उत्तराखण्ड विद्यालयी शिक्षा परिषद्, रामनगर (नैनीताल)	इण्टरमीडिएट परीक्षा (उत्तराखण्ड)	'अ' 12 पन्ने

केन्द्र संख्या की मुहर	केन्द्र व्यवस्थापक के हस्ताक्षर	नोट–परीक्षार्थी उत्तरपुस्तिका के किसी भी भाग में अपना

ANSWER - I

a)

ans- principle value of $\tan^{-1}(-\sqrt{3})$

$\Rightarrow \tan^{-1}(-\sqrt{3}) = -\underbrace{\tan^{-1}(\sqrt{3})}_{\pi/3}$

ii) $\boxed{\tan^{-1}(-\sqrt{3}) = -\pi/3}$ ans

b)

ans - given matrix : $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5 & 12 \end{bmatrix}$

* no. of rows in a matrix = 2

* number of columns in a matrix = 4

iv) $\boxed{\text{ORDER OF A MATRIX} = 2\times4}$ ans

c)

ans- given function : $F(x) = \cos(\sqrt{x})$

* Differentiate both sides wrt x →

$\Rightarrow F'(x) = \frac{d}{dx}\cos(\sqrt{x})$

$\Rightarrow F'(x) = -\sin\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$ (using, chain rule of differentiation).

iii) $\boxed{F'(x) = -\frac{1}{2\sqrt{x}}(\sin\sqrt{x})}$ ans d>

ans- given, at $r = 6$ cm, the rate of change of the Area of a circle with respect to its radius (r). $= dA/dr$

* Area of circle $(A) = \pi r^2$

and $\frac{dA}{dr} = \frac{d}{dr}(\pi r^2)$

$\Rightarrow \frac{dA}{dr} = 2\pi r$

at $r = 6$ cm $\rightarrow$ $\boxed{\frac{dA}{dr} = 12\pi \text{ cm}^2}$ ans (IV)

e>

ans- given Integral : $I = \int \frac{\sec^2 x \cdot dx}{\operatorname{cosec}^2 x}$

$\Rightarrow I = \int \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{1}\right) \cdot dx$

$\Rightarrow I = \int \frac{\sin^2 x \cdot dx}{\cos^2 x} = \int \tan^2 x \cdot dx$

also, $\tan^2 x = \sec^2 x - 1$: Putting in integral.

$\Rightarrow I = \int (\sec^2 x - 1) \cdot dx$

$\Rightarrow I = \int \sec^2 x \cdot dx - \int dx$

(1) $\boxed{I = \tan x - x + c}$ ans

F)

ans- given integral: $I = \int x^2 . e^{x^3} . dx$

now, putting → $K = x^3$: Differentiating both sides wrt x

$\frac{dK}{dx} = 3x^2$ and $\frac{dK}{3} = x^2 . dx$ → put in integral.

$\Rightarrow I = \int e^K . \frac{dK}{3}$

$\Rightarrow I = \frac{1}{3} \int e^K . dK = \frac{1}{3} e^K + c$

(iii) $\boxed{\text{Integral} = \frac{1}{3} e^{x^3} + c}$ ans

8)

ans- given: direction ratios of line are -18, 12, -4

now, we know if a, b and c are direction cosines.

Then, direction cosines are $l = \frac{a}{\sqrt{a^2+b^2+c^2}}$, $m = \frac{b}{\sqrt{a^2+b^2+c^2}}$ and $n = \frac{c}{\sqrt{a^2+b^2+c^2}}$

$\Rightarrow l = \frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$, $m = \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$ and $n = \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$

$\Rightarrow l = \frac{-18}{2 \times 11}$, $m = \frac{12}{2 \times 11}$, $n = \frac{-4}{2 \times 11}$

$\Rightarrow l = \frac{-9}{11}$, $m = \frac{6}{11}$, $n = \frac{-2}{11}$

$\boxed{\text{Direction cosines} : \left(\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}\right) \text{ (iii)}}$ ans h)

ans- In a general solution of a differential equation, the number of arbitrary constants is equal to its order.

* Since, we have a D.E of order 3.

↓

$\boxed{\text{number of arbitrary constant} = 3}$ ans

i)

ans- given vectors, $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$

$\Rightarrow |a| = \sqrt{1^2+2^2} = \sqrt{5}$ units

and

$\Rightarrow |b| = \sqrt{1^2+2^2} = \sqrt{5}$ units.

Since, $|a| = |b|$: Equal magnitude and Equal vectors. X

↓

C) ~~Both A and R~~ are correct and R is in correct. ~~explaination of Assertion.~~ C) (iii) ans

j)

ans- $F: Z \rightarrow Z$, $F(x) = 2x-3$.

at $F(x_1) = F(x_2)$: $2x_1 - 3 = 2x_2 - 3$

$x_1 = x_2 \Rightarrow$ one-one Function

codomain $\in Z$ and Range $\in Z$: Codomain = Range : Onto Function

↓

A) Both A and R are correct and R is correct explaination of Assertion. ans

ANSWER-2

ans- given, $x = a\sec\theta$ and $y = b\tan\theta$

now, $y = b\tan\theta$ → differentiate both sides wrt θ.

$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta}(b\tan\theta)$

$\Rightarrow \frac{dy}{d\theta} = b\cdot\sec^2\theta$ - (i)

now, $x = a\sec\theta$ → differentiate both sides wrt θ

$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta}(a\sec\theta)$

$\Rightarrow \frac{dx}{d\theta} = a.\sec\theta\tan\theta$ - (ii)

eq^n(i) ÷ eq^n(ii) → $\boxed{\frac{dy}{dx} = \frac{b}{a}\frac{\sec\theta}{\tan\theta} = \frac{b}{a}\operatorname{cosec}\theta}$ ans

ANSWER-3

ans- given Function: $y = \sin x + e^{2x}$

* differentiate the Function wrt x.

$\Rightarrow \frac{dy}{dx} = \cos x + e^{2x}(2)$

* again, differentiate both sides wrt x.

$\Rightarrow \frac{d^2y}{dx^2} = -\sin x + e^{2x}(2)(2)$

$\boxed{\frac{d^2y}{dx^2} = 4e^{2x} - \sin x}$ → Second order derivative.

ans

ANSWER - 4

ans- given integral : $I = \int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 . dx$

using , $(a-b)^2 = a^2+b^2-2ab$

$\Rightarrow \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2$ → putting in the integral.

$\Rightarrow I = \int \left(x + \frac{1}{x} - 2\right) dx$

$\Rightarrow I = \int x \cdot dx + \int \frac{dx}{x} - 2\int dx$

$$\boxed{I = \frac{x^2}{2} + \ln|x| - 2x + c}$$ ans

ANSWER - 5

ans- given differential equation : $\frac{dy}{dx} = (1+x^2)(1+y^2)$

$\Rightarrow \int \frac{dy}{1+y^2} = \int (1+x^2) dx$ → integrating both sides.

$\Rightarrow \int \frac{dy}{1+y^2} = \int dx + \int x^2 dx$

using, $\left\{ \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c \right\}$

$\Rightarrow \boxed{\tan^{-1} y = x + \frac{x^3}{3} + c}$ → general Solution of a differential Equation.

ans

ANSWER - 6

ans- given vector : $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$.

* Directions ratios : (2, -3, 4)

* Direction cosines : $l = \dfrac{2}{\sqrt{2^2+4^2+(-3)^2}}$, $m = \dfrac{-3}{\sqrt{2^2+4^2+(-3)^2}}$ and $n = \dfrac{4}{\sqrt{2^2+4^2+(-3)^2}}$

$\Rightarrow$ Direction cosines : $l = \dfrac{2}{\sqrt{29}}$, $m = \dfrac{-3}{\sqrt{29}}$ and $n = \dfrac{4}{\sqrt{29}}$

$\boxed{\text{Direction cosines : } \left(\dfrac{2}{\sqrt{29}}, \dfrac{-3}{\sqrt{29}}, \dfrac{4}{\sqrt{29}}\right)}$ ans

ANSWER - 7

ans- $P(A) = \dfrac{5}{12}$, $P(B) = \dfrac{1}{2}$ and $P(A \cap B) = \dfrac{1}{3}$

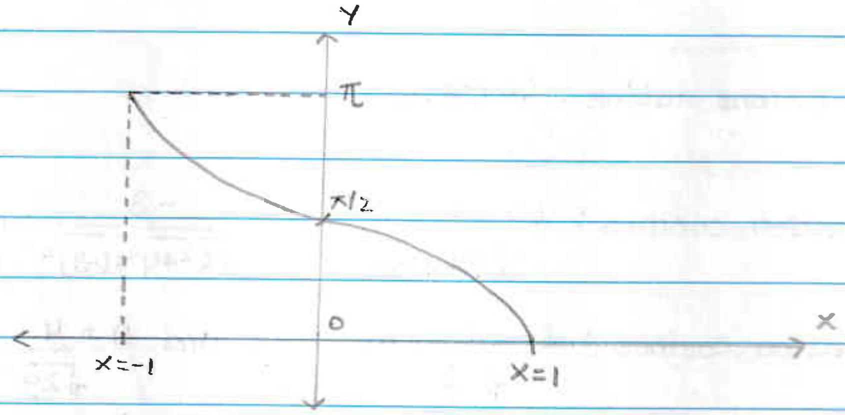
now, $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A|B) = \dfrac{1/3}{1/2}$

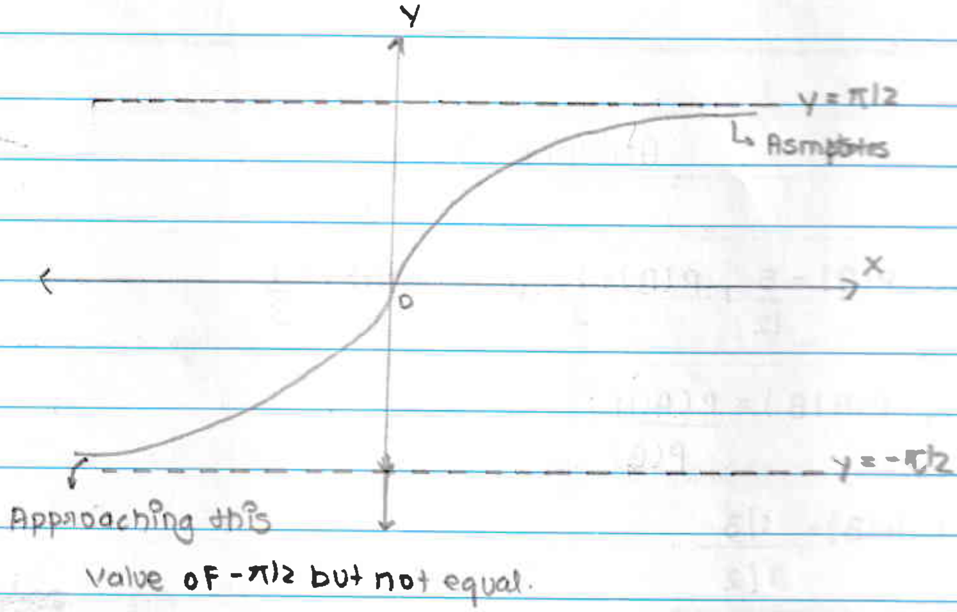
$\boxed{P(A|B) = \dfrac{2}{3}}$ ans

ANSWER-8

* graph of the Function $y = \cos^{-1}x$, Domain $\in [-1,1]$, Range $\in [0,\pi]$



* graph of the Function $y = \tan^{-1}x$, Domain $\in R$, Range $\in (-\pi/2, \pi/2)$



ANSWER - 9

ans. given Function : $F(x) = -2x^3 - 9x^2 - 12x + 1$

* Differentiate the Function wrt x.

$\Rightarrow F'(x) = -6x^2 - 18x - 12$ —①

now, we know, For Function to be strictly increasing, $F'(x) > 0$

$\Rightarrow F'(x) = -6x^2 - 18x - 12 > 0$

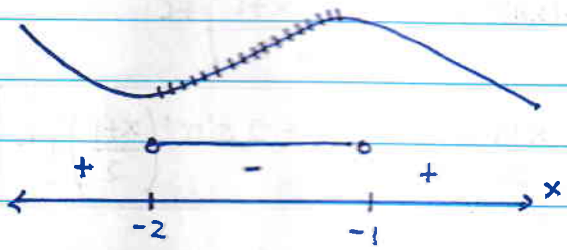
$\Rightarrow -(6x^2 + 18x + 12) > 0$

$\Rightarrow 6x^2 + 18x + 12 < 0$

$\Rightarrow 6x^2 + 6x + 12x + 12 < 0$

$\Rightarrow 6x(x+1) + 12(x+1) < 0$

$\Rightarrow (6x+12)(x+1) < 0 \rightarrow x = -1$ and -2



$\boxed{\text{Function is strictly increasing in } (-2, -1)}$ ans

P.T.O

ANSWER - 10

ans- given integral : $I = \int \sqrt{3-2x-x^2}\,.dx$

now, $3-2x-x^2 = 4-1-2x-x^2$

$= 4-(x^2+2x+1)$

$= 4-(x+1)^2$

$= (2)^2-(x+1)^2$

$\Rightarrow 3-2x-x^2 = (2)^2-(x+1)^2$: putting in integral

$\Rightarrow I = \int \sqrt{4-(x+1)^2}$

using, $\left\{ \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c \right\}$

$\Rightarrow I = \frac{x+1}{2}\sqrt{4-(x+1)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x+1}{2}\right) + c$

$$\boxed{\text{Integral} = \frac{x+1}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right) + c}$$ ans

ANSWER - 11

ans- given vectors, $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c} = \hat{i}-2\hat{j}+\hat{k}$

$\Rightarrow 2\vec{a} = 2\hat{i}+2\hat{j}+2\hat{k}$

$\Rightarrow 3\vec{c} = 3\hat{i}-6\hat{j}+3\hat{k}$

now, $2\vec{a}-\vec{b}+3\vec{c} = (2\hat{i}+2\hat{j}+2\hat{k}) - (2\hat{i}-\hat{j}+3\hat{k}) + (3\hat{i}-6\hat{j}+3\hat{k})$

$2\vec{a}-\vec{b}+3\vec{c} = \cancel{2\hat{i}}+2\hat{j}+2\hat{k} - \cancel{2\hat{i}}+\hat{j}-\cancel{3\hat{k}} + 3\hat{i}-6\hat{j}+\cancel{3\hat{k}}$

$\Rightarrow 2\vec{a}-\vec{b}+3\vec{c} = 3\hat{i}-3\hat{j}+2\hat{k}$ - ①

* now, the unit vector parallel/along to the vector $2\vec{a}-\vec{b}+3\vec{c}$ can be obtained by dividing this vector by its magnitude

$\Rightarrow$ unit vector = $\dfrac{(2\vec{a}-\vec{b}+3\vec{c})}{\sqrt{|2\vec{a}|^2+|\vec{b}|^2+|3\vec{c}|^2}}$

$\Rightarrow$ unit vector = $\dfrac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{(3)^2+(-3)^2+(2)^2}}$

$\Rightarrow$ $\boxed{\text{unit vector} = \frac{1}{\sqrt{22}}(3\hat{i}-3\hat{j}+2\hat{k})}$ ans

ANSWER – 12

* total scientists attendend seminar = 1000
* Asian scientists = 400
* Indian among asian scientists = 80

↓

* Probability that a randomly choosen scientist is from India, if given that the choosen scientist is from Asia.

$\Rightarrow P(\text{Asia}) = \dfrac{400}{1000}$ –(i)

$\Rightarrow P(\text{Indian} \cap \text{Asia}) = \dfrac{80}{1000}$ –(ii)

$\Rightarrow P(\text{India} \mid \text{Asia}) = \dfrac{P(\text{Indian} \cap \text{Asian})}{P(\text{Asian})}$

$\Rightarrow P(I|A) = \dfrac{80/1000}{400/1000} = \dfrac{\cancel{80}}{\cancel{400}_{5}} = \dfrac{1}{5}$

$\boxed{P(I|A) = \frac{1}{5}}$ ans

ANSWER-13

ans- $A = R - \{3\}$ and $B = R - \{1\}$, $F : A \to B$, $F(x) = \dfrac{x-2}{x-3}$

now, let $F(x_1) = F(x_2)$

$\Rightarrow \dfrac{x_1 - 2}{x_1 - 3} = \dfrac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow \cancel{x_1x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1x_2} - 2x_1 - 3x_2 + \cancel{6}$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow 3x_2 - 2x_2 = 3x_1 - 2x_1$

$\Rightarrow x_2 = x_1 \to$ ONE - ONE FUNCTION - (i)

also, codomain $\in R - \{1\}$ - (ii)

$\Rightarrow y = \dfrac{x-2}{x-3} \to$ range $\in R - \{1\}$ - (iii)

$\Rightarrow y(x) - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \dfrac{3y-2}{y-1}$, Range $\in R - \{1\}$

From eq^n (ii) and (iii) $\to$ codomain = Range : ONTO FUNCTION - (iv)

From eq^n (i) and (iv) $\to$ $\boxed{\text{Function is one-one, onto}}$ ans

ANSWER-14

given matrices : $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$

* LHS = $A(B+C)$: This operation / Algebra of matrices is not possible, because for the addition of matrices there order should be same.

* RHS = AB+BC.

$AB = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 14 & 12 \\ 8 & 10 & -6 \\ -8 & 7 & -9 \end{bmatrix}$

$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$

Again, we can't add AB, BC because both matrices are of different order.

P.T.O

ANSWER - 15

ans- given Function: $(x-a)^2+(y-b)^2=c^2$

* differentiating the Function wrt x.

$\Rightarrow 2(x-a)+2(y-b)\cdot\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{2(a-x)}{2(y-b)}$

$\Rightarrow \frac{dy}{dx}=\frac{a-x}{y-b}$ - (x)

Now, $\left(\frac{dy}{dx}\right)^2=\left(\frac{a-x}{y-b}\right)^2=\frac{(x-a)^2}{(y-b)^2}$

$\Rightarrow \left(\frac{dy}{dx}\right)^2=\frac{c^2-(y-b)^2}{(y-b)^2}$ - (I)

* Differentiate again, $\frac{dy}{dx}$ wrt x.

$\Rightarrow \frac{d^2y}{dx^2}=\frac{-1(y-b)-(a-x)dy/dx}{(y-b)^2}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{b-y-(a-x)\left(\frac{a-x}{y-b}\right)}{(y-b)^2}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{-(y-b)^2-(a-x)^2}{(y-b)^3}\Rightarrow -\frac{((y-b)^2+(x-a)^2)}{(y-b)^3}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{-c^2}{(y-b)^3}$ - (II)

From eqⁿ (i) and (ii) →

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[\frac{\cancel{(y-b)^2}+c^2-\cancel{(y-b)^2}}{(y-b)^2}\right]^{3/2}}{\frac{-c^2}{(y-b)^3}}$$

$$= \frac{c^3}{(y-b)^3} \times \frac{(y-b)^3}{-c^2} \Rightarrow -c$$

$$\Downarrow$$

$$\boxed{\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = -c}$$ ans

constant independent of a and b.

ANSWER - 16 (OR)

ans- given integral : $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

⇒ we know, $\cot x = \frac{\cos x}{\sin x}$ → putting in integral.

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}}$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}\cdot dx}{\sqrt{\sin x}+\sqrt{\cos x}} \quad -(1)$$

now, using $\left\{ \int_a^b F((a+b)-x) = \int_a^b F(x)\cdot dx \right\}$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin(\pi/2 - x)}\,.dx}{\sqrt{\sin(\pi/2-x)} + \sqrt{\cos(\pi/2-x)}}$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}\cdot dx}{\sqrt{\sin x} + \sqrt{\cos x}} \quad -(ii)$$

on adding eq^n (i) and (ii) $\rightarrow$

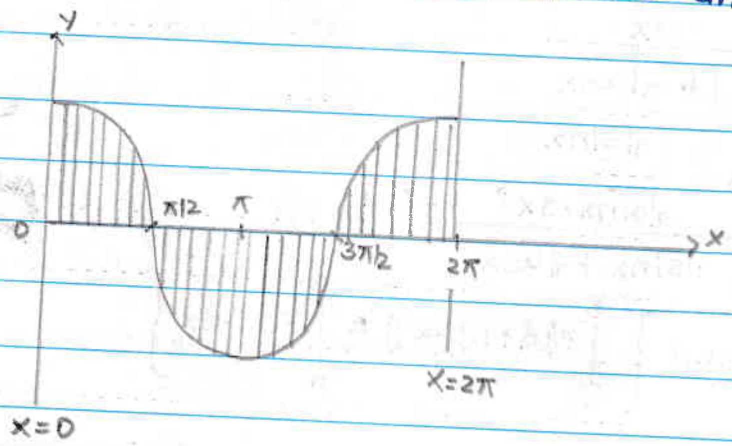
$$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}\ .\ dx}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} dx = \frac{2\times\pi}{2\times 3} - \frac{\pi}{6}$$

$$\Rightarrow \boxed{2I = \frac{\pi}{6} \quad \text{and} \quad I = \frac{\pi}{12}} \ \text{ans}$$

ANSWER - 17

* given curves : y = cos x and line x = 0 and x = 2π.



$$* \text{ AREA} = \int_0^{2\pi} y \cdot dx$$

$$* \text{ AREA} = \int_0^{\pi/2} \cos x \cdot dx + \left| \int_{\pi/2}^{3\pi/2} \cos x \cdot dx \right| + \int_{3\pi/2}^{2\pi} \cos x \cdot dx.$$

$$\Rightarrow \text{AREA} = \sin x \Big|_0^{\pi/2} + \left| \sin x \Big|_{\pi/2}^{3\pi/2} \right| + \sin x \Big|_{3\pi/2}^{2\pi}$$

$$\Rightarrow \text{AREA} = \sin \pi/2 - \sin 0 + \left| \sin\left(\frac{3\pi}{2}\right) - \sin\frac{\pi}{2} \right| + \sin 2\pi - \sin\left(\frac{3\pi}{2}\right)$$

$$\Rightarrow \text{AREA} = 1 - 0 + \left| -1 - 1 \right| + 0 - (-1)$$

$$\Rightarrow \text{AREA} = 1 + 1 + |-2|$$

$$\Rightarrow \text{AREA} = 1 + 1 + 2$$

$$\downarrow$$

$$\boxed{\text{AREA} = 4 \text{ sq. units.}}$$ ans

Hence, the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is 4 sq. units.

P.T.O

ANSWER-18

* vector Equation of a line passing through the point $(1,2,-4)$ and $\perp$ to lines :

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

* position vector of a point $= \hat{i} + 2\hat{j} - 4\hat{k}$ — (i)

* parallel vector of line 1 : $\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$
* parallel vector of line 2 : $\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$.
* IF the line is perpendicular to both L_1 an L_2, then $(\vec{b_3} = \vec{b_1} \times \vec{b_2})$

$$\Rightarrow \vec{b_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = \hat{i}(80-56) - \hat{j}(-15-21) + \hat{k}(24+48)$$
$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$\Rightarrow$ parallel vector $= 24\hat{i} + 36\hat{j} + 72\hat{k}$ — (ii)

Now, vector equation of line : $\vec{r} = \underset{\text{Posn vector}}{\vec{a}} + \lambda \underset{\text{Parallel vector}}{\vec{b}}$

$$\boxed{\begin{array}{c} \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(24\hat{i} + 36\hat{j} + 72\hat{k}) \\ \text{OR} \\ \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \end{array}}$$ ans

ANSWER - 19

given, system of Equations →

$2x+3y+3z=5$

$x-2y+z=-4$

$3x-y-2z=3$

now, using : $AX=B$ and $X=A^{-1}.B$ -(i)

and, $A=\begin{bmatrix}2 & 3 & 3\\ 1 & -2 & 1\\ 3 & -1 & -2\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B=\begin{bmatrix}5\\ -4\\ 3\end{bmatrix}$

⇒ we know, $\text{adj}(A)=(C_{ij}(A))^T$

⇒ $C_{ij}(A)=\begin{bmatrix}5 & 5 & 5\\ 3 & -13 & -11\\ 9 & 1 & -7\end{bmatrix}$ and $\text{adj}(A)=\begin{bmatrix}5 & 5 & 5\\ 3 & -13 & -11\\ 9 & 1 & -7\end{bmatrix}^T$

⇒ $\text{adj}(A)=\begin{bmatrix}5 & 3 & 9\\ 5 & -13 & 1\\ 5 & -11 & -7\end{bmatrix}$ -(ii)

now, $|A|=2(4+1)-3(-2-3)+3(-1+6)$

$|A|=2(5)-3(-5)+3(5)$

$|A|=10+15+15$ and $|A|=40$ -(iii)

From eq^n (ii) and (iii) → $A^{-1}=\dfrac{\text{adj}(A)}{|A|}=\dfrac{1}{40}\begin{bmatrix}5 & 3 & 9\\ 5 & -13 & 1\\ 5 & -11 & -7\end{bmatrix}$ -(iv)

NEXT SHEET-

उत्तराखण्ड विद्यालयी शिक्षा परिषद्, रामनगर (नैनीताल)	इण्टरमीडिएट परीक्षा 'ब' (उत्तराखण्ड) 06 पन्ने

From eqn ① and ⓘⓥ →

$\Rightarrow X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & -11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ 48 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 6/5 \end{bmatrix}$$

↓

on comparing : $\boxed{x = 1, y = 2 \text{ and } z = 6/5}$ ans

ANSWER-20

* curve passing through the point (0,1).

IF the slope of tangent to the curve at any point (x,y) is equal to the sum of the x-coordinate and product of x coordinate and y-coordinate.

$\Rightarrow$ Slope of tangent $= \frac{dy}{dx}$

$\Rightarrow$ x coordinate $= x$

$\Rightarrow$ product of x coordinate and y-coordinate $= xy$.

ATQ, $\frac{dy}{dx} = x + xy$ → given differential Equation.

P.T.O

$\Rightarrow \frac{dy}{dx} = x + xy$

$\Rightarrow \frac{dy}{dx} = x(1+y)$

$\Rightarrow \int \frac{dy}{(1+y)} = \int dx \cdot x$ → integrating both sides.

$\Rightarrow \boxed{\ln|1+y| = \frac{x^2}{2} + c}$ → general solution of a DE.

now, if the curve is passing through the point (0,1), then it should be satisfying the graph curve.

$\Rightarrow \ln|1+1| = 0 + c$ and $c = \ln|2|$

↓

$\boxed{\ln|1+y| = \frac{x^2}{2} + \ln(2)}$ ans

→ Equation of the curve.

ANSWER - 21

given points : A (+1, −2, −8), B (5, 0, −2) and C (11, 3, 7) ~~are~~ ~~collinear~~ and we have to find the ratio in which B divides line AC.

* Position vector of A = $+\hat{i} - 2\hat{j} - 8\hat{k}$.
* position vector of B = $5\hat{i} + 0\hat{j} - 2\hat{k}$.
* position vector of C = $11\hat{i} + 3\hat{j} + 7\hat{k}$.

now, the position vector of $\vec{AB} = 4\hat{i}+2\hat{j}+6\hat{k}$

$\Rightarrow$ Position vector of $\vec{BC} = 6\hat{i}+3\hat{j}+9\hat{k}$

$\Rightarrow$ position vector of $\vec{AC} = 10\hat{i}+5\hat{j}+15\hat{k}$.

and $|AB| = \sqrt{56}$ units $= 2\sqrt{14}$ -(i)

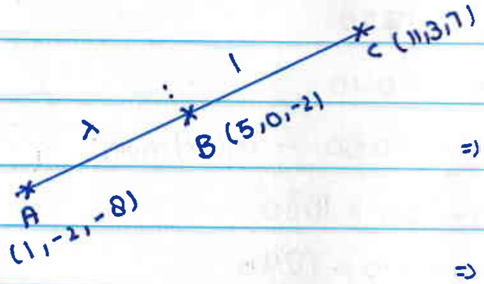
$\Rightarrow$ $|BC| = \sqrt{126}$ units and $|AC| = \sqrt{350}$ units. $= 5\sqrt{14}$. -(iii)

$\sqrt{126} = 3\sqrt{14}$ -(ii)

From eq^n (i) (ii) and (iii) $\rightarrow$ $\boxed{|AC| = |AB|+|BC| \rightarrow \text{Points are collinear.}}$ ans

* let the ratio in which Point B divides line AC be $\lambda : 1$.

~~ANSWER~~ $\Rightarrow$



$\Rightarrow$ $5 = \dfrac{11\lambda+1}{\lambda+1}$ and $0 = \dfrac{3\lambda-2}{\lambda+1}$

$\Rightarrow$ $5\lambda+5 = 11\lambda+1$

$\Rightarrow$ $5-1 = 11\lambda-5\lambda$

$\Rightarrow$ $4 = 6\lambda$ and $(\lambda = 2/3)$

$\boxed{\text{then the ratio is } (2/3 : 1) \text{ OR } (2:3)}$ ans

P.T.O

ANSWER - 22

* objective Function : $z = 2000 + 10x - 70y$

* given constraints : $x+y \leq 8$, $x+y \geq 4$, $x \geq 5$, $y \leq 5$, $x, y \geq 0$

now, $x+y=8 \rightarrow x=0$ at $y=8$ and $x+y=4 \rightarrow x=4$ at $y=0$

$y=0$ at $x=8$ $y=4$ at $x=0$

conner points : $(0,5), (0,4), (4,0), (5,0)$ $(3,5)$ $(5,3)$.

$\Rightarrow$ $x+y=8 \rightarrow$ at $y=5$: $x=3$

$\Rightarrow$ $x+y=8 \rightarrow$ at $x=5$: $y=3$

From the above graph $\rightarrow$

at $x=0, y=5$: $z = 2000 + (-350) = 1650 \rightarrow$ minimum

at $x=0, y=4$: $z = 2000 - 280 = 1720$

at $x=4, y=0$: $z = 2000 + 40 = 2040$

at $x=5, y=0$: $z = 2000 + 50 = 2050 \rightarrow$ maximum

at $x=3, y=5$: $z = 2000 + 30 - 350 = 1680$

at $x=5, y=3$: $z = 2000 + 50 - 210 = 1840$

$\boxed{\text{minimum is } 1650 \text{ at } (0,5) \text{ and maximum is } 2050 \text{ at } (5,0)}$

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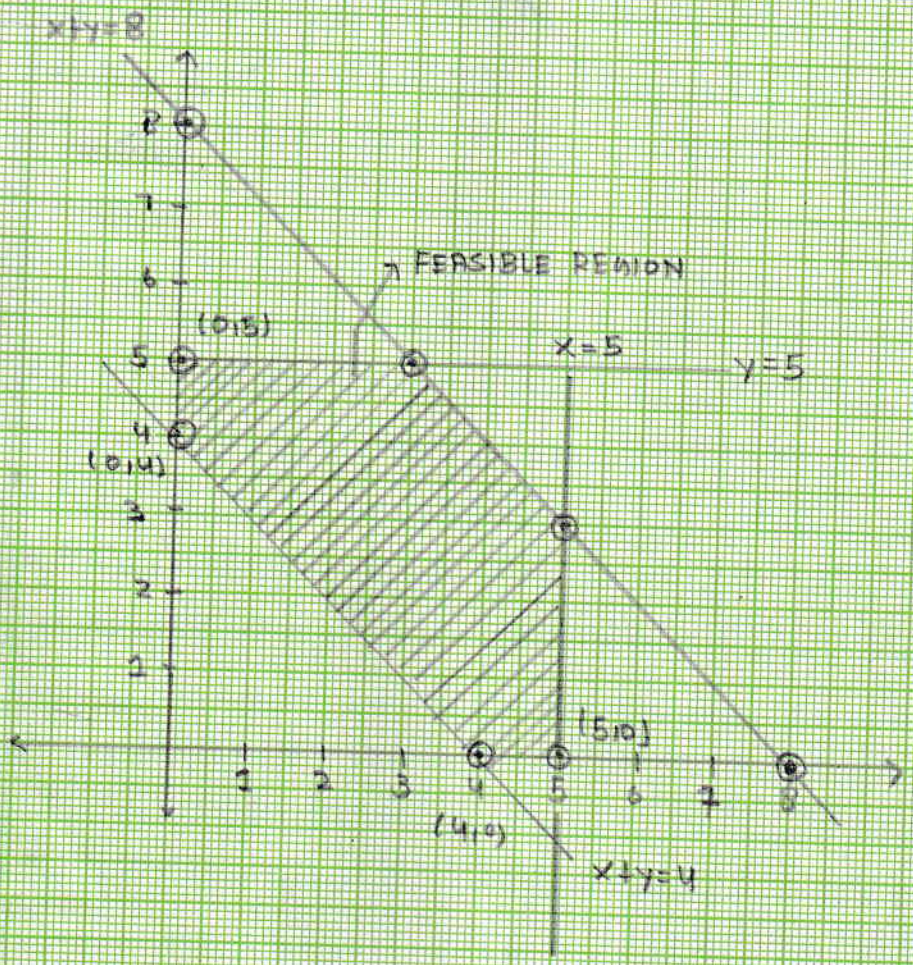

ANSWER- 23

total vehicles = 2000 + 4000 + 6000 = 12000

⇒ Scooter drivers = 2000

⇒ car drivers = 4000

⇒ truck drivers = 6000.

⇒ E: choosing a scooter driver → $P(E) = 2000/12000 = 1/6$

⇒ F: choosing a car driver → $P(F) = 4000/12000 = 2/6$

⇒ G: choosing a truck driver → $P(G) = 6000/12000 = 3/6$

⇒ A|E: Accident when Scooter driver → $P(A|E) = 0.01 = 1/100$

⇒ A|F: Accident when car driver → $P(A|F) = 0.03 = 3/100$

⇒ A|G: Accident when truck driver → $P(A|G) = 0.15 = 15/100$.

* Probability that he is a scooter driver if he meets the accident.

Using, Baye's theorem.

$$\Rightarrow P(E|A) = \frac{P(E) \cdot P(A|E)}{P(E) \cdot P(A|E) + P(F) \cdot P(A|F) + P(G) \cdot P(A|G)}$$

$$\Rightarrow P(E|A) = \frac{1/6 \times 1/100}{1/6 \times 1/100 + 2/6 \times 3/100 + 3/6 \times 15/100}$$

$$\Rightarrow P(E|A) = \frac{1}{1+6+45}$$

and

$$\boxed{P(E|A) = \frac{1}{52}}$$ ans

P.T.O.

ANSWER - 24

let the length and breadth be x and y

given depth = 2m

* volume = $32 \, m^3$ and $xy(2) = 32$

$xy = 16$ - ①

* cost of Base = $2000\,xy = 32000$

* cost of 4 wall = $1500\,(4x + 4y)$

$= 6000\,(x + y)$

⇒ Total cost = $32000 + 6000(x + y)$

⇒ $C = 32000 + 6000\left(\frac{16}{y} + y\right)$ → Differentiate w.r.t y

⇒ $\frac{dc}{dy} = 0 + 6000\left(\frac{-16}{y^2} + 1\right) = 0$

⇒ $y = 4, -4^{\times}$

* again differentiate → $\frac{d^2c}{dy^2} = \frac{32}{y^3}$ → always +ve minimum

$(x = 4, y = 4.)$

i) ans- $\boxed{\text{length} = 4m \text{ and width} = 4m}$ a

ii) an- Total cost $= 32000 + 6000(8)$

$= 32000 + 48000$

⇓

$\boxed{\text{minimum Total cost} = 80000 \text{ Rs}}$ a

Roll no -