Harbans Puzzles

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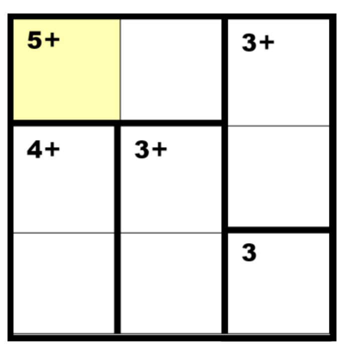
> Mathematical games, puzzles and stories involving numbers are useful to enable children to make connections between the logical functioning of their everyday lives to that of mathematical thinking and to build upon their everyday understandings. (NCERT, 2006)

According to Collins Dictionary, a Puzzle is- "a toy, problem, or other contrivance designed to amuse by presenting difficulties to be solved by ingenuity or patient effort". Most puzzles present a perplexing task that is not easily solved. At the same time, they are designed to be fun and engaging, so that the puzzle solver will invest time and energy in figuring out the solution. Mathematical puzzles can engage students in rich mathematical explorations and logical reasoning in the classroom (Evered, 2001). In this article, we will illustrate the extensions and variations of a Japanese Puzzle- 'KenKen' (which we have named Harbans Puzzle) and then discuss students' responses to these puzzles.

KenKen Puzzles were invented by a Japanese teacher-Tetsuya Miyamoto which he used as a learning tool for his students. KenKen means Cleverness and on these lines, Harbans Puzzle is an acronym for 'Har Banda Samajhdaar' in the local language, which means that everyone has the potential to do anything.

Getting started with KenKen Puzzle

To play with these puzzles, take a grid of size 3 × 3 or 4 × 4 or 5 × 5 …….



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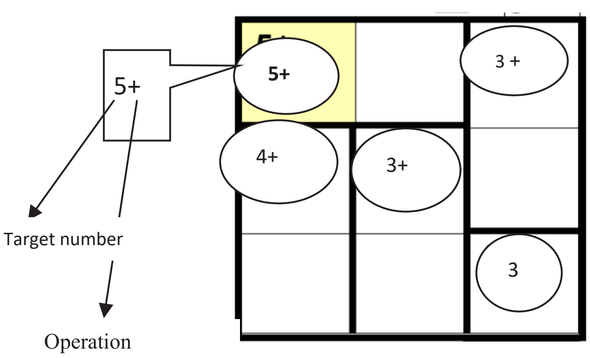
The target will be to fill in the whole grid with numbers, making sure no number is repeated in any row or column. In this 3 × 3 grid, we will use three numbers 1, 2, 3 only. In a 4 × 4 puzzle, we use the numbers 1, 2, 3, 4; in a 5 × 5, we use the numbers 1, 2, 3, 4, 5, and so on. The heavily-outlined area is called a cage (Group Box). The top left corner of each group box has a 'Target number' and one math operation. The numbers you enter (in any order) into the squares of the cage must combine to produce the target number using the math operation indicated (+, –, × or ÷). A cage with one square is named as 'freebie'(single box). Just fill in the number that is given in the box. Remember! Numbers cannot be repeated within the same row or column, just as in 'Sudoko'.

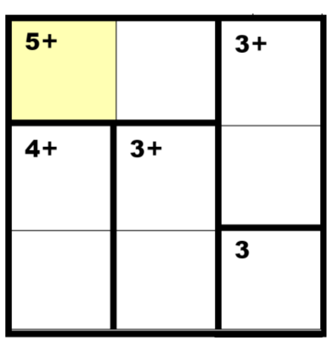
In the beginning, let us take the 3 x 3 grid shown above. In the first cage, the math operation to be used is **addition**, and the numbers must add up to 5. Since this group box has 2 squares, the possibilities are 2 and 3, in either order (3 + 2 or 2 + 3 = 5).



Let us Start

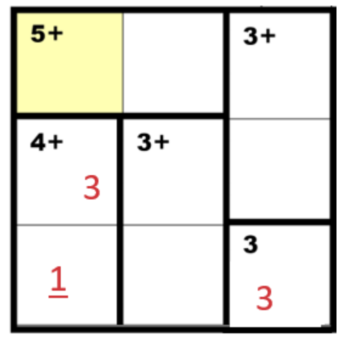
1. Enter 3 in the Freebie (Single Box). It's always best to begin with your singles.

Step 1



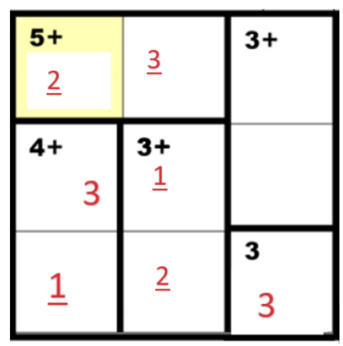
2. The lower-left group box must be filled in with a 1 down and 3 up in order to get 4 as 3 is already filled in the last row as shown in the figure.

Step 2



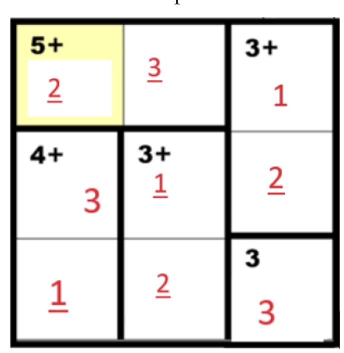
3. Each row and column must have a 1, 2 and 3 in it. The bottom row already has 1 and 3, so 2 is the only choice in the middle square of the lowest row and in the top left square. Then the middle square of the top row must be 3 and this completes the top cage.

Step 3



4. We can now enter 1 and 2 in the first and second squares of the right most column using the same logic.

Step 4



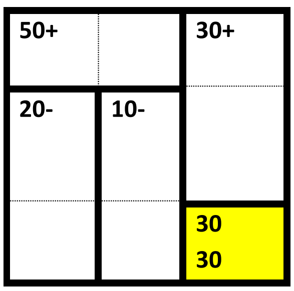
Let us Move to Harbans Puzzles

To extend a KenKen puzzle, we may use a random set of numbers (Consecutive or non- consecutive) and operations on them. In a 3 × 3 puzzle, you may use numbers such as 0, 1, 2 or 1, 2, 3 or 10, 20, 30 or three integers/prime numbers/ even numbers/ odd numbers - these need not be equally spaced. Similarly, we may take any random set of four numbers for a 4 × 4 puzzle, and so on. We would illustrate it using some examples.

Illustration 1

In this 3 × 3 puzzle, we will use the numbers 10, 20, 30 and operations - addition and subtraction.

Enter a 30 in the Single square. It's always best to begin with your singles.



Then using the same rules as of KenKen Puzzle, we can fill the grid using 10, 20 and 30 with the given operation to get the target numbers.

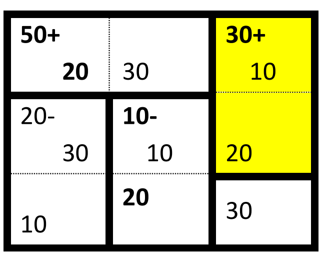


Illustration 2

Use three consecutive numbers from 1 to 5 (think and choose the numbers).

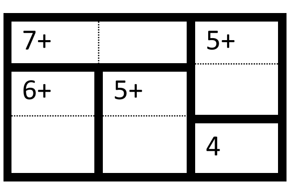


Illustration 3

Use three consecutive even numbers from 1 to 15.

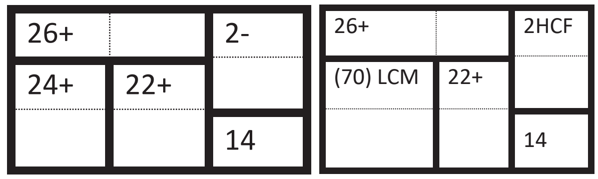


Illustration 4 Use 3 Prime Numbers from 1 to 10.

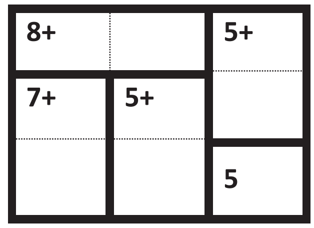


Illustration 5 Use 3 consecutive integers between -5 and 5

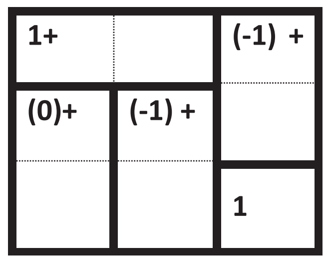
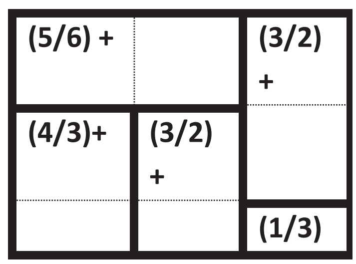


Illustration 6 Use 3 Fractions 1/1, 1/2, 1/3



Ilustration 7 Use 3 Fractions 1/1, 1/2, 1/3.

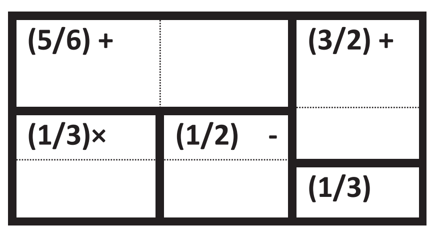


Illustration 8 (i) Choose the numbers to be used and consider them as x, y and z; here the operation is highlighted and the result is written before it. (ii) Use 3 consecutive integers -5 to 5

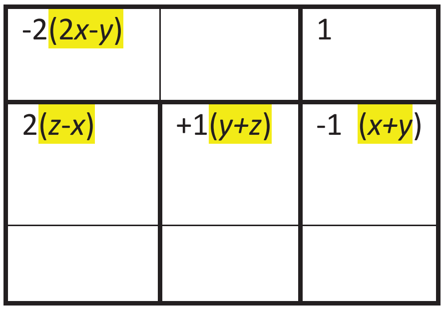


Illustration 9 Choose the numbers that can be used to solve this puzzle.

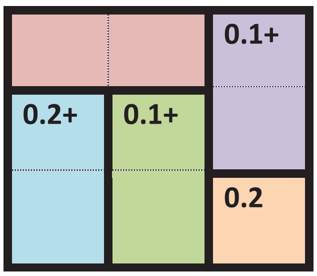
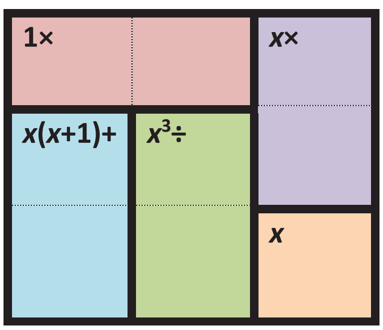
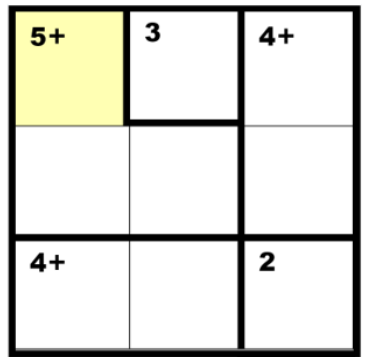
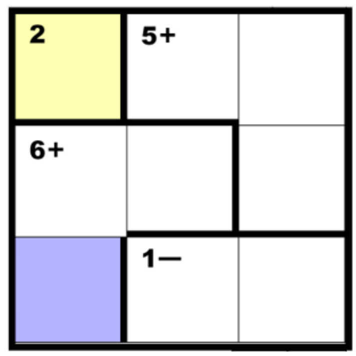


Illustration 10 Use $1/x$, x, x^2



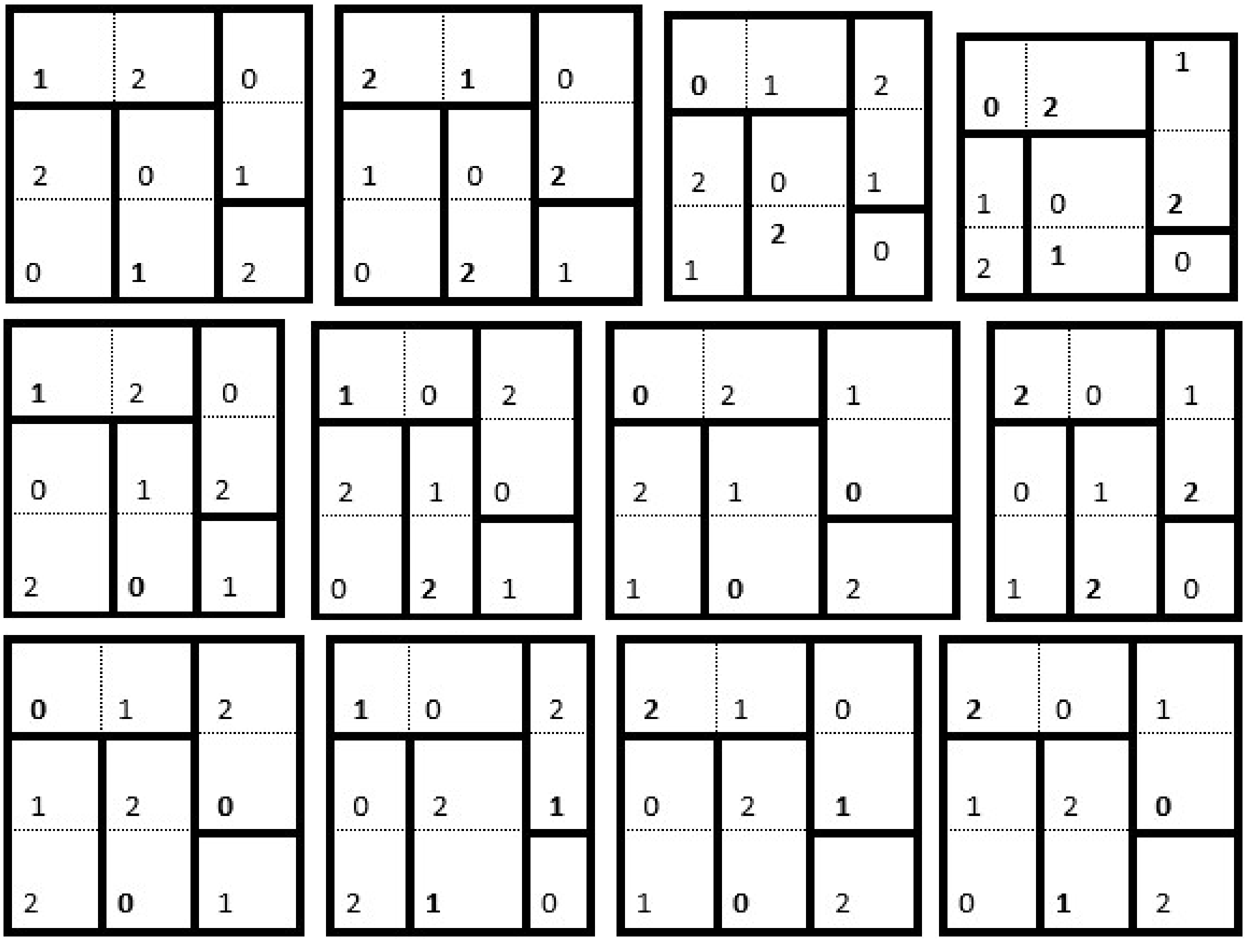
Here we can see how Harbans puzzles are different from KenKen puzzles as, in Harbans puzzles, a variety of number sets with different operations can be used, such as multiples of 10, even numbers, odd numbers, decimals, fractions, whole numbers, integers, variables, and so on. Further, the name 'cage' is replaced by 'Group box' and 'Freebie' is replaced by 'single box' to communicate in a simple language. The rule that no number will be repeated in any row or column is the same in both *Harbans* puzzles and *KenKen* puzzles and the symbolic representation is also the same, i.e., the target number is to be written on the top of a box with the operation. Further, we can use various types of caging having different number of cells. Shown below are examples of puzzles having cages with more than two cells.





Exploring the hidden patterns and potential of the puzzle to use it with students of all ages.

When we were exploring the puzzles, we came across a number of patterns while creating and solving these puzzles. We realized that such kind of puzzles work as low floor high ceiling tasks (i.e., they move in increasing order of difficulty) and also promote the computational thinking skills of students. To support our statement, we would illustrate the patterns observed in 3 × 3 grids and the possibilities to create 3 ×3 grid puzzles. Let us take an example of filling up 3 numbers 0, 1, 2 in a 3 × 3 grid; we can observe and can also compute using permutation that the possible number of arrangements is 12.



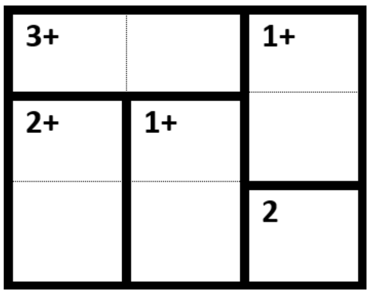
We can observe that the first square can be filled in 3 possible ways, second and fourth squares can be filled in 2 possible ways, the rest of the squares are left with a single choice only. Therefore, total number of possibilities is 3 × 2 × 2 = 12.

I	II	III
IV	V	VI
VII	VIII	IX

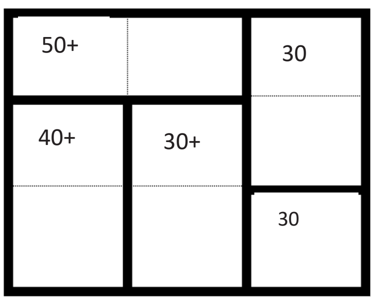
With just one grid, an ample number of puzzles can be created by varying the cages and operations. When these puzzles were given to students of different grades, we encountered different ways that students used to create and solve such puzzles. Many students of third and fourth grades used 'guess and error' strategy to write the target number to create the puzzle and rectify their errors while solving it. Another way to create the puzzle that two third graders used was 'solution to puzzle strategy,' i.e., they started with a solution (but did not write it in their puzzle sheet) to reach the target numbers and the operations used. For example, they decided to put 1, 2, 3 in the first row of a 3 × 3 grid and then subtracted 2 from 3 to get the target number as 1. This way they created a cage having 2 cells with a target number 1 and 'minus' as operation. Such opportunities made them think analytically and logically, which are important mathematical processes.

Further, while creating the puzzle, it was observed that just by taking one puzzle, we can create ample number of variations.

Suppose we have one puzzle where 0, 1, 2 numbers are to be filled, and we want to create a puzzle using 10, 20, 30 instead of 0, 1, 2. Observe the first target number, it is the sum of 1 and 2, i.e., second and third number. So, we would replace 3^+ by 50^+ as 50 is sum of second and third number. Similarly, 1 can be obtained by 1 + 0, i.e., first and second number, so 1 would be replaced by 30, i.e., sum of 10 and 20 and so on, as shown below.



So, our new Puzzle would be -



In this way we can take any three numbers (integers, fractions, etc.) that is to be filled in the grid, we can just replace the target numbers by using the same process as mentioned above. You can notice that all the illustrations (1 to 10) given above have been created using one puzzle only. Target Numbers have been replaced keeping the cages same.

Further, in the 3 × 3 grid, we can observe patterns in diagonal numbers and once those patterns are noticed, the puzzles can be solved in a minimum number of steps.

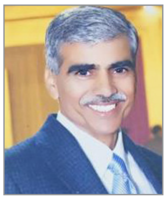
Explorations for Students

These puzzles can easily be accommodated in the school curriculum as such puzzles can be created for most of the school maths topics given from primary to secondary grades. These puzzles provide scope for problem solving and also problem posing in the classroom. Students can be given such puzzles as a practice material too which, though essential, is often boring and repetitive. Apart from this, there is a lot of scope to create and solve new puzzles, explore patterns and find more complex mathematical connections through these puzzles. Some of the questions for explorations would be-

- How many steps are used in solving the puzzle?
- What are the minimum number of steps to solve the puzzle?
- Is it always feasible to get the solution with the target numbers given?
- Do we always have a unique solution for a puzzle or more than one solution can exist?
- What are the possible number of arrangements in a 4 × 4 grid? Can we generalize to get the possible number of arrangements for an $n \times n$ grid?

References

1. Evered, L. (2001). Riddles, Puzzles, and Paradoxes: Having Fun with Serious Mathematics. Mathematics Teaching in the Middle School, 6 (8), 458-61.
2. NCERT (2006). National Position Paper on Teaching of Mathematics. NCERT: New Delhi. http://www.kenkenpuzzle.com/



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