q} + \vec{c} \cdot \vec{r}$ is equal to

(A) 0
(B) 1
(C) 2
(D) 3

58. For any vector $\vec{a}$, the value of

$(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to

(A) $4|\vec{a}|^2$
(B) $2|\vec{a}|^2$
(C) $|\vec{a}|^2$
(D) $3|\vec{a}|^2$

59. $\int e^{2x} \sinh x \, dx$ is given by

(A) $\frac{1}{3} e^{2x} (\sinh x - \cosh x) + c$
(B) $\frac{1}{5} e^{2x} (-\sinh x + \cosh x) + c$
(C) $\frac{2}{3} e^{2x} (\sinh x - \cosh x) + c$
(D) $\frac{e^{2x}}{3} (2 \sinh x - \cosh x) + c$

(c is constant of integration)

60. The singular solution of $y = px + p - p^2$ is

(A) $y = \frac{1}{2}(x + 1) - \frac{1}{4}(x + 1)^2$
(B) $y = cx + c - c^2$
(C) $2y = \frac{x^2}{2} + x$
(D) $4y = (x + 1)^2$

61. The differential equation $\frac{dy}{dx} = \frac{-4x^3y}{x^4 + y^4}$ is

(A) exact

(B) not exact, integrating factor is $\frac{1}{5x^4y + y^5}$

(C) not exact, integrating factor is $\frac{1}{4x^4y - y^5}$

(D) not exact, integrating factor is $e^{\int 4x^3 y\, dx}$

62. If $u = \varphi(x - y, y - z, z - x)$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} =$

(A) $x + y + z$ (B) 0

(C) $2(x + y + z)$ (D) $-(x + y + z)$

(φ is differentiable function of x, y and z)

63. $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}} =$

(A) 0 (B) -1

(C) 1 (D) $\frac{1}{2}$

64. Which of the following inequality is true for $x > 0$?

(A) $\frac{x}{1+x^2} < \tan^{-1} x < x$ (B) $\frac{x}{1+x^2} > \tan^{-1} x > x$

(C) $3 < \tan^{-1} x < 4 + x$ (D) $\frac{1}{2} > \tan^{-1} x < -\frac{1}{2}$

65. If $f(x) = \begin{cases} x \cdot \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

then

(A) $f(x)$ is continuous but not derivable at $x = 0$

(B) $f(x)$ is continuous and derivable at $x = 0$

(C) $f(x)$ is neither continuous nor derivable at $x = 0$

(D) $f(x)$ is not continuous but derivable at $x = 0$

66. The value of $\int_0^\infty \frac{x^2\,dx}{(1+x)^5}$ is

(A) $\frac{1}{10}$

(B) $\frac{1}{6}$

(C) $\frac{1}{12}$

(D) $\frac{1}{24}$

67. Let $f(x, y, z) = \begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$, then

(A) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z}$

(B) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = -\frac{\partial f}{\partial z}$

(C) $\frac{\partial f}{\partial x} = -\frac{\partial f}{\partial y} = \frac{\partial f}{\partial z}$

(D) $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$

68. Let $u = r^3$ and $x^2 + y^2 + z^2 = r^2$. Then the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ is

(A) 9r

(B) 12r

(C) 3r $(x + y + z)$

(D) 18r

69. Let $f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then the Taylor series expansion of f(x) about $x = 0$ agrees with f(x)

(A) in a small neighbourhood of 0 (B) at $x = 0$ only

(C) at every $x \geq 0$ (D) at every $x \leq 0$

70. $\lim_{\theta \to \frac{\pi}{2}} (1 - 5 \cot \theta)^{\tan \theta}$ is

(A) -5 (B) e^{-5}

(C) e^5 (D) 5

71. For $x \geq 1$, $P_n = \int_1^e (\ell n\, x)^n \, dx$, then $P_{10} - 90P_8$ is equal to

(A) -9 (B) 10 e

(C) $-9e$ (D) 10

72. For continuous function f(x),

let $\Sigma_1 = \int_{\sin^2 t}^{1+\cos^2 t} x\, f(x(2-x))\, dx$ and $\Sigma_2 = \int_{\sin^2 t}^{1+\cos^2 t} f(x(2-x))\, dx$, then $\frac{I_1}{I_2} =$

(A) 2 (B) $\frac{1}{2}$

(C) 1 (D) 3

73. If $y = 2 \cos x(\sin x - \cos x)$, then $y_{10}(0) =$

(A) 2^5 (B) 2^{10}

(C) 0 (D) -2^{10}

74. The curve whose subtangent is of constant length p is

(A) $x = ce^{-\frac{p}{4}}$

(B) $xy = p^2$

(C) $yp = x$

(D) $y = ce^{\frac{x}{p}}$

75. Area enclosed by the Astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, $a > 0$ is

(A) $\frac{\pi a^2}{8}$ square unit

(B) $\frac{3\pi a^2}{8}$ square unit

(C) $\frac{\pi a^2}{4}$ square unit

(D) $\frac{\pi a^2}{3}$ square unit

76. Let $u(x, y) = \frac{x^2y^2}{x+y}$, $x + y \neq 0$. Then $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$

(A) does not exist

(B) value is 6u

(C) value is 0

(D) value is $\frac{u}{3}$

77. $\lim\limits_{(x,y)\to(0,0)} \frac{\sqrt{(x^2y^2+1)}-1}{x^2+y^2}$

(A) does not exist

(B) value is 0

(C) value is 1

(D) ∞

78. Consider the curve $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$. Normal at $\theta = \frac{\pi}{2}$ is

(A) $x + y - \frac{a\pi}{2} = 2a$

(B) $x - y - \frac{a\pi}{2} = 0$

(C) $x + y + \frac{a\pi}{2} = 0$

(D) $x - y - \frac{3a\pi}{2} = 0$

79. What does the equation $x \cos \alpha + y \sin \alpha = 5$ represent for different values of α ?

(A) Normal to an ellipse

(B) Tangents to a circle

(C) Pair of tangents to a parabola

(D) Polar of a circle at a fixed point on it.

80. Let Z be a differentiable function of x and y. Let $x = e^u + e^{-v}$, $y = e^{-u} - e^v$. Then $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$ is changed to

(A) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} - 2$

(B) $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x}$

(C) $2x\frac{\partial z}{\partial x} + 3y\frac{\partial z}{\partial y}$

(D) $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$

81. If $\int \frac{x^2 + \sin^2 x}{1 + x^2} \cdot \sec^2 x \, dx = f(x) + c$, then f(x) is

(A) $\tan x + \tan^{-1} x$

(B) $\tan^{-1} x - \tan x$

(C) $\tan x - \tan^{-1} x$

(D) $-\tan x - \tan^{-1} x$

82. A balloon leaves the ground 50 ft away from an observer. If it rises vertically at the rate of 6 ft / sec, it is receding from the point of observation, when it is 120 ft above the ground with a speed of

(A) $\frac{72}{15}$ ft / sec

(B) $\frac{73}{15}$ ft / sec

(C) $\frac{73}{13}$ ft / sec

(D) $\frac{72}{13}$ ft / sec

83. The singular solution of the equation $y = px - p^2$, $p = \frac{dy}{dx}$, represents

(A) a circle

(B) a straight line

(C) a pair of straight lines

(D) a parabola

84. If $f(x) = \int_{-3}^{x} |x+2|\,dx$, then

(A) $f(x)$ is only continuous everywhere in $[-2, 2]$
(B) $f(x)$ is not continuous in $[-2, 2]$
(C) $f(x)$ is differentiable for all $x \in [-2, 2]$
(D) $f(x)$ is not differentiable in $[-2, 2]$

85. If a, b, c be non-zero real numbers such that

$\int_{0}^{1} (1 + \cos^8 x)(ax^2 + bx + c)\,dx = \int_{0}^{2} (1 + \cos^8 x)(ax^2 + bx + c)\,dx$, then the quadratic equation $ax^2 + bx + c = 0$ has

(A) no root in (0, 2)
(B) at least one root in (1, 2)
(C) at least one root in (0,1)
(D) two imaginary roots.

86. If $y = \tan^{-1}\left[\dfrac{\log_e\left(\frac{e}{x^3}\right)}{\log(ex^3)}\right] + \tan^{-1}\left[\dfrac{\log_e(ex^3)}{\log\left(\frac{e}{x^3}\right)}\right]$, then $\dfrac{d^2y}{dx^2}$ is equal to

(A) 1
(B) 0
(C) – 1
(D) e

87. The area bounded by the curves $y = \log_e x$ and $y = (\log_e x)^2$ is

(A) 2 + e
(B) 2 – e
(C) 3 – e
(D) 3 + e

88. Let $f(x) = x^3 - x^2 + x + 1$ and $g(x) = \begin{cases} = \max\{f(t), 0 \le t \le x\} & \text{for } 0 \le x \le 1 \\ = 3 - x & \text{for } 1 < x \le 2 \end{cases}$, then

(A) $g(x)$ is continuous in $[0, 2]$, except at $x = 1$
(B) $g(x)$ is differentiable in $[0, 2]$
(C) $g(x)$ is continuous in $[0, 2]$, not differentiable in $[0, 2]$ except at $x = 1$
(D) $g(x)$ is both continuous and differentiable in $[0, 2]$

89. If $\sin^{-1}$ a is the acute angle between the curves $x^2 + y^2 = 4x$ and $x^2 + y^2 = 8$ at (2, 2), then a =

(A) 1

(B) 0

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}}{2}$

90. A tangent to the graph of $y = f(x)$ at $x = a$ makes an angle $\frac{\pi}{3}$ with y-axis and a tangent at $x = 6$ makes an angle $\frac{\pi}{4}$ with x-axis, then $\int_a^b f''(x)\,dx =$

(A) $1 - \frac{1}{\sqrt{3}}$

(B) $-\frac{\pi}{12}$

(C) $\frac{\pi}{12}$

(D) $\sqrt{3} - 1$

91. If $\int_0^{\frac{\pi}{2}} \sin^6 x\,dx = \frac{5\pi}{32}$, then the value of $\int_{-\pi}^{\pi} (\sin^6 x + \cos^6 x)\,dx$ is equal to

(A) $\frac{5\pi}{8}$

(B) $\frac{5\pi}{16}$

(C) $\frac{5\pi}{2}$

(D) $\frac{5\pi}{4}$

92. Consider the equation $f(x) \equiv \tan x + x - 1 = 0$ in [0, 1]

(A) $\exists$ no solution in (0, 1)

(B) $\exists$ at least one solution in (0, 1)

(C) $\exists$ infinitely many solutions in (0, 1)

(D) $f'(x) = 0$ has a solution in (0, 1)

93. Given $1 + x \le e^x \le \frac{1}{1-x}$, $0 \le x < 1$. Let $I = \int_0^{\frac{1}{3}} e^{x^2}\,dx$, then

(A) $\frac{28}{81} < I < \frac{3}{8}$

(B) $\frac{14}{41} < I < \frac{1}{4}$

(C) $\frac{1}{2} < I < \frac{3}{4}$

(D) $\frac{2}{3} < I < \frac{5}{3}$

94. Let $f(t) = \frac{t+1}{t-1}$, $t \ne 1$, then f (f (2010)) equals

(A) $\frac{2011}{2009}$

(B) 2010

(C) 2009

(D) $\frac{2010}{2000}$

95. The orthogonal trajectories of the family $y^2 = ax$ is

(A) $2x^2 + y^2 = c^2$

(B) $x^2 + 2y^2 = c^2$

(C) $x^2 + y^2 = d^2$

(D) $xy = c^2$

96. The solution of the differential equation

$\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$, $y(0) = 0$ is

(A) $y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{x}{12} - 1$

(B) $y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{x}{3} - 1$

(C) $y = \frac{\sin 3x}{9} - e^x - \frac{x^4}{12} - \frac{x}{3} + 1$

(D) $y = \frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{x}{3} + 1$

97. The differential equation $(1 + x^2y^3 + \alpha x^2y^2)\,dx + (2 + x^3y^2 + x^3y)\,dy = 0$ is exact if $\alpha =$

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 2

(D) 3

98. If $x\frac{dy}{dx} + y - e^x = 0$, $y(a) = b$ then

(A) $\lim_{x \to 1} y(x) = e + 2ab - e^a$

(B) $\lim_{x \to 1} y(x) = e^2 + ab - e^{-a}$

(C) $\lim_{x \to 1} y(x) = e - ab + e^a$

(D) $\lim_{x \to 1} y(x) = e + ab - e^a$

99. $\lim_{x \to 0} \frac{\sin x}{x}$

(A) can be evaluated by L. Hopital's Rule

(B) cannot be evaluated by L. Hopital's Rule

(C) $\lim_{x \to 0+} \frac{\sin x}{x} = 1$, $\lim_{x \to 0-} \frac{\sin x}{x} = -1$

(D) $\frac{\sin x}{x}$ is not continuous at $x = 0$ whatever be f(0)

100. $f(x, y) = x + y + (x + y + 1)^2$

(A) Minimum at $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(B) no extrema exists

(C) Extrema at $\left(\frac{1}{3}, \frac{1}{3}\right)$

(D) Maxima at $\left(\frac{1}{2}, \frac{1}{2}\right)$

SPACE FOR ROUGH WORK