Mathsticks December, 2024

A newsletter dedicated to Mathematics teachers (all levels) Issue 12
Mathematics Education Department, SCERT, Haryana

इस Newsletter की ख़ास विशेषताएं हैं -

• रिसर्च से सीख (Learnings from the Research)

आपकी गणित अधिगम सम्बन्धी समझ को गहरा करने के लिए, प्रत्येक अंश में, रिसर्च के नए-नए पहलू

• कक्षा -कक्ष में करने के लिए

प्रत्येक अंश में आपकी गणित अधिगम प्रक्रिया को बेहतर करने के लिए रिसर्च पर आधारित कुछ सुझाव

• Take the challenge

चुनौती पूर्ण कार्य/समस्या / Puzzle जिसे किसी भी स्तर के शिक्षक या शिक्षार्थी कर सकते हैं। इसके हल व करने की प्रक्रिया आप हमसे mathsedu.scert@gmail.com पर साझा कर सकते हैं।

• आपके अनुभव (Learning from the classroom experiences)

आप अपने सुझाव, टिप्पणी ,अनुभव हमें इस लिंक https://forms.gle/ZoGd7HnLikqnLBGYA या mathsedu.scert@gmail.com पर साझा कर सकते हैं।

महत्वपूर्ण निर्देश 👉 हर सप्ताह, कम से कम एक चुनौती / पज़ल अपने स्कूल के नोटिस बोर्ड पर लगायें।

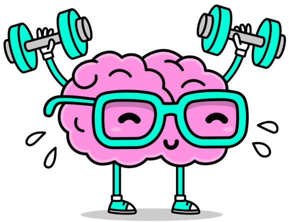
बच्चों को इन्हें करने के लिए प्रोत्साहित करें। उन्हें स्वयं से जूझने दें।

हल बता कर उनकी brain growth को न रोकें।

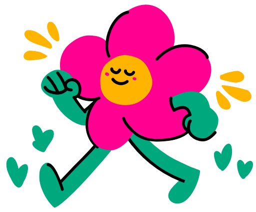
आप या आपके विद्यार्थी जब इस टास्क पर काम करें तो इन्हें अपने ब्लॉक के BRP /ABRC/ DIET MENTOR से अवश्य सांझा करें।

यहां सही उत्तर मायने नहीं रखता मायने रखता है कि आपने कैसे किया, आप की समस्या समाधान की यात्रा कैसी थी ,

Growth Mindset Statements



I believe in me



*Effort is my path to mastery.

* I am in control of my own success.

* Mistakes are opportunities to grow.

* I can always improve with practice.

* I am open to new ideas and different perspectives.

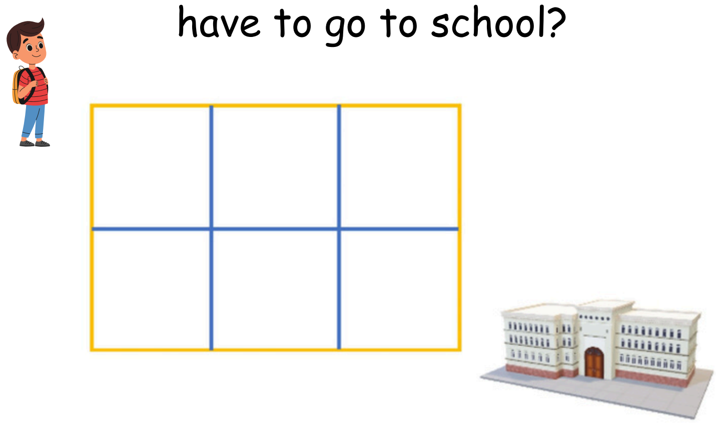
* Learning from others is a valuable part of my journey.

* Challenges help me to improve.

* I am capable of achieving great things.

Take the Challenge

Rahul is a student in class 8. He has various options for getting to his school. However, there are some conditions for choosing a path. He can only move directly to the right or directly down. He can't move left, up, or diagonally. So, how many different options does he have to go to school?

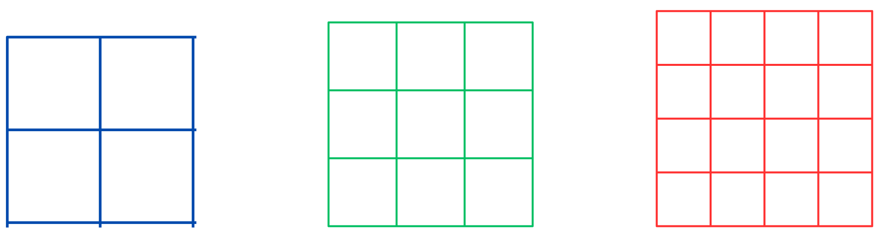


Why This Challenge ?

The purpose of this challenge is to build problem solving strategies and logical reasoning. It is helpful in developing the recursive thinking, visualization and pattern-seeking abilities in students. The students may be asked about the patterns they noticed while solving this challenge.

Major Questions ? ? ?

* What do you notice & wonder?
* How many paths there are for different dimension grids? Like



* Can you find a pattern for an m×n grid?
* Can you make a visual proof ?

Puzzle

Some time ago, I observed a case in a criminal court. A man was accused of stealing valuable jewels and attempting to flee with them when a clever police officer apprehended him. During cross-examination, the lawyer for the accused asked the police officer how he managed to catch up with the accused, who was already twenty-seven steps ahead when he began to pursue him. "Yes, Sir," the officer replied. "He takes eight steps for every five of mine!"

"But then, officer," the lawyer pressed, "how did you ever catch him if that was the case?"

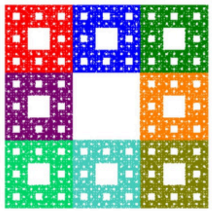
The officer responded, "I have a longer stride. Two of my steps equal the length of his five. Therefore, the number of steps I needed was fewer than his, which allowed me to reach the spot where I captured him."

A member of the jury, particularly skilled at quick calculations, did some figuring and determined the number of steps the police officer must have taken. Can you also figure out how many steps the officer needed to catch up with the thief?

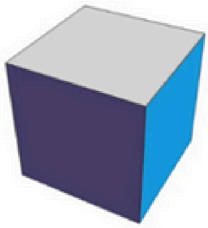
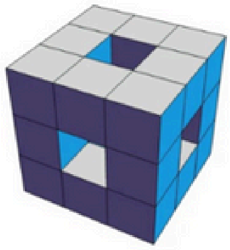
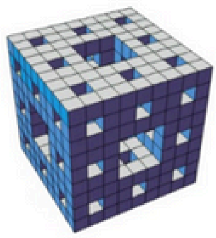
A Teacher's Exploration

The Menger Sponge

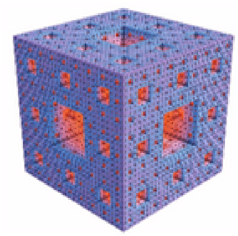
The Menger Sponge is a 3-dimensional fractal, a generalization of the Sierpiński Carpet (a 2-dimensional fractal), and it is one of the most famous examples of a 3D fractal. It was first described by Karl Menger in 1926. It is created through a process of iterative removal of smaller cubes from a larger cube.



Sierpiński Carpet

Formation of Menger Sponge		*Changes in Volume and TSA*
Start with a solid cube		$Volume = V$ $TSA = S$
First Iteration Divide the cube into 27 smaller cubes. Remove the middle cube from the center and the 6 cubes from the middle of each face of the original cube. This leaves 20 smaller cubes.		$Volume = \left(\frac{20}{27}\right)V$ $TSA = \frac{20}{9}S$
Second Iteration For each of the remaining 20 smaller cubes, repeat the same process: divide each cube into 27 smaller cubes and remove the middle cube and the 6 face-centered cubes.		$Volume = \left(\frac{20}{27}\right)^2 V$ $TSA = \left(\frac{20}{9}\right)^2 S$

A Teacher's Exploration

After infinite iterations **At each subsequent level, apply the same rule to each of the remaining cubes from the previous iteration.**		$Volume = \lim_{n\to\infty}\left(\frac{20}{27}\right)^n V \to 0$ $TSA = \lim_{n\to\infty}\left(\frac{20}{9}\right)^n S \to \infty$

Now, let us find the fractal dimension of ***Menger Sponge***. As we have already read in previous article in Mathsticks newsletter for self-similar fractals like the ***Menger Sponge***, the fractal dimension D is given by the formula:

$$D = \frac{\log N}{\log r} \quad or \quad \frac{\log(number\ of\ self - similar\ cubes)}{\log(magnification\ factor)}$$

Where:

- N is the number of self-similar cubes (in the case of the Menger Sponge first iteration, N=20).
- r is the magnification factor (in this case, r=3 because each smaller cube is 1/3 the size of the original cube).

$$D = \frac{\log(number\ of\ self-similar\ cubes)}{\log(magnification\ factor)} = \frac{\log 20}{\log 3} \approx 2.73$$

So, the dimension of Menger Sponge is somewhere between 2 and 3. The value is less than 3, meaning it doesn't completely fill a 3-dimensional space.

Learnings From The Classroom

For students in classes VI to VIII, grasping the concept of linear equations in one variable can often be challenging. Therefore, I am proposing an idea to explain how to solve equations involving one variable when two equations are provided, and we need to determine the value of that variable. To clarify the concept, it is presented as a conversation between Student and Teacher.

Teacher: Do you know how to solve linear equations in one variable practically?

Student: No, I don't. I only know one way to solve linear equations in a single variable.

Teacher: Let me illustrate this with an example of a linear equation in a single variable.

Solve for x: $3x + 5 = x + 13$

Student: How do we solve this ?

Teacher: First , explain how you would solve it?

Student: I possess just one method:

$$3x + 5 = x + 13$$
$$3x - x = 13 - 5$$
$$2x = 8$$
$$x = 8/2 = 4$$

Teacher: To solve this practically, we draw two identical rectangles and write each of the equation inside them. We can solve it step by step as demonstrated.

Step 1: Representing the Equation Visually

Let's take the equation: $3x + 5 = x + 13$

We represent this equation using rectangles:

x	x	x	5
x	13		

The first rectangle (top) shows: 3x+ 5

3 boxes with x and 1 box with 5.

The second rectangle (bottom) shows: x + 13

1 box with x and 1 box with 13.

Learnings from the classroom

Step 2: Canceling Equal Parts

Now, cancel out one 'x' from both the top and bottom rectangles.
We are now left with:

x	x	5
13		

So, the equation becomes

$2x + 5 = 13$

Step 3: Subtracting Equal Parts Again

Here, we can represent 13 in second box as 8+5.

x	x	5
8		5

Now, subtract 5 from both sides to keep the balance.
We are left with:

x	x
8	

Splitting 8 into 4+4, we obtain:

x	x
4	4

So each x must be 4.

Step 4: Final Result

$x = 4$

Student: This is the best method to understand and solve linear equations in one variable!

So, by utilizing diagrams and equal-sized rectangles, students can visualize how both sides of the equation balance. This approach establishes a solid conceptual foundation and minimizes the necessity for rote memorization. Particularly for students in classes VI to VIII, visual methods render learning interactive, engaging, and more comprehensible.